A PARAMETRIC MODEL CALIBRATED WITH A PHYSICALLY
BASED MODEL FOR RUNOFF PREDICTION
FROM UNGAGED STREAMS

by

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ABSTRACT

Recent developments in the numerical solution of the governing partial differential equations for overland and channel flow (represented by sets of intersecting planes) should make possible models which predict runoff from ungaged streams. However, these models (physically based models) are complex and require much computer time. Parametric models exist which have the advantage of being relatively simple, and once calibrated may be used efficiently and inexpensively. This study developed a procedure for calibrating a parametric model against a physically based model utilizing base areas of one acre and one square mile, with the expectation that base areas can be combined to model real watersheds.

Simulation experiments with the physically based model showed that, for the one-acre base area, the dominant parameters relate to the slope and friction of the planes, whereas for the square-mile area, the dominant parameters relate to the channel properties. The simple model-fitting parameters were the cell storage ratio, K , for the one-acre area, and K plus a lag factor, L , for the square-mile area. These parameters decreased more or less exponentially with increasing rainfall intensity and with changes in the physical parameters that produced high peaks and fast runoff.

CHAPTER 1

INTRODUCTION

Predictions of runoff are required for many aspects of planning, design, and management of water resource systems. For gaged watersheds, runoff can be accurately measured and records maintained. Most small, and many large, watersheds are ungaged and information about runoff due to a given precipitation input must be estimated. In most cases, such estimates are based on comparisons with records of gaged watersheds. In recent years, modeling techniques offer an alternatives means of obtaining such estimates.

Models of the hydrologic system may be divided into two general categories: physically based models and input-output (I-O) parametric models. Physically based models are those which attempt to simulate the complex physical processes occurring on a watershed. These models are based on the numerical solutions of the governing kinematic, nonlinear, partial differential equations describing overland and open channel flow.

A parametric model is one which ignores the complex, intermediate processes occurring on a watershed and simply modifies an input signal into the output runoff by means of a transfer relation (Porter and McMahon, 1971). Such models are empirically developed for a particular system by calibrating the model with known relations of input to output for that system.

Models of each group have particular advantages and disadvantages. Physically based models better represent the real system than do parametric models. If one has confidence in the construction of such a model for a given ungaged watershed, then calibration of the model is not required in order to make good predictions of runoff from that watershed. However, the physically based runoff model is a recent innovation in the field of hydrology and work has yet to be completed that would establish such a degree of confidence in these models. Also, since these models are complex, their solution requires much computer time and, consequently, they are quite expensive to use.

Parametric models have the advantage of being relatively simple, and once calibrated may be used efficiently and inexpensively. Their disadvantage is that the initial calibration requires an input-output data base for the particular system being studied.

Conceivably, there exist two possible ways of obtaining the input-output data base required for simple model calibration: 1) actual field data as measured on gaged watersheds, or 2) synthetic data as generated from a physically based model. Actual field data are often limited or inadequate to provide the kind and amount of data required for calibration. Also, using actual field data requires that the watershed already be gaged. The alternative to using actual field data is to calibrate a simple parametric model to a complex physically based model, using the complex model as an intermediary between the real watershed and the simple model. In this way, a complete range of synthetic data can be generated with which to calibrate the simple model. Once the parametric

model has been calibrated to the physically based model, the parametric model may be run repeatedly, saving both time and money, to predict the complete output set for the watershed under consideration. In other words, the simple model will be able to estimate runoff from an ungaged watershed, either directly or (more likely) after some brief calibration against synthetic I-O data produced for the watershed by the complex model. The overall scheme of calibration is illustrated in Figure 1.

One approach to modeling runoff from watersheds is to calibrate a simple model to a complex model for a relatively small watershed area (base area) and representing the output response of larger watersheds as several base area responses routed together by a channel routing routine. The advantage of such an approach is that several simplifying assumptions can be made regarding the base area (i.e., constant input, physical homogeneity), which in turn simplify the calibration procedure.

The objective of this study was to calibrate a simple, parametric model to a complex, physically based model for base areas of one acre and one square mile. A quasi-linear, spatially distributed, cell model (Diskin and Simpson, 1976) was selected as the simple model. A non-linear, kinematic cascade model developed by Kibler and Woolhiser (1970) and adapted by Chery (1976) was selected as the physically based representation of the surface-channel flow system. Base areas of one acre and one square mile were chosen because this was felt to be the smallest and largest watershed areas that would satisfy the definition of a base area, as discussed in Chapter 3.

Figure 1. Scheme of Calibrating the Parametric Model with the Physically Based Model.

The calibration of the parametric model to the kinematic model was conducted in three steps. First, an objective function was developed to compare model output responses for best fit. Second, a sensitivity analysis of the kinematic model parameters was conducted. The results of the analysis allow for reduction in number of kinematic model parameters, which in turn simplified the calibration procedure. The third step consisted of the actual calibration, by trial and error, of the simple model to the complex model.

CHAPTER 2

MODEL SELECTION

The Physically Based Model

A nonlinear, kinematic cascade model developed by Kibler and Woolhiser (1970) and adapted by Chery (1976) was selected for two reasons; first, it is a major assumption of this study that the physically based model selected for use is an acceptable representation of the natural watershed system. The kinematic watershed model (Chery, 1976) has parameters that represent the geometry, roughness, and infiltration properties of a real watershed and, therefore, it may be used to predict runoff from a watershed system for which there is no sampling of the output. Secondly, a version of this model (as prepared by Chery, 1976) is available at the Agricultural Research Service office in Tucson, Arizona, thus making the program, and any assistance relating to operation, readily available.

Description of the Physically Based Model

The physically based, kinematic model follows what Kibler and Woolhiser (1970) have described as the "reductionist" approach. The complex topography and geometry of natural watersheds is replaced by a combination of simpler planes and channel sections with simpler cross-sections. The kinematic model is a sequence of discrete overland flow

planes or channel segments in which the kinematic wave equations are used to describe the unsteady flow (Figure 2).

A greatly reduced, but still distributed, representation of complex rainfall phenomenon is used as the input to the model. From this input, rainfall excess is determined by an infiltration routine. The generated excess is then routed over the infiltrating planes and through the channel network of the model. Discharge from the planes is fed laterally into channels where it is conveyed via a cascade of channels to some downstream gaging point (Kibler and Woolhiser, 1970). The output at this point is the watershed output as predicted by the physically based model.

The model incorporates some additional developments in surface-flow modeling. Until recently, almost all mathematical watershed studies have assumed a known rainfall excess as input. Nes and Hendriks (1971, p. 129) remarked: "The separation of precipitation into precipitation excess and infiltration has not yet been achieved." However, in that same year, this necessary component was successfully modeled by Smith and Woolhiser (1971). Subsequent work of Smith (1972b) and Smith and Chery (1973) made possible the construction of a watershed model without the assumption of rainfall excess. This crucial process has been included in the model but was not used in this study. The calibration procedure of this study dealt exclusively with excess precipitation.

In conjunction with the continuously infiltrating planes and channels, the kinematic model uses a transitional friction relation. The transitional depth is that depth at which flow changes from laminar to

Figure 2. Plane-Channel Configuration of the Kinematic Model.

turbulent. The process that determines a transitional depth and uses both friction laws is referred to as the transitional friction relation (Chery, 1976). The use of this relation in the model allows for an improved representation of the real flow system.

Flow from triangular planes and a channel with distributed lateral inflow were also innovations in the adapted model. The use of triangular planes, combined with the use of rectangular planes, allows for an improved representation of the actual watershed configuration (Chery, 1976). The model was also expanded in the modification to handle a large number of planes and channels cascaded together, thus making it possible to model large watersheds.

The input information needed for the complex kinematic model, as outlined by Chery (1976), is listed below:

1. Precipitation input: Accumulated records of rainfall depth (inches) versus time (minutes). Measurements at uneven intervals can be accommodated.
2. Plane parameters:
 - a. Length (ft).
 - b. Width (ft).
 - c. Slope.
 - d. Friction (Darcy-Weisbach friction factor).
 - e. Critical Reynolds number.
 - f. Kinematic viscosity, ν (ft²/sec).
 - g. Numbers of segments for solution of flow on the planes.
 - h. Time increment for numerical solution, Δt (min).

3. Channel parameters:
 - a. Length (ft).
 - b. Bottom width (ft).
 - c. Bottom slope.
 - d. Side slope.
 - e. Friction (Chezy C).
 - f. Exponent of hydraulic radius in the flow equation.
 - g. Time increment, Δt (min).
 - h. Number of segments for solution of flow along channels.
4. Infiltration parameters: They are common for both planes and channels and they may be specified for each one defined in the model.
 - a. α , (ALF) exponent parameter in infiltration equation.
 - b. F, (FMIN) final infiltration rate.
 - c. CIN, normalizing time parameter.
 - d. BIN, coefficient in functional relation between dimensionless soil water at time of ponding and dimensionless rainfall rate.
 - e. SIN, initial soil water content by volume.
 - f. SOIN, maximum soil water content by volume.
 - g. ROCK, percentage of rock in the soil.

Method of Computation

The kinematic model is based on the numerical solution of the governing nonlinear, partial differential, kinematic wave equations describing overland and open channel flow. Woolhiser and Liggett (1967)

established that the kinematic approximation is an "adequate" model of overland flow when a dimensionless parameter, K^* , called the kinematic flow number, is greater than approximately 20. The kinematic flow number (Woolhiser Number) is defined as:

$$K^* = \frac{S_o L_o}{H_o F_o^2}$$

where S_o is the bed slope, L_o is the length of the flow plane, H_o is the normal depth of the downstream boundary at the maximum steady-state flow rate, and F_o is the Froude number at the downstream boundary at the same flow rate (Chery, 1976).

Kinematic is a term used for flow that can be described by the equation of continuity and a state-discharge relation, such as the Chezy or Manning friction formula. In this model, the Chezy relation was used:

$$V_o = C H_o^m S_o^{m+1} \quad (1)$$

where V_o is the velocity at normal depth, H_o ; S_o is the bed slope; and C is the Chezy turbulent resistance coefficient or Chezy laminar resistance coefficient (C_1). Equation (1) describes turbulent flow when $m = 1/2$ and laminar flow when $m = 2$. By dimensional analysis of the momentum equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = g(S_o - S_f) - \frac{q}{h} (u - v) \quad (2)$$

where

u, v = velocity,

h = depth,

x = space coordinate,

t = time,

q = lateral flow rate,

g = gravity, and

S_f = slope of the energy line,

Woolhiser and Liggett (1967) demonstrated that, for large K^* , the Chezy relation may be used to define the energy or stage-discharge relation:

$$u = C S_o^{\frac{m+1}{3}} h^m \quad (3)$$

By defining

$$\alpha = C S_o^{\frac{m+1}{3}}$$

Equation (3) becomes $u = \alpha h^m$. Combining this equation with the continuity equation:

$$\frac{\partial(hu)}{\partial x} + \frac{\partial h}{\partial t} = q$$

the kinematic wave equation used in the model is derived:

$$\frac{\partial h}{\partial t} + \alpha(m+1) h^m = \frac{\partial h}{\partial x} = q(x, t) \quad (4)$$

The computer program solves the kinematic wave equation numerically. A numerical solution of the amount of flow is made for certain distances (solution lengths) on the planes and along the channels, and for certain time intervals. The solution lengths are specified in the model by dividing the planes and channels into a particular number of segments. The time increment for numerical solution is read directly into the model. An explicit, finite-difference solution method called the single-step Lax-Wendroff (Kibler and Woolhiser, 1970) is used to solve the numerical equations.

Rainfall excess was calculated by the soil-equivalent model developed by Smith (1972b), and Smith and Chery (1973). The dimensionless infiltration rate (f_*) may be expressed as a function of dimensionless infiltrated volume:

$$(f_* - 1) = (1 - \alpha)(Q_*' - Q_{0*})^{-\alpha/(1-\alpha)}$$

where

$f_* = f/f_\infty$, dimensionless infiltration rate;

f = infiltration rate, L/T;

f_∞ = infiltration rate at $t = \infty$ which is also equivalent to

$K \times$ (saturated conductivity of porous media), L/T;

α = exponent parameter in the infiltration equation (Smith, 1972a, Equation 5):

$$f = f_\infty + A(t - t_0)^{-\alpha} \quad \text{for } t > t_p$$

or in dimensionless terms (Smith, 1972a, Equation 12):

$$f_* - 1 = (1 - \alpha)(t_* - t_{0*})^{-\alpha}$$

in which

t = time (T),

t_0 = reference time (T),

A = coefficient parameter,

$t^* = t/T_0$ (dimensionless time), and

t_{0*} = dimensionless reference time;

Q'_* = dimensionless soil water accumulated at rate $(r_* - f_{\infty*})$ up to time t_{p*} , where $r_* = R/f_{\infty}$ (dimensionless rainfall rate); and

Q_{0*} = dimensionless reference soil water storage term analogous to t_{0*} .

The development and application of this relation is detailed by Smith and Chery (1973).

The two main subroutines in the computer model are KINTRI and CHANNEL. The KINTRI subroutine makes the numerical kinematic solution of flow on a triangular or rectangular plane, and the CHANNEL subroutine uses the kinematic numerical method to solve for the flow in the defined channels.

Flow is routed through the sequence of planes and channels by a set of coded instructions in a control program. The process begins with reading the rainfall data and solving for the flow from the first plane. This solution is coded and stored for use in the solution of flow in

channels between intersecting planes or channels on the edge of planes. Additional controls determine in which arrays the computer outflow is to be stored (Chery, 1976). The computed outflow of the last channel in sequence is the total outflow of the watershed being studied. This outflow is printed out as a time series of data by the computer program.

For the remainder of this report, the complex, physically based, kinematic model shall be referred to as the complex model.

The Parametric Model

A quasi-linear, spatially distributed, cell model (Diskin and Simpson, 1976) was selected as the simple input-output parametric model to be used in this research. The reasons for selecting this particular model were two-fold: 1) the model is well-suited for the purposes of this research because it uses a small number of parameters and is easy to operate; and 2) the model was recently developed at the University of Arizona Department of Hydrology and Water Resources, therefore making the program, and any assistance needed in understanding and using the program, readily available.

Description of the Parametric Model

The quasi-linear parametric model is very similar to the combination of linear reservoirs and linear channels proposed by Dooge (1959). The major difference is that the Diskin model allows for a spatial distribution of input to the reservoirs, whereas the Dooge model treats the rainfall as a spatially lumped input. The spatial distribution is accounted for by dividing a watershed area (Figure 3a) into a number of

Figure 3. Design Sketch for the Quasi-Linear, Spatially Distributed Cell Model. -- From Diskin and Simpson (1976).

subareas (Figure 3b), each of which is represented by a cell in the model (Figure 3c). The number and size of the divisions is governed by the number of different rainfall intensities falling on a watershed and the area that each intensity covers. A cell may be assigned any percentage of the total watershed, but the sum of all cells must equal 100 percent.

The computer program, as originally written, allows for a maximum of 20 cells, but this number may be increased by a simple change in format. The input to a given cell is composed of two parts: 1) the rainfall over the area represented by the cell; and 2) the sum of the outputs of upstream cells, if any, directly connected to the given cell. Each cell may receive up to four inputs from upstream cells in addition to precipitation input. All inputs are combined to produce one output per cell (Figure 4).

The model response can be delayed in time by the use of a lag factor, L . The lag factor is read into the computer model as an integer which shifts the computed output hydrograph by " L " number of time increments. A time increment is specified by the modeler as the time between sequential output data points computed by the model.

Each cell in the model is assumed to act as a linear reservoir characterized by a cell storage ratio, K . Diskin and Simpson (1976, p. 5) define the storage ratio as "the ratio between the quantity of water in storage in the cell and the rate of outflow or discharge from the cell." The model is quasi-linear because K is not constant, but is a function of total excess rainfall (Diskin and Simpson, 1976). It is this value of storage constant which is of primary concern when calibrating

Figure 4. Input-Output Scheme to a Single Cell of the Parametric Model.

the model (in this study, the parameter K is related to the intensity of excess rainfall, rather than the total amount).

The model accounts for temporal as well as spatial distribution of rainfall by reading in the input data according to a specified time interval. The model also has the capability to calculate excess rainfall for each cell individually by means of a simple percentage loss rate function. However, this presupposes that the percentage of lost input is already known. The model does not compute this amount.

The parameters of the parametric model are:

1. K, cell storage ratio (t).
2. L, lag factor.
3. N, number of cells within the base area.
4. ALF, area weight of cells.
5. PER, percent of excess rainfall.
6. RTE, routing scheme of flow through the cells.

Method of Computation

The computer program developed for this model is based on the numerical convolution routine initially developed and used by Nash (1957). The storage equation:

$$y(t) = 1/K S(t) \quad (5)$$

where K is the cell storage ratio (T), S(t) is storage in a cell (L), and y(t) is the output signal with respect to time (L/t), is combined with a continuity equation:

$$x(t) - y(t) = dS/dt \quad (6)$$

where $x(t)$ is the input signal with respect to time, L/t . The solution of Equations (5) and (6) for $y(t)$ with $x(t)$ being a unit impulse produces the impulse response function:

$$h(t) = (1/K) e^{-t/K} \quad (7)$$

The impulse response function is the unknown parameter in the convolution integral which relates cell output to cell input by the function:

$$y(t) = \int_0^t h(\tau) x(t - \tau) d\tau \quad (8)$$

where $t-\tau$ is the input τ period of time before time t , and $h(t)$ is the impulse response function which describes the distribution in time of an instantaneous pulse of input.

The impulse response function of the Diskin model represents the cell as a linear reservoir. The model is written for a cascade of up to twenty linear reservoirs, all having a fixed storage ratio for a given storm event (i.e., any given value of excess rainfall) (Figure 5). This function of an integer number of reservoirs in series may be expressed as:

$$h(t) = \frac{1}{K(n-1)!} \left(\frac{t}{K} \right)^{n-1} e^{-t/K} \quad (9)$$

where n is the number of reservoirs or cells.

In addition to a convolution routine, there exists a routing routine which ensures that the n reservoirs are processed in their

Figure 5. Cascade of N Linear Reservoirs.

correct order and combines the outputs of upstream cells to form inputs to the immediate downstream cells. Due to the branching structure of the cells, the input, and therefore the output, of a particular cell is influenced only by the cells immediately upstream.

Computation of total watershed outflow begins with the most upstream cells whose only input is rainfall. The next cells processed are those receiving input from only the most upstream cells. These cells receive two inputs, the rainfall over their area and the delayed output of the upstream cells. This process continues as specified by the routing routine until the last cell, located at the watershed outlet, is reached. The output of this final cell is the total runoff from the watershed as predicted by the simple model.

For the remainder of this report, the simple, quasi-linear, spatially distributed cell model shall be referred to as the simple model.

A complete listing of the computer program and a list of symbols are presented in Appendix A and Appendix B, respectively.

CHAPTER 3

DEFINITION OF THE OBJECTIVE FUNCTION AND THE BASE AREA

Objective Function

This investigation is concerned with the comparison of two time series of data: the "observed" output hydrograph of the complex model and the computed output hydrograph of the simple model (see Figure 6). Calibration of the simple model to the complex model requires a means by which these outputs can be quantitatively compared in order to determine the optimal simple model parameters. An objective function is defined that makes the desired comparison.

There is little information available on the merits of different objective functions. Assertions have frequently been made concerning their relative merits and their effect on the optimal parameter values; however, these assertions generally have not been supported by data (Johnston and Pilgrim, 1976). A variety of statistical measures and tests have been used to express the degree of agreement between computed and observed outputs. A few of the more commonly used objective functions are discussed by Aitken (1973). For the purpose of this research, objective functions used by Diskin and Simpson (1976) were adopted after modification described below.

The concept of determining optimal model parameters requires that the objective function be compatible with the intended use or purpose of

Figure 6. Use of the Objective Function, S, to Determine Best Fit between the Observed Response of the Complex Model (---) and the Computed Response of the Simple Model (—) for Three Values of the Cell Storage Ratio, K.

the model (Singh, 1976). The modeler has the capability to bias the model depending on his definition of the objective function. If the intent of the model is to study floods, then greater weight would be placed on the height of the peaks and time to peak. If, on the other hand, the modeler is interested in recharge to groundwater, he probably would emphasize duration of flow. For the purposes of this research, it was decided to emphasize the height of peak and time to peak, while still trying to preserve an overall good fit between the two hydrographs.

In order to make the comparison between the simple and complex model hydrographs, the following statistical parameters are defined:

1. Normalized root mean square difference in discharge between the two hydrographs (S_1):

$$S_1 = \frac{\left[\frac{1}{N} \sum_{i=1}^N (\Delta Y)^2 \right]^{1/2}}{\frac{1}{N} \sum_{i=1}^N q_o} \quad (10)$$

2. Normalized maximum difference at any time between the two hydrographs (S_2):

$$S_2 = \frac{\text{MAX} |q_o - q_c|}{q_{op}} \quad (11)$$

3. Normalized difference between magnitude of peaks of the two hydrographs (S_3):

$$S_3 = \frac{|q_{op} - q_{cp}|}{q_{op}} \quad (12)$$

4. Normalized difference in time of peak between the two hydrographs (S_4):

$$S_4 = \frac{|t_{op} - t_{cp}|}{t_{op}} \quad (13)$$

where

ΔY = difference between the computed and observed output at a given time,

N = number of output data points,

q_o = observed output at a given time,

q_c = computed output at a given time,

q_{op} = observed peak output,

q_{cp} = computed peak output,

t_{op} = time to observed peak, and

t_{cp} = time to computed peak

As a result of normalization, the statistical parameters are made dimensionless.

The objective function is to be defined as a particular combination of the statistical parameters given in Equations (10) through (13). A trial and error searching technique, guided by subjective judgment, was used to arrive at a combination compatible with the intended purpose. The objective function is of the form:

$$S = \frac{1}{a_1 + a_2 + a_3 + a_4} (a_1 S_1 + a_2 S_2 + a_3 S_3 + a_4 S_4) \quad (14)$$

where each "a" is an integer coefficient or weighting factor. Watershed areas of various shapes (triangular, square, and rectangular), with various values of the plane and channel parameters, and with various intensities of rainfall input, were tested against various values of the coefficients a_1 , a_2 , a_3 , and a_4 . For each test, the goodness of fit between observed and computed hydrographs as determined (subjectively) by eye was compared with the value of S for different combinations of coefficients. After many such tests, the objective function was defined as:

$$S = \left(\frac{1}{10} \right) (S_1 + S_2 + 6S_3 + 2S_4) \quad (15)$$

A comparison of observed and computed hydrographs for three values of the parameter K (all other variables held constant), along with the values of the objective function, S, is shown in Figure 6. The minimum S, 0.067, indicates that a K of 0.16 yields the best fit between the two hydrographs.

The computer program for the simple model contains a routine which computes values of the statistical parameters and the value of the objective function. The calibration of the simple model to the complex model will consist of numerous trial and error runs in search of the optimal fit. For the entire study, the minimum of the objective function

(Equation 15) identifies the best fit between the observed and computed hydrographs.

Base Area

For a given precipitation input, each set of parameters (characterizing watershed properties) of the complex model produces a unique output hydrograph. Through a trial and error calibration procedure, the parameters of the simple model can be adjusted so as to produce an output response similar to that of the complex model. The objective of the calibration procedure is to establish a relationship between the parameters of the two models such that simple model parameters producing an output similar to that of the complex model can be quickly and accurately determined.

If all 23 of the physical parameters were to be treated as variables, the task of finding practical, workable relationships between the physical parameters of the complex model and the fitting parameters of the simple model would be overwhelming. The concept of a base area is introduced in order to identify those physical parameters whose influence on output is dominant, and those whose influence is slight. That is, for each base area, a sensitivity analysis was performed by means of which dominant physical parameters were identified and subsequently treated as variables. All other parameters were treated as constants and assigned typical values. Some of the physical parameters relate to the geometry of the particular base area and their values are implicitly determined by the base area definition. The size of the base area determines which parameters are dominant.

A base area is defined as a sub-unit of a watershed having the following properties: 1) a fixed area and boundary shape (the same for both models); 2) a fixed arrangement of intersecting planes and channels, except that the values of certain slope and friction parameters may be varied (for complex model), and a fixed number and arrangement of interconnected sub-cells, except that the values of the fitting parameters K and L may be varied (simple model); and 3) rainfall input constant in space but may vary in discrete time steps (the same for both models). The output response of a real watershed would be represented by the combined output of two or more base areas routed together.

With regard to the size of the base area, it is reasonable to assume that the smallest size would be one to several acres and the largest would be one square mile. The maximum size would be set by the assumptions of uniform input and physical "homogeneity" of the plane and channel parameters, while the minimum size would be set by the density and distribution of available field data and the desire not to represent a large watershed with an unmanageable number of base areas [a typical convective storm in the southwest covers an area of 40 square miles (Osborn, Lane, and Kagan, 1974), and assuming that the storm is contained within a single watershed, a model would then require about 40 square-mile base areas].

In this report, the largest and smallest base areas were studied: 1) a $40,000 \text{ ft}^2$ square area, approximately one acre (Figure 7); and 2) a square mile area (Figure 7). An $80,000 \text{ ft}^2$ rectangular area, approximately two acres (Figure 7) also was studied but was discarded in favor

Figure 7. Design Sketch of the (a) One-Acre and (b) Two-Acre Base Areas for Use in the Complex Model.

of the one-acre square because the square area gave better correlation between the complex and simple model responses.

CHAPTER 4

CALIBRATION PROCEDURE AND RESULTS

The calibration procedure seeks to establish a relationship between the simple model and complex model parameters such that, for a particular input, the simple model will accurately predict the output response of the complex model. The complex model was run to produce outputs from pulse inputs of various intensities for various sizes, shapes, slopes, and roughnesses of intersecting planes and their related channels. The simple model was similarly pulsed and its fitting parameters adjusted, by trial and error, until the best fit between the output responses of the two models was achieved. A relationship was sought between physical parameters and fitting parameters describing similar output responses.

Calibration began with the base areas of one and two acres to test the fitting of the simple model output to that of the complex model, and to study the optimization of simple model parameters for best fit. Then the study moved to the fitting procedure for a square mile base area.

One- and Two-Acre Base Areas

The base areas of approximately one and two acres are shown in Figure 7. It was thought that a one- or two-acre watershed could be accurately represented by two planes and a single channel.

Before the actual calibration could be conducted, the number of physical parameters and the input was reduced through sensitivity analysis and personal judgment. The parameters of the complex model may be divided into three categories: 1) plane parameters, 2) channel parameters, and 3) infiltration parameters. In addition, the input of precipitation, both in time and space, must be specified.

The response of both the simple and the complex models is a function of input; therefore, a range of inputs was tested. To simplify calibration, all precipitation inputs were short-duration pulses of constant intensity. Infiltration parameters were not considered in the calibration procedure. Instead, the simple model was calibrated to the complex model only for precipitation excess.

The length and width of the planes and channels were set according to the definition of the base area (see Figure 7). It was felt that a channel width of two feet is representative of a one- or two-acre watershed.

Of the five plane parameters to be considered (length and width excluded), it was determined that slope and friction were the only parameters having a significant influence on model output. Tests conducted with the number of segments on the plane indicated that there is little connection between the number of segments and output response. Figure 8 shows that the output responses for 10, 15, and 19 segments are nearly identical. As a result of these findings, the number of segments on the planes of both the one- and two-acre base areas was held constant at 10, yielding a solution length of 10 feet.

Figure 8. Response of the Complex Model to Changes in the Number of Segments on the Planes.

A critical Reynolds number of 20 and a time increment for numerical solution of the kinematic equations of 0.5 minute were found by Chery (1976) to be adaptable to most all applications of the complex model. These values were held constant throughout the investigation.

An interaction between plane slope and friction is apparent and a combination of the two parameters was sought. The product of the inverse of slope, expressed as a percent, and the friction, expressed as a Darcy-Weisbach friction factor, gives a number, called the S-F number. Figure 9 shows several such slope-friction combinations for a range of S-F numbers. The product of different slope and friction values yielding the same S-F numbers were found to produce similar output responses for the case of no infiltration. Output from the acre size base area for slope-friction combinations yielding S-F numbers of 50, 200, and 500 are shown in Figure 10.

Tests were conducted to determine the degree of influence the plane parameters (slope and friction) and the channel parameters (bottom slope, side slope, and friction) have on the model response. Tests for different values of the channel parameters (Figure 11) were compared to the results obtained for different values of plane slope and friction (S-F numbers) shown in Figure 10. When compared to the results where the S-F number was varied, it was judged that similar output responses resulted from tests where the channel parameters were varied. As a result of these findings, plane parameters (slope and friction) are dominant and were the only parameters considered in the calibration procedure of the one-acre base area.

Figure 9. Matrix of Slope and Friction Combined into a Single Variable for Use in the Complex Model. -- Numbers along the diagonals from the upper left to the lower right combine to produce equal S-F numbers (in parentheses) obtained from the product of inverse of slope as a percent and friction as a Darcy-Weisbach number.

Figure 10. Complex Model Output Response from Slope-Friction Combinations Yielding S-F Numbers of 50, 200, and 500 for the One-Acre Base Area.

Figure 11. Response of the One-Acre Base Area to Changes in Channel Bottom Slope (S_c), Side Slope (S_s), and Friction (C).

Initial tests of the complex model were run to produce outputs for an S-F number of 50, where slope and friction were set at 14% and 700, respectively. Subsequent tests were conducted for S-F numbers of 8 (slope = 30%, friction = 240) and 500 (slope = 10%, friction = 5,000). Channel bottom slope, side slope, and friction were kept constant at 5%, 10%, and 40 (Chezy C), respectively. Output responses of both base areas to two-minute pulse inputs, for intensities of 0.5, 1.0, 3.0, 6.0, and 9.0 inches per hour, and for three S-F numbers, were obtained by the complex model.

The simple model was similarly pulsed and, by trial and error, the difference between the simple and complex model output responses was minimized. Design sketches of the one- and two-acre base areas for use in the simple model are shown in Figure 12. Four of the six possible fitting parameters of the simple model were used in matching the output responses of the two models. These are: 1) cell storage ratio, K; 2) number of cells, N; 3) weighted area of cells, ALF; and 4) the routing of flow through the cells, RTE.

The cell storage ratio is the primary fitting parameter. In achieving a best fit of a simple model response to a response of the complex model, K was determined by making an initial estimate and then adjusting its value until a best fit, as determined by the objective function, was obtained. An optimum value of K was found for all combinations of inputs and parameters run in the complex model.

The base areas (both the one- and two-acre) were divided into 1, 2, and 4 equal-area cells, routed together as shown in Figure 12, to

Figure 12. Design Sketch of the One- and Two-Acre Base Areas (Depicting One, Two, or Four Subcells) for Use in the Simple Model.

determine the influence of N on the fit between the output responses of the two models. The results of these tests are shown in Table 1 and Figure 13. The 4-cell case achieved the best fit at all input intensities. As a result of these findings, N was set equal to 4 for all subsequent tests conducted on the one- and two-acre base areas.

The weighting parameter (ALF) was ranged through seven values for the four cells to determine its effect on fit between the output responses of the two models. The 4 cells were assigned different percentages of the total base area such that cells nearest the outflow became progressively smaller (Figure 14). Results of the weighted area tests are shown in Table 2 and Figure 15. The equal area case (case II) produced the best results at low intensities (1.0 in/hr) for both base areas. At higher intensities (3.0 and 9.0 in/hr), best fits were produced by weighted area Case VI and IV for the one-acre and two-acre base areas, respectively.

The results of tests with various values of N and ALF indicated that these parameters have little influence on the simple model response. Since weighted area case II performed best at the low intensities, and since there was little difference in performance between weighted area cases II through VII at the higher intensities, weighted area case II (equal-area cells) was used for all tests conducted on the one- and two-acre base areas. When working with different size base areas, the area would be calibrated one time for N equal-area cells and this value would be held constant for all subsequent tests on that particular area.

Table 1. Comparison of Fit between the Simple and Complex Model Output Responses for One, Two, and Four Equal-Area Subcells of the One- and Two-Acre Base Areas.

Number of Subcells	Objective Function (S)			
	Input =	1 in/hr	3 in/hr	9 in/hr
<u>One-Acre Base Area</u>				
1		.306	.293	.311
2		.328	.322	.321
4		.197	.275	.305
<u>Two-Acre Base Area</u>				
1		.295	.286	.321
2		.314	.307	.332
4		.214	.242	.251

Figure 13. Comparison of Fit for the One, Two, and Four Equal-Area Subcells of the One- and Two-Acre Base Areas.

Weighted area cases of the 200 ft. by 200 ft. base area

Figure 14. Seven Weighted Area Cases of Four Subcells of the One-Acre Base Area for Use in the Simple Model.

Table 2. Comparison of Fit between the Simple and Complex Model Output Responses for Seven Weighted Area Cases.

Input Intensity (in/hr)	Objective Function (S) for the Seven Weight Cases							
	K	I	II	III	IV	V	VI	VII
<u>One-Acre Base Area</u>								
1	.25	.717	.197	.342	.291	.294	.441	.210
3	.04	.628	.275	.183	.164	.164	.152	.200
9	.02	.525	.305	.260	.227	.220	.233	.219
<u>Two-Acre Base Area</u>								
1	.25	.714	.214	.253	.331	.331	.458	.277
3	.05	.658	.242	.255	.199	.197	.200	.207
9	.02	.599	.251	.268	.251	.246	.271	.292

Figure 15. Comparison of Fit for the Seven Weighted Area Cases of the One- and Two-Acre Base Areas.

At all inputs and for all weighted area cases, the responses from the one-acre square base area achieved better fits than the responses from the two-acre rectangular base area. As a result, all future calibration was conducted on the one-acre square base area.

A general calibration procedure was developed for the one-acre base area as shown in Figure 16. As a result of being able to combine slope and friction on the planes into a single variable, the cell storage ratio could be expressed as a function of pulse intensity and S-F number. The two outer curves of Figure 16 ($S-F = 8$ and $S-F = 500$) establish the upper and lower limit of the simple model. Similar curves could be found for S-F numbers between 8 and 500.

Square-Mile Base Area

The square-mile base area is thought to be the largest area that can be represented by a uniform input and single values of the plane and channel parameters. The procedures developed with the one- and two-acre areas were used as a basis for investigating the calibration of this larger area.

A design sketch of the square-mile base area for use in the complex model is shown in Figure 17. The overland flow length on the 104 planes was set equal to 100 feet, the same as in the one- and two-acre base areas, in the expectation of finding the same slope-friction relationship. The identical fit of the two curves shown in Figure 18 for two slope-friction combinations yielding the same S-F number demonstrates that the S-F relationship does hold.

Figure 16. Family of Calibration Curves (S-F Numbers of 8, 50, and 500) for the One-Acre Base Area.

Figure 17. Configuration of the Square Mile Watershed Adopted as the Base Area of the Complex Model.

Figure 18. Identical Response of the Square-Mile Base Area to Two Slope-Friction Combinations Having the Same S-F Number.

Tests with S-F numbers of 50, 500, and 2,000 found that similar output responses resulted (Figure 19), indicating that the plane parameters have little influence on model response. The channel parameters dominated the response from the system. As a result, only the input and channel parameters had to be considered when calibrating for the square-mile base area.

Five of the eight channel parameters were held constant in the calibration procedure. Channel lengths were set according to the definition of the base area. Four patterns of progressively increasing channel bottom widths (from the headwaters to the watershed outflow), as shown in Figure 20, were compared. The similar responses (Figure 21) indicate that channel bottom width has little influence on the complex model output response; therefore, width pattern IV was used in all tests. The number of segments of the channels and the time increment for numerical solution of the kinematic channel flow equations were assumed to have negligible influence on the output response as a result of findings with the one-acre base area. The time increment was set equal to 0.5 minute. The following pattern of channel segmentation was used: the 52 side channels were divided into 18 segments for a solution length of 147 feet, each of the 25 sections composing the central channel (except the last 100 feet) were divided into 4 segments for a solution length of 50 feet, and the final 100 feet was divided into 3 segments for a solution length of 33 feet. The parameter E is the exponent in the Chezy velocity relation:

$$V = C(RS)^E$$

Figure 19. Response of the Square-Mile Base Area to Three S-F Numbers.

Channel Width Patterns

	Width Pattern			
	I (ft)	II (ft)	III (ft)	IV (ft)
Channel section A	.5	1	2	3
Channel section B	1	2	4	6
Channel section C	2	4	8	12

Figure 20. Channel Bottom Width Patterns in the Square-Mile Base Area of the Complex Model.

Figure 21. Complex Model Output Response to Four Channel Bottom Width Patterns (See Figure 20) of the Square-Mile Base Area.

where V is the velocity, C is the Chezy coefficient, R is the roughness, and S is the slope (Chow, 1964). In this relation, E is usually equal to 0.5. The value of $E = 0.5$ was constant in this study.

The three remaining channel parameters are the bottom slope, side slope, and friction (Chezy C). The complex model was run to produce output responses to 10-minute pulse inputs, for intensities of 0.5, 1.0, 3.0, 6.0, and 9.0 inches per hour, and for the nine sets of the above three channel parameters shown in Table 3.

The simple model was similarly pulsed and, by trial and error, its parameters were adjusted in order to fit its output responses to that of the complex model. When working with the larger base area, it was found that the output is delayed more so than in the smaller base areas. Therefore, it was necessary to include a lag factor, L , as one of the five simple model parameters (same four parameters used in fitting the one- and two-acre responses plus the lag factor) used in matching responses from the square-mile base area.

K and L were the primary fitting parameters. An optimum value of K and L was found for each individual combination of input and parameters run in the complex model.

Tests with 4, 9, 12, and 18 equal-area cells determined that 12 cells ($N=12$), routed together as shown in Figure 22, achieved the best fit between the output responses of the two models for all intensities tested. Equal-area cells were chosen as a result of the findings with the one- and two-acre base areas. These three parameters, N , ALF , and

Table 3. Nine Channel Parameter Sets Used in Calibrating the Simple Model to the Complex Model for the Square-Mile Base Area.

Channel	Vary Bottom Slope (S_c)	Vary Channel Side Slope (S_s)	Vary Channel Friction (Chezy C) (C)
	<u>Test A-1</u>	<u>Test B-1</u>	<u>Test C-1</u>
	* $S_c = 1\%$	$S_c = 5\%$	$S_c = 5\%$
	$S_s = 10\%$	* $S_s = 1\%$	$S_s = 10\%$
	C = 10	C = 10	*C = 10
	<u>Test A-2</u>	<u>Test B-2</u>	<u>Test C-2</u>
	* $S_c = 2\%$	$S_c = 5\%$	$S_c = 5\%$
	$S_s = 10\%$	* $S_s = 10\%$	$S_s = 10\%$
	C = 10	C = 10	*C = 20
	<u>Test A-3</u>	<u>Test B-3</u>	<u>Test C-3</u>
	* $S_c = 5\%$	$S_c = 5\%$	$S_c = 5\%$
	$S_s = 10\%$	* $S_s = 20\%$	$S_s = 10\%$
	C = 10	C = 10	*C = 40

Figure 22. Twelve Equal-Area Subcells and Flow Routing Scheme of the Square-Mile Base Area for Use in the Simple Model.

RTE, were held constant for all calibration done on the square-mile base area.

The results of the calibration procedure, relating K and L to the channel parameters and inputs, are shown by the calibration curves of Figures 23 through 30. K expressed as a function of intensity and channel bottom slope, intensity and channel side slope, and intensity and channel friction (Chezy C) are shown in Figures 23, 24, and 25, respectively. Figure 26 shows K as a function of intensity and all three channel parameters.

The lag factor expressed as a function of intensity and bottom slope, intensity and side slope, and intensity and friction are shown in Figures 27, 28, and 29, respectively. Figure 30 shows L expressed as a function of intensity and all three channel parameters.

An example of how good the fits were between the simple and complex model responses is shown by the middle curve of Figure 6.

Figure 23. Family of Calibration Curves for the Square-Mile Base Area for Three Values of Channel Bottom Slope ($S_c = 1\%$, 2% , and 5%) versus the Cell Storage Ratio, K .

Figure 24. Family of Calibration Curves for the Square-Mile Base Area for Three Values of Channel Side Slope ($S_s = 5\%$, 10% , and 100%) versus the Cell Storage Ratio, K.

Figure 25. Family of Calibration Curves for the Square-Mile Base Area for Three Values of Channel Friction (Chezy $C = 10, 20$, and 40) versus the Cell Storage Ratio, K .

Figure 26. Family of Calibration Curves for the Square-Mile Base Area for Different Values of Channel Bottom Slope (S_C), Side Slope (S_S), and Friction (C) versus the Cell Storage Ratio, K .

Figure 27. Family of Calibration Curves for the Square-Mile Base Area for Three Values of Channel Bottom Slope ($S_c = 1\%$, 2% , and 5%) versus the Lag Factor, L.

Figure 28. Family of Calibration Curves for the Square-Mile Base Area for Three Values of Channel Side Slope ($S_s = 5\%$, 10% , and 100%) versus the Lag Factor, L.

Figure 29. Family of Calibration Curves for the Square-Mile Base Area for Three Values of Channel Friction (Chezy C = 10, 20, and 40) versus the Lag Factor, L.

Figure 30. Family of Calibration Curves for the Square-Mile Base Area for Different Values of Channel Bottom Slope (S_c), Side Slope (S_s), and Friction (C) versus the Lag Factor, L .

CHAPTER 5

DISCUSSION OF RESULTS

This investigation has shown that it is possible to obtain good fits between the responses of the complex model and the simple model for the one-acre and square-mile base areas, for the range of inputs and sets of parameters tested. A general calibration procedure, whereby the simple model can represent the output response of the complex model, has been developed for the one-acre base area. A general calibration procedure was not developed for the square-mile base area. Instead, the simple model was calibrated to the complex model for seven particular combinations of three channel parameters.

A general calibration procedure was not developed for the square-mile base area because it was not possible to reduce the number of complex model variables in the calibration procedure. For the one-acre base area, the plane parameters (slope and friction) governed the shape of the model response. By being able to represent slope and friction as a single variable (S-F number), a general relationship was established between K, input intensity, and the S-F number. When working with the square-mile base area, three channel parameters (bottom slope, side slope, and friction) were found to have the dominant influence on output. It is possible that a functional relationship among these three channel parameters can be found so that combinations may be characterized by one or two numbers, instead of by three numbers. Some work along these lines

was performed, but the results are negative and are not reported. Nevertheless, the dominant relations were demonstrated and the results for selected values of the parameters were given in Chapter 4 (Figures 23 through 30). A general calibration procedure requires further development.

It was possible to reduce the six simple model parameters to two primary fitting parameters, K and L. The remaining parameters had little influence on the shape of the model response; therefore, they could be calibrated one time for a particular size base area and held constant in all subsequent tests on that area.

The relationships established between the parameters of the two models and the inputs, as shown in Figures 23 through 30, follow a definite pattern. For example, a pattern exists between K and L, and input intensity for the square-mile base area. As the intensity increases, the peak of the output response rises and occurs at an earlier time. In order to fit the simple model to a higher, earlier peak, both the K and L must be decreased. The amount of decrease is large at low intensities (0.5, 1.0, and 3.0 in/hr) and quickly levels off at higher intensities (greater than 3.0 in/hr). It appears that, at low intensities, the complex model response is altered considerably by a small change in intensity, whereas a small change in intensity at high intensities has little effect on the response. Thus, in order to fit the simple model to the large changes in response occurring at the low intensities, large changes in K and L must be made. A similar relationship occurs on the one-acre base area, the only difference being that

the lag factor is not required to fit the output response from this small area.

A relationship also exists between K and L and the physical parameters. On the one-acre base area, K is related to the S-F number. Increasing the S-F number (lower slope, increase friction) results in a very low, delayed peak of the output response. In order to fit such a response, K must be increased. On the square-mile base area, a similar relationship exists between K and L and the 3 channel parameters. Increasing bottom slope and side slope, or decreasing friction, results in a higher, earlier hydrograph peak, implying that K and L must be decreased in order to fit the simple model to it.

Limitations of the simple model were realized when trying to fit very peaked or very flat output responses. For example, on the one-acre base area, responses resulting from a low intensity combined with a high S-F number (very flat response) or a high intensity combined with a low S-F number (very peaked response) could not be matched by the simple model. Despite the limitations, the simple model can be calibrated to a broad range of responses. Only the extreme responses cannot be matched.

When infiltration is incorporated in a model of a realistic storm event, it may be found that this causes a greater difference in the responses of Figure 19 for changes in S-F number. Infiltration is greater on a plane having a low slope and high friction than it is on a plane having a high slope and a low friction. The reason is that a plane with a low slope and high friction will hold precipitation on its surface longer, thus giving the water more time to infiltrate. For the same

reason, infiltration is greater at low intensities than it is at high intensities. These same principles also hold true in the channels. Depending on the circumstances of the particular problem, infiltration could significantly influence the shape of the output response, which in turn would alter the relationships established through the calibration procedure.

The calibration curves make it possible to predict the output response of the complex model without having to run the complex model. The procedure for using the calibration curves of the square-mile base area would be as follows. The rainfall input intensity (averaged for 10-minute intervals) and the channel parameters (average bottom slope, side slope, and friction) would be estimated from available data for the square-mile watershed. These values would then be used to determine a K and L value for each 10-minute period from the calibration curves (assuming the channel parameters conform to one of the seven combinations tested). The value of K and L would be used with the simple model square-mile base area, as shown in Figure 17, to produce an output response for each 10-minute period. The response for storms lasting longer than ten minutes would be represented by the summation of the required number of ten-minute responses, and the responses from areas of many square miles would be represented by the response of several square-mile base areas routed together.

It was less expensive in terms of computer cost to use the simple model rather than the complex model. The average cost per run for 20 runs of the complex model was \$14.70, whereas 20 runs of the simple

model averaged \$.30 per run. Thus, computer costs of the complex model were nearly 50 times those of the simple model.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

The eventual objective of this research coupled with future research will be to develop a procedure by which the simple model can efficiently and inexpensively predict runoff from ungaged streams. Due to the innovation of the base area in this investigation, this objective would be accomplished in two steps. First, the simple model must be calibrated to the complex model for a given base area, and second, a procedure must be developed by which base area responses can be routed together to represent the response from a real watershed.

This study was concerned with the first objective. This objective was not fully realized, but several significant contributions did result, establishing a foundation upon which future research can build. First, the base area concept was defined as one means of reducing the number of variables in the calibration procedure. This eliminates the need to have to calibrate an infinite number of different sized watersheds. Second, it has been shown that the simple model can accurately represent a broad range of output responses of the complex model. Third, a relationship was identified between slope and friction on the planes, making it possible to represent these two parameters as a single variable. Fourth, a procedure for identifying the principle physical and fitting parameters has been developed, and these parameters have been identified for a one-acre and a square-mile base area, for particular

sets of parameters and inputs. Last, a procedure for calibrating the simple model to the complex model was developed, and the one-acre and square-mile watersheds were calibrated for a range of inputs and for particular sets of the physical parameters.

The calibration procedure developed for the one-acre base area is more generalized than that for the square-mile base area. However, in both instances, possible limitations to the general application of the procedure exist. These possible limitations may be summarized as follows:

1. Design of base areas: The simulation experiments performed on the base areas with the complex model used only one pattern each of planes and channels for the two base-area sizes. In each case, the pattern chosen was believed to be a typical and reasonable representation of a watershed sub-unit. Different patterns undoubtedly would provide somewhat different responses to the input pulses. In other words, the sensitivity analysis referred to in the discussion of base areas in Chapter 4 was incomplete in this respect.
2. Rainfall input: Only excess rainfall was considered in this investigation. Therefore, in order to apply the simple model to a real watershed, an independent means for estimating excess runoff must be available.
3. Relation of fitting parameters to physical parameters: In the case of the square-mile base area, the fitting parameters, K and L , were expressed as discrete functions of particular combinations

of three physical parameters, S_c , S_s , and C , and of the intensity of the input pulse. In cases where these physical parameters do not match the values for the tested combinations, the values of the fitting parameters would have to be estimated by extrapolation or interpolation and, hence, are subject to error on this account.

4. Routing routines: Both the complex model and the simple model have routing routines by which the discharge from individual base areas may be interconnected and combined. However, these routines were not tested against one another and it is not known to what extent they may be compatible or incompatible.

Until these possible limitations can be tested and resolved, it will not be possible to conclude whether a general calibration procedure of the simple to the complex model for the one-acre and square-mile base area can be realized. If such a procedure can be developed, it will then remain to develop a procedure for routing base area responses together to represent the responses from real watersheds. Only when this has been accomplished will the ultimate objective be realized.

In order to reach this ultimate objective, the following recommendations for future research are submitted:

1. Research needed to complete calibration of base areas:
 - a. Study other patterns or configurations of planes and channels within the one-acre and square-mile base area.

- b. Investigate the accuracy of and methods for representing a base area with a single value of slope and friction of the planes, and bottom slope, side slope, and friction in the channels.
 - c. Pursue the possibility of combining the three dominant channel parameters into one or two variables in order that a general calibration procedure for the square-mile base area might be realized.
 - d. Calibrate other size base areas (between one acre and one square mile) that may be better represented by a uniform input and physical "homogeneity."
2. Research needed to apply the parametric model to real watersheds:
- a. Develop a procedure for routing base area output responses together to represent the output response from real watersheds.
 - b. Validate the calibration procedure in the field.
 - c. Pursue the possibility of going directly from the real watershed to the simple model, eliminating the physically based model as an intermediary.
3. Research needed to evaluate the error introduced by fitting a linear simple model to a non-linear complex model:
- a. Investigate the effect of preceding rainfall on the cell storage ratio, K . In this report, for a given set of physical parameters, an optimum K was found as a function of rainfall intensity for 2-minute and 10-minute isolated

pulses. It is possible that the optimum value will be affected by the amount and intensity of preceding rainfall.

APPENDIX A

PARAMETRIC MODEL COMPUTER LISTING

```

PROGRAM CELMOD      73/74      OPT=0 TRACE      PTN 4.6+428

1  PROGRAM CELMOD(INPUT,OUTPUT,PLOT,TAPE 99=PLOT)
   DIMENSION IC(20),ID(20),YZ(20),IPT(20),ALP(20),KC(20,4)
   DIMENSION U(300),R(100),X(300),YC(20,300),TITLE(20),NYA(20)
   DIMENSION Y(300), Q(300), RJ(20,100), PER(20)
   PRINT 102
5  NC= NUMBER OF CELLS, AK= CELL STORAGE RATIO(T), DT=
   COMPUTATION TIME INTERVAL(T) IWT= 0 FOR EQUAL WEIGHTS OF
   CELLS, IWT= 1 FOR GIVEN SET OF WEIGHTS, IWT= 2 FOR
   INDIVIDUAL WEIGHTS TO EACH STORM EVENT, IDT= 1 OR 0 FOR
10  DT=DTA(DTA= PRINTOUT TIME INTERVAL(T)), OTHERWISE IDT=
   DTA/DT, JPT= NUMBER OF INTERNAL POINTS OF PRINTOUT.
   READ 110,NC,AK,DT,IWT,IDT,JPT
   IF (IDT.LT.1) IDT=1
   DTA=IDT*DT
   PRINT 110,NC,AK
15  PRINT 110,IDT,DT,DTA
   PRINT 100
   IF (IWT.NE.1) GO TO 11
   C  ASSIGN AREA WEIGHTS(ALF) TO EACH CELL IF IWT= 1 OR 2
   READ 113, (ALP(J),J=1,NC)
   11 ANC=NC
   DO 10 J=1,NC
   C  READ IN ROUTING ROUTINE OF FLOW THROUGH THE CELLS. IC=
   C  CELL NUMBER STARTING WITH THE MOST UPSTREAM CELL. ID=
   C  DOWNSTREAM CELL TO WHICH OUTPUT OF CELL IC IS DIRECTED.
25  KC= UPSTREAM CELLS FROM WHICH CELL IC RECEIVES INPUT.
   C  (CELLS MUST BE NUMBERED SEQUENTIALLY(IE. A HIGHER ORDER
   C  CELL CANNOT DISCHARGE INTO A LOWER ORDER CELL), CELL MUST
   C  RECEIVE ALL INPUTS FROM UPSTREAM CELLS BEFORE IT CAN
30  DISCHARGE, MAXIMUM OF 4 UPSTREAM INPUTS TO A SINGLE CELL).

```



```

35      READ 111, IC(J), ID(J), (KC(J,I), I=1,4)
      IF (IWT.EQ.0) ALP(J)=1.0/ANC
      PRINT 111, IC(J), ID(J), (KC(J,I), I=1,4), ALP(J)
10  CONTINUE
      PRINT 100
      IF (JPT.LT.1) GO TO 28
      IPT= CELL(S) AT WHICH AN OUTPUT PRINTOUT IS DESIRED.
      READ 115, (IPT(J), J=1, JPT)
      DO 26 I=1, NC
      YZ(I)=0.0
      IF (I.GT.JPT) IPT(I)=0
      IF (I.GT.JPT) GO TO 26
      YZ(I)=IPT(I)
      IPT(I)=0
26  CONTINUE
      DO 27 I=1, NC
      II=YZ(I)
      IF (II.EQ.0) GO TO 27
      IF (II.GT.NC) GO TO 27
      IPT(II)=II
27  CONTINUE
      PRINT 115, (IPT(I), I=1, NC)
28  PRINT 100
      AU=0.0
      UZ=0.0
      UF=UZ
      DO 15 I=1, 300
      T=I*DT
      TA=(I-1)*DT
      U(I)=(EXP(-TA/AK)-EXP(-T/AK))/DT
      AU=AU+(U(I)+UF)*DT/2.0
      NU=I+1
      UF=U(I)
      IF (UF.LT.0.00005.OR.AU.GT.0.99995) GO TO 17
15  CONTINUE
17  U(NU)=0.0

```

```

70      SU=AU+U(NU-1)*DT/2.0
        T=NU*DT
        PRINT 105
        PRINT 100
        PRINT 112,T,SU
        PRINT 100
        UZ=UZ/SU
        UF=UZ
        T=0.0
        AU=0.0
        PRINT 112,T,UF,AU
        DO 14 I=1,NU
          T=I*DT
          U(I)=U(I)/SU
          AU=AU+(U(I)+UF)*DT/2.0
          UF=U(I)
          PRINT 112,T,UF,AU
14      CONTINUE
        PRINT 103
        CALL PLOTA (DT,NU,U,UZ,300)
        READ IN TITLE OF DATA TO BE COMPARED
        16 READ 125, TITLE
            PRINT 100
            PRINT 125, TITLE
            IF (IWT.NE.2) GO TO 19
        C  READ IN CELL AREA WEIGHTS(ALF) FOR EACH INDIVIDUAL STORM
        C  EVENT IF IWT= 2.
        READ 113, (ALF(I),I=1,NC)
        PRINT 101, (ALF(I),I=1,NC)
        19 DO 23 I=1,100
            R(I)=0.0
        23 CONTINUE
            DO 24 I=1,300
                Q(I)=0.0
            24 CONTINUE

```

100

```

C      NR= NUMBER OF TIME INTERVALS THE INPUT IS DIVIDED INTO.
      READ 110, NR
      IF (NR.EQ.0) GO TO 99
C      R(I)= THE INPUT(L/T) FOR EACH OF THE NR TIME INTERVALS.
      READ 113, (R(I), I=1, NR)
      PRINT 100
      PRINT 106
      PRINT 100
      PRINT 112, (Q(I), I=1, 5)
C      NQ= NUMBER OF TIME INTERVALS THE OUTPUT TO BE COMPARED IS
C      DIVIDED INTO.
      READ 110, NQ
C      Q(I)= THE AMOUNT OF OUTPUT(L/T) FROM EACH TIME INTERVAL.
      READ 113, (Q(I), I=1, NQ)
      NPT=MAXO(NR, NQ)
      AR=0.0
      AQ=0.0
      QF=0.0
      DO 40 I=1, NPT
      T=I*DTA
      AR=AR+R(I)*DTA
      AQ=AQ+(Q(I)+QF)*DTA/2.0
      QF=Q(I)
      PRINT 112, T, Q(I), AQ, R(I), AR
      40 CONTINUE
      PRINT 102
C      PER(J)= THE PERCENT OF RUNOFF IN EACH CELL(J) THAT BECOMES
C      OUTPUT.
      READ 120, (PER(J), J=1, NC)
      120 FORMAT (20F4.2)
      NRA=NR
      IF (IDT.GT.0) NR=NR*IDT
      DO 30 J=1, NC
      PRINT 111, IC(J)

```

```

C      RJ(J,I)= SAME AS R(I) EXCEPT THIS SPECIFIES THE INPUT(L/I)
C      TO EACH CELL INDIVIDUALLY.  INCLUDE A SEPARATE CARD FOR
C      FOR EACH CELL.
140  READ 113, (RJ(J,I), I=1,NRA)
      IF (IDT.LT.2) GO TO 45
      DO 42 I=1,NRA
        X(I)=RJ(J,I)
42  CONTINUE
      DO 43 I=1,NRA
        DO 43 K=1,IDT
          L=IDT*(I-1)+K
          RJ(J,L)=X(I)
43  CONTINUE
45  DO 12 I=1,300
      X(I)=0.0
12  CONTINUE
      IT=0
      DO 13 I=1,4
        IT=IT+KC(J,I)
13  CONTINUE
      IF (IT.GT.0) GO TO 18
      NX=NR
      DO 25 I=1,NR
        X(I)=RJ(J,I)*ALF(J)*DT*PER(J)
25  CONTINUE
      GO TO 21
18  NX=NR
      DO 31 M=1,4
        MC=KC(J,M)
        IF (MC.EQ.0) GO TO 31
        NYB=NYA(MC)
        IF (NX.LT.NYB) NX=NYB
        YP=YZ(MC)
      DO 33 I=1,NYB
        X(I)=X(I)+(YC(MC,I)+YP)*DT/2.0
        YP=YC(MC,I)
170

```

```

175 IF (M.GT.1.OR.I.GT.NR) GO TO 33
    X(I)=X(I)+RJ(J,I)*ALP(J)*DT*PER(J)
33 CONTINUE
31 CONTINUE
21 NY=NU+NX-1
    IF (NY.GT.300) NY=300
    AX=0.0
    AY=0.0
    T=0.0
    YP=X(1)*UZ
    YZ(J)=YP
    IF (IPT(J).GT.0) PRINT 112,T,YP,AY
    NT=0
    NZ=0
    DO 20 N=1,NY
    T=N*DT
    IB=1
    IE=N
    IP(N.GT.NU) IB=N-NU+1
    IF(N.GT.NX) IE=NX
    AX=AX+X(N)
    YN=0.0
    DO 22 I=IB,IE
    IN=N+1-I
    YN=YN+X(I)*U(IN)
22 CONTINUE
    IF (N.LT.NX) YN=YN+UZ*X(N+1)
    NYA(J)=N
    AY=AY+(YN+YP)*DT/2.0
    YP=YN
    YC(J,N)=YN
    NZ=NZ+1
    IF (NZ.LT.IDT) GO TO 35
    NT=NT+1
    Y(NT)=YN
    NZ=0
205

```

```

210      35 IF (IPT(J).GT.0) PRINT 112,T,YN,AY,X(N),AX
        IF (YN.LT.0.00002) GO TO 29
        20 CONTINUE
        29 PRINT 100
        PRINT 115, IC(J),NX,NY,NYA(J),NT
        PRINT 104,AY,AX
        PRINT 100
        IF (IPT(J).EQ.0) GO TO 30
        YA=YZ(J)
        NPT=NT
        CALL PLOTA (DTA,NPT,Y,Y,YA,300)

220      30 CONTINUE
        NY=NYA(NC)
        PRINT 100
        NPT=MAX0(NT,NQ)
        SME=0.0
        SMA=0.0
        SMY=0.0
        DMX=0.0
        YMX=0.0
        QMX=0.0
        PRINT 102
        PRINT 125,TITLE
        PRINT 100
        PRINT 108
        PRINT 109
        SMQ=0.0
        C READ K WHICH EQUALS THE LAG FACTOR. LAG IS READ IN AS
        C AN INTEGER NUMBER OF COMPUTATION TIME INTERVALS(DTA).
        READ 666, K
        666 FORMAT (I8)
        NPTK = NPT + K
        DO 79 I=1,NPT
            Y(NPTK-I+1) = Y(NPT-I+1)
        79 CONTINUE
        DO 80 I= 1,K

```

```

245      Q(NPT+1) = 0.0
      Y(I) = 0.0
80 CONTINUE
      DO 60 I=1,NPT
      TA=I*DTA
      DQY= (Y(I)-Q(I))
      PRINT 112, TA, Q(I), Y(I), DQY
      DQY= ABS(DQY)
      IF (YMX .LT. Y(I)) IYMX=I
      IF (QMX .LT. Q(I)) IQMX=I
      IF (DMX .LT. DQY) DMX=DQY
      YMX=Y(IYMX)
      QMX= Q(IQMX)
      SMA=SMA+DQY
      SME=SME+DQY*DQY
      SMY=SMY+Y(I)
      SMQ=SMQ+ Q(I)
255      CONTINUE
      SMA=SMA/NPT
      SME=SQRT(SME/NPT)
      PDF=ABS(YMX-QMX)
      S3=DMX/QMX
      S4= PDF/QMX
      DMAXT= FLOAT(IABS(IQMX-IYMX)) * DTA
      S5=DMAXT/(FLOAT(IQMX) * DTA)
      SUMAV= SMQ/NQ
      S1=SME/SUMAV
      S6=(S1+S3+(6*S4)+(2*S5))/10
      PRINT 100
      PRINT 101, SME, SMA, DMX, PDF
      PRINT 100
      PRINT 121, S1, S3, S4, S5, S6
121      FORMAT(" S1=",F8.4," S3=",F8.4," S4=",F8.4," S5=",F8.4," S6=",
1          F8.4)
      PRINT*, " RAINFALL INPUT=", RJ(1,1)

```

```

280 PRINT 102
    PRINT 125, TITLE
    PRINT 100
    QZ=YZ(NC)
    CALL PLOTA(DTA, NPTK, Q, Y, QZ, 300)
    PRINT*, " SHIFT=", K
    GO TO 16
285
    99 STOP
    100 FORMAT (10H ****+****/1X)
    101 FORMAT (10F8.4)
    102 FORMAT (1H1)
    103 FORMAT (35H1 UNIT HYDROGRAPH FOR SINGLE CELL)
    104 FORMAT (5X,2F10.4)
    105 FORMAT (35H UNIT HYDROGRAPH FOR SINGLE CELL)
    106 FORMAT (35H RAINFALL AND RUNOFF DATA )
    108 FORMAT (30H OBS. COMP.)
    109 FORMAT (39H TIME FLOW FLOW DIFF./1X)
    110 FORMAT (I8,2F8.3,3I8)
    111 FORMAT (6I4,F8.4)
    112 FORMAT (F12.2,6F9.4)
    113 FORMAT (10F8.0)
    115 FORMAT (20I4)
    125 FORMAT (20A4)
    END
290
295
300

```



```

1  SUBROUTINE PLOTA(DT,NP,Q, Y,YZ,NN)
   DIMENSION Q(NN), Y(NN),X(101),S(11)
   DATA P/1H./,C/1H*//,O/1HO//,B/1H /
   SU=YZ
5  DO 1 I=1,NP
     UM=AMAX1(Q(I), Y(I))
     IF (UM.GT.SU) SU=UM
1  CONTINUE
     SL=ALOG10(SU)
     IF (SL.LT.0.0) SL=SL-1
     EM=INT(SL)+1
     U=10.0**EM
     U2=U/2
     U5=U/5
15  IF (SU.LE.U2) U=U2
     IF (SU.LE.U5) U=U5
     U=U/100
     DO 2 I=1,11
       S(I)=(I-1)*10*U
2  CONTINUE
     PRINT 40
     PRINT 41, S
     DO 5 I=1,101
       X(I)=B
5  CONTINUE
     DO 3 I=1,11
       II=(I-1)*10+1
       X(II)=P
3  CONTINUE
     XJ=0.0
     K=INT(YZ/U+0.5)+1
     XK=X(K)
     X(K)=C
     PRINT 42,XJ,X
35  X(K)=XK

```

```

DO 4 J=1,NP
PRINT 44,X
XJ=J*DT
L=INT(Q(J)/U+0.5)+1
K=INT( Y(J)/J+0.5)+1
XL=X(L)
XK=X(K)
X(L)=O
X(K)=C
PRINT 42,XJ,X
X(L)=XL
X(K)=XK
4 CONTINUE
PRINT 41, S
PRINT 43
RETURN
40 FORMAT (20X,18H * COMPUTED FLOW/20X,18H O OBSERVED FLOW//1X)
41 FORMAT (14X,11F10.3)
42 FORMAT (11X,F7.2,2X,61A1,40A1)
43 FORMAT (1H1)
44 FORMAT (20X,61A1,40A1)
END

```

40

45

50

55

APPENDIX B

LIST OF SYMBOLS

<u>Symbol</u>	<u>Description</u>	<u>Dimension</u>
a	Acceleration.	LT^{-2}
a_n	Weighting factor.	--
A	Coefficient parameter in the infiltration equation.	--
ALF	Area weight of cells.	--
C	Chezy turbulent resistance coefficient.	L^{1-m_T-1}
C_1	Chezy laminar resistance coefficient.	$L^{-1}T^{-1}$
E	Exponent in velocity relation.	--
f	Infiltration rate.	LT^{-1}
f_*	Dimensionless infiltration rate.	--
f_∞	Final infiltration rate.	LT^{-1}
$f_{\infty*}$	Dimensionless final infiltration rate.	--
F_0	Froude number.	--
g	Gravity	LT^{-2}
h	Flow depth.	L
h(t)	System impulse response function with respect to time.	--
H_0	Steady-state normal depth at downstream boundary.	L
I	Input rate.	L^3T^{-1}, LT^{-1}

<u>Symbol</u>	<u>Description</u>	<u>Dimension</u>
k_*	Kinematic flow number (Woolhiser number).	--
K	Cell storage ratio.	T
L	Lag factor.	T
L_0	Length of flow plane.	L
m	Mass.	M
m	Exponent in velocity-stage relation, or parameter used as an integer counter.	--
n	An integer parameter.	--
N	Number of cells.	--
N	Number of output data points.	--
PER	Percent of excess rainfall.	--
q	Flow rate per unit area.	LT^{-1}
q_c	Computed output at a given time.	LT^{-1}
q_o	Observed output at a given time.	LT^{-1}
q_{cp}	Computed peak output.	LT^{-1}
q_{op}	Observed peak output.	LT^{-1}
Q	Discharge rate.	L^3T^{-1}
Q_{RM}	Real, measured watershed discharge.	L^3T^{-1}, LT^{-1}
Q_s	Predicted discharge of parametric model.	L^3T^{-1}, LT^{-1}
Q_{o*}	Dimensionless soil water storage volume.	--
Q'_o	Accumulated soil water in excess of f up to time t_{p*} .	--
R	Roughness.	L
R_o	Hydraulic radius.	L

<u>Symbol</u>	<u>Description</u>	<u>Dimension</u>
RTE	Routing scheme of flow through the cells of the simple model.	--
S	Storage.	L^3
S or S_o	Slope or bed slope.	LL^{-1}
S	Objective function.	--
S_c	Channel bottom slope	--
S_f	Slope of energy line.	--
S_s	Channel side slope.	--
S_1	Normalized root mean square difference in discharge.	--
S_2	Normalized maximum difference at any time.	--
S_3	Normalized difference between magnitude of peaks.	--
S_4	Normalized difference in time of peak.	--
$S(t)$	Storage in cell with respect to time.	L
t	Time.	T
t_p	Time of ponding.	T
t_{p^*}	Dimensionless ponding time.	--
t_o	Reference time.	T
t_{o^*}	Dimensionless reference time.	--
t_*	Dimensionless time.	--
t_{cp}	Time to computed peak.	T
t_{op}	Time to observed peak.	T
u	Velocity.	LT^{-1}

<u>Symbol</u>	<u>Description</u>	<u>Dimension</u>
v	Velocity.	LT^{-1}
V_o	Velocity at normal depth.	LT^{-1}
x	Space coordinate.	L
$y(t)$	Output signal with respect to time.	LT^{-1}
α	Parameter in kinematic velocity-stage (h) relation, exponent in infiltration equation.	--
τ	Time function parameter.	T

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