POLARIZATION CONVERSION CUBE CORNER
RETRO-REFLECTOR

by

Karlton Crabtree

A Dissertation Submitted to the Faculty of the

COLLEGE OF OPTICAL SCIENCES

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

2 0 1 0

THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Karlton Crabtree entitled Polarization Conversion Cube Corner Retro-reflector and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

Russell Chipman

Date: 07 April 2010

Thomas D. Milster

Date: 07 April 2010

J. Scott Tyo

Date: 07 April 2010

Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copies of the dissertation to the Graduate College.
I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

Dissertation Director: Russell Chipman

Date: 07 April 2010

STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his or her judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: _____ Karlton Crabtree

ACKNOWLEDGEMENTS

I would like to thank my advisor Russell Chipman for innumerable contributions over the years, Steve McClain for providing timely feedback of excellent clarity, and all members of the polarization lab for comments and suggestions.

TABLE OF CONTENTS

LIST OF FIGURES	8
LIST OF TABLES	12
ABSTRACT	13
CHAPTER 1 Introduction	14
1.1 Introduction	14
1.2 The Electro-Magnetic Field, the Polarization State, and Orthogonality	14
1.2.1 Orthogonality of Electric Fields and Polarization States	16
1.2.2 Linear Polarization Beam Splitter	17
1.2.3 Circular Polarization Beam Splitter	19
1.2.4 PBS with elliptical eigenstates	21
1.3 Polarization Change on Reflection	23
1.4 Polarization Change from Multiple Reflections	26
1.4.1 Polarization of Reflection from a Hemisphere	27
1.5 Comments on Polarization Properties of sub-wavelength surface-relief gratings	27
1.5.1 Example of Properties of surfaces with SWG	29
1.6 Originality	32
CHAPTER 2 Polarization Ray Tracing	34
2.1 Introduction	34
2.2 Geometrical Ray Tracing	35
2.3 The Polarization Ray Trace Matrix	36
2.4 Determination of the Jones Matrix of a Surface Interaction	40
2.5 Algorithm for Tracing Polarized Rays	41
2.6 Examples of PRT Matrix Calculation	43
2.6.1 Example Calculation of a Ray interacting with a Plane Mirror at 45° Angle of Incidence	43
2.6.2 Three reflection x-y translation prism system	45
2.7 Example: Fast Parabola	51
2.8 Polarization in the entrance pupil from a single point emitter	53
2.8.1 Polarizer in a diverging or converging beam	54
2.9 Polarization Pupil Maps and Polarization Aberrations	55

TABLE OF CONTENTS – *Continued*

2.9.1	Jones Pupil Example: Kodak Brownie Camera	57
2.10	Wavefront Splitting	62
2.11	Effects of multilayer coatings	64
2.11.1	Thin films and optical path length	64
2.11.2	Effect of coating on position and direction of refracted ray . .	65
2.11.3	Considerations for non-planar interfaces	67
CHAPTER 3	Cube-Corner Retroreflector Polarization	70
3.1	Introduction	70
3.2	Cube-Corner Retroreflector Geometry	70
3.3	PRT matrix of a Cube-Corner Retroreflector	73
3.3.1	Example: path (1,2,3) of N-BK7 CCR at normal incidence . .	74
3.4	Mueller Matrix of Cube-Corner Retroreflector	76
3.4.1	Example: path (1,2,3) of NBK7 CCR at normal incidence . .	78
3.5	Summary	81
CHAPTER 4	Polarization Conversion Cube-Corner Retroreflector	82
4.1	Introduction	82
4.2	Definition of PCCCR	82
4.3	Properties of isotropic CCR with SWG anisotropic surfaces	85
4.3.1	Examples of SWG PCCCR	86
4.4	Properties of CCR having isotropic surfaces with both diattenuation and retardance	90
4.4.1	Properties of Dielectric TIR CCR	92
4.4.2	Properties of Metal Coated CCR	94
4.5	CCR having three identical reflecting surfaces with arbitrary elliptical retardance	95
4.6	CCR having three different isotropic reflecting surfaces	98
4.7	Future Work	101
CHAPTER 5	Polarization Aberrations of Spherical Surfaces with SWG . .	104
5.1	Introduction	104
5.2	One Dimensional SWG as an Anti-Reflection Coating	104
5.2.1	Polarization variation of 1-D AR SWG	105
5.2.2	Polarization aberrations of 1-D AR SWG on a spherical surface	108
5.2.3	Conclusion	111
APPENDIX A	Parameters needed for calculation of Mueller matrices of CCR retroreflectors	112
A.1	Ray Propagation Vectors	112

TABLE OF CONTENTS – *Continued*

A.2	s-polarization Vectors	116
A.3	pInc-polarization Vectors	120
A.4	pExit-polarization Vectors	126
A.5	AOI on each surface	132
A.6	Azimuthal angle on each surface	132
A.7	Rotation angles between each surface (ac=arccos)	134
APPENDIX B Parameters for all paths through a CCR at normal incidence .		135
B.1	The six paths through a CCR at normal incidence	135
B.2	Direction vectors	136
B.3	s-polarized vectors	137
B.4	p_{inc} -polarized vectors	139
B.5	p_{exit} -polarized vectors	141
B.6	Angle of Incidence	143
B.7	Azimuthal Angle	143
B.8	Rotation Angles	144
APPENDIX C List of Variables Names		145
REFERENCES		148

LIST OF FIGURES

1.1	Polarizing beam splitter with linear eigenstates	18
1.2	Polarizing beam splitter with circular eigenstates	19
1.3	Configuration for PBS with arbitrary elliptical eigenstates given by polar angles θ and χ . The angles for the retarders at the left and right of the figure are measured from the x-axis towards the y-axis. The angles for the retarders at the top and bottom are measured from the z-axis toward the y-axis.	22
1.4	Incident and reflected polarization coordinate systems. The electric field oscillations remain in the same plane before and after reflection, but the polarization orientation changes due to the change in coordinate system.	24
1.5	45° Light reflecting from a mirror at shallow angle of incidence	25
1.6	Space helix for circularly polarized light. Red is incident, blue is reflected, and green is transmitted. The reflected and transmitted helices are mirrors of each other. The handedness of the helix clearly changes on reflection.	26
1.7	Radial cross sections of a hemisphere with retroreflected rays	28
1.8	Experimental apparatus for RCWA validation. Polarization state generator and analyzer are configured at normal incidence to a right prism. Plate with SWG on one side is attached to prism with index matching fluid.	30
1.9	SWG is rotated around surface normal to vary the azimuthal angle. . . .	31
1.10	Comparison of the RCWA calculations and experimental Mueller matrix measurements of a Thorlabs GT25-08 grating in TIR at 45°AOI. Red is experimental data, blue is RCWA simulation.	31
2.1	Traditional geometric ray trace calculates the OPL for each ray segment	34
2.2	Polarization Ray Tracing calculates a polarization matrix $\mathbf{P}$ for each propagation and for each interface	35
2.3	Ray intercepting a plane mirror. All parameters used in the calculation are shown. d_q is the distance along the ray to the surface, $\vec{\mathbf{o}}_q$ is an arbitrary point on the surface, $\vec{\mathbf{r}}_q$ is an arbitrary point on the incident ray, and $\hat{\mathbf{n}}_q$ is the surface normal vector	42
2.4	Ray reflecting from a plane mirror	44
2.5	Three prism system. s-vectors in red, p-vectors in blue, one particular electric field state in green. Rays are shown in black.	45

LIST OF FIGURES – *Continued*

2.6	System with incident and exiting local coordinates displayed. In this case, the incident and exiting local coordinate chosen are parallel. . .	48
2.7	Three prism system with coordinate systems chosen such that the Jones matrix is an identity matrix.	49
2.8	Cross section of rays on a fast parabola. Polarization orientation shown by red lines.	51
2.9	Electric field vectors converging on focal point of parabola assuming uniform (1,0,0) incident. This pattern represents electric field vectors converging from a full sphere. “Front“ marks the hemisphere toward the open end of the parabola.	52
2.10	Electric field vectors emitted by a dipole oscillator.	53
2.11	Transmission axis of polarizer in three dimensions.	56
2.12	Optical layout of reversed landscape lens, as used in the Kodak Brownie camera. The object is an infinite distance to the left, and the image is to the right. Optical system is F/16.	58
2.13	Variation in the $\hat{s}, \hat{p}$ basis with pupil position. This pattern is the dipole basis, looking along the axis.	58
2.14	Phase terms of the Jones matrix pupil map. Two different coordinate system choices are shown: s,p coordinates on the left, and x,y coordinates on the right. s,p coordinates, in this example, provide plots that are easier to interpret.	59
2.15	Phase terms of the Jones matrix pupil map due only to the effects of the thin film coating.	59
2.16	Phase terms of the Jones matrix pupil map showing only the polarization dependent phase terms.	60
2.17	Polarization independent amplitude and phase in the pupil.	61
2.18	Amplitude terms of the Jones matrix pupil map showing only the polarization dependent portion.	62
2.19	Phase terms of the Jones matrix pupil map showing only the polarization dependent portion.	63
2.20	Figure showing multiple reflections occurring inside a single layer thin film coating.	64
2.21	Figure showing effective reflecting surface inside a multilayer coating.	66
2.22	Two layer reflective coating with several ray paths shown.	66
2.23	Figure showing lens with a thick coating. Solid ray is path accounting for coating, dashed ray is ignoring the coating.	68
2.24	Figure showing lens with a thick coating. Solid ray is path accounting for coating, dashed ray is ignoring the coating.	68

LIST OF FIGURES – *Continued*

2.25	Each interface in the stack creates a separate reflection with a defocus equal to the layer thickness.	69
2.26	Each interface in the stack creates a separate reflection. Since the thickness of the coating varies with radius, each layer has different radius, making all focal points equal.	69
3.1	A Cube-Corner Retroreflector is the result of cutting one corner off a cube whose interior is reflective.	71
3.2	A corner cube retroreflector showing the vertex V, the center of the front face O, the other three corners, A, B, and C, and the centers of the edges of the front face, E, F, and G.	71
4.1	Experimental configuration for determining polarization coupling. . .	83
4.2	All possible Mueller matrices for a TIR CCR with SWG surfaces. . .	84
4.3	Minimum linear polarization coupling for all possible TIR CCR with SWG surfaces. The diagonal white line shows isotropic surfaces. . .	86
4.4	CCR tip having gratings of period 175 nm with a depth of 240 nm and a duty cycle of 0.75	88
4.5	Profiles of the SWG surfaces considered.	89
4.6	Angle of Incidence and Azimuthal angles for the various plots that follow.	89
4.7	Intensity reflectivity and MLPC as the wavelength is varied.	89
4.8	Isotropic CCR specified by diattenuation and retardance on each surface.	91
4.9	Minimum linear polarization coupling for isotropic CCR having diattenuation and retardance on each surface.	92
4.10	Mueller matrix of an isotropic CCR as a function of retardance of each surface.	93
4.11	Mueller matrix of all possible hollow bare metal CCR	94
4.12	Minimum linear polarization coupling for CCR having surfaces with elliptical retardance. Regions shown have 90% MLPC. Green planes are the cross sections shown in figures 4.13 and 4.14.	95
4.13	Mueller matrix of a CCR having surfaces with linear retardance at arbitrary orientation.	96
4.14	Mueller matrix of a CCR having surfaces with circular retardance magnitude 0.8165π	97
4.15	Minimum linear polarization coupling for CCR having three different isotropic surfaces. Regions shown have MLPC 0.7, with the peak of each region being 0.75.	99

LIST OF FIGURES – *Continued*

4.16	Mueller matrix of an isotropic CCR with surface 2 having zero retardance.	100
4.17	Mueller matrix of an isotropic CCR with surface 2 having $\frac{\pi}{2}$ retardance.	101
4.18	Mueller matrix of an isotropic CCR with surface 2 having π retardance.	102
5.1	1-D anti-reflection SWG of Santos & Bernardo	104
5.2	Intensity Transmittance of SWG	105
5.3	Phase of SWG	106
5.4	Pattern showing grating lines on surface	107
5.5	Plane of incidence on surface is radially oriented	107
5.6	Magnitude (Amplitude Transmittance) of the polarization aberration function.	109
5.7	Phases of the polarization aberration function.	110
5.8	Retardance Magnitude (deg)	110
5.9	Retardance Orientation (deg)	111

LIST OF TABLES

4.1	Intensity reflection coefficient and MLPC as a function angle of incidence and azimuthal angle for each of three PCCCR solutions. The color scaling is the same in all figures.	90
5.1	Phase Shifts of SWG at 24° AOI	106

ABSTRACT

This document presents the polarization conversion cube-corner retroreflector (PCCCR). The PCCCR is a cube-corner retroreflector which transforms the electric field as follows: the major axis is rotated by 90° and the handedness is reversed.

Since the polarization properties of a CCR are dependent on the polarization properties of each surface, exploration of the space of Mueller matrices is organized by surface type. The Mueller matrix of CCR having each of several surface types is calculated, including the traditional hollow metal and solid glass CCR types.

PCCCR only occur for non-isotropic surface types. Four particular surface polarization properties are found which produce PCCCR. Three examples of PCCCR are presented using sub-wavelength grating surfaces. Several other interesting CCR are presented, including a 45° polarization rotator.

CHAPTER 1

Introduction

1.1 Introduction

Cube-corner retroreflectors (CCR) are used extensively in the world today, for purposes as diverse as precision markers for measurements of large mirrors to elements which enhance visibility of signs and lane markers on streets. This work introduces a new class of CCR, the polarization conversion CCR (PCCCR) which performs a transformation of the electric field that has not been seen in passive devices to date. The PCCCR rotates the major axis of the electric field 90° and the handedness is reversed.

This document is structured into five major parts. The first chapter introduces several concepts which are important for understanding the PCCCR. The second chapter introduces a ray tracing formalism used for calculating properties of the PCCCR. The third chapter discusses calculation of the polarization properties of CCR. The fourth chapter shows the polarization properties of several types of CCR, including some PCCCR. Finally, the last chapter presents an example of angular dependence of a sub-wavelength grating using the polarization aberrations of a spherical surface as an example system.

1.2 The Electro-Magnetic Field, the Polarization State, and Orthogonality

Light is an electro-magnetic field that oscillates in a plane transverse to the Poynting vector. In isotropic media, the positive propagation direction of the phase front coincides with the Poynting vector. In this work, as all bulk media considered are simple, linear, and isotropic. For these media, the wave vector and Poynting vector can be opposite ($\hat{\mathbf{k}} = -\hat{\mathbf{S}}$) in certain situations such as negative index media, but

are usually equal ($\hat{\mathbf{k}} = -\hat{\mathbf{S}}$). In this work only the latter case is considered, so the propagation direction and the Poynting vector are treated synonymously.

When defining the polarization state, a local right-handed rectilinear coordinate system $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ is used, where the $+\hat{\mathbf{z}}$ direction is co-aligned with the ray propagation vector. The $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ coordinates are chosen based on what is most convenient for a particular problem. Common choices are the $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}$ coordinates of a particular surface or an orientation defined by an instrument that is performing a measurement. Here the $\hat{\mathbf{x}}$ axis will sometimes be referred to as horizontal and the $\hat{\mathbf{y}}$ axis will be referred to as vertical.

The polarization state of a ray is conventionally defined by the variation of the electric field vector at a particular plane, as a function of time, with the ray approaching the observer who, in this convention, is looking along the $-\hat{\mathbf{z}}$ direction. While the temporal variation of a particular electric field in the plane is unique, the description of this variation - the name given to the polarization state - has multiple conventions. In this document, the orientation of the major axis of the polarization ellipse is measured in the counter-clockwise direction from the $+\hat{\mathbf{x}}$ direction. The handedness of the polarization ellipse is right if the time evolution of the tip of the electric field vector is a clockwise rotation.

While the polarization state is defined in a local coordinate system, the electric field oscillates as a function of both space and time, and is a three dimensional vector. The electric field associated with a particular ray is transverse to that ray. When discussing the properties of the electric field, one can examine the evolution of the tip of the electric field vector in time, as seen from a particular global observer. Alternately, one can look at the variation in space at fixed time in some particular coordinate system. This last approach is desirable for some of the discussions which follow, and will be called the spatial picture of the electric field.

In the spatial picture, an elliptical electric field variation traces out a helix in space, much like the thread of a screw, and handedness is defined by the direction in which the helix rotates when the thumb is aligned with the axis of the helix. This definition of handedness is independent of the observer position. The orientation of

the ellipse will be defined in whichever global coordinate system is in use for the problem under discussion.

1.2.1 Orthogonality of Electric Fields and Polarization States

Mathematically, two vectors are defined to be orthogonal if the inner product of vector $\vec{x}$ with vector $\vec{y}$ equals zero. This definition can be applied to polarization states in the form of Jones or Stokes vectors. The same definition can be applied to a vector field, such as the full time and space variant electric field. In the case of the electric field, the test determines whether the two vector fields are orthogonal at all points in both space and time.

This mathematical definition is applicable, but not always useful. In the real world, polarizing beam splitters (PBS) are used to separate beams having different electric field components. Unfortunately, PBSs do not follow the mathematical orthogonality convention. Consider first some examples of electric field orthogonality, then some examples of PBSs.

Consider two linearly polarized electric fields.

$$\vec{\mathbf{E}}_1 = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} e^{i(+kz-\omega t)} \quad (1.1)$$

$$\vec{\mathbf{E}}_2 = \begin{pmatrix} \cos(\theta + \frac{\pi}{2}) \\ \sin(\theta + \frac{\pi}{2}) \\ 0 \end{pmatrix} e^{i(+kz-\omega t)} \quad (1.2)$$

The scalar product $\vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2^* = 0$ demonstrates orthogonality. Reversing the propagation direction of one or both electric field vectors does not change the orthogonality.

Consider now right and left hand circularly polarized electric fields propagating in both the positive and negative $\hat{\mathbf{z}}$ directions.

$$\vec{\mathbf{E}}_{R+z} = \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} e^{i(+kz-\omega t)} \quad (1.3)$$

$$\vec{\mathbf{E}}_{L+z} = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{i(+kz-\omega t)} \quad (1.4)$$

$$\vec{\mathbf{E}}_{R-z} = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{i(-kz-\omega t)} \quad (1.5)$$

$$\vec{\mathbf{E}}_{L-z} = \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} e^{i(-kz-\omega t)} \quad (1.6)$$

First consider co-propagating fields. For co-propagating fields, the two handednesses are orthogonal.

$$\vec{\mathbf{E}}_{R+z} \cdot \vec{\mathbf{E}}_{L+z}^* = \vec{\mathbf{E}}_{R-z} \cdot \vec{\mathbf{E}}_{L-z}^* = 0 \quad (1.7)$$

Also, for co-propagating fields, two fields of the same handedness have constant irradiance.

$$\vec{\mathbf{E}}_{R+z} \cdot \vec{\mathbf{E}}_{R+z}^* = \vec{\mathbf{E}}_{L+z} \cdot \vec{\mathbf{E}}_{L+z}^* = 2 \quad (1.8)$$

For counter propagating fields ($\vec{\mathbf{k}}_1 = -\vec{\mathbf{k}}_2$), right and left circular are not orthogonal, but form a standing wave along the propagation axis.

$$\vec{\mathbf{E}}_{R+z} \cdot \vec{\mathbf{E}}_{R-z}^* = 2e^{2zik} \quad (1.9)$$

Also, for counter-propagating fields, two fields of the same handedness are orthogonal, independent of the relative phase shift between the two fields.

$$\vec{\mathbf{E}}_{R+z} \cdot \vec{\mathbf{E}}_{R-z} = \vec{\mathbf{E}}_{L+z} \cdot \vec{\mathbf{E}}_{L-z} = 0 \quad (1.10)$$

Given the difficulties with keeping track of what electric fields are orthogonal, it is worthwhile to examine the behavior of electric fields at a polarizing beam splitter.

1.2.2 Linear Polarization Beam Splitter

Most polarizing beam splitters (PBS) have linear eigenstates, as shown in figure 1.1. The most common types have a thin film dielectric coating that transmits the

Figure 1.1: Polarizing beam splitter with linear eigenstates

p-polarization and reflects the s-polarization. The plane of oscillation of the electric field for each of these states is the same regardless of the direction of propagation. Also, as shown earlier, these two polarizations are orthogonal regardless of whether they are co-propagating or counter-propagating.

In the figure, the electric field 1 is

$$\vec{\mathbf{E}}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{i(kz - \omega t)} \quad (1.11)$$

and electric field 2 is

$$\vec{\mathbf{E}}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{i(kz - \omega t)} \quad (1.12)$$

Figure 1.2: Polarizing beam splitter with circular eigenstates

and electric field 3 is

$$\vec{\mathbf{E}}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{i(-kx - \omega t)} \quad (1.13)$$

1.2.3 Circular Polarization Beam Splitter

With the addition of quarter wave linear retarders (QWLR), the linear polarizing beam splitter described can be converted to a circular polarizing beam splitter (CPBS). This beam splitter transmits right hand circularly polarized electric fields and reflects left hand circularly polarized fields. Despite the non-orthogonality of right and left hand circularly polarized fields when counter-propagating, the CPBS still transmits right circular and reflects left circular after the direction of propagation has been reversed. The electric fields in the various spaces are listed.

For the transmitted ray, the electric field in region 1 is

$$\vec{\mathbf{E}}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} e^{i(kz-\omega t)} \quad (1.14)$$

and electric fields 2 and 3 are

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{i(kz-\omega t)} \quad (1.15)$$

and electric field 4 is

$$\vec{\mathbf{E}}_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} e^{i(kz-\omega t)} \quad (1.16)$$

For the reflected ray, the electric field in region 5 is

$$\vec{\mathbf{E}}_5 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{i(kz-\omega t)} \quad (1.17)$$

and electric field 6 is

$$\vec{\mathbf{E}}_6 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{i(kz-\omega t)} \quad (1.18)$$

and electric field 7 is

$$\vec{\mathbf{E}}_7 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{i(kx-\omega t)} \quad (1.19)$$

The addition of another QWLR on the output port could re-convert field 7 to left circular polarization again.

Now, reverse the transmitted ray. In region 4, the electric field is

$$\vec{\mathbf{E}}_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{i(-kz-\omega t)} \quad (1.20)$$

and electric field 2 and 3 is

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{i(-kz-\omega t)} \quad (1.21)$$

and electric field 1 is

$$\vec{\mathbf{E}}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{i(-kz-\omega t)} \quad (1.22)$$

This has shown that circular polarizations are transmitted or reflected based on handedness, independent of propagation direction.

1.2.4 PBS with elliptical eigenstates

Consider a more general elliptical polarizing beam splitter (EPBS). Suppose a PBS cube transmits horizontally polarized electric fields, and the desired device will transmit an elliptically polarized electric field with orientation θ and ellipticity χ . An EPBS will completely transmit one electric field, and completely reflect one electric field. These two electric fields are mathematically orthogonal when co-propagating. Defining these two electric fields in the spatial picture, the EPBS always transmits the same electric field, regardless of 180° changes in propagation direction. The EPBS can be thought of as a general polarization separator. Any electric fields with a particular orientation and ellipticity in global coordinates can be separated from a second electric field. This second electric field is not mathematically orthogonal to the first electric field, but represents a physically more meaningful parameter.

A conceptually simple design for this PBS with elliptical eigenstates (EPBS) has two linear retarders on each port. Consider for now, only the two retarders on the port where light enters the system. On this port, the orientation of all components is measured from the x-axis rotating toward the y-axis. Let the first retarder encountered be a quarter wave linear retarder (QWLR) with fast axis θ . The electric field emerges linearly polarized having orientation $\theta + \chi$. Let the second linear retarder be a half wave linear retarder (HWLR) with fast axis $\frac{\theta + \chi}{2}$. The

Figure 1.3: Configuration for PBS with arbitrary elliptical eigenstates given by polar angles θ and χ . The angles for the retarders at the left and right of the figure are measured from the x-axis towards the y-axis. The angles for the retarders at the top and bottom are measured from the z-axis toward the y-axis.

electric field with orientation θ and ellipticity ψ emerges horizontally polarized, transmitting through the PBS cube.

Consider now the two linear retarders on the PBS face where transmitted light exits. For an ideal PBS cube, a rotation about the s-axis by 180° has no effect on the polarization properties. Therefore, the angles for the retarders on the exit face can be found by rotating the incident pair of linear retarders about the PBS cube s-axis by 180° . When listed in the same coordinate system as the incident retarders, the exiting half wave linear retarder has orientation $\frac{\pi+\theta+\chi}{2}$ and the exiting quarter wave linear retarder has orientation $\frac{\pi}{2} + \theta$. The angles for the other two faces lie in a different plane, but can also be found by rotating the PBS. This PBS with elliptical eigenstates is shown in figure 1.3.

Due to the complexities involved in the mathematical orthogonality of electric fields, it is far more practical to use this elliptical PBS as a basis for orthogonality. If one configures the PBS to transmit one electric field, the orthogonal electric field can be defined as the field that is reflected. This definition is also beneficial due to independence from choice of global coordinate system.

1.3 Polarization Change on Reflection

Since chapters 3 and 4 discuss a reflective device, a discussion of the transformation of the electric field on reflection is in order. When an electric field is incident on an ideal reflective material, the electric field oscillations drive the electrons in the medium along a path determined by the projection of the electric field ellipse onto the surface. The excited electrons then re-radiate; therefore the radiated electric field, when projected onto the reflecting surface, has the same pattern as the incident field.

For normal incidence reflection, the major axis of the electric field ellipse lies in the same plane for both the incident and exiting rays. Figure 1.4 shows a not-quite-normal incidence reflection (to make it easier to separate the incident and exiting electric fields). The polarization state of the incident and exiting fields are not the

Figure 1.4: Incident and reflected polarization coordinate systems. The electric field oscillations remain in the same plane before and after reflection, but the polarization orientation changes due to the change in coordinate system.

same, due to the coordinate reversal in definition of the polarization state. Figure 1.4 also shows the coordinate system defining the incident and exiting polarization states. Since the $+\hat{\mathbf{x}}$ direction of the local coordinate system has changed, relative to the global coordinate system, and the observer has moved to have the light approaching the observer both before and after reflection, the exiting polarization orientation is the negative of the incident polarization orientation. This also leads to the requirement, for the Mueller and Jones calculi, to change the mathematical description of a particular system component, depending on which direction the light is propagating.

Figure 1.5: 45° Light reflecting from a mirror at shallow angle of incidence

For light reflected at grazing incidence, the projections of the incident and exiting electric field still must form the same ellipse on the reflecting plane. In this case, this results in the orientation of the oscillations changing from positive to negative on reflection. As the coordinate system used and the observer position for defining the exiting polarization state are negligibly different than the incident, the orientation of the polarization state also changes from positive to negative, which is the same as the normal incidence case.

The handedness of the polarization ellipse also changes on reflection.^{1,2}

Figure 1.6: Space helix for circularly polarized light. Red is incident, blue is reflected, and green is transmitted. The reflected and transmitted helices are mirrors of each other. The handedness of the helix clearly changes on reflection.

1.4 Polarization Change from Multiple Reflections

In the preceding section, it was established that for grazing incidence reflection, the plane of the major axis of the ellipse switches between positive and negative orientation and the handedness of the ellipse changes. Therefore, if the light undergoes two grazing incidence reflections, with negligible direction change, the electric field ellipse will be the same as the incident electric field. Despite this occurring at grazing incidence, there is a small change in direction. Therefore, after a very large number of reflections, the propagation direction of the light could reverse. This section will examine the electric field after the direction reversal.

Consider first the case where there are $2N$ grazing incidence reflections during

the direction reversal. Clearly the handedness of the electric field oscillations will be the same as the incident handedness. If the directional change were negligible, the major axis orientation would also be the same as the incident orientation. There is a gradual change in direction; alternately, a gradual rotation about the axis given by $(\hat{\mathbf{k}}_{incident} \otimes \hat{\mathbf{k}}_{exiting})$. This will cause a gradual rotation of the electric field about the same axis, and by the same amount. The handedness of the electric field is unchanged by rotation. For a 180° change in propagation direction, the orientation of the major axis of the electric field ellipse changes from $+\theta$ to $-\theta$. For the case of $2N+1$ reflections, the result can be found by taking the result for an even number of reflections and adding an additional reflection, which changes the handedness and again flips the orientation, resulting in the major axis of the electric field ellipse remaining in the same plane as the incident electric field ellipse.

1.4.1 Polarization of Reflection from a Hemisphere

One example that demonstrates the changes in the electric field with number of reflections is a perfect hemisphere, as shown in figure 1.7. If the whole hemisphere is illuminated, certain rays will return anti-parallel to the incident rays. These rays are located at radii $r = R - R\sin[\pi]/(2n)$. For a hemisphere with a radius of 10, the first few rays that retroreflect are located at radii 0., 2.92893, 5., 6.17317, 6.90983, 7.41181, 7.77479, 8.0491, 8.26352, 8.43566. These radii each correspond to an integer number of reflections, 1,2,3,..., so each successive retroreflected ray will show a different set of electric field transformations.

1.5 Comments on Polarization Properties of sub-wavelength surface-relief gratings

Most materials used in optics are isotropic, that is the index of refraction is the same in all directions. In anisotropic media, however, the effective index of refraction depends on the direction of the electric field vector under consideration. Surface relief gratings on isotropic media create surfaces with anisotropic behavior.

For anisotropic surfaces, the polarization properties are dependent both on the

Figure 1.7: Radial cross sections of a hemisphere with retroreflected rays

angle of incidence and the angle of the plane of incidence with respect to the direction of anisotropy, called the azimuthal angle. For media with surface relief gratings, the surface properties undergo sudden and dramatic changes including, but not limited to Wood's anomalies as a function of angle of incidence, azimuthal angle, and wavelength.

Unfortunately, there exists no closed form solution to accurately predict the polarimetric behavior of gratings whose period is similar to the wavelength. There are some approximate closed form theories.³⁻⁶ These theories have been shown to produce reasonable approximations for the magnitude of various diffracted orders, and simultaneously produce incorrect result for the phase shifts. Since the relative phase shift is vitally important when conducting polarization analysis, these closed

form effective medium theories are not usable for polarization analysis. There are also a number of rigorous numerical techniques for determination of the properties, including rigorous coupled wave analysis (RCWA).

Rigorous coupled wave analysis (RCWA) is a numerical analysis technique developed in the mid 1990s.⁷⁻¹⁶ The accuracy is dependent on several factors but has been proven, in the limit of infinite computation time, to be an exact solution to the interaction of infinite planar electric fields incident on infinite planar surfaces with periodic structures. For surfaces on which the grating is the same for more than a few periods, RCWA has been shown to produce a useful approximation to the actual behavior of a real, non-infinite incident field.¹⁷⁻¹⁹

In this work, isotropic media with surface relief gratings are used for the device in chapter 4. For the application discussed, gratings which have only zero order diffraction terms propagating are desirable, which will be referred to as non-diffracting. (ignoring diffraction into evanescent modes) For this reason, most of the gratings are much smaller than the design wavelength, making them sub-wavelength gratings (SWG). Note that SWG are not always non-diffracting, as in example 1.5.1, and that in some cases gratings with a period slightly larger than the wavelength can be non-diffracting. All calculations shown in this document which involve SWG were calculated by RCWA.

1.5.1 Example of Properties of surfaces with SWG

To validate the RCWA code used for calculations, an experiment was performed. A Thorlabs GT25-08 grating was obtained, having 830 grooves/mm with a 29.87° blaze angle and an area of 25 x 25 mm in epoxy on a glass substrate. The glass side of this grating was index-matched to the hypotenuse of a N-BK7 right prism. The prism was placed in a dual-rotating retarder polarimeter operating at 1550 nm such that the incident and exiting beams were at normal incidence to the other two faces of the prism, as shown in figure 1.8. This allowed measurement of the grating in TIR with an angle of incidence of 45° . The azimuthal angle of the grating could be rotated by rotating the grating about the surface normal vector of the prism

Figure 1.8: Experimental apparatus for RCWA validation. Polarization state generator and analyzer are configured at normal incidence to a right prism. Plate with SWG on one side is attached to prism with index matching fluid.

hypotenuse, as shown in figure 1.9. Measurements were taken for azimuthal angles from 0° to 180° in intervals of 2.5° . The index of refraction of the epoxy used in grating is not given by Thorlabs. Therefore, the refractive index of the simulation was varied to give the best fit to the measured data, with $n=1.5$ found to be a good fit. A graphical comparison of the experimental and theoretical results is shown in figure 1.10. The largest difference is diagonal depolarization. This is believed to be due, at least in part, to the polarimeter. This demonstrates adequate agreement between the RCWA code used and the experimental result.

Figure 1.9: SWG is rotated around surface normal to vary the azimuthal angle.

Figure 1.10: Comparison of the RCWA calculations and experimental Mueller matrix measurements of a Thorlabs GT25-08 grating in TIR at 45° AOI. Red is experimental data, blue is RCWA simulation.

1.6 Originality

Chapter 1 contains primarily background material which is relevant for the other chapters. First, the electric field and polarization state are defined and compared, followed by a large section discussing orthogonality of electric fields. This chapter also contains an original measurement of the Mueller matrix of a particular sub-wavelength grating as a function of azimuthal angle, used for validating the calculations performed.

Chapter 2 begins with a definition of three-dimensional polarization ray trace matrices, followed by an algorithm for tracing rays using the polarization ray trace matrices, both of which have been discussed by other authors.^{20,21} The polarization of a fast parabola has been addressed,^{22,23} but not with this approach. The author has been unable to locate other work discussing the transmitted electric field of an ideal wire grid polarizer in a converging beam. Finally, the discussion of the Jones matrix pupil of a landscape lens is original.

Chapter 3 contains three major parts, geometry, PRT matrix calculation, and Mueller matrix calculation. The geometry of a cube-corner retroreflector has been addressed by other authors,²⁴⁻³⁰ as has the calculation of Mueller matrices,³¹⁻³³ but the calculation of PRT matrices of cube-corner retroreflectors is original.

Chapter 4 is entirely original work, with five major sections, four of which have never been addressed. Isotropic cube-corner retroreflectors are the only type to have been addressed by previous authors. This work approaches these calculations differently than previous authors. The other four, entirely original sections are: definition of polarization conversion, cube-corner retroreflectors with three-different isotropic surfaces, cube-corner retroreflectors with elliptically retarding surfaces, and cube-corner retroreflectors with SWG surfaces.

Chapter 5 is entirely original, and discusses the polarization aberration function of a spherical surface with a sub-wavelength grating used as an anti-reflection coating.

Appendix A and Appendix B are listings of equations related to propagating rays through cube-corner retroreflectors. The fundamental approach used to generate

these equations has been published before, along with examples of the equations, but the author is unaware of any other complete listings of these equations.

CHAPTER 2

Polarization Ray Tracing

2.1 Introduction

The first part of this chapter presents the formalism used in chapter 3. The second major section of this chapter presents a discussion of the issues associated with defining polarization state and polarization coordinate systems. While most of the algorithms in this chapter have been published before,^{21,34–47} these are reproduced here for completeness.

Figure 2.1: Traditional geometric ray trace calculates the OPL for each ray segment

Figure 2.2: Polarization Ray Tracing calculates a polarization matrix $\mathbf{P}$ for each propagation and for each interface

2.2 Geometrical Ray Tracing

Traditional geometrical ray tracing involves the calculation of surface intercepts, ray direction vectors, and optical path length (OPL) for rays passing through an optical system. (figure 2.1) These algorithms are well documented in the literature.^{48–60} To obtain polarization information, it is necessary to calculate the polarization effects at each interface as well as on propagation. (figure 2.2) Polarization can be traced by tracking the electric field, the polarized irradiance, or by constructing matrices that describe each ray path. In this work, polarization matrices will be constructed for each ray path. For the systems considered here, having no anisotropic materials, the polarization does not change during propagation, only at interfaces, so the polarization matrices for propagation are ignored.

2.3 The Polarization Ray Trace Matrix

Several mathematical techniques have been developed for describing polarization transformations, with the Jones calculus (electric fields) and the Mueller calculus (irradiances) being the most widely applied. These are difficult to interpret due to a description based on local coordinate systems, which may not be the same for the ray segments incident on and exiting from a surface. For some problems, maintaining a single global coordinate system simplifies interpretation of the results. Here a three dimensional form of the electric field will be used, with a three dimensional extension of the Jones calculus used for transformations of this field.

The electric field vector is written

$$\vec{\mathbf{E}} = \begin{pmatrix} A_x e^{i\phi_x} \\ A_y e^{i\phi_y} \\ A_z e^{i\phi_z} \end{pmatrix} e^{i(\frac{2\pi n}{\lambda_0} \hat{\mathbf{k}} \cdot \vec{\mathbf{x}} - \omega t)} \quad (2.1)$$

where A_x , A_y , and A_z are the amplitudes of the field, ϕ_x , ϕ_y , and ϕ_z are the phases of the field components, $\hat{\mathbf{k}}$ is the wave vector, $\vec{\mathbf{x}}$ is the field position, ω is the frequency, and t is the time, and λ_0 is the vacuum wavelength. Note that the electric field of a ray is always transverse to the ray direction vector, making the quantity a time and space variant two-dimensional vector embedded within the three-dimensional space.

The Polarization Ray Trace (PRT) matrix is a complex valued 3rd order 2nd rank tensor which describes the transformation of a three-dimensional electric field vector in global Cartesian coordinates. These matrices can describe the polarization effects due to both propagation and interfaces. A global, right handed Cartesian coordinate system is used for describing the electric field. A PRT matrix can convert any electric field propagating in any direction into any other electric field propagating in any other direction.

Many optical surfaces and media have a simple description when expressed in one particular local coordinate basis. For this reason, the PRT matrix is often constructed from the product of three matrices. The first is a unitary rotation from the global coordinate system into the local coordinate system. The second describes

the interaction in the local coordinate basis. It is referred to here as $\mathbf{J3}$ since the matrix is a three-dimensional form of the Jones matrix of the surface. The third is a unitary rotation from the local coordinate basis into the global coordinate system. This is written as

$$\mathbf{P}_{interface} = \mathbf{O}_{q_{exit}} \cdot \mathbf{J3}_q \cdot \mathbf{O}_{q_{inc}} \quad (2.2)$$

For surfaces of isotropic media, the unitary transformation matrices are real. The letter $\mathbf{O}$ has been used historically for these coordinate transformation matrices. The combined PRT matrix for a system can be constructed by taking the inner product of the PRT matrices for the individual elements.

$$\mathbf{P}_{system} = \prod_{q=N,-1}^1 \mathbf{P}_q = \mathbf{P}_N \cdot \mathbf{P}_{N-1} \cdot \dots \cdot \mathbf{P}_2 \cdot \mathbf{P}_1 \quad (2.3)$$

While the choice of local coordinate basis is arbitrary, most optical surfaces have eigenstates along the s & p polarizations. Consider an isotropic interface such as an ordinary lens or mirror surface. The natural basis for this interaction is $\hat{\mathbf{s}}$, $\hat{\mathbf{p}}$, and $\hat{\mathbf{k}}$ unit vectors. The properties of the PRT matrix must be such that, $\hat{\mathbf{s}}$ is an eigenvector ($\mathbf{P} \cdot \hat{\mathbf{s}} = t_s \hat{\mathbf{s}}$), where t_s is the complex valued amplitude transmission (or reflection) coefficient for the s-polarization state. $\mathbf{P} \cdot \hat{\mathbf{p}}_{inc} = t_p \hat{\mathbf{p}}_{exit}$ gives an eigen-like relationship between $\hat{\mathbf{p}}_{inc}$ and $\hat{\mathbf{p}}_{exit}$ where t_p is the complex valued amplitude transmission coefficient for the p-polarized state. $\hat{\mathbf{p}}_{inc}$ and $\hat{\mathbf{p}}_{exit}$ are generally not parallel, and therefore not eigenvectors, but still an obviously useful coordinate pair. Similarly, the ray direction vector also has an eigen-like relationship $\mathbf{P} \cdot \hat{\mathbf{k}}_{inc} = t_k \hat{\mathbf{k}}_{exit}$. As there is no amplitude transmission coefficient produced by the Fresnel or thin film equations for the component along the direction of propagation, the value t_k can be chosen arbitrarily, with obvious choices of zero and one. A choice of one makes $\hat{\mathbf{k}}_{exit} = \mathbf{P} \cdot \hat{\mathbf{k}}_{inc}$. A choice of zero makes $\hat{\mathbf{k}}_{inc}$ an eigenvector of the matrix. Zero is used in this document.

Given that the polarization effects of an isotropic surface are calculated in local $\hat{\mathbf{s}}$, $\hat{\mathbf{p}}$, $\hat{\mathbf{k}}$ coordinates, and the PRT matrix is in global coordinates, it is necessary to convert between the two sets of coordinate systems. To rotate from global coordinate

to local coordinates, a matrix consisting of the projection of each global coordinate basis onto each local coordinate basis is used. This matrix is an orthogonal matrix, given by

$$\mathbf{O}_{inc_q} = \begin{pmatrix} s_{qx} & s_{qy} & s_{qz} \\ p_{inc_{qx}} & p_{inc_{qy}} & p_{inc_{qz}} \\ k_{inc_{qx}} & k_{inc_{qy}} & k_{inc_{qz}} \end{pmatrix} \quad (2.4)$$

An alternative way of writing the terms collects the terms into column vectors for the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ components

$$\mathbf{O}_{inc_q} = \begin{pmatrix} \vec{\mathbf{v}}_{1_{inc_q}} & \vec{\mathbf{v}}_{2_{inc_q}} & \vec{\mathbf{v}}_{3_{inc_q}} \end{pmatrix} \quad (2.5)$$

where

$$\vec{\mathbf{v}}_{1_{inc_q}} = \begin{pmatrix} s_{qx} \\ p_{inc_{qx}} \\ k_{inc_{qx}} \end{pmatrix} \quad \vec{\mathbf{v}}_{2_{inc_q}} = \begin{pmatrix} s_{qy} \\ p_{inc_{qy}} \\ k_{inc_{qy}} \end{pmatrix} \quad \vec{\mathbf{v}}_{3_{inc_q}} = \begin{pmatrix} s_{qz} \\ p_{inc_{qz}} \\ k_{inc_{qz}} \end{pmatrix} \quad (2.6)$$

Similarly, to convert from local coordinate back into global coordinates, a rotation of the local coordinates onto the global coordinates are used. This is also an orthogonal matrix, given by

$$\mathbf{O}_{exit_q} = \begin{pmatrix} s_{qx} & p_{exit_{qx}} & k_{exit_{qx}} \\ s_{qy} & p_{exit_{qy}} & k_{exit_{qy}} \\ s_{qz} & p_{exit_{qz}} & k_{exit_{qz}} \end{pmatrix} \quad (2.7)$$

The terms in this matrix can be collected into row vectors for the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ components

$$\mathbf{O}_{exit_q} = \begin{pmatrix} \vec{\mathbf{v}}_{1_{exit_q}} \\ \vec{\mathbf{v}}_{2_{exit_q}} \\ \vec{\mathbf{v}}_{3_{exit_q}} \end{pmatrix} \quad (2.8)$$

where

$$\begin{aligned} \vec{\mathbf{v}}_{1_{exit_q}} &= (s_{qx}, p_{exit_{qx}}, k_{exit_{qx}}) \\ \vec{\mathbf{v}}_{2_{exit_q}} &= (s_{qy}, p_{exit_{qy}}, k_{exit_{qy}}) \\ \vec{\mathbf{v}}_{3_{exit_q}} &= (s_{qz}, p_{exit_{qz}}, k_{exit_{qz}}) \end{aligned} \quad (2.9)$$

Note that $\hat{\mathbf{p}}_{\text{exit}_q}$ and $\hat{\mathbf{p}}_{\text{inc}_q}$, and similarly $\hat{\mathbf{k}}_{\text{exit}_q}$ and $\hat{\mathbf{k}}_{\text{inc}_q}$ are generally not parallel, so despite the similarities, $\mathbf{O}_{\text{exit}_q}$ is generally not the transpose of $\mathbf{O}_{\text{inc}_q}$

With the coordinate rotations established, the PRT matrix for surface q is

$$\mathbf{P}_q = \mathbf{O}_{\text{exit}_q} \cdot \mathbf{J}\mathbf{3}_q \cdot \mathbf{O}_{\text{inc}_q} \quad (2.10)$$

where $\mathbf{J}\mathbf{3}$ is a Jones matrix is calculated in the local coordinate system ($\hat{\mathbf{s}}, \hat{\mathbf{p}}$ here). Since the Jones matrix is a 2nd order tensor and a 3rd order tensor is required, the Jones matrix is padded with zeros. The symbol $\mathbf{J}\mathbf{3}$ is chosen to distinguish the three-dimensional form of the local coordinate Jones matrix.

$$\mathbf{J}\mathbf{3} = \begin{pmatrix} j_{ss} & j_{ps} & 0 \\ j_{sp} & j_{pp} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.11)$$

Note that for $t_k = 1$, the lower right element of $\mathbf{J}\mathbf{3}$ would equal one. For isotropic interfaces, j_{sp} and j_{ps} are zero, resulting in a $\mathbf{J}\mathbf{3}$ which is a complex valued diagonal matrix. The PRT matrix product for this surface is therefore

$$\mathbf{P} = \begin{pmatrix} s_{qx} & p_{\text{exit}_{qx}} & k_{\text{exit}_{qx}} \\ s_{qy} & p_{\text{exit}_{qy}} & k_{\text{exit}_{qy}} \\ s_{qz} & p_{\text{exit}_{qz}} & k_{\text{exit}_{qz}} \end{pmatrix} \cdot \begin{pmatrix} j_{ss} & j_{ps} & 0 \\ j_{sp} & j_{pp} & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} s_{qx} & s_{qy} & s_{qz} \\ p_{\text{inc}_{qx}} & p_{\text{inc}_{qy}} & p_{\text{inc}_{qz}} \\ k_{\text{inc}_{qx}} & k_{\text{inc}_{qy}} & k_{\text{inc}_{qz}} \end{pmatrix} \quad (2.12)$$

The above single surface PRT matrix elements can also be written

$$p_{ij} = \vec{v}_{i_{\text{exit}_q}} \cdot \mathbf{J}\mathbf{3}_q \cdot \vec{v}_{j_{\text{inc}_q}} \quad (2.13)$$

For cases where the most convenient local coordinate system is not $\hat{\mathbf{s}}, \hat{\mathbf{p}}, \hat{\mathbf{k}}$, the same basic procedure can be followed, substituting the desired local coordinate system. As noted earlier, coordinate transformation matrices are, in general unitary. Equation 2.14 is an example of a PRT for such a system, with $\hat{\mathbf{k}}_{\text{inc}} = \hat{\mathbf{k}}_{\text{exit}} = (0, 0, 1)$, exhibiting right and left handed circular polarization as eigenstates, with complex transmission (or reflection) coefficients given by t_R and t_L .

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ -i & i & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} t_R & 0 & 0 \\ 0 & t_L & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & i & 0 \\ 1 & -i & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.14)$$

Once the PRT matrix for the system has been calculated, it is often desirable to go back to a 2-dimensional representation of the polarization state, in order to simplify understanding of the polarization properties of the system. While many systems have a “natural” coordinate system, the actual coordinate system can be any member of the set of 2-D coordinate systems which are orthogonal to the direction of propagation in that space. Once the incident and exiting coordinate systems are chosen, the transformation is simply

$$\mathbf{J}\mathbf{3}_{system} = \mathbf{O}_{gl_{exit}} \cdot \mathbf{P}_{system} \cdot \mathbf{O}_{lg_{inc}} \quad (2.15)$$

where the subscript gl is short for global to local and the subscript lg is short for local to global. $\mathbf{O}_{lg_{inc}}$ is calculated in a similar fashion to equation 2.4 but using the desired $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ orthogonal to the $\hat{\mathbf{k}}_{in}$ instead of $\hat{\mathbf{s}}, \hat{\mathbf{p}}$.

2.4 Determination of the Jones Matrix of a Surface Interaction

Calculation of the Jones matrix of a ray intercept at a surface is clearly crucial in the PRT calculus. Appropriate methods for calculation vary, depending on the type of surface. This section summarizes the calculation of Jones matrices for some common surface types. In all cases discussed here, the calculation is performed in local $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ surface coordinates.

For uncoated interfaces between two isotropic media, the Fresnel coefficients can be used to calculate the Jones matrix. For this type of interface, the eigenpolarizations of the interaction are the local $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ surface coordinates, so the off-diagonal terms in the Jones matrix are zero. The Fresnel equations are widely known and will not be reproduced here.

For isotropic media with thin film interference filters, the thin film equations can be used to calculate the amplitude reflection and transmission coefficients. These interfaces, like the uncoated isotropic media, have polarization eigenstates aligned with the local surface $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ coordinates. The thin film equations are widely known and will not be reproduced here.

For surfaces with SWG, the calculation can still be done in surface $\hat{\mathbf{s}}$, $\hat{\mathbf{p}}$ coordinates, though these are not, in general, eigenstates of the surface, so off-diagonal terms will appear in the Jones matrix. In addition, these surface interactions depend on the azimuthal angle, not just the angle of incidence. No exact closed form solution exists for determining the polarization properties of these surfaces, but the algorithms discussed in section 1.5 can be used to obtain the elements of the Jones matrix of the interaction.

2.5 Algorithm for Tracing Polarized Rays

There are several universal parameters for rays: ray direction, initial position, wavelength, OPL. Many additional parameters are needed in some ray traces but not others. Examples include polarization state, Gaussian beam propagation, and surface order. Tracing a polarized ray through a series of sequential surfaces requires several steps. First, the geometric parameters are calculated: OPL to the next surface, and position where the ray strikes the surface, normal vector of the surface at that position. Second, the polarization parameters are calculated. These two steps are then repeated for all surfaces in the list.

For calculating a ray intercept with a surface, there are many algorithms published for various surface types. One example is a planar surface intercept, shown in equations 2.16 and 2.17.

$$d_q = \frac{(\vec{\mathbf{O}}_q - \vec{\mathbf{x}}_q) \cdot \hat{\mathbf{n}}_q}{\hat{\mathbf{k}}_{inc_q} \cdot \hat{\mathbf{n}}_q} \quad (2.16)$$

where d_q is the distance along the ray to the surface, $\vec{\mathbf{O}}_q$ is a point on the surface, $\vec{\mathbf{x}}_q$ is a point on the ray, and $\hat{\mathbf{n}}_q$ is the surface normal vector, as shown in figure 2.3. The position of the intercept is then

$$\vec{\mathbf{x}}_{(q+1)} = d_q \hat{\mathbf{k}}_{inc_q} + \vec{\mathbf{x}}_q \quad (2.17)$$

and the OPL in waves is given by $\tau_q = d_q/\lambda$. Intersection calculations for many surface types can be found in the literature.

Figure 2.3: Ray intercepting a plane mirror. All parameters used in the calculation are shown. d_q is the distance along the ray to the surface, $\vec{o}_q$ is an arbitrary point on the surface, $\vec{r}_q$ is an arbitrary point on the incident ray, and $\hat{\mathbf{n}}_q$ is the surface normal vector

Next the local polarization basis for the surface is calculated. Here, only surfaces with an $\hat{\mathbf{s}}_q, \hat{\mathbf{p}}_q$ basis will be considered, where

$$\hat{\mathbf{s}}_q = N \left(\hat{\mathbf{k}}_{inc_q} \times \hat{\mathbf{n}}_q \right) \quad (2.18)$$

$$\hat{\mathbf{p}}_{inc_q} = N \left(\hat{\mathbf{k}}_{inc_q} \times \hat{\mathbf{s}}_q \right) \quad (2.19)$$

where N is the normalization operator $\hat{\mathbf{v}} = N(\vec{\mathbf{v}}) = \frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$. The reflected ray direction vector is given by

$$\hat{\mathbf{k}}_{exit_q} = N \left(\hat{\mathbf{k}}_{inc_q} - 2 \left(\hat{\mathbf{n}}_q \cdot \hat{\mathbf{k}}_{inc_q} \right) \hat{\mathbf{n}}_q \right) \quad (2.20)$$

and refracted ray direction is given by

$$\hat{\mathbf{k}}_{exit_q} = N \left(\frac{1}{n_t} \left(n_i \hat{\mathbf{k}}_{inc_q} + \Gamma \hat{\mathbf{n}}_q \right) \right) \quad (2.21)$$

where

$$\Gamma = -n_i \hat{\mathbf{k}}_{inc_q} \cdot \hat{\mathbf{n}}_q \sqrt{n_i^2 \left(\hat{\mathbf{k}}_{inc_q} \cdot \hat{\mathbf{n}}_q \right)^2 + (n_t^2 - n_i^2)} \quad (2.22)$$

The p-basis state of the exiting ray is calculated in a similar fashion as the incident,

$$\hat{\mathbf{p}}_{exit_q} = N \left(\hat{\mathbf{k}}_{exit_q} \times \hat{\mathbf{s}}_q \right) \quad (2.23)$$

The angle of incidence (AOI), which is required to obtain the polarization properties of the surface, is given by

$$AOI_q = \arccos \left(\hat{\mathbf{n}}_q \cdot \hat{\mathbf{k}}_q \right) \quad (2.24)$$

The projection of the p-basis state onto the plane tangent to the surface at the intercept point is also important, and is given by

$$\hat{\mathbf{p}}_{pl_q} = N \left(\hat{\mathbf{s}}_q \times \hat{\mathbf{n}}_q \right) \quad (2.25)$$

If the surface is anisotropic, the azimuthal angle is calculated from

$$\psi = \arcsin \left(\hat{\mathbf{p}}_{pl_q} \cdot \widehat{\mathbf{an}}_q \right) \quad (2.26)$$

where $\widehat{\mathbf{an}}$ is the surface Anisotropy vector, which is assumed to lie in the plane of the surface, and defines the directionality of the anisotropy. Finally, the Jones matrix of the surface is calculated by an appropriate algorithm and padded with zeros to form **J3**.

There is now sufficient information to construct the PRT matrix of the surface using the algorithms in section 2.3. With $\mathbf{P}_q$ calculated, the above steps are repeated for the next surface.

2.6 Examples of PRT Matrix Calculation

2.6.1 Example Calculation of a Ray interacting with a Plane Mirror at 45° Angle of Incidence

Consider a plane mirror (figure 2.4) having position $\vec{\mathbf{o}} = \{0, 0, 0\}$ and surface normal $\hat{\mathbf{n}} = \left\{ \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right\}$. A ray is in air at position $\vec{\mathbf{x}} = \{0, 0, -1\}$ with ray direction vector

Figure 2.4: Ray reflecting from a plane mirror

$\hat{\mathbf{k}} = \{0, 0, 1\}$. The surface intercept is given by equation 2.16, and is $\{0, 0, 0\}$ in this case. The various local coordinate basis vectors become

$$\begin{aligned}
 \hat{\mathbf{s}} &= \{1, 0, 0\} \\
 \hat{\mathbf{p}}_{inc} &= \{0, 1, 0\} \\
 \hat{\mathbf{p}}_{exit} &= \{0, 0, 1\} \\
 \hat{\mathbf{k}}_{exit} &= \{0, -1, 0\}
 \end{aligned} \tag{2.27}$$

For a metallic first surface mirror having a complex index of refraction of $0.5 + 10i$ at 45° angle of incidence, the Fresnel coefficients are $r_s = -0.961538 - 0.27467i$ and $r_p = 0.849112 + 0.528212i$. Therefore, as shown in equation 2.12, the PRT matrix

Figure 2.5: Three prism system. s-vectors in red, p-vectors in blue, one particular electric field state in green. Rays are shown in black.

can be written

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -0.96 - 0.27i & 0 & 0 \\ 0 & 0.85 + 0.53i & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.28)$$

$$\mathbf{P} = \begin{pmatrix} -0.96 - 0.27i & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0.85 + 0.53i & 0 \end{pmatrix} \quad (2.29)$$

2.6.2 Three reflection x-y translation prism system

Consider three right prisms oriented as shown in figure 2.5. In this system, the incident and exiting ray direction vectors are parallel, but spatially separated. There

are nine surfaces, but the ray is incident on the six transmitting surfaces at normal incidence and have no polarization effect. For the reflecting surfaces, suppose they also have no polarization effect: the surface is 100% reflective and has zero retardance. The Fresnel coefficients are $r_s = r_p = 1.0 + 0i$. Despite the total lack of polarizing effects at every single interface, the PRT matrix for this system of prisms is not the identity matrix.

For the system shown in figure 2.5, the normal vectors for the reflective surfaces are given by

$$\begin{aligned}\hat{\mathbf{n}}_1 &= \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \\ \hat{\mathbf{n}}_2 &= \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix} \\ \hat{\mathbf{n}}_3 &= \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}\end{aligned}\tag{2.30}$$

and the ray direction vectors are given by

$$\begin{aligned}\hat{\mathbf{k}}_{inc} &= \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \\ \hat{\mathbf{k}}_1 &= \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \\ \hat{\mathbf{k}}_2 &= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \\ \hat{\mathbf{k}}_{exit} &= \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}\end{aligned}\tag{2.31}$$

Using the equations from section 2.5, the s-polarization vectors (red in figure 2.5) for each reflective surface are

$$\begin{aligned}\hat{\mathbf{s}}_1 &= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \\ \hat{\mathbf{s}}_2 &= \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \\ \hat{\mathbf{s}}_3 &= \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\end{aligned}\tag{2.32}$$

and the p-polarization vectors (blue in figure 2.5) are

$$\begin{array}{cc} \text{Incident} & \text{Exiting} \\ \hat{\mathbf{p}}_1 & \begin{pmatrix} 0 & 0 & -1 \end{pmatrix} & \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \\ \hat{\mathbf{p}}_2 & \begin{pmatrix} -1 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \\ \hat{\mathbf{p}}_3 & \begin{pmatrix} 0 & -1 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \end{array}\tag{2.33}$$

resulting in PRT matrices for each surface given by

$$\begin{aligned}\mathbf{P}_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \\ \mathbf{P}_2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \\ \mathbf{P}_3 &= \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\end{aligned}\tag{2.34}$$

which result in a system PRT matrix of

$$\mathbf{P}_{system} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \mathbf{Q}_{system}\tag{2.35}$$

This matrix is not the identity matrix, despite the lack of polarization effects at all surfaces. It must, therefore, represent changes in polarization state due only to the geometry of the system. This geometrical transformation matrix is designated $\mathbf{Q}$, with $\mathbf{Q}$ being chosen because this formalism uses $\mathbf{O}$ and $\mathbf{P}$, with $\mathbf{Q}$ being the next letter in the alphabet. The $\mathbf{Q}$ matrix can be calculated for any ray in any optical system by performing the PRT matrix calculations while using an identity Jones matrix for the creation of **J3**.

If we wish to return to a two-dimensional description of the polarization state, using equation 2.15, it is necessary to choose a local coordinate basis perpendicular to the incident ray and another one perpendicular to the exiting ray. For the system shown in figure 2.5, two choices for the local coordinate systems are obvious. Since the entering and exiting rays are parallel, one choice is to use the same local coordinate system for both incident and exiting sides of the system. The second obvious choice is to choose a coordinate system for the incident ray, and use the $\mathbf{Q}$ matrix to calculate the exiting coordinate system, eliminating the geometric effects from the polarization.

Figure 2.6: System with incident and exiting local coordinates displayed. In this case, the incident and exiting local coordinate chosen are parallel.

First, consider the case where the incident and exiting coordinate systems are to be identical, shown in figure 2.6, similar to the typical geometry for a polarization measurement. While the incident and exiting coordinate systems have the global coordinate ray propagation vector of $+\hat{\mathbf{y}}$, it becomes $+\hat{\mathbf{z}}$ in both incident and exiting local coordinate systems. The global $+\hat{\mathbf{z}}$ vector will become $+\hat{\mathbf{y}}$ in the local coordinate system. If we wish to remain in a right handed Cartesian coordinate system, the global $+\hat{\mathbf{x}}$ vector must become $-\hat{\mathbf{x}}$ in the local coordinate system. To regain a Jones matrix in these coordinates, the transformation used in equation 2.36 is used,

$$\mathbf{J}_{\text{system}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (2.36)$$

Figure 2.7: Three prism system with coordinate systems chosen such that the Jones matrix is an identity matrix.

$$\mathbf{J}\mathbf{3}_{system} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.37)$$

which results in a 3x3 matrix where the last row and last column are zero, and the other four elements are the Jones matrix in the chosen local coordinates. This Jones matrix appears to have circular retardance, but the rotation is an effect of the system geometry. If a different coordinate system for the Jones matrix is chosen, the rotation due to the geometry can be eliminated from the Jones matrix.

Suppose we use the above coordinate system (global $+\hat{\mathbf{y}}$ becomes local $+\hat{\mathbf{z}}$, global $+\hat{\mathbf{z}}$ becomes local $+\hat{\mathbf{y}}$, and global $+\hat{\mathbf{x}}$ becomes local $-\hat{\mathbf{x}}$) as the incident coordinate system, but choose the exiting coordinate system according to the transformation

given by the $\mathbf{Q}$ matrix (figure 2.7).

$$\begin{aligned}\hat{\mathbf{x}}_{exit} &= \mathbf{Q} \cdot \hat{\mathbf{x}}_{inc} \\ \hat{\mathbf{y}}_{exit} &= \mathbf{Q} \cdot \hat{\mathbf{y}}_{inc} \\ \hat{\mathbf{z}}_{exit} &= \mathbf{Q} \cdot \hat{\mathbf{z}}_{inc}\end{aligned}\tag{2.38}$$

The new exiting coordinate system will therefore transform the global $+\hat{\mathbf{y}}$ direction into a local $+\hat{\mathbf{z}}$ direction, the global $+\hat{\mathbf{x}}$ direction into a local $+\hat{\mathbf{y}}$ direction, and the global $+\hat{\mathbf{z}}$ direction into a local $+\hat{\mathbf{x}}$ direction. This resulting Jones matrix is given by equations 2.39 and 2.40.

$$\mathbf{J}\mathbf{3}_{system} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\tag{2.39}$$

$$\mathbf{J}\mathbf{3}_{system} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\tag{2.40}$$

The $\mathbf{Q}$ matrix for a ray can always be converted to an identity Jones matrix through the correct coordinate transformation. The PRT matrix can have the geometrical effects, as defined by the $\mathbf{Q}$ matrix, removed by this coordinate system. Note that the $\mathbf{Q}$ matrix does not account for cross-coupling of polarization effects of various surfaces, such as linear retardance at two different angles combining to create a component of circular retardance. The cube-corner retroreflectors discussed in chapters 3 and 4 are examples of such a system.

A second convention for $\mathbf{Q}$ matrices exists.⁶¹ In this alternate convention, when calculating the $\mathbf{Q}$ matrix for a reflecting surface, the Jones identity matrix is not used, but rather

$$\mathbf{J}\mathbf{3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\tag{2.41}$$

The downside of this definition, in systems with an odd number of reflections, is that the exiting coordinate system, calculated according to equation 2.38, is left

Figure 2.8: Cross section of rays on a fast parabola. Polarization orientation shown by red lines.

handed for a right handed incident coordinate system. The convention used in this document preserves the handedness of the coordinate system.

An additional consideration, if measurements are to be made, is the geometry of the measurement system. Often results are easier to interpret if they are in the same coordinate system as the metrology equipment.

2.7 Example: Fast Parabola

One example system where the polarization is hard to interpret in a two-dimensional local coordinate system is a parabolic reflector whose length along the axis of symmetry is essentially infinite. A cross section of only the region near the focus is shown in figure 2.8. For this reflector, rays approach the focal point from all possible angles; the reflected wavefront forms a complete sphere, subtending 4π steradians at the focal point (assuming a perfect infinite parabola with a plane wave incident).

Consider the case where the incident plane wave is linearly polarized along the $\hat{\mathbf{y}} = (0, 1)$ axis (p-polarization), in the plane of the page in figure 2.8. Let the incident local coordinate system be defined with $-\hat{\mathbf{z}}$ along the propagation direction, $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}$ polarization eigenstates of the surface as the Jones basis. For the rays shown, the polarization remains in the plane of the page after reflection. Consider the ray interaction marked A. The three dimensional electric field of this ray before interaction is $(0, 1, 0)$, and after interaction is $(0, 0, 1)$. The Jones vector, due to the change in local coordinates during the reflection, remains $(0, 1)$ after the reflec-

Figure 2.9: Electric field vectors converging on focal point of parabola assuming uniform $(1,0,0)$ incident. This pattern represents electric field vectors converging from a full sphere. “Front“ marks the hemisphere toward the open end of the parabola.

tion. The ray marked B has electric field vector $(0, 1, 0)$ before the interaction and $\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ after the interaction, but again has Jones vector $(0, 1)$ both before and after interaction. The ray marked C reflects at normal incidence, so any linearly polarized electric field remains in the same plane before and after reflection, $(0, 1, 0)$ in global coordinates. In local coordinates, the Jones vector is $(0, 1)$ both before and after the interaction, as in the case of the plane mirror. In this particular situation, the Jones vector for the electric field is $(0, 1)$ for all ray segments shown, despite the continuously changing basis in which they are measured.

A complete map of the electric field approaching the focal point from 4π steradians is shown on an orthographic projection in figure 2.9. The front of the pattern, facing the open end of the parabola, has only gradual changes from the horizontal polarization incident. The back of the pattern shows a clear singularity with the vector field rotating twice around the singularity. This is predicted by the winding number theorem, which states that any vector field on a sphere must have at least two singularities, about which the vector field undergoes a full rotation. In this pattern, the two singularities are superimposed.

Figure 2.10: Electric field vectors emitted by a dipole oscillator.

2.8 Polarization in the entrance pupil from a single point emitter

An entrance pupil contains a distribution of electric fields resulting from a single point emitter. There are two electric field distributions commonly used for the entrance pupil distribution. One obvious choice, based on basic electromagnetism, is a dipole emission pattern, shown in orthographic projection in figure 2.10. In this situation the magnitude and orientation of the electric field vector associated with each ray exiting the entrance pupil is determined by the physical properties of the emission of a perfect dipole oscillator.

While the dipole radiation pattern has a simple tie-in with physics, it is not representative of the electric field distribution incident on most optical systems. Consider the Kodak Brownie camera, consisting of a meniscus lens with the stop placed between the lens and the focal plane. The optical layout is shown in figure 2.12. The entrance pupil has a radius of infinity, while the exit pupil has a finite radius. Assuming that a uniform linearly polarized scene is viewed, the electric field of all rays in the entrance pupil is identical, while the electric fields in the exit pupil rotate to conform to the spherical shape. The non-polarizing transformation of the

electric fields here is exactly the same as a section of the front of the transformation of the electric fields in the infinite parabola. This arrangement of electric fields in the pupil is the most often useful.

2.8.1 Polarizer in a diverging or converging beam

It is noteworthy that the electric field distribution for a linearly polarized beam after passing through an ideal lens is different than the electric field distribution for a spherical wavefront passing through a linear polarizer. Consider an ideal wire grid polarizer.

Let the polarizer be a dummy surface with normal vector

$$\hat{\mathbf{n}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.42)$$

and let it reflect any electric field component along the surface anisotropy vector

$$\widehat{\mathbf{an}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (2.43)$$

Assume arbitrary incident ray vector $\hat{\mathbf{k}}$

$$\hat{\mathbf{k}} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (2.44)$$

the transmission vector of the polarizer must be perpendicular to the ray vector $\hat{\mathbf{k}}$ and the reflection axis $\widehat{\mathbf{an}}$

$$\hat{\mathbf{t}} = \hat{\mathbf{k}} \times \widehat{\mathbf{an}} = \frac{1}{\sqrt{x^2 + z^2}} \begin{pmatrix} -z \\ 0 \\ x \end{pmatrix} \quad (2.45)$$

The transmitted electric field always lies in the x-z plane, regardless of the incident ray direction. The orthogonal transformation matrix $\mathbf{O}$ is given by

$$\mathbf{O} = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{z}{\sqrt{x^2+z^2}} & 0 & \frac{x}{\sqrt{x^2+z^2}} \\ x & y & z \end{pmatrix} \quad (2.46)$$

Since this polarizer exists on a dummy surface, the local coordinates are the same on both sides of the surface, so $\mathbf{O}_{exit} = \mathbf{O}_{inc}^{-1}$ in this particular case, giving a PRT matrix of

$$\mathbf{P} = \frac{1}{x^2 + z^2} \begin{pmatrix} z^2 & 0 & -xz \\ 0 & 0 & 0 \\ -xz & 0 & x^2 \end{pmatrix} \quad (2.47)$$

Plotting the transmission eigenstate as a function of incident angle is shown in figure 2.11. This pattern follows the dipole coordinate system, not the double pole coordinates that arise when a collimated polarized beam is passed through an ideal lens.

2.9 Polarization Pupil Maps and Polarization Aberrations

The previous section considered how to define the electric field vector in a pupil. In the optical design of imaging systems, the OPD is calculated between the entrance and exit pupils, and a map of the pupil variation, called the wavefront aberration function, is created. This variation of OPD across the pupil is used for many purposes, including: plots for visual inspection, fits to the Seidel or Zernike aberration expansion, FFT to determine the PSF, optical transfer function, and Strehl ratio. For a polarization critical system, the polarization transformation that occurs varies across the pupil and must be understood.

Historically, a Jones matrix pupil map is used for this purpose.⁶²⁻⁶⁴ The Jones matrix pupil map consists of eight pupil maps, four for the amplitudes of each matrix element, and four for the phases of each matrix element. While not necessary, it is often desirable to extract any global phase (OPL) and amplitude (apodization)

Figure 2.11: Transmission axis of polarizer in three dimensions.

terms from the remaining polarization terms, for a total of 10 total pupil maps. This is shown in example 2.9.1. Regardless of other details, the Jones matrix is defined in a local coordinate system perpendicular to the ray vector. Therefore, the local coordinate system must be different for different parts of a spherical pupil. Both dipole and double pole coordinate systems are among the possibilities.

The double pole coordinates system has minimal variation in the coordinate basis for any low to moderate NA system. It is also the natural choice if using an ideal lens to create a collimated ray set. This is the most often employed technique.

The dipole coordinate system, if oriented so the axis of rotation is perpendicular to the chief ray, is advantageous if one is looking at a source whose behavior closely

mimics a dipole oscillator, if measuring single scattering from a small volume of gas for example. If the scatter region lies at the center of the entrance pupil, the polarization and apodization of the pupil should precisely match the light scattered from a single molecule, assuming the dipole axis chosen matches the molecule vibrational axis.

If one has a rotationally symmetric coordinate system with an on-axis field, the dipole coordinate system, with the axis of rotation aligned with the propagation vector, is a coordinate system where the eigenpolarizations of every surface are aligned with the coordinate basis. This simplifies calculation of diattenuation and retardance aberrations considerably. A downside is that there is a singularity at the center of the field, although the coordinate system for that ray can be chosen arbitrarily. Off-axis field points do not have consistent eigenpolarization on all surfaces, so this coordinate system choice would make less sense. This may also be useful for some off-axis systems that are simply a portion of a rotationally symmetric optical system.

Once the Jones pupil has been defined, one can fit the results to polarization extensions of either the Seidel or Zernike aberration expansions.

2.9.1 Jones Pupil Example: Kodak Brownie Camera

The Jones pupil contains all geometrical and polarization information about the system at a particular field point. Some methods of display of this information are more informative than others. As an example, consider a reversed landscape lens system, similar to that used in the famous Kodak Brownie camera series. This lens consists of a meniscus lens followed at some distance by the aperture stop, as seen in figure 2.12. Consider only the on-axis field for simplicity. Since this is a rotationally symmetric system, the Jones matrices will be in the surface s,p coordinates. Therefore, the basis vectors in which the Jones matrix is written will vary throughout the pupil as shown in figure 2.13. The Jones matrix will be considered in polar form. First examine the phase portion of the Jones pupil maps. The phases for each element are shown in figure 2.14. Due to the rotational symmetry of the system,

Figure 2.12: Optical layout of reversed landscape lens, as used in the Kodak Brownie camera. The object is an infinite distance to the left, and the image is to the right. Optical system is F/16.

Figure 2.13: Variation in the $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ basis with pupil position. This pattern is the dipole basis, looking along the axis.

Figure 2.14: Phase terms of the Jones matrix pupil map. Two different coordinate system choices are shown: s,p coordinates on the left, and x,y coordinates on the right. s,p coordinates, in this example, provide plots that are easier to interpret.

Figure 2.15: Phase terms of the Jones matrix pupil map due only to the effects of the thin film coating.

Figure 2.16: Phase terms of the Jones matrix pupil map showing only the polarization dependent phase terms.

and the matching rotational symmetry of the optical system, the off-diagonal terms of the matrix are zero. In this view, the s-s and p-p terms appear the same, indicating they are dominated by the geometrical phase errors, which are not polarization dependent. Figure 2.15 shows these charts again with the geometrical phase term removed, and only the effects due to the thin film coating on the surfaces shown. This is still dominated by the average phase, with the polarization dependent effects being too small to be visible. Figure 2.16 removes all phase terms which are common to all polarizations, and shows only the polarization dependent deviations from the mean phase.

By removing all polarization independent phase terms, a designer can easily see the polarization dependence of the phase terms, which in this case are extremely small. Polarization independent amplitude terms can also be separated and plotted separately. In those cases, the polarization dependent pupil map can be displayed most easily in three parts. The polarization independent amplitude and phase terms,

Figure 2.17: Polarization independent amplitude and phase in the pupil.

as shown in figure 2.17. The polarization dependent amplitude terms, as shown in figure 2.18, and the polarization dependent phase terms as shown in figure 2.19.

Figure 2.18: Amplitude terms of the Jones matrix pupil map showing only the polarization dependent portion.

2.10 Wavefront Splitting

Numerous processes can create multiple wavefronts exiting an optical system from a single input wavefront. Many systems recombine wavefronts as well.

For many systems, an entering wavefront is split into two or more exiting wavefronts, each of which travels separately, never overlapping. These systems are simple to handle since each path can be treated separately, as in zoom systems. An example is a system that uses a Wollaston prism to detect horizontal and vertical polarization components. Since each beam goes to a different detector, the two beams can be handled separately.

For other systems, such as a Twyman-Green interferometer, the optical system is configured such that the two exiting wavefronts have similar shapes and small OPD. For these systems, the wavefronts are combined coherently, creating an interference pattern. Jones or PRT vectors are suitable descriptions for the wavefronts

Figure 2.19: Phase terms of the Jones matrix pupil map showing only the polarization dependent portion.

from these systems.

In some systems, the wavefronts recombine incoherently. A good example is a ghost image formed from a double reflection in an optical window where the window thickness is many times the coherence length of the source. For these systems, the Mueller/Stokes description is suitable for performing the recombination.

For systems with short, but non-zero, coherence lengths, the recombination can become more complex. A Mireau interferometer is a white light interference microscope that produces a narrow range of black and white fringes, surrounded by a region of colored fringing, surrounded by a region of illumination without visible interference. One approach that has been used successfully is performing independent ray traces of the interferometer for each of many wavelengths. For each individual wavelength, the reference and test wavefronts are combined coherently. After all the wavelengths are traced, the different wavelengths are combined incoherently. Another example where partial coherence must be accounted for is ghost imaging in

Figure 2.20: Figure showing multiple reflections occurring inside a single layer thin film coating.

narrowband optical systems. If the coherence length is on the order of the optical path length between the ghost image and the main image, then some fringing may be visible. The same approach used with the Mireau interferometer may work in this case.

2.11 Effects of multilayer coatings

Thin films are responsible for several errors in a classical polarization ray trace. The first of these is loss of the traditional meaning of the OPL.

2.11.1 Thin films and optical path length

Consider a single layer coating as shown in figure 2.20. The electric field undergoes an infinite series of reflections inside the coating layer, with each subsequent surface interaction reducing the field amplitude. In traditional optical systems, coatings are used with essentially continuous wave illumination. Therefore, the infinity of exiting wavefronts sum coherently.

$$\vec{\mathbf{E}}_{trans} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 + \dots \quad (2.48)$$

For a single layer quarter wave thickness anti-reflection coating, this series of electric fields simplifies to

$$\vec{\mathbf{E}}_{\text{trans}} = \sum_{n=0}^{\infty} \vec{\mathbf{E}}_{\text{inc}} t_1 t_2 (r_1 r_2)^n e^{i2\pi n} \quad (2.49)$$

where t_1 and t_2 are the transmission coefficients of the two sides of the stack and r_1 and r_2 are the reflection coefficients. Note how each consecutive transmitted field in the sequence has an OPL exactly one wavelength longer than the previous. This makes it very simple to determine the phase of the wavefront modulus one wave, since that is the same for each field. There is no hope of determining a specific OPL of the transmitted wavefront, since there are an infinite number of different OPLs. Coatings with non-quarter wave layers have an equation similar to equation 2.49. Multilayer stacks also show a multiplicity of OPLs, but a single, well defined phase.

2.11.2 Effect of coating on position and direction of refracted ray

One of the original assumptions in thin film theory was that the thickness of the film is negligible. As multilayer stacks have become more complex, this assumption is not always valid.

One effect occurs because rays are typically traced to the surface of the substrate. The phase resulting from a thin film calculation may be referenced to either the top or bottom of the thin film stack. If it is referenced to the bottom of the stack, then the phase shift calculated can simply be added to the OPL. If the thin film calculation is referenced to the top of the stack, then the thickness of the stack must be subtracted from the OPL calculated. This makes the phase due to the coating

$$\phi = \phi_{\text{calculated}} - \frac{2\pi d}{\lambda_0} n_i \cos \theta_i \quad (2.50)$$

where d is the coating thickness, n_i is the refractive index of the incident medium and θ_i is the angle of incidence on the coating.

An additional issue arising from thin film stacks is the displacement of a ray relative to the original position due to the path taken through the thin film. This effect is especially pronounced for enhanced reflectivity coatings for EUV optical

Figure 2.21: Figure showing effective reflecting surface inside a multilayer coating.

Figure 2.22: Two layer reflective coating with several ray paths shown.

systems, as a very small reflection occurs at each layer interface. The most common technique for finding this displacement is to calculate an effective reflecting surface, as shown in figure 2.21. Note that the effective depth of the reflection is somewhere in the middle of the stack, which leads to a shift between the incident and exiting ray positions.

There are several techniques for calculating the position of the effective reflecting surface.⁶⁵ The conceptually simplest is to trace a ray through the coating, taking both the reflection and transmission at each layer interface, as shown in figure 2.22. All rays emerging from the coating can be summed, with the fraction of the incident power as a weighting function, to predict the position and direction of the exiting

ray. The point of intersection of the incident and exiting rays can then be found, which is the effective depth of the reflection.

A second approach that is more computationally efficient is to use the dispersion of the coating to calculate the depth of the effective reflecting surface. The dispersion can be either the chromatic dispersion or the angular dispersion. This approach is approximate, and not applicable to all coatings.³⁵

$$z_{eff} = \frac{1}{2\cos\theta_0} \frac{\partial}{\partial |k|} \phi(\vec{k}, \hat{n}) = -\frac{1}{2k_0\cos\theta_0} \frac{\partial^2 \phi}{\partial \theta^2} \quad (2.51)$$

where θ is the angle of incidence and ϕ is the phase shift of the coating. An approach that is not commonly used, but would produce an unquestionably correct result would be to use a rigorous electromagnetic wave solver to determine the properties of a wavefront transmitted through or reflected from a multilayer stack.

2.11.3 Considerations for non-planar interfaces

For non-planar interfaces, there are additional considerations. A thick multilayer film on a non-planar interface will also change the direction, not just position of a ray relative to an uncoated interface of the same shape. Figure 2.23 is an exaggerated sketch showing the change in position and direction of a ray due to a thick film. In this figure the solid ray shows the real path, and the dashed ray shows the ray path assuming the film can be neglected. Figure 2.24 shows reflections from two sides of an enhanced reflectivity coating leaving the surface in different directions. Both of these effects are due to the local change in slope of the surface, and do not occur with planar interfaces.

An additional consideration, primarily for enhanced reflectivity coatings, is the direction of the wavefronts that result from reflection at successive layers of the multilayer stack. Consider a plane wave incident on a uniform thickness coating on a parabola as shown in figure 2.25. Ignoring for the moment the changes due to refraction through each layer, an incident plane wave will create a wavefront having the same curvature but different defocus. To create coincident focal points, the thickness of the layers must be non-uniform.⁶⁵⁻⁶⁷ If each layer has the correct radial

Figure 2.23: Figure showing lens with a thick coating. Solid ray is path accounting for coating, dashed ray is ignoring the coating.

Figure 2.24: Figure showing lens with a thick coating. Solid ray is path accounting for coating, dashed ray is ignoring the coating.

Figure 2.25: Each interface in the stack creates a separate reflection with a defocus equal to the layer thickness.

Figure 2.26: Each interface in the stack creates a separate reflection. Since the thickness of the coating varies with radius, each layer has different radius, making all focal points equal.

gradient, shown in figure 2.26, then the focal point is the same for all layers.

CHAPTER 3

Cube-Corner Retroreflector Polarization

3.1 Introduction

This chapter presents the formalism for polarization analysis of a CCR. First the geometry of a cube-corner retroreflector (CCR) is reviewed, then calculation of the PRT and Mueller matrices associated with a simple uncoated CCR are shown. The review of these fundamentals prepares for the discussion of the more complex Polarization Conversion Cube Corner Retroreflectors in chapter 4.

3.2 Cube-Corner Retroreflector Geometry

The geometry of a CCR has been published many times, with many different approaches and conventions.²⁴⁻³³ No prior publication has discussed the parameters required for CCR with anisotropic surfaces. This section presents a discussion of the CCR geometry, with anisotropic surfaces considered.

A cube-corner retro-reflector is literally the corner of a cube, having three mutually perpendicular reflecting faces, shown in figure 3.1. In the symmetric construction, the lines representing the intersections of each pair of reflecting faces are chosen to be the same length, designated a . Given that the three reflecting faces are mutually perpendicular, each of the three reflecting faces is necessarily a right triangle. The fourth (front) face of the CCR is an equilateral triangle, forming a tetrahedron. Designate point V as the point of mutual intersection of the three reflecting faces (the vertex of the CCR), point O the center of the front face, and points A, B, C the corners of the front face, each of which lies on the intersection of one pair of reflecting faces. Designate points D, E, F the midpoints of lines BC, BA, AC. Figure 3.2 shows the relevant points on a CCR. The relative positions of

Figure 3.1: A Cube-Corner Retroreflector is the result of cutting one corner off a cube whose interior is reflective.

Figure 3.2: A corner cube retroreflector showing the vertex V, the center of the front face O, the other three corners, A, B, and C, and the centers of the edges of the front face, E, F, and G.

all these points are fully determined by the lines of intersections of the mutually perpendicular reflecting faces having length a . The lengths of all line segments are listed in equation 3.1.

Line Segment	Length
VA, VB, VC	a
AB, AC, BC	$\sqrt{2}a$
OA, OB, OC	$\sqrt{\frac{2}{3}}a$
OD, OE, OF	$\frac{1}{\sqrt{6}}a$
VD, VE, VF	$\frac{1}{\sqrt{2}}a$
OV	$\frac{1}{\sqrt{3}}a$

(3.1)

Choosing a right handed rectilinear coordinate system, axis OV is defined as the $+\hat{\mathbf{z}}$ direction, axis OA is defined as the $+\hat{\mathbf{x}}$ direction, and the $+\hat{\mathbf{y}}$ direction is the cross product of $+\hat{\mathbf{z}}$ and $+\hat{\mathbf{x}}$. The coordinates for these points are given in equation 3.2.

	x	y	z
Point O	0	0	0
Point A	$\sqrt{\frac{2}{3}}a$	0	0
Point B	$-\frac{a}{\sqrt{6}}$	$-\frac{a}{\sqrt{2}}$	0
Point C	$-\frac{a}{\sqrt{6}}$	$\frac{a}{\sqrt{2}}$	0
Point V	0	0	$\frac{a}{\sqrt{3}}$

(3.2)

For convenience, define surface ABCA as the front face, surface BCVB as surface #1, surface ACVA as surface #2, and surface ABVA as surface #3. The surface normal vectors for these surfaces are listed in equation 3.3

	x	y	z
Surface Normal Vector of Front Face	0	0	1
Surface Normal Vector #1	$\sqrt{\frac{2}{3}}a$	0	$-\frac{a}{\sqrt{3}}$
Surface Normal Vector #2	$-\frac{a}{\sqrt{6}}$	$-\frac{a}{\sqrt{2}}$	$-\frac{a}{\sqrt{3}}$
Surface Normal Vector #3	$-\frac{a}{\sqrt{6}}$	$\frac{a}{\sqrt{2}}$	$-\frac{a}{\sqrt{3}}$

(3.3)

For anisotropic surface types, it is necessary to denote the direction of the anisotropy on each surface. This vector, called the surface anisotropy vector, lies in the plane of

the surface, and for surfaces with SWG, is defined parallel to the grating lines. For the CCR discussed in this chapter, the surface anisotropy vectors are the directions given by VE, VF, and VG, listed in equation 3.4.

$$\begin{array}{rcccl}
 & x & y & z & \\
 \text{Surface Anisotropy Vector \#1} & \frac{a}{\sqrt{6}} & 0 & \frac{a}{\sqrt{3}} & \\
 \text{Surface Anisotropy Vector \#2} & -\frac{a}{2\sqrt{6}} & \frac{a}{2\sqrt{2}} & \frac{a}{\sqrt{3}} & \\
 \text{Surface Anisotropy Vector \#3} & -\frac{a}{2\sqrt{6}} & -\frac{a}{2\sqrt{2}} & \frac{a}{\sqrt{3}} &
 \end{array} \tag{3.4}$$

A CCR has three reflective faces. Since a ray must strike each face once to retro-reflect, there are six orders in which the faces can be encountered. These six possible ray paths each have, in general, different polarization properties. These six paths through the CCR are identified by the order in which they strike the reflective faces, path (1,2,3), for example, first strikes face #1, then face #2, and last face #3.

3.3 PRT matrix of a Cube-Corner Retroreflector

To determine the polarization properties of a single sub-aperture of a CCR, a Jones matrix or PRT matrix is sufficient. Here calculation of each ray path as a PRT matrix will be described.

Fundamentally, there are five interactions that occur as a ray retro-reflects from a CCR, with one PRT matrix per interaction.

$$\mathbf{P}_{path} = \mathbf{P}_{refraction2} \cdot \mathbf{P}_{reflection3} \cdot \mathbf{P}_{reflection2} \cdot \mathbf{P}_{reflection1} \cdot \mathbf{P}_{refraction1} \tag{3.5}$$

each of which is calculated according to the algorithms in chapter 2. This chapter will restrict its analysis to CCR at normal incidence. For arbitrary angle of incidence, the calculations are the same, but use the parameters tabulated in appendix A. At normal incidence, in isotropic media with isotropic surfaces, there is no polarization dependence at the surface, reducing the CCR to three interactions.

$$\mathbf{P}_{path} = \mathbf{P}_{reflection3} \cdot \mathbf{P}_{reflection2} \cdot \mathbf{P}_{reflection1} \tag{3.6}$$

3.3.1 Example: path (1,2,3) of N-BK7 CCR at normal incidence

As an example, the PRT matrices for path (1,2,3) are calculated for a normally incident ray, onto an uncoated N-BK7 CCR. First, using equations 2.20 and 2.21, the ray propagation vectors at each step are determined.

$$\begin{array}{ccccc}
 & \hat{\mathbf{k}}_1 & \hat{\mathbf{k}}_2 & \hat{\mathbf{k}}_3 & \hat{\mathbf{k}}_4 \\
 x & 0 & \frac{2\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & 0 \\
 y & 0 & 0 & \sqrt{\frac{2}{3}} & 0 \\
 z & 1 & \frac{1}{3} & -\frac{1}{3} & -1
 \end{array} \tag{3.7}$$

The $\hat{\mathbf{s}}$ vectors are calculated according to equation 2.18, and are listed in equation 3.8. Note that the incident and exiting coordinate systems, not being surface interactions, technically do not have an $\hat{\mathbf{s}}$ vector. A coordinate basis is needed, however, and $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}$ are used to denote the coordinate basis chosen.

	Incident					Exiting
	Coordinate	Reflection 1	Reflection 2	Reflection 3		Coordinate
	System					System
x	1	0	$-\frac{1}{2\sqrt{3}}$	$-\frac{\sqrt{3}}{2}$		-1
y	0	1	$\frac{1}{2}$	$\frac{1}{2}$		0
z	0	0	$\sqrt{\frac{2}{3}}$	0		0

(3.8)

The $\hat{\mathbf{p}}_{inc}$ vectors are calculated according to equation 2.19, and are listed in equation 3.9.

	Incident					Exiting
	Coordinate	Reflection 1	Reflection 2	Reflection 3		Coordinate
	System					System
x	0	-1	$-\frac{1}{6}$	$\frac{1}{6}$		0
y	1	0	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2\sqrt{3}}$		1
z	0	0	$\frac{\sqrt{2}}{3}$	$\frac{2\sqrt{2}}{3}$		0

(3.9)

The $\hat{\mathbf{p}}_{exit}$ vectors are calculated according to equation 2.23, and are listed in equation 3.10.

	Incident Coordinate System	Reflection 1	Reflection 2	Reflection 3	Exiting Coordinate System	
x	0	$-\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{2}$	0	(3.10)
y	1	0	$-\frac{1}{2\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	1	
z	0	$\frac{2\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	0	

The angle of incidence (AOI) on each surface is calculated according to equation 2.24, and listed in equation 3.11. Note that the 180° angle of incidence on the exiting refraction is due to the definition of the surface normal for the front surface as given in equation 3.3.

	Refraction 1	Reflection 1	Reflection 2	Reflection 3	Refraction 2	
AOI	0	$\arccos\left(\frac{1}{\sqrt{3}}\right)$	$\arccos\left(\frac{1}{\sqrt{3}}\right)$	$\arccos\left(\frac{1}{\sqrt{3}}\right)$	π	(3.11)

Finally, the Fresnel equations are used to calculate the complex amplitude reflection coefficients for the surface. Given a refractive index of 1.5161, the refractive index of N-BK7 at the sodium d line, the Fresnel coefficients for an AOI of $\arccos\left(\frac{1}{\sqrt{3}}\right)$ are $r_s = 0.180685 - 0.983541i$ and $r_p = -0.570929 - 0.821i$. When plugged into equation 2.11, the result is equation 3.12.

$$\mathbf{J}\mathbf{3} = \begin{pmatrix} 0.180685 - 0.983541i & 0 & 0 \\ 0 & -0.570929 - 0.821i & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.12)$$

The $\mathbf{O}_{inc}$ matrices are found according to equation 2.4, and shown in equation 3.13.

$$\mathbf{O}_{inc} \begin{pmatrix} \text{Reflection 1} \\ \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \text{Reflection 2} \\ \begin{pmatrix} -\frac{1}{2\sqrt{3}} & \frac{1}{2} & \sqrt{\frac{2}{3}} \\ -\frac{1}{6} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{3} \\ \frac{2\sqrt{2}}{3} & 0 & \frac{1}{3} \end{pmatrix} \\ \text{Reflection 3} \\ \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{2\sqrt{3}} & \frac{2\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & \sqrt{\frac{2}{3}} & -\frac{1}{3} \end{pmatrix} \end{pmatrix} \quad (3.13)$$

The $\mathbf{O}_{exit}$ matrices are found according to equation 2.7, and shown in equation 3.14.

$$\mathbf{O}_{exit} \begin{matrix} & \text{Reflection 1} & & \text{Reflection 2} & & \text{Reflection 3} \\ \begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{3} & 0 & \frac{2\sqrt{2}}{3} \\ \frac{2\sqrt{2}}{3} & 0 & \frac{1}{3} \end{pmatrix} & \begin{pmatrix} -\frac{1}{2\sqrt{3}} & \frac{1}{2} & \sqrt{\frac{2}{3}} \\ \frac{5}{6} & -\frac{1}{2\sqrt{3}} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & \sqrt{\frac{2}{3}} & -\frac{1}{3} \end{pmatrix} & \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \end{matrix} \quad (3.14)$$

Finally, the PRT matrices for each surface are constructed as shown in equation 2.2, and shown in equation 3.15.

$$\begin{aligned} \mathbf{P}_1 & \begin{pmatrix} 0.570929 + 0.821i & 0 & 0 \\ 0 & -0.0602285 + 0.327847i & 0 \\ 0 & 0.170352 - 0.927291i & 0 \end{pmatrix} \\ \mathbf{P}_2 & \begin{pmatrix} 0.0626345 - 0.0135451i & 0.22114 + 0.497465i & -0.177157 + 0.0383113i \\ -0.070935 + 0.197103i & -0.0674466 - 0.615059i & 0.200634 - 0.557491i \\ 0.0531054 + 0.245567i & 0.446295 + 0.348712i & -0.150205 - 0.694568i \end{pmatrix} \\ \mathbf{P}_3 & \begin{pmatrix} 0.0879367 - 0.806072i & -0.160646 + 0.307385i & -0.269138 - 0.387023i \\ -0.160646 + 0.307385i & -0.0975608 - 0.451135i & -0.466161 - 0.670343i \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (3.15)$$

And the matrices are multiplied together to get the PRT matrix for the ray path shown in equation 3.16.

$$\mathbf{P}_{path(1,2,3)} = \begin{pmatrix} 0.172388 - 0.0880335i & 0.558071 + 0.388681i & 0 \\ 0.226331 + 0.122512i & 0.344941 + 0.539466i & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.16)$$

3.4 Mueller Matrix of Cube-Corner Retroreflector

Cube-corners are often made in arrays, intended for use with incoherent illumination. Each of the CCR in the array has six different ray paths. Since the detection systems rarely separate the different ray paths, the observed Mueller matrix is the mean of the Mueller matrices of the six paths. In some cases it is more convenient to

calculate the Mueller matrix of each path directly, rather than by converting from a PRT matrix.

The Mueller matrix for one path through a CCR is given by the product of Mueller matrices listed in equation 3.17.

$$\mathbf{MM}_{path} = \mathbf{R}(\theta_1) \cdot \mathbf{F}_1 \cdot \mathbf{R}(\theta_2) \cdot \mathbf{T}_3 \cdot \mathbf{R}(\theta_3) \cdot \mathbf{T}_2 \cdot \mathbf{R}(\theta_4) \cdot \mathbf{T}_1 \cdot \mathbf{R}(\theta_5) \cdot \mathbf{F}_2 \cdot \mathbf{R}(\theta_6) \quad (3.17)$$

Beginning with the incident ray in the incident coordinate system, the interactions are: rotation from the incident coordinate system into $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ coordinates of the front face, refraction into the CCR at the front face, rotation into the $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ coordinates of the first reflecting surface, reflection from the first reflecting surface, rotation into the $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ coordinates of the second reflecting surface, reflection from the second reflecting surface, rotation into the $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ coordinates of the third reflecting surface, reflection from the third reflecting surface, rotation into the $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ coordinates of the front surface, refraction out of the front surface, rotation into the exiting coordinate system. There are six rotation matrices $\mathbf{R}$. The matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ are the polarization effects from refraction into and out of the CCR. The matrices $\mathbf{T}_1$, $\mathbf{T}_2$, and $\mathbf{T}_3$ are the polarization effects from the three reflections.

The rotation Mueller matrix is given by equation 3.18 and the surface interaction Mueller matrix is given by equation 3.19.

$$\mathbf{R}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3.18)$$

$$\mathbf{MM}_q = \frac{1}{2} \begin{pmatrix} |r_s|^2 + |r_p|^2 & |r_s|^2 - |r_p|^2 & 0 & 0 \\ |r_s|^2 - |r_p|^2 & |r_s|^2 + |r_p|^2 & 0 & 0 \\ 0 & 0 & 2|r_s||r_p|\cos(\delta) & 2|r_s||r_p|\sin(\delta) \\ 0 & 0 & -2|r_s||r_p|\sin(\delta) & 2|r_s||r_p|\cos(\delta) \end{pmatrix} \quad (3.19)$$

where $\delta = \arg(r_s) - \arg(r_p)$. The parameters needed are the complex amplitude transmission and reflection coefficients for all surfaces, and the rotation angles between each surface. To obtain these parameters, the propagation vector must be

calculated for each ray segment. Then the s-polarization vector for each surface must be calculated. The rotation angles are given by the angle between s vectors on successive surfaces. The complex reflection and transmission coefficients depend on the angle of incidence and azimuthal angle. All of the above parameters can be calculated using the equations in chapter 2. After the Mueller matrices for the six paths have been calculated, the mean of the Mueller matrices can be taken to determine the average properties of the CCR.

$$\mathbf{MM}_{mean} = \frac{1}{6} (\mathbf{MM}_{123} + \mathbf{MM}_{231} + \mathbf{MM}_{312} + \mathbf{MM}_{321} + \mathbf{MM}_{132} + \mathbf{MM}_{213}) \quad (3.20)$$

3.4.1 Example: path (1,2,3) of NBK7 CCR at normal incidence

As an example, the Mueller matrix for path (1,2,3) are calculated for a normally incident ray, for an uncoated NBK-7 CCR.

The ray propagation vectors at each step are determined.

$$\begin{array}{ccccc} & \hat{\mathbf{k}}_1 & \hat{\mathbf{k}}_2 & \hat{\mathbf{k}}_3 & \hat{\mathbf{k}}_4 \\ x & 0 & \frac{2\sqrt{2}}{3} & \frac{\sqrt{2}}{3} & 0 \\ y & 0 & 0 & \sqrt{\frac{2}{3}} & 0 \\ z & 1 & \frac{1}{3} & -\frac{1}{3} & -1 \end{array} \quad (3.21)$$

The $\hat{\mathbf{s}}$ vectors are calculated according to equation 2.18.

	Incident Coordinate System	Reflection 1	Reflection 2	Reflection 3	Exiting Coordinate System	
x	1	0	$-\frac{1}{2\sqrt{3}}$	$-\frac{\sqrt{3}}{2}$	-1	(3.22)
y	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0	
z	0	0	$\sqrt{\frac{2}{3}}$	0	0	

The rotation matrices are calculated as the angle between the s-vectors on successive surfaces.

	Rotation 1	Rotation 2	Rotation 3	Rotation 4	
Rotation Angle	0	$\frac{\pi}{3}$	$\frac{\pi}{3}$	0	(3.23)

The surface interaction matrices are calculated from the Fresnel coefficients $r_s = 0.180685 - 0.983541i$ and $r_p = -0.570929 - 0.821i$ and from equation 3.19.

Multiplying the matrices together gives

$$\mathbf{MM}_{123} = \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.838872 & -0.311394 & 0.446462 \\ 0 & -0.311394 & 0.398206 & 0.862824 \\ 0 & -0.446462 & -0.862824 & 0.237077 \end{pmatrix} \quad (3.24)$$

The list of Mueller matrices for all six paths are given in equation 3.25.

$$\begin{aligned}
\mathbf{MM}_{123} &= \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.838872 & -0.311394 & 0.446462 \\ 0 & -0.311394 & 0.398206 & 0.862824 \\ 0 & -0.446462 & -0.862824 & 0.237077 \end{pmatrix} \\
\mathbf{MM}_{231} &= \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & -0.0889364 & 0.224276 & -0.970459 \\ 0 & -0.847064 & -0.529602 & -0.0447649 \\ 0 & -0.523997 & 0.81806 & 0.237077 \end{pmatrix} \\
\mathbf{MM}_{312} &= \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & -0.0889364 & -0.847064 & 0.523997 \\ 0 & 0.224276 & -0.529602 & -0.81806 \\ 0 & 0.970459 & 0.0447649 & 0.237077 \end{pmatrix} \\
\mathbf{MM}_{321} &= \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.838872 & 0.311394 & -0.446462 \\ 0 & 0.311394 & 0.398206 & 0.862824 \\ 0 & 0.446462 & -0.862824 & 0.237077 \end{pmatrix} \\
\mathbf{MM}_{132} &= \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & -0.0889364 & 0.847064 & -0.523997 \\ 0 & -0.224276 & -0.529602 & -0.81806 \\ 0 & -0.970459 & 0.0447649 & 0.237077 \end{pmatrix} \\
\mathbf{MM}_{213} &= \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & -0.0889364 & -0.224276 & 0.970459 \\ 0 & 0.847064 & -0.529602 & -0.0447649 \\ 0 & 0.523997 & 0.81806 & 0.237077 \end{pmatrix}
\end{aligned} \tag{3.25}$$

Following equation 3.20, the mean of the six paths is given by equation 3.26.

$$\mathbf{MM}_{mean_{NBK7}} = \begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.220333 & 0 & 0 \\ 0 & 0 & -0.220333 & 0 \\ 0 & 0 & 0 & 0.237077 \end{pmatrix} \quad (3.26)$$

The mean Mueller matrix is diagonal due to the symmetry in the CCR. CCR with symmetry breaking characteristics, such as CCR having different polarization properties on each surface can have non-diagonal mean Mueller matrices. The depolarization results from the different orientations of the electric field eigenstates for each path through the CCR.

3.5 Summary

This chapter has presented techniques for calculating the polarization matrix of a particular path through a CCR using either the PRT matrices or Mueller matrices.

CHAPTER 4

Polarization Conversion Cube-Corner Retroreflector

4.1 Introduction

Most publications approach the question of CCR polarization properties by defining a physical CCR of some sort, calculating the polarization properties of each reflecting surface, then calculating the polarization properties of the CCR. A different approach will be taken here. This chapter focuses on finding designs for polarization conversion corner cubes (PCCCR) by examining possible surface polarization properties. The possible polarization properties of each surface type will be described, then the Mueller matrix of the CCR class calculated. Finally, after determining the polarization properties that produce a PCCCR, the question of how to create a surface having those polarization properties will be addressed. While the focus is finding designs for PCCCR, other interesting CCR emerge from this analysis.

4.2 Definition of PCCCR

A polarization conversion cube corner retroreflector (PCCCR) is a CCR where the orientation of the major axis of the exiting electric field is rotated 90° relative to the incident orientation, and the exiting handedness is opposite the incident. An alternate description is in terms of the apparatus in figure 4.1: all of the flux incident on the CCR is found at the detector, for arbitrary elliptical eigenstates of the PBS. As discussed in chapter 1, the coordinate system in which a Mueller matrix is described is different for the incident and exiting rays on a CCR. As a result, the Mueller matrix for a PCCCR is equation 4.1, while the Mueller matrix for an ideal

Figure 4.1: Experimental configuration for determining polarization coupling.

mirror is equation 4.2.

$$\mathbf{MM}_{PCCR} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (4.1)$$

$$\mathbf{MM}_{mirror} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (4.2)$$

One useful merit function for evaluating how close a particular CCR is to being a PCCR is the minimum linear polarization coupling (MLPC). Consider again the apparatus in figure 4.1. Suppose the beam splitter transmits vertical linearly polarized light. If the polarization is rotated by 90° , all of the returned light is reflected by the PBS. If the incident light is completely depolarized by the CCR, then the polarization coupling is $\frac{1}{2}$. If the electric field vector is unchanged, then the polarization coupling is zero - none of the light is reflected by the PBS. As the CCR is rotated about the axis of symmetry, the flux fraction at the detector will generally

Figure 4.2: All possible Mueller matrices for a TIR CCR with SWG surfaces.

vary. Minimum linear polarization coupling gives the smallest flux fraction at the detector for any CCR orientation.

$$MLPC(\mathbf{MM}) = \text{Min}[(1, -\cos(2 * \theta), \sin(2 * \theta), 0) \cdot \mathbf{MM} \cdot (1, \cos(2 * \theta), \sin(2 * \theta), 0) / 2] \quad (4.3)$$

Many other metrics are possible, ranging from mean polarization coupling to the Frobenius distance in Mueller space between the CCR in question and the ideal PCCR. Minimum linear polarization coupling is sufficient for this chapter.

4.3 Properties of isotropic CCR with SWG anisotropic surfaces

In this section, the anisotropic surfaces considered are SWG surfaces that are non-diffracting and invariant under 180° rotations about the surface normal. Since the azimuthal angles are 90° , 0° , -90° successively on the three surfaces, the polarization properties of these three surfaces may be different, despite having identical SWG.

Under the assumptions used, the polarization properties of the first and third surfaces encountered are the same, but the polarization properties of the second surface are different. Considering only SWG that do not spoil the TIR condition for CCR with rays incident at normal incidence to the front face, having the SWG aligned with the $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ planes of the reflecting surfaces. Since the first and third surfaces encountered have identical properties, and TIR surfaces do not have diattenuation, there are two free parameters, the retardance of the first surface δ_1 , and the retardance of the second surface δ_2 .

Figure 4.2 shows the Mueller matrix for a TIR CCR having SWG surfaces, where the vertical axis is the retardance of a surface for an azimuthal angle of $\pm 90^\circ$ (first and third surfaces) and the horizontal axis shows the retardance of the surface at an azimuthal angle of 0° (second surface). This figure clearly shows that the mirror matrix occurs for all combinations of $\pm\pi$ retardance, which are the four corners of the plot. Note for zero retardance, all linear states are depolarized (black portions of the m_{11} and m_{22} elements), while circular polarizations are returned in the same handedness. There is a region of retardances where all circular polarizations are depolarized (black portions of the m_{33} element)

Figure 4.3 shows the minimum linear polarization coupling (MLPC) as a function of the two retardances. This shows zero coupling in the regions where the retardances are $\pm\pi$. The white line through the figure shows the position of isotropic TIR surfaces ($\delta_2 = \delta_1$) in this plot, which will be treated in section 4.4.1. Two PCCCR occur on the line where the retardance $\delta_2 = -\delta_1$. While approximate values of the two retardances can be read directly from the plot, the closed form Mueller matrix simplifies dramatically with the additional restriction, and is given by equation 4.4,

Figure 4.3: Minimum linear polarization coupling for all possible TIR CCR with SWG surfaces. The diagonal white line shows isotropic surfaces.

where unlisted elements of the Mueller matrix are zero.

$$\begin{aligned}
 m_{00} &= 1 \\
 m_{11} &= -\frac{1}{16} \sin^2\left(\frac{\delta}{2}\right) (60 \cos(\delta) + 9 \cos(2\delta) + 35) \\
 m_{22} &= -m_{11} \\
 m_{33} &= \frac{1}{16} (7 \cos(\delta) + 6 \cos(2\delta) + 9 \cos(3\delta) - 6)
 \end{aligned} \tag{4.4}$$

Solving this equation for the values in equation 4.1 gives the exact retardance values for a PCCCR Mueller matrix; $\delta_1 = \pm \arccos(1/3)$.

4.3.1 Examples of SWG PCCCR

Three surface designs for PCCCR are presented. Each was located by choosing a grating shape then calculating many combinations of depth and period until a solution was found. The first is a PCCCR at 633 nm wavelength made of SF-57 glass with rectangular SWG having a period of 175 nm, a depth of 240 nm, and a

duty cycle of 0.75. The second is a PCCCR at 650nm wavelength made of PMMA with sinusoidal SWG having a period of 190 nm, a depth of 180 nm, and a coating thickness of 270nm. The third is a PCCCR at 650nm wavelength made of PMMA with trapezoidal SWG having a period of 275 nm and a depth of 400 nm. These three grating shapes are shown in figure 4.5. An example of what the rectangular grating would look like on the very tip of a CCR is shown in figure 4.4.

Consider the performance variation of these three PCCCR for rays incident at non normal incidence. For simplicity, the front face is considered to be a perfectly transmissive surface, exhibiting neither diattenuation nor retardance. The properties of each PCCCR will be considered for a complete cone of rays incident. Figure 4.6 shows the angle of incidence and azimuthal angle patterns on the front face of the CCR that are used in producing the figures in table 4.1.

The first line of table 4.1 shows the reflection coefficient for each of the three PCCCR over the incident light cone. The rectangular gratings have a much larger region of near total reflection due to the higher refractive index of the substrate; the refraction into the CCR reduces the angle cone more than the others, and the critical angle on the reflecting faces is larger. The sinusoidal grating has a much smaller region of TIR due to the lower refractive index, but the entire region shows perfect TIR, there are no resonant interactions with the grating causing diffraction. The reflection pattern for the trapezoidal SWG is quite interesting. Within the region where TIR would be expected for an unmodified substrate the interaction of the grating with the incident light causes a significant amount of loss due to diffraction into non-zero order terms.

The second line of table 4.1 shows the minimum linear polarization coupling for the three PCCCR over the incident light cone. The trapezoidal gratings show the smallest region of high polarization coupling. Despite the higher index, the rectangular gratings have a region of high coupling not much larger than the trapezoidal gratings. The sinusoidal gratings show the largest region of high polarization coupling. If only moderate polarization coupling is required, then the rectangular gratings have the largest region. The low index PCCCR have a fundamental

Figure 4.4: CCR tip having gratings of period 175 nm with a depth of 240 nm and a duty cycle of 0.75

constriction on the moderate coupling region by the more rapid loss of TIR with AOI.

Now consider the performance variation of the three PCCCR with wavelength, shown in figure 4.7. Each of the three PCCCR shows a sudden change in reflectivity, similar in appearance to TIR failure. The cause in this case is resonant interaction between the electric field and the SWG surfaces. The resonant interaction defines the minimum useful wavelength for a PCCCR. The MLPC for the three gratings as a function of wavelength is quite varied. The rectangular and trapezoidal SWG have a very small wavelength range of high coupling. These PCCCR are most useful with laser sources, but for some applications would be acceptable for use with a filtered LED source. The sinusoidal SWG shows much slower deterioration with wavelength, approximately 200 nm of bandwidth have greater than 95% coupling.

Figure 4.5: Profiles of the SWG surfaces considered.

Figure 4.6: Angle of Incidence and Azimuthal angles for the various plots that follow.

Figure 4.7: Intensity reflectivity and MLPC as the wavelength is varied.

Table 4.1: Intensity reflection coefficient and MLPC as a function angle of incidence and azimuthal angle for each of three PCCCR solutions. The color scaling is the same in all figures.

4.4 Properties of CCR having isotropic surfaces with both diattenuation and retardance

As noted earlier, the polarization properties of the reflective surfaces determine the polarization effect of a CCR. For isotropic surface types, the changing azimuthal angle on different surfaces has no effect, meaning that for CCR with three identical reflecting surfaces, all three surfaces have the same Mueller matrix in local coordinates. For these types of surfaces, there are two polarization effects that can occur, diattenuation and retardance, both of which are aligned with the $\hat{s}$, $\hat{p}$ planes of the surfaces. The Mueller matrices for all possible combinations of these two parameters are plotted in figure 4.8, with diattenuation magnitude on the vertical axis and retardance magnitude on the horizontal axis. This figure shows the rapidly increasing losses as diattenuation is increased. This is due to the 60° rotations of the polarization basis between surfaces. If one assumes that each surface is a perfect polarizer, Malus' law gives the throughput $\frac{1}{2} \cos(60^\circ)^2 \cos(-60^\circ)^2 = \frac{1}{32}$. Dielectric TIR CCR are the bottom horizontal line on this plot, having zero loss. Figure 4.9 shows the MLPC for isotropic CCR, again with diattenuation magnitude on the

Figure 4.8: Isotropic CCR specified by diattenuation and retardance on each surface.

vertical axis and retardance magnitude on the horizontal axis. Inspection of this figure shows that there are no PCCCR in this class of CCR. The largest MLPC is 0.5, and occurs where linear polarizations are completely depolarized.

The two most common types of CCR are subsets of this category, and therefore worth discussing despite the lack of PCCCR. These are the TIR dielectric CCR, with zero diattenuation, and Metal coated CCR, which has both diattenuation and retardance.

Figure 4.9: Minimum linear polarization coupling for isotropic CCR having diattenuation and retardance on each surface.

4.4.1 Properties of Dielectric TIR CCR

Isotropic dielectric CCR are a subset having only retardance on each surface, forming the bottom line of pixels in figure 4.8. These CCR include uncoated glass CCR and CCR with dielectric thin films on the reflecting faces. For CCR with three identical surfaces, there is only one free parameter for determining the polarization properties of the CCR. The closed form mean Mueller matrix simplifies well and is printed in equation 4.5. δ is the retardance of the surface and unlisted elements of the Mueller matrix are zero.

$$\begin{aligned}
 m_{00} &= 1 \\
 m_{11} &= \frac{1}{16} \sin^2\left(\frac{\delta}{2}\right) (-4 \cos(\delta) + \cos(2\delta) - 21) \\
 m_{22} &= -m_{11} \\
 m_{33} &= \frac{1}{16} (15 \cos(\delta) + 6 \cos(2\delta) + \cos(3\delta) - 6)
 \end{aligned} \tag{4.5}$$

Figure 4.10: Mueller matrix of an isotropic CCR as a function of retardance of each surface.

Figure 4.10 shows all possible CCR as the retardance is varied. Three things about this plot stand out. First, at zero retardance, all incident linear polarizations are completely depolarized, while the handedness of incident circular polarizations is preserved. Second, at 180° of retardance on each surface, the polarization properties of the CCR are the same as a perfect plane mirror at normal incidence. Third, there is a large region of stability surrounding the mirror-like CCR. One example of a CCR having zero retardance on each surface at 633 nm is a LAF-2 ($n=1.74$) CCR with a 262 nm thick layer of SiO_2 ($n=1.46$) on the reflecting surfaces. One example of an CCR having π retardance on each surface at 587 nm is a NBK-7 ($n=1.5161$) CCR with a 130 nm thick layer of $n=4.2$ on the reflecting surfaces. Uncoated CCR have retardance between 38° for an index of 1.4 and 68° for an index of 4.0.

Figure 4.11: Mueller matrix of all possible hollow bare metal CCR

4.4.2 Properties of Metal Coated CCR

Metal coated CCR are desirable in some applications since they do not suffer from TIR failure at high angles of incidence. Metal coated CCR are undesirable since they exhibit absorption and diattenuation, reducing the throughput. For bare metal interfaces, the diattenuation and retardance are dependent only on the real and imaginary parts of the index of refraction. Here hollow CCR will be considered, although the polarization properties of solid CCR with metal coatings are similar. Figure 4.11 shows all possible metals by real and imaginary refractive index components, with the real part of the index of refraction on the vertical axis and the imaginary part on the horizontal axis. From this figure, it is clear that no metal coated CCR comes close to the Mueller matrix for a PCCCR. For small real com-

Figure 4.12: Minimum linear polarization coupling for CCR having surfaces with elliptical retardance. Regions shown have 90% MLPC. Green planes are the cross sections shown in figures 4.13 and 4.14.

ponent of the refractive index and large imaginary component, the metal coated CCR approaches the Mueller matrix for an ideal mirror. For most metals, the losses due to the diattenuation and absorption are large. Note that the mapping from the complex index of refraction to particular retardance and diattenuation values is many-to-one.

4.5 CCR having three identical reflecting surfaces with arbitrary elliptical retardance

each of In this section, CCR with all three reflecting surfaces having identical elliptical retardance are considered. These parameters are independent of azimuthal

Figure 4.13: Mueller matrix of a CCR having surfaces with linear retardance at arbitrary orientation.

angle, so the Mueller matrices, in local coordinates, are the same for all three surfaces. This type of surface is not, strictly speaking, an isotropic surface, since the retardance and diattenuation are not aligned with the $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ planes. Figure 4.12 shows the regions having MLPC greater than 0.9. The three axes are the components of the retardance parameters; δh is the horizontal retardance, $\delta 45$ is the 45° retardance, and δr is the right circular retardance. The two green planes show the positions of the cross sections in which PCCCR lie. The cross section at the bottom has a circular retardance of zero. The Mueller matrices of this cross section are shown in figure 4.13. The second cross section has a circular retardance of 0.8165π . The Mueller matrices of this cross section are shown in figure 4.14.

Figure 4.14: Mueller matrix of a CCR having surfaces with circular retardance magnitude 0.8165π .

First consider figure 4.13. In this figure, the circular retardance magnitude on each surface is zero. The vertical axis shows the horizontal retardance component, and the horizontal axis shows the 45° retardance component. Since this is a TIR CCR, the m_{00} term is unity for all retardances.

Several interesting CCR are shown in this figure. First, a PCCCR is found at a retardance parameters of $(0, \pi, 0)$. Second, a mirror matrix is found at retardance parameters of $(\pi, 0, 0)$. Finally, a 45° polarization rotator, whose Mueller matrix is

given in equation 4.6 is found at retardance parameters of $\frac{\pi}{\sqrt{2}}(1, 1, 0)$.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (4.6)$$

In figure 4.14, the circular retardance magnitude is 0.8165π . Like figure 4.13, the vertical axis shows the horizontal retardance component, and the horizontal axis shows the 45° retardance component. Again, since this is a TIR CCR, the m_{00} term is unity for all retardances.

In this figure, a PCCCR is found at retardance parameters of $(0.5773\pi, 0, 0.8165\pi)$, and a mirror Mueller matrix can be found at retardance parameters of $(0, 0.5773\pi, 0.8165\pi)$. Visual inspection of the figure suggests that an ideal depolarization matrix should be located near the cross-section shown. Numerical exploration shows this occurs at retardance parameters of $(0.697407\pi, 0.719899\pi, 0.669488\pi)$.

4.6 CCR having three different isotropic reflecting surfaces

Prior to this point, all CCR discussed had all three reflecting faces identical. In this section, TIR CCR with different isotropic coatings on each surface are considered. For isotropic TIR surfaces, the retardance is aligned with the s,p planes. The retardances for each surface will be given in order as (ret1,ret2,ret3). For this class of CCR, numerical exploration found no PCCCR. Figure 4.15 shows all regions in this space for which the MLPC is greater than 0.7. The maximum value is 0.75.

Despite the lack of PCCCR, there are a couple of CCR worth pointing out. First, in figure 4.16, the point with retardances $(\pi, 0, \pi)$, the Mueller matrix is the identity matrix. Since this is in reflection, the 45° component of the electric field is reversed. Therefore, the electric field major axis is flipped about the $\hat{y}$ axis as defined in chapter 3, and the handedness is preserved. A second point of mention, shown in figure 4.18, is retardances (π, π, π) , which produces the ideal mirror matrix,

Figure 4.15: Minimum linear polarization coupling for CCR having three different isotropic surfaces. Regions shown have MLPC 0.7, with the peak of each region being 0.75.

Figure 4.16: Mueller matrix of an isotropic CCR with surface 2 having zero retardance.

returning the major axis of the electric field in the same orientation while reversing the handedness.

The most interesting point, shown in figure 4.17, retardances $(\pi, \frac{\pi}{2}, \pi)$, has the Mueller matrix of a quarter wave linear retarder.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad (4.7)$$

For this matrix, both the 45° component and the handedness of the electric field are reversed. What is most interesting is the fact that all six paths through the CCR

Figure 4.17: Mueller matrix of an isotropic CCR with surface 2 having $\frac{\pi}{2}$ retardance.

return the same, non-diagonal matrix, despite the different orders in which the reflecting faces are struck. This is possible since the rotations between the incident coordinate system and the local coordinates of each surface are different, as are the exiting rotations. The combination of all these differences, in this case, results in the same Mueller matrix for each path.

4.7 Future Work

While this chapter has presented the PCCCR found in several types of CCR, there are several potential CCR types that were not covered. The most glaring of these omissions is the treatment of CCR with different SWG on each reflecting surface. Given the properties of SWG surfaces, a CCR with three different SWG, all of

Figure 4.18: Mueller matrix of an isotropic CCR with surface 2 having π retardance.

which were aligned with $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ planes, provides a six-dimensional space to explore. A space of this dimensionality cannot be easily explored visually, but searching for a particular Mueller matrix in the space is feasible. CCR with gratings that are not aligned with the $\hat{\mathbf{s}}, \hat{\mathbf{p}}$ planes also remains to be investigated. Also not explored were CCR having different elliptical retardance on each reflecting face and CCR having elliptical diattenuation.

An additional possibility is the addition of wedge shaped polarization rotators to the front face, one for each sub-aperture. There are six different paths through a CCR, and each corresponds to one-sixth of the front face at normal incidence. For surfaces whose properties are independent of azimuthal angle, the only difference between the paths is the rotations. Therefore, if different rotators were placed for

each of the six paths, it would be possible to reduce the number of different Mueller matrices associated with the CCR to two. Note that the regions of the front face corresponding to each path change with angle of incidence and azimuthal angle on the front face, so the angular tolerances of such a CCR would be very limited.

CHAPTER 5

Polarization Aberrations of Spherical Surfaces with SWG

5.1 Introduction

SWG are useful in a variety of roles. The best known is the use of SWG as an anti-reflection coating. An ideal anti-reflection SWG creates an effective medium layer which is a gradient index from the index of the substrate to the index of the surrounding medium. It is also possible to use non-gradient layers where the effective index is the same as the index of refraction which is desired for an ordinary single layer thin film. Another, less common use of SWGs is as compensators for polarization aberrations elsewhere in the system. By manipulating the orientation, period, and depth of the grating, it is possible to significantly alter the retardance and diattenuation of a surface. This work was originally published in Applied Optics volume 46.⁶⁸

5.2 One Dimensional SWG as an Anti-Reflection Coating

Polarization aberrations of thin film coatings are well understood, but the polarization aberrations of surfaces with SWG have not been studied in detail. This section

Figure 5.1: 1-D anti-reflection SWG of Santos & Bernardo

Figure 5.2: Intensity Transmittance of SWG

presents an example of polarization aberrations due to a one dimensional SWG anti-reflection coating. The coating chosen was published by Santos & Bernardo.⁶⁹ A cross section of this grating is shown in figure 5.1.

5.2.1 Polarization variation of 1-D AR SWG

To understand the polarization aberrations of this grating when placed on a lens surface, examination of the properties of the grating as a function of angle of incidence is helpful. First, consider the transmittance of a surface with this SWG at various angles of incidence.

When the plane of incidence is parallel to the grating, the intensity transmission is 99.5%, with very little polarization dependence to at least a 50°AOI. When the plane of incidence is perpendicular to the grating there is some polarization dependence, though it is only about 1/6th that of the uncoated interface. Above 28° AOI, resonant interactions between the grating and the incident field begin to occur, causing a rapid reduction in transmittance. This limits the useful numerical aperture of the antireflection SWG.

The phase shift on transmission of an uncoated non-absorbing dielectric interface is zero for all AOI. The grating induces some optical path difference (phase shift) even at normal incidence. Comparing the normal incidence OPD when the plane of incidence is parallel and perpendicular to the grating lines, the s, p components

Figure 5.3: Phase of SWG

	s-phase (waves)	p-phase (waves)	Retardance (waves)
POI $\parallel$ to SWG	0.3037	0.1785	0.1252
POI $\perp$ to SWG	0.1662	0.2986	-0.1324

Table 5.1: Phase Shifts of SWG at 24° AOI

have reversed phase values, since the polarization component oriented along the grating lines has changed. This causes the retardance to become negative for plane of incidence (POI) perpendicular to the grating lines. As the AOI increases, the two retardance functions differ.

The phase shifts and retardance at 24° AOI for each polarization and grating orientation are given in table 5.1. The phases are given in terms of the s and p-components, but which of these is parallel to the grating lines depends on the POI. Therefore, the s-component parallel to the grating should be compared against the p-component perpendicular to the grating. This shows the change in the relative orientation of the POI and the gratings produces a 0.005 wave change in the electric field component perpendicular to the gratings, and a 0.008 wave change in the component parallel to the gratings. This causes a 0.013 wave difference in the retardance magnitude between the two grating orientations.

Figure 5.4: Pattern showing grating lines on surface

Figure 5.5: Plane of incidence on surface is radially oriented

5.2.2 Polarization aberrations of 1-D AR SWG on a spherical surface

Consider a $f/1.7$ (maximum AOI 35°) spherical lens surface of index 1.512 with a plane wave incident along the axis. The grating forms parallel lines on the surface, as shown in figure 5.4. The linear grating breaks the rotational symmetry of the system, making the wavefront aberrations astigmatic for an on-axis field point. The plane of incidence is oriented radially on the interface, shown in figure 5.5, so the angle between the grating and the plane of incidence varies with position. As a ray moves around the edge of the pupil, the orientation of the plane of incidence smoothly changes from parallel to the grating lines to perpendicular to the grating lines yielding non-radially symmetric aberration patterns. Figures 5.6 and 5.7 show the polarization aberration function (PAF) of this lens surface. Figure 5.6 shows the amplitude terms, while figure 5.7 shows the wavefront phases .

First consider the wavefront aberration for x-polarized incident light which remains x-polarized (xx term). There is a small amount of apodization due to changes in AOI. The wavefront aberration has more than 0.8 waves of piston, and nearly 0.03 waves peak-valley of astigmatism, on axis. For y-polarized incident and exiting light, the apodization is the same, but the wavefront aberration is modified. The wavefront piston has increased to 0.96 waves and the orientation of the astigmatism has rotated by 90 and the magnitude is reduced to 0.014 waves peak-valley. The difference between the wavefront piston of these two terms yields 0.13 waves of retardance on axis.

Now consider the cross-coupled terms yx and xy . For x-polarized incident light which becomes y-polarized, the wavefront aberration has 0.4 waves of piston and 0.1 waves of astigmatism. For y-polarized incident light exiting as x-polarized, the wavefront aberrations are 0.4 waves of piston and 0.1 waves of defocus, with only a slight hint of astigmatism. For both cross terms, the amplitude terms have the Maltese cross pattern, with maximum amplitude of 3% at the edge of the diagonal in the pupil.

It is useful to examine the retardance over the pupil, the difference in the wave-

Figure 5.6: Magnitude (Amplitude Transmittance) of the polarization aberration function.

front aberration between the two eigenpolarizations at each point. This pupil map of retardance can be characterized by substituting retardance in place of OPD in the wavefront aberration expansion, retaining the same names for patterns with the same mathematical form. Retardance defocus, for example, has quadratic radial dependence and no angular dependence. The retardance pupil map at the design wavelength of 550nm has 0.13 waves of retardance piston, 0.01 waves peak-valley of retardance astigmatism, and 0.01 waves of retardance defocus (figure 5.8). Figure 5.9 shows the variation in the retardance orientation. The orientation remains very close to the grating orientation, with small orientation changes forming a pattern similar to the Maltese cross.

Figure 5.7: Phases of the polarization aberration function.

Figure 5.8: Retardance Magnitude (deg)

Figure 5.9: Retardance Orientation (deg)

5.2.3 Conclusion

The polarization dependent wavefront aberrations of a one-dimensional SWG on a spherical surface have been analyzed. The resulting wavefront aberrations are dominated by astigmatism. The magnitude of the wavefront aberrations are less than 0.1 waves. Other SWG patterns could potentially reduce the magnitude of the aberrations still further.

APPENDIX A

Parameters needed for calculation of Mueller matrices of CCR retroreflectors

This appendix is a tabulation of the relevant parameters for a CCR where the ray incident is not at normal incidence. The tabulated values include all geometrical parameters needed to calculate the Mueller matrix of a CCR. These are the ray propagation vector $\hat{\mathbf{k}}$, the local coordinate basis $\hat{\mathbf{s}}$, $\hat{\mathbf{p}}_{inc}$, $\hat{\mathbf{p}}_{exit}$, the AOI, the azimuthal angle, and the polarization basis rotation angle between each pair of surfaces.

A.1 Ray Propagation Vectors

A CCR may be hollow or solid. For solid CCR, the first and last surface interactions are refractions, while the other interactions are reflections. Let the incident ray direction vector be $\hat{\mathbf{k}}_i = (k_{x_i}, k_{y_i}, k_{z_i})$. To simplify the expressions occurring after the refraction, the substitutions listed in equation A.1 are used.

$$(x, y, z) = \frac{n_i}{n_t} \left(k_{x_i}, k_{y_i}, \sqrt{\frac{n_t^2}{n_i^2} - (k_{x_i}^2 + k_{y_i}^2)} \right) \quad (\text{A.1})$$

The ray propagation vectors, for arbitrary incident ray, are:

$$\begin{array}{ccc}
 & \text{Path } \{1, 2, 3\} & \text{Path } \{2, 3, 1\} \\
 \hat{\mathbf{k}}_1 & \begin{pmatrix} k_{x_i} \\ k_{y_i} \\ k_{z_i} \end{pmatrix} & \begin{pmatrix} k_{x_i} \\ k_{y_i} \\ k_{z_i} \end{pmatrix} \\
 \hat{\mathbf{k}}_2 & \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\
 \hat{\mathbf{k}}_3 & \begin{pmatrix} \frac{1}{3}(2\sqrt{2}z - x) \\ y \\ \frac{1}{3}(2\sqrt{2}x + z) \end{pmatrix} & \begin{pmatrix} \frac{1}{3}(2x + \sqrt{3}y - \sqrt{2}z) \\ \frac{x+\sqrt{2}z}{\sqrt{3}} \\ \frac{1}{3}(-\sqrt{2}x + \sqrt{6}y + z) \end{pmatrix} \\
 \hat{\mathbf{k}}_4 & \begin{pmatrix} \frac{1}{3}(-2x + \sqrt{3}y + \sqrt{2}z) \\ \frac{x+\sqrt{2}z}{\sqrt{3}} \\ \frac{1}{3}(\sqrt{2}x + \sqrt{6}y - z) \end{pmatrix} & \begin{pmatrix} \frac{1}{3}(x - 2\sqrt{2}z) \\ -y \\ \frac{1}{3}(-2\sqrt{2}x - z) \end{pmatrix} \\
 \hat{\mathbf{k}}_5 & \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} & \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \\
 \hat{\mathbf{k}}_6 & \begin{pmatrix} -k_{x_i} \\ -k_{y_i} \\ -k_{z_i} \end{pmatrix} & \begin{pmatrix} -k_{x_i} \\ -k_{y_i} \\ -k_{z_i} \end{pmatrix}
 \end{array} \tag{A.2}$$

	Path {3, 1, 2}	Path {3, 2, 1}	
$\hat{\mathbf{k}}_1$	$\begin{pmatrix} k_{x_i} \\ k_{y_i} \\ k_{z_i} \end{pmatrix}$	$\begin{pmatrix} k_{x_i} \\ k_{y_i} \\ k_{z_i} \end{pmatrix}$	
$\hat{\mathbf{k}}_2$	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	
$\hat{\mathbf{k}}_3$	$\begin{pmatrix} \frac{1}{3} (2x - \sqrt{3}y - \sqrt{2}z) \\ -\frac{x+\sqrt{2}z}{\sqrt{3}} \\ \frac{1}{3} (-\sqrt{2}x - \sqrt{6}y + z) \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} (2x - \sqrt{3}y - \sqrt{2}z) \\ -\frac{x+\sqrt{2}z}{\sqrt{3}} \\ \frac{1}{3} (-\sqrt{2}x - \sqrt{6}y + z) \end{pmatrix}$	(A.3)
$\hat{\mathbf{k}}_4$	$\begin{pmatrix} \frac{1}{3} (-2x - \sqrt{3}y + \sqrt{2}z) \\ -\frac{x+\sqrt{2}z}{\sqrt{3}} \\ \frac{1}{3} (\sqrt{2}x - \sqrt{6}y - z) \end{pmatrix}$	$\begin{pmatrix} \frac{1}{3} (x - 2\sqrt{2}z) \\ -y \\ \frac{1}{3} (-2\sqrt{2}x - z) \end{pmatrix}$	
$\hat{\mathbf{k}}_5$	$\begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$	$\begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$	
$\hat{\mathbf{k}}_6$	$\begin{pmatrix} -k_{x_i} \\ -k_{y_i} \\ -k_{z_i} \end{pmatrix}$	$\begin{pmatrix} -k_{x_i} \\ -k_{y_i} \\ -k_{z_i} \end{pmatrix}$	

$$\begin{array}{ccc}
& \text{Path } \{1, 3, 2\} & \text{Path } \{2, 1, 3\} \\
\hat{\mathbf{k}}_1 & \begin{pmatrix} k_{x_i} \\ k_{y_i} \\ k_{z_i} \end{pmatrix} & \begin{pmatrix} k_{x_i} \\ k_{y_i} \\ k_{z_i} \end{pmatrix} \\
\hat{\mathbf{k}}_2 & \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\
\hat{\mathbf{k}}_3 & \begin{pmatrix} \frac{1}{3}(2\sqrt{2}z - x) \\ y \\ \frac{1}{3}(2\sqrt{2}x + z) \end{pmatrix} & \begin{pmatrix} \frac{1}{3}(2x + \sqrt{3}y - \sqrt{2}z) \\ \frac{x+\sqrt{2}z}{\sqrt{3}} \\ \frac{1}{3}(-\sqrt{2}x + \sqrt{6}y + z) \end{pmatrix} \\
\hat{\mathbf{k}}_4 & \begin{pmatrix} \frac{1}{3}(-2x - \sqrt{3}y + \sqrt{2}z) \\ -\frac{x+\sqrt{2}z}{\sqrt{3}} \\ \frac{1}{3}(\sqrt{2}x - \sqrt{6}y - z) \end{pmatrix} & \begin{pmatrix} \frac{1}{3}(-2x + \sqrt{3}y + \sqrt{2}z) \\ \frac{x+\sqrt{2}z}{\sqrt{3}} \\ \frac{1}{3}(\sqrt{2}x + \sqrt{6}y - z) \end{pmatrix} \\
\hat{\mathbf{k}}_5 & \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} & \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \\
\hat{\mathbf{k}}_6 & \begin{pmatrix} -k_{x_i} \\ -k_{y_i} \\ -k_{z_i} \end{pmatrix} & \begin{pmatrix} -k_{x_i} \\ -k_{y_i} \\ -k_{z_i} \end{pmatrix}
\end{array} \tag{A.4}$$

A.2 s-polarization Vectors

Several substitutions are needed

$$\begin{aligned}
\text{sub0} &= x^2 + y^2 \\
\text{sub1} &= (x + \sqrt{2}z)^2 + 3y^2 \\
\text{sub2} &= 5x^2 - 2\sqrt{3}y(x + \sqrt{2}z) - 2\sqrt{2}xz + 3y^2 + 4z^2 \\
\text{sub3} &= 5x^2 + 2\sqrt{3}xy - 2\sqrt{2}xz + 3y^2 + 2\sqrt{6}yz + 4z^2 \\
\text{sub4} &= x^2 + 2\sqrt{2}xz + 3y^2 + 2z^2 \\
\text{sub5} &= k_{x_i}^2 + k_{y_i}^2 \\
\text{sub6} &= 5x^2 - 2x(\sqrt{3}y + \sqrt{2}z) + 3y^2 - 2\sqrt{6}yz + 4z^2 \\
\text{sub7} &= 25x^4 - 20\sqrt{2}x^3z + 6x^2(3y^2 + 8z^2) - 4\sqrt{2}xz(9y^2 + 4z^2) + 9y^4 + 16z^4
\end{aligned} \tag{A.5}$$

The s-polarization vectors are:

	Path {1, 2, 3}	Path {2, 3, 1}	
Incident Coordinate System	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	
Refraction 1	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	
Reflection 1	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub1}}} \\ \frac{x+\sqrt{2}z}{\sqrt{\text{sub1}}} \\ -\frac{\sqrt{2}y}{\sqrt{\text{sub1}}} \end{pmatrix}$	$\begin{pmatrix} \frac{-\sqrt{2}y-\sqrt{3}z}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x-z}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{3}x+y}{\sqrt{\text{sub3}}} \end{pmatrix}$	
Reflection 2	$\begin{pmatrix} -\frac{4x+2\sqrt{3}y+\sqrt{2}z}{\sqrt{6}\sqrt{\text{sub3}}} \\ \frac{z-\sqrt{2}x}{\sqrt{\text{sub3}}} \\ -\frac{\frac{x}{\sqrt{3}}+y+2\sqrt{\frac{2}{3}}z}{\sqrt{\text{sub3}}} \end{pmatrix}$	$\begin{pmatrix} \frac{-4x+2\sqrt{3}y-\sqrt{2}z}{\sqrt{6}\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x-z}{\sqrt{\text{sub2}}} \\ \frac{-\sqrt{3}x-3y+2\sqrt{6}z}{3\sqrt{\text{sub2}}} \end{pmatrix}$	(A.6)
Reflection 3	$\begin{pmatrix} \frac{\sqrt{2}y-\sqrt{3}z}{\sqrt{\text{sub2}}} \\ \frac{z-\sqrt{2}x}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{3}x-y}{\sqrt{\text{sub2}}} \end{pmatrix}$	$\begin{pmatrix} \frac{y}{\sqrt{\text{sub1}}} \\ \frac{-x-\sqrt{2}z}{\sqrt{\text{sub1}}} \\ \frac{\sqrt{2}y}{\sqrt{\text{sub1}}} \end{pmatrix}$	
Refraction 2	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	
Exiting Coordinate System	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	

	Path {3, 1, 2}	Path {3, 2, 1}	
Incident Coordinate System	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	
Refraction 1	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	
Reflection 1	$\begin{pmatrix} \frac{\sqrt{3}z-\sqrt{2}y}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x-z}{\sqrt{\text{sub2}}} \\ \frac{y-\sqrt{3}x}{\sqrt{\text{sub2}}} \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}z-\sqrt{2}y}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x-z}{\sqrt{\text{sub2}}} \\ \frac{y-\sqrt{3}x}{\sqrt{\text{sub2}}} \end{pmatrix}$	
Reflection 2	$\begin{pmatrix} \frac{x+\sqrt{2}z}{\sqrt{3}\sqrt{\text{sub4}}} \\ -\frac{\sqrt{3}y}{\sqrt{\text{sub4}}} \\ \frac{\sqrt{2}x+2z}{\sqrt{3}\sqrt{\text{sub4}}} \end{pmatrix}$	$\begin{pmatrix} \frac{4x+2\sqrt{3}y+\sqrt{2}z}{\sqrt{6}\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x-z}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{3}x-3y-2\sqrt{6}z}{3\sqrt{\text{sub3}}} \end{pmatrix}$	(A.7)
Reflection 3	$\begin{pmatrix} \frac{\sqrt{2}y+\sqrt{3}z}{\sqrt{\text{sub3}}} \\ \frac{z-\sqrt{2}x}{\sqrt{\text{sub3}}} \\ \frac{-\sqrt{3}x-y}{\sqrt{\text{sub3}}} \end{pmatrix}$	$\begin{pmatrix} \frac{y}{\sqrt{\text{sub1}}} \\ \frac{-x-\sqrt{2}z}{\sqrt{\text{sub1}}} \\ \frac{\sqrt{2}y}{\sqrt{\text{sub1}}} \end{pmatrix}$	
Refraction 2	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	
Exiting Coordinate System	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	

	Path {1, 3, 2}	Path {2, 1, 3}	
Incident Coordinate System	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	
Refraction 1	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{k_{x_i}}{\sqrt{\text{sub5}}} \\ 0 \end{pmatrix}$	
Reflection 1	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub1}}} \\ \frac{x+\sqrt{2}z}{\sqrt{\text{sub1}}} \\ -\frac{\sqrt{2}y}{\sqrt{\text{sub1}}} \end{pmatrix}$	$\begin{pmatrix} \frac{-\sqrt{2}y-\sqrt{3}z}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x-z}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{3}x+y}{\sqrt{\text{sub3}}} \end{pmatrix}$	
Reflection 2	$\begin{pmatrix} \frac{4x-2\sqrt{3}y+\sqrt{2}z}{\sqrt{6}\sqrt{\text{sub2}}} \\ \frac{z-\sqrt{2}x}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x+\sqrt{6}y-4z}{\sqrt{6}\sqrt{\text{sub2}}} \end{pmatrix}$	$\begin{pmatrix} -\frac{x+\sqrt{2}z}{\sqrt{3}\sqrt{\text{sub4}}} \\ \frac{\sqrt{3}y}{\sqrt{\text{sub4}}} \\ -\frac{\sqrt{2}x+2z}{\sqrt{3}\sqrt{\text{sub4}}} \end{pmatrix}$	(A.8)
Reflection 3	$\begin{pmatrix} \frac{\sqrt{2}y+\sqrt{3}z}{\sqrt{\text{sub3}}} \\ \frac{z-\sqrt{2}x}{\sqrt{\text{sub3}}} \\ \frac{-\sqrt{3}x-y}{\sqrt{\text{sub3}}} \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{2}y-\sqrt{3}z}{\sqrt{\text{sub2}}} \\ \frac{z-\sqrt{2}x}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{3}x-y}{\sqrt{\text{sub2}}} \end{pmatrix}$	
Refraction 2	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	
Exiting Coordinate System	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{y}{\sqrt{\text{sub0}}} \\ \frac{x}{\sqrt{\text{sub0}}} \\ 0 \end{pmatrix}$	

A.3 pInc-polarization Vectors

$$\begin{array}{lcl}
& \text{Path } \{1, 2, 3\} & \\
\text{Incident Coordinate System} & \begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} & \\
\text{Refraction 1} & \begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} & \\
\text{Reflection 1} & \begin{pmatrix} \frac{-z(x+\sqrt{2}z)-\sqrt{2}y^2}{\sqrt{\text{sub1}}} \\ \frac{y(\sqrt{2}x-z)}{\sqrt{\text{sub1}}} \\ \frac{\text{sub0}+\sqrt{2}xz}{\sqrt{\text{sub1}}} \end{pmatrix} & \\
\text{Reflection 2} & \begin{pmatrix} \frac{4x^2-x(\sqrt{3}y+\sqrt{2}z)+3y^2+2\sqrt{6}yz-z^2}{3\sqrt{\text{sub3}}} \\ \frac{-\sqrt{3}x^2-xy-z(\sqrt{2}y+\sqrt{3}z)}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x^2+2\sqrt{6}xy-5xz+3\sqrt{2}y^2+\sqrt{3}yz+2\sqrt{2}z^2}{3\sqrt{\text{sub3}}} \end{pmatrix} & (A.9) \\
\text{Reflection 3} & \begin{pmatrix} \frac{5x^2+\sqrt{3}xy+\sqrt{2}xz-2\sqrt{6}yz+z^2}{3\sqrt{\text{sub2}}} \\ \frac{2\sqrt{3}x^2-3xy-2\sqrt{6}xz+3\sqrt{3}y^2-3\sqrt{2}yz+\sqrt{3}z^2}{3\sqrt{\text{sub2}}} \\ \frac{2\sqrt{2}x^2-x(2\sqrt{6}y+z)+z(4\sqrt{2}z-\sqrt{3}y)}{3\sqrt{\text{sub2}}} \end{pmatrix} & \\
\text{Refraction 2} & \begin{pmatrix} \frac{xz}{\sqrt{\text{sub0}}} \\ \frac{yz}{\sqrt{\text{sub0}}} \\ -\sqrt{\text{sub0}} \end{pmatrix} & \\
\text{Exiting Coordinate System} & \begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i}+yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} &
\end{array}$$

$$\begin{array}{l}
\text{Incident Coordinate System} \\
\text{Refraction 1} \\
\text{Reflection 1} \\
\text{Reflection 2} \\
\text{Reflection 3} \\
\text{Refraction 2} \\
\text{Exiting Coordinate System}
\end{array}
\begin{array}{l}
\text{Path } \{2, 3, 1\} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{\sqrt{3}xy - \sqrt{2}xz + y^2 + z^2}{\sqrt{\text{sub3}}} \\ \frac{-\sqrt{3}x^2 - xy - z(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x^2 - xz + y(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \\ \frac{x^2 - x(3\sqrt{3}y + \sqrt{2}z) + 5z^2}{\sqrt{\text{sub3}}} \end{pmatrix} \\
\begin{pmatrix} \frac{3\sqrt{\text{sub2}}}{2\sqrt{3}x^2 - 3xy - 2\sqrt{6}xz + 3\sqrt{3}y^2 - 3\sqrt{2}yz + \sqrt{3}z^2} \\ \frac{3\sqrt{\text{sub2}}}{4\sqrt{2}x^2 + xz + z(2\sqrt{2}z - 3\sqrt{3}y)} \\ \frac{3\sqrt{\text{sub2}}}{-2\sqrt{2}x^2 - 5xz - \sqrt{2}(3y^2 + z^2)} \\ \frac{3\sqrt{\text{sub1}}}{y(z - \sqrt{2}x)} \\ \frac{\sqrt{\text{sub1}}}{-x^2 + \sqrt{2}xz + 3y^2 + 4z^2} \\ \frac{3\sqrt{\text{sub1}}}{-x^2 + \sqrt{2}xz + 3y^2 + 4z^2} \end{pmatrix} \\
\begin{pmatrix} \frac{xz}{\sqrt{\text{sub0}}} \\ \frac{yz}{\sqrt{\text{sub0}}} \\ -\sqrt{\text{sub0}} \end{pmatrix} \\
\begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix}
\end{array}
\quad (A.10)$$

$$\begin{array}{l}
\text{Incident Coordinate System} \\
\text{Refraction 1} \\
\text{Reflection 1} \\
\text{Reflection 2} \\
\text{Reflection 3} \\
\text{Refraction 2} \\
\text{Exiting Coordinate System}
\end{array}
\begin{array}{l}
\text{Path } \{3, 1, 2\} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{-x(\sqrt{3}y+\sqrt{2}z)+y^2+z^2}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{3}x^2-xy+z(\sqrt{3}z-\sqrt{2}y)}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x^2-xz+y(\sqrt{2}y-\sqrt{3}z)}{\sqrt{\text{sub2}}} \end{pmatrix} \\
\begin{pmatrix} \frac{-\sqrt{2}x^2-x(\sqrt{6}y+4z)-3\sqrt{2}y^2+\sqrt{3}yz-2\sqrt{2}z^2}{3\sqrt{\text{sub4}}} \\ -\frac{\sqrt{2}x^2+xz-\sqrt{2}z^2}{\sqrt{3}\sqrt{\text{sub4}}} \\ \frac{\text{sub4}-2\sqrt{3}xy+\sqrt{6}yz}{3\sqrt{\text{sub4}}} \end{pmatrix} \\
\begin{pmatrix} \frac{5x^2-\sqrt{3}xy+\sqrt{2}xz+2\sqrt{6}yz+z^2}{3\sqrt{\text{sub3}}} \\ -\frac{2\sqrt{2}x^2+\sqrt{6}xy-4xz+3\sqrt{2}y^2+2\sqrt{3}yz+\sqrt{2}z^2}{\sqrt{6}\sqrt{\text{sub3}}} \\ \frac{2\sqrt{2}x^2+2\sqrt{6}xy-xz+\sqrt{3}yz+4\sqrt{2}z^2}{3\sqrt{\text{sub3}}} \end{pmatrix} \\
\begin{pmatrix} \frac{xz}{\sqrt{\text{sub0}}} \\ \frac{yz}{\sqrt{\text{sub0}}} \\ -\sqrt{\text{sub0}} \end{pmatrix} \\
\begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i}+yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix}
\end{array}
\quad (A.11)$$

$$\begin{array}{l}
\text{Incident Coordinate System} \\
\text{Refraction 1} \\
\text{Reflection 1} \\
\text{Reflection 2} \\
\text{Reflection 3} \\
\text{Refraction 2} \\
\text{Exiting Coordinate System}
\end{array}
\begin{array}{l}
\text{Path } \{3, 2, 1\} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{-x(\sqrt{3}y+\sqrt{2}z)+y^2+z^2}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{3}x^2-xy+z(\sqrt{3}z-\sqrt{2}y)}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x^2-xz+y(\sqrt{2}y-\sqrt{3}z)}{\sqrt{\text{sub2}}} \end{pmatrix} \\
\begin{pmatrix} \frac{x^2+3\sqrt{3}xy-\sqrt{2}xz+5z^2}{3\sqrt{\text{sub3}}} \\ -\frac{2\sqrt{2}x^2+\sqrt{6}xy-4xz+3\sqrt{2}y^2+2\sqrt{3}yz+\sqrt{2}z^2}{\sqrt{6}\sqrt{\text{sub3}}} \\ \frac{4\sqrt{2}x^2+xz+z(3\sqrt{3}y+2\sqrt{2}z)}{3\sqrt{\text{sub3}}} \end{pmatrix} \\
\begin{pmatrix} \frac{-2\sqrt{2}x^2-5xz-\sqrt{2}(3y^2+z^2)}{3\sqrt{\text{sub1}}} \\ \frac{y(z-\sqrt{2}x)}{\sqrt{\text{sub1}}} \\ \frac{-x^2+\sqrt{2}xz+3y^2+4z^2}{3\sqrt{\text{sub1}}} \end{pmatrix} \\
\begin{pmatrix} \frac{xz}{\sqrt{\text{sub0}}} \\ \frac{yz}{\sqrt{\text{sub0}}} \\ -\sqrt{\text{sub0}} \end{pmatrix} \\
\begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i}+yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix}
\end{array}
\quad (A.12)$$

$$\begin{array}{lcl}
& \text{Path } \{1, 3, 2\} & \\
\text{Incident Coordinate System} & \begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} & \\
\text{Refraction 1} & \begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} & \\
\text{Reflection 1} & \begin{pmatrix} \frac{-z(x+\sqrt{2}z)-\sqrt{2}y^2}{\sqrt{\text{sub1}}} \\ \frac{y(\sqrt{2}x-z)}{\sqrt{\text{sub1}}} \\ \frac{\text{sub0}+\sqrt{2}xz}{\sqrt{\text{sub1}}} \end{pmatrix} & \\
\text{Reflection 2} & \begin{pmatrix} \frac{4x^2+\sqrt{3}xy-\sqrt{2}xz+3y^2-2\sqrt{6}yz-z^2}{3\sqrt{\text{sub2}}} \\ \frac{\sqrt{3}x^2-xy+z(\sqrt{3}z-\sqrt{2}y)}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x^2-2\sqrt{6}xy-5xz+3\sqrt{2}y^2-\sqrt{3}yz+2\sqrt{2}z^2}{3\sqrt{\text{sub2}}} \end{pmatrix} & (A.13) \\
\text{Reflection 3} & \begin{pmatrix} \frac{5x^2-\sqrt{3}xy+\sqrt{2}xz+2\sqrt{6}yz+z^2}{3\sqrt{\text{sub3}}} \\ -\frac{2\sqrt{2}x^2+\sqrt{6}xy-4xz+3\sqrt{2}y^2+2\sqrt{3}yz+\sqrt{2}z^2}{\sqrt{6}\sqrt{\text{sub3}}} \\ \frac{2\sqrt{2}x^2+2\sqrt{6}xy-xz+\sqrt{3}yz+4\sqrt{2}z^2}{3\sqrt{\text{sub3}}} \end{pmatrix} & \\
\text{Refraction 2} & \begin{pmatrix} \frac{xz}{\sqrt{\text{sub0}}} \\ \frac{yz}{\sqrt{\text{sub0}}} \\ -\sqrt{\text{sub0}} \end{pmatrix} & \\
\text{Exiting Coordinate System} & \begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i}+yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} &
\end{array}$$

$$\begin{array}{lcl}
& \text{Path } \{2, 1, 3\} & \\
\text{Incident Coordinate System} & \begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} & \\
\text{Refraction 1} & \begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} & \\
\text{Reflection 1} & \begin{pmatrix} \frac{\sqrt{3}xy - \sqrt{2}xz + y^2 + z^2}{\sqrt{\text{sub3}}} \\ \frac{-\sqrt{3}x^2 - xy - z(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x^2 - xz + y(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \end{pmatrix} & \\
\text{Reflection 2} & \begin{pmatrix} \frac{-\sqrt{2}x^2 + \sqrt{6}xy - 4xz - 3\sqrt{2}y^2 - \sqrt{3}yz - 2\sqrt{2}z^2}{3\sqrt{\text{sub4}}} \\ \frac{\sqrt{2}x^2 + xz - \sqrt{2}z^2}{\sqrt{3}\sqrt{\text{sub4}}} \\ \frac{x^2 + 2x(\sqrt{3}y + \sqrt{2}z) + 3y^2 - \sqrt{6}yz + 2z^2}{3\sqrt{\text{sub4}}} \end{pmatrix} & (A.14) \\
\text{Reflection 3} & \begin{pmatrix} \frac{5x^2 + \sqrt{3}xy + \sqrt{2}xz - 2\sqrt{6}yz + z^2}{3\sqrt{\text{sub2}}} \\ \frac{2\sqrt{3}x^2 - 3xy - 2\sqrt{6}xz + 3\sqrt{3}y^2 - 3\sqrt{2}yz + \sqrt{3}z^2}{3\sqrt{\text{sub2}}} \\ \frac{2\sqrt{2}x^2 - x(2\sqrt{6}y + z) + z(4\sqrt{2}z - \sqrt{3}y)}{3\sqrt{\text{sub2}}} \end{pmatrix} & \\
\text{Refraction 2} & \begin{pmatrix} \frac{xz}{\sqrt{\text{sub0}}} \\ \frac{yz}{\sqrt{\text{sub0}}} \\ -\sqrt{\text{sub0}} \end{pmatrix} & \\
\text{Exiting Coordinate System} & \begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} &
\end{array}$$

A.4 pExit-polarization Vectors

$$\begin{array}{l}
\text{Incident Coordinate System} \\
\text{Refraction 1} \\
\text{Reflection 1} \\
\text{Reflection 2} \\
\text{Reflection 3} \\
\text{Refraction 2} \\
\text{Exiting Coordinate System}
\end{array}
\begin{array}{l}
\text{Path } \{1, 2, 3\} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{zk_{x_i}}{\sqrt{\text{sub5}}} \\ \frac{zk_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub5}}} \end{pmatrix} \\
\begin{pmatrix} \frac{-2\sqrt{2}x^2 - 5xz - \sqrt{2}(3y^2 + z^2)}{3\sqrt{\text{sub1}}} \\ \frac{y(z - \sqrt{2}x)}{\sqrt{\text{sub1}}} \\ \frac{-x^2 + \sqrt{2}xz + 3y^2 + 4z^2}{3\sqrt{\text{sub1}}} \end{pmatrix} \\
\begin{pmatrix} \frac{x^2 + 3\sqrt{3}xy - \sqrt{2}xz + 5z^2}{3\sqrt{\text{sub3}}} \\ -\frac{2\sqrt{2}x^2 + \sqrt{6}xy - 4xz + 3\sqrt{2}y^2 + 2\sqrt{3}yz + \sqrt{2}z^2}{\sqrt{6}\sqrt{\text{sub3}}} \\ \frac{4\sqrt{2}x^2 + xz + z(3\sqrt{3}y + 2\sqrt{2}z)}{3\sqrt{\text{sub3}}} \end{pmatrix} \\
\begin{pmatrix} \frac{-x(\sqrt{3}y + \sqrt{2}z) + y^2 + z^2}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{3}x^2 - xy + z(\sqrt{3}z - \sqrt{2}y)}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x^2 - xz + y(\sqrt{2}y - \sqrt{3}z)}{\sqrt{\text{sub2}}} \end{pmatrix} \\
\begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} \\
\begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix}
\end{array}
\quad (A.15)$$

$$\begin{array}{l}
\text{Incident Coordinate System} \\
\text{Refraction 1} \\
\text{Reflection 1} \\
\text{Reflection 2} \\
\text{Reflection 3} \\
\text{Refraction 2} \\
\text{Exiting Coordinate System}
\end{array}
\begin{array}{l}
\text{Path } \{2, 3, 1\} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{z k_{x_i}}{\sqrt{\text{sub5}}} \\ \frac{z k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub5}}} \end{pmatrix} \\
\begin{pmatrix} \frac{5x^2 - \sqrt{3}xy + \sqrt{2}xz + 2\sqrt{6}yz + z^2}{3\sqrt{\text{sub3}}} \\ -\frac{2\sqrt{2}x^2 + \sqrt{6}xy - 4xz + 3\sqrt{2}y^2 + 2\sqrt{3}yz + \sqrt{2}z^2}{\sqrt{6}\sqrt{\text{sub3}}} \\ \frac{2\sqrt{2}x^2 + 2\sqrt{6}xy - xz + \sqrt{3}yz + 4\sqrt{2}z^2}{3\sqrt{\text{sub3}}} \end{pmatrix} \\
\begin{pmatrix} \frac{4x^2 + \sqrt{3}xy - \sqrt{2}xz + 3y^2 - 2\sqrt{6}yz - z^2}{3\sqrt{\text{sub2}}} \\ \frac{\sqrt{3}x^2 - xy + z(\sqrt{3}z - \sqrt{2}y)}{\sqrt{\text{sub2}}} \\ \frac{\sqrt{2}x^2 - 2\sqrt{6}xy - 5xz + 3\sqrt{2}y^2 - \sqrt{3}yz + 2\sqrt{2}z^2}{3\sqrt{\text{sub2}}} \end{pmatrix} \\
\begin{pmatrix} \frac{-z(x + \sqrt{2}z) - \sqrt{2}y^2}{\sqrt{\text{sub1}}} \\ \frac{y(\sqrt{2}x - z)}{\sqrt{\text{sub1}}} \\ \frac{\text{sub0} + \sqrt{2}xz}{\sqrt{\text{sub1}}} \end{pmatrix} \\
\begin{pmatrix} \frac{x k_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{y k_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} \\
\begin{pmatrix} \frac{x k_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{y k_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix}
\end{array}
\quad (A.16)$$

$$\begin{array}{lcl}
& \text{Path } \{3, 1, 2\} & \\
\text{Incident Coordinate System} & \left(\begin{array}{c} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{array} \right) & \\
\text{Refraction 1} & \left(\begin{array}{c} \frac{z k_{x_i}}{\sqrt{\text{sub5}}} \\ \frac{z k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub5}}} \end{array} \right) & \\
\text{Reflection 1} & \left(\begin{array}{c} \frac{5x^2 + \sqrt{3}xy + \sqrt{2}xz - 2\sqrt{6}yz + z^2}{3\sqrt{\text{sub2}}} \\ \frac{2\sqrt{3}x^2 - 3xy - 2\sqrt{6}xz + 3\sqrt{3}y^2 - 3\sqrt{2}yz + \sqrt{3}z^2}{3\sqrt{\text{sub2}}} \\ \frac{2\sqrt{2}x^2 - x(2\sqrt{6}y + z) + z(4\sqrt{2}z - \sqrt{3}y)}{3\sqrt{\text{sub2}}} \end{array} \right) & \\
\text{Reflection 2} & \left(\begin{array}{c} \frac{-\sqrt{2}x^2 + \sqrt{6}xy - 4xz - 3\sqrt{2}y^2 - \sqrt{3}yz - 2\sqrt{2}z^2}{3\sqrt{\text{sub4}}} \\ \frac{\sqrt{2}x^2 + xz - \sqrt{2}z^2}{\sqrt{3}\sqrt{\text{sub4}}} \\ \frac{x^2 + 2x(\sqrt{3}y + \sqrt{2}z) + 3y^2 - \sqrt{6}yz + 2z^2}{3\sqrt{\text{sub4}}} \end{array} \right) & (A.17) \\
\text{Reflection 3} & \left(\begin{array}{c} \frac{\sqrt{3}xy - \sqrt{2}xz + y^2 + z^2}{\sqrt{\text{sub3}}} \\ \frac{-\sqrt{3}x^2 - xy - z(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x^2 - xz + y(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \end{array} \right) & \\
\text{Refraction 2} & \left(\begin{array}{c} \frac{x k_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{y k_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub0}}} \end{array} \right) & \\
\text{Exiting Coordinate System} & \left(\begin{array}{c} \frac{x k_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{y k_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub0}}} \end{array} \right) &
\end{array}$$

$$\begin{array}{l}
\text{Incident Coordinate System} \\
\text{Refraction 1} \\
\text{Reflection 1} \\
\text{Reflection 2} \\
\text{Reflection 3} \\
\text{Refraction 2} \\
\text{Exiting Coordinate System}
\end{array}
\begin{array}{l}
\text{Path } \{3, 2, 1\} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{z k_{x_i}}{\sqrt{\text{sub5}}} \\ \frac{z k_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub5}}} \end{pmatrix} \\
\begin{pmatrix} \frac{5x^2 + \sqrt{3}xy + \sqrt{2}xz - 2\sqrt{6}yz + z^2}{3\sqrt{\text{sub2}}} \\ \frac{2\sqrt{3}x^2 - 3xy - 2\sqrt{6}xz + 3\sqrt{3}y^2 - 3\sqrt{2}yz + \sqrt{3}z^2}{3\sqrt{\text{sub2}}} \\ \frac{2\sqrt{2}x^2 - x(2\sqrt{6}y + z) + z(4\sqrt{2}z - \sqrt{3}y)}{3\sqrt{\text{sub2}}} \\ \frac{4x^2 - x(\sqrt{3}y + \sqrt{2}z) + 3y^2 + 2\sqrt{6}yz - z^2}{3\sqrt{\text{sub3}}} \\ \frac{-\sqrt{3}x^2 - xy - z(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x^2 + 2\sqrt{6}xy - 5xz + 3\sqrt{2}y^2 + \sqrt{3}yz + 2\sqrt{2}z^2}{3\sqrt{\text{sub3}}} \end{pmatrix} \\
\begin{pmatrix} \frac{-z(x + \sqrt{2}z) - \sqrt{2}y^2}{\sqrt{\text{sub1}}} \\ \frac{y(\sqrt{2}x - z)}{\sqrt{\text{sub1}}} \\ \frac{\text{sub0} + \sqrt{2}xz}{\sqrt{\text{sub1}}} \end{pmatrix} \\
\begin{pmatrix} \frac{x k_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{y k_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} \\
\begin{pmatrix} \frac{x k_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{y k_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{x k_{x_i} + y k_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix}
\end{array}
\quad (A.18)$$

$$\begin{array}{l}
\text{Incident Coordinate System} \\
\text{Refraction 1} \\
\text{Reflection 1} \\
\text{Reflection 2} \\
\text{Reflection 3} \\
\text{Refraction 2} \\
\text{Exiting Coordinate System}
\end{array}
\begin{array}{l}
\text{Path } \{1, 3, 2\} \\
\begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} \\
\begin{pmatrix} \frac{zk_{x_i}}{\sqrt{\text{sub5}}} \\ \frac{zk_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub5}}} \end{pmatrix} \\
\begin{pmatrix} \frac{-2\sqrt{2}x^2 - 5xz - \sqrt{2}(3y^2 + z^2)}{3\sqrt{\text{sub1}}} \\ \frac{y(z - \sqrt{2}x)}{\sqrt{\text{sub1}}} \\ \frac{-x^2 + \sqrt{2}xz + 3y^2 + 4z^2}{3\sqrt{\text{sub1}}} \\ \frac{x^2 - x(3\sqrt{3}y + \sqrt{2}z) + 5z^2}{3\sqrt{\text{sub2}}} \end{pmatrix} \\
\begin{pmatrix} \frac{2\sqrt{3}x^2 - 3xy - 2\sqrt{6}xz + 3\sqrt{3}y^2 - 3\sqrt{2}yz + \sqrt{3}z^2}{3\sqrt{\text{sub2}}} \\ \frac{4\sqrt{2}x^2 + xz + z(2\sqrt{2}z - 3\sqrt{3}y)}{3\sqrt{\text{sub2}}} \\ \frac{\sqrt{3}xy - \sqrt{2}xz + y^2 + z^2}{\sqrt{\text{sub3}}} \\ \frac{-\sqrt{3}x^2 - xy - z(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \\ \frac{\sqrt{2}x^2 - xz + y(\sqrt{2}y + \sqrt{3}z)}{\sqrt{\text{sub3}}} \end{pmatrix} \\
\begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} \\
\begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix}
\end{array}
\quad (A.19)$$

$$\begin{array}{lcl}
& \text{Path } \{2, 1, 3\} & \\
\text{Incident Coordinate System} & \begin{pmatrix} \frac{k_{x_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ \frac{k_{y_i} k_{z_i}}{\sqrt{\text{sub5}}} \\ -\sqrt{\text{sub5}} \end{pmatrix} & \\
\text{Refraction 1} & \begin{pmatrix} \frac{zk_{x_i}}{\sqrt{\text{sub5}}} \\ \frac{zk_{y_i}}{\sqrt{\text{sub5}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub5}}} \end{pmatrix} & \\
\text{Reflection 1} & \begin{pmatrix} \frac{5x^2 - \sqrt{3}xy + \sqrt{2}xz + 2\sqrt{6}yz + z^2}{3\sqrt{\text{sub3}}} \\ -\frac{2\sqrt{2}x^2 + \sqrt{6}xy - 4xz + 3\sqrt{2}y^2 + 2\sqrt{3}yz + \sqrt{2}z^2}{\sqrt{6}\sqrt{\text{sub3}}} \\ \frac{2\sqrt{2}x^2 + 2\sqrt{6}xy - xz + \sqrt{3}yz + 4\sqrt{2}z^2}{3\sqrt{\text{sub3}}} \\ -\frac{\sqrt{2}x^2 - x(\sqrt{6}y + 4z) - 3\sqrt{2}y^2 + \sqrt{3}yz - 2\sqrt{2}z^2}{3\sqrt{\text{sub4}}} \end{pmatrix} & \\
\text{Reflection 2} & \begin{pmatrix} \frac{3\sqrt{\text{sub4}}}{\sqrt{2}x^2 + xz - \sqrt{2}z^2} \\ \frac{\sqrt{3}\sqrt{\text{sub4}}}{\text{sub4} - 2\sqrt{3}xy + \sqrt{6}yz} \\ \frac{3\sqrt{\text{sub4}}}{-x(\sqrt{3}y + \sqrt{2}z) + y^2 + z^2} \\ \frac{\sqrt{\text{sub2}}}{\sqrt{3}x^2 - xy + z(\sqrt{3}z - \sqrt{2}y)} \\ \frac{\sqrt{\text{sub2}}}{\sqrt{2}x^2 - xz + y(\sqrt{2}y - \sqrt{3}z)} \end{pmatrix} & (A.20) \\
\text{Reflection 3} & \begin{pmatrix} \frac{\sqrt{\text{sub2}}}{xk_{z_i}} \\ \frac{\sqrt{\text{sub0}}}{yk_{z_i}} \\ \frac{\sqrt{\text{sub0}}}{xk_{x_i} + yk_{y_i}} \\ -\frac{\sqrt{\text{sub0}}}{xk_{z_i}} \\ \frac{\sqrt{\text{sub0}}}{yk_{z_i}} \\ -\frac{\sqrt{\text{sub0}}}{xk_{x_i} + yk_{y_i}} \end{pmatrix} & \\
\text{Refraction 2} & \begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} & \\
\text{Exiting Coordinate System} & \begin{pmatrix} \frac{xk_{z_i}}{\sqrt{\text{sub0}}} \\ \frac{yk_{z_i}}{\sqrt{\text{sub0}}} \\ -\frac{xk_{x_i} + yk_{y_i}}{\sqrt{\text{sub0}}} \end{pmatrix} &
\end{array}$$

A.5 AOI on each surface

	Path {1, 2, 3}	Path {2, 3, 1}	
Refraction 1	$\cos^{-1}(k_{z_i})$	$\cos^{-1}(k_{z_i})$	
Reflection 1	$\cos^{-1}\left(\frac{\sqrt{2x-z}}{\sqrt{3}}\right)$	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	
Reflection 2	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} - \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	(A.21)
Reflection 3	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} - \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{\sqrt{2x-z}}{\sqrt{3}}\right)$	
Refraction 2	$\cos^{-1}(z)$	$\cos^{-1}(z)$	

	Path {3, 1, 2}	Path {3, 2, 1}	
Refraction 1	$\cos^{-1}(k_{z_i})$	$\cos^{-1}(k_{z_i})$	
Reflection 1	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} - \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} - \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	(A.22)
Reflection 2	$\cos^{-1}\left(\frac{\sqrt{2x-z}}{\sqrt{3}}\right)$	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	
Reflection 3	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{\sqrt{2x-z}}{\sqrt{3}}\right)$	
Refraction 2	$\cos^{-1}(z)$	$\cos^{-1}(z)$	

	Path {1, 3, 2}	Path {2, 1, 3}	
Refraction 1	$\cos^{-1}(k_{z_i})$	$\cos^{-1}(k_{z_i})$	
Reflection 1	$\cos^{-1}\left(\frac{\sqrt{2x-z}}{\sqrt{3}}\right)$	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	(A.23)
Reflection 2	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} - \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{\sqrt{2x-z}}{\sqrt{3}}\right)$	
Reflection 3	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	$\cos^{-1}\left(-\frac{x}{\sqrt{6}} - \frac{y}{\sqrt{2}} - \frac{z}{\sqrt{3}}\right)$	
Refraction 2	$\cos^{-1}(z)$	$\cos^{-1}(z)$	

A.6 Azimuthal angle on each surface

	Path {1, 2, 3}	
Reflection 1	$\arcsin\left(\frac{x+\sqrt{2}z}{\sqrt{x^2+2\sqrt{2}xz+3y^2+2z^2}}\right)$	(A.24)
Reflection 2	$\arcsin\left(\frac{\sqrt{\frac{3}{2}}(\sqrt{3}x+y)}{\sqrt{5x^2+2\sqrt{3}xy-2\sqrt{2}xz+3y^2+2\sqrt{6}yz+4z^2}}\right)$	
Reflection 3	$\arcsin\left(\frac{\sqrt{3}x+3y-2\sqrt{6}z}{\sqrt{6}\sqrt{5x^2-2x(\sqrt{3}y+\sqrt{2}z)+3y^2-2\sqrt{6}yz+4z^2}}\right)$	

$$\begin{aligned}
& \text{Path } \{2, 3, 1\} \\
\text{Reflection 1} & \arcsin \left(\frac{-\sqrt{3}x+3y+2\sqrt{6}z}{\sqrt{6}\sqrt{5x^2+2\sqrt{3}xy-2\sqrt{2}xz+3y^2+2\sqrt{6}yz+4z^2}} \right) \\
\text{Reflection 2} & \arcsin \left(\frac{\sqrt{3}y-3x}{\sqrt{10x^2-4x(\sqrt{3}y+\sqrt{2}z)+6y^2-4\sqrt{6}yz+8z^2}} \right) \\
\text{Reflection 3} & -\arcsin \left(\frac{x+\sqrt{2}z}{\sqrt{x^2+2\sqrt{2}xz+3y^2+2z^2}} \right)
\end{aligned} \tag{A.25}$$

$$\begin{aligned}
& \text{Path } \{3, 1, 2\} \\
\text{Reflection 1} & \arcsin \left(\frac{-\sqrt{3}x-3y+2\sqrt{6}z}{\sqrt{6}\sqrt{5x^2-2x(\sqrt{3}y+\sqrt{2}z)+3y^2-2\sqrt{6}yz+4z^2}} \right) \\
\text{Reflection 2} & -\arcsin \left(\frac{y}{\sqrt{\frac{1}{3}(x+\sqrt{2}z)^2+y^2}} \right) \\
\text{Reflection 3} & \arcsin \left(\frac{\sqrt{3}x-3y-2\sqrt{6}z}{\sqrt{6}\sqrt{5x^2+2\sqrt{3}xy-2\sqrt{2}xz+3y^2+2\sqrt{6}yz+4z^2}} \right)
\end{aligned} \tag{A.26}$$

$$\begin{aligned}
& \text{Path } \{3, 2, 1\} \\
\text{Reflection 1} & \arcsin \left(\frac{-\sqrt{3}x-3y+2\sqrt{6}z}{\sqrt{6}\sqrt{5x^2-2x(\sqrt{3}y+\sqrt{2}z)+3y^2-2\sqrt{6}yz+4z^2}} \right) \\
\text{Reflection 2} & -\arcsin \left(\frac{\sqrt{\frac{3}{2}}(\sqrt{3}x+y)}{\sqrt{5x^2+2\sqrt{3}xy-2\sqrt{2}xz+3y^2+2\sqrt{6}yz+4z^2}} \right) \\
\text{Reflection 3} & -\arcsin \left(\frac{x+\sqrt{2}z}{\sqrt{x^2+2\sqrt{2}xz+3y^2+2z^2}} \right)
\end{aligned} \tag{A.27}$$

$$\begin{aligned}
& \text{Path } \{1, 3, 2\} \\
\text{Reflection 1} & \arcsin \left(\frac{x+\sqrt{2}z}{\sqrt{x^2+2\sqrt{2}xz+3y^2+2z^2}} \right) \\
\text{Reflection 2} & \arcsin \left(\frac{\sqrt{\frac{3}{2}}(\sqrt{3}x-y)}{\sqrt{5x^2-2x(\sqrt{3}y+\sqrt{2}z)+3y^2-2\sqrt{6}yz+4z^2}} \right) \\
\text{Reflection 3} & \arcsin \left(\frac{\sqrt{3}x-3y-2\sqrt{6}z}{\sqrt{6}\sqrt{5x^2+2\sqrt{3}xy-2\sqrt{2}xz+3y^2+2\sqrt{6}yz+4z^2}} \right)
\end{aligned} \tag{A.28}$$

$$\begin{aligned}
& \text{Path } \{2, 1, 3\} \\
\text{Reflection 1} & \arcsin \left(\frac{-\sqrt{3}x+3y+2\sqrt{6}z}{\sqrt{6}\sqrt{5x^2+2\sqrt{3}xy-2\sqrt{2}xz+3y^2+2\sqrt{6}yz+4z^2}} \right) \\
\text{Reflection 2} & \arcsin \left(\frac{y}{\sqrt{\frac{1}{3}(x+\sqrt{2}z)^2+y^2}} \right) \\
\text{Reflection 3} & \arcsin \left(\frac{\sqrt{3}x+3y-2\sqrt{6}z}{\sqrt{6}\sqrt{5x^2-2x(\sqrt{3}y+\sqrt{2}z)+3y^2-2\sqrt{6}yz+4z^2}} \right)
\end{aligned} \tag{A.29}$$

A.7 Rotation angles between each surface (ac=arccos)

	Path {1, 2, 3}	Path {2, 3, 1}
Rotation 1	0	0
Rotation 2	$\text{ac} \left(\frac{-(x+\sqrt{2}z)k_{x_i}-yk_{y_i}}{\sqrt{\text{sub4}}\sqrt{\text{sub5}}} \right)$	$\text{ac} \left(\frac{(z-\sqrt{2}x)k_{x_i}-(\sqrt{2}y+\sqrt{3}z)k_{y_i}}{\sqrt{\text{sub3}}\sqrt{\text{sub5}}} \right)$
Rotation 3	$\text{ac} \left(-\frac{2\sqrt{3}x^2-6xy+\sqrt{6}xz+3\sqrt{2}yz-2\sqrt{3}z^2}{\sqrt{6}\sqrt{\text{sub3}}\sqrt{\text{sub4}}} \right)$	$\text{ac} \left(\frac{\sqrt{2}x^2+4xz+\sqrt{2}(2z^2-3y^2)}{\sqrt{2}\sqrt{\text{sub7}}} \right)$
Rotation 4	$\text{ac} \left(\frac{\sqrt{2}x^2+4xz+\sqrt{2}(2z^2-3y^2)}{\sqrt{2}\sqrt{\text{sub7}}} \right)$	$\text{ac} \left(\frac{-2x^2-x(2\sqrt{3}y+\sqrt{2}z)+z(\sqrt{6}y+2z)}{\sqrt{2}\sqrt{\text{sub4}}\sqrt{\text{sub6}}} \right)$
Rotation 5	$\text{ac} \left(\frac{-\sqrt{2}x^2+xz+y(\sqrt{3}z-\sqrt{2}y)}{\sqrt{\text{sub0}}\sqrt{\text{sub6}}} \right)$	$\text{ac} \left(-\frac{\text{sub0}+\sqrt{2}xz}{\sqrt{\text{sub0}}\sqrt{\text{sub4}}} \right)$
Rotation 6	0	0

(A.30)

	Path {3, 1, 2}	Path {3, 2, 1}
Rotation 1	0	0
Rotation 2	$\text{ac} \left(\frac{zk_{x_i}-\sqrt{2}xk_{x_i}+\sqrt{3}zk_{y_i}-\sqrt{2}yk_{y_i}}{\sqrt{\text{sub5}}\sqrt{\text{sub6}}} \right)$	$\text{ac} \left(\frac{zk_{x_i}-\sqrt{2}xk_{x_i}+\sqrt{3}zk_{y_i}-\sqrt{2}yk_{y_i}}{\sqrt{\text{sub5}}\sqrt{\text{sub6}}} \right)$
Rotation 3	$\text{ac} \left(-\frac{\sqrt{6}x^2+x(3\sqrt{2}y+\sqrt{3}z)-z(3y+\sqrt{6}z)}{\sqrt{3}\sqrt{\text{sub4}}\sqrt{\text{sub6}}} \right)$	$\text{ac} \left(\frac{\sqrt{2}x^2+4xz+\sqrt{2}(2z^2-3y^2)}{\sqrt{2}\sqrt{\text{sub7}}} \right)$
Rotation 4	$\text{ac} \left(-\frac{\sqrt{6}x^2-3\sqrt{2}xy+\sqrt{3}xz+3yz-\sqrt{6}z^2}{\sqrt{3}\sqrt{\text{sub3}}\sqrt{\text{sub4}}} \right)$	$\text{ac} \left(-\frac{2x^2-2\sqrt{3}xy+\sqrt{2}xz+\sqrt{6}yz-2z^2}{\sqrt{2}\sqrt{\text{sub3}}\sqrt{\text{sub4}}} \right)$
Rotation 5	$\text{ac} \left(\frac{-\sqrt{2}x^2+xz-y(\sqrt{2}y+\sqrt{3}z)}{\sqrt{\text{sub0}}\sqrt{\text{sub3}}} \right)$	$\text{ac} \left(-\frac{\text{sub0}+\sqrt{2}xz}{\sqrt{\text{sub0}}\sqrt{\text{sub4}}} \right)$
Rotation 6	0	0

(A.31)

	Path {1, 3, 2}	Path {2, 1, 3}
Rotation 1	0	0
Rotation 2	$\text{ac} \left(\frac{-(x+\sqrt{2}z)k_{x_i}-yk_{y_i}}{\sqrt{\text{sub4}}\sqrt{\text{sub5}}} \right)$	$\text{ac} \left(\frac{(z-\sqrt{2}x)k_{x_i}-(\sqrt{2}y+\sqrt{3}z)k_{y_i}}{\sqrt{\text{sub3}}\sqrt{\text{sub5}}} \right)$
Rotation 3	$\text{ac} \left(-\frac{2\sqrt{3}x^2+6xy+\sqrt{6}xz-3\sqrt{2}yz-2\sqrt{3}z^2}{\sqrt{6}\sqrt{\text{sub4}}\sqrt{\text{sub6}}} \right)$	$\text{ac} \left(-\frac{\sqrt{6}x^2-3\sqrt{2}xy+\sqrt{3}xz+3yz-\sqrt{6}z^2}{\sqrt{3}\sqrt{\text{sub3}}\sqrt{\text{sub4}}} \right)$
Rotation 4	$\text{ac} \left(\frac{\sqrt{2}x^2+4xz+\sqrt{2}(2z^2-3y^2)}{\sqrt{2}\sqrt{\text{sub7}}} \right)$	$\text{ac} \left(-\frac{\sqrt{6}x^2+x(3\sqrt{2}y+\sqrt{3}z)-z(3y+\sqrt{6}z)}{\sqrt{3}\sqrt{\text{sub4}}\sqrt{\text{sub6}}} \right)$
Rotation 5	$\text{ac} \left(\frac{-\sqrt{2}x^2+xz-y(\sqrt{2}y+\sqrt{3}z)}{\sqrt{\text{sub0}}\sqrt{\text{sub3}}} \right)$	$\text{ac} \left(\frac{-\sqrt{2}x^2+xz+y(\sqrt{3}z-\sqrt{2}y)}{\sqrt{\text{sub0}}\sqrt{\text{sub6}}} \right)$
Rotation 6	0	0

(A.32)

APPENDIX B

Parameters for all paths through a CCR at normal incidence

B.1 The six paths through a CCR at normal incidence

$$\begin{aligned}
 \mathbf{MM}_{123} &= \mathbf{R}(-120^\circ) \cdot \mathbf{T}_3 \cdot \mathbf{R}(60^\circ) \cdot \mathbf{T}_2 \cdot \mathbf{R}(-60^\circ) \cdot \mathbf{T}_1 \cdot \mathbf{R}(-60^\circ) \\
 \mathbf{MM}_{231} &= \mathbf{R}(120^\circ) \cdot \mathbf{T}_1 \cdot \mathbf{R}(60^\circ) \cdot \mathbf{T}_3 \cdot \mathbf{R}(-60^\circ) \cdot \mathbf{T}_2 \cdot \mathbf{R}(180^\circ) \\
 \mathbf{MM}_{312} &= \mathbf{R}(0^\circ) \cdot \mathbf{T}_2 \cdot \mathbf{R}(60^\circ) \cdot \mathbf{T}_1 \cdot \mathbf{R}(-60^\circ) \cdot \mathbf{T}_3 \cdot \mathbf{R}(60^\circ) \\
 \mathbf{MM}_{321} &= \mathbf{R}(-60^\circ) \cdot \mathbf{T}_1 \cdot \mathbf{R}(-60^\circ) \cdot \mathbf{T}_2 \cdot \mathbf{R}(60^\circ) \cdot \mathbf{T}_3 \cdot \mathbf{R}(-120^\circ) \\
 \mathbf{MM}_{132} &= \mathbf{R}(180^\circ) \cdot \mathbf{T}_2 \cdot \mathbf{R}(-60^\circ) \cdot \mathbf{T}_3 \cdot \mathbf{R}(60^\circ) \cdot \mathbf{T}_1 \cdot \mathbf{R}(120^\circ) \\
 \mathbf{MM}_{213} &= \mathbf{R}(60^\circ) \cdot \mathbf{T}_3 \cdot \mathbf{R}(-60^\circ) \cdot \mathbf{T}_1 \cdot \mathbf{R}(60^\circ) \cdot \mathbf{T}_3 \cdot \mathbf{R}(0^\circ)
 \end{aligned} \tag{B.1}$$

B.2 Direction vectors

	$\{1, 2, 3\}$	$\{2, 3, 1\}$	$\{3, 1, 2\}$	$\{3, 2, 1\}$	$\{1, 3, 2\}$	$\{2, 1, 3\}$
$\hat{\mathbf{k}}_1$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
$\hat{\mathbf{k}}_2$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
$\hat{\mathbf{k}}_3$	$\begin{pmatrix} \frac{2\sqrt{2}}{3} \\ 0 \\ \frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{2}}{3} \\ \sqrt{\frac{2}{3}} \\ \frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{2}}{3} \\ -\sqrt{\frac{2}{3}} \\ \frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{2}}{3} \\ -\sqrt{\frac{2}{3}} \\ \frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{2\sqrt{2}}{3} \\ 0 \\ \frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{2}}{3} \\ \sqrt{\frac{2}{3}} \\ \frac{1}{3} \end{pmatrix}$
$\hat{\mathbf{k}}_4$	$\begin{pmatrix} \frac{\sqrt{2}}{3} \\ \sqrt{\frac{2}{3}} \\ -\frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{2\sqrt{2}}{3} \\ 0 \\ -\frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{2}}{3} \\ -\sqrt{\frac{2}{3}} \\ -\frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{2\sqrt{2}}{3} \\ 0 \\ -\frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{2}}{3} \\ -\sqrt{\frac{2}{3}} \\ -\frac{1}{3} \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{2}}{3} \\ \sqrt{\frac{2}{3}} \\ -\frac{1}{3} \end{pmatrix}$
$\hat{\mathbf{k}}_5$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$
$\hat{\mathbf{k}}_6$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

(B.2)

B.3 s-polarized vectors

	$\{1, 2, 3\}$	$\{2, 3, 1\}$	$\{3, 1, 2\}$
Incident	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
Coordinate			
System			
Refraction 1	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
Reflection 1	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$
Reflection 2	$\begin{pmatrix} -\frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2\sqrt{3}} \\ -\frac{1}{2} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \sqrt{\frac{2}{3}} \end{pmatrix}$
Reflection 3	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$
Refraction 2	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$
Exiting	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$
Coordinate			
System			

(B.3)

	$\{3, 2, 1\}$	$\{1, 3, 2\}$	$\{2, 1, 3\}$	
Incident	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	
Coordinate				
System				
Refraction 1	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	
Reflection 1	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	
Reflection 2	$\begin{pmatrix} \frac{1}{2\sqrt{3}} \\ -\frac{1}{2} \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 0 \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	(B.4)
Reflection 3	$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	
Refraction 2	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	
Exiting	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	
Coordinate				
System				

B.4 p_{inc} -polarized vectors

	$\{1, 2, 3\}$	$\{2, 3, 1\}$	$\{3, 1, 2\}$
Incident	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
Coordinate			
System			
Refraction 1	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
Reflection 1	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$
Reflection 2	$\begin{pmatrix} -\frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2\sqrt{3}} \\ -\frac{1}{2} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \sqrt{\frac{2}{3}} \end{pmatrix}$
Reflection 3	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$
Refraction 2	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$
Exiting	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$
Coordinate			
System			

(B.5)

	$\{3, 2, 1\}$	$\{1, 3, 2\}$	$\{2, 1, 3\}$	
Incident	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	
Coordinate				
System				
Refraction 1	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	
Reflection 1	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	
Reflection 2	$\begin{pmatrix} \frac{1}{2\sqrt{3}} \\ -\frac{1}{2} \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 0 \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	(B.6)
Reflection 3	$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	
Refraction 2	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	
Exiting	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	
Coordinate				
System				

B.5 p_{exit} -polarized vectors

	$\{1, 2, 3\}$	$\{2, 3, 1\}$	$\{3, 1, 2\}$
Incident	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
Coordinate			
System			
Refraction 1	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
Reflection 1	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$
Reflection 2	$\begin{pmatrix} -\frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{2\sqrt{3}} \\ -\frac{1}{2} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\sqrt{3}} \\ 0 \\ \sqrt{\frac{2}{3}} \end{pmatrix}$
Reflection 3	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$
Refraction 2	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$
Exiting	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$
Coordinate			
System			

(B.7)

	$\{3, 2, 1\}$	$\{1, 3, 2\}$	$\{2, 1, 3\}$	
Incident	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	
Coordinate				
System				
Refraction 1	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	
Reflection 1	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{pmatrix}$	
Reflection 2	$\begin{pmatrix} \frac{1}{2\sqrt{3}} \\ -\frac{1}{2} \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{2\sqrt{3}} \\ \frac{1}{2} \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 0 \\ -\sqrt{\frac{2}{3}} \end{pmatrix}$	(B.8)
Reflection 3	$\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	$\begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$	
Refraction 2	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	
Exiting	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$	
Coordinate				
System				

B.6 Angle of Incidence

	Path {3, 2, 1}	Path {1, 3, 2}	Path {2, 1, 3}	
Refraction 1	0	0	0	
Reflection 1	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	
Reflection 2	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	
Reflection 3	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	
Refraction 2	π	π	π	(B.9)

	Path {3, 2, 1}	Path {1, 3, 2}	Path {2, 1, 3}	
Refraction 1	0	0	0	
Reflection 1	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	
Reflection 2	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	
Reflection 3	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$	
Refraction 2	π	π	π	(B.10)

B.7 Azimuthal Angle

	Reflection 1	Reflection 2	Reflection 3	
Path {1, 2, 3}	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$	
Path {2, 3, 1}	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$	
Path {3, 1, 2}	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$	
Path {3, 2, 1}	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$	
Path {1, 3, 2}	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$	
Path {2, 1, 3}	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$	(B.11)

B.8 Rotation Angles

	$\{1, 2, 3\}$	$\{2, 3, 1\}$	$\{3, 1, 2\}$	$\{3, 2, 1\}$	$\{1, 3, 2\}$	$\{2, 1, 3\}$
Rotation 1	0	0	0	0	0	0
Rotation 2	0	0	0	0	0	0
Rotation 3	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$
Rotation 4	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$
Rotation 5	0	0	0	0	0	0
Rotation 6	0	0	0	0	0	0

(B.12)

APPENDIX C

List of Variables Names

Scalar quantities

A various electric field amplitudes

d physical distance between two points (surface intercepts)

r reflectance

t transmittance

t time

τ optical distance between two points in waves

λ wavelength

Vectors

$\widehat{\mathbf{a}}\mathbf{n}$ surface anisotropy vector

$\vec{\mathbf{E}}$ electric field vector

$\hat{\mathbf{k}}$ wavefront propagation direction

$\hat{\mathbf{n}}$ surface normal vector

$\vec{\mathbf{o}}$ point on surface

$\hat{\mathbf{p}}_{\text{inc}}$ unit vector in the plane of incidence of a surface and perpendicular to the incident ray direction

$\hat{\mathbf{p}}_{\text{exit}}$ unit vector in the plane of incidence of a surface and perpendicular to the exiting ray direction

$\hat{\mathbf{p}}_{\text{pl}}$ line of intersection of the plane of incidence with the plane tangent to the surface at the point of ray intersection

$\hat{\mathbf{s}}$ unit vector for the local s-basis state

$\hat{\mathbf{t}}$ polarizer transmission axis in global coordinates

$\vec{\mathbf{v}}$ another vector

$\hat{\mathbf{x}}$ unit vector for the global x-coordinate

$\vec{x}$ point on a ray (point of intersection with a surface)

$\vec{r}$ point on a ray (point of intersection with a surface)

$\hat{y}$ unit vector for the global y-coordinate

$\hat{z}$ unit vector for the global z-coordinate

Angles

θ orientation of a polarization component, azimuthal angle. Anything which determines local orientation.

θ_i angle of incidence

θ_r angle of refraction or reflection

χ ellipticity angle

ψ azimuthal angle

δ retardance magnitude

ϕ various phase terms

Matrices

F refraction matrix

J Jones matrix

J3 Jones matrix padded with zeros to become 3rd order

MM Mueller matrix

P PRT matrix

Q geometric transformation PRT matrix

R rotation matrix

T reflection matrix

Matrix elements

i subscript for matrix row number

j subscript for matrix column number

j element of Jones matrix

m element of Mueller matrix

p element of PRT matrix

q subscript denoting surface number

Path Ordering

Path 123

Path 231

Path 312

Path 321

Path 132

Path 213

Substitutions

sub0

sub1

sub2

sub3

sub4

sub5

sub6

sub7

REFERENCES

- [1] WILLIAM SWINDELL. Handedness of polarization after metallic reflection of linearly polarized light. *J. Opt. Soc. Am.*, 61(2):212–215, 1971.
- [2] W. SWINDELL. Handedness of polarization after total reflection of linearly polarized light. *J. Opt. Soc. Am.*, 62(2):295–295, 1972.
- [3] Charles W. Haggans, Lifeng Li, and Raymond K. Kostuk. Effective-medium theory of zeroth-order lamellar gratings in conical mountings. *J. Opt. Soc. Am. A*, 10(10):2217–2225, 1993.
- [4] Hisao Kikuta, Yasushi Ohira, Hayao Kubo, and Koichi Iwata. Effective medium theory of two-dimensional subwavelength gratings in the non-quasi-static limit. *J. Opt. Soc. Am. A*, 15(6):1577–1585, 1998.
- [5] Michael A. Golub and Asher A. Friesem. Effective grating theory for resonance domain surface-relief diffraction gratings. *J. Opt. Soc. Am. A*, 22(6):1115–1126, 2005.
- [6] Philippe Lalanne and Dominique Lemerrier-Lalanne. Depth dependence of the effective properties of subwavelength gratings. *J. Opt. Soc. Am. A*, 14(2):450–458, 1997.
- [7] M. G. Moharam and T. K. Gaylord. Rigorous coupled-wave analysis of planar-grating diffraction. *J. Opt. Soc. Am.*, 71(7):811–818, 1981.
- [8] M. G. Moharam and T. K. Gaylord. Coupled-wave analysis of reflection gratings. *Appl. Opt.*, 20(2):240–244, 1981.
- [9] M. G. Moharam and T. K. Gaylord. Diffraction analysis of dielectric surface-relief gratings. *J. Opt. Soc. Am.*, 72(10):1385–1392, 1982.
- [10] M. G. Moharam and T. K. Gaylord. Three-dimensional vector coupled-wave analysis of planar-grating diffraction. *J. Opt. Soc. Am.*, 73(9):1105–1112, 1983.
- [11] M. G. Moharam, T. K. Gaylord, G. T. Sincerbox, H. Werlich, and B. Yung. Diffraction characteristics of photoresist surface-relief gratings. *Appl. Opt.*, 23(18):3214–3220, 1984.
- [12] M. G. Moharam and T. K. Gaylord. Rigorous coupled-wave analysis of metallic surface-relief gratings. *J. Opt. Soc. Am. A*, 3(11):1780–1787, 1986.

- [13] Nicolas Chateau and Jean-Paul Hugonin. Algorithm for the rigorous coupled-wave analysis of grating diffraction. *J. Opt. Soc. Am. A*, 11(4):1321–1331, 1994.
- [14] M. G. Moharam, Drew A. Pommet, Eric B. Grann, and T. K. Gaylord. Stable implementation of the rigorous coupled-wave analysis for surface-relief gratings: enhanced transmittance matrix approach. *J. Opt. Soc. Am. A*, 12(5):1077–1086, 1995.
- [15] M. G. Moharam, Eric B. Grann, Drew A. Pommet, and T. K. Gaylord. Formulation for stable and efficient implementation of the rigorous coupled-wave analysis of binary gratings. *J. Opt. Soc. Am. A*, 12(5):1068–1076, 1995.
- [16] Song Peng and G. Michael Morris. Efficient implementation of rigorous coupled-wave analysis for surface-relief gratings. *J. Opt. Soc. Am. A*, 12(5):1087–1096, 1995.
- [17] Shun-Der Wu and Elias N. Glytsis. Finite-number-of-periods holographic gratings with finite-width incident beams: analysis using the finite-difference frequency-domain method. *J. Opt. Soc. Am. A*, 19(10):2018–2029, 2002.
- [18] K. Hirayama, E. N. Glytsis, and T. K. Gaylord. Rigorous electromagnetic analysis of diffraction by finite-number-of-periods gratings. *J. Opt. Soc. Am. A*, 14(4):907–917, 1997.
- [19] Jon M. Bendickson, Elias N. Glytsis, Thomas K. Gaylord, and David L. Brundrett. Guided-mode resonant subwavelength gratings: effects of finite beams and finite gratings. *J. Opt. Soc. Am. A*, 18(8):1912–1928, 2001.
- [20] J. P. McGuire, Jr. *Image Formation and Analysis in Optical Systems with Polarization Aberrations*. PhD thesis, UNIVERSITY OF ALABAMA IN HUNTSVILLE., 1990.
- [21] Garam Yun, Karlton Crabtree, and Russell A. Chipman. Properties of the polarization ray tracing matrix. volume 6682, page 66820Z. SPIE, 2007.
- [22] Peter Varga and Peter Török. Focusing of electromagnetic waves by paraboloid mirrors. ii. numerical results. *J. Opt. Soc. Am. A*, 17(11):2090–2095, 2000.
- [23] Peter Varga and Peter Török. Focusing of electromagnetic waves by paraboloid mirrors. i. theory. *J. Opt. Soc. Am. A*, 17(11):2081–2089, 2000.
- [24] Edson R. Peck. Polarization properties of corner reflectors and cavities. *J. Opt. Soc. Am.*, 52(3):253–253, 1962.
- [25] Jian Liu and R. M. A. Azzam. Polarization properties of corner-cube retroreflectors: theory and experiment. *Appl. Opt.*, 36(7):1553–1559, 1997.

- [26] Marija S. Scholl. Ray trace through a corner-cube retroreflector with complex reflection coefficients. *J. Opt. Soc. Am. A*, 12(7):1589–1592, 1995.
- [27] W. H. Steel. Polarization-preserving retroreflectors. *Appl. Opt.*, 24(21):3433–3434, 1985.
- [28] David A. Thomas and J. C. Wyant. Determination of the dihedral angle errors of a corner cube from its twyman-green interferogram. *J. Opt. Soc. Am.*, 67(4):467–472, 1977.
- [29] Atsushi Minato, Satoru Ozawa, and Nobuo Sugimoto. Optical design of a hollow cube-corner retroreflector for a geosynchronous satellite. *Appl. Opt.*, 40(9):1459–1463, 2001.
- [30] Meixiao Shen, Shaomin Wang, Laigui Hu, and Daomu Zhao. Mode properties produced by a corner-cube cavity. *Appl. Opt.*, 43(20):4091–4094, 2004.
- [31] Sergio E. Segre and Vincenzo Zanza. Mueller calculus of polarization change in the cube-corner retroreflector. *J. Opt. Soc. Am. A*, 20(9):1804–1811, 2003.
- [32] Ralph Kalibjian. Output polarization states of a corner cube reflector irradiated at non-normal incidence. *Optics & Laser Technology*, 39(8):1485 – 1495, 2007.
- [33] Ralph Kalibjian. Stokes polarization vector and mueller matrix for a corner-cube reflector. *Optics Communications*, 240(1-3):39 – 68, 2004.
- [34] Russell A. Chipman. Progress in polarization ray tracing. volume 2265, pages 141–151. SPIE, 1994.
- [35] Joachim Wesner, Frank Eisenkramer, Joachim Heil, and Thomas Sure. Improved polarization ray tracing of thin-film optical coatings. volume 5524, pages 261–272. SPIE, 2004.
- [36] Wei B. Chen, Pei F. Gu, Zeng R. Zheng, and Wei Lu. Polarization analysis of projection display system. volume 5638, pages 378–386. SPIE, 2005.
- [37] Russell A. Chipman. Mechanics of polarization ray tracing. *Optical Engineering*, 34(6):1636–1645, 1995.
- [38] Conrad Wells. Polarization sensitivity modeling of reflective imaging systems. volume 2265, pages 239–244. SPIE, 1994.
- [39] Gorden Videen. Polarization opposition effect and second-order ray tracing. *Appl. Opt.*, 41(24):5115–5121, 2002.
- [40] Eugene Waluschka. Polarization ray tracing for space optics. *Opt. Photon. News*, 17(3):42–47, 2006.

- [41] J. Paul Lesso, Alan J. Duncan, Wilson Sibbett, and Miles J. Padgett. Aberrations introduced by a lens made from a birefringent material. *Appl. Opt.*, 39(4):592–598, 2000.
- [42] Stephen C. McClain, Lloyd W. Hillman, and Russell A. Chipman. Polarization ray tracing in anisotropic optically active media. ii. theory and physics. *J. Opt. Soc. Am. A*, 10(11):2383–2393, 1993.
- [43] Stephen C. McClain, Lloyd W. Hillman, and Russell A. Chipman. Polarization ray tracing in anisotropic optically active media. i. algorithms. *J. Opt. Soc. Am. A*, 10(11):2371–2382, 1993.
- [44] Gorden Videen, Karri Muinonen, and Kari Lumme. Coherence, power laws, and the negative polarization surge. *Appl. Opt.*, 42(18):3647–3652, 2003.
- [45] Jr. James P. McGuire and Russell A. Chipman. Polarization aberrations. 1. rotationally symmetric optical systems. *Appl. Opt.*, 33(22):5080–5100, 1994.
- [46] E. D. Evans and Jr. S. A. Collins. Expressions for tracing ray-associated electromagnetic fields through optical systems. *J. Opt. Soc. Am. A*, 8(6):841–849, 1991.
- [47] Russell Chipman. Polarization aberration functions in three dimensions. In *Frontiers in Optics*, page FThL1. Optical Society of America, 2009.
- [48] L. A. Whitehead. Simplified ray tracing in cylindrical systems. *Appl. Opt.*, 21(19):3536–3538, 1982.
- [49] P. W. Ford. New ray tracing scheme. *J. Opt. Soc. Am.*, 50(6):528, 1960.
- [50] G. H. SPENCER and M. V. R. K. MURTY. General ray-tracing procedure. *J. Opt. Soc. Am.*, 52(6):672–676, 1962.
- [51] Omar García-Lievanos, Sergio Vazquez-Montiel, Jorge Castro-Ramos, and Juan Hernández-Cruz. Optical design with aspheric surfaces and exact ray tracing: An analytic method. In *International Optical Design*, page ME9. Optical Society of America, 2006.
- [52] Mark G. Nicholson, Kenneth E. Moore, and Ingolf Hörsch. Ray-tracing cad objects. In *International Optical Design*, page TuA1. Optical Society of America, 2006.
- [53] WILLIAM A. ALLEN and JOHN R. SNYDER. Ray tracing through uncentered and aspheric surfaces. *J. Opt. Soc. Am.*, 42(4):243–249, 1952.

- [54] Ronald Alpiar. Algebraic ray tracing on a digital computer. *Appl. Opt.*, 8(2):293–304, 1969.
- [55] M. HERZBERGER. Automatic ray tracing. *J. Opt. Soc. Am.*, 47(8):736–736, 1957.
- [56] Jose Sasian. Review of ray-tracing tricks. In *Frontiers in Optics*, page FWN1. Optical Society of America, 2005.
- [57] Donald P. Feder. Differentiation of ray-tracing equations with respect to construction parameters of rotationally symmetric optics. *J. Opt. Soc. Am.*, 58(11):1494, 1968.
- [58] P. W. FORD. Aspheric ray trace. *J. Opt. Soc. Am.*, 56(2):209–211, 1966.
- [59] Orestes N. Stavroudis. Simpler derivation of the formulas for generalized ray tracing. *J. Opt. Soc. Am.*, 66(12):1330–1333, 1976.
- [60] V. M. PAPADOPOULOS. Ray tracing through a lens of circular cross section. *J. Opt. Soc. Am.*, 48(9):667–667, 1958.
- [61] Garam Yun and Russell A. Chipman. Retardance in three-dimensional polarization ray tracing. volume 7461, page 74610S. SPIE, 2009.
- [62] Michael Totzeck, Paul Gräupner, Tilmann Heil, Aksel Göhnermeier, Olaf Dittmann, Daniel Krähmer, Vladimir Kamenov, Johannes Ruoff, and Donis Flagello. Polarization influence on imaging. *Journal of Microlithography, Microfabrication, and Microsystems*, 4(3):031108, 2005.
- [63] Qiaolin Zhang, Hua Song, and Kevin Lucas. Polarization aberration modeling via jones matrix in the context of opc. volume 6730, page 67301Q. SPIE, 2007.
- [64] Donis Flagello, Bernd Geh, Steve Hansen, and Michael Totzeck. Polarization effects associated with hyper-numerical-aperture (> 1) lithography. *Journal of Microlithography, Microfabrication, and Microsystems*, 4(3):031104, 2005.
- [65] Matthieu Bal, Florian Bociort, and Joseph J. M. Braat. The influence of multilayers on the optical performance of extreme ultraviolet projection systems. volume 4832, pages 149–157. SPIE, 2002.
- [66] Matthieu F. Bal, Mandeep Singh, and Joseph J. M. Braat. Optimization of multilayer reflectors for extreme ultraviolet lithography. *Journal of Microlithography, Microfabrication, and Microsystems*, 3(4):537–544, 2004.
- [67] Chen Liang, Michael R. Descour, Jose M. Sasian, and Scott A. Lerner. Multilayer-coating-induced aberrations in extreme-ultraviolet lithography optics. *Appl. Opt.*, 40(1):129–135, 2001.

- [68] Karlton Crabtree and Russell A. Chipman. Subwavelength-grating-induced wavefront aberrations: a case study. *Appl. Opt.*, 46(21):4549–4554, 2007.
- [69] Jo ao M. dos Santos and Luís M. Bernardo. Antireflection structures with use of multilevel subwavelength zero-order gratings. *Appl. Opt.*, 36(34):8935–8938, 1997.