INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

University Microfilms International
A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
313/761-4700 800/521-0600

Order Number 9220691

Solid matrix Christiansen filters

Milanovic, Zoran, Ph.D.

The University of Arizona, 1992

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

SOLID MATRIX CHRISTIANSEN FILTERS

by

Zoran Milanovic

**A Dissertation Submitted to the Faculty of the
COMMITTEE ON OPTICAL SCIENCES (GRADUATE)
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA**

1 9 9 2

THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

As members of the Final Examination Committee, we certify that we have
read the dissertation prepared by Zoran Milanovic
entitled Solid Matrix Christiansen Filters

and recommend that it be accepted as fulfilling the dissertation
requirement for the Degree of Doctor of Philosophy

H. Angus Macleod
Dr. H. Angus Macleod

February 28, 1992
Date

James J. Burke
Dr. James J. Burke

February 28, 1992
Date

Michael R. Jacobson
Dr. Michael R. Jacobson

February 28, 1992
Date

Date

Date

Final approval and acceptance of this dissertation is contingent upon
the candidate's submission of the final copy of the dissertation to the
Graduate College.

I hereby certify that I have read this dissertation prepared under my
direction and recommend that it be accepted as fulfilling the dissertation
requirement.

H. Angus Macleod
Dissertation Director Dr. H. Angus Macleod

February 28, 1992
Date

STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under the rules of the Library.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his or her judgement the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: Zoran Milanovic

This dissertation is dedicated to my family.

Acknowledgements

I would like to thank my advisor, H. Angus Macleod for his help in my research and studies at OSC. I learned how to view and practice science from him. Michael R. Jacobson deserves my deepest gratitude for guiding me through my research and working alongside me in the fabrication of our filters. He was always available to discuss this project and proofread articles and manuscripts. I would like to thank Professor James J. Burke for serving on my committee and editing this dissertation. I acknowledge Dr. Bart Wilburn for his sponsorship of the Christiansen filter project and thank Dr. Raymond V. Wick and Colonel Larell K. Smith for accepting and advising me in the Air Force Senior Palace Knight Program. In spite of some difficulties at the outset of this program, I still think it is an excellent investment in America's future. I would also like to thank Dr. Charles Stein, Dr. Charles Miglionico and Mr. Robert Roybal of the Air Force Phillips Laboratory for the SEM micrographs.

The number of people I have become acquainted with, worked with, and studied with in my careers at Chicago, UCLA, XonTech, Inc., and Arizona is too enormous to list on one page. They have all influenced my education and development and I would like to thank every one of them.

Finally, I would like to thank my parents, Vukosava and Dragutin, my sister, Olga, and my brother, Dragan, for their patience during my long years in school. The road has been a long one and sometimes I wondered why I chose to take it, but what else would I have done with my life?

Table of Contents

List of Illustrations	8
List of Tables	11
Abstract	12
Section 1: Symbol List	14
Section 2: Introduction	15
Section 3: The Principle of Christiansen Filters	20
Section 4: Theoretical Considerations	27
Single Scattering Theory	28
Mie Theory	37
Clewel's Theory of Light Scattering by Pigment Particles	50
Clarke's Theory for the Christiansen Filter	54
Section 5: Candidate Materials Search	67
Section 6: Index Dispersion Verification	76
Section 7: Plastics and Molding Technology	91
Optical Polymers	99
Plastics Molding Technology and Injection Molding	106
Section 8: Fabrication of Solid Matrix Christiansen Filters	114
Section 9: Optical Characterization and Theoretical Modeling	151
Section 10: Conclusions and Suggestions for Further Study	177
Appendix A: Christiansen Filter Combinations	179

	7
Appendix B: Fabrication Parameters	197
List of References	207

List of Illustrations

Figure 1 The Christiansen filter concept	.21
Figure 2 Existence of a spectral bandpass	.22
Figure 3 Typical optical arrangement to remove scattered light	.23
Figure 4 The coordinate system for light scattering by an arbitrary particle	.31
Figure 5 Spherical polar coordinate system centered on a particle	.40
Figure 6 Forward scattering by a sphere	.49
Figure 7 Types of single scattering	.51
Figure 8 The scattering medium divided into thin, random phase changing screens	.55
Figure 9 The geometry of the multiple scattering medium	.57
Figure 10 The geometry of a ray path through a spherical particle	.61
Figure 11 Dispersion curves of three optical glasses and two optical plastics . . .	.74
Figure 12 Geometry for critical angle refractometry	.77
Figure 13 The principle of the Pulfrich refractometer	.79
Figure 14 Pulfrich refractometer for measuring n as a function of λ	.83
Figure 15 Measured dispersion curves for three brands of SAN compared with data from Lytle	.87
Figure 16 Measured dispersion curves for Styron PS compared with data from Lytle	.88
Figure 17 Measured dispersion curves for Lexan 121 and Lexan 141 compared with data from Lytle and General Electric	.89

- Figure 18 (a.) Addition polymerization of polyethylene (b.) Formation of alkyd resin by condensation polymerization95
- Figure 19 Chemical manufacture of polystyrene, methacrylate, and acrylonitrile 104
- Figure 20 Diagram of a typical injection molding machine 109
- Figure 21 Transmission spectra of samples of (a.) Styron (b.) Monsanto SAN (c.) General Electric Lexan 141 112
- Figure 22 Top: Electron micrograph of SK11 glass milled at OSC Bottom: Electron micrograph of BAK4 glass milled at EGP118
- Figure 23 Diagram of the sample compartment of the Varian 2415 Spectrophotometer 120
- Figure 24 Front view of our three - cavity mold on a standard 20 by 25 cm mold base123
- Figure 25 Transmittance spectra of the first successful injection molded Christiansen filters126
- Figure 26 Illustration of tensile stress and cavity formation around glass particles during cooling 127
- Figure 27 Transmission spectra of CFs that contain coupling agents (a.) SK5/SAN (b.) BAK4/SAN (c.) SK11/SAN 132
- Figure 28 Temperature dependence of the transmittance spectrum of sample (a.) 13-4 BAK4/SAN (b.) 13-11 BAK4/SAN 133
- Figure 29 Run 23-1 without dye (solid line) Required band edge (dashed line) . 138
- Figure 30 (a.) Transmittance spectra of each Macrolex dye and their combination molded into SAN (b.) The effect upon T when less compounded material or more dye are added to SK11/SAN CFs (c.) Another example of the effects of adding dye and mineral oil 140
- Figure 31 (a.) The effect upon transmission when the concentration of compounded SK11/SAN is increased (b.) The effect upon transmission when the concentration of gray dye is increased 142
- Figure 32 (a.) A comparison between samples with the same amount of dye but

different amounts of compound (b.) An attempt to produce diffuse visible scattering by adding mineral oil 143

Figure 33 Transmission spectra of the best samples from the last injection-molding run 145

Figure 34 Micrographs of SK11 glass particles before (top) and after (bottom) one month of tumble milling 146

Figure 35 Plasma sprayed SK5 glass particles magnified (a.) 100X (b.) 250X ... 150

Figure 36 The decrease in spectral transmission as filters are stacked (a.) SK5/SAN (b.) SK11/SAN 153

Figure 37 The insensitivity of the bandpass location to angle of incidence (a.) SK5/SAN (b.) SK11/SAN (c.) SK5/PS 154

Figure 38 The decrease in spectral bandwidth when the detector aperture is decreased (a.) SK5/SAN (b.) SK11/SAN (c.) SK11/SAN with dye 157

Figure 39 Scattering of incoherent light by a green SK5/PS CF (a.) at 500 nm (b.) at 900 nm 159

Figure 40 Scattering of HeNe laser light by a (a.) SK11/SAN CF (b.) SK5/SAN CF 160

Figure 41 Scanning electron micrographs of glass particles on the surfaces of two SK5/SAN CFs 162

Figure 42 (a.) SEM photograph showing polymer - glass adhesion by a coupling agent (b.) Internal cross-section showing no voids 163

Figure 43 Path length through a cylinder 169

Figure 44 Least squares fit of the Clewell equation to measured spectra of two SK5/SAN CFs 175

Figure 45 Least squares fit of the Clewell equation to measured spectra of two SK11/SAN CFs 176

List of Tables

Table 1	Potential glass - plastic Christiansen filters	70
Table 2	Refractive indices for SK6, K7, BK7 and measured values for BK7 ...	85
Table 3	Measured refractive indices for 6 optical plastics	85
Table 4	Optical properties of PMMA, PS, PC, and NAS	100
Table 5	Conversion of mesh designations to particles sizes	116
Table 6	Contents of compounded batches	136
Table 7	Calculation of absorption coefficient versus incidence angle	156
Table 8	Physical parameters for five CFs	166
Table 9	Predicted and measured values of spectral bandwidth	166
Table 10	Determined values for K from the Clarke theory	166
Table 11	K values for the CFs when a cylindrical particle model is used	172
Table 12	Comparison of K values from the spherical particle and cylindrical particle model	172
Table A1	Potential glass - glass Christiansen filters	179
Table A2	Potential Christiansen filters from thin film optical materials	184
Table A3	Potential Christiansen filter combinations from glasses, plastics, and thin film optical materials	191

ABSTRACT

Christiansen filters are scattering filters composed of two materials with dispersion curves [for refractive index] that intersect at one wavelength. These filters transmit unscattered light at the intersection wavelength and incoherently scatter light of other wavelengths. Unlike filters based on optical interference, the transmitted wavelength does not depend on the angle of light incidence on the filter. These filters have been fabricated with solid host matrices by mixing finely ground optical glass powders with pellets of optical grade resins and injection molding the combination into planar filters of various sizes. This method of fabrication can produce filters of any shape or size, limited only by the skill of the mold designer and the size of the molding machine.

This dissertation discusses the fabrication, characterization and modeling of solid matrix Christiansen filters. After a brief proposal of our research objectives to motivate this study, the concepts behind and history of these filters are reviewed. A multiple scattering theory of Christiansen filters is discussed and a computer-based search for compatible materials is summarized. After verifying the refractive index dispersions of the selected polymer matrices, a detailed description of the technology and fabrication of these filters follows. Spectral transmission and other measurements are employed to characterize the optical performance of the filters and to compare the filters to theoretical models. We have produced blue, bandpass filters with

transmittances ranging from 0.25 to 0.40 at 392 nm; green, bandpass filters with transmittances of 0.20 centered at 510 nm; and near-infrared, edge filters which begin at 700 nm and reach maximum transmittances ranging from 0.30 to 0.50 at 800 nm. The dissertation concludes with recommendations for future improvements and suggestions for possible applications. The results from the computer-based material search and tables of fabrication parameters are included in two appendices.

Section 1: Symbol List

CF(s)	Christiansen filter(s)
SCF(s)	Solid Christiansen filters(s)
SMCF(s)	Solid matrix Christiansen filter(s)
λ (s)	wavelength(s)
λ_c	Christiansen wavelength
θ_o	Angle of incidence
UV	Ultra-violet
NIR	Near-infrared
VIS	Visible
IR	Infrared
T	Transmission, transmittance
n, N	Index of refraction
μm	Micron
nm	Nanometer
FWHM	Full width at half maximum
x	Size parameter
α	Particle radius
$\Delta n'$	$d/d\lambda$ ($n_{\text{medium}} - n_{\text{particle}}$)
L	Filter thickness
c	Particle concentration
SK5, SK11, BAK4, K7, SF6, BK7	Schott glass designations
SAN	Styrene acrylonitrile
NAS	Styrene methacrylate
PS	Polystyrene
PC	Polycarbonate
PMMA	Polymethyl methacrylate
CCD	Charged-coupled device
IM	Injection molding
MO	Mineral oil
ESO	Epoxidized soybean oil
dB	decibel

Section 2: Introduction

The focus of the research project that generated this dissertation was the development of a method to fabricate large aperture, optical bandpass filters which scatter rather than specularly reflect the rejected wavelengths [λ 's]. The filters were to be as large as a typical architectural window - several square feet or more - and not necessarily flat. The spectral locations of the transmission bandpasses were to be independent of the angle of incidence of the incoming light [θ_o]. Equally important, the fabrication technique had to be economical.

For one specific application, the filter was to transmit near-infrared [NIR] radiation and scatter light evenly across the visible [VIS] spectrum, producing a gray or milky white appearance. The desired specifications were 80% specular transmission [T_{specular}] in the 0.8 to 1.5 μm region and less than 10% specular T_{specular} in the VIS. We also investigated other possible bandpass regions.

There are many types of optical filters that are employed in science and industry. Optical filters reflect, transmit, absorb, and polarize light in narrow or broad regions of the electromagnetic spectrum. They operate according to principles of reflection, refraction, diffraction, interference, polarization, and scattering. An excellent review of optical filters has been written by Dobrowolski¹ in the *Handbook of Optics*.

The most common types of optical filters manufactured today are based on thin

film interference and absorption. The thin film filter operates by optical interference from the interfaces formed by layers of thin films that are deposited under vacuum conditions upon suitable substrates. Unfortunately, the rejected light from these filters is specularly reflected. Most important, the optical path differences among the film layers decrease as the angle of incidence θ_0 , measured from the normal to the layers increases. This results in the filter bandpass shifting to shorter wavelengths for oblique angles of incidence². Moreover, the size of an interference filter is limited by the capacity of the deposition chamber and the uniformity of the filter is constrained by the source-substrate geometry, since depositing a film uniform in thickness over a large surface is a challenge in itself³. These limitations made interference filters unsuitable for our sponsor's applications.

Absorption filters utilize the bulk absorption spectra of their constituent materials to filter out radiation. They are constructed with colored glasses, dyed gelatins, thick films, plastics, powders, liquids, and even gases⁴. Since they operate by absorption instead of interference, they do not suffer from bandpass shifts. However, since they absorb rather than scatter light, they do not degrade image contrast at wavelengths outside of the bandpass as much as scattering filters. We felt that, if leakage of undesired wavelengths was unavoidable, imaging at those wavelengths should be prevented. Fortunately for our purposes, there are absorption dyes that produce sharp spectral band edges when added to organic polymers⁵. We would eventually employ some of these dyes in our filters to improve spectral characteristics,

but the primary mechanism for our filters could not be absorption.

The type of filter we chose to investigate is traditionally known as a Christiansen filter [CF]. It works on the basis of light scattering by small particles suspended in a host medium. First described by C. Christiansen⁶, these filters have solved a number of optical problems over the century since their inception. In fact, before the development of modern thin film deposition techniques, CFs were the most convenient narrow transmission bandpass filters available⁷. They can be constructed relatively easily and their spectral bandpasses are insensitive to oblique angles of incidence.

Conventionally, CFs were made of glass particles suspended in clear organic liquids. Our approach was to develop techniques for producing solid matrix Christiansen filters [SMCFs], in which particles are dispersed in a rigid, solid medium. We concentrated on mixing finely ground optical glass powders with optical grade plastic resins and molding them into different shapes and sizes. This method appeared to be the most practical approach toward meeting the requirements for spectral profile and economic feasibility, especially for mass production. To our knowledge, nobody has tried to produce SMCFs by injection molding with organic polymer hosts.

The sections of this dissertation are arranged and defined as follows.

Section 1: Symbols List

Section 2: Introduction This section explains the reasoning behind our choice to study and develop solid matrix Christiansen filters.

Section 3: The Principle of Christiansen Filters This section describes the mechanism

of Christiansen filters and their historical development.

Section 4: Theoretical Considerations This section introduces the theory behind small-angle Mie scattering. A multiple scattering theory of Christiansen filters postulated by R.H. Clarke is reviewed. A theory on particle scattering by D.H. Clewell is also discussed.

Section 5: Candidate Materials Search A computerized search of refractive index dispersion data for possible solid Christiansen filter combinations is described. The feasibility of constructing filters from discovered combinations is also discussed.

Section 6: Index Dispersion Verification Refractive index dispersion data of candidate polymers are measured with a simple Pulfrich refractometer and compared to published values. A discussion of the sources of errors associated with these measurements is included.

Section 7: Plastics and Molding Technology This section is an introduction to plastic materials, their uses, and their manufacturing technology. Injection molding is described in detail.

Section 8: Fabrication of Solid Matrix Christiansen Filters This section is a detailed guide to the fabrication of polymer-based SMCFs. Spectral transmission measurements are shown to illustrate problems in fabrication. Solutions to these problems are described.

Section 9: Optical Characterization and Theoretical Modeling Spectral transmission measurements are shown and discussed. Also included are scanning electron micro-

graphs of near-surface features. Measurements that verify the Christiansen effect and test the Clarke theory are shown and discussed. A modification of the Clarke theory is attempted to provide a better model for these filters. The Clewell theory is also discussed.

Section 10: Conclusions and Suggestions for Future Study

Appendix A: Tables of possible solid filter combinations.

Appendix B: Tables of fabrication parameters.

Section 3: The Principle of Christiansen Filters

Christiansen filters [CFs] reject light by scattering from small particles suspended in the host matrix of the filter. As illustrated in Figure 1, small transparent particles randomly dispersed in a transparent host medium will scatter light except at λ_c , the wavelength at which the refractive indices [n's] of the particles and the host medium are equal. At λ_c , the filter is optically homogeneous and will not scatter light. In fact, scattering is vanishingly small for a narrow range of wavelengths around λ_c , the bandpass of the filter. Figure 2 illustrates the intersection of the index dispersion curves and the Gaussian-shaped spectral bandpass of an optimal CF. If the dispersion curves do not intersect, the device will scatter light of all wavelengths. Unlike the bandpass of an interference filter, the bandpass of a CF does not shift as the light incidence angle is changed; hence the filter can work with converging or diverging beams. To remove scattered light from a beam, an optical arrangement as shown in Figure 3 is typically employed. The center wavelength λ_c is known as the Christiansen wavelength.

In 1884, the Danish physicist C. Christiansen attempted to index match various powders in mixtures of two liquids. He reported that a perfect match was possible for only one wavelength due to the difference in the refractive index dispersions of the powder and liquid.⁶ Until the 1950's, Christiansen filters were used for wavelengths ranging from the infrared to the ultraviolet. The advent of dielectric narrowband

Figure 1. The Christiansen filter concept.

Figure 2. Existence of a spectral bandpass due to an intersection between the dispersion curves of the host medium and the particles.

Figure 3. Typical optical arrangement to remove scattered light from a beam transmitted by a Christiansen filter.

interference filters has since made CFs commercially obsolete; nevertheless CFs were applied in research ranging from astrophysics to biology.

An early study of CFs to develop a theory of their operation was conducted by Sethi⁸ in 1920. Sogani⁹ made observations of the Christiansen effect in chromatic emulsions. A first, comprehensive review of CFs was written by E.D. McAlister¹⁰ in 1935. In this article, he lists limitations to solid-liquid filters such as the need for an additional optical system to remove unwanted wavelengths and the sensitivity of the peak wavelength to temperature changes. The refractive index $[n]$ of the liquid matrix undergoes larger changes than that of the solid particles when the combination is heated or cooled. McAlister alleviated this problem by constructing large filters with heat sinking vanes to control temperature changes. He used these filters in experiments on the photosynthesis of plants. By adding borosilicate crown glass particles to a carbon disulphide-benzene mixture and altering the concentration of CS_2 , McAlister produced 5 narrowband filters that span the visible spectrum.

Denmark and Cady¹¹ studied filters composed of glass particles suspended in ethyl salicylate. They measured the bandwidths at 10 wavelengths for 6 sizes of particles and arrived at an optimum particle size for the minimum bandwidth. Barnes and Bonner¹² experimented with particles of quartz, magnesium oxide, calcite and marble in organic liquids to produce the Christiansen effect at infrared wavelengths. They produced a powder filter composed of quartz in air that had a transmission band at $7.35\ \mu m$. They suggested the use of CFs in spectrometers because there are

no overlapping diffraction orders as in grating spectrometers. Henry¹³ investigated powder filters in the IR by dispersing small particles on transparent plates. He tried to model the scattering region between the Rayleigh and Mie regions.

An array of 10 CFs spanning the UV, VIS, and IR wavelengths was used by Abbot, Stebbins and Aldrich¹⁴ on the 100-inch Mt. Wilson reflecting telescope to record stellar spectra. These filters had bandwidths measuring 0.015 μm FWHM. Cloupeau and Klarsfeld¹⁵ utilized the transmission peak shifts from temperature changes to visualize thermal fields in porous media. They photographed the differently colored, heated regions of a flat cell filled with glass particles in chlorobenzene. Wojak, Czarnetzki, and Dobeles¹⁶ produced tunable liquid CFs for the 230-370 nm region. One filter was constructed of lithium fluoride particles in hexane that showed 14% transmission at 250 nm with 6.5 nm FWHM.

Auerbach¹⁷ listed several combinations that can form solid-liquid CFs. He also mentioned that filters can be formed with glass particles embedded in plastic. This article was published thirty years before this dissertation project! A study of glass particles in liquid styrene was conducted by Balasubramanian¹⁸ in the first phase of our project. Tuning a liquid CF by applying pressure changes to the liquid [which alters its density and refractive index] was demonstrated by Matovich¹⁹. He constructed a 15 nm half-width filter with 70% transmission that could be rapidly scanned across the visible spectrum. Carlon²⁰ simulated the Christiansen effect in atmospheric dusts by exploding particles to form clouds in front of a light source. Gerritsen²¹

suggested replacing the particles with a relief hologram in a liquid medium to act as a tunable diffraction grating. The grating would be inoperative at λ_c . Yeh²² proposed fabricating multilayer stacks of Christiansen combinations to produce a narrowband filter free of the sensitivity to angle of incidence. He was awarded a patent for this idea.

A final study to mention is an investigation of the Christiansen effect in the IR by Redfield and Baum²³. They made use of the rapidly changing refractive index of boron nitride near its 7.3 μm absorption band to provide an index dispersion intersection. They observed transmission peaks at wavelengths short of 7.3 μm when BN powders were combined with potassium compounds. They observed peaks on the long wavelength sides of both absorption bands [7.3 and 12.3 μm] when they pressed BN powder under high pressure into thallium bromide-iodide [KRS-5]. This process produced compacted disks which were the first documented solid matrix Christiansen filters. A brief review of CFs by Dobrowolski²⁴ is contained in the *Handbook of Optics*.

Section 4: Theoretical Considerations

Before a theory for a scattering medium can be devised, it is important to determine the nature of the scattering; whether it be single, double, or multiple scattering. For single and double scattering, the size, shape and orientation of the scattering elements are important factors in determining the intensities and directions of the scattered radiation.²⁵ This intractable problem has been studied extensively for over a hundred years by famous and obscure mathematicians. A review of the entire body of literature on just the single scattering problem would fill a large volume.

In the case of multiple scattering, the effects of shape and orientation of the scattering elements on the scattered radiation are much less obvious than in the single case. It is a more difficult problem to solve in detail, so the theories tend to concentrate on global variables such as the average total phase change or the average total scattered energy rather than local variables such as the electric fields inside and outside the scatterers. This problem has also been studied extensively in many ramifications. In the literature, it is referred to as multiple scattering and the problem of radiative transfer.²⁶

In Section 9, I will present evidence that suggests our filters exhibit multiple scattering. A simple test suggested by van de Hulst²⁷ will show that the optical depths of our filters are too large for single scattering. Other measurements will show the spectral transmission decreasing as the optical depth is increased. If the filters were exhibiting single scattering, the transmission peaks would not decrease as fast as

shown when the optical path lengths increase. These remarks will be explained and expanded further in Section 9 when the transmission measurements are discussed.

In light of the preceding remarks, this dissertation will not delve deeply into single scattering theory. A summary of Mie theory will be given to highlight the parameters involved. An empirical formula for the spectral transmission through a single scattering medium will follow. The section will close with a detailed description of a multiple scattering theory devised by R.H. Clarke because:

- the theory was specifically designed for the Christiansen filter whose salient feature is a spectral bandpass of unscattered radiation.
- the theory predicts the transmission spectrum which is the primary characteristic feature for an optical filter.

Many theoretical papers on scattering calculate the spatial distribution of scattered intensities or scattering cross sections rather than the transmission spectrum. The following discussion on single scattering (Mie) theory is based on *Absorption and Scattering of Light by Small Particles* by Bohren and Huffman.²⁵

Single Scattering Theory

The formulation of the basic problem of single scattering is as follows: Given a particle of specified size, shape, and optical properties that is illuminated by an arbitrarily polarized monochromatic wave, determine the electromagnetic field at all points inside the particle and at all points of the homogeneous (assumed) medium surrounding the particle.²⁸ Since an arbitrary electromagnetic field can be decomposed into Fourier components which are plane waves,²⁹ we can limit this discussion

to plane harmonic waves. Theoretically, the solution to any arbitrary field is obtained by the superposition of the plane wave solutions. The field inside the particle is denoted by $(\mathbf{E}_1, \mathbf{H}_1)$, the bold type indicating vector fields. The field $(\mathbf{E}_2, \mathbf{H}_2)$ in the medium surrounding the particle is the superposition of the incident field $(\mathbf{E}_i, \mathbf{H}_i)$ and the scattered field $(\mathbf{E}_s, \mathbf{H}_s)$:

$$\mathbf{E}_2 = \mathbf{E}_i + \mathbf{E}_s, \quad \mathbf{H}_2 = \mathbf{H}_i + \mathbf{H}_s$$

where

$$\mathbf{E}_i = \mathbf{E}_0 \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t), \quad \mathbf{H}_i = \mathbf{H}_0 \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t)$$

and $\mathbf{k}$ is the appropriate wave vector for the surrounding medium.

The fields must satisfy the Maxwell equations

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = -i\omega\epsilon\mathbf{E}$$

at all points where ϵ and μ are continuous. Taking the curl of the last two equations and using the vector identity

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A},$$

we obtain the vector wave equations

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0, \quad \nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0, \quad (4.1)$$

where $k^2 = \omega^2 \epsilon \mu$ and $\nabla^2 \mathbf{A} = \nabla \cdot (\nabla \mathbf{A})$. Note that $\nabla \mathbf{A}$ is a dyadic which when operated on by the divergence operator $\nabla \cdot$ yields a vector.

The conditions placed on $\mathbf{E}$ and $\mathbf{H}$ at the boundary of the particle are that the

tangential components of $\mathbf{E}$ and $\mathbf{H}$ are continuous across a boundary separating media with different properties. This requirement is a sufficient condition for energy conservation across the boundary. Mathematically, these conditions are written

$$[\mathbf{E}_2(\mathbf{x}) - \mathbf{E}_1(\mathbf{x})] \times \mathbf{n} = 0 \quad (\mathbf{x} \text{ on } S) \quad (4.2)$$

$$[\mathbf{H}_2(\mathbf{x}) - \mathbf{H}_1(\mathbf{x})] \times \mathbf{n} = 0$$

where $\mathbf{n}$ is the outward directed normal to the surface S of the particle. The basic task is to find solutions to the Maxwell equations, both inside and outside the particle, which satisfy (4.2) at the boundary between the particle and the surrounding medium. An arbitrarily polarized wave can be expressed as the superposition of two orthogonal polarization states (such as horizontal and vertical; right circular and left circular; etc). Thus, we need to solve each scattering problem twice for a given direction of propagation to determine the scattering of an arbitrarily polarized plane wave.

Now consider the illumination of a particle by a plane wave. The coordinate system is depicted in Figure 4. The direction of propagation of the incident light defines the forward direction (z axis). The scattering direction $\mathbf{e}_r$ and the forward direction $\mathbf{e}_z$ define the scattering plane as shown in Figure 4. The incident electric field $\mathbf{E}_i$, which lies in the xy plane, can be resolved into components parallel ($E_{\parallel}$) and perpendicular ($E_{\perp}$) to the scattering plane:

$$\mathbf{E}_i = (E_{0\parallel}\mathbf{e}_{\parallel} + E_{0\perp}\mathbf{e}_{\perp})\exp(ikz - i\omega t) = E_{\parallel}\mathbf{e}_{\parallel} + E_{\perp}\mathbf{e}_{\perp}$$

where $k = 2\pi N_2/\lambda$ is the wave number in the medium, N_2 is the medium refractive index and λ is the vacuum wavelength of the incident light. The following vector

Figure 4. The coordinate system for light scattering by an arbitrary particle.

relations follow from Figure 4:

$$\mathbf{e}_{\perp i} = \sin(\phi)\mathbf{e}_x - \cos(\phi)\mathbf{e}_y$$

$$\mathbf{e}_{\parallel i} = \cos(\phi)\mathbf{e}_x + \sin(\phi)\mathbf{e}_y$$

$$\mathbf{e}_{\perp i} \times \mathbf{e}_{\parallel i} = \mathbf{e}_z \quad (\text{right-handed triad})$$

$$\mathbf{e}_{\perp i} = -\mathbf{e}_\phi$$

$$\mathbf{e}_{\parallel i} = \sin(\theta)\mathbf{e}_r + \cos(\theta)\mathbf{e}_\theta.$$

The orthonormal basis vectors $(\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\phi)$ are associated with a spherical polar coordinate system (r, θ, ϕ) defined at the particle. If E_{xi} and E_{yi} denote the x and y components of the incident field, then

$$E_{\parallel i} = \cos(\phi)E_{xi} + \sin(\phi)E_{yi}$$

$$E_{\perp i} = \sin(\phi)E_{xi} - \cos(\phi)E_{yi}.$$

For large distances from the origin ($kr \gg 1$, far-field region), the scattered electric field $\mathbf{E}_s$ is approximately transverse and has the asymptotic form³⁰

$$\vec{E}_s \sim \frac{e^{ikr}}{-ikr} \vec{A}$$

where $\mathbf{e}_r \cdot \mathbf{A} = 0$. The scattered field in the far-field region may be written

$$\mathbf{E}_s = E_{\parallel s}\mathbf{e}_{\parallel s} + E_{\perp s}\mathbf{e}_{\perp s}$$

$$\mathbf{e}_{\parallel s} = \mathbf{e}_\theta, \quad \mathbf{e}_{\perp s} = -\mathbf{e}_\phi, \quad \mathbf{e}_{\perp s} \times \mathbf{e}_{\parallel s} = \mathbf{e}_r$$

where $\mathbf{e}_{\parallel s}$ and $\mathbf{e}_{\perp s}$ are parallel and perpendicular to the scattering plane.

Due to the linearity of the boundary condition (4.2), the amplitude of the field scattered by an arbitrary particle is a linear function of the amplitude of the incident field. The relation between incident and scattered fields is then written:

$$\begin{pmatrix} E_{\parallel s} \\ E_{\perp s} \end{pmatrix} = \frac{e^{ik(r-z)}}{-ikr} \begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix} \begin{pmatrix} E_{\parallel i} \\ E_{\perp i} \end{pmatrix} \quad (4.3)$$

The elements S_j ($j=1,2,3,4$) of the amplitude scattering matrix depend on the scattering angle θ and the azimuthal angle ϕ . The real and imaginary parts of the amplitude scattering matrix elements are rarely measured for all values of θ and ϕ . This would require measuring the amplitude and phase of the light scattered in all directions for two incident polarization states; measuring the phase is difficult.

The quantities that are typically measured and discussed are the Stokes parameters. This is done by interposing various polarizers between the particle and detector and recording the resulting irradiances as a function of θ and ϕ . The procedures for these measurements are described in several references.^{31,32} The scattered Stokes parameters are related to the scattered fields as follows:

$$\begin{aligned} I_s &= \langle E_{\parallel s} E_{\parallel s}^* + E_{\perp s} E_{\perp s}^* \rangle \\ Q_s &= \langle E_{\parallel s} E_{\parallel s}^* - E_{\perp s} E_{\perp s}^* \rangle \\ U_s &= \langle E_{\parallel s} E_{\perp s}^* + E_{\perp s} E_{\parallel s}^* \rangle \\ V_s &= i \langle E_{\parallel s} E_{\perp s}^* - E_{\perp s} E_{\parallel s}^* \rangle \end{aligned} \quad (4.4)$$

where the multiplicative factor $k/2\omega\mu$ is omitted. The relation between the incident and scattered Stokes parameters is obtained by using (4.3) in (4.4):

$$\begin{pmatrix} I_s \\ Q_s \\ U_s \\ V_s \end{pmatrix} = \frac{1}{k^2 r^2} \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix} \begin{pmatrix} I_i \\ Q_i \\ U_i \\ V_i \end{pmatrix} \quad (4.5)$$

$$S_{11} = \frac{1}{2}(|S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2),$$

$$S_{12} = \frac{1}{2}(|S_2|^2 - |S_1|^2 + |S_4|^2 - |S_3|^2),$$

$$S_{13} = \text{Re}(S_2 S_3^* + S_1 S_4^*),$$

$$S_{14} = \text{Im}(S_2 S_3^* - S_1 S_4^*),$$

$$S_{21} = \frac{1}{2}(|S_2|^2 - |S_1|^2 - |S_4|^2 + |S_3|^2),$$

$$S_{22} = \frac{1}{2}(|S_2|^2 + |S_1|^2 - |S_4|^2 - |S_3|^2),$$

$$S_{23} = \text{Re}(S_2 S_3^* - S_1 S_4^*),$$

$$S_{24} = \text{Im}(S_2 S_3^* + S_1 S_4^*),$$

$$S_{31} = \text{Re}(S_2 S_4^* + S_1 S_3^*),$$

$$S_{32} = \text{Re}(S_2 S_4^* - S_1 S_3^*),$$

$$S_{33} = \text{Re}(S_1 S_2^* + S_3 S_4^*),$$

$$S_{34} = \text{Im}(S_2 S_1^* + S_4 S_3^*),$$

$$S_{41} = \text{Im}(S_2^* S_4 + S_3^* S_1),$$

$$S_{42} = \text{Im}(S_2^* S_4 - S_3^* S_1),$$

$$S_{43} = \text{Im}(S_1 S_2^* - S_3 S_4^*),$$

$$S_{44} = \text{Re}(S_1 S_2^* - S_3 S_4^*).$$

The 4×4 matrix in (4.5) [*the scattering matrix*] is the Mueller matrix for scattering by a single particle. Only seven of the elements are independent, corresponding to the

four $|S_j|$ of Equation (4.3) and the three differences in phase between them. Thus, there must be nine independent relations among the S_{ij} for single scattering.

The Stokes parameters of the light scattered by a screen of dilute, randomly separated particles are the sum of the Stokes parameters of the light scattered by the individual particles. Therefore, the total scattering matrix of the screen is the sum of the individual scattering matrices. This assumes that the linear dimensions of the screen are small compared with the distance at which the scattering is observed. In this general case, there are 16 nonzero independent matrix elements, although symmetry considerations could reduce this number.

The logical path in most scattering studies is to predict the scattered Stokes parameters based on a model of the scattering elements, measure the appropriate parameters and compare them to the predictions. Before moving on to Mie theory, a few words on extinction, cross sections, and absorption are in order.

Extinction is defined as the attenuation of a beam of radiation traversing a medium by both scattering and absorption mechanisms.³³ For example, the energy extinction rate, W_{ext} , of a particle in a nonabsorbing medium is the sum of the energy absorption rate W_a and the energy scattering rate W_s of the particle.

$$W_{\text{ext}} = W_a + W_s \quad (4.6)$$

where $W_a = -\int_A \mathbf{S} \cdot \mathbf{e}_r dA$, $W_s = \int_A \mathbf{S}_s \cdot \mathbf{e}_r dA$,

$\mathbf{S}$ is the Poynting vector of the incident radiation which is integrated over the surface A of an imaginary sphere around the particle and $\mathbf{S}_s$ is the Poynting vector of the scattered field.

Suppose the incident electric field is x-polarized such that $E_i = E e_x$. Since the medium is nonabsorbing, W_a is independent of the radius r of the imaginary sphere around the particle. Choose r such that the far-field approximation can be taken:

$$E_s \sim \frac{e^{ik(r-z)}}{-ikr} X E, \quad H_s \sim \frac{k}{\omega \mu} e_r \times E_s$$

X is the vector scattering amplitude. By evaluating the integrals for W_{ext} , it can be shown that, as $kr \rightarrow \infty$,

$$W_{ext} = I_i \frac{4\pi}{k^2} \text{Re}\{(\mathbf{X} \cdot \mathbf{e}_x)_{\theta=0}\} \quad (4.7)$$

where I_i is the incident irradiance. The ratio of W_{ext} to I_i is the extinction cross section (C_{ext}) and has dimensions of area.

$$C_{ext} = \frac{W_{ext}}{I_i} = \frac{4\pi}{k^2} \text{Re}\{(\mathbf{X} \cdot \mathbf{e}_x)_{\theta=0}\}.$$

It follows from (4.6) that C_{ext} can also be written as:

$$C_{ext} = C_{abs} + C_{scat}.$$

In terms of geometrical optics, it is said that the particle "casts a shadow" of area C_{ext} ; however this shadow may be greater or less than the particle's geometrical shadow depending upon the type of scattering and the distance of the detector. Equation (4.7) is one form of the optical theorem which expresses the fact that extinction depends only on the scattering amplitude in the forward direction ($\theta=0$). Extinction is physically interpreted as a manifestation of interference between the incident light and the forward scattered light. By conservation of energy, the light

removed from the incident beam by the interference is accounted for by scattering in all directions and by absorption in the particle.

Other terms employed in the literature are the efficiency factors for extinction, scattering, and absorption:

$$Q_{ext} = \frac{C_{ext}}{G}, \quad Q_{sca} = \frac{C_{sca}}{G}, \quad Q_{abs} = \frac{C_{abs}}{G}$$

where G is the particle cross-sectional area projected onto the plane perpendicular to the direction of propagation of the incident light. In addition to defining an absorption cross section C_{abs} , absorption can be characterized by the well-known Beer's law which relates the incident and transmitted intensities in a medium by:

$$I_t(z) = I_i \exp(-\alpha z). \quad (4.8)$$

The factor α is the absorption coefficient, z is the distance into the medium along the direction of propagation, and I_i and I_t are the incident and transmitted intensities, respectively. Beer's law must be incorporated into the mathematical formalism when particles in an absorbing medium are considered. We are now ready to consider Mie theory.

Mie Theory

In 1908, Gustav Mie developed a scattering theory in an attempt to understand the colors exhibited by small particles of gold suspended in water. Although he was not the first to study absorption and scattering by a sphere, his derivations are straightforward enough to associate his name to the general solution of the problem. The following is a direct quote from Bohren and Huffman³⁴: "The mathematics, di-

vorced from the physical phenomena, can be somewhat boring". This dissertation will not rederive the Mie theory in detail, but will sketch the solution in terms of the previously described parameters and discuss the observable features.

The solution begins with the vector wave equations (4.1). A new vector function $\mathbf{M}$ is constructed from a scalar function ψ and an arbitrary constant vector $\mathbf{c}$:

$$\mathbf{M} = \nabla \times (\mathbf{c}\psi).$$

Note that the divergence of $\mathbf{M}$ vanishes because $\mathbf{M}$ is the curl of a vector. Substituting $\mathbf{M}$ into (4.1), we find

$$\nabla^2 \mathbf{M} + k^2 \mathbf{M} = \nabla \times [\mathbf{c}(\nabla^2 \psi + k^2 \psi)].$$

$\mathbf{M}$ satisfies the vector wave equation if ψ is a solution of the scalar wave equation:

$$\nabla^2 \psi + k^2 \psi = 0. \quad (4.9)$$

If we construct another vector function:

$$\mathbf{N} = \frac{\nabla \times \mathbf{M}}{k},$$

then

$$\nabla \cdot \mathbf{N} = 0, \quad \nabla^2 \mathbf{N} + k^2 \mathbf{N} = 0,$$

and

$$\nabla \times \mathbf{N} = k\mathbf{M}.$$

Therefore, $\mathbf{M}$ and $\mathbf{N}$ satisfy all of the required properties of an electromagnetic wave if ψ is a solution of the scalar wave equation (4.9). Scalar function ψ is called the generating function for the vector harmonics $\mathbf{M}$ and $\mathbf{N}$ and vector $\mathbf{c}$ is called the guiding or pilot vector.

The choice of ψ is dictated by the symmetry in the problem. For scattering by a sphere, we choose functions ψ that satisfy equation (4.9) in spherical polar coordinates (r, θ, ϕ) (see Figure 5). We choose the radius vector $\mathbf{r}$ as the pilot vector and define $\mathbf{M}$ as:

$$\mathbf{M} = \nabla \times (\mathbf{r}\psi)$$

so that $\mathbf{M}$ is a solution of the vector wave equation in spherical polar coordinates.

The scalar wave equation written in spherical coordinates is:

$$\frac{1}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{\sin \theta} \left(\frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{\partial^2 \psi}{\partial \phi^2} \right) + k^2 r^2 \psi \right] = 0 \quad (4.10)$$

Substituting a solution of the form

$$\psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$$

into (4.10) yields three separated equations:

$$\frac{d^2 \Phi}{d\phi^2} + m^2 \Phi = 0 \quad (4.11)$$

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left[\sin \theta \frac{d\Theta}{d\theta} \right] + \left[n(n+1) - \frac{m^2}{\sin^2 \theta} \right] \Theta = 0 \quad (4.12)$$

$$\frac{d}{dr} \left[r^2 \frac{dR}{dr} \right] + [k^2 r^2 - n(n+1)] R = 0. \quad (4.13)$$

It is convenient to introduce the dimensionless variable $\rho = kr$ and the function $Z = R\rho^{1/2}$ into equation (4.13) to get

$$\rho \frac{d}{d\rho} \left(\rho \frac{dZ}{d\rho} \right) + \left[\rho^2 - \left(n + \frac{1}{2} \right)^2 \right] Z = 0. \quad (4.14)$$

Figure 5. Spherical polar coordinate system centered on a spherical particle with radius α .

The linearly independent solutions to equation (4.11) are

$$\Phi_e = \cos(m\phi), \quad \Phi_o = \sin(m\phi)$$

where the subscripts e and o denote even and odd, respectively. The solutions to (4.12) are the associated Legendre functions of the first kind $P_n^m(\cos(\phi))$ of degree n and order m . The solutions to equation (4.14) are the Bessel functions of the first and second kinds, J_ν and Y_ν . It then follows that linear, independent solutions to (4.13) are the spherical Bessel functions:

$$j_n(\rho) = \sqrt{\frac{\pi}{2\rho}} J_{n+\frac{1}{2}}(\rho),$$

$$y_n(\rho) = \sqrt{\frac{\pi}{2\rho}} Y_{n+\frac{1}{2}}(\rho).$$

Any linear combination of j_n and y_n is also a solution to equation (4.13). Two such combinations are the spherical Bessel functions of the third kind, otherwise known as the spherical Hankel functions:

$$h_n^{(1)}(\rho) = j_n(\rho) + iy_n(\rho),$$

$$h_n^{(2)}(\rho) = j_n(\rho) - iy_n(\rho).$$

The generating functions that satisfy the scalar wave equation in spherical polar coordinates (4.10) can now be written:

$$\Psi_{emn} = \cos(m\phi) P_n^m(\cos\theta) z_n(kr) \quad (4.15)$$

$$\Psi_{omn} = \sin(m\phi) P_n^m(\cos\theta) z_n(kr)$$

where z_n is any of the four spherical Bessel functions j_n , y_n , $h_n^{(1)}$, or $h_n^{(2)}$. The vector spherical harmonics generated by Ψ_{emn} and Ψ_{omn} are

$$\mathbf{M}_{emn} = \nabla \times (\mathbf{r} \Psi_{emn}), \quad \mathbf{M}_{omn} = \nabla \times (\mathbf{r} \Psi_{omn}),$$

$$\mathbf{N}_{emn} = \frac{\nabla \times \mathbf{M}_{emn}}{k}, \quad \mathbf{N}_{omn} = \frac{\nabla \times \mathbf{M}_{omn}}{k}.$$

The next step is to expand the incident plane wave ($\mathbf{E}_i, \mathbf{H}_i$), the field inside the sphere ($\mathbf{E}_1, \mathbf{H}_1$) and the scattered field ($\mathbf{E}_s, \mathbf{H}_s$) in vector spherical harmonics and impose the continuity conditions (4.2) at the boundary between the sphere and the surrounding medium. If we skip through the mathematics, the incident plane wave can be written as

$$\mathbf{E}_i = E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} (\mathbf{M}_{o1n}^{(1)} - i \mathbf{N}_{e1n}^{(1)}) \quad (4.16)$$

$$\mathbf{H}_i = \frac{-k}{\omega \mu} E_0 \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} (\mathbf{M}_{e1n}^{(1)} + i \mathbf{N}_{o1n}^{(1)})$$

where the superscript (1) on $\mathbf{M}$ and $\mathbf{N}$ indicates that the radial dependence is specified by the spherical Bessel function j_n . The expansion of the field inside the sphere is

$$\mathbf{E}_1 = \sum_{n=1}^{\infty} E_n (c_n \mathbf{M}_{o1n}^{(1)} - i d_n \mathbf{N}_{e1n}^{(1)}) \quad (4.17)$$

$$\mathbf{H}_1 = \frac{-k_1}{\omega \mu_1} \sum_{n=1}^{\infty} E_n (d_n \mathbf{M}_{e1n}^{(1)} + i c_n \mathbf{N}_{o1n}^{(1)})$$

where k_1 is the wave number in the sphere, μ_1 is the permeability of the sphere and $E_n = i^n E_0 (2n+1)/n(n+1)$. The superscript (1) refers to the function $j_n(k_1 r)$. The expansion of the scattered field is

$$\mathbf{E}_s = \sum_{n=1}^{\infty} E_n (i a_n \mathbf{N}_{e1n}^{(3)} - b_n \mathbf{M}_{o1n}^{(3)}) \quad (4.18)$$

$$\mathbf{H}_s = \frac{k}{\omega \mu} \sum_{n=1}^{\infty} E_n (i b_n \mathbf{N}_{o1n}^{(3)} + a_n \mathbf{M}_{e1n}^{(3)})$$

where the superscript (3) refers to the Bessel function $h_n^{(1)}(kr)$.

The boundary conditions are now applied:

$$(\mathbf{E}_i + \mathbf{E}_s - \mathbf{E}_1) \times \mathbf{e}_r = (\mathbf{H}_i + \mathbf{H}_s - \mathbf{H}_1) \times \mathbf{e}_r = 0.$$

For a given n , there are four unknown coefficients, a_n , b_n , c_n , and d_n . The boundary conditions in component form supply four equations for these unknowns:

$$E_{i\theta} + E_{s\theta} = E_{1\theta}, \quad E_{i\phi} + E_{s\phi} = E_{1\phi},$$

$$H_{i\theta} + H_{s\theta} = H_{1\theta}, \quad H_{i\phi} + H_{s\phi} = H_{1\phi},$$

$$r = a.$$

The solutions for the coefficients inside the sphere are

$$c_n = \frac{\mu_1 j_n(x) \left[x h_n^{(1)}(x) \right]' - \mu_1 h_n^{(1)}(x) [x j_n(x)]'}{\mu_1 j_n(mx) \left[x h_n^{(1)}(x) \right]' - \mu h_n^{(1)}(x) [mx j_n(mx)]'}$$

$$d_n = \frac{\mu_1 m j_n(x) \left[x h_n^{(1)}(x) \right]' - \mu_1 m h_n^{(1)}(x) [x j_n(x)]'}{\mu m^2 j_n(mx) \left[x h_n^{(1)}(x) \right]' - \mu_1 h_n^{(1)}(x) [mx j_n(mx)]'}$$

and the scattering coefficients are

$$a_n = \frac{\mu m^2 j_n(mx) [x j_n(x)]' - \mu_1 j_n(x) [mx j_n(mx)]'}{\mu m^2 j_n(mx) \left[x h_n^{(1)}(x) \right]' - \mu_1 h_n^{(1)}(x) [mx j_n(mx)]'} \quad (4.19)$$

$$b_n = \frac{\mu_1 j_n(mx) [x j_n(x)]' - \mu j_n(x) [mx j_n(mx)]'}{\mu_1 j_n(mx) \left[x h_n^{(1)}(x) \right]' - \mu h_n^{(1)}(x) [mx j_n(mx)]'}$$

where the size parameter x and the relative refractive index m are

$$x = k\alpha = \frac{2\pi N\alpha}{\lambda}, \quad m = \frac{N_1}{N}. \quad (4.20)$$

The factor α is the radius of the sphere and N_1 and N are the refractive indices of the particle and medium, respectively. The prime indicates differentiation with respect to the argument in parentheses.

The scattering coefficients can be simplified if we take the permeabilities of the particle and the medium to be equal and substitute the Riccati-Bessel functions:

$$\Psi_n(\rho) = \rho j_n(\rho), \quad E_n(\rho) = \rho h_n^{(1)}(\rho).$$

We then get

$$a_n = \frac{m \Psi_n(mx) \Psi_n'(x) - \Psi_n(x) \Psi_n'(mx)}{m \Psi_n(mx) E_n'(x) - E_n(x) \Psi_n'(mx)}, \quad (4.21)$$

$$b_n = \frac{\Psi_n(mx) \Psi_n'(x) - m \Psi_n(x) \Psi_n'(mx)}{\Psi_n(mx) E_n'(x) - m E_n(x) \Psi_n'(mx)}.$$

These coefficients vanish when m equals 1. This makes sense since there is no scattered field when the particle disappears.

Two useful functions to define are the angle dependent functions:

$$\Pi_n = \frac{P_n^1}{\sin(\theta)} \quad , \quad T_n = \frac{dP_n^1}{d\theta} . \quad (4.22)$$

By applying most of the previous definitions, we can write the components of the incident and scattered fields. Assuming that the incident wave is x-polarized, the incident field components are:

$$E_{i\theta} = \frac{\cos(\phi)}{\rho} \sum_{n=1}^{\infty} E_n (\Psi_n \Pi_n - i \Psi'_n T_n) ,$$

$$H_{i\theta} = \frac{k}{\omega \mu} \tan(\phi) E_{i\theta} ,$$

$$E_{i\phi} = \frac{\sin(\phi)}{\rho} \sum_{n=1}^{\infty} E_n (i \Psi'_n \Pi_n - \Psi_n T_n) ,$$

$$H_{i\phi} = \frac{-k}{\omega \mu} \cot(\phi) E_{i\phi}$$

where $\rho = kr$. The scattered field components are:

$$E_{s\theta} = \frac{\cos(\phi)}{\rho} \sum_{n=1}^{\infty} E_n (i a_n E'_n T_n - b_n E_n \Pi_n) , \quad (4.23a.)$$

$$E_{s\phi} = \frac{\sin(\phi)}{\rho} \sum_{n=1}^{\infty} E_n (b_n E_n T_n - i a_n E'_n \Pi_n) , \quad (4.23b.)$$

$$H_{s\theta} = \frac{k}{\omega \mu} \frac{\sin(\phi)}{\rho} \sum_{n=1}^{\infty} E_n (i b_n E'_n T_n - a_n E_n \Pi_n) , \quad (4.23c.)$$

$$H_{s\phi} = \frac{k}{\omega \mu} \frac{\cos(\phi)}{\rho} \sum_{n=1}^{\infty} E_n (i b_n E'_n \Pi_n - a_n E_n T_n) . \quad (4.23d.)$$

By evaluating the integrals for W_s and W_{ext} in equation (4.6), we can write the scattering cross section as:

$$C_{sca} = \frac{W_s}{I_i} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1) (|a_n|^2 + |b_n|^2) , \quad (4.24)$$

and the extinction cross section as:

$$C_{ext} = \frac{W_{ext}}{I_i} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1) \operatorname{Re} [a_n + b_n] . \quad (4.25)$$

It appears in equations (4.23a-d) that we lost the $e^{ikr}/-ikr$ factor from equation (4.3); however, it can be shown³⁵ that $h_n^{(1)}(kr)$ asymptotically approaches $(-i)^n e^{ikr}/ikr$ for $kr \gg n^2$ (subscript n). The function $h_n^{(1)}(\rho)$ appears in the definition for $E_n(\rho)$. Assuming that the series expansions of equations (4.23a-d) are uniformly convergent and we can terminate each series after n_c terms, the transverse components of the scattered electric fields are:

$$E_{s\theta} \sim E_o \frac{e^{ikr}}{-ikr} \cos(\phi) S_2(\cos(\theta))$$

$$E_{s\phi} \sim -E_o \frac{e^{ikr}}{-ikr} \sin(\phi) S_1(\cos(\theta))$$

where

$$S_1 = \sum_n \frac{2n+1}{n(n+1)} (a_n \Pi_n + b_n T_n) , \quad (4.26a.)$$

$$S_2 = \sum_n \frac{2n+1}{n(n+1)} (a_n T_n + b_n \Pi_n) . \quad (4.26b.)$$

The relation between the incident and the scattered field amplitudes (4.3) can now be written as:

$$\begin{pmatrix} E_{\parallel s} \\ E_{\perp s} \end{pmatrix} = \frac{e^{ik(r-z)}}{-ikr} \begin{pmatrix} S_2 & 0 \\ 0 & S_1 \end{pmatrix} \begin{pmatrix} E_{\parallel i} \\ E_{\perp i} \end{pmatrix}, \quad (4.27)$$

and the Stokes parameters (4.5) are:

$$\begin{pmatrix} I_s \\ Q_s \\ U_s \\ V_s \end{pmatrix} = \frac{1}{k^2 r^2} \begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{11} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & -S_{34} & S_{33} \end{pmatrix} \begin{pmatrix} I_i \\ Q_i \\ U_i \\ V_i \end{pmatrix}, \quad (4.28)$$

where

$$\begin{aligned} S_{11} &= \frac{1}{2}(|S_2|^2 + |S_1|^2), & S_{12} &= \frac{1}{2}(|S_2|^2 - |S_1|^2), \\ S_{33} &= \frac{1}{2}(S_2^* S_1 + S_2 S_1^*), & S_{34} &= \frac{1}{2i}(S_1 S_2^* - S_2 S_1^*). \end{aligned}$$

If the incident light is polarized parallel to the scattering plane, the scattered Stokes parameters are

$$I_s = \frac{(S_{11} + S_{12})}{k^2 r^2} I_i, \quad Q_s = I_s, \quad U_s = V_s = 0,$$

and the scattered light is also polarized parallel to the scattering plane. If the incident light is polarized perpendicular to the scattering plane, the parameters are

$$I_s = \frac{(S_{11} - S_{12})}{k^2 r^2} I_i, \quad Q_s = -I_s, \quad U_s = V_s = 0$$

and the scattered light is also polarized perpendicular to the scattering plane.

Define $i_{\parallel}$ as the scattered irradiance per unit incident irradiance if the incident light is polarized parallel to the scattering plane:

$$i_{\parallel} = S_{11} + S_{12} = |S_2|^2.$$

Define $i_{\perp}$ as the same as $i_{\parallel}$ except that the incident light is polarized perpendicular to the scattering plane:

$$i_{\perp} = S_{11} - S_{12} = |S_1|^2.$$

Figure 6 is derived from Bohren and Huffman³⁶ and shows $i_{\parallel}$ and $i_{\perp}$ as functions of the scattering angle θ for size parameter $x = 3$ and complex refractive index $m = 1.33 + i10^{-8}$. This corresponds to a droplet of water with radius $0.26 \mu\text{m}$ that is illuminated by visible light of wavelength $0.55 \mu\text{m}$. We see strong scattering in the forward direction that is 100 times greater than scattering in the backward direction. This asymmetry is even more pronounced as the size parameter increases. This plot shows the predominance of forward scattering even for rather small spheres.

At this point, the reader may feel frustrated after reading pages of equations just to see a well-known result. My intent was to demonstrate the mathematical complexity for even the simplest cases such as single scattering from a sphere. Since the glass particles in our CFs are much larger than the visible wavelengths, small angle forward scattering is predominant for wavelengths away from the Christiansen wavelength. Unlike the spherical particles of Mie theory, the glass particles are best

Figure 6. Forward scattering by a sphere with $x = 3$ and $m = 1.33 + i10^{-8}$ (derived from Bohren and Huffman³⁶)

described as multi-faceted slivers of irregular shapes. The sizes range from 20 μm to over 100 μm . In addition to the complex shapes and various sizes, the particles may be too close to each other to apply the far-field approximations in Mie theory. Bohren and Huffman estimate that it takes x (size parameter) number of terms for the scattering cross section series (4.24) to converge³⁷. For our particles, x is approximately 350. Therefore, simulating the multiple scattering in our CFs by tracing the scattered intensities from particle to particle would require many computations. In fact, it may be unrewarding since the large size parameters would cause scattering at very small angles. Figure 7 defines the single scattering regions.

Clewell's Theory of Light Scattering by Pigment Particles

D.H. Clewell³⁸ deduced an empirical formula for the scattering coefficient of a system of particles. He assumed single scattering from each particle and that the mass concentration per unit volume remains constant, regardless of the sizes of the particles. The single scattering assumption implies that the scattering from N_s particles is N_s times the scattering from one particle. Clewell did not attempt a rigorous theoretical justification of his equation; he just constructed an equation that reduced to familiar expressions from Rayleigh scattering theory. If d represents the particle diameter, n_o is the index of the medium, and n_r is the index of the particle relative to the medium, the total scattering coefficient for a particle illuminated by an unpolarized beam is

- $|m|X \ll 1$ OR $[2\pi|N_p|]\alpha \ll \lambda$ DEFINES RAYLEIGH SCATTERING.
 SCATTERED IRRADIANCE $\sim \frac{\alpha^6}{\lambda^4} (1 + \cos^2 \theta)$ INCIDENT IRRADIANCE.
 θ IS THE SCATTERING ANGLE.
- $|m - 1| \ll 1$ OR $|N_p| \sim |N|$ AND $kd|m - 1| \ll 1$ DEFINES RAYLEIGH-GANS SCATTERING.
 THE SCATTERING MATRIX ELEMENTS PER UNIT VOLUME CAN BE INTEGRATED OVER ANY PARTICLE SHAPE.
- $|m|X \geq 1$ OR $\alpha \geq \lambda$ DEFINES MIE (AEROSOL) SCATTERING.
 PREDOMINANTLY FORWARD SCATTERING.
- $|m|X \gg 1$ REQUIRES GEOMETRICAL OPTICS.
 USE THE GENERALIZED SNELL'S LAW AND THE FRESNEL FORMULAS AT EACH INTERFACE BETWEEN THE PARTICLE AND THE MEDIUM.

Figure 7. Definitions for the different single scattering regions.

$$S' \sim \frac{\gamma d^6}{\left(\frac{\lambda}{n_o}\right)^4}, \quad (4.29)$$

when d/λ is small and

$$S' \sim \frac{\gamma d^4}{\left(\frac{\lambda}{n_o}\right)^2}, \quad (4.30)$$

when $n_r \sim 1$ and d is of the same order of magnitude as λ . The factor $\gamma = 9[(n_r^2 - 1)/(n_r^2 + 2)]^2$.

To make the mass concentration independent of d , the coefficient for the system of particles is S' divided by volume (d^3). Thus,

$$S \sim \frac{\gamma d^3}{\left(\frac{\lambda}{n_o}\right)^4}; \quad \frac{d}{\lambda} \ll 1, \quad (4.31)$$

and

$$S \sim \frac{\gamma d}{\left(\frac{\lambda}{n_o}\right)^2}; \quad n_r \approx 1. \quad (4.32)$$

Equation (4.31) is the famous d^3/λ^4 dependence of Rayleigh scattering by small particles.

Clewell states that, when large particles are made finer in size, scattering increases;

$$S \sim 1/d. \quad (4.33)$$

He makes the point that equation (4.31) predicts that scattering also increases as

small particles are made larger. Therefore, there must be a scattering maximum between the regions of validity for equations (4.31) and (4.33). Several experiments^{39,40} show that this scattering maximum occurs when the particle diameter is approximately $\lambda/2$, a size that coincides with the ultimate resolving power of a microscope. From diffraction theory, a particle with diameter d in a medium of index n_o will diffract a ray of light into a cone whose half-angle θ is given by

$$\sin(\theta) = 0.61 \lambda/n_o d$$

and the maximum scattering will occur when $\theta = \pi/2$ or

$$0.61 \lambda/n_o d = 1. \quad (4.34)$$

Clewell's empirical equation, which approximately reduces to relations (4.31-34), is:

$$S \sim \frac{[(n_r)^2 - 1] / [(n_r)^2 + 2]^2}{d \left[\left(\frac{0.61 \lambda}{n_o d} \right)^2 - 1 \right]^2 + \frac{\lambda^2}{n_o^2 d [(n_r - 1)^2 c + 1]}} \quad (4.35)$$

where c is a suitable constant. Assuming no absorption in the system, the transmittance T is related to S by the relation

$$T = e^{-SL} \quad (4.36)$$

where L is the thickness of the suspension. Clewell shows that a good fit to experimental data⁴¹ of scattering as a function of particle size can be obtained with equations (4.35) and (4.36).

A multiple scattering theory for the Christiansen filter devised by R.H. Clarke⁴² is described next. Unlike Mie theory, Fourier and statistical analyses are used to

calculate the transmission spectrum rather than deriving the scattering intensities by solving for the fields everywhere. This theory is mathematically simpler and directly applicable to our filters.

Clarke's Theory for the Christiansen Filter

Very close to the Christiansen wavelength λ_c (see Fig. 2), the difference in refractive index between the particle and medium will be of the order of a few parts in a thousand. The particle sizes will be typically tens of wavelengths. The effect of each particle on the wave incident upon it can be described as a change in path length with little change in amplitude or direction. Suppose the system is contained between two parallel planes as shown in Figure 8. If the contained region is subdivided into laterally infinite slabs, each of these slabs introduces a degradation of spatial phase coherence into the traversing wave. The wave crossing the entrance plane will have the same phase at all points in the plane, but the phase of each wave crossing the slabs will be affected by the number of intervening particles or by the different path lengths through the particles or by a combination of both. The wave emerging from the first slab will be nearly uniform in amplitude but will have a random spatial variation of phase. Thus, the region between the entrance and exit planes can be treated as a series of thin, random phase changing screens.

Fejer⁴³ has calculated the scattering of a thick medium containing weak refractive index irregularities by tracing the scattered wave through a stack of phase changing screens. Each slab produces an angular spectrum of plane waves which is incident upon the next slab. Most of the energy is carried by a plane wave traveling in the

Figure 8. The Clarke theory assumes that the scattering medium is divided into thin slabs that behave as thin, random phase changing screens.

same direction as the incident wave. The rest of the energy is scattered around this principal direction. The angular spectrum emerging from the exit plane is calculated from the cumulative effect of each successive thin slab. Fejer considered a medium with randomly situated, spherical scatterers. An important condition in the analysis is that the width of each slab must be thin enough so that the rms phase change across the slab is much less than 1 radian but thick enough to contain enough scatterers so that the scattered waves from different slabs are randomly phased with respect to each other.

Bramley⁴⁴ simplified Fejer's approach by showing that the same angular power spectrum for the emerging field can be obtained if one considers only phase variations in parallel ray paths through the region. Clarke uses Bramley's method to investigate the angular distribution of the transmitted power for the CF as a function of particle size, particle concentration, path length through the suspension, and rate of change of the refractive index difference. An historical note is that Fejer and Bramley were studying radio wave propagation through an irregular ionosphere.

The geometry of the problem is shown in Figure 9. The random medium is composed of weak, statistically homogeneous irregularities in refractive index and is contained between two parallel planes at $z=0$ and $z=L$. According to Bramley, if a plane wave is incident at $z=0$, we only need to consider changes of phase along paths such as PP' which are parallel to the direction of propagation of the incident wave. If the refractive index within the region is $n(x,y,z)$ and the plane wave enters with zero phase, then the phase across the exit plane ($z=L$) is

Figure 9. The geometry of the multiple scattering medium.

$$\phi(x, y) = -k_o \int_0^L n(x, y, z) dz, \quad (4.37)$$

where k_o is the free space phase constant and an $\exp(i\omega t)$ time dependence has been dropped for convenience.

The exit plane is then a radiating aperture with a field distribution

$$E(x, y) = E_o \exp[i \phi(x, y)], \quad (4.38)$$

where E_o is the incident amplitude. The two-dimensional (2-d) angular plane wave spectrum $A(S_1, S_2)$ is the 2-d Fourier transform of $E(x, y)$. Essentially, the angular spectrum is the decomposition of radiation from an aperture into plane wave amplitudes as a function of direction cosines. It is given by

$$A(S_1, S_2) = \frac{1}{\lambda^2} \iint_{-\infty}^{\infty} E(x, y) \exp[i k_o (S_1 x + S_2 y)] dx dy,$$

where $\lambda = 2\pi/k_o$ is the free space wavelength and S_1 and S_2 are the direction cosines [$S_1 = \sin(\theta)\cos(\phi)$; $S_2 = \sin(\theta)\sin(\phi)$ in the usual spherical coordinate geometry].

For the aperture distribution of Equation (4.38),

$$A(S_1, S_2) = \frac{E_o}{\lambda^2} \iint_{-\infty}^{\infty} \exp[i \phi(x, y)] \exp[i k_o (S_1 x + S_2 y)] dx dy, \quad (4.39)$$

where the average value is

$$\begin{aligned}
\langle A(S_1, S_2) \rangle &= \frac{E_o}{\lambda^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \exp [i \phi(x, y)] \rangle \exp [i k_o (S_1 x + S_2 y)] dx dy \\
&= E_o C(1) \delta(S_1) \delta(S_2) . \quad (4.40)
\end{aligned}$$

$C(u)$ is the characteristic function of the random variable ϕ and $\delta(\cdot)$ is the delta function. The average radiated field is a plane wave propagating in the same direction as the incident wave and is the coherent part of the transmitted field [the average value of the incoherent part is zero]. The transmission coefficient is the transmitted coherent power divided by the incident power:

$$T = \frac{|\langle A(S_1, S_2) \rangle|^2}{E_o^2} = |C(1)|^2 \quad (4.41)$$

The first order statistics of ϕ must be known to calculate T . If ϕ obeys a Gaussian distribution with zero mean and variance σ_ϕ^2 , then

$$T = \exp(-\sigma_\phi^2) \quad (4.42)$$

and the coherent transmission is 4.34 σ_ϕ^2 dB (decibels).

Clarke argues that adding the incoherent power to the coherent power to get the total power would only add a minor correction to estimates of filter bandwidth. The incoherent power is determined from the second order statistics of ϕ . If the aperture of the observing instrument is small, most of the scattered radiation is not detected and the incoherent power is small when compared with the coherent power in the forward direction at λ_c . What remains to be determined is the phase variance through the suspension.

Consider the geometry of a ray path through a spherical particle in Figure 10. If the particles are randomly positioned and a ray path intersects a particle, the lateral displacement of the ray from the particle center will be a random variable and, hence, the ray path length ζ through the particle will also be a random variable. Assuming that the displacements ξ and ν in Figure 10 are both independent, uniformly distributed random variables bounded by $\pm\alpha$ (α is the radius of the sphere), Clarke shows that the average path length is

$$\langle \zeta \rangle = \left(\frac{\pi^2 \alpha}{8} \right) = 1.235 \alpha \quad (4.43)$$

with standard deviation:

$$\sigma_{\zeta} = 0.506 \alpha . \quad (4.44)$$

If the relative particle concentration (particle volume/total volume) is c for a CF whose width along the beam direction is L , the average length of particulate material traversed by a ray is

$$l = cL . \quad (4.45)$$

Therefore, the average number of particles intersected by a ray is

$$\langle N_s \rangle = \frac{l}{\langle \zeta \rangle} = \frac{8 c L}{\pi^2 \alpha} . \quad (4.46)$$

It was assumed that the particles have the same radii. If not, replace α in (4.46) with the average radius $\langle \alpha \rangle$. Clarke attributes the statistical phase fluctuations to fluctuations in the particle number (N_s), the path length through each particle (ζ), and the particle size ($\langle \alpha \rangle$). We will examine each of these types of fluctuations in turn.

Figure 10. The geometry of a ray path through a spherical particle.

The number N_* along any line through an assembly of randomly positioned particles will have a Poisson distribution which is defined by the parameter $\langle N_* \rangle$. The characteristic function of a Poisson-distributed random variable dN_* , where d is a constant, is

$$C(u) = \langle \exp(iudN_*) \rangle = \exp\{\langle N_* \rangle [\exp(iud) - 1]\} . \quad (4.47)$$

The phase of the rays at the exit plane of the CF relative to their phase if no particles were present is

$$\phi = N_* \langle \zeta \rangle k_o \Delta n = N_* \left(\frac{\pi^2 \alpha}{8} \right) \left(\frac{2\pi}{\lambda} \right) \Delta n \quad (4.48)$$

Thus, the constant is (from 4.48)

$$d = \pi^3 \alpha \Delta n / 4\lambda , \quad (4.49)$$

where Δn is the difference in refractive index between the particles and the medium.

From (4.40) and (4.47), the average transmitted field is

$$C(1) = \langle \exp(i\phi) \rangle = \exp\{\langle N_* \rangle [\exp(id) - 1]\} . \quad (4.50)$$

The average number $\langle N_* \rangle$ will be large and $d \sim 1$, so equations (4.41) and (4.50) give the transmission coefficient as

$$T \doteq \exp(-\langle N_* \rangle d^2) \quad (4.51)$$

and the power attenuation will be

$$4.343 \langle N_* \rangle d^2 = 211.5 [cL\alpha(\Delta n)^2 / \lambda^2] \text{ dB} . \quad (4.52)$$

Close to the Christiansen wavelength λ_c , Δn can be written in terms of the differential rate of change with respect to wavelength ($\Delta n'$) and the wavelength departure from λ_c :

$$\Delta n = \Delta n' \Delta \lambda ,$$

where

$$\Delta n' = \left(\frac{d}{d\lambda} \right) (n_{medium} - n_{particle}) ; \quad \Delta \lambda = (\lambda - \lambda_c) .$$

The power attenuation is

$$211.5 \text{ cL}\alpha (\Delta n')^2 (\Delta \lambda / \lambda_c)^2 , \quad (4.53)$$

and the 3-dB bandwidth is

$$(\Delta \lambda)_{3dB} = [0.238 / (\text{cL}\alpha)^{1/2} (\Delta n')] \lambda_c . \quad (4.54)$$

Now suppose that the fluctuations in the overall path length are caused by the ray passing through different parts of the spherical particles. In this case, the final phase of the ray is the sum of a large number of random, independent paths through individual particles. By the Central Limit theorem⁴⁵, the phase across the output plane will be Gaussian-distributed. Assuming the average phase across the plane equal to zero, the variance is given by

$$\sigma_\phi^2 = \langle N \rangle \sigma_\zeta^2 k_o^2 (\Delta n)^2 . \quad (4.55)$$

Equations (4.55), (4.46), and (4.44) give

$$\sigma_\phi^2 = 8.2 \text{ cL}\alpha (\Delta n)^2 / \lambda^2 , \quad (4.56)$$

and the transmission loss is $4.343 \sigma_\phi^2$ dB. This loss is one-sixth of the loss given by the fluctuation in particle number (4.53). Thus, it only has a second order effect on the phase variance.

The last possible source of a variance in the phase is a fluctuation in particle size. The effect on the final phase can be isolated by assuming that the ray encoun-

ters the average number $\langle N \rangle$ of particles and the path length through each particle is the mean value for particles of a certain radius. Then, the phase fluctuation of the emerging rays will be due solely to random changes in radius from particle to particle. This means we replace the radius α in (4.43) and (4.46) with the average radius $\langle \alpha \rangle$:

$$\langle \zeta \rangle = \pi^2 \langle \alpha \rangle / 8, \quad \langle N \rangle = 8cL / \pi^2 \langle \alpha \rangle. \quad (4.57)$$

If we define

$$\sigma_{\alpha}' = \sigma_{\alpha} / \langle \alpha \rangle \quad (4.58)$$

where σ_{α} is the standard deviation in particle radius, the variance of the total path length through $\langle N \rangle$ particles is

$$\langle N \rangle (\sigma_{\alpha}')^2 \langle \zeta \rangle^2 = \langle N \rangle (\sigma_{\alpha}')^2 \left(\frac{\pi^2 \langle \alpha \rangle}{8} \right)^2 \quad (4.59)$$

The phase is also Gaussian-distributed from the Central Limit theorem, so the phase variance across the output plane is given by equation (4.55):

$$\sigma_{\phi}^2 = \langle N \rangle (\sigma_{\alpha}')^2 \left(\frac{\pi^2 \langle \alpha \rangle}{8} \right)^2 k_o^2 (\Delta n)^2 = \left(\frac{\pi^4 (\sigma_{\alpha}')^2}{2} \right) \left[\frac{cL \langle \alpha \rangle (\Delta n)^2}{\lambda^2} \right]. \quad (4.60)$$

The transmission loss is $4.343 \sigma_{\phi}^2$ dB. When this loss is compared to (4.52), the loss from particle number fluctuations only, they are in ratio $\sigma_{\phi}^2:1$, so size fluctuations may not be a second order effect.

In summary, the transmission coefficient for the coherent power of a plane wave passing through a CF is of exponential form [from (4.42)]:

$$T = \exp[-kcL \langle \alpha \rangle (\Delta n')^2 (\Delta \lambda / \lambda_c)^2]. \quad (4.61)$$

The factor c is the relative particle concentration (dimensionless), L is the width of

the CF, $\langle \alpha \rangle$ is the average particle radius, $\Delta n'$ is the derivative of Δn ($n_{\text{medium}} - n_{\text{particle}}$) with respect to wavelength, λ_c is the Christiansen wavelength and $\Delta \lambda$ is the wavelength shift. The constant k depends on the mechanism which produces degradation in phase coherence:

- particle number fluctuations: $k_n = (\pi^4 / 2) = 48.7$ (4.62a)

- path length through individual spherical particles: $k_p = 8.2$ (4.62b)

- particle radius fluctuations: $k_r = 48.7 (\sigma_{\alpha'})^2$ (4.62c)

For a uniform distribution of particle sizes, we can use

$$(\sigma_{\alpha'})^2 = \frac{1}{3}[(\alpha_{\max} - \alpha_{\min}) / (\alpha_{\max} + \alpha_{\min})] . \quad (4.63)$$

The fractional bandwidth between the half-power points is

$$\left[\frac{(\Delta \lambda)_{3dB}}{\lambda_c} \right] = \left(\frac{2.77}{k} \right)^{\frac{1}{2}} \left[\frac{1}{(cL\alpha)^{\frac{1}{2}} (\Delta n')} \right] \quad (4.64)$$

This dissertation will compare predicted 3 dB bandwidths with measured values from our CFs. An attempt will be made to alter the Clarke theory to accommodate cylindrical particle shapes and compare the estimated bandwidths with the measured data. The Clewell formulas [(4.35) and (4.36)] will be checked against transmission measurements at $\lambda_s \neq \lambda_c$.

Note that to minimize the bandpass width in (4.64), the derivative needs to be maximized. This implies that the intersection between the dispersion curves of the host medium and particles should be as orthogonal or steep as possible. Plastic polymers are relatively dispersive - $dn/d\lambda$ is large in the visible λ 's - when compared

to most silica-based glasses. In the NIR region, both plastics and glasses remain transparent and become much less dispersive ($dn/d\lambda \rightarrow 1$). A significant drawback to this research is that the location or even mere existence of a spectral bandpass is heavily dependent upon material availability. A computerized search for dispersion curve intersections was conducted and is the subject of the next section.

Section 5: Candidate Materials Search

A search for possible Christiansen filter components was conducted on a computer. Refractive index data for the following materials were entered into the computer: 12 optical plastics files covering polycarbonate, polystyrene, SAN, acrylic, NAS, and plexiglass;^{46,47} 296 optical glass files from the Schott catalog;⁴⁸ and 79 files of optical materials tabulated by Palik⁴⁹ and by the Handbook of Optics.⁵⁰ The wavelength ranges of the data files were 0.3 - 1.0 μm for the plastics, 0.3 - 3.0 μm for the glasses, and 0.3 - 30.0 μm for the other optical materials.

The 74,691 comparisons between the 387 files compelled us to use a computer to search for intersections among the index dispersion data. The computer was a 33 Mhz IBM compatible 80386 machine with 4 Mbytes of RAM and 200 Mbytes of hard disk memory. In order to take advantage of CPU speed and RAM capacity and minimize writing to the hard disk, Microsoft SmartdriveTM was installed to recover 3 Mbytes of extended memory. All of the index files were combined into one large file which was loaded into extended RAM with the searching program. Information about discovered intersections was also stored in RAM during program execution. After the search was completed, the intersection data was then dumped to the hard disk.

The search program operates in the following manner. Two index data files are compared to each other by reading the wavelengths of one file and linearly interpolating the index dispersion data of the other file at those wavelengths. The differences in the indices of the two files are calculated at each wavelength and the

signs of the differences are tracked. A change in the index difference sign indicates an intersection of the dispersion curves. The names of the materials, the wavelength range in which the intersection occurs, and the change in index difference with respect to wavelength $[d(n_p - n_h)/d\lambda]$ is stored in memory for each detected intersection. The latter quantity is useful for estimating the full width at half maximum (FWHM) of the transmission bandpass (see Eq. 4.64). To conserve RAM space, only the Schott dispersion coefficients of the glass files are read by the program which then generates the dispersion data during each comparison.

The search took 8 hours to complete and uncovered 12,792 intersections! Additional programs were written to sort the intersections according to material type, location of intersection wavelength, and FWHM. Table 1 lists the glass-plastic intersections, Table A-1 lists intersections among the glass files, Table A-2 lists intersections among the other optical materials and Table A-3 contains the intersections with the narrowest FWHM for all of the files. These tables list the candidate materials increasing in FWHM for the following ranges; 0.3 - 0.4 μm , 0.4 - 0.5 μm , 0.5 - 0.6 μm , 0.6 - 0.7 μm , 0.7 - 0.8 μm , 0.8 - 0.9 μm , 0.9 - 1.0 μm , 1.0 - 3.0 μm , 3.0 - 5.0 μm , 5.0 - 8.0 μm , 8.0 - 10.0 μm , 10.0 - 12.0 μm , 12.0 - 30.0 μm . The number of intersections in each wavelength range is limited to 25. The values for FWHM were estimated with reference to the average measured FWHM of our SK5 - SAN Christiansen filters described in Section 8. Thus, these values assume the same concentration, particle size, and filter thickness used for our SK5 - SAN CFs and are only approximate. The Christiansen wavelengths of the combinations in these tables

were assumed to be the midpoints of the listed wavelength ranges. Table 1 is included in this section while tables A-1 - A-3 are relegated to Appendix A.

The first item one notices when examining these tables is that the FWHM values increase with the wavelength. The smallest widths occur in the deep purple to ultraviolet wavelengths. This is due to the steep rise in index with decreasing wavelength immediately before the anomalous dispersion region coincident with a region of high absorption. Steeper index dispersion lead to more orthogonal intersections and smaller widths; all optical materials have absorption regions in the ultraviolet spectrum. The steep index dispersions associated with absorption bands evoke a word of caution about these tables. No allowance was made for absorption bands in the calculation of these tables; some of the narrowest bandpasses probably lie in absorption regions of one or both materials which could render the filter opaque. This may not be a problem if only one material is absorbing. If the absorbing material can be made into particles and embedded in the non-absorbing material, the filter would exhibit a transmission bandpass.

Table 1 contains many combinations in the 0.35 to 0.4 μm region (intersections lying below 0.35 μm were excluded). As implied earlier, we chose to study the SK5-SAN glass-plastic CF. A better choice would have been any of the glass-polycarbonate filters since PC transmits more light in the NIR than SAN, but we decided to develop SAN and polystyrene matrix filters. For our NIR edge filter, we chose the SK11-SAN combination with an intersection near 0.79 μm . In retrospect,

Table 1. Potential glass - plastic Christiansen filters

MATERIAL #1	MATERIAL #2	WAVELENGTH RANGE 10 ⁻⁶ m		FWHM 10 ⁻⁶ m
POLYCARBONATE	SK19	.3650	.4047	.04855
POLYCARBONATE	SK6	.3650	.4047	.04903
POLYCARBONATE	SK9	.3650	.4047	.04978
POLYCARBONATE	SK7	.3800	.4000	.05204
POLYCARBONATE	SK3	.3800	.4000	.05251
POLYCARBONATE	SK4	.3800	.4000	.05289
POLYCARBONATE	SK2	.3800	.4000	.05401
POLYCARBONATE	SK1	.3800	.4000	.05416
POLYCARBONATE	SK8	.3800	.4000	.05483
POLYCARBONATE	BAF52	.3650	.4047	.05689
POLYCARBONATE	BAF7	.3650	.4047	.05691
POLYCARBONATE	BAF4	.3650	.4047	.05967
POLYCARBONATE	BAF5	.3800	.4000	.06214
POLYCARBONATE	BASF5	.3650	.4047	.06159
POLYSTYRENE	SK19	.3650	.4047	.06170
POLYSTYRENE	SK6	.3650	.4047	.06246
POLYSTYRENE	SK9	.3650	.4047	.06370
POLYCARBONATE	F8	.3650	.4047	.06671
POLYSTYRENE	SK4	.3800	.4000	.06860
POLYSTYRENE	SK1	.3800	.4000	.07075
POLYSTYRENE	SK8	.3800	.4000	.07189
SAN	PSK51	.3800	.4000	.07428
POLYSTYRENE	BAF52	.3650	.4047	.07581
POLYSTYRENE	BAF7	.3650	.4047	.07586
POLYSTYRENE	BAF4	.3650	.4047	.08084
SAN	SK12	.3800	.4000	.08409
POLYSTYRENE	BAF5	.3800	.4000	.08500
SAN	BALF4	.3800	.4000	.09590
SAN	SK5	.3600	.3650	.09022
SAN	LGSK2	.3650	.3800	.09800
POLYCARBONATE	SK14	.4047	.4200	.09442
POLYCARBONATE	SSK51	.4000	.4047	.10686
POLYSTYRENE	SSK51	.4047	.4358	.11577
POLYSTYRENE	SK7	.4047	.4200	.11724
POLYSTYRENE	SK3	.4000	.4047	.11612
POLYSTYRENE	SK2	.4000	.4047	.12215

TABLE 1 (continued)

POLYCARBONATE	KZFSN9	.4047	.4200	.13059
POLYSTYRENE	PSK52	.4400	.4600	.16090
POLYCARBONATE	PSK52	.4200	.4358	.15464
POLYCARBONATE	KZFS6	.4358	.4800	.16964
POLYSTYRENE	SK14	.4358	.4400	.19937
POLYSTYRENE	KZFSN9	.4400	.4600	.25397
SAN	BAK6	.4400	.4600	.26280
POLYCARBONATE	SK13	.4861	.5000	.33620
SAN	BALF51	.4400	.4600	.30947
SAN	BAK1	.4600	.4800	.35520
POLYSTYRENE	TIF5	.4358	.4800	.40898
POLYSTYRENE	F8	.4400	.4600	.47407
SAN	LF1	.4047	.4200	.46676
SAN	BALF3	.4861	.5000	.70373
SAN	BALF2	.4861	.5000	.81572
POLYCARBONATE	LF2	.4861	.5000	.92715
SAN	KZF4	.4861	.5000	1.03716
POLYCARBONATE	LGSK2	.5890	.6000	.23669
POLYCARBONATE	SK5	.5200	.5400	.26676
LEXAN	PSK51	.5600	.5800	.29040
LEXAN	SK13	.4861	.5893	.28395
POLYCARBONATE	BALF6	.5200	.5400	.30425
POLYSTYRENE	SK13	.5600	.5800	.33283
LEXAN	SK5	.5400	.5600	.32240
POLYCARBONATE	BALF50	.5200	.5400	.31453
POLYCARBONATE	BALF7	.5200	.5400	.31648
POLYCARBONATE	BAF6	.5200	.5400	.33546
POLYCARBONATE	BAFN6	.5200	.5400	.33688
LEXAN	BALF6	.5400	.5600	.36817
POLYSTYRENE	KZFS6	.5600	.5800	.39104
LEXAN	BALF50	.5400	.5600	.38079
LEXAN	BALF7	.5400	.5600	.38308
POLYCARBONATE	PSK51	.5461	.5600	.43248
LEXAN	BAF6	.5200	.5400	.46298
LEXAN	BAFN6	.5200	.5400	.46569
SAN	BAF2	.5200	.5400	.50373
NAS	SK11	.5890	.6000	.59833
NAS	BAK3	.5600	.5800	.57958
LEXAN	LGSK2	.5800	.5893	.67866

TABLE 1 (continued)

SAN	PSK2	.5461	.5600	.67639
SAN	BAK4	.5461	.5600	.83999
NAS	LF8	.5600	.5800	.89349
POLYCARBONATE	SK12	.6000	.6200	.23619
POLYCARBONATE	BAF3	.6200	.6400	.25829
POLYCARBONATE	LF3	.6200	.6400	.27088
POLYCARBONATE	LF5	.6200	.6400	.27478
POLYCARBONATE	TIF4	.6000	.6200	.30234
POLYSTYRENE	PSK51	.5890	.6510	.52566
NAS	BALF1	.6200	.6400	.61776
POLYSTYRENE	SK5	.6200	.6400	.67949
POLYSTYRENE	BALF6	.6200	.6400	.69788
POLYSTYRENE	BALF50	.6200	.6400	.72247
POLYSTYRENE	BALF7	.6200	.6400	.72479
POLYSTYRENE	BAF6	.6200	.6400	.76918
POLYSTYRENE	BAFN6	.6200	.6400	.77573
POLYSTYRENE	LF2	.5890	.6510	.83736
LEXAN	SK12	.6328	.6563	.96813
POLYSTYRENE	BALF7	.6400	.6439	1.84466
SAN	LF6	.6000	.6200	2.61475
POLYSTYRENE	LGSK2	.7400	.7600	2.25458
SAN	SK11	.7800	.8000	2.96069
SAN	BAK3	.7400	.7600	4.39526
POLYCARBONATE	BALF4	.9200	.9400	7.79149

the BALF4 - Polycarbonate filter near $0.93\ \mu\text{m}$ may have been a better choice since the intersection is farther from the visible wavelengths than the intersection for SK11-SAN; this filter would scatter visible light more efficiently. Nevertheless, we investigated SK5-SAN, SK11-SAN, SK5 - Polystyrene (green) and BAK4-SAN (yellowish-green). The dispersion curves of these combinations are shown in Figure 11.

Table A-1 in Appendix A contains data for narrowband, visible wavelength CFs composed of glass particles in glass matrices. Such filters could withstand more heating from intense sources than could the glass-plastic filters. Manufacture would be difficult; perhaps cooled particles could be injected into a molten glass matrix while the matrix is quickly cooled to solidification. Crazing would probably occur throughout the filter, however Table A-1 is available to anyone who is willing to try.

Table A-2 is the result from searching among the known optical constants of 79 optical or coating materials. We were interested in CF combinations with bandpasses occurring in the atmosphere's "optical windows" [$3\text{-}5\ \mu\text{m}$, $8\text{-}12\ \mu\text{m}$]. The table lists 57 possible combinations which must be checked for high absorption in those "window" regions. Manufacture of these filters would be subject to much experimentation; perhaps powders could be pressed between thin sheets. One application could be the passive detection of infrared sources in the atmosphere (aircraft, missiles, etc.) that emit in the "optical windows". For example, Wolfe⁵¹ shows intense radiant emissions from the tail of an aircraft between 3.5 and $4.1\ \mu\text{m}$ and between 4.35 and $4.45\ \mu\text{m}$. Table A-2 lists an Iridium - Lead Sulfide intersection between 3.54 and $3.75\ \mu\text{m}$, an

Figure 11. Dispersion curves of three optical glasses and two optical plastics.

Iridium - Germanium intersection between 3.2 and 3.54 μm and a Silver - Silicon Carbide intersection between 4.13 and 4.43 μm . Iridium and silver are highly absorbing and could be powdered. Lead Sulfide and Silicon Carbide have extinction coefficients k approximately⁵² 5×10^{-4} , which is probably still too absorbing (21% transmission through a 1 mm thick sample). Germanium has a k value less than 10^{-5} between 3.2 and 3.5 μm and would make an excellent matrix (>96% transmission through a 1 mm thick sample). Table A-3 lists the narrowest bandpasses among all 387 materials. Most of them probably lie in absorption bands. Having chosen our glass-plastic CFs, we needed to verify the index data for the plastic matrices. This is the topic of the next section.

Section 6: Index Dispersion Verification

During our research, we recognized the necessity of confirming the optical properties of our polymers and qualifying new polymers for CF applications. Optical dispersion data is not generally available for polymers from commercial suppliers. At most, they list indices at three wavelengths only. Limits upon funding led us to construct a Pulfrich refractometer from parts found in our laboratory rather than purchasing a commercial instrument. Difficulties in developing a reliable instrument limited the number of measurements, but we were able to confirm the dispersion properties of optical grade plastic samples under certain conditions. This chapter will review critical angle refractometry, describe our Pulfrich refractometer, and discuss our measurements.

When Snell's law,

$$N \sin(\chi) = N' \sin(\chi'), \quad (6.1)$$

is applied to the case of grazing incidence ($\chi = 90^\circ$), we see that χ' has a real value if N' is greater than N . A critical angle χ' can then be defined as

$$\sin(\chi') = N/N'. \quad (6.2)$$

Referring to Figure 12, let N_s be the refractive index of the sample to be tested; let N_p be the index of the block with prism angle A ; and let N_a be the index of the final medium into which the light emerges. Considering a grazing incident ray and applying Snell's law at the two surfaces in turn:

Figure 12. Geometry of critical angle refractometry.

$$N_s \sin(90^\circ) = N_p \sin(\chi) \quad (6.3)$$

$$N_p \sin(\chi-A) = N_s \sin(\beta) \quad (6.4)$$

$$N_s/N_p = \sin(\chi) \quad (6.5)$$

$$\sin(\chi-A) / \sin(\beta) = N_s/N_p \quad (6.6)$$

Transforming equation (6.5)⁵³,

$$N_s/N_p = \sin(\chi) = \sin(A+\chi-A) = \sin(A)\cos(\chi-A) + \cos(A)\sin(\chi-A), \quad (6.7)$$

and substituting (6.6),

$$\frac{N_s}{N_p} = \sin(A) \sqrt{1 - \left(\frac{N_s}{N_p} \sin(\beta) \right)^2} + \cos(A) \frac{N_s \sin(\beta)}{N_p} . \quad (6.8)$$

The exit medium is usually air so $N_s = 1$ and

$$N_s = \sin(A) \sqrt{N_p^2 - \sin^2(\beta)} + \cos(A) \sin(\beta) . \quad (6.9)$$

In a Pulfrich refractometer, the testing block is polished such that angle A is 90° to high accuracy. Equation (6.9) then simplifies to

$$N_s = \sqrt{N_p^2 - \sin^2(\beta)} . \quad (6.10)$$

The principle of the Pulfrich refractometer appears in Figure 13. The substance whose index is to be measured is placed on top of or clamped to a glass block of known refractive index (the Pulfrich block). In the case of a solid to be measured, a thin layer of liquid whose refractive index is greater than the solid's index is placed between the two surfaces. This is done to prevent total internal reflection of the rays by an air layer.

If light enters the sample from position L , that entering above the line LO will

Figure 13. The principle of the Pulfrich refractometer.

enter the Pulfrich block and follow a path like NP. A telescope placed at P would see a band of light with a sharp boundary edge on the upper side. The limit of this band of light corresponds to the grazing rays; any ray entering below line LO will not be able to enter the Pulfrich block at all. The angle β of this boundary is directly related to the critical angle which depends solely on the indices of the block and the sample (Equation 6.10).

Traditional Pulfrich refractometers usually contain a hydrogen tube and/or sodium lamp as the light source and a telescope mounted on a rotary table with vernier scaling to measure the exit angle. A table with calibrated values of index versus angle is provided so that the sample index can be read quickly. The telescope is usually autocollimating so that the normal to the exit face of the Pulfrich block (the "zero" angle line) may be obtained by back reflection. Some instruments include a system for water circulation and a thermometer in order to provide temperature control of the sample.

Most instrument makers agree that the contacting surfaces should be well-polished and thoroughly cleaned before measurements are taken. The edge at which the sample and block join should be very sharp. The surfaces should be accurately parallel to each other and interference (Fizeau) fringes should be observed in the index matching fluid when illuminated by monochromatic light. These fringes should be made as broad as possible by pressing on the sample and not more than six fringes should be observed for optimum measurement conditions. Martin⁵⁴ recommends that the index of the matching fluid be greater than but close to the index of

the sample. He calculates that a one minute error in the angle measurement would cause an error in the fourth decimal place of the sample index.

Our measurement requirements posed several problems. We were interested in measuring the plastic indices for the red and NIR wavelengths so a telescope would be unacceptable. The temperature of the plastic sample under measurement is important since the thermo-optic coefficients (dn/dT) of plastics are typically one to two orders of magnitude greater than those of glasses. Finally, the index matching fluid should not chemically attack nor soften the plastic sample. This last problem would affect some of our measurements.

It would be easy to purchase a precision engineered Pulfrich refractometer from a reputable manufacturer (if one can be found) and achieve very accurate results; it is quite another story trying to assemble one from spare parts found in a laboratory! The basic challenge of all optical instruments is choosing a light source that emits the desired wavelengths with the required intensity, selecting optical elements that sufficiently transmit the spectral range and direct the light to the proper positions and finding a detector with enough responsivity at the needed wavelengths to produce a strong, recordable signal. After much trial and error, we achieved a VIS-NIR Pulfrich refractometer that measured indices to an error in the third decimal place under optimum conditions.

A diagram of our Pulfrich refractometer is shown in Figure 14. The light source is a 300 Watt tungsten-halogen General Electric multi-mirror projector lamp housed in an air-cooled chamber that is clamped to an optical bench. Two condenser lenses

and a third lens collect and focus the beam onto the entrance slit of a long collimator. The collimated light illuminates a polished Pulfrich block made of SF6 (Schott designation) glass ($n_D \approx 1.8$). The edge between the contacting and exit surfaces was polished to an angle of 90 ± 0.03 degrees. An aluminum clamp with nylon screws holds the sample against the block. The refracted light is detected by a Fairchild CCD122 linear CCD array containing 1728 elements ($13\mu \times 13\mu$ size). This detector enabled us to measure indices at both VIS and NIR wavelengths. The digital outputs from the CCD were recorded on floppy disks and displayed on a monochrome monitor by an IBM PC. The Pulfrich block and detector were mounted on a massive Societe Genovaise rotary table with an angle measuring vernier (to seconds of degree) assembly. A series of narrow bandpass (FWHM ≈ 10 nm) filters that span the VIS and NIR from Microcoatings, Inc. were placed in the beam before the collimator to obtain the dispersion data. The entire assembly before the Pulfrich block was covered with thick, black cloth to prevent white background light from entering the detector.

The measurement of angle β is the key to determining the index. Since we could not use an autocollimating telescope, the light incident upon the sample had to be well collimated to get the zero angle line. Ideally, to measure β , the rotation axis of the detector arm should be aligned with the point where the light exits the Pulfrich block (Point N in Figure 13). To account for angular errors, we clamped a small reference glass (K7) next to the sample to be measured. Measuring the angle β for the reference glass and calculating the expected angle gave us the inherent angular

Figure 14. Pulfrich refractometer for measuring n as a function of λ . Light from a tungsten-halogen lamp is condensed, collimated, and passed through a narrowband filter. The beam is then directed at the sample which is clamped to a Pulfrich prism. The critical angle boundary is detected by a CCD array attached to a rotary arm. The angle of the boundary is read from the rotary table vernier assembly.

correction for the system.

The procedures for our measurements were as follows:

- Index matching fluid ($n_D \approx 1.7$) was applied to the cleaned surface of the Pulfrich block.
- Both the reference glass and the sample were clamped to the block with a "small space" between them. Their leading edges were aligned with a ruler.
- The CCD detector was pushed close to the block and centered in the beam. The CCD was shifted horizontally to place the shadow edge of the "small space" in the middle of the array. This located the edge of block and set the zero angle line. Care was taken not to disturb the block after this step.
- Data was taken for the reference glass with each narrowband filter and then repeated for the plastic sample. The detector was rotated so that the critical refraction edge fell upon the same element of the CCD that set the zero angle line. A thermometer was placed upon the sample to measure its temperature.
- The difference between the measured angle and the calculated angle for the reference glass was used to correct the measured angle for the sample at each wavelength.

This technique was tested with a BK7 glass sample. Table 2 shows the published indices for the Pulfrich block (SF6), the reference glass (K7), and the test sample (BK7) at each wavelength⁵⁵. Temperature correction would affect the fifth decimal place only. The fifth column shows the measured indices for the BK7 sample. We see

Table 2. Refractive indices for SF6, K7, and BK7 (Schott and measured)

λ (nm)	n (SF6)	n (K7)	n (BK7)	n (BK7)meas
589	1.804954	1.511045	1.516739	1.516087
600	1.803275	1.510592	1.516294	1.515955
650	1.796791	1.508746	1.514521	1.513764
700	1.791744	1.507243	1.513065	1.511958
900	1.779379	1.503144	1.509001	1.509194

Table 3. Measured refractive indices for six optical plastics.

λ (nm)	Lustran	Tyrol	Exp	Styron	Lex 121	Lex141
500	1.5777	1.5774	N/A	1.5944	N/A	N/A
550	1.5704	1.5702	1.5692	1.5866	1.5806	N/A
589	1.5667	1.5660	1.5644	1.5825	1.5756	1.5609
600	1.5649	1.5651	1.5637	1.5817	1.5745	1.5599
650	1.5626	1.5625	1.5604	1.5779	1.5706	1.5565
700	1.5603	1.5608	1.5593	1.5754	1.5679	1.5544
900	1.5525	1.5549	1.5543	1.5674	N/A	1.5476

errors in the fourth and third decimal places with an average error of -0.0005324.

Encouraged by this result, we measured the dispersion for samples of styrene-acrylonitrile (SAN), polystyrene (PS), and polycarbonate (PC). Table 3 shows our measured results for Monsanto Lustran™ (SAN), Dow Tyril™ (SAN), Dow Exp™ (SAN), Dow Styron™ (PS), GE Lexan 121™ (PC), and GE Lexan 141™ (PC). The temperature for all of these measurements was 24°C. Figure 15 plots the measured SAN data with dispersion data for generic SAN tabulated by Lytle⁵⁶. We see good agreement between the data sets. Figure 16 shows the measured data from two PS samples plotted with Lytle data for PS. Our measurements are almost 0.01 below the Lytle curve. Our PC measurements are even farther away from Lytle data. Figure 17 plots our data with Lytle results and data from GE's brochure for these Lexan™ types.

We noted that the PS and PC samples bonded to the Pulfrich block during the measurements. In fact, it was difficult to remove them afterwards. It appears that the index matching liquid chemically reacted with the polymers causing them to adhere to the Pulfrich block. This effect probably altered the index measurements by creating a layer inhomogeneous in index next to the block. The Lustran™ (SAN) and Tyril™ (SAN) did not bond to the glass and we obtained good measurements. The Exp™ (SAN) bonded slightly but did not affect our results very much. The thermo-optic coefficients for these polymers are approximately $100 \times 10^{-6}/^{\circ}\text{C}$. To account for the discrepancies, the temperature difference between our measurements and the published results must be approximately 100°C. There was no temperature listed for the

Figure 15. Measured dispersion curves for three brands of SAN compared with data from Lytle.

Figure 16. Measured dispersion curves for Styron PS compared data from Lytle.

Figure 17. Measured dispersion curves for Lexan 121 and Lexan 141 compared with data from Lytle and General Electric.

Lytle data but the GE data for Lexan™ was taken at 20°C and is similar to the Lytle data. Therefore, temperature correction cannot account for the measurement differences.

We managed to verify the dispersion data for the SAN polymer used in our CFs. To measure other polymers in our Pulfrich refractometer, non-reacting index matching fluids are required. Measuring the optical dispersion of the glass that went into our filters was unnecessary; it was provided by the glass supplier whenever we ordered glass. The next section is a primer on plastics and the technology for manufacturing plastic products.

Section 7: Plastics and Molding Technology

In this section, I review the amazing world of plastics in general and optical plastics in particular. These materials have truly changed the appearance of the entire world and have been an economic boon to mankind. I cannot begin to adequately cover the science, immense technology and applications that have evolved in the plastics industry. What I propose to do is introduce the basics of polymer science, describe optical polymers and list the methods for forming plastic products.

What are plastics? The American Society for Testing Materials has provided this description: "A plastic is a material that contains as an essential ingredient an organic substance of large molecular weight, is solid in its finished state, and at some stage in its manufacture or in its processing into finished articles, can be shaped by flow". The term "resin" may be used synonymously with "plastic" but it usually refers to the primary ingredients before final processing and fabrication⁵⁷.

Most plastics are synthetically derived from petroleum, coal, natural gas, salt, air, and water. A few occur naturally, for example, rubber, gutta percha, and some types of vegetable waxes. The molecules of plastics are composed primarily of carbon, hydrogen, oxygen, nitrogen and, in some inorganic-organic combinations, silicon (silicones). The properties of plastics depend largely upon the size, shape, and organization of the molecules of which they are composed. In the early stages of manufacture, most plastics are monomers, that is, they consist of small, single molecules. Under the influence of heat and pressure or of chemical catalysts, these small molecules

combine to form complex molecular chains which become solid or semi-solid structures (polymers)⁵⁸.

Before 1930, most industrial goods and household products were made of metals, wood, papers, glass, leathers, and vulcanized natural rubber. Since then, plastics have replaced these materials in many applications and have created vast new markets of their own. Conventional engineering designs for load-bearing elements are based on allowable deflection by yielding or failure by breaking. The fundamental measurement for deflection under load is the elastic modulus, E (Young's modulus), defined as the tensile stress required to produce 100% strain. Most materials yield long before 100% strain, so the ratio of the measured stress for small strain is used for E . Steels have moduli of 29 million lb/sq-in and yield strengths of 60 thousand lb/sq-in. This is 60 times the modulus and 8 times the yield strength of polystyrene (PS), a common rigid plastic. In absolute terms, plastics cannot compare to steels as load-bearing elements; however, if deflection in bending and material weight are important in a design, the materials become comparable. Stiffness in bending depends upon modulus and volume; if weight is also a design consideration, then the ratio of modulus to density (E/d) or strength to density (S/d) are more relevant parameters. The ratio E/d for PS is only 1/8 that of steel and S/d for PS is equivalent to that of steel.

A drawback to plastics is that they tend to creep under load at or above room temperatures. More simply put, this means that plastics subjected to stresses at 1/3 or more of their yield strengths will gradually stretch (flow) in the direction of the stress over time. Metals and ceramics also creep, but at much higher temperatures⁵⁹.

The biggest advantage to plastics is their ability to be processed into almost any shape with any desired surface finish with little or no finishing after manufacture. To make a smooth sphere out of steel or wood, much time and great care would be required in finishing that single sphere. Once an equivalent amount of time is spent on a tool-steel mold cavity, tens of thousands of near perfect spheres could be molded using plastics, thus becoming very economical. Also, excess plastic material can be quickly reground and processed again⁶⁰.

Other advantages of plastics are that they do not rust nor do they absorb much moisture. They have excellent resistance to inorganic acids, bases, and water solutions of salts. Some plastics even resist some organic solvents! Plastics also make good electrical and thermal insulators. For example, the elastic and damping qualities of synthetic rubbers are useful for energy-absorbing mountings and springs⁶¹.

There are two types of plastics, thermosetting and thermoplastic. Thermosetting materials become fluid upon heating and can be shaped under pressure in a mold. After further heating, a chemical change, catalyzed by additives, causes the molded material to harden. Finally, after a prescribed time, the material can be removed from the mold without distortion even if still hot. What happens is that separated molecular groups in the plastic become chemically bonded together (cross-linked) to form insoluble three-dimensional networks. Such cross-linked plastics cannot be remelted or reshaped again once they are fully cured⁶². Popular thermosetting plastics are epoxies, polyesters, silicones, and urethanes⁶³.

Thermoplastic materials always remain capable of being softened by heat and

of hardening on cooling even after many cycles. They can be remelted and reshaped repeatedly since they do not cross-link to any great extent⁶⁴. Commercial thermoplastics include nylon, polycarbonate, polystyrene and vinyls⁶⁵.

What exactly is a polymer? A molecule is a group of atoms which have strong bonds among themselves but weaker bonds to adjacent molecules. Examples of small molecules are water and carbon dioxide. Polymers contain thousands to millions of bonded atoms in large molecules (macromolecules). Polymers are made by joining a large number of small, identical molecules called monomers into large chains through a chemical process called polymerization. A simple example is the polymerization of ethylene (C_2H_4). Ethylene is a good monomer candidate because it has a weak double bond between the carbon atoms which makes it very reactive. Under the influence of heat, light or chemical agents (called initiators), this bond becomes activated enough to start a chain reaction of self-addition of the ethylene molecules. The result is the production of a high molecular weight material chemically identical to ethylene, known as polyethylene, the polymer of ethylene⁶⁶ (Figure 18a.).

Polymerization of two monomers (copolymerization) forms a copolymer. Three monomers copolymerize into a terpolymer, and so on. SAN, a plastic component of some of our CFs, is a copolymer of styrene and acrylonitrile. ABS is a terpolymer of acrylonitrile, butadiene, and styrene. Copolymers combine the properties of the monomers into more versatile materials⁶⁷.

A polymer molecule consists of a repetition of a basic unit called a mer (derived from the term monomer, "one" mer). The number of mer repetitions in a polymer

Figure 18. (a.) The formation of polyethylene (addition polymerization).
 (b.) The formation of alkyd resin (condensation polymerization).

is called the degree of polymerization, DP. This number can be quite high. The difference in behavior between ordinary organic compounds and polymers is due mainly to the shape and large size of the polymer molecules. Common organic materials such as alcohols, ethers, sugars, etc., consist of small molecules having molecular weights less than 1000. The molecular weights of polymers vary from twenty to hundreds of thousands. These high molecular weight, high DP polymers are sometimes referred to as, appropriately enough, high polymers⁶⁸.

There are two general types of polymerization reactions; addition or chain polymerization and condensation or step polymerization. Addition polymerization is a reaction which results in the bonding of two or more molecules without the elimination of any by-product molecules. These reactions can involve one or more molecular species. An example of addition polymerization is the reaction for polyethylene discussed earlier. Compounds with triple bonds may react to form double bonds or two single bonds with other monomers. This is a common method for producing copolymers⁶⁹. Examples of addition polymers are polystyrene, polyacrylonitrile, SAN, and polymethyl methacrylate (PMMA)⁷⁰.

Condensation polymerization is a reaction in which the bonding together of molecules results in the elimination of an element, such as nitrogen or hydrogen, or of a simple compound, such as water, hydrogen chloride, or sodium chloride. Depending upon how multi-functional the reactants are, a small molecule, a larger molecule, or a plastic polymer can be formed. For example, phthalic anhydride and glycerol react to form the alkyd resin which is made of long chains of repeating units

(see Figure 18b.). The remaining OH group could then react with another acid molecule to put on a side chain which would cross-link to another growing chain. Cross-linking can occur in polymers from a combination of both types of polymerization⁷¹. Examples of condensation polymers are polycarbonate, polyester, and polyurethane⁷².

Polymers have three basic types of molecular structure: straight chain, branched chain and cross-linked. The long, linear chains formed by addition polymerization may be bundled together in a haphazard manner. The atoms in the chains are strongly connected by valence bonds. The bonds between the chains in straight chain polymers are weaker secondary bonds or van der Waals bonds that are effective only where the chains come into contact. This unoriented, haphazard structure produces an amorphous material. Amorphous polymers do not have narrowly defined melting points and are usually hard and brittle at room temperatures. They are characterized by two temperatures: the melting temperature (T_m) and the glass transition temperature (T_g). The glass transition temperature is defined as the central point in the temperature band where the polymer goes from a hard, "glassy" state to a soft "rubbery" state. Examples of amorphous plastics are polystyrene, acrylics, poly-vinyl chloride (PVC) and SAN^{73,74}.

If the chains are parallel, the polymer tends to be crystalline. These polymers have no defined glass transition temperature, are not brittle at room temperatures, and have sharp melting points. Examples of crystalline polymers are polyethylene, nylon, and polyesters. Some polymers have both crystalline and amorphous regions.

All straight chain polymers, whether amorphous or crystalline, are thermoplastics.

The two general rules are:

- the longer the chains, the lower the solubility of the polymer in organic solvents, and the higher the melting point;
- the higher the crystallinity, the lower the solubility, and the higher the melting point also.

Plastic processors can alter solubility and melting points by copolymerization⁷⁵.

Side chains branch out from the main polymer chain. This usually lowers the crystallinity, the strength and the melting point of the polymer. These are called branched chain polymers.

If the polymer chains are linked by side chains or by valence bonding, then we have a cross-linked polymer. Cross-linking increases the strength and the melting point and lowers solubility. This structure defines the thermosetting plastics⁷⁶.

There are many chemical modifiers that are added to plastics to improve performance. Plasticizers are added to thermoplastics to increase flexibility and improve flow characteristics. Organic peroxides are added to initiate cross-linking between chains and stabilizers are incorporated to prevent or slow degradation due to heat, radiation or oxidation. There are flame retardants, smoke suppressants, foaming agents, colorants, fragrances, and even preservatives to impart resistance to deterioration from microorganisms⁷⁷. An important additive that we employed in our filters is a coupling agent which increased the bonding between the plastic and the glass particles. This additive will be described later in more detail. The remainder of

this section will describe the properties of common optical plastics, review the molding techniques employed today, and discuss optical transmission measurements of the particular plastics that we used.

Optical Polymers

The ten most common optical plastics are polymethyl methacrylate (PMMA, acrylic), polystyrene (PS), polycarbonate (PC), styrene acrylonitrile copolymer (SAN), PS-PMMA copolymer (NAS), polymethylpentene (TPX), amorphous polyamide (Nylon), polysulfone, allyl diglycol carbonate (CR-39), and epoxies. The first five are general use thermoplastics while the last five are used for special applications. All of these plastics degrade somewhat both physically and optically when exposed to UV radiation. All become opaque between 300 and 400 nm. In the visible and NIR regions, all of the major optical plastics transmit well up to approximately 1.5 μm . For longer wavelengths, more careful selection is required. TPX has higher NIR-IR transmission than the other plastics. Polystyrene becomes transparent again at 100 μm and has been used for long-wave IR applications. All of these plastics exhibit absorption and reflection losses in the visible with PMMA having the lowest values. A summary of the optical properties of PMMA, PS, PC, and NAS is shown in Table 4⁷⁸. The five most common optical thermoplastics will now be discussed in more detail.

PMMA (acrylic) has been commercially available since 1935 as a molding material⁷⁹ and is the most popular optical plastic. It is the hardest, most scratch-resistant and most mechanically stable of all optical plastics. Light absorption is less than 0.5% per inch, so its transparency is greater than that of most optical glasses⁸⁰.

Table 4. Optical properties of PMMA, PS, PC, and NAS.

Properties	PMMA	PS	PC	NAS
n_D .589 μm	1.491(7)	1.590(3)	1.586(0)	1.564
n_F .486 μm	1.497(8)	1.604(1)	1.593(4)	1.574(4)
n_C .656 μm	1.489(2)	1.584(9)	1.576	1.558(3)
Abbe (V)	57.2	30.8	34.0	35
dn/dt $\times 10^{-5}/^\circ\text{C}$	-12.5	-12.0	-14.3	-14.0
Haze(%)	< 2	< 3	< 3	< 3
Trans- mittance*	0.92	0.87 - 0.92	0.85 - 0.91	90
Critical Angle($^\circ$)	42.1	39.0	39.1	39.8

Luminous Transmittance (0.125-in. thickness)

PMMA can withstand prolonged exposure to weather and sunlight without yellowing or stress cracking. UV absorbers are added to prevent deterioration from UV radiation. Acrylics are resistant to most inorganic detergents but are subject to attack by alcohols, ketones, and chlorinated hydrocarbons. The undesirable features are higher thermal expansion than other optical plastics, great flammability (requiring flame retardants), and higher pressures required in injection molding. PMMA has been used in signs, insulated skylights, transparent covers, automobile taillights, and eyeglasses. It has been used since World War II in aircraft canopies and windows. PMMA's low refractive index and large dispersion make it a lightweight substitute for crown glass lenses⁸¹.

Polystyrene (poly-vinyl benzene) is one of the oldest known polymers, being discovered in 1839, but not developed for industry until 1935⁸². During World War II, supplies of natural rubber were denied to the Allies by the Japanese occupation of Malaysia and other rubber-producing areas of the Far East. As a substitute, butadiene-styrene synthetic rubber was manufactured on a massive scale. After the war, natural rubber became available again and the styrene manufacturing units were sold cheaply to the chemical industry⁸³. Thus began the plastics revolution in the U.S.A.. Today, PS is one of the most popular and widely used thermoplastics in the industry. The U.S. domestic market alone accounts for almost 2 billion pounds annually⁸⁴. The plastic is amorphous with a softening point of approximately 100°C, making it one of the easiest plastics to mold. It is cheap to manufacture and has excellent transparency, low water absorption, and low electrical conductivity. Unfor-

tunately, the plastic is brittle (low impact strength), susceptible to UV degradation, subject to stress crazing prior to fracture and flammable⁸⁵. Fortunately, styrene monomer can be readily incorporated in copolymers for special engineering applications. Addition of blowing agents such as pentane and careful heating produces styrofoam. Applications for PS are too numerous to list, ranging from molded pieces such as cups and utensils to packing peanuts⁸⁶. For optical applications, its high index and lower dispersion make it an acceptable "flint" to PMMA's "crown" in plastic lens achromats. Unfortunately, PS scratches more easily than PMMA⁸⁷.

SAN is a styrene acrylonitrile copolymer typically containing 65%-70% styrene. It was developed to double the impact strength and increase the softening point (+10°C) of regular PS. SAN has a combination of the better impact properties of acrylic and the good molding characteristics of PS with lower cost (SAN is cheaper than PMMA)⁸⁸. SAN also has higher resistance to some organic solvents, can withstand more heat and is less likely to suffer from stress cracking than PS. On the negative side, SAN can absorb up to 0.6% moisture during storage and needs to be UV stabilized. It is a rather hard plastic and care should be taken when handling broken pieces. It has been used in refrigerator shelving, vacuum cleaner parts, automobile dashboard components, etc⁸⁹. For optical purposes, SAN has low haze and a light straw color in its natural form. It has an index ($n_D \approx 1.567$) near PS, but it is more scratch resistant⁹⁰.

NAS is a copolymer of 70% PS and 30% PMMA. It has physical and optical properties similar to SAN. NAS has been used in optics more extensively than SAN,

serving as a third material in color corrected imaging optics⁹¹.

The last optical plastic to be discussed is polycarbonate (PC), a high performance, amorphous thermoplastic commercially introduced in 1958. Since then, it has gained acceptance as a premier engineering plastic. PC is characterized by high impact resistance (16 ft-lb/in) that is superior to zinc and aluminum die castings. Molded PC 1/8 inch thick can stop a .22 caliber bullet, so PC is used in some bullet-proof windows. It is transparent, has 89% visible transmittance and exhibits less than 1% haze. PC has high electrical insulation properties, good heat and flame resistance, and low moisture absorption. It has a remarkable resistance to creep and can be molded to tight dimensional tolerances. Amazingly enough, PC can retain most of these characteristics through a wide temperature range, -60°F to 250°F! The only weaknesses are limited resistance to chemical attack and scratches and a tendency to yellow after long-term UV exposure. Utilization of special coatings and UV stabilizers can overcome the latter two limitations. PC is resistant to most acids, animal and vegetable compounds, and some other organic compounds, but is attacked by alkalis, esters, ketones, and aromatic hydrocarbons. PC is the material of choice for laminating compact and optical memory disks; it can adhere to metal surfaces. It is used in electronic components, appliance parts, vehicle hoods, boat propellers, and vandal resistant glazing⁹². PC has been used on the outside surfaces of supersonic aircraft. The optical properties of PC are similar to PS; however PC cannot be polished as easily. Lenses made of PC need to be entirely injection molded⁹³.

The chemical manufacture of PS, methacrylate, acrylonitrile, and PC is outlined

Polystyrene:

Methacrylate polymer:

Acrylonitrile (one of several ways):

Figure 19. Chemical manufacture of polystyrene, methacrylate, and acrylonitrile.

Polycarbonate:

Figure 19. (cont.) Chemical manufacture of polycarbonate.

in Figure 19. The starting points for their manufacture are hydrocarbon gases usually obtained from petroleum cracking in refineries⁹⁴. An historical note is that phosgene (carbonyl chloride) used in the manufacture of PC was also used as a poison gas in World War I.

The next topic of this section is plastics molding technology with emphasis on injection molding. Chemical companies produce resin pellets or molding powders which are shipped to processors who transform the materials into finished plastic items.

Plastics Molding Technology and Injection Molding

The technology of molding plastic into finished products developed along several lines. Simple solid shapes can be cast. Casting consists of pouring liquid plastic into a mold, adding a catalyst, and allowing the plastic to harden. In compression molding, the resin is placed in the cavity of a hot mold and pressure is applied through the upper or plunger part of the mold to force the softened plastic to conform to the contour of the mold. This method is well suited for thermosetting plastics. Transfer molding is a variation of compression molding to mold thermosetting plastics with inserts. These inserts would be moved out of position by unliquified molding powder in compression molding. In transfer molding, enough molding powder for one piece is melted at a temperature just below that resulting in polymerization and is forced or transferred from the heating pot into the hot mold cavity by a plunger⁹⁵.

Extrusion is a process of forcing plastic material through a die to continuously produce such products as rods, tubes, films, bars, filaments and a variety of other

elongated shapes with uniform cross-sections. In this process, the thermoplastic molding material is fed from a hopper into a hot cylinder. The heat softens the material and it is forced by one or more spiral screws through the cylinder and out through a die orifice. The die forms the cross-sectional shape of the continuous plastic mass as it passes through. The extruded product is conveyed through either the air or a water-cooling bath to harden it. In extrusion blow molding, hot plastic tubing from an extruder enters an open mold attached to a revolving turntable. The mold closes, air is blown in through the tubing, expanding it against the mold, and the mold opens, ejecting the product. This is a common method for producing plastic bottles⁹⁶.

Plastic films or sheets can be formed by extrusion or calendering. Calendering is a process where plastic material, with any added plasticizer or colorant, is passed between a series of heated rollers, finally emerging from the last pair of rollers as film or sheeting. Variation of these methods are numerous⁹⁷.

The method employed for our filters was injection molding (IM), the most popular form of processing. Injection molding is a widespread technology for manufacturing an almost innumerable range of consumer and industrial products. IM products are so ubiquitous that it is hard to believe that the broad commercial application of this technology is less than 50 years old. IM manufactures relatively complex shapes with excellent dimensional tolerances and economically yields from hundreds to thousands of units per hour. For CFs, IM promised to produce homogeneous, bubble-free, smooth-surfaced filters rapidly and reliably. The major expense, aside

from the cost of the large machines which we saved by renting machine time at a local molding company, is the cost of the mold. Even simple molds cost several thousand dollars and multi-cavity molds for mass production can cost tens of thousands. However, once those investments are made, great quantities of an item can be produced for very small marginal costs. We achieved much more success with IM than any other method and have concentrated most of our efforts on IM from 1988 through 1990.

Figure 20 presents a typical reciprocating screw IM machine. Injection molding begins with specially prepared thermoplastic resins and additives that are mixed and poured into a hopper which feeds into a heated chamber containing a screw plunger. Heat from band heaters attached to the chamber and pressure from the rotating screw are applied to the plastic until it becomes soft enough to allow viscous flow. The screw then forces the mixture under high pressure into cavities of a tightly clamped, often heated mold. Pressure and temperature are maintained until the resin conforms to the mold and cools to rigidity. Once the entrance to the mold (sprue bush) has solidified, the screw retreats and starts rotating to pick up the next volume of material (shot). When the resin is hard enough (often determined by a pre-set cooling time), the mold opens and drive pins eject the product. The resulting product is often ready for use without any additional processing. Several cavities connected by channels (runners) can be machined into large molds for producing multiple or different products in the same injection cycle. IM machines are rated in size by the maximum number of ounces or grams of heated polymer (usually PS) that can be

Figure 20. Diagram of a typical injection molding machine. Resin is poured into a hopper which feeds into a heated chamber containing a screw plunger. Heat from band heaters attached to the chamber and pressure from the rotating screw are applied to the plastic until it is soft enough to allow viscous flow. The screw then forces the molten resin under high pressure into cavities of a tightly clamped, often heated mold. Pressure and temperature are maintained until the resin conforms to the mold and cools to rigidity. When the resin is hard enough, the mold opens and drive pins eject the product.

shot into the mold in one complete stroke of the injection ram. A machine's capability also can be rated by the clamp force (in tons) available to hold the mold closed during injection. The clamp force is related to the shot capacity and is cited when a machine is rated as "90-ton" for example^{98,99}.

We employed several reciprocating screw machines, the most popular type of IM machine. Here, a rotating screw acts like a pump, moving plastic horizontally through a heated extruder barrel where it becomes molten and viscous. The plastic is forced along by the screw and accumulates in front of it. A valve closes the output nozzle and a check valve is engaged to prevent backflow of the plastic around the screw. As the pressure of the accumulating plastic increases, it forces the screw to retreat until the appropriate shot size is collected. Setting the retreat distance thus determines the shot size. Then the nozzle valve opens and the entire screw is pushed forward by hydraulic pressure to ram the plastic through the nozzle into the mold. Immediately before the injection action, a moving platen contacts the mold and applies the clamping force necessary to hold the plastic under the high injection pressure. High injection pressure is required to completely fill the mold cavity and insure uniform distribution of any additives or colorants. After solidification, the platen releases the mold and the product is ejected. Most IM machines can be run fully automatically to achieve high production levels¹⁰⁰.

IM is very sensitive to the plastic's temperature and viscosity. Mold design requires great skill. Some of the important parameters to monitor for successful IM are shot size, screw cylinder temperature(s), mold temperature, injection pressure,

holding pressure (applied after injection until the sprue bush or gate is sealed), back pressure (pressure applied from behind the screw to keep it from pushing backwards as it rotates; it is steadily countered by the pressure of the accumulating plastic), injection time (= filling time + holding time), mold cooling time, shot charging time, injection speed, screw rotation speed, and cycle time. Many defects in the final product can be eliminated by the proper determination of these parameters¹⁰¹.

We improved our CFs by having the glass powder **compounded** into the resin pellets before we molded them into filters. In compounding, polymers and polymeric materials are modified with additives through melting. Pellets, powders, fibers, additives, and other ingredients are mixed and melted together in a reactor, extended into long, thin fibers a few millimeters in diameter and then chopped into pellets. Typically, single screw extruders or twin screw (co- or counter-rotating intermeshing screw) extruders are employed to mix the heated material. Compounding is routine practice when coloring agents or strengthening glass fibers are required¹⁰².

In closing this section, I present spectral transmission measurements of PS, SAN, and LEXAN 141™ (PC) taken on a dual-beam Cary 2415 spectrophotometer. They are all shown in Figure 21 for comparison. The PS and SAN samples were 0.062 inches thick while the PC sample was 0.125 inches thick. All of the spectra show 80% or greater transmission in the 400-700 nm region. The PC shows an increase to ~90% in the 700-1100 nm NIR region. There are four absorption bands common to all of the spectra: 1.1-1.2 μm , 1.3-1.5 μm , 1.6-1.8 μm , and 2.1-2.2 μm . The PC and SAN show another dip at ~1.9 μm . These spectra match well with results found else

Figure 21. Transmission spectra of samples of (A.) Styron (PS), 0.062 inch thick; (B.) Monsanto SAN, 0.062 inch thick; (C.) General Electric Lexan 141 (PC), 0.125 inch thick.

where¹⁰³.

The absorption bands appearing in the region below 3.0 μm involve stretching vibrations of hydrogen-containing bonds or combinations of such stretching vibrations with other modes of vibration of the molecules. Only a few of these vibrations are fundamental; in general, the NIR spectra consist of overtones of fundamental absorptions in the 3-6 μm region. For example, the 1.6-1.8 μm band and the 1.1-1.2 μm band are the first and second overtones of C-H stretching vibrations, respectively. The 2.1-2.2 band may be a combination of a C-H stretching vibration and another vibration (such as C=O stretching in PC)¹⁰⁴.

The 1.9 μm band in SAN is associated with a C $\equiv$ N and CH₂ combination vibration¹⁰⁵. In summary, the numerous dips in the spectra of Figure 20 are combinations or overtones of fundamental vibrations lying beyond 3.0 μm .

Now that we have reviewed the theories and technologies behind our Christiansen filters, it is time to describe their fabrication. This is the subject of the next section.

Section 8: Fabrication of Solid Matrix Christiansen Filters

Before fabrication of our CFs could begin, we needed to obtain powders of the glass types that our studies predicted would make possible filters with our chosen optical plastics. We could not expect that these glasses would be available commercially as powders and had to find a way to grind the glass.

The four largest suppliers of pure optical glass in the world are the Schott, Ohara, Corning and Hoya Corporations. They supply glass as unfinished ingots, slabs or gobs, certifying the optical, mechanical and chemical characteristics of each shipment. Price, quantity and delivery time depend upon the quantity and availability of the glass ordered. Each company has distribution offices on both East and West Coasts, which carry relatively complete stocks of their own products, but prices can be very high and minimum order policies are common. On the other hand, there are literally hundreds of glass jobbers, fabricators, custom manufacturers, and other dealers in glass in the U.S.; typically, they have odd pieces of glass from several manufacturers that may be purchased at reduced prices. We were interested in obtaining the two low-dispersion glasses mentioned in Section 5, SK5 and SK11. Glass Fab Inc. of Rochester, NY sold us odd pieces of these glasses, such as the corners left after round blanks are cut from rectangular slabs, at a 25% discount from standard prices.

Preparation of the glass particles proved initially to be difficult. The particles must be of nearly uniform size and very clean because the filters will not work prop-

erly if the particles are highly contaminated. At first, the glass was ground in-house with a hard alumina mortar and pestle from Coors Porcelain and sized with different meshes of standard stainless steel sieves. This proved to be a very slow, tedious, and inefficient process. Next, we purchased a bar jar mill from U.S. Stoneware [Garland, TX] and an automated Ro-Tap™ [Wyler Co., Gastonia, NC] sieve shaker. The glass was manually crushed to shards of millimeter size and loaded into a 5-liter ceramic jar. Balls were added to the jar to do the milling and the jar was rotated rapidly on the mill's rollers until an appropriate powder consistency was reached. It was discovered that the milling time might be decreased if the glass shards were heated to 200°C in an oven and then rapidly cooled by immersion in room temperature water before loading them into the jar. However, the milling could take days if a large quantity of particles with mean diameters less than 90 μm was required. After milling, the powder was loaded into a canister partitioned with sieves of different mesh sizes. The canister was then shaken in the sieve shaker to sort the glass particles according to mesh size. This process is cost-effective only if small amounts of glass are needed for a few trial filters; for larger production runs or more extensive experimentation, it is not very efficient.

Eventually, we found a vendor willing to clean, mill, and sieve our glass, Electro-Glass Products [EGP] of Mammoth, PA. We were only required to break the glass into one-inch shards before shipping. This proved to be the most efficient way to produce large amounts of glass powder. Table 5 lists the particle sizes according to mesh designations. We were provided with particles ranging from coarse sizes greater than

Table 5. Conversion of mesh designations to particles sizes.

Mesh Size Number	Particle Size
400 +	< 37 μm
325 - 400	37 - 44 μm
270 - 325	44 - 53 μm
230 - 270	53 - 63 μm
200 - 230	63 - 74 μm
170 - 200	74 - 88 μm

88 μm to fine sizes near 20 μm . Figure 22 contains electron micrographs of glass milled by our jar mill and by EGP; the particles produced by EGP are smaller, but both produced highly irregular shapes. The problem with irregular shapes and surfaces will be discussed later.

As we learned to mold solid matrix CFs, we evaluated our progress by measuring their spectral transmission T_λ . The results of these measurements drove further experimentation so I will be displaying transmission spectra as I describe CF fabrication. The Optical Measurement Laboratory at the Optical Sciences Center [OSC] has a dual-beam Cary 2415 UV-VIS-NIR spectrophotometer that is manufactured by Varian Associates. Since these filters operate by small-angle multiple scattering, caution must be taken when measuring T_λ . If particle size, α , is significantly larger than λ or if the intersection angle between the dispersion curves is small, the non-Christiansen λ s are scattered by a small angle, θ_s , so the filtering effect becomes stronger as the detector moves away from the filter. The larger the distance between filter and detector, the less likely the detector will intercept the scattered λ s. The optics in the Cary 2415 are designed to collect all of the light passing through the sample chamber and the distance between the entrance window into the chamber and the detector is fixed. To measure the Christiansen effect, the detector was covered with a pinhole size aperture to simulate a distant detector. The filter sample was placed next to the chamber entrance window to limit the detected light to that transmitted specularly into a cone with an apex angle smaller than 1° . Since T_λ is measured with respect to air, represented by an empty reference chamber in a two-

Figure 22. Top: Electron micrograph of SK11 glass milled at OSC.
Bottom: Electron micrograph of BAK4 glass milled at EGP.

beam instrument, a similar aperture covered the detector in the reference beam. Before measurements are taken, the spectrophotometer is calibrated by recording a baseline transmission spectrum without the sample in the compartment. The spectrophotometer determines the signal imbalance between the sample detector and the reference detector and automatically adjusts the detectors when measurements are taken. The measurement configuration is illustrated in Figure 23.

We experimented with casting solid CFs with no success. The conventional method practiced by hobbyists is to pour liquid styrene monomer into molds and add initiators to promote polymerization. Delimiters are routinely added to styrene monomer to prevent premature polymerization and must be defeated for polymerization to occur. Unfortunately, solidification took hours to occur and the glass particles settled to the bottom of the mold. The delimiter can be washed out by chemical separation, but we had neither the expertise nor the equipment to accomplish this. One monomer supplier suggested we use a strong initiator, benzoyl peroxide, and carefully heat the solution. This could have been the most spectacular experiment of the project since benzoyl peroxide is a strong, explosive oxidizer that is normally stored under refrigeration. To make matters worse, the flash point for liquid styrene is 86°F, not much above room temperature, and styrene vapors are toxic when inhaled. A very small amount of benzoyl peroxide was added to a small amount of styrene in a Pyrex beaker and slowly heated by a hot plate in a chemical hood. The solution went up in a small puff of smoke before any solidification was observed. The beaker appeared to be irreversibly stained by the experience. Lack of a temperature-

Figure 23. Diagram of the sample compartment of the Varian 2415 Spectrophotometer.

controlled heater discouraged us from pursuing this approach any further.

We next attempted to mold CFs in a small, heated manual press. An aluminum mold for the press was machined and polished to produce filters with smooth surfaces. SK11 powder and SAN resin pellets were pressed and heated but the resulting filters contained air bubbles and an uneven distribution of glass particles. Our final attempt at an alternate molding technique was to dissolve a mixture of SK11 glass powder and SAN pellets with tetrahydrofuran (THF) and press mold the combination into a filter. This was done to avoid heating the plastic and producing air bubbles. The result was a flexible structure in which the pellets were incompletely dissolved. We observed no Christiansen effect and concluded that either the solution formed by THF had altered the optical properties of the SAN or the glass was not sufficiently embedded in the SAN for scattering to work. Success with injection molding led us to abandon these alternate techniques.

Our first IM experiments were done at Dyna-Day Plastics [DDP] in Tucson. The standard arrangement was for the company to provide an operator who would control the machine under our supervision. After Dyna-Day closed down in the spring of 1989, we were able to reach an agreement with Tucson Mold Inc. (TMI) to let us mold our filters at their plant under the same arrangement.

At DDP, the filters were molded by 85- and 110-ton VistaTM automated IM systems built by Cincinnati Milacron. At TMI, we molded with 88-ton ToyoTM and 90-ton PlastarTM semi-automated IM machines. For the DDP machines, we modified a steel mold that once produced lids for plastic bottles. This saved us the cost of a new

mold, typically \$5000 to \$10,000; the modified mold had four cavities and one inlet that formed disks 2.75 inches in diameter and 0.062 inches thick. (Dimensions are given in English units since all design and fabrication work was implemented in these units.) When we moved to TMI, we had another mold fabricated to take advantage of their in-house mold-making capabilities and our better appreciation for what was necessary. The new mold had three cavities, a square 2.75 inches on a side and 0.062 inches thick, a disk 2.75 inches in diameter and 0.062 inches thick, and an equilateral prism 1 inch long with 0.5 inch faces. This mold is illustrated in Figure 24. We chose to make some square samples in case we wanted to tile them together to form larger effective filters. The equilateral prisms were for making refractive index measurements. In January, 1990, our contract monitor, Dr. Bart Wilburn, requested that we mold larger samples to fit aircraft windows, so we had another mold machined to form windows 7.625 inches long by 5.75 inches wide by 0.25 inches thick.

The fabrication process began by acquiring enough glass powder and plastic resin for a run using the methods described earlier. We concentrated on glass preparation, which involved the addition of coupling agents beyond the powdering and sifting procedures, but delegated the resin arrangements to TMI. When we scheduled an all-day molding session at the factory, we would ask them to order two 50 lb sacks of resin pellets. TMI personnel would heat the resin from the sacks for 24 hours in special ovens to remove water. This would prevent water bubbles from forming in our samples during molding and also eliminate a streaking effect known as "splay". The enormous variations in relative humidity in Tucson necessitate active drying to control

Figure 24. Front view of our three-cavity mold on a standard 20 by 25 cm mold base.

this variable.

Early in the day's run, following arrival at the plant, we would start mixing glass and plastic materials according to predetermined ratios in small plastic bags with the help of a digital, 0.1 gm-sensitive scale. As molding continued, we would alter the glass-plastic ratio by weighing them separately on the scale and combining them in a bag. Mixing the materials at the plant allowed us to adjust the molding parameters after observing the results. Usually several ounces of pure resin was extruded through the screw to remove contamination from previous runs before the bags with glass and plastic were poured into the hopper. The mixture was backed with more pure resin; we molded samples until we could no longer see any glass powder in them. As each sample was ejected, it was immediately labeled and inspected before a tungsten-halogen lamp for color, clarity and uniformity. Much time was spent perfecting the glass-plastic ratios and the machine parameters such as barrel temperature, cycle time, mold temperature, cooling time, and backing pressure. Appendix B of this dissertation contains a list of the material proportions for our injection molded filters.

We tried to mold with several different polymers; styrene butadiene, NAS, PS and SAN; our best results were with Monsanto Lustran™ SAN copolymer and Dow Chemical Styron™ PS. As discussed in Section 5, we chose to mold SK5/SAN, SK11/SAN, and SK5/PS filters. These combinations were predicted to produce blue, red/NIR and green filters, respectively. We concentrated most of our efforts on the SK11/SAN combination since it came closest to our NIR specifications. The remainder of this section is a chronological summary of our results as we identified and

solved problems, preserving earlier solutions. Thus, our final filters are the results of several techniques applied together.

One of the first successful injection molded solid matrix CFs produced the transmittance spectra in Figure 25. The SK5/SAN filter has a broad peak near 425 nm and the asymmetry about the peak is due largely to strong absorption by the host SAN in the near UV. The SK11/SAN combination behaves like an edge filter with the half-maximum point near 650 nm. We made a videotape of outdoor scenery through this filter using a NIR camera with a VIS blocking filter and a regular VCR camera. The tape shows placard signs whose lettering is discernible in the NIR but not in the VIS. However, we needed to improve the transmission characteristics of the SK11/SAN NIR filter. The spectrum in Figure 25 shows a significant T_{VIS} rising to around 0.50 at 800 nm; ideally, we would like this T_{NIR} to be around 0.80.

A major, rather frustrating problem was noticed during molding. As samples of certain combinations cooled down after ejection from the machine, their colors changed from highly saturated to less intense hues. For example, BaK4/SAN filters would transmit intense hues of violet or green while hot, but lose this intensity as they cooled down. We attribute this to a loss of stable, tenacious contact between the glass particles and the plastic host. Voids would form around the particles and affect the optical properties of the filter. This is illustrated in Figure 26.

The problem centers around the greater coefficient of thermal expansion, α , of plastics with respect to glass. For example, at room temperature, the coefficients for two of the glasses and the plastics are

Figure 25. Transmittance spectra of one of the first successful injection - molded Christiansen filters.

Figure 26. Illustration of tensile stress and cavity formation around the glass particles during cooling.

$$\alpha[\text{SK11}] = 7.6 \times 10^{-6}/^{\circ}\text{C}$$

$$\alpha[\text{SAN}] = 65-68 \times 10^{-6}/^{\circ}\text{C}$$

$$\alpha[\text{SK5}] = 6.5 \times 10^{-6}/^{\circ}\text{C}$$

$$\alpha[\text{PS}] = 50-83 \times 10^{-6}/^{\circ}\text{C}$$

Note that these optical plastics have coefficients an order of magnitude greater than those of the glasses; this is typical of many polymers. Over the temperature range encountered during and after molding, from $T_{\text{mold}} \approx 260^{\circ}\text{C}$ to $T_{\text{room}} \approx 20^{\circ}\text{C}$, we drop roughly 240°C . When applied over the different coefficients, this translates to a strain of about 1.5×10^{-3} , or 0.15%; this is enough of a difference to craze the resin as it goes into tensile stress around the glass particles.

At first, we tried adding mineral oil (MO) or epoxidized soybean oil (ESO) to our samples during molding. These oils are employed in the industry to improve flow in the molten resin as it is forced into the mold. We hoped that this would fill the voids at the glass-plastic interfaces formed from crazing. In practice, both oils helped in homogenizing the mixture so that clumps of glass particles would not appear in our filters. MO gave a desirable milky-white appearance to our filters by scattering from oil droplets; however neither oil prevented the color change during cooling. Occasionally, when we added too much oil, the reciprocating screw would become too slick to pick up material which would solidify around the screw. Since the screw was rotating through the material and not pushing it along, there would be insufficient resisting pressure to retreat the screw and start injection. These machine "jams" would delay our molding runs until the operators could remove the material stuck in the barrel.

Several discussions with Dr. Edwin P. Plueddemann of Dow-Corning and Dr. Herbert Friedlander, an independent consultant, have indicated that crazing has been

a difficult problem for the manufacture of fiberglass and other plastics strengthened by the addition of long fibers to the resin. Coupling agents are additives that are designed to increase the bond strength to prevent cracking and crazing. They must bond tenaciously to both glass and resin, displace any water layers, and spread the forces around each glass particle; they must also withstand the shear forces of IM. The more popular coupling agents are based on silicones or titania.

The history of coupling agents began in 1940 when glass fibers were first used as reinforcement in organic resins. The specific strength-to-weight ratios of early glass-resin composites were greater than those of aluminum or steel, but they lost much of their strength during prolonged exposure to moisture. This loss in strength was attributed to the debonding of resin from glass by the intrusion of water. Since organofunctional silicones are hybrids of silica and of organic materials related to resins, they were first tested as coupling agents to improve bonding of organic resins to mineral surfaces. "Coupling agents" have come to be defined as materials that improve the chemical resistance (especially to water) of the bond across the interface between an organic polymer and an inorganic substance. Research was spurred on first by the military and government agencies and later by demand in the plastics industry for new, versatile composites¹⁰⁶.

Organosilanes are able to bridge the gap between resins and inorganic substrates because of their bifunctional nature; one end of the molecule is reactive with the inorganic substrate while the other is reactive or compatible with the organic matrix. Their empirical formula is written as

where X represents a hydrolyzable group typically acyloxy, alkoxy, or chlorine with methoxy or ethoxy being most common. R is an organic radical, most often three carbon atoms in length which joins the silicon atom to an organofunctional group, Y. This hydrolytically stable carbon-silicon bond is responsible for the ability of the coupling agent to retain improved bonding properties under harsh environmental (high moisture) conditions. The selection of functional group Y is determined by its reactivity or compatibility with the resin matrix. In the popular coupling agents, there are usually three hydrolyzable groups and one organic group¹⁰⁷.

Normally, the silane coupling agent is hydrolyzed before application (mixed with distilled water or methanol). The hydrolyzable groups form reactive silanols and HX:

The silanols condense with other silanols found on the surface of siliceous fillers and reinforcements^{108,109}.

Several mechanisms have been proposed to explain silane-glass adhesion¹¹⁰. They are:

- Cross-linked sheath around glass.
- Removal of water from glass surface.
- Physical adsorption to glass.
- Hydrogen bonding to glass.
- Covalent bonding to glass.
- Protection of glass: eliminate microcracks, prevent flaws, corrosion, and water

access.

For the silane-resin adhesion:

- Improve resin wetting.
- Increase surface roughness.
- Stress-transferring boundary layer.
- Water barrier.
- Improve "compatibility" or diffusional bonding.
- Covalent bonding.

Dr. Plueddemann kindly sent us a total of three Dow-Corning coupling agents to try. We also received an agent from Petrarch Systems. They must be applied - dissolved in methyl alcohol - to the glass powder; the treated powder is then thoroughly dried before mixing with resin for molding. The first Dow-Corning agent was a widely marketed silane, Z-6032 (trimethoxysilane) which has been found to be a good adhesion promoter for PS¹¹¹. The Petrarch agent was an aminosilane which has been found to promote adhesion between mica and PMMA and between titania and PS¹¹². However, neither agent improved our IM samples significantly. Transmittance spectra of SK5/SAN (blue), BaK4/SAN (yellowish-red), and SK11/SAN (red-NIR) samples from these runs are shown in Figure 27. The transmittance spectra of BaK4/SAN CFs at different temperatures reached in the spectrophotometer sample chamber are shown in Figure 28. In these case, the spectrum broadens toward the blue as temperature increases, thus partially accounting for the violet or green hues we observed immediately after molding. This effect may also be due to a change in the

Figure 27. Transmittance spectra of SMCFs that contain no coupling agents.

(A.) SK5/SAN bandpass filters.

(B.) BaK4/SAN edge filters.

(C.) SK11/SAN edge filters.

See Appendix B. for detailed conditions.

Figure 28. (A.) Temperature dependence of the transmittance of sample 13-4 - BaK4/SAN. (B.) Temperature dependence of the transmittance of sample 13-11 - BaK4/SAN. Temperatures are measured adjacent to the samples in the sample compartment of the Varian 2415 spectrophotometer. See Appendix B. for details on these samples.

index of refraction of SAN as temperature increases; this would shift the entire dispersion curve of SAN with respect to the curve of BaK4 which would change the spectral transmission of the CF.

The second and third coupling agents sent by Dr. Plueddemann are experimental formulations intended for IM applications. They feature bonds that slip along rather than break free of the interfaces during the stress of molding. For application, they are diluted in distilled water rather than alcohol. Additional information about these coupling agents - labeled 7169-45B and X1-6124 - is regarded as proprietary. In Appendix B, the 7169-45B is denoted as DC2 and X1-6124 is named DC3; DC4 is a 1:1 mix of X1-6124 and the Dow Corning silane, Z-6032. The 7169-45B coupling agent generated noticeable improvements with very little color change during cooling. A filter molded with this coupling agent is labeled 15-5 in the SK11/SAN spectrum of Figure 27(C). Note the slightly sharper band edge when compared to the other filters since the coupling agent prevented the shift toward the yellow. The X1-6124 worked best with no color change as the filters cooled. There appeared to be no difference between DC3 and DC4 in preventing crazing. For the remainder of the project, we treated the glass powder with the DC3 formulation.

Having solved the crazing problem, we tackled another problem with IM. We noticed a strong tendency for the glass and the resin to remain unmixed or to separate partially in the barrel of the machine before injection into the mold. This results in a variation in the effective glass:resin ratio in the molded samples. In particular, when we molded SK11 glass and SAN polymer, only the samples at the end of the

run had the desirable fine-grained milky appearance. We tried grinding the resin to a fine flour-like consistency, like the glass powder, rather than using the resin in pellet form. This had no effect on filter quality. We next tried adding prepared glass-resin mixtures in spoonfuls to the barrel, separated by cups of pure resin, to promote even dilution. This did not help; the colors were simply less saturated. We then tried molding with higher glass:resin ratios. This also produced less saturated colors. The solution was suggested to us by John Taylor, the floor manager at TMI. He recommended that we have our glass and plastic compounded before we molded them into CFs. Compounding was described in Section 7.

We contacted a compounder recommended by Mr. Taylor, Compounding Technologies Inc. [CTI] in Corona, CA, and arranged for them to blend 14 pounds of our less critical, coarse mesh SK11 glass with 28 pounds of Lustran™ SAN. We molded the blended product with almost unqualified success. The samples had uniform glass content throughout most of the molding runs. CTI refused to compound with coupling agents, so we switched to a competitor willing to add the agent, Modified Plastics, Inc. [MPI] of Santa Ana, CA. When we molded MPI compound, we achieved uniform filters with no color change. When we began molding the thicker windows, our compounded material was too glass rich to allow much light transmission. We had to have another batch with a lower glass/resin ratio compounded. The batches are compared in Table 6. The only drawback to compounding was that MPI required a minimum of 40 lbs of material before they could compound. We did not have enough of the finer glass powder and had to ship the coarse glass powder.

Table 6. Contents of compounded batches

Batch	SAN (kg)	SK11 (kg)	Resin:Glass Ratio	Particle Size	Coupling Agent
CTI - 1	14.4	7.2	2 : 1	coarse	none
MPI - 1	14.2	7.1	2 : 1	coarse	DC3
MPI - 2	19.4	4.85	4 : 1	74-44 μm	DC3

The greater variance in particle size make these filters more difficult to analyze and model.

Figure 29 is the transmittance spectrum of a window sized filter (Run 23-1) made from compounded material treated with the best coupling agent. This spectrum demonstrates the remaining problems with our NIR filter. Recall that the objective was to produce a specularly transmitting window in the NIR - $\lambda > 800$ nm - that scattered vigorously in the VIS with minimal absorption to generate a bright, lightly gray appearance. We were left with the dilemma of trading off specular transmission in the NIR for sufficient scattering in the VIS. Reducing specular VIS transmission by adding more glass also reduced NIR transmission. Several attempts to adjust process temperatures, glass:resin ratios, coupling agents, mineral oil modifiers, and other parameters led to only minor improvements in this impasse. We needed to generate a sharper band edge between the VIS and NIR. Discussions with Dr. Sukant Tripathy of the Plastics Engineering Department at the University of Lowell led us to investigate VIS absorbing dyes in our filters to obtain sharper band edges, at the expense of VIS scattered radiation, which undermined the sponsor's directive to limit absorption in favor of scattering. We decided to maintain a low glass content to preserve the acceptable NIR transmission and to add dyes that absorb in the VIS to enhance the Christiansen effect in that part of the spectrum. The addition of dyes would maintain the angular independence that is characteristic of CFs.

After calling several dye producers, we contacted Dr. Jack Graff of the Mobay Corporation in Pittsburgh, PA. He was very helpful and sent us free samples of con-

Figure 29. Run 23-1 without dye (solid line). Required band edge (dashed line).

concentrated dyes that we could mix together to form a gray. The dyes are designated Macrolex Red 5B, Orange 3G, and Green G. Red 5B and Green G are anthraquinone dyestuffs while Orange 3G belongs to the perinone chemical family. Orange 3G acts as a colorant in automobile taillights. The chemical formulas are proprietary and all of these dyes are soluble in SAN, PC, and PS. These dyes are so concentrated that amazingly small amounts can color hundreds of pounds of plastic. We formed master batches for each of 1/2% concentrated dye in 462 grams of SAN. The weights of dyes listed in Appendix B refer to these master batches.

Figure 30(a) shows the transmittance spectrum of each dye separately molded into SAN and the spectrum for a smoky gray produced by combining the dyes. We see the appearance of a band edge at 700 nm that is produced by the green dye. Figure 30(b) shows the effect of adding the gray dye to a mixture of compounded SK11/SAN and pure SAN. We see the production of a steeper band edge at the expense of transmission losses. This figure also plots the spectrum if we add less compounded material to the dye-SAN mixture. We see more transmission in the NIR than in the VIS region. Figure 30(c) presents another example of adding dye to block the VIS and create an edge. These results show that we have some control over transmission in both regions by adding these dyes. This figure also shows the effect of adding mineral oil [MO] to the compounded glass-dye-SAN mixture. We see an overall reduction in transmittance and the suppression of the band edge.

During the next series of runs, we systematically varied the concentration of the compounded SK11/SAN pellets with respect to pure SAN and kept the dye concen-

Figure 30: (a) T spectra of each Macrolex dye and their combination molded into SAN. (b) The effect upon T when less compounded material or more dye are added to SK11/SAN filters. (c) Another example of the effect of adding dye and the effect of adding mineral oil.

tration constant. A graph comparing these mixtures appears as Figure 31(a). As the compound concentration increases from 0 to 30 grams in 900 grams of SAN, the VIS transmittance and NIR transmittance both decrease due to scattering. Clearly, the addition of compounded material completely levels the VIS transmittance from 12 grams upward, generating the desired gray color. At this level, $T_{\text{NIR}} = 0.55$ is reached. Figure 31(b) compares four runs with constant concentration of compound and increasing levels of dye. While none of the four runs has very high T_{NIR} , increasing the dye level steepens the edge of the filter noticeably by blocking the VIS with virtually no losses in T_{NIR} .

These findings led us to compare three samples with different amounts of glass compound and the same high concentration of dye in Figure 32(a). While sample 26-13 displays a wonderful contrast between the visible and near-IR ($T=0.07$ at 700 nm vs. $T=0.90$ at 850 nm), the lack of compounded glass allows T to rise again, giving it unwanted color. Sample 26-16, with 24 grams, has a flat VIS spectrum, but T_{NIR} is suppressed to 0.45. Our best sample of this series is 26-17, where 12 grams of compound permit a T_{NIR} of 0.62 while T_{VIS} is flat and remains below 0.10. The major drawback for these samples is their lack of diffuse scattering in the visible, a characteristic generated mainly by the addition of more glass powder and/or mineral oil [MO]. Our sample 26-18 represents an attempt to produce this feature and to maintain the sharp edge, but the near-IR transmittance is inevitably reduced to values around 0.33, which is well below project specifications. This curve appears as Figure 32(b).

Figure 31(a): The effect upon transmission when the concentration of compounded SK11/SAN is increased in the NIR filter.

Figure 31(b): The effect upon transmission when the concentration of gray dye is increased in the NIR filter.

Figure 32(a): A comparison between samples with the same amount of dye but different amounts of SK11/SAN compound.

Figure 32(b): An attempt to produce diffuse visible scattering by adding mineral oil to a SK11/SAN filter with dye.

The next figure represents the best of our final IM run under the program. Figure 33 shows the transmittance spectrum of a sample from run 27-8. This filter contains enough glass and MO to have a smooth, milky-white appearance; unfortunately T_{NIR} is not sufficiently large to meet our required 80%. Samples from runs 27-13 and 27-7 contain the same concentration of glass and MO but with increasing amounts of gray dye. Again, we see T_{VIS} become flatter with increasing amounts of dye. Samples from runs 27-4 and 27-6 exhibit the minimum amount of scattering of VIS that is required. These filters do not have the desired milky-white appearance; nor do they meet our T_{NIR} requirements. Ironically, if we just add a relatively heavy dose of gray dye to pure resin with little or no glass, we almost achieve our transmittance requirements. The spectrum of such a sample is labeled Run 27-2 in Figure 33. This sample absorbs VIS and has virtually no scattering to speak of. It appears that the glass particles absorb too much NIR light for these filters to meet the specifications. Based on these results, we feel that we cannot meet the transmittance and scattering (appearance) requirements simultaneously with this choice of glass, resin, and dye. The upper limit we reached for $T_{800\text{ nm}}$ is in the 0.5 to 0.6 range.

When examined under an optical microscope, the glass particles we prepare for our CFs display irregular, angular shapes, reminiscent of macroscopic shards of broken glass. Figure 34 contains a micrograph of representative particles. This is the result of the mechanical milling process that produces the glass powders. We tried several techniques to produce smooth, spherical particles. Our hope was that smooth-

Figure 33. Transmittance spectra of the best samples from the last injection-molding run.

Figure 34. Micrographs of SK11 glass particles before (top) and after (bottom) one month of tumble milling.

ing the particles would reduce light absorption in the NIR and would allow better contact and adhesion between the particles and the plastic host. One technique appeared promising, but we were unable to pursue it to the point of including spherized particles in our CFs.

One technology that we briefly explored was sol-gel processing. A paper by Fegley and Barringer [1984] reports on the production of small SiO_2 particles. These sol-gel particles tend to be spherical and highly uniform in size and shape. In principle, the range of particle sizes can be controlled by adjusting process parameters, such as reaction time and temperature. Also, the refractive index, n , and its dispersion, presumably could be modified by doping the starting materials with oxides of other indices. Dr. Brian Fabes of the Material Science & Engineering Department and the Arizona Materials Laboratory [AML] kindly agreed to introduce us to these techniques. We obtained the required starting material, tetraethylorthosilicate alkoxide [TEOS] and visited his laboratory where we performed a simple gelation experiment to produce a silica gel. Unfortunately, ensuing discussions led us to believe that the particle yield will be too low - many liters of the fairly expensive TEOS precursor to produce just hundreds of grams of particles - to be viable, since the gels must be dilute to avoid coalescence of the glass microspheres. This small yield becomes a serious problem since, as stated earlier, compounders will not process less than 40 pounds of material. For example, we must send at least 12 pounds of glass powder per compounding order, which costs a minimum of \$300. An attempt was made to cast a small amount of SiO_2 - TiO_2 sol-gel powder in PS, but no

Christiansen effect was observed in the result. Due to our other avenues of research, we did not explore this method further; it does bear further investigation.

During one of his visits to the Optical Sciences Center, our contract monitor, Dr. Bart Wilburn, related his observation that Perkin-Elmer in Danbury, CT (now Hughes-Danbury) has a small "blast furnace" capable of melting and perhaps spherizing glass particles. Essentially, this furnace is a tower enclosing a rising blast of hot air into which glass particles can be dropped. As they fall and heat rapidly in the counterflow, the particles may melt and form droplets in response to their surface tension. Even though the tower was built for other purposes, Perkin-Elmer's Dr. Matthew Magida agreed to try the experiment on some SK5 glass powder that we sent. This experiment failed to produce spherization; we attribute this to the short dwell time of the particles in the tower's 800°C hot zone.

Dr. Magida suggested two other approaches to spherization: a molten salt melting process and a plasma spray method. They used a thermomechanical analyzer to determine the softening point of the SK5 glass to be 745°C. In the molten salt process, the glass particles are heated to 665-730°C, just below their softening point in a powder bed consisting of 75 wt.% NaCl and 25wt.% LiCl. The heating times range from 2 to 24 hours. Afterwards, the particles are rinsed in DI water to remove the salt. As a result, they noticed that the particles appeared slightly less angular in an optical microscope. This process shows promise if extended to higher temperatures, if emulsified enough to prevent agglomeration, and if we can be assured that chemical interchange with the salt bath had been avoided.

Plasma spray involves the rapid heating of glass particles beyond their softening point in the heated plasma of a torch flame. As molded particles, they form droplets and solidify as spheres. For example, their 32.5 kW plasma gun permits heating for milliseconds to over 15,000°C in a mixture of Argon and Hydrogen gas. The resulting particles were either dumbbell or spherical in shape and brownish in color; Figure 35 contains a micrograph of these particles as seen through an optical microscope. The brownish color indicates reduction and/or devitrification. We feel that this method is the most promising approach. These experiments were performed by J. Richmond, M. Borden, and M. Magida at Perkin-Elmer [Danbury, CT] [now Hughes-Danbury Optical], who also mentioned that it may be possible to synthesize glasses with specific refractive indices and dispersions from solutions. However, this would require considerable research into and understanding of glass chemistry. Perkin-Elmer's impending sale of this division to Hughes and our concentration on IM ended further collaboration with this group in mid-1989. We are very grateful for their efforts.

As our final tactic, we tumbled coarsely sized SK5 glass in our jar mill continuously for two months. We hoped that the action of glass grinding upon glass would round and smooth the shard-like particles. We saw no improvements in shape or texture when we compared these particles to unmilled particles from microscope observations. The best methods appear to be the molten salt and plasma spray techniques. The final section of this dissertation will analyze and model the performance of our solid matrix Christiansen filters.

(A.)

(B.)

Figure 35. Plasma sprayed SK5 glass particles magnified (A.) 100 X and (B.) 250 X.

Section 9: Optical Characterization and Theoretical Modeling

In this section, I will present evidence that our CFs exhibit small angle multiple scattering. The independence of the spectral bandpass location to the angle of incidence of the incident light will be demonstrated. I will test the validity of the Clarke and Clewell theories described in Section 4 and attempt to determine if changing the particle shape in the Clarke theory will improve our prediction of bandpass width. The structure of the interfaces between the particles and the plastic matrix will be shown with scanning electron micrographs.

As mentioned in Section 4, it is important to determine the nature of the scattering in the CFs; whether it is single, double or multiple scattering; in order to correctly describe their optical characteristics. A medium exhibiting single particle scattering will follow a simple relation: The light intensity scattered by M_p particles will be M_p times the intensity scattered by a single particle. This proportionality holds only if the radiation incident on each particle is the original beam of intensity I_o . If the original beam is sufficiently reduced by scattering, many particles receive only scattered light and the condition of multiple scattering prevails.¹¹³ This changes the character and complexity of the scattering theory.

Van de Hulst²⁷ suggests criteria for extinction to test for the absence of multiple scattering based on the attenuation of incident intensity I_o : The intensity of a beam (I_t) passing through the sample is reduced to $e^{-\tau}$ times I_o ,

$$I_t = I_o e^{-\tau}$$

where τ is the optical depth of the sample along the beamline. The criteria are:

- If $\tau < 0.1$ [$T(\%) > 90.5\%$], single scattering dominates.
- If $0.1 < \tau < 0.3$ [$90.5\% > T(\%) > 74.1\%$], a correction for double scattering is appropriate.
- If $0.3 < \tau$ [$74.1\% > T(\%)$], multiple scattering exists.

Obviously τ depends upon the thickness of the filter and the particle density. Figure 36 shows the decrease in spectral transmission as similar filters are stacked upon each other. Figure 36a contains the spectra for a representative SK5-SAN blue CF; we see a 50% decrease in transmission as each filter is added. The average increase in optical depth τ is 0.749. Figure 36b contains the spectra for a SK11-SAN red-NIR CF with an average increase in τ of 0.938. The decrease in the peak transmission in both graphs indicates absorption too, but the decrease in transmission is faster than would be expected from simple absorption, especially in the red-NIR CF. These values are typical for all of our filters and indicate multiple scattering in them.

An important requirement for our CFs demanded by our sponsor was that the bandpass location not shift when the angle of the incident beam with respect to the filter surface is changed. Figure 37 shows the effect upon bandpass location when the filters are tilted with respect to beam direction. The graphs are for blue, red-NIR, and green [SK5-PS] CFs, respectively. The listed angles denote the tilt away from normal incidence [0° = filter surface normal to beam direction]. We see the bandpasses remain stationary as the angle is increased as predicted. Unfortunately, the transmission peaks decrease and the bandpasses narrow as the filters are tilted. This

Figure 36. The decrease in spectral transmission as filters are stacked (van de Hulst test); (A.) SK5/SAN (B.) SK11/SAN.

Figure 37. The insensitivity of the bandpass location to angle of incidence; (A.) SK5/SAN, (B.) SK11/SAN, (C.) SK5/PS.

is due to increased scattering as the path lengths, h , through the filters are increased by the tilting. We can verify that this is not due solely to increased absorption if we calculate the absorption coefficient, Ξ , for a homogeneous slab with the index of SAN as a function of the incident angle. The absorption coefficient (Ξ_a) can be obtained from the formula:¹¹⁴

$$T_{slab} = \frac{(1-R)^2 \exp(-\Xi_a h)}{1-R^2 \exp(-2\Xi_a h)}$$

where T is the transmittance through the slab, R is the reflectance from the slab obtained from the Fresnel formulas, and h is the path length through the slab. If we substitute the peak transmittances of the blue CF in Figure 37a and solve for Ξ_a as a function of tilt angle, we obtain the values listed in Table 7. The absorption coefficient changes as the filter is tilted which indicates inhomogeneities in the filter. The particles may not be uniformly distributed throughout the filter since Ξ_a decreases with tilt angle.

Figure 38 illustrates the effect on the filter bandwidth when the size of the detector aperture is decreased. Square apertures were constructed from two pairs of carefully spaced razor blades and placed over the spectrophotometer detector windows before measurements were taken. The spectra in Figure 38 are labeled according to the angle subtended by the aperture from the filter. We see the difference in the spectral bandwidth of a blue filter when apertures are added in Figure 38a. The bandpass appears Gaussian and centered on λ_c with the apertures in place.

The short wavelength side of the bandpass is an absorption edge so transmission

Table 7. Calculation showing a varying absorption coefficient as a function of incidence angle for a SK5/SAN CF.

Angle	h (cm)	R	T	Ξ_a (cm ⁻¹)
0°	0.158750	0.05271	0.359	5.773715
20°	0.168939	0.05301	0.348	5.605823
40°	0.207234	0.05900	0.325	4.838851
60°	0.317500	0.10462	0.235	3.868015
70°	0.464153	0.18748	0.170	2.928021

Figure 38. The decrease in spectral bandwidth when the detector aperture is decreased; (A.) SK5/SAN, (B.) SK11/SAN, (C.) SK11/SAN with dye.

drops off sharply. We see a non-uniform decrease in transmission on the long wavelength side which indicates that some scattered wavelengths are being rejected by the apertures. This illustrates the importance of baffles in any optical system employing these filters. Apparently, the apertures were smaller than the beamwidth of the source in the spectrophotometer which caused the decrease in peak transmission. Figure 38b shows the aperture effect upon a red-NIR filter. However, since this is an edge filter, the approach to a Gaussian peak is not apparent. Figure 38c contains the spectra of one of our NIR CFs with visible absorbing dyes for five apertures (2, 1, 0.5, 0.2, 0.1 degrees) and no aperture. We see little effect upon the spectral edge. This points to dye absorption as the principal mechanism for light rejection, and not scattering.

More evidence of the Christiansen effect is seen in Figures 39 and 40. Collimated light was directed at CFs and the scattered intensities were recorded with a linear CCD array. Light from a tungsten-halogen lamp was passed through a narrow-band interference filter transmitting 500 nm (FWHM $\approx$ 10 nm) and directed at one of our green SK5-PS CFs ($\lambda_c \approx$ 510 nm). Then the 500 nm filter was replaced with a 900 nm filter. Figure 39a shows the CCD outputs with and without the CF at 500 nm. We see scattering that leads to a divergence of one degree on each side. Figure 39b shows the CCD outputs much farther away from λ_c at 900 nm. Scattering here results in a beam that diverges three degrees on each side. Beam divergence due to small angle scattering can also be seen in Figure 40. A HeNe laser ($\lambda =$ 633 nm) was directed at a red-NIR filter (edge begins at 600-650 nm) and at a blue filter (λ_c

Figure 39. Scattering of incoherent light by a green SK5/PS CF; (A.) minor scattering at 500 nm, (B.) much more scattering at 900 nm.

Figure 40. Scattering of HeNe laser light; (A.) minor scattering by a red SK11/SAN CF, (B.) much scattering by a blue SK5/SANCF.

= 392 nm). The red-NIR CF spreads the beam only 1.25 degrees beyond the unscattered beam half-width of 0.75 degree (Figure 40a). The blue filter scatters the laser light to 8 degrees on either side (Figure 40b). This filter contains our smallest particles; those that passed through all of the meshes and were designated less than 37 μm in size. Apparently, this filter contains some rather small particles which scatter the light to larger angles.

The variance in the size of the particles embedded in this blue filter (12-6) can be seen in the scanning electron micrograph in Figure 41a. The magnification in this photograph is 200x; we see irregularly shaped particles ranging in size from 2 μm to 40 μm . These particles were near the surface of the filter. Figure 41b is a micrograph of a similar CF. Note the shard-like shapes of the particles which would complicate a theoretical prediction of optical performance that depended upon particle shape. Figure 42a shows the effect of the coupling agent we added to improve polymer-glass adhesion. This is a micrograph of a surface particle in a blue SK5-SAN filter with coupling agent. As surface tension pulled the plastic away from the glass particle, we see adhesion points along the interface toward the right side of the micrograph. Figure 42b shows an internal cross-section of a red-NIR SK11-SAN filter with coupling agent. The streaks were formed by the cutting instrument. We see the glass particles completely embedded in the plastic matrix with no apparent voids along the interfaces.

The data presented in this section confirmed or strongly supported general statements about the optical characteristics of our CFs. The remainder of this section

Figure 41. Scanning electron micrographs of glass particles on the surfaces of two SK5/SAN CFs.

(A.)

163

(B.)

Figure 42. (A.) Polymer-glass adhesion is seen in this SEM photograph of a surface particle on a SK5/SAN CF with coupling agent.
(B.) Internal cross-section shows no voids around particles.

will examine the validity of the theories described in Section 4. The evidence of multiple scattering from the van de Hulst test, the large size of the glass particles when compared to the wavelengths (large size parameter), and the irregular, non-spherical particle shapes should convince the reader of the futility of applying Mie theory to these CFs. The complexity of the scattering requires the statistical approach of the Clarke theory or the simple, physical approach of Clewell's formula. These will be studied next.

The Clarke formula for the FWHM of the bandpass of a CF is

$$(\Delta\lambda)_{3dB} = \lambda_c \sqrt{\frac{2.77}{K} \left(\frac{1}{\sqrt{cL\alpha} (\Delta n')} \right)} \quad (4.64)$$

where λ_c = Christiansen wavelength where dispersion curves intersect,

c = relative particle concentration (dimensionless),

L = filter length,

α = particle radius,

$\Delta n'$ = derivative of index difference with respect to wavelength,

K = constant depending upon the mechanism for degrading phase coherence,

i.e. $K_n = 48.7$ due to particle number density fluctuations,

$K_p = 8.2$ due to fluctuations in the path length through individual spherical particles,

$K_r = 48.7 (\sigma'_\alpha)^2$ due to particle radius fluctuations,

$(\sigma'_\alpha)^2 = \frac{1}{3} [\alpha_{\max} - \alpha_{\min} / \alpha_{\max} + \alpha_{\min}]$.

Table 8 lists the parameters for two red-NIR CFs, two blue CFs and one green CF. Table 9 lists three values for the FWHM from the three values of K and the measured FWHM. The derivative $\Delta n'$ was taken from the dispersion intersection study of Section 5. L was measured directly and α was determined from the mesh size designation for the particular glass powder. The peak transmission wavelength of the CFs with Gaussian-shaped bandpasses was identified with λ_c . The λ_c determined by the intersection search was employed for the red-NIR edge filters. The particle volume concentration c was determined by dividing the weight of the glass in the filter by the glass density to get the glass volume. The glass volume was then divided by the filter volume to get c . The weight of the glass in a filter was determined by subtracting the weight of a filter of pure plastic from the weight of the CF. Incidentally, if we estimate the total area of the 17 largest particles in the micrograph of Figure 41a and divide by the area of the micrograph, we would get a value for c that is less than 1% different than the value determined by the previous method.

Most of the predicted bandwidths do not approach the measured values. The two filters with the closest values are numbered 14-15 and 15-7 which are 16% and 20% in error, respectively, when K_r is used. Coincidentally, these filters have a smaller variance in particle size than the three other filters [$37\text{-}44\text{ }\mu\text{m}$ vs. $< 37\text{ }\mu\text{m}$]. Apparently, irregularly shaped particles can almost be regarded as fluctuations in particle radius. Table 10 computes the K values obtained from the measured or theoretical (from Section 5) bandwidths. We see consistency in the K values for filters 12-6, 14-15, and 15-7 but these values do not match Clarke's K values.

Table 8. Physical parameters for five CFs.

Filter	Type	λ_c (μm)	L (μm)	2α (μm)	$\Delta n'$ (μm^{-1})	c
12-6	blue	0.392	1587.5	37	0.18	0.03445
14-15	blue	0.392	1587.5	40.5	0.18	0.02952
12-5	red-NIR	0.790	1587.5	37	0.01195	0.03163
15-7	red-NIR	0.790	1587.5	40.5	0.01195	0.02109
18-8	green	0.510	1587.5	37	0.10581	0.02952

Table 9. Predicted and measured values of spectral bandwidths.

Filter	Type	$\Delta\lambda_{3\text{dB}} K_n$	$\Delta\lambda_{3\text{dB}} K_p$	$\Delta\lambda_{3\text{dB}} K_r$	measured
12-6	blue	0.016328	0.039793	0.032400	0.082222
14-15	blue	0.016857	0.041081	0.099323	0.085555
12-5	red-NIR	0.517035	1.260021	1.025961	2.961 theo
15-7	red-NIR	0.605256	1.475016	3.566099	2.961 theo
18-8	green	0.039035	0.095130	0.077458	0.1435

Table 10. Determined values for K from the Clarke theory.

Filter	Type	Determined K Value
12-6	blue	1.920630
14-15	blue	1.890685
12-5	red-NIR	1.485196
15-7	red-NIR	2.035269
18-8	green	3.603660
		$K_{\text{ave}} = 2.187088$
		$\sigma_K = 0.818723$

The Clarke theory assumes spherical particle shapes and estimates the average path length through the particles as

$$\langle \zeta \rangle = 1.235 \alpha \quad [\alpha \text{ is the average radius of the spheres}]$$

with a standard deviation of

$$\sigma_{\zeta} = 0.506 \alpha.$$

These relations were calculated analytically from the spherical geometry. An attempt was made to replace the spherical particles with cylindrical particles in the theory and reexamine the measured bandwidths. The average path length through a right cylinder as a function of the cylinder length to radius ratio was estimated by a Monte Carlo computer simulation. From this simulation, an approximate functional relationship between the average path length, the cylinder length, and the cylinder radius was deduced and substituted into the Clarke formula.

Before describing the Monte Carlo simulation of the cylindrical particles, I will give the results of a Monte Carlo calculation of the average path length through a sphere. This was done to evaluate the random number generator I used¹¹⁵ and determine the sufficient number of trials to run. Random positions for two points on a unit sphere [$\alpha = 1$] were generated and the distance between them was calculated. The average distance for 1,000,000 pairs of points was

$$\langle \zeta \rangle_{1,000,000} = 1.321212$$

and the standard deviation was

$$(\sigma_{\zeta})_{1,000,000} = 0.507121.$$

With the radius of the sphere increased to 10, the results for 10,000 trials were

$$\langle \zeta \rangle_{10,000} = 13.220529$$

and

$$(\sigma_{\zeta})_{10,000} = 5.082117.$$

From these results, we see that these statistics are both proportional to the radius and the difference between 1,000,000 and 10,000 trials is less than 0.2%. The standard deviation is fairly close to Clarke's result (0.4% error) but the average value is 7% greater than Clarke's average. There may be a small correlation between the outputs from the random number generator. I decided to run 10,000 trials for each of the cylinder simulations.

Figure 43 illustrates the factors to consider when calculating the average path length through a cylinder. The problem is not as symmetrical as in the spherical case and is not easily reduced to integrals. As the radius increases and the length shrinks (disk shape), we would expect the average path length to approach the cylinder length. As the radius decreases and the length increases (rod shape), we would expect the average path length to approach the cylinder diameter. A function which has these characteristics is

All paths through a sphere can be treated as chords referred to the center. The average path length can be determined analytically.

There is no obvious reference point for paths through a cylinder. The geometry has no symmetry and it is difficult to solve for the average path length analytically.

DISK SHAPE

Average path length $\longrightarrow 2Z$
when $R \gg Z$.

ROD SHAPE

Average path length $\longrightarrow 2R$
when $R \ll Z$.

Figure 43. The problems in estimating the average path length through a cylinder.

$$\frac{RZ}{R+Z} \rightarrow Z \text{ for } R \gg Z,$$

$$\frac{RZ}{R+Z} \rightarrow R \text{ for } R \ll Z.$$

A series of Monte Carlo calculations was conducted for cylinder radii and half-heights ranging from 0.1 to 30.0. The point pairs were chosen as follows:

- The surface areas of the top, bottom, and side of the cylinder were calculated and the fractions with respect to the total surface area were formed.
- A number was randomly chosen from the [0,1] range to locate the points on the top, bottom, or side depending upon the fraction determined previously.
- For cylindrical coordinates [r,θ,z], if the point lay on the top or bottom, z is constant, r is chosen from [0,R], and θ is chosen from [0,2π].
- If the point lay on the side, r is constant, z is chosen from [-Z,Z], and θ is chosen from [0,2π].
- 10,000 point pairs were generated, the distances between the point pairs were calculated, and the average distance was computed.
- Each average was divided by the function

$$\frac{\frac{RZ}{\frac{R}{\pi} + Z}}{\pi} \quad (9.1)$$

to see if a proportionality constant could be deduced from all of the Monte Carlo cases.

The average value of the average path lengths from this series of runs was 1.082443 and the standard deviation was an unfavorable 0.195837.

Substituting (9.1) for α in the Clarke formula and solving for K, we get

$$K = \left(\frac{\lambda_c}{(\Delta\lambda)_{3dB} (\Delta n')} \right)^2 \left[\frac{\frac{R}{\pi} + Z}{2.77 cLRZ} \right]. \quad (9.2)$$

Table 11 lists the new K values for cylindrical particles from (9.2) using the parameters of the five filters listed in Table 8. The cylinder half-heights (z) were chosen to be multiples of the particle radius ($z = \alpha, 2\alpha, 3\alpha$). A comparison of the statistics in the derived K values from the spherical and three cylindrical models is given in Table 12. We see smaller values of K for cylindrical shapes but no improvement in modeling the bandwidth. The standard deviations of the K values for all shapes are approximately 1/3 the average values. Thus, no improvement was gained by considering cylindrical instead of spherical particles.

The Clarke theory correctly predicts the Gaussian bandpasses of our blue CFs and the dependence of the bandwidth on the derivative of the index difference ($\Delta n'$) but is limited in accounting for the fluctuations in particle size and shape.

The Clewell formulas (4.35 and 4.36) were tested for their ability to predict spectral transmittance away from λ_c . Unfortunately, the particles in our filters are much larger than the wavelengths ($d > \lambda$) so that the scattering matrix S (4.35) reduces to

Table 11. K values for the CFs when a cylindrical particle model is used.

Filter	Type	R (μm)	Z (μm)	K
12-6	Blue	18.5	18.5	0.329990
			37.0	0.290152
			55.5	0.276872
14-15	Blue	20.25	20.25	0.324846
			40.5	0.285629
			60.75	0.272556
12-5	Red-NIR	18.5	18.5	0.255177
			37.0	0.224370
			55.5	0.214102
15-7	Red-NIR	20.25	20.25	0.349687
			40.5	0.307471
			60.75	0.293398
18-8	Green	18.5	18.5	0.619158
			37.0	0.544410
			55.5	0.519493

Table 12. Comparison of K values from the spherical particle and cylindrical particle models.

	Sphere	$z = R$	$z = 2R$	$z = 3R$
	1.920630	0.329990	0.290152	0.276872
	1.890685	0.324846	0.285629	0.272556
	1.485196	0.255177	0.224370	0.214102
	2.035269	0.349687	0.307471	0.293398
	3.603660	0.619158	0.544410	0.519493
Average	2.187088	0.375772	0.330406	0.315284
σ_{n-1}	0.818723	0.140668	0.123686	0.118025

$$S \sim \frac{\left[\frac{(n^2-1)}{(n^2+1)} \right]^2}{d}$$

where $n = n_{\text{particle}} / n_{\text{plastic}}$ and d = diameter of the particle. We performed a least-squares fit of the function

$$T(\lambda) = \exp(-\Gamma S(\lambda) L)$$

to the measured spectra of the two blue and two red-NIR CFs of Table 11. The variable Γ is an adjustable fitting parameter and the index values were taken from the dispersion data of Sections 5 and 6. Figures 44 and 45 contain plots of the measured spectra and the least-squares fits. The dispersion data predicted different λ_c s than those measured so the fitted curves were shifted to the measured λ_c s. The least-squares fits of Figure 44 were done over the range $0.365 \mu\text{m}$ to $0.80 \mu\text{m}$ but only data for $0.365 \mu\text{m}$ to $0.60 \mu\text{m}$ is plotted. We see that the Clewell formulas can predict the gross features of the bandpasses but each filter has a different fitting parameter Γ .

To summarize, both the Clarke and Clewell theories can predict gross features of the transmittance spectrum of our CFs, but neither can make accurate predictions of the bandwidth or the scattering loss. The particle shapes and sizes proved to be too varied for these theories handle.

In this section, we have presented evidence of multiple scattering in our CFs. The independence of the bandpass locations to incident angle was demonstrated and

the structure of the filters was observed in SEM micrographs. Finally, two theories were evaluated as predictors of spectral characteristics.

Figure 44. Least squares fit of the Clewell equation to measured spectra of two SK5/SAN CFs.

Figure 45. Least squares fit of the Clewell equation to measured spectra of two SK11/SAN CFs.

Section 10: Conclusions and Suggestions for Further Study.

We have fabricated solid Christiansen filters composed of glass particles in polymer matrices with bandpasses in the blue, green, and red-NIR regions of the spectrum. These filters can be made as large as the size of the injection molding machine will allow. They also can be mass produced very economically. We have characterized their spectral transmission properties and their operation over a range of incident angles.

With suitable baffling to reject the scattered wavelengths, these filters can be employed in optical systems that do not have strict imaging requirements or high power applications. Since the thermo-optic coefficient ($dn/dT(C)$) of the polymer matrix is typically ten times higher than that of glass, one could attempt to construct a thermal optical switch with these filters. A liquid matrix CF would be better suited for this application. Not only can we construct a large aperture, planar filter, but a large aperture, lightweight lens can also be molded. This lens could not only filter light but also focus it. As discussed in Section 5, other optical materials could also be combined to produce bandpasses in the IR spectrum once fabrication problems are solved.

As for further development of glass-plastic CFs, switching to PC matrices would improve transmission by 10% over PS matrices. We did not meet our sponsor's request for 80% transmission at 800 nm in our red-NIR CF. If smaller, smoother, spherical particles with no internal fractures could be produced, absorption by the particles could be reduced and transmission would increase. The plasma spray

method for spherizing particles mentioned in Section 8 looks encouraging and more experimentation is needed. More dispersion data on optical polymers would be appreciated. A systematic search for index matching fluids that do not melt polymers is necessary if dispersion data is to be obtained from a Pulfrich refractometer. Finally, a better theory of multiple scattering in CFs is an item for theoretical study.

APPENDIX A.

TABLE A-1

MATERIAL #1	MATERIAL #2	WAVELENGTH RANGE 10 ⁻⁶ m		FWHM 10 ⁻⁶ m
LASF6	SF58	.3800	.4000	.01061
SF57	LASFN31	.3800	.4000	.01269
SF57	LASFN15	.3800	.4000	.01316
LASF1	SF14	.3600	.3800	.01370
SF14	LASFN30	.3600	.3800	.01374
LASF12	SFL6	.3800	.4000	.01472
LASF12	SF6	.3800	.4000	.01566
SF13	LAFN28	.3600	.3800	.01525
SF11	LASF33	.3600	.3800	.01546
LASF3	SFL56	.3800	.4000	.01632
LAKN7	TIF6	.3600	.3800	.01593
LAF25	SF14	.3800	.4000	.01681
SFL56	LASF33	.3600	.3800	.01648
LAK17	SF4	.3600	.3800	.01651
LAK17	SF61	.3600	.3800	.01654
LAK17	SF63	.3600	.3800	.01678
LASF3	SF56	.3800	.4000	.01778
SF14	LAF22A	.3800	.4000	.01790
LAF22	SF14	.3800	.4000	.01797
LAF21	SF4	.3600	.3800	.01709
LAF24	SF10	.3600	.3800	.01755
SF10	LAFN24	.3600	.3800	.01756
SF54	LAFN28	.3600	.3800	.01756
BAF12	TIF6	.3600	.3800	.01784
LAF10	SF4	.3600	.3800	.01793
LAF10	SF61	.3600	.3800	.01797
SF4	LAFN10	.3600	.3800	.01797
SKN18	TIF6	.3800	.4000	.01909
LAF12	TISF1	.3800	.4000	.01912
LAKN19	SF10	.3800	.4000	.01923
LASF3	SF11	.4000	.4200	.01765
LAK17	SF14	.4000	.4200	.01768
LAF21	SF14	.4000	.4200	.01804
LASF1	SF11	.4200	.4400	.01901
SF11	LASFN30	.4200	.4400	.01906
LASF11	SF11	.4200	.4400	.01933
LAF10	SF14	.4000	.4200	.01855
SF14	LAFN10	.4000	.4200	.01855
LASF1	SFL56	.4200	.4400	.01984
SFL56	LASFN30	.4200	.4400	.01989
LASF11	SFL56	.4200	.4400	.02018
LAK17	SF55	.4000	.4200	.02006

TABLE A-1 (continued)

LASF1	SF56	.4200	.4400	.02127
SF56	LASFN30	.4200	.4400	.02133
LASF11	SF56	.4200	.4400	.02166
TISF1	LAK31	.4000	.4200	.02107
LAK24	TISF1	.4000	.4200	.02109
LAF13	SF14	.4200	.4400	.02215
LAKN14	TISF1	.4000	.4200	.02117
LAF10	SF55	.4000	.4200	.02118
SF55	LAFN10	.4000	.4200	.02119
LAK3	TISF1	.4000	.4200	.02143
LAKN13	TISF1	.4000	.4200	.02145
SF61	LAFN28	.4000	.4200	.02163
TISF1	LAK20	.4000	.4200	.02167
LAF24	SF13	.4200	.4400	.02310
SF13	LAFN24	.4200	.4400	.02310
SF14	LAFN28	.4600	.4800	.02542
LAF13	SF55	.4000	.4200	.02277
LAKN19	SF13	.4400	.4600	.02522
LASF7	SF58	.5400	.5600	.02457
LASF9	SF57	.5200	.5400	.02902
SF57	LASFN9	.5200	.5400	.02903
LAK17	SF11	.5400	.5600	.03487
LASF8	SFL6	.5200	.5400	.03370
LAF21	SF11	.5400	.5600	.03519
LAK17	SFL56	.5400	.5600	.03613
LASF3	SF6	.5400	.5600	.03630
LAF21	SFL56	.5400	.5600	.03647
LASF8	SF6	.5200	.5400	.03534
SFL6	LASF33	.5600	.5800	.03898
LAK17	SF56	.5400	.5600	.03811
LAF21	SF56	.5400	.5600	.03849
LAF10	SF11	.5800	.6000	.04208
SF6	LASF33	.5600	.5800	.04072
LAKN19	SF63	.5000	.5200	.03673
SF13	LAK28	.5200	.5400	.03821
LAK33	SF63	.5000	.5200	.03680
SF61	LAFN24	.5000	.5200	.03704
LAF24	SF61	.5000	.5200	.03704
SF6	SFL6	.5600	.5800	.04183
LAFN2	SF13	.5200	.5400	.03913
SF13	LAF2	.5200	.5400	.03914
LAF10	SFL56	.5800	.6000	.04357
SF6	SFL6	.5400	.5600	.04079
SF10	LAK16A	.5000	.5200	.03804
LAKN16	SF10	.5000	.5200	.03806
LAKN19	SF61	.5200	.5400	.03976
SF53	LAK16A	.5000	.5200	.03940
LAKN16	SF53	.5000	.5200	.03942

TABLE A-1 (continued)

SFL6	LASFN30	.6000	.6200	.04356
SF6	LASFN30	.6000	.6200	.04533
LASF1	SFL6	.6200	.6400	.04698
LASF1	SF6	.6200	.6400	.04885
LASF11	SFL6	.6400	.6600	.05085
LASF11	SF6	.6400	.6600	.05286
SF11	SF56	.6000	.6200	.04979
SF6	SFL6	.6400	.6600	.05754
SF6	LASF32	.6600	.6800	.05947
SFL56	LAF22A	.6600	.6800	.05965
SF4	LAK33	.6000	.6200	.05545
LAF24	SF14	.6800	.7000	.06452
SF14	LAFN24	.6800	.7000	.06454
LAF24	SF55	.6800	.7000	.07038
SF55	LAFN24	.6800	.7000	.07040
LAK28	SF63	.6600	.6800	.07002
LASF11	LASF32	.6400	.6600	.06856
SSK3	TIF6	.6200	.6400	.06814
LAK10	SF18	.6200	.6400	.06888
SF15	LAK31	.6200	.6400	.07023
LAK24	SF15	.6200	.6400	.07026
LAKN14	SF15	.6200	.6400	.07038
LAK23	TISF1	.6800	.7000	.07778
BAFN10	TISF1	.6800	.7000	.07914
TISF1	BAF53	.6800	.7000	.07917
SF5	TISF1	.6600	.6800	.07783
SK19	TIF6	.6600	.6800	.07805
SK6	TIF6	.6600	.6800	.07815
SK9	TIF6	.6600	.6800	.07832
LAKN19	LAF11A	.6200	.6400	.07497
LAKN19	SF14	.7200	.7400	.07294
SF56	SFL56	.7000	.7200	.07505
SF14	LAK33	.7600	.7800	.08262
LAKN19	SF55	.7400	.7600	.08446
LAFN2	SF63	.7000	.7200	.08078
SF61	LAK28	.7400	.7600	.08955
SF55	LAK33	.7800	.8000	.09522
LAK17	LAF9	.7600	.7800	.09283
SF13	LAK16A	.7800	.8000	.09716
LAKN16	SF13	.7800	.8000	.09717
SK4	TIF6	.7000	.7200	.08943
BAF13	TISF1	.7400	.7600	.09615
SF3	LAK16A	.7600	.7800	.09954
LAKN16	SF3	.7600	.7800	.09956
SF61	LAF26	.7800	.8000	.10271
LAFN3	SF18	.7200	.7400	.09640
SF18	LAF3	.7200	.7400	.09647
LASF8	LASFN30	.7400	.7600	.10399

TABLE A-1 (continued)

LAKN13	SF15	.7400	.7600	.10508
LAK3	SF15	.7400	.7600	.10509
LASF1	LASF8	.7600	.7800	.11013
SF15	LAK20	.7600	.7800	.11180
TIF6	KZFSN4	.7800	.8000	.11765
LASF33	LASFN30	.7200	.7400	.11071
LAK23	SF5	.7000	.7200	.11103
LASF1	LASF33	.7400	.7600	.11731
LAKN14	BASF55	.7000	.7200	.11290
LAK22	SF50	.7000	.7200	.11605
SF50	LAKN22	.7000	.7200	.11606
LAK22	SF9	.7000	.7200	.12300
LASF12	SF57	.8800	.9000	.09198
LAFN8	SF13	.8000	.8200	.10430
LAF9	LAF21	.8000	.8200	.10457
LAFN2	SF61	.8200	.8400	.11321
SF61	LAF2	.8200	.8400	.11328
SF54	LAK16A	.8000	.8200	.11118
LAKN16	SF54	.8000	.8200	.11120
SK8	TIF6	.8000	.8200	.12194
BASF51	SF10	.8600	.8800	.13342
BAFN11	TISF1	.8600	.8800	.13386
TISF1	BAF54	.8600	.8800	.13397
TIF6	KZFS1	.8200	.8400	.13199
SK1	TIF6	.8600	.8800	.14361
LASF8	LASF33	.8400	.8600	.14057
LAF12	SFN64	.8600	.8800	.14753
LAF20	SF52	.8800	.9000	.16698
BAF13	SF5	.8000	.8200	.15206
BASF12	SF5	.8600	.8800	.17986
LAKN6	SF2	.8200	.8400	.17563
SSK1	F11	.8000	.8200	.17503
SK5	TIFN5	.8000	.8200	.17708
LAK21	SF16	.8400	.8600	.18772
PSK51	TIF5	.8200	.8400	.18387
SSKN8	F11	.8200	.8400	.18618
SSK1	FN11	.8200	.8400	.18632
SSKN8	FN11	.8200	.8400	.18780
LAKN6	SF12	.8800	.9000	.20227
SF16	LAKL21	.8600	.8800	.19883
PSK53	F7	.8200	.8400	.19377
PSK51	TIFN5	.8600	.8800	.20664
SF11	LAFN28	.9800	1.0000	.13730
SFL56	LAFN28	.9800	1.0000	.14137
LAK10	SF10	.9200	.9400	.15280
BASF6	TISF1	.9200	.9400	.15657
LAK10	SF53	.9400	.9600	.16490

TABLE A-1 (continued)

LAK25	SF15	.9000	.9200	.16222
LAK8	SF18	.9400	.9600	.17366
LAKN9	SF15	.9400	.9600	.17969
LASF8	LASF11	.9600	.9800	.18375
BAF50	SF52	.9800	1.0000	.21207
LAF20	SF8	.9800	1.0000	.21865
LAFN7	LAK28	.9200	.9400	.21024
SF50	BASF57	.9200	.9400	.21596
BAF9	SF2	.9000	.9200	.21763
SF9	BASF57	.9200	.9400	.22817
BALF50	TIF5	.9400	.9600	.25245
BALF6	TIFN5	.9400	.9600	.25413
SK16	F1	.9200	.9400	.25414
SSK4	F13	.9200	.9400	.25874
LAKN13	BASF13	.9600	.9800	.26987
LAK3	BASF13	.9600	.9800	.27003
BALF7	TIF5	.9800	1.0000	.27576
F1	SK55	.9400	.9600	.26735
LAFN2	LAF26	.9400	.9600	.26864
SSK50	F13	.9400	.9600	.27177
LAF2	LAF26	.9800	1.0000	.29361
BAFN11	SF5	1.0000	1.0200	.24544
SK3	TIF6	1.0000	1.0200	.20169
SF56	LAFN28	1.0000	1.0200	.15454
LAKN13	BASF55	1.0200	1.0400	.25694
BAF9	SF12	1.0200	1.0400	.28143
SF11	SFL56	1.0200	1.0400	.15891
LAK3	BASF55	1.0200	1.0400	.25711
SFN64	BASF64	1.0400	1.0600	.22626
LAK31	SFN64	1.0400	1.0600	.22438
SF5	BAF54	1.0400	1.0600	.26764
LAK24	SFN64	1.0400	1.0600	.22446
LAKN18	BASF53	1.0400	1.0600	.23383
LAKN13	BASF14	1.0600	1.0800	.28939
LAK3	BASF14	1.0600	1.0800	.28957
LAKN14	SFN64	1.0600	1.0800	.23403
LAK21	SF2	1.0800	1.1000	.32238
BASF55	LAK20	1.0800	1.1000	.29085
LAKN18	BASF54	1.0800	1.1000	.25511
LASF11	LASF33	1.1000	1.1200	.27718
BASF13	BASF55	1.1200	1.1400	.32018
BAF50	SF8	1.1400	1.1600	.30460
SF4	LAK28	1.2400	1.2600	.28290
LAF13	SFL56	1.2600	1.2800	.24716
SF3	SF13	1.2600	1.2800	.28517
LASF9	LASFN9	1.3000	1.3200	.33623
SK7	TIF6	1.3200	1.3400	.37377
SK2	TIF6	1.3600	1.3800	.39776

TABLE A-1 (continued)

LAF13	SF11	1.3600	1.3800	.28425
SF4	LAK28	1.4000	1.4200	.37092

TABLE A-2

MATERIAL #1	MATERIAL #2	WAVELENGTH RANGE 10^{-6}m		FWHM 10^{-6}m
MOLYBDENUM	AMORPHOUS SILICON	.3100	.3263	.00071
CADMIUM TELLURIDE	OSMIUM	.3179	.3263	.00073
TUNGSTEN	AMORPHOUS SILICON	.3100	.3179	.00072
GALLIUM ARSENIDE	INDIUM PHOSPHIDE	.3803	.3850	.00089
GALLIUM PHOSPHIDE	AMORPHOUS SILICON	.3924	.4000	.00094
CADMIUM TELLURIDE	LEAD SULFIDE	.3444	.3647	.00085
CADMIUM TELLURIDE	HEX ZINC SULFIDE	.3179	.3263	.00077
CADMIUM TELLURIDE	OSMIUM	.3874	.3999	.00096
OSMIUM	INDIUM ARSENIDE	.3647	.3712	.00092
MOLYBDENUM	LEAD SULFIDE	.3594	.3647	.00091
GALLIUM ARSENIDE	AMORPHOUS SILICON	.3351	.3444	.00086
SILVER	CALCIUM FLUORIDE	.3024	.3061	.00079
SILVER	GLASS SILICON DIOXIDE	.3024	.3061	.00080
SILVER	FUSED SILICA	.3024	.3061	.00080
SILVER	VYCOR	.3024	.3061	.00080
GERMANIUM	TITANIUM DIOXIDE	.3180	.3270	.00087
CADMIUM TELLURIDE	TITANIUM DIOXIDE	.3580	.3880	.00105
MOLYBDENUM	TITANIUM DIOXIDE	.3757	.3875	.00108
MOLYBDENUM	TUNGSTEN	.3024	.3100	.00087
NICKEL	LEAD SULFIDE	.3024	.3100	.00089
CADMIUM TELLURIDE	TUNGSTEN	.3757	.3875	.00113
CADMIUM TELLURIDE	OSMIUM	.3542	.3647	.00112
OSMIUM	SILICON CARBIDE	.3542	.3647	.00116
TUNGSTEN	INDIUM ARSENIDE	.3351	.3397	.00109
GALLIUM PHOSPHIDE	GERMANIUM	.3100	.3212	.00103
TUNGSTEN	TITANIUM DIOXIDE	.3580	.3880	.00125
OSMIUM	ARSENIC SULFIDE	.3700	.3757	.00140
CADMIUM TELLURIDE	ARSENIC SULFIDE	.3700	.3800	.00142
CADMIUM TELLURIDE	MOLYBDENUM	.3647	.3757	.00140
TUNGSTEN	ARSENIC SULFIDE	.3024	.3100	.00129
GERMANIUM	LEAD SULFIDE	.4133	.4275	.00105
GALLIUM ARSENIDE	GERMANIUM	.4920	.4959	.00134
OSMIUM	LEAD SULFIDE	.3936	.4133	.00112
CADMIUM TELLURIDE	INDIUM ARSENIDE	.4217	.4305	.00122
GERMANIUM	SILICON	.4881	.4959	.00142
GALLIUM PHOSPHIDE	INDIUM ARSENIDE	.4525	.4558	.00132
GOLD	LEAD SELENIDE	.4133	.4428	.00136
TUNGSTEN	INDIUM ARSENIDE	.4217	.4275	.00144

TABLE A-2 (continued)

GALLIUM PHOSPHIDE	INDIUM ANTIMONIDE	.4959	.5000	.00176
NICKEL	LEAD SELENIDE	.4428	.4592	.00194
GALLIUM ARSENIDE	INDIUM PHOSPHIDE	.4000	.4052	.00174
GOLD	RHODIUM	.4133	.4275	.00197
PLATINUM	LEAD SELENIDE	.4592	.4769	.00224
GOLD	SODIUM FLUORIDE	.4428	.4769	.00220
GOLD	LITHIUM FLUORIDE	.4600	.4769	.00225
GOLD	VYCOR	.4428	.4769	.00221
GOLD	CALCIUM FLUORIDE	.4428	.4769	.00221
GOLD	KDP	.4428	.4769	.00221
GOLD	QUARTZ CRYSTAL	.4428	.4769	.00221
GOLD	ADP	.4428	.4769	.00221
GOLD	MAGNESIUM FLUORIDE	.4428	.4678	.00219
GOLD	CRYSTAL SILICON DIOXIDE	.4428	.4678	.00219
GOLD	FUSED SILICA	.4428	.4678	.00220
GOLD	GLASS SILICON DIOXIDE	.4428	.4678	.00220
GERMANIUM	INDIUM PHOSPHIDE	.4428	.4525	.00216
GOLD	BARIUM FLUORIDE	.4428	.4658	.00220
GOLD	POTASSIUM CHLORIDE	.4428	.4600	.00219
GOLD	SODIUM CHLORIDE	.4428	.4600	.00219
RHODIUM	POTASSIUM CHLORIDE	.4000	.4133	.00198
RHODIUM	CALCIUM FLUORIDE	.4000	.4133	.00201
GOLD	METAL ALUMINUM	.4959	.5061	.00116
GERMANIUM	INDIUM ARSENIDE	.5081	.5123	.00141
COPPER	METAL ALUMINUM	.5391	.5500	.00159
GALLIUM PHOSPHIDE	MOLYBDENUM	.5000	.5166	.00158
MOLYBDENUM	CRYSTAL ARSENIC SULFIDE	.5000	.5128	.00159
CADMIUM TELLURIDE	LEAD SELENIDE	.5500	.5750	.00198
GERMANIUM	AMORPHOUS SILICON	.4960	.5081	.00186
GERMANIUM	LEAD SULFIDE	.4959	.5081	.00186
MOLYBDENUM	INDIUM PHOSPHIDE	.5166	.5298	.00198
OSMIUM	LEAD SULFIDE	.5636	.5904	.00220
OSMIUM	AMORPHOUS SILICON	.5636	.5904	.00268
GALLIUM ARSENIDE	INDIUM ANTIMONIDE	.5344	.5391	.00290
MOLYBDENUM	TUNGSTEN	.4959	.5061	.00312
TUNGSTEN	CRYSTAL ARSENIC SULFIDE	.5128	.5166	.00359
IRIDIUM	SILVER CHLORIDE	.5166	.5391	.00454
IRIDIUM	SILICON MONOXIDE	.4959	.5166	.00440
IRIDIUM	THALLIUM CHLORIDE	.5460	.5636	.00536
IRIDIUM	CALCIUM MOLYBDATE	.4959	.5086	.00488
IRIDIUM	CUBIC ZINC SULFIDE	.5767	.5904	.00576
IRIDIUM	SILICON NITRIDE	.4959	.5166	.00504
IRIDIUM	LITHIUM NIOBATE	.5636	.5770	.00588
PLATINUM	SILICON MONOXIDE	.4959	.5166	.00542
IRIDIUM	HEX ZINC SULFIDE	.5750	.5904	.00675
IRIDIUM	STRONTIUM TITANATE	.5904	.6000	.00694
PLATINUM	CALCIUM MOLYBDATE	.4959	.5086	.00617
NICKEL	MUSCOVITE MICA	.5539	.4935	.00648

TABLE A-2 (continued)

IRIDIUM	CUBIC DIAMOND CARBON	.5904	.6000	.00761
PLATINUM	SILVER CHLORIDE	.5166	.5390	.00686
TUNGSTEN	INDIUM PHOSPHIDE	.5904	.6048	.00800
GALLIUM PHOSPHIDE	TUNGSTEN	.5299	.5438	.00729
GALLIUM PHOSPHIDE	LEAD SELENIDE	.5900	.6100	.00206
OSMIUM	INDIUM ANTIMONIDE	.5961	.6078	.00243
OSMIUM	LEAD SELENIDE	.6358	.6525	.00269
TUNGSTEN	LEAD SELENIDE	.6048	.6199	.00285
METAL ALUMINUM	LEAD FLUORIDE	.6888	.7000	.00383
METAL ALUMINUM	MAGNESIUM OXIDE	.6888	.7000	.00383
METAL ALUMINUM	MUSCOVITE MICA	.6960	.6316	.00367
GALLIUM ARSENIDE	LEAD SELENIDE	.6262	.6526	.00354
METAL ALUMINUM	SAPPHIRE	.6888	.7000	.00386
METAL ALUMINUM	RUBY	.6888	.7000	.00386
METAL ALUMINUM	POTASSIUM IODIDE	.6702	.6888	.00400
METAL ALUMINUM	CALCITE	.6702	.6888	.00401
METAL ALUMINUM	SPINEL	.6702	.6888	.00407
METAL ALUMINUM	CALCIUM TITANATE	.6560	.6680	.00455
METAL ALUMINUM	SODIUM NITRATE	.6560	.6680	.00455
METAL ALUMINUM	CRYSTAL SILICON DIOXIDE	.6563	.6702	.00457
METAL ALUMINUM	POTASSIUM BROMIDE	.6500	.6702	.00455
METAL ALUMINUM	POTASSIUM CHLORIDE	.6500	.6600	.00452
METAL ALUMINUM	SODIUM CHLORIDE	.6600	.6702	.00459
METAL ALUMINUM	ADP	.6500	.6702	.00457
METAL ALUMINUM	KDP	.6500	.6702	.00457
METAL ALUMINUM	QUARTZ CRYSTAL	.6500	.6702	.00458
METAL ALUMINUM	GLASS SILICON DIOXIDE	.6438	.6500	.00507
METAL ALUMINUM	FUSED SILICA	.6438	.6500	.00507
METAL ALUMINUM	MAGNESIUM FLUORIDE	.6199	.6358	.00495
METAL ALUMINUM	SODIUM FLUORIDE	.6199	.6358	.00496
METAL ALUMINUM	CALCIUM FLUORIDE	.6358	.6500	.00509
METAL ALUMINUM	VYCOR	.6358	.6500	.00510
METAL ALUMINUM	LITHIUM FLUORIDE	.6358	.6400	.00507
MOLYBDENUM	LEAD SELENIDE	.6199	.6526	.00606
METAL ALUMINUM	CADMIUM SULFIDE	.7293	.7500	.00255
METAL ALUMINUM	THALLIUM BROMIDE	.7293	.7500	.00257
METAL ALUMINUM	KRS-6	.7293	.7500	.00257
METAL ALUMINUM	THALLIUM CHLORIDE	.7293	.7500	.00259
METAL ALUMINUM	STRONTIUM TITANATE	.7293	.7500	.00259
METAL ALUMINUM	CUBIC ZINC SULFIDE	.7293	.7500	.00260
METAL ALUMINUM	LEAD MOLYBDATE	.7293	.7500	.00261
METAL ALUMINUM	LITHIUM NIOBATE	.7293	.7500	.00261
METAL ALUMINUM	HEX ZINC SULFIDE	.7293	.7500	.00262
METAL ALUMINUM	CUBIC DIAMOND CARBON	.7293	.7500	.00262
METAL ALUMINUM	SILICON MONOXIDE	.7085	.7293	.00286
METAL ALUMINUM	SILVER CHLORIDE	.7085	.7293	.00288
METAL ALUMINUM	SILICON NITRIDE	.7085	.7293	.00289

TABLE A-2 (continued)

METAL ALUMINUM	CALCIUM MOLYBDATE	.7085	.7293	.00290
METAL ALUMINUM	SILICON CARBIDE	.7500	.7749	.00362
NICKEL	METAL ALUMINUM	.7293	.7500	.00371
OSMIUM	INDIUM PHOSPHIDE	.7560	.7749	.00388
IRIDIUM	METAL ALUMINUM	.7749	.7999	.00478
METAL ALUMINUM	TITANIUM DIOXIDE	.7600	.7749	.00471
METAL ALUMINUM	ARSENIC SULFIDE	.7600	.7749	.00506
RHODIUM	METAL ALUMINUM	.7500	.7749	.00646
RHODIUM	TITANIUM DIOXIDE	.7400	.7600	.00756
RHODIUM	ARSENIC SULFIDE	.7400	.7600	.00756
RHODIUM	ARSENIC SULFUR	.7400	.7600	.00773
NICKEL	CUBIC ZINC SULFIDE	.7293	.7514	.00778
GALLIUM ARSENIDE	MOLYBDENUM	.7749	.8051	.00897
NICKEL	KRS-6	.7293	.7514	.00842
TUNGSTEN	SILICON	.7560	.7653	.00868
RHODIUM	KRS-5	.7400	.7749	.00880
MOLYBDENUM	SILICON	.7847	.7948	.00919
METAL ALUMINUM	THALLIUM BROMIDE	.8750	.8856	.00299
METAL ALUMINUM	LITHIUM NIOBATE	.8750	.8856	.00300
METAL ALUMINUM	CADMIUM SULFIDE	.8750	.8856	.00301
METAL ALUMINUM	CUBIC ZINC SULFIDE	.8750	.8856	.00302
METAL ALUMINUM	HEX ZINC SULFIDE	.8750	.8856	.00302
METAL ALUMINUM	KRS-6	.8750	.8856	.00302
METAL ALUMINUM	STRONTIUM TITANATE	.8750	.8800	.00301
METAL ALUMINUM	LEAD MOLYBDATE	.8750	.8856	.00302
METAL ALUMINUM	THALLIUM CHLORIDE	.8856	.9000	.00320
OSMIUM	PLATINUM	.8103	.8265	.00297
CADMIUM TELLURIDE	PLATINUM	.8200	.8266	.00305
NICKEL	METAL ALUMINUM	.8500	.8551	.00342
OSMIUM	RHODIUM	.8265	.8550	.00357
GALLIUM PHOSPHIDE	OSMIUM	.7897	.8103	.00356
OSMIUM	CRYSTAL ARSENIC SULFIDE	.7897	.8103	.00356
OSMIUM	RHODIUM	.8266	.8856	.00391
METAL ALUMINUM	ARSENIC SULFIDE	.8600	.8750	.00422
METAL ALUMINUM	CUBIC DIAMOND CARBON	.8500	.8750	.00422
METAL ALUMINUM	SILICON CARBIDE	.8500	.8750	.00423
METAL ALUMINUM	TITANIUM DIOXIDE	.8600	.8750	.00447
CADMIUM TELLURIDE	PLATINUM	.8266	.8400	.00432
OSMIUM	ARSENIC SELENIDE	.8030	.8103	.00451
NICKEL	OSMIUM	.8551	.8856	.00542
IRIDIUM	OSMIUM	.8266	.8550	.00530
IRIDIUM	METAL ALUMINUM	.8266	.8500	.00553
OSMIUM	CRYSTAL ARSENIC SULFIDE	.8000	.8333	.00603
MOLYBDENUM	INDIUM PHOSPHIDE	.8266	.8856	.00658
RHODIUM	METAL ALUMINUM	.7999	.8250	.00665
OSMIUM	SILICON CARBIDE	.8550	.8856	.00754
GALLIUM PHOSPHIDE	MOLYBDENUM	.8856	.9000	.00784

TABLE A-2 (continued)

METAL ALUMINUM	SILICON NITRIDE	.9000	.9250	.00353
METAL ALUMINUM	CALCIUM MOLYBDATE	.9000	.9250	.00354
METAL ALUMINUM	SILVER CHLORIDE	.9000	.9250	.00354
METAL ALUMINUM	SPINEL	.9250	.9500	.00373
METAL ALUMINUM	SILICON MONOXIDE	.9000	.9250	.00364
METAL ALUMINUM	SODIUM CHLORIDE	.9250	.9500	.00375
METAL ALUMINUM	CRYSTAL SILICON DIOXIDE	.9250	.9500	.00375
METAL ALUMINUM	QUARTZ CRYSTAL	.9250	.9500	.00376
METAL ALUMINUM	CALCITE	.9250	.9460	.00375
METAL ALUMINUM	SAPPHIRE	.9250	.9500	.00376
METAL ALUMINUM	POTASSIUM BROMIDE	.9250	.9500	.00376
METAL ALUMINUM	POTASSIUM IODIDE	.9250	.9500	.00376
METAL ALUMINUM	KDP	.9250	.9500	.00376
METAL ALUMINUM	ADP	.9250	.9500	.00376
METAL ALUMINUM	LEAD FLUORIDE	.9250	.9500	.00376
METAL ALUMINUM	MAGNESIUM OXIDE	.9250	.9500	.00378
MOLYBDENUM	RHODIUM	.8856	.9537	.00475
MOLYBDENUM	PLATINUM	.8856	.9537	.00485
IRIDIUM	MOLYBDENUM	.8856	.9537	.00552
RHODIUM	TUNGSTEN	.9184	.9537	.00706
MOLYBDENUM	NICKEL	.9537	.9919	.00760
RHODIUM	INDIUM PHOSPHIDE	.9750	.9837	.00861
MOLYBDENUM	ARSENIC SELENIDE	.9000	.9200	.00803
PLATINUM	TUNGSTEN	.9184	.9537	.00829
CADMIUM TELLURIDE	MOLYBDENUM	.9322	.9464	.00851
MOLYBDENUM	SILICON CARBIDE	.9537	1.0330	.01085
MOLYBDENUM	KRS-5	.9537	1.0330	.01089
PLATINUM	INDIUM PHOSPHIDE	.9700	.9750	.01068
OSMIUM	CUBIC ZINC SULFIDE	.9184	.9537	.01065
GALLIUM PHOSPHIDE	RHODIUM	.8856	.9537	.01065
COPPER	SILVER	1.8790	1.9070	.00922
GOLD	SILVER	1.8790	1.9070	.00935
GOLD	SILVER	1.9070	1.9370	.00959
COPPER	SILVER	1.9070	1.9370	.00973
TUNGSTEN	SILICON CARBIDE	1.4760	1.4940	.00798
TUNGSTEN	SELENIUM ARSENIC	1.4760	1.4940	.00805
TUNGSTEN	ARSENIC SELENIDE	1.4420	1.4590	.00921
PLATINUM	RHODIUM	2.3840	2.4550	.01665
TUNGSTEN	ARSENIC SULFIDE	1.4940	1.5120	.01102
TUNGSTEN	ARSENIC SULFUR	1.4940	1.5120	.01109
NICKEL	PLATINUM	2.3840	2.4550	.01786
TUNGSTEN	IRTRAN 4	1.4940	1.5000	.01105
PLATINUM	LEAD SULFIDE	2.3840	2.4000	.01882
TUNGSTEN	TITANIUM DIOXIDE	1.4940	1.5000	.01195
PLATINUM	INDIUM ANTIMONIDE	2.3840	2.4550	.01960
GOLD	SILVER	1.6530	1.6750	.01384
RHODIUM	LEAD SELENIDE	2.9520	3.0240	.02500
PLATINUM	METAL ALUMINUM	2.4800	2.6380	.02225

TABLE A-2 (continued)

GOLD	SILVER	1.6310	1.6530	.01478
CADMIUM TELLURIDE	TUNGSTEN	1.4590	1.4760	.01331
TUNGSTEN	CRYSTAL ARSENIC SULFIDE	1.4250	1.4420	.01365
PLATINUM	INDIUM ANTIMONIDE	1.1810	1.2400	.01303
GERMANIUM	PLATINUM	1.2400	1.3050	.01378
PLATINUM	LEAD SULFIDE	1.2400	1.3000	.01424
PLATINUM	LEAD SELENIDE	1.3050	1.3780	.01539
PLATINUM	SILICON	1.0330	1.0780	.01244
TUNGSTEN	METAL ALUMINUM	1.7220	1.7710	.02075
GALLIUM ARSENIDE	PLATINUM	.9919	1.0330	.01204
PLATINUM	AMORPHOUS SILICON	1.0330	1.0780	.01258
OSMIUM	TUNGSTEN	1.5900	1.6300	.01923
PLATINUM	LEAD SELENIDE	4.9590	5.0000	.01518
IRIDIUM	LEAD SELENIDE	3.8750	4.0000	.03233
RHODIUM	LEAD SELENIDE	3.0240	3.1000	.03776
METAL ALUMINUM	LEAD TELLURIDE	3.8150	3.8500	.06276
METAL ALUMINUM	LEAD SULFIDE	3.0000	3.1000	.05095
IRIDIUM	TELLURIUM	4.0000	5.0000	.08559
METAL ALUMINUM	LEAD SELENIDE	3.2630	3.4440	.06641
IRIDIUM	LEAD TELLURIDE	4.4280	4.5920	.09065
IRIDIUM	LEAD SULFIDE	3.5420	3.7500	.08798
RHODIUM	LEAD TELLURIDE	4.1330	4.2750	.10309
IRIDIUM	GALLIUM ANTIMONIDE	3.1000	3.5000	.09142
GERMANIUM	IRIDIUM	3.2000	3.5420	.09552
IRIDIUM	SILICON	3.1000	3.3033	.09153
IRIDIUM	INDIUM ANTIMONIDE	3.1000	3.5420	.09765
IRIDIUM	INDIUM ARSENIDE	3.1000	3.5420	.09798
IRIDIUM	AMORPHOUS SILICON	3.2820	3.4170	.10091
SILVER	INDIUM ARSENIDE	4.7690	5.1660	.16287
PLATINUM	GALLIUM ANTIMONIDE	3.8750	4.1330	.13136
GALLIUM PHOSPHIDE	PLATINUM	3.1000	3.2630	.10483
PLATINUM	ARSENIC SELENIDE	3.1000	3.2630	.10521
PLATINUM	CRYSTAL ARSENIC SULFIDE	3.1000	3.2630	.10529
GALLIUM ARSENIDE	SILVER	4.7690	5.1660	.16607
SILVER	SILICON	4.7690	5.0000	.16367
SILVER	TUNGSTEN	3.4440	3.6470	.12126
RHODIUM	METAL ALUMINUM	3.3060	3.5420	.11920
GOLD	INDIUM ARSENIDE	4.5920	4.9590	.16688
OSMIUM	SILVER	3.8750	4.1330	.14039
COPPER	INDIUM ARSENIDE	4.7680	5.1660	.17625
GOLD	GALLIUM ARSENIDE	4.5920	4.7680	.16697
SILVER	SILICON CARBIDE	4.1330	4.4280	.15292
MOLYBDENUM	LEAD SELENIDE	5.9040	6.1090	.11874
MOLYBDENUM	TELLURIUM	5.9040	6.0000	.12054
SILVER	LEAD TELLURIDE	6.1990	6.5260	.14554
MOLYBDENUM	LEAD TELLURIDE	6.5260	6.7020	.15262
PLATINUM	LEAD SELENIDE	5.0000	5.1660	.11903

TABLE A-2 (continued)

COPPER	LEAD TELLURIDE	6.5260	6.7020	.17074
SILVER	TELLURIUM	5.6360	6.0000	.15618
GOLD	LEAD TELLURIDE	6.3580	6.5260	.17573
SILVER	LEAD SELENIDE	5.6360	6.1990	.16636
MOLYBDENUM	NICKEL	5.3910	5.6360	.16532
SILVER	LEAD SULFIDE	5.1660	5.6360	.16245
GERMANIUM	SILVER	5.1660	5.4000	.15998
SILVER	GALLIUM ANTIMONIDE	5.1660	5.6360	.16411
COPPER	LEAD SELENIDE	5.6360	6.1990	.18013
COPPER	NICKEL	5.1660	5.6350	.16690
SILVER	AMORPHOUS SILICON	5.1660	5.3300	.16491
TUNGSTEN	LEAD SELENIDE	6.8880	7.2930	.22395
COPPER	TELLURIUM	5.6350	6.0000	.18384
TUNGSTEN	LEAD TELLURIDE	7.2930	7.7490	.23801
TUNGSTEN	TELLURIUM	6.8880	7.0000	.22131
MOLYBDENUM	LEAD SULFIDE	5.3910	5.6360	.17609
GOLD	LEAD SELENIDE	5.6360	5.9040	.18535
SILVER	INDIUM ANTIMONIDE	5.1660	5.5000	.17134
GERMANIUM	MOLYBDENUM	5.4000	5.6360	.17750
MOLYBDENUM	GALLIUM ANTIMONIDE	5.1660	5.3910	.17164
MOLYBDENUM	AMORPHOUS SILICON	5.1660	5.3300	.17065
GOLD	NICKEL	5.1660	5.3910	.17316
COPPER	LEAD SULFIDE	5.1660	5.6350	.17813
GOLD	TELLURIUM	5.3910	5.9040	.18747
COPPER	GERMANIUM	5.1660	5.4000	.17554
GERMANIUM	SILICON DIOXIDE	9.0000	9.4000	.09767
OSMIUM	SILICON DIOXIDE	9.5000	10.0000	.11049
GERMANIUM	SILICON DIOXIDE	9.5000	9.8000	.11794
CADMIUM TELLURIDE	SILICON DIOXIDE	8.8560	9.5370	.11242
GALLIUM ARSENIDE	SILICON DIOXIDE	9.5000	9.5370	.11642
GALLIUM PHOSPHIDE	SILICON DIOXIDE	9.5000	10.0000	.11944
CADMIUM TELLURIDE	GLASS SILICON DIOXIDE	9.3020	9.5240	.18333
NICKEL	LEAD TELLURIDE	8.2660	8.8560	.43320
GALLIUM ARSENIDE	SILICON DIOXIDE	9.5370	10.3300	.52544
GOLD	MOLYBDENUM	9.1840	9.9190	.69319
GALLIUM PHOSPHIDE	OSMIUM	8.2650	9.0000	1.31380
OSMIUM	INDIUM PHOSPHIDE	8.2650	8.8530	1.30647
GALLIUM ARSENIDE	OSMIUM	9.5370	10.3300	1.53239
CADMIUM TELLURIDE	SILICON MONOXIDE	10.0000	10.5000	.26821
CADMIUM TELLURIDE	SILICON DIOXIDE	10.0000	10.2600	.48586
MOLYBDENUM	RHODIUM	11.2700	12.4000	.60728
CADMIUM TELLURIDE	SILICON DIOXIDE	10.0000	10.5000	.54673
PLATINUM	TUNGSTEN	10.3300	12.4000	.65008
CADMIUM TELLURIDE	SILICON MONOXIDE	11.0000	11.5000	.79052
OSMIUM	LEAD SULFIDE	11.7600	12.0000	.86024
OSMIUM	INDIUM ARSENIDE	10.0000	10.3300	1.47087
OSMIUM	INDIUM ANTIMONIDE	11.1100	12.0600	1.73553

TABLE A-2 (continued)

OSMIUM	ARSENIC SULFIDE	8.2650	12.4000	1.59969
GERMANIUM	OSMIUM	8.2650	12.4000	1.61083
OSMIUM	AMORPHOUS SILICON	11.2100	12.2900	1.84465
OSMIUM	SILICON	8.2650	12.4000	1.62212
OSMIUM	GALLIUM ANTIMONIDE	8.2650	12.0000	1.64252
GALLIUM PHOSPHIDE	SILICON CARBIDE	12.0000	13.0000	.06908
GALLIUM PHOSPHIDE	SILICON DIOXIDE	12.5500	12.6000	.08361
GALLIUM PHOSPHIDE	SILICON DIOXIDE	12.6500	12.7000	.11975
CADMIUM TELLURIDE	SILICON DIOXIDE	12.7000	12.7500	.17163
GERMANIUM	SILICON CARBIDE	12.5000	13.0000	.07030
GALLIUM ARSENIDE	SILICON CARBIDE	12.4000	13.7800	.41480
CADMIUM TELLURIDE	SILICON CARBIDE	12.5000	13.7800	.41380
GERMANIUM	SILICON CARBIDE	15.5000	16.6700	5.74527
GALLIUM PHOSPHIDE	LITHIUM NIOBATE	17.2400	17.5400	.19568
CADMIUM TELLURIDE	LITHIUM NIOBATE	17.2400	17.5400	.19802
GALLIUM ARSENIDE	LITHIUM NIOBATE	17.5400	17.7100	.14326
GERMANIUM	LITHIUM NIOBATE	17.5400	17.8600	.14417
CADMIUM TELLURIDE	TITANIUM DIOXIDE	19.0000	19.2000	.25891
GALLIUM PHOSPHIDE	TITANIUM DIOXIDE	19.2000	19.4000	.16695
GALLIUM ARSENIDE	TITANIUM DIOXIDE	19.2000	19.4000	.16895
GERMANIUM	TITANIUM DIOXIDE	19.4000	19.6000	.13236
GERMANIUM	LITHIUM NIOBATE	19.2300	20.0000	.92740
GALLIUM ARSENIDE	LITHIUM NIOBATE	20.0000	20.6600	1.02798
GALLIUM PHOSPHIDE	LITHIUM NIOBATE	20.0000	20.8300	1.14381
CADMIUM TELLURIDE	LITHIUM NIOBATE	20.0000	20.8300	1.01876
GERMANIUM	TITANIUM DIOXIDE	20.8000	21.0000	.46828
GALLIUM ARSENIDE	TITANIUM DIOXIDE	21.2000	21.4000	.58371
GALLIUM PHOSPHIDE	SILICON DIOXIDE	21.0000	22.0000	1.21719
CADMIUM TELLURIDE	TITANIUM DIOXIDE	21.6000	21.8000	.62958
GALLIUM PHOSPHIDE	TITANIUM DIOXIDE	21.6000	21.8000	.69789
GALLIUM ARSENIDE	SILICON DIOXIDE	20.6600	22.9600	.39482
CADMIUM TELLURIDE	SILICON DIOXIDE	21.7400	22.2200	1.53814
GERMANIUM	SILICON DIOXIDE	22.8000	23.0000	.41211
GALLIUM ARSENIDE	LEAD SULFIDE	22.2200	24.0000	3.08251
GALLIUM ARSENIDE	SILICON DIOXIDE	23.0000	23.5000	.73235

TABLE A-3

MATERIAL #1	MATERIAL #2	WAVELENGTH RANGE 10^{-6}m		FWHM 10^{-6}m
GOLD	RHODIUM	.4133	.4275	.00197
PLATINUM	LEAD SELENIDE	.4592	.4769	.00224
GOLD	SODIUM FLUORIDE	.4428	.4769	.00220
GOLD	LITHIUM FLUORIDE	.4428	.4769	.00221
GOLD	VYCOR	.4428	.4769	.00221

TABLE A-3 (continued)

GOLD	FK54	.4428	.4600	.00217
GOLD	CALCIUM FLUORIDE	.4428	.4769	.00221
GOLD	FK51	.4428	.4600	.00217
GOLD	FK52	.4428	.4600	.00217
FK50	GOLD	.4428	.4600	.00217
GOLD	KDP	.4428	.4769	.00221
FK6	GOLD	.4428	.4600	.00217
FK5	GOLD	.4428	.4600	.00217
GOLD	PK51	.4428	.4600	.00217
FK1	GOLD	.4428	.4600	.00217
FK3	GOLD	.4428	.4600	.00217
GOLD	QUARTZ CRYSTAL	.4428	.4769	.00221
GOLD	ADP	.4428	.4769	.00221
GOLD	MAGNESIUM FLUORIDE	.4428	.4678	.00219
BK10	GOLD	.4428	.4600	.00217
PK50	GOLD	.4428	.4600	.00217
PK1	GOLD	.4428	.4600	.00217
BK4	GOLD	.4428	.4600	.00218
BK3	GOLD	.4428	.4600	.00218
BK5	GOLD	.4428	.4600	.00218
GOLD	CRYSTAL SILICON DIOXIDE	.4428	.4678	.00219
PK2	GOLD	.4428	.4600	.00218
BK50	GOLD	.4428	.4600	.00218
UBK7	GOLD	.4428	.4600	.00218
BK7	GOLD	.4428	.4600	.00218
OSMIUM	AMORPHOUS SILICON	.5636	.5904	.00268
GALLIUM ARSENIDE	INDIUM ANTIMONIDE	.5344	.5391	.00290
MOLYBDENUM	TUNGSTEN	.4959	.5061	.00312
INDIUM ANTIMONIDE	SILICON	.5391	.5438	.00345
TUNGSTEN	CRYSTAL ARSENIC SULFIDE	.5128	.5166	.00359
INDIUM ARSENIDE	LEAD SULFIDE	.4862	.5276	.00401
INDIUM ARSENIDE	LEAD SULFIDE	.5344	.5438	.00462
IRIDIUM	SILVER CHLORIDE	.5166	.5391	.00454
IRIDIUM	SILICON MONOXIDE	.4959	.5166	.00440
IRIDIUM	CUBIC ZINC SULFIDE	.5636	.5904	.00510
IRIDIUM	THALLIUM CHLORIDE	.5391	.5636	.00492
TUNGSTEN	LEAD SELENIDE	.6048	.6199	.00285
INDIUM ANTIMONIDE	LEAD SULFIDE	.6458	.6526	.00317
LEAD SELENIDE	AMORPHOUS SILICON	.6526	.6888	.00343
INDIUM ARSENIDE	LEAD SELENIDE	.6526	.6666	.00346
LEAD SELENIDE	SILICON	.6458	.6526	.00357
SFL6	METAL ALUMINUM	.6888	.7000	.00383
SF6	METAL ALUMINUM	.6888	.7000	.00383
SF11	METAL ALUMINUM	.6888	.7000	.00383
SFL56	METAL ALUMINUM	.6888	.7000	.00383
SF56	METAL ALUMINUM	.6888	.7000	.00383
METAL ALUMINUM	LEAD FLUORIDE	.6888	.7000	.00383

TABLE A-3 (continued)

SF14	METAL ALUMINUM	.6888	.7000	.00383
SF55	METAL ALUMINUM	.6888	.7000	.00383
METAL ALUMINUM	MAGNESIUM OXIDE	.6888	.7000	.00383
LAF9	METAL ALUMINUM	.6888	.7000	.00383
SF4	METAL ALUMINUM	.6888	.7000	.00384
SF61	METAL ALUMINUM	.6888	.7000	.00384
LASF32	METAL ALUMINUM	.6888	.7000	.00384
LASF8	METAL ALUMINUM	.6888	.7000	.00384
LAF11A	METAL ALUMINUM	.6888	.7000	.00384
LAFN11	METAL ALUMINUM	.6888	.7000	.00384
LASF33	METAL ALUMINUM	.6888	.7000	.00384
METAL ALUMINUM	MUSCOVITE MICA	.6960	.6316	.00367
LAFN7	METAL ALUMINUM	.6888	.7000	.00384
LAF22	METAL ALUMINUM	.6888	.7000	.00384
LAF22A	METAL ALUMINUM	.6888	.7000	.00384
LAF13	METAL ALUMINUM	.6888	.7000	.00384
LASF12	METAL ALUMINUM	.6888	.7000	.00384
LASF3	METAL ALUMINUM	.6888	.7000	.00384
GALLIUM ARSENIDE	LEAD SELENIDE	.6262	.6526	.00354
METAL ALUMINUM	KRS-5	.7500	.7749	.00355
SF58	METAL ALUMINUM	.7000	.7085	.00331
SF57	METAL ALUMINUM	.7000	.7085	.00332
LASFN18	METAL ALUMINUM	.7000	.7085	.00332
LASF7	METAL ALUMINUM	.7000	.7085	.00333
LASFN9	METAL ALUMINUM	.7000	.7085	.00333
LASF9	METAL ALUMINUM	.7000	.7085	.00333
LASFN15	METAL ALUMINUM	.7000	.7085	.00333
LASF13	METAL ALUMINUM	.7000	.7085	.00333
LASF5	METAL ALUMINUM	.7000	.7085	.00333
LASFN31	METAL ALUMINUM	.7000	.7085	.00333
METAL ALUMINUM	STRONTIUM TITANATE	.7200	.7600	.00350
METAL ALUMINUM	SILICON CARBIDE	.7500	.7749	.00362
METAL ALUMINUM	CUBIC ZINC SULFIDE	.7293	.7514	.00352
METAL ALUMINUM	CUBIC DIAMOND CARBON	.7000	.7500	.00346
NICKEL	METAL ALUMINUM	.7293	.7500	.00371
OSMIUM	INDIUM PHOSPHIDE	.7560	.7749	.00388
METAL ALUMINUM	THALLIUM BROMIDE	.6500	.7500	.00394
METAL ALUMINUM	THALLIUM CHLORIDE	.6500	.7500	.00399
IRIDIUM	METAL ALUMINUM	.7749	.7999	.00478
METAL ALUMINUM	TITANIUM DIOXIDE	.7600	.7749	.00471
METAL ALUMINUM	ARSENIC SULFUR	.7600	.7749	.00481
METAL ALUMINUM	ARSENIC SULFIDE	.7600	.7749	.00506
METAL ALUMINUM	SILICON NITRIDE	.6199	.8266	.00586
RHODIUM	METAL ALUMINUM	.7500	.7749	.00646
INDIUM ANTIMONIDE	LEAD SELENIDE	.7749	.7847	.00672
METAL ALUMINUM	SILICON MONOXIDE	.6888	.7749	.00633
OSMIUM	RHODIUM	.8266	.8856	.00391

TABLE A-3 (continued)

METAL ALUMINUM	ARSENIC SULFIDE	.8500	.8750	.00420
METAL ALUMINUM	CUBIC DIAMOND CARBON	.8500	.8750	.00422
METAL ALUMINUM	SILICON CARBIDE	.8500	.8750	.00423
METAL ALUMINUM	KRS-5	.8500	.8750	.00431
METAL ALUMINUM	ARSENIC SULFUR	.8500	.8750	.00432
METAL ALUMINUM	TITANIUM DIOXIDE	.8500	.8750	.00444
CADMIUM TELLURIDE	PLATINUM	.8266	.8400	.00432
OSMIUM	ARSENIC SELENIDE	.8030	.8103	.00451
METAL ALUMINUM	CADMIUM SULFIDE	.8500	.9000	.00539
METAL ALUMINUM	HEX ZINC SULFIDE	.8000	.9000	.00525
METAL ALUMINUM	KRS-6	.8000	.9000	.00526
NICKEL	OSMIUM	.8551	.8856	.00542
IRIDIUM	OSMIUM	.8266	.8550	.00530
IRIDIUM	METAL ALUMINUM	.8266	.8500	.00553
GALLIUM PHOSPHIDE	OSMIUM	.7749	.8266	.00591
OSMIUM	CRYSTAL ARSENIC SULFIDE	.8000	.8333	.00603
MOLYBDENUM	INDIUM PHOSPHIDE	.8266	.8856	.00658
RHODIUM	METAL ALUMINUM	.7999	.8250	.00665
OSMIUM	TITANIUM DIOXIDE	.8550	.8856	.00746
OSMIUM	ARSENIC SULFIDE	.8550	.8856	.00746
OSMIUM	SILICON CARBIDE	.8550	.8856	.00754
GALLIUM PHOSPHIDE	MOLYBDENUM	.8856	.9000	.00784
CADMIUM TELLURIDE	RHODIUM	.8670	.8731	.00772
OSMIUM	ARSENIC SULFUR	.8550	.8856	.00775
METAL ALUMINUM	LEAD FLUORIDE	.9000	.9500	.00477
MOLYBDENUM	RHODIUM	.8856	.9537	.00475
MOLYBDENUM	PLATINUM	.8856	.9537	.00491
IRIDIUM	MOLYBDENUM	.8856	.9537	.00552
METAL ALUMINUM	SILVER CHLORIDE	.9000	1.0000	.00592
RHODIUM	TUNGSTEN	.9184	.9537	.00706
MOLYBDENUM	NICKEL	.9537	.9919	.00760
CADMIUM TELLURIDE	MOLYBDENUM	.8856	.9537	.00737
MOLYBDENUM	CRYSTAL ARSENIC SULFIDE	.8856	.9537	.00738
RHODIUM	INDIUM PHOSPHIDE	.9750	.9837	.00861
MOLYBDENUM	ARSENIC SELENIDE	.9000	.9200	.00803
PLATINUM	TUNGSTEN	.9184	.9537	.00829
COPPER	SILVER	1.8790	1.9070	.00922
GOLD	SILVER	1.8790	1.9070	.00935
GOLD	SILVER	1.9070	1.9370	.00959
COPPER	SILVER	1.9070	1.9370	.00973
TUNGSTEN	SILICON CARBIDE	1.4760	1.4940	.00798
TUNGSTEN	SELENIUM ARSENIC	1.4760	1.4940	.00805
TUNGSTEN	ARSENIC SELENIDE	1.4420	1.4590	.00921
PLATINUM	RHODIUM	2.3840	2.4550	.01665
TUNGSTEN	ARSENIC SULFIDE	1.4940	1.5120	.01102
TUNGSTEN	CUBIC DIAMOND CARBON	1.4940	1.5120	.01103
TUNGSTEN	KRS-5	1.4940	1.5120	.01107

TABLE A-3 (continued)

TUNGSTEN	ARSENIC SULFUR	1.4940	1.5120	.01109
NICKEL	PLATINUM	2.3840	2.4550	.01786
TUNGSTEN	IRTRAN 4	1.4940	1.5000	.01105
PLATINUM	LEAD SULFIDE	2.3840	2.4000	.01882
TUNGSTEN	TITANIUM DIOXIDE	1.4940	1.5000	.01195
PLATINUM	INDIUM ANTIMONIDE	2.3840	2.4550	.01960
GERMANIUM	PLATINUM	2.3840	2.4550	.01995
GOLD	SILVER	1.6530	1.6750	.01384
RHODIUM	LEAD SELENIDE	2.9520	3.0240	.02500
PLATINUM	METAL ALUMINUM	2.4800	2.6380	.02225
LEAD SELENIDE	TELLURIUM	4.4280	4.5090	.03566
IRIDIUM	LEAD SELENIDE	3.8750	4.0000	.03233
SILICON DIOXIDE	GLASS SILICON DIOXIDE	8.5500	8.6000	.04046
SILICON DIOXIDE	GLASS SILICON DIOXIDE	8.6000	8.6500	.04838
LITHIUM NIOBATE	GLASS SILICON DIOXIDE	9.0910	9.1740	.05664
SILICON DIOXIDE	TITANIUM DIOXIDE	9.0910	9.1740	.05722
SILICON DIOXIDE	CALCIUM FLUORIDE	9.0910	9.1740	.05905
SILICON DIOXIDE	IRTRAN 3	9.0910	9.1740	.05906
SILICON DIOXIDE	BARIUM FLUORIDE	9.0910	9.1740	.05917
SILICON DIOXIDE	SODIUM CHLORIDE	9.0910	9.1740	.05921
POTASSIUM CHLORIDE	SILICON DIOXIDE	9.0910	9.1740	.05922
SILICON DIOXIDE	SODIUM FLUORIDE	9.0910	9.1740	.05924
SILICON DIOXIDE	POTASSIUM BROMIDE	9.0910	9.1740	.05925
SILICON CARBIDE	GLASS SILICON DIOXIDE	9.1740	9.2590	.07572
LITHIUM FLUORIDE	GLASS SILICON DIOXIDE	9.1000	9.1740	.08165
SILICON DIOXIDE	SILICON MONOXIDE	9.0090	9.0910	.08100
SILICON CARBIDE	TITANIUM DIOXIDE	12.2000	12.4000	.08886
SILICON CARBIDE	SILICON DIOXIDE	12.3500	12.4000	.07953
SILICON CARBIDE	LITHIUM FLUORIDE	12.3500	12.4200	.09256
SILICON CARBIDE	ARSENIC SULFIDE	11.9100	12.9900	.07751
SILICON CARBIDE	KRS-6	12.0000	13.0000	.06897
SILICON CARBIDE	IRTRAN 4	12.0000	13.0000	.06898
SILICON CARBIDE	IRTRAN 2	12.0000	13.0000	.06908
SILICON CARBIDE	SILICON MONOXIDE	12.0000	13.0000	.06970
GALLIUM PHOSPHIDE	SILICON CARBIDE	12.0000	13.0000	.06908
GALLIUM PHOSPHIDE	SILICON DIOXIDE	12.5500	12.6000	.08361
INDIUM PHOSPHIDE	SILICON DIOXIDE	12.5500	12.6000	.08364
ARSENIC SULFIDE	SILICON DIOXIDE	12.5500	12.6000	.08378
INDIUM PHOSPHIDE	SILICON DIOXIDE	12.6000	12.6500	.10129
SILICON CARBIDE	KRS-5	12.4000	13.0000	.07004
SILICON CARBIDE	SODIUM CHLORIDE	12.4000	13.0000	.07011
GERMANIUM	SILICON CARBIDE	12.5000	13.0000	.07030
CUBIC ZINC SULFIDE	LITHIUM NIOBATE	15.5000	17.7100	.13472
INDIUM PHOSPHIDE	LITHIUM NIOBATE	16.5300	17.7100	.13872
CADMIUM TELLURIDE	LITHIUM NIOBATE	16.6700	17.7100	.14000
LITHIUM NIOBATE	KRS-5	17.0000	17.6000	.14083

TABLE A-3 (continued)

GALLIUM ARSENIDE	LITHIUM NIOBATE	17.5400	17.7100	.14326
INDIUM ARSENIDE	LITHIUM NIOBATE	17.5400	17.7100	.14352
INDIUM ANTIMONIDE	LITHIUM NIOBATE	17.5400	17.8500	.14395
LEAD SULFIDE	LITHIUM NIOBATE	17.5400	17.8600	.13822
AMORPHOUS SILICON	LITHIUM NIOBATE	17.5400	17.8600	.14311
GERMANIUM	LITHIUM NIOBATE	17.5400	17.8600	.14417
SILICON	LITHIUM NIOBATE	17.5400	17.8600	.14417
LEAD SELENIDE	LITHIUM NIOBATE	17.5400	17.8600	.14375
SILICON CARBIDE	LITHIUM NIOBATE	17.5400	17.8600	.14095
ARSENIC SULFIDE	TITANIUM DIOXIDE	18.8700	19.2300	.16644

APPENDIX B: Fabrication Parameters**Results for Injection Molding on 11/18/88**

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	RATIO [G/R]	OIL gm
9-1	SK11	63-90	70	NAS	56	1.2	-
9-2	SK11	63-90	77	NAS	54	1.4	12
9-3	SK5	63-90	85	NAS	62	1.4	-
9-4	SK11	63-90	70	NAS	56	1.2	-
9-5	SK11	63-90	75	P-359	60	1.2	8
9-6	SK5	63-90	82	P-359	61	1.2	10
9-7	SK5	63-90	82	P-359	61	1.3	-
9-8	SK11	63-90	80	KR01	60	1.3	-
9-9	SK11	63-90	81	KR01	61	2.0	8
9-10	SK11	425-500	60	KR01	60	1.0	7
9-11	SK5	63-90	100	KR01	77	1.3	-
9-12	SK5	63-90	84	KR01	61	1.4	9
9-13	SK11	63-90	80	TYRIL	60	1.3	-
9-14	SK11	63-90	77	TYRIL	90	0.9	12
9-15	SK11	63-90	36	TYRIL	70	0.5	13 ABORT

G/R = glass/resin ratio, TYRIL = styrene acrylonitrile resin from Dow-Corning, KR01 = styrene-butadiene copolymer from Phillips 66; NAS and P-359 = styrene methyl methacrylate copolymers from Richardson.

Results for Injection Molding on 11/21/88

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	RATIO [G/R]	OIL gm
10-1	SK11	63-90	25	TYRIL	50	0.5	6
10-2	SK11	106-125	73	TYRIL	50	1.4	14 ABORT
10-3	SK11	106-125	79	TYRIL	41	1.9	3
10-4	SK11	125-180	50	TYRIL	30	1.7	5
10-5	SK11	106-125	51	TYRIL	28	1.8	10 ABORT

G/R = glass/resin ratio, TYRIL = styrene acrylonitrile resin from Dow-Corning.

Results for Injection Molding on 12/20/88

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	RATIO [G/R]	MO gm
11-1	SK5	53-63	59	LUSTRAN	42	1.41	
11-2	SK5	53-63	60	LUSTRAN	45	1.33	7
11-3	SK5	53-63	69	DOW-EXP	40	1.72	
11-4	SK11	63-90	66	DOW-EXP	44	1.50	
11-5	SK11	63-90	60	LUSTRAN	45	1.50	
11-6	SK11	63-90	51	LUSTRAN	44	1.16	10

G/R = glass to resin ratio, LUSTRAN = styrene acrylonitrile copolymer from Monsanto Lustran, MO = mineral oil.

Results for Injection Molding on 6/8/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	RATIO [G/R]	ESO gm
12-1	SK5	170-200	50	LUSTRAN	50	1.0	-
12-2	BAK4	170-200	100	LUSTRAN	100	1.0	-
12-3	SK11	170-200	50	LUSTRAN	50	1.0	-
12-4	SK11	170-200	50	LUSTRAN	50	1.0	4
12-5	SK11	400+	50	LUSTRAN	50	1.0	-
12-6	SK5	400+	50	LUSTRAN	50	1.0	-
12-7	SK5	400+	50	LUSTRAN	50	1.0	4
12-8	BAK4	400+	50	LUSTRAN	50	1.0	-
12-9	BAK4	400+	50	LUSTRAN	50	1.0	4
12-10	SK11	400+	20	LUSTRAN	20	1.0	-
12-11	SK5	400+	20	LUSTRAN	20	1.0	-
12-12	BAK4	400+	20	LUSTRAN	40	0.5	-

G/R = glass to resin ratio, ESO = Epoxidized soybean oil.

Results for Injection Molding on 7/6/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	ESO gm	MO gm	CA
13-1	SK11	400+	50	LUSTRAN	50	-	-	DC-1%
13-2	BAK4	400+	50	LUSTRAN	50	-	-	DC-1%
13-3	BAK4	400+	50	LUSTRAN	50	6	-	DC-1%
13-4	BAK4	400+	50	LUSTRAN	50	-	5	DC-1%
13-5	SK5	400+	50	LUSTRAN	50	-	-	DC-1%
13-6	SK5	400+	10	LUSTRAN	50	5	-	DC-1%
13-7	SK5	400+	20	LUSTRAN	50	-	-	DC-1%
13-8	BAK4	400+	10	LUSTRAN	50	5	-	DC-1%
13-9	BAK4	400+	25	LUSTRAN	50	5	-	DC-1%
13-10	BAK4	400+	25	LUSTRAN	50	-	5	-
13-11	BAK4	400+	35	LUSTRAN	50	5	5	DC-1%
13-12	SK11	400+	25	LUSTRAN	50	-	-	DC-1%
13-13	SK11	400+	10	LUSTRAN	50	-	15	DC-1%
13-14	SK11	400+	35	LUSTRAN	50	-	10	DC-1%
13-15	SK11	400+	50	LUSTRAN	50	-	10	DC-1%
13-16	SK11	400+	50	LUSTRAN	50	10	-	DC-1%
13-17	SK11	400+	60	LUSTRAN	30	10	-	-

G/R = glass to resin ratio, ESO = epoxidized soybean oil, MO = mineral oil, CA = coupling agent, DC-1% = Dow Corning silane coupling agent treated glass - 1% in alcohol.

Results for Injection Molding on 7/28/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	DPO gm	MO gm	CA
14-1	SK5	400+	50	LUSTRAN	50	-	5	DC-1%
14-2	SK5	400+	50	LUSTRAN	100	-	-	PA-1%
14-3	SK5	400+	50	LUSTRAN	50	-	5	PA-1%
14-4	SK11	400+	50	LUSTRAN	50	-	-	PA-1%
14-5	SK11	400+	50	LUSTRAN	50	-	5	PA-1%
14-6	SK11	400+	50	LUSTRAN	50	-	5	DC-5%
14-7	SK11	400+	50	LUSTRAN	50	5	-	DC-5%
14-8	SK5	400+	50	LUSTRAN	50	5	-	DC-5%
14-9	SK11	325-400	30	LUSTRAN	30	3	-	-
14-10	SK5	325-400	30	LUSTRAN	30	3	-	-
14-11	NULL RUN	-	-	LUSTRAN	30	3	-	-
14-12	NULL RUN	-	-	LUSTRAN	30	-	3	-
14-13	SK11	325-400	30	LUSTRAN	15	2	-	-
14-14	SK11	400+	50	LUSTRAN	25	3	-	PA-5%
14-15	SK5	325-400	30	LUSTRAN	15	3	-	-
14-16	SK5	400+	50	LUSTRAN	25	3	-	PA-5%
14-17	SK5	400+	10	LUSTRAN	30	3	-	PA-5%
14-18	PRISM RUN	-	-	LUSTRAN	-	-	-	-

G/R = glass to resin ratio, ESO = epoxidized soybean oil, MO = mineral oil, CA, coupling agent, DPO = Dow-Corning 705 diffusion pump oil, DC-x% = Dow Corning silane coupling agent Z-6302 treated glass - x% in alcohol; PA-x% = Petrarch silane treated glass - x% in alcohol.

Results for Injection Molding on 8/29/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	MO gm	DPO gm	CA
15-1	SK11	106-125	50	LUSTRAN	44	8	Not SK11!	
15-2	SK11	170-200	51	LUSTRAN	50	8	-	DC1-1%
15-3	SK11	125-180	50	LUSTRAN	50	10	Not SK11!	
15-4	SK11	325-400	50	LUSTRAN	35	10	-	DC2-1%
15-5	SK11	325-400	50	LUSTRAN	65	10	-	DC2-1%
15-6	SK11	325-400	20	LUSTRAN	60	5	-	DC2-1%
15-7	SK11	325-400	20	LUSTRAN	25	2	-	DC2-1%
15-8	SK11	170-200	25	KR01	32	5	-	DC2-1%
15-9	SK11	170-200	25	PS	50	10	-	DC2-1%
15-10	SK11	125-180	20	PS	50	5	-	DC2-1%
15-11	PRISM RUN	-	-	PS	-	-	-	-
15-12	SK11	212-425	50	LUSTRAN	65	10	Not SK11!	
15-13	PRISM RUN	-	-	LUSTRAN	15	2	-	DC2-1%

G/R = glass to resin ratio, PS = polystyrene, ESO = epoxidized soybean oil, MO = mineral oil, CA, coupling agent, DPO = Dow-Corning 705 diffusion pump oil, DC1 - x% = Dow Corning Z-6032 silane coupling agent treated glass - x% in alcohol; DC2 - x% = Dow Corning 7169-45B coupling agent x% in water.

Results for Injection Molding on 10/11/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	MO gm	DPO gm	CA
16-1	SK11	325-400	50	LUSTRAN	50	10	-	DC2-1%
16-2	SK11	325-400	50	STYRON	50	10	-	DC2-1%
16-3	SK5	400+	30	STYRON	60	10	-	DC1-5%
16-4	SK5	400+	40	LUSTRAN	30	10	-	DC1-5%
16-5	SK5	400+	40	LUSTRAN	30	12.5	-	DC1-5%

Results for Injection Molding on 10/13/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	MO gm	DPO gm	CA
17-1	SK11	325-400	25	LUSTRAN	50	10	-	DC2-1%
17-2	SK11	325-400	25	LUSTRAN	50	13.4	-	DC2-1%
17-3	SK11	325-400	25	LUSTRAN	50	8	-	DC2-1%
17-4	SK11	325-400	40	LUSTRAN	40	10	-	DC2-1%
17-5	SK11	325-400	40	LUSTRAN	40	10	-	DC2-1%
17-6	SK11	325-400	40	LUSTRAN	40	10	-	DC2-1%
17-7	SK11	325-400	25	LUSTRAN	50	12	-	DC2-1%
17-8	SK11	325-400	25	LUSTRAN	50	12	-	DC2-1%
17-9	SK11	325-400	25	LUSTRAN	50	10	-	DC2-1%
17-10	SK11	325-400	50	LUSTRAN	100	24	-	DC2-1%
17-11	SK11	325-400	50	LUSTRAN	100	25	-	DC2-1%
17-12	SK11	325-400	50	LUSTRAN	100	24	-	DC2-1%

G/R = glass to resin ratio, PS = polystyrene, ESO = epoxidized soybean oil,
 MO = mineral oil, CA, coupling agent, DPO = Dow-Corning 705 diffusion pump oil,
 DC1 - x% = Dow Corning Z-6032 silane coupling agent treated glass - x% in alcohol;
 DC2 -x% = Dow Corning 7169-45B coupling agent x% in water,
 LUSTRAN - ground resin, STYRON - ground resin

Results for Injection Molding on 10/19/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	MO gm	ESO gm	CA
18-1	SK5	400+	50	STYRON	50	10	-	DC2-1%
18-2	SK5	400+	25	STYRON	50	4	-	DC2-1%
18-3	SK5	400+	25	STYRON	50	2	-	DC2-1%
18-4	SK5	400+	20	STYRON	50	-	-	DC2-1%
18-5	-	-	-	STYRON	50	6	-	-
18-6	SK5	400+	25	STYRON	50	10	-	DC2-1%
18-7	SK5	400+	25	STYRON	50	10	-	DC2-1%
18-8	SK5	400+	50	STYRON	50	10	-	DC2-1%
18-9	SK5	400+	50	STYRON*	50	10	-	DC2-1%
18-10	SK5	325-400	53	LUSTRAN	53	10	-	DC2-1%
18-11	SK5	325-400	52	LUSTRAN	52	-	10	DC2-1%
18-12	SK5	325-400	50	LUSTRAN	50	-	10	DC2-1%

G/R = glass to resin ratio, PS = polystyrene, ESO = epoxidized soybean oil, MO = mineral oil,
 CA, coupling agent, DPO = Dow-Corning 705 diffusion pump oil, DC1 - x% = Dow Corning Z-
 6032 silane coupling agent treated glass - x% in alcohol; DC2 -x% = Dow Corning 7169-
 45B coupling agent x% in water, STYRON = ground resin.

Results for Injection Molding on 11/8/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	MO gm	ESO gm	CA
19-1	SK11	400+	25	LUSTRAN	50	-	-	DC3-1%
19-2	SK11	400+	25	LUSTRAN	50	8	-	DC3-1%
19-3	SK11	400+	25	LUSTRAN	50	8	-	DC4-1%
19-4	SK11	400+	10	LUSTRAN	50	10	-	DC3-1%
19-5	SK11	400+	50	LUSTRAN	100	20	-	DC3-1%
19-6	SK5	400+	25	LUSTRAN	50	10	-	DC3-1%
19-7	SK5	400+	30	STYRON	60	13	-	DC4-1%
19-8	SK5-11	400+	15/15	STYRON	75	13	-	DC3-1%
19-9	SK5	400+	50	LUSTRAN	15	10	-	DC4-1%
19-10	SK5	400+	50	FINA	25	10	-	DC3-1%
19-11	SK5	400+	50	STYRON	25	11	-	DC3-1%

G/R = glass to resin ratio, PS = polystyrene, ESO = epoxidized soybean oil, MO = mineral oil, CA, coupling agent, DPO = Dow-Corning 705 diffusion pump oil, DC3 - x% = Dow Corning phenyl coupling agent treated glass - x% in water; DC4 - x% = Dow Corning [6072:phenyl coupling agent - 1:1] treated glass x% in water, STYRON = ground resin.

Results for Injection Molding on 12/13/89

NUMBER	GLASS	SIZE μm	WT gm	RESIN	WT gm	MO gm	ESO gm	CA
20-1	SK11	400+	50	LUSTRAN	50	10.5	-	DC3-1%
20-2	SK11	400+	20	LUSTRAN	50	11	-	DC3-1%
20-3	SK11	400+	10	LUSTRAN	50	3	-	DC3-1%
20-4	SK11	400+	INC	LUSTRAN	80	10	-	DC3-1%
20-5	SK11	400+	50	LUSTRAN	50	10	-	DC4-1%
20-6	SK11	400+	50	LUSTRAN	50	10	-	DC4-1%
20-7	SK11	400+	57	LUSTRAN	57	11	-	DC4-1%

G/R = glass to resin ratio, PS = polystyrene, ESO = epoxidized soybean oil, MO = mineral oil, CA, coupling agent, DPO = Dow-Corning 705 diffusion pump oil, DC3 - x% = Dow Corning phenyl coupling agent treated glass - x% in water; DC4 - x% = Dow Corning [6072:phenyl coupling agent - 1:1] treated glass x% in water, STYRON = ground resin.

Results for Injection Molding on 1/9/90

NUMBER	COMPOUND	SIZE	WT	RESIN	WT	MO	Shape	CA
		μm	gm		gm	gm	[R/S]	
21-1	CTI-1	PELLET	50	-	-	11	R	-
21-2	CTI-1	PELLET	50	-	-	8	R	-
21-3	CTI-1	PELLET	100	-	-	5.5	R	-
21-4	CTI-1	PELLET	50	-	-	5.5	S	-
21-5	CTI-1	PELLET	50	LUSTRAN	50	-	S	-
21-6	CTI-1	PELLET	25	LUSTRAN	50	6	S	-
21-7	CTI-1	PELLET	50	LUSTRAN¶	50	-	S	DC4-1%

G/R = glass to resin ratio, CTI-1 = Compounding Technologies Inc. - SK11 coarse glass in SAN resin, 1:2 ratio, MO = mineral oil, CA = coupling agent, DPO = Dow-Corning 705 diffusion pump oil, DC3 - x% = Dow Corning phenyl coupling agent treated glass - x% in water; DC4 -x% = Dow Corning [6072:phenyl coupling agent - 1:1] treated glass x% in water, ¶LUSTRAN = ground resin.

Results for Injection Molding on 2/21/90

NUMBER	COMPOUND	SIZE	WT	RESIN	WT	MO	Shape	CA
		μm	gm		gm	gm	[R/S/W]	
22-1	-	-	-	LUSTRAN	large	-	S	Test run
22-2	MPI-1	PELLET	50	LUSTRAN	50	-	S	Yes
22-3	CTI-1	PELLET	50	LUSTRAN	100	10	S	No
22-4	MPI-1	PELLET	30	LUSTRAN	100	5	S	Yes
22-5	MPI-1	PELLET	30	LUSTRAN	100	-	S	Yes
22-6	MPI-1	PELLET	15	LUSTRAN	100	-	S	Yes
22-7	MPI-1	PELLET	8	LUSTRAN	100	-	S	Yes
22-8	MPI-1	PELLET	4	LUSTRAN	100	-	S	Yes
22-9	MPI-1	PELLET	8	LUSTRAN	100	3	S	Yes
22-10	MPI-1	PELLET	15	LUSTRAN	100	6	S	Yes
22-11	-	-	-	STYRON	large	-	W	Test Run
22-12	-	-	-	LUSTRAN	large	-	W	Yes
22-13	MPI-1	PELLET	42	LUSTRAN	693	-	W	Yes
22-14	MPI-1	PELLET	14	LUSTRAN	231	5	W	Yes
22-15	MPI-1	PELLET	10	LUSTRAN	231	3	W	Yes
22-16	MPI-1	PELLET	10	LUSTRAN	231	5	W	Yes
22-17	MPI-1	PELLET	7.5	LUSTRAN	231	5	W	Yes

G/R = glass to resin ratio MO = mineral oil ESO = epoxidized soybean oil

CA = coupling agent PS = polystyrene SAN = styrene acrylonitrile

STYRON = PS from Dow Chemical FINA - PS resin from FINA

LUSTRAN = SAN from Monsanto

R = round cavity on Mold 1 S = square cavity on Mold 1 W = large window cavity on Mold 2 ¶ = Indicates ground resin

CTI-1 = Compounding Technologies Inc. - SK11 coarse glass in SAN resin, 1:2 ratio

MPI-1 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:x ratio, with DC-4 coupling agent.

DC3-x% = Dow Corning phenyl coupling agent treated glass - x% in water,

DC4-x% = Dow Corning [6072:phenyl coupling agent - 1:1] treated glass x% in water

Results for Injection Molding on 2/26/90

NUMBER	COMPOUND	SIZE	WT	RESIN	WT	MO	Shape	CA
		μm	gm		gm	gm	[R/S/W]	
23-1	MPI-1	PELLET	20	LUSTRAN	924	7	W	Yes
23-2	MPI-1	PELLET	20	LUSTRAN	924	12	W	Yes
23-3	MPI-1	PELLET	20	LUSTRAN	924	7	W	Yes
23-4	MPI-1	PELLET	20	LUSTRAN	924	8	W	Yes
23-5	MPI-1	PELLET	20	LUSTRAN	924	12	W	Yes
23-6	MPI-1	PELLET	20	LUSTRAN	924	12	W	Yes
23-7	MPI-1	PELLET	40	LUSTRAN	924	13	W	Yes
23-8	MPI-1	PELLET	20	LUSTRAN	462	10	W	Yes
23-9	MPI-1	PELLET	60	LUSTRAN	462	20	W	Yes
23-10	MPI-1	PELLET	60	LUSTRAN	924	12	W	Yes

G/R = glass to resin ratio

MO = mineral oil ESO = epoxidized soybean oil

CA = coupling agent

PS = polystyrene SAN = styrene acrylonitrile

STYRON = PS from Dow Chemical FINA - PS resin from FINA

LUSTRAN = SAN from Monsanto

R = round cavity on Mold 1

S = square cavity on Mold 1

W = large window cavity

on Mold 2 ¶ = indicates ground resin

CTI-1 = Compounding Technologies Inc. - SK11 coarse glass in SAN resin, 1:2 ratio

MPI-1 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:x ratio, with

DC-4 coupling agent.

DC3-x% = Dow Corning phenyl coupling agent treated glass - x% in water,

DC4-x% = Dow Corning [6072:phenyl coupling agent - 1:1] treated glass x% in water

Results for Injection Molding on 4/27/90

NUMBER	COMPOUND	SIZE	WT	RESIN	WT	MO	Shape	CA
		μm	gm		gm	gm	[R/S/W]	
24-1	-	-	-	LUSTRAN	large	-	S	Test run
24-2	MPI-1	PELLET	50	LUSTRAN	50	-	S	Yes
24-3	CTI-1	PELLET	50	LUSTRAN	100	10	S	No
24-4	MPI-1	PELLET	30	LUSTRAN	100	5	S	Yes

G/R = glass to resin ratio

MO = mineral oil ESO = epoxidized soybean oil

CA = coupling agent

PS = polystyrene SAN = styrene acrylonitrile

STYRON = PS from Dow Chemical FINA - PS resin from FINA

LUSTRAN = SAN from Monsanto

R = round cavity on Mold 1

S = square cavity on Mold 1

W = large window cavity

on Mold 2 ¶ = indicates ground resin

CTI-1 = Compounding Technologies Inc. - SK11 coarse glass in SAN resin, 1:2 ratio

MPI-1 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:x ratio, with

DC-4 coupling agent.

DC3-x% = Dow Corning phenyl coupling agent treated glass - x% in water,

DC4-x% = Dow Corning [6072:phenyl coupling agent - 1:1] treated glass x% in water

Results for Injection Molding on 7/3/90

NUMBER	COMPOUND	SIZE/FORM μm	WT gm	RESIN	WT gm	RATIO [G/R]	MO gm	MOLD [R/S/W]	CA
25-1	RED	-	2.0	LUSTRAN	924	-	-	W	No
25-2	ORANGE	-	2.0	LUSTRAN	924	-	-	W	No
25-3	GREEN	-	2.0	LUSTRAN	924	-	-	W	No
25-4	GRAY	-	2.4	LUSTRAN	924	-	-	W	No
25-5	GRAY	-	4.8	LUSTRAN	924	-	-	W	No
25-6	MPI-2	PELLET	46.2	LUSTRAN	462	0.143	-	W	Yes
25-7	MPI-2	PELLET	46.2	LUSTRAN	878	0.013	-	W	Yes
25-8	MPI-2	PELLET	46.2	LUSTRAN	878	0.013	-	W	Yes
25-9	MPI-2	PELLET	24	LUSTRAN	900	0.006	-	W	Yes
25-10	MPI-2	PELLET	46.2	LUSTRAN	878	0.013	-	W	Yes
25-11	MPI-2	PELLET	34.5	LUSTRAN	890	0.009	-	W	Yes
25-12	MPI-2	PELLET	24	LUSTRAN	900	0.006	-	W	Yes
25-13	MPI-2	PELLET	34.5	LUSTRAN	890	0.009	9	W	Yes
25-14	SK-11	COARSE	50	LUSTRAN	50	1.0	-	W	No
25-15	SK-11	MILLED	50	LUSTRAN	50	1.0	-	W	No
25-16	SK-11	COARSE	100	LUSTRAN	800	0.125	-	W	Yes
25-17	SK-11	MILLED	100	LUSTRAN	800	0.125	-	W	Yes
25-18	MPI-2	PELLET	30	LUSTRAN	895	0.008	-	W	Yes

G/R = glass to resin ratio

MO = mineral oil

CA = coupling agent

SAN = styrene acrylonitrile

LUSTRAN = SAN from Monsanto

¶ = indicates ground resin

R = round cavity on Mold 1
on Mold 2

S = square cavity on Mold 1

W = large window cavity

CTI-1 = Compounding Technologies Inc. - SK11 coarse glass in SAN resin, 1:2 ratio

MPI-1 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:2 ratio, with
DC-4 coupling agent.MPI-2 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:4 ratio, with
DC-4 coupling agent.

DC3-x% = Dow Corning phenyl coupling agent treated glass - x% in water,

DC4-x% = Dow Corning [6072:phenyl] coupling agent - 1:1] treated glass x% in water

Results for Injection Molding on 8/24/90

NUMBER	COMPOUND	SIZE μm	WT gm	RESIN	WT gm	DYE [R:O:G]	Shape [C/S/W]	Comment
26-1	-	-	-	LUSTRAN	900	2-4-4	W	Test run
26-2	MPI-2	PELLET	6	LUSTRAN	900	2-4-4	W	
26-3	MPI-2	PELLET	12	LUSTRAN	900	2-4-4	W	
26-4	MPI-2	PELLET	18	LUSTRAN	900	2-4-4	W	
26-5	MPI-2	PELLET	24	LUSTRAN	900	2-4-4	W	
26-6	MPI-2	PELLET	30	LUSTRAN	900	2-4-4	W	
26-7	MPI-2	PELLET	20	LUSTRAN	900	2-4-4	W	
26-8	MPI-2	PELLET	22	LUSTRAN	900	2-4-4	W	
26-9	MPI-2	PELLET	24	LUSTRAN	900	1-2-2	W	
26-10	MPI-2	PELLET	34	LUSTRAN	900	1-2-2	W	
26-11	MPI-2	PELLET	24	LUSTRAN	900	3-6-6	W	
26-12	MPI-2	PELLET	14	LUSTRAN	900	3-6-6	W	
26-13	-	-	-	LUSTRAN	900	5-12-12	W	Test run
26-14	SK11	MILLED	100	LUSTRAN	200	-	W	Test run
26-15	SK11-CA*	MILLED	100	LUSTRAN	200	-	W	
26-16	MPI-2	PELLET	24	LUSTRAN	900	5-12-12		W
26-17	MPI-2	PELLET	12	LUSTRAN	900	5-12-12		W
26-18	MPI-2	PELLET	24	LUSTRAN	900	2-4-4	W	9 g MO

G/R = glass to resin ratio
 SAN = styrene acrylonitrile
 C = circular cavity on Mold 1
 on Mold 2

CA = coupling agent

LUSTRAN = SAN from Monsanto ¶ = indicates ground resin
 S = square cavity on Mold 1 W = large window cavity

MO = mineral oil

R = Macrolex Red 5B dye

O = Macrolex Orange 3G dye

G = Macrolex Green G dye

CTI-1 = Compounding Technologies Inc. - SK11 coarse glass in SAN resin, 1:2 ratio

MPI-1 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:2 ratio, with
 DC-4 coupling agent.

MPI-2 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:4 ratio, with
 DC-4 coupling agent.

DC4-x% = Dow Corning [6072:phenyl coupling agent - 1:1] treated glass x% in water

Results for Injection Molding on 9/12/90

NUMBER	COMPOUND	SIZE μm	WT gm	RESIN	WT gm	DYE [R:O:G]	Shape [C/S/W]	Comment
27-1	MPI-2	PELLET	12	LUSTRAN	900	5-12-12	W	Redo 26-17
27-2	MPI-2	PELLET	6	LUSTRAN	900	7-14-14	W	
27-3	MPI-2	PELLET	6	LUSTRAN	900	6-12-12	W	
27-4	MPI-2	PELLET	18	LUSTRAN	900	5-12-12	W	
27-5	MPI-2	PELLET	6	LUSTRAN	900	5-12-12	W	
27-6	MPI-2	PELLET	12	LUSTRAN	900	6-13-12	W	9 g MO
27-7	MPI-2	PELLET	24	LUSTRAN	900	5-12-12	W	10 g MO
27-8	MPI-2	PELLET	24	LUSTRAN	900	1-2-2	W	10 g MO
27-9	MPI-2	PELLET	18	LUSTRAN	900	2-4-4	W	10 g MO
27-10	MPI-2	PELLET	18	LUSTRAN	900	3-6-6	W	10 g MO
27-11	MPI-2	PELLET	6	LUSTRAN	900	5-12-12	W	
27-12	MPI-2	PELLET	12	LUSTRAN	900	2-4-4	W	10 g MO
27-13	MPI-2	PELLET	24	LUSTRAN	900	3-6-6	W	10 g MO
27-14	MPI-2	PELLET	18	LUSTRAN	900	5-12-12	W	10 g MO
27-15	MPI-2	PELLET	12	LUSTRAN	900	4-8-8-	W	10 g MO

G/R = glass to resin ratio

CA = coupling agent

MO = mineral oil

SAN = styrene acrylonitrile

LUSTRAN = SAN from Monsanto ¶ = indicates ground resin

C = circular cavity on Mold 1
on Mold 2

S = square cavity on Mold 1 W = large window cavity

R = Macrolex Red 5B dye

O = Macrolex Orange 3G dye

G = Macrolex Green G dye

CTI-1 = Compounding Technologies Inc. - SK11 coarse glass in SAN resin, 1:2 ratio

MPI-1 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:2 ratio, with

DC-4 coupling agent.

MPI-2 = Modified Plastics, Inc. SK11 coarse glass in SAN resin, 1:4 ratio, with

DC-4 coupling agent.

DC4-x% = Dow Corning [6072:phenyl coupling agent - 1:1] treated glass x% in water

REFERENCES

1. J.A. Dobrowolski, "Coatings and Filters" in Handbook of Optics, W.G. Driscoll and W. Vaughan, eds. (New York: McGraw-Hill Publishing Co., 1978), pp. 8-1 - 8-124.
2. H.A. Macleod, Thin-Film Optical Filters (New York: McGraw-Hill Publishing Co., 1989), p. 265.
3. Macleod, pp. 412 - 420.
4. Dobrowolski, p. 8-12.
5. Dobrowolski, p. 8-32.
6. C. Christiansen, Ann. Phys. Chem. **23**, 298(1884).
7. U. Wojak, U. Czarnetzki, and H.F. Dobeles, Appl. Opt. **26**, 4788(1987).
8. N.K. Sethi, Proc. Ind. Assoc. Cult. Sci. **6**, 121(1920).
9. C.M. Sogani, Phil. Mag. S.7. **1**, 321(1926).
10. E.D. McAlister, Smithsonian Inst. Misc. Collections **93**, No. 7 (1935).
11. H.S. Denmark and W.M. Cady, JOSA **25**, 330(1935).
12. R.B. Barnes and L.G. Bonner, Phys. Rev. **49**, 732(1936).
13. R.L. Henry, JOSA **38**, 775(1948).
14. C.G. Abbot, JOSA **25**, 42(1935).
15. M. Cloupeau and S. Klarsfeld, Appl. Opt. **12**, 198(1973).
16. Wojak et.al., p. 4788.
17. L. Auerbach, Am. J. Physics **25**, 440(1957).
18. K. Balasubramanian, "Structural Inhomogeneity and Anisotropy in Optical Filters and Thin Films; Applications to Optical Storage Media", Ph.D. Dissertation, Optical Sciences Center, The University of Arizona, (1988).

19. E. Matovich, ISA J. **12**, 53(1965).
20. H.R. Carlon, Appl. Opt. **18**, 3610(1979).
21. H.J. Gerritsen, Appl. Opt. **25**, 2382(1986).
22. P. Yeh, Opt. Comm. **35**, 9(1980).
23. D. Redfield and R.L. Baum, JOSA **51**, 184(1961).
24. Dobrowolski, p. 8-103 - 8-106.
25. C.F. Bohren and D.R. Huffman, Absorption and Scattering of Light by Small Particles (New York: John Wiley and Sons, Inc., 1983), p. 57.
26. H.C. van de Hulst, Light Scattering by Small Particles (New York: Dover Publications, Inc., 1981), p. 6.
27. Ibid.
28. Bohren et. al., p. 57.
29. Op. cit., pp. 22 - 23.
30. Op. cit., p. 62.
31. J.M. Greenberg, N.E. Pedersen, and J.C. Pedersen, J. Appl. Phys. **32**, 233(1961).
32. B.W. Bell, "The Entire Scattering Matrix for a Single Fiber", M.S. Thesis, Optical Sciences Center, The University of Arizona, (1981).
33. van de Hulst, p. 3.
34. Bohren et. al., p. 82.
35. Op. cit., pp. 93 - 94.
36. Op. cit., p. 114.
37. Op. cit., p. 126.
38. D.H. Clewell, JOSA **31**, 521(1941).

39. G.F.A. Stutz and A.H. Pfund, *Ind. Eng. Chem.* **19**, 51(1927).
40. B.A. Keen and A.W. Porter, *Proc. Roy. Soc.* **A89**, 370(1913).
41. Clewell, p. 526.
42. R.H. Clarke, *Appl. Opt.* **7**, 861(1968).
43. J.A. Fejer, *Proc. Roy. Soc. Ser. A.* **220**, 455(1953).
44. E.N. Bramley, *Proc. Roy. Soc. Ser. A.* **225**, 515(1954).
45. B.R. Frieden, Probability, Statistical Optics, and Data Testing (Berlin: Springer-Verlag, 1983), pp. 87 - 89.
46. J.D. Lytle, "Refractive Indices of Optical Polymers", M.U. Engineering internal memo, (1981).
47. M. Leonard, ed., The Handbook of Plastic Optics (Cincinnati, Ohio: U.S. Precision Lens, Inc., 1983), p. 19.
48. Schott Glass Technologies Inc., Optical Glass (Federal Republic of Germany: 1989).
49. E.D. Palik, ed., Handbook of Optical Constants of Solids (Orlando, Fl: Academic Press, Inc., 1985).
50. Dobrowolski, pp. 7-75 - 7-110.
51. W.L. Wolfe and G.J. Zissis, eds., The Infrared Handbook, (Ann Arbor, Mich: Environmental Research Institute of Michigan, 1985) pp. 2-79,80.
52. Palik, pp. 529,590.
53. L.C. Martin, Optical Measuring Instruments (London: Blackie and Son Limited, 1924), p. 140.
54. Op. cit., pp. 141-142, 147.
55. Schott Glass Technologies Inc.
56. Lytle.

57. H.R. Simonds and J.M. Church, A Concise Guide to Plastics (New York: Reinhold Publishing Corp., 1963), pp. 2-3.
58. Op. cit., pp. 4-5.
59. R. Juran, ed., Modern Plastics Encyclopedia (New York: McGraw-Hill Inc., 1988), p. 5.
60. Op. cit., pp. 5-6.
61. D.H. Morton-Jones, Polymer Processing, (London: Chapman and Hall, 1989), p. 1.
62. J. Bown, Injection Moulding of Plastic Components (London: McGraw-Hill Book Company (UK) Limited, 1979), p. 65.
63. Simonds et. al., p. 6.
64. Bown, p. 65.
65. Simonds et. al., p. 4.
66. M. Chanda and S.K. Roy, Plastics Technology Handbook (New York: Marcel Dekker, Inc., 1987), p. 1.
67. Bown, p. 70.
68. Chanda et. al., p. 2.
69. L.K. Arnold, Introduction to Plastics (Ames, Iowa: The Iowa State University Press, 1968), pp. 7 - 8.
70. Chanda et. al., pp. 10 - 11.
71. Arnold, pp. 8 - 9.
72. Chanda et. al., pp. 14 - 15.
73. Arnold, pp. 11 - 13.
74. Morton-Jones, pp. 13 - 14.
75. Arnold, pp. 11 - 13.

76. Op. cit., pp. 13 - 14.
77. Juran, pp. 127 - 178.
78. Leonard, pp. 18 - 21.
79. J.S. Walker and E.R. Martin, Injection Moulding of Plastics (London: Iliffe Books Ltd., 1966), p. 41.
80. Leonard, pp. 25 - 26.
81. Juran, pp. 13 - 14.
82. Walker et. al., p. 34.
83. A.W. Birley and M.J. Scott, Plastics Materials: Properties and Applications (Glasgow, U.K.: Leonard Hill, 1982), p. 43.
84. Juran, p. 77.
85. Birley et. al., pp. 43 - 45.
86. Juran, pp. 76 - 79.
87. Leonard, p. 26.
88. Birley et. al., pp. 49 - 50.
89. Juran, pp. 82 - 84.
90. Leonard, p. 28.
91. Op. cit., p. 26 - 27.
92. Juran, p. 42 - 43.
93. Leonard, p. 27.
94. Simonds et. al., pp. 73, 139 - 142.
95. Arnold, pp. 38 - 39, 46 - 47.
96. Op. cit., pp. 47 - 49.

97. Op. cit., p. 50.
98. Birley et. al., pp. 12 - 13.
99. Morton-Jones, p. 152.
100. Birley et. al., pp. 12 - 13.
101. Toshiba Machine Textbook 5: Molding (Tokyo: Toshiba Machine Co., Ltd., 1988), pp. 22 - 27.
102. Juran, pp. 330 - 332, 340.
103. Leonard, p. 20.
104. R.T. Conley, Infrared Spectroscopy (Boston: Allyn and Bacon, Inc., 1972), pp. 244 - 247.
105. H.W. Siesler and K. Holland-Moritz, Infrared and Raman Spectroscopy of Polymers (New York: Marcel Dekker, Inc., 1980), p. 325.
106. E.P. Plueddemann, Silane Coupling Agents (New York: Plenum Press, 1991), pp. 1 - 4.
107. Juran, p. 140.
108. Op. cit., pp. 140 - 141.
109. E.P. Plueddemann, ed., Interfaces in Polymer Matrix Composites (New York: Academic Press, 1974), p. 135.
110. Op. cit., pp. 136 - 137.
111. Plueddemann, p. 147.
112. Plueddemann, ed., pp. 157 - 158.
113. van de Hulst, p. 5.
114. Bohren et.al., p. 38.
115. W.H. Press, B.P. Flannery, S.A. Teukolsky and W.T. Vetterling, Numerical Recipes (Fortran) (Cambridge: Cambridge University Press, 1989), p. 196 - 197.