Polarization Phase-Shifting Point-Diffraction
Interferometer

by

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I look up to the mountains –
does my help come from there?
My help comes from the Lord,
who made the heavens and the earth!

He will not let you stumble and fall;
the one who watches over you will not sleep.
Indeed, he who watches over Israel
never tires and never sleeps.

The Lord himself watches over you!
The Lord stands beside you as your protective shade.
The sun will not hurt you by day, nor the moon at night.
The Lord keeps you from all evil
and preserves your life.
The Lord keeps watch over you as you come and go,
Both now and forever.

- Psalms 121

To Jordan,

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ABSTRACT

A new instrument, the polarization phase-shifting point-diffraction interferometer (PPSPDI), is developed utilizing a birefringent pinhole plate. The interferometer uses polarization to separate the test and reference beams, interfering what begin as orthogonal polarization states. The instrument combines the robust nature of Linnik's original point-diffraction interferometer with the ability to phase-shift for interferogram analysis. The instrument is compact, simple to align, vibration insensitive and can phase-shift without moving parts or separate reference optics.

This dissertation describes the theory, design, application and manufacturing considerations of the PPSPDI. The original PDI design is expanded to include polarization and phase-shifting. The discussion includes the properties of the birefringent material used as well as various fabrication methods used for creating the pinhole. A new model is developed to determine the quality of the diffracted reference wavefront from the pinhole as a function of pinhole size and aberrations of the test optic. The operation and performance of the interferometer are also presented along with a detailed error analysis and performance limits of the design.

INTRODUCTION

As technology increases, so does the need for faster, more accurate metrology equipment. With the need for higher accuracy, the physical limitations of current interferometers are becoming problematic. Overcoming these limitations using current techniques means building more complicated systems or increasing computation time. New interferometer designs make it is possible to increase the speed and accuracy of interferometric measurements while maintaining a relatively simple system.

To date, despite their obvious advantages for optical testing, common path interferometers, such as the scatterplate, Fresnel zone plate and point-diffraction interferometers, have been largely neglected for use in phase-shifting interferometry. This is primarily due to the difficulty of phase-shifting a common path interferometer. The common-path design provides significantly increased environmental stability and decreased system complexity. Unfortunately, the common-path design also causes problems separating the test and reference beams. With both beams traversing the same path, adding phase in one beam without adding it to the other becomes difficult, thus making phase-shifting difficult.

This dissertation describes a novel method to phase-shift a point diffraction interferometer (PDI). Using a birefringent pinhole plate, the interferometer uses polarization to separate the test and reference beams, interfering what began as orthogonal polarization components. By modulating only one of the orthogonal states,

one can phase-shift the PDI without requiring moving parts or separate reference optics. This work begins with a look at previous attempts to phase-shift the PDI followed by a brief review of polarization and birefringence and a discussion of the conventional non-phase-shifting PDI. Then, a new design and mathematical model are presented that take into account polarization and phase-shifting. Next, the fabrication of the pinhole plate used in the interferometer is covered showing the various methods used and the advantages or disadvantages of each. Error analysis is then performed for the major sources of error in the PPSPDI. Following this, a mathematical simulation is used to determine the quality of the diffracted reference wavefront from the pinhole as a function of pinhole size and test optic aberrations. Finally, the performance of the interferometer is determined with suggestions for future improvements.

HISTORY OF THE PDI

The point diffraction interferometer (PDI) was first invented by Linnik in 1933 as a simple interferometer for directly evaluating optical components (Linnik 1933 via Malacara 1992). In 1972, Smartt and Strong rediscovered the PDI for use in optical testing (Smartt 1972). In 1975, Smartt and Steel further developed the concept by using a partially transmitting pinhole plate (Smartt 1975). Smartt et. al. later integrated the PDI into both an interference microscope and an alignment and testing tool for large-scale telescope mirrors (Smartt 1979, Smartt 1985).

In 1978, Koliopoulos et. al. developed an infrared PDI for use in testing high energy laser systems (Koliopoulos 1978). The pinhole used in the interferometer is etched into a thin gold film deposited on a thin silicon wafer substrate. The final interferogram is imaged onto a pyroelectric vidicon.

With the advent of phase-shifting techniques in the early 1980's, Underwood et. al. created a modified PDI with the ability to take phase measurement data. The interferometer utilizes a small, mirrored spot on one surface of a polarization beamsplitter cube to create the reference beam (Underwood 1982). On passing through the beamsplitter cube, the test and reference beams are orthogonally polarized. Next, an electro-optic modulator is used to phase-shift one of the two beams before passing through a final analyzer and onto the detector array. Unfortunately, the system requires a

custom-made, very complex beamsplitter cube to operate and has a very low optical throughput.

A variation of the PDI created by Kwon in 1984 uses a pinhole fabricated into a transmission grating to produce three simultaneous phase shifted interferograms (Kwon 1984). The wavefront phase from the test optic is determined from the three interferograms, but three separate detectors were required and the phase difference between the interferograms was set during the construction of the grating. This design has the advantage of acquiring the interferograms simultaneously allowing for high speed applications.

In 1987, Kadono et. al. developed a phase-shifting PDI using a series of polarization optics (Kadono 1987). The pinhole is constructed in a linear polarizer, followed by a quarter wave plate, half wave plate and a final linear polarizer. Phase shifting is accomplished by rotating the half wave plate about the optical axis. This technique gives an accuracy of $\lambda/40$, but is limited to measuring very slow optics because one must align the incident light nearly normal to the pinhole-etched polarizer.

In 1991, Wang et. al. demonstrated a PDI interferometer using polarization optics to phase shift (Wang 1991). The test beam is focused onto a pair of polarizers, the second of which has a pinhole. This combination creates orthogonal, linearly polarized test and reference beams with a phase difference. A Fresnel rhomb converts the linearly polarized

beams into elliptically polarized beams of left-handed and right-handed orientations, producing an interferogram on the camera. A polarization regulator placed near the source controlled the phase difference between the test and reference beams allowing for phase shifting.

In 1994, Kadono et. al. developed a second phase shifting PDI using a liquid crystal cell as a phase modulator (Kadono 1994). As a voltage is applied to the cell, the liquid crystal molecules change orientation creating a change in the optical path through the cell. A very small circular area of the transparent electrode on the liquid crystal cell is removed creating the pinhole for the PDI. Without the electrodes, the liquid crystals inside the pinhole do not change orientation with applied voltage; consequently, they do not change the phase inside the pinhole. So, it is possible to independently phase shift the test beam with respect to the reference beam by applying a voltage to the liquid crystal cell.

In 1996, Mercer et. al. developed a similar liquid crystal phase shifting interferometer, which uses a glass microsphere as the diffracting aperture (Mercer 1996). A liquid crystal cell with an embedded small glass microsphere serves as the pinhole plate for the PDI. The light from the test optic is focused onto the microsphere. The portion incident on the microsphere is diffracted and serves as the reference beam. The remainder of the light passes through the liquid crystal cell and serves as the test beam. As the voltage is varied across the liquid crystal cell, the refractive index changes which increases the

optical path, changes the phase of the transmitted beam, and phase shifts the interferometer.

In 1996, Medecky created a variation on the PDI used for phase shifting (Medecky 1996). A transmission grating is inserted after the test lens. The grating divides the incident light into two beams with a small angular separation creating two separate foci. The normal pinhole plate used in a PDI is replaced with an opaque mask containing a small pinhole and a larger window centered on the two foci. The light incident on the pinhole creates the reference wave while the window passes the test beam undisturbed allowing the two wavefronts to overlap and interfere at the camera. The grating is translated by one grating period to introduce the phase shifts necessary for phase measurements. While this design provides a simple method for phase shifting, it requires laterally separating the test and reference beams inducing high density tilt fringes in the final interferogram. This limits the interferometers ability to test anything other than well corrected systems.

Finally, in 2002, Totzeck et. al. created a phase shifting PDI based on separating the test and reference beams by polarization (Totzeck 2002). Light from the test optic, assumed to be linear, is focused onto a pinhole etched through a half wave plate made from mica. The polarization state of the reference beam remains the same while the test beam's polarization state rotates by 90° . Immediately after the pinhole plate, the test and reference beams are in orthogonal polarization states. Next, a variable retarder introduces

an adjustable phase shift between the test and reference beams while a final polarizer interferes the two beams. The major disadvantage to this design is the fabrication of the mica half wave plate and subsequent pinhole. Because of a tendency for the mica sheets to rip, achieving the correct thickness of the mica plate is extremely difficult as well as maintaining homogenous retardance across the plate. The best result achieved for this instrument is 168° with a pinhole diameter of $100\text{ }\mu\text{m}$.

POLARIZATION REVIEW

Polarization is an important optical concept and the basis for the Polarization Phase-Shifting Point-Diffraction Interferometer (PPSPDI). The ability to polarize a source and manipulate the polarized light is what allows phase-shifting of the point-diffraction interferometer. This chapter focuses on polarization and its manipulation that includes linear and circular polarization, biaxial materials, polarization optics to include wave plates and electro-optic modulators, and Jones calculus. The chapter starts with a review of the basics of polarization (Hecht 1998, Born 1997). The rest of the chapter focuses on the manipulation of polarization. A review of biaxial materials is necessary to understand the operation of the birefringent pinhole plate (Hodgkinson 1997, Born 1997, Yariv 1976). Next, reviewing polarization optics, such as wave plates, provides insight into the modulation of polarization in the PPSPDI (Yariv 1976, Bass 1995). Finally, Jones calculus, a mathematical description for polarization, is discussed (Hecht 1998, Shurcliff 1966).

POLARIZATION

In the simplest of terms, polarization is an easy method for describing the transverse oscillations of the electric field in time and space. When viewed along the axis of propagation, an electric field is considered completely polarized when its oscillations vary predictably in time. An electric field whose oscillations vary randomly in time is considered completely unpolarized.

Consider a harmonic plane wave described by

$$\vec{E}(\vec{r}, t) = E_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \delta) \quad (2.1)$$

Assuming the z-axis is the direction of propagation and knowing the field is transverse, the plane wave is expressed as an orthogonal combination of optical disturbances or

$$\begin{aligned} \vec{E}(z, t) &= \vec{E}_x(z, t) + \vec{E}_y(z, t) \\ &= \hat{i}E_{0x} \cos(kz - \omega t) + \hat{j}E_{0y} \cos(kz - \omega t + \varepsilon) \end{aligned} \quad (2.2)$$

The relative amplitudes, E_{0x} and E_{0y} , along with the phase difference, ε , determines the amplitude and direction of the electric field oscillation in time and space. The first case of interest occurs when ε is zero or a multiple of 2π . In this case, equation 2.2 is

$$\vec{E}(z, t) = (\hat{i}E_{0x} + \hat{j}E_{0y}) \cos(kz - \omega t) \quad (2.3)$$

and the waves are considered in-phase. The resultant wave has a fixed amplitude equal to $(\hat{i}E_{0x} + \hat{j}E_{0y})$, traces out a line in the x-y plane as it oscillates and is classified as linearly polarized. The same holds true when ε is equal to π with the exception that the amplitude now becomes $(\hat{i}E_{0x} - \hat{j}E_{0y})$, and the line traced out in the x-y plane is perpendicular to the first case.

The second case arises when the relative amplitudes, E_{0x} and E_{0y} , are equal and previously defined ε is a whole number multiple of $\pi/2$. When this happens the x and y components of the wave are

$$\begin{aligned} \vec{E}_x(z, t) &= \hat{i}E_0 \cos(kz - \omega t) \\ \vec{E}_y(z, t) &= \pm \hat{j}E_0 \sin(kz - \omega t) \\ E_{0x} &= E_{0y} = E_0 \end{aligned} \quad (2.4)$$

with the sign of $\vec{E}_y(z,t)$ dependent on the actual value of ε . For ε equal to $\pi/2+2m\pi$, where $m=0, \pm 1, \pm 2, \dots$, $\vec{E}_y(z,t)$ is negative, while for ε equal to $-\pi/2+2m\pi$, with $m=0, \pm 1, \pm 2, \dots$, $\vec{E}_y(z,t)$ is positive. The resultant total wave is

$$\vec{E}(z,t) = E_0[\hat{i} \cos(kz - \omega t) \pm \hat{j} \sin(kz - \omega t)] \quad (2.5)$$

As the wave oscillates, the amplitude of $\vec{E}$ remains constant but its orientation does not. When viewed along the z-axis, the wave appears to trace out a circle in the x-y plane as a function of time or space and is circularly polarized. If the sign of $\vec{E}_y(z,t)$ is positive, the wave rotates clockwise when viewed head-on and is right-circularly polarized. If the sign of $\vec{E}_y(z,t)$ is negative, the wave rotates counterclockwise and is left-circularly polarized.

Mathematically, both linear and circular polarizations are special cases of the more general case of elliptical polarization. In elliptical polarization, the electric field both rotates and changes magnitude as the wave oscillates. Furthermore, the magnitudes of the x and y components of the field as well as the phase difference are all arbitrary unlike linear and circular polarization. Viewed head-on, the electric field traces out an ellipse in the x-y plane. Taking the original definitions of $\vec{E}_x$ and $\vec{E}_y$, modifying the equations and squaring the result gives the following equation of an ellipse

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2 \left(\frac{E_x}{E_{0x}}\right) \left(\frac{E_y}{E_{0y}}\right) \cos(\varepsilon) = \sin^2(\varepsilon) \quad (2.6)$$

The relative magnitudes of the x and y components and the phase difference determine the size and shape of the ellipse as well as its orientation. Figure 2-1 shows the three cases of polarization.

Figure 2-1: States of Polarization: a) Linear, b) Circular and c) Elliptical viewed along the axis of propagation

BIAXIAL MATERIALS

Many materials are optically anisotropic, especially crystalline materials. As such, the optical properties of the materials are not the same in all directions, e.g. the index of refraction, causing birefringence. It is this property that is exploited in making the pinhole plate for the PPSPDI. Therefore, it is important to understand the physical properties of biaxial materials.

Biaxial materials are uniquely optically anisotropic meaning that the electric vector, $\vec{E}$, and the electric displacement vector, $\vec{D}$, are no longer in the same direction. In isotropic media, there is a direct relationship between $\vec{D}$ and $\vec{E}$, specifically $\vec{D} = \epsilon \vec{E}$. For anisotropic media, this relationship is replaced by one where each component of $\vec{D}$ is linearly related to each component of $\vec{E}$ or

$$\begin{aligned} D_x &= \epsilon_{xx}E_x + \epsilon_{xy}E_y + \epsilon_{xz}E_z \\ D_y &= \epsilon_{yx}E_x + \epsilon_{yy}E_y + \epsilon_{yz}E_z \\ D_z &= \epsilon_{zx}E_x + \epsilon_{zy}E_y + \epsilon_{zz}E_z \end{aligned} \quad (2.7)$$

The nine quantities $\epsilon_{xx}, \epsilon_{yy}, \dots$ are the dielectric constants of the material and make up the dielectric tensor. It is possible to choose the x, y and z-axes of the material so that all the off-diagonal elements of the tensor vanish, leaving

$$\begin{aligned} D_x &= \epsilon_{xx}E_x = \epsilon_0 n_x^2 E_x \\ D_y &= \epsilon_{yy}E_y = \epsilon_0 n_y^2 E_y \\ D_z &= \epsilon_{zz}E_z = \epsilon_0 n_z^2 E_z \end{aligned} \quad (2.8)$$

These directions are called the principle dielectric axes of the material. The apparent consequence of optical anisotropy for a material is that the material's index of refraction along each of the principle axes is different. To take a close look at this, the energy density can be written as

$$w_e = \frac{1}{2} \left(\frac{D_x^2}{\epsilon_{xx}} + \frac{D_y^2}{\epsilon_{yy}} + \frac{D_z^2}{\epsilon_{zz}} \right) \quad (2.9)$$

Making the substitutions

$$x = \frac{D_x}{\sqrt{2\varepsilon_0 w_e}}, \quad y = \frac{D_y}{\sqrt{2\varepsilon_0 w_e}}, \quad z = \frac{D_z}{\sqrt{2\varepsilon_0 w_e}} \quad (2.10)$$

results in

$$1 = \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} \quad (2.11)$$

which is the well known index ellipsoid. The ellipsoid's semi-axes coincide with the directions of the principle axes and its lengths are equal to the principle refractive indices as shown in figure 2-2. Using the index ellipsoid, one can find the index of refraction along any line of propagation through the material. The waves direction of propagation through the material forms a vector $\vec{A}$. There exists a plane through the ellipsoid perpendicular to $\vec{A}$ that passes through the center of the ellipse and represents all possible polarization states for the wave. The magnitude of the vector $\vec{B}$ along the direction of polarization from the center of the ellipse to the surface is equal to the index for the wave shown in Figure 2-2. So, the phase velocity for the wave propagating along a given direction in the material depends on its direction of propagation and polarization.

Figure 2-2: Index ellipsoid

Likewise, in a biaxial material, the index of refraction is different for all three principle axes

$$n_x \neq n_y \neq n_z \quad (2.12)$$

When viewed along any one of the principle axes, the index ellipsoid appears as an ellipse with principle radii equal to the indices of refraction of the other two principle axes. This property makes biaxial materials useful for manipulating polarization. The most common way to change the state of polarization is to manipulate the relative phase difference between the orthogonal components of the wave. In a biaxial material, this is done by altering the optical path for the two components of the incident wave. Since the principle axes have different indices of refraction, the optical path along the principle axes is different. The optical path difference is

$$\Delta d = d_a - d_b = (n_a - n_b)z \quad (2.13)$$

where a and b are the axes of the wave's orthogonal components and z is the physical thickness of the crystal. The thickness of the material is then used to control the change in polarization state of an incident wave. The importance of this property becomes apparent in the next section on polarization optics and when the design of the PSSPDI pinhole plate is discussed.

WAVE PLATES AND POLARIZATION OPTICS

Polarization optics are defined as any optics that are used for the express purpose of altering the polarization state of the incident wave, typically in the form of wave plates. As previously discussed, the most common way to change the wave's polarization state is to manipulate the relative phase difference between the orthogonal components of the wave. Because they are more common, uniaxial materials are typically used to make wave plates. In a uniaxial material, two of the principle axes, one being the axis of propagation, have the same index of refraction. The index of refraction shared by two of the principle axes is designated as the ordinary index, n_o , while the index of the third principle axes is designated as the extraordinary index, n_e . The orthogonal components of the beam divided along the principle axes see two different path lengths and a total path difference through the material of

$$\Delta d = d_o - d_e = (n_o - n_e)z \quad (2.14)$$

giving a total phase difference between the components of

$$\delta = \delta_o - \delta_e = (n_o - n_e)kz \quad (2.15)$$

The two most commonly used wave plates are the half-wave and quarter-wave plate, having phase differences of π and $\pi/2$ respectively. A quarter-wave plate changes linear polarization into elliptical and vice versa. A half wave plate takes linear polarization incident at an angle θ to the fast axis of the plate and rotates the polarization by 2θ . For example, linear polarization incident at 45° is rotated by 90° but remains linear.

Another common polarization optic is the electrooptic modulator consisting of an electrooptic crystal sandwiched between electrodes through which a voltage is applied across one of its principle axes shown in Figure 3-3. Slight deformations in the crystal lattice along the direction of the applied voltage cause the dielectric tensor to change, rotating the principle axes of the crystal. The direct effect of changing the principle axes is a corresponding change in the index of refraction along these axes. The new index ellipsoid is diagonalized yielding

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z'^2}{n_{z'}^2} = 1 \quad (2.16)$$

where x' , y' and z' are the new principle axes with their associated indices of refraction $n_{x'}$, $n_{y'}$ and $n_{z'}$. This rotation is typically a very small angle for most electrooptic materials. For an ADP crystal, the rotation is only on the order of a few milliradians even for high voltages.

Fig 2-3: Diagram of an Electrooptic Modulator

Viewing the electrooptic above as a phase modulation for an optical wave at the output of the crystal shows that electrooptic modulators are useful as variable retarders. Assume the voltage is applied across the y-axis of the crystal, the propagation is along the z-axis, and the input wave has components along both the x and y-axes. The component along the y-axis exits the crystal at $z=L$ as

$$E_y(z,t) = E_y \cos(kz - \omega t + \phi_y) \quad (2.17)$$

where

$$\phi_y = \frac{2\pi}{\lambda} n_y L \quad (2.18)$$

The component along the x-axis exits the crystal at $z=L$ as

$$E_x(z,t) = E_x \cos(kz - \omega t + \phi_x) \quad (2.19)$$

where

$$\phi_x = \frac{2\pi}{\lambda} (n_x + \Delta n_x) L \quad (2.20)$$

The total phase difference between the components consists of a natural phase difference, ϕ_0 , without an applied voltage

$$\phi_0 = \frac{2\pi}{\lambda} (n_y - n_x) L \quad (2.21)$$

and an electrically induced phase term

$$\Delta\phi_x = \frac{2\pi}{\lambda} \Delta n_x L \quad (2.22)$$

for the polarization along the x-axis. The electrically induced phase term is

$$\Delta\phi_x = \frac{\pi}{\lambda} n_x^3 r_{41} V \left(\frac{L}{d} \right) \quad (2.23)$$

that is a function of the aspect ratio of the crystal L/d and the applied voltage V . r_{41} is the associated linear electrooptic tensor element for the x-axis. The phase difference induced between the two components of the incident wave changes linearly with applied voltage, this is known as the linear electrooptic effect. By changing the voltage applied, the phase difference between the two components of incident wave is varied. This is the method for phase shifting the PPSPI.

JONES CALCULUS

For simple systems, the effect of polarization optics and wave plates on the polarization state of the incident beam is easily predictable. As systems become increasingly complicated, a more formal approach to the propagation of polarization is necessary. For completely polarized light, one can use Jones calculus which is a matrix approach where the incident field and all optical elements are represented by matrices. The incident field is a 2×1 matrix while all optical elements are 2×2 matrices. The incident beam is expressed in terms of its Jones vector that separates the two orthogonal components

$$\vec{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} \quad (2.24)$$

Each optical component in the system is expressed as a 2×2 matrix. Propagation of the incident beam is accomplished by multiplying the incident wave matrix by the matrix for each element in the system as shown below

$$\begin{bmatrix} E_x \text{ Output} \\ E_y \text{ Output} \end{bmatrix} = \begin{bmatrix} \text{Final Element} \end{bmatrix} \dots \begin{bmatrix} 2^{nd} \text{ Element} \end{bmatrix} \begin{bmatrix} 1^{st} \text{ Element} \end{bmatrix} \begin{bmatrix} E_x \text{ Input} \\ E_y \text{ Input} \end{bmatrix} \quad (2.25)$$

Table 3-1 lists the Jones matrices for several polarization states and optical elements.

Optical Element or Polarization State	Associated Jones Matrix
Horizontal Linear Polarization State	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
Linear Polarization State at +45°	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
Right Circular Polarization	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$
Horizontal Linear Polarizer	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Vertical Linear Polarizer	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Half Wave Plate Fast Axis Vertical	$\begin{bmatrix} e^{i\pi} & 0 \\ 0 & 1 \end{bmatrix}$
Half Wave Plate Fast Axis Horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix}$
Quarter Wave Plate Fast Axis Vertical	$\begin{bmatrix} e^{i\frac{\pi}{2}} & 0 \\ 0 & 1 \end{bmatrix}$
Quarter Wave Plate Fast Axis Horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix}$
Arbitrary Retarder of Retardance ϕ Fast Axis Vertical	$\begin{bmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{bmatrix}$

Table 2-1: Jones Matrices for Several Polarization States and Optical Elements

In addition, any optical element is rotated by multiplying its matrix by a rotation matrix

$$P_{\theta} = S(-\theta)P_0S(\theta) \quad (2.26)$$

with

$$S(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \quad (2.27)$$

For the very general case of an arbitrary retarder of retardance ϕ at an angle θ , the associated Jones matrix is

$$\begin{aligned}
 R_\theta &= \begin{bmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{bmatrix} \begin{bmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \\
 &= \begin{bmatrix} e^{i\phi} \cos^2(\theta) + \sin^2(\theta) & e^{i\phi} \cos(\theta) \sin(\theta) - \sin(\theta) \cos(\theta) \\ e^{i\phi} \cos(\theta) \sin(\theta) - \sin(\theta) \cos(\theta) & e^{i\phi} \sin^2(\theta) + \cos^2(\theta) \end{bmatrix}
 \end{aligned} \tag{2.28}$$

with θ measured from the horizontal axis.

CONVENTIONAL PDI

The conventional PDI is a simple common-path interferometer capable of directly measuring optical wavefronts for metrology and optical testing. The PDI's primary advantage is its common-path design, where the test and reference beams travel the same or almost the same path. This design makes the PDI extremely useful where environmental isolation is not possible or the reduction in the number of precision optics is required. This chapter discusses the operation of the conventional PDI both qualitatively and through scalar diffraction theory.

QUALITATIVE DESCRIPTION OF THE CONVENTIONAL PDI

The PDI is a simple two-beam interferometer where the reference beam is created from a portion of the test beam by diffraction via a small pinhole in a semitransparent coating. The operation of the PDI is shown in Figure 3-1.

Fig 3-1. Conventional PDI

Light from the laser is sent into a spatial filter whose pinhole acts as a point source for the test lens. In the above diagram, the spatial filter pinhole is positioned at twice the front focal distance from the test lens, producing a focus spot at twice the back focal distance of the test lens. The experimental setup could be modified such that collimated light is

incident on the test lens producing a focus spot at the back focal distance of the test lens. The spatial filter pinhole is chosen sufficiently small so that the size of the focus spot formed by the test lens is solely due to the diffraction limit of the test lens plus aberrations. The PDI plate, which is created by placing a small pinhole in a semitransparent coating, is placed at the focus spot of the test lens. The diameter of the pinhole created is approximately half the unaberrated Airy disk diameter of the test lens (Wyant 1979), or

$$d_{pinhole} \approx (1.22\lambda) f / \# \text{ working}_{test\ optic} \quad (3.1)$$

This requirement sets the lower limit on the f-number of a particular optic that can be tested for a given pinhole diameter. A 5 μ m diameter pinhole can be used to test optics with working f-numbers of 6.5 and larger. The pinhole is aligned so that it is coincident with a portion, usually the center, of the focus spot formed by the test lens. The portion of light incident on the pinhole is diffracted by the aperture into a spherical wavefront that serves as the reference wavefront for the interferometer. The diameter constraint of the pinhole produces a reference wavefront with only minimal amplitude and phase variations (Wyant 1979). The remainder of the light from the test lens is attenuated but is otherwise transmitted unaffected through the semitransparent region surrounding the pinhole shown in Figure 3-2. With the PDI pinhole smaller than the Airy disk radius of the test lens, the angular subtense of the diffracted reference beam will always be larger than the angular subtense of the test beam ensuring the entire optic is tested.

Figure 3-2: Operation of the PDI Plate

The test and reference wavefronts pass through the imaging optics, which form an image of the test lens superimposed with interference fringes on the CCD camera. For good fringe contrast, the test beam is carefully attenuated so that the relative intensities of the test and reference beams are similar. Typical transmittances of the PDI plate are .01 to .1 (Steel 1983).

SCALAR DIFFRACTION THEORY REPRESENTATION

A theoretical model of the conventional PDI is derived from scalar diffraction theory. This model serves as a starting point for describing the PPSPDI presented later, and shows the origin of both the test and reference beams incident on the image plane; however the effect of lateral misalignment of the PDI pinhole is not presented. The PDI schematic in Figure 3-3 indicates the planes used in the analysis.

Fig. 3-3: Schematic of the PDI for scalar diffraction model

For this scalar diffraction model to be valid, four assumptions are made (Goodman 1996)

1. Assume the geometric lens law, $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, for the test optic is satisfied.
2. Neglect petzval curvature across the image plane
3. Neglect complex proportionality constants.
4. Place the PDI pinhole coincident with the center of the test optic focus spot.

The optical field just behind the test lens is described by the lens pupil function

$$U(x_1, y_1) = \text{cyl}\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)} \quad (3.2)$$

where d_1 is the diameter of the lens and $W(x_1, y_1)$ is the aberration function for the test

lens. The cylinder function, $\text{cyl}\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right)$, represents a function equal to zero

everywhere except for $\sqrt{x_1^2 + y_1^2} < d_1/2$, where the function equals one (Gaskill, 1978).

In propagating to the test lens focus and neglecting petzval curvature at the focus plane, the impulse response of the test lens

$$h(x_2, y_2) = \frac{1}{\lambda^2 z_1 z_2} \iint U(x_1, y_1) e^{-i \frac{2\pi}{\lambda z_2} (x_1 x_2 - y_1 y_2)} dx_1 dy_1 \quad (3.3)$$

with z_1 and z_2 equal to the object and image distances for the test lens, simply Fourier transforms the lens pupil function (Goodman 1996). The optical field just prior to the PDI plate is represented by the Fourier transform of the test lens pupil function

$$U^-(x_2, y_2) = FT \left\{ cyl \left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1} \right) e^{ikW(x_1, y_1)} \right\} \quad (3.4)$$

The optical field is then multiplied by the transmittance function of the PDI plate given by

$$\tau_{PDI} = t_{plate} + (1 - t_{plate}) cyl \left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2} \right) \quad (3.5)$$

with t_{plate} equal to the transmittance of the semitransparent coating and d_2 equal to the diameter of the pinhole, resulting in

$$\begin{aligned} U^+(x_2, y_2) &= \tau_{PDI} \cdot U^-(x_2, y_2) \\ &= [t_{plate} + (1 - t_{plate}) cyl \left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2} \right)] \cdot FT \left\{ cyl \left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1} \right) e^{ikW(x_1, y_1)} \right\} \end{aligned} \quad (3.6)$$

Finally the imaging lens images the plane of the test lens onto the camera. The final image is a scaled and possibly magnified copy of the wavefront at the test lens, modified by the PDI plate and convolved with the impulse response of the imaging lens. In the limit that the imaging lens diameter is very large, the impulse response becomes a delta

function that samples but does not smooth the incoming wavefront. The imaging lens has the effect of Fourier transforming the field just after the PDI plate, $U^+(x_2, y_2)$. Neglecting petzval curvature across the image and a complex proportionality constant, the field in the image plane is

$$U(x_3, y_3) = T_{PDI} ** FT\{U^-(x_2, y_2)\} \quad (3.7)$$

$$U(x_3, y_3) = T_{PDI} ** U(x_1, y_1)$$

where T_{PDI} is the Fourier Transform of the transmittance function of the PDI plate. So the field at the camera is the optical field just behind the test lens convolved with the Fourier transform of the transmittance function of the PDI plate. Multiplied out this becomes

$$U(x_3, y_3) = t_{plate} cyl\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) e^{ikW(x_3, y_3)} + \quad (3.8)$$

$$(1 - t_{plate}) \frac{d_2^2 \pi}{4} Somb\left(\frac{d_2 \sqrt{x_3^2 + y_3^2}}{\lambda z_3}\right) ** cyl\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) e^{ikW(x_3, y_3)}$$

where $Somb\left(\frac{\sqrt{x^2 + y^2}}{d}\right) = 2J_1\left(\frac{\pi\sqrt{x^2 + y^2}}{d}\right) / \left(\frac{\pi\sqrt{x^2 + y^2}}{d}\right)$, d is a constant and $J_1(x)$ is a

first order Bessel function. This equation shows that the interferogram is formed by the coherent addition of an attenuated, aberrated test wavefront and a reference wavefront created by the convolution of the aberrated wavefront with the Fourier transform of the pinhole in the PDI plate. The test and reference beams are given by the first and second terms in the above equation respectively. The test beam is an attenuated copy of the original test lens pupil function while the reference beam is a significantly smoothed

version of the same test lens pupil function. The intensity is the true quantity measured at the camera, so at the final image plane, the intensity is

$$I(x_3, y_3) = \left| t_{plate} \text{cyl}\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) e^{ikW(x_3, y_3)} + (1 - t_{plate}) \frac{d_2^2 \pi}{4} \text{Somb}\left(\frac{d_2 \sqrt{x_3^2 + y_3^2}}{\lambda z_3}\right) ** \text{cyl}\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) e^{ikW(x_3, y_3)} \right|^2 \quad (3.9)$$

which is simply the square modulus of the electric field at the camera.

In the limit that the PDI pinhole diameter approaches zero, the sombrero function is approximated by a constant while the reference wave becomes a smooth, continuous function. For relatively slowly varying aberration functions $W(x_3, y_3)$ the reference wave becomes spherical (Mercer, 1996).

POLARIZATION PHASE-SHIFTING POINT-DIFFRACTION INTERFEROMETER

The Polarization Phase-Shifting Point-Diffraction Interferometer (PPSPDI) is a modification of the conventional PDI utilizing polarization changes in the incident beam to induce a phase-shift. The PPSPDI retains the common-path design and advantages of the conventional PDI while the novel PDI filter allows for phase shifting and increased accuracy in phase measurement. The difference in the design lies in the construction of the PDI filter. In the conventional PDI, the filter is a partially transmitting pinhole plate, but the PDI filter in the PPSPDI is a pinhole etched into a thin film half-wave plate. This chapter discusses the operation of the PPSPDI qualitatively, and through both scalar diffraction theory and Jones calculus.

QUALITATIVE DESCRIPTION OF PPSPDI

The laboratory experiment is illustrated in Fig. 4-1.

Fig 4-1: Operation of the PPSPDI

Light from the laser operating at $\lambda=632.8\text{nm}$ passes through the combination of a polarizer and half-wave plate. The polarizer oriented at 0° is used to isolate a single,

linear polarization state while the half-wave plate at 22.5° rotates the polarization state so its output is linearly polarized at 45° . The polarizer and half-wave plate combination ensures selection of linear polarization at any desired angle by a rotation of the half-wave plate. Following this combination is an electrooptic modulator with electrodes oriented vertically (0°), that is used as the phase-shifter for the interferometer. When a voltage is applied to the modulator, the index ellipsoid of the crystal inside the modulator rotates producing a change in the index of refraction of the crystal in the plane perpendicular to the electrodes. The linear input state at 45° is propagated through the modulator by decomposing it into its orthogonal horizontal and vertical states, p and s, which correspond to the eigenmodes of the modulator. In propagating through the crystal, the two orthogonal states will encounter a constant natural phase difference without an applied field due to the nature of the crystal and an electrically induced phase difference that increases linearly with the applied field. Because of these two facts, the horizontal component, p, encounters a larger optical path through the crystal and is given an extra phase, δ that is changed by varying the field applied to the modulator shown in Figure 4-1. Recombining the orthogonal components, the output of the modulator is in general elliptically polarized. Following the modulator is a spatial filter with a $5\mu\text{m}$ pinhole. The spatial filter acts as a point source and is positioned at twice the focal distance in front of the test lens to simulate a 4-f imaging system with a magnification of -1 . The spatial filter pinhole is small so that the size of the focus spot formed by the test lens is due solely to the diffraction limit of the lens plus aberrations. The test lens forms an aberrated focus spot on the PDI plate, located at twice the focal distance behind the test

lens. The pinhole plate is constructed by etching the pinhole through a birefringent silicon thin film half-wave plate with the fast axis oriented at 45° to the orthogonal components of the incident beam, s and p. As with the conventional PDI, the diameter of the pinhole created is approximately half the unaberrated Airy disk diameter of the test lens (Wyant 1979), or

$$\begin{aligned} d_{pinhole} &\approx (1.22\lambda) \frac{f}{\# \text{ working}_{test \text{ optic}}} \\ &\approx (1.22\lambda) \cdot 2 \frac{f}{\#_{test \text{ optic}}} \end{aligned} \quad (4.1)$$

where the working f-number of the test lens in a 4-f imaging system is twice the actual lens f-number. Again, the clear pinhole is aligned so it is coincident with the center of the focus spot formed by the test lens. The portion of light incident on the pinhole does not encounter the thin film half-wave plate and is diffracted into a spherical reference wavefront retaining the elliptical polarization state of the incident beam. The thin film half-wave plate transmits and attenuates the remainder of the light from the test lens, rotates the s and p orthogonal states by 90° , and the s and p states emerge orthogonal but flipped in orientation shown in Figure 4-2. This wavefront retains the aberrations of the incident wavefront and serves as the test wavefront for the interferometer.

Fig 4-2: Operation of the PPSPDI Plate

An analyzer with transmission axis horizontal placed after the PDI plate isolates one set of the orthogonal components from the test and reference beams. Moreover, the analyzer produces two interfering wavefronts, the test and reference wavefronts, with a variable phase difference between them. By varying the voltage in the electrooptic modulator, the phase difference between the test and reference wavefronts is varied causing phase shifting. Both wavefronts then pass through imaging optics which image the plane of test lens onto a rotating ground glass plate. Interference fringes are superimposed on the image of the test lens on the rotating plate. The rotating ground glass plate reduces the coherence of the system; thereby reducing the spurious fringes from the protective glass plate in front of the CCD chip in the camera. The camera optics then image the interference pattern produced on the ground glass plate onto the CCD.

OPERATION OF THE HALF-WAVE PLATE

The most important component in the PPSPDI is the thin film half-wave plate used to create the pinhole filter; however its operation shown in Figure 4-3 is not completely obvious.

Fig 4-3: Operation of the Thin Film Half-Wave Plate

The elliptically polarized light incident on the thin film is decomposed into horizontal and vertical orthogonal components, E_x and E_y , and the fast axis of the film is oriented at 45° . As the light is incident on the thin film, each of the two components is decomposed into the two eigenmodes of the film along the fast and slow axes.

$$\begin{aligned} E_x &= E_{xFA} + E_{xSA} \\ E_y &= E_{yFA} + E_{ySA} \end{aligned} \quad (4.2)$$

where subscript FA denotes the fast axis and SA the slow axis. In propagating through the film, the components along the slow axis encounter a 180° or $e^{i\pi}$ phase lag relative to the fast axis.

$$\begin{aligned}
E_x &= E_{xFA} + E_{xSA}e^{i\pi} = E_{xFA} - E_{xSA} \\
E_y &= E_{yFA} + E_{ySA}e^{i\pi} = E_{yFA} - E_{ySA}
\end{aligned}
\tag{4.3}$$

Upon exiting the half-wave plate thin film, the components along the fast and slow axes are recombined. From Fig 4-3, it is observed that

$$\begin{aligned}
E_{x \text{ Final}} &= E_{y \text{ Initial}} \\
E_{y \text{ Final}} &= E_{x \text{ Initial}}
\end{aligned}
\tag{4.4}$$

The incident horizontal and vertical orthogonal components are rotated by 90° in passing through the half-wave plate thin film.

The effect of the rotation of the orthogonal horizontal and vertical components upon passing through the half-wave thin film becomes important when observing both the test and reference beams in the interferometer. Assuming that the electrooptic modulator adds additional phase to the horizontal component, the reference beam retains the phase shift on the horizontal component. On the other hand, the test beam passes through the thin film half-wave plate, which both rotates its polarization state by 90° and causes the phase shift to be on the vertical component. This allows isolation of either the horizontal or vertical components of the two beams and production of phase shifting interferograms.

SCALAR DIFFRACTION THEORY REPRESENTATION

A theoretical model of the PPSPDI is derived from scalar diffraction theory in a similar manner to the conventional PDI in the previous chapter. This model shows the origin of both the test and reference beams as well as the origin of the phase shift; however,

misalignment of the PDI pinhole is again not presented. The PPSPDI schematic in Figure 4-4 indicates the planes used in the analysis.

Fig. 4-4: Schematic of PPSPDI for Scalar Diffraction Model

For the scalar diffraction model to be valid, four assumptions are made (Goodman 1996)

1. Assume the geometric lens law, $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, for the test optic is satisfied.
2. Neglect Petzval curvature across the image plane.
3. Neglect complex proportionality constants.
4. Place the PDI pinhole coincident with the center of the test optic focus spot.

In this model, the horizontal and vertical orthogonal components of the input polarization state are traced separately through the optical system. The electrooptic modulator produces a variable $e^{i\delta}$ additional phase in one of the components that this model assumes is the horizontal component, E_x . The optical field just behind the test lens for both polarization states is described by the lens pupil function

$$\begin{aligned}
U_x(x_1, y_1) &= E_x \text{cyl}\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)} e^{i\delta} \\
U_y(x_1, y_1) &= E_y \text{cyl}\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)}
\end{aligned} \tag{4.5}$$

where d_1 is the diameter of the test lens and $W(x_1, y_1)$ is the aberration function for the test lens. The cylinder function, $\text{cyl}\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right)$, represents a function equal to zero everywhere except for $\sqrt{x_1^2 + y_1^2} < d_1/2$ where the function equals one (Gaskill 1978). In propagating to the test lens focus and neglecting Petzval curvature at the focus plane, the impulse response of the test lens Fourier transforms the lens pupil function for each of the orthogonal components (Goodman 1996). The optical field just prior to the PDI plate is represented by the Fourier transform of the test lens pupil function

$$\begin{aligned}
U_x^-(x_2, y_2) &= \mathfrak{F}\left\{E_x \text{cyl}\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)} e^{i\delta}\right\} \\
U_y^-(x_2, y_2) &= \mathfrak{F}\left\{E_y \text{cyl}\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)}\right\}
\end{aligned} \tag{4.6}$$

The optical field is then multiplied by the transmittance function of the PDI plate given by

$$\begin{aligned}
\tau_{PDI, x} &= t_{plate, FA} + t_{plate, SA} e^{i\pi} + (1 - t_{plate, x}) \text{cyl}\left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2}\right) \\
\tau_{PDI, y} &= t_{plate, FA} + t_{plate, SA} e^{i\pi} + (1 - t_{plate, y}) \text{cyl}\left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2}\right)
\end{aligned} \tag{4.7}$$

where d_2 is the diameter of the pinhole and t_{plate} is the transmittance of the film for light polarized in the specified direction. The fast and slow axes of the thin film half-wave plate used to create the pinhole filter are oriented at 45° to the x and y orthogonal components. In propagation through the PDI filter, the horizontal and vertical polarization states must be further decomposed into components along the fast and slow axes. After propagating through the film, the individual components along the fast and slow axes are recombined. In propagating through the PDI plate, the optical field becomes

$$\begin{aligned}
 U_x(x_2, y_2) = & t_{plate,FA} \frac{E_x}{\cos(45^\circ)} \Im\left\{cyl\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)} e^{i\delta}\right\} \\
 & + t_{plate,SA} \frac{E_x}{\cos(45^\circ)} \Im\left\{cyl\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)} e^{i\delta}\right\} e^{i\pi} \\
 & + (1 - t_{plate,x}) cyl\left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2}\right) \Im\left\{E_x cyl\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)} e^{i\delta}\right\} \\
 U_y(x_2, y_2) = & t_{plate,FA} \frac{E_y}{\cos(45^\circ)} \Im\left\{cyl\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)}\right\} \\
 & + t_{plate,SA} \frac{E_y}{\cos(45^\circ)} \Im\left\{cyl\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)}\right\} e^{i\pi} \\
 & + (1 - t_{plate,y}) cyl\left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2}\right) \Im\left\{E_y cyl\left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1}\right) e^{ikW(x_1, y_1)}\right\}
 \end{aligned} \tag{4.8}$$

The net result of the thin film half-wave plate is a rotation of the orthogonal components by 90° giving a final result for the optical field after propagating through the PDI plate of

$$\begin{aligned}
U_x^+(x_2, y_2) &= t_{plate,y} \Im \{ E_y \text{cyl} \left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1} \right) e^{ikW(x_1, y_1)} \} \\
&\quad + (1 - t_{plate,x}) \text{cyl} \left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2} \right) \Im \{ E_x \text{cyl} \left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1} \right) e^{ikW(x_1, y_1)} e^{i\delta} \} \\
U_y^+(x_2, y_2) &= t_{plate,x} \Im \{ E_x \text{cyl} \left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1} \right) e^{ikW(x_1, y_1)} e^{i\delta} \} \\
&\quad + (1 - t_{plate,y}) \text{cyl} \left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2} \right) \Im \{ E_y \text{cyl} \left(\frac{\sqrt{x_1^2 + y_1^2}}{d_1} \right) e^{ikW(x_1, y_1)} \}
\end{aligned} \tag{4.9}$$

An analyzer with transmission axis either horizontal or vertical placed after the PDI plate isolates one set of orthogonal components exiting the PDI plate. Imaging optics then image the plane of the test lens onto a rotating ground glass plate. The image on the ground glass plate is a scaled and possibly magnified copy of the wavefront at the test lens, modified by the PDI plate and analyzer and then convolved with the impulse response of the imaging lens. In the limit that the imaging lens diameter is very large, the impulse response becomes essentially a delta function that samples but does not smooth the incoming wavefront. The imaging lens has the effect of Fourier transforming the field just after the PDI plate and analyzer combination. Neglecting Petzval curvature across the image and a complex proportionally constant, the field at the ground glass plate is

$$\begin{aligned}
U(x_3, y_3) &= T_{PDI+Analyzer}^{**} FT \{ U^-(x_2, y_2) \} \\
&= T_{PDI+Analyzer}^{**} U(x_1, y_1)
\end{aligned} \tag{4.10}$$

where T_{PDI} is the Fourier transform of the transmittance function of the PDI plate and the effect of the transmittance function of the analyzer is to allow transmission of only one

orthogonal component of the field. Assuming that the analyzer allows transmission of only the horizontally polarized component, the field at the rotating ground glass plate is

$$U_x(x_3, y_3) = t_{plate,y} \text{cyl}\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) e^{ikW(x_3, y_3)} + (1 - t_{plate,x}) \frac{d_2^2 \pi}{4} \text{Somb}\left(\frac{d_2 \sqrt{x_3^2 + y_3^2}}{\lambda z_3}\right) ** \text{cyl}\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) e^{ikW(x_3, y_3)} e^{i\delta} \quad (4.11)$$

where $\text{Somb}\left(\frac{\sqrt{x^2 + y^2}}{d}\right) = 2J_1\left(\frac{\pi \sqrt{x^2 + y^2}}{d}\right) / \left(\frac{\pi \sqrt{x^2 + y^2}}{d}\right)$ where d is a constant and $J_1(x)$

is a first order Bessel function. This equation shows that the interferogram is formed by the coherent addition of an attenuated aberrated test wavefront and a reference wavefront formed by the convolution of the aberrated wavefront with the Fourier transform of the pinhole in the PDI plate. The test and reference beams are the first and second terms in the above equation respectively with the $e^{i\delta}$ variable phase shift term in the reference beam. The test beam is an attenuated copy of the original test lens pupil function while the reference beam is a significantly smoothed version of the same test lens pupil function. The CCD camera images the interferogram produced at the rotating ground glass plate. Since intensity is the true quantity measured by the camera, the intensity at the ground glass plate is

$$I(x_3, y_3) = \left| t_{plate,y} \text{cyl}\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) e^{ikW(x_3, y_3)} + (1 - t_{plate,x}) \frac{d_2^2 \pi}{4} \text{Somb}\left(\frac{d_2 \sqrt{x_3^2 + y_3^2}}{\lambda z_3}\right) ** \text{cyl}\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) e^{ikW(x_3, y_3)} e^{i\delta} \right|^2 \quad (4.12)$$

which is the square modulus of the field at the ground glass plate.

As before, in the limit that the PDI pinhole diameter approaches zero, the sombrero function is approximated by a constant, and the reference wave becomes a smooth, continuous function. For relatively slowly varying functions $W(x_3, y_3)$ the reference wave becomes spherical (Mercer 1996). The effect of pinhole errors is presented in a later chapter.

JONES MATRIX REPRESENTATION

One can model the PPSPDI using Jones matrices. For completely polarized systems such as the PPSPDI, the system elements are represented by 2x2 matrices with the incident field represented by a 2x1 matrix. Propagation of the beams is accomplished by multiplying the incident field by the matrix for each element in the system as shown below

$$\begin{bmatrix} E_x \text{ Output} \\ E_y \text{ Output} \end{bmatrix} = \begin{bmatrix} \text{Final Element} \end{bmatrix} \dots \begin{bmatrix} 2^{nd} \text{ Element} \end{bmatrix} \begin{bmatrix} 1^{st} \text{ Element} \end{bmatrix} \begin{bmatrix} E_x \text{ Input} \\ E_y \text{ Input} \end{bmatrix} \quad (4.13)$$

The Jones matrices necessary to simulate the PPSPDI are given in table 4-1.

Optical Element or Polarization State	Jones Matrix
Linear Polarization State at +45°	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
Horizontal Linear Polarizer	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Vertical Linear Polarizer	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Half Wave Plate Fast Axis Vertical	$\begin{bmatrix} e^{i\pi} & 0 \\ 0 & 1 \end{bmatrix}$
Rotation Matrix	$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$
Arbitrary Retarder of Retardance ϕ Fast Axis Vertical	$\begin{bmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{bmatrix}$
Test Surface	$\begin{bmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{bmatrix}$
Electrooptic Modulator with Retardance On the Vertical Component	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix}$

Table 4-1: Jones Matrices for Modeling the PPSPDI

For the Jones matrix model to be valid, the test and reference beams must propagate through the system separately. Both the test and reference beams begin as a single linearly polarized beam at 45° passing through the electrooptic modulator that adds a temporal phase to the vertical orthogonal component.

$$\begin{aligned}
 \text{Test} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
 \text{Reference} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
 \end{aligned} \tag{4.14}$$

The beams then pass through the test optic, which adds a spatial phase, Δ , to both components. The spatial phase added to the test and reference beams is directly related to the optical path difference or surface error on the test optic. The test and reference beams are represented by

$$\begin{aligned}\text{Test} &= \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \text{Reference} &= \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\end{aligned}\quad (4.15)$$

Both beams are then focused onto the PDI plate. The test beam passes through the half wave plate thin film while the reference beam passes through the pinhole. The reference beam in passing through the pinhole loses the spatial phase associated with the test optic but retains the temporal phase from the electrooptic modulator. So, only the test beam is multiplied by the matrix associated with the half wave plate while the reference beam loses the matrix associated with the test optic.

$$\begin{aligned}\text{Test} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \text{Reference} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\end{aligned}\quad (4.16)$$

Finally, both the test and reference beams pass through an analyzer with its transmission axis either horizontal or vertical. If the transmission axis is assumed to be horizontal, the test and reference beams become

$$\begin{aligned}
 \text{Test} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
 \text{Reference} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
 \end{aligned} \tag{4.17}$$

Multiplied out the test and reference beams become

$$\begin{aligned}
 \text{Test} &= \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i(\delta+\Delta)} \\ 0 \end{bmatrix} \\
 \text{Reference} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}
 \end{aligned} \tag{4.18}$$

The total amplitude in the interferogram is simply the addition of the test and reference beams.

$$\text{Total Amplitude} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 + e^{i(\delta+\Delta)} \\ 0 \end{bmatrix} \tag{4.19}$$

The total intensity in the interferogram measured at the camera is the square modulus of the amplitude

$$\text{Total Intensity} = 1 + \cos(\delta + \Delta) \tag{4.20}$$

where Δ is the phase associated with the optical path difference in the test optic and δ is the variable phase shift from the electrooptic modulator. The phase from the test optic, Δ , is ideally determined by phase shifting the above intensity and entering the values into a phase shifting algorithm. Using a four step algorithm

$$\begin{aligned}
I_1 &= 1 + \cos(\Delta) & \delta &= 0 \\
I_2 &= 1 - \sin(\Delta) & \delta &= \frac{\pi}{2} \\
I_3 &= 1 - \cos(\Delta) & \delta &= \pi \\
I_4 &= 1 + \sin(\Delta) & \delta &= \frac{3\pi}{2}
\end{aligned} \tag{4.21}$$

$$\begin{aligned}
\text{Phase} &= \text{ArcTan} \frac{I_4 - I_2}{I_1 - I_3} \\
&= \text{ArcTan} \frac{1 + \sin(\Delta) - (1 - \sin(\Delta))}{1 + \cos(\Delta) - (1 - \cos(\Delta))} \\
&= \text{ArcTan}(\tan(\Delta)) \\
&= \Delta
\end{aligned} \tag{4.22}$$

So, by varying the phase produced by the electrooptic modulator, the surface phase of the test optic is determined from the resulting interferograms as expected.

BIREFRINGENT THIN FILMS AND PINHOLE CONSTRUCTION

The most important component in the PPSPDI is the novel pinhole filter that is constructed by etching a pinhole into a half-wave plate birefringent thin film. This chapter begins by discussing the deposition and properties of the novel birefringent thin films. The second half of the chapter covers techniques used to etch the films and the successes or failures of each.

BIREFRINGENT THIN FILM DEPOSITION AND PROPERTIES

The pinhole filter in the PPSPDI is created from a birefringent silicon thin film with a biaxial index structure. The locations of the three orthogonal principle dielectric axes and associated indices of refraction are fixed by the deposition geometry and symmetry.

Birefringent thin films are deposited in much the same way as isotropic films. In isotropic films, the evaporant material is heated in a vacuum with either an electron beam gun or heated coil, and the evaporant atoms travel from the source to the substrate where they condense. The substrate, oriented at an angle θ to the evaporation source, is stationary and the film grows with a tilted columnar microstructure as shown in Figure 5-1. Limited mobility of the evaporant atoms along with self-shadowing cause the columnar structure growth. The condensing atoms are unable to move far enough to fill vacant positions in the shadow of existing material (Hodgkinson 1997).

Fig 5-1: Characteristic Tilted Columnar Structure of Thin Films Deposited at Oblique Incidence (Hodgkinson 1997)

In the case of the birefringent films, the substrate is not stationary during the deposition process. Evaporant atoms condense on the substrate shown in Figure 5-2, but after every few nanometers of deposition, the substrate is rotated by 180 degrees. A crystal sensor in the chamber monitors the deposition thickness to ensure proper rotation of the substrate. In-situ optical measurements of the films can be taken through windows in the vacuum chamber.

Fig. 5-2: Deposition Apparatus for Birefringent Films

Rotating the substrate causes the columnar microstructure to grow normal to the substrate illustrated in Figure 5-3.

Fig 5-3: Characteristic Normal Columnar Structure For Bi-Directional Deposited Birefringent Films (Hodgkinson 1997)

The material used for the films is silicon that is not birefringent in typical oblique deposition. The birefringence in the thin films is caused by the normal columnar microstructure. This birefringence due to the structure of the film as opposed to the material itself is termed form birefringence. For form birefringence, the columns are much thicker or bunched preferentially in the direction perpendicular to the deposition plane, and typically both effects are present. This form birefringence depends on both the column shape and packing density of the columns to cause direction-related variations in the refractive index. This deposition geometry causing the columnar structure to grow normal to the substrate, sets the orientations of the three principle dielectric axes: normal to the substrate, perpendicular to the deposition plane, and parallel to the deposition plane (Hodgkinson 1997). The direction perpendicular to the deposition plane, which has a greater packing density of columns, has a large index of refraction, n_e , and is considered the slow axis for the retarder. The direction parallel to the deposition plane has a lower index of refraction, n_o , and is the fast axis for the retarder. The relationship describing the retardance of the film in waves for a given wavelength λ is

$$R = (n_e - n_o) \frac{t}{\lambda} = \Delta n \frac{t}{\lambda} \quad (5.1)$$

where the thickness of the film, t , determines the retardance. For a half-wave plate this gives a film thickness of

$$t = \frac{\lambda}{2\Delta n} \quad (5.2)$$

The index difference, Δn , is not constant with wavelength and is given by a dispersion equation unique for each film material. Δn is a function of wavelength and for the silicon films is

$$\Delta n_{\lambda} = \Delta n_{633} \left(1 + c \left(\frac{1}{\lambda^2} - \frac{1}{633^2} \right) \right) \quad (5.3)$$

where c was experimentally determined as -268332 and Δn_{633} is .2816 (Hodgkinson 2001). This gives a characteristic dispersion curve that increases with wavelength given in Figure 5-4.

Fig 5-4: Dispersion Relation For Silicon Birefringent Thin Films

Consequently, one may determine the index difference between the fast and slow axes for any wavelength and thus the total retardance at any wavelength for any known film thickness. Now, one may achieve the desired retardance by depositing a film of a specific, required thickness.

The retardance of the wave plate is monitored in-situ as the film is deposited using perpendicular incidence ellipsometry (Hodgkinson 1997). This was the method used when the half-wave films for the PPSPDI were deposited. The light beam is circularly polarized when it reaches the film shown in Figure 5-5. After passing through the sample, the beam encounters a rotating polarizer and a spectral bandpass filter before being detected by a photomultiplier. The signal detected at the photomultiplier oscillates as a function of two variables: the angle between the axis of the rotating polarizer and the p polarized portion of the light leaving the sample, ξ , and the total phase retardance of the film including interference, Δ . From this signal, Δ is determined while the film is deposited. Once the correct total phase difference is reached, deposition is stopped.

Fig 5-5: Perpendicular Incidence Ellipsometry

In a similar manner, the principle refractive indices are measured using two ellipsometers in transmission mode at different angles in the deposition plane as the film is deposited. This is a significantly more difficult process, as the gradient of the smoothed retardation versus thickness plot is necessary to eliminate the effects of interference (Hodgkinson 1998). Each ellipsometer provides phase retardation versus thickness information as the

film is being deposited, but at different angles. Two different angles are required to allow two independent phase retardation versus thickness measurements to be taken. The smoothed versions of these plots give differences in the indices of refraction for the different ellipsometer angles. With two independent values at two different angles, the principle indices of refraction parallel and perpendicular to the deposition plane are determined. The modulation in the curves due to interference effects, removed for determination of the first two indices, allows the determination of the third principle index of refraction perpendicular to the surface of the film. The inherent problem with this measurement is that one must determine the angle of the columnar microstructures in order to calculate the indices. The tangent rule

$$\tan(\Psi) = \frac{1}{2} \tan(\theta) \quad (5.4)$$

which approximates the angle of the column structures, Ψ , based on the deposition angle, θ , is often unreliable for biaxial films so one must determine the actual angle using an SEM after deposition is completed. Having to determine the column angle after deposition defeats the purpose of the in-situ measurement.

ARGON ION MILLING

The first method used to etch the pinhole in the films was argon ion milling that uses a photolithographic process to etch features into the film illustrated in Figure 5-6.

Fig 5-6: Photolithographic Process for Argon Ion Milling

The process begins by spin coating the thin film with a layer of Shipley 1813 photoresist. The thin film is coated by spinning the sample at 500rpm for 5 seconds and then 5000rpm for 30 seconds yielding a uniform photoresist coating. The thickness of the coating is unimportant as long as it can withstand the etching process. The photoresist coating is baked for 3 minutes at 100°C in an oven to harden the photoresist. Using a mask aligner, a chrome-on-glass contact mask is used to expose the pinhole pattern into the photoresist. The sample is then submerged in developer for approximately 30 seconds that removes the exposed photoresist creating the pinhole pattern in the unexposed photoresist. After rinsing in deionized water to remove the developer, the photoresist is again baked in an oven at 100°C for 5 minutes to harden the photoresist and remove any remaining water before the etching process. This remaining photoresist serves as a shield for the film under it during the etching process.

In an argon ion mill, the sample with photoresist mask is placed on the cathode of the system inside a vacuum chamber pictured in Figure 5-7. The anode of the system consists of an argon ion gun and a high voltage bias grid that provides additional acceleration to the ions. Argon ions are produced by the ion gun and initially accelerated by the bias grid. Upon leaving the anode the ions are further accelerated due to the bias applied between the anode and cathode. The ions travel to the cathode and strike the sample with very high momentum causing atoms of the sample to dislodge. This bombardment of the sample with high velocity argon ions is the physical etch process used to create the pinhole features. Because of the highly directional nature of the ions in the electric field, the etch is also highly directional leading to very accurate feature profiles (Hakola 2003). The photoresist mask must withstand more ion bombardment than the thin film for the etch process to work correctly. Only the portion of the film not protected by the photoresist is etched by the argon ions, the remaining film is left unaffected. Once the film is etched, the remaining photoresist is removed by rinsing the sample with acetone leaving the bare film with etched features.

Fig 5-7: Argon Ion Mill

The biggest problem in using argon ion milling is heat. As the ions are accelerated, they become very hot. Consequently, as the ions strike the sample, not only do they etch the surface but heat it as well. Table 5-1 lists the minimum etching parameters for proper ion mill to operation. In turn, these parameters are used to minimize the heating of the sample. Even with the minimum energy, this heating of the sample caused the photoresist to burn long before the film etched, therefore, argon ion milling is not a viable option for etching the pinhole.

Parameter	Value
RF Power	250 W
Beam Current	160 mA
Beam Voltage	250 V
Acceleration Voltage	300V
Neutralization Current	175 mA
Etch Source Gas	10.0 sccm
FBN1 Gas	5.0 sccm
Chamber Gas	0.0 sccm
Arm Angle	0.0 deg
Platen Rotation Speed	20.0 rpm

Table 5-1: Argon Ion Mill Parameters

REACTIVE ION BEAM ETCHING

The second etching method attempted was reactive ion beam etching. Unlike argon ion milling, this process is both chemical and physical. The etchant is chosen such that its chemical reaction with the sample surface produces a volatile species. The etchant strikes the sample, chemically reacts with the surface and by forming a volatile compound removes a small portion of the surface of the sample.

Reactive ion beam etching follows a very similar process to argon ion milling in the preparation of the sample. A layer of photoresist is spun onto the film and the pinhole pattern is created in the photoresist using the same contact mask photolithographic process.

Fig 5-8: Reactive Ion Beam Etch Process

Once the mask is patterned onto the sample, the sample is placed on one of the electrodes of the reactive ion etcher shown in Figure 5-8. Next, the chamber is pumped to form a vacuum and a small concentration of etchant gas is inserted into the system. For the silicon films, the etchant was chosen to be tetrafluoromethane, CF_4 . As an electric field is applied across the electrodes, a plasma is formed as electrons ejected from the negative electrode start the process of breaking the tetrafluoromethane into free radicals

A voltage bias develops between the plasma formed by the free radicals and the electrodes causing the free radicals to move toward the electrode with the film to be

etched. Striking the film, the fluorine radicals react with the silicon producing the volatile compound

that comes off the surface removing a molecule of silicon (Akinwande 2003). The surface is etched essentially molecule by molecule. Since tetrafluoromethane was chosen to etch silicon specifically, the photoresist is left untouched, accordingly, only the film not protected by the photoresist mask is etched. Once the etching is completed the photoresist is removed by washing the sample in acetone leaving only the film with the pinhole etched in it.

Reactive ion beam etching using tetrafluoromethane will etch the silicon films but not with the required feature profiles. Because the process is chemical as well as physical, etching occurs fast in all directions. The etching process tends to undercut the mask leaving a sloping sidewall to the feature instead of a vertical feature profile. Because of the undercutting of the mask, reactive ion beam etching will not produce features with walls vertical enough for optimal operation of the PPSPDI. Figure 5-9 gives the profile of the features produced during a partial etch that have walls with an angle of about 60 degrees from vertical. With a full etch depth of 1 micron, the walls have an angle of about 45 degrees. The profile in the figure was taken using a WYKO NT2000 interference microscope on a sample that was etched with tetrafluoromethane for 70 minutes. This angled profile will produce a pinhole with a tapered bore, that will introduce wavefront errors of more than .01 waves into the reference beam for only a 20 degree taper when

used in a PDI (Goldberg 1996). Because of the angled profile, reactive ion beam etching is not a viable option for etching the pinholes for the PPSPDI.

Fig 5-9: Edge Profile of Reactive Ion Beam Etched Thin Film

FOCUSED ION BEAM ETCHING

Focused ion beam etching was the third method used to etch the pinhole in the silicon thin film. A focused ion beam etcher operates similar to a scanning electron microscope since both instruments take charged particles from a source, focus them into a beam, and then scan across small areas of a sample. A focused ion beam etcher differs from an electron microscope by using gallium ions to form its beam, instead of electrons. Since gallium ions are orders of magnitude more massive than electrons, a FIBE's ion beam mills the sample surface instead of imaging it (Street 2002). This etching method provides a way for a maskless and resistless pinhole etch process.

The FIBE uses a very sharp tungsten needle wetted with gallium. The tip of the needle is subjected to a high voltage causing the ejection of gallium ions and acceleration toward the sample. The gallium ions are focused by electromagnetic fields into a highly focused beam and steered to a specific spot on the sample (Freidrich 2002). The kinetic energy of the ions as they strike the sample causes the ejection of atoms from the sample through a sputtering process illustrated in Figure 5-10. A wide variety of shapes are possible by varying the scan rate, pattern and energy of the ion beam as well as the dwell time of the beam at any given spot.

Fig 5-10: Focused Ion Beam Etching

The focused ion beam etching of the silicon films was done by Integrated Reliability Corporation of San Diego, California. Because the FIBE uses a highly focused beam of

ions, there is no need for the photoresist mask used in the previous two etch methods. The desired etch pattern is programmed into the computer controlling the FIBE that positions and scans the ion source to etch the features while leaving the remainder of the film untouched. The FEI FIB 200 system used has a lateral ion beam resolution of 12-nm, that for a 5-micron pinhole gives a maximum etch error of .25%. It is possible to etch the feature profile to within 3 degrees of vertical, a marked improvement over 45-degree profile achieved with the RIBE method. An integrated high magnification microscope was used to image the etched pinhole pictured in Figure 5-11.

Fig 5-11: Focused Ion Beam Etched
Pinhole in Silicon Thin Film

Focused ion beam etching produces the best result of all etch methods attempted and is the best choice as the process to create the pinholes for the PPSPDI. While requiring the fewest steps to complete, it produced features with very high lateral resolution and near vertical edge profiles.

ERROR ANALYSIS OF THE PPSPDI

In this chapter, mathematical models are developed to investigate the systematic errors in the PPSPDI. Because the test and reference beams are separated by polarization state, the PPSPDI is exceptionally sensitive to errors in the alignments and retardances of its various polarization optics. The three most important of these are the retardance of the thin film used to construct the pinhole filter, angle of the fast axis of the film, and the angle of the final analyzer used to obtain the interference between the test and reference wavefronts. In turn, these errors cause discrepancies in the phase shifts, intensity modulations between phase steps and error in the final measured phase.

NONIDEAL JONES MATRIX PROPAGATION

In Chapter 4, the ideal Jones matrix representation for the PPSPDI was presented. This model assumed that the film used to create the pinhole plate is a perfect half-wave plate oriented at exactly 45° and the transmission axis of the final analyzer was precisely horizontal. Unfortunately, this is not the case since not only is the thin film not a half-wave plate, but the diattenuation of the film is nonzero. The film used in the PPSPDI was measured on an Axometrics Muller Matrix Polarimeter and found to have a retardance of $160^\circ \pm .5^\circ$ and diattenuation of $20\% \pm .5\%$. As a result of the diattenuation, the film absorbs differently along the fast and slow axes in addition to the error in the retardance. It is also expected that the alignment of the film will not be exactly 45° and neither will the orientation of the transmission axis of the final analyzer be exactly

horizontal. Consequently, it is imperative to develop a new Jones matrix model that takes into account the possible error sources.

For new model validity, the test and reference beams must propagate through the system separately just as in the ideal Jones matrix case. Both the test and reference beams begin as a single linearly polarized beam at 45° passing through an electrooptic modulator that adds a temporal phase to the vertical orthogonal component. The mathematical equations are

$$\begin{aligned} \text{Test} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \text{Reference} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} A \\ A \end{pmatrix} \end{aligned} \quad (6.1)$$

where δ is the temporal phase shift introduced by the modulator and A is a beam balance constant determined by the properties of the pinhole filter. Next, the beams pass through the test optic, where a spatial phase, Δ , is added to both components. The spatial phase added to the test and reference beams is directly related to the optical path difference or surface error on the test optic. The test and reference beams are represented by

$$\begin{aligned} \text{Test} &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \text{Reference} &= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} A \\ A \end{pmatrix} \end{aligned} \quad (6.2)$$

Other than the beam balance factor, the two beams are identical up to this point. After the test optic, the beams are focused onto the PDI plate. The test beam passes through the thin film retarder with retardance ϕ and fast axis oriented at an angle θ from vertical

while the reference beam passes through the pinhole. The diattenuation in the thin film is 20%, meaning the slow axis of the film transmits 80% of the light transmitted by the fast axis. The reference beam in passing through the pinhole loses the spatial phase associated with the test optic but retains the temporal phase from the electrooptic modulator. Accordingly, only the test beam is multiplied by the matrices associated with the rotated thin film while the reference beam loses the matrix associated with the test optic. This is the point where the beam balance factor, A , becomes important. A is determined by the amount of light diffracted by the pinhole and absorption of the thin film, and it measures how much light is diffracted by the pinhole into the reference beam as opposed to transmitted by the film in the test beam. The closer the balance between the beams, the closer this value approaches unity. Experimentally, the transmission of the film along the fast axis is about 55%. For a moderately aberrated system, 25% of the incident light is assumed incident on the pinhole, which is half the Airy disk size and centered in the focus spot of the test optic. For these values, A was assigned a value of .6; therefore, the Jones matrix representation of the beams after the pinhole plate is

$$\begin{aligned} \text{Test} &= \frac{1}{\sqrt{2}} \underbrace{\begin{pmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix}}_{\text{Rotation Matrix } (-\theta)} \underbrace{\begin{pmatrix} .8e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}}_{\substack{\text{Retarder of} \\ \text{Retardance } \phi \text{ and} \\ \text{.8 Diattenuation}}} \underbrace{\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}}_{\text{Rotation Matrix } (\theta)} \begin{pmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \text{Reference} &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} .6 \\ .6 \end{pmatrix} \end{aligned} \quad (6.3)$$

Finally, the test and reference beams pass through an analyzer with transmission axis oriented at an angle ψ from horizontal. This gives the final Jones matrix representation of the test and reference beam amplitudes at the camera of

$$\begin{aligned}
\text{Test} &= \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\psi)^2 & \cos(\psi)\sin(\psi) \\ \cos(\psi)\sin(\psi) & \sin(\psi)^2 \end{pmatrix} \begin{pmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix} \begin{pmatrix} .8e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix} \\
&\quad \cdot \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\
\text{Reference} &= \frac{1}{\sqrt{2}} \begin{pmatrix} \cos(\psi)^2 & \cos(\psi)\sin(\psi) \\ \cos(\psi)\sin(\psi) & \sin(\psi)^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} .6 \\ .6 \end{pmatrix}
\end{aligned} \tag{6.4}$$

The four amplitude components from the test and reference beams are added together to give the total amplitude at the camera that is multiplied by its complex conjugate giving the total intensity. The general form of the intensity as a function of all five variables, δ , Δ , ϕ , θ and ψ , is too long and complex for written presentation, but is included in the appendix in the Mathematica program “Non-Ideal Jones Matrix.nb”. For each error analysis case presented, all variables except the one under analysis are set to their ideal values allowing an independent investigation of each error source.

RETARDANCE OF THE THIN FILM

As previously discussed, the thin film used to create the pinhole filter is not a half-wave plate. The error resulting from the wrong retardance is investigated by setting the angle of the retarder, θ , to -45° and the transmission axis of the final analyzer to horizontal. This gives a much simpler expression for the intensity at the camera as a function of δ , Δ and ϕ only

$$\begin{aligned}
\text{Intensity}(\delta, \Delta, \phi) &= .59 + .09\cos(\delta) + .03\cos(\Delta) + .03\cos(\delta + \Delta) \\
&\quad + .2\cos(\delta - \phi) - .2\cos(\delta + \phi) + .24\cos(\Delta + \phi) - .24\cos(\delta + \Delta + \phi)
\end{aligned} \tag{6.5}$$

Substituting values of 0° , 90° , 180° and 270° in for δ facilitates using the above intensity in a four-step phase-shifting algorithm and yields four phase-stepped intensities as a function of Δ and ϕ

$$\begin{aligned}
 I_1 &= .68 + .6\cos(\Delta) & \delta &= 0^\circ \\
 I_2 &= .59 + .3\cos(\Delta) + .24\cos(\Delta + \phi) - .3\sin(\Delta) & \delta &= 90^\circ \\
 &\quad + .4\sin(\phi) + .24\sin(\Delta + \phi) \\
 I_3 &= .5 + .48\cos(\Delta + \phi) & \delta &= 180^\circ \\
 I_4 &= .59 + .3\cos(\Delta) + .24\cos(\Delta + \phi) + .3\sin(\Delta) & \delta &= 270^\circ \\
 &\quad - .4\sin(\phi) - .24\sin(\Delta + \phi)
 \end{aligned} \tag{6.6}$$

Plots of the four intensities are shown in Figure 6-1 for $\phi = 180^\circ$ and illustrate the intensity modulations between phase steps solely due to the diattenuation of the films. Changing the value of ϕ will change both the heights of the four intensities and shifts the peaks relative to each other causing 90° phase steps to no longer shift the frames by 90° . The intensity modulations and phase-shift errors will cause errors when calculating the output phase from the four interferograms.

Fig 6-1: Four Phase-Stepped Intensities as a Function of Test Surface Phase

Using the four phase-shifted intensities given above in the four-step phase-shifting routine gives an equation for the output phase at the camera as a function of the retardance of the thin film and the surface phase from the test optic

$$\text{Output Phase} = \text{ArcTan} \left[\frac{.6\text{Sin}(\Delta) - .8\text{Sin}(\phi) - .48\text{Sin}(\Delta + \phi)}{.18 + .6\text{Cos}(\Delta) - .48\text{Cos}(\Delta + \phi)} \right] \quad (6.7)$$

In an ideal situation, the output phase is identical to the input phase from the test optic, Δ , as shown in Chapter 4; however, this is not the case. The error between the input phase from the test optic and the output phase at the camera is a function of both the retardance of the thin film and the input phase. To calculate the phase error, set ϕ to a value and vary Δ between 0 and 2π allowing for all possible phases from the test optic. The difference between the input phase and output phase for each value of Δ is the phase error, and the peak-to-valley and rms phase error are then determined for each value of ϕ . Since each input phase is used once, one may consider the peak-to-valley and rms errors to come from a uniform distribution of input phase and representative of actual data. This assumption is entirely reasonable since any interferogram with multiple fringes has a roughly uniform phase distribution between 0 and 2π . Figure 6-2 gives the peak-to-valley and rms phase error for ϕ ranging from 160° to 200° .

Fig 6-2: Peak-to-Valley and RMS Phase Error vs. Thin Film Retardance

With $\phi = 180^\circ$, the only source of error is the diattenuation of the film. With only the 20% diattenuation as a source of error, the peak-to-valley and rms phase error are .053 and .019 waves respectively. Even with a perfect half-wave thin film, the diattenuation

still limits the accuracy of the measured surface phase to approximately $\lambda/20$ peak-to-valley and $\lambda/50$ rms. The simulation is included in the appendix as the Mathematica program “Non-Ideal Jones Retardance.nb” and the IDL program “variableretardance.pro”.

ANGULAR ALIGNMENT OF THE THIN FILM

Angular alignment of the thin-film retarder used to create the pinhole is the second major source of error for the PPSPDI. Errors due to the wrong angular alignment of the film are determined by setting the film retardance to 180° and the transmission axis of the final analyzer to horizontal. This gives an intensity expression at the camera in terms of δ , Δ and θ

$$\begin{aligned}
 Intensity(\delta, \Delta, \theta) = & .18 - .48\cos(\Delta)\cos^2(\theta) + .32\cos^4(\theta) \\
 & - 1.08\cos(\delta + \Delta)\cos(\theta)\sin(\theta) + 1.44\cos(\delta)\cos^3(\theta)\sin(\theta) \\
 & + .6\cos(\Delta)\sin^2(\theta) + .82\cos^2(\theta)\sin^2(\theta) \\
 & - 1.8\cos(\delta)\cos(\theta)\sin^3(\theta) + .5\sin^4(\theta)
 \end{aligned} \tag{6.8}$$

Substituting values of 0° , 90° , 180° and 270° in for δ makes possible the use of the above intensity expression in a four-step phase-shifting algorithm that results in four phase-stepped intensities as a function of Δ and θ

$$\begin{aligned}
I_1 &= .59 - .09\cos(2\theta) - .18\cos(\theta)\sin(\theta) + .06\cos(\Delta) - .54\cos(\Delta)\cos(2\theta) - 1.08\cos(\Delta)\cos(\theta)\sin(\theta) + .405\sin(4\theta) & \delta=0^\circ \\
I_2 &= .59 + .06\cos(\Delta) - .09\cos(2\theta) - .54\cos(\Delta + 2\theta) & \delta=90^\circ \\
I_3 &= .59 - .09\cos(2\theta) + .18\cos(\theta)\sin(\theta) + .06\cos(\Delta) - .54\cos(\Delta)\cos(2\theta) + 1.08\cos(\Delta)\cos(\theta)\sin(\theta) - .405\sin(4\theta) & \delta=180^\circ \\
I_4 &= .59 + .06\cos(\Delta) - .09\cos(2\theta) - .54\cos(\Delta - 2\theta) & \delta=270^\circ
\end{aligned} \tag{6.9}$$

Using these intensities in a four-step phase-shifting routine gives an equation for the output phase at the camera as a function of the angle of the thin film, θ , and the surface phase from the test optic, Δ

$$\text{Output Phase} = \text{ArcTan} \left[\frac{.54\cos(\Delta + 2\theta) - .54\cos(\Delta - 2\theta)}{(-.36\cos(\theta)\sin(\theta) - 2.16\cos(\Delta)\cos(\theta)\sin(\theta) + .81\sin(4\theta))} \right] \tag{6.10}$$

The difference between the input phase from the test optic, Δ , and the output phase at the camera is a function of both the angular alignment of the film and the input phase. As in the previous case, θ is set to a value and Δ is varied from 0 to 2π allowing for all possible phases from the test optic to determine the phase error. Subtracting the input phase from the output phase allows one to calculate the peak-to-valley and rms phase error for each value of θ . Figure 6-3 gives the peak-to-valley and rms phase error for θ ranging from -37° to -47° .

Fig 6-3: Peak-to-Valley and RMS Phase Error vs. Thin Film Alignment Angle

The minimum peak-to-valley and rms phase errors of 1.7×10^{-4} and 6.07×10^{-5} waves were found at -41.8° rotation of the thin film. With an ideal film retardance of 180° , the effect of the diattenuation is corrected by rotating the thin film to -41.8° instead of -45° . The

simulation is included in the appendix as the Mathematica program “Non-Ideal Jones Angle.nb” and the IDL program “thetaerror180.pro”.

ALIGNMENT OF THE ANALYZER

Angular alignment of the transmission axis of the final analyzer is the last major source of error investigated. To determine the error due solely to the analyzer ϕ is set to 180° and aligned at $\theta = -45^\circ$. The only remaining source of error is the alignment of the transmission axis of the analyzer. The result is an intensity expression at the camera in terms of δ , Δ and ψ . The intensity expression is too long and complex to presented directly in the text but is included in the appendix in the program “Non-Ideal Jones Analyzer.nb”.

Substituting values of 0° , 90° , 180° and 270° for δ in the intensity expression permits using the intensity in a four-step phase-shifting algorithm giving four phase-stepped intensities as a function of Δ and ψ .

$$\begin{aligned}
I_1 &= 1.02 + .9\cos(\Delta) - .15\cos(\Delta - 4\psi) - .34\cos(4\psi) & \delta=0^\circ \\
&\quad - .15\cos(\Delta + 4\psi) - .6\sin(\Delta - 2\psi) \\
&\quad + 2.72\cos(\psi)\sin(\psi) + .6\sin(\Delta + 2\psi) \\
I_2 &= .635 + .33\cos(\Delta) - .27\cos(\Delta - 4\psi) - .045\cos(4\psi) & \delta=90^\circ \\
&\quad - .57\sin(\Delta - 2\psi) + 1.36\cos(\psi)\sin(\psi) \\
&\quad + .03\sin(\Delta + 2\psi) \\
I_3 &= .25 - .24\cos(\Delta) - .12\cos(\Delta - 4\psi) + .25\cos(4\psi) & \delta=180^\circ \\
&\quad - .12\cos(\Delta + 4\psi) \\
I_4 &= .635 + .33\cos(\Delta) - .045\cos(4\psi) - .27\cos(\Delta + 4\psi) & \delta=270^\circ \\
&\quad - .03\sin(\Delta - 2\psi) + 1.36\cos(\psi)\sin(\psi) \\
&\quad + .57\sin(\Delta + 2\psi)
\end{aligned} \tag{6.11}$$

Using these intensities in a four-step phase-shifting routine gives an equation for the output phase at the camera as a function of the alignment angle of the analyzer and the phase of the test optic

$$\text{Output Phase} = \text{ArcTan} \left[\frac{.27\cos(\Delta - 4\psi) - .27\cos(\Delta + 4\psi) + 1.08\cos(2\psi)\sin(\Delta)}{.77 + 1.14\cos(\Delta) - .03\cos(\Delta - 4\psi) - .59\cos(4\psi) - .03\cos(\Delta + 4\psi) - .6\sin(\Delta - 2\psi) + .6\sin(\Delta + 2\psi) + 2.72\cos(\psi)\sin(\psi)} \right] \tag{6.12}$$

The difference between the input phase, Δ , and the calculated output phase at the camera is a function of both the input phase and the angle of the transmission axis of the final analyzer in the system. Like the other two cases, ψ is set to a value and Δ is varied from 0 to 2π to determine the error for all possible phases from the test optic for each value of ψ . Subtracting the output phase from the input phase for all the Δ values allows

determination of the peak-to-valley and rms phase error for each value of ψ . Figure 6-4 gives the peak-to-valley and rms phase error for ψ ranging from 0° to -9.5° .

Fig 6-4: Peak-to-Valley and RMS Phase Error vs. Final Analyzer Angle

The minimum peak-to-valley and rms phase errors of .006 and .0003 waves are found at -4.4° . With a film retardance of 180° , rotating the transmission axis of the analyzer to -4.4° from horizontal can significantly reduce the effect of the diattenuation. The simulation is included in the appendix as the Mathematica program “Non-Ideal Jones Analyzer.nb” and the IDL program “analyzererror180.pro”.

The simulations show two possible ways of correcting the phase error due to the diattenuation in the films: rotate the film to -41.8° instead of -45° from vertical or rotate the transmission axis of the final analyzer to -4.4° from horizontal. While both methods work well at correcting the phase error due to the diattenuation, rotating the retarder to -41.8° provides the best result.

ACTUAL PPSPDI FILM WITH 160° RETARDANCE

The thin-film used in the PPSPDI was determined to have a retardance of 160° instead of 180° as desired for optimal system operation. To determine the error due only to this incorrect retardance, θ is set to -45° and the transmission axis of the final analyzer to horizontal. The only source of error outside of the diattenuation in the film is the incorrect retardance. Setting ϕ to 160° gives an intensity expression at the camera in terms of δ and Δ

$$\begin{aligned} \text{Intensity}(\delta, \Delta) = & .59 + .09\text{Cos}(\delta) + .074\text{Cos}(\Delta) + .525\text{Cos}(\delta + \Delta) \\ & + .137\text{Sin}(\delta) - .082\text{Sin}(\Delta) + .082\text{Sin}(\delta + \Delta) \end{aligned} \quad (6.13)$$

A value of $\phi = 160^\circ$ changes the heights of the four intensity values for $\delta = 0^\circ, 90^\circ, 180^\circ$ and 270° and shifts the peaks relative to each other compared with the first simulation on thin film retardance with $\phi = 180^\circ$. This shift of the intensity peaks causes the 90° phase steps to no longer shift the frames by 90° . Figure 6-5 shows the four intensity values as a function of Δ for $\delta = 0^\circ, 90^\circ, 180^\circ$ and 270° illustrating the intensity modulations between frames and the incorrect phase shifts.

Fig 6-5: Incorrect Phase Stepped Intensities
As a Function of Surface Phase for $\phi = 160^\circ$

For $\phi = 160^\circ$, the four phase steps should be $0^\circ, 107^\circ, 198^\circ$ and 271° for the four frames to all be 90° out of phase. Intellwave, the software used to control the PPSPDI, can easily accommodate the nonlinear phase steps. Intellwave applies a voltage ramp to the electrooptic modulator and watches the fringes to determine 90° changes in phase at the camera, regardless of the actual change in phase, δ , inside the modulator. At the same time, Intellwave is also able to overcome the intensity modulations between frames by contrast stretching the interferograms prior to calculating the phase. Figure 6-6 shows the

four phase-shifted intensities after the proper phase steps have been applied and the intensities have been contrast stretched.

Fig 6-6: Correctly Phase-Shifted Intensities with Contrast Stretching for $\phi = 160^\circ$

After setting the correct phase steps and contrast stretching, the result is four 90° phase stepped intensities that can be used in a four-step phase shifting routine.

$$\begin{aligned}
 I_1 &= .5 + .5\cos(\Delta) & \delta &= 0^\circ \\
 I_2 &= .5 + .0715\cos(\Delta) + .246\cos\left(\frac{107\pi}{180} + \Delta\right) - .264\sin(\Delta) & \delta &= 107^\circ \\
 I_3 &= .5 - .234\cos(\Delta) - .266\cos\left(\frac{179\pi}{90} + \Delta\right) + .0118\sin(\Delta) & \delta &= 198^\circ \\
 I_4 &= .5 + .0837\cos(\Delta) + .337\cos\left(\frac{271\pi}{180} + \Delta\right) & \delta &= 271^\circ \\
 & - .270\cos\left(\frac{431\pi}{180} + \Delta\right) - .0922\sin(\Delta)
 \end{aligned} \tag{6.14}$$

Using these four intensities in a phase shifting algorithm gives an equation for the output phase at the camera as a function of the input phase of the test optic

$$Output\ Phase = ArcTan \left[\frac{.0122Cos(\Delta) - .246Cos(\frac{107\pi}{180} + \Delta) + .337Cos(\frac{271\pi}{180} + \Delta) - .270Cos(\frac{431\pi}{180} + \Delta) + .172Sin(\Delta)}{.734Cos(\Delta) + .266Cos(\frac{179\pi}{90} + \Delta) - .0118Sin(\Delta)} \right] \quad (6.15)$$

The difference between the input phase from the test optic and the calculated output phase at the camera is a function of the input phase only. In this case, Δ is allowed to vary between 0 and 2π to allow for all possible phases from the test optic. Subtracting the output phase from the input phase allows the peak-to-valley and rms phase errors to be calculated. Figure 6-7 gives the difference between the input phase and the calculated output phase in waves for all input phase values between 0 and 2π .

Fig 6-7: Difference Between Input and Calculated Phase for $\phi = 160^\circ$ With Phase Step Correction and Contrast Stretching

This gives a peak-to-valley and rms phase error of 2.63×10^{-5} and 9.33×10^{-6} waves respectively. Allowing Intelliwave to set the correct phase steps and stretch the contrast of the four intensities gives almost ideal operation of the PPSPDI even with the incorrect thin film retardance. The simulation is included in the appendix as the Mathematica program “Non-Ideal Jones Retardance for 160.nb” and the IDL program “errorfor160plate.pro”.

It must be remembered that the simulation above is an ideal case. The simulation looks at only a single pixel with perfect phase steps and contrast stretching. The phase error from the actual operation of the PPSPDI will not be this small. The instrument is not able to phase shift as accurately nor contrast stretch as well as the simulation. With average phase shifts of around 91° and rms phase shift errors of about 2° across all the pixels in all frames, the PPSPDI operates with an rms phase error of .032 waves.

QUALITY OF THE REFERENCE WAVEFRONT

Just as any point-diffraction interferometer, the PPSPDI uses no reference optics; instead, it relies on diffraction from a pinhole to create a spherical reference wavefront. Ultimately, the quality of this reference wavefront will limit the accuracy of the PDI interferometer. Lawrence Livermore National Laboratory conducted research into the quality of the PDI's reference wavefront, but in each simulation the wavefront incident on the pinhole is assumed to be a uniformly illuminated plane wave (Goldberg 1996). This is simply not a valid assumption since the amplitude and phase incident on the pinhole are neither uniform nor planar and depend on the aberrations of the test optic. This chapter focuses on the quality of the reference wavefront for a PDI as a function of both incident aberrations from the test lens and pinhole size. Coma, spherical aberration and astigmatism are investigated for pinhole sizes ranging from 50% to 150% of the unaberrated Airy disk diameter of the test optic.

SIMULATION THEORY

This simulation begins in the same place as the scalar diffraction theory presented earlier, i.e. the test lens. In this case, the test lens pupil function is represented by a cylinder function multiplied by an exponential phase term to include aberrations.

$$U(x_p, y_p) = \text{cyl}\left(\frac{\sqrt{x_p^2 + y_p^2}}{d_1}\right) \cdot e^{i2\pi W(x_p, y_p)} \quad (7.1)$$

where $W(x_p, y_p)$ is the aberration function in units of waves with pupil coordinates x_p and y_p ranging from 0 at the center to 1 at the edge of the cylinder

$$W(x_p, y_p) = W_{11}y_p + W_{20}(x_p^2 + y_p^2) + W_{40}(x_p^2 + y_p^2)^2 + W_{31}(x_p^2 + y_p^2)y_p + W_{22}y_p^2 + W_{20}(x_p^2 + y_p^2) + W_{11}y_p \quad (7.2)$$

describing all first- and third-order aberrations with field dependence removed; thereby allowing the simulation to accommodate all possible aberrations or combination of aberrations. The optical field from the test lens is brought to focus at the PDI plate where the focus spot formed is the Fourier transform of the lens pupil function. For this simulation, the pinhole is assumed to have perfect transmission while the area outside the pinhole has zero transmission. This allows investigation of the reference wavefront independently of the test wavefront since the test wavefront is not transmitted by the area around the pinhole. The field is multiplied by the pinhole, represented by a second cylinder function with diameter d_2 , that has the effect of spatial filtering the field prior to the pinhole, removing the high frequencies and allowing transmission of only the lower frequencies.

$$U(x_2, y_2) = \mathfrak{F}\left\{cyl\left(\frac{\sqrt{x_2^2 + y_2^2}}{d_1}\right) \cdot e^{i2\pi W(x_2, y_2)}\right\} \cdot cyl\left(\frac{\sqrt{x_2^2 + y_2^2}}{d_2}\right) \quad (7.3)$$

After the pinhole, the optical field propagates to the far field, mathematically represented by a second Fourier transform of the field just after the pinhole. This is the same effect as using a lens to image the field onto a camera just like the actual operation of the interferometer. Neglecting multiplicative constants, the optical field at the image plane is

$$U(x_3, y_3) = \text{cyl}\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_1}\right) \cdot e^{i2\pi W(x_3, y_3)} ** \mathfrak{F}\left\{\text{cyl}\left(\frac{\sqrt{x_3^2 + y_3^2}}{d_2}\right)\right\} \quad (7.4)$$

where $**$ is a two-dimensional convolution across x and y . The propagation of the field from the test lens to the image plane is shown in Figure 7-1.

Fig 7-1: Simulated Propagation of Reference Wavefront

The simulation takes the optical field at the test lens including aberrations, Fourier transforms it, multiplies it by the pinhole and inverse Fourier transforms. The amplitude and phase of the optical field are recorded at each step.

There are two planes of interest in the simulation: the plane just prior to the pinhole and the final image plane. Determining the amplitude and phase of the optical field prior to the pinhole will verify under what conditions, if any, the current assumption of a uniform plane wave striking the pinhole is valid. The variation in phase across the final image plane gives the error in the reference wavefront. The larger the variations in the phase, the worse the quality of the reference wavefront for the PPSPDI.

In the simulation, aberration values ranging from 0 to 6 waves of spherical aberration, coma and astigmatism are applied to the test lens. In each case, the pinhole size is allowed to vary from 50% to 150% of the Airy disk diameter. Each aberration can then be analyzed separately as to its effect on the quality of the reference wavefront as a function of aberration magnitude and pinhole size. The simulation is included in the appendix in the IDL program “referencewavequality.pro”.

SPHERICAL ABERRATION

Spherical aberration is the first aberration used in the simulation, and with spherical aberration only, the lens pupil function becomes

$$U(x_p, y_p) = \text{cyl}\left(\frac{\sqrt{x_p^2 + y_p^2}}{d_1}\right) \cdot e^{i2\pi W_{40}(x_p^2 + y_p^2)^2} \quad (7.5)$$

where W_{40} is the number of waves of spherical aberration and the Fourier transform gives the optical field just prior to the pinhole. Figure 7-2 shows the amplitude and phase of the field in front of the pinhole for 2 waves of spherical aberration where the effect of the spherical aberration is radially symmetric so only the $y_p = 0$ profile is shown.

Fig 7-2: Amplitude and Phase at the Pinhole
For 2 Waves of Spherical Aberration

The red lines show the portions of the amplitude and phase incident on the pinhole with diameter equal to 50% of the Airy disk diameter. With this diameter and amount of

spherical aberration, there is very little variation in amplitude and phase across the pinhole so the assumption of an incident plane wave is valid. This assumption for the 50% pinhole does not hold true for all values of spherical aberration as the variations in phase and amplitude change as the amount of spherical aberration is varied. Likewise, as the pinhole size increases to 100% or 150% of the Airy disk diameter, shown by the blue and green lines respectively, the variations in phase and amplitude increase and the incident plane wave assumption is also no longer valid. While there are special cases where the plane wave assumption is valid, in general when the test wavefront incident on the pinhole contains even small amounts of spherical aberration the incident plane wave assumption is invalid regardless of pinhole size.

After multiplication by the pinhole, the optical field is Fourier transformed to simulate propagation to the final image plane. The peak-to-valley error of the phase of the optical field across the final image plane is calculated and gives the maximum error in the reference wavefront for the PPSPDI as a function of pinhole diameter and amount of spherical aberration. Figure 7-3 shows the reference wavefront error as a function of the amount of spherical aberration included in the simulation for pinhole diameters ranging from 50% to 150% of the unaberrated Airy disk diameter of the test optic.

Fig 7-3: Reference Wavefront Error vs. Spherical Aberration For Various Pinhole Sizes

While the wavefront error oscillates with increasing spherical aberration for all cases, the average value and amplitude of the oscillation changes with pinhole size. For larger pinholes, the wavefront error increases dramatically up to about 1 wave of added spherical aberration, then levels off approaching .5 waves at 6 waves of added spherical. There is a significant decrease in the amplitude of the oscillations with increasing incident spherical aberration. For smaller pinholes, there is an initial peak in the amplitude of the oscillation at .7 waves of added spherical aberration. Beyond this point, there is a decrease in both the amplitude and average value of the oscillation with increasing spherical aberration.

For a pinhole half the Airy disk diameter, the maximum wavefront error of .004 waves occurs with .7 waves of spherical aberration added to the incident wavefront. This is the point of maximum variation in the amplitude and phase incident on the pinhole shown by the red lines in Figure 7-4. For pinhole diameters of 75% and 100% of the Airy disk, given by the blue and green lines in Figure 7-4 respectively, the maximum wavefront errors increase to .033 and .188 waves at .7 waves of added spherical aberration. This is the point of maximum variation in amplitude and phase for these larger pinhole sizes as well. It is apparent that for .7 waves of incident spherical aberration the amplitude and phase across the pinhole for all diameters less than the Airy disk is neither uniform nor planar and again invalidates the assumption of a uniformly illuminated plane wave striking the pinhole.

Fig 7-4: Amplitude and Phase at the Pinhole
For .7 Waves of Spherical Aberration

Adding an equal but opposite amount of defocus to the spherical aberration to maximize the Strehl ratio reduces the reference wavefront error for pinholes up to the size of the test

lens Airy disk diameter. The addition of the defocus has the effect of minimizing the rms wavefront error at the pinhole. For a pinhole equal to the Airy disk diameter of the test lens, the maximum reference wavefront error is .188 waves at .7 waves of incident spherical aberration. By adding defocus, the maximum error in the reference wavefront is reduced to .046 waves at 2.5 waves of spherical aberration and -2.5 waves of defocus. This is a factor of four decrease in the reference wavefront error. For pinholes larger than the Airy disk diameter of the test lens, the effect of adding defocus to the incident wavefront is negligible.

As shown above, the larger the variations in amplitude and phase across the pinhole, the larger the error in the diffracted reference wavefront. While this is not an unexpected result, it does give insight into why it is a common practice to create the PDI pinhole where its diameter is no more than half the unaberrated Airy disk diameter of the test optic. The smaller the pinhole diameter, the smaller the variations in amplitude and phase across the pinhole are regardless of the incident aberrations. For a pinhole half the Airy disk diameter of the test lens, the variations in amplitude and phase caused by spherical aberration cause a maximum error in the reference wavefront of only .004 waves.

ASTIGMATISM

Astigmatism is the second aberration used in the simulation and looking only at astigmatism the lens pupil function becomes

$$U(x_p, y_p) = \text{cyl}\left(\frac{\sqrt{x_p^2 + y_p^2}}{d_1}\right) \cdot e^{i2\pi W_{22} y_p^2} \quad (7.6)$$

Where W_{22} is the number of waves of astigmatism added to the simulation. This function is Fourier transformed to give the optical field just prior to the pinhole. Figure 7-5 shows the amplitude and phase of the field in front of the pinhole for 2 waves of astigmatism. Although astigmatism has its maximum effect along the y-axis, both $x_p=0$ and $y_p=0$ profiles are shown as the effects on each axis is different.

Fig 7-5: X and Y Profiles of Amplitude and Phase at the Pinhole for 2 Waves of Astigmatism

The red lines show the portions of the amplitude and phase incident on a pinhole with diameter equal to 50% of the Airy disk diameter of the test optic. With this diameter and amount of astigmatism, the amplitude is not uniform along either the x or y profiles and only along the x profile is the phase approximated as planar. Once again, the immediate conclusion is the incident plane wave assumption is invalid. As the pinhole diameter increases to 100% or 150% of the Airy disk shown by the blue and green lines in Figure 7-5 respectively, even the possible assumption for the x profile phase is lost. In no way can one make the assumption of a plane wave incident on the pinhole for 2 waves of astigmatism regardless of the size of the pinhole. In fact, this holds true for all values of astigmatism and pinhole size.

After multiplication by the pinhole, the optical field is Fourier transformed to simulate propagation to the final image plane. The peak-to-valley error of the phase of the optical field across the final image plane is calculated and gives the maximum error in the reference wavefront for the PPSPDI as a function of pinhole diameter and amount of astigmatism. Figure 7-6 shows the reference wavefront error as a function of the amount of astigmatism included in the simulation for pinhole diameters ranging from 50% to 150% of the unaberrated Airy disk diameter of the test optic.

Fig 7-6: Reference Wavefront Error vs. Astigmatism
For Various Pinhole Sizes

In this case, there appears to be two distinct functional forms for the wavefront error for pinholes less than the Airy disk diameter and pinholes equal to or greater than the Airy disk diameter. While again the wavefront error oscillates with increasing aberration for all pinhole sizes, for pinholes equal to the Airy disk diameter and greater, the wavefront error increases sharply to about 1 wave of added astigmatism. Then the amplitude of the oscillation decreases with the average value of the wavefront error approaching .5 waves at 6 waves of added astigmatism. For the smaller pinholes, the functional form is very similar to the form found for spherical aberration with an initial peak around .7 waves of added astigmatism and then a decrease in both the amplitude and average value of the oscillation of the wavefront error with increasing astigmatism.

For a pinhole half the Airy disk diameter, the maximum reference wavefront error of .011 waves occurs with .7 waves of astigmatism added to the incident wavefront from the test optic. Like the case for spherical aberration, this is the point of maximum variation in the amplitude and phase incident on the pinhole shown by the red lines in Figure 7-7. For a pinhole diameter 75% of the Airy disk given by the blue lines in Figure 7-7, the maximum reference wavefront error increases to .09 waves for .7 waves of incident astigmatism.

Fig 7-7: X and Y Profiles of Amplitude and Phase at the Pinhole for .7 Waves of Astigmatism

With the sharp increase in error in the diffracted reference wavefront with increasing pinhole diameter, the importance of constructing the PDI pinhole with diameter no larger than half the Airy disk diameter of the test optic is apparent. While a maximum of .011 waves of error in the reference wavefront for the 50% Airy disk pinhole is not spectacular, it is still significantly better than the .09 waves for the 75% pinhole and .5 waves for pinholes larger than the Airy disk diameter.

COMA

Coma is the last aberration used in the simulation. With coma only, the lens pupil function becomes

$$U(x_p, y_p) = \text{cyl}\left(\frac{\sqrt{x_p^2 + y_p^2}}{d_1}\right) \cdot e^{i2\pi W_{31}(x_p^2 + y_p^2)y_p} \quad (7.7)$$

where W_{31} is the number of waves of coma added for the simulation. This function is Fourier transformed to give the optical field just prior to the pinhole. Figure 7-8 shows the amplitude and phase of the field in front of the pinhole for 2 waves of coma as an example. The effect of the added coma is not symmetric so both the $x_p=0$ and $y_p=0$ profiles are shown.

Fig 7-8: X and Y Profiles of Amplitude and Phase at the Pinhole for 2 Waves of Coma

The red lines show the portions of the amplitude and phase incident on the pinhole with diameter equal to 50% of the Airy disk diameter of the test optic. Even with two waves of coma in the incident wavefront, the phase across the 50% pinhole is planar. The variations in amplitude, especially across the y profile, keep the uniformly illuminated plane wave approximation from being valid. As the pinhole size increases to 100% and 150% of the Airy disk diameter, shown by the blue and green lines respectively, the variations in amplitude increase along both profiles. Under no circumstances of pinhole

size or incident amount of coma was the uniformly illuminated plane wave approximation valid.

After multiplication by the pinhole, the optical field is Fourier transformed to simulate propagation to the final image plane. The peak-to-valley error of the phase of the optical field across the final image plane is calculated and gives the maximum error in the diffracted reference wavefront for the PPSPDI as a function of pinhole diameter and amount of coma added to the simulation. Figure 7-9 shows the reference wavefront error as a function of the amount of coma included in the simulation for pinhole diameters ranging from 25% to 150% of the unaberrated Airy disk diameter of the test optic. The pinhole diameter is reduced to 25% of the Airy disk diameter to successfully moderate the errors induced by the added coma.

Fig 7-9: Reference Wavefront Error vs. Coma
For Various Pinhole Sizes

For coma there appears to be only a single functional form for all pinhole sizes. There is an initial increase in the error to a peak occurring from .7 waves of coma for the 25% pinhole to 1.2 waves for the 150% pinhole followed by the now expected oscillation of the wavefront error with increasing aberration. The amplitude of the oscillation decreases as the amount of added coma increases. Unlike the previous two cases, the average value of the oscillations does not change with increasing aberration but remains relatively constant. For a pinhole one quarter the Airy disk diameter, the maximum wavefront error in the diffracted wave is .026 waves at .7 waves of incident coma. For a pinhole half the Airy disk diameter, the maximum error increases to .14 waves at the same .7 waves of incident coma, the point of maximum variation in amplitude and phase

at the pinhole for these pinhole sizes. This error is significantly larger than was found for both spherical aberration and astigmatism for the same pinhole size.

From these results, the diffracted reference wavefront is much more susceptible to errors due to coma in the incident beam than to either spherical aberration or astigmatism. For both spherical aberration and astigmatism, the variations in amplitude and phase across the pinhole are symmetric. This is not the case with coma, as the shift in the central peak of the amplitude with increased aberration causes the amplitude variations across the pinhole to be nonsymmetric. This difference could account for the increased sensitivity of the reference wavefront to coma versus spherical aberration and astigmatism.

PPSPDI PERFORMANCE, LIMITATIONS AND IMPROVEMENTS

This chapter finally gets to the point. How well does the PPSPDI actually work? Performance is typically analyzed by examining both the accuracy and repeatability of the instrument, and this will be no different. The accuracy will be determined by comparing a measurement from the PPSPDI with a measurement from a commercial fizeau interferometer. Comparing these measurements will verify the output of the PPSPDI. By subtracting two measurements taken by the PPSPDI the repeatability of the interferometer can be determined. Finally the limitations of the PPSPDI are discussed with suggestions for possible improvements.

PPSPDI SETUP

Before the performance of the PPSPDI can be presented, the specifics of the actual measurements must be discussed. The PPSPDI was set up as previously shown in Figure 4-1 with the spatial filter positioned at twice the focal distance in front of the test lens to simulate a 4-f imaging system. The lens tested is a 42 mm diameter biconvex singlet with 200 mm focal length giving an f-number of 4.75. In the 4-f configuration, the lens is operating at an effective f-number of 9.5. Using a HeNe laser at $.6328\ \mu\text{m}$, the diffraction limited Airy disk formed at the rear focus of the lens has a diameter of $14.7\ \mu\text{m}$. The pinhole used in the PPSPDI was etched into a silicon thin film retarder with a retardance of 160° and 20% diattenuation. The pinhole diameter of $5\ \mu\text{m}$, 34% of the test lens Airy disk diameter, was used to minimize errors in the reference wavefront

improving the performance of the PPSPDI. Through its phase shift calibration routine, Intellwave was able to set the nonuniform phase shifts, discussed in the error analysis chapter, required to shift the frames by 90° . This is an automated process and the actual shift in phase between frames inside the electrooptic modulator is not known. The phase measurement of the test lens is done using the 5B Schmidt/Creath algorithm, which uses five frames to reduce errors due to incorrect phase shifts (Schmidt 1995). The five interferograms are shown in Figure 8-1.

Fig 8-1: Five Interferograms For F/4.75 Biconvex Lens

Since there are intensity modulations between the frames, once Inteliwave acquires the five frames, each frame is contrast stretched and normalized so that each frame has the same contrast before the phase information is calculated. The blue circles on the interferograms show the area of the lens tested. Only about the center 70%, or 34 mm, of the lens could be tested due to the combination of high frequency fringes near the edges of the interferograms and low contrast in interferogram #2. The combination of these two factors made obtaining phase data outside the blue circles impossible even with the software compensation for contrast.

MEASUREMENT ACCURACY

The accuracy of the PPSPDI is determined by comparing the results for the test lens using the PPSPDI to the results of the same lens obtained using a commercial Wyko 6000 phase-shifting fizeau interferometer. Figure 8-2 shows the measurements of the lens obtained by the two interferometers.

Measurement with the Wyko 6000 Phase-Shifting Fizeau Interferometer

Measurement with the Polarization Phase-Shifting Point-Diffraction Interferometer

Fig 8-2: Comparison of Measurements of the Test Lens Using a Wyko 6000 Phase-Shifting Fizeau Interferometer and the PPSPDI

From the above pictures, the general shape of the test lens compares well. Table 8-1 lists the seidel coefficients for spherical aberration, coma and astigmatism for the measurements on both instruments and the difference.

	Wyko 6000	PPSPDI	Difference
Spherical Aberration (waves)	-5.182	-5.189	.007
Coma (waves)	.363	.192	.171
Astigmatism (waves)	.329	.185	.144

Table 8-1: Comparison of Seidel Coefficients for Measurements on the Wyko 6000 Phase-Shifting Fizeau Interferometer and the PPSPDI

The coefficients for spherical aberration compare favorably. While the coefficients for coma and astigmatism do not compare as well, the differences can be attributed to misalignment and tilt of the measured portion of the test lens from one instrument to the other.

The difference in the astigmatism of .144 waves is caused by a difference in the tilt of the test lens between the two measurements. Assuming there is no tilt on the test lens during the PPSPDI measurement, the difference of .144 waves is due to a tilt in the test lens when the measurement was made on the Wyko interferometer. Using Zemax, the test lens was simulated in a 4-f imaging configuration and allowed to tilt. .144 waves of astigmatism is induced at a tilt angle of .61°.

Since the entire aperture of the lens was not tested, it is possible that the areas tested on the two instruments are slightly shifted. This would account for the difference in the coma values of .171 waves. The amount of misalignment necessary to induce this coma is determined by taking the difference of two shifted spherical aberration terms and set it equal to the coma difference.

$$W_{40}(x_p^2 + y_p^2)^2 - W_{40}((x_p + \Delta)^2 + y_p^2)^2 = \text{Coma Difference} \quad (8.1)$$

With x_p and y_p equal to one, Δ is the percentage of the measurement area radius that one of the measurements must be shifted to induce the coma. Substituting the values for W_{40} and the coma difference the equation becomes

$$-5.189 \cdot (1^2 + 1^2)^2 - (-5.189) \cdot ((1 + \Delta)^2 + 1)^2 = .171 \quad (8.2)$$

This equation gives a solution of .0041 or .41%. With a measurement area radius of 17 mm, the shift in the two areas necessary to induce the .171 waves of coma is 70 μm .

INTERFEROMETER REPEATABILITY

The most important requirement when determining the performance of an interferometer is the repeatability of the instrument. The repeatability gives an indication as to whether the interferometer results can be trusted from one measurement to the next. The repeatability is determined by subtracting two consecutive measurements. Figure 8-3 shows the subtraction of the two measurements.

Fig 8-3: Subtraction of Two Consecutive Measurements

The resulting rms error is .032 waves. This gives a system repeatability of just under $\lambda/30$. Careful inspection of the above plot shows that the peaks in the error occur in what appear to be concentric rings occurring at roughly double the frequency of the fringes in the original interferograms. The most obvious cause of this error in the repeatability is phase shift errors in the two measurements. The average phase shift across all frames in both measurements was found to be 90.85° with an rms error of 2.1° . A second possible cause for the error in the repeatability is the contrast stretching done to the original

interferograms before the surface phase is calculated. While stretching the contrast of the desired fringes, the contrast stretching also amplifies the noise in the interferograms. Since each interferogram is stretched by a different amount, the noise in each will be amplified by different amounts. Interferogram #2 is the most likely culprit for the repeatability error since it required the most contrast stretching and therefore ended up with the largest amplified noise.

LIMITATIONS AND IMPROVEMENTS

This final section will take a look at the limitations of the PPSPDI and suggest possible improvements to the problems encountered in operating the interferometer. The two biggest limitations in the operation of the PPSPDI are alignment error and the birefringent thin films used to create the pinhole filter.

Alignment error can be a big problem in operating the PPSPDI. Great care must be taken when aligning the spatial filter, test lens and pinhole plate to ensure that all three are aligned precisely along the optic axis of the interferometer. As was demonstrated in the previous section on instrument accuracy, even a small tilt in the test lens or shift in the measurement area produces significant artificial aberrations.

The most significant limitation in the PPSPDI is the birefringent thin film used to create the half-wave plate pinhole filter. The wrong retardance along with significant diattenuation caused major problems in getting the PPSPDI to operate correctly. The

fluctuations in intensity and nonuniform phase steps caused by the incorrect retardance and diattenuation taxed the abilities of both the operator and controlling software. The films cannot presently be deposited accurately enough nor are they environmentally stable enough for the PPSPDI to become a viable commercial instrument. Research must be done into improving the accuracy of the deposition process and environmental stability of the films. The only solution to reducing the diattenuation is to consider different materials for the thin films. Other materials have lower diattenuation, but the birefringence will be lower as well. Manufacturing concerns for etching deeper pinholes will have to be considered when looking at the new materials.

SUMMARY

This dissertation describes the development of a new interferometer, the Polarization Phase-Shifting Point-Diffraction Interferometer (PPSPDI). The PPSPDI uses a custom pinhole plate made by etching a pinhole in a birefringent thin film to separate the test and reference beams based on their polarization states. Light from the test optic is brought to focus on the pinhole in the film. The portion of light incident on the pinhole is diffracted creating the reference wavefront but retains its original polarization state. The remainder of the light passes through the film, which is designed to be a half-wave plate, and its polarization state is rotated by 90° . This allows the interferometer to interfere what were originally orthogonal polarization components. An electrooptic modulator placed near the laser source induces a variable phase shift between these orthogonal components allowing for phase shifting.

The pinhole plate was manufactured by etching a pinhole into a bidirectionally deposited thin film. Ideally these films can be deposited to provide retarders of any specific retardance. For the PPSPDI the films were designed to be half-wave plates. Several methods were used to attempt to etch the pinhole including argon ion milling, reactive ion beam etching and focused ion beam etching. The focused ion beam etching technique proved the most successful giving an etch resolution of 12 nm and feature profiles vertical to 3° . Unfortunately, not only were the films not half-wave plates but

included diattenuation as well. The best of the supplied films was used having a retardance of 160° and 20% diattenuation.

The incorrect retardance and diattenuation in the film caused significantly more problems than expected. A phase shift in the electrooptic modulator of 90° no longer guaranteed a phase shift of 90° in the interferogram. The nonuniform phase shifts in the modulator required to shift the frames correctly caused problems in calibrating the instrument. Intensity modulations across the five frames, again due to the films, limited what could be tested.

Considering the problems with the thin films, the PPSPDI's performance exceeded expectations. The measurement of the test lens on the PPSPDI compared favorably with the result for the same lens on the commercial interferometer. Subtracting two consecutive measurements demonstrated a measurement repeatability of better than $\lambda/30$. Correcting the problems with the thin films would significantly improve the performance of the PPSPDI.

APPENDIX A

Non-Ideal Jones Matrix.nb

Non-Ideal Jones Matrix.nb
 Written by Mark Neal
 26 March 03

This program performs the Jones matrix propagation to obtain a general form for the intensity at the camera for the PPSPDI. This program does not give values to any of the variables except for the beam balance factor A which is given a value of .6. This program goes with the section "Nonideal Jones Matrix Propagation" in the chapter on error analysis.

The first few commands ensure that all the variables are real.

```
Conjugate[phi] ^= phi;
Conjugate[theta] ^= theta;
Conjugate[delta] ^= delta;
Conjugate[Delta] ^= Delta;
Conjugate[psi] ^= psi;
Conjugate[A] ^= A;

A = .6;
```

The next expression is the Jones matrix propagation for the test beam of the PPSPDI as given in the error analysis chapter for the nonideal propagation.

```
TestBeam =
MatrixForm[
Simplify[ $\frac{1}{\sqrt{2}} * \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} * \begin{pmatrix} \cos[-\theta] & \sin[-\theta] \\ -\sin[-\theta] & \cos[-\theta] \end{pmatrix} * \begin{pmatrix} .8 e^{i \phi} & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} \cos[\theta] & \sin[\theta] \\ -\sin[\theta] & \cos[\theta] \end{pmatrix} * \begin{pmatrix} e^{i \Delta} & 0 \\ 0 & e^{i \Delta} \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & e^{i \delta} \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \end{pmatrix}]]$ 

$$\begin{pmatrix} \frac{e^{i \Delta} \cos[\psi] (\sin[\theta] (0.8 e^{i (\delta + \phi)} \cos[\theta - \psi] + \sin[\theta - \psi]) + \cos[\theta] (0.8 e^{i \phi} \cos[\theta - \psi] - e^{i \delta} \sin[\theta - \psi]))}{\sqrt{2}} \\ \frac{e^{i \Delta} (\sin[\theta] (0.8 e^{i (\delta + \phi)} \cos[\theta - \psi] + \sin[\theta - \psi]) + \cos[\theta] (0.8 e^{i \phi} \cos[\theta - \psi] - e^{i \delta} \sin[\theta - \psi])) \sin[\psi]}{\sqrt{2}} \end{pmatrix}$$

```

This expression is the reference beam for the PPSPDI

```
ReferenceBeam = MatrixForm[ $\frac{1}{\sqrt{2}} * \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & e^{i \delta} \end{pmatrix} * \begin{pmatrix} A \\ A \end{pmatrix}]$ 

$$\begin{pmatrix} \frac{0.6 \cos[\psi]^2 + 0.6 e^{i \delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} \\ \frac{0.6 \cos[\psi] \sin[\psi] + 0.6 e^{i \delta} \sin[\psi]^2}{\sqrt{2}} \end{pmatrix}$$

```

The total amplitude at the camera is just the addition of all four components of the test and reference beams (an x and y component for each beam).

$$\begin{aligned} \text{TotalAmplitude} = & \text{TestBeam}[[1, 1]] + \text{TestBeam}[[1, 2]] + \text{ReferenceBeam}[[1, 1]] + \text{ReferenceBeam}[[1, 2]] \\ & \left\{ \frac{e^{i\Delta} \cos[\psi] (\sin[\theta] (0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi]) + \cos[\theta] (0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi]))}{\sqrt{2}} + \right. \\ & \frac{e^{i\Delta} (\sin[\theta] (0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi]) + \cos[\theta] (0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi])) \sin[\psi]}{\sqrt{2}} + \\ & \left. \frac{0.6 \cos[\psi]^2 + 0.6 e^{i\delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} + \frac{0.6 \cos[\psi] \sin[\psi] + 0.6 e^{i\delta} \sin[\psi]^2}{\sqrt{2}} \right\} \end{aligned}$$

The total intensity at the camera is the total amplitude multiplied by its complex conjugate.

$$\begin{aligned} \text{Intensity} = & \text{Simplify}[\text{Expand}[\text{TotalAmplitude} * \text{Conjugate}[\text{TotalAmplitude}]]] \\ & \{ \cos[\theta]^2 (\cos[\theta - \psi]^2 ((0.32 + 0. i) + (0. + 0. i) \cos[2\psi] + (0.64 + 0. i) \cos[\psi] \sin[\psi]) + \\ & e^{-i(\delta+\phi)} (e^{2i\delta} + e^{2i\phi}) \cos[\theta - \psi] \sin[\theta - \psi] (-0.4 \cos[\psi]^2 - 0.8 \cos[\psi] \sin[\psi] - 0.4 \sin[\psi]^2) + \\ & \sin[\theta - \psi]^2 (0.5 \cos[\psi]^2 + 1. \cos[\psi] \sin[\psi] + 0.5 \sin[\psi]^2)) + \\ & e^{-i(\delta+\Delta+\phi)} \cos[\theta] (0.32 e^{i(\Delta+\phi)} (1 + e^{2i\delta}) \cos[\theta - \psi]^2 \sin[\theta] (1. \cos[\psi]^2 + 2. \cos[\psi] \sin[\psi] + 1. \sin[\psi]^2) + \\ & \cos[\theta - \psi] (0.24 e^{i\delta} (1 + e^{2i(\Delta+\phi)}) \cos[\psi]^3 + (0.48 e^{i\delta} + 0.24 e^{2i\delta} + 0.24 e^{2i(\Delta+\phi)} + 0.48 e^{i(\delta+2(\Delta+\phi))}) \cos[\psi]^2 \sin[\psi] + \\ & (0.24 e^{i\delta} + 0.48 e^{2i\delta} + 0.48 e^{2i(\Delta+\phi)} + 0.24 e^{i(\delta+2(\Delta+\phi))}) \cos[\psi] \sin[\psi]^2 + 0.24 (e^{2i\delta} + e^{2i(\Delta+\phi)}) \sin[\psi]^3) - \\ & 0.3 e^{i\phi} \sin[\theta - \psi] (1. (1 + e^{2i(\delta+\Delta)}) \cos[\psi]^3 + 2. (1 + 0.5 e^{i\delta} + 1. e^{2i(\delta+\Delta)} + 0.5 e^{i(\delta+2\Delta)}) \cos[\psi]^2 \sin[\psi] + \\ & 1. e^{i\delta} (1 + e^{2i\Delta}) \sin[\psi]^3) + \\ & \cos[\psi] \sin[\psi] (3.33333 e^{i\Delta} (1 + e^{2i\delta}) \sin[\theta] \sin[\theta - \psi] + 1. (1 + 2. e^{i\delta} + 1. e^{2i(\delta+\Delta)} + 2. e^{i(\delta+2\Delta)}) \sin[\psi])) + \\ & e^{-i(\delta+\Delta+\phi)} (0.18 e^{i(\delta+\Delta+\phi)} \cos[\psi]^4 + \cos[\psi]^3 (0.24 (1 + e^{2i(\delta+\Delta+\phi)}) \cos[\theta - \psi] \sin[\theta] + \\ & 0.18 e^{i\phi} (1.66667 e^{i\delta} (1 + e^{2i\Delta}) \sin[\theta] \sin[\theta - \psi] + 1. e^{i\Delta} (1 + e^{i\delta}) (1 + e^{i\delta}) \sin[\psi])) + \\ & \cos[\psi] \sin[\psi] (0.64 e^{i(\delta+\Delta+\phi)} \cos[\theta - \psi]^2 \sin[\theta]^2 + \cos[\theta - \psi] \sin[\theta] (0.8 e^{i\Delta} (1 + e^{2i(\delta+\phi)}) \sin[\theta] \sin[\theta - \psi] + \\ & 0.24 (1 + 2. e^{i\delta} + 1. e^{2i(\delta+\Delta+\phi)} + 2. e^{i(\delta+2(\Delta+\phi))}) \sin[\psi]) + 0.3 e^{i\phi} (3.33333 e^{i(\delta+\Delta)} \sin[\theta]^2 \sin[\theta - \psi]^2 + \\ & (1. e^{i\delta} + 2. e^{2i\delta} + 2. e^{2i\Delta} + 1. e^{i(\delta+2\Delta)}) \sin[\theta] \sin[\theta - \psi] \sin[\psi] + 0.6 e^{i\Delta} (1 + 2. e^{i\delta} + 1. e^{2i\delta}) \sin[\psi]^2)) + \\ & \sin[\psi]^2 (0.32 e^{i(\delta+\Delta+\phi)} \cos[\theta - \psi]^2 \sin[\theta]^2 + \cos[\theta - \psi] \sin[\theta] (0.4 e^{i\Delta} (1 + e^{2i(\delta+\phi)}) \sin[\theta] \sin[\theta - \psi] + \\ & 0.24 e^{i\delta} (1 + e^{2i(\Delta+\phi)}) \sin[\psi]) + 0.3 e^{i\phi} (1.66667 e^{i(\delta+\Delta)} \sin[\theta]^2 \sin[\theta - \psi]^2 + 1. (e^{2i\delta} + e^{2i\Delta}) \\ & \sin[\theta] \sin[\theta - \psi] \sin[\psi] - 0.833333 e^{i\Delta} (1. (1 + e^{2i\delta}) \sin[2\theta] \sin[\theta - \psi]^2 - 0.72 e^{i\delta} \sin[\psi]^2))) + \\ & \cos[\psi]^2 (0.32 e^{i(\delta+\Delta+\phi)} \cos[\theta - \psi]^2 \sin[\theta]^2 + \cos[\theta - \psi] \sin[\theta] (0.4 e^{i\Delta} (1 + e^{2i(\delta+\phi)}) \sin[\theta] \sin[\theta - \psi] + \\ & 0.48 (1 + 0.5 e^{i\delta} + 1. e^{2i(\delta+\Delta+\phi)} + 0.5 e^{i(\delta+2(\Delta+\phi))}) \sin[\psi]) + \\ & 0.6 e^{i\phi} (0.833333 e^{i(\delta+\Delta)} \sin[\theta]^2 \sin[\theta - \psi]^2 + (1. e^{i\delta} + 0.5 e^{2i\delta} + 0.5 e^{2i\Delta} + 1. e^{i(\delta+2\Delta)}) \sin[\theta] \\ & \sin[\theta - \psi] \sin[\psi] - 0.416667 e^{i\Delta} (1. (1 + e^{2i\delta}) \sin[2\theta] \sin[\theta - \psi]^2 - 1.44 (1 + e^{i\delta} + e^{2i\delta}) \sin[\psi]^2)))) \} \end{aligned}$$

Non-Ideal Jones Retardance.nb

Non-Ideal Jones Retardance.nb

Written by Mark Neal

26 Mar 03

This program performs the Jones matrix propagation to obtain the form of the intensity at the camera and the output phase of the interferometer as a function of the retardance of the thin film. This program gives the ideal values to the alignment angles of the thin film and final analyzer. This program goes with the section "Retardance of the Thin Film" in the error analysis chapter.

The first few commands ensure that the variables are all real

```
<< Graphics`Legend`

Conjugate[φ] ^= φ;

Conjugate[θ] ^= θ;

Conjugate[δ] ^= δ;

Conjugate[Δ] ^= Δ;

Conjugate[ψ] ^= ψ;

Conjugate[A] ^= A;
```

The next expression is the Jones matrix representation for the test beam of the PPSPI given in the error analysis chapter.

```
TestBeam =
MatrixForm[
Simplify[ $\frac{1}{\sqrt{2}} * \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} * \begin{pmatrix} \cos[-\theta] & \sin[-\theta] \\ -\sin[-\theta] & \cos[-\theta] \end{pmatrix} * \begin{pmatrix} .8 e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} \cos[\theta] & \sin[\theta] \\ -\sin[\theta] & \cos[\theta] \end{pmatrix} * \begin{pmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \end{pmatrix}]]$ 

$$\begin{pmatrix} \frac{e^{i\Delta} \cos[\psi] (\sin[\theta] (0.8 e^{i(\delta+\phi)} \cos[\theta-\psi] + \sin[\theta-\psi]) + \cos[\theta] (0.8 e^{i\phi} \cos[\theta-\psi] - e^{i\delta} \sin[\theta-\psi]))}{\sqrt{2}} \\ \frac{e^{i\Delta} (\sin[\theta] (0.8 e^{i(\delta+\phi)} \cos[\theta-\psi] + \sin[\theta-\psi]) + \cos[\theta] (0.8 e^{i\phi} \cos[\theta-\psi] - e^{i\delta} \sin[\theta-\psi])) \sin[\psi]}{\sqrt{2}} \end{pmatrix}$$

```

This expression is the reference beam for the PPSPI

```
ReferenceBeam = MatrixForm[ $\frac{1}{\sqrt{2}} * \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} * \begin{pmatrix} A \\ A \end{pmatrix}$ 

$$\begin{pmatrix} \frac{A \cos[\psi]^2 + A e^{i\delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} \\ \frac{A \cos[\psi] \sin[\psi] + A e^{i\delta} \sin[\psi]^2}{\sqrt{2}} \end{pmatrix}$$

```

The total amplitude is just the addition of all four components of the test and reference beams (an x and y component for each beam)

$$\begin{aligned} \text{TotalAmplitude} = & \text{TestBeam}[[1, 1]] + \text{TestBeam}[[1, 2]] + \text{ReferenceBeam}[[1, 1]] + \text{ReferenceBeam}[[1, 2]] \\ & \left\{ \frac{e^{i\Delta} \cos[\psi] \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi] \right) \right)}{\sqrt{2}} + \right. \\ & \frac{e^{i\Delta} \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi] \right) \right) \sin[\psi]}{\sqrt{2}} + \\ & \left. \frac{A \cos[\psi]^2 + A e^{i\delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} + \frac{A \cos[\psi] \sin[\psi] + A e^{i\delta} \sin[\psi]^2}{\sqrt{2}} \right\} \end{aligned}$$

The total intensity at the camera is the total amplitude multiplied by its complex conjugate. This is where the ideal values for the thin film and analyzer angles are set. The thin film angle is set to -45 degrees and the analyzer is set to 0 degrees. The beam balance factor is set to .6.

$$\begin{aligned} \text{RetardanceIntensity}[\delta, \Delta, \phi] = & \text{Chop}[\text{ExpToTrig}[\text{Expand}[\text{TotalAmplitude} + \text{Conjugate}[\text{TotalAmplitude}]]] /. A \rightarrow .6 /. \theta \rightarrow \frac{-\pi}{4} /. \psi \rightarrow 0 \\ & \{ 0.59 + 0.09 \cos[\delta] + 0.3 \cos[\Delta] + 0.3 \cos[\delta + \Delta] + 0.2 \cos[\delta - \phi] - 0.2 \cos[\delta + \phi] + 0.24 \cos[\Delta + \phi] - 0.24 \cos[\delta + \Delta + \phi] \} \end{aligned}$$

The four phase stepped intensities are found by inserting the proper values of 0, 90, 180 and 270 degrees in for the phase step variable.

$$\begin{aligned} \text{Retardance1} = & \text{Chop}[\text{Simplify}[\text{RetardanceIntensity}[0, \Delta, \phi]]] \\ & \{ 0.68 + 0.6 \cos[\Delta] \} \\ \text{Retardance2} = & \text{Chop}[\text{Simplify}[\text{RetardanceIntensity}[\frac{\pi}{2}, \Delta, \phi]]] \\ & \{ 0.59 + 0.3 \cos[\Delta] + 0.24 \cos[\Delta + \phi] - 0.3 \sin[\Delta] + 0.4 \sin[\phi] + 0.24 \sin[\Delta + \phi] \} \\ \text{Retardance3} = & \text{Chop}[\text{Simplify}[\text{RetardanceIntensity}[\pi, \Delta, \phi]]] \\ & \{ 0.5 + 0.48 \cos[\Delta + \phi] \} \\ \text{Retardance4} = & \text{Chop}[\text{Simplify}[\text{RetardanceIntensity}[\frac{3\pi}{2}, \Delta, \phi]]] \\ & \{ 0.59 + 0.3 \cos[\Delta] + 0.24 \cos[\Delta + \phi] + 0.3 \sin[\Delta] - 0.4 \sin[\phi] - 0.24 \sin[\Delta + \phi] \} \end{aligned}$$

The four intensities are then put into a 4 step phase shifting routine to calculate the output phase from the interferometer as a function of the retardance of the film and input phase from the test optic. This is the equation used in the IDL program to find the error in the output of the interferometer.

$$\begin{aligned} \text{RetardancePhase} = & \text{Simplify}[\text{ArcTan}[\frac{\text{Chop}[\text{Retardance4} - \text{Retardance2}]}{\text{Chop}[\text{Retardance1} - \text{Retardance3}]]] \\ & \left\{ \text{ArcTan} \left[\frac{0.6 \sin[\Delta] - 0.8 \sin[\phi] - 0.48 \sin[\Delta + \phi]}{0.18 + 0.6 \cos[\Delta] - 0.48 \cos[\Delta + \phi]} \right] \right\} \end{aligned}$$

```
$DefaultFont = {"Arial", 17}
```

```
{Arial, 17}
```

This plots the four intensities as a function of input phase from the test optic for the perfect case of a half-wave plate.

```
Plot[{Retardance1 /.  $\phi \rightarrow \pi$ , Retardance2 /.  $\phi \rightarrow \pi$ , Retardance3 /.  $\phi \rightarrow \pi$ , Retardance4 /.  $\phi \rightarrow \frac{180 \pi}{180}$ },
  { $\Delta$ , 0, 2  $\pi$ }, AxesStyle  $\rightarrow$  Thickness[.004],
  PlotStyle  $\rightarrow$  {{AbsoluteThickness[2], RGBColor[1, 0, 0]},
    {AbsoluteThickness[2], Dashing[{.015, .015}], RGBColor[0, .5, 0]},
    {AbsoluteThickness[2], Dashing[{.025, .025}], RGBColor[0, 0, 1]},
    {AbsoluteThickness[2], Dashing[{.05, .05}], RGBColor[.5, .5, .5]}}},
  AxesLabel  $\rightarrow$  {"Surface Phase  $\Delta$ ", Intensity}, PlotLegend  $\rightarrow$  {"I1", "I2", "I3", "I4"},
  LegendPosition  $\rightarrow$  {.65, -.3}, LegendSize  $\rightarrow$  {.6, .6}, LegendShadow  $\rightarrow$  None]
```

VariableRetardance.pro

pro variableRetardance

;Written by Mark Neal on 26 Mar 03

;This program calculates the error due to the PDI plate being the wrong retardance
;and includes diattenuation. It is assumed that the retarder is at 45 degrees and
;the analyzer is horizontal. The analytic solution was obtained using a
;Mathematica program with the filename "Non-Ideal Jones Retardance.nb"

;Define the arrays used

device, decompose=0

k=100

step1=fltarr(k)

step2=fltarr(k)

step3=fltarr(k)

step4=fltarr(k)

phase=fltarr(k)

difference=fltarr(k)

Ideal=fltarr(k)

delta=fltarr(k)

i=complex(0,1)

surf=fltarr(k)

phi=fltarr(k)

maxx=fltarr(k)

minn=fltarr(k)

error=fltarr(k)

stddeviat=fltarr(k)

;Loop through retardance values

for m=0,k-1 do begin

phi(m)=double(160+(.4*m))*(!pi/180.0)

;Loop through input surface phase from 0 to 2pi. If perfect then the output of the program will
;be exactly this value

for n=0,k-1 do begin

surf(n)=double((2.0*!pi*(n))/k)

;This is the analytic solution from Mathematic for $(I_4 - I_2)/(I_1 - I_3)$. So, the output phase is the

;Arctangent of this value.

phase(n)=Atan((.6*Sin(surf(n))-.8*Sin(phi(m))-.48*Sin(surf(n)+phi(m)))/(.18+.6*cos(surf(n))\$
-.48*cos(surf(n)+phi(m))))

endfor

;Plot the unwrapped output phase and wrapped input phase in radians.

window,1

plot,surf,phase, Background=255,color=0,title='Measured Phase vs. Input Phase',\$

xtitle='Input Phase',ytitle='Measured Phase'

oplot,surf,Atan(Tan(surf)),color=0

```

;Unwrap the output phase to remove phase ambiguities.
for n=0,98 do begin
if (phase(n) gt 0.0) and (phase(n+1) gt 0.0) and (phase(n+1) gt phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) gt 0.0) and (phase(n+1) gt 0.0) and (phase(n+1) le phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)+!pi
if (phase(n) gt 0.0) and (phase(n+1) le 0.0) and (abs(abs(phase(n+1))+phase(n)) gt .8)$
then phase(n+1:*)=phase(n+1:*)+!pi
if (phase(n) le 0.0) and (phase(n+1) gt 0.0) and (abs(phase(n+1)+abs(phase(n))) gt .8)$
then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) le 0.0) and (phase(n+1) le 0.0) and (phase(n+1) gt phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) le 0.0) and (phase(n+1) le 0.0) and (phase(n+1) le phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)+!pi
endfor

;Plot the output phase in radians.
window,2
plot,surf,phase,Background=255,color=0,title='Measured Phase vs. Input Phase',$
xtitle='input phase',ytitle='Measured Phase'
oplot,surf,surf,color=0

;Plot the difference between the output phase and the input phase in radians from the surface.
difference=phase-surf
window,4
plot,surf,difference,Background=255,color=0,$
title='Difference Between Input Phase and Measured Phase',xtitle='Input Phase',ytitle='Difference'

;Find the maximum, minimum, max-min and standard deviation for the difference of the input and output
phase.
maxx(m)=max(difference)
minn(m)=min(difference)
error(m)=maxx(m)-minn(m)
stddeviat(m)=stddev(difference,/double,/nan)
;Print the peak to valley and rms error in waves for each value of retardance.
print,(180/!pi)*phi(m),' peak to valley error in waves ',(1/(2*!pi))*error(m)
print,(180/!pi)*phi(m),' rms error in waves ',(1/(2*!pi))*stddeviat(m)
endfor

;Plot the peak to valley and rms error in waves vs. retardance
window,5
plot,(180/!pi)*phi,(1/(2*!pi))*abs(error),Background=255,color=0,title='Peak to Valley Phase Error vs.$
Retarder Thickness',xtitle='Retarder Thickness (in degrees)',ytitle='Peak to Valley Error (in waves)'
window,6
plot,(180/!pi)*phi,(1/(2*!pi))*abs(stddeviat),background=255,color=0,title='RMS Phase Error vs.$
Retarder Thickness',xtitle='Retarder Thickness (in degrees)',ytitle='RMS Error (in waves)'

end

```


Non-Ideal Jones Angle.nb

Non-Ideal Jones Angle.nb

Written by Mark Neal

26 Mar 03

This program performs the Jones matrix propagation to obtain the form of the intensity at the camera and the output phase of the interferometer as a function of the angle of the thin film retarder used in the PPSPDI. This program uses the ideal values for the film retardance and the angle of the final analyzer. This program also gives the four phase-stepped intensities and the output phase of the interferometer as a function of the angle of the film. This program goes with the section "Angular Alignment of the Thin Film" in the error analysis chapter.

The first few commands ensure that the variables are all real.

`Conjugate[ $\phi$ ] ^=  $\phi$ ;`

`Conjugate[ $\theta$ ] ^=  $\theta$ ;`

`Conjugate[ $\delta$ ] ^=  $\delta$ ;`

`Conjugate[ $\Delta$ ] ^=  $\Delta$ ;`

`Conjugate[ $\psi$ ] ^=  $\psi$ ;`

`Conjugate[A] ^= A;`

This expression is the Jones matrix representation for the test beam of the PPSPDI from the error analysis chapter.

$$\begin{aligned} \text{TestBeam} = & \text{MatrixForm}\left[\text{Simplify}\left[\frac{1}{\sqrt{2}} * \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} \cdot \begin{pmatrix} \cos[-\theta] & \sin[-\theta] \\ -\sin[-\theta] & \cos[-\theta] \end{pmatrix} \cdot \begin{pmatrix} .8 e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & \sin[\theta] \\ -\sin[\theta] & \cos[\theta] \end{pmatrix} \cdot \right. \\ & \left. \begin{pmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right] \\ & \left(\frac{e^{i\Delta} \cos[\psi] \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta-\psi] + \sin[\theta-\psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta-\psi] - e^{i\delta} \sin[\theta-\psi] \right) \right)}{\sqrt{2}} \right. \\ & \left. \frac{e^{i\Delta} \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta-\psi] + \sin[\theta-\psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta-\psi] - e^{i\delta} \sin[\theta-\psi] \right) \right) \sin[\psi]}{\sqrt{2}} \right) \end{aligned}$$

This expression is the reference beam for the PPSPDI.

$$\begin{aligned} \text{ReferenceBeam} = & \text{MatrixForm}\left[\frac{1}{\sqrt{2}} * \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \cdot \begin{pmatrix} A \\ A \end{pmatrix} \right] \\ & \left(\frac{A \cos[\psi]^2 + A e^{i\delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} \right) \\ & \left(\frac{A \cos[\psi] \sin[\psi] + A e^{i\delta} \sin[\psi]^2}{\sqrt{2}} \right) \end{aligned}$$

The total amplitude is just the addition of the four components of the test and reference beams (an x and y component from each beam).

$$\begin{aligned} \text{TotalAmplitude} = & \text{TestBeam}[[1, 1]] + \text{TestBeam}[[1, 2]] + \text{ReferenceBeam}[[1, 1]] + \text{ReferenceBeam}[[1, 2]] \\ & \left\{ \frac{e^{i\Delta} \cos[\psi] \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi] \right) \right)}{\sqrt{2}} + \right. \\ & \frac{e^{i\Delta} \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi] \right) \right) \sin[\psi]}{\sqrt{2}} + \\ & \left. \frac{A \cos[\psi]^2 + A e^{i\delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} + \frac{A \cos[\psi] \sin[\psi] + A e^{i\delta} \sin[\psi]^2}{\sqrt{2}} \right\} \end{aligned}$$

The total intensity at the camera is the total amplitude multiplied by its complex conjugate. This is where the ideal values for the film retardance and analyzer angles are set. The thin film retardance is set to 180 degrees and the angle of the analyzer is set to 0 degrees. The beam balance factor is set to .6.

$$\begin{aligned} \text{AngleIntensity}[\delta, \Delta, \theta] = & \text{Chop}[\text{ExpToTrig}[\text{Expand}[\text{TotalAmplitude} * \text{Conjugate}[\text{TotalAmplitude}]]]] /. A \rightarrow .6 /. \phi \rightarrow \pi /. \psi \rightarrow 0 \\ & \left\{ 0.18 - 0.48 \cos[\Delta] \cos[\theta]^2 + 0.32 \cos[\theta]^4 - 1.08 \cos[\delta + \Delta] \cos[\theta] \sin[\theta] + \right. \\ & \left. 1.44 \cos[\delta] \cos[\theta]^3 \sin[\theta] + 0.6 \cos[\Delta] \sin[\theta]^2 + 0.82 \cos[\theta]^2 \sin[\theta]^2 - 1.8 \cos[\delta] \cos[\theta] \sin[\theta]^3 + \frac{\sin[\theta]^4}{2} \right\} \end{aligned}$$

The four phase stepped intensities are found by inserting the proper values of 0, 90, 180 and 270 degrees in for the phase step variable.

$$\begin{aligned} \text{Angle1} = & \text{Chop}[\text{FullSimplify}[\text{AngleIntensity}[0, \Delta, \theta]]] \\ & \{0.59 - 0.09 \cos[2\theta] - 0.18 \cos[\theta] \sin[\theta] + \cos[\Delta] (0.06 - 0.54 \cos[2\theta] - 1.08 \cos[\theta] \sin[\theta]) + 0.405 \sin[4\theta]\} \\ \text{Angle2} = & \text{Chop}[\text{FullSimplify}[\text{AngleIntensity}[\frac{\pi}{2}, \Delta, \theta]]] \\ & \{0.59 + 0.06 \cos[\Delta] - 0.09 \cos[2\theta] - 0.54 \cos[\Delta + 2\theta]\} \\ \text{Angle3} = & \text{Chop}[\text{FullSimplify}[\text{AngleIntensity}[\pi, \Delta, \theta]]] \\ & \{0.59 - 0.09 \cos[2\theta] + 0.18 \cos[\theta] \sin[\theta] + \cos[\Delta] (0.06 - 0.54 \cos[2\theta] + 1.08 \cos[\theta] \sin[\theta]) - 0.405 \sin[4\theta]\} \\ \text{Angle4} = & \text{Chop}[\text{FullSimplify}[\text{AngleIntensity}[\frac{3\pi}{2}, \Delta, \theta]]] \\ & \{0.59 + 0.06 \cos[\Delta] - 0.54 \cos[\Delta - 2\theta] - 0.09 \cos[2\theta]\} \end{aligned}$$

The four intensities are then put into a 4 step phase shifting algorithm to calculate the output phase from the interferometer as a function of the angle of the thin film and input phase from the test optic. This is the equation used in the IDL program to find the error in the output of the interferometer.

$$\begin{aligned} \text{AnglePhase} = & \text{Simplify}\left[\text{ArcTan}\left[\frac{\text{Chop}[\text{Angle4} - \text{Angle2}]}{\text{Chop}[\text{Expand}[\text{Angle1} - \text{Angle3}]]}\right]\right] \\ & \left\{ \text{ArcTan}\left[\frac{-0.54 \cos[\Delta - 2\theta] + 0.54 \cos[\Delta + 2\theta]}{(-0.36 - 2.16 \cos[\Delta]) \cos[\theta] \sin[\theta] + 0.81 \sin[4\theta]}\right] \right\} \end{aligned}$$

This plots the four intensities as a function of input phase from the test optic for the ideal alignment of the retarder at -45 degrees.

```
Plot[{Angle1 /.  $\theta \rightarrow \frac{-\pi}{4}$ , Angle2 /.  $\theta \rightarrow \frac{-\pi}{4}$ , Angle3 /.  $\theta \rightarrow \frac{-\pi}{4}$ , Angle4 /.  $\theta \rightarrow \frac{-\pi}{4}$ }, { $\Delta$ , 0, 2  $\pi$ },
PlotStyle -> {{RGBColor[1, 0, 0]}, {RGBColor[0, .5, 0]}, {RGBColor[0, 0, 1]}, {RGBColor[.5, .5, .5]}},
AxesLabel -> {Surface Phase  $\Delta$ , Intensity}]
```

thetaerror180.pro

```
pro thetaerror180
```

```
;Written by Mark Neal on 26 Mar 03
```

```
;This program calculates the error due to the PDI plate being at the wrong angle
;and includes diattenuation. It is assumed that the film has a retardance of 180 degrees and
;the analyzer is horizontal. The analytic solution was obtained using a
;Mathematica program with the filename "Non-Ideal Jones Angle.nb"
```

```
;Define the arrays used
```

```
device, decompose=0
```

```
k=100
```

```
num=fltarr(k)
```

```
denom=fltarr(k)
```

```
step3=fltarr(k)
```

```
step4=fltarr(k)
```

```
phase=fltarr(k)
```

```
difference=fltarr(k)
```

```
Ideal=fltarr(k)
```

```
delta=fltarr(k)
```

```
i=complex(0,1)
```

```
surf=fltarr(k)
```

```
theta=fltarr(k)
```

```
maxx=fltarr(k)
```

```
minn=fltarr(k)
```

```
error=fltarr(k)
```

```
stddeviat=fltarr(k)
```

```
;Loop through the angle of the retarder
```

```
for m=0,k-1 do begin
```

```
theta(m)=double(-46.7+(.1*m))*(pi/180.0)
```

```
;Loop through surface phase from 0 to 2pi. If perfect then the output of the program will
;be exactly this value.
```

```
for n=0,k-1 do begin
```

```
surf(n)=double((2.0*pi*(n))/k)
```

```
;This is the analytic solution from Mathematica for I4-I2.
```

```
num(n)=-.54*cos(surf(n)-2*theta(m))+.54*cos(surf(n)+2*theta(m))
```

```
;This is the analytic solution from Mathematica for I1-I3.
```

```
denom(n)=-.36-2.16*cos(surf(n))*cos(theta(m))*sin(theta(m))+.81*sin(4*theta(m))
```

```
;The output phase is the Arctangent of (I4-I2)/(I1-I3).
```

```
phase(n)=atan(num(n)/denom(n))
```

```
endfor
```

```
;Plot the wrapped output phase and wrapped input phase.
```

```
window,1
```

```
plot,surf,phase,Background=255,color=0,title='Measured Phase vs. Input Phase',$
```

```

xtitle='Input Phase',ytitle='Measured Phase'
oplot,surf,atan(tan(surf)),color=0

;Unwrap the output phase to remove the phase ambiguities.
for n=0,98 do begin
if (phase(n) gt 0.0) and (phase(n+1) gt 0.0) and (phase(n+1) gt phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) gt 0.0) and (phase(n+1) gt 0.0) and (phase(n+1) le phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)+!pi
if (phase(n) gt 0.0) and (phase(n+1) le 0.0) and (abs(abs(phase(n+1))+phase(n)) gt .8)$
then phase(n+1:*)=phase(n+1:*)+!pi
if (phase(n) le 0.0) and (phase(n+1) gt 0.0) and (abs(phase(n+1)+abs(phase(n))) gt .8)$
then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) le 0.0) and (phase(n+1) le 0.0) and (phase(n+1) gt phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) le 0.0) and (phase(n+1) le 0.0) and (phase(n+1) le phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)+!pi
endfor

;Plot the unwrapped output phase and incident phase.
window,2
plot,surf,phase,Background=255,color=0,title='Measured Phase vs. Input Phase',$
xtitle='input phase',ytitle='Measured Phase'
oplot,surf,surf,color=0

;Plot the difference between the output phase and the input phase from the surface.
difference=phase-surf
window,4
plot,surf,difference,background=255,color=0,title='Difference Between Measured Phase $
and Input Phase',ytitle='Phase Difference',xtitle='Input Phase'

;Calculate the maximum, minimum, max-min and standard deviation for the difference between the input
and output phase.
maxx(m)=max(difference)
minn(m)=min(difference)
error(m)=maxx(m)-minn(m)
stddeviat(m)=stddev(difference,/double,/nan)
;Print the retardance value and peak to valley and rms error.
print,(180/!pi)*theta(m),' peak to valley error in waves ',(1/(2*!pi))*error(m)
print,(180/!pi)*theta(m),' rms error in waves',(1/(2*!pi))*stddeviat(m)
endfor

;Plot the peak to valley and rms error versus angle.
window,5
plot,(180/!pi)*theta,(1/(2*!pi))*abs(error),background=255,color=0,title='Peak to $
Valley Phase Error vs. Retarder Angle',xtitle='Retarder Angle (in degrees)',$
ytitle='Peak to Valley Error (in waves)'
window,6
plot,(180/!pi)*theta,(1/(2*!pi))*abs(stddeviat),background=255,color=0,$
title='RMS Phase Error vs. Retarder Angle',$
xtitle='Retarder Angle (in degrees)',ytitle='RMS Phase Error (in waves)'

end

```

Non-Ideal Jones Analyzer.nb

Non-Ideal Jones Analyzer.nb

Written by Mark Neal

26 Mar 03

This program performs the Jones matrix propagation to obtain the form of the intensity at the camera and output phase of the interferometer as a function of the alignment angle of the final analyzer in the PPSPDI. This program gives the ideal values to the retardance and alignment angle of the thin film. This program goes with the section "Alignment of the Analyzer" in the error analysis chapter.

The first few commands ensure all the variable are real.

`Conjugate[ $\phi$ ] ^=  $\phi$ ;`

`Conjugate[ $\theta$ ] ^=  $\theta$ ;`

`Conjugate[ $\delta$ ] ^=  $\delta$ ;`

`Conjugate[ $\Delta$ ] ^=  $\Delta$ ;`

`Conjugate[ $\psi$ ] ^=  $\psi$ ;`

`Conjugate[A] ^= A;`

This expression is the Jones matrix representation of the test beam of the PPSPDI for non-ideal propagation.

`TestBeam =`

$$\text{MatrixForm}\left[\text{Simplify}\left[\frac{1}{\sqrt{2}} \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} \cdot \begin{pmatrix} \cos[-\theta] & \sin[-\theta] \\ -\sin[-\theta] & \cos[-\theta] \end{pmatrix} \cdot \begin{pmatrix} .8 e^{i \phi} & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & \sin[\theta] \\ -\sin[\theta] & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} e^{i \Delta} & 0 \\ 0 & e^{i \Delta} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i \delta} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]\right]$$

$$\begin{pmatrix} \frac{e^{i \Delta} \cos[\psi] \left(\sin[\theta] \left(0.8 e^{i (\delta + \phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i \phi} \cos[\theta - \psi] - e^{i \delta} \sin[\theta - \psi] \right) \right)}{\sqrt{2}} \\ \frac{e^{i \Delta} \left(\sin[\theta] \left(0.8 e^{i (\delta + \phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i \phi} \cos[\theta - \psi] - e^{i \delta} \sin[\theta - \psi] \right) \right) \sin[\psi]}{\sqrt{2}} \end{pmatrix}$$

This expression is the reference beam for the PPSPDI.

$$\text{ReferenceBeam} = \text{MatrixForm}\left[\frac{1}{\sqrt{2}} \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i \delta} \end{pmatrix} \cdot \begin{pmatrix} A \\ A \end{pmatrix}\right]$$

$$\begin{pmatrix} \frac{A \cos[\psi]^2 + A e^{i \delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} \\ \frac{A \cos[\psi] \sin[\psi] + A e^{i \delta} \sin[\psi]^2}{\sqrt{2}} \end{pmatrix}$$

The total amplitude is just the addition of all four components for the test and reference beams (an x and y component for each beam).

$$\begin{aligned} \text{TotalAmplitude} = & \text{TestBeam}[[1, 1]] + \text{TestBeam}[[1, 2]] + \text{ReferenceBeam}[[1, 1]] + \text{ReferenceBeam}[[1, 2]] \\ & \left\{ \frac{e^{i\Delta} \cos[\psi] \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi] \right) \right)}{\sqrt{2}} + \right. \\ & \frac{e^{i\Delta} \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi] \right) \right) \sin[\psi]}{\sqrt{2}} + \\ & \left. \frac{A \cos[\psi]^2 + A e^{i\delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} + \frac{A \cos[\psi] \sin[\psi] + A e^{i\delta} \sin[\psi]^2}{\sqrt{2}} \right\} \end{aligned}$$

The total intensity at the camera is the total amplitude multiplied by its complex conjugate. The ideal values are set for the thin film. The retardance is set to 180 degrees and alignment angle is set to 45 degrees. The beam balance factor is set to .6.

$$\begin{aligned} \text{Intensity}[\delta, \Delta, \psi] = & \text{Simplify}[\text{TotalAmplitude} * \text{Conjugate}[\text{TotalAmplitude}]] /. A \rightarrow .6 /. \phi \rightarrow \pi /. \theta \rightarrow \frac{-\pi}{4} \\ & \left\{ \frac{1}{\sqrt{2}} \left(e^{-i(\pi+\delta+\Delta)} \left(0.6 \cos[\psi]^2 + \frac{e^{i\Delta} (\cos[\psi] + \sin[\psi]) (-0.8 \cos[\frac{\pi}{4} + \psi] + e^{i\delta} \sin[\frac{\pi}{4} + \psi])}{\sqrt{2}} + \right. \right. \right. \\ & \left. \sin[\psi] \left(-0.565685 e^{i(\pi+\delta+\Delta)} \cos[\frac{\pi}{4} + \psi] + 0.6 e^{i\delta} \sin[\psi] + \frac{e^{i\Delta} \sin[\frac{\pi}{4} + \psi]}{\sqrt{2}} \right) + \right. \\ & \left. \left. \cos[\psi] \left(-0.565685 e^{i(\pi+\delta+\Delta)} \cos[\frac{\pi}{4} + \psi] + 0.6 (1 + e^{i\delta}) \sin[\psi] + \frac{e^{i\Delta} \sin[\frac{\pi}{4} + \psi]}{\sqrt{2}} \right) \right) \right\} \\ & \left(0.424264 e^{i(\pi+\delta+\Delta)} \cos[\psi]^2 + \frac{(\cos[\psi] + \sin[\psi]) (0.565685 e^{i\delta} \cos[\frac{\pi}{4} + \psi] - 0.707107 \sin[\frac{\pi}{4} + \psi])}{\sqrt{2}} + \right. \\ & \left. \sin[\psi] \left(-0.4 \cos[\frac{\pi}{4} + \psi] + 0.424264 e^{i(\pi+\delta)} \sin[\psi] + 0.5 e^{i(\pi+\delta)} \sin[\frac{\pi}{4} + \psi] \right) + \right. \\ & \left. \left. \cos[\psi] \left(-0.4 \cos[\frac{\pi}{4} + \psi] - 0.707107 \left(0.6 e^{i\Delta} (1 + e^{i\delta}) \sin[\psi] + \frac{e^{i\delta} \sin[\frac{\pi}{4} + \psi]}{\sqrt{2}} \right) \right) \right) \right\} \end{aligned}$$

The four phase stepped intensities are found by inserting the proper values of 0, 90, 180 and 270 in for the phase step variable

$$\begin{aligned} \text{Analyzer1} = & \text{Chop}[\text{FullSimplify}[\text{ExpToTrig}[\text{Expand}[\text{Intensity}[0, \Delta, \psi]]]]] \\ & \{1.02 + 0.9 \cos[\Delta] - 0.15 \cos[\Delta - 4\psi] - 0.34 \cos[4\psi] - 0.15 \cos[\Delta + 4\psi] - 0.6 \sin[\Delta - 2\psi] + 2.72 \cos[\psi] \sin[\psi] + 0.6 \sin[\Delta + 2\psi]\} \\ \text{Analyzer2} = & \text{Chop}[\text{FullSimplify}[\text{Expand}[\text{ExpToTrig}[\text{Intensity}[\frac{\pi}{2}, \Delta, \psi]]]]] \\ & \{0.635 + 0.33 \cos[\Delta] - 0.27 \cos[\Delta - 4\psi] - 0.045 \cos[4\psi] - 0.57 \sin[\Delta - 2\psi] + 1.36 \cos[\psi] \sin[\psi] + 0.03 \sin[\Delta + 2\psi]\} \\ \text{Analyzer3} = & \text{Chop}[\text{FullSimplify}[\text{Expand}[\text{ExpToTrig}[\text{Intensity}[\pi, \Delta, \psi]]]]] \\ & \{0.25 - 0.24 \cos[\Delta] - 0.12 \cos[\Delta - 4\psi] + 0.25 \cos[4\psi] - 0.12 \cos[\Delta + 4\psi]\} \\ \text{Analyzer4} = & \text{Chop}[\text{FullSimplify}[\text{Expand}[\text{ExpToTrig}[\text{Intensity}[\frac{3\pi}{2}, \Delta, \psi]]]]] \\ & \{0.635 + 0.33 \cos[\Delta] - 0.045 \cos[4\psi] - 0.27 \cos[\Delta + 4\psi] - 0.03 \sin[\Delta - 2\psi] + 1.36 \cos[\psi] \sin[\psi] + 0.57 \sin[\Delta + 2\psi]\} \end{aligned}$$

The four intensities are then put into a four step phase shifting algorithm to calculate the output phase from the interferometer as a function of the angle of the analyzer and the input phase from the test optic. This is the equation used in the IDL program to find the error in the output of the interferometer.

$$\text{AnalyzerPhase} = \text{ArcTan}\left[\frac{\text{Simplify}\left[\frac{\text{Chop}[\text{FullSimplify}[\text{Analyzer4} - \text{Analyzer2}]]}{\text{Chop}[\text{FullSimplify}[\text{Analyzer1} - \text{Analyzer3}]]}\right]}{\left\{\text{ArcTan}\left[\frac{0.27 \cos[\Delta - 4\psi] - 0.27 \cos[\Delta + 4\psi] + 1.08 \cos[2\psi] \sin[\Delta]}{0.77 + 1.14 \cos[\Delta] - 0.03 \cos[\Delta - 4\psi] - 0.59 \cos[4\psi] - 0.03 \cos[\Delta + 4\psi] - 0.6 \sin[\Delta - 2\psi] + 2.72 \cos[\psi] \sin[\psi] + 0.6 \sin[\Delta + 2\psi]}\right]\right\}}\right]$$

This plots the four intensities as a function of input phase from the test optic for the ideal case with the analyzer aligned at 0 degrees.

```
Plot[{Analyzer1 /.  $\psi \rightarrow 0$ , Analyzer2 /.  $\psi \rightarrow 0$ , Analyzer3 /.  $\psi \rightarrow 0$ , Analyzer4 /.  $\psi \rightarrow 0$ }, { $\Delta$ , 0, 2  $\pi$ },
PlotStyle -> {{RGBColor[1, 0, 0]}, {RGBColor[0, .5, 0]}, {RGBColor[0, 0, 1]}, {RGBColor[.5, .5, .5]}},
AxesLabel -> {Surface Phase  $\Delta$ , Intensity}]
```

analyzererror180.pro

```
pro analyzererror180
```

```
;Written by Mark Neal on 26 Mar 03
```

```
;This program calculates the error due to the final analyzer being at the wrong angle in the PPSPDI
;and includes diattenuation in the film. It is assumed that the film has 180 degrees retardance and is
;oriented at -45 degrees. The analytic solution was obtained using a
;Mathematica program with the filename "Non-Ideal Jones Analyzer.nb"
```

```
;Define the arrays used.
```

```
device, decompose=0
```

```
k=100
```

```
num=fltarr(k)
```

```
denom=fltarr(k)
```

```
phase=fltarr(k)
```

```
difference=fltarr(k)
```

```
Ideal=fltarr(k)
```

```
delta=fltarr(k)
```

```
i=complex(0,1)
```

```
surf=fltarr(k)
```

```
theta=fltarr(k)
```

```
maxx=fltarr(k)
```

```
minn=fltarr(k)
```

```
error=fltarr(k)
```

```
stddeviat=fltarr(k)
```

```
;Loop through the different analyzer angles
```

```
for m=0,k-1 do begin
```

```
theta(m)=double(-9.3+.1*m)*(!pi/180.0)
```

```
;Loop through the input surface phase from 0 to 2pi. If perfect then the output of the program will
```

```
;be exactly this value
```

```
for n=0,k-1 do begin
```

```
surf(n)=double((2.0*!pi*(n))/k)
```

```
;This is the analytic solution from Mathematica for I4-I2.
```

```
num(n)=.27*cos(surf(n)-4*theta(m))-.27*cos(surf(n)+4*theta(m))+1.08*cos(2*theta(m))*sin(surf(n))
```

```
;This is the analytic solution from Mathematica for I1-I3.
```

```
denom(n)=.77+1.14*cos(surf(n))-.03*cos(surf(n)-4*theta(m))$
```

```
-.59*cos(4*theta(m))-.03*cos(surf(n)+4*theta(m))$
```

```
-.6*sin(surf(n)-2*theta(m))+2.72*cos(theta(m))*sin(theta(m))+.6*sin(surf(n)+2*theta(m))
```

```
;This calculates the phase -> ArcTan((I4-I2)/(I1-I3))
```

```
phase(n)=Atan(num(n)/denom(n))
```

```
endfor
```

```
;Plot the wrapped output phase and input phase
```

```
window,1
```

```

plot,surf,phase,Background=255,color=0,title='Measured Phase vs. Input Phase',$
xtitle='input phase',ytitle='Measured Phase'
oplot,surf,atan(tan(surf)),color=0

;Unwrap the output phase to remove the phase ambiguities.
for n=0,98 do begin
if (phase(n) gt 0.0) and (phase(n+1) gt 0.0) and (phase(n+1) gt phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) gt 0.0) and (phase(n+1) gt 0.0) and (phase(n+1) le phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)+!pi
if (phase(n) gt 0.0) and (phase(n+1) le 0.0) and (abs(abs(phase(n+1))+phase(n)) gt .8)$
then phase(n+1:*)=phase(n+1:*)+!pi
if (phase(n) le 0.0) and (phase(n+1) gt 0.0) and (abs(phase(n+1)+abs(phase(n))) gt .8)$
then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) le 0.0) and (phase(n+1) le 0.0) and (phase(n+1) gt phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) le 0.0) and (phase(n+1) le 0.0) and (phase(n+1) le phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)+!pi
endfor

;Plot the unwrapped output and input phase.
window,2
plot,surf,phase,Background=255,color=0,title='Measured Phase vs. Input Phase',$
xtitle='Input Phase',ytitle='Measured Phase'
oplot,surf,surf,color=0

;Calculate and plot the difference between the output and input phase from the surface.
difference=phase-surf
window,3
plot,surf,difference,background=255,color=0,title='Difference Between Measured Phase and Input Phase',$
ytitle='Phase Difference',xtitle='Input Phase'

;Find the maximum, minimum, max-min and standard deviation for the difference of the input and output
phase.
maxx(m)=max(difference)
minn(m)=min(difference)
error(m)=maxx(m)-minn(m)
stddeviat(m)=stddev(difference,/double,/nan)
;Print the peak to valley and rms error in waves for each value of analyzer angle.
print,(180/!pi)*theta(m),' peak to peak error in waves ',error(m)
print,(180/!pi)*theta(m),' rms error in waves ',(1/(2*!pi))*stddeviat(m)
endfor

;Plot the peak to valley and rms error in waves vs. analyzer angle.
window,4
plot,(180/!pi)*theta,abs(error),background=255,color=0,title='Peak to Valley Error vs. Analyzer Angle',$
xtitle='Analyzer Angle',ytitle='Peak-to-Valley Error (in waves)'
window,5
plot,(180/!pi)*theta,(1/(2*!pi))*abs(stddeviat),background=255,color=0,title='RMS Phase Error$
vs.Analyzer Angle',xtitle='Analyzer Angle',ytitle='RMS Error (in waves)

end

```

Non-Ideal Jones Retardance for 160.nb

Non-Ideal Jones Retardance for 160.nb

Written by Mark Neal

26 Mar 03

This program performs the Jones matrix propagation to obtain the intensity at the camera and output phase of the interferometer for a thin film with a retardance of 160 degrees like the one actually used in the PPSPDI. This program sets the angle of the retarder and angle of the final analyzer to their ideal values. This program goes with the section "Actual PPSPDI Film with 160 Degrees Retardance" in the error analysis chapter.

The first few commands ensure that all the variables are real.

```
<< Graphics`Legend`
```

```
Conjugate[φ] ^= φ;
```

```
Conjugate[θ] ^= θ;
```

```
Conjugate[δ] ^= δ;
```

```
Conjugate[Δ] ^= Δ;
```

```
Conjugate[ψ] ^= ψ;
```

```
Conjugate[A] ^= A;
```

This expression is the Jones matrix representation for the test beam of the PPSPDI given in the error analysis chapter. Values for the variables have not been set.

```
TestBeam =
```

$$\text{MatrixForm}\left[\text{Simplify}\left[\frac{1}{\sqrt{2}} * \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} \cdot \begin{pmatrix} \cos[-\theta] & \sin[-\theta] \\ -\sin[-\theta] & \cos[-\theta] \end{pmatrix} \cdot \begin{pmatrix} .8 e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & \sin[\theta] \\ -\sin[\theta] & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} e^{i\Delta} & 0 \\ 0 & e^{i\Delta} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]\right]$$

$$\left\{ \frac{e^{i\Delta} \cos[\psi] \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta-\psi] + \sin[\theta-\psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta-\psi] - e^{i\delta} \sin[\theta-\psi] \right) \right)}{\sqrt{2}}, \right.$$

$$\left. \frac{e^{i\Delta} \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta-\psi] + \sin[\theta-\psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta-\psi] - e^{i\delta} \sin[\theta-\psi] \right) \right) \sin[\psi]}{\sqrt{2}} \right\}$$

This is the expression for the reference beam for the PPSPDI.

$$\text{ReferenceBeam} = \text{MatrixForm}\left[\frac{1}{\sqrt{2}} * \begin{pmatrix} \cos[\psi]^2 & \cos[\psi] * \sin[\psi] \\ \cos[\psi] * \sin[\psi] & \sin[\psi]^2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix} \cdot \begin{pmatrix} A \\ A \end{pmatrix}\right]$$

$$\left\{ \frac{A \cos[\psi]^2 + A e^{i\delta} \cos[\psi] \sin[\psi]}{\sqrt{2}}, \right.$$

$$\left. \frac{A \cos[\psi] \sin[\psi] + A e^{i\delta} \sin[\psi]^2}{\sqrt{2}} \right\}$$

The total amplitude is just the addition of all four components of the test and reference beams (an x and y component for each beam).

$$\begin{aligned} \text{TotalAmplitude} = & \text{TestBeam}[[1, 1]] + \text{TestBeam}[[1, 2]] + \text{ReferenceBeam}[[1, 1]] + \text{ReferenceBeam}[[1, 2]] \\ & \left\{ \frac{e^{i\Delta} \cos[\psi] \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi] \right) \right)}{\sqrt{2}} + \right. \\ & \frac{e^{i\Delta} \left(\sin[\theta] \left(0.8 e^{i(\delta+\phi)} \cos[\theta - \psi] + \sin[\theta - \psi] \right) + \cos[\theta] \left(0.8 e^{i\phi} \cos[\theta - \psi] - e^{i\delta} \sin[\theta - \psi] \right) \right) \sin[\psi]}{\sqrt{2}} + \\ & \left. \frac{A \cos[\psi]^2 + A e^{i\delta} \cos[\psi] \sin[\psi]}{\sqrt{2}} + \frac{A \cos[\psi] \sin[\psi] + A e^{i\delta} \sin[\psi]^2}{\sqrt{2}} \right\} \end{aligned}$$

The total intensity at the camera is the total amplitude multiplied by its complex conjugate. The ideal values for the alignment angles of the analyzer and thin film are set to 0 degrees and -45 degrees respectively. The beam balance factor is set to .6. The retardance of the film is still left as a variable.

$$\begin{aligned} \text{RetardanceIntensity}[\delta_, \Delta_, \phi_] = & \text{Chop}[\text{ExpToTrig}[\text{Expand}[\text{TotalAmplitude} * \text{Conjugate}[\text{TotalAmplitude}]]]] /. A \rightarrow .6 /. \theta \rightarrow \frac{-\pi}{4} /. \psi \rightarrow 0 \\ & \{0.59 + 0.09 \cos[\delta] + 0.3 \cos[\Delta] + 0.3 \cos[\delta + \Delta] + \\ & 0.2 \cos[\delta - \phi] - 0.2 \cos[\delta + \phi] + 0.24 \cos[\Delta + \phi] - 0.24 \cos[\delta + \Delta + \phi]\} \end{aligned}$$

This is the intensity with the retardance of the film set to 160 degrees.

$$\begin{aligned} \text{Intensity} = & \text{Chop}[\text{FullSimplify}[\text{RetardanceIntensity}[\delta, \Delta, \frac{160\pi}{180}]]] \\ & \{0.59 + 0.09 \cos[\delta] + 0.0744738 \cos[\Delta] + \\ & 0.525526 \cos[\delta + \Delta] + 0.136808 \sin[\delta] - 0.0820848 \sin[\Delta] + 0.0820848 \sin[\delta + \Delta]\} \end{aligned}$$

This is a plot of the four phase stepped intensities using phase shift values of 0, 90, 180 and 270. From the plot it can be seen that these shifts do not shift the intensities by 90 degrees.

$$\begin{aligned} \text{Plot}[\{ & \text{RetardanceIntensity}[0, \Delta, \frac{160\pi}{180}], \text{RetardanceIntensity}[\frac{\pi}{2}, \Delta, \frac{160\pi}{180}], \\ & \text{RetardanceIntensity}[\pi, \Delta, \frac{160\pi}{180}], \text{RetardanceIntensity}[\frac{3\pi}{2}, \Delta, \frac{160\pi}{180}]\}, \{\Delta, 0, 2\pi\}, \\ \text{AxesStyle} \rightarrow & \text{Thickness}[.004], \\ \text{PlotStyle} \rightarrow & \{\{\text{AbsoluteThickness}[2], \text{RGBColor}[1, 0, 0]\}, \\ & \{\text{AbsoluteThickness}[2], \text{Dashing}[\{.015, .015\}], \text{RGBColor}[0, .5, 0]\}, \\ & \{\text{AbsoluteThickness}[2], \text{Dashing}[\{.025, .025\}], \text{RGBColor}[0, 0, 1]\}, \\ & \{\text{AbsoluteThickness}[2], \text{Dashing}[\{.05, .05\}], \text{RGBColor}[\.5, .5, .5]\}\}, \\ \text{AxesLabel} \rightarrow & \{\text{"Surface Phase } \Delta", \text{"Intensity"}\}, \text{PlotLegend} \rightarrow \{\text{"0"}, \text{"90"}, \text{"180"}, \text{"270"}\}, \\ \text{LegendPosition} \rightarrow & \{.65, -.3\}, \text{LegendSize} \rightarrow \{.6, .6\}, \text{LegendShadow} \rightarrow \text{None} \end{aligned}$$

Two intensities can be plotted against each other to see at what phase shift value the intensities are 90 degrees out of phase. For this plot the first and fourth steps are plotted and show that the correct shift for the fourth intensity is 271 degrees. The second and third intensities can be set the same way.

```
Plot[{RetardanceIntensity[0, Δ,  $\frac{160 \pi}{180}$ ], RetardanceIntensity[ $\frac{3 \pi}{2} + \frac{1 \pi}{180}$ , Δ,  $\frac{160 \pi}{180}$ ]},
{Δ, 0, 4 π}, PlotStyle → {{RGBColor[1, 0, 0]}, {RGBColor[0, .5, 0]}},
GridLines → {{ $\frac{\pi}{2}$ , π,  $\frac{3 \pi}{2}$ , 2 π,  $\frac{5 \pi}{2}$ , 3 π,  $\frac{7 \pi}{2}$ }, Automatic}]
```

The four correctly phase stepped intensities are found by inserting the proper phase steps of 0, 97, 198 and 270 degrees into the intensity equation with the retardance of the film set to 160 degrees. This gives four intensities as a function of the input phase from the test lens only.

```
Retardance1 = Chop[Simplify[RetardanceIntensity[0, Δ,  $\frac{160 \pi}{180}$ ]]]
{0.68 + 0.6 Cos[Δ]}

Retardance2 = Chop[Simplify[RetardanceIntensity[ $\frac{\pi}{2} + \frac{17 \pi}{180}$ , Δ,  $\frac{160 \pi}{180}$ ]]]
{0.694517 + 0.0870344 Cos[Δ] + 0.3 Cos[ $\frac{107 \pi}{180} + \Delta$ ] - 0.321756 Sin[Δ]}

Retardance3 = Chop[Simplify[RetardanceIntensity[ $\pi + \frac{18 \pi}{180}$ , Δ,  $\frac{160 \pi}{180}$ ]]]
{0.462129 - 0.210843 Cos[Δ] - 0.24 Cos[ $\frac{179 \pi}{90} + \Delta$ ] + 0.0106203 Sin[Δ]}

Retardance4 = Chop[Simplify[RetardanceIntensity[ $\frac{3 \pi}{2} + \frac{1 \pi}{180}$ , Δ,  $\frac{160 \pi}{180}$ ]]]
{0.454783 + 0.0744738 Cos[Δ] + 0.3 Cos[ $\frac{271 \pi}{180} + \Delta$ ] - 0.24 Cos[ $\frac{431 \pi}{180} + \Delta$ ] - 0.0820848 Sin[Δ]}
```

This plots the four intensities for comparison.

```
Plot[{Retardance1, Retardance2, Retardance3, Retardance4}, {Δ, 0, 4 π},
GridLines -> {{ $\frac{\pi}{2}$ , π,  $\frac{3\pi}{2}$ , 2 π,  $\frac{5\pi}{2}$ , 3 π,  $\frac{7\pi}{2}$ }, Automatic}]
```

The next several lines find the maximum and minimum values of the four intensities referred to as frames. These will be used later to contrast stretch the intensities.

```
Frame1 = Table[0.68~ + 0.599999 Cos[Δ], {Δ, 0°, 720°, 1°}];
```

```
max1 = Max[Frame1]
```

```
1.28
```

```
min1 = Min[Frame1]
```

```
0.080001
```

```
Frame2 = Table[0.6945167 + 0.0870344 Cos[Δ] + 0.2999999 Cos[ $\frac{107\pi}{180}$  + Δ] - 0.3217559 Sin[Δ],
{Δ, 0°, 720°, 1°}];
```

```
max2 = Max[Frame2]
```

```
1.30316
```

```
min2 = Min[Frame2]
```

```
0.0858695
```

```
Frame3 = Table[0.4621288 - 0.2108431 Cos[Δ] - 0.2399999 Cos[ $\frac{179\pi}{90}$  + Δ] + 0.0106202 Sin[Δ],
{Δ, 0°, 720°, 1°}];
```

```
max3 = Max[Frame3]
```

```
0.912826
```

```
min3 = Min[Frame3]
```

```
0.011432
```

```
Frame4 = Table[0.4547834 + 0.0744737 Cos[Δ] + 0.2999999 Cos[ $\frac{271\pi}{180}$  + Δ] - 0.2399999 Cos[ $\frac{431\pi}{180}$  + Δ] -
0.0820848 Sin[Δ], {Δ, 0°, 720°, 1°}];
```

```
max4 = Max[Frame4]
```

```
0.899577
```

```
min4 = Min[Frame4]
```

```
0.00998963
```

This is a plot of the first frame after contrast stretching

These are the four intensities after contrast stretching

$$\text{Retardance1} = \frac{\text{Retardance1} - \text{min1}}{\text{max1} - \text{min1}}$$

$$\{0.833335 (0.599999 + 0.6 \cos[\Delta])\}$$

$$\text{Retardance2} = \frac{\text{Retardance2} - \text{min2}}{\text{max2} - \text{min2}}$$

$$\left\{0.821494 \left(0.608647 + 0.0870344 \cos[\Delta] + 0.3 \cos\left[\frac{107\pi}{180} + \Delta\right] - 0.321756 \sin[\Delta]\right)\right\}$$

$$\text{Retardance3} = \frac{\text{Retardance3} - \text{min3}}{\text{max3} - \text{min3}}$$

$$\left\{1.10939 \left(0.450697 - 0.210843 \cos[\Delta] - 0.24 \cos\left[\frac{179\pi}{90} + \Delta\right] + 0.0106203 \sin[\Delta]\right)\right\}$$

$$\text{Retardance4} = \frac{\text{Retardance4} - \text{min4}}{\text{max4} - \text{min4}}$$

$$\left\{1.12412 \left(0.444794 + 0.0744738 \cos[\Delta] + 0.3 \cos\left[\frac{271\pi}{180} + \Delta\right] - 0.24 \cos\left[\frac{431\pi}{180} + \Delta\right] - 0.0820848 \sin[\Delta]\right)\right\}$$

The next two commands plot the contrast and properly phase shifted intensities to compare with the plot of the four intensities before the correct phase steps and contrast stretching have been applied.

```
$DefaultFont = {"Arial", 13}

{Arial, 13}

Plot[{Retardance1, Retardance2, Retardance3, Retardance4}, {\Delta, 0, 2 \pi}, AxesStyle \to Thickness[.004],
PlotStyle \to {{AbsoluteThickness[2], RGBColor[1, 0, 0]},
{AbsoluteThickness[2], Dashing[{.015, .015}], RGBColor[0, .5, 0]},
{AbsoluteThickness[2], Dashing[{.025, .025}], RGBColor[0, 0, 1]},
{AbsoluteThickness[2], Dashing[{.05, .05}], RGBColor[.5, .5, .5]}},
AxesLabel \to {"Surface Phase \Delta", "Intensity"}, PlotLegend \to {"I1", "I2", "I3", "I4"},
LegendPosition \to {.55, -.3}, LegendSize \to {.6, .6}, LegendShadow \to None]
```

The four intensities are then put into a four step phase shifting routine to calculate the output phase from the interferometer as a function of the input phase from the test lens. So, the difference between the output phase and input phase can be calculated.

$$\text{RetardancePhase} = \text{Chop}\left[\text{Simplify}\left[\text{ArcTan}\left[\frac{\text{Chop}\left[\text{Simplify}\left[\text{Retardance4} - \text{Retardance2}\right], 10^{-5}\right]}{\text{Chop}\left[\text{Simplify}\left[\text{Retardance1} - \text{Retardance3}\right], 10^{-6}\right]}\right]\right]\right]$$

$$\left\{\text{ArcTan}\left[\frac{0.012219 \cos[\Delta] - 0.246448 \cos\left[\frac{107\pi}{180} + \Delta\right] + 0.337235 \cos\left[\frac{271\pi}{180} + \Delta\right] - 0.269788 \cos\left[\frac{431\pi}{180} + \Delta\right] + 0.172048 \sin[\Delta]}{0.733909 \cos[\Delta] + 0.266254 \cos\left[\frac{179\pi}{90} + \Delta\right] - 0.011782 \sin[\Delta]}\right]\right\}$$

This is a plot of both the input and output phase. They are very close to the same.

`Plot[{RetardancePhase, ArcTan[Tan[Δ]]}, {Δ, 0, 2 π}]`

This is a plot of the difference between the input and output phase for input phase values from 0 to 2π . The vertical scale is in radians.

`Plot[RetardancePhase - ArcTan[Tan[Δ]], {Δ, 0, 2 π}]`

errorfor160plate.pro

```
pro errorfor160plate
```

```
;Written by Mark Neal on 26 Mar 03
```

```
;This program calculates the error due to the retardance of the film being 160 degrees. This simulation
;includes diattenuation in the film. It assumes a retardance of 160 degrees with the film oriented at -45
;degrees. The analyzer angle is set to 0 degrees. The analytic solution was obtained using a
;Mathematica program with the filename "Non-Ideal Jones Retardance for 160.nb"
```

```
;Define the arrays used.
```

```
device, decompose=0
```

```
k=100
```

```
num=fltarr(k)
```

```
denom=fltarr(k)
```

```
phase=fltarr(k)
```

```
difference=fltarr(k)
```

```
Ideal=fltarr(k)
```

```
delta=fltarr(k)
```

```
i=complex(0,1)
```

```
surf=fltarr(k)
```

```
;Loop through the input surface phase from 0 to 2pi. If perfect then the output of the program will
;be exactly this value
```

```
for n=0,k-1 do begin
```

```
surf(n)=double((2.0*pi*(n))/k)
```

```
;This is the analytic solution from Mathematica for I4-I2.
```

```
num(n)=.012219*cos(surf(n))-246448*cos(((107*pi)/180)+surf(n))+
.337235*cos(((271*pi)/180)+surf(n))-269788*cos(((431*pi)/180)+surf(n))$
+.172048*sin(surf(n))
```

```
;This is the analytic solution from Mathematica for I1-I3.
```

```
denom(n)=.733909*cos(surf(n))+.266254*cos(((179*pi)/90)+surf(n))-0.11782*sin(surf(n))
```

```
;This calculates the phase -> ArcTan((I4-I2)/(I1-I3))
```

```
phase(n)=atan(num(n)/denom(n))
```

```
endfor
```

```
;Plot the wrapped output phase and input phase
```

```
window,1
```

```
plot,surf,phase,Background=255,color=0,title='Measured Phase vs. Input Phase',$
```

```
xtitle='input phase',ytitle='Measured Phase'
```

```
oplot,surf,atan(tan(surf)),color=0
```

```
;Unwrap the output phase to remove the phase ambiguities.
```

```
for n=0,98 do begin
```

```
if (phase(n) gt 0.0) and (phase(n+1) gt 0.0) and (phase(n+1) gt phase(n))$
```

```
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)-!pi
```

```
if (phase(n) gt 0.0) and (phase(n+1) gt 0.0) and (phase(n+1) le phase(n))$
```

```

and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)+!pi
if (phase(n) gt 0.0) and (phase(n+1) le 0.0) and (abs(abs(phase(n+1))+phase(n)) gt .8)$
then phase(n+1:*)=phase(n+1:*)+!pi
if (phase(n) le 0.0) and (phase(n+1) gt 0.0) and (abs(phase(n+1)+abs(phase(n))) gt .8)$
then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) le 0.0) and (phase(n+1) le 0.0) and (phase(n+1) gt phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)-!pi
if (phase(n) le 0.0) and (phase(n+1) le 0.0) and (phase(n+1) le phase(n))$
and (abs(abs(phase(n+1))-abs(phase(n))) gt .8) then phase(n+1:*)=phase(n+1:*)+!pi
endfor

;Plot the unwrapped output and input phase.
window,2
plot,surf,phase,Background=255,color=0,title='Measured Phase vs. Input Phase',$
xtitle='Input Phase',ytitle='Measured Phase'
oplot,surf,surf,color=0

;Calculate and plot the difference between the output and input phase from the surface in waves.
difference=(1/(2*!pi))*(phase-surf)
window,3
plot,surf,difference,/ynozero,pos=[.20, 0.1, .95, .9],background=255,color=0,title='Difference Between
Measured Phase and Input Phase',$
ytitle='Phase Difference (in waves)',xtitle='Input Phase'

;Find the maximum, minimum, max-min and standard deviation for the difference of the input and output
phase.
maxx=max(difference)
minn=min(difference)
error=maxx-minn
stddeviat=stddev(difference,/double,/nan)
;Print the peak to valley and rms error in waves for each value of analyzer angle.
print,maxx
print,minn
print,'peak to peak error in waves ',error
print,'rms error in waves ',stddeviat

end

```

referencewavequality.pro

pro referencewavequality
device, decompose=0

;Written by Mark Neal on 26 Mar 03

;This program is used to investigate the quality of the reference wave in a PDI
;as a function of pinhole size and aberrations on the incident wave. The program allows the
;person using it to vary the incident aberrations and the size of the pinhole. This way, all aberrations
;and pinhole sizes can be investigated. The phase and amplitude of the beam is calculated at the plane
;prior to the pinhole and in the final image plane. So, the phase and amplitude at the pinhole, size of the
;pinhole and error in the reference wave can be correlated together.

```
;Sets the array sizes
ObjectEdge=2048
LensFunction=complexarr(ObjectEdge,ObjectEdge)
Wtotal=complexarr(ObjectEdge,ObjectEdge)
Aberration=complexarr(ObjectEdge,ObjectEdge)
Transform=complexarr(ObjectEdge,ObjectEdge)
TransformPhase=complexarr(ObjectEdge,ObjectEdge)
AfterPDI=complexarr(ObjectEdge,ObjectEdge)
cylinder1=fltarr(ObjectEdge,ObjectEdge)
cylinder2=fltarr(ObjectEdge,ObjectEdge)
cylinder3=fltarr(ObjectEdge,ObjectEdge)
PDIpinhole=fltarr(objectEdge,ObjectEdge)
FinalImage=complexarr(ObjectEdge,ObjectEdge)
FinalPhase=fltarr(ObjectEdge,ObjectEdge)
PerfectLens=fltarr(ObjectEdge,ObjectEdge)
PerfectTransform=fltarr(ObjectEdge,ObjectEdge)
BigPinhole=fltarr(ObjectEdge,ObjectEdge)
PerfectImage=fltarr(ObjectEdge,ObjectEdge)
PerfectPhase=fltarr(ObjectEdge,ObjectEdge)
i=complex(0,1.0)
lambda=632.8E-9
k=2*pi/lambda
```

```
;Sets the cylinder function for the lens function
radius=64
LensFunction=dist(ObjectEdge,ObjectEdge)
LensFunction=shift(LensFunction,ObjectEdge/2,ObjectEdge/2)
index=where(LensFunction le radius)
LensFunction(index)=1.0
index=where(LensFunction gt radius)
LensFunction(index)=0.0
```

```
;Define the aberration values (put various values in here – in waves).
W200=0.0
W111=0.0
W020=0.0
W040=0.0
W131=0.0
```

```

W222=0.0
W220=0.0
W311=0.0

;Create the aberration function
for n=0,2*radius-1 do begin
xpupil=1.0*(n-radius)/radius
x=ObjectEdge/2-radius+n

for j=0,2*radius-1 do begin
ypupil=1.0*(j-radius)/radius
y=ObjectEdge/2-radius+j

Wtotal(x,y)=W200+W111*ypupil+W020*(xpupil^2+ypupil^2)+W040*((xpupil^2+ypupil^2)^2)$
+W131*(xpupil^2+ypupil^2)*ypupil+(W222*(ypupil^2))+W220*(xpupil^2+ypupil^2)+W311*ypupil

endfor
endfor

index=where(LensFunction eq 1.0)
Aberration(index)=Exp(i*2*pi*Wtotal(index))

LensPhase=fltarr(ObjectEdge,ObjectEdge)
imaginarypart=imaginary(Aberration)
realpart=float(Aberration)
LensPhase=Atan(imaginarypart,realpart)
window,1
plot,LensPhase(0:ObjectEdge-1,ObjectEdge/2),background=255,color=0,title='Horizontal Test Lens Phase'
window,2
plot,LensPhase(ObjectEdge/2,0:ObjectEdge-1),background=255,color=0,title='Vertical Test Lens Phase'

;Fourier Transforms the Aberration Function to propagate to focus
Transform=FFT(shift(Aberration,ObjectEdge/2,ObjectEdge/2),-1)
Transform=shift(Transform,ObjectEdge/2,ObjectEdge/2)
Window,4
plot,Transform(ObjectEdge/2-100:ObjectEdge/2+100,ObjectEdge/2),$
pos=[.15, 0.1, .95, .9],background=255,color=0,title='X Profile of FFT at the Pinhole',$
xtitle='X position (pixels)',ytitle='Amplitude'
Window,5
plot,Transform(ObjectEdge/2,ObjectEdge/2-100:ObjectEdge/2+100),$
pos=[.15, 0.1, .95, .9],background=255,color=0,title='Y Profile of FFT at the Pinhole',$
xtitle='Y position (pixels)',ytitle='Amplitude'

TransformPhase=(1/(2*pi))*Atan(Imaginary(Transform),Float(Transform))
window,6
plot,TransformPhase(ObjectEdge/2-100:ObjectEdge/2+100,ObjectEdge/2),$
background=255,color=0,title='X Profile Phase at the Pinhole'
window,7
plot,TransformPhase(ObjectEdge/2,ObjectEdge/2-100:ObjectEdge/2+100),$
background=255,color=0,title='Y Profile Phase at the Pinhole'

;Phase Unwrap the Phase at the Pinhole Over a Region Approximately 5 Times the Diameter of the Pinhole
htransformphase=fltarr(200)

```

```

htransformphase=TransformPhase(ObjectEdge/2-100:ObjectEdge/2+100,ObjectEdge/2)
initialcenter=htransformphase(100)
for n=0,199 do begin
if (htransformphase(n) gt 0.0) and (htransformphase(n+1) gt 0.0) and (htransformphase(n+1)$
    gt htransformphase(n)) and (abs(abs(htransformphase(n+1))-abs(htransformphase(n))) gt .6) $
    then htransformphase(n+1:*)=htransformphase(n+1:*)-1.0
if (htransformphase(n) gt 0.0) and (htransformphase(n+1) gt 0.0) and (htransformphase(n+1)$
    le htransformphase(n)) and (abs(abs(htransformphase(n+1))-abs(htransformphase(n))) gt .6)$
    then htransformphase(n+1:*)=htransformphase(n+1:*)+1.0
if (htransformphase(n) gt 0.0) and (htransformphase(n+1) le 0.0) and$
    (abs(abs(htransformphase(n+1))+htransformphase(n)) gt .6)$
    then htransformphase(n+1:*)=htransformphase(n+1:*)+1.0
if (htransformphase(n) le 0.0) and (htransformphase(n+1) gt 0.0) and$
    (abs(htransformphase(n+1)+abs(htransformphase(n))) gt .6)$
    then htransformphase(n+1:*)=htransformphase(n+1:*)-1.0
if (htransformphase(n) le 0.0) and (htransformphase(n+1) le 0.0) and (htransformphase(n+1)$
    gt htransformphase(n)) and (abs(abs(htransformphase(n+1))-abs(htransformphase(n))) gt .6)$
    then htransformphase(n+1:*)=htransformphase(n+1:*)-1.0
if (htransformphase(n) le 0.0) and (htransformphase(n+1) le 0.0) and (htransformphase(n+1)$
    le htransformphase(n)) and (abs(abs(htransformphase(n+1))-abs(htransformphase(n))) gt .6)$
    then htransformphase(n+1:*)=htransformphase(n+1:*)+1.0
endfor
finalcenter=htransformphase(100)
difference=finalcenter-initialcenter
htransformphase=htransformphase-difference
window,8
plot,htransformphase,background=255,color=0,title='X Profile Phase at the Pinhole',$
xtitle='X position (pixels)',ytitle='Phase (waves)'

vtransformphase=fltarr(200)
vtransformphase=TransformPhase(ObjectEdge/2,ObjectEdge/2-100:ObjectEdge/2+100)
initialcenter=vtransformphase(100)
for n=0,199 do begin
if (vtransformphase(n) gt 0.0) and (vtransformphase(n+1) gt 0.0) and (vtransformphase(n+1)$
    gt vtransformphase(n)) and (abs(abs(vtransformphase(n+1))-abs(vtransformphase(n))) gt .8)$
    then vtransformphase(n+1:*)=vtransformphase(n+1:*)-1.0
if (vtransformphase(n) gt 0.0) and (vtransformphase(n+1) gt 0.0) and (vtransformphase(n+1)$
    le vtransformphase(n)) and (abs(abs(vtransformphase(n+1))-abs(vtransformphase(n))) gt .8)$
    then vtransformphase(n+1:*)=vtransformphase(n+1:*)+1.0
if (vtransformphase(n) gt 0.0) and (vtransformphase(n+1) le 0.0) and$
    (abs(abs(vtransformphase(n+1))+vtransformphase(n)) gt .8)$
    then vtransformphase(n+1:*)=vtransformphase(n+1:*)+1.0
if (vtransformphase(n) le 0.0) and (vtransformphase(n+1) gt 0.0) and$
    (abs(vtransformphase(n+1)+abs(vtransformphase(n))) gt .8)$
    then vtransformphase(n+1:*)=vtransformphase(n+1:*)-1.0
if (vtransformphase(n) le 0.0) and (vtransformphase(n+1) le 0.0) and (vtransformphase(n+1)$
    gt vtransformphase(n)) and (abs(abs(vtransformphase(n+1))-abs(vtransformphase(n))) gt .8)$
    then vtransformphase(n+1:*)=vtransformphase(n+1:*)-1.0
if (vtransformphase(n) le 0.0) and (vtransformphase(n+1) le 0.0) and (vtransformphase(n+1)$
    le vtransformphase(n)) and (abs(abs(vtransformphase(n+1))-abs(vtransformphase(n))) gt .8)$
    then vtransformphase(n+1:*)=vtransformphase(n+1:*)+1.0
endfor
finalcenter=vtransformphase(100)

```

```

difference=finalcenter-initialcenter
vtransformphase=vtransformphase-difference
window,9
plot,vtransformphase,background=255,color=0,title='Y Profile Phase at the Pinhole',xtitle='Y position
(pixels)',ytitle='Phase (waves)'

;create cylinder function for PDI pinhole
;pinholeradius=4 for 25% of the Airy disk
;pinholeradius=9 for 50% of the Airy disk
;pinholeradius=14 for 75% of the Airy disk
;pinholeradius=19 for 100% of the Airy disk
;pinholeradius=24 for 125% of the Airy disk
;pinholeradius=29 for 150% of the Airy disk
pinholeradius=9
cylinder1=dist(ObjectEdge,ObjectEdge)
cylinder1=shift(cylinder1,ObjectEdge/2,ObjectEdge/2)
index=where(cylinder1 le pinholeradius)
cylinder1(index)=1.0
index=where(cylinder1 gt pinholeradius)
cylinder1(index)=0.0

;Multiply the FT of the lens function by the PDI pinhole
index=where(cylinder1 eq 1.0)
AfterPDI(index)=Transform(index)

;Fourier Transform to propagate to far field
FinalImage=FFT(shift(AfterPDI,ObjectEdge/2,ObjectEdge/2),1)
FinalImage=shift(FinalImage,ObjectEdge/2,ObjectEdge/2)
index=where(abs(FinalImage) le .01)
FinalImage(index)=0.0+0.0*i
FinalImaginary=imaginary(FinalImage)
FinalReal=float(FinalImage)
FinalPhase=(Atan(FinalImaginary,FinalReal))

;Now create perfect simulation with perfect lens and infinite pinhole
radius=64
PerfectLens=dist(ObjectEdge,ObjectEdge)
PerfectLens=shift(PerfectLens,ObjectEdge/2,ObjectEdge/2)
index=where(PerfectLens le radius)
PerfectLens(index)=1.0
index=where(PerfectLens gt radius)
PerfectLens(index)=0.0

;Simulate infinite pinhole by not multiplying by anything at focus, just FT and inverse FT.
PerfectTransform=FFT(shift(PerfectLens,ObjectEdge/2,ObjectEdge/2),-1)
PerfectTransform=shift(PerfectTransform,ObjectEdge/2,ObjectEdge/2)
PerfectImage=FFT(shift(PerfectTransform,ObjectEdge/2,ObjectEdge/2),1)
PerfectImage=shift(PerfectImage,ObjectEdge/2,ObjectEdge/2)

;Threshold to get rid of noise outside of overlap region when calculating phase.
index=where(abs(PerfectImage) le .01)
PerfectImage(index)=0.0+0.0*i
PerfectPhase=(Atan(imaginary(PerfectImage),float(PerfectImage)))

```

;Find the difference between the aberrated case and ideal case for the y=0 horizontal slice in the final
;image plane and phase unwrap it. Then reset the center to its initial value and the difference across
;the result is the reference wavefront error along the y=0 plane.

```
hphase=ftarr(2*radius+1)
hphase=(1/(2*pi))*(FinalPhase(ObjectEdge/2-radius:ObjectEdge/2+radius,ObjectEdge/2)-$
PerfectPhase(ObjectEdge/2-radius:ObjectEdge/2+radius,ObjectEdge/2))
initialcenter=hphase(radius)
for n=0,2*radius-1 do begin
if (hphase(n) gt 0.0) and (hphase(n+1) gt 0.0) and (hphase(n+1) gt hphase(n))$
and (abs(abs(hphase(n+1))-abs(hphase(n))) gt .8) then hphase(n+1:*)=hphase(n+1:*)-1.0
if (hphase(n) gt 0.0) and (hphase(n+1) gt 0.0) and (hphase(n+1) le hphase(n))$
and (abs(abs(hphase(n+1))-abs(hphase(n))) gt .8) then hphase(n+1:*)=hphase(n+1:*)+1.0
if (hphase(n) gt 0.0) and (hphase(n+1) le 0.0) and (abs(abs(hphase(n+1))+hphase(n)) gt .8)$
then hphase(n+1:*)=hphase(n+1:*)+1.0
if (hphase(n) le 0.0) and (hphase(n+1) gt 0.0) and (abs(hphase(n+1)+abs(hphase(n))) gt .8)$
then hphase(n+1:*)=hphase(n+1:*)-1.0
if (hphase(n) le 0.0) and (hphase(n+1) le 0.0) and (hphase(n+1) gt hphase(n))$
and (abs(abs(hphase(n+1))-abs(hphase(n))) gt .8) then hphase(n+1:*)=hphase(n+1:*)-1.0
if (hphase(n) le 0.0) and (hphase(n+1) le 0.0) and (hphase(n+1) le hphase(n))$
and (abs(abs(hphase(n+1))-abs(hphase(n))) gt .8) then hphase(n+1:*)=hphase(n+1:*)+1.0
endfor
finalcenter=hphase(radius)
difference=finalcenter-initialcenter
hphase=hphase-difference
window,10
plot,hphase,/ynozero,pos=[.20, 0.1, .95, .9],background=255,color=0,title='X Profile Error',xtitle='X
position (pixels)',ytitle='Error (waves)'
```

;Find the difference between the aberrated case and ideal case for the x=0 vertical slice in the final
;image plane and phase unwrap it. Then reset the center to its initial value and the difference across
;the result is the reference wavefront error along the x=0 plane.

```
vphase=ftarr(2*radius+1)
vphase=(1/(2*pi))*(FinalPhase(ObjectEdge/2,ObjectEdge/2-radius:ObjectEdge/2+radius)-$
PerfectPhase(ObjectEdge/2,ObjectEdge/2-radius:ObjectEdge/2+radius))
initialcenter=vphase(radius)
for n=0,2*radius-1 do begin
if (vphase(n) gt 0.0) and (vphase(n+1) gt 0.0) and (vphase(n+1) gt vphase(n))$
and (abs(abs(vphase(n+1))-abs(vphase(n))) gt .8) then vphase(n+1:*)=vphase(n+1:*)-1.0
if (vphase(n) gt 0.0) and (vphase(n+1) gt 0.0) and (vphase(n+1) le vphase(n))$
and (abs(abs(vphase(n+1))-abs(vphase(n))) gt .8) then vphase(n+1:*)=vphase(n+1:*)+1.0
if (vphase(n) gt 0.0) and (vphase(n+1) le 0.0) and (abs(abs(vphase(n+1))+vphase(n)) gt .8)$
then vphase(n+1:*)=vphase(n+1:*)+1.0
if (vphase(n) le 0.0) and (vphase(n+1) gt 0.0) and (abs(vphase(n+1)+abs(vphase(n))) gt .8)$
then vphase(n+1:*)=vphase(n+1:*)-1.0
if (vphase(n) le 0.0) and (vphase(n+1) le 0.0) and (vphase(n+1) gt vphase(n))$
and (abs(abs(vphase(n+1))-abs(vphase(n))) gt .8) then vphase(n+1:*)=vphase(n+1:*)-1.0
if (vphase(n) le 0.0) and (vphase(n+1) le 0.0) and (vphase(n+1) le vphase(n))$
and (abs(abs(vphase(n+1))-abs(vphase(n))) gt .8) then vphase(n+1:*)=vphase(n+1:*)+1.0
endfor
finalcenter=vphase(radius)
difference=finalcenter-initialcenter
vphase=vphase-difference
```

```

window,11
plot,vphase,/ynozero,pos=[.20, 0.1, .95, .9],background=255,color=0,title='Y Profile Error',xtitle='Y
position (pixels)',ytitle='Error (waves)'

;Calculate the maximum and minimum values for the error along both the x=0 and y=0 final
;image planes. The max-min will be the wavefront error for the two planes. These will be the
;planes of maximum error so the larger of the two errors will be the max wavefront error. Be
;careful as the error for astigmatism was the max in the y=0 minus the min in the x=0 plane since
;the centers of the curves was the same but one axis went positive from the center and the other
;went negative. This last few lines might need to be changed depending on the aberration and the
;resulting errors.
vmaximum=max(vphase)
vminimum=min(vphase)
hmaximum=max(hphase)
hminimum=min(hphase)

print,'vertical min, max and error'
print,vminimum
print,vmaximum
print,vmaximum-vminimum
print,' '
print,'horizontal min, max and error'
print,hminimum
print,hmaximum
print,hmaximum-hminimum
end

```


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