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INELASTIC BEHAVIOR OF SINGLE ANGLE COLUMNS

by

Saleh Hamed Alsayed

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF CIVIL ENGINEERING AND ENGINEERING MECHANICS

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY
WITH A MAJOR IN CIVIL ENGINEERING

In the Graduate College
THE UNIVERSITY OF ARIZONA

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As members of the Final Examination Committee, we certify that we have read
the dissertation prepared by Saleh Hamed Alsayed
entitled Inelastic Behavior of Single Angle Columns

and recommend that it be accepted as fulfilling the dissertation requirement
for the Degree of Doctor of Philosophy.

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SIGNED: _____

Saleh Alsayed

To my parents, my wife Hayat, and
my children Albara and Ala'

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ABSTRACT

The study examines the behavior of pinned-end, centrally loaded columns of monosymmetric and asymmetric cross sections, with emphasis on angle shapes. The investigation covers flexural and flexural-torsional buckling in the elastic and inelastic ranges, with the aim of developing a rational method of predicting the buckling load for cross sections with low torsional rigidity and single or no axes of symmetry.

The computer program that was developed takes into account the effect of residual stresses. The properties of the cross section were determined in the laboratory and utilized in the computer model. Full-scale column tests were run to verify the theoretical model.

The results shows that equal-legged angles with low width-to-thickness ratio have flexural and flexural-torsional buckling loads that are less than 2% different. It is therefore suitable to continue using a flexural buckling solution for such shapes. This is also true for equal-legged angles with a high width-to-thickness ratio that fail in the elastic range, but in the inelastic range the flexural-torsional buckling load was about 11% less than the flexural buckling load.

When the angle is unequal-legged, the flexural-torsional buckling load is always smaller than the corresponding flexural buckling load, in both the elastic and the inelastic ranges. The average difference between the flexural and flexural-torsional load for unequal-legged angle ranges from 3% in the elastic range to 10% in the inelastic range. The average ratio of the experimental results to the minimum of the theoretical results was 0.95 and the coefficient of variation was 0.053.

Comparison with the results of other researchers show that it is possible to formulate an empirical formula that can be used in designing columns that are made of monosymmetric or asymmetric cross sections. However, due to the scarcity of data at this stage, it is recommended that the development of such a formula be postponed until additional test data are available. Moreover, in designing any cross section that does not have two axes of symmetry, it is advisable to check the possibility of flexural and flexural-torsional buckling.

CHAPTER 1

INTRODUCTION

Thin-walled members are easy to fabricate, cut, and maintain. They can be joined together or to other parts of the structure with minimal effort. One of the most common of such members is the angle, which is used extensively for a variety of purposes in steel-framed structures. Thus, they are utilized as primary load-carrying elements in the form of columns and truss components, for example, as well as for secondary purposes such as lintels, railings and the like. However, in spite of their extensive usage, design criteria for angular members are still lacking in important areas. This is particularly evident when angles are used as columns, and the buckling considerations of today are still based on elastic performance data.

Depending on the length and the cross section configuration, when angles as well as some other types of cross sections are used as columns, they may fail in one of four primary modes:

1. Flexural buckling about the minor principal axis.
2. Twist buckling about the shear center.
3. Combined flexural and torsional buckling.

4. Local buckling of one or more of the component plate. This may occur before or after the column has failed by squashing.

These modes of failure are illustrated in Figures 1.1-a through 1.1-d. However, due to the fact that for an angle cross section the shear center and the centroid do not coincide, and the torsional rigidity is small, the interaction of flexural and torsional buckling may be the governing mode.

A typical non-dimensional column curve which illustrates the relationship between strength and slenderness ratio is shown in Figure 1.2. In this figure, p_{cr} is the flexural buckling load, p_y is the yield load, ℓ is the length of the column, and r is the minimum radius of gyration of the cross section. For large values of ℓ/r , the column will buckle at a load that is equal to or close to the Euler (elastic) load. When ℓ/r gets smaller, the flexural buckling load decreases from the Euler value to one that reflects yielding of some fibers in the cross sections caused by the presence of residual stresses.

This flexural phenomenon is a typical characteristic of columns made of doubly symmetric cross sections. However, for asymmetric cross sections, such as unequal-legged angles, the elastic flexural-torsional buckling load is usually smaller than the corresponding flexural buckling

Figure 1.1. Modes of failure for thin-walled angular members. -- (a) Flexural buckling; (b) torsional buckling; (c) Lateral-Torsional Buckling; and (d) Local plate buckling.

Figure 1.2. Typical flexural buckling column curve for steel members.

load. For cross sections with one axis of symmetry such as equal-legged angles, flexural buckling may or may not be smaller than the corresponding flexural-torsional one. Moreover, in the inelastic range there is no certainty whether the flexural or the flexural-torsional buckling will dominate the mode of failure.

Extensive work has been done on the elastic flexural buckling characteristics for various cross sections. Inelastic buckling studies are fewer in number, and most of these have dealt with the behavior and strength of doubly symmetric sections such as wide flange shapes. Research on the maximum strength of singly symmetric shapes and on asymmetric shapes is virtually nonexistent.

Current structural steel design specifications use a single column curve for all shapes of hot-rolled cross sections. The curve is based on the assumption that the flexural buckling load is the one that governs the strength of the column. The possibility of failure by flexural-torsional buckling is not recognized, and some specifications even go so far as to caution against the use of single angle compression members. In spite of this, current construction makes extensive use of such members. It is therefore evident that specific data on the inelastic behavior and strength of angle columns are needed, particularly in the complex area of inelastic twist buckling.

It is the purpose of this study to offer a mathematical model for the elastic and inelastic buckling of an angle cross section under the application of a concentric axial load. Because of the nonlinearity in the inelastic range, a computer program that is based on an incremental procedure has been developed. This takes into account the effect of residual stresses and the reduction in stiffness due to the spread of yielding as the load is increased. To substantiate the analytical study, full-scale column tests were carried out for angles of different cross sections and lengths, and detailed evaluations were made for the correlation between tests and theory. It is demonstrated that the solution will provide criteria that may be used to develop design rules.

CHAPTER 2

PREVIOUS INVESTIGATIONS

2.1 Flexural Buckling

Buckling as a measure of column strength has been known since the eighteenth century. Leonard Euler [1] was the first to determine that an initially straight, simply supported column will remain straight until the critical load is reached. This is given by:

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} \quad (1)$$

in which

P_{cr} = the critical (buckling) load

E = modulus of elasticity

I = moment of inertia

ℓ = length of the column

Subsequent investigations showed that Equation 1, which is known as the Euler flexural buckling equation, is valid only when buckling occurs in the elastic range. In other words, it is applicable only when the material exhibits homogeneous and isotropic behavior. As will be seen, this in fact means that the Euler equation is valid

only for very long columns, and then only in an approximate fashion since the theory is based on perfect member straightness.

Short to intermediate length columns tend to fail in the inelastic domain, where the cross section is part elastic, part plastic. The realization of this effect led Engesser and Considere [2,3] to modify the Euler equation into one that reflects the partial plastification of the cross section at buckling. Thus the E that appears in Equation (1) is replaced by the effective modulus of elasticity, also called the tangent modulus of elasticity, E_t , to account for the behavior of the material and the full cross section beyond the proportional limit.

Although the critical stress obtained by the tangent modulus equation was found to compare well with column test result, Engesser subsequently revised his theory to incorporate the elastic unloading that occurs in certain fibers of the cross section after buckling. In this way he introduced the use of two elastic moduli in the cross section of the column. This approach has since been named the reduced or double modulus buckling theory. Basically, Engesser proposed that strain increases in some fibers and decreases in others during buckling; therefore, two values of the modulus of elasticity must be used.

Inasmuch as the tangent modulus concept and the reduced modulus concept are the essential bases for inelastic flexural buckling [4], the actual behavior of the column was not fully understood until 1946, when Shanley [5] proposed a model which proved that the reduced modulus load is the upper bound and the tangent modulus load is the lower bound of the column strength.

The tangent modulus and the reduced modulus theories are based on the assumption that the column is perfectly straight. They can account for the effect of the non-linearities of the material and the residual stresses, but their shortcoming lies in the fact that they cannot account for the out-of-straightness of the column and the eccentricity of the load [6].

This deficiency of the tangent modulus and the reduced modulus theories led investigators to develop a theory for the true behavior of columns, namely, the maximum strength theory in which the effect of all factors that influence the strength of the column are incorporated. Bjorhovde [7,8] and Batterman and Johnston [9] are some of the early researchers who devoted their attention to the development of this theory. Due to the complexity of the theory, no closed-form solution can be obtained, and a numerical solution is the only feasible approach. As illustrated in Figure 2.1, the tangent modulus theory can

Figure 2.1. Load-deflection relationship for a concentrically loaded column (7).

give reliable results when the geometric imperfections are small; but with the increasing combined effect of the out-of-straightness and the residual stress, the maximum strength theory is the only realistic approach [6]. Consequently, having developed the theory, Bjorhovde [6,7,10] recognized that using a single curve to represent the strength of all types of columns may underestimate or overestimate the strength of many types of columns. To overcome this crucial representation of column strength, he proposed the use of more than one column curve, where each curve was assigned to represent the strength of implanted column types. As a result, the prediction of the column strength can be very much improved. The resulting set of multiple column curves, now known as SSRC curves 1, 2 and 3, is illustrated in Figure 2.2. [4,8].

Much research has been conducted on the flexural strength of columns, taking into account various material properties, manufacturing methods, imperfections, and boundary conditions. Reference 14 gives an extensive and detailed review of these studies.

2.2 Residual Stresses

Residual stresses in steel members are the stresses that result from plastic deformations which take place during and after the manufacturing and fabrication operations. The mechanism of plastic deformations has been

Figure 2.2. SSRC column curves (7).

discussed previously [11,12]. Although the effect of residual stresses on steel columns was noticed in 1908 [13], Osgood [14] was one of the first to formulate the general problem of residual stress effects on columns.

Extensive studies of residual stresses and their effects on the behavior and strength of steel members was performed at Lehigh University, of which references [4,7,11,12,15,16,17] give some examples. For example, Huber and Beedle [12] developed the basic column equation that includes the effect of residual stresses. It was shown that these stresses are the primary cause of the reduction of the strength of the columns of intermediate length, due to the earlier appearance of yielding in parts of the cross section.

The difference in behavior of columns with and without residual stresses can be realized by performing tension coupon tests for the steel itself and stub column tests for the full column cross section. When the coupon test is conducted, the stress-strain curve is similar to the one given in Figure 2.3. In other words, typical structural steel exhibits a linear stress-strain relationship up to the yield stress (point A in the figure). Subsequently, the strain increases with no increase in stresses. The material, therefore, becomes perfectly plastic upon yielding.

(a)

(b)

Figure 2.3. Typical stress strain curve for coupon test. -- (a) Actual relationship; and (b) Idealized relationship.

In contrast, if the compression is performed on the full cross section, then the stress-strain curve is similar to the one given in Figure 2.4, where the linearity between stress and strain vanishes at an applied stress equal to the difference between the yield stress and the maximum compressive residual stress (point B in the figure). Therefore, due to the presence of residual stresses, the proportional stress (σ_p) can be expressed as

$$\sigma_p = \sigma_y - \sigma_r$$

where

σ_y = yield stress

σ_r = residual stress

σ_p = proportional limit stress

More details on the effects of residual stresses on the strength of columns are readily available in the literature [4,15,16,18]. Johnston [19] and Bjorhovde [7] extended the effect of residual stresses to include the inelastic domain using the tangent modulus concept and the maximum strength concept, respectively.

Previous studies [11,12,20] have shown that residual stresses are known to be caused by one or more of the following:

1. Uneven cooling of the steel after rolling or other heat input will cause most of the residual stresses

Figure 2.4. Typical stress strain curve for stub column test.

in hot-rolled sections, as certain parts will cool before others in the section.

2. Welding and flame-cutting will cause additional residual stresses, due to the localized heat input created by the fabrication operation.
3. Cold-forming, such as straightening, will also cause residual stresses to develop.

In order to predict the practical influence of residual stresses, their actual values and distributions are measured or computed. Measurement is the most accurate, and a number of methods can be used. These are generally classified according to the degree of specimen destruction that takes place. The methods can be categorized as:

1. Non-destructive.
2. Semi-destructive.
3. Destructive.

The sectioning approach of the group 3 is the most common. References 17 and 20 give a detailed description of this and other methods.

Extensive work was done at Lehigh University on the measurement of residual stresses in rolled and welded shapes and plates. The results have been summarized by Beedle and Tall [15]. However, for an angle cross section the only study that has been reported is the one by Nuttal and Adams

[21]. They measured the distribution of the residual stresses in small thickness angles and reported peak values of approximately $0.27\sigma_y$ at the heel of a 4 x 4 x 3/8 (in) angle. Symmetrical residual stresses were not observed even for the equal-legged angles. However, neither distributions nor peak values of residual stresses have been reported in the literature.

2.3 Flexural-Torsional Buckling

Under the application of an axial load, columns sometimes tend to fail by simultaneous twisting and flexural buckling. This phenomenon is known as flexural-torsional buckling. Wagner [22] is considered to be the first to report on the elastic torsional buckling of an open thin-walled cross section. He discussed the principle of torsional buckling for most types of thin-walled sections, assuming that during buckling, the shear center and the center of gravity would coincide. Tests were also conducted [23] on plain and aluminum alloy angles to confirm the theory proposed by Wagner. However, later investigations [24] showed that the assumptions are not necessarily true.

Ostenfeld [24] was the first to present an exact solution for buckling by torsion and flexure of some rolled sections. However, the solution approach he used was very complicated, and therefore did not get much attention. Considering the displacement of the shear center as a

coordinate instead of the displacement of the centroid, led to a significant simplification in the derivation of the governing equations. Bleich and Bleich [25], Kappus [26], and Lundquist [27] were some of the early investigators who reported on the new development of the flexural-torsional problem. The theory can be found in the references [24,28,29].

The application of the theory of flexural-torsional buckling in the elastic range is relatively straightforward. Moreover, some researchers have worked out simplifications, such as ready-to-use charts, for certain types of cross sections to minimize the mathematical computations required by designers [30,31,32]. In contrast, the application of the theory in the inelastic domain, where the residual stresses play a significant role, is complex and can only be performed in terms of approximations.

Neal [33] solved the governing differential equation for rectangular cross-section beams, using a finite difference approach. He calculated the flexural stiffness (EI) of a partially yielded beam on the basis of the tangent modulus concept, while the shear modulus, G , was found on the basis of the incremental theory of plasticity [34]. According to his work, the torsional rigidity, GK_T , is not affected by the spread of yielding. Wittrick [35] applied this theory to a narrower, rectangular, simply supported beam with

incremental increases in the stress-strain relationship. He also considered only the elastic core of the section in calculating the bending stiffness and the whole section in evaluating the torsional stiffness.

Fukumoto and Galambos [36] extended the work to I-beams under the action of axial load and non-uniform moment, with the consideration of the effect of a linear distribution of residual stresses. They showed that residual stresses have a great influence on the inelastic critical moments.

In 1970, Nuttal and Adams [21] investigated pinned-end axially loaded columns of double-angle cross section. A computer model was developed to incorporate the effect of the measured residual stresses in calculating the effective load and the differential warping term, $\bar{K}$. It was found that the difference between the flexural-torsional buckling load and the flexural buckling load was always within 2%.

In 1971, Usami and Galambos [37] presented theoretical and experimental investigations on eccentrically loaded single angles. The sizes used in the tests were $2 \times 2 \times 1/4$ and $3 \times 2 \times 1/4$ (inches). The angle ends welded to a T-shape to simulate the chord of a steel truss. It was noticed that the load-carrying capacity of the angles was affected by the method of loading. The correlation between

the theoretical and the experimental results was found to be acceptable.

In 1972, Kennedy and Murty [38] reported on the behavior of axially loaded, equal-legged angle columns. The method of analysis was based on the assumption that the peak value of the compressive residual stresses was $0.5 \sigma_y$. Further, flexural-torsional buckling was considered for long columns, while a modified inelastic flexural buckling solution was considered for short columns. Under these assumptions, one of the 5 short columns that was tested failed by plate buckling, while the others failed by inelastic flexural buckling. The correlation between theoretical and experimental results was reported to be within 5 percent.

In 1986, Kitipornchai and Lee [39,40] published the results of a study on single angle cross sections. The sizes tested ranged from 64 x 64 x 5 to 102 x 76 x 6.5 mm (2-1/2 x 2-1/2 x 3/16 to 4 x 3 x 1/4 (in)). All specimens were simply supported flexurally and fixed torsionally. Residual stresses were assumed to be similar to the distribution shown in Figure 2.5. It was found that short specimens of equal-leg angle columns buckled flexurally with a predicted correlation of approximately 5%, whereas short specimens of unequal-leg angle columns buckled in a

Figure 2.5. Assumed residual stress distribution for a single angle [37].

flexural-torsional mode with a predicted correlation of approximately 20 percent.

2.4 Plate Buckling

Local or plate buckling is accompanied by changes of the shape of the cross section. This is illustrated in Figure 1.1-d. Since the angle cross section is composed of plate elements, failure of any of the plates may lead to an overall failure of the column. The theory of plate buckling in the elastic range for various shapes of plates and different boundary conditions has been well documented in the literature [4,24,28].

In the inelastic range, there is no closed-form solution available. Approximation solutions have, therefore, received a great deal of attention from many reseachers [41,42,43]. In formulating the problem, most investigators have relied on one of two techniques. The first is the equilibrium method in which the equations of equilibrium are formulated on the deflected shape. The second approach is an energy method, where the principle of virtual work is used to formulate the problem.

Due to the complexity of the local buckling phenomenon, neither of the two methods has been found to give a precise solution. In recent years, however, due to the availability of high-speed computers, the discretization

technique has led to a satisfactory agreement between theoretical and experimental results [44,45,46].

An approximation solution for the interaction between the flexural and local buckling of an I-shaped column in the elastic range has been investigated utilizing the finite strip method [47,48]. However, a solution for the interaction between local and flexural-torsional buckling in the elastic or in the inelastic range has not been found [22].

Local buckling criteria will not be investigated in this paper. However, some stub column specimens were tested as part of the overall project. The results that were found will therefore be discussed in light of the local buckling behavior that was observed.

CHAPTER 3

DEVELOPMENT OF THE THEORY

3.1 Introduction

Steel members are generally thought of as having torsional rigidity, such that pure flexural behavior controls the strength. This may hold true for closed or doubly symmetric open cross section; however, in the case of thin-walled open cross sections, the torsional rigidity can be very small and, hence, torsional buckling may control the strength. Furthermore, when the cross section is thin-walled and does not have two axes of symmetry, as illustrated in Figure 3.1, torsional and flexural buckling will interact. This interaction may produce a buckling strength that is smaller than both the bending and the torsional strength. In this chapter, the interaction between flexural and torsional buckling will be investigated, including the elastic and inelastic range of behavior. The development of the theory will provide response characteristics of members of a general thin-walled cross section, incorporating material non-linearities.

Figure 3.1. Asymmetric thin-walled member.

3.2 Differential Equations of Equilibrium

For axially loaded columns, as shown in Figure 3.2, that are made of cross sections similar to the one shown in Figure 3.1, a second order analysis [29] gives the following equilibrium differential equations:

$$\beta_x v'' + Pv - P\phi x_o - P\phi u - P\phi^2 y_o = 0 \quad (2)$$

$$\beta_y u'' + Pu + P\phi y_o + P\phi v - P\phi^2 x_o = 0 \quad (3)$$

$$C_w \phi''' - (C_T + \bar{K}) - P\phi' - Px_o v' + Py_o u' + P(u'v - \phi u' x_o - uv' - \phi v' y_o) = 0 \quad (4)$$

where u and v are the x and y deflections of the shear center, ϕ is the angle of rotation of the cross section about the shear center; and x_o , y_o are the x and y coordinates of the shear center (Figure 3.3). All derivatives are with respect to the axial direction of the member, Z . Other terms are defined as:

$$\beta_x = EI_x = \text{bending rigidity about the } x\text{-axis} \quad (5)$$

$$\beta_y = EI_y = \text{bending rigidity about the } y\text{-axis} \quad (6)$$

$$C_w = EI_\omega = \text{Warbling torsional rigidity} \quad (7)$$

$$C_T = Gk_t = \text{St. Venant torsional rigidity} \quad (8)$$

$$\bar{K} = \int_A \sigma a^2 dA \quad (9)$$

Figure 3.2. Pin-ended axially loaded column.

Figure 3.3. Definition of displacement terms u , v , and ϕ .

in which

E = modulus of elasticity

G = shear modulus of elasticity

I_x, I_y = principal moment of inertia about x- and y-axis,
respectively

I_ω = warping moment of inertia

k_T = St. Venant torsional constant

$$a^2 = (x_0 - x)^2 + (y_0 - y)^2 \quad (10)$$

x, y = coordinates of any point on the cross section

$$\sigma = \sigma_a + \sigma_r \quad (11)$$

σ_a = applied stresses

σ_r = residual stresses

In the derivation of the equilibrium equations, it was assumed that [29]:

1. The axial load is the only load on the column, and is applied at the centroid of the cross section.
2. The cross section retains its shape during buckling.
3. The cross section is constant along the length of the column (i.e., the member is prismatic).
4. The column is initially straight and free of imperfections.

In addition, the following assumptions are necessary for the development of the theory in this paper.

1. The column is simply supported at both ends and is free to rotate about the x-axis as well as the y-axis.
2. The residual stresses are uniform along the length of the column.
3. Strain hardening is not considered.
4. The tangent modulus concept is considered, i.e., no strain reversal occurs prior to buckling, and the column is initially perfectly straight.
5. The residual stresses are constant throughout the thickness of the cross section.
6. Displacements u , v , and ϕ are small.

Moreover, the sign convention that is used in the following treats compressive stresses as negative and tensile stresses as positive.

Since small deflection theory is assumed, all higher order terms in Equations (2) through (4) can be dropped with no significant error. This will simplify the equations and make them linear with respect to the deformations. Thus, the equilibrium equations can be written as:

$$\beta_x v'' + Pv - P\phi x_O = 0 \quad (12)$$

$$\beta_y u'' + Pu + P\phi y_O = 0 \quad (13)$$

$$C_w \phi''' - (C_T + \bar{K})\phi' - Px_O v' + Py_O u' = 0 \quad (14)$$

Due to the second order approach that forms the basis of the above, these differential equations are coupled (dependent). Therefore, the solution for flexural buckling cannot be separated from torsional buckling. Consequently, an acceptable solution is possible only when the three equations are solved simultaneously.

3.3 Buckling of Pin-ended Column

In the case of a pin-ended column (Figure 3.2), Equations (12) through (14) will be satisfied if u , v , and ϕ are assumed as:

$$u = c_1 \sin \pi z / \ell \quad (15)$$

$$v = c_2 \sin \pi z / \ell \quad (16)$$

$$\phi = c_3 \sin \pi z / \ell \quad (17)$$

where c_1 , c_2 , and c_3 are constants, and ℓ is the length of the column. When the displacements according to Equations (15) through (17) and their derivatives are substituted into Equations (12) and (13) and the derivative of Equation (14), the following simultaneous homogeneous equations are obtained:

$$(P_x - P)c_1 + (0)c_2 - (Px_O)c_3 = 0 \quad (18)$$

$$(0)c_1 + (P_y - P)c_2 - (Py_O)c_3 = 0 \quad (19)$$

$$(P_{x_0})c_1 - (P_{y_0})c_2 + \left(\frac{C_w\pi^2}{\ell^2}\right) + c_T + \bar{K})c_3 = 0 \quad (20)$$

where

$$P_x = \frac{\pi^2 EI_x}{\ell^2} \quad (21)$$

$$P_y = \frac{\pi^2 EI_y}{\ell^2} \quad (22)$$

which can be expressed in matrix form as:

$$\begin{bmatrix} (P_x - P) & (0) & (-P_{x_0}) \\ (0) & (P_y - P) & (-P_{y_0}) \\ (P_{x_0}) & (-P_{y_0}) & \left(\frac{C_w\pi^2}{\ell^2} + c_T + \bar{K}\right) \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = 0$$

The only non-trivial solution for the homogeneous equations is obtained when the coefficient matrix is singular (i.e., its determinant is equal to zero). This condition gives the following buckling equation:

$$(p - p_x) \left[(p - p_y) \left(\frac{C_w\pi^2}{\ell^2} + c_T + \bar{K} \right) + p_y^2 \right] + p_x^2 (p - p_y) = 0 \quad (23)$$

Equation (23) represents the characteristic equation of a centrally loaded pin-ended column. Therefore, eigenvalues of the equation give the buckling strengths of the column. The smaller of the three eigenvalues indicates both the mode of failure and the buckling strength of the column.

However, the cross sectional configuration plays a significant role in defining that root. For instance, when the cross section has two axes of symmetry, as in a wide flange shape, then $x_o = y_o = 0$ and Equation (23) simplifies to:

$$\frac{\bar{K}}{P} (P - P_x) (P - P_y) (P - P_z) = 0 \quad (24)$$

where

$$P_z = - \frac{P}{\bar{K}} \left(C_T + \frac{\pi^2 C_w}{\ell^2} \right)$$

The governing buckling load is given by Equation (24) as the lowest of the flexural buckling loads P_x and P_y , and the twist buckling load P_z . This means that for this type of cross section, buckling would result from pure bending or pure torsion, and the cross sectional shape and dimensions will define the mode of failure.

On the other hand, when the cross section has one axis of symmetry, such as a channel, then either $x_o = 0$ if the axis of symmetry is the y-axis, or $y_o = 0$ if the axis of symmetry is the x-axis. Letting the y-axis be the axis of symmetry (i.e., $x_o = 0$), Equation (23) then reduces to:

$$(P - P_x) \left[(P - P_y) \left(\frac{C_w \pi^2}{\ell^2} + C_T + \bar{K} \right) + P^2 y_o^2 \right] = 0 \quad (25)$$

One of the three roots in this equation is $P = P_x$, which is a pure flexural buckling strength. The other two roots are:

$$P = \frac{M}{2} \left[-1 \pm \sqrt{1 + \frac{4}{M}} \right] \quad (26)$$

where

$$M = \frac{1}{y_o^2} \left[\bar{K} + C_T + \frac{C_w \pi^2}{l^2} \right]$$

These two roots are functions of both the flexural and torsional modes, i.e., they represent the flexural-torsional buckling strength. The cross sectional shape and dimensions control the lowest of the three roots and thus the mode of failure.

Finally, when the cross section has no axis of symmetry, such as an unequal-legged angle, Equation (23) cannot be resolved into simple terms. In addition, all three roots of the equation are of the flexural-torsional buckling type and the smaller of the three roots, which represents the critical buckling load of the column, is always less than P_x , P_y , and P_z .

At this point, it should be noted that the material properties and the domain of the applied stresses were not specified either in deriving the equilibrium equations or in setting up the proposed solution, although elastic behavior

is implied. However, Equations (23) through (26) can be made valid for both the elastic and the inelastic ranges if the proper terms for each case are substituted into the corresponding equation. This will be detailed in the following.

Evaluation of the coefficients of the equilibrium equations listed in Equations (5) to (11) will be outlined for both the elastic and inelastic stages of behavior. As an example, the discussion will use a steel angle as the representative cross section. However, for materials other than steel and shapes other than angles, a similar technique to the one developed in the following may be used, incorporating suitable modifications for the properties of the material and the cross-sectional shape.

3.4 Evaluation of the Coefficients and the Strength in the Elastic Range

In the elastic range, columns are long enough to assure that buckling will commence while all fibers in the cross section are stressed to below the proportional limit of Figure 2.4. Therefore, every coefficient that appears in Equations (5) through (11) has a unique value. In the evaluations of these coefficients, the bending stiffnesses β_x and β_y and the torsional rigidity C_T can be found in texts on strength of material [1,2,3]. The other coefficients are determined as follows:

3.4.1 Warping Rigidity, EI_ω

For a cross section made of plate elements, the warping moment of inertia, I_ω , can be defined as [2]:

$$I_\omega = \frac{1}{3} \sum_{i=0}^{i=n} (\omega_{ni}^2 + \omega_{ni}\omega_{nj} + \omega_{nj}^2) t_{ij} b_{ij} \quad (27)$$

where

ω_n = normalized unit warping

t = thickness of the plate element

b = length of the plate element

However, since the shear center for an angle cross section is located at the intersection of the centerline of the two legs, as seen in Figure 3.4, ω_n will always be zero. As a result, the angle cross section has zero warping rigidity. Consequently, the term C_w is dropped from Equation (23).

3.4.2 Differential Warping Constant, $\bar{K}$

$\bar{K}$ is defined in Equation (9); it results from the differential warping of two adjacent cross sections. The term σ that appears in Equation (9) can be expressed as:

$$\sigma = \sigma_a + \sigma_r \quad (28)$$

where

σ_a = the applied stress

σ_r = the residual stress

Figure 3.4. Determination of shear center coordinates. -- (a) Elastic; and (b) Inelastic.

In the elastic range, however, residual stresses have no effect on the strength of the column [6]. Therefore, in this range the stress equation can be reduced to:

$$\sigma_{\text{total}} = \sigma_a = \text{constant for a given axial load.}$$

Consequently, Equation (9) reduces to:

$$\bar{K} = \sigma_a \int_A [(x_o - x)^2 + (y - y_o)^2] dA \quad (29)$$

Performing the integration and noticing that $\sigma_a = -\frac{P}{A}$

$$\bar{K} = -Pr_o^2 \quad (30)$$

where

$$r_o^2 = (I_x + I_y)/A + x_o^2 + y_o^2 \quad (31)$$

Considering all the changes that have been introduced into Equation (23), the solution of the equilibrium equations for an axially loaded pin-ended column, made of a single angle cross section, can be expressed as:

$$\begin{aligned} (P_{cr} - P_x)(P_{cr} - P_y)(C_T - P_{cr}r_o^2) + P_{cr}^2 y_o^2 (P_{cr} - P_x) \\ + P_{cr}^2 x_o^2 (P_{cr} - P_y) = 0 \end{aligned} \quad (32)$$

Once the cross-sectional properties and the column length are known, the buckling strength of the column can then be evaluated.

3.5 Inelastic Behavior

Buckling of short to intermediate length columns is expected to occur at an applied load that may be relatively close to the yield load. However, due to the presence of the residual stresses, some fibers in the cross section will yield before buckling. Because of this, the subsequent effective distribution of the stress will not be uniform. Further, according to the assumption that the tangent modulus concept is considered and the assumed stress-strain relationship given by Figure 3.5, the modulus of elasticity for the yielded portion will be zero. Therefore, yielding lowers the rigidity of the cross section. This is a well-known effect of residual stresses in a member that is subjected to axial compressive stresses.

Any increase in the applied load of a partially yielded cross section will result in further yielding and a corresponding decrease in the rigidity of the cross section. This process of increasing the load and weakening the cross section continues until failure occurs. Moreover, since the coefficients of Equation (23) are highly dependent on the net distribution of the stresses in the cross section, their values are no longer constants. Thus, they must be modified

Figure 3.5. Assumed stress-strain relationship for steel.

to account for the inelastic behavior of the member. The evaluation of the coefficients for the inelastic range is given in the following section.

3.6 Evaluation of the Coefficients in the Inelastic Range

3.6.1 Bending Stiffnesses β_x and β_y

Considering the yielded portions of the cross section to be perfectly plastic, the modulus of elasticity of these portions will be zero. Thus, when the tangent modulus concept is applied, the bending stiffnesses will only be due to the part of the section that remains elastic. Thus:

$$\beta_{xe} = EI_{ex} \quad (33)$$

$$\beta_{ye} = EI_{ey} \quad (34)$$

where I_{ex} and I_{ey} are the moments of inertia of the elastic core of the cross section about the principal x- and y-axes, respectively. Therefore, for any given applied stresses or strains, the effective bending stiffnesses can be determined if the remaining elastic part of the cross section is defined.

It should be emphasized here that the residual stresses are not uniformly distributed throughout the cross section. Consequently, the lengths of the yielded portions of the two legs of the angle are not equal. The implication

is that the lengths of the remaining elastic part of the two legs are not in proportion. Therefore, for each successive increase in the external load, there will be a new location for the centroid and a new orientation for the principal axes. These two parameters must be predetermined in order to calculate the effective bending stiffnesses.

3.6.2 Shear Center Coordinates x_0 and y_0

As pointed out in Section 3.4, the shear center for an angle cross section is located at the intersection of the centerlines of the two legs, based on the assumption that the shape can be regarded as thin-walled. According to the assumption that the cross section will retain its shape until buckling occurs, the intersection of the centerlines and, in turn, the shear center, will retain their locations.

However, as explained in Section 3.6.1, the centroid of the cross section will change its location in accordance with the change in the applied stress. Therefore, even though the shear center location is fixed, its coordinates, x_0 and y_0 (measured relative to the centroid) (Figure 3.5) will vary.

3.6.3 Warping Stiffness, C_w

As indicated in Section 3.4, the angle cross section will always have a zero warping stiffness regardless of the level of the applied load. Thus:

$$C_w = 0$$

3.6.4 Torsional Rigidity, C_T

The St. Venant torsional stiffness in the elastic range is defined by [1, 3]:

$$C_T = \left[\frac{1}{3} \sum_{i=1}^n b t^3 \right] G \quad (35)$$

However, for a partially yielded cross section, researchers such as Neal [4] believe that Equation (35) can be used for both the elastic and inelastic stage. Others, such as Lay [29] and Haaijer [5] argue that the inelastic effect has to be considered. Thus, a reduced value must be used for the yielded portions.

Regardless of the concept that is applied, the final result will not be significantly affected. This is due to the fact that for relatively short columns, which is the case for the inelastic range, the St. Venant torsion has a small influence. However, the first approach has been used in the majority of recent reports [7, 8, 9] and is the concept that will be considered in this study.

3.6.5 Differential Warping Term,

$$\bar{K} = \int_A \sigma a^2 dA$$

The coefficient K is defined by Equation (9). However, in the inelastic range, both the distribution of the stresses and the shear center coordinates change. A

general closed-form equation for $\bar{K}$ therefore cannot be developed. Nevertheless, for a given cross section, residual stress distribution, and applied stress, the integration can be performed for this specific pattern and, hence, $\bar{K}$ can be obtained.

For the purpose of illustration, a partially yielded cross section is shown in Figure 3.6 in which y_1 , y_2 , x_2 and x_1 represent the length of the part that is yielded at the tip of the vertical leg, the heel of the angle (y- and x-direction), and at the tip of the horizontal leg, respectively. The yielded portions are chosen to be at the tips and at the heel of the angle because this is where the compressive residual stresses are maximum.

The shaded area in Figure 3.6 denotes the net stress distribution in the section. According to the assumed stress-strain relationships as given in Figure 3.5, the net stress cannot exceed the yield stress. Therefore, the stress in the yielded fibers will be equal to the yield stress. Elsewhere, the stress will be a function of the applied stress and the residual stress distribution.

In order to evaluate $\bar{K}$ for the given pattern, it is easier to calculate the contribution of each stress separately. Thus:

$$\bar{K} = \int_A \sigma_a a^2 dA + \int_A \sigma_r a^2 dA - \int_{\bar{a}} \sigma_{rr} a^2 d\bar{a} \quad (36)$$

Figure 3.6. Distribution of stresses for partially yielded angle.

in which $\int \sigma_a a^2 dA$ is the part due to the applied stress (see Figure 3.7); $\int \sigma_r a^2 dA$ is the part due to the residual stress (see Figure 3.8); and $\int \sigma_{rr} a^2 d\bar{a}$ is the part that must be subtracted in the regions where the summation of the applied stress and the residual stress exceeds the yield stress (see Figure 3.9). Consequently, the first integration of Equation (36) is:

$$\sigma_a \int_A a^2 dA = \sigma_a \int_A [(x_o - \bar{x})^2 + (y_o - \bar{y})^2] dA$$

where $\bar{x}$ and $\bar{y}$ are principal centroidal coordinates. Performing the integration yields

$$\begin{aligned} \sigma_a \int_A a^2 dA = \sigma_a \{ & (I_x + I_y) + A(x_o^2 + y_o^2 + d_x^2 + d_y^2) \\ & - 2t[\ell_x k_x (y_o \sin\theta - x_o \cos\theta) \\ & + \ell_y k_y (x_o \sin\theta + y_o \cos\theta)] \} \end{aligned} \quad (37)$$

where k_x and k_y are, respectively, the x and y movements of the centroid due to yielding. All other terms are defined in Figure 3.7.

Since the residual stresses are not uniformly distributed throughout the whole cross section, it is more logical to perform the integration of the second term of Equation (36) separately for each individual leg. Thus:

Figure 3.7. Applied stress distribution and cross-sectional geometry.

$$\int_A \sigma_r a^2 dA = t \int_{-y_t}^{y_b} \sigma_r a^2 dy + t \int_{x_t}^{x_c} \sigma_r a^2 dx \quad (38)$$

Moreover, the first term of the right-hand side of Equation (38) can be written as (Figure 3.8):

$$\begin{aligned} t \int_{-y_t}^{y_b} \sigma_r a^2 dy &= t \left\{ \int_{-y_t}^{-r_v} (C_1 y + C_2) [(x_o - \bar{x})^2 + (y_o - \bar{y})^2] dy \right. \\ &+ \int_{-r_v}^{y_b-t} (C_3 y + C_4) [(x_o - \bar{x})^2 + (y_o - \bar{y})^2] dy \\ &\left. + \int_{y_b-t}^{y_b} C_5 [(x_o - \bar{x})^2 + (y_o - \bar{y})^2] dy \right\} \end{aligned} \quad (39)$$

in which $\bar{x}$, $\bar{y}$ are the principal coordinates of an arbitrary point and can be written in terms of the centroidal coordinates as follows:

$$\bar{x} = x \cos\theta + y \sin\theta \quad (40)$$

$$\bar{y} = y \cos\theta - x \sin\theta$$

The other terms in Equation (39) can be defined as

$$\begin{aligned} C_1 &= -\frac{1}{h_1} (\sigma_{rc1} + \sigma_{rtl}) \\ C_2 &= \frac{-r_v \sigma_{rc1} - y_t \sigma_{rtl}}{h_1} \\ C_3 &= \frac{\sigma_{rtl} + \sigma_{rc2}}{(y_b - t) + r_v} \end{aligned} \quad (41)$$

Figure 3.8. An assumed residual stress distribution.

$$C_4 = \frac{r_v^\sigma r_{c2} - (y_b - t)^\sigma r_{t1}}{(y_b - t) + r_v}$$

$$C_5 = \sigma_{rc2}$$

For the purpose of illustrating the integration process, consider the first term of the right-hand side of Equation (39), which, after substitution for $\bar{x}$ and $\bar{y}$ from Equation (40), can be written as:

$$t \int_{-y_t}^{-r_v} (C_1 y + C_2) [[(x_o - (x \cos \theta + y \sin \theta))^2 + [y_o - (y \cos \theta - x \sin \theta)]^2] dy$$

After performing the integration, this simplifies to:

$$\begin{aligned} & t \{ [x_o^2 + y_o^2 - 2x_o \bar{x}_c \cos \theta + 2y_o \bar{x}_c \sin \theta + \bar{x}_c^2] \\ & \quad \left[\frac{C_1}{2} (r_v^2 - y_t^2) + C_2 (-r_v + y_t) \right] \\ & \quad - 2(x_o \sin \theta + y_o \cos \theta) \left[\frac{C_1}{3} (-r_v^3 + y_t^3) + \frac{C_2}{2} (r_v^2 - y_t^2) \right] \\ & \quad + \frac{C_1}{4} (r_v^4 - y_t^4) + \frac{C_2}{3} (-r_v^3 + y_t^3) \end{aligned} \quad (42)$$

The same method can be used to evaluate other terms of Equation (39) and the second term of the right-hand side of Equation (38). Although the details of the integrations are not shown, the following expression is obtained for Equation (38):

$$\begin{aligned}
\int_A \sigma_r(a^2) dA &= t \{ [x_o^2 + y_o^2 - 2\bar{x}_c(x_o \cos\theta - y_o \sin\theta - \frac{1}{2} \bar{x}_c)] \\
&+ [\frac{C_1}{2}(r_v^2 - y_t^2) + C_2(-r_v + y_t) + \frac{C_3}{2}((y_b - t)^2 - r_v^2) \\
&+ C_4(y_b - t + r_v) + C_5(t)] - 2(x_o \sin\theta + y_o \cos\theta) \\
&+ [\frac{C_1}{3}(-r_v^3 + y_t^3) + \frac{C_2}{2}(r_v^2 - y_t^2) + \frac{C_3}{3}((y_b - t)^3 + r_v^3) \\
&+ \frac{C_4}{2}((y_b - t)^2 - r_v^2) + \frac{C_5}{2}t(2y_b - t) + [\frac{C_1}{4}(r_v^4 - y_t^4) \\
&+ \frac{C_2}{3}(-r_v^3 + y_t^3) + \frac{C_3}{4}((y_b - t)^4 - r_v^4) + \frac{C_4}{3}((y_b - t)^3 + r_v^3) \\
&+ \frac{C_5}{3}(y_b - (y_b - t)^3)] + [x_o^2 + y_o^2 - 2\bar{y}_b(x_o \sin\theta + y_o \cos\theta - \frac{1}{2} \bar{y}_b)] \\
&+ [\frac{k_1}{2}(d_h^2 - x_t^2) + k_2(-d_h + x_t) + \frac{k_3}{2}(x_b^2 - d_h^2) + k_4(x_b + d_h)] \\
&- 2(x_o \cos\theta - y_o \sin\theta) [\frac{k_1}{3}(-d_h^3 + x_t^3) + \frac{k_2}{2}(d_h^2 - x_t^2) \\
&+ \frac{k_3}{3}(x_b^3 + d_h^3) + \frac{k_4}{2}(x_b^2 - d_h^2)] + [\frac{k_1}{4}(d_h^4 - x_t^4) + \frac{k_2}{3}(-d_h^3 + x_t^3) \\
&+ \frac{k_3}{4}(x_b^4 - d_h^4) + \frac{k_4}{3}(x_b^3 + d_h^3)] \} \quad (43)
\end{aligned}$$

in which

$$k_1 = -(\sigma_{rc3} + \sigma_{rt2})/h_2$$

$$k_2 = -[d_h \sigma_{rc3} + x_t \sigma_{rt2}]/h_2 \quad (44)$$

$$k_3 = (\sigma_{rt2} + \sigma_{rc2}) / (x_b + d_h)$$

$$k_4 = [d_h \sigma_{rc2} - x_b \sigma_{rt2}] / (x_b + d_h)$$

At this point the only part that is left in evaluating $\bar{K}$ is the third term of the right-hand side of Equation (36), which can be found using the technique that was used to evaluate $\int_{rr} \sigma_{rr}^2 d\bar{a}$. As before, the integration details are not shown, but the result of performing the integration of the third part of Equation (36) with the aid of Figure 3.9 can be shown to be equal to:

$$\begin{aligned} \int_{\bar{a}} \sigma_{rr} (a^2) d\bar{a} = & t \{ [x_o^2 + y_o^2 - 2\bar{x}_c (x_o \cos\theta - y_o \sin\theta - \frac{1}{2} \bar{x}_c)] \\ & [\frac{n_1}{2}(D_1^2 - y_t^2) + n_2(-D_1 + y_t) + \frac{n_3}{2}((y_b - t)^2 - D_2^2) \\ & + n_4(y_b - t - D_2) + n_5(t)] - 2(x_o \sin\theta + y_o \cos\theta) \\ & [\frac{n_1}{3}(-D_1^3 + y_t^3) + \frac{n_2}{2}(D_1^2 - y_t^2) + \frac{n_3}{3}((y_b - t)^3 - D_2^3) \\ & + \frac{n_4}{2}((y_b - t)^2 - D_2^2) + \frac{n_5}{2}t(2y_b - t)] + \frac{n_1}{4}(D_1^4 - y_t^4) \\ & + \frac{n_2}{3}(-D_1^3 + y_t^3) + \frac{n_3}{4}((y_b - t)^4 - D_2^4) + \frac{n_4}{3}((y_b - t)^3 - D_2^3) \\ & + \frac{n_5}{3}(y_b^3 - (y_b - t)^3) + [x_o^2 + y_o^2 - 2\bar{y}_b(x_o \sin\theta + y_o \cos\theta - \frac{1}{2} \bar{y}_b)] \} \end{aligned}$$

Figure 3.9. Locations and values of stresses that exceed the yield stress.

$$\begin{aligned}
& + \left[\frac{m_1}{2}(D_3^2 - x_t^2) + m_2(-D_3 + x_t) + \frac{m_3}{2}(x_b^2 - D_4^2) + m_4(x_b + D_4) \right] \\
& - 2(x_0 \cos\theta - y_0 \sin\theta) \left[\frac{m_1}{3}(-D_3^3 + x_t^3) + \frac{m_2}{2}(D_3^2 - x_t^2) \right. \\
& + \frac{m_3}{3}(x_b^3 + D_4^3) + \frac{m_4}{2}(x_b^2 - D_4^2) \left. \right] + \frac{m_1}{4}(D_3^4 - x_t^4) \\
& + \frac{m_2}{3}(-D_3^3 + x_t^3) + \frac{m_3}{4}(x_b^4 - D_4^4) + \frac{m_4}{3}(x_b^3 + D_4^3) \} \quad (45)
\end{aligned}$$

where

$$\begin{aligned}
n_1 &= -(\sigma_{rc1} + \sigma_{app} - \sigma_y)/y_1, & n_2 &= n_1 D_1 \\
n_3 &= \frac{(\sigma_{rc2} + \sigma_{app} - \sigma_y)}{y_2 - t} \\
n_4 &= -n_3 D_2, & n_5 &= \sigma_{rc2} + \sigma_{app} - \sigma_y \\
m_1 &= -\frac{(\sigma_{rc3} + \sigma_{app} - \sigma_y)}{x_1}, & m_2 &= m_1 D_3 \\
m_3 &= \frac{(\sigma_{rc2} + \sigma_{app} - \sigma_y)}{x_2}, & m_4 &= m_3 D_4
\end{aligned}$$

Hence, for a given cross section, $\bar{K}$ can be obtained by simply subtracting Equation (45) from the result of adding Equation (37) to Equation (43). Having calculated all the coefficients in the inelastic stage, one can substitute them into Equation (32) to solve for the unknown in the equation.

In the inelastic range, however, it is more appropriate to solve for the length of the column. Equation (32) can be rewritten as:

$$\begin{aligned}
& (P_{cr} - \frac{\pi^2 EI_{ex}}{\ell^2}) (P_{cr} - \frac{\pi^2 EI_{ey}}{\ell^2}) (C_T + \bar{K}) + P_{cr}^2 y_O^2 (P_{cr} - \frac{\pi^2 EI_{ex}}{\ell^2}) \\
& + P_{cr}^2 x_O^2 (P_{cr} - \frac{\pi^2 EI_{ey}}{\ell^2}) = 0
\end{aligned} \tag{46}$$

Let

$$\begin{aligned}
C_1 &= \pi^2 EI_{ex} \\
C_2 &= \pi^2 EI_{ey} \\
C_3 &= C_T + \bar{K}
\end{aligned} \tag{47}$$

After substitution from Equation (47) into Equation (46), performing the multiplications, and rearranging the terms, Equation (46) can be expressed as:

$$\begin{aligned}
& [P_{cr}^2 C_3 + P_{cr}^3 (x_O^2 + y_O^2)] - \frac{1}{\ell^2} [P_{cr} C_3 (C_1 + C_2) \\
& P_{cr}^2 (x_O^2 C_2 + y_O^2 C_1)] + \frac{C_1 C_2 C_3}{\ell^4} = 0
\end{aligned} \tag{48}$$

Multiplying both sides of Equation (48) by ℓ^4 , the final shape of the solution equation for an axially loaded, single angle column in terms of the length is obtained. That is:

$$\begin{aligned}
& [P_{cr}^2 C_3 + P_{cr}^3 (x_O^2 + y_O^2)] \ell^4 - [P_{cr} C_3 (C_1 + C_2) + P_{cr}^2 (x_O^2 C_2 \\
& + y_O^2 C_1)] \ell^2 + C_1 C_2 C_3 = 0
\end{aligned} \tag{49}$$

This fourth-order polynomial equation can be solved to find the critical flexural-torsional length for any given load. The result can then be compared with that pertaining to flexural buckling. This can be done as follows.

For any applied stress the total load on the cross section is $P_{\text{total}} = \int \sigma dA$, where σ is defined by Equation (11). The coefficients of the equation can then be calculated in accordance with Section 3.4. Once this is done, the corresponding critical flexural-torsional length, ℓ_1 , is obtained by solving the polynomial equation. The corresponding critical flexural length, ℓ_2 , can be found by substitution into:

$$\ell_2 = \sqrt{\frac{\pi^2 EI_{ey}}{P}} \quad (50)$$

The smaller of ℓ_1 and ℓ_2 will define the critical length and the mode of failure for the given load.

However, since the procedure is cumbersome and not practical, it is easier and more practical to solve the problem using a high-speed digital computer. This method is detailed in Chapter 4.

CHAPTER 4

COMPUTER MODEL

The complexity of the methods of predicting the inelastic flexural-torsional buckling load or the corresponding length, as derived in Chapter 3, makes the solutions impractical, especially for routine use. The most efficient method for solving the problem involves the use of numerical integration techniques. For this purpose, a computer program was developed for determining the column lengths that correspond to flexural and flexural-torsional buckling. It was decided to solve the problem by finding the column length that corresponds to a certain buckling load rather than using the traditional approach of computing the load itself. This is particularly convenient for the complex solution of the inelastic buckling problem.

The cross section is discretized into small segments, as illustrated in Figure 4.1, and the properties at the center of gravity of each segment are considered to represent those of the whole segment. This is an accurate approach as long as the segments are kept sufficiently small. Then the balanced residual stress at the center of

Figure 4.1. Discretization of the cross section.

gravity is read by the program, along with the other representative material and cross-sectional data. These are the yield stresses of the steel, σ_y , as obtained from coupon test or a stub column test. The yield stress is treated as a constant value for the column [49]. The modulus of elasticity, E_y , is taken to be equal to 29×10^3 ksi [4,50] for the elastic regions of the cross section and equal to zero for the inelastic portions. Poisson's ratio, μ , is set equal to 0.3. Furthermore, the assumptions that were given at the beginning of Chapter 3 prevail.

The numerical integration is then performed. It consists of two main loops. In the first loop the cross sectional properties for the elastic core are evaluated, and in the second loop the coefficients of Equation (49) are found. The steps of the integration process can be summarized as follows:

1. Assume a uniform axial strain ϵ_a . The total strain on the segment, ϵ_{t1} , is then equal to

$$|\epsilon_{t1}| = \epsilon_a + \epsilon_r \quad (51)$$

where ϵ_r = residual strain in the segment given as $\epsilon_r = \frac{\sigma_r}{E}$, assuming that elastic unloading will take place in the segment if the residual stress is released. This is consistent with the linearly elastic-perfectly plastic stress-strain curve for

the steel, and also agrees with the approaches used in a similar problem [8].

2. If a segment yields, it is regarded as having lost any further strength and stiffness, and is therefore no longer an effective part of the cross section. Otherwise, its location and contribution to the area is included in the computation.
3. Steps 1 and 2 are repeated until all segments have been considered.
4. Compute the cross-sectional properties; I_x , I_y , A , x_0 , y_0 , x , and y where all terms are defined in Chapter 3.
5. Repeat step 1, but this time with ϵ_{t2} considered to be equal to or less than the yield strain, ϵ_y . Thus

$$\epsilon_{t2} = \epsilon_a + \epsilon_r \leq \epsilon_y \quad (52)$$

6. Calculate the contribution of the segment to the actual load, p_T , and to the differential warping term, $\bar{K}$.
7. Repeat steps 5 and 6 until all segments are considered.
8. Substitute the coefficients into Equation (49) and solve the fourth-order polynomial equation to obtain the length ℓ_1 , which corresponds to the flexural-torsional buckling load, P_T .

9. Calculate the ℓ_2 corresponding to the flexural load P_T using the following equation

$$\ell_2 = \sqrt{\frac{\pi^2 EI_{ye}}{P_T}} \quad (53)$$

where I_{ye} = effective minor principal moment of inertia of the cross section (i.e., moment of inertia of the elastic portion of the shape).

10. Increase the applied strain and repeat steps 1 through 9 to obtain the length ℓ_1 and ℓ_2 that correspond to the larger axial load.

The procedure is repeated until a complete column curve can be prepared. A flow chart for the program is given in Figure 4.2; the program itself is listed in Appendix B.

Figure 4.2. Flow chart for the computer program.

Figure 4.2--Continued

CHAPTER 5

MATERIAL PROPERTIES

In order to substantiate the theoretical investigation of this study, full-scale column tests for angles of different cross sections and lengths have been carried out as part of the project program. Details of these tests and their results are given in Chapters 6 and 7. Further, material properties were obtained from tests conducted on the five different sizes of angle cross sections that were to be used for the column tests.

The steel was ASTM A36 [51], and all sizes but the 3 x 3 x 3/8 were supplied from the same heat, as evidenced by the mill test certificate. Chemical analyses were not performed; however, the mill test data indicated compliance with the standard. In order to determine the detailed properties of the material, the following tests were conducted.

5.1 Mechanical Properties

The dimensions of the specimens were selected according to ASTM Standard A370 [51] using the full thickness of the material (i.e., the leg thickness of the angle)

and a width of 1/2 in. over a 2 in. gage length, as shown in Figure 5.1. The tests were performed with a Tinius Olsen 200 kip universal testing machine, using a testing speed of approximately 5 kips/min in the elastic range and 1 kip/min in the inelastic range, as mandated by the standard. Strains were measured using a mechanical strain gage, and the testing machine loads were used to determine the stresses. It is noted that the machine had been calibrated recently.

In order to obtain a better representation of the material properties, 3 to 5 test specimens were cut from each angle, and the average data determined accordingly. This is common procedure when establishing material properties through tests.

The results of the laboratory tests are summarized in Tables 5.1 through 5.5. In the tables, σ_y represents the yield stress, as defined by the yield plateau in the stress strain curve [4,50]; ϵ_y denotes the yield strain; σ_u is the ultimate tensile strength; and ϵ_B represents the elongation at fracture. The overall average yield stress, σ_{yav} , for the material in the angles was 44.63 ksi, and the average value of the ultimate tensile strength was 67.95 ksi. These are within the acceptable range of the ASTM specifications for A36 steel [51], where the specified minimum yield stress

Figure 5.1. Tension test specimen.

Table 5.1. Tension test results for angle L3 x 3 x 3/8.

Specimen No.	σ_y (ksi)	ϵ_y	σ_u (ksi)	ϵ_B
1	48.00	0.00166	81.06	0.35150
2	48.00	0.00166	72.53	0.33150
3	44.00	0.00152	77.33	0.36500
Averages	46.67	0.00161	76.97	0.34930

Locations of test specimen for L3 x 3 x 3/8

Table 5.2. Tension test results for angle L5 x 3 x 3/8.

Specimen No.	σ_y (ksi)	ϵ_y	σ_u (ksi)	ϵ_B
1	45.33	0.00156	64.00	0.43641
2	44.80	0.00154	64.27	0.40000
3	45.17	0.00156	66.67	0.33900
Averages	45.10	0.00155	64.98	0.36180

Location of test specimens for L5 x 3 x 3/8

Table 5.3. Tension test results for angle L5 x 5 x 3/8.

Specimen No.	σ_y (ksi)	ϵ_y	σ_u (ksi)	ϵ_B
1	44.80	0.00154	71.47	0.35100
2	42.67	0.00147	69.33	0.36200
3	48.73	0.00151	76.27	0.33650
4	43.73	0.00151	70.40	0.30000
Averages	44.83	0.00151	71.87	0.33738

Locations of test specimens for L5 x 5 x 3/8

Table 5.4. Tension test results for angle L4 x 4 x 5/8.

Specimen No.	σ_y (ksi)	ϵ_y	σ_u (ksi)	ϵ_B
1	42.88	0.00148	61.44	0.41200
2	43.20	0.00149	62.72	0.40100
3	38.40	0.00132	58.88	0.41900
4	47.36	0.00163	62.40	0.37850
Averages	42.96	0.00148	61.36	0.40263

Locations of test specimens for L4 x 4 x 5/8

Table 5.5. Tension test results for angle L6 x 4 x 3/4.

Specimen No.	σ_Y (ksi)	ϵ_Y	σ_u (ksi)	ϵ_B
1	42.21	0.00146	72.80	0.37000
2	48.00	0.00166	64.53	0.36600
3	42.67	0.00147	65.33	0.38000
4	45.80	0.00158	64.00	0.36400
5	44.67	0.00154	65.67	0.37300
Averages	44.67	0.00154	66.47	0.3706

Locations of test specimens for L6 x 4 x 3/4

is 36 ksi and the ultimate tensile strength range is 58-80 ksi.

5.2 Residual Stress Measurements

Longitudinal residual stresses were measured using the method of sectioning [4,17]. This is described in detail in the literature and only the essential characteristics of the method will be described in the following [17,20].

The test piece was first marked into strips and holes were drilled on each side of a strip. The distance between the two holes on each side of a strip was carefully measured; the nominal length was 5", to accommodate the Whittemore gage; the actual length varied slightly. The test piece of the angle was then cold-sawed from the rest of the specimen, and the strips were subsequently cut using a band-saw. The change in the length between the two holes was measured and used to calculate the residual stress that existed in the strip before slicing, assuming elastic unloading after release of restrained deformation.

The width of each strip was kept at 1/2 in., except for the first, second, and sometimes the third (starting from the heel) element. These were selected in accordance with the thickness and the length of the fillet of the angle. An example of strip marking before and after cutting is shown in Figures 5.2 and 5.3, respectively. The test

Figure 5.2. Strip marking on the specimen.

Figure 5.3. Residual stress strips after slicing.

specimens were 10 in. long and the Wittemore gage that was used to measure the released deformation had a 5 inch gage length.

Measurements of strains before and after slicing were carried out at 75°F. Any error due to bending deformations caused by the removal of the sections from the test piece were checked and found to be negligible. However, for zero applied load, equilibrium requires that the following three conditions be satisfied:

$$\int_A \sigma_r dA = 0 \quad (54)$$

$$\int_A \sigma_r x dA = 0 \quad (55)$$

$$\int_A \sigma_r y dA = 0 \quad (56)$$

where σ_r indicates the residual stress. These were checked using the measurement data, and it was found that inaccuracies in the strain measurements and the coarseness of the discretization of the cross section caused the equilibrium requirements to be violated. A computer program was therefore developed to make the necessary revision to the residual stress distribution, to satisfy the equilibrium conditions for a tolerance of 1.0 percent. A flow chart for the program is given in Fig. 5.4. The program listing is given in Appendix A.

Figure 5.4. Flow chart for the computer program to adjust the measured residual stresses to satisfy the equilibrium conditions.

As expected, it was found that the maximum compressive residual stresses were located at the heel and the tips of the angle legs, and the maximum tensile residual stresses were found at mid-length of each leg. The maximum measured compressive residual stress was 14.9 ksi, which is equal to 32% of the yield stress that was obtained from the tension coupon test, and the maximum measured tensile residual stress was 17 ksi ($0.4 \sigma_y$). Of interest is the fact that no symmetrical residual stresses were observed even for the equal-legged angles, confirming the findings of other researchers [21]. The distributions of the measured residual stresses before and after equilibration are given in Figures 5-5 through 5-9.

5.3 Stub Column Tests

Stub column tests were performed for all five cross sections with specimen dimensions and their locations in the stock selected according to the procedures of the Structural Stability Research Council [4]. The cutting of the specimens was done using a cold saw; minor end surface blemish effects were eliminated using thin sheets of copper inserted between the testing machine bearing plates and the ends of the specimen during the actual testing. The actual length, width, and thickness were measured to obtain the exact cross sectional area and the length of the specimen.

Figure 5.5. Residual stress distribution in an angle L3 x 3 x 3/8.

Figure 5.6. Residual stress distribution in an angle L5 x 3 x 3/8.

Figure 5.7. Residual stress distribution in an angle L5 x 5 x 3/8.

Figure 5.8. Residual stress distribution in an angle L4 x 4 x 5/8.

Figure 5.9. Residual stress distribution in an angle L6 x 4 x 3/4.

Alignment for all specimens was checked by imposing a load on the angle corresponding to a uniform axial stress of approximately $0.15 \sigma_y$, unloading, and adjusting the position of the stub column until readings differed by no more than 4%. These strain gages were located at mid-height of the specimen. Details of the stub column specimen and its instrumentation are shown in Figures 5.10 and 5.11. Vertical shortening of the angle and angle legs, as well as any horizontal and vertical movements at mid-height were recorded by using electrical transducers, Spring Retained Linear Position Sensor Module (SRLPSM), connected to a Hewlett Packard 3054A data acquisition system. Strains at mid-height were also recorded using 3 lead wire strain gages. The maximum slenderness ratio for all specimens was approximately 15. In order to detect the progress of yielding during testing, all specimens were whitewashed. The loading rate was approximately 5 kips/min in the elastic range and 2 kips/min in the inelastic range. SRLPSM and strain gage readings were made by the computer for increments of 2 kips.

Except for the 5 x 5 x 3/8 angle, the measured values of the maximum compressive residual stresses for the tested specimens are in good agreement with the corresponding values obtained from stub column tests, as indicated by the proportional limit. For example, the maximum

a.

b.

Figure 5.10. Instrumentation of the stub column specimen. --
(a) Locations for 6(SRLPSM) transducers; and
(b) Locations for 10 strain gages.

Figure 5.11. Stub column specimen under testing.

measured compressive residual stress for the 5 x 3 x 3/8 angle was 11.82 ksi, while the corresponding value obtained from the stub column test was 11.73. Moreover, the average yield stress obtained from the tension coupon tests is very close to the yield stress obtained from the stub column tests.

The results of the stub column tests that were conducted are given in Figures 5.12 through 5.16. The results are shown in a non-dimensional form; the ordinate represents the average axial stress vs. yield stress, $\frac{\sigma}{\sigma_y}$, while the abscissa gives average cross-sectional strains. A further discussion of the results is given in Chapter 6.

Figure 5.12.. Stub column curve for angle L3 x 3 x 3/8.

Figure 5.13. Stub column curve for angle L5 x 3 x 3/8.

Figure 5.14. Stub column curve for angle L5 x 5 x 3/8.

Figure 5.15. Stub column curve for angle L4 x 4 x 5/8.

Figure 5.16. Stub column curve for angle L6 x 4 x 3/4.

CHAPTER 6

EXPERIMENTAL PROGRAM AND RESULTS

6.1 Test Arrangement and Equipment

The test program was designed to study the behavior of axially loaded angle columns in the elastic and inelastic ranges. This would allow comparison of the results with those obtained by the computer model, which had been developed as part of the research program (see Chapters 3 and 4).

In addition to the stub column and other property tests that were developed as part of the overall experimental program (these are described in Chapter 5), the testing program also was planned to incorporate three full-size, pinned-end columns for each cross-sectional size. The maximum principal slenderness ratios for each cross section were selected as approximately 50, 100 and 150, to cover both the elastic and inelastic ranges of behavior. However, the short specimens ($L/r = 50$) of the angle sizes $6 \times 4 \times 3/4$ and $4 \times 4 \times 5/8$ and of the intermediate specimen ($L/r = 100$) for the angle size $6 \times 4 \times 3/4$ were excluded from the test program, because the failure loads for these could not be accommodated by the available testing equipment. The rest of the specimens were cold-sawed from

stock pieces to the appropriate lengths. Table 6.1 gives the data for all the specimens that were made.

Due to the fact that all specimens were to be tested as pinned-end columns, the critical section was expected to be near the mid-height of the column. Therefore, horizontal and vertical movements and strains were measured at mid-height for all specimens using SRLPSM transducers and strain gages. Depending on the size of the cross section, 4 to 6 transducers were attached to each specimen. Thus, horizontal and vertical movements and the rotation of the cross section and, in turn, of the center of gravity of the angle could be determined. Hence, the validity of the assumption that the column has a tendency to bend and twist simultaneously could be checked.

Further, three strain gages for each specimen were used to measure the axial strain at the toes and the heel of the angle. Consequently, any unusual behavior of the column under the application of the axial load could be reported. In addition to that, 5 strain gages were used in the transverse direction for the stub column specimens, to monitor any local buckling tendency. An additional transducer was used to measure the overall shortening of the specimen during the test, and all transducers and strain gages were connected to the data acquisition equipment. Details of the instrumentation are shown in Figs. 5.9 and 6.1.

Table 6.1 Column test specimen data.

Specimen No.	Specimen Size	Actual l/r	B/t
S1	3 x 3 x 3/8	49.4	8.0
M1	3 x 3 x 3/8	103.3	8.0
L1	3 x 3 x 3/8	156.3	8.0
S2	5 x 3 x 3/8	44.3	13.3
M2	5 x 3 x 3/8	99.8	13.3
L2	5 x 3 x 3/8	140.3	13.3
S3	5 x 5 x 3/8	81.2	13.3
M3	5 x 5 x 3/8	111.9	13.3
L3	5 x 5 x 3/8	154.8	13.3
M4	4 x 4 x 5/8	112.8	6.4
L4	4 x 4 x 5/8	150.0	6.4
L5	6 x 4 x 3/4	147.4	8.0

Fig. 6.1. Strain gage locations in column test specimen.

The short specimens of the 3 x 3 x 3/8 and the 5 x 3 x 3/8 angles were tested in the vertical position using a Tinius Olsen 200 kip universal testing machine. This is illustrated in Fig. 6.2. All other specimens were tested in the horizontal position using a calibrated hydraulic jack and other arrangement details as shown in Figs. 6.3 and 6.4. It was decided that this would be satisfactory, since the self-weight of the specimens is relatively small and will not have a significant influence on the buckling load.

During testing, the specimen end fixture allowed bending and twisting to take place freely at both ends. This was achieved by using rollers which were lubricated prior to each test, to ensure frictionless conditions. Details of the end fixtures are shown in Fig. 6.5.

The column specimens were carefully aligned in the test setup, first by welding the ends of the angles to the adjacent vertical plates, as shown in Fig. 6.5. in such a way that the centers of gravity of the plates and the angle cross section would coincide. A second alignment was then made by leveling the adjustable table, as shown in Fig. 6.4. The adjustable table is a thick, movable steel plate that is used mainly for two purposes. First, it is needed to hold the vertical plates at a 90° angle to the horizontal, and thus can be welded to the test specimen in that position. Secondly, the adjustable table is needed in the alignment of

Fig. 6.2. Testing a specimen in Tinius Olson 200 kip testing machine.

Fig. 6.3. Testing a specimen with a hydraulic jack.

Fig. 6.4. Horizontal column test setup, using a hydraulic jack.

Fig. 6.5. End fixture for pinned-end condition.

the specimens to move the vertical plates and the specimen up or down until the centers of gravity of the plates and the hydraulic jack coincide. Consequent to that, the loading line was to pass from the center of the hydraulic jack to the center of gravity of the test specimen. Due to the actual location of the end supports, the effective length of the specimen would be the center-to-center distance between the rollers of the end fixtures.

6.2 Test Procedure

Before testing of a column, the specimen was prepared and aligned as explained in Section 6.1. After providing for a satisfactory alignment, a small load (about $1/15$ to $1/20$ of the estimated failure load) was kept applied to the specimen to preserve the aligned condition. This was considered to be the initial load on the tested member, and all measurement devices were initialized at this load level.

For specimens that were tested in the Tinius Olsen testing machine, a loading rate of 5 kips/min was used in the elastic range and approximately 1.5 kips/min in the inelastic range. This rate of loading would allow for enough data points to be collected. The onset of yielding of each specimen was first estimated prior to the start of testing by determining the proportional limit stress. This equals the yield stress minus the measured maximum compressive residual stress of the specimen. This means that as long as the

applied stress was smaller than the proportional stress, the behavior would be elastic. The converse is true for the inelastic range. Secondly, the start of the inelastic range was noticed by the reduction of the speed of testing as the stress passed from the elastic to the inelastic range. This is due to the fact that in the inelastic range the column deforms more rapidly than when in the elastic range.

The readings were recorded using a Hewlett-Packard Series 200 computer; they were made for every 2 kips in the elastic range and for every 1 kip in the inelastic range. For the specimens that were tested with the hydraulic jack system, the load increment and the testing rate were determined individually. The maximum load and scale available with the hydraulic pump were the two major factors that controlled both of the above. In general, the load increment ranged from 3 to 8 kips, depending on the size of the specimen. Each load increment was maintained until the readings for that increment were made.

The incremental loading process and the recording of the corresponding readings were repeated until the maximum load of the member was reached. However, due to the fact that the specimens were simply supported at both ends and were loaded axially, it was felt that it would be safer not to load the specimens far beyond the maximum load. The column load-deflection curves that are given therefore do

not show an appreciable post-buckling behavior. More importantly, the main purpose of the tests was to obtain the buckling load and the mode of failure of the column, and not the behavior of the column after buckling. Halting the test shortly after the maximum load had been reached therefore had no effect on the primary focus of the study.

6.3 Test Results

The load-deflection curves for the various column test specimens are shown in Figs. 6.6 through 6.10. The load-data have been non-dimensionalized as P/P_y , where P is the applied load and P_y is the yield load. They are plotted against the overall axial strain, $\frac{\Delta L}{L}$.

As determined in the analytical study (see Chapter 3), all columns except one buckled at a load equal to or less than the yield load (i.e., no significant strain hardening occurred during testing). However, the short specimen of the 3 x 3 x 3/8 angle appeared to have developed strain hardening before buckling, as indicated by the curve in Fig. 6.6. It is noted that the yield load for this angle was 98.5 kips; the buckling load was 110 kips. The elastic buckling load was 246 kips. However, in any testing program it is reasonable to find that not all of the results will fall within certain limits. Due to the low slenderness ratio of the short specimen of the 3 x 3 x 3/8 angle, it was expected that the failure load would be close to the yield

Fig. 6.6. Load vs. axial strain deformation for $L3 \times 3 \times 3/8$.

Fig. 6.7. Load vs. axial strain deformation for L5 x 3 x 3/8.

Fig. 6.8. Load vs. axial strain deformation for L5 x 5 x 3/8.

Fig. 6.9. Load vs. axial strain deformation for L4 x 4 x 5/8.

Fig. 6.10. Load vs. axial strain deformation for L6 x 4 x 3/4.

load. However, the test result was approximately 10% higher than what was expected. This deviation may be attributed to unavoidable human error.

All of the tests showed that the relationship between P/P_y and the overall axial strain started out being linear and then became increasingly non-linear as the applied load got closer to P_y . This is due to the combined effect of the residual stresses and the increasing deflection, producing local yielding in the cross section. The deviation from linearity is particularly noticeable in the load deflection curves for the short specimens, as indicated in Figs. 6.6 through 6.10.

It was not always possible to record many data points for the post-buckling region of the specimens, particularly due to the suddenness of the failure. Thus, some of the curves in Figs. 6.6 through 6.10 appear to have a sudden cutoff.

The test results indicate that columns of large slenderness ratio are more deformable than those of small L/r . In other words, long columns experienced large deflections after buckling without any significant decrease in the applied load. This is due to the fact that for large slenderness ratios, a very large axial displacement is required to cause yielding in the member.

Load-strain curves for the columns are given in Figs. 6.11 through 6.13, where the non-dimensional load ratio P/P_y is used along with the strains that were measured by the strain gages at mid-height. The locations of the individual strain gages in the cross section are also shown in the figures. At the onset of loading, the strain was uniformly distributed over the cross section, and the three strain gages therefore give almost identical curves (see Fig. 6.11). However, as the applied load increases beyond the proportional limit, the resulting strains and, in turn, the resulting curves from the three strain gages diverge. This can be attributed to the combined effect of the non-uniformly distributed residual stresses and the bending that takes place in the column.

At this point, it is important to recall that up to the instant before buckling, all strain gages in the axial direction of the column are under compression; but when the column buckles, a sudden change in the strain distribution occurs, due to the lateral translation of the column. Consequently, some fibers in the cross section undergo an increase in the compressive strain, whereas others develop tension. This can be seen more clearly in the longer column as illustrated by the curves in Fig. 6.13. Similar results were obtained for all of the specimens. As a result, only

Fig. 6.11. Load-strain relationship at mid-height for test specimen No. S2 (L5 x 3 x 3/8 and L/r = 44.3).

Fig. 6.12. Load-strain relationship at mid-height for test specimen No. M2 (L5 x 3 x 3/8 and L/r = 99.8).

Fig. 6.13. Load-strain relationship at mid-height for test specimen No. L2 (L5 x 3 x 3/8 and L/r = 140.3).

the curves for the $5 \times 3 \times 3/8$ have been presented, since the data for the other members are very similar.

Careful attention was paid to the occurrence of any form of local buckling. To assist in this endeavor, all specimens were whitewashed prior to testing. This would aid in determining the onset of any local buckling along with local yielding. In addition, the strain gage readings would indicate local buckling. However, there was no evidence of this. A typical relationship between P/P_y and transverse strain are shown in Fig. 6.14.

In consequence, all specimens failed due to overall buckling. However, the behavior was different from that of the stub columns. The relationship between P/P_y and the transverse direction strains are shown in Figs. 6.15 through 6.24, indicating clearly that local buckling did occur in some of the stub columns. For instance, Fig. 6.15 gives the transverse strains at the opposite sides of the tip of one leg of the angle $3 \times 3 \times 3/8$. At the start of loading, both strain gages are under tension. The state of tension continues until local buckling occurs, whereupon the strain on one side of the angle leg reverses direction, and on the other side the tension increases. No local buckling was observed in the other leg, and the strain gages attached to this side continued to show tension until failure occurred (see Fig. 6.16).

Fig. 6.14. Load-transverse relationship at mid-height for test specimen No. S2
($L_5 \times 3 \times 3/8$ and $L/r = 44.3$).

Fig. 6.15. Load vs. transverse strain for the vertical leg of the stub column
L3 x 3 x 3/8.

Fig. 6.16. Load vs. transverse strain for the horizontal leg of the stub column L3 x 3 x 3/8.

Fig. 6.17. Load vs. transverse strain for the vertical leg of the stub column
L5 x 3 x 3/8.

Fig. 6.18. Load vs. transverse strain for the horizontal leg of the stub column L5 x 3 x 3/8.

Fig. 6.19. Load vs. transverse strain for the vertical leg of the stub column L5 x 5 x 3/8.

Fig. 6.20. Load vs. transverse strain for the horizontal leg of the stub column L5 x 5 x 3/8.

Fig. 6.21. Load vs. transverse strain for the vertical leg of the stub column
L4 x 4 x 5/8.

Fig. 6.22. Load vs. transverse strain for the horizontal leg of the stub column L4 x 4 x 5/8.

Fig. 6.23. Load vs. transverse strain for the vertical leg of the stub column L6 x 4 x 3/4.

Fig. 6.24. Load vs. transverse strain for the horizontal leg of the stub column L6 x 4 x 3/4.

Of particular interest here is the fact that even for the equal-legged angle $3 \times 3 \times 3/8$, local buckling occurred in one leg only. This verifies that the residual stress distribution is not symmetrically distributed in the equal-legged angle. In a similar fashion, local buckling in the other sizes of angles is indicated by the curves in Figs. 6.16 through 6.24.

The computed vertical and horizontal movements of the centroids, as well as the rotations of the cross sections at the mid-height of the specimens for the intermediate and long columns are shown in Figures 6.25 through 6.42. These indicate that all angles experienced simultaneous twist and flexure at mid-height during the testing. Bending and twisting are represented in the figures by the translation of the centroids and the angles of rotation of the cross sections, respectively. (The vertical and horizontal movements of the centroid and the angle of rotation of the cross section are defined in Fig. 3.1.) These results verify that, due to the low torsional stiffness and the lack of symmetry in the angle cross section, there is a substantial tendency toward twisting and bending. The results also show that the translations and the rotation increase with an increase in the length and/or the slenderness ratio of the specimen. (It is noted that due to the sensitivity of the SRLPSM transducers, any

variation that occurred during testing was recorded. Some of the test curves are therefore not smooth.)

Fig. 6.25. Center of gravity movements for test specimen No. M1 ($L_3 \times 3 \times 3/8$ and $L/r = 103.3$).

Fig. 6.26. Cross-sectional rotation of test specimen M1, measured relative to original center of gravity ($L3 \times 3 \times 3/8$ and $L/r = 103.3$).

Fig. 6.27. Center of gravity movements for test specimen No. L1 ($L3 \times 3 \times 3/8$ and $L/r = 156.3$).

Fig. 6.28. Cross-sectional rotation for test specimen No. L1, measured relative to original center of gravity ($L3 \times 3 \times 3/8$ and $L/r = 156.3$).

Fig. 6.29. Center of gravity movements for test specimen No. M2 ($L5 \times 3 \times 3/8$ and $L/r = 99.8$).

Fig. 6.30. Cross-sectional rotation for test specimen No. M2, measured relative to original center of gravity ($L5 \times 3 \times 3/8$ and $L/r = 99.8$).

Fig. 6.31. Center of gravity movements for test specimen No. L2 (L5 x 3 x 3/8 and $L/r = 140.3$).

Fig. 6.32. Cross-sectional rotation for test specimen No. L2, measured relative to original center of gravity ($L5 \times 3 \times 3/8$ and $L/r = 140.3$).

Fig. 6.33. Center of gravity movements for test specimen No. M3 (L5 x 5 x 3/8 and $L/r = 111.9$).

Fig. 6.36. Cross-sectional rotation for test specimen No. L3, measured relative to original center of gravity ($L5 \times 5 \times 3/8$ and $L/r = 154.8$).

Fig. 6.37. Center of gravity movements for test specimen No. M4 (L4 x 4 x 5/8 and $L/r = 112.8$).

Fig. 6.38. Cross-sectional rotation for test specimen No. M4, measured relative to original center of gravity ($L_4 \times 4 \times 5/8$ and $L/r = 112.8$).

Fig. 6.39. Center of gravity movements for test specimen No. L4 (L4 x 4 x 5/8 and $L/r = 150.0$).

Fig. 6.40. Cross-sectional rotation for test specimen No. L4, measured relative to original center of gravity (L4 x 4 x 5/8 and $L/r = 150.0$).

Fig. 6.41. Center of gravity movements for test specimen No. L5 ($L_6 \times 4 \times 3/4$ and $L/r = 147.4$).

Fig. 6.42. Cross-sectional rotation for test specimen No. L5, measured relative to original center of gravity ($L6 \times 4 \times 3/4$ and $L/r = 147.4$).

CHAPTER 7

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

7.1 Theoretical Buckling Load Predictions

The results of the analytical study using the computer model are presented in the form of traditional non-dimensional column curve in Figs. 7.1 to 7.5. The ordinate is the axial load ratio, P/P_y , and the abscissa is the maximum principal slenderness ratio.

Each of the figures gives three column curves and the test results for the respective member. Thus, the uppermost curve is the Euler curve, the middle one (which appears as a dotted line) is the minimum flexural buckling curve, and the lowest one is the flexural-torsional buckling curve. The test results are indicated by the symbols x in the figures.

Comparing the theoretical flexural and flexural-torsional buckling loads, it appears for some specimens the flexural-torsional buckling loads are always less than the flexural buckling loads, while for others the flexural-torsional loads are smaller only in the inelastic range. It therefore seems more reasonable to classify the

Fig. 7.1. Comparison of experimental and theoretical strength data for L3 x 3 x 3/8.

Fig. 7.2. Comparison of experimental and theoretical strength data for L5 x 3 x 3/8.

Fig. 7.3. Comparison of experimental and theoretical strength data for L5 x 5 x 3/8.

Fig. 7.4. Comparison of experimental and theoretical strength data for L4 x 4 x 5/8.

Fig. 7.5. Comparison of experimental and theoretical strength data for L6 x 4 x 3/4.

results into two categories, as follows. The first one includes the equal-legged angles and the second one the unequal-legged angles, and the strength and the behavior characteristics are clearly different.

In the first group, the two curves that represent the flexural buckling and the flexural-torsional buckling modes coincide in the elastic range (i.e., in the elastic range the flexural-torsional buckling load is equal to the minimum flexural buckling load). The two buckling loads being equal in the elastic range is as expected, because one of the possible roots of the flexural-torsional stability equation of the equal-legged is the minimum flexural buckling load (see Section 3.3). However, in the inelastic range the two curves are separated, and the flexural-torsional buckling load is always the lower one. This is because at this stage of loading, the cross section twists and bends simultaneously. The difference between the two loads is about 3% for the 3 x 3 x 3/8 and the 4 x 4 x 5/8 angles, and about 10% for the 5 x 5 x 3/8 angle. The major difference between these cross sections is the width-thickness ratio of the angle legs, which equals 8 for the 3 x 3 x 3/8 angle, 6.2 for the 4 x 4 x 5/8 angle, and 13.33 for the 5 x 5 x 3/8 angle. On the basis of these results, even though a general conclusion should not be drawn, due to the limited number of tests, it appears that

in the elastic range the flexural and the flexural-torsional buckling approaches give the same load for equal-legged angle, and also for the inelastic range when the angle has a low width-to-thickness ratio. However, for an equal-legged angle with a high width-to-thickness ratio, the flexural torsional buckling solution governs in the inelastic range.

In the second group, which includes the unequal-legged angles (see Figures 7.2 and 7.5), the flexural-torsional buckling load is always less than the corresponding flexural one. This is true for both the elastic and the inelastic ranges. However, it was pointed out in section 3.3 that when the angle has no axis of symmetry, the flexural-torsional buckling is smaller than P_x , P_y , and P_z . Further, Figs. 7.2 and 7.5 show that the deviation between the two curves increases in the inelastic range. For example, the maximum difference between the flexural and the flexural-torsional buckling loads for the 5 x 3 x 3/8 angle is 5% in the elastic range and 11% in the inelastic range. The corresponding values for the 6 x 4 x 3/4 angle are 4% and 7%. In addition, the difference between the flexural and flexural-torsional loads is greater for the 5 x 3 x 3/8 angle than for the 6 x 4 x 3/8 angle. This may be due to the magnitude of the width-to-thickness ratio: for the first angle it is 13.33; for the second it is 8.0.

It was intended to provide a detailed comparison of these results with those of other researchers, but it appears that such information is not currently available. The results of this study, therefore, take an added importance, especially to the extent that they indicate strength and behavior trends for an important class of members where other data do not exist. The practical implications of this may be significant, in that they point to the development of new design criteria. However, additional research should be conducted to add to the data base, before any formal design recommendations are made.

7.2 Further Evaluations of Experimental and Theoretical Results

Figures 7.1 through 7.5 show the column test data and the theoretical results. In general, it can be seen that there is good agreement between the analytical and the experimental findings. Further, it was found that the long columns tend to fail by flexural buckling, and the short ones by flexural-torsional buckling. However, for the long columns the buckling loads obtained by the analytical solution are slightly higher than those obtained in the tests. The lower experimental values may be attributed in part to testing errors such as inaccuracies in alignment and column imperfections. These types of errors were not considered in the computer model.

Another consideration is that since the short specimen of the 3 x 3 x 3/8 angle reached strain hardening before buckling, the failure load was 110 kips as compared to the yield load of 98.5. Therefore, the data point 'x' in Fig. 7.1 appears above the line represents the yield load. However, from a structural point of view, the member is unserviceable once it has reached the yield point. It can be seen from Fig. 7.1 that yielding of the short specimen of the 3 x 3 x 3/8 angle started at a load equal to 94.6 kips, which is equal to 0.96 of P_y . Therefore, the symbol o, which represents the failure load of the specimen after modification, appears in the figure at the 0.96 P_y position.

A more detailed comparison between the experimental and the computer model results is given in Table 7.1. In this table, the experimental loads are given in Column 3 and the computed flexural and flexural-torsional buckling loads are given in columns 4 and 5, respectively. The smaller value of the two computer results was used to indicate the mode of failure of that specimen. This is given in Column 6 of Table 7.1. The ratio of the experimental failure load to the minimum value of the corresponding theoretical failure load is shown as a percentage in Column 7. The average value for the specimens that failed by flexural-torsional buckling is 0.97 and for those failed by the flexural buckling is 0.93. The overall average value of the

Table 7.1. Comparison between analytical and experimental results.

Angle	L/r (1)	P _y (2)	Measured Load (3)	Flexural Buck (4)	F. T. Buck. (5)	Type of Failure (6)	<u>Experimental Load</u> Min. Theor. Load
3 x 3 x 3/8	49.40	98.47	94.10	90.35	88.65	FT ¹	1.06
3 x 3 x 3/8	103.27	98.47	51.80	56.0	56.00	F ^{2,3}	0.93
3 x 3 x 3/8	156.30	98.47	23.32	24.35	24.35	F	0.96
5 x 3 x 3/8	44.34	128.99	114.80	122.20	116.00	FT	0.98
5 x 3 x 3/8	99.77	128.99	68.20	80.40	72.10	FT	0.95
5 x 3 x 3/8	140.29	128.99	36.38	42.5	40.50	FT	0.90
5 x 5 x 3/8	81.18	157.87	112.67	126.50	111.95	FT	1.01
5 x 5 x 3/8	111.87	157.87	72.00	80.00	80.0	F	0.90
5 x 5 x 3/8	154.80	157.87	40.12	41.50	41.50	F	0.97
4 x 4 x 5/8	112.84	209.22	96.20	104.50	104.50	F	0.92
4 x 4 x 5/8	150.03	209.22	52.26	60.0	60.00	F	0.87
6 x 4 x 3/4	147.38	314.79	82.25	93.5	90.00	FT	0.91

¹ FT = Flexural-torsional buckling

² F = Flexural buckling

³ For singly symmetrical angles, the governing root of the stability equation is identical for flexural and flexural-torsional buckling. Flexural buckling has been chosen as the more representative buckling criterion.

(experimental/theory) ratio is 0.95 and the standard deviation is 0.05. This is an indication of the good agreement between the analytical and the experimental work.

It should be noted here that since the average of the (experimental/theory) ratio is less than 1.0, the theory is somewhat unconservative.

The good agreement between the computer model and the laboratory experiments verifies the assumptions made at the beginning of Chapter 3. Further, it shows that the computer model can be used to predict the behavior of other sizes of angle cross sections, even with different cross-sectional and material parameters.

CHAPTER 8

FURTHER DISCUSSION OF THE RESULTS

The experimental and the theoretical results have already been discussed in detail in Chapters 6 and 7. In this chapter, the discussion will focus on the differences and the similarities between the results of the current study and those of other researchers. In addition, the potential impact on current design criteria is considered in some detail.

An exhaustive survey of the existing literature was conducted in order to locate experimental and other earlier studies on angle cross sections. Briefly, it was found that very little work had been conducted on this important class of structural shapes. Of primary relevance are the recent investigations of Kennedy and Murty [38] and Kitipornchai and Lee [39,40].

Kennedy and Murty investigated equal-legged, pinned-end, single-angle axially loaded columns. In the analytical study it was assumed that plate buckling and flexural-torsional buckling are the same. Therefore, the calculated plate buckling load was considered to represent the two modes of failure. This assumption is not

necessarily correct, and the plate buckling load, according to the equation that was used by Kennedy, is constant for a given cross section regardless of the length of the column. Therefore, there is no recognition of the slenderness ratio in this solution. However, as has been shown by the current study, the occurrence of flexural-torsional buckling is strongly influenced by the slenderness ratio. A direct comparison of the analytical results of the Kennedy and the current study is therefore not realistic, since the bases are so different.

However, the experimental part of Kennedy's investigation was not affected by the analytical assumptions, and these results can be used for comparison with the test results of this study. This is shown in a nondimensional form in Fig. 8.1. The ordinate represents the ratio of the axial load to the yield load, P/P_y , and the abscissa is the non-dimensional slenderness term $\lambda_{\max} = (L/r)_{\min} \sqrt{\frac{\sigma_y}{\pi^2 E}}$.

In their theoretical evaluations, Kitipornchai and Lee [39] used only one equal and one unequal-legged angle and there is therefore not enough data to compare with the theoretical results of the current study. However, for the experimental study, Kitipornchai tested seven equal-legged and six unequal-legged angles. The specimens were loaded axially and the end fixtures were detailed in such a way that the end rotations were permitted, but twisting was

restrained. This restraint caused ϕ' to be zero at both ends of the tested specimen (i.e., no St. Venant torsional moment at the ends). Therefore, the ends of the specimens tested by Kitipornchai were free from torsion. As a result, the torsional rotations along the axis of the fixed-ended specimen, when comparing to the corresponding rotations of the simply supported one, were expected to be smaller. Thus the difference between the flexural-torsional buckling and the flexural buckling may be reduced due to the end restraint of twisting. However, the results of the investigation of Kitipornchai, along with the results of the current investigation, are shown in Figs. 8.1 and 8.2.

In general, the experimental results that are shown appear to correlate quite well, especially the findings of Kennedy and Murty and the current study. The results of Kitipornchai and Lee appear to have been shifted to the left (lower slenderness ratio) of other results in the case of the equal-legged angles and to the right in the case of unequal-legged angles. This deviation is most likely due to twisting end restraint that was imposed in the Kitipornchai tests.

This agreement between the test results of the current study and other studies verifies the experimental results of the current investigation. Since the analytical results and the experimental results of this investigation

Fig. 8.1. Comparison of test results for equal-legged angles.

Fig. 8.2. Comparison of test results for unequal-legged angles.

are closely related (see Chapter 7), this verification of the test results implies the validity of the computer model that was developed as part of the program of this project.

A comparison of the results obtained in this study with the current design specifications is of somewhat limited use. This is due to the fact that the AISC allowable stress design specification, the CRC basic column strength curve, and the SSRC multiple column curves are based on the assumption that flexural buckling is the governing mode of failure. This is correct insofar as buckling of doubly symmetric shapes is concerned, such as the common wide-flange type. The design criteria for angles and similar members are lacking specific information on the behavior and the strength, with the exception of the elastic torsional buckling rules of the new AISC LRFD specification. However, the results that have been obtained in this study point the way to an important and comprehensive design method for singly symmetric and asymmetric shapes where the complete range of behavior and strength have been accounted for.

CHAPTER 9

SUMMARY AND CONCLUSIONS

In this study an analytical formulation for predicting the buckling load of an axially loaded, pinned-end, angle cross section column has been formulated. The effect of residual stresses has been accounted for in the mathematical model. Thus, the movement of the centroid, the reduction in the moment of inertia, and the change in the warping constant, $\bar{K}$, due to the spread of yielding for each successive increase of loading in the inelastic range have been considered. The solution therefore is based on an inelastic formulation of the flexural and flexural-torsional stability of singly symmetric and asymmetric cross-section members, and incorporates certain second-order effects. To the author's knowledge, this is the first time such a solution has been obtained.

To improve the prediction of the buckling load, the actual residual stress distribution in the cross section was measured, and the material properties were determined from tension specimen and stub column tests.

Due to the complexity of the problem, a numerical integration technique to evaluate the critical lengths was

developed. To verify the analytical model, a series of full-scale column tests were conducted. The average of the ratio of the experimental buckling loads to the minimum theoretical buckling load was 0.95, the standard deviation was 0.05, and the coefficient of variation was 0.053. This good agreement between the analytical and the experimental results verifies that the mathematical model can be used with confidence in future applications for studying different characteristics of angle cross-section columns. Moreover, with some changes made in the computer programs, it can also be used for cross-sectional shapes other than angles, for which torsional considerations may be important.

It should be mentioned here that in modifying the theory and the program for cross sections other than angles, the warping moment of inertia I_ω must be incorporated. This term was eliminated in development of the theory in this study, since the warping resistance of angles is small to non-existent, depending on the angle leg thickness.

The buckling loads of columns of angle cross section can be categorized as follows:

1. When the cross section is of the unequal-legged angle type, the flexural-torsional buckling load is always less than the flexural one. This is true for both the elastic and inelastic range of loading and

for all b/t values. The difference between the two modes of failure ranges from 3 to 10%.

2. When the cross section is of the equal-legged angle type, with a low width-to-thickness ratio, the flexural and the flexural-torsional buckling loads are identical in the elastic range. The former is about 2% higher in the inelastic range. Therefore, for this type of cross section either method can be used for design purposes without any significant error.
3. When the cross section is of the equal-legged type, with a high width-to-thickness ratio, the flexural and flexural-torsional buckling loads are almost the same in the elastic range, but in the inelastic range the flexural-torsional mode of buckling is the governing one. The difference between the two approaches can be as high as 11%. Therefore, for design purposes the two buckling loads must be calculated and the governing one determined.

The theoretical and experimental work presented herein verify that for angle cross sections, flexural-torsional buckling load is often the governing mode of failure. However, to generalize the statement for other cross sectional shapes that are not doubly symmetric, more research is needed to determine the actual distribution of

residual stresses as well as the correlation between column strength and other column parameters.

Further, due to the increased availability of high yield stress rolled shapes, the usage of thin-walled cross section members in general and angle cross section members in particular is likely to be more widespread. The real behavior of these types of cross sections should therefore be considered. This has been achieved in the current study, where good correlation has been obtained between a theoretical model and experimental results.

It should be noted here that, in using the current design specifications for columns of asymmetric or mono-symmetric cross sections, it is advisable to check the possibility of flexural-torsional failure, as one of the applicable limit states. This can be used in designs based on load and resistance factor design as well as allowable stress design, although in the latter case a suitable factor of safety must be applied.

APPENDIX A

PROGRAM TO ADJUST RESIDUAL STRESSES
TO SATISFY EQUILIBRIUM REQUIREMENTS

PROGRAM TO ADJUST RESIDUAL STRESSES
TO SATISFY EQUILIBRIUM REQUIREMENTS

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PROGRAM TO ADJUST RESIDUAL STRESSES TO SATISFY EQUILIBRIUM
REQUIREMENTS
DIMENSION VESG(20),HESG(20),VD(20),HD(20),YV(20)
DIMENSION YH(20),HDD(20),XH(20),XV(20)
DIMENSION VRES(20),HRES(20)
COMMON /A/ RESF, MOMENT
REAL MOMENT
REAL IX,IY,IXY,IXP,IYP
REAL LV,LH
READ(3,*) , LV,LH,TH,AREA
WRITE(4,151) LV,LH,TH,AREA
151 FORMAT(10X,'LENGTH=',F5.2,5X,'WIDTH=',F5.2,5X,'THICKNESS=',
* F5.2,5X,'AREA=',F6.3)
C   NVE=NO. OF SEGMENT IN VERTICAL LEG
C   NHE=NO. OF SEGMENT IN HORIZONTAL LEG
READ(3,*) NVE,NHE
WRITE(4,152) NVE,NHE
152 FORMAT(///,10X,'# OF VERTICAL ELEMENT=',I2,/,10X,'# OF
* HORIZONTAL ELEMENT=',I2)
C   READ IN THE LENGTH OF EACH SEGMENT
READ(3,*) (VSEG(I),I=1,NVE)
WRITE(4,153)
153 FORMAT(//,30X,'LENGTH OF VERTICAL ELEMENTS ARE :')
WRITE(4,*) (VSEG(I),I=1,NVE)
READ(3,*) (HSEG(J),J=1,NHE)
WRITE(4,154)
154 FORMAT(30X,'LENGTH OF HORIZONTAL ELEMENTS ARE :')
WRITE(4,*) (HSEG(J),J=1,NHE)
DO 11 I=1,NVE
11  VD(I)=0.0
DO 12 I=1,NHE
12  HD(I)=0.0
C   TO CALCULATE THE DISTANCE BETWEEN S.C AND THE C.G OF EACH SEG
DUM1=0.0
DUM2=0.0
DO 13 M=1,NVE
VD(M)=DUM1+DUM2+0.5*VSEG(M)
DUM1=VD(M)
13  DUM2=VSEG(M)*0.5
DO 14 I=1,NHE
14  HD(I)=0.0
DUM3=0.0
DUM4=0.0
DO 15 J=1,NHE
HD(J)=DUM3+DUM4+HSEG(J)*0.50
DUM3=HD(J)

```

```

DUM4=HSEG(J)*0.5
15 HD(J)=HD(J)+VSEG(1)
C READ IN MOMENT OF INERTIA,C.G COORDINATES,THEANGLE OF
C INCLINATION OF THE PRINCIPAL AXES (IN RADIANS)
READ(3,*) IX,IY,X,Y,THEATA
WRITE(4,155) IX,IY,X,Y,THEATA
155 FORMAT(/,10X,'IX=',F6.3/,10X,'IY=',F6.3/,10X,'X=',F6.3
* ,/,10X,'Y=',F6.3/,10X,'THEATA=',F7.0)
C CALCULATE THE PRINCIPAL AXIS MOMENT OF INERTIA
IXY=((IX-IY)/2.0)*TAN(2.0*THEATA)
IXP=((IX+IY)/2.0)+SQRT(((IX-IY)/2.0)**2.0+IXY**2.0)
IYP=((IX-IY)/2.0)-SQRT(((IX-IY)/2.0)**2.0-IXY**2.0)
WRITE(4,156) IXP,IYP,IXY
156 FORMAT(/,10X,'IXP=',F6.3/,10X,'IYP=',F6.3/,10X,'IXY=',F7.3)
C TO LOCATE THE INTERSECTION OF THE TWO AXES WITH THE TWO
C LEGS
X=X-0.5*TH
Y=Y-0.5*TH
XX=X+Y*TAN(THEATA)
YY=Y+(X/(TAN(THEATA)))
C TO CALCULATE THE PERPENDICULAR DISTANCE FOR EACH
C SET FROM THE AXES.
C 1- THE DISTANCES FOR VERTICAL SEG FROM X-AXIS
XOUT=(Y/TAN(THEATA))-X
YOUT=XOUT*TAN(THEATA)
DO 18 L=1,NVE
18 YV(L)=(YOUT-VD(L))*COS(THEATA)
C 2- THE DISTANCES FOR HORIZONTAL SEG. FROM X-AXIS
DO 19 J=1,NHE
HDD(J)=HD(J)+XOUT
19 YH(J)=HDD(J)*SIN(THEATA)
C 3- THE DISTANCES FOR HORIZONTAL SEG. FROM Y-AXIS
DO 20 N=1,NHE
20 XH(N)=(XX-HD(N))*COS(THEATA)
DO 21 NN=1,NVE
21 XV(NN)=(YY-VD(NN))*SIN(THEATA)
C INPUT RESIDUAL STRESSES
READ(3,*) (VRES(JK),JK=1,NVE)
READ(3,*) (HRES(LK),LK=1,NHE)
WRITE(4,158)
158 FORMAT(1H1,/,20X,'THE ACTUAL RESIDUAL STRESSES ARE :')
WRITE(4,159) (VRES(NJ),NJ=NVE,1,-1)
159 FORMAT(/,10X,F7.3)
WRITE(4,161) VRES(1), (HRES(KI),KI=1,NHE)
161 FORMAT(/,10X,F7.3,5(2X,F7.3))
170 CALL RESID (VRES,YV,VSEG,NVE,TH)
CALL RRESID (HRES,YH,HSEG,NHE,TH)
FORCE=RESF
ST=FORCE/AREA

```

```

XMOMET=MOMENT
CALL RESID (HRES,XH,HSEG,NHE,TH)
CALL RSED (VRES,XV,VSEG,NVE,TH)
YMOMET=MOMENT
CALL EQUIL (VRES,YV,IXP,XMOMET,NVE)
DO 26 J=1,NHE
ALAL=ST+(XMOMET/IXP)*YH(J)
HRES(J)=HRES(J)-ALAL
26 CONTINUE
CALL EQUIL(HRES,XH,IYP,YMOMET,NHE)
DO 30 KL=1,NVE
ALVL=ST+(YMOMET/IYP)*XV(KL)
32 VRES(KL)=VRES(KL)-ALVL
30 CONTINUE
RUSH=ABS(FORCE)
IF(RUSH.LE. 0.01) GO TO 180
GO TO 170
180 WRITE(4,163) FORCE,ST,XMOMET,YMOMET
163 FORMAT(1H1,/,15X,
* 'THE OUT OF EQUILIB. FORCE & MOMENTS AFTER EQUILI
* . PROCESS ARE:',/,30X,'FORCE=',F8.4,/,30X,'STRESS=',
* F8.4,/,30X,'XMOMET='
* ,F8.4,/,30X,'YMOMET=',F8.4)
WRITE(4,159) (VRES(K),K=NVE,1,-1)
WRITE(4,161) VRES(1), (HRES(L),L=1,NHE)
STOP
END

```

```

SUBROUTINE EQUIL (RES,AAY,ENER,AMOME,N)
DIMENSION RES(20),AAY(20)
ALA=AMOME/ENER
DO 25 I=1,N
25 RES(I)=RES(I)-ALA*AAY(I)
CONTINUE
RETURN
END

```

```

SUBROUTINE RESID(RES,XZ,SEG,NE,TH)
COMMON /A/ RESF,MOMENT
DIMENSION RES(20),XZ(20),SEG(20)
REAL MOMENT
C INITIALIZE THE VALUES
RESSTR=0.0
RESF=0.0

```



```
MOMENT=0.0
DELTAM=0.0
DELTAF=0.0
ENTRY RRSED (RES,XZ,SEG,NE,TH)
DO 40 I=1,NE
RESSTR=RES (I)
DELTAF=RESSTR*SEG (I) *TH
RESF=RESF+DELTAF
DELTAM=DELTAF*XZ (I)
40 MOMENT=MOMENT+DELTAM
RETURN
END
```

APPENDIX B

PROGRAM ANALYTICAL STUDY FOR
ANGLE CROSS SECTION

PROGRAM ANALYTICAL STUDY FOR
ANGLE CROSS SECTION

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PROGRAM ANALYTICAL STUDY FOR ANGLE CROSS SECTION
DIMENSION RES1(50),RES2(50)
DIMENSION VSTRN(50),HSTRN(50)
DIMENSION VALUE1(50),VALUE3(50)
DIMENSION CHECK1(50),CHECK2(50)
DIMENSION VD(50),YPSLEV(50),XPSLEV(50)
DIMENSION IXE,IYE,IXPE,IYPE
C   READ NO. OF VERTICAL & HORIZ. ELEMENTS
   READ(7,*) NVSEG,VSEG,NHSEG,HSEG,T
   WRITE(8,305)NVSEG,VSEG,NHSEG,HSEG,T
305  FORMAT(1H1,/,15X,'NVSEG=',I3,/,15X,'VSEG=',F10.7,
*    /,15X,'NHSEG=',I3,/,15X,'HSEG=',F10.7,/,15X,'THICKNESS=',F7.5)

C   Y1=THE LENGTH OF THE VERTICAL LEG & X1=THE LENGTH OF THE H. LEG

   Y1=NVSEG*VSEG
   X1=NHSEG*HSEG
C   READ THE START APPLIED LOAD & STRAIN INCREMENT
   READ(7,*)P,STINCR
C   READ THE VALUE OF RESIDUAL STRESSES AT THE CENTER OF EACH SEG
   READ (7,*) (RES1(K),K=1,NVSEG)
   WRITE(8,701)
   WRITE (8,306) (RES1(M),M=1,NVSEG)
306  FORMAT(//,6(3X,F7.3))
701  FORMAT(1H1,/,15X,'THE VERTICAL RESIDUAL STRESSES ARE:')
   READ (7,*) (RES2(N),N=1,NHSEG)
   WRITE(8,702)
   WRITE(8,307) (RES2(J),J=1,NHSEG)
307  FORMAT(//,6(3X,F7.3))
702  FORMAT(///,10X,'THE HORIZ.RESIDUAL STRESSES ARE:')
   READ(7,*)FY,E
   WRITE(8,308)FY,E
308  FORMAT(///,10X,' YIELD STRESS=',F12.2,/,10X,'MOD. OF ELAST.=',
*    F12.1)
C   TO TRANSFER STRESSES INTO STRAINS
   DO 215 I=1,NVSEG
215  VSTRN(I)=-RES1(I)/E

   DO 216 J=1,NHSEG
216  HSTRN(J)=-RES2(J)/E
C
C   TO DEFINE THE LENGTH THAT DID NOT YIELD YET
C   STRAIN=THE APPLIED STRAIN
C   YSTRN=YIELD STRAIN

```

```

      READ(7,*) YSTRN, STRAIN
      WRITE(8,590) YSTRN, STRAIN
590  FORMAT(//,20X,'YIELD STRAIN=',F8.6,/,20X,'APPLIED STRAIN=',
      * F8.6)
      WRITE(8,850) STINCR
850  FORMAT(//,15X,'STRAIN INCREMENT=',F12.9)
      STRAIN=STRAIN-STINCR
      P=P-5.0
      A1=Y1-0.5*VSEG-0.5*T
      DELTAA=VSEG*T
      DISTA=A2
      B1=X1+0.5*T-0.5*HSEG
      DELTAB=HSEG*T
      DISTB=B1
C      START THE DO LOOP FOR CALCULATION THE COEFFICIENTS OF THE
      * POLYNOMIAL EQUATION
C      DO 2 INCREM=1,40
      STRAIN=STRAIN+STINCR
      P=P+5.0
      DO 217 L=1,NVSEG
      VALUE1(L)=VSTRN(L)=STRAIN
      IF(VALUE1(L) .LT. 0.0) GO TO 217
      IF(VALUE1(L) .GT. YSTRN) VALUE1(L)=YSTRN
217  CONTINUE
C
C      FO THE HORIZ. LEG
      DO 25 LM=1,NHSEG
      VALUE3(LM)=HSTRN(LM)+STRAIN
      IF(VALUE3(LM) .LT. 0.0) GO TO 225
      IF(VALUE3(LM) .GT. YSTRN) VALUE3(LM)=YSTRN
225  CONTINUE
C      TO LOCATE C.G OF THE ELASTIC PART
      DISTA=A1
      SUMA=0.0
      SUMD=0.0
      DO 805 I=1,NVSEG
      DISTA=DISTA-DUMV
      CHECK1(I)=ABS(VALUE1(I))
      IF(CHECK1(I) .EQ. YSTRN) GO TO 804
      SUMA=SUMA+DELTAA
      SUMD=SUMD+DELTAA*DISTA
804  DUMV=VSEG
805  CONTINUE
      DISTB=B1
      SUMK=0.0
      SUML=0.0
      DUMH=0.0
      DO 807 J=1,NHSEG
      DIST=DISTB-DUMH

```

```

      CHECK2(J)=ABS(VALUE3(J))
      IF(CHECK2(J) .EQ. YSTRN) GO TO 806
      SUMK=SUMK+DELTAB
      SUML=SUML+DISTB*DELTAB
806  DUMH=HSEG
807  CONTINUE
      SUMAR=SUMA+SUMK
      IF(SUMAR .EQ. 0.0) GO TO 111
      YAXIS=SUMD/SUMAR
      XAXIS=SUML=SUMAR
111  CONTINUE
      WRITE(8,800) INCREM
800  FORMAT(1H1,/,30X,'INCREM #',12)
      WRITE(8,851)P,STRAIN
851  FORMAT(/,20X,'APPLIED LOAD =' ,F8.3,10X,'APPLIED STRAIN=' ,F12.9)
      WRITE(8,181)YAXIS,XAXIS
282  FORMAT(/,10X,'YAXIS=' ,F10.5,5X,'XAXIS=' ,F10.5)

C      TO CALCULATE THE MOMENT OF INERTIA ABOUT THE X_AXIS
      DA=A1
      FUM1=0.0
      EIX1=0.0
      EIX2=0.0
      EIXYV=0.0
      DEIXY1=0.0
      IXE=0.0
      DEL1IX=0.0
      DEL2IX=0.0
      DO 813 M=1,NVSEG
      DA=DA-FUM1
      IF(CHECK1(M) .EQ. YSTRN) GO TO 813
      DEL1IX=VSEG*T*(DA-YAXIS)**2
      EIX1=EIX1+DEL1IX
      DEIXY1=VSEG*T*(YAXIS-DA)*XAXIS
      EIXYV=EIXYV+DEIXY1
813  FUM1=VSEG

      DO 814 LM=1,NSEG
      IF(CHECK2(LM) .EQ. YSTRN) GO TO 814
      DEL2IX=HSEG*T*YAXIS**2
      EIX2=EIX2+DEL2IX
814  CONTINUE
      IXE=EIX1+EIX2

C      TO CALCULATE THE MOMENT OF INERTIA ABOUT THE Y_AXIS
      DB=X1+0.5*T-0.5*HSEG
      FUM2=0.0
      EIY1=0.0

```

```

EIIY2=0.0
IYE=0.0
DELIY=0.0
DEL2IY=0.0
DEIXY2=0.0
EIXYH=0.0
EIXY=0.0
DO 8.5 NI=1,NHSEG
DB=DB-FUM2
IF(CHECK2(NI) .EQ. YSTRN) GO TO 815
DELIY=HSEG*T*(DB-XAXIS)**2
EIIY1=EIIY1+DELIY
DEIXY2=HSEG*T*(XAXIS-DB)*YAXIS
EIXYH=EIXYH+DEIXY2
815 FUM2=HSEG

EIXY=EIXYV+EIXYH

DO 816 KI=1,NVSEG
IF(CHECK1(KI) .EQ. YSTRN) GO TO 816
DEL2IY=VSEG*T*XAXIS**2
EIIY2=EIIY2+DEL2IY
816 CONTINUE
IYE=EIIY1+EIIY2
WRITE(8,183) IXE,IYE,EIXY
183 FORMAT(/,10X,'IXE=',F10.5,5X,'IYE=',F10.5,5X,'EIXY=',F10.5)

IF(IXE .EQ. 0.0 .AND. IYE .EQ. 0.0) GO TO 144
C TO CALCULATE THE ANGLE OF ROTATION AND XO AND YO
DIVE=ABS(IXE-IYE)
IF(DIVE .GT. .01) GO TO 913
TH=0.785398
GO TO 914
913 TANANG=(-2.0*EIXY/(IXE-IYE))
TH=0.5*ATAN(TANANG)
XTAN=XAXIS*TAN(TH)
914 IF(XTAN .GT. YXAIS) GO TO 917
XO=(XAXIS/COS(TH))+(YAXIS-XTAN)*SIN(TH)
YO=(YAXIS-XAXIS*TAN(TH))*COS(TH)
GO TO 918
917 XO=(XAXIS/COS(TH))-(XTAN-YAXIS)*SIN(TH)
YO=(XTAN-YAXIS)*COS(TH)
918 CONTINUE
      WRITE(8,915) TH,XO,YO
915 FORMAT(/,10X, 'ANGLE OF TOTATION=',F8.6,/,10X,'XI=',
* F10.5,/,10X,'YO=',F10.5)

```

G TO CALCULATE THE PRINCIPAL AXIS MOMENT OF INERTIA

```

CONST1=(IXE+IYE)*0.5
CONST2=SQRT(((IXE-IYE)*0.5)**2+EIXY**2)
IXPE=CONST1+CONST2
IYPE=CONST1-CONST2
WRITE(8,184) IXPE,IYPE
184  FORMAT(/,10X,'IXPE=',F10.5,5X,'IYPE=',F10.5)

YY1=Y1-0.5*T
YYY1=YY1-0.5*VSEG
VD(1)=YYY1
DO 603 II=2,NVSEG
603  VD(II)=VD(II-1)-VSEG
C    TO CALCULATE INTEGRATION OF SIGMA*((YO-Y)**2++(XI-X)**2)
C 1   FOR VERTICAL LEG
      YOUT=ABS(YAXIS)+ABS(XAXIS/TAN(TH))
      CROSS=YAXIS-XTAN
      DO 412 I=1,NVSEG
      IF(XTAN .LT. YAXIS) GO TO 415
      YPSLEV(I)=0VD(I)*COS(TH)+YO
      GO TO 412
415  YPSLEV(I)=- (VD(I)-CROSS)*COS(TH)
412  CONTINUE
      DO 420 J=1,NVSEG
      XPSLEV(J)=(YOUT-VD(J))*SIN(TH)
420  CONTINUE
      DUM31=0.0
      DUM33=0.0
      SIGMAV=0.0
      DUM44=0.0
      PV=0.0
      DO 422 K=1,NVSEG
      DUM31=(XO-XPSLEV(I))**2+(YO-YPSLEV(K))**2
      DUM33=(E*VALUE1(I)*VSEG*T)
      DUM44=DUM33*DUM31
      SIGMAV=SIGMAV+DUM44
      PV=PV+DUM33
422  CONTINUE
      WRITE(8,606) SIGMAV
606  FORMAT(/,15X,'SIGMAV=',F10.4)
C    X1=THE LENGTH OF THE HORIZONTAL LEG
      XX1=X1+0.5*T
      XXX1=XX1-0.5*HSEG
      HD(1)=XXX1
      DO 608 N=2,NHSEG
608  HD(N)=ND(N-1)-HSEG

```

```

C 2   FOR THE HORIZ. LEG
      XOUT=ABS (XAXIS) -T/2.0 +ABS (YAXIS*TAN (TH) )
      DBF= (ABS (XTAN) -ABS (YAXIS) ) / (TAN (TH) ) -T/2.0
      DO 421 L=1,NHSEG
      IF (XTAN .LT. YAXIS ) GO TO 432
      IF (DF .LT. 0.0) GO TO 432.
      YPSLEX (L) = (HD (L) -DF) *SIN (TH)
      GO TO 431
432   YPSLEX (L) =HD (L) *SIN (TH) + (CROSS + 0.5*T*TAN (TH) ) *COS (TH)
431   CONTINUE
      DO 435 MP=1,NHSEG
      XPSLEX (MP) = (XOUT-HD (MP) ) *COS (TH)
435   CONTINUE
      DUM40=0.0
      DUM42=0.0
      SIGMAH=0.0
      DUM45=0.0
      PH=0.0
      DO 436 M=1,NHSEG
      DUM40= (XO-XPSLEX (M) ) **2+ (YO-YPSLEX (M) ) **2
      DUM42= (E*VALUE3 (M) *HSEG*T)
      DUM45=DUM42*DUM40
      SIGMAH=SIGMAH+DUM45
436   PH=PH+DUM42
      WRITE (8,609) SIGMAH
609   FORMAT (//,15X, 'SIGMAH=',F10.4)
      SIGTOT=SIGMAV+SIGMAH
      WRITE (8,610) SIGTOT
610   FORMAT (//,15X, 'SIGTOT=',F10.4)

C      PT IS THE TOTAL FORCE ON THE CROSS SECTION = INTEGRATION OF
C      * SIG*DEL A

      PT=PV+PH
      WRITE (8,200) PT
200   FORMAT (//,15X, 'TOTAL FORCE (APPLIED + RESIDUAL) =' ,F7.3)
      G=0.5*E/ (1.0+0.3)
      AC= ( (Y1+X1) /3.0) *T**3
      CT=AC*G
      D1=9.869604*E*IXPE
      D2=9.869604*E*IYPE
      FD=X0**2+YO**2
      CK=CT-SIGTOT
      H1= (PT**3) *FD+ (PT**2) *CK
      W11= (PT**2) * (D1*YO**2+D2*XO**2)
      W25= (PT*CK) * (D1+D2)
      H2=- (W11+W25)
      H2=H2/H1
      H3=CK*D1*D2
      H0=H3/H1

```



```
WRITE(8,473) D1,D2
473  FORMAT(/,L5X,'DL=',E14.8,5X,'D2=',E14.8)
      FLEXL=SQRT(D2/PT)
      WRITE(8,588) FLEXL
588  FORMAT(15X,'THE FLEXURAL LENGTH IS =',F10.4)
      WRITE(8,474) G,AC,CT
474  FORMAT(/,10X,'G=',F10.4,5X,'AC=',F10.4,5X,'CT=',F10.4)
      WRITE(8,475) FD,CK
475  FORMAT(/,15X,'FD=',F10.5,5X,'CK=',F10.4)
      WRITE(8,476) H2,H0
476  FORMAT(/,10X,'H2=',E14.8,5X,'H0=',E14.8)

2    CONTINUE
144  STOP
      END
```

REFERENCES

1. Euler, L., "Sur La Force decolonnes," Memories de l'Academie de Berlin, Vol. 13, Berlin, Germany, 1759, p. 252; English translation of the letter by J. A. Van den Broek, Euler Classic Paper 'On the Strength of Columns," American Journal of Physics, 15 (January-February 1947), pp. 309-318.
2. Engesser, F., "Knickfragen," Schweiz, Bauztg. Vol. 25, No. 13, March 1895.
3. Considere, A., "Resistance des pieces comprimees," Congres International des Procedes de Construction, Paris, Vol. 3, 1891.
4. Johnston, B. G., Editor, "Structural Stability Research Council Guide to Stability Design Criteria for Metal Structures," 3rd Edition, John Wiley & Sons, Inc., New York, 1976.
5. Shanley, F. R., "Inelastic Column Theory," Journal of Aeronautical Science, Vol. 14, No. 5, May 1947, p. 261.
6. Bjorhovde, R., and Birkemoe, P. C., "Limit State Design of HSS Columns," Canadian Journal of Engineering, Vol. 6, No. 2, 1979 (pp. 276-291).
7. Bjorhovde, R., and Tall, L., "Maximum Column Strength and the Multiple Column Curve Concept," Fritz Engineering Laboratory Report No. 337-.29, Lehigh University, October 1971.
8. Bjorhovde, R., "Deterministic and Probabilistic Approaches to the Strength of Steel Columns," Ph.D. Dissertation, Lehigh University, Bethlehem, Pennsylvania, May 1972.
9. Batterman, R. H., and Johnson, B. G., "Behavior and Maximum Strength of Metal Columns," Journal of the Structural Division, ASCE, Vol. 93, No. ST2, April 1967 (pp. 205-230).

10. Bjorhovde, R., "The Philosophy of Column Design," Lehigh University, Fritz Engineering Laboratory Report No. 337.19, Lehigh University, December 1968.
11. Yang, C. H., Beedle, L. S., and Johnston, B. G., "Residual Stress and the Yield Strength of Steel Beam," Welding Journal, April 1952, 20s-229s.
12. Huber, A. W., and Beedle, L. S., "Residual Stresses and the Compressive Strength of Steel," Welding Journal, December 1954, pp. 589-612.
13. Howard, J. E., "Some Results of the Tests of Steel Columns, Proc. ASTM, Vol. 8, 1908, p. 336.
14. Osgood, W. R., "The Effect of Residual Stress on Column Strength," Proc. First National Congress, Appl. Mech., June 1951.
15. Beedle, L. S., and Tall, L., "Basic Columns Strength," Journal of the Structural Division, ASCE, Vol. 86, No. ST7, July 1960.
16. Yu, C. K., and Tall, L., "Significant and Application of Stub Column Test Results," Journal of Structural Division, ASCE, Vol. 97, No. ST7, July 1971.
17. Tebedge, N., Alpsten, G. A., and Tall, L., "Measurement of Residual Stresses: A Study of Methods," Fritz Engineering Laboratory, Report No. 337.8, Lehigh University, February 1971.
18. Tall, L., Editor, "Structural Steel Design," 2nd Edition, Ronald Press Company, New York, 1974.
19. Johnston, B. G., "Inelastic Buckling Gradient," Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineering, Vol. 90, No. EM6, December 1964.
20. Structural Stability Research Council, "Technical Memorandum No. 6: Determination of Residual Stresses," 4th Edition, to be published by Wiley-Interscience in 1987.
21. Nuttal, N. J., and Adams, P. F., "Flexural and Lateral-Torsional Buckling Strength of Double Angle Struts," Structural Engineering Report No. 30, University of Alberta, Edmonton, Alberta, Canada, Fall 1970.

22. Wagner, H., Verdrehung and Knickung von offenen Profilen," 25th Anniversary Publication, Technische Hochschule Danzing, 1929 (Translated in NACA Technical Memorandum, No. 807, October 1936).
23. Wagner, H., and Pretschner, W., "Verdrehung and knickung von offenen profilen, "Luftfahrtforschung, Vol. 11, No. 6, December 1934 (Translated in NACA Technical Memorandum, No. 784, January 1936).
24. Bleich, F., "Buckling Strength of Metal Structures," McGraw-Hill, N.Y., 1952.
25. Bleich, F., and Bleich, H., "Bending, Torsion, and Buckling of Bars Composed of Thin Walls," Preliminary Publication, Second Congress of the International Association for Bridge and Structural Engineering, Berlin, Germany, 1936.
26. Kappus, R., "Drillknicken zentrisch Gedruckter stabemit offenem profil im elastischen Bereich," Luftfahrtforschung, Vol. 14, No. 9, Munchen, Germany, September 1937 (Translated in NACA Technical Memorandum, No. 851, March 1938).
27. Lundquist, E., and Fligg, C., "A Theory for Primary Failure of Straight Centrally Loaded Columns," NACA Technical Report No. 582, 1937.
28. Timoshenko, S., and Gere, J., "Theory of Elastic Stability," 2nd ed., McGraw-Hill Book Company, New York, 1961.
29. Galambos, T. V., "Structural Members and Frames," Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1968.
30. Chajes, A., and Winter, G., "Torsional-Flexural Buckling of Thin-walled Members," Journal of the Structural Division, ASCE, Vol. 91, No. ST4, Proc. Paper 4442, August 1965 (pp. 103-124).
31. Pekoz, T. B., and Winter, G., "Torsional-flexural Buckling of Thin-walled Section under Eccentric Load," Journal of the Structural Division, ASCE, Vol. 92, No. ST2, April 1966.

32. Goldberg, J., Bogdanoff, j., and Glauz, W., "Lateral and Torsional Buckling of Thin-walled Beams," Publications, International Association for Bridge and Structural Engineering, Vol. 24, Zurich, Switzerland, 1964 (pp. 91-100).
33. Neal, B. G., "The Lateral Instability of Yielded Mild Steel Beams of Rectangular Cross Section," Philosophical Transactions of the Royal Society of London, Vol. 242A, January 1950 (pp. 126-242).
34. Prager, W., and Hodge, P. G., "The Theory of Perfectly Plastic Solids," John Wiley & Sons, Inc., New York, N.Y., 1951.
35. Wittrick, W. H., "Lateral Instability of Rectangular Beams of Strain-hardening Material under Uniform Bending," Journal of Aeronautical Sciences, Vol. 19, No. 12, December 1952.
36. Fukumoto, Y., and Galambos, T. V., "Inelastic Lateral-Torsional Buckling of Beam-columns," Journal of the Structural Division, ASCE, Vol. 92, No. ST2, April 1966.
37. Usami, T., and Galambos, T. V., "Eccentrically Loaded Single Angle Columns," Publications, International Association for Bridge and Structural Engineering, Vol. 31-II, Zurich, Switzerland, 1971.
38. Kennedy, J. B., and Murty, M. K., "Buckling of Steel Angle and Tee Struts," Journal of the Structural Division, ASCE, Vol. 98, No. STII, November 1972.
39. Kitipornchai, S., and Lee, H. W., "Inelastic Buckling of Single Angles, Tee and Double Angle Struts," Journal of Constructional Steel Research, Vol. 6, No. 1, 1986 (pp. 3-20).
40. Kitipornchai, S., and Lee, H. W., "Inelastic Experiments on Angle and Tee Struts," Journal of Constructional Steel Research, Vol. 6, No. 3, 1986 (pp. 219-236).
41. Haaijer, G., and Thurlimann, B., "On Inelastic Buckling in Steel," Journal of Engineering Mechanics Division, ASCE, Vol. 84, No. EM2, April 1958.

42. Bijlaard, P. P., "Some Contribution to the Theory of Elastic and Plastic Stability," Publications, International Association for Bridge and Structural Engineering, Vol. 8, Zurich, Switzerland, 1947 (pp. 17-80).
43. Haaijar, G., and Thurlmann, B., "Inelastic Buckling in Steel," Transaction, ASCE, Vol. 125, Part I, 1960.
44. Dave, J. L., and Kulak, G. L., "Local Buckling of W Shapes used as Columns, Beams, and Beam-columns," Structural Engineering Report No. 95, University of Alberta, Edmonton, Alberta, Canada, March 1981.
45. Hancock, G. J., "Local Distortional and Lateral Buckling of I-beam," Journal of Structural Division, ASCE, Vol. 104, No. ST11, November 1978.
46. Rajasekaran, S., and Murray, D. W., "Coupled Local Buckling in Wide-flange Beam Columns," Journal of the Structural Division, ASCE, Vol. 99, No. ST9, June 1973.
47. Mulligan, G. P., and Pekoz, T., "Locally Buckled Thin-walled Columns," Journal of Structural Engineering, ASCE, Vol. 110, No. ST11, November 1984.
48. Hancock, G. J., "Interaction Buckling in K-section Columns," Journal of the Structural Division, ASCE, Vol. 107, No. ST1, January 1981.
49. Johnston, B. G., and Opila, F., "Compression and Tension Tests of Structural Alloys," Proceedings, ASTM, Vol. 41, 1941.
50. American Institute of Steel Construction, "Manual of Steel Construction," 8th Edition, AISC, Chicago, Illinois, 1980.
51. ASTM Standard, Section 1, American Society for Testing Materials, Volume 01.04, 1983.