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THE EFFECT OF COGNITIVE STRATEGY TRAINING ON VERBAL MATH
PROBLEM SOLVING PERFORMANCE OF LEARNING DISABLED
ADOLESCENTS

The University of Arizona

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THE EFFECT OF COGNITIVE STRATEGY TRAINING
ON VERBAL MATH PROBLEM SOLVING PERFORMANCE
OF LEARNING DISABLED ADOLESCENTS

by
Marjorie Montague

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF SPECIAL EDUCATION
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

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THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

As members of the Final Examination Committee, we certify that we have read
the dissertation prepared by Marjorie Montague

entitled THE EFFECT OF COGNITIVE STRATEGY TRAINING ON VERBAL
MATH PROBLEM SOLVING PERFORMANCE OF LEARNING
DISABLED ADOLESCENTS

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SIGNED: Maioire Montague

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ABSTRACT

This study investigated the effect of an eight-step cognitive strategy on verbal math problem solving performance of six learning disabled adolescents. The research was conducted in an applied setting by the investigator, the students' learning disabilities teacher. The cognitive strategy was designed to enable students to read, understand, carry out, and check verbal math problems that are encountered in the general math curriculum at the secondary level.

A multiple baseline across individuals design permitted demonstration of the effectiveness of the strategy. Conditions of the experiment included baseline, treatment, generalization, maintenance, and, for two students, retraining. During treatment, students received strategy acquisition training over three sessions. When the students demonstrated verbalization of the eight strategy steps from memory, strategy application practice and testing commenced. Utilization of the strategy and improved performance were measured by scores on tests of two-step verbal math problems. The number of correct responses and the number of minutes taken to complete each test were recorded on graphs.

Visual analysis of the data indicated that this eight-step cognitive strategy appeared to be an effective intervention for this sample of students who had deficits in verbal math problem solving. Overall, the students demonstrated improved performance on two-step

verbal math problems with four of the six students generalizing the use of the strategy to three-step problems. Four students maintained improved performance over a two-week lapse in instruction and practice. Substantial increases were noted for the amount of time required to complete the verbal math problem solving tests immediately following strategy acquisition training. Completion time rapidly stabilized to an acceptable level.

This study has implications for an alternative teaching methodology that focuses on cognitive strategy training to improve verbal math problem solving for learning disabled youngsters. Future research could offer evidence of the applicability of cognitive strategy training to other populations and further delineate the characteristics of students who do and do not benefit from cognitive strategy intervention.

CHAPTER 1

INTRODUCTION

The Education for All Handicapped Children Act of 1975 (PL 94-142) included in its definition of learning disabilities "a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations" (Federal Register, 1977a, p. 42478). One of the criteria specified by the regulations for determining the presence of a learning disability is that the "child does not achieve commensurate with his or her age and ability when provided with appropriate educational experiences in the areas of oral expression, listening comprehension, written expression, basic reading skill, reading comprehension, mathematics calculation or mathematics reasoning" (Federal Register, 1977b, p. 65082). Although it is recognized that learning disabled individuals may demonstrate mathematical deficits, learning disabilities are most often associated with reading deficits (Reid and Hresko, 1981a).

Mathematics, traditionally viewed as a basic skill along with reading and writing, has not been given the appropriate consideration in regular education (National Council of Teachers of Mathematics, 1980) nor in remedial education (Skrtic, 1980). Mathematics

instruction, especially for the disabled population, has focused predominately on arithmetical computation rather than application or, more specifically, mathematical reasoning and problem solving. However, educators agree that "applied problem solving processes constitute an important part of the basic skills that are needed for mathematical literacy among average citizens" (Lesh, 1979, p. 1).

With the current emphasis on "back to basics" and stress on math and science competencies (Nation at Risk, 1983) and the recent call for a focus on problem solving by the National Council of Teachers of Mathematics (NCTM) in 1980, a reassessment of the goals and objectives for mathematics education is imminent. Notwithstanding that the ultimate goal of mathematics education is considered to be the improvement of students' problem solving performance (Days, Wheatley, and Kulm, 1979; Kilpatrick, 1969), most traditional approaches to mathematics instruction emphasize computation exercises (Carpenter et al., 1980; Reid and Hresko, 1981b). This emphasis may help to explain the dearth of empirical studies which go beyond mere computational exercises in mathematical problem solving in the developmental literature (Suydam, 1980) and in the disabilities literature (Cawley, 1981).

Cawley (1981) emphasized the need for a national effort focusing on mathematics and learning disabilities. Knowledge of normal children's problem solving development and the processes utilized in mathematical problem solving are vital to the understanding of deficient problem solvers' characteristics and the development of intervention programs to improve their performance. Understanding the processes is not a new endeavor. Polya's (1945) four-part problem solving model has

been regarded as a cornerstone by researchers and theorists in the field of mathematics education. Studying the components of problem solving, often referred to as strategies, techniques, or heuristics, may contribute to an overall picture of the cognitive processes involved and may provide a foundation upon which to build instructional programs. Problem solving research requires a multidisciplinary effort involving input from cognitive psychologists, mathematics educators, and special educators.

A survey of learning disabilities teachers conducted by McLeod and Armstrong (1982) indicated the need for commercial math programs based on the mainstream math curriculum with emphasis on systematic extended practice for the learning disabled youngster. The question arose as to when to abandon basic skill instruction for life skills instruction. Halpern (1981) advocated a math curriculum for the secondary learning disabled student that concentrates primarily on functional skills required for independent living. Included in her instructional suggestions are verbal problems and multiple part problems.

Competency in mathematical problem solving is essential for the learning disabled adolescent who is expected to complete mathematics requirements for high school graduation and function in everyday society. Functional life skills include the ability to apply the mathematical computation skills ideally acquired during the elementary years. Unfortunately little differentiation between elementary programming and secondary programming for learning disabled students is apparent even though the regular secondary level curriculum focuses on

content areas and application of skills rather than the acquisition of basic skills (Deshler, 1978).

Currently underway is an effort to provide students with training in strategies that may facilitate success with the secondary curriculum (Alley and Deshler, 1979). The Learning Strategies Model developed and researched at the Institute for Research in Learning Disabilities at the University of Kansas is designed to assist secondary learning disabled students in dealing effectively with the content of regular secondary classes. While many of these strategies focus on reading, researchers are looking into their application in teaching or learning communication skills (Van Reusen, 1983), written language (Moran, Schumaker, and Vetter, 1981), and verbal math problem solving (Smith and Alley, 1981).

The addition of an effective verbal math problem solving strategy to a growing repertoire of strategies for reading and written language may allow the learning disabled adolescent to participate more successfully in the mainstream and meet the academic demands of the secondary curriculum.

Background of the Problem

The background of the problem revolves around the verbal math problem solving of learning disabled adolescents. This discussion focuses on the need to establish a body of data-based literature that describes the characteristics of students who are deficient in verbal mathematical problem solving. It also addresses the need to provide appropriate instruction and service delivery that will enable the

learning disabled secondary student deficient in this area to function effectively in the regular classroom and everyday society.

Developmental dyscalculia is a term commonly used to describe the condition of mathematical deficiency. McEntire (1981) discussed three theoretical views of developmental dyscalculia: (1) the neurological-mental ability deficits; (2) educational task deficits; and (3) cognitive developmental differences. Each of these views represents an attempt to explain mathematical dysfunction in terms of internal deficits, external deficits, and deficiencies in interactions between learner and the learner's environment. The cognitive developmental view considers both internal and external variables and their interaction. Presently few empirical studies to substantiate the validity of these views are available in the literature.

Kosc (1981), operating within a neuropsychological framework, has classified several types and levels of developmental dyscalculia. He included in his classification system a dysfunction in problem solving referred to as ideognostical which is "manifested primarily by a disability in understanding mathematical ideas and relations and thus in doing mental calculation" (Kosc, 1981, p. 26).

Recently researchers have begun to investigate the characteristics of good and poor problem solvers (Lee and Hudson, 1981; Moroz, 1978; Robinson, 1973; Silver, 1979) and the factors associated with the process of verbal math problem solving (Days et al., 1979; Kantowski, 1981). It has been established that both conceptual knowledge and cognitive processes are important factors in problem solving (Webb, 1979). How these factors are related and how conceptual knowledge is

acquired remain important questions. Simon (1975) suggested that the acquisition of conceptual knowledge is dependent on heuristics, i.e., problem solving strategies or capabilities. Lesh (1979) maintained that the investigation of mathematical knowledge and abilities requires knowledge about the characteristics and capabilities of children and about the mathematical concepts under investigation. Understanding the cognitive processes involved in math problem solving will assist in isolating those characteristics of learners who are efficient or deficient.

Effective remedial intervention should be conducted only after the characteristics of the individual experiencing difficulty in math problem solving have been identified. More precise diagnostic instruments designed to measure the cognitive processes necessary to solve verbal math problems and methods designed to remediate identified disabilities must be developed. Assessment should include an examination of the strategies utilized by the student in solving mathematical problems while remediation should focus on more efficient utilization of those strategies already in the repertoire of the student and acquisition of additional strategies appropriate to the task. Further applied research in strategy training may provide the practitioner with the means to identify which strategies the learning disabled youngster has or has not acquired and the methods of teaching strategies appropriate to the task and the individual. This approach mirrors aptitude-treatment interaction which has typified remediation in special education (Kirk, 1972).

With the current trend to increase mathematics units necessary for graduation from high school (Nation at Risk, 1983) and the rise in the number of states requiring minimum competency testing (McCarthy, 1980), it is critical that secondary learning disabled students are provided with facilitating tools that promote success in secondary mathematics classes and provide preparation for post-secondary opportunities. The elementary model, which has for so long characterized secondary resource programs (Weiderholt, 1978), may not adequately serve the needs of the learning disabled adolescent. At some point, preferably at the junior high level, a transition from remediation in basic skills (e.g., computation) to application of those basic skills must occur. Programs that emphasize comprehension, reasoning, and problem solving may allow for more success in content area classes and vocational programs. A study skills approach or Learning Strategies Model (Alley and Deshler, 1979), which teaches the secondary learning disabled student to apply those skills he or she has acquired, seems more appropriate than continued training in basic skills. This approach may increase the likelihood of success in mainstream classes, particularly if the learned strategies can be maintained and generalized to similar situations and settings.

It is recognized that math problem solving strategies can be taught (Schoenfeld, 1979a; Smith and Alley, 1981). However, maintenance and generalization of the learned strategies continue to be problematic (Deshler et al., 1981). Keogh and Barkett (1979) concluded that, of three intervention approaches, cognitive training appeared to offer more possibility of transfer or generalization than either

medication or behavior modification. Techniques to test conditions under which the strategies are maintained and generalized must be built into research designs in order that findings are applicable to the needs of students and useful to the classroom teacher.

The goal of cognitive training, in this case strategy training, is to facilitate learning in an area in which the learning disabled student has demonstrated failure. Learning strategies may act as enablers if the student can select the appropriate strategy and apply it successfully. Program goals and objectives for individual learners may need to be reassessed with emphasis on cognitive intervention as a type of remedial program for those students who may benefit. Alley and Deshler (1979) suggested the learning strategies resource room model as the most practical service delivery model for teaching strategies. Direct instruction in well designed strategies may provide the means by which previously unsuccessful learning disabled students can demonstrate competency in verbal math problem solving.

Research leading to knowledge of characteristics of students who might benefit from strategy training in verbal math problem solving combined with tested techniques that fit those characteristics is essential. Expedient service delivery models that focus on acquisition and application of strategies appropriate to the needs of the individual learner within the secondary school setting should allow learning disabled students greater access to the regular program.

Statement of the Problem

Research in cognitive intervention that provides learning disabled adolescents with strategies to improve their academic performance in the mainstream and opportunities for success in the adult world is needed. Verbal math problem solving is one academic area that has been neglected in educational literature. The little research that has been conducted has not clarified the characteristics of learning disabled students who manifest a verbal math problem solving deficit, nor systematically investigated intervention effectiveness. Learning disabled students need to demonstrate proficiency in this area for academic as well as occupational success.

Statement of the Purpose

The purpose of this study is to investigate the effectiveness of an eight-step strategy designed to enable students to read, understand, carry out, and check verbal math problems that are encountered in the general math curriculum at the secondary level. This strategy incorporates many of the components of problem solving models (Kramer, 1970; Polya, 1945; Smith and Alley, 1981) and utilizes many of the instructional practices advocated by proponents of the Learning Strategies Model including verbal rehearsal, modeling, and corrective feedback.

In this study, cognitive strategy training combines the teaching techniques of modeling, corrective feedback, and verbal rehearsal with direct instruction in the verbal math problem solving techniques of paraphrasing, visualizing, detecting relevant information, locating the

question, hypothesizing, estimating, labeling, and checking. A cognitive strategy that is goal-oriented and appropriate for the task may provide the structure necessary for the learning disabled adolescent to achieve success in verbal math problem solving.

Questions to be Addressed

Using a multiple baseline design for introducing the verbal math problem solving strategy, this study will seek to answer the following questions:

1. Will the strategy used for verbal math problem solving improve the test performance on two-step verbal math problems to a criterion of at least seven correct responses maintained over four consecutive tests by learning disabled adolescents placed in grades 10-12?
2. Will the improved performance be maintained by the students at a criterion of at least seven correct responses after a two week elapsed period during which no instruction or testing occurred?
3. Does generalization of the strategy to three-step verbal math problems occur as measured by a performance criterion of at least five correct responses?
4. How does the time required to complete the verbal math problem solving tests vary across and within the experimental conditions?

Definition of Terms

Generically, a learning strategy has been defined as a "technique, principle, or rule that will facilitate the acquisition, manipulation, integration, storage, and retrieval of information across situations and settings" (Alley and Deshler, 1979, p. 13). Terms specific to this study are defined below in relation to the verbal math problem solving process.

Two-step verbal math problem--a written word problem which requires a solution involving any combination of two of the four basic math operations, i.e., addition, subtraction, multiplication, and division.

Three-step verbal math problem--a written word problem which requires a solution involving any combination of three of the four basic math operations, i.e., addition, subtraction, multiplication, and division.

Performance--the number of correct responses on ten-problem measures of verbal math problem solving.

Cognitive problem solving--the ability to arrive at the correct solution to a verbal math problem by utilizing a specified cognitive strategy.

Cognitive strategy--the process which is utilized to arrive at the correct solution to a verbal math problem.

CHAPTER 2

REVIEW OF THE LITERATURE

A multidisciplinary effort from researchers in the fields of general mathematics education, cognitive psychology, and special education is required to establish a foundation upon which curricula can be built. The literature reviewed is representative of related research from the three disciplines. This introduction describes the interrelation of the fields.

Mathematical problem solving has been defined in numerous ways by professionals in general mathematics education. Lesh (1979, p. 2) claimed that "applied problem solving occurs when ordinary people attempt to solve real problems in real (or at least realistic) situations." Kantowski (1980, p. 195) defined a problem as a "situation for which the individual who confronts it has no algorithm that will guarantee a solution" and requires a novel approach by the person. Meiring (1980) described a mathematical problem as a situation involving an initial state and a goal with some blockage between the two requiring motivation and the feeling that the problem is within the capability of the solver. Brownell (1942) stressed the complexity of a task if it is to be referred to as a problem and identified three requisites. A task is a problem if (1) it calls for a solution under certain specified conditions, (2) the person understands the task but does not see an

immediate strategy for its solution, and (3) the person is motivated to search for the solution.

Research in cognitive psychology has focused on the variables related to problem solving. Variables researched by Days et al. (1979) included subject variables (age, cognitive level, and mathematical experience) and task variables (problem structure, problem context, problem length, magnitude of the numbers, and placement of the question). These variables seem directly related to Flavell and Wellman's (1976) three classes of variables (person, task, and strategy) which are purported to interact in retrieval problems.

Metacognition and metamemory are two recent areas of research in learning and development that consider this interaction of variables. Metacognition refers to that "level of understanding in which the learner demonstrates capability or knowledge concerning one's own cognitive processes and product" (Flavell, 1976, p. 232) and metamemory refers to "intuitive knowledge and understanding of memory that most individuals possess" (Kail, 1979, p. 35). Flavell (1977) asserted that sensitivity to the demand of the task and a person's knowledge of the three classes of variables and their interactive effects on memory performance constitute the two major categories of metamemory. Research in this area of cognitive psychology may provide further insight into factors influencing problem solving.

The importance of problem solving in general (Resnick and Glaser, 1976) and verbal mathematical problem solving in particular (Cowan and Clary, 1978; Hiatt, 1979) to academic success and developmental growth is a current topic of interest among psychologists and

educators. Designing instructional programs to effectively teach problem solving as a process requires consideration of the underlying components of the cognitive processes and an understanding of the characteristics of the individual.

Special educators have been particularly interested in the problem solving abilities of learning disabled students. Learning disabled children and youth demonstrate a marked inability to perform efficiently on tasks that require reasoning and problem solving ability (Becker, Bender, and Morrison, 1978; Cawley et al., 1979; Havertape and Kass, 1978; Robson, 1977; Torgesen, 1980). Cawley et al. (1979, p. 25) emphasized the need for programmatic reform in the area of mathematical verbal problem solving for the learning disabled.

Problem solving should be an integral and necessary component of all programs for learning disabled youth. Much to our chagrin, problem solving in mathematics programs has yet to receive an appropriate level of attention from research workers, diagnosticians, or curriculum developers; yet problem solving can be used to promote language, reading comprehension, reasoning and evaluative skills, and many other areas of development.

Research that will promote an understanding of the factors involved in solving mathematical problems and provide educators and clinicians with methods of assessing and teaching this complex activity to learning disabled youngsters is critical to an overall understanding of problem solving ability.

Mathematical abilities research involving the learning disabled has been far less popular than that involving the gifted (Lesh, 1979). Recently, special educators have focused on the need to teach learning disabled adolescents how to learn by developing instructional programs

that stress information processing, organizational skills, study skills, application of information and problem solving strategies (Towle, 1982). Learning how to solve verbal math problems is an essential part of such programs.

Torgesen (1977, p. 30) maintained that the "child's use of active and efficient strategies for information processing depends not only on the level of his or her general cognitive awareness but also on purposes and goals in the situation." This view along with other cognitive theories (Flavell, 1977; Meichenbaum, 1977) has generated investigations into several of these "activating" strategies with learning disabled students including external and internal cueing, verbalization, visual imagery, extraneous information cueing, and self-questioning (Havertape and Kass, 1978; Torgesen, 1979; Wong, 1980). Many of these techniques may be appropriate for inclusion in larger scale instructional programs such as the Learning Strategies Model for learning disabled adolescents developed by Alley and Deshler (1979).

From the interrelated work of general math educators, cognitive psychologists, and special educators, four general observations emerge:

1. Psychologists and educators are interested in verbal math problem solving since research in the area may lead to the development of curricula.
2. Some learning disabled youngsters experience difficulty in problem solving, specifically mathematical problem solving.
3. Of late there has been interest in intervention for youngsters with mathematical problem solving disabilities.

4. Research that focuses on learning strategies in general is evident.

The following sections of the literature review focus on research in general problem solving, cognitive strategy training and heuristics, developmental trends in verbal math problem solving, and learning disabled students as math problem solvers. Furthermore, since maintenance and generalization of strategies are critical to the effectiveness of intervention programs, selected studies addressing these concerns are reviewed.

Problem Solving

Problem solving as a complex interaction presumably involving higher order mental processes has historically concerned theorists of learning and development (Bruner, Goodnow, and Austin, 1956; Flavell, 1977; Gagne, 1965; Kail, 1979; Piaget and Inhelder, 1958; Scandura, 1977). These theorists provide a base of knowledge and experience for further investigation of the problem solving process. Individual differences in the areas of cognitive abilities and cognitive styles have been under investigation (Johnson, 1972; Meichenbaum, 1977). Simultaneously, a systematic approach to an analysis of the components of the process and a methodology of teaching problem solving are developing (Gagne and Smith, 1962; Johnson, 1972; Meichenbaum, 1977; Ross and Ross, 1978; Scandura, 1977; Simon, 1978). Research with regard to the characteristics of general problem solving and to the characteristics of learning disabled problem solvers is reviewed.

Characteristics of Problem Solving

Traditionally defined as an initial unsuccessful attempt to reach a goal by a motivated individual (Erickson and Jones, 1978), problem solving, like other motivated behavior, has certain characteristics. These were described by Johnson (1972, p. 134) as "(1) goal orientation and continuity of action toward that goal and (2) change of activity after the goal is attained. Special characteristics, in contrast to routine activity are (3) intraindividual variability, (4) interindividual variability, (5) time required, and (6) the assumption of mediating activities."

Cameron (1976) listed three components of a problem solving task: (1) comprehension and recall of task instructions; (2) formulation of a decision rule or solution strategy; and (3) implementation of a decision rule. The forgetting of critical information was adjudged to be a factor in poor problem solving among four year olds (Bryant and Trabasso, 1971), seven year olds (Eimas, 1970), and adolescents (Siegler and Liebert, 1975). Among five year olds, deficient encoding seemed to be a factor in poor problem solving although instructions in encoding resulted in improved performance (Siegler, 1975).

Greeno (1978) hypothesized that general knowledge and specific knowledge of the problem solving process and direct attention to the goal are requisites for successful achievement of the goal in finding solutions. Resnick and Glaser (1976) identified the primary feature of problem solving as assembly (integration) which is an inventive process involving three components: problem solving, feature scanning, and goal analysis. Simon (1978) has developed a structured information

processing system (IPS) of problem solving which involves interaction between the information processing system, the problem solver, and the task environment and problem space. It is speculated that problem solving proceeds sequentially with consideration for memory capacity and search strategies.

Two studies reviewed typify those utilizing the IPS model. In the first (Simon and Reed, 1976), a computer simulation model was matched to human lab data for the missionaries and cannibals task to explain the effects upon problem performance of giving a hint and the effects of solving the problem a second time after one successful solution had been achieved. Results indicated that a strategy shift is made when the subject is given a subgoal; in this case, a strategy of balancing the number of missionaries and cannibals shifted to a means-end strategy which influenced the operations leading to the solution. The second study (Reed and Abramson, 1976) investigated how the effectiveness of subgoals interact with the structure of the problem space. The finding pointed to the usefulness of the subgoal on the size of the problem space. Reed, Ernst, and Banerji (1974, p. 445) discussed generalization of problem solving by positing the idea that the "subject's memory for subgoals, rather than an exact sequence of moves, may form the basis for transfer to an identical or similar problem."

There has been some concern about the applicability of information processing models to in-school instruction (Bell, 1979). Reasons for this view include the type of tasks utilized in the research, the applicability of laboratory research to applied settings, the focus on college age students as subjects and the general short time devoted to

these experiments. However, IPS research has provided a framework that has encouraged "meticulous descriptions of varieties of purposeful strategies, none of which could have developed within a pure behavioristic paradigm" (Erickson and Jones, 1978, p. 71). These strategies, in conjunction with other remedial techniques, may assist in the development of instructional programs to alleviate problem solving deficits of children with learning disabilities.

Characteristics of Learning Disabled Problem Solvers

Torgesen's (1980) view of the learning disabled as an inactive learner attributes poor general problem solving to the learning disabled student's difficulty in applying task-appropriate strategies, rather than to any presumed psychological processing deficit. This "production deficiency" may partly account for poor problem solving. In support of Torgesen's explanation, Robson (1977) studied upper elementary learning disabled students and their use of categories in problem solving situations. The findings suggested that the learning disabled have acquired an average amount of information and have organized it, but do not use it within the same context or in response to the same cues as the normal achiever does. A highly structured context and cueing were recommended as methods to assist learning disabled students in selecting appropriate strategies thereby using information efficiently.

Kennedy (1981) designed a study to illustrate how learning disabled students manipulate information to learn how to solve problems. The study focused on strategy utilization as a three stage process.

During all three stages, comprehension of task requirements, production of appropriate strategies, and successful execution of these strategies, no significant differences were detected between learning disabled students and their nonlearning disabled peers (ages 8.3-10.0). The author suggested that cueing of the need for a strategy may prove as effective as specific strategy training.

Pistono (1980), in a study with younger learning disabled (LD) students (6 to 7½ year old boys), obtained results that partially supported a characterization of LD students as "inactive" learners. Many LD subjects attempted strategic activity but failed to adequately allocate and control their strategies. However, he postulated a difference rather than a delay hypothesis explaining that neither production inefficiency nor processing dysfunction may be solely responsible for poor performance on similar problem solving tasks since an interaction may exist. Pistono suggested that a learned strategy incorporating appropriate cognitive processing techniques may positively affect problem solving performance.

Mellard and Alley (1981) suggested that learning disabled students may be active but inefficient problem solvers. Their study involving learning disabled (LD) adolescents' performance on discrimination learning tasks and a similar study by Phillips (1974) indicated that the mean trial of last error (TLE) is a possible discriminator of a problem solving deficit. The explanation of trial of last error includes the assumption that more efficient problem solvers will use more efficient solution strategies and be more efficient in their information processing. Fewer trials, then, will be required to solve

a problem. Efficiency is determined by calculating the TLE of all the student's problems. Problem solution time was correlated with TLE and was considered to be associated with efficiency. Mellard and Alley found that LD students require additional trials and time to solve the problems and have a longer latency period following a confirmed hypothesis. Feedback appeared to be irrelevant to success. They concluded that LD students not only lacked the ability to apply task appropriate strategies but lacked the strategic approaches of the NLD peers. Information processing differences between LD and NLD adolescents in coding, recoding, and recalling of information were also significant.

Self-regulatory skills are considered another characteristic essential to the problem solving process (Sternberg and Detterman, 1979). In a signal detection study, Deshler, Ferrell, and Kass (1978) found that learning disabled adolescents were less willing than their normal counterparts to call an element an error in material they produced themselves suggesting a need to teach self-monitoring skills to learning disabled students. Results in a study by Havertape (1976) showed significant differences between learning disabled and nonlearning disabled adolescents in self-instructional techniques they use while solving problems and in performance on selected tests. It was discovered that the learning disabled did not attend appropriately to directions for completing a task and guessed at solutions rather than selecting a useful and appropriate strategy.

An extension of the previous study (Havertape and Kass, 1978) regarding verbalized self-instructions by learning disabled (LD) adolescents yielded significant differences between the LD students and their

normal peers (NLD) in procedures used for problem solving. Twenty junior high and 20 senior high LD students were matched on age and intelligence with NLD and examined on four different tasks (comparison of food prices, detection of logical progression in number series, solving novel word problems, and writing completions of limericks). One-third of the LD compared with 88% of the NLD read the problems efficiently, 40% of the LD compared with 80% of the NLD verbalized comprehension of the problems, and 60% of the LD compared with 94% of the NLD responded to problems or proceeded in solutions using an organized plan rather than random or impulsive responses. Interpretations of the results suggest that poor performance among the LD may be due to the lack of attack strategies or inefficient use of those which they do have in their repertoire, and that their thought processes are haphazard although low academic skills may be partly responsible. No significant interactions were found for the different age groups. This study corroborates, to some degree, Robson's (1977) conclusions that learning disabled children acquire and categorically store information equally well with normal children but do not use this information within the same context or in response to the same cues as normal achievers.

Research on learning disabled students as problem solvers has indicated that as a group these students have difficulty utilizing task appropriate strategies, are inefficient in their use of strategies and need additional cueing to improve performance. Validating teaching techniques for problem solving among the learning disabled requires further investigations of both specific techniques and instructional packages. Webb (1979) advocated a holistic approach to instruction

whereby the effects of both conceptual knowledge and problem solving processes are considered.

Cognitive Strategy Training and Math Problem Solving

Cognitive strategy training for verbal mathematical problem solving appears to be the approach advocated by experts in the field. Strategies are "general skills or abilities that can be learned, are useful for a variety of problems, may be used singly or in combinations to solve a single problem, and give the individual the tools with which to begin or continue productive work on a problem" (Meiring, 1980. p. 7). A heuristic is defined as a "general suggestion or strategy, independent of any topic or subject matter, which helps problem solvers approach, understand, and/or efficiently marshall their resources in solving problems" (Schoenfeld, 1979b, p. 37). The terms strategies and techniques are used interchangeably in the literature. Examples include drawing a diagram, establishing subgoals, estimating, restating the problem in one's own words, working backwards, exploiting analogous problems, and identifying extraneous information. For the purpose of this study, a strategy is considered the global process which incorporates problem solving techniques.

Recently researchers in education and cognitive psychology have begun to examine processes or sets of steps used in finding solutions (Kantowski, 1981). This is often accomplished by using a "think-aloud" process which entails taping and interpreting the problem solvers' verbalizations during the process. Drawbacks to this method include the time consuming and cumbersome nature of the experiment and the

realization that students may not be accurately verbalizing the steps. Consequently, conclusions drawn from research of this nature must be interpreted cautiously.

There is agreement that correct solutions to mathematical problems involve setting up a plan for solution. Kantowski (1977) discovered that high ability ninth graders use goal-directed heuristics and, as problem solving improved, the tendency to use goal-directed heuristics to achieve solutions increased. Schoenfeld (1979b) suggested a managerial strategy which may generalize to other problem solving situations, i.e., utilizing a teaching process that focuses on a global problem solving strategy which accurately describes the principles followed by accomplished problem solvers. Pinpointing these principles and formulating methods of instruction remain difficult.

Salient Aspects of Cognitive Training

Several aspects of cognitive training have been shown to be critical to successful instruction of problem solving. Modeling strategies were recommended by Polya (1945, p. 30) who maintained that "experience in solving problems must be the basis on which the heuristic is built." Brown and Palinscar (1982), in a reading comprehension study involving four seventh grade students with learning problems, suggested using a "dialogue leader" who not only models the appropriate strategies but engages in active self-questioning and paraphrasing for the students to mimic.

Bennett (1981) improved the ability of learning disabled students to solve verbal math problems with a verbal mediation program

and recommended a highly structured program with direct teaching of the decision-making process involved in problem solving. Vos (1976) demonstrated that key organizers (diagrams, approximations, and charts) improved the problem solving performance of sixth, seventh, and eighth graders while Hall (1976) found that students with good estimation ability were superior in problem solving.

Visualization is another technique viewed as important (Forrest, 1981; Polya, 1945). Dunlap and Brennan (1979) discussed the importance of mental imagery in conjunction with the active and rapid manipulation of mathematical symbols. Pictorial representation or drawing a diagram has proved to be an effective technique (Carpenter et al., 1981; Nelson, 1975; Polya, 1945).

Research in cognitive strategy training for math problem solving is becoming more evident in developmental literature. The first recommendation of the National Council of Teachers of Mathematics (1980) was that problem solving must be the focus of school mathematics in the 1980's. Realization of that goal is dependent on careful planning and guidelines and a strong commitment on the part of educators. Insightful and practical applied research endeavors may provide the necessary information to make that goal a reality.

Development of Verbal Mathematical Problem Solving

Investigations of math problem solving as a process are relatively recent in the developmental literature (Suydam, 1980). Perhaps the call for a focus on problem solving by the National Council of Teachers of Mathematics (1980) and the results of the Math Assessment

of the National Assessment of Educational Progress (NAEP) will spur future research.

Carpenter et al. (1980) summarized the NAEP results. There appeared to be a decline in problem solving between the 1973 and 1978 NAEP results among 9, 13, and 17 year old students who scored very poorly on multi-step and complex one-step problems requiring analysis. The reading factor was controlled by the use of an audiotape. Students did well on "textbook" one-step problems when corresponding computational skills had been attained but demonstrated difficulties when presented with problems involving unfamiliar situations. A graphic component (pictorial representation) seemed to improve problem solving among 9 and 13 year olds. Difficulties with nonroutine problems appeared to result from the interpretation that problem solving simply involves choosing the appropriate arithmetical operation and applying it to numbers given in the problem. The focus on key words was viewed as a crutch for students and seemed to provide no foundation for developing analytic skills. The major conclusions were that students, in general, lack strategies for problem solving and need analytic techniques.

There seems to be consensus that problem solving strategies can be specifically taught and when taught, are used more frequently and with greater success (Driscoll, 1980; Meiring, 1980; Suydam and Weaver, 1977). Although some students will develop problem solving skills naturally (Ausubel, 1968), experts in the field concur that systematic planned instruction with practice built in is generally required for success (Kantowski, 1980; Polya, 1945). Kantowski (1980) cited three variables, including understanding the problem, planning, and

computational skills as important to problem solving. She warned, however, that skill in computation processes appears necessary for solving problems but does not guarantee successful problem solving. Instruction in setting up a plan for problem solving techniques appeared to have a positive effect on problem solving (Kantowski, 1977). Also, pupils' awareness of their own processes while solving problems had a positive effect on problem solving (Gurova, 1969).

Learner characteristics have been addressed in the developmental literature. Silver (1979) found that good and poor problem solvers differed on recall of verbal math problems. Good problem solvers recalled the structural features of the problems and poor problem solvers recalled only details. Findings of Robinson (1973) indicated that good problem solvers tended to use a formal strategy more often than the trial and error method preferred by poor problem solvers. Good problem solvers required more time for solutions. This study provided information regarding characteristics of problem solvers. IQ, reading comprehension, arithmetic concepts, arithmetic problem solving, self-esteem, low test anxiety, and a reflective nature were found to have high correlations with successful verbal math problem solving while high test anxiety and impulsivity had low correlations with successful problem solving.

Information regarding the development of problem solving skills and learner characteristics is essential to the development of instructional programs. Current literature supports that verbal math problem solving is a developmental skill that increases with age and that good problem solvers tend to utilize a systematic approach to problem solving.

Disabilities and Verbal Mathematical Problem Solving

It is generally agreed that a variety of subgroups exists within the total population of learning disabled children and adolescents. Harber (1981) noted the need for research to identify subgroups of the diverse population referred to as learning disabled. One subgroup which may emerge as a result of research is that group of learning disabled students who demonstrate poor verbal math problem solving performance. Research that addresses the relationship of problem solving to learning disabilities is reviewed.

Problem Solving Difficulties as a Correlate to Learning Disabilities

Researchers have begun to investigate an inability to solve verbal math problems as a possible correlate of learning disability. While some have focused on remedial aspects (Bennett, 1981; Cawley et al., 1979; Fafard, 1976; Smith and Alley, 1981), a few have looked at characteristics of problem solvers for purposes of identification of learning disabled students (Lee and Hudson, 1981; Levy, 1981; Skrtic, 1980). Initial teaching of the subskills required for competent mathematical problem solving and appropriate remedial techniques are dependent on knowledge of those learner characteristics which prevent success in problem solving.

A comparative study by Lee and Hudson (1981) was designed to yield both quantitative and qualitative differences in verbal math problem solving abilities between learning disabled (LD) and nonlearning disabled (NLD) seventh grade males matched on age, grade, sex, and

intelligence. On problems which were read orally and solved at two levels of difficulty, the 20 LD boys evidenced a lower mean problem solving score and significantly more errors in both arithmetic and oral reading than the NLD boys. Error pattern analysis revealed weaknesses for LD students in the areas of reasoning, selection of relevant information, determination of the correct and most efficient process, assessment of correctness of response, estimation, and judgment. Although the two groups were similar in number of computation errors, they differed significantly in oral reading errors. The study suggests that LD students may approach verbal math problem solving with less efficient methods for solution than NLD students and may experience deficits in reading and cognitive abilities which influence successful performance. It appears that computation disability may be only one area of concern in identifying mathematically learning disabled students.

Relationship of Reading and Verbal Math Problem Solving

The relationship between reading and verbal math problem solving has not been substantiated. Robinson (1973) and Knifong and Holton (1976) suggested that reading may not play as large a role in incorrect solutions as was assumed. In the later study (Knifong and Holton, 1976), 52% of the errors in word problems on the Metropolitan Achievement Test were attributed only to computational inaccuracies. Reading, along with many other factors, accounted for the remaining 48% of errors. Levy and Schenck (1981) investigated the interactive effect of arithmetic formats upon the verbal problem solving performance of 12 year old learning disabled youngsters and concluded that the learning

disabled children were able to read and perform mathematical computation at a level above which a verbal problem is written, but still had serious problems in reaching the solution.

In contrast, Lee (1979) found that reading achievement was a significant factor contributing to superior problem solving skills demonstrated by nonlearning disabled seventh graders. The results of these studies indicate that although reading may have some relationship to verbal math problem solving, other factors such as computational skills and strategy utilization may be more important.

Developmental Levels and Problem Solving

Developmental level appears to be related to problem solving performance in disabled learners. Grady (1975) found that average students functioning at Piaget's formal-operational level used more means-end heuristics (techniques) than concrete-operational students. A supporting study (Days et al., 1979) indicated that problem solving processes available to formal students may not be available to concrete students. The findings of Moroz (1978) suggested a developmental delay in learning disabled children (ages 7-12) in memory organization and developmental level when solving verbal math problems.

Skrtic (1980) supported Moroz's hypothesis with results that suggest that learning disabled junior high students are functioning at a concrete operations stage of Piaget's developmental sequence. This level of reasoning may be a learner characteristic which precludes successful grade level math performance. The study explored the relationship between the variability on the Classroom Test of Formal

Reasoning (CTFR) and variances associated with specific independent variables including sex and subtest scores from the Woodcock-Johnson Psychoeducational Battery (WJPB). A significant difference was found between the level of formal reasoning attained by learning disabled (LD) and nonlearning disabled (NLD) junior high students as indicated by their scores on the CTFR. Although mixed results were found in the attempt to identify mathematical deficiencies in ability and performance in the LD sample, the LD group performed significantly less well than the NLD group on five of the seven mathematics aptitude and achievement subtests on the WJPB (i.e., Visual Matching, Antonyms-Synonyms, Quantitative Concepts, Calculation, and Application Problems). The LD groups maintained this trend toward lower performance on the other two subtests (i.e., Analysis-Synthesis and Concept Formation) of the WJPB. Evidence of a substantial relationship between level of formal reasoning and mathematics aptitude and achievement was found. One of the implications of the findings is that "concrete and graphic modes of developmental mathematics instruction may be essential to efficient learning for LD students across grade levels" (Skrtic, 1980, p. 24).

Instructional Studies in Verbal Math Problem Solving

The mathematically learning disabled may require more extensive and more appropriate developmental instruction in math problem solving. Appropriate instructional programming for math problem solving is predicated on the identification of children who demonstrate a verbal math problem solving deficit. Few data-based studies address developmental

or remedial teaching methods to improve verbal math problem solving although a need exists (Halpern, 1981; McLeod and Armstrong, 1982).

Cawley and his colleagues (1979, p. 41) have developed an instructional program based on "the role of divergent problem solving activities in the educational experiences of the learning disabled child." They have created an interdisciplinary approach to the teaching of verbal math problem solving by utilizing iconic and inductive strategies and by incorporating activities designed to train cognitive processing skills, mathematical reasoning, reading, and language. Data are needed before any conclusions regarding the effectiveness of this approach can be reached.

Data are also required to determine the efficacy of an approach by Dunlap and House (1976) who advocated task analysis as an approach to teaching mathematical skills. In this instructional model, established criteria for evaluation of prerequisite skills on three levels provided a measure of a child's abilities. These levels are the enactive (acting out the answer with manipulatives), the iconic (drawing a representation of the solution), and the symbolic (actual mathematical computation). In the design of instructional programs for learning disabled students, it may be feasible to determine at which of these developmental levels the child has difficulty and progress from that point. Another possibility is to design a teaching program to assist the student with information organization by incorporating the strategies which are mental age appropriate.

In a clinical study (Blankenship and Lovitt, 1976), seven boys (ages 9-12) were taught techniques of rereading, writing, and

reexamining solutions to verbal math problems. The subjects had normal IQ's but were at least one year behind in reading and math. Results indicated evidence of a "rote computational" habit and trouble with extraneous information. They suggested the use of a systematic method of instruction with varying classes of problems.

Using the Learning Strategies Model proposed by Alley and Deshler (1979), Smith and Alley (1981) sought to find whether or not sixth grade students with learning disabilities could learn a verbal math problem solving strategy and apply it effectively. A multiple baseline design was utilized with three students who were taught to verbally rehearse the strategy until 100% criterion was reached. After verbalizing the strategy, each student applied the strategy to eight one-step verbal math problems. The results indicated an increase in percentage correct for each of three students after intervention and a generalization to grade level materials after learning the strategy with controlled level materials.

The little research that has been conducted in the area of verbal math problem solving with learning disabled students suggests that consideration must be given to appropriate developmental sequencing, task analysis, and cognitive strategies when designing instructional programs. Lee and Hudson (1981) noted four areas to be addressed when designing such programs. These included (a) understanding of the reading skills specific to verbal arithmetic problems, (b) determination of the correct process(es) to be utilized, (c) estimation of the answer, and (d) calculation.

To reiterate, verbal math problem solving is an important area of problem solving behavior for children in the educational setting and for the adolescent who is expected to acquire requisite life skills to function in everyday society. Studies with learning disabled students indicate that some students have a deficit in verbal math problem solving, and that this deficit is related to not only generalized reading abilities and computational skills but also to the level or stage of reasoning. Instructional research does hold promise for remediating verbal math problem solving deficits but more systematic instructional techniques need to be developed and/or validated.

Maintenance and Generalization of Strategies

Maintenance and generalization of learned strategies by learning disabled children are major factors in successful academic performance. Generalizability refers to the number of outcome domains and situations affected; durability refers to the maintenance of effects over time (Keogh and Glover, 1980). The difficulty that learning disabled youngsters experience in generalizing from one set of responses to another has been recognized in the literature (McLeskey, Rieth, and Polsgrove, 1980).

Producing nonspecific transfer of heuristic methods from miscellaneous training tasks to problems from disciplines such as mathematics raises questions as to how general or how specific the heuristic should be (Kilpatrick, 1969). Wilson (1967), in a study using normal subjects, predicted that subjects taught specific heuristics would perform better on training tasks but worse on transfer tasks than would

subjects taught general heuristics. Significant interactions suggested that a combination of heuristics during training facilitated performance on some of the transfer tasks and that general heuristics learned in the first training task were practiced on the second task, thereby facilitating transfer.

Using a clustering strategy with 9 to 13 year old learning disabled students, Hall (1979) found considerable evidence to support maintenance and generalization of a problem solving strategy. The learning disabled children in the sample were given appropriate, elaborate, and contextually relevant information which facilitated and improved their overall level of performance. Lloyd, Saltzman, and Kauffman (1981) drew two conclusions regarding generalization of skills by four learning disabled boys on basic multiplication and division facts. They found that generalization across tasks can be predicted when preskills and strategy training are provided and that rapid acquisition of related learning can be obtained by teaching a related strategy.

In a review of literature, Keogh and Barkett (1979) concluded that cognitive training appeared to offer a great possibility of transfer or generalization. Brown and Palinscar's review (1982, p. 14) indicated that "the results of training programs such as those used by Brown et al. (1979), Day (1980), Palinscar and Brown (1981) and Paris et al. (1981) suggest that combined packages that include metacognitive supplements to strategy training, either informed training or self-control training or both, result in satisfactory maintenance and generalization." They specify that concentration on self-questioning,

comprehension-inducing strategies that are of general use in a variety of settings is one way of avoiding the problem of transfer to everyday learning activities.

Strategies for training generalization (McLeskey et al., 1980) include modeling and prompting, self-control, and curriculum sequencing. Deshler et al. (1981) stressed the need to systematically program and instruct for generalization. Their list of techniques, adapted from Stokes and Baer (cited in Deshler et al., 1981), included (1) teaching learning disabled students to cue others for reinforcement, (2) using sufficient and diverse exemplars, (3) training loosely which is a tactic whereby training is conducted with relatively little control over the stimuli or the responses, (4) using intermittent reinforcement, and (5) telling students to generalize.

Deshler et al. (1980) investigated whether or not learning disabled adolescents can acquire and generalize their use of learning strategies using an eight-step instructional sequence. Six learning strategies were taught using a variety of multiple baseline designs. The six strategies were self-questioning, visual imagery, paragraph writing, monitoring writing errors, multipass, and a listening/notetaking strategy. Instruction on each strategy followed a standardized format including testing the student's current skill, description of the strategy, modeling the strategy for the student, verbal rehearsal, skill practice in ability level materials, and feedback. Major findings indicated that (1) learning disabled students can generalize the use of the strategies to tasks not previously practiced and (2) learning disabled students taught a reading strategy in ability level

materials can generalize their use of the strategy to grade level reading materials. Extending this same approach to verbal mathematical problem solving, Smith and Alley (1981) found that three sixth grade learning disabled students generalized the use of a learned strategy to grade level problems on a posttest which resulted in maintenance of the increased correct responses.

The importance of generalization and maintenance of learned strategies across settings and situations in academic environments cannot be underestimated. Successful academic performance of learning disabled adolescents in secondary mainstream classes is, after all, the goal of resource intervention programs.

Summary

The primary aim of this review was to consider research relevant to verbal math problem solving and learning disabilities. The implications of general problem solving and cognitive psychology provide the foundation upon which to view verbal math problem solving as it directly relates to both mathematics and learning disabilities.

Although general problem solving research has contributed to the development of strategies for problem solving, it is not considered directly applicable to classroom instruction. Many of the studies conducted, though, have influenced researchers in mathematics education who advocate cognitive strategy training as an instructional approach for teaching verbal math problem solving. These researchers have investigated learner characteristics and their importance to successful verbal math problem solving. Several factors including IQ, reading

level, computational skills, and developmental level have been identified as having some effect on verbal math problem solving performance.

Research in the field of learning disabilities has characterized learning disabled students as often experiencing difficulties in such cognitive activities as acquiring, selecting, and applying strategies appropriate to the task. This deficiency is evident in studies that specifically focus on verbal math problem solving. While there is a body of research that supports the use of cognitive strategies with learning disabled students, few studies have specifically addressed cognitive strategy training for verbal math problem solving. There is a need to empirically test instructional programs that emphasize cognitive strategy training for verbal math problem solving. Furthermore, maintenance and generalization of learned strategies are important considerations in the design of such programs. This study attempts to address this need.

CHAPTER 3

DESIGN OF THE STUDY

A multiple baseline design across individuals was selected for this intervention study. After establishing baselines for each subject, staggered application of the independent variable (verbal math problem solving strategy) across each set of three subjects permitted demonstration of the reliability of the dependent variable (tests of ten two-step verbal math problems).

Two experimental sets of three subjects each were included. Rather than extending treatment sequentially over all six subjects, two sets of three served as replications of the treatment effect. Time constraints and the unfeasibility of administering so many baseline tests led the researcher to limit extensions of baselines to three subjects. An intervention effect was demonstrated across subjects by showing that change in test performance accompanied introduction of the intervention at different points in time.

The experimental phases included (a) baseline, (b) treatment consisting of strategy training, (c) generalization to more complex verbal math problems, (d) maintenance of the learned strategy over time, and (e) retraining for subjects who did not reach criterion established for maintenance.

Subjects

Six adolescent students between the ages of 15 and 19 who had been diagnosed as learning disabled according to the criteria set by Sunnyside School District for purposes of diagnosis and placement were selected for the study. The experimenter first selected a population of students who were scheduled for small-group remediation classes and whose full-scale IQ scores were between 85-115 on the Wechsler Intelligence Scale for Children-Revised (WISC-R) (Wechsler, 1974) or the Wechsler Adult Intelligence Scale (WAIS) (Wechsler, 1955). Approximately 25 students met the two criteria. The researcher had established several other criteria for participation. These included a scaled score on the arithmetic subtest of the WISC-R or WAIS of at least one standard deviation below the mean, and a reading level of at least fourth grade as determined by the reading achievement cluster of the Woodcock-Johnson Psychoeducational Battery (WJPB) (Woodcock and Johnson, 1977). Other criteria included low math achievement as indicated by at least a three and one-half year discrepancy between grade equivalent score and grade placement on the math achievement cluster of the WJPB, and adequate computation of the four basic operations as observed during the administration of the calculation subtest of the WJPB. In addition, each student scored 40% or less on a measure of ten randomly selected two-step verbal math problems. Only eight of the 25 students met all criteria. Of these, one student refused to participate, and one had a schedule change that precluded participation.

The age range for the six remaining subjects (five males and one female) was 15-6 to 18-2. The subjects had IQ's ranging from

85-102 as measured by the WISC-R or WAIS. The reading cluster scores ranged from a grade equivalent of 4.9 to 11.4, and math cluster scores ranged from 5.4 to 7.2 as measured by the WJPB. Relevant subject data are listed in Table 1. Subjects were randomly assigned to groups and positions within the two groups.

The University of Arizona policy on human subjects was followed. Permission for conducting the experiment was obtained from the Director of Special Education of Sunnyside School District, the principal of Sunnyside High School, and the human subjects policy board of the district. Written parental and student approval for participation in the study was obtained.

Strategy Trainer

The strategy trainer was the researcher, a certified learning disabilities specialist. The researcher was employed as a learning disabilities specialist in Sunnyside School District and had been teaching in the district for six years. Consequently, this applied research was designed and conducted by the researcher who was the students' resource teacher. The researcher was responsible for the training and testing of the students.

Materials

Materials for the study included an outline and wall chart for strategy training, 18 tests of the two-step verbal math problem variety, one test of the three-step verbal math problem variety, practice problems of each type, six individual graphs for recording

Table 1. Subject variable data

Variable	Subjects						Mean
	1	2	3	4	5	6	
Age	15-11	16-6	16-8	16-10	15-6	18-2	16-7
Sex	M	M	F	M	M	M	
Grade	10.1	11.1	11.1	11.1	10.1	12.1	10.9
WJPB							
Reading Grade Score	9.1	6.2	6.8	4.9	6.0	11.4	7.4
Reading Standard Score	95	85	87	79	86	95	87.8
Math Grade Score	6.4	6.4	6.0	7.2	5.4	6.2	6.3
Math Standard Score	81	75	72	82	75	72	76.2
WAIS or WISC-R							
Verbal Scale Score	92	90	77	72	106	84	86.8
Performance Scale Score	111	112	102	104	86	108	103.8
Full Scale Score	102	101	86	85	95	94	93.8
Arithmetic Subtest	5	7	5	7	7	6	6.2

test scores, two group graphs for recording test scores, and two group graphs for recording the time taken to complete the tests.

Strategy Materials

A typed outline of the learning strategy as it appears in Appendix A and a wall chart listing the steps of the strategy as it appears in Appendix A were utilized.

Dependent Measure Materials

To form a verbal math problem pool, 200 verbal math problems of the two-step variety including any combination of the four basic mathematical operations, i.e., addition, subtraction, multiplication, and division, and 50 verbal math problems of the three-step variety involving any combination of the four basic operations were selected from secondary level general math textbooks available in the district (Bolster and Woodburn, 1977; Bolster, Woodburn, and Gipson, 1983; Clark et al., 1982; Fairbank and Schultheis, 1980; Geier and Lamm, 1978; Gerardi, Jones, and Foster, 1982, 1983; Lankford, 1981; Price et al., 1982; Shaw, Kane, and Merseeth, 1982; Stein, 1974). Ten items were selected randomly from the two-step problem pool for each test measure including the subject selection measure and the dependent measures during baseline, treatment, maintenance, and retraining phases of the experiment. Ten items were selected randomly from the three-step problem pool for the generalization test. Items were not returned to the pool.

Each verbal math problem was assigned a number. Ordering of test problems for the dependent variable depended on random drawing of the numbers. Random drawing also determined the order of the verbal

math problems utilized during the strategy acquisition and strategy application practice sessions. Each problem was placed on a half sheet of unlined paper 8 1/2 by 11 inches. Work space was provided. An example of the test format is presented in Appendix B.

Social validation of the verbal math problems was established. Four math teachers at Sunnyside High School each were given 20 randomly selected problems. Each teacher completed a questionnaire regarding similarity of the problems to actual textbook problems encountered in the classroom texts. Approximately 60 verbal math problems were reviewed by the participants in the social validation survey. Results of the survey indicated that the problems were representative of textbook verbal math problems. Representativeness of general math textbook problems was confirmed with 100% agreement among the participants.

Graphs

Individual graphs and group graphs for recording the data were maintained throughout the experiment by the investigator. The test scores were plotted on group and individual graphs that were constructed with the possible exam scores of 0-10 placed on the Y axis and the numbered test sessions on the X axis of the graph. The time taken to complete the tests was recorded on two group graphs. These were constructed with the possible time of 0-100 minutes on the Y axis and the numbered test sessions on the X axis.

Procedures

All of the participants received strategy training in verbal math problem solving in a learning disabilities resource setting during

regular school hours at Sunnyside High School. Both instruction and testing occurred during the students' regularly scheduled 50 minute resource classes. Experimental conditions included baseline, treatment, generalization, maintenance, and retraining when necessary.

Baseline Condition

Subjects were notified of their participation in the study prior to collection of baseline data. Baseline data were collected over consecutive days. Each subject was presented with a test consisting of ten two-step verbal math problems at the beginning of the 50 minute class period. Although essentially an untimed test, the experimenter recorded the time required for completion of the test by each subject. Subjects were informed of this variable but were told that the amount of time taken did not influence their scores. The subjects were told to turn in the test upon completion or at the end of class. Unfinished tests were completed on the following day. Each subject was instructed to complete the ten problems and to show all work. The student was informed that aid would be freely given by the experimenter in pronunciation and definition of vocabulary. However, the student had to ask for assistance. The students were instructed to circle their answers to the verbal math problems.

The test was scored by the examiner, and test results were made available to each student at the beginning of the first training session. All scores were determined by the number of correct answers. No partial credit was given.

Baseline data continued for all students until a stable baseline was evident for the first student in each group. Upon evidence of a

stable baseline or a descending trend for the first student in each of the two groups, intervention was introduced for the first student. Baseline data were collected continuously for the remaining two subjects in each group.

Treatment Condition

Treatment consisted of: (1) strategy acquisition training, (2) strategy application practice, and (3) testing sessions during which the dependent variable measures were collected. An explanation of the strategy is provided in this section.

Intervention for the first subject in each group was introduced after a stable baseline was established. Upon evidence of an upward trend and performance stability of at least seven correct responses on tests during dependent variable collection for the first subject in each group, strategy training was introduced for the second subject. During this time interval, baseline data collection continued for the third subject in each group. A similar procedure was followed for the third subject, i.e., strategy intervention was introduced after the second subject demonstrated stability.

Strategy Acquisition Training. At least three 50 minute training sessions were provided for the students to reach 100% criterion in verbalization of the strategy steps from memory. The strategy incorporated a combination of effective teaching techniques and recommended techniques for verbal math problem solving. The specific eight-step cognitive strategy utilized in this study was designed by the experimenter as an adaptation of the model proposed by Polya (1945), techniques

suggested by Kramer (1970), and a strategy developed by Smith and Alley (1981). The eight steps in the verbal math problem solving strategy are presented below:

1. Read the problem aloud. Ask the teacher to pronounce or define any word you do not know. (The teacher will pronounce and provide meanings for any words if the student asks.)

Example: In a high school there are 2878 male and 1943 female students enrolled. By how many students must the enrollment increase to make the enrollment 5000?

2. Paraphrase the problem aloud. State important information giving close attention to the numbers in the problem. Repeat the question part aloud. A self-questioning technique such as What is asked? or What am I looking for? is used to provide focus on the outcome.

Example: Altogether there are a certain number of kids in high school. There are 2878 boys and 1943 girls. The question is by how many students must the enrollment increase to make the total enrollment 5000. What is asked? How many more kids are needed to total 5000 in the school?

3. Visualize. Graphically display the information. Draw a representation of the problem.

Example:

4. State the problem. Complete the following statements aloud.

I have . . . I want to find Underline the important information in the problem.

Example: I have the number of boys and the number of girls who go to the school now. I want to find how many more kids it would take to make 5000.

5. Hypothesize. Complete the following statements aloud. If I . . . then . . . how many steps will I use to find the answer? Write the operation signs.

Example: If I add 2878 boys and 1943 girls, I'll get the number of kids now. Then I must subtract that number from 5000 to find out how many more must enroll. First add, then subtract. + - This is a two-step problem.

6. Estimate. Write the estimates. My answer should be around . . . or about (The skills of rounding and estimating answers should be reinforced at this step.) Underline the estimate.

Example: 2800 and 2000 are 4800. 4800 from 5000 is 200. My answer should be around 200.

7. Calculate. Show the calculation and label the answer. Circle the answer. Use a self-questioning technique such as, Is this answer in the correct form? (Change from cent sign to dollar sign and decimal point should be reinforced when solving money problems. Correct labels for the problems should be reinforced.)

Example:	2878	5000
	+1943	-4821
	4821	<u>179 students</u>

8. Self-check. Refer to the problem and check every step to determine accuracy of operation(s) selected and correctness of response and solution. Check computation for accuracy. (Skills of checking computation will be reinforced at this step.) Use the self-questioning technique by asking if the answer makes sense.

The acquisition steps used to train the cognitive strategy incorporated all but one of the acquisition steps of the Learning Strategies Model (Deshler et al., 1981). The acquisition steps included (a) analysis of current learning habit, (b) description of the new strategy, (c) modeling of the new strategy, (d) verbal rehearsal of the strategy steps, (e) student practice in classroom materials (secondary general mathematics), and (f) corrective feedback that was ongoing throughout the acquisition period. Student practice in controlled materials prior to grade level materials was not deemed necessary since a minimum reading level of at least fourth grade had been established for each subject, and the student could ask the experimenter to pronounce and provide definitions for any unfamiliar words. Also included was the technique of self-questioning that has been determined to be a significant component of cognitive training packages (Meichenbaum, 1977). The acquisition steps were incorporated into the strategy acquisition training sessions. The acquisition training sessions are described as a step-by-step procedure. Figure 1 presents a summary of training procedures utilized by the trainer, and each training session is described in the following paragraphs.

The trainer follows the step-by-step procedure.

- Session 1:
- (a) Discuss baseline scores, subject's method of solving problems, and improvement as the goal of instruction.
 - (b) Describe format of training.
 - (c) Read strategy to subject and answer questions.
 - (d) Model use of strategy using example cited.
 - (e) Model use of strategy using a randomly selected problem.
 - (f) Provide corrective feedback while subject solves problem.
 - (g) Provide corrective feedback while subject verbally rehearses strategy.
- Session 2:
- (a) Review strategy steps.
 - (b) Solve randomly selected problem while subject verbalizes steps.
 - (c) Monitor subject's verbal rehearsal of strategy.
 - (d) Repeat b.
 - (e) Repeat c.
- Session 3:
- (a) Monitor subject's verbal rehearsal of strategy.
 - (b) Provide corrective feedback while subject verbalizes steps and solves randomly selected problem.
 - (c) Repeat a.
 - (d) Repeat b.
 - (e) Test student in verbalization of strategy steps from memory.

Figure 1. Summary of strategy acquisition training

Session 1. The cognitive strategy was introduced to the student using the following steps:

1. The student was told that instruction was designed to improve his/her verbal math problem solving performance. Scores received during baseline were reviewed with emphasis on improving those scores on subsequent tests. Each was asked to describe the process that he/she had used to solve the problems on the baseline tests. The student was told that the strategy to be learned would improve his/her problem solving performance.

2. The format of the training was described, i.e., verbalization of the strategy steps from memory to 100% criterion for approximately three daily training sessions. The experimenter explained that after reaching criterion on verbalization of the strategy steps, practice and testing sessions would commence.

3. The experimenter read the eight strategy steps to the student. Questions elicited by the student were answered, and the experimenter clarified any step of the strategy that the student did not understand. The vocabulary of the strategy was defined. The importance of each step in the process was emphasized.

4. The experimenter modeled the strategy using the example cited in this section.

5. The experimenter then modeled the strategy using a randomly selected problem. The experimenter vocalized the strategy steps and described her actions.

6. The student then utilized the strategy to solve one randomly selected problem and was provided with corrective feedback. The student was told to vocalize thoughts and describe actions.

7. Strategy steps were practiced aloud for the remainder of the session.

Session 2. The cognitive strategy was reviewed and verbally rehearsed using the following steps:

1. Steps in the strategy were reviewed and verbalized by the student.

2. The experimenter modeled a randomly selected problem. This time the student recited the strategy steps while solving the problem with the experimenter.

3. The student verbally rehearsed the steps without solving a problem. Corrective feedback was provided by the experimenter.

4. Step 2 was repeated.

5. Step 3 was repeated.

Session 3. The cognitive strategy was practiced, and verbal rehearsal was tested using the following steps:

1. The student verbally rehearsed the eight-step strategy.

2. The student then solved one problem with a step-by-step recitation of the strategy. Corrective feedback was provided.

3. Verbal rehearsal of the strategy was repeated.

4. If time permitted, the student solved another problem while reciting the steps.

5. The student was tested in verbalization of the steps during the final five minutes of the class period. (One or two more sessions would have been provided, as necessary, to ascertain performance to criterion on verbal rehearsal of the strategy.)

Strategy Application Practice. After reaching criterion in verbalization of the strategy, strategy application practice was initiated. On the day before each testing session, a practice session was held. The practice session included the following steps:

1. The student verbalized the strategy steps.
2. The student engaged in verbal math problem solving practice using the strategy. The chart and outline listing the steps were available during the practice. Ongoing corrective feedback to ensure correct utilization of the strategy was provided.

Practice sessions were discontinued after two or three test sessions if the student maintained a score of seven or better. If the student scored below seven, practice sessions were reinstated.

Test Sessions. The test sessions were used to collect the dependent variable. Each test consisting of ten two-step verbal math problems was completed by the student using the same procedures as during baseline condition. In addition, the student was cued to use the strategy. The chart and outline were unavailable. No feedback was given. The test was timed. Unfinished tests were completed on the next day. Scores achieved were reported to the student on the day following the completion of the test.

Data including the number of correct responses on each test of verbal math problem solving and the number of minutes required to complete each test were collected and plotted on the graphs. The criterion for improved performance was at least seven correct responses over four consecutive tests. The experimenter selected this criterion since it reflects successful performance in regular and special education secondary classrooms. Generally, 70% is considered a "C" or average grade. It also reflects the criterion level selected by other studies utilizing learning strategy intervention (Clark et al., 1981; Schumaker et al., 1981, 1982). Sustained performance of an average score or better over four consecutive tests indicated acceptable performance.

Generalization and Maintenance

A test consisting of ten three-step verbal math problems was administered on the day following the final treatment test to determine generalization of the strategy to more complex verbal math problems. A criterion of five correct responses had been established as the level of acceptability.

Two weeks subsequent to generalization testing, a test consisting of ten two-step verbal math problems was administered to each subject to determine level of maintenance. If the subject failed to achieve at least seven correct answers on the maintenance test, the subject was retrained to 100% criterion on verbalization of the strategy steps and was retested. If the subject did not obtain a score of at least seven, application practice sessions were held. When a score of seven or better was achieved on a single test of ten two-step problems, this phase of the experiment was terminated.

Analysis of the Data

To determine interscorer reliability, 10% of the total tests were independently scored by a certified learning disabilities specialist employed by the district as well as the experimenter. Interscorer reliability was 1.00.

The following questions were considered when analyzing the data:

1. Will the strategy used for verbal math problem solving improve the test performance on two-step verbal math problems to a criterion of at least seven correct responses maintained over four consecutive tests by learning disabled adolescents placed in grades 10-12?
2. Will the improved performance be maintained by the students at a criterion of at least seven correct responses after a two week elapsed period during which no instruction or testing occurred?
3. Does generalization of the strategy to three-step verbal math problems occur as measured by a performance criterion of at least five correct responses?
4. How does the time required to complete the verbal math problem solving tests vary across and within the experimental conditions?

The questions were answered by graphically recording the number of correct responses on each test of verbal math problem solving and graphically recording the time required to complete each test during all phases of the experimental design. A line graph representing continuous data points was developed for each individual to demonstrate levels of change in baseline and treatment conditions. Two group graphs, one for each set of subjects, reflecting data collected on

test performance during testing sessions were developed to demonstrate reliability of treatment effect.

Treatment effect was determined by a visual analysis of the data. Data were graphically recorded for the tests of maintenance and generalization. Discussions are based on the graphic data that reflect an individual's performance. Error analysis of the final three verbal math problem tests for all subjects to detect patterns of response was conducted and is discussed.

Design Strengths and Limitations

The multiple baseline design demonstrates the effect of an intervention by showing that behavior change accompanies introduction of the intervention at different points in time (Kazdin, 1978). It allows stronger inference for experimental control than the traditional ABAB design since sequential introduction of interventions helps eliminate historical invalidating influences (Kazdin and Kopel, 1975). Successful demonstration of an intervention across subjects also helps to establish the external validity of research results (Kratochwill, 1978). The multiple baseline design is not limited by two major restrictions of the ABAB design, the irreversibility of behavior and the undesirability of reversing behavior. Other advantages of the multiple baseline design include the sensitivity to individual differences and the ability to make detailed observations of the effects of intervention as a result of the number of data points.

Kratochwill (1978), in discussing several limitations of the design, suggested that (1) inference may be weaker since replications

occur on different subjects, (2) a minimum of three replications must be conducted (a condition which was met in this experiment), and (3) the more replications across subjects, the greater likelihood that effect is due to intervention. Furthermore, no inferences about experimental effect can be drawn for populations other than the subjects involved in the study. In addition, any generalization made must consider the ordering and presentation of the verbal math problems and the conditions of the experiment.

CHAPTER 4

RESULTS

This chapter presents the performance results for six learning disabled adolescents using a verbal math problem solving cognitive strategy. Methodology consisted of a multiple baseline design with the dependent variable collected during baseline, treatment, generalization, maintenance, and, if necessary, retraining conditions.

Data are presented on four line graphs. Figures 2 and 3 display scores received on tests of verbal math problem solving. These figures include: (a) an X axis representing the numbered test sessions, (b) a Y axis representing the ten possible correct answers on each test, and (c) data points representing performance on each test.

Figures 4 and 5 present the results for the time taken to complete the tests. These figures include: (a) an X axis representing the numbered test sessions, (b) a Y axis representing 100 minutes in units of ten, and (c) data points representing the number of minutes taken to complete each test.

Question 1: Effect of Cognitive Strategy Training

Data related to Question 1 demonstrate the effect of cognitive strategy training on verbal math problem solving performance of learning disabled adolescents.

Figure 2. Number of correct responses on verbal math problem solving tests for subject 1 (S1), subject 2 (S2), and subject 3 (S3)

Figure 3. Number of correct responses on verbal math problem solving tests for subject 4 (S4), subject 5 (S5), and subject 6 (S6)

Figure 4. Number of minutes required for verbal math problem solving test completion by subject 1 (S1), subject 2 (S2), and subject 3 (S3)

Figure 5. Number of minutes required for verbal math problem solving test completion by subject 4 (S4), subject 5 (S5), and subject 6 (S6)

Question 1: Will the strategy used for verbal math problem solving improve the test performance on two-step verbal math problems to a criterion of at least seven correct responses over four consecutive tests by learning disabled adolescents placed in grades 10-12?

Visual analysis of Figure 2 indicates that S1 and S2 made substantial progress after intervention with S3 making questionable progress.

S1 achieved a baseline mean score of 3.3 correct responses and a treatment score of 7.8 correct responses, an increase of approximately five correct responses. S1 required four practice sessions and six tests to reach criterion of seven correct responses over four tests. It is concluded that cognitive strategy training was effective with S1.

S2 achieved a baseline mean score of 3.3 correct responses and a treatment mean score of 8.2 demonstrating approximately the same level of increased performance as S1, five correct responses. S2 required five practice sessions and nine tests to reach the established criterion. Upon visual inspection, it is concluded that cognitive strategy training was effective with S2.

S3 achieved a baseline mean score of 4.9 correct responses. Although S3 met all criteria for inclusion in the study including a score of two correct responses on the pretest for the two-step verbal math problem solving, her baseline performance was somewhat higher than the other subjects in the study and reflected erratic performance. S3 obtained a mean treatment score of 6.7 correct responses, an increase of approximately two correct responses, not as substantial an increase as the other two subjects. Although S3 did meet the established

criterion of seven correct responses over four consecutive tests, the criterion for acceptable performance, visual inspection of the graph does not show a substantial gain between baseline and treatment. S3 required two practice sessions and six tests to reach the established criterion. It is concluded that cognitive strategy training was not effective for S3.

Visual analysis indicates that two subjects (S1 and S2) made substantial progress after cognitive strategy training. However, all three subjects met the criterion of improved performance by scoring seven correct responses over four consecutive test sessions.

Visual analysis of Figure 3 indicates that all three subjects (S4, S5, and S6) made substantial progress after intervention. However, only S4 and S6 met the criterion of improved performance.

S4 achieved a baseline mean score of two correct responses and a mean treatment score of 8.2 correct responses, an increase of approximately six correct responses. S4 received three practice sessions and required five tests to reach criterion. It is concluded that cognitive strategy training was effective with S4.

Visual inspection of the graph indicates that S5 did not have one correct response during baseline with a mean score of zero. During treatment, S5 achieved a mean score of six, an increase of six correct responses, showing substantial improvement. Although S5 did not reach the criterion established for satisfactory performance during treatment, his improvement of six points demonstrates a substantial increase in the number of correct responses. Treatment consisted of five practice sessions and eight tests. Treatment was discontinued for S5 as a result of

the winter vacation. It is concluded that, although S5 did not reach the established criterion, cognitive strategy training was effective.

S6 achieved a baseline mean score of 1.4 correct responses and a treatment mean score of 7.8 correct responses, an increase of approximately six correct responses. S6 received one practice session and four tests to reach criterion established for improved performance. It is concluded that cognitive strategy training was effective with S6.

Visual analysis indicates that all three subjects (S4, S5, and S6) made substantial progress after intervention.

Results of data collected in response to Question 1 indicate improved performance on tests of two-step verbal math problems for five of the six subjects. All subjects except S3 demonstrated an increase of approximately five to six correct responses during treatment. Overall, the results support the effectiveness of this eight-step strategy training for improved performance of learning disabled adolescents on two-step verbal math problems.

Question 2: Maintenance of Improved Performance

Data related to Question 2 demonstrate maintenance of improved performance on tests of two-step verbal math problems.

Question 2: Will the improved performance be maintained by the students at a criterion of at least seven correct responses after a two week elapsed period during which no instruction or testing occurred?

Figure 2 illustrates that both S1 and S3 received scores of eight correct responses on the maintenance test, thereby meeting criterion. S2 had a score of five and did not maintain improved

performance. Further intervention in the form of a practice session was provided for S2. S2 scored nine correct responses on the subsequent test, thereby exceeding the criterion of at least seven correct responses on a single test following retraining.

Figure 3 illustrates that S4 and S6 both met criterion for maintenance by receiving scores of eight correct on the maintenance test. S5, with a score of five, did not reach the criterion established for maintenance although he did demonstrate improvement when compared to a baseline mean of zero. To reiterate, S5 was the only subject who did not reach criterion during treatment. Rather than reinstate treatment following the two week elapsed period (the winter break), the experimenter chose to administer the maintenance test since S5 had demonstrated a marked increase in the number of correct responses during treatment. S5's score on the maintenance test was five. Further intervention in the form of two practice sessions and two test sessions yielded scores of five and nine, respectively, during retraining, thereby meeting the criterion for retraining.

In summary, four of the six subjects reached the criterion established for maintenance. S2 required one practice session followed by a test session to reach the retraining criterion; S5 required two practice and two test sessions to meet the retraining criterion.

Question 3: Generalization to Three-Step Problems

Data related to Question 3 demonstrate generalization of the strategy to three-step verbal math problems.

Question 3: Does generalization of the strategy to three-step verbal math problems occur as measured by a performance criterion of at least five correct responses?

Figure 2 illustrates that S1 and S2, both with five correct responses, reached criterion established for generalization of the strategy to more complex problems. S3, with a score of four, did not reach criterion.

Figure 3 illustrates that S4, with a score of five correct, and S6, with a score of six correct, generalized the strategy by reaching the criterion established. S5, who did not reach criterion during the treatment or maintenance conditions, again failed to reach criterion with a score of zero on the generalization test.

In summary, four of the six subjects generalized the strategy to three-step verbal math problems as measured by a criterion of at least five correct.

Question 4: Variability of Test Time

Considerable variability of the time required to solve verbal math problems over all conditions is evidenced by the data represented in Figures 4 and 5. These data relate to Question 4.

Question 4: How does the time required to complete the verbal math problem solving tests vary across and within the experimental conditions?

Figure 4 illustrates the time required for S1, S2, and S3. S1 required a baseline mean time of 19.3 minutes and a mean time of 50.2 minutes during treatment, an increase of approximately 31 minutes. S1

required 45 minutes for the generalization test and 30 minutes for the maintenance test. S2, with a baseline mean time of 22.3 minutes, increased to a mean time of 36.8 minutes during treatment, an increase of approximately 14 minutes. The time required for the generalization test was 31 minutes, for the maintenance test, 30 minutes, and for the retraining test, 37 minutes. S3 required a baseline mean time of 28.3 minutes and a treatment mean time of 39.3 minutes, an increase of approximately 11 minutes during treatment. The generalization test took 37 minutes, and the maintenance test, 33 minutes.

Figure 5 illustrates the variability of time required for S4, S5, and S6. S4 required a baseline mean time of 23 minutes and a mean time of 44.6 minutes during treatment, an increase of approximately 21 minutes. S4 required 61 minutes for the generalization test and 23 minutes for the maintenance test. S5, with a baseline mean time of 13.7 minutes, increased to a mean time of 52 minutes during treatment, an increase of approximately 39 minutes. The time required for the generalization test was 46 minutes, for the maintenance test, 15 minutes, for the first retraining test, 40 minutes, and for the second retraining test, 31 minutes. S6 required a baseline mean time of 34.6 minutes and a treatment mean time of 61.5 minutes, an increase of approximately 27 minutes. The generalization test lasted 37 minutes while the maintenance test took 24 minutes.

In summary, all subjects demonstrated an increase in time required to complete the verbal math problem solving tests during treatment. Four subjects (S1, S4, S5, and S6) demonstrated a marked mean increase of approximately 20-40 minutes, while two subjects (S2 and S3)

demonstrated moderate increases of between 10 and 15 minutes. However, it must be noted that the initial one or two treatment tests required substantially more time than the remaining treatment tests. Thus, the mean score may be misleading. The mean increase does not appear as substantial if the average of minutes for completion of only the final two treatment tests is considered instead of the mean for all the treatment tests. The range for the average time taken to complete the final two treatment tests is 21-45.5 minutes as compared with the mean baseline range of 13.7-34.6 minutes, representing increases of between 8 and 13 minutes for all subjects.

Summary

A summary of the results obtained in relation to the research questions is provided in this section. The results included the following:

1. Five of the six subjects demonstrated a substantial (four to six correct responses) increase in the number of correct responses following cognitive strategy training, with five of the six subjects reaching the criterion established for improved performance.

2. Four subjects maintained improved performance as established. Retraining in strategy application for the remaining two subjects improved performance to nine correct responses (above criterion) on a single test. One subject required one retraining practice session; the other required two retraining practice sessions.

3. Four subjects demonstrated generalization of the strategy to more complex verbal math problems.

4. All six subjects demonstrated substantial increases in the amount of time required to complete the verbal math problem solving tests immediately following strategy acquisition training. A descending trend was noted for all subjects as treatment progressed. All subjects stabilized to a completion time of 50 minutes or less, a standard class period.

CHAPTER 5

CONCLUSIONS, DISCUSSION, AND IMPLICATIONS

Included in this chapter are an overview of the problem and procedures, a summary of results and conclusions, a discussion of the conclusions, implications of the study, and suggestions for future research.

Overview of the Problem and Procedures

Studies addressing mathematical disabilities, specifically verbal math problem solving, are becoming more evident in the literature. Methods of assessing and remediating mathematical problem solving deficits are currently under investigation in the fields of special education, regular education, and cognitive psychology. This study focused on the improvement of verbal mathematical problem solving for the learning disabled adolescent.

Alley and Deshler (1979) recently developed the Learning Strategies Model that emphasizes a cognitive approach to instruction. The acquisition of a cognitive strategy specifically addressing an identified deficit should facilitate the learning process for a learning disabled youngster. This study examined the effects of an eight-step cognitive strategy designed to facilitate verbal math problem solving for learning disabled students at the secondary level.

A multiple baseline across individuals design permitted demonstration of the effectiveness of the independent variable. Strategy training materials included an outline and wall chart listing the strategy steps. Tests of ten two-step verbal math problems were the dependent measures. Also included was a generalization test of ten three-step verbal math problems. Conditions of the experiment were baseline, treatment, generalization, maintenance, and, for two subjects, retraining. Baseline data were collected continuously over all subjects. Intervention was introduced for the first subject when a descending trend or stable baseline was evident. Intervention was staggered for the remaining individuals and was introduced upon evidence of an upward trend for the previous subject. Positive changes in the data series accompanied introduction of the intervention at different points in time for each individual.

Two sets of three subjects were included to provide for replication of the study. All subjects were adolescents identified as learning disabled according to criteria set by their school district. Each demonstrated average intelligence, a reading ability of at least fourth grade, a math ability at least three and one-half years below grade level, and low verbal math problem solving performance as indicated by a score on an informal measure. It is assumed that this sample of students represents a subgroup of learning disabled youth who are characterized by low verbal math problem solving.

Utilization of the strategy and improved performance were measured by scores on tests of two-step verbal math problems. The number of correct responses and the number of minutes taken to complete

each test were recorded on four group graphs. Two graphs reflected levels of change in the dependent variable for baseline, treatment, generalization, maintenance, and retraining conditions of the experiment. Variability of testing time was reflected by the data on the remaining two graphs. Transformation of data points into line graphs allowed for visual analysis of the data.

Summary of Results and Conclusions

Results of the experiment included the following:

1. Five of the six subjects demonstrated a substantial (four to six correct responses) increase in the number of correct responses following cognitive strategy training, with five of the six subjects reaching the criterion established for improved performance. Students' increases in mean performance from baseline to treatment ranged from two to six correct responses.
2. Four subjects maintained improved performance at a performance criterion of at least seven correct responses. Retraining in strategy application for the remaining two subjects improved performance to nine correct responses (above criterion) on a single test. One subject required one retraining practice session; the other required two retraining practice sessions.
3. Four subjects demonstrated generalization of the strategy to more complex verbal math problems by scoring at least five correct responses.
4. All six subjects demonstrated substantial increases in the amount of time required to complete the verbal math problem solving

tests immediately following strategy acquisition training. A descending trend was noted for all subjects as treatment progressed. All subjects stabilized to a completion time of 50 minutes or less, a standard class period.

Discussion

The results indicated that this eight-step cognitive strategy appears to be an effective intervention for verbal math problem solving for this sample of students who had deficits in verbal math problem solving. All subjects were able to verbalize the strategy from memory within three training sessions. All subjects increased the number of correct responses on verbal math problem solving tests following strategy acquisition training. Smith and Alley (1981) found similar results with three sixth graders who averaged an approximate 80% increase in solving one-step word problems after strategy training.

When designing cognitive strategy instruction for students, the teacher must consider both the cognitive and behavioral characteristics of students. Appropriate instructional components, performance patterns, and generalization and maintenance are also important issues. The following section discusses these considerations.

Student Characteristics

The six students selected for this study all had certain characteristics in common. All were classified as learning disabled, were attending a public high school, had average intelligence with at least fourth grade reading achievement, had basic knowledge of mathematical computation, and demonstrated a verbal math problem solving

deficit. Of the six subjects in the study, S5 was the single subject who failed to achieve criterion of at least seven correct responses over four consecutive tests for improved performance as well as failing to achieve criterion on the generalization and maintenance tests. Also, S5 was the only student who did not have one correct response during baseline condition. His strategy consisted of "just putting the numbers together and adding." This subject expressed frustration indicated by remarks such as "not another test" and "oh, Miss, can't I take that test tomorrow?" during baseline and treatment conditions. However, he did say that he was pleased with some of the scores during treatment. Both cognitive and behavioral characteristics may have influenced his performance and be relevant to the interpretation of the results for this study.

Cognitive Characteristics. The cognitive strategy for math problem solving utilized in this study may not be effective for all learning disabled students. Perhaps only students who demonstrate certain defined characteristics can benefit from this particular intervention. For example, five of the six subjects in this study had significantly higher performance than verbal scaled scores on the WISC-R or WAIS with the difference ranging from 19 to 32 points. S5, the only student who did not achieve criterion during all three conditions, had a verbal score which exceeded his performance score by 20 points. On this factor, S5 was different from the other, more successful, students in the study. One might hypothesize that the ability to acquire and select appropriate strategies could be different for

students with certain cognitive profiles and that, in the case of S5, modifications of the strategy may be necessary to ensure success.

Behavioral Characteristics. Motivational factors also influence performance. "Learned helplessness" is a term often used to explain poor motivation among learning disabled adolescents. Learned helplessness or the inability of individuals to view their own behavior as having no influence on consequent events, whether positive or negative (Grimes, 1981), may contribute to poor performance. In contrast to the other subjects, S5 often remarked that he "couldn't do these problems" even when he was successful. It appeared that he did not attribute successful performance to himself and lacked confidence in his ability. Awareness of improved performance due to strategy utilization may lead to increased self-esteem. When asked by the experimenter, all subjects said that they found the strategy useful and that they felt good when they scored well. Two students said that they were previously unable to solve math word problems and felt comfortable with the task after training. Behavioral characteristics such as low motivation or self-esteem should be considered when determining the appropriateness of intervention programs. Teachers need to be aware of the interaction of achievement and self-esteem. Successful cognitive strategy application could act as a reinforcer for some students but not others. If teachers can recognize the interaction of student differences with cognitive strategy training, then they can appropriately adjust programming.

Instructional Components

Providing the necessary time for strategy application practice is important in planning instructional programs. By staggering application of the independent variable, this design demonstrated that continued practice without strategy intervention, typical classroom procedure, does not result in improvement on verbal math problem solving tests. Extended drill and practice alone during baseline did not improve performance in this study. Several practice sessions also were given following strategy training for all students. These sessions when paired with strategy training improved performance. However, some students may require more or fewer practice sessions than others for success. Practical classroom application of this intervention would necessitate monitoring of a student's strategy utilization. Conceivably, at some point, students would no longer require feedback or reinforcement and would automatically cue themselves to use the strategy. Modifying the student's approach to verbal math problem solving by providing (a) a facilitating cognitive strategy, (b) direct instruction in the use of the learned strategy, (c) strategy application practice, (d) external cueing to use the strategy, and (e) corrective feedback improved students' performances.

Performance Patterns

Strategy Individualization. Each student in this study appeared to adapt the strategy for individual utilization. That is, fading of different strategy steps was evident for individual students. All students seemed to utilize all eight steps following strategy

acquisition, but, as testing progressed, became more selective in utilization. For example, S2 drew diagrams, hypothesized by showing operation symbols, estimated by writing the rounded numbers, and recorded the calculation check for only certain selected problems rather than all problems as testing progressed. The strategy appeared to have been adapted to fit the individual needs of the problem solver. Classroom teachers need to be aware of this individualized aspect of cognitive strategy utilization in order to tailor instruction for individual needs.

Time as a Variable. Time also was adjusted by the individual problem solver as testing progressed. The time taken to complete the tests increased following strategy acquisition training for all students. Four of the six students required more than one class period to complete the first test following acquisition training. All students demonstrated a return to a more acceptable completion time, i.e., within 50 minutes. This reduction may have been due to a decreased need to bring cognitive processes to a conscious level as strategy utilization became more automatic for each student. However, when compared to baseline, the final two treatment tests indicated a mild increase in time. Metacognitive awareness of the strategy utilization (Flavell, 1976) may allow students to become more reflective in their approach to problem solving and require more time. Also, increased time may be associated with awareness of process, greater care in solving problems, and a more organized approach to problem solving.

Error Patterns. Decreased time during later intervention test sessions did not appear to be associated with a greater number of errors. Error analysis of the final three treatment tests for all subjects revealed a variety of error types. Ten of the 40 errors or 25% were computation errors. The remaining 30 (75%) could be classified as process or operation errors. Only two errors appeared due to an incorrect choice of operations. Examples of other process or operation errors were reversal of operations, incorrect placement of numbers, omission of numbers, elimination of either step in a two-step problem, carelessness in placement of numbers in money problems, and errors in copying. Procedures to eliminate particular error types can be incorporated into instructional programs, thereby providing even greater verbal math problem solving success.

Generalization and Maintenance

Although cognitive intervention offers promise for generalizability and durability of behaviors (Keogh and Barkett, 1979), little research has been conducted to substantiate this belief.

A generalization test and a maintenance test were incorporated into the design of this study. Two of the six subjects failed to achieve the criterion of at least five correct responses established for the generalization test. The generalization test was administered on the day before a two-week winter vacation. Intraindividual factors such as anticipation of the vacation may have negatively influenced the outcome. Practice with a variety of problems and in a variety of situations might facilitate generalization. Instruction must consider

the student's ability to generalize. External cueing and internal cueing, as techniques, could help students know when to use the strategy. Direct instruction in generalization skills is essential for some learning disabled adolescents (Deshler et al., 1981).

Greater emphasis on certain steps in the problem solving process is recommended. One hundred percent of S3's errors and 60% of S5's errors were due to incorrect process(es) selection on the generalization test. Hypothesizing is the critical step necessary for correct process selection. Certain steps may need to be reinforced intermittently to provide focus on problem areas for certain students. Some students probably require direct instruction and practice before success on more difficult problems can be demonstrated. Task specific instruction, e.g., practice in three-step verbal math problems, might be required. Practice in determining the number of steps necessary to solve a variety of mixed problems could positively affect generalization and maintenance of the strategy.

For those students who demonstrate a decline in performance over time or a lack of maintenance of the acquired problem solving skill, one or two practice sessions may be sufficient to improve performance to criterion after a cognitive strategy has been learned and utilized successfully. Fleischner (1984) advocated teacher-directed instruction, established performance criteria, and distributed practice as instructional procedures to effect positive changes in verbal math problem solving performance. Distributed practice might possibly eliminate maintenance problems.

In summary, this study found several influential factors that must be considered when designing cognitive strategy training programs to teach verbal math problem solving. These include (a) the cognitive and behavioral characteristics of students, (b) instructional components, (c) performance as a function of the individual learner, problem solving time, and error habits, and (d) procedures to promote generalization and maintenance of the strategy.

Limitations

Traditionally, a major limitation of single subject research has been that findings may not be generalizable to individuals outside of the study. However, Kazdin (1982) noted that direct replication of an experiment, i.e., applying the same procedures across a number of different subjects, strengthens external validity. Since this study included a direct replication in its design, treatment effects may be generalizable to individuals with characteristics that are similar to those of the subjects in the study.

Any generalization made must consider the ordering and presentation of the verbal math problems and the conditions of the experiment such as the setting, time of day, and presentation. The multiple baseline design, in fact, may not be the optimal choice of design since the baseline condition can be quite extensive. The task may become aversive or frustrating, factors that can influence results.

Since this experiment utilized a cognitive strategy package consisting of eight separate components, it is difficult to determine which of the independent variable components are responsible for the

outcome. Extensions of the present study should address this limitation. Systematic replication that focuses on parceling out the critical factors is recommended.

Finally, durability of the learned strategy and generalization to other settings and situations must be tested more thoroughly to provide stronger conclusions.

Implications

The conclusions of this study have implications for the individual learning disabled student and the secondary programs that serve learning disabled adolescents.

This study indicates that students who demonstrate behaviors and characteristics similar to those students in the study may effectively utilize a strategy approach to learning verbal math problem solving. The strategy employed in this study can be an effective learning tool for the adolescent student who exhibits poor verbal math problem solving performance. Unlike most research, this study represented applied research and was conducted by the researcher (the students' regular learning disabilities teacher) during the students' regularly scheduled learning disabilities resource classes. The significance of this type of research is that results are directly applicable to classroom instruction. Teachers can duplicate instructional procedures with greater confidence that instruction will improve their students' achievement.

Matching the aptitude of the individual with the appropriate treatment is the primary goal of special education programming. More

sophisticated assessment devices to measure verbal math problem solving are needed to allow for this match. Many standardized tests that purport to measure this basic skill actually measure other skills as well. Prior knowledge (i.e., the number of inches in a foot), and quantitative concepts are often required for successful performance. Diagrams and picture problems interspersed with written word problems on certain tests may confound a verbal math problem solving score. Often scores on calculation are averaged with problem solving scores and are interpreted as a mathematics achievement score. Further definition of learners who manifest a verbal math problem solving deficit is dependent on the development of reliable and valid assessment instruments.

Cognitive strategy training as exemplified in this experiment could satisfy the programming needs of certain adolescents at the secondary level. If students are able to maintain a learned strategy and generalize its use to regular textbooks and classroom, then regular classroom teachers should react positively to this type of instruction. If students can learn to cue themselves in the use of the strategy, then they should become more independent learners and experience greater success in school.

Remediation models and resource programs primarily at the secondary level may be impacted as a result of this study and others that have focused on the application of learned strategies in applied settings. There is enough support of a learning strategies approach for teacher training programs to address such alternative methods for serving learning disabled adolescents in secondary schools. Teaching methodology, techniques and program design at this level should be

modified to include cognitive strategy training for those students who will benefit. Academic, vocational, and social opportunities in the mainstream may increase as a result.

In conclusion, this study adds to a growing body of literature suggesting that learning disabled adolescents at the secondary level could be served better as a result of this alternative instructional approach, cognitive strategy intervention.

Recommendations for Future Research

Future empirical studies in cognitive strategy training for verbal math problem solving should focus on generalization studies. Two additional areas for research include (1) maintenance, and (2) strategy component analysis.

Generalization

In this study, the students successfully learned to solve two-step verbal math problems with generalization to three-step problems for some students. Although all problems were taken directly from secondary general mathematics textbooks, the study was conducted in a resource setting. Future research studies should address the following four areas for generalization.

1. Levels of verbal math problem complexity should be researched with emphasis on generalization of the cognitive strategy to more complex problems, i.e., problems that require an increased number of steps for solution. Distributed practice in deciding the necessary number of steps for problem solution should facilitate generalization to problems of varying complexity.

2. Research studies addressing strategy generalization to textbooks found within resource and regular classroom settings are essential.

3. Strategy generalization to math classes for regular and mathematically disabled students is an important concern. Group research could offer evidence of the applicability of group cognitive strategy training for verbal math problem solving to regular and disabled students within the mainstream and further delineate the characteristics of students who do and do not benefit from cognitive strategy intervention.

4. Future research studies should address strategy generalization to higher level mathematics, i.e., algebra and geometry.

Maintenance

This study tested the maintenance of an eight-step cognitive strategy after a two-week period during which no instruction or formal practice occurred. Future studies utilizing strategy training should address long-term as well as short-term maintenance. Procedures for teaching students to cue themselves in the selection and utilization of strategies appropriate to the task may facilitate maintenance.

Strategy Component Analysis

This study demonstrated that a cognitive strategy package can facilitate verbal math problem solving for the learning disabled adolescent. However, it appeared that of the eight steps, certain ones, i.e., diagramming and estimating, faded more quickly than others. Hypothesizing, which was indicated by writing the signs for the

selected operations, was utilized throughout treatment by half of the subjects. Future studies should systematically investigate which components are most salient for successful performance, thereby validating a strategy that is not only effective but also efficient.

APPENDIX A

STEPS IN VERBAL MATH PROBLEM SOLVING

1. READ THE PROBLEM ALOUD
2. PARAPHRASE
 - Give important information.
 - Repeat question aloud.
 - What is asked? What am I looking for?
3. VISUALIZE
 - Draw a diagram.
4. STATE THE PROBLEM
 - I have . . . I want to find . . .
5. HYPOTHESIZE
 - If I . . . then . . .
 - How many steps?
6. ESTIMATE
 - Round the numbers.
7. CALCULATE
 - Label.
 - Circle.
8. SELF-CHECK
 - Check every step.
 - Check calculation.
 - Does the answer make sense?

APPENDIX B

VERBAL MATH PROBLEM SOLVING TEST FORMAT

James is making a deposit at his bank. He has checks for \$237.31 and \$176.15. He wants to keep \$50 in cash. What was his net deposit?

David's brother wants to buy an automobile costing \$4000. If he pays \$500 in cash, he could pay off the balance in 36 monthly installments of \$113 each. How much does he end up paying for the car?

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