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The origins of hot subdwarf stars

Saffer, Rex Anderson, Ph.D.

The University of Arizona, 1991

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THE ORIGINS OF HOT SUBDWARF STARS

by

Rex Anderson Saffer

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A Dissertation Submitted to the Faculty of the
DEPARTMENT OF ASTRONOMY
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

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2

As members of the Final Examination Committee, we certify that we have read
the dissertation prepared by Rex Anderson Saffer
entitled THE ORIGINS OF HOT SUBDWARF STARS

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SIGNED: Ray A. Saffin

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ABSTRACT

High signal-to-noise optical spectrophotometry of a sample of field subluminous B stars drawn largely from the Palomar Green ultraviolet excess survey is analyzed with a new grid of model atmospheres and synthetic spectra. The stellar effective temperatures, surface gravities, and photospheric helium abundances are determined simultaneously from a detailed analysis of hydrogen and helium absorption line profiles. The derived temperatures and gravities place the subluminous B stars in the theoretical H-R diagram along and bounded below by theoretical sequences of the zero-age extended horizontal branch, lending strong support to the hypothesis that these stars are composed of helium-burning cores of $\sim 0.5 M_{\odot}$ overlain by very thin layers of hydrogen ($\lesssim 0.02 M_{\odot}$). Various scenarios for their past evolutionary history are examined in the context of their probable future evolution into white dwarfs of lower than average mass. The derived distances above the Galactic plane support a scale height for the population of $z_0 = 285$ pc, consistent with the identification of their progenitor stars as members of the old disk population. Radial velocities of sdB and sdO stars are analyzed to infer their kinematic characteristics. The results for the sdB stars are inconclusive, but for the sdO stars the results also are consistent with the population belonging to the older part of the thin disk.

CHAPTER 1

INTRODUCTION

Hot subdwarfs, stars with temperatures exceeding 20,000 K and surface gravities somewhat higher than main sequence stars of the same temperature, are thought to be the direct progenitors of the white dwarfs, although they likely comprise only a small fraction of all stars which evolve directly to that state. It is believed that most white dwarfs are formed by low and intermediate mass stars evolving toward their final states after the ejection of a planetary nebula at the tip of the asymptotic giant branch. Yet while the birthrates of planetary nebulae and white dwarfs are estimated to be comparable, the most common objects found in color selected, magnitude limited samples such as the Palomar Green (Green, Schmidt, & Liebert 1986) and Kitt Peak Downes surveys (Downes 1986) are the hot subdwarfs. The subdwarf B (sdB) stars have hydrogen-dominated atmospheres, while the subdwarf O (sdO) stars have helium-enriched or helium-dominated atmospheres. An intermediate class (sdOB) also has been identified (Baschek & Norris 1975, Hunger et al. 1981); these stars have effective temperatures and photospheric helium abundances between those of the sdB and sdO stars.

The hot subdwarfs were first recognized as a distinct spectroscopic class of objects in the 1950's by J. L. Greenstein and G. Münch. Münch

(1958) performed one of the first detailed analyses of the spectrum of one of these stars. The first reasonably comprehensive description of the group is given in Greenstein (1960). He found from the surface gravities that these stars were intermediate between the main sequence and the white dwarfs; like the Population II main sequence stars which have smaller radii than their Population I counterparts, it was appropriate to refer to these stars as subdwarfs. Since there was evidence that they belonged to an old population, they were much too hot to be extensions of the Population II main sequence. Most of the early examples were identified from the list of isolated blue stars of Humason & Zwicky (1947), but Greenstein noted that they fall in an HR Diagram at fainter visual magnitudes and bluer colors than the normal horizontal branch of globular clusters. However, he also noted that some clusters appeared to have extended horizontal branches whose endpoints reached the regime of the field subdwarfs in a color magnitude diagram. Greenstein suggested that such stars in the clusters and the field may be dying objects in the process of transition to the white dwarf stage.

In this earliest incarnation, there were two recognized subclasses of hot subdwarfs – the sdB and the sdO stars. The former showed lines of hydrogen, broader of course than those of main sequence A stars, while lines of helium and in other elements were weak at best. The latter exhibited strong helium lines, though it was already evident from the six sdO stars

described in Greenstein (1960) that a wide variety of relative strengths of He I, He II and H lines could be exhibited. Heavier elements were also found in the spectrum of HZ 44; some 400 lines of various ions of nitrogen, silicon and neon were measured, while another sdO star showed carbon features.

In the following decade, other surveys for faint blue stars – especially at high galactic latitude – such as the Tonantzintla stars of Iriarte & Chavira (1957), Chavira (1958) and those of Feige (1958) provided valuable hunting grounds for Greenstein and others to enlarge greatly the sample of hot subdwarfs. The first extensive identification list, spectroscopic description and analysis was given in Greenstein (1966), utilizing color measurements and other information provided by the deceased D. L. Harris III. In addition to subdwarfs and white dwarfs, Greenstein found that some of these survey stars were of B type with what appeared to normal main sequence gravities, there were already a few objects which would later be called quasi-stellar sources or quasars, some were field horizontal branch stars, some had spectra like old novae and other interacting binaries which would later be called cataclysmic variables, and some showed unique peculiarities. Only rudimentary model atmospheres were available for these types of objects, and there was little effort then to derive quantitative atmospheric parameters. Despite the apparent linkage to the bluest horizontal branch stars in certain globular clusters, Greenstein (1965) did note that some sdO and other faint blue stars

exhibited small enough proper motions to belong to an old disk population and not the halo, and he predicted a luminosity function for hot subdwarfs based on a scale height of 1 kpc.

Newell (1973) presented analyses of faint blue halo stars, including the bluest stars in some globular clusters, but the first comprehensive analysis of the field hot subdwarfs and other faint blue stars was a giant compendium by Greenstein & Sargent (1974). An adequate summary of the landmark set of conclusions first advanced in this work is beyond the scope of this dissertation, the introduction here will focus only on those aspects of the work concerning the hot subdwarfs. In that work the concept of an extended horizontal branch (EHB) – and its identification at least in part with core-helium burning stars having little or no overlying hydrogen envelopes – was first developed, although a preliminary discussion appears in Greenstein (1971). Colors and spectral line profiles were analyzed using model atmospheres resulting in the first large set of derived effective temperatures, surface gravities and atmospheric abundances (primarily of H and He) for subdwarf B and O stars. Most sdB stars fell into a T_{eff} range of 25–35,000 K, while most sdO stars appeared to be hotter than 40,000 K and the T_{eff} values assigned to the hottest of these were lower limits since the colors represented only the Rayleigh-Jeans limit. While some sdO stars had parameters consistent with the hottest part of the EHB, they exhibited a wider range of $\log g$ and luminosity while the

sdB stars showed only a small dispersion in luminosity. In terms of stellar atmospheres parameters, Greenstein & Sargent (1974) found that the EHB stars shared a constant value of $\log(g\theta^4)$, where $\theta = 5040K/T_{\text{eff}}$.

In the past two decades, a number of hypotheses on the origin of the hot subdwarfs have been advanced:

1) Greenstein (1971) and Greenstein & Sargent (1974) first suggested that the sdB stars are the field counterparts of the extreme blue extension of the horizontal branch (extended horizontal branch; EHB) of very metal-poor globular clusters. This interpretation is supported by results of the analysis of extreme blue horizontal branch stars in the globular cluster NGC 6752 by Heber et al. (1986). Some of those stars proved to have atmospheric parameters like those of field sdB and sdOB stars belonging to the disk population, and one star was identified as an sdO with an effective temperature near 40,000 K . Other globular clusters are known with blue horizontal branches; all have $[\text{Fe}/\text{H}] \lesssim -1.5$ and are thought to have ages comparable to that of the Galactic halo. Globular clusters with a metal content closer to the solar value and which may be younger than the halo appear to lack blue horizontal branches (see, e.g., Armandroff 1988). Yet recent observations of the ultraviolet energy distributions of more metal-rich disk globular clusters (M. Rich, J. Liebert, & D. Minniti, private communication) have identified ultraviolet excesses at wavelengths shorter than

2000 Å in a handful of clusters. Similar ultraviolet excesses also have been observed in some metal-rich elliptical galaxies (Burstein et al. 1988). Whether those excesses are due to populations of EHB stars like those in NGC 6752 remains to be determined.

2) The hottest and most luminous subdwarfs might be the descendents of AGB stars whose evolutionary times are much longer than the time required for the ejection of a planetary nebula. For example, Schönberner & Drilling (1984) and Mendez et al. (1986) discuss sdO stars of low surface gravity which lie in the same region as low mass post-AGB theoretical tracks. The problem remains that the vast majority of sdB stars are found in regions of the H-R diagram which do not overlap with any theoretical post-AGB evolutionary tracks.

3) Close binary evolution could lead to post-RGB phases with effective temperatures and luminosities like hot subdwarfs, or to systems that evolve through mergers into hot subdwarfs. Iben & Tutukov (1986a,b) describe calculations of the formation of helium degenerates in which the first mass loss event occurs after the primary develops a degenerate helium core on the red giant branch, but before it reaches the mass and temperature required for core helium burning. Roche lobe detachment occurs when the primary's hydrogen-burning shell is quenched. Before beginning a final gravitational contraction toward its final state as a helium degenerate, the primary passes

through several short-lived phases in the region of the hot subdwarfs. If most of the mass lost from the primary escapes or is driven from the system, the secondary does not accrete appreciable mass, and its subsequent evolution to a second helium degenerate can form a close binary He-He white dwarf system, with most of the hydrogen from both stars having been lost from the system in common envelope events. Iben (1990) and Tutukov & Yungelson (1979) have proposed that most subdwarfs could be the product of mergers of He-He or He-CO pairs in these and other similar close binary systems. The mergers release enough gravitational energy to initiate helium burning in the surface layers of the merged products, which then propagates to the center in a series of helium flashes. Any resulting core helium-burning stars retaining little or no hydrogen in their outer layers would populate the extended horizontal branch in the region of the subluminous B stars, while some of those with more hydrogen could evolve through the H-R diagram in the vicinity of those subluminous O stars with the lowest effective temperatures and surface gravities.

In this dissertation, a number of approaches are taken to investigate the origins and evolutionary history of the hot subdwarfs. First, atmospheric parameters are derived for a subsample of the sdB stars defined in the Palomar Green (PG) Survey. Other objects admitted to the sample for analysis include bright and well-studied stars previously classified as sdB, as

well as some stars previously misidentified as blue horizontal branch B (BHB) stars or DA white dwarfs on the basis of low resolution or low signal-to-noise classification spectroscopy. Higher quality spectra of those objects revealed the presence of moderately broadened high Balmer lines extending to $n = 12$, characteristic of sdB stars. Preliminary results of this program are reported by Saffer et al. (1991). Second, the atmospheric parameters are used to infer the scale height of the sdB population, and the results are examined with the goal of characterizing the population and identifying the progenitor population. Third, a large set of radial velocities of samples of sdB and sdO stars are analyzed to determine the kinematic characteristics of the populations, again with the goal of identifying the progenitor population.

In Chapter 2 the observations are described, and optical spectra of the program objects are presented. In Chapter 3 model spectra of the sdB stars are presented and the procedure used to fit the observations and determine the atmospheric parameters are described. The radial velocity measurements are described and presented. Extensive discussions of the internal and external errors are presented. In Chapter 4 distributions in the H-R diagram of the derived atmospheric parameters of the sample are discussed, and their implications in terms of the evolutionary history of the sdB stars are examined. Extensive comparisons are made of the results presented here with those in the literature, including a critical analysis of previous determinations of the

atmospheric parameters derived from Strömgren colors. In Chapter 5 the derived atmospheric parameters are used to estimate the stellar distances and the Galactic disk scale height of the population. In Chapter 6 the radial velocities of the sdB and sdO samples are analyzed, and the kinematic characteristics of the samples are discussed. Monte Carlo simulations are used to examine the effects of sampling error on the derived parameters. This dissertation closes with a Summary in Chapter 7, followed by a compilation of all references. Tables and figures, in that order, follow text in the chapter where reference is made to them.

CHAPTER 2

OBSERVATIONS AND REDUCTIONS

2.1 Observations for Atmospheric Analyses

Most of the observations described here were obtained at the Steward Observatory Kitt Peak Station with the 2.3-m reflector equipped with the Boller & Chivens Cassegrain spectrograph and UV-flooded TI CCD. A 600 l/mm first-order grating was used behind a 4.5 arcsec by 4 arcmin long slit, which for every observation was rotated to the parallactic angle according to the calculations of Fillipenko (1982). The instrumentation provided wavelength coverage $\lambda\lambda 3650\text{--}5200$ at a spectral resolution of $\sim 6\text{\AA}$ FWHM. This spectral coverage is ideal for observations of the Balmer series in subdwarf B stars, covering $H\beta$ through the Balmer jump, while the spectral resolution is sufficiently high to show important details both in the line cores of the Balmer absorption profiles and in the narrower neutral helium lines often present in the spectra. The spectra were extracted from the two-dimensional frames and reduced to linear wavelength and intensity scales using standard reduction packages in the Image Reduction and Analysis Facility (IRAF). Some spectra were obtained at the National Optical Astronomy Observatory's Kitt Peak 2.1-m reflector equipped with the GOLDCAM spectrograph and TI CCD. This instrumentation provided spectral coverage and resolution similar to that of the 2.3-m instrumentation. A handful of spectra kindly made

available by Diana Foss were obtained using the 2.3-m Steward Observatory reflector equipped as described above but using a 1200 l/mm first order grating. The spectral coverage of those observations is from $H\gamma$ through the Balmer jump at 2-3 Å resolution. Spectra of all program objects are presented in Figures 2.1a-j.

The analysis described in Chapter 3 identifies 52 of the 92 program objects as classical sdB stars according to the criteria of Greenstein & Sargent (1974); the atmospheres are helium-depleted, the effective temperatures lie between about 24,000 K and 42,000 K , and the surface gravities exceed $\log g = 5.0$. An additional 15 program objects have surface gravities in excess of $\log g = 5.0$ but have less severely depleted atmospheric helium; some have near-solar or even larger helium abundances. Of the remaining 24 program objects having surface gravities less than $\log g = 5.0$, all but three resemble main sequence B stars at the intermediate spectral resolution employed here. Alternatively, some of these stars could be contaminants from the blue horizontal branch (BHB). Spectral features which would distinguish between these two classes are not well-resolved. For example, at the spectral resolution of the observations shown here, Mg I $\lambda 4481$ in main sequence B stars would be nearly invisible in the red wing of He I $\lambda 4471$. The last 3 objects in the sample show very narrow Balmer absorption up to $n = 16$, characteristic of giant stars, and their derived surface gravities are well below

$\log g = 4.0$.

The classical sdB spectra are characterized by strong blue continua and the presence of moderately Stark-broadened Balmer absorption lines. Helium appears to be underabundant, on average, by a factor of ten compared to the solar value. In most spectra having a spectral resolution of $\sim 6\text{\AA}$, He I $\lambda 4471$ and sometimes He I $\lambda 4026$ are detected weakly, while in the remainder there is no helium detected whatsoever. In most stars having effective temperatures exceeding $\sim 32,000\text{ K}$, He II $\lambda 4686$ is detected weakly, even in the most helium-depleted stars. This is reminiscent of the appearance of the sdOB stars, although those stars tend to have helium abundances closer to the solar value.

The 15 helium-strong high gravity stars have continua and Balmer absorption profiles similar to the classical sdB stars, and show in addition strong, but narrower, He I absorption lines. The hottest of these also show He II $\lambda 4686$ absorption. Hereafter, both the classical sdB stars and the helium-strong sdB stars shall be referred to as extended horizontal branch (EHB) stars.

The 24 lower-gravity stars show blue continua with deeper and narrower Balmer line profiles extending to $n = 14$. C II $\lambda 4267$ and Ca II “K” are often seen. The three narrow-lined objects show Balmer lines extending to $n = 16$. Apparently, moderate to significant reddening is present in

spectra of four of these low-gravity objects. Alternatively, the objects could be composite, with low-mass main-sequence companions. Ferguson et al. (1984) have shown that a large fraction of the hot subdwarfs in the PG catalog are composite. However, the spectra of those low-gravity stars shown a deficit in their blue and near-UV fluxes, as would be expected for reddening, instead of the red and near-IR excess in the fluxes expected if red companions were present. Neither do the spectra show absorption features characteristic of low-mass main sequence stars. Further observations at longer wavelengths will be necessary to distinguish between the possibilities.

2.2 Observations for Radial Velocity Measurements

Observations of the $H\alpha$ absorption line profiles of the sdB candidate stars were obtained at the Steward Observatory Kitt Peak Station with the 2.3-m reflector. The B & C Spectrograph was used with a first-order 1200 l/mm grating and UV-flooded TI CCD behind a 1.5 arcsec by 4 arcmin long slit. The slit was rotated to the parallactic angle on the sky for all observations. The spectral coverage was $\lambda\lambda 6200\text{--}6900$ with a spectral resolution of $2 \text{ \AA}$ FWHM. Wavelength calibrations were provided by exposures of an Fe/Ne comparison arc lamp taken at the telescope position of the objects on the sky. For sdO stars included in the kinematical analysis of Chapter 6, observations of the He II $\lambda 4686$ line were obtained at two different telescopes. The bulk of the observations were obtained at the Multiple Mirror

Telescope (MMT) using an echelle spectrograph and photon-counting Reticon. The wavelength coverage was $\lambda\lambda 4660\text{--}4710$ at a spectral resolution of 0.25 Å FWHM. Wavelength calibration was provided by exposures of a Th/Ar comparison lamp. The remaining observations were obtained at the Mayall 4-m telescope at Kitt Peak, using the RC spectrograph, second order 1200 l/mm grating # 380, UV fast camera, and TI CCD. Wavelength coverage was $\lambda\lambda 4500\text{--}4800$ at a spectral resolution of 1 Å FWHM. Exposures of a He/Ar comparison lamp provided wavelength calibration. The radial velocity zero-point calibrations for all observations were established by observations of the dusk and dawn twilight sky and of multiple observations of IAU radial velocity standards on each night. Figures 2.2a–c show representative spectra of the object line profiles. In Figure 2.2b, the object PG 1102+499 is of particular interest. At one epoch of observation, the line profile appeared normal (that is, single), while at a second epoch the line core was obviously doubled. The velocity separation of the line cores implies that the system is a short-period binary. Ongoing observations are taking place in an attempt to determine the system period and other orbital parameters.

Figure 2.1a. Observed spectra of subluminal B star candidates.

Figure 2.1b. Observed spectra of subluminous B star candidates.

Figure 2.1c. Observed spectra of subluminal B star candidates.

Figure 2.1d. Observed spectra of subluminal B star candidates.

Figure 2.1e. Observed spectra of subluminal B star candidates.

Figure 2.1f. Observed spectra of subluminal B star candidates.

Figure 2.1g. Observed spectra of subluminal B star candidates.

Figure 2.1h. Observed spectra of subluminal B star candidates.

Figure 2.1i. Observed spectra of subluminal B star candidates.

Figure 2.1j. Observed spectra of subluminal B star candidates.

Figure 2.2a. Observed 2.3-m CCD spectra of sdB H α line profiles.

Figure 2.2b. Observed MMT echelle spectra of sdO He II $\lambda 4686$ line profiles.

Figure 2.2c. Observed 4-m CCD spectra of sdO He II $\lambda 4686$ line profiles.

CHAPTER 3

ESTIMATION OF STELLAR PARAMETERS

3.1 Line Profile Fitting

3.1.1 Synthetic Spectra

Daou et al. (1990), Bergeron, Saffer, & Liebert (1991a, 1991b), and Bergeron, Wesemael, & Fontaine (1991c) have shown that it is possible to obtain reliable estimates of the atmospheric parameters T_{eff} and $\log g$ in DA white dwarfs by fitting simultaneously line profiles of the members of the Balmer series with appropriate model spectra. In the analysis described here, the model atmospheres were calculated in LTE and incorporate hydrogen-line blanketing. Theoretical spectra were calculated using the occupation probability formalism of Hummer & Mihalas (1988), as described in Bergeron et al. (1991c). At the surface gravities characteristic of white dwarfs, a proper treatment of the dissolution of the higher members of the Balmer series is crucial for the determination of the surface gravities of those stars. At the lower surface gravities characteristic of EHB stars, however, line dissolution is less important, becoming significant only in stars having the highest surface gravities (see below). The grid encompasses the following ranges of atmospheric parameters: $20,000 \text{ K} < T_{\text{eff}} < 40,000 \text{ K}$, $4.0 < \log g < 6.0$, and $0.00 < y \equiv N(\text{He})/N(\text{H}) < 0.10$. In Figures 3.1a–g, the effects of variations in

effective temperature, surface gravity, and helium abundance on the theoretical Balmer and helium line profiles are shown.

Figure 3.1a shows line profile variations with changes in effective temperature from 20,000 K to 40,000 K . For any given transition, the Balmer line equivalent widths decrease monotonically with increasing effective temperature in the EHB temperature regime. The line profiles are seen to be highly sensitive to variations in effective temperature.

Figure 3.1b shows line profile variations with changes in surface gravity for the same model grid used in Figure 3.1a. For changes in $\log g$ from 4.0 to 6.0 the lower Balmer line equivalent widths are seen to increase monotonically with increasing surface gravity due to the linear Stark broadening. However, in the higher Balmer lines of the models at high surface gravity, the effects of atomic level dissolution become important. This makes the rate of increase of the equivalent width of these lines slow with increasing $\log g$. A close inspection of the H9 profile (Figure 3.1c) even reveals that the equivalent width *decreases* in the highest surface gravity models. Similar but much larger effects are seen at the surface gravities of white dwarfs (Bergeron et al. 1991a).

Figures 3.1d - 3.1f show line profile variations as in Figures 3.1a - 3.1c, but in an atmosphere with helium abundance $y = 0.10$. Like the Balmer lines, the He I and He II lines are seen to show high sensitivity

to changes in effective temperature and surface gravity, providing further constraints on these atmospheric parameters. As can be seen by comparing Figures 3.1c and 3.1f, the presence of helium in the atmosphere provides a partial pressure beyond that in a pure hydrogen atmosphere at the same surface gravity, thus enhancing the effects of level dissolution. It seems likely that a treatment of level occupation probabilities is even more important in mixed-composition atmospheres with higher helium abundances, such as the hot, helium-rich subluminous O stars.

Figure 3.1g shows the variation of the helium line profiles with changes in helium abundance from $y = 0.01$ to 0.10. For the cool DA white dwarfs, it has been shown that variations in the Balmer line profiles due to changes in the amount of photospheric helium cannot be distinguished from variations with changes in surface gravity (Bergeron et al. 1991c). Those stars are too cool to show helium lines in their spectra even for very high helium abundance. However, in the EHB temperature regime, the helium lines are strong, providing a crucial constraint lacking in cool DA stars. The variations of the line profiles with changes in the atmospheric parameters thus differ both qualitatively and quantitatively for each parameter, and should provide sufficient constraints to estimate the atmospheric parameters from a single optical spectrum.

Regarding the use of model spectra calculated in LTE model

atmospheres for the analysis performed here, it is well-known that for hot stars the details of both absorption and emission line profiles are strongly affected by non-LTE processes (see, e.g., Kudritzki & Hummer (1990) for a review of quantitative spectroscopy of hot stars). However, some previous comparisons suggest that the use of LTE model atmospheres in the EHB regime is not seriously inappropriate. Wesemael et al. (1980) find good agreement for continuum fluxes in comparisons of LTE and non-LTE calculations at subdwarf surface gravities, although an overpopulation of the $n = 2$ level strengthens the Balmer line cores. Another comparison of LTE and non-LTE atmospheres and model spectra in the subdwarf B regime (S. Vennes, private communication) shows that the higher Balmer members have line profiles that differ from those calculated in LTE only in the core, and that the differences are small and not easily detected at the spectral resolution of the observations employed in this work.

Other factors which might affect the shapes of the line profiles are the presence of rapid rotation, or of strong magnetic fields. However, either of these processes affects the line profile shapes in a fundamentally different manner than do variations in the line profiles with variations in either effective temperature or surface gravity. Simulations in which the theoretical line profiles are convolved with rotational broadening functions show that for significant rotation velocities, the line profiles are perturbed to the extent

that a satisfactory fit to the observed spectra cannot be found for any choice of atmospheric parameters. Line profile variations due to the presence of magnetic fields first appear as flattening of the line cores as the Zeeman components split in low magnetic fields, after which Zeeman triplets become noticeably visible. At higher magnetic fields the spectra cease to resemble at all the zero-field spectra as the quadratic Zeeman effect adds to the magnetic broadening of the line profiles. In either case the result is the same as for significant rotation; the spectra cannot be satisfactorily fit by zero-field model line profiles for any choice of atmospheric parameters. In the sample analyzed here, only one star fails to be well-fit (PG 2218+051, discussed subsequently), and for the rest of the stars in the sample the conclusion is that these stars neither rotate rapidly, nor do they have strong magnetic fields in the line-forming regions of the atmosphere.

3.1.2 Least-squares Fitting Procedure

In the fitting procedure used by Bergeron et al. (1991a,b), the observed and theoretical Balmer line profiles are normalized to a linear continuum, and the atmospheric parameters T_{eff} and $\log g$ are determined with least-squares fitting techniques. Here the technique is extended to three dimensions, and the effective temperature, surface gravity, and helium abundance are determined simultaneously by fitting theoretical spectra to a single optical spectrum of the program object.

A non-linear least-squares algorithm (Press et al. 1986) is used to estimate the atmospheric parameters, and it is important to verify that a unique minimum in χ^2 exists over at least some subset of parameter space. The algorithm also must be provided with good initial estimates of the parameters; these can be provided by an inspection of suitable projections of the χ^2 hypersurface. As an example, the spectrum of PG 1232–136 (Figure 2.1d) suggests that the effective temperature, surface gravity, and helium abundance all have intermediate values in parameter space. Crude initial estimates might take the values $T_{\text{eff}} = 30,000 \text{ K}$, $\log g = 5.0$, and $y = 0.03$ based on the strength of the Balmer jump and the failure to detect He II $\lambda 4686$, the number of visible high Balmer lines, and the strength of the He I lines. Projections of the χ^2 hypersurface for this star onto the $T_{\text{eff}} - \log g$ and $\log g - N(\text{He})/N(\text{H})$ planes (Figures 3.2a and 3.2b) show that a well-defined minimum exists over a large fraction of parameter space. In both figures, the χ^2 minimum is normalized to unity, and the solid contours represent equally spaced levels from the minimum to a normalized χ^2 of 1.4. The dashed lines are scaled differently, representing equally spaced levels from the last solid contour to the maximum value of χ^2 within the parameter space. In this way, fine detail near the minimum as well as global structure over the parameter space may be examined simultaneously. The common scaling near the minimum also allows surfaces for different objects

to be compared against each other. The relative sizes of the regions near the minima are a measure of the relative internal errors of the estimated parameters.

Figures 3.2c – 3.2f show similar projections for PG 1317+123 and PG 1538+269. In each of the projections, the minima agree roughly with the crude parameter estimates based on the appearance of the spectra, and for most stars examined in this way, the initial estimates result in rapid convergence to a solution for which all lines are fitted satisfactorily. In the few cases where the initial estimates result in convergence to a secondary, local minimum at inappropriate values of the fitting parameters, the resulting fit is obviously and emphatically bad. In other words, a satisfactory fit is found if and only if the resulting parameter estimates lie in the global minimum of the χ^2 hypersurface. In contrast, Daou et al. (1990) have shown that for the ZZ Ceti stars there can be two closely spaced minima in effective temperature on either side of the maximum in Balmer line strengths near 12,000 K . For those stars, it is necessary to resort to ultraviolet energy distributions or colors in order to resolve the ambiguity.

Selected spectra and model fits are shown in Figures 3.3a–g. Representative spectra of classical sdB stars are shown in Figure 3.3a in order of increasing T_{eff} . The corresponding signal-to-noise ratios averaged over short segments of line-free continuum range from 80 to more than

100. The corresponding model fits (Figure 3.3b) are excellent; indeed, for many of the fits it is difficult to distinguish the model spectra from the data. With the exception of one star (PG 2218+051, discussed below), it was possible to achieve repeatable, rapidly converging solutions for a wide range of reasonable initial conditions. Figures 3.3c and 3.3d show spectra and model fits for high-gravity stars arranged in order of increasing atmospheric helium abundance, and Figures 3.3e and 3.3f show spectra and model fits for non-subdwarf contaminants of the sample. The object 2150+256 is not an sdB candidate, but is 16 Peg, a main sequence standard star classified as B3V. A high signal-to-noise spectrum was obtained and is shown here for the purpose of comparison with other low-gravity objects in the sample. The horizontal and vertical axes in Figure 3.3f have been expanded to show more clearly details of the fits in the narrow Balmer lines of these low-gravity stars. Note in some of these objects the weak Mg I $\lambda 4481$ absorption in the red wing of He I $\lambda 4471$. Some 26% of all stars selected as sdB candidates based on the PG spectral classifications fall in the non-subdwarf category, probably because at the low signal-to-noise ratios characteristic of followup classification spectroscopy of large, color-selected surveys, it is difficult to resolve line profile variations that discriminate changes in surface gravity, especially if only the lower, less gravity-sensitive Balmer lines are analyzed. It is important to include the higher Balmer lines in classification spectra of

hot, subluminous stars if clear separation of luminosity classes is desired at intermediate spectral resolution.

Figure 3.3g shows attempted fits for PG 2218+051, including in the analysis, respectively from left to right and top to bottom, the Balmer lines $H\beta - H9$, $H\beta - H8$, $H\gamma - H9$, $H\beta - H\epsilon$, $H\gamma - H8$, and $H\delta - H9$. In the three-parameter fits, the derived helium abundance always was near $N(\text{He})/N(\text{H}) = 0.15$, and in the fits which are described below, it was held fixed at that value. The observed profiles and best-fit model profiles for all six Balmer lines are shown even when fewer lines were included in the fits. Classified sdOA on the basis of its broad, strong Balmer absorption and strong He I absorption, the star has anomalously broad and somewhat flat-bottomed He II $\lambda 4686$ and He I line profiles. Whenever H9 is included in the analysis, a satisfactory fit for $H\beta$ and H8 cannot be found, with the model line core being too shallow for the observed $H\beta$ profile and too deep for the observed H8 profile. Further, some model helium line profiles appear too narrow compared to the observed profiles. Whenever H9 is excluded from the fit, the fits to the lower Balmer lines are better, but H9 itself is obviously misfit. Again, some model helium line profiles appear too narrow. High resolution observations of the $H\alpha$ line core are planned to investigate whether the star is rapidly rotating, as well as spectropolarimetric observations to detect possible magnetic fields of order 1 Megagauss or less. To be definite,

an average of the atmospheric parameters of the fits shown in Figure 3.3g that include H9 in the analysis have been adopted for this work. These estimates should be considered highly uncertain pending further analyses.

3.2 Atmospheric Parameter Error Estimation

3.2.1 Internal Error Estimation

In the least-squares fitting procedure, the spectra first are fitted with the weights set to unity. After convergence, the weights are calculated by computing the rms deviations of the data from the best-fit model, averaged over short segments of the spectrum. A second, weighted fit is then performed, and the resulting parameter estimates are adopted for the object. The weights then are propagated into the covariance matrix of the fit, from which the single-measurement internal error estimates are derived (cf. Press et al. 1986). In Figures 3.4a through 3.4c, the internal error distributions for the fitted atmospheric parameters are shown. For each parameter the distributions are quite similar, each having a broad, skewed shape with a maximum on the low-error side, and an extended high-error tail which always corresponds to those spectra with the lowest signal-to-noise ratios. It is useful in subsequent discussions to adopt a mean internal error for the derived atmospheric parameters. A preliminary value was chosen for each parameter so that approximately 2/3 of the model fits for the 92 program objects give individual measurement error estimates smaller than the adopted

mean errors: $\sigma_T = 1200\text{ K}$, $\sigma_{logg} = 0.16$, and $\sigma_y = 0.005$. Even if systematic errors bias the absolute accuracy of the derived parameters, their relative precision would be the highest for any sample of EHB stars analyzed to date.

The derived atmospheric parameters are more uncertain in some regimes. At low temperature and surface gravity, metals in the atmosphere are not depleted by diffusion and are blended with hydrogen and helium lines, and line cores are affected to some extent by departures from LTE. At the spectral resolution of the observations, weaker lines useful as luminosity indicators are not well detected. Some estimates extrapolated beyond the model grid parameter space are highly uncertain. Happily, most estimates lie inside the grid, or nearly so. More accurate determinations of atmospheric parameters for the lower-gravity stars await an analysis of higher resolution spectra with appropriate models. Still, the analysis performed here serves to cull low-gravity stars from the confirmed EHB sample and gives a rough estimate of their luminosities.

Previous analyses, for example those using equivalent width measurements, have been shown to be vulnerable to the choice of continuum placement and imperfect flux calibration. A series of simulations was performed to determine the effect on the derived parameters due to errors in flux calibration of the observations. A subsample of spectra which by inspec-

tion appear to have been calibrated satisfactorily compared to standard star spectra obtained on the same night were subjected to a number of perturbations, such as the imposition of random low frequency intensity variations to simulate incorrect or inappropriate choices among various quantum response curve fitting functions or orders of fit, variation of the continuum slope, and variation of the applied extinction correction.

The resulting spectra were analyzed in the same way as all other program objects, and the resulting estimated effective temperatures and surface gravities are shown in Figure 3.5. The larger filled circles represent the parameters derived from the unperturbed spectra, and the smaller symbols represent parameters derived from the perturbed spectra. Low frequency intensity variations proved to have no significant effect on the derived parameters, but extinction and color variations contribute about equally to the observed variations in the derived parameters. Compared to the preliminary mean internal errors of 1200 K and 0.16 dex, the effective temperature estimates have a mean rms dispersion of only 250 K , while the surface gravity estimates have a mean rms dispersion of 0.06 dex. The amplitude of the various perturbations imposed on the spectra was chosen to be about 10% peak-to-peak, the largest observed variations seen in the whole sample. But most spectra are flux calibrated to a relative precision of about 2% or better, and only a few spectra have variations as large as that assumed for

the simulations. Hence, this source of error is not expected to contribute significantly to the total.

3.2.2 External Error Estimation

Another independent estimate of the derived parameter uncertainties is possible through repeated measurements of any given object. While no program object in the sample was observed often enough to make a meaningful estimate of its parameter uncertainties, one of the flux standards, Feige 34, was observed many times throughout the course of the project. Feige 34 is a hot, atmospheric helium-enriched subdwarf O star, which nonetheless has a hydrogen-dominated atmosphere. Figure 3.6 shows a representative model fit to a typical spectrum, with the analysis having included only the lines H δ – H9. Other analyses have shown that this star has an effective temperature in excess of 60,000 K and $\log g$ near 6.0 (cf. Thejll, MacDonald, & Saffer 1991). Thus it is clearly inappropriate to analyze the star with the EHB model grid and uncritically adopt the resulting parameter estimates. The figure shows that, while a satisfactory fit is found for the 4 lines analyzed, the lower two Balmer lines obviously are inconsistent with the fit, and the resulting parameter estimates are not to be believed. Still, the parameters are only slightly extrapolated outside the parameter space, and the internal error estimates should be useful for the purposes of comparison with external errors. With these caveats in mind, 17 independent spectra of Feige

34 obtained on 5 different nights separated by at least several weeks were analyzed as described above. The resulting bivariate distribution of estimated effective temperature and surface gravity is shown in Figure 3.7. The axes have been scaled arbitrarily so that $\Delta \log g / \log g = \Delta T_{\text{eff}} / T_{\text{eff}}$, that is, so that a given unit of length in the figure represents the same percentage error for either parameter. There appears to be no correlation between the estimated parameters for the independent observations.

In the first line of text at the top of the figure, the unweighted means of the distribution shown in the figure are given, followed by the unweighted mean internal errors of the individual model fits, then the rms deviations of the distribution about the measured means. The agreement between the two independent error estimates is excellent, providing a measure of assurance that no other significant sources of random error contribute to the parameter uncertainties, and that the individual internal measurement errors as calculated from the covariance matrices of the fits may confidently be used in subsequent error propagation. The question of whether systematic errors result in inaccurate or biased estimates must be addressed by comparisons of the spectroscopic parameter estimates with those derived by other methods of analysis of well-studied subluminoous B stars.

Another source of possible error in the derived atmospheric parameters is variations in the atmospheric seeing. The slit width used for all

observations was 4.5 arcsec, and combined with other sources of instrumental broadening, this always gave well-exposed comparison lamp emission lines with $8\text{\AA}$ FWHM. The seeing almost never was that bad, with 1–2 arcsec seeing more common. These correspond more closely to 4–6 $\text{\AA}$ spectral resolution. The model grid must be convolved to the spectral resolution of the observations, since non-LTE effects in the centers of the absorption lines often result in sharp line cores with small widths compared to the instrumental resolution. In general, if spectra are analyzed with model grids convolved with a resolution element that has a FWHM that is too small (large) compared with the observations, the model line cores will be too deep (shallow), and effective temperatures and surface gravities that are too high (low) will be required to fit any given spectrum, at least for temperatures characteristic of EHB and main sequence B stars.

Figures 3.8a and 3.8b show comparisons for the entire sample of effective temperatures and surface gravities derived using model grids convolved to $4\text{\AA}$ and $6\text{\AA}$ resolution. There are systematic shifts in the expected directions, with accompanying dispersion. The shifts and dispersions both are of the order of the mean internal errors. To reduce this source of error, the adopted estimated parameters were taken from the model fit with the best-fit line cores. Although the line wings always were well fit, it almost always was possible to make an unambiguous choice, with one or the other

of the line core fits clearly being inferior. Excellent fits were obtained for all but one star previously described, PG 2218+051. The contribution of this source of error should therefore be diminished somewhat, but probably not altogether.

Based on the foregoing discussions, the quadrature sum of all sources of error other than that due to photon-counting statistics and noise introduced during the reduction process seems unlikely to be as large as the preliminary mean internal errors, or as small as half that. Therefore, 3/4 of the values for the preliminary mean internal errors for effective temperature and surface gravity were added in quadrature to those estimates. Accordingly, total mean internal errors for derived effective temperature and surface gravity adopted for this work are: $\sigma_T = 1500\text{ K}$, $\sigma_{\log g} = 0.20$. For the helium abundance, $\sigma_y = 0.01$, has been adopted, having conservatively been taken to be twice the preliminary mean internal error and in rough agreement with the detection limit.

3.3 Best-fit Atmospheric Parameter Estimates

In this work, stars were selected for observation and analysis if the PG spectral classification was given as sd, sdB, sdB-O, or sdOA. Three other classes of objects also appear in the sample: 1) Of all the DA white dwarfs selected from the catalog of McCook & Sion (1987) for the sample of Bergeron et al. (1991a,b), 10 stars proved upon inspection of high signal-to-noise blue

spectra to have high Balmer lines extending at least to $n = 12$. The analysis performed here shows that 9 of those stars have effective temperatures and surface gravities characteristic of subdwarf B stars, and the tenth has the temperature and gravity of a main sequence B star. 2) From a sample of blue horizontal branch B star (BHB) candidates observed by Diana Foss, 5 stars proved to be subdwarf B stars. 3) Five other known subdwarf B stars and one main sequence B3V standard not detected in the PG survey were chosen for observation and analysis. The subdwarfs were included because they have previously been analyzed in other investigations, and comparisons with other determinations of the atmospheric parameters of subdwarf B stars are desirable. The main sequence star 16 Peg was observed for the purpose of comparison with the non-subdwarf contaminants of the sample.

The program objects were chosen so as to give the most uniform sky coverage possible, and to sample a wide range of apparent B magnitudes (and hence distances above the Galactic plane). Uniform sky coverage is necessary for the kinematic analysis of EHB stars in Chapter 6, in which certain averages over the space motions of the sample are assumed to vanish. In Figure 3.9, the distribution of the 68 confirmed EHB stars in Galactic latitude and longitude is shown. The severe non-uniformity of the sky coverage will be improved by including in the sample stars with published radial velocities in the literature. In Figure 3.10, the solid line shows the differential number

distribution with apparent photographic B magnitude for the 68 confirmed EHB stars in the sample. The faintest magnitude bins are populated, so that for typical subdwarf luminosities, stars with perpendicular distances of as much as several kiloparsecs above the galactic disk are sampled. The dashed line shows the fraction of all PG subdwarf B stars in each magnitude bin included in the sample analyzed here. The fractional incompleteness is progressively more severe with increasing apparent magnitude. In order to limit the uncertainty in the correction for this incompleteness, it will prove necessary to restrict the scale height analysis of Chapter 5 to stars brighter than apparent B magnitude 15.4.

Table 3.1 presents the best-fit derived atmospheric effective temperatures, surface gravities, and helium abundances for the 68 stars in the sample confirmed as subdwarfs and having surface gravities in excess of $\log g = 5.0$. The columns labelled " B_{PG} " and " $PG Sp$ " give, respectively, the photographic B magnitude and spectral classification listed in the PG Catalog. Table 3.2 presents derived parameters for 24 non-subdwarf stars in the sample having surface gravities less than $\log g = 5.0$. These contaminants comprise 26% of the sample.

3.4 Radial Velocity Measurements

Radial velocities were measured from intermediate- to high-resolution spectra of $H\alpha$ and He II $\lambda 4686$ line profiles by fitting synthetic profiles to the

spectra with a non-linear least squares algorithm (see Figures 2.2a-c). The method is described by Saffer, Liebert, & Olszewski (1988). Internal error estimates were derived from the covariance matrices of the fits, which were weighted by the photon counting statistics of the spectra. These internal errors were dominated largely by the photon counting statistics and the considerable breadth of the Stark-broadened absorption line profiles in the object spectra. Contributions to the total velocity errors from wavelength calibration errors as estimated from measurements of the velocities of the night sky lines in the spectra were minimal.

External error estimates were provided by multiple observations of the radial velocity standards, and by multiple observations of one of the program objects, PG 0749+658. In the latter object, 9 independent measurements of the radial velocity were made. The individual internal measurement errors ranged from 15 km s^{-1} to 35 km s^{-1} , depending on the signal-to-noise ratio achieved in any one spectrum, with a mean internal measurement error of 22.5 km s^{-1} , while for the 9 individual radial velocity measurements, the error of the mean was 20.7 km s^{-1} . Thus, the agreement between the internal and external error estimates is excellent, and internal errors for the program objects may be used confidently in subsequent error analysis. Presentation and further discussion of the measured radial velocities is deferred to Chapter 6, where the radial velocities are used in the analysis of the kinematic

properties of the hot subdwarfs.

TABLE 3.1
PG CONFIRMED EHB ATMOSPHERIC PARAMETERS

Object	Name	B_{PG}	$T_3(K)$	$\log g$	y	$PG\ Sp$	Remark
0044+097	HD 4539		27.0	5.46	.005	NPG sdB	
0101+039	Feige 11	10.98	28.4	5.63	.005	sdB	
0111+177		15.85	30.9	5.57	.013	DA3	vis. dbl.
0112+142		14.70	38.0	5.82	.004	sdB	
0212+230		15.36	26.4	5.80	.004	sdB-O	
0221+217		15.62	33.6	5.87	.118	DA2	
0242+132		11.78	31.2	5.65	.023	sdOA	
0250+189		13.65	26.1	5.81	.001	sdB	
0319+054		14.92	30.5	5.61	.019	DA3	
0322+114		15.26	26.6	5.72	.002	sdB	
0342+026		11.07	26.2	5.67	.004	sdB	
0349+094		15.69	25.4	5.74	.002	sdB	
0749+658		10.75	24.6	5.54	.004	sdB-O	
0806+516		15.03	34.9	6.13	.027	sdB	
0816+313	Ton 313	15.64	32.4	5.99	.010	sdB	
0823+465		14.55	29.9	5.75	.002	sdB	
0839+399	K 345-30	13.87	36.1	5.91	.002	sd	vis. dbl.
0856+121		13.06	26.4	5.73	.001	sdB	
0909+275		10.74	35.4	6.02	.121	sdOA	
0910+621		15.54	28.8	5.57	.002	DA3	
0918+029		12.49	31.7	6.03	.008	sdB-O	
0919+272	Ton 13	11.95	31.9	5.97	.011	sdB	
0920+296	Ton 14	14.58	31.3	6.15	.052	sdOA	CB 5
0933+383	K 347-17	15.28	27.7	5.55	.006	DA2	
0940+171		16.13	27.9	5.69	.007	BHB	
0941+280	Ton 22	12.46	29.0	5.58	.001	BHB	
0947+639		14.78	30.5	5.60	.001	DA 3	
1012+007		14.82	32.2	5.99	.014	sdB	
1101+249	Feige 36	12.98	29.6	5.82	.013	sd	
1114+072	Feige 38	12.96	29.8	5.81	.006	sdB	
1154-070		13.52	28.2	5.58	.005	sd	
1230+052		12.45	28.3	5.72	.001	sdB	
1232-136		13.43	29.0	5.69	.034	sdOA	
1233+426	Feige 65	11.90	26.5	5.60	.005	sdB	
1234+253	Feige 66		33.4	6.20	.017	NPG sdB	

TABLE 3.1 – Continued

Object	Name	B_{PG}	$T_3(K)$	$\log g$	y	PG	Sp	Remark
1236+479	Ton 96	15.26	27.9	5.47	.004	DA2		
1255+547		13.40	33.4	5.81	.027	sdOA		
1256+278	HZ 38	14.14	34.4	5.93	.118	sdOA		
1303-114		13.53	31.9	5.97	.003	sd		
1313+132	Feige 75	14.48	25.6	5.41	.001	BHB		
1325+101		13.29	34.5	6.11	.021	sdB-O		
1343-101		11.99	28.9	5.65	.001	sdB-O		
1432+158		13.66	26.9	5.75	.008	sdB		
1433+240	Ton 209	10.71	29.6	5.57	.000	sdB		
1438-029		13.82	27.7	5.50	.005	BHB		
1442+342		14.40	29.9	5.80	.006	sd		
1538+269	Ton 245	13.18	25.2	5.30	.003	DA2		
1559+533		14.38	30.8	5.85	.007	sdOA		
1613+426	KUV	14.39	34.4	5.97	.022	sdOA		
1619+522		12.84	32.3	5.98	.011	sdB		
1623+386	KUV	15.43	41.6	6.10	.008	sd		vis. dbl.
1631-039	HD 149382		34.2	5.89	.026	NPG	sdB	
1636+216		14.61	34.3	5.76	.018	BHB		
1643+209		14.82	30.2	5.62	.023	sdB		
1708+409		15.09	28.5	5.33	.006	sdB		
1710+490		12.06	29.9	5.74	.006	sdB		
1716+426		13.69	27.4	5.47	.003	sdB		
1758+364	KUV		32.1	5.91	.015	NPG	sdB	
2059+013		13.95	32.4	5.80	.020	sdB		
2110+127		13.56	33.7	5.33	.004	sdB		composite
2128+096		14.25	39.4	5.87	.156	sdOA		
2128+112		15.73	33.5	5.58	.001	DA2		
2204+034		13.44	31.4	5.96	.018	sdB-O		
2218+051		15.30	36.5	6.20	.148	sdOA		anomalous
2313-021	Feige 108		34.5	6.01	.001	NPG	sdB	
2317+046	PB 5333	11.38	37.9	5.81	.002	sdB		y uncertain
2345+318		14.37	27.5	5.70	.006	sdB		
2349+001	PB 5562	12.02	29.3	5.77	.000	sdB		

TABLE 3.2
PG NON-SUBDWARF ATMOSPHERIC PARAMETERS

Object	Name	B_{PG}	$T_3(K)$	$\log g$	y	$PG\ Sp$	Remark
0229+064		11.50	22.0	4.65	.137	sdOA	Ca II "K"
0823+499		11.04	20.1	3.95	.133	sdOA	Ca II "K"
0832+675		14.23	29.2	3.99	.015	sdOA	
0833+698		13.27	21.6	3.83	.120	sdOA	Ca II "K"
0848+186		13.09	21.5	4.63	.106	sdOA	Ca II "K"
0855+293	CB 29	10.34	22.2	4.06	.076	BHB	
1208+224		14.33	26.0	3.55	.007	sdOA	giant
1223+058		15.99	20.0	4.51	.007	sdB	Ca II "K"
1224+671		11.64	29.3	4.86	.012	sdB-O	
1257+276	HZ 47	15.37	19.6	4.42	.025	sdOA	
1317+123	Feige 80	9.01	32.8	4.83	.006	sdO	Ca II "K"
1323-085		12.63	27.0	3.70	.009	sdB-O	giant; composite?
1400+388	PB 1207	12.05	21.6	4.75	.113	sdOA	Ca II "K"
1430+427		14.55	20.1	4.12	.012	DA2	
1607+173	KUV	10.24	27.6	4.92	.153	sdOA	Ca II "K"
1704+221		11.77	26.2	3.72	.015	sdB-O	giant; composite?
2135+044		13.97	32.1	4.79	.016	DA2	
2150+256	16 Peg		20.5	4.20	.025	NPG B3V	standard star
2214+183		13.57	18.6	4.26	.023	sd	composite?
2219+093		9.41	29.7	4.60	.021	sdO	Ca II "K"
2229+099		12.61	18.1	3.90	.085	sd	Ca II "K"
2301+259		12.16	21.3	4.21	.013	sdB	
2345+241		11.58	22.4	4.07	.021	sdB-O	Ca II "K"
2356+166		13.92	23.8	4.70	.148	sdB-O	Ca II "K"

Figure 3.1a. Normalized hydrogen line profiles showing the effects of effective temperature variations at fixed surface gravity in a pure hydrogen atmosphere.

Figure 3.1b. As for Figure 3.1a, but for line profile variations with changes in surface gravity at fixed effective temperature.

Figure 3.1c. As for Figure 3.1b, expanded to illustrate the effects of level depopulation of the higher members of the Balmer series.

Figure 3.1d. As for Figure 3.1a, but with atmospheric helium abundance $y \equiv N(\text{He})/N(\text{H}) = 0.10$.

Figure 3.1e. As for Figure 3.1b, but with atmospheric helium abundance $y \equiv N(\text{He})/N(\text{H}) = 0.10$.

Figure 3.1f. As for Figure 3.1c, but with atmospheric helium abundance $y \equiv N(\text{He})/N(\text{H}) = 0.10$.

Figure 3.1g. Variation of the helium line profiles with helium abundance variations at constant effective temperature and surface gravity.

Figure 3.2a. Contours of the projection of the χ^2 hypersurface for PG 1232-136 in the T_{eff} - $\log g$ plane.

Figure 3.2b. χ^2 contours for PG 1232-136 in the $\log g - N(\text{He})/N(\text{H})$ plane.

Figure 3.2c. χ^2 contours for PG 1317+123 (Feige 80) in the $T_{\text{eff}} - \log g$ plane, showing correlation of effective temperature and surface gravity.

Figure 3.2d. χ^2 contours for PG 1317+123 (Feige 80) in the $\log g - N(\text{He})/N(\text{H})$ plane.

Figure 3.2e. χ^2 contours for PG 1538+269 (Ton 245) in the $T_{\text{eff}} - \log g$ plane. Saddle points illustrate the need for good seed values for the atmospheric parameters.

Figure 3.2f. χ^2 contours for PG 1538+269 (Ton 245) in the $\log g - N(\text{He})/N(\text{H})$ plane. Saddle points illustrate the need for good seed values for the atmospheric parameters.

Figure 3.3a. Observed spectra of confirmed classical sdB stars, arranged in order of increasing effective temperature from top ($T_{\text{eff}} = 25,200 \text{ K}$) to bottom ($T_{\text{eff}} = 41,500 \text{ K}$).

Figure 3.3b. Model fits (dotted lines) to the observed spectra (solid lines) of Figure 3.3a.

Figure 3.3c. Observed spectra of classical sdB and sdOB stars, arranged in order of increasing atmospheric helium abundance from top ($y = 0.00$) to bottom ($y = 0.12$).

Figure 3.3d. Model fits (dotted lines) to the observed spectra (solid lines) of Figure 3.3c.

Figure 3.3e. Observed spectra of non-subdwarf stars having lower surface gravities. Non-subdwarf contaminants comprise 26% of the sample.

Figure 3.3f. Model fits (dotted lines) to the observed spectra (solid lines) of Figure 3.3e.

Figure 3.3g. Attempted model fits to the spectrum of PG 2218+051. It proves impossible to achieve satisfactory fits for all line profiles.

Figure 3.4a. Internal error distribution for T_{eff} derived from covariance matrices of the model fits.

Figure 3.4b. Internal error distribution for log g derived from covariance matrices of the model fits.

Figure 3.4c. Internal error distribution for $N(\text{He})/N(\text{H})$ derived from covariance matrices of the model fits.

Figure 3.5. Simulations of imperfect flux calibration. The large and small filled circles represent the atmospheric parameters derived, respectively, from unperturbed and perturbed spectra.

Figure 3.6. Representative model fit to an observed spectrum of Feige 34, using only the H δ - H9 line profiles, for the purpose of comparison with external errors derived from repeated measurements.

Figure 3.7. External error estimation from repeated observations of Feige 34.

Figure 3.8a. Variations in derived effective temperature due to variations in seeing.

Figure 3.8b. Variations in derived surface gravity due to variations in seeing.

Figure 3.9. Distribution of 68 confirmed EHB stars in Galactic longitude and latitude.

Figure 3.10. Differential number distribution of 68 confirmed EHB stars (solid line) with apparent photographic B magnitude, and the corresponding percentage incompleteness (dashed line) with respect to the Palomar Green Survey.

CHAPTER 4

EHB STARS IN THE HR DIAGRAM

4.1 Effective Temperatures and Surface Gravities

The derived effective temperatures and surface gravities for the entire sample of 92 sdB candidates are displayed in Figure 4.1. The open symbols distinguish ranges of photospheric helium abundance, and connected filled symbols denote the zero-age helium-burning main sequence (HeMS), zero-age extended horizontal branch (ZAEHB), zero-age horizontal branch (ZAHB), and zero-age hydrogen-burning main sequence (HMS) as indicated in the legend: diamonds – HeMS, Paczyński (1971); circles – HeMS and ZAEHB, Caloi (1972, 1989); triangles – ZAHB and ZAEHB, Sweigart (1987); and HMS, Allen (1973). The HeMS is labelled with the stellar mass in solar units. The hooked track on the lower part of the ZAEHB illustrates evolution of the stellar luminosity during the core helium-burning phase. The error bars show the conservatively adopted mean internal errors of 1500 K and 0.20 dex.

The stars represented in the figure can be divided into three broad classes: 1) Classical sdB and sdOB stars having effective temperatures between 20,000 and 40,000 K and surface gravities exceeding $\log g = 5.0$, and having helium-deficient photospheres ($N(\text{He})/N(\text{H}) \lesssim 0.02$). 2) Stars with temperatures and surface gravities characteristic of classical sdB and sdOB

stars but having normal or even enhanced helium abundances compared to the solar value. These stars tend to have the highest effective temperatures and lie near the helium main sequence, as well as the cooler and higher-gravity helium-rich sdO stars. 3) Lower gravity main sequence B and blue horizontal branch stars having photospheric helium abundances ranging from normal to mildly depleted compared to the solar value. At temperatures approaching 20,000 K , the theoretical horizontal branch and main sequence begin to overlap, making these stars difficult to distinguish from each other at the intermediate spectral resolution of the observations.

The confirmed EHB stars cluster in a well-defined sequence lying parallel to theoretical sequences of the zero age extended horizontal branch (ZAEHB; Caloi 1972, 1989) and zero age horizontal branch (ZAHB; Sweigart 1987). For all stars with $\log g \gtrsim 5.0$, every star resides within a sloping band with vertical cross-section ~ 0.5 dex and bounded below by the ZAEHB sequence with mass $\sim 0.5 M_{\odot}$. The theoretical sequences are parameterized by $q = M_c/M_*$, the ratio of the helium-burning core mass to total mass. For the $0.5 M_{\odot}$ sequence shown in Figure 4.1, $q = 0.92$ and 0.98 at $T_{\text{eff}} = 20,000$ and $26,000 K$, respectively. For $q \gtrsim 0.96$, corresponding to an envelope mass $M_e \lesssim 0.02 M_{\odot}$, it is not possible to sustain hydrogen shell burning as it is for horizontal branch stars, which derive a significant fraction of their luminosity from hydrogen shell burning. For the same reason, those stars are unable to

ascend the asymptotic giant branch after the end of the core helium-burning phase, represented in the figure as a hooked track on the EHB at about 32,000 K . Evolution to higher luminosity along the track takes place at a nearly constant rate during the approximately 10^8 year lifetime of the core helium-burning phase. The subsequent evolution of an EHB star in the H-R diagram should resemble that of a pure helium star of the same mass.

Grouped in the third class with the lower gravity stars, an apparent sequence of stars with surface gravities between the main sequence and the EHB sequence is evident, with the separation from those two sequences becoming quite pronounced at effective temperatures between 25,000 and 35,000 K . The EHB evolutionary tracks of Caloi (1989) suggest that the intermediate sequence might be composed of post-EHB stars. For EHB tracks with an initial helium-burning core mass of $0.5 M_{\odot}$ and envelope masses of 0.003, 0.005 and $0.01 M_{\odot}$, calculations of the phases immediately after core helium exhaustion (at the end of the hooked track in Figure 4.1) show a gradual rise in luminosity over a period of some 2×10^7 yr to a locus overlying the observed intermediate sequence. The ratio of this time scale to the core helium-burning time scale of $\sim 1.3 \times 10^8$ yr predicts that for the 68 confirmed EHB stars one would expect ~ 10 stars in post-EHB configurations occupying a sloping band roughly parallel to the EHB sequence and bounded by effective temperatures of 22,000–40,000 K and

surface gravities of $\log g = 4.2$ to $\log g = 5.1$. The observed intermediate sequence in Figure 4.1 is marginally consistent with these predictions, although there is a tendency for the stars to occupy the upper boundary of the post-EHB area, while the evolutionary calculations suggest that the area should be more uniformly populated. Following the gradual rise to the upper boundary of the post-EHB area, the stars rapidly evolve to high temperature and luminosity in a phase lasting only some 10^5 yr, after which they join the white dwarf cooling sequence. Thus, the region in the H-R diagram hotter than $\sim 40,000$ K and with surface gravity lower than $\log g \sim 5.0$ would not be expected to be populated, and no stars in the sample are found to lie there. A similar behavior in more massive horizontal branch stars after the exhaustion of central helium has been demonstrated by Strom et al. (1970) for the so-called “supra-horizontal branch” stars in the globular cluster M3.

4.2 The Field Horizontal Branch Gap

The EHB sequence is particularly well populated at surface gravities exceeding $\log g = 5.0$ and effective temperatures exceeding 25,000 K. Near $T_{\text{eff}} = 24,500$ K, the sequence ends abruptly. This cutoff is in good agreement with the boundary defined by the blue edge of the second Newell gap in the cumulative distribution of the density of field BHB stars vs. the reddening-free Strömgren index $[u - b]$ (Newell & Graham 1976). That study demonstrated the existence of two gaps, centered on $[u - b] \sim 0.95$ and $[u - b] \sim 0.25$, with

widths that, according to the temperature calibration of Philip & Newell (1975), correspond to effective temperature intervals of 12,800 K to 13,300 K and 20,000 K to 23,000 K , respectively. It should be noted that the observed cutoff is not related to the color selection criterion $(U - B) < -0.46$ imposed on objects detected and selected for followup spectroscopic classification in the PG survey. At effective temperatures exceeding 20,000 K , all stars having surface gravities exceeding those of main sequence stars have $(U - B) \lesssim -0.8$, so that even with the large uncertainty of 0.38 mag in the $(U - B)$ colors of the PG catalog, hot B stars are not selected against in the survey. Only for stars as late as B5 to B7 (15,000 K to 13,000 K) would the selection effect begin to become important. Even at those temperatures only the stars with the lowest surface gravities would be selected against.

Unambiguous evidence exists for similar gaps in the blue horizontal branches of four metal-poor globular clusters: NGC 6752 (Cannon 1981), NGC 288 (Buonanno et al. 1984), M15 (Buonanno, Corsi, & Fusi Pecci 1985), and ω Cen (Da Costa, Norris, & Villumsen 1986; hereafter DCNV). However, DCNV report that the midpoints of the gaps in those clusters have effective temperatures of 20,000 K , 12,900 K , 10,000 K , and 16,200 K , respectively. There appears to be no correlation between mean cluster metallicity and gap position. The cluster gaps might be identified with one of the Newell gaps for field BHB stars in NGC 6752 and NGC 288, but the field and

cluster gaps cannot be demonstrated to be related. The very existence of such gaps, either in the field or in clusters, is troublesome in terms of current stellar evolution theory. Evolutionary models do not predict gaps if the various physical processes which might affect HB morphology, such as age, mass loss, envelope helium and heavy metal abundances, and stellar rotation, are continuously distributed (cf. Rood & Crocker 1985). Bimodal cyanogen distributions are seen in the giant branch stars in some clusters having gaps in their blue horizontal branches, and CNO abundance is one candidate for the long-sought "second parameter" influencing cluster HB morphology (Norris et al. 1981; Norris 1981; Smith & Norris 1982, 1983). But DCNV find no evidence for a bimodal CN distribution in the giant branch of ω Cen, which demonstrably possesses a well-defined BHB gap.

For the most part, the gaps in globular cluster blue horizontal branches are not completely unpopulated by stars. Rood & Crocker (1985) and Crocker, Rood, & O'Connell (1986, 1988) have demonstrated bimodality in a number of globular cluster blue horizontal branches. They suggest the gaps are not places that stars have evolved away from, or where stars are forbidden, but rather are the underpopulated regions lying between the bimodal distributions of BHB stars. The mass distribution of BHB stars in M15 is best explained under this hypothesis by a pair of gaussian distributions with overlapping tails. The high mass (red) distribution is centered on

$0.67 M_{\odot}$ with $\sigma \sim 0.025 M_{\odot}$, and the low mass (blue) distribution is centered on $0.59 M_{\odot}$ with $\sigma \sim 0.01 M_{\odot}$ (Rood & Crocker 1985).

Figure 4.2 shows the number distribution with effective temperature of the confirmed EHB stars in the sample analyzed here. The low temperature cutoff at $\sim 25,000 K$ is very sharp; the falloff between $25,000 K$ and $27,000 K$ is consistent with a step function convolved with the effective temperature mean internal error bar shown in the figure. It is possible that gaps in globular cluster blue horizontal branches have similar sharp boundaries, but the precision of derived effective temperatures and masses in previous investigations has been insufficient to resolve the edges, if present.

The dearth of objects on the $0.5 M_{\odot}$ ZAHB tracks in Figure 4.1 at effective temperatures cooler than $20,000 K$ might be due to the omission of PG objects classified as blue horizontal branch stars from the sample analyzed here. Another possibility is that the cutoff is a selection effect due to the subdwarf classification criteria of the PG survey: weak hydrogen-lined candidates were rejected if absorption was detected at Ca II “K”. That criterion might have led to rejection of cooler EHB stars at surface gravities somewhat less than $\log g = 5.0$ and effective temperatures between $15,000$ and $25,000 K$. However, while it is true that none of the 68 confirmed EHB stars show detectable Ca II “K”, there are a dozen examples in the non-subdwarf contaminants that were classified as PG subdwarfs and that

show detectable Ca II “K” absorption with equivalent widths $\sim 1 \text{ \AA}$. These stars are identified in the “Remarks” column of Table 2. Not a single star falls on the theoretical EHB tracks in Figure 4.1. Only one star has an effective temperature between 23,000 and 24,500 K , and its surface gravity of $\log g = 4.70$ separates it cleanly from the EHB stars. Dr. Richard Green has provided the authors with the classification spectra of several hundred stars rejected from the subdwarf category because of the clear detection of Ca II “K” absorption. Careful inspection of these spectra identified only a handful of stars that have weak hydrogen lines, and at the same time show weak Ca II “K” while lacking altogether any hint of a “G” band or Mg I “b” absorption. Thus it seems quite unlikely that the low-temperature EHB cutoff is due to this selection effect.

The cutoff in the EHB distribution in Figure 4.1 also coincides with the position along the model evolutionary sequences where the overlying hydrogen-rich envelope has a mass $\sim 0.02 M_{\odot}$ ($q \sim 0.96$), the mass below which hydrogen shell-burning cannot be sustained. Further investigations of the atmospheric parameters of field blue horizontal branch stars are badly needed to determine the extent and nature of gaps in the field Population I horizontal and extended horizontal branches. Unfortunately, such analyses are complicated greatly by the confluence of the hydrogen-burning main sequence and the blue horizontal branch at cooler effective temperatures and are

beyond the scope of this work. Observations at higher spectral resolution and analyses using appropriate model grids will be required to extend the current investigation to cooler temperatures.

At the hot end, the EHB sequence is populated by hydrogen-rich stars at temperatures as high as 40,000 K , overlapping to some extent with low-luminosity sdO stars having helium enriched atmospheres. There are no stars found to have surface gravities higher than those of the theoretical helium main sequences (HeMS) of Paczyński (1971, upper track) and Caloi (1989, lower track) shown in Figure 4.1. This is further evidence in favor of the EHB interpretation of the subdwarf B stars, since the HeMS is the limiting case of the mass of the overlying hydrogen-rich envelope of a core helium-burning star going to zero.

4.3 Photospheric Helium Abundances

The helium abundances of all the program objects are plotted against effective temperature in Figure 4.3a, where the three broad subdivisions of the sample noted previously are distinguished by different symbols. The open circles represent classical sdB stars, having helium abundances $y \lesssim 0.02$, an arbitrary standard, but one consistent with the notion of depleted atmospheric helium at sdB surface gravities. High gravity stars with $y > 0.02$ are identified by open boxes and labeled “He-strong”, or at least closer to the solar value compared to classical sdB stars, and low gravity stars are identified by open

triangles. The figure shows a clear boundary in effective temperature at some 30,000 K , below which all high-gravity objects have helium-depleted atmospheres. A trend to higher helium abundance at higher temperatures is evident, and the distribution becomes modestly bifurcated at the highest effective temperatures ($T_{\text{eff}} \gtrsim 35,000 \text{ K}$), where approximately equal numbers of stars have either very high or very low helium abundance. Figure 4.3b shows a similar plot of y vs. $\log g$. The model grid is bounded above by $y = 0.10$, and for the highest derived helium abundances all that can be said is that they are significantly in excess of the solar value. An extension of the model grid to higher helium abundances is needed to more accurately determine the abundances for the most helium-rich component of the sample.

In Figure 4.4, the stars in the H-R diagram of Figure 4.1 with helium abundances $y > 0.02$ are seen to cluster in two well-separated groups. Many of these stars have PG spectral classifications of sdB-O or sdOA, that is, those stars of apparently higher gravity having moderate to strong He I absorption lines. Again a trend toward higher helium abundance at higher effective temperature is evident for high-gravity stars. As discussed subsequently, these helium abundance patterns are difficult to reconcile with time-dependent diffusion calculations, which predict depletion of all helium from the photosphere on very short time scales over the whole range of characteristic EHB effective temperatures.

4.4 Comparisons with Other Analyses

All previous analyses of EHB stars have been carried out in a similar manner: First, the effective temperature is estimated from an analysis of the ultraviolet energy distribution or from intermediate- or narrow-band photometry. The effective temperature then is held fixed while a Balmer line profile, typically $H\gamma$, is analyzed for the surface gravity. Finally, the equivalent width or detailed shape of the line profile of a strong helium line, like He I $\lambda 4471$, gives an estimate of the helium abundance. The last two steps may have to be iterated at higher helium abundances. Unfortunately, with the exception of the recent works of Moehler et al. (1990) and Moehler, Heber, & de Boer (1990), with which comparisons will be discussed in this section, most previous analyses of stars analyzed in this work either have quite uncertain surface gravity determinations or lack them altogether. The only other extensive body of accurate surface gravity determinations (see, e.g., Heber et al. 1984, Groth et al. 1985, and Heber 1986) is for southern hemisphere stars for which there is little overlap with this sample.

While most of the program objects selected for analysis in this work were chosen from the Palomar Green survey, the compilation of Kilkenny, Heber, & Drilling (1987) provides data on some 200 spectroscopically classified hot subdwarfs, 100 of which have been analyzed for effective temperature and surface gravity. Of the EHB component in this catalog, 13 stars having

19 independent effective temperature estimates were selected for observation and analysis here, and of these, 7 stars have 9 independent surface gravity estimates. These exclude the temperature and gravity estimates of Moehler, Heber, & deBoer (1991), which will be discussed subsequently. Table 3 summarizes these previous analyses, and the table and Figures 4.5a and 4.5b show comparisons of those results with the results of the analysis performed in this work. In Figure 4.5a, the derived effective temperatures are compared. There is excellent agreement between the spectroscopic temperature estimates derived here and estimates derived by other groups using the methods specified next to the various symbols in the legend. It is important to note that, save for the discrepant point in the lower left corner of the figure, the agreement is excellent without regard to the method of estimation of the effective temperature. That sole significantly discrepant point (for PG 0342+026) is taken from Lamontagne et al. (1987), in which the authors warn of potential uncertainties in having substituted the LWP camera of the IUE Observatory for the LWR camera used in previous investigations. To reiterate, all other previous estimates of effective temperatures of stars in common with this sample, excluding those of Moehler, Heber, & deBoer (1991), are in excellent agreement without regard to the method of analysis.

In Figure 4.5b, surface gravity estimates are compared. The agreement is not as good as for the effective temperatures, with other estimates

having slightly lower derived values. The most discrepant points are for the same star having the most discrepant temperature estimate in Figure 4.5a, and for an early, admittedly uncertain estimate of the surface gravity of UV 1758+364 by Giddings & Dworetzky (1978). There is no compelling explanation for the systematic shift of the remaining 7 points, but in terms of the mean internal error of the spectroscopic estimates the difference is only mildly significant.

4.5 Estimates from Spectroscopy vs. Colors

A much more extensive check of the internal consistency of the temperature and gravity estimates in this work is possible using Strömgren colors. Fontaine et al. (1987) have graciously provided the authors with the results of their sdB Strömgren photometry program, in which all PG sdB and sdOA stars with photographic B magnitudes brighter than 14.6 were observed. By calibrating a color index-effective temperature scale with the model grid used to analyze the spectra, it is possible to compare derived effective temperatures for a considerable portion of the sample using that Strömgren photometry.

The Strömgren colors $(b - y)$ and $(u - b)$ were calculated for the $y = 0.00$ model grid, using the sensitivity functions and the calibration of Olson (1974). For each effective temperature and surface gravity, the reddening-free index $[u - b] = (u - b) - 1.56(b - y)$ was calculated, and following

Bergeron et al. (1984), for each effective temperature an average value of $[u - b]$ was calculated from the two values of $[u - b]$ for $\log g = 5.0$ and 6.0 . For 54 stars in the sample having measured Strömgren colors, the resulting $[u - b]$ vs. T_{eff} relation then was interpolated for the stellar effective temperatures. Those derived effective temperatures were compared against effective temperatures derived from the same data, but using the $[u - b]$ vs. T_{eff} relation of Bergeron et al. (1984). The correlation is excellent over the common range of T_{eff} .

Figure 4.6a shows a comparison of effective temperatures derived in the 3-parameter fits with the effective temperatures derived from Strömgren colors, as described above. For 7 stars, the color temperature estimates are wildly discrepant, and these have been excluded from the comparison. Two of these stars have blue $(b - y)$ colors between -0.075 and -0.15 mag and with $(u - b)$ colors that are significantly more negative than the bulk of the stars with the comparable $(b - y)$ colors. Due to the steep color index-effective temperature relation at high temperatures, their positions in the two-color diagram above the $45,000\text{ K}$ theoretical reddening line correspond to derived effective temperatures as high as $60,000\text{ K}$. The accompanying photometric errors are of order 0.05 mag, and observational error could be responsible. The presence of unresolved red companions also could account for the discrepancy by making $(b - y)$ more positive, although the PG Catalog does

not identify the stars as composite, nor is there any record in the observing logs of the objects having been resolved on the acquisition camera monitor. The three very low gravity objects in the sample have color temperatures between 6,000 K and 12,000 K lower than their spectroscopic temperatures. It is inappropriate to have analyzed them with the temperature relation calibrated with high gravity model spectra, and they also have been excluded from the comparison. The last star, which has a color temperature some 10,000 K less than the spectroscopic temperature, shows a dramatic downturn in flux in the blue part of its spectrum, evidence of reddening or the presence of a companion.

Figure 4.6a shows that for the remaining stars, the agreement is excellent, except for the mild systematic trend at high T_{eff} . The most discrepant stars have a range of helium abundances, so that differences in the model atmosphere helium abundances do not appear to have significantly affected the estimates. The discrepancy may not be significant, since the $[u - b]$ vs. T_{eff} relation is very steep at high T_{eff} , and small differences in the measured and theoretical colors correspond to large differences in T_{eff} . Non-LTE effects might become significant at high T_{eff} , but as previously noted, Wesemael et al. (1980) found no significant difference in continuum fluxes in the EHB regime.

With color temperatures in hand, the spectra were fitted for surface

gravity by holding the effective temperature constant at the value of the color temperature determined above, and varying the surface gravity alone. The helium abundance was held fixed at the value derived from the 3-parameter fits, and all available absorption lines were analyzed. Figure 4.6b shows a comparison of the gravities thereby derived with the gravities derived from the 3-parameter fits. Not surprisingly, there is excellent agreement, with a mild systematic trend at the highest gravities caused by the systematic trend in T_{eff} . The deviations are positively correlated, as expected, since when the corresponding color temperatures are too high, and when T_{eff} is held fixed in the fits, the Balmer line equivalent widths are too small compared to the observations, and to compensate, the surface gravities must assume higher values. The rms deviation in effective temperature of the points about the diagonal line in Figure 4.6a is $\langle \Delta T_{\text{eff}} \rangle_{\text{rms}} = 1920 K$, only some 6% at $T_{\text{eff}} = 30,000 K$. The corresponding rms deviation induced in the surface gravities in Figure 4.6b is $\langle \Delta \log g \rangle_{\text{rms}} = 0.47$, or nearly a factor of 3 in the derived gravities and luminosities, highly disproportionate considering the excellent effective temperature correlation.

4.6 Comparisons with Moehler et al.

Moehler et al. (1990) and Moehler, Heber, & de Boer (1991, hereafter MHdB) have presented spectra and atmospheric analyses for 92 stars selected from the PG Catalog, including 37 confirmed EHB stars. In

those analyses, effective temperature estimates were derived from ultraviolet energy distributions or optical photometry. The surface gravities then were derived from an analysis of the $H\gamma$ profile made while holding T_{eff} fixed. In the sample presented here there is overlap with the MHdB sample only for 19 stars, and of those only 13 were analyzed by MHdB for the atmospheric parameters. Moehler et al. (1990) report Strömgren photometry and show spectra for the remaining six stars, but their effective temperature and surface gravity estimates are not reported.

Comparisons of derived effective temperatures for the stars in common with MHdB are shown in Figure 4.7a. There is a significant rms systematic difference of 2,750 K in the color temperature scales of this work and of MHdB in the range 25,000–35,000 K . The cooler MHdB estimates were derived either from their own photometry or that of Fontaine et al. (1987). For stars common to both photometric samples the reported colors are in good agreement, and it seems likely that the discrepancy lies in the $[u - b] - T_{\text{eff}}$ calibration. Indeed, MHdB used a calibration based on the models of Lester, Gray, & Kurucz (1986) with solar composition and $\log g = 4.5 - 5.0$. The difference between their calibration and the one used here can fully account for the discrepancy in the derived color temperatures. As shown previously, the color temperature estimates derived from the Olson (1974) calibration of the $y = 0.00$ model grid used here and the spectroscopic temperature estimates

derived from the 3-parameter fits agree closely between 25,000–35,000 K (Figure 4.6a). If the Lester et al. (1986) calibration had been used, that comparison would have shown a systematic offset similar to that seen in Figure 4.7a.

To compare surface gravities derived from the lower-resolution spectra analyzed here, effective temperatures were held fixed at the values cited by MHdB, and gravities were determined by fitting first $H\gamma$, then all available Balmer lines in each spectrum (Figure 4.7b). The resulting gravity estimates are significantly lower than the spectroscopic estimates, as expected considering the significantly lower estimated temperatures of MHdB. The rms difference in gravity is 0.55 dex, a very large discrepancy in terms of the derived stellar luminosity, and to a lesser extent the derived distances. If the theoretical EHB tracks corresponding to a core mass of $0.5 M_{\odot}$ shown in Figure 4.1 are displaced upwards by 0.55 dex, the intersection of those tracks with the helium main sequence tracks implies a significant increase in the derived EHB core mass to as much as $0.7 M_{\odot}$. This is uncomfortably high if the sdB stars are identified with the EHB, and if their immediate progenitors are red giant branch stars, which for an old disk population might be expected to be producing large numbers of $0.45\text{--}0.5 M_{\odot}$ cores at helium ignition.

When the effective temperatures were held fixed at the MHdB values, it proved possible to fit $H\gamma$ and $H\delta$ satisfactorily for all the stars in common

with MHdB, even when the higher Balmer lines were obviously poorly fit. The difference need only be as large as 2,000–3,000 K to induce quite large surface gravity differences of up to 0.8 dex compared to the estimates derived in 3-parameter fits. The spectra analyzed by MHdB covered $\lambda\lambda 4000\text{--}5000$ at 2.5 $\text{\AA}$ resolution, with the red wing of $H\beta$ and continuum not detected. The line is unsuitable for fitting because the continuum redward of the line cannot be established accurately, leaving only $H\gamma$ and $H\delta$ available for analysis. Spectral coverage $\lambda\lambda 3750\text{--}4750$ makes available for analysis $H\gamma - H9$ as gravity-sensitive lines while still covering the strong He II line at $\lambda 4686$, a useful diagnostic at higher effective temperature.

Figures 4.8a–c show selected spectra and best-fit models illustrating the dependence of the derived surface gravities on the agreement between the color and spectroscopic effective temperature estimates. For PG 1101+249 ($T_{\text{eff}}(\text{Color}) = 29,200\text{ K}$, $T_{\text{eff}}(\text{Spectroscopic}) = 29,600\text{ K}$), the estimated temperatures agree well. Holding the color temperature fixed, and analysing only $H\gamma$ (Figure 4.8a, upper panel), then analysing $H\beta - H9$ (middle panel), similar derived surface gravities are obtained. These both agree fairly well with the surface gravity estimate of the 3-parameter fit (lower panel), and in all three cases, a satisfactory fit can be found for all line profiles. For PG 2128+096 (Figure 4.8b), the estimated color temperature is some 3,000 K cooler than the estimated spectroscopic temperature, and the fits (as in

Figure 4.8a) show that when the color temperature is held fixed in the fit (upper and middle panels), the derived surface gravity is significantly less than the spectroscopic estimate, and a satisfactory fit for all the lines cannot be found. The model absorption line cores are too deep for the observations, especially in the higher Balmer lines, because the color temperature estimate is too cool. Figure 4.8c shows for PG 2345+318 the opposite situation; the color temperature estimate is hotter than the spectroscopic temperature estimate, the surface gravities derived at fixed temperature are higher than the spectroscopic gravity, and the model line cores in the figure (upper and middle panels) are too shallow for the observations. With the exception noted previously for PG 2218+051, for every object in the sample a satisfactory fit for all line profiles can be found with full 3-parameter fits. These comparisons suggest that the spectroscopic estimates have significantly better intrinsic relative precision than estimates derived in part from colors, and that the inclusion of the higher Balmer lines in the analysis is crucial if self-consistent solutions are to be obtained.

The foregoing rather lengthy comparisons and discussion suggest that atmospheric parameter estimates derived from optical photometry, especially surface gravity estimates derived from the analysis of a single lower Balmer line while holding the color temperature fixed, are vulnerable to large uncertainties compared to the relatively small uncertainties of only some 10% in the

adopted color temperatures. The simultaneous fitting of several high and low Balmer lines provides constraints to determine the atmospheric parameters with significantly higher precision. Of course, the absolute accuracy of the derived parameters depends on a host of other uncertainties, such as the adopted theoretical Strömgren color calibration, the details of the treatment of the level occupation probabilities, as well as any residual systematic trends in the calibration of the observations. However, within the framework of the observations and models used in the analysis here, consistent estimates for the atmospheric parameters are obtained in both the photometric and spectroscopic analyses.

4.7 Luminosity Evolution and Core Mass Dispersion

Greenstein & Sargent (1974) first identified the field sdB stars with the extended horizontal branch (EHB), and pointed out that for core helium-burning stars, $\log(g\theta^4)$ scales with $\log(L/L_\odot)$ and is constant for a given core mass, with $\theta = 5,040 K/T_{\text{eff}}$. If the uncertainties in the derived atmospheric parameters were small enough, any existing structure in the EHB luminosity distribution might be resolved, for example, the luminosity evolution during the approximately 10^8 year core helium-burning lifetime. In Figure 4.1, the morphology of the hooked track near 32,000 K and $\log g = 6.0$ suggests that, assuming a single value of the core mass, the density of stars should be higher at the top of the distribution. Alternatively, the dispersion due to

a distribution of core masses significantly larger than that due to luminosity evolution might be resolved.

In Figure 4.9, $\log(g\theta^4)$ is plotted against θ for 68 confirmed classical sdB and helium-strong sdB stars with derived surface gravities in excess of $\log g = 5.0$. The 1σ error bars shown in the figure were computed from the individual internal errors of the model fits, and the unweighted mean 1σ uncertainty of the individual measurements is about 0.16 in $\log(g\theta^4)$. The apparent trend to higher luminosity with increasing effective temperature is not significant. A weighted average gives a best-fit constant value of $\log(g\theta^4) = 2.64$, of the same order as the value of 2.38 determined by Greenstein & Sargent (1974). In Figure 4.10, the histogram shows the number distribution of $\log(g\theta^4)$ for the stars of Figure 4.9 and the accompanying mean 1σ error. The distribution is broad; in fact, its FWHM of ~ 0.4 in $\log(g\theta^4)$ is only marginally larger than the FWHM of a normal distribution having a 1σ standard deviation of 0.16. Hence most, if not all, of the width of the distribution is due to observational error. There is no significant structure to suggest that any underlying luminosity evolution has been resolved, which if present would result in a skew to higher density at smaller values of $\log(g\theta^4)$ (i.e., higher luminosity.) Neither does the distribution support a conservative 1σ dispersion in core mass larger than about $0.04 M_{\odot}$.

The narrow distribution of core masses about the $0.5 M_{\odot}$ EHB

tracks in Figure 4.1 constrains the range of initial masses of prospective EHB progenitors if they are red giant branch (RGB) stars which lose the bulk of their hydrogen-rich envelope at the tip of the RGB. Recent calculations by Sweigart, Greggio, & Renzini (1989, 1990, hereafter SGR) have explored the evolution of the RGB as a function of the initial mass of the turnoff stars in a stellar population. For low mass stars, there are profound differences in the morphology of the RGB, the luminosity at its tip, and the core mass at helium ignition compared to intermediate mass stars. The intermediate mass stars ignite helium quietly under nondegenerate conditions shortly after reaching the RGB and have a short-lived and low-luminosity RGB phase. The low mass stars ignite helium violently under degenerate conditions and have a more luminous and long-lived RGB phase. The SGR evolutionary sequences span a range of initial masses from 1.4 to $3.4 M_{\odot}$. The SGR “RGB phase transition”, marked by a rapid increase in the luminosity, extent, and lifetime of the RGB phase as the turnoff stars begin to ignite helium in a degenerate core, takes place over a narrow range of initial masses between 2.0 and $2.8 M_{\odot}$, depending on the initial helium and metal mass fractions Y and Z .

Of more interest in the context of the apparently narrow range of EHB core masses inferred here is the value of the core mass at helium ignition at the tip of the RGB. For initial masses below $\sim 2.0 M_{\odot}$, irrespective of the initial metal abundances, the core mass at helium ignition converges to

a value between 0.46 and $0.48 M_{\odot}$ for initial helium mass fractions of 0.30 and 0.20 , respectively. Sweigart & Gross (1978) found the core mass to be $\sim 0.47 M_{\odot}$ for an initial mass of $0.8 M_{\odot}$ and for $Y = 0.30$ and $Z = 0.01$, values appropriate for old disk stars. For $Z = 0.01$, and $Y = 0.20$ and 0.30 , the core mass at ignition reaches an absolute minimum of $\sim 0.33 M_{\odot}$ for initial masses of ~ 2.7 and $\sim 2.3 M_{\odot}$, respectively. For initial masses larger than these, the core mass increases monotonically. Thus, if disk RGB stars are the immediate progenitors of the field EHB stars, their initial masses are constrained to be $\lesssim 2 M_{\odot}$. For initial masses between 4 and $5 M_{\odot}$, the core masses at helium ignition also would be near $0.5 M_{\odot}$ (Castellani, Chieffi, & Straniero 1990), but there are too few of those more massive stars to account for the bulk of the EHB stars. The kinematics of EHB stars (Chapter 5, Saffer & Liebert 1991) also suggest that their progenitors are older, less massive disk stars.

It is interesting to note that several previous determinations of the masses of evolved stars and stellar remnants also have narrow derived mass distributions. Weidemann & Koester (1984) found for DA white dwarfs with effective temperatures between $8,000 K$ and $15,000 K$ a distribution with a narrow peak, an extended high mass tail, and a sharp low mass cutoff. Bergeron, Saffer, & Liebert (1991a,b) found a similar distribution for DA white dwarfs hotter than $15,000 K$, although their analysis, with its significantly

better mass resolution, identified a separate population of very low-mass white dwarfs that presumably derive from close binary evolution (see also Liebert, Bergeron, & Saffer 1991). If this group of stars is excluded from the comparison, however, all previously determined white dwarf mass distributions have remarkably similar shapes, with the salient feature being a very narrow central peak. Mendez (1987) found for 21 Galactic planetary nebula central stars a peaked mass distribution with 11 stars in the central two $0.05 M_{\odot}$ bins. Jacoby (1989) and Ciardullo et al. (1989) found it necessary to assume a planetary central star mass distribution centered on $0.61 M_{\odot}$ and having a gaussian dispersion of only $0.02 M_{\odot}$ in order to fit the luminosity function of the brightest planetary nebulae in the Local Group. However, there could be a selection effect, the “guillotine” effect, which selects against the most (least) massive planetary central stars because the evolutionary time scales for those stars are too short (long) for the nebulae to become bright enough for long enough to be detected. Still, neither very high- nor very low-mass central stars are known to exist in abundance.

4.8 Astrophysical Implications

The morphology of the EHB sequence shown in Figure 4.1 strongly supports the core helium-burning interpretation of the EHB and is inconsistent with the hypothesis that the majority of these stars are in post-AGB evolutionary states. If the interpretation of the sequence as one of

continuously decreasing envelope mass is correct, and if the narrow width of the distribution correctly implies a narrow range of core masses, evolution from RGB progenitors might account for the majority of the field EHB stars. The kinematics of the sample (Chapter 5, Saffer & Liebert 1991) strongly indicate membership in the old disk population, and upon completing RGB evolution such a population would contribute large numbers of helium-burning stars with core masses near $0.5 M_{\odot}$ to the field horizontal branch population. Only a small fraction of all RGB stars would have to lose the bulk of their hydrogen-rich envelopes to account for the observed birth rate of field EHB stars, assuming core helium-burning lifetimes. It is difficult, however, to reconcile this scenario with the observed characteristics of horizontal branches in disk globular clusters. Those clusters have metallicities much closer to the solar value than those in globular clusters with $[\text{Fe}/\text{H}] \lesssim -1.5$. On the basis of studies of both more metal-rich globular clusters and old galactic clusters, these stars would be expected to have very red core helium-burning horizontal branches and very sparse or non-existent blue horizontal branches and extended horizontal branches. This is indeed the case for horizontal branch stars in known old disk clusters like M 67 and NGC 188.

Alternate hypotheses for the origins of the EHB stars predict a larger mass dispersion for a core helium-burning sequence than is observed. Iben & Tutukov (1986a) have calculated the evolution of a close binary system that

first undergoes Roche lobe overflow when the primary has become a giant with a degenerate helium core with a mass of $0.3 M_{\odot}$. When the envelope around this degenerate core is lost, the binary detaches, and its subsequent evolution is driven by the contraction of the inert helium core. There are three stages of evolution with luminosities reaching or exceeding those of the EHB stars. The first and longest-lived of these stages lasts for about 5×10^6 yr, evolving at a nearly constant luminosity dominated by residual hydrogen burning at the base of the envelope. The other 2 stages last only for 3×10^5 yr and 5×10^3 yr, respectively. More massive remnants would evolve even more rapidly than this example. If one takes the longest-lived phase for the $0.3 M_{\odot}$ core and uses the sdB space density of Downes (1986; $2 \times 10^{-6} \text{ pc}^{-3}$), the resulting birthrate is $dn/dt \sim 2 \times 10^{-12} \text{ yr}^{-1} \text{ pc}^{-3}$, or about the same as the birthrate of all white dwarfs. This is too high if all EHB stars evolve directly to the white dwarf cooling sequence, since the vast majority of white dwarfs are believed to evolve directly from the central stars of planetary nebulae.

This hypothesis predicts that there would be a spectrum of remnant core masses, depending on the initial mass of the primary, mass ratio, and orbital separation in the way described by Iben & Tutukov (1986b). This picture is inconsistent with the narrow width of the EHB distribution in surface gravity. At $T_{\text{eff}} = 25,000 \text{ K}$, the $0.3 M_{\odot}$ core would have $\log g \sim 4.6$,

with the higher mass cores having even lower surface gravity. Only for very low mass cores would the evolutionary tracks overlap the observed EHB surface gravities. Those calculations also predict that the bulk of EHB stars should be radial velocity variables with amplitudes of many tens, or even hundreds, of km s^{-1} . Saffer & Liebert (1991) have made multiple measurements of radial velocities of nearly 50 of the stars in the sample presented here. Only 5 of these stars have radial velocities measured at different epochs which differ by more than 3 standard deviations ($\sigma \sim 15\text{--}30 \text{ km s}^{-1}$), inconsistent with this close binary hypothesis for the origin of the majority of EHB stars.

In related scenarios, Iben (1990) calculates the evolution of close binary systems leading to mergers of He-He and CO-He degenerate cores. These mergers can result in core helium-burning stars with masses ranging from $0.3 M_{\odot}$ to $0.7 M_{\odot}$. Such a dispersion in core mass would result in a smearing of the distribution in the theoretical H-R diagram in the direction parallel to the HeMS, inconsistent with the observed distribution. However, as the somewhat hotter sdO stars exhibit a considerably larger range of surface gravities than the EHB stars analyzed here (see, e.g., Hunger et al. 1981, Schönberner & Drilling 1984, and Groth et al. 1985), it is possible that some fraction of those stars might have evolved from both types of mergers, especially given the hydrogen-poor composition expected from prior common envelope evolution.

The distribution of photospheric helium abundances also helps to test the various origin hypotheses. In Figure 4.3a, the derived photospheric helium abundances for all program objects are plotted against effective temperature. Michaud et al. (1989) have made detailed non-LTE calculations of radiative acceleration on helium in model atmospheres appropriate to sdOB stars. For $N(\text{He})/N(\text{H}) = 0.1$, the radiative accelerations always are at least 10 times smaller than the gravitational acceleration in the line-forming region, so that helium should indeed diffuse below the photosphere and become underabundant. However, even at the abundances derived here, $N(\text{He})/N(\text{H}) \sim 0.01$, the radiative acceleration remains less than the gravitational acceleration, and diffusion should lead to abundances even lower than are observed. The observed abundances thus cannot be explained by diffusion, and other processes must be invoked. Mass loss is one attractive possibility; Michaud et al. (1985) have shown that the equilibrium abundances of heavy elements in sdOB stars can be modified significantly in the presence of a mass loss rate of only $2 \times 10^{-15} M_{\odot} \text{ yr}^{-1}$. Such calculations for helium have not yet been reported. Moreover, such low mass loss rates are completely undetectable observationally. The explanation for the observed helium underabundances in classical sdB and sdOB stars thus remains a completely open question.

TABLE 4.1
COMPARISONS WITH PREVIOUS DETERMINATIONS

Object	$T_3(K)$ (This Work)	$T_3(K)$ (Other)	$\log g$ (This Work)	$\log g$ (Other)	Reference
HD 4539	27.0	25.0 26.0 24.8	5.46	5.4 5.4	1 6 9
Feige 11	28.4	28.8 26.9	5.63	5.5	6 9
PG 0342+026	26.2	21.8	5.67	5.0	10
Feige 36	29.6	29.2			6
Feige 38	29.8	30.2			6
Feige 65	26.5	26.2	5.60	5.3	8
Feige 66	33.4	36.0 35.0 35.7	6.20	6.0	5 6 7
Feige 75	25.6	25.3			6
Ton 209	29.6	29.3			6
Ton 245	25.2	24.0			6
HD 149382	34.2	40.0 35.0	5.89	5.8 5.5	2 4
UV 1758+364	32.1	32.5 31.8	5.91	5.25 5.6	3 8
Feige 108	34.5	35.0			6

<u>Reference</u>	<u>Primary Temperature Diagnostic</u>
1) Baschek et al. 1972	Line wing shapes
2) Baschek & Norris 1975	Line wing shapes
3) Giddings & Dworetzky 1978	UV continuum, line wing shapes
4) Baschek et al. 1982a	H and He equivalent widths
5) Baschek et al. 1982b	Ionization equilibria
6) Bergeron et al. 1984	Stromgren colors
7) Wesemael et al. 1985	UV continuum
8) Lamontagne et al. 1985	Composite UV and optical continuum
9) Heber & Langhans 1986	UV continuum, Stromgren colors
10) Lamontagne et al. 1987	UV continuum

Figure 4.1. Derived effective temperatures and surface gravities for 92 sdB candidates (open symbols). As indicated in the legend, four ranges of helium abundance $y \equiv N(\text{He})/N(\text{H})$ are distinguished by different symbols.

Figure 4.2. Differential number distribution with effective temperature of 68 confirmed EHB stars.

Figure 4.3a. Atmospheric helium abundance plotted against effective temperature. A trend to higher helium abundance with higher effective temperature is evident.

Figure 4.3b. Atmospheric helium abundance plotted against surface gravity.

Figure 4.4. Derived effective temperatures and surface gravities, as in Figure 4.1, for stars having atmospheric helium number fractions exceeding 0.02.

Figure 4.5a. Comparison of effective temperature estimates of selected stars previously analysed in the literature with estimates derived in this work from detailed line profile analysis.

Figure 4.5b. Comparison of surface gravity estimates of selected stars previously analysed in the literature with estimates derived in this work from detailed line profile analysis.

Figure 4.6a. Internal comparison of effective temperatures derived using spectroscopy and colors.

Figure 4.6b. Internal comparison of surface gravities derived using spectroscopy and colors.

Figure 4.7a. Comparison of derived effective temperatures for 13 objects in common with the sample of Moehler et al. (1990a,b).

Figure 4.7b. Comparison of derived surface gravities for 13 objects in common with the sample of Moehler et al. (1990a,b).

Figure 4.8a. Comparison of derived surface gravities when color temperature and spectroscopic temperature estimates agree.

Figure 4.8b. Comparison of derived surface gravities when the color temperature estimate is significantly lower than the spectroscopic estimate.

Figure 4.8c. Comparison of derived surface gravities when the color temperature estimate is significantly higher than the spectroscopic estimate.

Figure 4.9. For 68 confirmed sdB stars having surface gravities in excess of $\log g = 5.0$, $\log(g\theta^4)$ is constant, as first proposed by Greenstein and Sargent (1974).

Figure 4.10. Number distribution of $\log(g\theta^4)$ for 68 confirmed sdB stars. The width of the distribution implies that the underlying spectrum of helium-burning core masses is unresolved.

CHAPTER 5

EHB DISTANCES AND POPULATION SCALE HEIGHT

5.1 Distance Determinations

Having determined the effective temperature and surface gravity of each confirmed EHB star, an interpolation of the grid of synthetic spectra yields the theoretical flux at the surface of the star. By comparing this with the flux observed at the earth, the angular diameter of the star is determined. Then if the mass of the star is known or can be estimated, the radius can be determined from the surface gravity, and the distance to the star can be found.

To convert the measured photometric magnitudes of the program objects into absolute fluxes, it first is necessary to define the average observed flux through any given filter for a star which has $m_\lambda = 0$ in that filter:

$$f_{0,x} = \frac{\int_S f(\lambda) S(\lambda) d\lambda}{\int_S S(\lambda) d\lambda} \quad (5.1)$$

where $f(\lambda)$ is the flux received at earth and x is Strömgren y , Johnson V , Greenstein multichannel v , or the photographic B magnitude B_{PG} given in the Palomar Green Survey when none of the first three magnitudes are available. The y and V filter functions were taken from Olson (1974) and Matthews & Sandage (1963), respectively, and the B_{PG} filter function was

constructed by convolving the transmission function of a Schott GG 13 UV blocking filter (new designation GG 385) with the long-wavelength sensitivity cutoff of the Kodak IIa-O spectroscopic plate emulsion. Figure 5.1 shows the various transmission functions. Johnson V is obtained from Greenstein multichannel v by a zero-point shift: $V = v + 0.05$. For the remainder of this discussion, all references to stellar fluxes and photometric magnitudes will be taken to mean the filter-weighted averages as defined above in equation (5.1).

For any given star in the sample, the observed magnitude in any filter is given by

$$m_x = -2.5 \log(f_x) + C_x, \quad (5.2)$$

while for a star with magnitude zero in the same filter,

$$m_{0,x} = 0 = -2.5 \log(f_{0,x}) + C_x. \quad (5.3)$$

Subtracting these two equations gives

$$m_x = -2.5 \log \left(\frac{f_x}{f_{0,x}} \right), \quad (5.4)$$

from which the observed flux at earth f_x is found if $f_{0,x}$ is known for each filter. The latter fluxes are calculated from the standard Kurucz model for Vega, scaled to the flux received at earth with the calibration of Hayes and Latham (1975):

$$f_{0,x} = f_{Vega,x} \times 10^{[m_{Vega,x}/2.5]}, \quad (5.5)$$

where again the fluxes and magnitudes are filter averages. The foregoing calibrations yield

$$\begin{aligned}
 f_{0,y} &= 3.606 \times 10^{-9} \\
 f_{0,V} &= 3.603 \times 10^{-9} \\
 f_{0,v} &= 3.603 \times 10^{-9} \\
 f_{0,B} &= 6.268 \times 10^{-9},
 \end{aligned}
 \tag{5.6}$$

where B is B_{PG} and the same calibration is used for Johnson V and Greenstein multichannel v after the conversion $V = v + 0.05$. The fluxes $f_{0,x}$ are given in units of $\text{erg cm}^{-2} \text{ sec}^{-1}$.

The stellar angular diameter is given by

$$\left(\frac{R}{d} \right)^2 = \frac{1}{4\pi} \frac{f_x}{f_{th,x}},
 \tag{5.7}$$

where R is the stellar radius, d its distance, f_x is given by equation (5.4), and $f_{th,x}$ is the theoretical flux at the stellar surface. The factor 4π relates the theoretical stellar fluxes to the Eddington fluxes of the grid of model spectra. The observed stellar magnitudes m_x were taken for most objects as Strömgren y from the photometry of Fontaine et al. (1987), and for the majority of the remaining objects as Johnson V (or converted Greenstein multichannel v) from the literature. For a handful of stars discovered in the PG survey and without published photoelectric photometry the value of B_{PG} cited in the PG catalog was used. A 3-dimensional grid of theoretical

fluxes $f_{th,x}$ was calculated from the grid of synthetic spectra used to analyze the stellar line profiles, in the same manner and with the same transmission functions as were $f_{0,x}$ from the standard Kurucz model. For each object the theoretical flux corresponding to its estimated atmospheric parameters was obtained by cubic spline interpolation on the flux grid.

The stellar radii were obtained from the estimated surface gravities by assuming a stellar mass of $0.5 M_{\odot}$. As shown previously, the distribution of EHB stars in the HR diagram along the theoretical zero-age extended horizontal branch sequences (see Figures 4.1 and 4.10) implies a mean mass of some $0.5 M_{\odot}$ with a very small dispersion. Then

$$R = R_{\odot} \left(\frac{M}{M_{\odot}} \frac{g_{\odot}}{g} \right)^{\frac{1}{2}}. \quad (5.8)$$

Combining equations (5.4), (5.6), (5.7), and (5.8) gives for the stellar distance

$$d = \left(\frac{4\pi G M f_{th,x}}{f_{0,x} g} \right)^{\frac{1}{2}} \exp[0.451(x - A_x)], \quad (5.9)$$

where G is the universal gravitational constant and with the interstellar extinction A_x having been taken into account. For their sample of 20 northern sdB stars, Moehler, Heber, and de Boer (1990, hereafter MHdB) found almost no extinction ($\langle A_V \rangle = 0.063$), and this term has been ignored in the distance calculations for the sample analyzed here.

Errors in the derived distances were calculated in the usual manner:

$$\sigma_d^2 = \sum_i \left(\frac{\partial d}{\partial a_i} \right)^2 \sigma^2(a_i), \quad (5.10)$$

with the a_i representing the parameters for which error estimates are possible, namely, M , $f_{th,x}$, g , and x . The 1σ error in the assumed stellar mass was taken as $0.1 M_{\odot}$, larger than the value of $0.05 M_{\odot}$ cited previously under the assumption that luminosity evolution contributes to the observed dispersion in the observed EHB distribution. If all the stars on the EHB are on the zero-age EHB, a 1σ dispersion in mass of $0.1 M_{\odot}$ can be tolerated, and the latter conservative estimate is adopted here for the purpose of estimating the distance errors. The individual measurement errors in the derived effective temperatures and surface gravities from the model fits were propagated into the calculation of the theoretical flux errors, and observed magnitude errors were taken from the various literature sources. Those magnitude errors generally were of the order of 0.05 mag or less except for the handful of photographic B_{PG} magnitudes taken from the PG catalog, these having standard errors of 0.29 mag. A comparison of the relative contribution to the total distance error by these 4 sources reveals that for the majority of confirmed EHB stars, the distance error induced by errors in observed photometric magnitudes exceeds by a factor of two that induced by the error in the assumed stellar mass, and these dominate by an order of magnitude the error induced by errors in theoretical fluxes and derived surface gravities. Reducing the adopted error in the assumed stellar masses to $0.05 M_{\odot}$ has little effect on the total estimated distance errors.

Having determined the stellar distances, the perpendicular distances z of the EHB stars from the Galactic plane are found from their Galactic latitudes:

$$z = d \sin(b). \quad (5.11)$$

In all subsequent discussions, any reference to the height z from the plane shall be taken to mean the absolute perpendicular distance $|z|$.

Table 4 gives for 68 confirmed EHB stars in the sample the adopted stellar magnitudes m_x and their 1σ errors, the filter x in which the stellar magnitudes were measured, the Galactic longitudes and latitudes l^{II} and b^{II} , and the derived heliocentric distances and perpendicular distances z from the Galactic plane and the 1σ errors in pc.

5.2 Scale Height Determination

If the sample of confirmed EHB stars analyzed here were complete to some reasonably faint limiting magnitude, the population scale height could be determined directly from the derived distances from the Galactic plane. As shown in Figure 12, this sample is seriously incomplete, especially at the fainter magnitudes. However, it is possible to use the observed sample to infer how many true EHB stars exist in the PG compilation in each of several PG spectral classes that contribute to the sample of EHB stars analyzed here.

In Table 1, the “*PG Sp*” column gives the spectral class of each object as cited in the PG catalog. There are 4 such classes that contribute to the sample of 68 confirmed EHB stars: 1) sd, sdB, and sdB-O, lumped together to form a “classical sdB” class, 2) sdOA, the “helium-strong” but hydrogen-dominated subdwarf class, 3) DA2 and DA3, objects misclassified due to the similarity of the appearance of the Balmer absorption lines in hot white dwarfs to those seen in sdB stars, and 4) BHB, objects classified as blue horizontal branch but which upon analysis prove to have surface gravities in excess of $\log g = 5.0$. In each of these classes, a certain percentage of the stars analyzed in this work prove to have subdwarf-like surface gravities, with the remainder having surface gravities outside the range $5.0 < \log g \lesssim 6.0$.

In the “classical sdB” class, about 80% of the stars observed here are confirmed to be EHB stars, with the remainder having surface gravities characteristic of BHB stars, main sequence B stars, or even B giants. The latter might well be in post-AGB or post-HB phases of evolution. In the BHB and sdOA classes, about 20% and 50%, respectively, of the stars observed and analyzed here are confirmed to be EHB stars, the remainder again having gravities like BHB or main sequence B stars. Finally, in a complete (to B_{PG} magnitude 15.6) sample of stars originally classified as DA white dwarfs (Bergeron et al. 1991a,b), 9 stars are found instead to be EHB stars. In what follows, the goal is to use these statistics to infer what

percentage of all the stars in the complete PG sample in each of these classes should be true EHB stars, and to use the fact that the EHB stars reside on a constant-luminosity track in the HR diagram to derive a mapping from photographic B_{PG} magnitude to distance from the Galactic plane. The resulting completeness-corrected sample of all (inferred) true EHB stars then is suitable for the determination of the population scale height.

5.2.1 Relation between B_{PG} and z

It will first be shown that there exists a mapping between the photographic B_{PG} magnitude and the distance z from the Galactic plane. On the surface this would appear to ignore the dependence of z on the value of the Galactic latitude b in equation (5.11). However, the PG survey is restricted to values of b exceeding $\pm 30^\circ$, so that $z \geq 0.5d$ in all cases. Further, since $\sin(b)$ is approaching a maximum as $b \rightarrow 90^\circ$, in most cases $z \sim d$ to within 20 to 40 percent. Another implicit simplifying assumption is that the condition of constant luminosity transforms to constant B_{PG} magnitude in the range $25,000 K < T_{\text{eff}} < 40,000 K$ for the EHB stars. This certainly is not the case, for the bolometric corrections for main sequence O and B stars with such temperatures differ by approximately one magnitude in the relevant range of T_{eff} (Allen 1973). The validity of this assumption will be addressed in the estimation of the distance errors resulting from the adoption of the z vs. B_{PG} relation.

Figure 5.2 shows $\log(|z|)$ plotted against m_x , along with the best-fit weighted linear least-squares regression

$$\log(|z|) = (0.215 \pm 0.005)m_x - (0.295 \pm 0.070). \quad (5.12)$$

The unweighted average of the individual internal error bars in Figure 5.2 is $\langle \Delta \log(|z|) \rangle = 0.07$, while the rms dispersion of the data points about the regression line is $\langle y - y_{fit} \rangle_{rms} = 0.13$, with $y = \log(|z|)$. The reduced goodness of fit measure is $\chi^2/\nu = 4.2$, with $\nu = 66$ for the number of degrees of freedom. Both the value of χ^2/ν and the value of the rms dispersion compared to the mean internal error reflect the additional dispersion induced in the relation by the variation in the Galactic latitude and the assumption that B_{PG} is constant for stars with constant luminosity in the sdB temperature range, as expected. Hence, for purposes of error propagation in subsequent analyses, the mean error in distances from the plane will conservatively be taken to be $\sigma_{\log(|z|)} = 0.15$ whenever equation (5.12) is used to estimate z from m_x . In practice, for the complete sample of PG contributor spectral classes, m_x will be taken to be the value of B_{PG} cited in the PG catalog. Since for hot subdwarfs with effective temperatures of about 30,000 K , the color $(B - V) \sim -0.1$, the error in the distance from the Galactic plane is dominated by the variation in Galactic latitude and B_{PG} itself, and equation (5.12) approximates $\log(|z|)$ equally well for any choice of the photometric filter functions shown in Figure 5.1.

5.2.2 *PG Misclassification and Incompleteness Corrections*

Prior to estimating the percentage of all stars in each contributing PG spectral class in the catalog that actually belong to the EHB, each class must be corrected for three categories of incompleteness. Stars fail to be detected in the survey due to 1) errors in the measured PG $(U - B)$ color, 2) B_{PG} magnitude errors, and 3) accidental losses due to plate defects, field crowding, etc.

To correct for color errors, the distribution of errors in $(U - B)$ is assumed to be gaussian with a 1σ deviation of 0.38 mag, as given in the PG Catalog. The compilation of hot subdwarf stars of Kilkenny, Heber, and Drilling (1987) lists Johnson $(U - B)$ colors for 21 subdwarf B stars; the mean color is -1.04 mag with a standard error of the mean of 0.1 mag. The color threshold for inclusion in the PG catalog is $(U - B) < -0.46$ mag, so that the threshold lies $+1.42\sigma$ from the mean of the much narrower sdB color distribution. The percentage loss due to color errors thus is just the probability $P(z > 1.42)$ for the standard normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$, found in standard tables of mathematical functions to be 7.8%, with the correction to the observed counts positive.

The correction for magnitude measurement errors assumes a gaussian error distribution with a standard deviation $\sigma_B = 0.29$, as given in the PG catalog. Eddington (1940) gives the correction to the differential counts at

any given limiting magnitude as

$$\Delta n = - \left(\frac{\sigma_B^2}{2} \right) \frac{\partial^2 n}{\partial B^2}. \quad (5.13)$$

Since the sdB stars in the PG catalog have a positive count slope $d(\log n)/dB = +0.32$, more stars are displaced into the brighter magnitude bin than are displaced into the fainter bin by random errors, and a net negative correction of -2.3% results.

The correction for accidental losses was calculated in the PG catalog from an independent list of white dwarfs and was found to be $+5.3\%$. This correction should be independent of subsample color and distribution of apparent B_{PG} magnitudes. Combining the three corrections gives a total correction for the number of stars in each of the 4 PG spectral classes contributing to the EHB of $\Delta n/n = +10.8\%$ at any given limiting magnitude. Having made these incompleteness corrections, the remaining task is to determine what percentage of the stars in each PG spectral class contribute to the EHB, after which the relation between B_{PG} magnitude and distance z from the Galactic plane will be used to assign each star a value of z for the scale height estimate.

The percentage of true EHB stars in each incompleteness-corrected PG spectral class is estimated from the subsample of stars in each class observed and analyzed here. There are two possible approaches, depending

on whether the differential or cumulative counts are analyzed. It first will be shown in what follows that effects involving the choice of bin size of the differential distributions induce large uncertainties in the derived population scale height. In fact, it is possible to obtain any scale height desired, from 150 to 300 pc, depending on the choice of bin size and the particular choice of misclassification correction (hereafter MC, for brevity). Then the analysis of the cumulative counts will be discussed, in which the question of binning never arises, and it will be shown that a unique solution is found.

5.2.3 Differential Count Analysis

In the analysis of the differential counts, it is necessary to adopt a binning scheme. Not only must the size of each bin be chosen, but it also must be determined if all bins may be assigned the same size. In Figure 5.3 the relative (logarithmic) errors in the distance from the Galactic plane for the 68 confirmed EHB stars are plotted against $\log(|z|)$. There is no apparent trend with increasing distance, and the mean value of the relative errors is $\langle \sigma_{\log(|z|)} \rangle = 0.07$. If the variance about the regression line in Figure 5.2 is taken into account, the mean value increases to 0.15. For either value, the size of the error in the distance from the plane is proportional to the distance, and if it is desired that Poisson counting statistics characterize the errors in the differential counts, the ratio of the distance errors to the size of the bins must be constant from bin to bin. Equivalently, the bin sizes

must be proportional to the distance assigned to the bin center. If the bin boundaries are chosen uniformly in the observed photometric magnitude, the corresponding distance bin boundaries increase exponentially with increasing magnitude, and the desired properties are obtained. Specifically, assume a relation of the form of equation (5.12):

$$z = 10^{a+bm_x}, \quad (5.14)$$

and let $m_x = m_i$, with $m_i = m_0 + i\Delta m$. Then

$$\begin{aligned} z_{i+1} &= 10^{a+b[m_0+(i+1)\Delta m]} \\ &= 10^{a+bm_0+b\Delta m} 10^{i\Delta m}, \\ z_i &= 10^{a+b[m_0+i\Delta m]} \\ &= 10^{a+bm_0} 10^{i\Delta m}. \end{aligned} \quad (5.15)$$

The relative size of the bins with respect to their midpoints then is

$$\begin{aligned} \frac{\Delta z_i}{z_{i,mid}} &= \frac{z_{i+1} - z_i}{(z_{i+1} + z_i)/2} \\ &= \frac{2(10^{a+bm_0+b\Delta m} - 10^{a+bm_0})}{10^{a+bm_0+b\Delta m} + 10^{a+bm_0}}, \end{aligned} \quad (5.16)$$

and this is a constant for any particular choice of magnitude bin size, with the common factors in which the index i appears having cancelled.

Once the bin sizes are chosen, three categories are defined for each PG spectral class: 1) all stars of that class listed in the PG catalog, 2) the

subsample of stars of that class observed and analyzed here, and 3) the stars in category 2 that prove to belong to the EHB based on their estimated surface gravities. Then in each category for each spectral class, the stars are binned into differential distributions, and for each spectral class the number of true EHB stars inferred in each distance bin is given by the number in category 1 multiplied by the ratio of the number of stars in categories 3 and 2, respectively. The estimated counts in each spectral class then are summed to obtain the total number of EHB stars inferred to exist among all stars detected in the PG survey in the spectral classes of interest. The resulting distribution is fit with a barometric spatial density law to estimate the population scale height.

If the bin sizes were uniform in the distance from the plane, the number of stars in any bin would be given by

$$n(z)dz = n_0 \exp\left(-\frac{z}{z_0}\right)dV \propto z^2 \exp\left(-\frac{z}{z_0}\right)dz, \quad (5.17)$$

or for the logarithmic counts in the i th bin of a histogram,

$$\ln n_i - 2 \ln z_i = -\frac{z_i}{z_0} + C, \quad (5.18)$$

again provided that dz were constant for all bins. But for the exponentially increasing bin sizes adopted here, $dz \propto z$, as shown in equation (5.16), thus the differential distribution of equation (5.17) gains a factor of z in the

volume element, and the correct description of the logarithmic differential counts is

$$\ln n_i - 3 \ln z_i = -\frac{z_i}{z_0} + C. \quad (5.19)$$

The scale height is given by the negative of the inverse of the slope of the linear regression to the data of equation (5.19). The fit is weighted by

$$w_i = \frac{1}{\sigma_i^2}; \quad \sigma_i = \frac{\Delta n_i}{n_i}. \quad (5.20)$$

The variances of the n_i do *not* obey Poisson statistics, since those counts are obtained by scaling down the counts in the distributions of all PG stars in each spectral class by a factor determined from the number of stars actually observed and analyzed here. The uncertainties in the inferred EHB counts are given solely by the sampling error of the number of stars in each bin that were observed and analyzed here in each spectral class:

$$\begin{aligned} \frac{\Delta n_{EHB}(z_i, Sp)}{n_{EHB}(z_i, Sp)} &= \frac{\Delta n_{OBS}(z_i, Sp)}{n_{OBS}(z_i, Sp)} \\ &= \frac{1}{\sqrt{n_{OBS}(z_i, Sp)}}, \end{aligned} \quad (5.21)$$

where the latter equality obtains from the Poisson statistics of the observed subsamples, and Sp denotes the spectral class. Alternatively, the MC may be made by assuming that the ratio of confirmed EHB stars to stars observed and analyzed here in any spectral class is independent of distance. In this case, the ratio is computed from the cumulative counts at some limiting

magnitude. The limiting magnitude for the last bin's upper boundary was fixed at $B_{PG} \leq 15.4$, the faintest magnitude for which there were stars observed and analyzed here in all of the spectral classes contributing to the EHB. Otherwise, the MC would have had to have been extrapolated to fainter magnitudes.

The distance from the plane corresponding to $B_{PG} = 15.4$ is $z_{\text{lim}} = 985$ pc from equation (5.12), and if the population scale height were too large, the ability to distinguish between various values for the scale height could be lost, since for very large scale heights, the stars accessible in the PG catalog all would reside within a single scale height, and the number density distribution would be very similar to a uniform density distribution. However, as will be shown below, the limiting magnitude adopted here probes to between 3 and 6 scale heights from the plane.

Figure 5.4 shows the differential counts for a particular choice of magnitude bins: the first bin is centered on $B_{PG} = 10.0$, and all bins have a width $\Delta B_{PG} = 1.0$ mag. The bin boundaries were transformed from magnitudes to perpendicular distance z via equation (5.12), and the uncertainties in the differential counts incorporate the sampling error of equation (5.21), with the MC having been determined bin by bin. The effect of varying the central distance of the first bin, the bin widths, and the choice of MC was explored in a systematic manner. Beginning with the

bin choice of Figure 5.4, and determining the MC bin by bin, a weighted linear regression was made to the logarithmic differential counts of equation (5.19). The scale height z_0 is given by the negative inverse of the slope of the regression, with a standard error derived from the covariance matrix of the regression. The magnitude bin sizes then were increased by 0.2 mag, the new differential counts were fitted, etc., until the bin size became so large that it only was possible to define two bins without exceeding the magnitude limit of $B_{PG} = 15.4$. Three data points are required in order to preserve one degree of freedom in the regression. Then the central magnitude of the first bin was increased by 0.5 mag, the bin size was reset to 1.0 mag, and the process was iterated until it no longer was possible to increase the first bin's central magnitude. The choice of determining the MC then was changed to the second, cumulative method, and the process was repeated.

Figures 5.5a (differential MC) and 5.5b (cumulative MC) show representative examples of the regressions and resulting estimates of the scale height z_0 . The variances of the individual data points appear to have been slightly overestimated, and if the rms dispersions about the regression lines had been used to estimate the uncertainties in the derived scale heights, those uncertainties would have been reduced somewhat. While individual determinations of z_0 have relatively small uncertainties, the scatter among those determinations is large. Figure 5.6 shows for all combinations

of beginning central magnitude, bin width, and MC the distribution of derived scale heights z_0 and their standard uncertainties plotted against the uncertainties. There is an obvious correlation; larger derived scale heights have larger uncertainties. This is a consequence of the inverse relation between the slope of the regression and the derived scale height. The correlation thus should not be interpreted causally, and the choice of σ_{z_0} for the horizontal axis was made only for convenience of display.

A Monte Carlo simulation was used to investigate the effect on the derived scale heights of distance errors due to propagation of the magnitude errors in the PG survey. Taking the published standard magnitude error from the PG catalog, the calculations resulting in the scale height estimates of Figure 5.6 were repeated, but for each choice of beginning magnitude, bin size, and MC, 100 trials were performed in which the PG B magnitudes were perturbed with a gaussian error distribution with a standard deviation of $\sigma_{B_{PG}} = 0.29$. Each resulting distribution of estimated scale heights gave an unweighted mean and rms dispersion for comparison with the individual scale height and error estimates of Figure 5.6. Figure 5.7 shows the distribution for the binning and MC choices of Figure 5.4. It is evident that the contribution of distance errors to variations in the derived scale heights is comparable to those of the counting statistics themselves. Therefore the two error sources were summed in quadrature to form the final error estimates for each derived

scale height for any given choice of binning and MC. Figure 5.8 shows the resulting distribution; it differs qualitatively from the distribution of Figure 5.6 only in its larger errors.

The results of the differential count analysis are unsatisfactory because of their indefiniteness. It is possible to obtain any desired scale height from 150 to 300 pc by a particular choice of bin size and MC. While the range spans only a factor of two in the scale height, adopting one end of the range over the other leads to very different conclusions about the age of the population and the nature of the progenitor population. Thus, an analysis of the cumulative count distribution was undertaken in an attempt to obtain a more definitive estimate of the EHB scale height.

5.2.4 Cumulative Count Analysis

Assuming the barometric number density distribution of equation (5.17), the cumulative counts to a given limiting magnitude and distance z from the plane by equation (5.12) are given by

$$\begin{aligned} N(< z) &= C \int_0^z t^2 \exp\left(-\frac{t}{z_0}\right) dt \\ &= D \left\{ 2 - \exp\left(-\frac{z}{z_0}\right) \left[\left(\frac{z}{z_0}\right)^2 + 2\frac{z}{z_0} + 2 \right] \right\}. \end{aligned} \tag{5.22}$$

To compare this to the inferred number of EHB stars in the PG catalog, the incompleteness and misclassification corrections in each PG spectral class

contributing to the EHB were accomplished in much the same manner as in the analysis of the differential counts, except that the corrections were applied cumulatively, that is, the MC was applied at each value of z in the cumulative number distributions. Figure 5.9 illustrates the correction for the “classical sdB” class. The upper curve shows the cumulative counts for all the stars in the PG catalog (N_{PG}) in this class with $B_{PG} < 15.4$. The two lower curves show the counts for the subsample of stars observed and analyzed here (N_{OBS}), and those proving to belong to the EHB (N_{EHB}). The fourth curve shows the misclassification-corrected counts (N_{PGEHB}) based on the ratio $R = N_{EHB}/N_{OBS}$.

After the corrected distributions for each spectral class were summed, the final cumulative distribution was normalized to the number of stars at $z_{lim} = 985$ pc. Then, a family of theoretical distributions of the form of equation (5.22) was calculated for various values of the scale height z_0 . Figure 5.10 shows the fractional cumulative distribution of all inferred PG EHB stars with $z < z_{lim}$, and the theoretical distributions for values of z_0 from 100 to 800 pc at 100 pc intervals. For the larger values of z_0 , it is clear that the sensitivity to variations in z_0 is being lost as the density distribution approaches that of a uniform distribution, but for values of z_0 near 300 pc, there remains sufficient sensitivity to distinguish variations of the order of 100 pc.

A quantitative measure to assess the likelihood that any particular choice of z_0 best describes the data set is provided by the one-sided Kolmogorov-Smirnov (KS) test. While the total number of stars in Figure 5.10 with $z < z_{\text{lim}}$ is 351, it is incorrect to use this as the number of objects N in the probability calculation in the KS test. Since only 82 objects with $B_{PG} \leq 15.4$ were observed and analyzed here, the counting statistics can be no better than Poisson statistics with $N = 82$. Using this for the number of objects in the KS test, a grid of theoretical distributions with z_0 ranging from 100 to 800 pc in steps of 10 pc was tested against the inferred EHB distribution of Figure 5.10. The result is shown in Figure 5.11, where the probability that the theoretical distribution with $z_0 = z$ best describes the data is plotted against z . The curve has been normalized so that the integral under the curve is unity. A parabola was fitted to seven points at the peak of the curve, and the value of z at the maximum was adopted as the most likely value for z_0 .

The 1σ uncertainty in the adopted value of z_0 was established by requiring that the integrals under the curve from $z = 0$ to $z = \sigma^-$ and from $z = \sigma^+$ to $z = \infty$ be equal to the equivalent integrals of the standard normal distribution with $\mu = 0$ and $\sigma = 1$, those integrals having a value of 0.159 at either end. The values for σ^- and σ^+ in the distribution of Figure 5.11 are unequal due to the asymmetry of the distribution. The final adopted

values for the EHB scale height and its errors are $z_0 = 285^{+120}_{-35}$ pc. The determination is satisfactory in the sense that there is a unique value, and there is no uncertainty introduced by the binning processes forced upon the analysis of the differential counts.

Again, a Monte Carlo simulation was used to investigate the effect of B_{PG} magnitude errors in the derived scale height. The error in equation (5.12), relating B_{PG} and z , has previously been adopted as $\Delta \log(|z|) = 0.15$. One thousand trials were performed in which each distance z associated with the data points of Figure 5.10 was perturbed by a gaussian random distribution with a standard deviation $\sigma_z = 0.15z$. Figures 5.12 and 5.13 show representative examples of the effect on the cumulative number and probability distributions of Figures 5.10 and 5.11, respectively. Each of the peaks of the 1000 perturbed probability distributions was fitted with a parabola to determine the most probable value of z_0 , and the resulting number distribution of derived scale heights is shown in Figure 5.14. The rms dispersion of the distribution is only 14 pc, thus the error in the scale height derived from the cumulative counts of Figure 5.10 is dominated completely by the width of the probability distribution in Figure 5.11.

The derived scale height of $z_0 = 285$ pc is similar to that found for white dwarfs, planetary nebulae, and main sequence K stars in the solar neighborhood (cf. Boyle 1989, Mihalas & Binney 1981), and suggests that

the progenitors of the field EHB stars are disk turnoff stars. As has been shown previously, the narrow range of inferred masses for the EHB stars also is consistent with the identification of disk stars as the progenitors of the EHB. Other determinations of the field EHB scale height are similar, with a moderate dispersion in the adopted values. MHdB found $z_0 = 250$ pc, while Heber (1986) found $z_0 = 190\text{--}220$ pc. The value and 1σ limits adopted for z_0 here would rule out the latter values at a high confidence level. On the other hand, Green, Schmidt and Liebert (1988) derived a scale height of 325 pc by adopting the space density in the plane derived by Downes (1986) and matching the space densities of their high-latitude PG sample to the local density. That value is in good agreement with the one adopted here. All of these values for z_0 support the hypothesis that the field EHB stars belong to the older component of the thin disk population (cf. Gilmore and Wyse 1985).

TABLE 5.1
PG CONFIRMED EHB DISTANCES AND HEIGHTS

Object	m_x (mag)	σ_m (mag)	x	l^{II} (deg)	b^{II} (deg)	d (pc)	σ_d (pc)	z (pc)	σ_z (pc)
0044+097	10.31	.03	y	121.3	-52.8	171	33	136	26
0101+039	12.09	.02	y	129.1	-58.4	336	50	286	43
0111+177	15.85	.29	B_{PG}	130.3	-44.5	2667	359	1869	252
0112+142	16.26	.05	v	131.5	-47.9	2365	621	1714	450
0212+230	15.36	.29	B_{PG}	146.6	-35.6	1361	308	792	179
0221+217	15.87	.29	B_{PG}	149.7	-36.0	2091	204	1229	120
0242+132	13.24	.01	y	160.8	-40.8	614	82	401	53
0250+189	14.11	.02	y	158.8	-35.0	640	114	367	65
0319+054	15.04	.05	V	176.5	-41.0	1420	135	931	89
0322+114	15.26	.29	B_{PG}	171.9	-36.1	1434	291	844	171
0342+026	10.96	.05	y	184.4	-38.4	176	44	109	27
0349+094	15.69	.29	B_{PG}	179.2	-32.8	1627	317	881	172
0749+658	12.14	.07	y	150.4	+30.9	334	51	171	26
0806+516	15.21	.05	V	167.2	+32.9	957	163	520	89
0816+313	15.64	.29	B_{PG}	191.2	+31.4	1573	370	819	193
0823+465	14.53	.03	y	173.5	+35.2	948	88	546	50
0839+399	14.39	.05	y	182.1	+37.7	881	127	539	78
0856+121	13.50	.03	y	216.4	+33.6	534	145	295	80
0909+275	12.25	.01	y	199.3	+41.8	282	30	188	20
0910+621	15.54	.29	B_{PG}	152.9	+40.2	2134	448	1377	289
0918+029	13.42	.08	y	229.4	+34.2	437	65	246	37
0919+272	12.77	.01	y	200.4	+43.8	350	35	242	24
0920+296	14.80	.05	y	197.2	+44.4	710	75	497	52
0933+383	15.47	.03	y	185.0	+48.1	1708	214	1271	159
0940+171	16.13	.29	B_{PG}	216.0	+45.4	2347	1215	1671	865
0941+280	12.46	.29	B_{PG}	200.7	+48.6	515	99	386	74
0947+639	14.78	.29	B_{PG}	148.4	+43.4	1550	128	1065	88
1012+007	14.90	.03	y	241.6	+44.0	920	139	639	97
1101+249	12.79	.05	y	212.7	+65.8	389	34	355	31
1114+072	13.06	.05	y	250.4	+59.8	448	45	387	39
1154-070	14.37	.05	y	280.0	+53.1	1011	229	808	183
1230+052	13.29	.02	y	291.0	+67.4	526	85	485	78
1232-136	13.34	.03	y	296.9	+48.7	570	46	428	34
1233+426	12.04	.01	y	133.7	+74.4	318	36	306	35
1234+253	10.56	.09	y	245.1	+86.2	101	24	101	24

TABLE 5.1 – Continued
PG CONFIRMED EHB DISTANCES AND HEIGHTS

Object	m_x (mag)	σ_m (mag)	x	l^{II} (deg)	b^{II} (deg)	d (pc)	σ_d (pc)	z (pc)	σ_z (pc)
1236+479	15.52	.03	y	128.8	+69.3	1931	190	1806	177
1255+546	13.53	.03	y	120.9	+62.6	623	98	553	87
1256+278	14.25	.05	y	47.4	+88.1	777	105	776	105
1303-114	13.96	.05	y	308.7	+51.0	604	69	470	54
1313+132	14.57	.01	y	326.1	+74.7	1230	294	1186	283
1325+101	14.02	.01	y	331.1	+70.7	570	103	538	97
1343-101	13.76	.05	y	324.2	+50.1	723	107	554	82
1432+158	13.90	.01	y	12.8	+63.3	640	95	572	85
1433+239	12.57	.01	y	30.5	+66.3	469	84	429	77
1438-029	13.82	.29	B_{PG}	348.2	+49.8	1000	334	764	255
1442+342	15.13	.05	y	55.7	+64.6	1182	133	1068	120
1538+269	13.87	.04	y	42.4	+52.5	995	111	789	88
1559+533	14.39	.05	y	83.0	+46.5	817	112	592	81
1613+425	14.33	.05	y	67.1	+46.2	769	79	555	57
1619+522	13.30	.01	y	80.4	+43.9	446	39	309	27
1623+386	16.07	.05	v	61.4	+44.2	1656	415	1128	283
1631-039	8.95	.04	y	11.8	+27.8	70	7	32	3
1636+216	15.08	.05	V	39.8	+38.4	1361	387	845	240
1643+209	15.81	.05	v	39.6	+36.6	1980	266	1153	155
1708+409	15.22	.05	v	65.4	+35.9	1985	393	1137	225
1710+490	12.90	.01	y	75.4	+36.0	453	35	266	20
1716+426	13.97	.01	y	67.6	+34.6	928	94	527	53
1758+364	11.37	.05	y	62.6	+25.4	197	29	84	12
2059+013	15.05	.06	y	50.7	-27.9	1235	223	578	104
2110+127	12.93	.01	y	62.8	-23.6	831	136	332	54
2128+096	14.74	.05	y	63.0	-28.9	1149	118	555	57
2128+112	15.46	.05	v	64.6	-27.9	1955	438	894	200
2204+034	14.24	.03	y	64.3	-39.8	686	105	439	67
2218+051	15.30	.29	B_{PG}	69.1	-41.2	1183	127	779	84
2313-021	12.98	.03	y	76.8	-55.9	395	50	327	41
2317+046	12.87	.02	y	84.8	-51.0	508	199	394	155
2345+318	14.18	.01	y	107.6	-28.9	787	103	380	50
2349+001	13.28	.01	y	93.1	-58.9	510	149	437	127

Figure 5.1. Filter transmission functions used to calculate $f_{0,x}$ and $f_{th,x}$ as described in the text.

Figure 5.2. Best-fit weighted linear least-squares regression of $\log(|z|)$ against m_x .

Figure 5.3. Relative (logarithmic) errors in the derived distances from the Galactic plane for 68 confirmed EHB stars.

Figure 5.4. Inferred differential counts of PG EHB stars. The bins were transformed from magnitude bins beginning with $B_{PG} = 10.0$ and having a width of 1.0 mag, and the misclassification correction was made differentially (bin by bin).

Figure 5.5a. Representative weighted linear regressions to the logarithmic differential counts for different binning choices. The misclassification corrections were made differentially (bin by bin).

Figure 5.5b. Representative weighted linear regressions to the logarithmic differential counts for different binning choices. The misclassification corrections were made cumulatively, as described in the text.

Figure 5.6. Distribution of derived scale heights for all choices of binning and misclassification correction (triangles – differential MC, squares – cumulative MC).

Figure 5.7. Variation in the derived scale height for the binning of Figure 5.4 in 100 trials in which the B_{PG} magnitudes were perturbed by a gaussian error distribution with a standard deviation $\sigma_{B_{PG}} = 0.29 \text{ mag}$.

Figure 5.8. Distribution of the perturbed scale heights for all choices of binning and misclassification correction as in Figure 5.6, but with errors due to counting statistics and distance errors having been combined in quadrature.

Figure 5.9. Illustration of the misclassification correction procedure for the “classical sdB” spectral class. See the text for details.

Figure 5.10. Fractional cumulative count distribution for all inferred PG EHB stars (solid curve). Theoretical distributions $f_{\text{cum}}(<z)$ for values of the scale height z_0 from 100 to 800 pc at 100 pc intervals are overplotted for comparison (dotted curves).

Figure 5.11. Probability distribution $P(z_0 = z)$ as determined from one-sided KS tests of the theoretical cumulative distributions of Figure 5.10 against the inferred counts of PG EHB stars. The adopted value for the scale height is $z_0 = 285^{(+120)}_{(-35)} \text{ pc}$.

Figure 5.12. Representative perturbed cumulative counts, illustrating the effects of assuming 15% errors in the transformed distances from the plane.

Figure 5.13. Representative perturbed probability distributions, illustrating the effects of assuming 15% errors in the transformed distances from the plane.

Figure 5.14. Distribution of derived scale heights from 1000 z -perturbed probability distributions. The rms dispersion of the distribution is only 14 pc.

CHAPTER 6

KINEMATICS OF HOT SUBDWARF STARS

6.1 Solar Motion Solutions

The motion of the Sun relative to the centroid of velocities of a population of stars in the solar neighborhood may be determined from an analysis of the population's radial velocities, proper motions, or both. For the hot subdwarfs, the radial velocities determined from the intermediate- and high-resolution spectroscopy described in Chapters 2 and 3 and listed in Tables 5 and 6 will be used. For the sdB stars, the velocities determined from the sample observed here were measured from the $H\alpha$ profile in those hydrogen-dominated stars, while for the sdO stars the He II $\lambda 4686$ profile defining the class was used. These were supplemented with velocities found in the literature (Slettebak & Brundage 1971; Graham & Slettebak 1973; Greenstein & Sargent 1974; Baschek & Norris 1975; Drilling & Heber 1987, 1989).

Figures 6.1 and 6.2 show the distribution on the celestial sphere of the sdB and sdO samples used in the kinematic analysis. Stars with radial velocities listed here in Tables 5 and 6 are represented by filled circles, while those found in the literature are represented by filled triangles. Clearly, the sdB sample is poorly distributed on the sky, with scarcely any stars populating the zone between $b = +30^\circ$ and $b = -30^\circ$. There also is a pronounced deficit

of stars in the southern Galactic hemisphere. The poor coverage is a result of constructing the sdB sample largely from the PG Catalog, which is a high-galactic latitude survey that avoids the Galactic plane, and which in addition seldom ventures below the declination $\delta = 0^\circ$ line shown in the figures. The sdB sample is perfectly adequate for the purpose of determining the scale height of the population, as was done in Chapter 5, but for a kinematic analysis it is very inadequate. In retrospect, it would have been better to expand the sample to include stars in the plane, for example from the KPD catalog of Downes (1986). Future work is planned to remedy this deficiency.

The dearth of sdB stars in the Galactic plane results in a poor sampling of the u and v space motions, and for v this is further compounded by the gaping hole in the vicinity of $l^{II} = 270^\circ$ and extending far into the southern hemisphere. Only the w space motion is well-sampled, but since that component is a well-known tracer of the age of the stellar population, it may be that some useful information can be gleaned in spite of the poor overall sky coverage. A search of the literature nearly doubled the number of sdB stars having measured radial velocities, but unfortunately none of these were in the plane, and only a handful were in the southern hemisphere. The sdO sample is much more well distributed on the sky, thanks in large part to the measurements of southern sdO stars of Drilling & Heber (1987, 1989).

If radial velocities alone are used in the kinematic analysis, the distances to the stars need not be known. Conventionally, the analysis is performed in the equatorial rectangular coordinate system, since the observational quantities of interest, especially the proper motions, are always given in this system. However, for the analysis of the radial velocities this is not imperative, and since the object of the analysis is to determine the peculiar motion and velocity ellipsoid of the population in the Galactocentric rectangular coordinate system, the relevant equations will be developed in that system.

The coordinate system is centered on the Sun, but does not move with the Sun. Rather, the system moves with the velocity centroid of the hot subdwarf populations, so that the Sun itself has a non-zero velocity in this system. The positive x-axis points toward the Galactic center, the positive y-axis points in the direction of the rotation of the Sun about the Galactic center, and the positive z-axis points toward the north Galactic pole (Figure 6.3). For the i th star in the population, the position vector is given by

$$\mathbf{r}_i = d_i \mathbf{g}_i, \quad \text{where} \quad \mathbf{g}_i = \begin{pmatrix} \cos(l_i) \cos(b_i) \\ \sin(l_i) \cos(b_i) \\ \sin(b_i) \end{pmatrix}, \quad (6.1)$$

d_i is the stellar distance, and l_i and b_i are the Galactic longitude and latitude (Figure 6.3). The peculiar motions of the star and the Sun in the coordinate system are $\mathbf{V}_i = (u_i, v_i, w_i)$ and $\mathbf{V}_\odot = (u_\odot, v_\odot, w_\odot)$,

respectively. The radial velocity ρ_i of the star then is related to the peculiar motions $\mathbf{V}_i$ and $\mathbf{V}_\odot$ by

$$\rho_i = (\mathbf{V}_i - \mathbf{V}_\odot) \cdot \mathbf{g}_i. \quad (6.2)$$

Since the solar motion is determined with respect to the velocity centroid of the stellar population, the observed stellar space velocities must average to zero, i.e., $\langle u \rangle = \langle v \rangle = \langle w \rangle = 0$, where for example $\langle u \rangle = (\sum_{i=1}^N u_i)/N$ by definition. Anticipating the summation over all stars in the population in subsequent steps, it is evident that the coefficients g_{ij} in equation (6.2) are different from star to star, so that the average $\langle u \rangle$ is not the same as the direction cosine-weighted average $(\sum_{i=1}^N u_i g_{i1})/N$. However, for a sufficiently large sample that is uniformly distributed on the sky, these averages of the space velocities may be assumed to vanish. The validity of this assumption will be explored subsequently for the samples of sdB and sdO stars analyzed here, and its effect on the uncertainty in the derived solar motion will be estimated.

Ignoring the terms V_{ij} on the left side of equation (6.2), the equation reduces for each star in the sample to

$$u_\odot g_{i1} + v_\odot g_{i2} + w_\odot g_{i3} = \rho_i. \quad (6.3)$$

This is a system of N equations in three unknowns, and the optimal solution is given by the method of least squares. Minimizing the squares of the

residuals δ_i weighted by the errors σ_i in the measured radial velocities,

$$\delta_i^2 = \frac{(g_{i1}u_{\odot} + g_{i2}v_{\odot} + g_{i3}w_{\odot} - \rho_i)^2}{\sigma_i^2}, \quad (6.4)$$

leads to a linear system of equations in the unknown components of the solar motion: $\mathbf{A} \mathbf{V}_{\odot} = \mathbf{b}$, with

$$A_{jk} = \sum_{i=1}^N \frac{g_{ij}g_{ik}}{\sigma_i^2} \quad \text{and} \quad b_k = - \sum_{i=1}^N \frac{\rho_i g_{ik}}{\sigma_i^2}. \quad (6.5)$$

The solutions for the sdB and sdO stars are given by $\mathbf{V}_{\odot} = \mathbf{A}^{-1}\mathbf{b}$:

$$\begin{aligned} u_{\odot} &= -3.1 \pm 3.3 \text{ km s}^{-1}, \\ v_{\odot} &= 5.0 \pm 4.1 \text{ km s}^{-1}, \quad (\text{sdB}) \end{aligned} \quad (6.6)$$

$$\begin{aligned} w_{\odot} &= -4.7 \pm 2.7 \text{ km s}^{-1}, \\ u_{\odot} &= -5.9 \pm 0.8 \text{ km s}^{-1}, \\ v_{\odot} &= 35.6 \pm 0.7 \text{ km s}^{-1}, \quad (\text{sdO}) \\ w_{\odot} &= 5.8 \pm 0.8 \text{ km s}^{-1}. \end{aligned} \quad (6.7)$$

The 1σ errors are derived from the covariance matrices of the least squares fits and reflect only the uncertainties in the measured radial velocities at this point. The low errors for the sdO solar motion solution reflect the considerably lower errors of the sdO radial velocities (see Tables 5 and 6).

The poor sampling of the sdB population is evident in the derived solar motions. The values do not resemble at all the characteristic values of

any population of disk or halo stars. The values for $u_{\odot}$ and $v_{\odot}$ are more similar to those for extremely young disk and spiral arm objects like main sequence O and B stars, while the value for $w_{\odot}$ has the wrong sign altogether for disk stars. However, the better sampling of the w velocities compared to u and v still could be useful in the analysis of the velocity dispersions to come. The solar motion with respect to the sdO population is much more easily interpreted. It resembles most closely the solar motion with respect to planetary nebulae and white dwarfs, and it is consistent with the notion that these stars belong to an older disk population.

The validity of the previous assumption, that the sdO sample is sufficiently well-distributed so that the direction cosine-weighted averages of the space motions vanish, remains to be examined. Indeed, it will be shown next that the sampling errors due to the small number of stars in both the sdB and sdO samples dominate the errors in the derived solar motions. If the space motions $\mathbf{V}_i$ themselves could be calculated from knowledge of the radial velocities, distances, and proper motions, the assumption that the space motion averages vanish could be tested directly. In fact, the solar motion itself could be calculated directly. Without that knowledge, it is necessary to resort to simulations to test the assumption. Two different methods now are described, one using the sampling of the sdB and sdO stars of Figures 6.1 and 6.2, and the second using randomly chosen samples on the sky.

The sdO solar motion solution suggests that the population belongs to the older part of the thin disk. The velocity dispersions for such disk stars have been found to lie in the range 15–50 km s⁻¹, approximately, depending on the space motion component. Hence, a Monte Carlo simulation was used in which velocities were selected from a triaxial velocity ellipsoid, with means $\mu = 0$ and standard deviations $\sigma_{u,v,w}$ in the ratios 45:35:20, these being the ratios of velocity dispersions found for the planetary nebulae. At each position on the celestial sphere of the stars in the sdB and sdO samples (Figures 6.1 and 6.2), three velocities were selected to form simulated u, v , and w distributions. The observed radial velocities then were analyzed in a manner much like that described previously, but without discarding the terms averaging the space motions weighted by the direction cosines. The terms were included explicitly in the b_k of equation (6.5) for each trial:

$$b_k = - \sum_{i=1}^N \frac{\rho_i g_{ik}}{\sigma_i^2} \rightarrow - \sum_{i=1}^N \frac{(\rho_i - \mathbf{V}_i \cdot \mathbf{g}_i) g_{ik}}{\sigma_i^2}. \quad (6.8)$$

The solar motion then was calculated as before, and the experiment was performed repeatedly.

In Figure 6.4, the results for 1001 trials are shown for the 49 sdB stars of Figure 6.1. The means of the distributions are $(\langle u_\odot \rangle, \langle v_\odot \rangle, \langle w_\odot \rangle)_{sdB} = (-3.1, +4.6, -4.6)$ km s⁻¹, and comparing with the values given in equations (6.6), it appears that assuming that the direction cosine-weighted space motion averages of equation vanish does in fact lead to unbiased estimates for

the solar motion. However, the mean square dispersions are quite large, $(\langle u_{\odot}^2 \rangle^{1/2}, \langle w_{\odot}^2 \rangle^{1/2}, \langle w_{\odot}^2 \rangle^{1/2})_{sdB} = (13.4, 19.1, 8.0) \text{ km s}^{-1}$. The distributions in Figure 6.5 for the sdO sample behave similarly: $(\langle u_{\odot} \rangle, \langle v_{\odot} \rangle, \langle w_{\odot} \rangle)_{sdO} = (-6.5, +36.0, +5.6) \text{ km s}^{-1}$, $(\langle u_{\odot}^2 \rangle^{1/2}, \langle w_{\odot}^2 \rangle^{1/2}, \langle w_{\odot}^2 \rangle^{1/2})_{sdO} = (16.5, 12.5, 10.2) \text{ km s}^{-1}$ (compare with equations (6.7)). For the sdO solar motion, the sampling error is an order of magnitude larger than the internal errors of the solution due only to the radial velocity errors.

A second set of simulations was performed to investigate the dependence of the the sampling error on the sample size. In these trials, a sample of stars was uniformly distributed in solid angle on the celestial sphere. At each position, the velocity ellipsoid used in the previous simulation was used to generate the random space motions. A value for the solar motion of $(u_{\odot}, v_{\odot}, w_{\odot}) = (-30, 0, +30) \text{ km s}^{-1}$ was arbitrarily chosen, and the radial velocity at each position was generated via equation (6.2). Radial velocity errors were chosen at each position from a uniform distribution on the interval $[5, 20] \text{ km s}^{-1}$, representative of the errors in the sdB and sdO samples of Figures 6.1 and 6.2. The average space motions were assumed to vanish, and the radial velocity distribution was analysed as in equations (6.3–6.5) to recover the assumed solar motion. The experiment was repeated for total numbers of longitude–latitude pairs $N_{l,b}$ ranging from 100 to 1100 on the celestial sphere. The results are shown in Figure 6.6, where it seen that while

the dispersions in the recovered solar motions diminish with increasing sample size, as expected, the errors in the derived solar motions remain dominated by the sampling error until the sample size $N_{l,b}$ exceeds some 400–500 stars.

These results suggest that unless the sample size is substantially larger than those that have been analyzed to date in the literature, it is a waste of time to obtain extremely high signal-to-noise or high resolution spectra in order to derive precise radial velocities, since sampling errors will dominate the solar motion errors, even with poorly determined velocities. The simulations provide the means to predict the precision of the measured radial velocities that will be required to achieve a given uncertainty in the solar motion solution for a given sample size.

6.2 Velocity Ellipsoids

With the solar motion with respect to the two subdwarf populations determined, equation (6.2) may be rewritten as

$$\rho_i + \mathbf{V}_\odot \cdot \mathbf{g}_i = \mathbf{V}_i \cdot \mathbf{g}_i. \quad (6.9)$$

Squaring both sides, and ignoring cross-terms of the form $u_i v_i g_{i1} g_{i2}$, etc., on the grounds that these terms will vanish upon subsequent summation over the stars in the samples (again to be verified by simulation), then for each star the square of equation (6.9) reduces to

$$(\rho_i + \mathbf{V}_\odot \cdot \mathbf{g}_i)^2 = \sum_{j=1}^3 (V_{ij} g_{ij})^2. \quad (6.10)$$

It now is possible to form a system of three equations by multiplying through by g_{ik}^2 and summing over all stars, yielding for the k th equation

$$\sum_{i=1}^N (C_i g_{ik})^2 = \sum_{i=1}^N \sum_{j=1}^3 (V_{ij})^2 (g_{ij} g_{ik})^2, \quad (6.11)$$

where $C_i = \rho_i + \mathbf{V}_\odot \cdot \mathbf{g}_i$.

The j th term in the sum on the right side of equation (6.11) is a direction cosine-weighted sum of a square of a velocity distribution component W_j :

$$\sum_{i=1}^N (W_{ij})^2 (g_{ij} g_{ik})^2. \quad (6.12)$$

Now *define* the direction cosine-weighted average

$$\langle W^2 \rangle_{jk} \equiv \frac{\sum_{i=1}^N (W_{ij})^2 (g_{ij} g_{ik})^2}{\sum_{i=1}^N (g_{ij} g_{ik})^2}, \quad (6.13)$$

whereupon the term in expression (6.12) becomes

$$\langle W^2 \rangle_{jk} \sum_{i=1}^N (g_{ij} g_{ik})^2. \quad (6.14)$$

Rewriting the right hand sides of equations (6.11) in this manner yields a system of equations in the space velocity dispersion estimators $\langle V^2 \rangle_{jk}$.

It will be shown by Monte Carlo simulation that for $j = 1$, these direction cosine-weighted averages converge to the space velocity dispersion $\langle u^2 \rangle^{1/2}$, and likewise for $\langle v^2 \rangle^{1/2}$ (for $j = 2$), and $\langle w^2 \rangle^{1/2}$ (for $j = 3$),

provided that the samples of stars are sufficiently large and uniformly distributed on the celestial sphere. Then equations (6.11) reduce to a linear system in the velocity dispersions, weighted again by the errors in the radial velocities: $\mathbf{A}'\mathbf{X}' = \mathbf{b}'$, with

$$A'_{jk} = \sum_{i=1}^N \frac{(g_{ij}g_{ik})^2}{\sigma_i^2}, \quad X'_k = \langle V_k^2 \rangle, \quad \text{and} \quad b'_k = \sum_{i=1}^N \frac{(C_i g_{ik})^2}{\sigma_i^2}. \quad (6.15)$$

The solutions for the sdB and sdO stars are given by $\mathbf{X}' = \mathbf{A}'^{-1}\mathbf{b}'$:

$$\begin{aligned} \langle u^2 \rangle^{1/2} &= 32.7 \pm 5.0 \text{ km s}^{-1}, \\ \langle v^2 \rangle^{1/2} &= 29.1 \pm 6.1 \text{ km s}^{-1}, \quad (\text{sdB}) \end{aligned} \quad (6.16)$$

$$\begin{aligned} \langle w^2 \rangle^{1/2} &= 44.2 \pm 3.7 \text{ km s}^{-1}, \\ \langle u^2 \rangle^{1/2} &= 44.9 \pm 1.3 \text{ km s}^{-1}, \\ \langle v^2 \rangle^{1/2} &= 38.8 \pm 0.9 \text{ km s}^{-1}, \quad (\text{sdO}) \\ \langle w^2 \rangle^{1/2} &= 31.9 \pm 1.0 \text{ km s}^{-1}. \end{aligned} \quad (6.17)$$

The 1σ errors are due only to the contribution of the radial velocity errors.

The sdB velocity dispersions again reflect the poor sampling on the sky, with the ratios $\langle u^2 \rangle^{1/2} : \langle v^2 \rangle^{1/2} : \langle w^2 \rangle^{1/2}$ resembling no known disk or halo population. In this case, the value of $\langle w^2 \rangle^{1/2}$ is larger than either of the other two dispersions, and for all known Galactic disk stellar populations the ratio $\langle u^2 \rangle^{1/2} : \langle w^2 \rangle^{1/2}$ is about 2:1. The value of $\langle w^2 \rangle^{1/2}$ is almost certainly overestimated, while $\langle u^2 \rangle^{1/2}$ and $\langle v^2 \rangle^{1/2}$ are underestimated for

the sdB sample of Figure 6.1, because of the lack of stars in the sample in the Galactic plane. However, $\langle w^2 \rangle^{1/2}$ at least provides an upper bound on the velocity dispersion perpendicular to the plane, although it is of little practical use. For comparison, spheroidal component metal poor F, G, and K subdwarfs have $\langle w^2 \rangle^{1/2} \sim 50 \text{ km s}^{-1}$. Thus, the sdB sample proves to be completely useless for inferring anything about the population's age or probable progenitor population. The sdO velocity dispersions are very similar again to those found for planetary nebulae and white dwarfs, and the conclusions from this and from the solar motion solution are consistent: the sdO population in the solar neighborhood appears to belong to the older part of the thin disk.

To demonstrate that the direction cosine-weighted averages of equation (6.13) are unbiased estimators of the space motion velocity dispersions, a series of simulations using the velocity ellipsoid of the previous simulations were performed. At the positions of the stars in the sdB and sdO samples of Figures 6.1 and 6.2, simulated u , v , and w distributions were generated. A solar motion identically equal to zero was assumed, and radial velocities were generated via equation (6.2). The radial velocity errors were generated from the same representative error distribution used previously. The radial velocities then were analyzed as in equations (6.10–6.15), and the experiment was repeated. For sample sizes of up to 1100 stars, the results are shown

in Figures 6.7 and 6.8. The direction cosine-weighted averages $\langle W^2 \rangle_{jk}$ prove to be unbiased estimators of the space motion velocity dispersions, but with variations about the mean of the order of the dispersions of the distributions of solar motion estimators. Thus, the sampling errors again exceed the errors induced by the radial velocity errors by an order of magnitude.

The sampling errors decrease for increasing sample sizes (Figure 6.9), where the ratios of the velocity dispersion estimates to the standard deviations of the parent space motion distributions are shown. The percentage errors in the derived dispersions again are dominated by the sampling errors for sample sizes smaller than $N_{l,b} = 400$ –500 stars, and the time spent obtaining high resolution and high signal-to-noise spectra for the purpose of measuring very precise radial velocities would have been much more efficiently spent obtaining many spectra of lower quality. Again the simulations provide a quantitative measure of the precision in the radial velocities required to achieve a certain precision in the derived velocity dispersions for a sample of a given size.

TABLE 6.1
PG SDB CANDIDATE RADIAL VELOCITIES

Object	<i>PG Sp</i>	UT Date (d/m/y)	$V_r(\text{H}\alpha)$	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
0101+039 Feige 11	sdB-O	02/01/89	-85.4	18.9			variable
		22/08/89	+89.1	15.7			
		04/12/89	+32.3	22.2			
		06/12/89	-106.7	15.0			
		09/01/90	+83.2	17.6			
0112+142	sdB	02/01/89	-163.6	28.8			variable
		22/08/89	-85.4	18.8			
		23/08/89	-149.4	17.9			
		06/12/89	-14.2	19.6			
		09/01/90	-142.0	17.2			
0154+204	sdB-O	02/01/89	+13.0	30.9			
0154+182	sdB-O	02/01/89	+86.5	25.8			
0212+230	sdB-O	31/12/88	-23.6	50.0	-78.2	15.4	
		01/01/89	-79.5	35.5			
		23/08/89	-85.1	18.2			
0250+189	sdB	31/12/88	-19.0	28.5	+4.4	11.4	
		31/12/88	-9.9	35.2			
		01/01/89	-8.9	30.7			
		24/08/89	+16.1	14.7			
0322+114	sdB	31/12/88	+25.3	19.0	+28.5	9.3	
		01/01/89	+25.8	17.9			
		06/12/89	+31.6	13.2			
0342+026	sdB-O	31/12/88	+21.3	10.6	+12.6	7.0	
		01/01/89	+6.0	19.7			
		02/01/89	+1.0	15.0			
		23/08/89	+10.7	14.6			
0349+094	sdB	31/12/88	+47.1	21.3	+30.4	10.8	
		01/01/89	+24.1	18.3			
		04/12/89	+24.9	17.3			

TABLE 6.1 - Continued
PG SDB CANDIDATE RADIAL VELOCITIES

Object	<i>PG Sp</i>	UT Date (d/m/y)	$V_r(\text{H}\alpha)$	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
0749+658	sdB	31/12/88	-43.9	25.6	-27.3	6.9	
		01/01/89	-36.9	23.6			
		02/01/89	-48.2	21.5			
		30/04/89	+6.9	35.7			
		30/04/89	-9.8	17.6			
		22/08/89	-9.3	22.7			
		23/08/89	-49.8	15.9			
		04/12/89	-8.2	22.0			
		06/12/89	-21.9	17.3			
0806+516	sdB-O	31/12/88	-46.5	50.4	+5.6	14.4	
		01/01/89	-31.7	33.6			
		30/04/89	+37.0	30.2			
		04/12/89	+13.4	20.3			
0816+313 Ton 313	sdB-O	31/12/88	-56.4	23.2	-52.7	14.9	
		01/01/89	-69.6	26.6			
		06/12/89	-28.0	28.4			
0823+465	sdB	31/12/88	+7.9	19.9	+23.7	9.9	
		01/01/89	+16.0	23.4			
		04/12/89	+32.9	13.1			
0839+399 K345-30	sdB	31/12/88	+62.6	39.6	-9.3	13.2	
		01/01/89	-35.2	38.9			
		06/12/89	-15.8	15.0			
0856+121	sdB	04/12/89	+91.0	16.0	+97.0	10.2	
		06/12/89	+101.1	13.2			
0918+029	sdB-O	01/03/89	+113.8	25.0	+79.4	16.3	
		06/12/89	+53.9	21.5			
0919+272 Ton 13	sdB-O	01/03/89	-14.9	41.2	-33.2	16.9	
		02/03/89	+26.2	41.4			
		06/12/89	-52.9	20.8			
1012+007	sdB-O	30/04/89	-112.9	24.7			variable
		04/12/89	-69.6	19.1			
		06/12/89	+63.6	20.2			

TABLE 6.1 - Continued
PG SDB CANDIDATE RADIAL VELOCITIES

Object	<i>PG Sp</i>	UT Date (d/m/y)	$V_r(\text{H}\alpha)$	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
1101+249 Feige 36	sdB-O	01/03/89 02/03/89 30/04/89 06/12/89	-30.4 +117.9 -123.7 -33.2	16.5 23.6 23.6 14.4			variable
1114+072 Feige 38	sdB-O	01/01/89 01/03/89 02/03/89	+15.3 +2.6 +0.0	20.5 13.0 12.8	+3.6	8.3	
1154-070A	sdB-O	01/03/89 04/12/89 06/12/89	-4.8 -36.0 +42.2	23.8 20.5 25.1	-4.7	13.2	
1223+058	sdB-O	31/12/88	+131.4	21.5			low g
1224+671	sdB-O	01/01/89 01/03/89 04/12/89 06/12/89	-57.7 -34.5 -32.1 -44.9	18.3 14.9 13.7 11.5	-41.1	7.0	low g
1230+052	sdB	31/12/88 01/01/89 01/03/89	-24.6 -90.5 -75.5	19.2 26.2 21.6	-57.1	12.6	
1233+426 Feige 65	sdB-O	01/03/89 30/04/89 01/05/89	+55.5 +67.1 +72.8	17.4 27.5 32.6	+61.2	13.4	
1303-114	sdB-O	30/04/89 01/05/89 01/05/89	-44.4 -9.9 +32.7	27.2 21.4 22.6	-3.2	13.5	
1323-085	sdB-O	01/03/89 01/05/89 01/05/89 04/12/89	-50.8 -37.6 -52.2 -65.3	22.2 17.5 12.8 13.9	-53.2	7.8	very low g
1325+101	sdB-O	01/03/89 01/05/89	+7.5 +14.8	33.2 30.0	+11.5	22.3	

TABLE 6.1 - Continued
PG SDB CANDIDATE RADIAL VELOCITIES

Object	PG Sp	UT Date (d/m/y)	$V_r(\text{H}\alpha)$	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
1343-101	sdB	01/03/89	+48.2	30.0	+48.9	17.4	
		01/03/89	+71.7	35.8			
		01/05/89	+37.0	26.5			
		04/12/89	-112.0	21.0			
1432+158	sdB-O	01/03/89	+110.4	22.4			probable variable
		01/05/89	+30.7	19.4			
1442+342	sdB-O	01/03/89	-19.0	5.2			
1623+386 KUV	sd	30/04/89	-90.9	41.7			
1643+209	sdB-O	30/04/89	+75.8	32.1	+52.3	12.8	
		01/05/89	+30.0	19.1			
		23/08/89	+68.5	20.5			
1704+221	sdB-O	06/07/89	-65.1	17.0	-62.1	9.2	very low g
		23/08/89	-64.7	13.9			
		24/08/89	-54.7	17.8			
1708+409	sdB-O	23/08/89	-200.8	18.2	-197.4	13.1	
		24/08/89	-193.7	18.8			
1716+426	sdB-O	06/07/89	-98.2	24.4			variable
		23/08/89	+81.1	22.6			
		24/08/89	+59.6	19.6			
2059+013	sdB	22/08/89	+16.3	26.8	-7.6	14.3	
		23/08/89	-1.9	37.2			
		24/08/89	-40.0	33.0			
		06/12/89	-11.9	23.4			
2110+127	sdB	22/08/89	+3.1	10.4	+6.8	5.2	
		23/08/89	+8.6	12.2			
		24/08/89	-1.7	11.7			
		04/12/89	+13.1	8.7			
2128+096	sdOA	07/09/87	+20.7	26.9	+23.0	19.4	
		08/09/87	+26.7	35.7			
		08/12/87	+23.5	45.2			

TABLE 6.1 - Continued
PG SDB CANDIDATE RADIAL VELOCITIES

Object	PG Sp	UT Date (d/m/y)	$V_r(\text{H}\alpha)$	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
2135+044	sdB	24/08/89	-21.8	15.9	-39.0	7.3	low g
		24/08/89	-14.8	18.5			
		04/12/89	-56.1	13.0			
		06/12/89	-45.3	13.0			
2204+034	sdB-O	23/08/89	+0.0	22.5	-10.1	13.7	
		24/08/89	+1.7	23.2			
		04/12/89	-38.1	25.8			
2214+183	sd	22/08/89	+5.7	15.3	-17.6	8.1	low g
		06/12/89	-16.3	15.5			
		06/12/89	-33.2	17.1			
		09/01/90	-32.9	17.1			
2229+099	sd	23/08/89	-23.2	15.6	-19.3	9.9	low g
		24/08/89	+0.5	17.8			
		04/12/89	-35.2	18.5			
2301+259	sdB	23/08/89	-125.4	16.8	-130.5	9.4	low g
		24/08/89	-140.9	16.3			
		04/12/89	-125.2	15.9			
2345+318	sdB-O	23/08/89	-121.0	15.4			variable
		24/08/89	-108.9	17.2			
		04/12/89	+114.2	14.4			
2356+166	sdB-O	24/08/89	-2.4	13.6	-7.4	8.1	low g
		04/12/89	-18.0	14.7			
		06/12/89	-3.2	14.0			

TABLE 6.2
PG & KPD SDO RADIAL VELOCITIES

Object	<i>PG Sp</i>	UT Date (d/m/y)	V_r ($\lambda 4686$)	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
0005+511 KPD	sdO	13/05/87 08/12/87 01/01/88	+35.2 +40.0 +33.0	6.2 3.4 14.0	+38.4	2.9	emission
0038+198	sdOC	07/09/87 08/09/87 08/12/87 03/01/88	-54.9 -56.5 -58.2 -49.7	11.5 6.5 10.8 16.3	-56.0	4.8	emission
0039+134	sdOB	07/09/87 08/09/87 03/01/88	-29.2 -37.5 -39.5	25.3 14.4 29.6	-36.1	11.5	
0113+259	sdOC	07/09/87 08/09/87	+31.7 +52.0	39.0 24.3	+46.3	20.6	
0208+015 Feige 19	sdOB	07/09/87 08/09/87 02/01/88	+27.5 +28.7 +28.7	20.5 15.0 28.4	+28.3	11.1	
0216+032 PHL 1256	sdOC	07/09/87 08/09/87 02/01/88 03/01/88	-38.0 -37.5 -39.2 -40.2	9.5 4.5 12.8 10.8	-38.0	3.6	emission
0217+155	sdOB	03/01/88	+30.2	51.1			
0240+046	sdOB	07/09/87 08/12/87 02/01/88	+45.5 +90.2 +46.2	27.2 25.1 39.6	+65.4	16.7	
0319+458 KPD	sdOB	08/12/87 02/01/88 03/01/88	-48.5 -35.0 -52.0	22.1 33.2 34.7	-46.0	16.3	
0549+198 KPD	sdOC	08/12/87	-9.0	27.3			
0552+190 KPD	sdOC	08/12/87	-22.0	21.3			

TABLE 6.2 - Continued
PG & KPD SDO RADIAL VELOCITIES

Object	<i>PG Sp</i>	UT Date (d/m/y)	V_r ($\lambda 4686$)	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
0553+179 KPD	sdOC	08/12/87 02/01/88	-1.7 -19.7	27.3 50.8	-5.7	24.0	
0720-000 KPD		07/03/88	+88.7	43.9			
0823+546	sdOC	08/12/87 02/01/88	-128.5 -118.0	24.4 39.6	-125.6	20.0	
0838+132	sdOB	08/12/87 02/01/88 03/01/88	+54.5 +49.7 +49.2	29.7 34.2 38.9	+51.6	19.4	
0921+310 Ton 414	sdOB	18/05/89 18/05/89	-26.6 +1.0	29.0 17.1	-6.1	14.7	CB 7 He I $\lambda 4713$
0934+553 GD 299	sdO	08/12/87 01/01/88 02/01/88 03/01/88	+63.0 +64.2 +71.5 +64.2	17.2 34.9 19.5 28.3	+66.1	11.1	
0949-101	sdOB	07/03/88	+128.7	34.3			
0950+13	PNN	08/12/87	+20.5	10.9			emission
0952+518 GD 300	sdO	08/12/87 01/01/88 02/01/88 03/01/88	+6.0 +11.0 +21.7 +37.2	21.7 41.0 31.7 34.5	+15.8	14.8	
0953+024	sdOC	07/03/88 18/05/89 18/05/89	+27.0 +16.5 +31.7	37.0 14.4 14.7	+24.2	9.9	He I $\lambda 4713$
0958-118	sdOB	03/01/88 07/03/88 18/05/89	+40.7 +31.5 +11.7	46.1 51.1 13.3	+15.0	12.4	
1011+649	sdOB	03/01/88 07/03/88 17/05/89 18/05/89 18/05/89	-80.0 -67.5 -85.8 -71.5 -62.4	35.2 47.1 17.2 18.0 17.8	-78.4	11.4	He I $\lambda 4713$

TABLE 6.2 - Continued
PG & KPD SDO RADIAL VELOCITIES

Object	PG Sp	UT Date (d/m/y)	V_r ($\lambda 4686$)	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
1020+694	sdOB	08/12/87	-156.7	28.0	-168.1	11.3	
		03/01/88	-178.5	42.3			
		17/05/89	-181.6	17.4			
		18/05/89	-156.0	20.1			
1034+001	DO h	13/05/87	+59.7	8.7	+58.7	7.4	emission
		02/01/88	+52.2	20.8			
		03/01/88	+59.5	19.9			
1036+433 Feige 34	sdO	08/12/87	-2.5	10.5	+0.3	7.6	
		01/01/88	+4.2	22.7			
		02/01/88	-1.2	15.5			
		03/01/88	+10.7	20.6			
1038+510	sdOB	07/03/88	+118.9	33.3	+122.0	11.0	
		18/05/89	+99.4	24.0			
		18/05/89	+129.5	13.4			He I $\lambda 4713$
1047-066	sdOB	07/03/88	+288.8	32.6	+302.2	12.9	
		18/05/89	+304.7	14.0			
1102+499	sdOC	02/01/88	-30.2	39.9	-60.9	8.8	single
		07/03/88	+12.5	23.9			doubled
		07/03/88	-111.0	36.3			doubled
		17/05/89	-81.8	12.9			single
		18/05/89	-44.1	12.6			single
1127+018	sdOD	02/01/88	+33.0	32.4	+20.3	6.6	
		03/01/88	+29.2	38.1			
		17/05/89	+20.5	10.9			
		17/05/89	+18.8	8.7			He I $\lambda 4713$
1134+144 EG 81	sdO	07/03/88	+101.7	36.8	+95.7	7.8	
		18/05/89	+81.1	14.5			
		18/05/89	+101.7	9.6			He I $\lambda 4713$
1159-035	1159	02/01/88	+50.7	15.4	+49.1	4.6	emission
		17/05/89	+54.4	8.2			
		17/05/89	+46.1	19.3			C IV $\lambda 4658$
		18/05/89	+45.2	7.2			
		18/05/89	+48.5	12.8			C IV $\lambda 4658$

TABLE 6.2 - Continued
PG & KPD SDO RADIAL VELOCITIES

Object	PG Sp	UT Date (d/m/y)	V_r ($\lambda 4686$)	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
1230+066	sdOB	03/01/88	+25.7	52.1	-12.0	7.8	
		07/03/88	-14.0	36.7			
		17/05/89	-6.6	18.1			
		17/05/89	-14.9	14.7			He I $\lambda 4713$
		18/05/89	-28.7	18.1			
		18/05/89	-4.2	14.7			He I $\lambda 4713$
1239+177 Feige 67	sdO	03/01/88	+25.0	26.7	+23.5	22.4	
		07/03/88	+20.0	41.3			
1317+127 Feige 80	COMP	03/01/88	+52.5	18.9	+23.0	5.8	sdO + A?
		17/05/89	+20.0	7.8			
		18/05/89	+19.6	9.9			
1348+368	sdOB	13/05/87	+57.2	12.3			emission
1544+252 Ton 803	sdOB	13/05/87	-65.2	46.9			
1545+035	sdOB	07/03/88	+32.2	46.7	+21.5	5.6	
		17/05/89	+22.5	7.8			
		18/05/89	+20.0	8.3			
1554+221	sdOB	07/03/88	-24.0	32.2	-22.7	7.0	
		17/05/89	-17.3	14.0			
		18/05/89	-24.5	8.3			
1856+230 KPD		08/09/87	+10.5	25.3	+19.4	6.3	
		16/05/89	+38.1	8.4			
		18/05/89	-8.2	10.5			
1903+256 KPD		07/09/87	+22.0	26.7	+24.3	15.3	
		08/09/87	+7.7	40.7			
		16/05/89	+30.0	20.9			
1931+291 KPD		08/09/87	-22.7	29.0	-24.9	6.0	
		07/03/88	-51.0	48.0			
		18/05/89	-21.7	14.8			
		18/05/89	-25.2	6.8			He I $\lambda 4713$
1952-233 LSE-21		07/09/87	-13.5	12.7	-7.2	9.3	
		08/09/87	+0.2	13.8			

TABLE 6.2 - Continued
PG & KPD SDO RADIAL VELOCITIES

Object	<i>PG Sp</i>	UT Date (d/m/y)	V_r ($\lambda 4686$)	σ_{V_r}	$\langle V_r \rangle$	$\langle \sigma_{V_r} \rangle$	Remark
2055+311 KPD		07/09/87 08/09/87 08/12/87	-15.2 -19.7 -31.7	22.4 17.1 27.5	-20.7	12.2	
2129+150	sdOB	07/09/87 08/09/87 18/05/89 18/05/89	+7.7 +20.5 +5.6 +20.2	22.5 25.8 10.3 12.1	+12.0	7.1	He I $\lambda 4713$
2158+082	sdO	07/09/87 08/09/87 02/01/88 03/01/88	-76.2 -85.7 -79.2 -69.7	32.1 39.0 26.3 45.9	-78.3	16.8	
2213-006 PB 5121	sdOB	07/09/87 08/09/87 18/05/89 18/05/89	-42.0 -59.0 -59.4 -32.8	20.8 18.3 10.1 8.4	-45.1	5.8	He I $\lambda 4713$
2215+150	sdOC	07/09/87 08/09/87 18/05/89 18/05/89	-28.0 -24.5 -19.8 -21.8	39.3 29.2 18.2 9.6	-21.9	8.0	He I $\lambda 4713$
2259+518 KPD		07/09/87 08/09/87 02/01/88	-18.7 -21.5 +0.5	25.4 26.9 17.3	-9.1	12.6	
2321+213	sdOD	07/09/87 08/09/87	-31.5 -62.0	41.8 36.5	-48.1	26.1	
2322+495 KPD		07/09/87 08/09/87 08/12/87	-75.5 -88.5 -79.5	29.1 19.8 20.1	-82.4	12.7	
2352+180	sdOB	07/09/87 08/09/87 08/12/87	-17.0 -13.7 -22.0	16.9 22.1 25.4	-17.1	11.9

Figure 6.1. Distribution of 49 sdB stars on the celestial sphere with measured radial velocities. The zone with $|b| < 30^\circ$ is very sparsely populated, and the sample provides very poor sky coverage.

Figure 6.2. Distribution of 76 sdO stars on the celestial sphere with measured radial velocities. The sample provides significantly better sky coverage than the sdB sample.

Figure 6.3. Galactocentric rectangular coordinate system used in the kinematic analysis.

Figure 6.4. Solar motion solutions with respect to the sdB sample, incorporating simulations of the sampling error. The dispersions of the distributions are 8, 19, and 13 km s⁻¹ from top to bottom.

Figure 6.5. Solar motion solutions with respect to the sdO sample, incorporating simulations of the sampling error. The dispersions of the distributions are 10, 13, and 17 km s⁻¹ from top to bottom.

Figure 6.6. Simulations demonstrating decreases in the fluctuations of the solar motion estimates as the density and uniformity of the sky coverage is increased.

Figure 6.7. Simulated velocity dispersions derived from the sdB sampling. The dispersion estimators prove to be unbiased, but with dispersions of 6, 14, and 11 km s^{-1} from top to bottom.

Figure 6.8. Simulated velocity dispersions derived from the sdO sampling. The dispersion estimators prove to be unbiased, but with dispersions of 8, 9, and 13 km s⁻¹ from top to bottom.

Figure 6.9. Simulations demonstrating decreases in the fluctuations of the velocity dispersion estimates as the density and uniformity of the sky coverage is increased.

CHAPTER 7

SUMMARY

In this dissertation, the results of a program to determine the atmospheric parameters of subdwarf B stars using a detailed analysis of the Balmer and helium line profiles has been presented. The method yields the most precise determinations to date of effective temperatures and surface gravities for the EHB sequence in the ranges $25,000\text{ K} < T_{\text{eff}} < 40,000\text{ K}$ and $5.0 < \log g < 6.0$. The photospheric helium abundances, determined simultaneously with T_{eff} and $\log g$ by fitting model spectra to optical spectrophotometry, range from completely depleted (formally, $N(\text{He})/N(\text{H}) \equiv 0$) to significantly larger than the solar value. The derived parameters T_{eff} and $\log g$ beautifully fit the theoretical sequences of the extended horizontal branch of Caloi (1972, 1989) and Sweigart (1987) and give strong support to the interpretations of the EHB proposed by Greenstein & Sargent (1974) and Heber (1986) for a plausible helium-burning core mass of $\sim 0.5\ M_{\odot}$.

Comparisons both with previous determinations in the literature, especially determinations of T_{eff} , and with alternative methods of estimating T_{eff} and $\log g$ for the sample analyzed spectroscopically here, suggest that the spectroscopic method is far more precise than other methods. Specifically, previous analyses in which the effective temperature is estimated from intermediate-band colors are shown to give rise to rms differences with the

spectroscopic temperature estimates of order 3,000 K . These temperature errors then are shown to induce very large errors of up to a factor of three in the derived surface gravities. It is shown that an analysis which includes line profile fits for the higher members of the Balmer series is imperative if internal inconsistency in the derived atmospheric parameters is to be achieved.

Taking into account estimates of the uncertainties in the derived atmospheric parameters, the width of the EHB sequence in $\log(g\theta^4)$, only 0.4 dex, is consistent with the observational error, so that any underlying core mass distribution appears unresolved and must be assumed to be quite narrow. The position and morphology of the EHB sequence in the theoretical H-R diagram are inconsistent with other proposed evolutionary histories of the EHB stars. Photospheric helium is underabundant by a factor of ten, on average, for the majority of the EHB stars, although at effective temperatures higher than $\sim 30,000\ K$ helium abundances rise. The observed abundances are higher than predicted by detailed diffusion calculations, and other diffusive processes, such as that due to mass loss, must be invoked.

The derived atmospheric parameters, together with an assumption that the mean mass of the field EHB stars is $0.5\ M_{\odot}$, yield estimates of the heliocentric distances of the stars. Combined with the values of Galactic latitude, the perpendicular distances of the stars from the Galactic plane are found. If the sample of confirmed EHB stars were complete to

some reasonably faint limiting magnitude, the scale height of the population could be determined directly. The sample is seriously incomplete, but it is representative, so that the characteristics of the stars in the sample may be used to infer the number of true EHB stars contained in the complete, magnitude-limited Palomar Green catalog. The scale height of the EHB population then is calculated from this larger sample, appropriately corrected for various forms of incompleteness.

The traditional method of binning the stars by the distance from the Galactic plane and analysing the differential counts is shown to be unreliable, in that a large range of derived values of the scale height may be obtained depending on the particular choice of binning scheme and the details of the correction for spectral misclassification. An approach which analyzes the cumulative count distribution is introduced, and it is shown that a unique derived value for the scale height is obtained. Monte Carlo simulations are employed to show that errors in the derived distances to the stars do not affect the value of the scale height significantly, and that the error is dominated by the spectral classification correction. The final adopted value for the field EHB scale height is $z_0 = 286^{+120}_{-35}$ pc, consistent with membership of the population in the older part of the thin disk.

Radial velocities for samples of subluminoous O and B stars are analyzed to obtains solutions for the solar motion and the velocity dispersions

of the samples. The sdB sample is shown to have an insufficiently uniform distribution on the celestial sphere for the analysis to yield useful results. Those results point out what steps need to be taken in future work to remedy the deficiency. The sdO sample proves to be well-distributed on the celestial sphere, and the solar motion and velocity dispersion solutions also indicate that the population belongs to the older part of the thin disk.

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