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Correlation based analysis of a generalized target description

Marshall, David Fielding, M.S.

The University of Arizona, 1993

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**CORRELATION BASED ANALYSIS OF A
GENERALIZED TARGET DESCRIPTION**

by

David Fielding Marshall

A Thesis Submitted to the Faculty of the
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

In Partial Fulfillment of the Requirements

For the Degree of

MASTER OF SCIENCE

WITH A MAJOR IN ELECTRICAL ENGINEERING

In the Graduate College

THE UNIVERSITY OF ARIZONA

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STATEMENT BY AUTHOR

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"Graduate students are supposed to suffer. Are you suffering yet?" -Donald Dudley

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For my Parents, Lucia and Ruff

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ABSTRACT

This thesis presents a discrete time implementation and analysis of a generalized impulse response model. This model is a generalization of the weighted sum of time delayed delta functions typically used to describe the impulse response of scattering targets.

The target chosen for the analysis is the rigid acoustic sphere. The coefficients of the generalized model are used to calculate estimates of the radius of the sphere, which is known in advance for testing purposes. The accuracy of the radius estimates indicates the accuracy of the model coefficients. The generalized model is shown to be superior from the standpoint of radius estimation.

An estimator for the time of arrival of a signal of unknown but deterministic form is derived. It is based upon a generalized likelihood ratio test whose structure accomodates the generalized model. This estimator performs well in high levels of noise.

CHAPTER 1

INTRODUCTION

1.1 Conventional and Generalized Target Descriptions

The conventional model of the impulse response of a target is a weighted sum of time delayed delta functions. When an input signal is convolved with an impulse response of this form, the resulting output is a series of amplitude scaled and time delayed duplicates of the input. Not all targets actually behave in this fashion however. For example, Kennaugh and Moffatt [1] calculate several impulse responses that contain step functions. Altes [2] suggests that a model containing differentiated and integrated delta functions, as well as delta functions themselves, may provide a better approximation to the impulse response of a target. It is the analysis of this generalized model that is the focus of this thesis.

Evaluation of either model requires a set of return data from a target. In this thesis, the target is the acoustically rigid sphere. The impulse response of the sphere has a well known asymptotic series approximation [8] in the frequency domain. This facilitates the evaluation of the generalized model. The classic Mie series [5] for the

impulse response of the sphere is used to produce a synthetic data set. This impulse response is taken to be the true representation of the sphere.

1.2 A Potential Application for the Generalized Model

Let us first consider the conventional model for a target. It is completely characterized by its weight coefficients and time delays. For example, a three delay conventional model has six parameters. These are the weights h_1 , h_2 , and h_3 and the time delays τ_1, τ_2 , and τ_3 . Taking these all together as a six-tuple, one may view the parameters as coordinates in a six dimensional pattern space. These coordinates specify a vector whose tip marks the position of the model in this space.

Now consider a three delay generalized model, where at each time delay the model contains one differentiated, one ordinary, and one integrated delta function. This three delay model is characterized by twelve parameters, three weights at each time delay and the three time delays themselves. Looking at this situation in the way just described for the conventional model, we see that the dimension of the space has increased to twelve. The increase in dimensionality of the pattern space can improve classification of targets by increasing the number of features used in class segregation. As suggested by the discussion in the previous section, the new features extracted by parameterizing a generalized model should be meaningful to the identification of targets.

1.3 Organization Of Thesis

Chapter 2 of this thesis is a detailed look at the mathematical structure of the generalized model and its relationship to the conventional model. The overall structure is centered around the correlation between actual measured echo data and a set of specially chosen analysis signals. A minimum mean square error solution for the model coefficients is given. This solution leads to a system of linear equations where the matrix and column vector entries are specified by integrals. These integrals are shown to be inverse Fourier transforms evaluated at appropriate times. This means that standard signal processing routines may be used to calculate the values of the matrix and vector entries when the modeling problem is converted to discrete time format. The conventional model is shown to be a special case of the generalized model. An important distinction is made between the weight coefficient associated with a conventional (zeroth order) model and the zeroth order weight within a generalized model. This effect of this distinction appears in the experimental results given in Chapter 4.

Chapter 3 introduces the target that is to be used in the analysis. It is the rigid (or hard) acoustic sphere. Its importance as a target is based upon the fact that it has a known impulse response approximation (an asymptotic series in negative powers of $j\omega$) whose terms involve integrations and whose coefficients contain information about the radius of the sphere. Since we are calculating the coefficients of a generalized model whose terms include integrations, we may compare those model

coefficients with corresponding ones from the asymptotic series. Performing this comparison leads to radius estimates that indicate the accuracy of the generalized model coefficients. The chapter concludes with a description of the analysis signal set that is used throughout this thesis.

Chapter 4 begins with a determination, based upon the dimensionality of the signal set, of the maximum model order to be evaluated. It is shown that increasing the order of the model beyond the dimension of the analysis signal set is to no advantage. The chapter concludes with the presentation of results from the evaluation of several orders of generalized models. These results demonstrate important features of the generalized model. One of these features is the accuracy improvement in the zeroth order coefficient when passing from the conventional to the generalized model. Another feature is the decrease in stability of the model as the order of the model is increased toward the dimensionality of the analysis signal set.

Chapter 5 is centered around the application of the generalized model to marking the arrival of different temporal components of an echo from a target. A detector for an unknown but deterministic signal is derived using the maximum likelihood principle. A time of arrival estimator is formed from the detector by viewing the likelihood ratio in the detection scheme as a function of time. The performance of this estimator is demonstrated for a few selected signal to noise ratios.

Chapter 6 summarizes the results in this thesis and draws a few conclusions from them.

CHAPTER 2

IMPULSE RESPONSE MODELS FOR TARGET DESCRIPTION

2.1 Introduction

Historically, the impulse response of a scatterer has been modeled as a weighted sum of impulse functions. We call this model the “conventional model”. Altes in [2] proposed a new model for the impulse response which is more general in that it includes differentiated and integrated impulse functions. Henceforth this will be called the “generalized” model.

In this chapter a comparison between these two models is made in terms of time and frequency domain characteristics. The conventional model is discussed in Section 2.2. Its time and frequency domain representations are given, and the equation for the conventional model of an echo is derived. The generalized model is discussed in Section 2.3. A matrix equation for the generalized model coefficients is derived. Also, the equations specifying the elements of the matrices are interpreted from the viewpoint of simple inverse Fourier transform manipulations.

2.2 The Conventional Model

The conventional model represents the impulse response of a scattering target as a linear combination of weighted and time-delayed delta functions. The time domain impulse response, $h_c(t)$, and its Fourier transform $H_c(\omega)$ are given as

$$h_c(t) = \sum_{m=1}^M h_m \delta(t - \tau_m) \quad (2.1)$$

and

$$H_c(\omega) = \sum_{m=1}^M h_m e^{-j\omega\tau_m} \quad (2.2)$$

respectively. In Equations (2.1) and (2.2), m is the index corresponding to time delay τ_m , and h_m is the weight coefficient associated with that part of the impulse response occurring at time $t = \tau_m$.

Consider a bandpass signal $u(t)$ with Fourier transform $U(\omega)$. Transmit this signal toward a target represented by Equations (2.1) and (2.2). Denote the model of the echo by $\bar{e}(t)$ in the time domain and $\bar{E}(\omega)$ in the frequency domain. Since the system is linear the modeled echo $\bar{e}_c(t)$ can be written as [3, 4]

$$\begin{aligned} \bar{e}_c(t) = u(t) * h_c(t) &= \int_{-\infty}^{\infty} u(t - \xi) \sum_{m=1}^M h_m \delta(\xi - \tau_m) d\xi \\ &= \sum_{m=1}^M h_m \int_{-\infty}^{\infty} u(t - \xi) \delta(\xi - \tau_m) d\xi. \end{aligned} \quad (2.3)$$

Equation (2.3) is identically zero except when $(\xi - \tau_m) = 0$. Therefore Equation (2.3) becomes

$$\bar{e}_c(t) = \sum_{m=1}^M h_m u(t - \tau_m). \quad (2.4)$$

Figure 2.1: Conventional Model Schematic

Examination of Equation (2.4) reveals the conventional model $\bar{e}(t)$ of the echo is a weighted linear combination of time delayed versions of the input signal $u(t)$. Other than a constant multiplier, no alteration is made to the shape of the input signal. Since the conventional model is a subset of the generalized model, the solution for the conventional model weights is deferred until after the generalized model is discussed. Figure 2.1 is a diagram of the conventional model represented by Equation (2.4).

2.3 The Altes Generalized Model [1]

Altes proposed a generalization of the model described in Section 2.2. Within the impulse response model $h(t)$, at any time delay $t = \tau_m$, there is not only a delta function but a weighted linear combination of differentiated and integrated delta functions. That is to say, model the impulse response by

$$h(t) = \sum_{m=1}^M \sum_{n=-N}^N h_{nm} \delta^{(n)}(t - \tau_m) \quad (2.5)$$

where m is the time delay index, and n is the index of the order of differentiation or integration. When n is positive, differentiation is indicated. When $n = 0$, a simple reflection is indicated. Finally, when n is negative, integration is indicated.

The term-by-term Fourier transform of Equation (2.5) yields two components, one of which is a dc term and another which contains no dc component. Throughout this work the analysis signals will be bandpass, hence the dc term has no effect. If we write

$$H(\omega) = \sum_{m=1}^M \sum_{n=-N}^N h_{nm}(j\omega)^n e^{-j\omega\tau_m} + K(\omega) \quad (2.6)$$

where $K(\omega)$ is the dc term, we have $H(0) = 0$ and $K(\omega) = 0$ for $\omega \neq 0$. Thus we represent the Fourier transform of Equation (2.6) as

$$H(\omega) = \sum_{m=1}^M \sum_{n=-N}^N h_{nm}(j\omega)^n e^{-j\omega\tau_m}. \quad (2.7)$$

Figure 2.2 shows the structure of the Altes model description.

Figure 2.2: Generalized Model Schematic

2.3.1 Determining the Altes Weight Coefficients

To fully model the echo it is necessary to determine the weights h_{nm} and the time delays τ_m . In this section it is shown how to determine the weights h_{nm} for a fixed and known time delay τ_m . The derivation follows closely the one given by Altes [2].

Recall the frequency domain version of the generalized impulse response model:

$$H(\omega) = \sum_{m=1}^M \sum_{n=-N}^N h_{nm}(j\omega)^n e^{-j\omega\tau_m}. \quad (2.8)$$

When a signal $u(t)$ with Fourier transform $U(\omega)$ is transmitted toward a target, the frequency response of the echo is modeled as:

$$\bar{E}(\omega) = H(\omega)U(\omega) = \sum_{m=1}^M \sum_{n=-N}^N h_{nm}[(j\omega)^n U(\omega)] e^{-j\omega\tau_m}. \quad (2.9)$$

The h_{nm} 's and τ_m 's are parameters to be estimated.

To simplify the analysis we redefine the quantities in Equation (2.9). Let

$$\varphi_n = \frac{1}{2\pi} \int_{-\infty}^{\infty} | (j\omega)^n U(\omega) |^2 d\omega \quad (2.10)$$

be the energy of the n^{th} -order signal component. Then

$$V_n(\omega) = (j\omega)^n U(\omega) / \sqrt{\varphi_n} \quad (2.11)$$

is an n^{th} -order energy-normalized signal component. Also, if we define

$$w_{nm} = h_{nm} \sqrt{\varphi_n} \quad (2.12)$$

it follows that the modeled echo can be written as

$$\bar{E}(\omega) = \sum_{m=1}^M \sum_{n=-N}^N w_{nm} V_n(\omega) e^{-j\omega\tau_m}. \quad (2.13)$$

Given $U(\omega)$ and the true echo $E(\omega)$, the goal is to define the coefficients w_{nm} so that $\bar{E}(\omega)$ is a good representation of $E(\omega)$. The mean square error between the echo model and the true echo is the quality measure for the model.

Consider the estimation of the w coefficients for a given known time delay, say τ_m . Restricting the analysis to this single time delay removes the summation over M , and Equation (2.13) becomes

$$\bar{E}_m(\omega) = \sum_{n=-N}^N w_{nm} V_n(\omega) e^{-j\omega\tau_m}. \quad (2.14)$$

Define the mean square error as

$$\epsilon(\tau_m) = \frac{1}{2\pi} \int_{-\infty}^{\infty} | E(\omega) - \bar{E}_m(\omega) |^2 d\omega. \quad (2.15)$$

Substituting Equation (2.14) for $\bar{E}_m(\omega)$ in the previous equation gives

$$\epsilon(\tau_m) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| E(\omega) - \sum_{n=-N}^N w_{nm} V_n(\omega) e^{-j\omega\tau_m} \right|^2 d\omega. \quad (2.16)$$

The procedure is to take the partial derivative of ϵ with respect to a particular w_{km}

and set it equal to zero. Expanding the integrand of Equation (2.16) gives

$$\begin{aligned} & \left| E(\omega) - \sum_{n=-N}^N w_{nm} V_n(\omega) e^{-j\omega\tau_m} \right|^2 = \\ & |E(\omega)|^2 - E^*(\omega) \left\{ \sum_{n=-N}^N w_{nm} V(\omega) e^{-j\omega\tau_m} \right\} \\ & - E(\omega) \left\{ \sum_{n=-N}^N w_{nm} V(\omega) e^{-j\omega\tau_m} \right\}^* \\ & + \left\{ \sum_{n=-N}^N w_{nm} V(\omega) e^{-j\omega\tau_m} \right\} \left\{ \sum_{n=-N}^N w_{nm} V(\omega) e^{j\omega\tau_m} \right\}^* \end{aligned} \quad (2.17)$$

Consider the terms in Equation (2.17) individually. First we have

$$\frac{\partial}{\partial w_{km}} [|E(\omega)|^2] = 0 \quad (2.18)$$

since the true echo does not depend upon any of the model parameters. Next we have

$$\frac{\partial}{\partial w_{km}} \left\{ E^*(\omega) \left[\sum_{n=-N}^N w_{nm} V_n(\omega) e^{j\omega\tau_m} \right] \right\} = E^*(\omega) V_k(\omega) e^{j\omega\tau_m} \quad (2.19)$$

where the partial derivative is evaluated simply by picking out the term in the sum corresponding to subscript k . The following partial derivative

$$\frac{\partial}{\partial w_{km}} \left\{ E(\omega) \left[\sum_{n=-N}^N w_{nm} V_n(\omega) e^{-j\omega\tau_m} \right]^* \right\} = E(\omega) V_k^*(\omega) e^{-j\omega\tau_m} \quad (2.20)$$

is found similarly. The last partial derivative

$$\begin{aligned}
& \frac{\partial}{\partial w_{km}} \left\{ \left[\sum_{n=-N}^N w_{nm} V_n(\omega) e^{j\omega\tau_m} \right] \left[\sum_{n=-N}^N w_{nm} V_n(\omega) e^{j\omega\tau_m} \right]^* \right\} \\
&= V_k^*(\omega) e^{j\omega\tau_m} \left[\sum_{n=-N}^N w_{nm} V_n(\omega) e^{-j\omega\tau_m} \right] + \\
&\quad V_k(\omega) e^{-j\omega\tau_m} \left[\sum_{n=-N}^N w_{nm} V_n^*(\omega) e^{-j\omega\tau_m} \right] \\
&= \sum_{n=-N}^N w_{nm} V_n(\omega) V_k^*(\omega) + \sum_{n=-N}^N w_{nm} V_n^*(\omega) V_k(\omega) \\
&= 2\Re \left[\sum_{n=-N}^N w_{nm} V_n(\omega) V_k^*(\omega) \right] \tag{2.21}
\end{aligned}$$

is evaluated using the product rule for derivatives and the fact that $z + z^* = 2\Re[z]$

where $\Re[\]$ indicates taking the real part. Upon combining these terms we have

$$\frac{\partial \epsilon}{\partial w_{km}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ -2\Re[E(\omega) V_k^*(\omega) e^{j\omega\tau_m}] + 2\Re \left[\sum_{n=-N}^N w_{nm} V_n(\omega) V_k^*(\omega) \right] \right\} d\omega = 0. \tag{2.22}$$

This implies

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \Re \left[E(\omega) V_k^*(\omega) e^{j\omega\tau_m} \right] d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Re \left[\sum_{n=-N}^N w_{nm} V_n(\omega) V_k^*(\omega) \right] d\omega$$

or

$$\begin{aligned}
& \Re \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} E(\omega) V_k^*(\omega) e^{j\omega\tau_m} d\omega \right] \\
&= \sum_{n=-N}^N w_{nm} \Re \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} V_n(\omega) V_k^*(\omega) d\omega \right]. \tag{2.23}
\end{aligned}$$

Defining

$$a_{kn} = \Re \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} V_n(\omega) V_k^*(\omega) d\omega \right] \quad (2.24)$$

and

$$r_k = \Re \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} E(\omega) V_k^*(\omega) e^{j\omega\tau_m} d\omega \right] \quad (2.25)$$

Equation (2.23) may be rewritten as

$$\mathbf{A}\mathbf{w} = \mathbf{r} \quad (2.26)$$

where $\mathbf{r} = [r_{-N}, \dots, r_0, \dots, r_N]^T$, $\mathbf{w} = [w_{-Nm}, \dots, w_{0m}, \dots, w_{Nm}]^T$, and $\mathbf{A}$ is an $2N$ by $2N$ matrix with elements a_{kn} . If $\mathbf{A}$ is invertible it follows from Equation (2.26) that

$$\mathbf{w} = \mathbf{A}^{-1} \mathbf{r} \quad (2.27)$$

Once the transmitted signal spectrum $U(\omega)$ is specified, the elements of $\mathbf{A}$ are known. When an echo is received, $\mathbf{r}$ is fixed. Closer examination of Equation (2.24) reveals that a_{kn} is simply the real part of the inverse Fourier transform of $V_n(\omega) V_k^*(\omega)$ evaluated at time $t = 0$. This is true regardless of which time delay τ_m is being considered. Similarly, r_k is the real part of the inverse Fourier transform of $E(\omega) V_k^*(\omega)$ evaluated at the time delay of interest, $t = \tau_m$. If a real signal is used for $u(t)$, the $\Re[\]$ may be dropped from Equations (2.24) and (2.25), and both are ordinary inverse Fourier transforms.

To summarize, the Altes model coefficients may be found in the following way. First, an appropriate bandpass signal $u(t)$ with spectrum $U(\omega)$ is specified. Second,

the matrix $\mathbf{A}$ is filled by evaluating the integrals given by Equation (2.24). Third, a target is excited by $U(\omega)$ and the return $E(\omega)$ is measured. Fourth, the r_k specified by Equation (2.25) are calculated. Finally, Equation (2.27) is evaluated for the weight vector $\mathbf{w}$. If desired, the scaling implied by Equation (2.12) may be applied to $\mathbf{w}$ to calculate the elements of $\mathbf{h} = [h_{-Nm}, \dots, h_{0m}, \dots, h_{Nm}]^T$.

2.4 Chapter Summary

The conventional model is just a subset of the generalized model. This is easily seen by setting $N = 0$ in Equation (2.14). As a result, the minimum mean square solution for the conventional model weight w_{0m} follows exactly the one for the generalized model weights. The summations have only one term and Equations (2.26) and (2.27) become scalar equations. One can not simply pick off the zeroth order coefficient from a higher order model and call that the conventional model solution however. This is because in higher order models the solution for the zeroth order coefficients is coupled to higher order information by Equation (2.26). This higher order information is not available in a true conventional model solution. On the other hand, if one has the machinery in place to construct a generalized model it is a simple matter to pick out the zeroth order terms from the matrix and vectors in Equation (2.26) and use these to write the appropriate scalar equation.

CHAPTER 3

THE SPHERE AS A TARGET

3.1 Introduction

The objectives of this chapter are to justify the choice of a hard acoustic sphere as a target and to present sample backscatter responses for the transmitted signal used in the analysis.

Three main topics are covered in Section 3.2. First, the Mie series for the frequency response of the acoustically rigid sphere is discussed. This series provides an accurate representation of the target for generating synthetic return data. Second, the asymptotic series approximation for the early time backscatter from the rigid sphere is discussed. A comparison is made between the asymptotic and Mie series responses. Third, in Section 3.2.4 the connection between the generalized model and the asymptotic series is made. Restricting the model to integrating terms only provides the basis for comparison between generalized model and asymptotic series coefficients. In this restricted case, the coefficients of the model may be compared to those from the asymptotic series in a straightforward manner.

In Section 3.3 the transmitted signal and sample backscatter are discussed. The parameters that control the nature of the input signal are introduced. The different temporal components of the backscatter are defined and their relationships to the input signal parameters are described.

Figure 3.1: The Target Sphere

3.2 Theoretical Evaluation of the Sphere

3.2.1 The Mie Series Representation

Consider a rigid sphere of radius a which is centered at the origin of a coordinate system as shown in Figure 3.1. The input to this physical system is a plane wave traveling in the negative z -direction. The far field ($kr \gg 1$) expression for the scattered velocity potential V^s is [5, 6]

$$rV^s(k = \frac{\omega}{c}, r, \theta) \approx -i \frac{e^{-ikr}}{k} \sum_{n=0}^{\infty} (-1)^n (2n+1) \frac{j'_n(ka)}{h_n^{(2)}(ka)} P_n(\cos\theta) \quad (3.1)$$

where the potential has been normalized with respect to radial distance r . Equation (3.1) is called the Mie series response for the hard acoustic sphere. In Equation (3.1) θ is the angle between the ray from the origin to the observer and the z -axis, k is the wavenumber, ω is the radian frequency, and c is the velocity of propagation in the medium. Also, P_n is a Legendre polynomial of order n , j'_n is the derivative of a

spherical Bessel function of order n , and $h_n^{(2)}$ is the derivative of a spherical Hankel function of the second kind and order n [7]. The primes denote differentiation with respect to the argument ka . Equation (3.1) may be modified by multiplication with a phase factor chosen so the first observation of the scattered field occurs at time $t = 0$. If θ is set equal to zero then, $\cos(\theta) = 1$, and $P_n(1) = 1$. In this case the output resulting from excitation by $U(\omega)$ is

$$E(\omega) = rV^s(k, r, \theta = 0)U(\omega)e^{j(2a-r)\frac{\omega}{c}}. \quad (3.2)$$

This output is independent of the distance between the observer and the origin [8].

An equivalent time domain representation is

$$e(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} rV^s(k, r, 0)U(\omega)e^{j(2a-r)\frac{\omega}{c}}e^{j\omega t}d\omega. \quad (3.3)$$

To test the Altes model, simulated data as given by Equation (3.2) must be compared to a modeled echo given by Equation (2.13). To use Equation (3.2) the Mie series must be evaluated over a range of frequencies.

For the arrangement in Figure 3.1, with $\theta = 0$, the Mie series is

$$rV^s(k = \frac{\omega}{c}, r, \theta) \approx -i \frac{e^{-ikr}}{k} \sum_{n=0}^{\infty} (-1)^n (2n+1) \frac{j'_n(ka)}{h_n^{(2)}(ka)}. \quad (3.4)$$

The sum of this infinite series must be approximated for any value of ka that is of interest. To illustrate the procedure used in approximating the sum we establish some notation. Denote a sequence by

$$c_1, c_2, \dots, c_{50}, \dots = \{c_n\}. \quad (3.5)$$

Figure 3.2: Complex Valued Frequency Response

This plot shows the frequency response given by Equation (3.4) with $\theta = 0$ and $a = 1$. The spiral winds in a clockwise fashion as frequency increases. Because of conjugate symmetry, only the first 2049 points are shown.

plot of this bandlimited frequency response. Taking the inverse fast Fourier transform of the complex frequency domain data gives the bandlimited impulse response function of time shown in Figure 3.6.

Examination of Figure 3.6 immediately reveals that there is something to be gained by deviating from the conventional impulse response model given by Equation (2.1). Equation (2.1) is a sum of delayed and weighted delta functions. The impulse response

Figure 3.3: Real Part of Frequency Response

Figure 3.4: Imaginary Part of Frequency Response

Figure 3.5: Frequency Response Magnitude

This is a magnitude plot of the complex valued frequency response shown in Figure 3.2. Comparison with Figure 3.3 reveals that at high frequencies the real part of the response dominates and is approximately equal to one half.

Figure 3.6: Bandlimited Impulse Response

This is the inverse Fourier transform of the data shown in Figure 3.2. The first portion of this impulse response produces the reflection part of the echo, and the small bump near $t = 15ms$ produces the creeping wave. These terms are defined and illustrated in Subsection 3.3.2, Sample Backscatter.

of the sphere begins with with a component that closely resembles a delta function, but departs from that description soon thereafter. The portion immediately following time zero resembles a decaying exponential. The small component located just after $t = 0.015$ sec resembles a double sided decaying exponential.

3.2.3 The Asymptotic Series Representation

An asymptotic representation for the early time backscatter from an acoustically rigid sphere is [5]

$$E_{Asy}(\omega) \approx \frac{a}{2} \left[1 - \frac{3/2}{(j\omega)(a/c)} + \frac{5/2}{(j\omega)^2(a/c)^2} - \frac{25/4}{(j\omega)^3(a/c)^3} + \frac{22}{(j\omega)^4(a/c)^4} + O[(\omega a/c)^{-5}] \right] U(\omega) \quad (3.7)$$

where a is the radius of the sphere and c is the velocity of propagation. The accuracy of the approximation improves with increasing frequency, and the five term expression given is accurate for $(\omega a/c)$ as low as 10 [8]. (For the problem at hand $c = 330 \text{ meters/second}$ and $a = 1$, so the asymptotic series should be accurate for frequencies greater than approximately 600Hz .) Figure 3.7 is a plot of the complex valued function in Equation (3.7). To compare this asymptotic series response with that from the Mie series it is necessary to separate the portion of the Mie series frequency response that is due to the early time portion ($t < 12\text{ms}$) of the impulse response shown in Figure 3.6. Zeroing out the other part of Figure 3.6 and taking the transform of what is left gives the response shown in Figure 3.8. Figures 3.9 and 3.10 are overlays of these responses and their magnitudes respectively. These figures reveal the close correspondence between the asymptotic series response and the Fourier transform of the early time portion of the impulse response. The difference at high frequencies exhibited in Fig. 3.10 is not due to discrepancies between the Mie and asymptotic series. Rather, it is a computational effect caused by truncation of the impulse response. Comparing Figures 3.2 and 3.7 reveals that the full Mie series

Figure 3.7: The Asymptotic Series Response

response and the asymptotic series response both converge to the real number $1/2$ as frequency increases.

3.2.4 Restriction of the Model to Integrating Terms

If one has the Mie series for the response of the sphere, of what utility is the asymptotic series approximation? Comparison of the asymptotic series with the Altes model reveals the answer. Recall that for any time delay τ_m the Altes model represents the impulse response of the target as a linear combination of differentiated and integrated delta functions. In the frequency domain, for time delay $\tau_m = 0$,

Figure 3.8: Early Time Mie Series Response

Figure 3.9: Asymptotic and Early Time Mie Series Responses

The smooth line is the asymptotic series response, while the wavy line is the early time response portion of the Mie series. These complex valued curves trace roughly left to right as frequency increases.

Figure 3.10: Magnitudes from Figure 3.9

The smooth line is the magnitude of the asymptotic series response and the wavy line is the magnitude of the early time portion of the Mie series response.

Equation (2.7) becomes

$$H_m(\omega) = \sum_{n=-N}^N h_{nm}(j\omega)^n. \quad (3.8)$$

The summation over m is not present because only one specific delay is being considered. The subscript m is added to the left hand side of Equation (3.8) to emphasize this fact. Expanding Equation (3.8) and reordering the terms obtains

$$\begin{aligned} H_m(\omega) = & h_{Nm}(j\omega)^N + h_{N-1,m}(j\omega)^{N-1} + \dots \\ & + h_{0m} + h_{-1m}(j\omega)^{-1} + \dots + h_{-Nm}(j\omega)^{-N} \end{aligned} \quad (3.9)$$

Examine the right hand side of Equation (3.9) beginning with the term h_{0m} . It is a polynomial in negative powers of $(j\omega)$. This is exactly the form of Equation (3.7). So if the Altes model is restricted to integrating terms only (here $n = 0, \dots, -4$), equating the coefficients of similar terms results in

$$\begin{aligned} h_{0m} &= (a/2) & h_{-1m} &= -\frac{a(3/2)}{2(a/c)} & h_{-2m} &= \frac{a(5/2)}{2(a/c)^2} \\ h_{-3m} &= -\frac{a(25/4)}{2(a/c)^3} & h_{-4m} &= \frac{a(22)}{2(a/c)^4}. \end{aligned} \quad (3.10)$$

Recall that the coefficients h_{km} can be found by the method given in Section (2.3.1). Also, c is a known constant. The only remaining variable in Equation (3.10) is a , the radius of the sphere. So, once the h_{km} are calculated, simple algebraic manipulations produce several estimates of the radius of the sphere. When the experiment is performed with a sphere of known radius these estimates indicate the accuracy of the Altes model coefficients h_{km} . From this point the analysis is restricted to these integrating terms.

3.2.5 Section Summary

This section described the rigid acoustic sphere as a target. The Mie series from Section 3.2.1 provides an analytical representation of the frequency response of the sphere. This was used to calculate numerical values for a discrete time simulation. The asymptotic series discussed in Section 3.2.3 provides a basis for testing the accuracy of the generalized target model. It is a frequency domain series in negative powers of $j\omega$, and is an approximation for early time backscatter from the sphere. The negative powers of $j\omega$ represent integrations in the time domain. Also, the coefficients of the series contain information about the radius of the sphere. So when a generalized target model restricted to integrating terms is made for the early time backscatter, the coefficients of the generalized model and the asymptotic series may be compared. This provides for making radius estimates which indicate the accuracy of the generalized model coefficients.

3.3 Input Signal Description and Sample Backscatter

3.3.1 The Input Signal

In previous work Weyker and Dudley [6, 8] used the following signal:

$$u(t) = \begin{cases} 0 & t < 0 \\ \sin 2\pi f_c t \sin^2 \alpha t & 0 \leq t \leq \pi/\alpha \\ 0 & \pi/\alpha < t \end{cases} \quad (3.11)$$

Note that $u(t)$ is a sinusoid modulated by a sine squared envelope. Figure 3.11 is an example of $u(t)$ with $f_c = 1200\text{Hz}$ and $\alpha = 300$. Figure 3.12 is the corresponding magnitude of the Fourier transform $U(\omega)$. While $u(t)$ is not strictly bandlimited it has very little energy at dc, so the assumption of a bandpass signal is not unreasonable. The parameter α controls the duration of the envelope and f_c (referred to as center, carrier, or analysis frequency) governs the number of oscillations present in the non-zero portion of the input signal. As will be seen, the parameter α figures heavily in separating different temporal components of the echo from one another. The choice of carrier frequency, f_c , influences the accuracy and sensitivity of the generalized model coefficients.

Figure 3.11: Test signal $u(t)$ with $f_c = 1200Hz$ and $\alpha = 300$.

Figure 3.12: Frequency spectrum $|U(\omega)|$ of Figure 3.11.

3.3.2 Sample Backscatter

The immediate purpose is to develop familiarity with the structure of an echo from the target sphere. Recall the test signal $u(t)$ from Figure 3.11. Convolving this input with the bandlimited impulse response of Figure 3.6 produces the output shown in Figure 3.13. For this case $f_c = 1200Hz$ and $\alpha = 300$.

The first part of the echo ($t < 11ms$) is the reflection. Its peak amplitude is approximately one half. This is reasonable since the input $u(t)$ has a peak amplitude equal to one and it is being convolved with an impulse-like function of peak amplitude one half.

The second part of the echo is the creeping wave. As the small bump at $t \approx 15ms$ in Figure 3.6 would suggest, the amplitude of the creeping wave is much reduced from the input amplitude. Although it is hard to tell now, the shape of the creeping wave (other than amplitude) differs more from the input than does the shape of the reflection. This is reasonable since the part of the impulse response which produces the creeping wave looks much less like a delta function than does the early time part.

Figure 3.13: Sample Echo

This is the echo produced by using the input $u(t)$ from Equation (3.11) with $f_c = 1200Hz$ and $\alpha = 300$.

Figure 3.14: Sample Echo

This is the echo produced by using the input $u(t)$ from Equation (3.11) with $f_c = 2200Hz$ and $\alpha = 300$.

Figure 3.14 shows the echo when f_c is set to $2200Hz$. Note how much smaller the creeping wave is compared to the one in Figure 3.13. This indicates that the part of the impulse response which produces the creeping wave acts somewhat like a low pass filter. This supports the feasibility of an integrating term generalized model for the echo.

Figure 3.15: Sample Echo

This is the echo produced by using the input $u(t)$ from Equation (3.11) with $f_c = 600Hz$ and $\alpha = 100$.

Figure 3.15 shows the echo resulting from using $u(t)$ with $f_c = 6Hz$ and $\alpha = 100$. Now the duration of the input pulse is longer, and the reflection overruns the creeping wave. This perturbs any radius estimates made by comparison to the asymptotic series coefficients since the asymptotic series does not take creeping waves into account [8]. Since the delay is related to the circumference of the sphere, and hence its radius, it is important to set α to an appropriate value. For the analysis in this thesis $\alpha = 300$ is used with the $1m$ radius sphere.

CHAPTER 4

REFLECTION MODEL ANALYSIS

4.1 Determination of the Model Order N

Recall the integrating-terms-only generalized target description as restricted to the zero time delay epoch, e.g.

$$h(t) = \sum_{n=0}^{-N} h_{n1} \delta^{(n)}(t); \quad \tau_1 = 0 \quad (4.1)$$

The modeled echo, $\bar{E}(\omega) = H(\omega)U(\omega)$, is the product of the frequency responses of the scatterer and the transmitted signal. Thus,

$$\bar{E}(\omega) = \sum_{n=0}^{-N} w_{n1} V_n(\omega) = H(\omega)U(\omega) \quad (4.2)$$

where $V_n(\omega)$ is the Fourier transform of the energy normalized n^{th} order integral of the transmitted signal $u(t)$. Equation (4.2) indicates that the model spectrum $\bar{E}(\omega)$ is an expansion of the true echo spectrum $E(\omega)$ in terms of the components $V_n(\omega)$. Likewise, in the time domain the model echo $\bar{e}(t)$ is an expansion of the real echo $e(t)$ in terms of the components $v_n(t) = \mathcal{F}^{-1}\{V_n(\omega)\}$. That is

$$\bar{e}(t) = \sum_{n=0}^{-N} w_{n1} v_n(t) . \quad (4.3)$$

As was discussed in Chapter 3, only integrals of $u(t)$ are considered. Given $U(\omega)$ and the true echo $E(\omega)$, the objective is to select the weights w_{n1} ; $n = 0, \dots, -N$ such that the mean square error between E and $\bar{E}$ is minimized.

The set $\{v_n\}$ may be viewed as a set of vectors in a function space. The dimension of $\{v_n\}$ will determine the largest magnitude for N that can be used. Hence, it is of interest to determine the dimension of this signal set. The well known Gram-Schmidt orthogonalization process can be used to investigate this question. Given an original set of vectors, say $\{v_n\}_{n=0}^{-N}$, this process finds a minimum orthonormal set of vectors $\{\phi_k\}$ that spans the same space as the original set. The number of functions in this minimum set is the dimension of the space spanned by either set [12]-[15].

Roughly speaking, this process consists of three steps: projection, subtraction, and normalization. Select v_0 , the original vector corresponding to the 0^{th} integration of the input signal. Since at this point there is nothing to project against or subtract from, v_0 is simply divided by $\|v_0\|$ (Euclidean norm in the discrete time implementation) to produce the normalized vector

$$\phi_0 = \frac{v_0}{\|v_0\|} \quad (4.4)$$

which is the first in the minimum set.

Now select a second vector, say v_1 , from the original set. Take the inner product between this vector and ϕ_0 . Subtract the result from v_1 giving the difference vector

$$d_1 = v_1 - \langle v_1, \phi_0 \rangle \cdot \phi_0 \quad (4.5)$$

To produce the second vector for the minimum set, normalize d_1 yielding

$$\phi_1 = \frac{d_1}{\|d_1\|} . \quad (4.6)$$

Proceeding in this fashion produces the expressions for the k^{th} difference vector and the k^{th} vector in the minimum orthonormal set. These are

$$d_k = v_k - \sum_{n=0}^{k-1} \langle v_k, \phi_n \rangle \phi_n \quad (4.7)$$

and

$$\phi_k = \frac{d_k}{\|d_k\|} \quad (4.8)$$

respectively.

In the present situation the point of this exercise is not just to arrive at the orthonormal set $\{\phi_n\}$. The objective is to determine the value of N that should be used. Indeed, the procedure produces five different ϕ_n (for $n = 0, \dots, 4$) from the original five v_n . So the dimension of the space spanned by the set $\{v_n\}_{n=0}^{-N}$ is in fact five. Remember, the echo $e(t)$ is to be coordinatized with respect to the original $\{v_n\}_{n=0}^{-N}$, not a new set of orthonormal functions. Given this framework, it is the set of difference vectors $\{d_n\}$ that become important. For instance, if $d_1 = v_1 - \langle v_1, \phi_0 \rangle \phi_0$ is close to zero the vectors v_1 and v_0 lie close to one another in the signal space. If the vectors whose inner products (correlations) fill the elements of this matrix are too similar the matrix may become ill-conditioned. For instance, if the elements of the matrix A differ from one another by a very small amount, say 10^{-16} , inverting A is unreasonable.

Figure 4.1: Input Signal $u(t)$ with $f_c = 1200Hz$ and $\alpha = 300$

To illustrate the Gram-Schmidt process for the class of input signals described in Section 3.3.1 we start with $u(t)$ for which $f_c = 1200Hz$ and $\alpha = 300$. This signal is shown in Figure 4.1. Since the asymptotic series (see Equation 3.10) gives five coefficients to compare with those from the generalized model, we use $u(t)$ and its first four integrals as the original set of vectors from which an orthonormal set is produced.

Figure 4.2: The first difference function d_1

Proceeding as previously outlined, we obtain the first difference function d_1 shown in Figure 4.2. This difference function is approximately one-fourth the size of the original signal $u(t)$. It is on the scale 10^0 .

Figure 4.3: The second difference vector d_2

Continuing in this fashion produces the second, third, and fourth difference functions shown in Figures 4.3 through 4.5. For reference, Figure 4.6 is the superposition of all the difference functions.

Figure 4.4: The third difference vector d_3

Note that the second and third difference functions are on the order of 10^{-2} while the fourth difference function is on the order of 10^{-3} . This downward trend in the magnitude of the difference vectors as the order of integration increases indicates that past $N = 4$ any new function is very nearly a replication of one already in the set. Essentially, the signal $u(t)$ is five dimensional. Thus, N is constrained to be from 0 to 4 in all future work.

Figure 4.5: The fourth difference vector d_4

Figure 4.6: Superposition of the difference vectors

4.2 Matrix Structure for the Order N Model

Recall the overall modeling problem for the zero time delay integrating terms-only-case. A test signal $u(t)$ is selected. It is convolved with an impulse response $h(t)$. The resulting output $e(t)$ is expanded in the form

$$\bar{e}(t) = \sum_{n=0}^{-N} w_{n1} v_n(t); \tau_1 = 0 \quad (4.9)$$

where $v_n(t)$ is the energy normalized n^{th} order derivative of $u(t)$. Since N is being restricted to $0 \leq N \leq 4$, the model $\bar{e}(t)$ is an expansion of $e(t)$ in terms of integrated (negative order differentiation) versions of the input signal $u(t)$.

When $N = 0$ the model is simply $\bar{e}(t) = w_{0,1} v_0(t)$. This is the zeroth order or conventional model. It is the standard against which the various orders of the generalized model are compared. For this situation Equation (2.26) becomes

$$a_{00} w_{0,1} = r_{0,1} \quad (4.10)$$

where

$$a_{00} = \frac{1}{2\pi} \int_{-\infty}^{\infty} V_0(\omega) V_0^*(\omega) d\omega \quad (4.11)$$

and

$$r_{0,1} = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(\omega) V_0^*(\omega) d\omega \quad (4.12)$$

When $w_{0,1}$ is calculated a zeroth order radius estimate may be made using the zeroth order asymptotic series coefficient for comparison. (Refer to equation 3.11)

When $N = 1$ the model becomes

$$\bar{e}(t) = \sum_{n=0}^{-1} w_{n,1} v_n(t). \quad (4.13)$$

As in the $N = 0$ case, $w_{0,1}$ may be used to make a zeroth order estimate of the radius. On the other hand $w_{-1,1}$ can not be used to make a radius estimate since the matching order coefficient in the asymptotic series contains no radius information. The inclusion of the extra coefficient may influence the quality of the model however.

This is because Equation (2.26) becomes

$$\begin{bmatrix} a_{-1,-1} & a_{-1,0} \\ a_{0,-1} & a_{0,0} \end{bmatrix} \begin{bmatrix} w_{-1,1} \\ w_{0,1} \end{bmatrix} = \begin{bmatrix} r_{-1,1} \\ r_{0,1} \end{bmatrix} \quad (4.14)$$

and the resulting model may well fit the reflection better than the conventional one.

When $N = 2$ the inclusion of $w_{-2,1}$ allows the possibility of making a negative second order estimate of the radius. For this case Equation (2.26) becomes

$$\begin{bmatrix} a_{-2,-2} & a_{-2,-1} & a_{-2,0} \\ a_{-1,-2} & a_{-1,-1} & a_{-1,0} \\ a_{0,-2} & a_{0,-1} & a_{0,0} \end{bmatrix} \begin{bmatrix} w_{-2,1} \\ w_{-1,1} \\ w_{0,1} \end{bmatrix} = \begin{bmatrix} r_{-2,1} \\ r_{-1,1} \\ r_{0,1} \end{bmatrix} \quad (4.15)$$

Notice the growing number of cross terms in the $\mathbf{A}$ matrix. This is where the self-similarity problem may appear. The solution to the matrix equation can become sensitive to small perturbations in $\mathbf{r}$.

The descriptions of the models for $N = 3$ and $N = 4$ follow in a similar fashion with the added possibilities of negative third and negative fourth order radius estimates respectively.

Note: Henceforth the adverb *negative* will be dropped from phrases specifying a model order. This makes the terminology less cumbersome and causes no confusion since only negative-order models are being considered.

4.3 Radius Estimates and Mean Square Error Results

4.3.1 Formulas for Radius Estimates

As discussed in Section 4.2, radius estimates of several different orders may be calculated from the coefficients of a single model. These estimates differ in the order of the information they contain. A zeroth order (conventional) model can provide only one estimate since the model contains only one weight w to calculate with. A fifth order model can provide four estimates: zeroth, second, third, and fourth order. No first order estimate is possible since the first order terms contain no explicit information about the radius of the sphere.

Comparing Equations (2.10), (2.12), and (3.10) reveals the operations necessary to produce the various order radius estimates. The zeroth order radius estimate $\hat{a}_0$ is given by

$$\hat{a}_0 = 2w_{0,1}/\sqrt{\varphi_0} \quad (4.16)$$

where φ_0 is the energy of the zeroth order signal component. The second order radius estimate $\hat{a}_{-2}$ is given by

$$\hat{a}_{-2} = \frac{5c^2\sqrt{\varphi_{-2}}}{4w_{-2,1}} \quad (4.17)$$

where c is the velocity of propagation and φ_{-2} is the energy of the second order signal component. Similarly, the third and fourth order estimates are given by

$$\hat{a}_{-3} = \sqrt{\frac{-25c^3\sqrt{\varphi_{-3}}}{8w_{-3,1}}} \quad (4.18)$$

and

$$\hat{a}_{-4} = \left(\frac{11c^4 \sqrt{\varphi_{-4}}}{w_{-4,1}} \right)^{1/3} \quad (4.19)$$

respectively.

4.3.2 Radius Estimates for the Noise Free Case

The analysis of the generalized model begins with the noise free case. Here, the interaction between the order of a model and the solution for its coefficients is studied in a deterministic fashion. The noise free case provides a performance benchmark against which to compare the noisy case results that are presented later.

Unless stated otherwise the following plotting conventions hold through the rest of this chapter. Each graph shows one parameter of interest, be it a radius estimate, a mean square error value, or something similar. A particular graph may display this single parameter as extracted from several models however. Quantities derived from fourth order ($N = 4$) models are plotted with a “plus” symbol. Quantities from third order models are plotted with the character “x”. Second, first, and zeroth order models use the character “o”, stars, and dash-dots respectively.

Figure 4.7: Zeroth Order Radius Estimates, No Noise

Note that the second, third, and fourth order models provide a better zeroth order radius estimate than the conventional and first order models.

4.3.2.1 Zeroth Order Radius Estimates

Figure 4.7 is a plot of zeroth order radius estimates as given by Equation (4.16). Note the magnified ordinate scale. All model orders are represented since all of the models contain first order coefficients. As is true for all of the analysis in this thesis, the actual radius of the sphere is one meter. The dash-dot line shows the radius estimates produced by the conventional zeroth order model for a range of carrier frequencies. As frequency increases the accuracy improves. This is expected

since the radius estimates are derived from the asymptotic series representation of the sphere, and the accuracy of this representation improves at high frequencies. As expected, the radius estimates from the first order model fall exactly upon those from the conventional model. As far as radius estimates go, adding the first order term does nothing to improve the accuracy of the zeroth order coefficient.

In contrast, the second, third, and fourth order terms do contribute to the accuracy of the zeroth order radius estimate. Recall the distinction set forth in Chapter 2 between the zeroth order coefficient that specifies a conventional model and the zeroth order term that is within a generalized model. Adding the higher order information improves the accuracy of the zeroth order coefficient. Even if no higher order radius estimates are desired, adding these terms to the model is worthwhile. The best performance is provided by the second and third order models however. The degradation in accuracy when going from the third order model to the fourth is the first indication of the sensitivity of the solution for the higher order coefficients when the Dudley/Weyker signal set is used.

Figure 4.8: Second Order Radius Estimates, No Noise

4.3.2.2 Second Order Radius Estimates

Figure 4.8 is a plot of the second order radius estimates as given by Equation (4.17). The second, third, and fourth order models are represented here since they do contain second order information. The second and third order models provide an accurate estimate of the diameter of the sphere for a wide range of analysis frequencies. In this case they do in fact provide identical estimates. As the plot shows, the fourth order model fails to provide an accurate second order estimate.

Figure 4.9: Third Order Radius Estimates, No Noise

Note that the plot symbols “x” and “+” for the third and fourth order models are superimposed.

4.3.2.3 Third Order Radius Estimates

Figure 4.9 is a plot of the third order radius estimates as given by Equation (4.18). Only the third and fourth order models contain the third order information necessary to produce this estimate. These estimates are accurate over a wide range of carrier frequencies. However, these estimates are far less accurate than those given by zeroth or first order estimators.

Figure 4.10: Fourth Order Radius Estimates, No Noise

4.3.2.4 Fourth Order Radius Estimates

Figure 4.10 is a plot of fourth order radius estimates given by Equation (4.19). Even though noise did not corrupt the data used to make these estimates, the estimator fails. The data indicates the physically impossible case of a sphere with negative radius. Combining this with the fact that adding the fourth order term did not improve upon any of the estimates made by the third order model disqualifies the fourth order model from further consideration. Using the Dudley-Weyker signal limits the model to order $N \leq 3$. This result indicates that the Dudley-Weyker signal

set is too self-similar for accurate use in high order radius estimators. A different signal set must be used to analyze models of order greater than $N = 3$. Seemingly, the only way to improve upon this situation is to start with an input analysis signal whose integrals are not as self similar as those for the Dudley-Weyker signal. (Refer to Section 4.1 for clarification.)

4.3.3 Radius Estimates for the Noisy Case

4.3.3.1 Preliminaries

The performance of the radius (and hence model parameter) estimators in the presence of noise is discussed in this section. Since the noise that is added to the true echo is random in nature, we cannot meaningfully use a single radius estimate. An *average* radius estimate is significant however. Here, the term average refers to the sample average or sample mean defined by:

$$\text{sample mean, } \hat{a} = \frac{1}{K} \sum_{k=1}^K a_k \quad (4.20)$$

where K is the number of samples in the population for which the average is being calculated [16, 17]. For the work in this thesis, $K = 500$. We must define the population being averaged. Three things are fixed for each of these averages: the order N of the model, the order n of the radius estimate under consideration, and the analysis or carrier frequency f_c . Given these three constants, the experiment of generating a radius estimate is repeated K times, each time with a different realization of white Gaussian noise. The resulting set of a_k , $k = 1, \dots, K$ is the population over which Equation (4.20) is defined. Similarly, we define the sample variance (or simply variance for brevity) as:

$$\text{var}(a) = \frac{1}{K-1} \sum_{k=1}^K (a_k - \hat{a})^2. \quad (4.21)$$

Furthermore, we must define the signal to noise ratio. It is ten times the natural logarithm of the average square signal amplitude divided by the variance of the noise.

When $e(t)$ is the signal of interest,

$$SNR = 10 \log \frac{\frac{1}{T} \int_0^T e^2(\xi) d\xi}{\sigma_n^2} \quad (4.22)$$

where σ_n^2 is the noise variance.

4.3.3.2 Radius Estimate Results for 80 dB SNR

Figure 4.11 is a plot of zeroth order radius estimates made from noisy echoes with $SNR = 80dB$. Comparison with Figure 4.7 reveals that even this low level of noise perturbs the estimates from the fourth order model to a small degree. The lower order models show little change in the accuracy of their zeroth order estimates.

Figure 4.12 shows the variances of these estimates. The zeroth and first order models have the lowest variance, and this variance is relatively constant with frequency. Although they are higher, the variances of the estimates from the higher order models are still much less than one.

Figure 4.11: Average Zeroth Order Radius Estimates, SNR = 80dB

Note that the estimates from the second and third order models overlap.

Figure 4.12: Variance of Zeroth Order Radius Estimates, SNR = 80dB

Lower points: zeroth and second order model results
 Middle points: second and third order model results
 Top points: fourth order model results

Figure 4.13: Average Second Order Radius Estimates, SNR = 80dB

Figure 4.13 shows the average second order radius estimates as produced by the higher order models. The second and third order models produce identical estimates over the frequency range shown. The fourth order model does not fare so well however. It is significantly in error over the whole frequency range. As Figure 4.14 shows, there is a marked increase in the variance when going from the first order estimate to the second, regardless of which model order is being evaluated.

Figure 4.14: Variance of Second Order Radius Estimates, SNR = 80dB

Figure 4.15: Average Third Order Radius Estimates, SNR = 80dB

Figure 4.15 shows the third order radius estimates as produced by the third and fourth order models. The results are identical, so only one plot symbol is used. Obviously, even at the relatively good signal to noise ratio of 80dB, the third order estimate has problems. A complex valued radius is not only in error, but is hard to imagine in any case.

Figure 4.16: Average Fourth Order Radius Estimates, SNR = 80dB

Figure 4.16 is a plot of average fourth order radius estimates, produced at a signal to noise ratio of $80dB$. The results are not much different from those shown in Figure 4.10, which is to say they are inaccurate.

Figure 4.17: Average Zeroth Order Radius Estimates, SNR = 40dB

4.3.3.3 Radius Estimate Results for 40 dB SNR

Figure 4.17 is a plot of the average zeroth order radius estimates, with the SNR equal to 40dB. The estimates produced by the zeroth through third order models are relatively unperturbed and are still accurate over a wide range of analysis frequencies. The zeroth order estimate from the fourth order model is showing some instability however. The sensitivity of the fourth order model is readily apparent since its zeroth order estimate, which is the most robust for any given model order, is beginning to fail.

Figure 4.18: Variance of Zeroth Order Radius Estimates, SNR = 40dB

Lower points: zeroth and second order model results
 Middle points: second and third order model results
 Top points: fourth order model results

Figure 4.18 shows the variances of the zeroth order radius estimates from Figure 4.17. Comparison with Figure 4.12 reveals that when the SNR degraded from 80dB to 40dB, the corresponding variances increased by a factor of 10^4 .

Figure 4.19: Average Second Order Radius Estimates, SNR = 40dB

Figure 4.19 shows the second order radius estimates for a 40dB SNR. Figure 4.20 is a plot of the corresponding variances. The estimates are inaccurate and the sample variances range upward to nearly one hundred. Obviously, at this signal to noise ratio, any estimates of order greater than zero are unacceptable.

Figure 4.20: Variance of Second Order Radius Estimates, SNR = 40dB

4.3.4 Mean Square Error Results

Figure 4.21 is a plot of the mean square error between the generalized model of the reflection component of the echo and the actual reflection component. In this case the models were produced without any noise corrupting the backscatter. This provides a baseline for comparison. As is readily apparent from the graph, the addition of even one term to the model produces a significant reduction in the MSE. Adding a second order term produces a smaller improvement, but adding more terms than this has little obvious effect.

Changing the signal to noise ratio to $80dB$ increases the mean square error for the high order models slightly, and then only at higher frequencies. Examining the plot closely reveals that in the frequency neighborhood where the error increased, the fourth order model error rose more than the third order model error. Similarly, the error for the third order model grew more than that for the second.

The plot for the $40dB$ SNR case shows an increase in error across the frequency range for the high order models. The growth in mean square error produced by the increased noise level is largest in the highest order model. This points out the sensitivity of the model to noise when the signal set becomes self similar. At this noise level the generalized model of any order still provides a result superior to that achieved by the conventional zeroth order model.

Figure 4.21: Mean Square Error, No Noise

Top curve: zeroth order model results

Middle curve: first order model results

Bottom curve: superposition of second, third, and fourth order results

Figure 4.22: Average Mean Square Error, SNR = 80dB

Figure 4.23: Average Mean Square Error, SNR = 40dB

4.4 Chapter Summary

The generalized model of the rigid acoustic sphere can indeed be superior to the conventional model. The radius estimate data shows the generalized model can identify the integration coefficients that are explicit in the asymptotic series representation of the sphere. The mean square error data shows that a linear combination of integrated versions of the transmitted signal is superior to a scaled version of the transmitted signal when trying to provide a fit to the early time return from the sphere.

This superiority is not without qualifications however. While the higher order models provide superior mean square error performance in any case, the radius estimates can become unstable under noisy conditions. This is especially true for high order estimates from high order models. This indicates that under noisy conditions, and for the signal set being used, the model coefficients stray from the “ideal” of matching the asymptotic series coefficients. This is not an unexpected result, since the algorithm for finding the coefficients is based upon a minimum mean square error fit between the model and the echo from the target. Other than that, no restriction is placed on the coefficients.

With these qualifications, it appears that the second order model provides the most gain with the least problems. Its mean square error figures are among the lowest recorded for any of the measurement situations tried. Among the generalized models that are able to provide radius estimates of order higher than zero, its estimates and

hence its coefficients are the most stable. In direct comparison with the conventional model, the zeroth order radius estimates show that the inclusion of the first and second order information improves the accuracy of the zeroth order coefficient.

These results compare favorably with those found by Dudley and Weyker in [6]. The approach taken in [6] did not lead to accurate radius estimates based upon terms of order two and higher, even in the no noise case. The approach taken in this thesis led to accurate second and third order estimates of the radius when the third order model was used. In the $80dB$ SNR case, the approach used here was still able to produce accurate second order radius estimates. In lower signal to noise ratios, the methods used here and in [6] were similar in that both of them found low order coefficients satisfactorily, while failing to accurately identify high order coefficients.

CHAPTER 5

TIME OF ARRIVAL ESTIMATION

5.1 Introduction

Until now, the focus of this thesis was answering the question: “How well does the generalized model describe the characteristics of a known target ?” Here the phrase *generalized model* refers not only to the mathematical representation as proposed by Altes [2] but the method (Section 2.3.1) for determining its parameters as well. The goal of this chapter is to *introduce* the use of the generalized model to determine the time of arrival of the creeping wave discussed in Chapter Three.

In [2], Altes indicates a method for detecting the arrival of an echo in noise. It involves a statistic derived from a likelihood ratio test. The test statistic is similar to the common matched filter, except that the measurement vector serves as its own reference vector since the actual form of the signal being detected is unknown. That is, instead of having

$$\lambda = \bar{r}^T R_n^{-1} \bar{s} \tag{5.1}$$

where $\bar{r}$ is the measurement vector, R_n^{-1} is the inverse of the covariance matrix of the noise, and $\bar{s}$ is the signal to be detected, we have

$$\lambda = \bar{r}^T R_n^{-1} \bar{r} . \quad (5.2)$$

In this chapter, a generalization of λ is derived by modifying the derivation of a test statistic for detecting unknown but non-random signals as given by Whalen in [18]. This modification is explained in the next section.

The value of a test statistic may be computed at any given time, and the result compared with some threshold to decide whether or not the signal is present at that time. This is not the present goal however. Here it is assumed that a signal will be present at some point within the time interval during which measurements are taken. The goal is to decide when this signal arrives. In this light, we may consider the test statistic λ to be a function of time. In the discrete time situation, we calculate the value of λ for each time increment within the measurement interval. The resulting set of values specifies the function $\lambda(t)$ on the interval. The term *lambda curve* denotes a function of this form. Generally speaking, local maxima in these lambda curves may indicate the arrival of a signal. Not every one of these maxima indicates an arrival however. Some maxima are caused by the nature or shape of the analysis signals being used rather than arrivals due to different temporal components of the echo.

To resolve this problem, we examine the nature of the local maxima which may be caused by effects other than signal arrivals. We show that simple filtering of a lambda curve can remove the “spurious” maxima, revealing the time of arrival.

Since this chapter is intended to be an introduction to the use of the generalized model in estimating the time of arrival of the creeping wave, the scope of the chapter is limited compared to the previous work. First, the only model being evaluated is the (negative) second order one. This is reasonable since, given the results of the previous chapter, it is the model most likely to be used. Second, only one analysis frequency ($f_c = 1200Hz$) is used. This specification is not too restrictive since the concepts being discussed are either independent of analysis frequency or hold at any given analysis frequency with minor modifications.

5.2 Detecting a Signal of Unknown Form

This derivation is based upon the one given in chapter eleven of Whalen [18].

Consider deciding between the following hypotheses

$$\begin{aligned} H_1 : \quad \bar{r} &= H\bar{s} + \bar{n} \\ H_0 : \quad \bar{r} &= \bar{n} \end{aligned} \tag{5.3}$$

where $\bar{n}$ is a zero mean Gaussian vector and $\bar{s}$ is an unknown but deterministic vector. The dimensions of H , $\bar{s}$, and $\bar{n}$ are $K \times M$, $M \times 1$, and $K \times 1$ respectively. This problem amounts to deciding whether or not the signal vector $\bar{s}$ is present. Given these assumptions it is reasonable, though not necessarily optimum, to apply the maximum likelihood principle. That is, in the likelihood ratio

$$\lambda = \frac{\max_s \{p_1(\bar{r}|\bar{s})\}}{p_0(\bar{r})} \tag{5.4}$$

a maximum likelihood estimate of $\bar{s}$ is used as though it were the actual value of $\bar{s}$. In Equation (5.4), $p_1(\bar{r}|\bar{s})$ is the probability density function of $\bar{r}$ conditioned on the signal $\bar{s}$ being present, and $p_0(\bar{r})$ is the probability density function of $\bar{r}$ conditioned on the signal being absent.

In [18], Whalen derives a maximum likelihood estimate of the form

$$\hat{\bar{s}} = (H^T R_n^{-1} H)^{-1} H^T R_n^{-1} \bar{r} \tag{5.5}$$

which leads to the statistic given by Equation (5.2). This maximum likelihood estimate is dependent upon $H^T R_n^{-1} H$ being non-singular however. The derivation that follows removes this restriction.

Since

$$p_1(\bar{r}|\bar{s}) = \frac{1}{(2\pi)^{K/2}|R_n|^{1/2}} e^{[-\frac{1}{2}(\bar{r}-H\bar{s})^T R_n^{-1}(\bar{r}-H\bar{s})]}, \quad (5.6)$$

the maximum likelihood estimate is the vector $\hat{\bar{s}}$ that minimizes the quadratic form

$$(\bar{r} - H\bar{s})^T R_n^{-1}(\bar{r} - H\bar{s}) \quad (5.7)$$

where R_n^{-1} is the covariance matrix of the noise. The minimizing vector $\hat{\bar{s}}$ is

$$\hat{\bar{s}} = H^\dagger \bar{r} \quad (5.8)$$

where $H^\dagger$ denotes the Moore-Penrose pseudoinverse of H [19].

When the usual multivariate normal distributions are substituted into Equation (5.4), and after some manipulation, we see that

$$\lambda = e^{[-\frac{1}{2}(H\bar{s})^T R_n^{-1}(H\bar{s})] + [\bar{r}^T R_n^{-1}(H\bar{s})]} \quad (5.9)$$

or taking logarithms

$$\log \lambda = -\frac{1}{2}(H\bar{s})^T R_n^{-1}(H\bar{s}) + \bar{r}^T R_n^{-1}(H\bar{s}). \quad (5.10)$$

If the vector $\bar{s}$ were known, then the second part of Equation (5.10), $\bar{r}^T R_n^{-1}(H\bar{s})$, would be used as the test statistic. It follows that in the present case this form, with $\bar{s}$ replaced by the maximum likelihood estimate $\hat{\bar{s}}$, is used as the test statistic. Let λ denote this test statistic, where

$$\begin{aligned} \lambda &= \bar{r}^T R_n^{-1}(H\hat{\bar{s}}) \\ &= \bar{r}^T R_n^{-1}(HH^\dagger \bar{r}) \\ &= \bar{r}^T R_n^{-1}(HH^\dagger) \bar{r}. \end{aligned} \quad (5.11)$$

In the case where the null space of H contains only the zero vector, the product $HH^\dagger$ reduces to an identity matrix and we have the same result given in Equation (5.2). Note: For the analysis signal set used in this thesis, the product $HH^\dagger$ reduces (within computational precision) to an identity matrix.

How does this relate to the second order generalized model? The relationship is made clear by performing the following substitutions. Replace the column vector $\bar{s}$ in Equation (5.3) with the actual echo e . That is, the echo is the unknown but deterministic signal. It is unknown because in this scenario we do not claim to know the parameters of the target. It is deterministic because we assume that the target itself is unchanging. The matrix H is formed by filling its rows with the analysis signal set. For a second order integrating terms only model we put the second integral signal $v_{-2}(t)$ in the first row, the first integral signal $v_{-1}(t)$ in the second row, and the zeroth integral signal $v_0(t)$ in the third row. The first, second, and third rows of the column vector $\bar{n}$ are filled by the correlation of the noise with the second, first, and zeroth order analysis signals respectively. The column vector $\bar{r}$ then corresponds in form to the $\mathbf{r}$ described in Section 2.3.1. Calculating the value of λ for each time within the measurement interval produces the lambda curve to be used in time of arrival estimation.

5.3 Noise Free Time of Arrival Results

The target-observer relationship is the same here as it was in Chapter Three. Therefore the time of arrival of the creeping wave is known. So any result obtained from a particular time of arrival estimator may be directly compared to this known value.

The method for finding the time of arrival for the creeping wave involves three steps. First, a signal is transmitted toward the sphere and the resulting echo is measured. Second, the test statistic given by Equation (5.11) is calculated for each time instant within the interval of interest. Third, the resulting (λ) curve is examined for maxima that may indicate the arrival of a creeping wave.

Figure 5.1: Echo from the Sphere

5.3.1 Reference Localization of the Creeping Wave

Figure 5.1 shows the relationship between the temporal components of the echo from the sphere. Beginning at time zero is the reflection that was analyzed in the previous chapters. Next, beginning at approximately 0.016 seconds is the creeping wave. The precise localization of the start of the creeping is the point of interest. Figure 5.2 is a magnified view of the creeping wave. Figure 5.3 is a magnified view of the beginning of the creeping wave. It reveals that the beginning of the creeping wave occurs at time $t = 0.0158$ seconds.

Figure 5.2: The Creeping Wave

Figure 5.3: Early Part of the Creeping Wave

Figure 5.4: Lambda Curve

5.3.2 Lambda Curves

Now that a reference time for the arrival of the creeping wave is established, we may examine the use of the lambda curves in finding this time. Figure 5.4 shows the lambda curve produced from processing the echo shown in Figure 5.1. Note the peak at time $t = 0$. This shows that the lambda curve places the arrival of the reflection component of the echo at time $t = 0$. This is a positive indication since

the mathematical representation of the target is configured to produce the reflection component starting at time zero. Not much of the lambda curve is visible near the known time of arrival for the creeping wave however. This is expected since the creeping wave is small compared to the reflection component.

Figure 5.5: Lambda Curve near the Creeping Wave

Figure 5.5 shows the lambda curve in the neighborhood of the creeping wave. Once again the result is good since the maximum value for the part of the curve that is shown occurs at the correct time, namely $t = 0.0158$ seconds. The explanation for jagged appearance of the curve is simple. Examination of Equation (5.11) reveals that at any given time the statistic is essentially a weighted correlation sum. Since the return from the sphere is oscillatory in nature, and since it is being correlated against itself, the jagged shape of the lambda curve is to be expected.

Despite the fact that the lambda curve in its present state gives an accurate localization of the creeping wave, a further improvement is possible. Figure 5.6 shows the magnitude of the Fourier transform of the lambda curve plotted versus frequency. The frequency content of the lambda curve is localized near the origin and near $f = 2400\text{Hz}$, or twice f_c . It is the activity near $2f_c$ that gives the lambda curve in Figure 5.5 its jagged appearance. This fact suggests that simple filtering of the lambda curve may remove “spurious” oscillations while leaving “desirable maxima” undisturbed. A zero-phase filter will not cause any time shifts in what remains of the lambda curve after filtration. So if the higher frequency information is not necessary for locating the arrival of a creeping wave, the filtered lambda curve will have a maximum at the correct time. Simple brick wall filtering (cutoff at 1200Hz) removes the high frequency information as shown in Figure 5.7.

Figure 5.6: Magnitude of Fourier Transform of the Lambda Curve

Figure 5.7: Filtered Version of Figure 5.6

Figure 5.8: Filtered Lambda Curve near the Creeping Wave

Figure 5.8 is the lambda curve that results from taking the inverse Fourier transform of the filtered frequency domain data shown in Figure 5.7. It is plotted for times near the arrival of the creeping wave.

Figure 5.9: Superposition of Figures 5.5 and 5.8

Figure 5.9 is the superposition of Figures 5.5 and 5.8. Note that the desired effect takes place. The spurious oscillations are removed while the location of the maximum in the lambda curve is not shifted in time. The filtered lambda curve accurately marks the time of arrival of the creeping wave.

5.4 Time of Arrival Results for the Additive Noise Case

The accuracy of the time of arrival estimate provided by the filtered lambda curve has been established for the noise free case. Here the effects of additive uncorrelated Gaussian noise on the estimate are shown.

Figure 5.10 is a plot of a creeping wave in noise with a local SNR of $26dB$. It is the last of the realizations used in a forty trial test of the filtered lambda curve as time of arrival estimator. Figure 5.11 is a plot of the time of arrival estimates for forty trials in this level of noise. The sample average of the estimates is $t = 0.015825$ seconds, close to the true value of $t = 0.0158$ seconds. Most of the estimates are correct, and those estimates that are wrong are in error by no more than one time sample. (Remember that the sampling rate is set to $10kHz$ for all of the processing being done.) The errors are in the direction of being late. That is, the noise tends to obscure the creeping wave in such a way as to make the peak in the filtered lambda curve occur after the correct time.

Figure 5.10: Creeping Wave in Noise, $\text{SNR} = 26\text{dB}$

Figure 5.11: Time of Arrival Estimates for $\text{SNR} = 26\text{dB}$

Figure 5.12: Creeping Wave in Noise, $\text{SNR} = 6\text{dB}$

Figure 5.12 is a plot of a creeping wave in noise with a local SNR of 6dB . The arrival time of the creeping wave is very difficult to determine from the illustration without prior knowledge of the actual time. The noise is not very far down in power from the signal here.

Figure 5.13: Time of Arrival Estimates for $\text{SNR} = 6\text{dB}$

Figure 5.13 is a plot of the time of arrival estimates for forty trials in this level of noise. For this data set the sample average is $t = 0.0159$ seconds, still close to the true value. As in the previous set of estimates made in the presence of noise, most of the errors are “late”. There are a few “early” errors here though. Still, given the poor signal to noise ratio, the performance of the estimator appears to be satisfactory. No gross errors were made in this series of trials.

Figure 5.14: Creeping Wave in Noise, $\text{SNR} = 1\text{dB}$

As a severe test, a set of trials was run with a local SNR of 1dB . As Figure 5.14 shows, it is difficult to pick out the creeping wave by inspection. Figure 5.15 is a plot of the time of arrival estimates. Amazingly enough, the estimator still does not make any gross errors and manages to provide an average estimate of $t = 0.0159$ seconds.

Figure 5.15: Time of Arrival Estimates for $\text{SNR} = 1\text{dB}$

5.5 Chapter Summary

In this chapter we described the detection of a signal of unknown form through the use of a likelihood ratio test. The problem was cast into a form suitable for interpretation involving the generalized target model. A generalized version of the non-coherent matched filter was derived through the use of the Moore-Penrose pseudoinverse. The use of the resulting detector as an estimator for time of arrival was described. The chapter closes with the presentation of simulation results that support the use of the generalized model and the time of arrival estimator as described. The support comes from the fact that even in severely noisy conditions, the estimator was able to accurately locate the arrival of the creeping wave.

CHAPTER 6

SUMMARY AND CONCLUSION

The generalized target description suggested by Altes [2] was analyzed in this thesis. This generalized model characterizes a target as a series of time delayed linear combinations of differentiators and integrators, rather than just reflectors as in the conventional model. A detailed discussion of the mathematical structure of the model, as well as an algorithm for calculating its parameters, was given in Chapter 2.

To test the model it was necessary to simulate a target and generate synthetic echo data. Chapter 3 described the target: a rigid sphere. A rigid sphere was chosen as a target for two reasons. First, there exists an accurate mathematical representation of the impulse response of the sphere, the Mie series. This provided a good target with which the synthetic echo data was created. Second, there is an asymptotic series approximation for the early time response from the rigid sphere. This asymptotic series is a frequency domain expression in negative powers of $j\omega$. These terms represent increasing orders of integration in the time domain. The coefficients of the terms contain information about the radius of the sphere. This relationship provided the means to test the quality of the generalized model coefficients that were calculated

according to the algorithm of Chapter 2. The coefficients were used to estimate the radius of the sphere, and since the radius was known a priori, the accuracy of the radius estimates indicated the accuracy of the calculated model coefficients. Chapter 3 concludes with an introduction to the Dudley-Weyker signal set.

Chapter 4 began with a discussion of the effects a particular analysis signal may have on the model coefficients that are calculated through its use. A study of the dimensionality of the Dudley-Weyker signal set was undertaken, and the limitations of the signal set were described. Next, the matrix structure of the discrete time implementation of the generalized model was described. Chapter 4 concluded with a detailed study of radius estimates and mean square error (between model and actual echo) figures as produced by different order models in the presence of varying levels of noise.

Chapter 5 gave an introduction to the use of the generalized model in estimating the time of arrival of echo components. A test statistic for detection of deterministic signals of unknown form was derived. This test statistic, evaluated at each instant within a measurement interval and interpreted as a function of time, was used to estimate the time of arrival of the unknown signal. The chapter concluded with several demonstrations of the performance of the estimator.

Within certain limitations the generalized target model proved to be a satisfactory representation of a scatterer, in this case the rigid sphere. With its order suitably chosen, the generalized model provided not only more information about the target in

terms of higher order coefficients, but improvement in the accuracy of the zeroth order coefficient that is also present in the conventional model. Finally, a time of arrival estimator based upon the structure of the generalized model was shown to be accurate and potentially quite robust in the presence of noise. All of these results suggest that conservative application of the generalized model provides an improvement over the conventional model that is now widely used.

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