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THE INFLUENCE OF A THERMALLY BLOOMED ATMOSPHERE ON
TARGET IMAGE QUALITY

The University of Arizona

PH.D. 1981

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THE INFLUENCE OF A THERMALLY BLOOMED ATMOSPHERE
ON TARGET IMAGE QUALITY

by

David Alan Nahrstedt

A Dissertation Submitted to the Faculty of the
COMMITTEE ON OPTICAL SCIENCES (GRADUATE)
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

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As members of the Final Examination Committee, we certify that we have read
the dissertation prepared by David Alan Nahrstedt

entitled The Influence of a Thermally Bloomed Atmosphere on Target Image
Quality

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TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS	vi
LIST OF TABLES	viii
ABSTRACT	ix
1. INTRODUCTION	1
Previous Work	2
Problem Approach	5
Outline of Remaining Chapters	6
2. THERMAL BLOOMING	8
Background	8
Forward Propagation Model for Thermal Blooming	14
The Reduced Wave Equation	14
Calculation of Density and Refractive Index Change	16
Screen Placement	20
The HELPRP Propagation Code	25
3. INCOHERENT IMAGE RECONSTRUCTION-PART I	28
Definition and Conditions	28
Point Spread Function Generation	35
Point Source Considerations	35
4. INCOHERENT IMAGE RECONSTRUCTION-PART II	52
Target (Object) Considerations	52
Geometry	52
Size	52
Irradiance Distribution	53
The Imaging Algorithm	53
Advantages	60
5. ANALYSIS AND RESULTS	62
Scenario Parameters	62
Image Stability and Merit Functions	71

TABLE OF CONTENTS--Continued

	Page
Distortion	71
Relative Intensity	90
Spot Size or Blur	93
Suggestions for Future Research	95
6. SUMMARY AND CONCLUSIONS	99
APPENDIX A: CALCULATION OF CHANGE IN AIR DENSITY BY LINEARIZATION OF THE HYDRODYNAMIC EQUATIONS . . .	101
APPENDIX B: <u>HELPRP</u> PROPAGATION CODE LISTING	106
APPENDIX C: <u>IMAGE</u> RECONSTRUCTION PROGRAM LISTING	142
REFERENCES	162

LIST OF ILLUSTRATIONS

Figure	Page
1-1. Tracking through thermal blooming	3
2-1. Thermal blooming in quiescent air	11
2-2. Thermal blooming in a constant wind	13
2-3. DF laser ($\lambda=3.8 \mu\text{m}$) absorption model (McClatchey and Selby, 1974)	21
2-4. Atmospheric wind velocity distribution model (USAF Handbook of Geophysics, 1960)	22
2-5. Atmospheric temperature distribution model (USAF Handbook of Geophysics, 1960)	23
2-6. HELPRP forward propagation flow chart	26
3-1. Typical bloomed point spread functions received by tracker	30
3-2. Bloomed PSF widths (at tracker) as a function of position about far field centroid	33
3-3. Typical bloomed point spread functions at final image plane	36
3-4. Point spread function generation	38
3-5. Point spread function generation: initial, forward propagation	39
3-6. Point spread function generation: back propagation	40
3-7. Point source irradiance profile	43
3-8. Plane wave approximation (at tracker)	44
3-9. Vacuum tilt parameters	45
3-10. Point source tilt approximation	47

LIST OF ILLUSTRATIONS--Continued

Figure	Page
3-11. Displacement of bloomed PSF centroid from center of tracker as a function of distortion number	49
4-1. Imaging algorithm	54
4-2. Typical target image reconstruction	56
5-1. Bloomed far field irradiance distributions	66
5-2. Vacuum target image	72
5-3. Bloomed target images	73
5-4. Return wavelength sensitivity	80
5-5. Geometric image distortion	82
5-6. Geometrical distortion as a function of distortion number	89
5-7. Relative intensity as a function of distortion number . . .	92
5-8. Bloomed far field irradiance and target image width as a function of distortion number	94
5-9. RMS normalized irradiance difference as a function of distortion number	96

LIST OF TABLES

Table	Page
3-1. Baseline cases for vacuum tilt approximation study	37
5-1. Scenarios studied	65

ABSTRACT

An assessment is made of the impact of a thermally bloomed atmosphere on target image quality. The steady-state phase perturbations due to blooming and the effects of the distortion on return wavefronts are determined using a physical optics propagation code. The wavelength of the return radiation used to sense the distortions is shown to be an important consideration in the stability of the return image. The return targets images for several realistic scenarios are reconstructed based on the incoherent point spread function for each isoplanatic region of the object (target) plane. Special requirements of the back propagation algorithm are discussed with respect to "point source" geometry, tilt, tilt sensitivity, and isoplanatism criterion. The wave optics approach is shown to agree with the experimental results in regards to image blur and distortion. The stability of the reconstructed images is discussed using merit functions defining the amount of blur, distortion, and degradation in peak irradiance. The merit functions are shown to correlate well with the scenario distortion number used to define the degree of blooming induced in the forward propagation.

CHAPTER 1

INTRODUCTION

Several processes occur when laser radiation propagates through the atmosphere. The nature of these processes and the degree to which they occur depend largely on the energy density of the radiation and the constituents of the propagation medium. In the case of low power propagation, the effects of the radiation are linear, i.e., the medium itself is not effected by the presence of the radiation. However, as the power of the laser beam is increased, a larger fraction of its energy is absorbed by the medium. The absorbed energy results in a rise in temperature corresponding to the local irradiance, which in turn decreases the density and refractive index of the medium. This alters the initial energy distribution of the beam which results in a change in the temperature gradient. The irradiance distribution of the beam is continually updated in this manner until, under the appropriate conditions, steady state is reached. These atmospheric processes cause a thermal defocusing or "blooming" of the laser beam which results in a lower average irradiance at the focal (target) plane.

Tracking a target in a shared-aperture, high-energy laser (HEL) system is necessary in cases where a minimum amount of energy must be delivered to the target in a given time period in order to disable or destroy it. Since the tracker "views" the target through some portion

of the thermally bloomed atmosphere, the location and image of the target and subsequent accuracy in tracking, is deteriorated in two respects. One effect is blurring or spreading in the image due to induced phase curvature perturbations on the return wavefront. The other effect is image distortion due to tilt perturbations. Both effects are nonlinear with respect to the transmitted laser power and are the result of thermally-induced index variations both transverse to and along the propagation path. As the back propagated wavefront from the target intercepts portions of the bloomed region, the tracking error from these effects can become significant, leading to poor tracking system performance. This is illustrated in Figure 1-1.

The importance of knowing the appearance of the target a priori is in being able to predict and possibly compensate for the effects of the phase distorting medium. The objective of this study is to investigate the degradation and stability of a target image with respect to the system's impulse response and parameters which characterize the high energy laser engagement scenario. This method of describing the effects of blooming in the forward propagation has been found to be effective in determining the degradation in peak irradiance at the target plane.

Previous Work

Relatively little has been published in the literature regarding imaging through a thermally bloomed atmosphere. The main reason for this is that blooming, in its relation to high-energy lasers as weapon systems, has established itself as an industry problem rather than an academic one.

Fig. 1-1. Tracking through thermal blooming.

The earliest (and only) paper published in the literature which discusses the influence of blooming on return images is by Gebhardt (1972). Gebhardt showed in a series of scaled laboratory experiments the effects of phase curvature and tilt perturbations induced by a collimated CO₂ beam. He described the image degradation in terms of a warping effect analogous to deflection of the beam and a demagnification in the central portion of the image. An additional overall effect was that of image blur analogous to a "low pass" filter.

In his analysis of the experiment, Gebhardt approximated the effects of blooming by assuming the induced distortion and tilt was concentrated in a small region of the propagation path, i.e., a single, negative thick lens. This approach has been adopted by several others in analyzing blooming, for example, Hermann (1977) and Fleck, Morris, and Feit (1976). The focal length of the lens and separation of the principal planes and therefore the degree of distortion and blur, was correlated with the scenario distortion number. A similar technique is used in computer modeling of the propagation, namely, the concept of discrete phase screens placed at appropriate locations along the path.

Gebhardt found that significant image distortion occurs even for small values of the distortion number, where the deflection and irradiance degradation are relatively small. This was also shown by Pringle, Horwitz, and Politopoulis (1978), particularly in the case of low slew, self-defense scenarios. Pringle's study discussed tracking with images aberrated by blooming as well as thermal bowing, which creates additional blur in the image due to bending or bowing of the mirrors in the optical

train. The effects of blurring and distortion were simulated by convolving each point source in a grid at the target plane with a modified point spread function. Each point spread function, determined by a first-order back propagation technique, was approximated by a two-dimensional Gaussian distribution. Each Gaussian included an additional Zernike sum corresponding to phase perturbations due to blooming for an on-axis target point which was established in the initial, forward propagation.

Pringle showed the effect of nonisoplanatism by plotting the magnitude of the axis of the modified Gaussian as a function of position of the point source location in the target plane. The rate of change of the size of the Gaussian is an approximate indication of the regions of isoplanicity, i.e., for small changes between target positions within the isoplanatic patch, the point spread function will remain fairly constant.

Problem Approach

Gebhardt and Pringle have established a base from which to complete a more thorough study of imaging through a bloomed medium. This study includes:

- (i) The development of a full, wave optics, FFT (Fast Fourier Transform) algorithm for back propagation to sense the phase distortions induced in the forward propagation.
- (ii) The development of an incoherent imaging algorithm for the reconstruction of target images as viewed by a full, spectrally-shared aperture-tracker.

- (iii) An examination of the assumptions made in generating the incoherent point spread functions used in the image reconstruction.
- (iv) An examination of image blur, distortion, and peak irradiance degradation as a function of the scenario distortion number.

Outline of Remaining Chapters

A qualitative description of thermal blooming is given in Chapter 2. The induced phase change due to blooming is derived for a segment of the propagation path. This is followed by a description of the propagation code, HELPRP, which calculates and stores the phase deformation at each screen location. Also discussed is the effect of the back propagated wavefront on phase screen placement determined in the forward propagation.

Chapters 3 and 4 discuss the point spread function (PSF) generation and imaging algorithm used to perform the incoherent image reconstruction of targets. Several important considerations are presented regarding the point source geometry, tilt, tilt sensitivity, and wavelength of the return radiation. Nonisoplanicity of the bloomed PSFs is discussed in the context of Pringle's criterion and its limitations. This is followed by a detailed discussion of the imaging algorithm, how this algorithm relates to the PSF generation, and how the algorithm is used with other propagation codes. The algorithm's advantages, assumptions, and limitations are outlined.

Chapter 5 presents the results of the image reconstruction for various levels of blooming in some specific scenarios. Total image stability with respect to blur, distortion, and degradation in peak irradiance is shown to correlate well with the forward propagation distortion number. This is followed by suggestions for future work in extending the analysis and simulation.

A summary of the study and discussion of important conclusions is given in Chapter 6.

CHAPTER 2

THERMAL BLOOMING

The basic sequence of processes which occur in the atmosphere that result in thermal blooming was briefly discussed in the last chapter. These processes are described in more detail in this chapter, with comments regarding the assumptions and limitations made in modeling the theory and computer simulation.

Background

Thermal blooming is a nonlinear process due to absorption of the laser radiation and subsequent heating of the medium, inducing wavefront phase distortions. The interaction between the energy distribution of the beam and the refractive index distortions can, under the appropriate conditions, result in a steady-state situation. The time for this to occur depends on the thermal characteristics of the absorbing gas (e.g., time constants associated with the molecular relaxation time), and is a function of the laser power and transverse wind velocity.

In order to understand these processes and accurately model them, it is necessary to borrow from several areas such as fluid dynamics, thermodynamics, geometrical and physical optics, and numerical methods. To simplify the model, it is first necessary to establish some ground rules. The following assumptions parallel those outlined by Hogge (1974)

and are usually made in describing the behavior of the thermal blooming process:

- (1) The high-energy laser beam has an unperturbed irradiance that is defined by a Gaussian profile. This is not always necessary since many "ideal" irradiance distributions, e.g., the uniform distribution used in this study, will approach a Gaussian due to diffraction as they propagate.
- (2) The propagation medium passes across the beam at a constant, uniform velocity. The velocity is constrained to a plane perpendicular to the direction of propagation and with a magnitude somewhat less than the speed of sound.
- (3) There is no turbulence of the propagation medium, only laminar flow across the beam. This refers to mechanical, as well as atmospheric turbulence.
- (4) Heating of the atmosphere due to absorption of energy from the beam is assumed to occur instantaneously.
- (5) The hydrodynamic processes in the atmosphere are essentially isobaric so that instantaneous temperature changes result in instantaneous density and refractive index changes. "Instantaneous" in this sense refers to a time scale which is similar to the time it takes for pressure differences to propagate at the speed of sound across the beam diameter.

Ignoring the second assumption for the time being, consider the pure blooming case where there is no transverse wind. Referring to Figure 2-1, the temperature of the quiescent air rises as a result of absorption and as a function of the beam's irradiance profile. This causes the air near the warmest part of the beam to become less dense as it flows away from the center at the speed of sound. Consequently, the index of refraction profile which is proportional to the local air density, decreases, resulting in a longer optical path near the edges of the beam. The result is the effect of a negative lens. This defocusing or thermal blooming has been observed in numerous scaled laboratory experiments. (See, for example, Gebhardt and Smith, 1972.)

Now consider the case where the air is no longer stationary. For example, natural winds, system-induced winds (laser aboard an aircraft), and beam slewing. The net effect of these winds is air movement across the laser giving rise to a steady-state irradiance profile, depending on the velocity of the wind and medium characteristics. Although the time-dependent problem is significant, especially with respect to adaptive optics correction and multiple-pulse propagation, only the steady-state aspect of the blooming problem will be considered in this analysis.

(The possibility of a zero net wind at some point along the propagation path is rare but can occur, resulting in a "stagnation zone." This is the result of beam slew and wind in the same direction so that at some point along the path, the tangential velocity associated with the slew is equal to that of the wind. The result is zero net wind,

Fig. 2-1. Thermal blooming in quiescent air.

with the air mass remaining in the beam. This effect has been looked at by various workers [see, for example, Hogge and Butts, 1973]. However, since a no slew condition is imposed in the cases studied, the possibility of a stagnation zone is eliminated.)

Assume that the cross wind is constant in magnitude and flowing in the - x-direction. The effect of the wind is best seen by considering a small volume of air as it passes across the beam, as shown in Figure 2-2. As the air mass passes across the beam it becomes warmer. The air density decreases as the temperature rises. However, due to the direction of the wind, the density and index profile that results is no longer symmetrical about the transverse axis perpendicular to the wind. Instead, an index profile evolves that is proportional to the integrated irradiance across the beam. Thus, for a Gaussian distribution, the resulting index profile is similar in form to the "error function," as shown in the figure. The resulting irradiance becomes skewed into the wind. In cases of mild to moderate blooming, it has been shown (Gebhardt and Smith, 1971) that the bloomed centroid position relative to the optical axis is proportional to α/v , where α is the absorption coefficient and v , the magnitude of the velocity along the tilt axis. At high wind speeds, the accumulated temperature difference across the beam may not be enough to cause significant phase changes. The resulting irradiance may be near diffraction-limited with the only effect being tilt. However, for sufficiently slow wind speeds such that the air volume rises in temperature, enough to create large phase changes, the irradiance distribution will continue to bloom.

Fig. 2-2. Thermal blooming in a constant wind.

From this qualitative description of the blooming process, it can be seen that the beam transit time, t_{tr} , where

$$t_{tr} = \text{beam diameter/wind velocity}$$

is an important parameter in determining the threshold for blooming. If the time constant associated with the atmospheric processes is given by τ , then for $t_{tr} \ll \tau$, very little energy will be absorbed and blooming will be negligible. For $t_{tr} \gtrsim \tau$, blooming will be the main source of phase distortion. These two regimes will clearly be seen in the cases studied.

Forward Propagation Model for Thermal Blooming

This section derives some of the basic equations that characterize thermal blooming in steady-state. The purpose of the analysis is to present the assumptions normally made in reducing the wave equation to a form which depends on the atmospheric parameters which contribute to blooming, namely, density and index. The resulting change in index is expressed in terms of the total phase change over an increment of the propagation path. The phase change derived is used in the propagation algorithm to calculate the phase distorting screens.

The Reduced Wave Equation

The reduced form of the wave equation used in simulating the effects of blooming is given by (Hogge, 1972):

$$\nabla_T^2 U - 2ik_0 n_0 \frac{\partial U}{\partial z} + k_0^2 [n^2 - n_0^2] U = 0 \quad (2.1)$$

where $U(\vec{r}, z; t=0)e^{-ik_0 n_0 z}$ corresponds to the Fresnel integral solution at time $t=0$. The other variables in Eq. (2.1) are:

$\vec{r}$ = radius vector of position transverse to the direction of propagation

n_0 = nominal index of refraction prior to heating

n = index after heating

k_0 = wavenumber = $2\pi/\lambda$

The assumptions made in deriving Eq. (2.1) include:

- (1) The differential operator ∇_T^2 is an approximation to the Laplacian, i.e., the field U is assumed slowly varying along the propagation path (+ z -direction). Hence, the subscript "T" for transverse coordinates only:

$$\nabla_T^2 \triangleq \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (2.2)$$

- (2) The dielectric constant of the air is both spatially and temporally invariant, even though density changes occur due to heating. At optical frequencies, these changes are slow enough to neglect any time derivatives relative to changes in the field along the propagation path.
- (3) The absorption coefficient of the propagation medium is independent of the beam irradiance, i.e., the "small signal" absorption value is assumed. This assumption neglects the effect of gain saturation or bleaching.

- (4) The laser is assumed to be a stabilized device, emitting radiation that is quasi-monochromatic. In relation to (3), even though there may be a small spectral bandwidth about the nominal frequency, it is assumed that all of the power is associated with the single wavelength, thus eliminating the need for an averaged or power weighted absorption model.
- (5) The field amplitude does not change significantly in the time it takes for the light to propagate from the transmitter to the target.

These assumptions form the basis for most physical optics propagation codes used in determining the effects of blooming, as well as turbulence and the use of adaptive optics. While some of the assumptions may seem unrealistic, the results of the codes have been consistent with experimental models (Gebhardt and Smith, 1971, 1972).

Calculation of Density and Refractive Index Change

The governing equations used in determining the change in air density as a result of the absorption of energy are given by (Hogge, 1972; Avizonis et al., 1972):

- (1) Conservation of mass:

$$\frac{d}{dt} \rho + \rho \vec{\nabla} \cdot \vec{v} = 0 \quad (2.3)$$

- (2) Conservation of momentum:

$$\rho \frac{d\vec{v}}{dt} + \vec{\nabla} p = 0 \quad (2.4)$$

(3) Energy balance:

$$\rho \frac{dE}{dt} + p \vec{\nabla} \cdot \vec{v} = \rho \dot{Q} \quad (2.5)$$

(4) Internal energy of a fluid:

$$E = C_v T \quad (2.6)$$

(5) Equation of state for an ideal gas:

$$p/\rho = (C_p - C_v) T \quad (2.7)$$

where:

ρ = gas density

v = velocity of the gas particle

p = gas pressure

E = internal energy of gas per unit mass

T = temperature of gas or fluid prior to heating

C_p = specific heat at constant pressure

C_v = specific heat at constant volume

$\dot{Q}$ = energy deposited per unit mass per second in gas.

In order to derive the change in air density as a function of absorbed energy, it is necessary to make the following assumptions:

- (1) Variations about the nominal index are small enough to make the approximation:

$$n^2 - n_0^2 \approx 2n_0 \Delta n \quad (2.8)$$

where $n = n_0 + \Delta n$, $\Delta n \ll n_0$. In air at STP, $\Delta n \approx 2.92 \times 10^{-4}$.

(2) Use of the Gladstone-Dale relationship:

$$\frac{n-1}{\rho} = \frac{n_0-1}{\rho_0} \quad (2.9)$$

where ρ_0 is the quiescent air density prior to absorption and heating and ρ is the density afterwards.

(3) The velocity of the wind is such that

$$v_0^2 \ll c_\infty^2$$

where c_∞ is the speed of sound in a quiescent fluid. This assumption is generally true and simplifies the derived equation for the first-order density perturbation.

The first-order effects of blooming on index and phase are determined by expressing the density, pressure and wind velocity in a series expansion about their initial values and equating like terms after combining Eqs. (2.5) through (2.7). It is basically a linearization of the given hydrodynamic equations and is referred to as such since only terms through first-order are retained. The resulting steady-state solution for the air density is given by (see Appendix A):

$$\frac{\Delta \rho}{\rho_0}(x, y, z) = - \frac{(\gamma-1)}{\gamma v_0 p_0} \int_{-\infty}^x \rho_0 \dot{Q}(\beta, y, z) d\beta \quad (2.10)$$

where $\gamma \triangleq c_p/c_v$ and $\rho \triangleq \rho_0 + \Delta\rho$, $\Delta\rho \ll \rho_0$. The heat deposition term, $\rho_0 \dot{Q}$, is a function of the laser wavelength and absorption-thermalization process of the propagation medium. In the case of instantaneous heating (as was previously assumed):

$$\rho_0 \dot{Q}(x,y,z) = \alpha(z) I(x,y,z) \quad (2.11)$$

where $\alpha(z)$ is the absorption coefficient and $I(x,y,z)$, the beam irradiance distribution at position z along the propagation path. Thus, Eq. (2.10) can be rewritten as:

$$\Delta\rho(x,y,z) = - \frac{(\gamma-1)\rho_0 \alpha(z)}{\gamma v_0 p_0} \int_{-\infty}^x I(\beta,y,z) d\beta \quad (2.12)$$

From Eq. (2.8),

$$\Delta n(x,y,z) = (n_0-1) \frac{\Delta\rho}{\rho_0} \quad (2.13)$$

Using this result and Eq. (2.9), the index perturbation corresponding to Eq. (2.12) is given by

$$\Delta n(x,y,z) = \frac{(n_0-1) \alpha(z)}{\rho_0 c_p T v_0} \int_{-\infty}^x I(\beta,y,z) d\beta \quad (2.14)$$

Eq. (2.14) describes analytically what is occurring in Figure 2-2. As the volume of air passes across the beam's irradiance distribution, it accumulates energy through absorption. The result is an index perturbation in the transverse direction that is proportional to the change in density. The index, in turn, modifies the initial

irradiance distribution at the point $z'=z+\Delta z$. From this it is easy to visualize the nonlinear feedback process that occurs between the index and irradiance that qualitatively describes the blooming.

The wavefront phase error due to the change in index is determined by integrating the index perturbation over the propagation path increment, Δz :

$$\Delta\phi(x,y;\Delta z) = k \int_z^{z+\Delta z} \int_{-\infty}^x \Delta n(\beta,y,\xi) d\beta d\xi \quad (2.15)$$

or,

$$\Delta\phi(x,y;\Delta z) = \frac{2\pi(n_o-1)}{\lambda c_p \rho_o} \int_z^{z+\Delta z} \int_{-\infty}^x \frac{\alpha(\xi) I(\beta,y,\xi)}{T(\xi) [v(\xi) + \dot{\theta}\xi]} d\beta d\xi \quad (2.16)$$

where the path length dependence (and thus, altitude dependence in the case of slant paths) of absorption, temperature and effective wind are left in the ξ -integration. The absorption, natural wind, and temperature profiles used in this study are shown in Figures 2-3 through 2-5. The "effective" wind is defined as the vector sum of the transmitter velocity (including "natural" wind) and the slew contribution ($\dot{\theta}\xi$, where $\dot{\theta}$ is in radians per second).

Screen Placement

Thermal blooming is a distributed process, i.e., the phase perturbations extend continuously from the transmitter to the target plane. The distortions are distributed along the propagation path depending on the laser power, atmospheric absorption and wind speed

Fig. 2-3. DF laser ($\lambda=3.8 \mu\text{m}$) absorption model (McClatchey and Selby, 1974).

Fig. 2-4. Atmospheric wind velocity distribution model (USAF Handbook of Geophysics, 1960).

Fig. 2-5. Atmospheric temperature distribution model (USAF Handbook of Geophysics, 1960).

distribution, degree of focusing, initial irradiance distribution, and whether the beam is slewed. In the computer simulation, propagation through the bloomed atmosphere is approximated by transmission through several absorbing, phase distorting screens which are the "lumped constant" approximation to the continuous representation. The phase distribution at each screen is given by Eq. (2.16). The incident complex field is modified by the addition of the perturbed phase corresponding to the previous path increment, and propagated as though in a vacuum to the next screen. The purpose of this section is to discuss the placement of the screens and how it is effected by the back propagated field in the scenarios to be studied.

The general rule is that for diverging or collimated beams, the portion of the optical path that contributes most to the phase distortions is at, or very near, the transmitter. Beam slewing also tends to weight the distortions toward the transmitter plane. For focused beams, the distortions may occur well away from the transmitter, especially for large aperture systems where the distortions occur near the target plane. To accurately simulate the blooming, the screens are placed where the greatest distortions occur.

The magnitude and distribution of the bloomed phase distortions is a function of the same characteristics of the incident irradiance. Because of this dependence, if only a few screens (say about 3 or 4) are used to model the atmosphere, the screen placement to model the back propagation may differ from the forward locations. The placement is especially questionable in cases of severe blooming with a heavily

biased distribution resulting from, for example, a ground to space propagation or a low altitude (high absorption), short range scenario.

This concern is alleviated in this investigation by selecting appropriate, yet realistic scenarios for which the blooming is fairly evenly distributed along the propagation path. This is accomplished by assuming a nearly collimated beam (low Fresnel number) with little or no slew but sufficiently high transverse wind speed. The field is propagated through a medium characterized by constant absorption, i.e., a horizontal propagation. The situation is similar to an HEL system aboard an aircraft which is tracking a long distance target. The specific set of scenario variables is discussed in the next chapter. The assumptions made in the geometry of the back propagated wavefront will be seen to satisfy the requirements of the code, the physics of the propagation, and screen placement in both directions.

The HELPRP Propagation Code

The HELPRP (High Energy Laser ProPagation) diffraction code was used in the study. It is a fixed-mesh, three-dimensional optical program for the simulation of a high-energy laser in a thermal blooming environment. It has undergone extensive modifications from its original form (Mevers et al., 1977) in order to perform the back propagation, sensor imaging, and phase decomposition for use in an adaptive optics simulation. A listing of the code is given in Appendix B.

Propagation in the bloomed atmosphere is outlined in Figure 2-6. The phase distortions are determined over a 64 x 64 element grid which contains the complex field as it propagates from screen to screen. At

Fig. 2-6. HELPRP forward propagation flow chart.

each screen, the phase of the field is modified by the bloomed phase and propagated using the two-dimensional transfer function:

$$H(f_x, f_y) = \exp[ikz(1 - \lambda^2 f_x^2 - \lambda^2 f_y^2)^{\frac{1}{2}}] \quad (2.17)$$

where f_x and f_y are the spatial frequencies associated with the x- and y-axis, respectively. The FFT used in the code to perform the transitions from the spatial to the frequency domain and back is quite efficient and is also used in the imaging algorithm to perform convolutions.

After the phase screens are generated, they are stored for use in the back propagation. The return wavefront passes through the screens, sensing the effects of the bloomed phase, resulting in blur and distortion in the final image plane. This procedure is discussed in detail in the next chapter.

CHAPTER 3

INCOHERENT IMAGE RECONSTRUCTION-PART I

This chapter and the next discuss the theory and procedure used in determining the image of a target distorted by a thermally bloomed atmosphere as viewed by a full, spectrally-shared aperture-tracker. Classical techniques of incoherent imaging are used in reconstructing the target's image. The approximations made in the theory and in the imaging algorithm are discussed as well as limitations of the program.

Definition and Conditions

It is well known from linear systems theory that the image of an object in an incoherent image forming system is given by the two-dimensional convolution of the object irradiance distribution and the system's impulse response or point spread function (PSF):

$$I(x,y) = O(x,y) ** h(x,y) \quad (3.1)$$

where:

$I(x,y)$ = image irradiance distribution

$O(x,y)$ = object irradiance distribution

$h(x,y)$ = system point spread function

and "**" denotes a two-dimensional convolution.

Conditions need to be satisfied before applying Eq. (3.1). The object (or target in this application) is assumed to be a collection of

independent point radiators. In the case of active illumination, each point source results from reflecting the incident laser light in varying amounts depending on the object's geometry and reflectivity. In this sense, the reflected radiation is spatially incoherent. In the case where the target is treated as a blackbody, emitting thermal radiation in a wide spectral bandwidth but peaked in the 8 to 12 μm region, the return radiation can be considered as passive. In this sense, the object is also treated as a collection of point sources whose phases are uncorrelated so that no interference occurs between wavefronts from different points on the target. Target images in this study will be determined as though the object illumination was passive for a return radiation wavelength of 10 μm .

The single, two-dimensional convolution implies that the system impulse response does not change spatially with respect to geometry within the angle subtended by the object, i.e., the image forming system is stationary. In a thermally bloomed environment, index variations are sufficient in the direction transverse to propagation so as to cause changes in the structure of the point spread function for relatively small displacements of the point source in the object plane. The shift-invariant or so-called "isoplanatic regions" (or "patches") may be dimensionally similar to the diffraction limited spot size in cases of severe blooming. Back propagated wavefronts received by the tracker, corresponding to several different areas of a typically bloomed crescent generated in the forward propagation, are shown in Figure 3-1. The

Fig. 3-1. Typical bloomed point spread functions received by tracker.

imaging system used to determine these return irradiance distributions is discussed below.

The nonisoplanatic nature of the phase distortions will result in making the use of several PSFs necessary to reconstruct an extended target image. The number of PSFs depends on the size of the target and degree of blooming. In order to determine the size of the isoplanatic regions, it is necessary to establish a tolerance criterion for a characteristic parameter of the PSFs. Pringle et al. (1978) have suggested the following method, which has been shown to be adequate in specifying PSFs in cases where up to a 70% reduction in peak irradiance resulted due to blooming alone. Second-order moments for each PSF are determined in the direction parallel (x) and perpendicular (y) to the wind, and used as a measure of the PSF change with respect to point source location at the object plane. A similar approach will be used in this discussion, with the normalization of the moments being with respect to the vacuum propagation widths, i.e.,

$$\sigma_{x,N}^2 \triangleq \left(\frac{A_{VAC}}{A_B} \right) \frac{\iint h_B(x,y) (x-x_B)^2 dx dy}{\iint h_{VAC}(x,y) (x-x_{VAC})^2 dx dy} \quad (3.2)$$

$$\sigma_{y,N}^2 \triangleq \left(\frac{A_{VAC}}{A_B} \right) \frac{\iint h_B(x,y) (y-y_B)^2 dx dy}{\iint h_{VAC}(x,y) (y-y_{VAC})^2 dx dy} \quad (3.3)$$

where:

$$A_{VAC} \triangleq \iint h_{VAC}(x,y) dx dy \quad (3.4)$$

$$A_B \triangleq \iint h_B(x,y) dx dy \quad (3.5)$$

The subscripts "B" and "VAC" refer to the bloomed and vacuum cases, respectively. The centroid coordinates of the bloomed and vacuum PSFs are located at (x_B, y_B) and (x_{VAC}, y_{VAC}) . The unnormalized form of Eqs. (3.2) and (3.3) has been suggested by Hogge (1972) as an effective method of defining beam spread.

The rate of change of $\sigma_{x,N}$ and $\sigma_{y,N}$ as a function of point source location is a measure of isoplanacity, i.e., local isoplanatism defined as the region of the target where $\sigma_{x,N}$ and $\sigma_{y,N}$ remain fairly constant. These two quantities are shown in Figure 3-2 for the degree of blooming in Figure 3-1. The widths correspond to the irradiance distribution after imaging by a tracker (sensor). Several target images were reconstructed using different sizes of isoplanatic regions to determine total image sensitivity (or error) as a function of patch size. An approximate 15% variation in the normalized PSF width to define a given isoplanatic region was arbitrarily decided upon for use as a maximum criterion for the cases studied.

The above method should be approached with caution because two PSFs can result in similar widths but differ in shape, thus defining two slightly different target images. This can be especially true in cases of severe far-field blooming where the back propagated point sources intercept regions where large transverse phase gradients exist, causing distortion in the PSF. However, because of the decreased sensitivity of the phase distortions sensed by the 10 μm radiation and clipping of the wavefront by the tracker, the bloomed, imaged PSFs show much less distortion. The resulting symmetry about the x- and y-axis is sufficient to

x = distance along x-axis through bloomed crescent centroid
($x=0$ at centroid)

σ_x = x-component of bloomed crescent spot size = 0.35m

σ_{VAC} = x-component of vacuum PSF spot size = 0.19m

$\sigma_{\text{PSF},x}$ = x-component of bloomed PSF spot size

$\sigma_{\text{PSF},y}$ = y-component of bloomed PSF spot size

(a)

Fig. 3-2. Bloomed PSF widths (at tracker) as a function of position about far field centroid.

(a) Along x-axis through centroid.

y = distance along y -axis through bloomed crescent centroid
($y=0$ at centroid)

σ_y = y -component of bloomed crescent spot size = 0.42m

σ_{VAC} = y -component of vacuum PSF spot size = 0.19m

$\sigma_{PSF,x}$ = x -component of bloomed PSF spot size

$\sigma_{PSF,y}$ = y -component of bloomed PSF spot size

(b)

Fig. 3-2.--Continued

(b) Along y -axis through centroid.

justify the use of PSF width as a criterion. This is shown in Figure 3-3.

Point Spread Function Generation

As discussed in Chapter 2, the HELPRP propagation code is used to perform both the forward and backward propagations. Scenarios have been chosen which result in sufficient resolution at each phase screen for the fixed-size mesh. This is accomplished by choosing the range so that the far field diffraction spot diameter is approximately the same size (or slightly less) than the transmitting aperture diameter, i.e.,

$$R \approx D^2/2.44\lambda \quad (3.6)$$

or about 100 km for $\lambda = 3.8 \mu\text{m}$ and the aperture diameter, $D = 1 \text{ m}$. Possible scenarios are listed in Table 3-1. They are sufficiently general and with realistic characteristics to justify the use of a fixed-mesh code. Figures 3-4 through 3-6 outline and illustrate the forward and reverse propagation algorithms used in generating the bloomed PSFs based on these scenarios.

Point Source Considerations

Ideally, a true point source (two-dimensional delta function) is used to determine a system PSF. However, when using an FFT propagator, the Fourier transform of the point source immediately fills the entire mesh, thus introducing a propagation error because the guard band requirements are no longer satisfied. (The guard band is taken to be 40 - 60% of the mesh width.) To eliminate the problem, the complex field

Fig. 3-3. Typical bloomed point spread functions at final image plane.

Table 3-1. Baseline cases for vacuum tilt approximation study.

Constants:Wavelength (Forward Propagation) = 3.8 μm Wavelength (Back Propagation) = 10 μm

Wind Speed = 100 m/sec

Fresnel Number = 3.833

<u>Set</u>	<u>Range, km</u>	<u>Diameter, m</u>	<u>Transmission</u>
I	27.0	0.5	0.934
II	107.85	1.0	0.760
III	431.38	2.0	0.334

	<u>Cases For Each Set</u>					
<u>Case</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
Power, MW	0.1	0.2	0.4	0.8	1.6	3.2

Fig. 3-4. Point spread function generation.

Fig. 3-5. Point spread function generation: initial, forward propagation.

Fig. 3-6. Point spread function generation: back propagation.

defining a point source is approximated by a small Gaussian amplitude distribution given by:

$$U_{PS}(r; z=R) = e^{-[r^2/\omega_o^2 + i\phi_o(r; z=R)]} \quad (3.7)$$

where:

$$r = \sqrt{x^2 + y^2}$$

$$\omega_o = \text{beam waist}$$

$$\phi_o(r; z=R) = \text{phase}$$

The two variables ω_o and ϕ_o are important approximations and will each be discussed in detail.

The value of ω_o is chosen such that when the field given by Eq. (3.7) is back propagated along the optical axis through a vacuum, the irradiance of the Gaussian beam has fallen to its e^{-1} value at the edge of the tracking aperture. Thus,

$$I(r=D/2; z=0) = e^{-2r^2/\omega^2(r,0)} = 0.37 \quad (3.8)$$

or, $\omega(r,0) = D/\sqrt{2}$, where $\omega(r,0)$ is the beam spot size at the tracker.

The value of the beam waist in order to satisfy Eq. (3.8) is given by (Kogelnik and Li, 1966):

$$\begin{aligned} \omega_o &\triangleq \frac{\lambda R}{\pi \omega} \bigg|_{\omega=D/\sqrt{2}} \\ &= \frac{\sqrt{2} \lambda R}{\pi D} \end{aligned} \quad (3.9)$$

The point source irradiance distribution with characteristic width as given by Eq. (3.9) is shown in Figure 3-7. Also shown is the diffraction profile for the initial, forward propagation (both normalized to unity).

A true point source back propagated in a vacuum to the receiving aperture would result in a uniform irradiance distribution. The approximation given by Eq. (3.7) results in a scaled Gaussian. The imaged irradiance distributions of these two functions must be compared to determine the validity of the Gaussian "point source" approximation. Figure 3-8 shows the effect of the truncated Gaussian as an approximation for the uniform distribution at the tracking aperture. The received wavefront is clipped due to the finite diameter of the tracker, assumed to be the same size as the transmitting aperture. The clipped wavefront is focused at a distance equal to the target range to achieve uniform sampling with the fixed-mesh. It can be seen that the width and shape of the two imaged wavefronts is nearly the same (within about 6%).

The tilt of the phase associated with the Gaussian point source is chosen such that a ray normal to the phasefront will intersect the center of the transmitting aperture in a vacuum, as shown in Figure 3-9. In Eq. (3.7), the tilted phasefront is given by:

$$\phi_o(x,y) \triangleq \psi_{x-TILT} + \psi_{y-TILT} \quad (3.10)$$

where:

$$\psi_{x-TILT} = k x \sin \theta_{x-TILT} \quad (3.11)$$

Fig. 3-7. Point source irradiance profile.

Fig. 3-8. Plane wave approximation (at tracker).

- (a) Clipped, focused plane wave.
 (b) Truncated, focused Gaussian.

Fig. 3-9. Vacuum tilt parameters.

$$\psi_{y\text{-TILT}} = k y \sin \theta_{y\text{-TILT}} \quad (3.12)$$

The angles $\theta_{x\text{-TILT}}$ and $\theta_{y\text{-TILT}}$ are the projections of the ray relative to the optical axis in the xz- and yz-planes, respectively. The variables x and y are referenced to a coordinate system centered at $(x = x_c, y = y_c, z = R)$, where (x_c, y_c) is the location of the bloomed irradiance centroid at the target plane ($z = R$). From the figure, the tilt angles are given by,

$$\theta_{x\text{-TILT}} = \tan^{-1} \left(\frac{x_c}{R} \right) \quad (3.13)$$

$$\theta_{y\text{-TILT}} = \tan^{-1} \left(\frac{y_c}{R} \right) \quad (3.14)$$

Due to the nature of the phase distortions in the atmosphere, the forward propagation results in a wavefront that, in general, does not follow a straight path to the focal plane. By reciprocity, the same can be implied about point sources propagating back through the blooming towards the tracker, as shown in Figure 3-10. The validity of assuming a vacuum derived tilt in directing the point source in order to sense the proper phase distortions on back propagation is questionable and will be dealt with below. Physically, each incoherent, radiating point on the target produces a full, spherical wavefront. The calculation to be carried out must select the actual portion of the return wavefront which remains after clipping by the tracking aperture. This will involve some assumptions in artificially weighting the return wavefront, which will be shown to produce a correct result with negligible error.

Fig. 3-10. Point source tilt approximation.

Figure 3-11 shows the result of the tilt approximation for various levels of blooming. The scenarios examined are given in Table 3-1. The ordinate in the figure shows the (absolute) displacement between the centroid of the bloomed PSF (prior to clipping) and the center of the tracking aperture. The function plotted along the abscissa is the distortion number, N_D . This parameter will be discussed in detail in Chapter 5. For now, it will suffice to define N_D as simply a measure of blooming which is proportional to the induced phase change.

The point source (Gaussian approximation) used in generating Figure 3-11 is, in all cases, located at the centroid of the bloomed irradiance distribution in the target plane. Even in cases of severe blooming ($N_D \gtrsim 10$), the displacement or error is less than 10% of the tracker diameter. This implies that the phase distortions, sensed along the direction resulting from the vacuum tilt approximation, are nearly the same as those for the exact ray path.

The size of the tracking aperture relative to the transmitter depends on several conditions such as the laser wavelength and power, and necessary signal-to-noise with respect to energy in the return wavefront. For simplicity, the two are made equal in this study. This represents a realistic situation, namely, a full, spectrally-shared aperture-tracker. The spectral-sharing is the result of using, for example, a grating to cover the entire transmitting aperture or a relayed image of the aperture. The return $10\text{ }\mu\text{m}$ radiation is reflected while the $3.8\text{ }\mu\text{m}$ is transmitted. This represents an idealization of the actual sensing package. Making the tracker equal to the transmitter diameter

Fig. 3-11. Displacement of bloomed PSF centroid from center of tracker as a function of distortion number.

results in only a small change in the centroid location of the back propagated wavefront before and after clipping. Since the centroid calculation is weighted by the irradiance distribution, the centroid location will remain constant only if the irradiance distribution stays symmetrical or has decreased significantly away from the centroid. In general, blooming will cause the irradiance distribution to become skewed, as shown in Figure 3-1. This implies that the centroid location remains fairly constant because the irradiance contribution beyond the boundary of the tracking aperture is negligible. Thus, the energy, and therefore, the wavefront information loss is small.

Determination of the exact tilt angle of the point source to place the bloomed PSF centroid in the center of the tracking aperture requires a search about the vacuum tilt. In general, the vacuum tilt will be too small since the wavefront bends into the wind. A search for the exact angle will determine the tilt sensitivity, i.e., centroid movement in the tracker as a function of point source tilt. This sensitivity was determined for the cases in Table 3-1, with the x-tilt varied up to 20% beyond the vacuum value ($\approx \mu$ radians). The variation in centroid displacement was found to be around 0.11 m/ μ rad. The constant and relatively small sensitivity over such a wide range of blooming implies that the vacuum tilt is sufficient in sensing the phase distortions (as was suggested by Figure 3-11).

Finally, one method of checking the validity of the back propagation algorithm is to see if the vacuum PSFs are stationary, or shift-invariant in the final image plane, i.e.,

$$h_{\text{VAC}}(x,y) = A \cdot h_{\text{VAC}}(x-x_0, y-y_0) \quad (3.15)$$

The scaling factor "A" is to account for the decrease in peak irradiance as the field angle, and therefore range, increases. Eq. (3.15) implies that for the vacuum case, the imaging system is isoplanatic over the entire field. Several vacuum PSFs were compared. The widths (spot sizes) were found to be nearly equal except near the extreme edge of the field, where the width increased by about 18%. The "field" in this case is defined as the inside edge of the guard band, where in the extreme case, 60% of the grid width is allocated to the wavefront. The result is some skewness of the irradiance distribution as it propagates by means of the FFT. For the cases studied, only the last one begins to violate the guard band requirements in generating the point source grid about the bloomed irradiance centroid. The effects of this will be seen in Chapter 5.

CHAPTER 4

INCOHERENT IMAGE RECONSTRUCTION-PART II

Target (Object) Considerations

Three parameters define a target: geometry or shape, size, and irradiance distribution. Each of these variables will be discussed in this section with respect to the chosen value or function used in the simulation.

Geometry

The shape of the target area is assumed to be circular. This is the simplest yet most realistic geometry to study as well as being one which is convenient in comparing bloomed and vacuum spot size as a criteria for image degradation. When used with the proper diameter and irradiance profile, the circular geometry also provides an opportunity to compare the coherent, forward propagation irradiance with the incoherent, back propagated image.

Size

The target centroid is coincident with the bloomed far field irradiance centroid. The radius of the target is confined to about twice the spot size (radius) along the wind axis of the bloomed distribution. This corresponds to about a 1.15 m diameter, which is slightly larger than the diffraction spot diameter of the transmitter, and subtending

an angle of about $10 \mu\text{rad}$ at the tracker. The assumed width represents a reasonable sized target, e.g., a missile diameter or vulnerable region.

Irradiance Distribution

Even though the imaging algorithm can be used with any target irradiance distribution, in the case of blackbody radiation, the irradiance distribution must correspond to the return or sensing wavelength. In this study, it is assumed that the target area to be imaged is at a uniform temperature and therefore, of uniform irradiance. Using either a uniform or Gaussian distribution to define the target is convenient because it is used as a weighting or probability distribution function for determining spot size as a characteristic parameter of the return images.

The Imaging Algorithm

The program, IMAGE, reconstructs the return target image using the PSFs generated in HELPRP. The algorithm is based on the incoherent imaging principles discussed in the previous chapter. A flow chart showing the basic steps in the reconstruction is shown in Figure 4-1. A listing of the program is given in Appendix C.

The driver of IMAGE is used to:

- (1) Choose the type of image (bloomed or vacuum)
- (2) Define the field or grid increment size
- (3) Define the number of isoplanatic regions

Fig. 4-1. Imaging algorithm.

- (4) Define the target and area of the target to be imaged
- (5) Determine the type of plot to display the image or partial image (two-dimensional contour, normalized number array, gray scale, or three-dimensional contour with the Calcomp plotter)
- (6) Call for the calculation of various parameters to characterize an array, e.g., spot size or centroid.

The target used can be generated within the code or a disk file containing a specific irradiance distribution can be read.

The sequence of steps used in the reconstruction is straightforward. The PSF corresponding to a given isoplanatic region of the target is read from a disk file. This region of the target is transferred to the center of a working array and a two-dimensional convolution between the target area and PSF is performed using an FFT. The resulting blurred and distorted image of the target patch is added to a total image array after adding the x and y tilt due to blooming. The process is repeated for all of the isoplanatic regions of interest in the target plane, each time summing the resultant image with the previous partial image. A typical sequence of image reconstruction is shown in Figure 4-2.

The two approximations made in the imaging algorithm are:

- (1) Displacements between the vacuum and bloomed PSF centroids are rounded to the closest grid increment. For a 64 x 64 array, this corresponds to about $1\frac{1}{2}\%$ of the total field, or a possible error of less than 1% per axis in the position of the partial image. This is felt to be tolerable in view of the resolution.

(a)

Fig. 4-2. Typical target image reconstruction.

(a) After 6 isoplanatic regions.

(b)

Fig. 4-2.--Continued

(b) After 9 regions.

(c)

Fig. 4-2.--Continued

(c) After 14 regions.

(d)

Fig. 4-2.--Continued

(d) After 16 regions (final image).

Smaller shifts would require interpolation of the grid at an increased cost with little gain in the accuracy of the total image.

- (2) As discussed in the previous chapter, the isoplanatic regions are determined with respect to PSF widths. The criterion was chosen such that finer sampling of the phase distortions would not appreciably increase the accuracy in reconstructing the total image. Also, it is assumed that additional tilt due to blooming within an isoplanatic region is constant. In view of the amount of distortion observed in cases of severe blooming (to be discussed), the validity of this assumption depends on the necessary accuracy needed in the total image. If the vulnerable region on a target is very small, finer sampling within the isoplanatic region may be justified.

Advantages

The imaging algorithm was developed to operate independently of the type of propagator used to generate the bloomed PSFs. This was done so that the image reconstruction can be based on the PSFs from a propagation code which has special capabilities. These capabilities might include the additional effects of atmospheric turbulence, thermal loading distortion in the optical train and pointer-tracker, or adaptive optics (deformable mirror). In addition, the use of a real, rather than ideal wavefront from an HEL device may need to be used to generate the initial phase screens. Phase distortions for real wavefronts can be significantly different than for the ideal case. The resulting image based on the back

propagated PSFs could be considerably more accurate in simulating the actual image in cases of strong blooming.

The imaging algorithm is based on a fixed-size mesh. However, some propagation codes use a variable-sized mesh to contain the beam for constant sampling. This does not pose a problem providing the mesh spacing is known so that either an expansion or reduction of the PSF array can be performed in order for the grid to remain compatible with the target distribution.

CHAPTER 5

ANALYSIS AND RESULTS

This chapter discusses the characteristics of the reconstructed target image for various levels of blooming. Image stability with respect to blur, distortion, and degradation in peak irradiance is shown to correlate well with the forward propagation, scenario distortion number.

Scenario Parameters

Four dimensionless parameters have evolved as a result of the development used in generating scaling laws to describe the effects of blooming in the forward propagation. These parameters uniquely define an engagement scenario and have been found useful in estimating the far field irradiance degradation in cases of ideal beams producing mild to moderate blooming (Gebhardt and Smith, 1971; Breaux, 1976; Nahrstedt, 1977a, 1977b). Scaling laws are "derived" by fitting the general form of a simplified, reduced equation to experimental results or the results of a full, wave optics propagation code, such as HELPRP.

The four parameters used to define the scenarios in this study are:

$$N_A = \text{absorption number} = \alpha R \quad (5.1)$$

$$N_F = \text{Fresnel or focusing number} = 2\pi a^2 / \lambda R \quad (5.2)$$

$$N_S = \text{slew number} = \dot{\theta}R/v \quad (5.3)$$

$$N_D = \text{distortion number} = \frac{-2(\partial n/\partial T)P\alpha R^2}{n_o \rho_o c_p v a^3} \quad (5.4)$$

where:

λ = laser wavelength

α = absorption coefficient of propagation medium for wavelength

a = transmitting aperture radius

$\dot{\theta}$ = slew rate

v = transverse wind speed

P = laser power

n_o = nominal index of propagation medium

ρ_o = nominal density of propagation medium

c_p = specific heat at constant pressure of propagation medium

$\frac{\partial n}{\partial T}$ = variation in index with respect to temperature of medium.

In cases where the parameter is a function of altitude, the integrated or average quantity should be used, e.g.,

$$N_A = \int_0^R \alpha(z) dz \quad (5.5)$$

where:

$z = h \sin \theta_{el}$ (h = altitude, θ_{el} = elevation angle)

$h(z = 0)$ = transmitter altitude

$h(z = R)$ = target altitude.

The scenarios studied are listed in Table 5-1. The absorption and Fresnel number have been held constant and a no slew condition imposed:

$$N_A = 0.274$$

$$N_F = 3.833$$

$$N_S = 0$$

The parameter I_{rel} shown in Table 5-1 is a measure of degradation in irradiance due to blooming and is discussed below.

The distortion number is varied by continually doubling the power level from 0.1 to 3.2 megawatts (MW). This results in

$$0.33 \lesssim N_D \lesssim 10.68 \quad (5.6)$$

corresponding from near vacuum propagation to severe blooming (beam break-up). The bloomed far field irradiance distributions for each of the cases is shown in Figure 5-1, (a) through (f).

The reasons for selecting the above values for these parameters are:

- (1) The Fresnel number is constrained by the fixed-size mesh of the propagation code. In order to achieve sufficient wavefront resolution and fairly constant sampling, only a limited range of acceptable, yet realistic scenarios exist.
- (2) Some of the effects of beam slewing on phase screen placement were discussed in Chapter 2. However, slew has been eliminated as an additional variable in this study since it only serves to

Table 5-1. Scenarios studied.

Constants:Wavelength (Forward Propagation) = 3.8 μm Wavelength (Back Propagation) = 10 μm

Wind Speed = 100 m/sec

Fresnel Number = 3.833

Slew Number = 0

Absorption Number = 0.274

Grid Increment = 4.76 cm

<u>Case</u>	<u>Power, MW</u>	<u>N_D</u>	<u>I_{rel}[*]</u>
1	0.1	0.333	0.884
2	0.2	0.667	0.789
3	0.4	1.335	0.657
4	0.8	2.670	0.480
5	1.6	5.339	0.305
6	3.2	10.679	0.241

*Forward Propagation

(a)

(b)

Fig. 5-1. Bloomed far field irradiance distributions.

(a) Case 1: $N_D = 0.33$.(b) Case 2: $N_D = 0.67$.

(c)

Fig. 5-1.--Continued(c) Case 3: $N_D = 1.34$.

(d)

Fig. 5-1.--Continued

(d) Case 4: $N_D = 2.67$.

(e) Case 5: $N_D = 5.34$.

complicate the problem further. Changing the power level in a constant transverse wind will introduce the necessary amounts of blooming, as well as insure a steady-state condition.

- (3) The purpose of this study is to investigate the relationship between image degradation and degree of blooming. As an important first step in this direction using a full, wave optics approach, the degree of blooming should be specified by a single parameter, preferably one of significance in defining the forward propagation and which has been used successfully in generating scaling laws in similar applications.

Image Stability and Merit Functions

The reconstructed vacuum and bloomed images of the target are shown in Figures 5-2 and 5-3, (a) through (f), for the cases in Table 5-1. Each of the proposed merit functions defining an image parameter will be discussed.

Distortion

The incoherent return images do not show nearly the same distortion as in the forward propagation. This is because the phase distortions due to blooming are generated at 3.8 μm and sensed at 10 μm , producing smaller tilt perturbations in the return wavefront. This is easily seen by examining the phase difference, $\Delta\phi$, along a ray path for two different wavelengths. Let

$$\Delta\phi_F = \frac{2\pi}{\lambda_F} (n_F - 1)z \quad (5.7)$$

Fig. 5-2. Vacuum target image.

(a)

Fig. 5-3. Bloomed target images.

(a) Case 1: $N_D = 0.33$.

(b)

Fig. 5-3.--Continued

(b) Case 2: $N_D = 0.67$.

(c)

Fig. 5-3.--Continued(c) Case 3: $N_D = 1.34$.

(d)

Fig. 5-3.--Continued

(d) Case 4: $N_D = 2.67$.

(e)

Fig. 5-3.--Continued

(e) Case 5: $N_D = 5.34$.

(f)

Fig. 5-3.--Continued

(f) Case 6: $N_D = 10.68$ (reconstruction marginal).

$$\Delta\phi_B = \frac{2\pi}{\lambda_B} (n_B - 1)z \quad (5.8)$$

where n is the index, z is the propagation distance, and the subscripts, F and B , refer to the forward and back propagations, respectively. For $\lambda_F = 3.8 \mu\text{m}$ and $\lambda_B = 10 \mu\text{m}$, the dispersion is negligible in air (Wolfe and Zissis, 1978), i.e., $n_F \approx n_B$. Thus,

$$\Delta\phi_B = \frac{\lambda_F}{\lambda_B} \Delta\phi_F \approx 0.38 \Delta\phi_F \quad (5.9)$$

where the (λ_F/λ_B) coefficient can be considered as a "phase scaling" term. If sensing is being done at a comparable wavelength, the effect of the distortions is seen immediately in cases of strong blooming, as shown in Figure 5-4, which compares $3 \mu\text{m}$ with $10 \mu\text{m}$ sensing.

The conclusion that sensing should be accomplished at a much longer wavelength to reduce the effect of phase distortions is misleading. The use of a longer wavelength laser to deliver a specified amount of energy to the target may induce less blooming than one of a shorter wavelength, due to the spectral dependence of absorption. Thus, even though the wavelength may be comparable to that sensed, the induced distortions may be sufficiently small to compensate for the increased sensitivity.

One method of checking the back propagation is to compare the bloomed and vacuum PSF irradiance centroid locations. The purpose is similar to the use of a Hartmann screen in optical testing to determine wavefront distortion. A grid of point sources imaged by the system

(a)

(b)

Fig. 5-4. Return wavelength sensitivity.

(a) Sensing at $10\mu\text{m}$.

(b) Sensing at $3\mu\text{m}$.

under test and the displacement of the PSFs from their "ideal" position is an indication of the types of aberrations present in the wavefront.

The result of this concept is shown in Figure 5-5, (a) through (f). The object is a square grid composed of 16 point sources located at the intersection points. The grid is centered about the centroid of the bloomed irradiance distribution in each case. The point sources are individually back propagated through either a vacuum or bloomed atmosphere to the tracker, where the wavefront is clipped and focused to the image plane. The centroids of the imaged PSFs produce the intersection points of the grid corresponding to vacuum or bloomed propagation. Also shown in each figure as a reference is the intersection point of the optical axis of the tracking aperture.

As mentioned in Chapter 2, for low laser power levels (or conversely, high wind speeds), the effect of the induced index gradient is mostly tilt. This is shown in Figure 5-5 (a) and (b) for low values of the distortion number. As the amount of absorbed energy increases, so does the degree of distortion. The demagnification in the central portion of the images and resulting pincushion distortion is identical to the experimental results of Gebhardt (1972). From these figures, it can be seen why the vulnerable area of a target may be difficult to track for any length of time, especially if the blooming is constantly changing due to variations in absorption and wind.

The distortion in these figures can be expressed in the form of a merit function. The function is simply the root-mean-squared (rms) difference between points on the vacuum grid and the bloomed image grid.

----- VACUUM
———— BLOOMED

(a)

Fig. 5-5. Geometric image distortion.

(a) Case 1: $N_D = 0.33$.

----- VACUUM
——— BLOOMED

(b)

Fig. 5-5.--Continued

(b) Case 2: $N_D = 0.67$.

----- VACUUM

———— BLOOMED

(c)

Fig. 5-5.--Continued

(c) Case 3: $N_D = 1.34$.

----- VACUUM
———— BLOOMED

(d)

Fig. 5-5.--Continued

(d) Case 4: $N_D = 2.67$.

----- VACUUM

———— BLOOMED

(e)

Fig. 5-5.--Continued

(e) Case 5: $N_D = 5.34$.

(f)

Fig. 5-5.--Continued

(f) Case 6: $N_D = 10.68$.

The rms geometrical distortion along the x-axis is defined as:

$$D_{\text{rms}}(x) \triangleq \frac{1}{N} \left[\sum_{i=1}^N (x_{i,B} - x_{i,v})^2 \right]^{1/2} \quad (5.10)$$

where:

$x_{i,B}$ = x-axis position of the i th bloomed centroid

$x_{i,v}$ = x-axis position of the i th vacuum centroid

N = total number of points within grid ($i \leq N$).

A similar expression can be defined for the y-axis:

$$D_{\text{rms}}(y) \triangleq \frac{1}{N} \left[\sum_{i=1}^N (y_{i,B} - y_{i,v})^2 \right]^{1/2} \quad (5.11)$$

A plot of D_{rms} for each axis as a function of the distortion number is shown in Figure 5-6. The vertical shift between the x and y components is due mainly to the additional x-tilt from the wavefront bending into the wind. The curves generated by the data points are quite well-behaved for $N_D \lesssim 8$. Beyond this, the blooming becomes so severe that the far field irradiance begins to break up (occurring at about $N_D \approx 10$), as shown in Figure 5-1 (f) for Case 6. Also, the off-axis point source is just passed the inside border of the guard band so that the back propagation is introducing some error due to spreading from diffraction beyond the edge of the mesh. This is shown as a slight compression of the right side portion of the grid. For this reason, Figure 5-3 (f) has been labeled as a marginal reconstruction.

Fig. 5-6. Geometrical distortion as a function of distortion number.

Relative Intensity

Relative intensity is a measure of the degradation in peak irradiance at a focal plane. It is similar in concept to a Strehl ratio except that it can be defined for any initial irradiance distribution, not just a point source. It is defined as

$$I_{rel} = \frac{I_{pk}(\text{bloomed})}{I_{pk}(\text{vacuum}) e^{-\alpha R}} \quad (5.12)$$

where:

$I_{pk}(\text{bloomed})$ = peak far field irradiance through blooming

$I_{pk}(\text{vacuum})$ = peak far field irradiance through vacuum

$e^{-\alpha R}$ = transmission over range, R.

Since the reduction in peak irradiance due to absorption is removed,

I_{rel} represents the loss due to blooming only.

Use of I_{rel} as a merit function can be misleading because as the blooming increases and peak irradiance decreases, the spread of the distribution is generally not uniform, i.e., the distortion of the irradiance distribution for a given I_{rel} is not unique. However, it can be seen from the bloomed target images that due to phase scaling, there exists far more rotational symmetry than in the forward propagation distributions. This symmetry provides some justification for using I_{rel} as an image quality parameter, particularly when combined with the geometrical distortion function.

I_{rel} has been shown (Breux, 1976; Gebhardt, 1976) to relate to the distortion number by a function of the form:

$$I_{rel} = \frac{1}{1 + c_1 N_D^{c_2}} \quad (5.13)$$

where c_1 and c_2 are curve-fitted constants depending on the laser transmitter irradiance distribution (truncated Gaussian, uniform, or infinite Gaussian). The degree of correlation between N_D and I_{rel} in Eq. (5.13) usually decreases as the blooming increases. This is because the above dependence on N_D is an approximation to a decaying exponential and is derived from the energy balance and paraxial wave equation in much the same way as the phase change due to blooming, given in Eq. (2.16), was determined. That is, only linear terms were retained in the series expansion of the important atmospheric parameters effected by the absorption of energy. For strong blooming, the approximation is subject to error and higher-order effects should be considered.

Figure 5-7 shows the forward and back propagated I_{rel} based on Eq. (5.12) for the study cases. The forward propagation ($I_{rel,FP}$) corresponds to the distributions in Figure 5-1. The back propagation ($I_{rel,BP}$) is for the target images. The curves connecting the two data sets are based on Eq. (5.13) and the result of the least squares curve-fitting program, CURFIT (Southwell, 1978). The constants for each data set are:

Forward Propagation: $c_1 = 0.406$

$c_2 = 0.954$

Back Propagation: $c_1 = 0.356$

$c_2 = 0.500$

Fig. 5-7. Relative intensity as a function of distortion number.

In both cases, the fit is good except in the forward propagation where the phase distortions become severe ($N_D \geq 10$).

The effect of phase scaling is also apparent in Figure 5-7. Because the sensed phase distortions are less, there is more symmetry to the bloomed PSFs. This results in a less distorted target image with a higher peak irradiance.

Spot Size or Blur

The concept of spot size was discussed in Chapters 3 and 4. It is frequently used as a measure of energy distribution or spread. The x and y components of the bloomed image spot sizes are defined as:

$$\sigma_x^2 \triangleq \frac{1}{I_0} \int \int (x-x_0)^2 I_B(x,y) dx dy \quad (5.14)$$

$$\sigma_y^2 \triangleq \frac{1}{I_0} \int \int (y-y_0)^2 I_B(x,y) dx dy \quad (5.15)$$

where (x_0, y_0) is the image centroid location with respect to the optical axis. The normalization factor is given by:

$$I_0 \triangleq \int \int I_B(x,y) dx dy \quad (5.16)$$

where $I_B(x,y)$ is the bloomed target image irradiance distribution.

When the bloomed spot size is normalized with respect to the corresponding vacuum width, the ratio becomes a merit function defining the amount of blur. Figure 5-8 shows the relative increase in blur size as a function of distortion number for both the forward propagated, bloomed crescent and incoherent target image.

Fig. 5-8. Bloomed far field irradiance and target image width as a function of distortion number.

An astigmatism-like aberration is apparent in the forward propagation. This results in a roughly, elliptical irradiance distribution at the target plane. These distortions are similarly seen but to a lesser extent in the return PSFs, as was shown in Figure 3-2 and the total images, shown in Figure 5-3. Note that while the width of the crescent begins to increase rapidly for $N_D \gtrsim 2$, the target images also increase but at a much slower rate indicating some stability.

Not only do the widths of the target images increase in a stable manner, so does the behavior of the irradiance distribution within the central portion of the image. This is shown in Figure 5-9. The function, ϵ_N , represents the rms irradiance difference between the bloomed and vacuum images, both normalized to unity and with their centroids coincident. The difference is taken over the area of the vacuum image, as defined by its spot size (constant in this case since the range, wavelength, and tracker diameter is held constant). The slope of ϵ_N as a function of distortion number is significant. For $N_D \gtrsim 4$, the difference between the bloomed and vacuum irradiances has stabilized or saturated so that further increases in blooming will have only a small effect, at least within the same area. Thus, the width of the image, as well as its irradiance distribution, remains fairly stable for what would normally be considered as strong blooming.

Suggestions for Future Research

The algorithms developed as tools in this study and the proposed merit functions used to define image quality and stability can be used

Fig. 5-9. RMS normalized irradiance difference as a function of distortion number.

in support of some of the present work efforts in HEL propagation.

These areas include:

- (1) Adaptive optics: The degradation in a return image irradiance distribution is an indication of the stability that can be achieved in an adaptive optics system. An out-going wave, multidither system uses the energy in the return image (or glint) as a criterion in closing the correction loop. The tracker in a return-wave system will examine all or part of the received wavefront and scale the conjugate phase to compensate for the induced phase distortions. In both systems, knowing the nature of the return wavefront in terms of distortion, blur and signal degradation will help determine the type of sensor, hillclimb algorithm, and scaling of the compensating phase.
- (2) Ground-to-space application: Before it is possible to place an HEL system in orbit, it is appropriate to simulate the system in a ground-to-space demonstration. Although the problem of blooming may be reduced by a high-altitude transmitter and/or shorter wavelength, the effects of atmospheric turbulence can be severe. Tracking in this application must be precise due to the small angular subtense of the target. If long tracking times are required, path lengths through the atmosphere at low elevation angles can result in severe image degradation. Use of the imaging algorithm in simulating the effects of blooming, turbulence, and device jitter would help in determining potential problems in tracking.

- (3) Wavelength sensitivity: A parametric trade-off analysis is needed between the effects of blooming at various levels for existing laser lines and the quality of the return images sensed at different wavelengths. This problem was briefly mentioned in discussing phase scaling. The effect of turbulence at the shorter wavelengths would have to be included.
- (4) Special scenarios: Short range, high absorption cases including the effects of slew such as in tracking helicopters and other low-altitude targets, present problems due to an uneven distribution of the bloomed phase distortions. Vulnerable areas may be difficult to track or discriminate. The effect of screen location in modeling the back propagation in these scenarios needs further study before conclusions can be made regarding image stability and the effect of adaptive optics.
- (5) Tracker size and location: The effect of separate and subaperture trackers on imaging and adaptive optics correction for blooming, needs to be studied. In the case of a separate tracker, its position relative to the wind direction and transmitter will effect the return image due to different phase distortions being sensed. For subaperture trackers in strong blooming, the return wavefront may lose too much information due to the tracker's limited size, resulting in an inaccurate image reconstruction and conjugate phase, leading to an unstable correction system.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The purpose of this study was to investigate the relationship between return target image stability and parameters which characterize the strength of the induced phase distortions due to thermal blooming.

The results show that, under certain conditions, characteristics of an incoherent target image can be determined from information regarding the forward propagation only. In particular, the degree of blur, distortion, and degradation in peak irradiance has been shown to be a well-behaved function of the scenario distortion number over a wide variation in blooming. A large part of the correlation is due to the scaling of the phase perturbations sensed by the return radiation.

The approach leading to these conclusions was somewhat involved. The first step was to obtain an understanding of the atmospheric processes resulting in blooming and derive the relationships necessary in simulating the initial, forward propagation. The concept of phase screens and the importance of their location in the simulation was discussed.

The theory of incoherent imaging was outlined with the assumptions concerning isoplanicity and the generation of point spread functions placing special requirements on the back propagation algorithm. The results of the code were integrated with an image reconstruction

program which generated the target images as viewed in the infrared by a full, spectrally-shared aperture-tracker. Target images were reconstructed for several realistic scenarios producing a broad range of blooming. The effect of the wavelength of the radiation used in sensing the bloomed phase distortions was found to be an important consideration in the reconstruction.

The reconstructed images were analyzed with respect to image quality criteria used in both high-energy laser propagation and image processing. The stability of the target image as defined by its blur, distortion, and relative intensity was found to correlate well with the distortion number, used in defining the forward propagation.

Finally, future work was proposed which could utilize the reconstruction algorithm and stability criteria defined in the analysis. The results presented in the study represent a first step using a full, wave optics approach to imaging through blooming. But applying these concepts and tools to some of the present-day problems in laser propagation, shows promise, especially in the areas of adaptive optics and target tracking.

APPENDIX A

CALCULATION OF CHANGE IN AIR DENSITY BY LINEARIZATION OF THE HYDRODYNAMIC EQUATIONS

The hydrodynamic equations used in deriving the change in air density are given by:

$$\frac{d}{dt} \rho + \rho \vec{\nabla} \cdot \vec{v} = 0 \quad (\text{A.1})$$

$$\rho \frac{d\vec{v}}{dt} + \vec{\nabla} p = 0 \quad (\text{A.2})$$

$$\rho \frac{dE}{dt} + p \vec{\nabla} \cdot \vec{v} = \rho \dot{Q} \quad (\text{A.3})$$

$$E = C_v T \quad (\text{A.4})$$

$$\frac{p}{\rho} = (C_p - C_v) T \quad (\text{A.5})$$

The variables are defined in Chapter 2. The density, pressure and wind velocity are expanded about their nominal values, substituted into Eqs. (A.1) through (A.3), and like powers in the expansion are equated:

$$\text{Let:} \quad \rho \approx \rho_o + \epsilon \Delta \rho \quad (\text{A.6})$$

$$p \approx p_o + \epsilon \Delta p \quad (\text{A.7})$$

$$\vec{v} \approx \vec{v}_0 + \epsilon \Delta \vec{v} \quad (\text{A.8})$$

(Second order and higher effects, assumed negligible, should always be checked.)

Conservation of mass (Eq. A.1):

$$\frac{d}{dt} (\rho_0 + \epsilon \Delta \rho) + (\rho_0 + \epsilon \Delta \rho) \vec{v} \cdot (\vec{v}_0 + \epsilon \Delta \vec{v}) = 0 \quad (\text{A.9})$$

$$\epsilon^0: \frac{d}{dt} \rho_0 + \rho_0 \vec{v} \cdot \vec{v}_0 = 0 \quad (\text{A.10})$$

$$\epsilon^1: \frac{d\Delta\rho}{dt} + \rho_0 \vec{v} \cdot \Delta \vec{v} = 0 \quad (\text{A.11})$$

where it is assumed that $\rho_1 \vec{v} \cdot \vec{v}_0 \ll \rho_0 \vec{v} \cdot \Delta \vec{v}$ in Eq. (A.11):

Conservation of momentum (Eq. A.2):

$$(\rho_0 + \epsilon \Delta \rho) \frac{d}{dt} (\vec{v}_0 + \epsilon \Delta \vec{v}) + \vec{v} (p_0 + \epsilon \Delta p) = 0 \quad (\text{A.12})$$

$$\epsilon^0: \rho_0 \frac{d\vec{v}_0}{dt} + \vec{v} p_0 = 0 \quad (\text{A.13})$$

$$\epsilon^1: \rho_0 \frac{d\Delta \vec{v}}{dt} + \vec{v} \Delta p = 0 \quad (\text{A.14})$$

where it is assumed that $\Delta \rho \frac{d\vec{v}_0}{dt} \ll \rho_0 \frac{d\Delta \vec{v}}{dt}$ in Eq. (A.14).

Energy balance (Eq. A.3):

From Eq. (A.4) and (A.5)

$$E = \frac{C_v}{C_p - C_v} \frac{p}{\rho} \quad (\text{A.15})$$

Thus,

$$\frac{dE}{dt} = \frac{1}{\gamma-1} \left(\frac{1}{\rho} \frac{dp}{dt} - \frac{p}{\rho^2} \frac{d\rho}{dt} \right) \quad (\text{A.16})$$

where $\gamma \equiv C_p/C_v$.

From Eq. (A.1):

$$\vec{\nabla} \cdot \vec{v} = - \frac{1}{\rho} \frac{d\rho}{dt} \quad (\text{A.17})$$

Combining Eqs. (A.3), (A.16) and (A.17):

$$\frac{dp}{dt} - \gamma \frac{p}{\rho} \frac{d\rho}{dt} = \rho(\gamma-1)\dot{Q} \quad (\text{A.18})$$

Substituting in the expansions:

$$\frac{d}{dt} (p_o + \epsilon \Delta p) - \gamma \frac{(p_o + \epsilon \Delta p)}{(\rho_o + \epsilon \Delta \rho)} \frac{d}{dt} (\rho_o + \epsilon \Delta \rho) = (\rho_o + \epsilon \Delta \rho) (\gamma-1) \dot{Q} \quad (\text{A.19})$$

$$\epsilon^0: \frac{dp_o}{dt} - \gamma \frac{p_o}{\rho_o} \frac{d\rho_o}{dt} = 0 \quad (\text{A.20})$$

$$\epsilon^1: \frac{d\Delta p}{dt} - \gamma \frac{p_o}{\rho_o} \frac{d\Delta \rho}{dt} = \rho_o (\gamma-1) \dot{Q} \quad (\text{A.21})$$

where it is assumed that the terms $\Delta p \frac{dp_o}{dt}$ and $\gamma \Delta p \frac{d\rho_o}{dt}$ are negligible and the $\rho \dot{Q} (= \alpha I)$ term is considered to be a first order and higher effect only.

The time derivative can be written as:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}_o \cdot \vec{\nabla} \quad (\text{A.22})$$

Thus, Eqs. (A.11), (A.14) and (A.21) become:

$$\vec{v}_0 \cdot \vec{\nabla} \Delta \rho + \rho_0 \vec{\nabla} \cdot \Delta \vec{v} = 0 \quad (A.23)$$

$$\rho_0 \vec{v}_0 \cdot \vec{\nabla} \Delta \vec{v} + \vec{\nabla} \Delta p = 0 \quad (A.24)$$

$$\vec{v}_0 \cdot \vec{\nabla} \Delta \rho - \gamma \frac{p_0}{\rho_0} \vec{v}_0 \cdot \vec{\nabla} \Delta \rho = (\gamma-1) \rho_0 \dot{Q} \quad (A.25)$$

where time derivatives of the first order terms in the expansion are assumed negligible. Assuming that ρ_0 and p_0 are constant and the wind is constant in magnitude and in the x-direction, then Eqs. (A.23) through (A.25) become:

$$v_0 \frac{\partial \Delta \rho}{\partial x} + \rho_0 \frac{\partial \Delta v}{\partial x} = 0 \quad (A.26)$$

$$\rho_0 v_0 \frac{\partial \Delta v}{\partial x} + \frac{\partial \Delta p}{\partial x} = 0 \quad (A.27)$$

$$v_0 \frac{\partial \Delta p}{\partial x} - c_\infty^2 v_0 \frac{\partial \Delta \rho}{\partial x} = (\gamma-1) \rho_0 \dot{Q} \quad (A.28)$$

where $c_\infty^2 \triangleq \gamma p_0 / \rho_0$ (c_∞ being the speed of sound in quiescent air).

Solving for $\frac{\partial \Delta v}{\partial x}$ in Eq. (A.26), substituting into Eq. (A.27) and combining with Eq. (A.28) results in:

$$v_0 \frac{\partial}{\partial x} (v_0^2 - c_\infty^2) \Delta \rho = (\gamma-1) \rho_0 \dot{Q} \quad (A.29)$$

Assuming $v_o^2 \ll c_\infty^2$, then

$$\frac{\Delta \rho}{\rho_o} = - \frac{(\gamma-1)}{\gamma} \frac{1}{p_o} \frac{1}{v_o} \int_{-\infty}^x \rho_o \dot{Q}(\beta, y, z) d\beta \quad (\text{A.30})$$

which is Eq. (2.10) in Chapter 2.

APPENDIX B

HELPRP PROPAGATION CODE LISTING

PROGRAM HELPRP

74/175 OPT=2

FTN 4.8+528

```

1      PROGRAM HELPRP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
*      TAPE21,TAPE22,TAPE23,TAPE24,TAPE25,TAPE26,
*      TAPE14,TAPE15=5000,TAPE16,TAPE17=5000)
5      REAL      INCHES      ,KM      ,METERS      ,KW      ,MW
      COMPLEX    C      ,CDUM      ,EYE      ,TRAMP      ,CS
      LOGICAL    LTRID      ,LGAMP      ,LPLOTI      ,LCLEAN      ,LCQ2
      B      ,LMVGTR      ,LJANRY      ,LPFAST      ,LVERT      ,LREFI
      C      ,LCOP10      ,LCOP11      ,LCQ2SW      ,LPLOTP      ,LTURB
      D      ,LPRINT
10     COMMON/REAL/ DLX      ,DLZ      ,ELRAD      ,ELSEP      ,FLDSZ
      B      ,FOCLLN      ,PI      ,WVL      ,RANGE      ,KAY
      C      ,INCHES      ,KM      ,METERS      ,CM
      D      ,POWER      ,DISTNO      ,SLEWNO      ,VORBIT      ,STREHL
      E      ,FREQ      ,CAVE      ,CRMS      ,PHVAR      ,EO
15     F      ,APRAD      ,SRUFF      ,HTRANS      ,VMAX      ,VTRANS
      G      ,VTARG      ,VWIND      ,WVPR      ,RELHUM      ,ELANG
      H      ,WATTS      ,KW      ,MW      ,OMEGA      ,ACUT
      I      ,RBUKIT      ,FLDMG      ,WAVEL      ,TRANS      ,TRPOW
      J      ,TRBI      ,XAVE      ,YAVE      ,SPOTSZ      ,HTARGET
20     K      ,TURBJTR      ,TRAKJTR      ,BMJTR      ,RO      ,TURBTST
      COMMON/INT / IMLOOP      ,INUM      ,INTVL      ,MSIZE      ,NEL
      B      ,NRINGS      ,NSCRN      ,NSIZE      ,NSTEP      ,NHOP
      C      ,IFFER      ,MID      ,NHI      ,NAPDPR      ,NROSPT
      D      ,NPTS      ,ITAPE      ,NPLOTS      ,NSEED      ,NDZ
25     E      ,NTIME      ,NSIZ2      ,NPLOT      ,NSCALE
      F      ,ICD      ,NCS      ,NCF      ,IPRINT      ,ITR
      G      ,MA      ,I      ,J      ,NIT
      COMMON/CMPLX/EYE
      COMMON/LOG /LTRID      ,LGAMP      ,LPLOTI      ,LCLEAN      ,LCQ2
30     B      ,LMVGTR      ,LJANRY      ,LPFAST      ,LVERT      ,LREFI
      C      ,LCOP10      ,LCOP11      ,LCQ2SW      ,LPLOTP      ,LTURB
      D      ,LPRINT
      COMMON/ARRAY/ELXPOS(220)      ,ELYPOS(220)      ,NPEL(220)
      B      ,TRAMP(220)      ,TRPHAS(220)      ,TRPWR(220)
35     C      ,C(64,64)      ,CDUM(64)      ,RDUM(64)
      D      ,IDUM(64)      ,SFFT(16)      ,FAC(33)
      E      ,ZPOS(21)      ,XSPACE(6)      ,YSPACE(6)
      F      ,DELZ(21)      ,ROSPMG(10)      ,PHAX(10)
      G      ,ROXPOS(10)      ,ROYPOS(10)      ,ROSPWD(10)
40     H      ,ACQOL(21)      ,STORE(64,64)      ,CS(64,64)
      I      ,TEMP(21)      ,HEIGHT(21)      ,TAU(21)
      J      ,ALPHA(21)      ,AH2D(21)      ,ACQ2(21)
      K      ,PX(220)      ,PY(220)      ,PB(220)
      L      ,AL(500)      ,H(500)      ,TMP(500)
45     M      ,RAD(500)      ,VW(500)      ,Z(500)
      N      ,DIST(21)      ,DZ(500)      ,ARATIO(500)
      O      ,X(64)      ,Y(64)      ,ZK(22)      ,ACDEF(22)      ,SDME(22)
      P      ,ZZ(22)      ,V(64,64)      ,SZ(22)      ,BC(22)      ,CP(22,22)
      Q      ,S(64,64)
50     REXP(X)=EXP(AMAX1(-600.,AMIN1(600.,X)))
      ABSQ(C)=REAL(C)**2+AIMAG(C)**2
      NAMELIST/INPUT/ ACUT      ,APRAD      ,EO      ,ELANG
      B      ,ELRAD      ,ELSEP      ,FLDSZ      ,FOCLLN      ,FREQ      ,HTRANS
      C      ,HTARGET      ,MSIZE      ,NAPDPR      ,NDZ      ,NEL      ,NHOP
55     D      ,NPLOT      ,NRINGS      ,NROSPT      ,NSEED      ,NTIME      ,PHVAR
      E      ,RANGE      ,RBUKIT      ,RELHUM      ,ROSPMG      ,POWER      ,ROSPWD
      F      ,ROXPOS      ,ROYPOS      ,SRUFF      ,TRAKJTR      ,TURBTST      ,VMAX

```


PROGRAM HEL PRP

74/175 OPT=2

FTN 4.8+528

```

60      G ,VORBIT ,VTARG ,VTRANS ,VWIND ,WAVEL ,WVPR
      H ,LCO2 ,LCO2SW ,LCOP10 ,LCOP11 ,LGAMP ,LJANRY
      I ,LMVGTR ,LPFAST ,LPLOTI ,LPLOTP ,LREFI ,LS1
      J ,LTRID ,LTURB ,LVERT ,NCS ,NCF ,NIT
      K ,ICD ,ITR
      L ,IPRINT ,LPRINT
      CALL DEFAULT
65      READ (5,INPUT)
      ELANG=ASIN((HTARGET-HTRANS)/RANGE)
      ELANG=ELANG*180./PI
      WRITE (6,INPUT)
70      CALL SETUP
      CALL MSTTR
      CALL UNITY
      CALL CIRCLE
      CALL TRAPWR
      CALL TOTPWR
75      CALL LENS
      CALL PROPST
      CALL MBPF
      CALL STRCSQ
      CALL MPRNT
80      KNT=0
      REWIND 14
      REWIND 15
      REWIND 16
      REWIND 17
85      IVAC=1
      IVAC=0
      ICASU=4
      JCASU=4
      DO 1843 ICAS=1,ICASU
90      DO 1844 JCAS=1,JCASU
      KNT=KNT+1
      IF(KNT.GT.1) NSCRN=NHOP+1
      XSHFT=-.295
      ROXPOS(1)=8.*DLX*FLOAT(ICAS)+(XSHFT-20.*DLX)
95      ROYPOS(1)=-8.*DLX*FLOAT(JCAS)+(20.*DLX)
      CALL REFLCT
      CALL STRCSQ
      CALL MPRNT
      CALL MBPB
100      CALL STRCSQ
      CALL MPRNT
      CALL BAKLENS
      CALL STRCSQ
      CALL MPRNT
105      CALL MVPF
      CALL STRCSQ
      CALL MPRNT
      WRITE(6,173) ROXPOS(1),ROYPOS(1)
173  FORMAT(//,* ROXPOS(1) = *,E10.3,4X,*ROYPOS(1) = *,E10.3)
110      IF(XAVE.LT.0.) ICT=IFIX(XAVE/DLX-0.5)
      IF(XAVE.GE.0.) ICT=IFIX(XAVE/DLX+0.5)
      IF(YAVE.LT.0.) JCT=+IFIX(YAVE/DLX-0.5)
      IF(YAVE.GE.0.) JCT=+IFIX(YAVE/DLX+0.5)
      XAVE=-XAVE

```

PROGRAM HELPRP

74/175 OPT=2

FTN 4.8+528

```

115      YAVE=-YAVE
        IF(IVAC.EQ.1) WRITE(14) XAVE
        IF(IVAC.EQ.1) WRITE(14) YAVE
        IF(IVAC.EQ.0) WRITE(16) XAVE
        IF(IVAC.EQ.0) WRITE(16) YAVE
120      DO 29 I=1,64
        DO 30 J=1,64
        IF((I+ICT).LT.1.OR.(I+ICT).GT.64.OR.
          * (J+JCT).LT.1.OR.(J+JCT).GT.64) GO TO 31
        V(I,J)=STORE(I+ICT,J+JCT)
125      31 CONTINUE
        30 CONTINUE
        29 CONTINUE
        DO 2113 I=2,64
        DO 2113 J=1,64
130      S(I,J)=V(J,66-I)
        2113 CONTINUE
        DO 427 I=2,64
        DO 428 J=2,64
        STORE(I,J)=S(66-I,66-J)
135      428 CONTINUE
        427 CONTINUE
        CALL MPRNT
        DO 429 I=1,64
        DO 430 J=1,64
140      S(I,J)=STORE(I,J)
        430 CONTINUE
        429 CONTINUE
        IF(IVAC.EQ.1) WRITE(15) S
        IF(IVAC.EQ.0) WRITE(17) S
145      976 CONTINUE
        1844 CONTINUE
        1843 CONTINUE
        600 WRITE (6,601)
        601 FORMAT(*0 END OF PROGRAM*)
150      STOP
        END

```

SUBROUTINE MSUB

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE MSUB
      REAL      INCHES ,KM      ,METERS ,KW      ,MW
      COMPLEX   C      ,CDUM    ,EYE     ,TRAMP   ,CS
      LOGICAL   LTRID ,LGAMP    ,LPLOTI ,LCLEAN ,LCO2
5      B      ,LMVGTR ,LJANRY ,LPFAST ,LVERT ,LREFI
      C      ,LCOP10 ,LCOP11 ,LCO2SW ,LPLOTP ,LTURB
      D      ,LPRINT
      COMMON/REAL/ DLX ,DLZ ,ELRAD ,ELSEP ,FLDSZ
10     B      ,FOCLLN ,PI      ,MVL     ,RANGE ,KAY
      C      ,INCHES ,KM      ,METERS ,CM
      D      ,POWER  ,DISTNO ,SLEWNO ,VORBIT ,STREHL
      E      ,FREQ   ,CAVE   ,CRMS   ,PHVAR ,EO
      F      ,APRAD ,SRUFF ,HTRANS ,VMAX  ,VTRANS
15     G      ,VTARG ,VWIND ,WVPR   ,RELHUM ,ELANG
      H      ,WATTS ,KW      ,MW      ,OMEGA ,ACUT
      I      ,RBUKIT ,FLDMG ,WAVEL  ,TRANS ,TRPOW
      J      ,TRBI  ,XAVE   ,YAVE   ,SPOTSZ ,HTARGET
      K      ,TURBJTR ,TRAKJTR ,BMJTR ,RO    ,TURBTST
20     COMMON/INT / IMLOOP ,INUM ,INTVL ,MSIZE ,NEL
      B      ,NRINGS ,NSCRN ,NSIZE  ,NSTEP ,NHOP
      C      ,IFFER  ,MID   ,NHI    ,NAPDPR ,NROSPT
      D      ,NPTS   ,ITAPE ,NPLOTS ,NSEED ,NDZ
      E      ,NTIME  ,NSIZ2 ,NPLOT  ,NSCALE
25     F      ,ICD   ,NCS   ,NCF    ,IPRINT ,ITR
      G      ,MA     ,I     ,J      ,NIT
      COMMON/CHPLX/EYE
      COMMON/LOG /LTRID ,LGAMP ,LPLOTI ,LCLEAN ,LCO2
30     B      ,LMVGTR ,LJANRY ,LPFAST ,LVERT ,LREFI
      C      ,LCOP10 ,LCOP11 ,LCO2SW ,LPLOTP ,LTURB
      D      ,LPRINT
      COMMON/ARRAY/ELXPOS(220) ,ELYPQS(220) ,NPQL(220)
      B      ,TRAMP(220) ,TRPHAS(220) ,TRPWR(220)
      C      ,C(64,64) ,CDUM(64) ,RDUM(64)
35     D      ,IDUM(64) ,SFFT(16) ,FAC(33)
      E      ,ZPOS(21) ,XSPACE(6) ,YSPACE(6)
      F      ,DELZ(21) ,ROSPHG(10) ,PHAX(10)
      G      ,ROXPOS(10) ,ROYPOS(10) ,ROSPWD(10)
      H      ,ACDOL(21) ,STORE(64,64) ,CS(64,64)
40     I      ,TEMP(21) ,HEIGHT(21) ,TAU(21)
      J      ,ALPHA(21) ,AH20(21) ,ACQ2(21)
      K      ,PX(220) ,PY(220) ,PB(220)
      L      ,AL(500) ,H(500) ,TMP(500)
      M      ,RAD(500) ,VW(500) ,Z(500)
45     N      ,DIST(21) ,DZ(500) ,ARATIO(500)
      O      ,X(64) ,Y(64) ,ZK(22) ,ACDEF(22) ,SOME(22)
      P      ,ZZ(22) ,V(64,64) ,SZ(22) ,BC(22) ,CP(22,22)
      Q      ,S(64,64)
      DIMENSION FX(65)
      REXP(X)=EXP(AMAX1(-600.,AMIN1(600.,X)))
50     ABSQ(C)=REAL(C)**2+AIMAG(C)**2
      ENTRY MSTTR
      CALL BLMSET
      CALL YNEWT
      CALL STTRCA
55     RETURN
      ENTRY MVPF
      CALL PRPFWD

```

SUBROUTINE MSUB

74/175 OPT=2

FTN 4.8+528

```
        DO 100 N=1,NHOP
        CALL VACSCN
60      95  CALL FRMSPC
        CALL PROP
        CALL FRMFLD
        100 CONTINUE
        RETURN
65      ENTRY MPRNT
        CALL PRNTI
        CALL BEAMSZ
        CALL TOTPHR
        CALL BUCKET
70      IF(LPLOTI) CALL STRI
        IF(LPLOTI) CALL INVERT
        IF (LPLOTI.OR.LPLOTI) CALL FRAME1
        IF(LPLOTI) CALL PLOT3D
        RETURN
75      ENTRY MBPF
        CALL PRPFWD
        DO 200 N=1,NHOP
        CALL VBLOOM
        CALL PRPSCN
80      CALL FRMSPC
        CALL PROP
        CALL FRMFLD
        200 CONTINUE
        RETURN
85      ENTRY MBPB
        CALL PRPBAK
        DO 300 N=1,NHOP
        CALL FRMSPC
        CALL PROP
90      CALL FRMFLD
        CALL SCNBK
        300 CONTINUE
        RETURN
        END
```

SUBROUTINE ZERNK

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE ZERNK
      REAL      INCHES ,KM      ,METERS ,KW      ,MW
      COMPLEX   C      ,CDUM ,EYE    ,TRAMP ,CS
      LOGICAL   LTRID ,LGAMP ,LPLOTI ,LCLEAN ,LCD2
5      B      ,LMVGTR ,LJANRY ,LPFAST ,LVERT ,LREFI
      C      ,LCOP10 ,LCOP11 ,LCO2SW ,LPLOTP ,LTURB
      D      ,LPRINT
      COMMON/REAL/ DLX      ,DLZ      ,ELRAD ,ELSEP ,FLDSZ
10     B      ,FOCLLN ,PI      ,WVL    ,RANGE ,KAY
      C      ,INCHES ,KM      ,METERS ,CM      ,STREHL
      D      ,POWER  ,DISTNO ,SLEWNO ,VORBIT ,EO
      E      ,FREQ   ,CAVE   ,CRMS   ,PHVAR ,VTRANS
15     F      ,APRAD ,SRUFF  ,HTRANS ,VMAX   ,ELANG
      G      ,VTARG ,VWIND  ,WVPR   ,RELHUM ,ACUT
      H      ,WATTS ,KW      ,MW      ,OMEGA ,TRPOW
      I      ,RBUKIT ,FLDMG ,WAVEL  ,TRANS  ,HTARGET
      J      ,TRBI   ,XAVE   ,YAVE   ,SPOTSZ ,TURBTST
      K      ,TURBJTR, TRAKJTR, BMJTR ,RO     ,NEL
20     COMMON/INT / IMLOOP ,INUM   ,INTVL ,MSIZE ,NHOP
      B      ,NRINGS ,NSCRN ,NSIZE  ,NSTEP ,NROSP
      C      ,IFFER  ,MID    ,NHI    ,NAPDPR ,NDZ
      D      ,NPTS   ,ITAPE ,NPLOTS ,NSEED ,NROSP
      E      ,NTIME  ,NSIZ2 ,NPLOT  ,NSCALE ,ITR
25     F      ,ICD   ,NCS   ,NCF    ,IPRINT ,NIT
      G      ,MA     ,I     ,J      ,NIT
      COMMON/CMPLX/EYE
      COMMON/LOG /LTRID ,LGAMP ,LPLOTI ,LCLEAN ,LCD2
      B      ,LMVGTR ,LJANRY ,LPFAST ,LVERT ,LREFI
      C      ,LCOP10 ,LCOP11 ,LCO2SW ,LPLOTP ,LTURB
30     D      ,LPRINT
      COMMON/ARRAY/ELXPOS(220) ,ELYPPOS(220) ,NPOL(220)
      B      ,TRAMP(220) ,TRPHAS(220) ,TRPWR(220)
      C      ,C(64,64) ,CDUM(64) ,ROUM(64)
35     D      ,IDUM(64) ,SFFT(16) ,FAC(33)
      E      ,ZPOS(21) ,XSPACE(6) ,YSPACE(6)
      F      ,DELZ(21) ,ROSPHG(10) ,PMAX(10)
      G      ,ROXPOS(10) ,ROYPOS(10) ,ROSPWD(10)
      H      ,ACDOL(21) ,STORE(64,64) ,CS(64,64)
40     I      ,TEMP(21) ,HEIGHT(21) ,TAU(21)
      J      ,ALPHA(21) ,AH20(21) ,ACD2(21)
      K      ,PX(220) ,PY(220) ,PB(220)
      L      ,AL(500) ,H(500) ,TMP(500)
      M      ,RAD(500) ,VW(500) ,Z(500)
      N      ,DIST(21) ,DZ(500) ,ARATID(500)
45     O      ,X(64) ,Y(64) ,ZK(22) ,ACDEF(22) ,SOME(22)
      P      ,ZZ(22) ,V(64,64) ,SZ(22) ,BC(22) ,CP(22,22)
      Q      ,S(64,64)
      R=APRAD
      RX = R
50     XZ = X(I)
      YZ = Y(J)
      XSQ = XZ**2
      YSQ = YZ**2
      X4 = XSQ**2
      Y4 = YSQ**2
55     RS = XSQ+YSQ
      RSS = RS**2

```

SUBROUTINE ZERNK

74/175 OPT=2

FTN 4.8+528

```

        RRS = R**2
        RRS4 = RRS**2
60      RRS6 = RRS**3
        PR2 = PI*RRS
        PR4 = PR2*RRS
        PR6 = PR4*RRS
        PR8 = PR6*RRS
65      PR10 = PR8*RRS
        PR12 = PR10*RRS
        PR14 = PR12*RRS
        ZC1 = SQRT(1./PR2)
        ZC2 = SQRT(4./PR4)
70      ZC3 = ZC2
        ZC4 = SQRT(3./PR6)
        ZC5 = SQRT(24./PR6)
        ZC6 = SQRT(6./PR6)
        ZC7 = SQRT(8./PR8)
75      ZC8 = ZC7
        ZC9 = ZC7
        ZC10 = ZC7
        ZC11 = SQRT(5./PR10)
        ZC12 = SQRT(10./PR10)
80      ZC13 = SQRT(40./PR10)
        ZC14 = ZC12
        ZC15 = SQRT(160./PR10)
        ZC16 = SQRT(12./PR12)
        ZC17 = ZC16
85      ZC18 = ZC16
        ZC19 = ZC16
        ZC20 = ZC16
        ZC21 = ZC16
        ZC22 = (7./PR14)
90      ZK(1) = ZC1
        ZK(2) = ZC2*XZ
        ZK(3) = ZC3*YZ
        ZK(4) = ZC4*(2.*RS-RRS)
        IF(NCF .LE. 4) GO TO 90
95      ZK(5) = ZC5*XZ*YZ
        ZK(6) = ZC6*(XSQ-YSQ)
        C78 = 3.*RS-2.*RRS
        ZK(7) = ZC7*YZ*C78
        ZK(8) = ZC8*XZ*C78
100     ZK(9) = ZC9*YZ*(3.*RS-4.*YSQ)
        ZK(10) = ZC10*XZ*(4.*XSQ-3.*RS)
        ZK(11) = ZC11*(6.*RSS-6.*RRS*RS+RRS4)
        IF (NCF .LE. 11) GO TO 90
        C23 = 4.*RS-3.*RRS
105     ZK(12) = ZC12*(XSQ-YSQ)*C23
        ZK(13) = ZC13*XZ*YZ*C23
        ZK(14) = ZC14*(X4-6.*XSQ*YSQ+Y4)
        ZK(15) = ZC15*XZ*YZ*(XSQ-YSQ)
        C67 = 10.*RSS-12.*RS*RRS+3.*RRS4
110     ZK(16) = ZC16*XZ*C67
        ZK(17) = ZC17*YZ*C67
        ZK(18) = ZC18*XZ*(5.*X4-10.*XSQ*YSQ-15.*Y4-4.*XSQ*RRS
        +12.*YSQ*RRS)
        ZK(19) = ZC19*YZ*(15.*X4+10.*XSQ*YSQ-5.*Y4-12.*XSQ*RRS

```

SUBROUTINE ZERNK

74/175 OPT=2

FTN 4.8+528

```

115      B  +4.*YSQ*RRS)
          ZK(20) = ZC20*XZ*(X4-10.*XSQ*YSQ+5.*Y4)
          ZK(21) = ZC21*YZ*(5.*X4-10.*XSQ*YSQ+Y4)
          ZK(22) = ZC22*(20.*RSS*RS-30.*RSS*RRS+12.*RS*RRS4-RRS6)
          90  IF (MA .EQ. 0) RETURN
120      DO 113 IM=NCS,NCF
          113  ZK(IM) = ZK(IM) * BC(IM)
          RETURN
          ENTRY CPROD
          IF (ITR .EQ. 1) GO TO 211
125      DO 799 M=NCS,NCF
          DO 799 L=NCS,NCF
          799  CP(L,M) = ZK(L)*ZK(M) + CP(L,M)
          RETURN
130      211  DO 123 M=NCS,NCF
          123  CP(M,M) = ZK(M)**2 + CP(M,M)
          RETURN
          END

```

SUBROUTINE DECCMP

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE DECCMP
      REAL      INCHES      ,KM      ,METERS      ,KW      ,MW
      COMPLEX   C           ,CDUM     ,EYE         ,TRAMP     ,CS
      LOGICAL   LTRID      ,LGAMP     ,LPLOTI      ,LCLEAN    ,LCO2
5      B        ,LMVGTR     ,LJANRY    ,LPFAST     ,LVERT     ,LREFI
      C        ,LCOP10     ,LCOP11    ,LCO2SW      ,LPLOTP    ,LTURB
      D
      COMMON/REAL/ DLX      ,DLZ      ,ELRAD      ,ELSEP     ,FLDSZ
10     B        ,FOCLLN    ,PI        ,WVL        ,RANGE     ,KAY
      C        ,INCHES     ,KM        ,METERS     ,CM        ,
      D        ,POWER      ,DISTND    ,SLEWNO     ,VORBIT     ,STREHL
      E        ,FREQ       ,CAVE      ,CRMS       ,PHVAR     ,EO
      F        ,APRAD      ,SRUFF     ,HTRANS     ,VMAX      ,VTRANS
15     G        ,VTARG     ,VWIND     ,WVPR       ,RELHUM    ,ELANG
      H        ,WATTS      ,KW        ,MW         ,OMEGA     ,ACUT
      I        ,RBUKIT     ,FLDMG     ,WAVEL      ,TRANS     ,TRPOW
      J        ,TRBI       ,XAVE      ,YAVE       ,SPOTSZ    ,HTARGET
      K        ,TURBJTR    ,TRAKJTR   ,BMJTR      ,RO        ,TURBTST
20     COMMON/INT / INLOOP ,INUM      ,INTVL      ,MSIZE     ,NEL
      B        ,NRINGS     ,NSCRN     ,NSIZE      ,NSTEP     ,NHOP
      C        ,IFFER      ,MID       ,NHI        ,NAPOPR    ,NRDSPT
      D        ,NPTS       ,ITAPE     ,NPLOTS     ,NSEED     ,NDZ
      E        ,NTIME      ,NSIZ2     ,NPLOT      ,NSCALE    ,
      F        ,ICD        ,NCS       ,NCF        ,IPRINT    ,ITR
25     G        ,MA        ,I         ,J          ,NIT
      COMMON/CMPLX/EYE
      COMMON/LOG /LTRID    ,LGAMP     ,LPLOTI      ,LCLEAN    ,LCO2
      B        ,LMVGTR     ,LJANRY    ,LPFAST     ,LVERT     ,LREFI
      C        ,LCOP10     ,LCOP11    ,LCO2SW      ,LPLOTP    ,LTURB
30     D
      COMMON/ARRAY/ELXPOS(220)      ,ELYPOS(220)      ,NPOL(220)
      B        ,TRAMP(220)      ,TRPHAS(220)      ,TRPWR(220)
      C        ,C(64,64)       ,CDUM(64)         ,RDUM(64)
      D        ,IDUM(64)       ,SFFT(16)         ,FAC(33)
35     E        ,ZPOS(21)       ,XSPACE(6)        ,YSPACE(6)
      F        ,DELZ(21)       ,ROSPMG(10)       ,PMAX(10)
      G        ,ROXPOS(10)     ,ROYPOS(10)       ,ROSPWD(10)
      H        ,ACDOL(21)     ,STORE(64,64)      ,CS(64,64)
      I        ,TEMP(21)      ,HEIGHT(21)       ,TAU(21)
40     J        ,ALPHA(21)     ,AH2D(21)        ,ACD2(21)
      K        ,PX(220)       ,PY(220)        ,PB(220)
      L        ,AL(500)       ,H(500)         ,TMP(500)
      M        ,RAD(500)      ,VW(500)        ,Z(500)
      N        ,DIST(21)     ,DZ(500)        ,ARATIO(500)
45     O        ,X(64)       ,Y(64)        ,ZK(22)   ,ACDEF(22),SOME(22)
      P        ,ZZ(22)      ,V(64,64)     ,SZ(22)   ,BC(22)   ,CP(22,22)
      Q        ,S(64,64)
      REXP(X)=EXP(AMAX1(-600.,AMIN1(600.,X)))
      ABSQ(C)=REAL(C)**2 + AIMAG(C)**2
50     IC = IABS (ICD)
      NN1 = 1
      ITER=ITR
      IS = 1
      JS = 1
55     MX=2**MSIZE
      NY=2**MSIZE
      MA = 0

```


SUBROUTINE DECOMP 74/175 OPT=2

FTN 4.8+328

```

        DO 10 J = 1, 22
          BC(J) = 0.
60      SZ(J) = 0.
          ZK(J) = 0.
          DO 5 I = 1, 22
            CP(I,J)=0.
10      CONTINUE
65      R = APRAD
          RX = R
          JR=(R/DLX) + 1
          J1=MID-JR
          I1=J1
70      J2=MID + JR
          I2=J2
          DO 100 J=J1,J2
            YZ = Y(J) / R
            DO 100 I=I1,I2
75      XZ = X(I) / RX
            RAD2 = XZ **2 + YZ **2
            RS = V(I,J)
            IF ( RAD2 .GT. 1. ) RS = 0.
            IF ( IC .NE. 1 ) RS = S(I,J) * V(I,J)
80      CALL ZERNK
            DO 29 IZ= NCS,NCF
              SZ(IZ) = RS * ZK(IZ) + SZ(IZ)
29      CONTINUE
            IF ( RAD2 .GT. 1. ) GO TO 40
85      CALL CPROD
40      CONTINUE
100     CONTINUE
          DO 200 N = NN1, ITER
            DO 110 I = NCS, NCF
90      SOME(I) = 0.
            IF ( N .EQ. NN1 ) GO TO 121
            DO 120 L = NCS, NCF
            DO 120 M = NCS, NCF
            IF ( L .EQ. M ) GO TO 120
95      SOME(L) = CP(L,M)*BC(M) + SOME(L)
120     CONTINUE
121     DO 130 M = NCS, NCF
            IF (CP(M,M) .EQ. 0.) GO TO 130
            BC(M) = (SZ(M)-SOME(M))/CP(M,M)
100     130 CONTINUE
200     CONTINUE
          WRITE (6, 201) (BC(L), L = NCS, NCF)
201     FORMAT (*O B-COEFFICIENTS*/ (1X, 1P8E13.5) )
          IF (ICD .LT. 0) RETURN
105      IF (LPRINT) WRITE(6,9001)
9001     FORMAT (*O*, 15X, *VALUES OF ZERNIKES AT ALL X,Y*)
          CT = 0.
          DELDZS = 0.
          ZSQS = 0.
110      DO 250 J=J1,J2
            YZ = Y(J) / R
            K = 0
            DO 250 I=I1,I2
              XZ = X(I) / RX

```

SUBROUTINE DECOMP

74/175 OPT=2

FTN 4.8+528

```

115      RSQ = XZ **2 + YZ **2
          K = K + 1
          MA = 1
          CALL ZERNK
          ZZ(K)=0.
120      DO 220 IZ = NCS, NCF
220      ZZ(K)=ZK(IZ) + ZZ(K)
230      IF (.NOT. LPRINT) GO TO 240
          IXP = I - K + 1
          IF (K .EQ. 32 .OR. I .EQ. MX)
125      *      WRITE (6, 9002) IXP, (ZZ(L), L = 1, K, 2)
240      IF ( RSQ .GT. 1. ) GO TO 250
          CT = CT + 1.
          DELOZS=(V(I,J) - ZZ(K))**2 + DELOZS
          ZSQS = V(I,J) **2 + ZSQS
130      250 CONTINUE
9002      FORMAT (1X,I6,2X,16E7.3)
          DEVN = SQRT(DELOZS / ZSQS)
          ERMS= SQRT( DELOZS / CT )
          WRITE (6,202) ERMS
135      202      FORMAT (*ORMS DEVIATION **, 1PE13.5)
          WRITE(6,203) DEVN
          203      FORMAT(*O   DEVN = *, 1PE13.5)
          RETURN
          END

```

SUBROUTINE ASUB

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE ASUB
      C
      REAL      INCHES ,KM      ,METERS ,KW      ,MW
      COMPLEX   C      ,CDUM    ,EYE     ,TRAMP   ,CS
5      LOGICAL  LTRID   ,LGAMP   ,LPLOTI  ,LCLEAN  ,LCO2
      B         ,LMVGTR ,LJANRY ,LPFAST  ,LVERT   ,LREFI
      C         ,LCOP10 ,LCOP11 ,LCO2SW  ,LPLOTP  ,LTURB
      D
      COMMON/REAL/ DLX ,DLZ ,ELRAD ,ELSEP ,FLDSZ
10     B         ,FOCLLN ,PI      ,WVL    ,RANGE ,KAY
      C         ,INCHES ,KM      ,METERS ,CM
      D         ,POWER  ,DISTNO ,SLEWNO ,VORBIT ,STREHL
      E         ,FREQ   ,CAVE    ,CRMS    ,PHVAR  ,EO
15     F         ,APRAD ,SRUFF   ,HTRANS  ,VMAX   ,VTRANS
      G         ,VTARG  ,VWIND   ,WVPR    ,RELHUM ,ELANG
      H         ,WATTS  ,KW      ,MW      ,OMEGA  ,ACUT
      I         ,RBUKIT ,FLDMG   ,WAVEL   ,TRANS  ,TRPDW
      J         ,TRBI   ,XAVE    ,YAVE    ,SPOTSZ ,HTARGT
      K         ,TURBJTR,TRAKJTR,BMJTR   ,RO      ,TURBTST
20     COMMON/INT / IMLOOP ,INUM   ,INTVL  ,MSIZE  ,NEL
      B         ,NRINGS ,NSCRN   ,NSIZE  ,NSTEP  ,NHOP
      C         ,IFFER  ,MID     ,NHI     ,NAPDPR ,NROSPT
      D         ,NPTS   ,ITAPE   ,NPLOTS  ,NSEED  ,NDZ
      E         ,NTIME  ,NSIZ2   ,NPLOT   ,NSCALE ,
25     F         ,ICD   ,NCS     ,NCF     ,IPRINT ,ITR
      G         ,MA     ,I       ,J       ,NIT
      COMMON/CMPLX/EYE
      COMMON/LOG /LTRID ,LGAMP ,LPLOTI ,LCLEAN ,LCO2
30     B         ,LMVGTR ,LJANRY ,LPFAST  ,LVERT   ,LREFI
      C         ,LCOP10 ,LCOP11 ,LCO2SW  ,LPLOTP  ,LTURB
      D
      COMMON/ARRAY/ELXPDS(220) ,ELYPDS(220) ,NPDL(220)
      B         ,TRAMP(220) ,TRPHAS(220) ,TRPWR(220)
      C         ,C(64,64) ,CDUM(64) ,RDUM(64)
35     D         ,IDUM(64) ,SFFT(16) ,FAC(33)
      E         ,ZPDS(21) ,XSPACE(6) ,YSPACE(6)
      F         ,DELZ(21) ,ROSPMG(10) ,PMAX(10)
      G         ,ROXPDS(10) ,ROYPDS(10) ,ROSPWD(10)
      H         ,ACDOL(21) ,STORE(64,64) ,CS(64,64)
40     I         ,TEMP(21) ,HEIGHT(21) ,TAU(21)
      J         ,ALPHA(21) ,AH2D(21) ,ACD2(21)
      K         ,PX(220) ,PY(220) ,PB(220)
      L         ,AL(500) ,H(500) ,TMP(500)
      M         ,RAD(500) ,VW(500) ,Z(500)
45     N         ,DIST(21) ,DZ(500) ,ARATIO(500)
      O         ,X(64) ,Y(64) ,ZK(22) ,ACDEF(22) ,SOME(22)
      P         ,ZZ(22) ,V(64,64) ,SZ(22) ,BC(22) ,CP(22,22)
      Q         ,S(64,64)
      DATA KDIM/64/
      NAMELIST/ERR/IFFER
      ABSQ(C)=REAL(C)**2+AIMAG(C)**2
      REXP(X)=EXP(AHAX1(-600.,AMIN1(600.,X)))
      ENTRY DEFAULT
50     CM      = .01
      KM      = 1000.
      KW      = 1.E3
      METERS = 1.
55

```

SUBROUTINE ASUB

74/175 OPT=2

FTN 4.8+528

```

        MW      = 1.E6
        INCHES  = 1.
60      PI      = 3.1415926
        WATTS   = 1.
        EYE     = (0.,1.)
        ACUT    = 1.
        ALPHA(1) = 1.E-6
65      APRAD   = 62.5 * CM
        EO      = 0.05
        ELANG   = 0.
        ELRAD   = 62.5 * CM
        ELSEP   = 0.
70      FLDSZ   = 3.0 * METERS
        FOCLLN  = 125. * KM
        FREQ    = 0.5
        HTRANS  = 12. * KM
        HTARGET = 12. * KM
75      MSIZE   = 6
        NAPDPR  = 12
        NDZ     = 9
        NEL     = 1
        NHDP    = 4
80      NPLOT   = 1
        NRINGS  = 0
        NROSPT  = 1
        NSEED   = 1
        NTIME   = 0
85      PHVAR   = 1.
        RANGE   = 125. * KM
        RBUKIT  = 1.
        RELHUM  = 0.5
        ROSPMG(1) = 1.
90      POWER   = 1.25 * MW
        ROSPWD(1) = 20. * CM
        ROXPOS(1) = 0.
        ROYPOS(1) = 0.
        SRUFF   = 10.E-5
95      TEMP(1) = 290.
        TRAKJTR = 0.
        TURBTST = 0.05
        VMAX    = 50.
        VORBIT  = 7800.
100     VTARG   = 400.
        VWIND   = 1.
        VTRANS  = 10.
        WAVEL   = 3.8E-6 * METERS
        WVPR    = 0.3 * 17.5
105     LCD2    = .FALSE.
        LCD2SW  = .FALSE.
        LCOP10  = .FALSE.
        LCOP11  = .FALSE.
        LGAMP   = .FALSE.
110     LJANRY  = .FALSE.
        LMVGTR  = .TRUE.
        LPFAST  = .TRUE.
        LPLOTI  = .FALSE.
        LPLOTP  = .FALSE.

```

SUBROUTINE ASUB

74/175 OPT=2

FTN 4.8+528

```

115      LREFI = .FALSE.
      LTRID = .FALSE.
      LTURB = .FALSE.
      LVERT = .FALSE.
      ICD = 1
120      ITR = 1
      NCS = 1
      NCF = 6
      NIT = 1
      IPRINT = 0
125      LPRINT = .FALSE.
      RETURN
      ENTRY SETUP
      WVL = WVEL
      KAY = 2.*PI/WVL
130      NSIZE = 2**MSIZE
      NSIZ2 = NSIZE*NSIZE
      MID = NSIZE/2 + 1
      NHI = NSIZE/2
      DLX = FLDSZ / (NSIZE - 1)
135      IF (ELANG.EQ.0) LVERT = .FALSE.
      ELANG = ELANG*PI/180.
      IF (LVERT) RANGE = HTARGT/SIN(ELANG)
      IF (LVERT) FCCLN=RANGE
      IF (LVERT) VTARG = VORBIT*SIN(ELANG)
140      OMEGA=(VTARG-VTRANS)/RANGE
      DO 1000 N=1,20
      ZPOS(N)= (N-1)*RANGE/NHOP
      ZPOS(N+1) = N*RANGE/NHOP
      DELZ(N)= ZPOS(N+1)-ZPOS(N)
145      1000 CONTINUE
      DO 1010 I=1,NSIZE
      DO 1011 J=1,NSIZE
      X(I)=(I-MID)*DLX
      Y(J)=(J-MID)*DLX
150      1011 CONTINUE
      1010 CONTINUE
      RETURN
      ENTRY YNEWT
      DO 1100 I=1,220
      TRAMP(I) = 1.
155      1100 IF (LTRID) TRAMP(I) = I
      DO 1104 I=1,3
      MI = MOD(I,2)
      XSPACE(I) = 2. - I
160      XSPACE(7-I) = 2. - I
      YSPACE(I) = -1 + 0.5*MI
      1104 YSPACE(7-I) = 1. - 0.5*MI
      ELYPOS(1) = 0.
      ELXPOS(1) = 0.
165      ELXPOS(2) = 0.
      ELYPOS(2) = 0.
      N = 2
      IF (NRINGS .EQ. 0) GO TO 1120
      DO 1110 NR=1,NRINGS
      ELYPOS(N) = ELYPOS(N) + ELSEP
170      DO 1108 N=1,6

```

SUBROUTINE ASUB

74/175 OPT=2

FTN 4.8+528

```

      DO 1106 NN=1,NR
      N = N + 1
      ELYPOS(N) = ELYPOS(N-1) + YSPACE(M)*ELSEP
175      ELXPOS(N) = ELXPOS(N-1) + XSPACE(M)*ELSEP*SQRT(3.)/2.
      1106 CONTINUE
      1108 CONTINUE
      1110 CONTINUE
      1120 NEL = N-1
180      IMLOOP = 0
      NSCRN = 1
      RETURN
      ENTRY STTRCA
      RADSQ = ELRAD**2
185      DO 1210 I=1,NSIZE
      DO 1208 J=1,NSIZE
      1208 C(I,J) = 1.E-30
      1210 CONTINUE
      DO 1215 N=1,NEL
190      1215 NPEL(N) = 0
      DO 1240 N=1,NEL
      IMIN = (ELXPOS(N) - ELRAD)/DLX - 1. + MID
      IMAX = (ELXPOS(N) + ELRAD)/DLX + 1. + MID
      JMIN = (ELYPOS(N) - ELRAD)/DLX - 1. + MID
195      JMAX = (ELYPOS(N) + ELRAD)/DLX + 1. + MID
      DO 1230 I=IMIN,IMAX
      DO 1228 J=JMIN,JMAX
      DUMR = ((I-MID)*DLX-ELXPOS(N))**2 + ((J-MID)*DLX-ELYPOS(N))**2
      IF (DUMR.GT.RADSQ) GO TO 1228
200      C(I,J) = TRAMP(N)
      NPEL(N) = NPEL(N) + 1
      1228 CONTINUE
      1230 CONTINUE
      1240 CONTINUE
205      RETURN
      END

```

SUBROUTINE BSUB

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE BSUB
      REAL      INCHES ,KM      ,METERS ,KW      ,MW
      COMPLEX   C      ,CDUM    ,EYE     ,TRAMP   ,CS
      LOGICAL   LTRID   ,LGAMP   ,LPLOTI  ,LCLEAN  ,LCO2
5      B        ,LMVGTR ,LJANRY ,LPFAST  ,LVERT   ,LREFI
      C        ,LCOP10 ,LCOP11 ,LCO2SW  ,LPLOTP  ,LTURB
      D        ,LPRINT
      COMMON/REAL/ DLX ,DLZ ,ELRAD ,ELSEP ,FLOSZ
10     B        ,FOCLLN ,PI      ,WVL    ,RANGE ,KAY
      C        ,INCHES ,KM      ,METERS ,CH
      D        ,POWER  ,DISTNO ,SLEWNO ,VORBIT ,STREHL
      E        ,FREQ   ,CAVE   ,CRMS   ,PHVAR  ,EO
      F        ,APRAD  ,SRUFF  ,HTRANS ,VMAX   ,VTRANS
15     G        ,VTARG ,VWIND ,WVPR   ,RELHUM ,ELANG
      H        ,WATTS  ,KW      ,MW      ,OMEGA  ,ACUT
      I        ,RBUKIT ,FLDMG  ,WAVEL  ,TRANS  ,TRPOW
      J        ,TRBI   ,XAVE   ,YAVE   ,SPOTSZ ,HTARGT
      K        ,TURBJTR ,TRAKJTR ,BMJTR ,RO      ,TURBTST
      COMMON/INT / IMLOOP ,INUM  ,INTVL  ,MSIZE  ,NEL
20     B        ,NRINGS ,NSCRN ,NSIZE  ,NSTEP  ,NHOP
      C        ,IFFER  ,MID   ,NHI    ,NAPDPR ,NROSPT
      D        ,NPTS   ,ITAPE ,NPLOTS ,NSEED  ,NOZ
      E        ,NTIME  ,NSIZ2 ,NPLOT  ,NSCALE
      F        ,ICD    ,NCS   ,NCF    ,IPRINT ,ITR
25     G        ,MA     ,I     ,J     ,NIT
      COMMON/CMPLX/EYE
      COMMON/LOG /LTRID   ,LGAMP   ,LPLOTI  ,LCLEAN  ,LCO2
      B        ,LMVGTR ,LJANRY ,LPFAST  ,LVERT   ,LREFI
      C        ,LCOP10 ,LCOP11 ,LCO2SW  ,LPLOTP  ,LTURB
30     D        ,LPRINT
      COMMON/ARRAY/ELXPOS(220) ,ELYPOS(220) ,NPOL(220)
      B        ,TRAMP(220) ,TRPHAS(220) ,TRPWR(220)
      C        ,C(64,64) ,CDUM(64) ,RDUM(64)
      D        ,IDUM(64) ,SFFT(16) ,FAC(33)
35     E        ,ZPOS(21) ,XSPACE(6) ,YSPACE(6)
      F        ,DELZ(21) ,ROSPMG(10) ,PHAX(10)
      G        ,ROXPOS(10) ,ROYPDS(10) ,ROSPWD(10)
      H        ,ACCOOL(21) ,STORE(64,64) ,CS(64,64)
      I        ,TEMP(21) ,HEIGHT(21) ,TAU(21)
40     J        ,ALPHA(21) ,AH2D(21) ,ACO2(21)
      K        ,PX(220) ,PY(220) ,PB(220)
      L        ,AL(500) ,H(500) ,TMP(500)
      M        ,RAD(500) ,VW(500) ,Z(500)
      N        ,DIST(21) ,DZ(500) ,ARATIO(500)
45     O        ,X(64) ,Y(64) ,ZK(22) ,ACDEF(22) ,SOME(22)
      P        ,ZZ(22) ,V(64,64) ,SZ(22) ,BC(22) ,CP(22,22)
      Q        ,S(64,64)
      DATA KDIN/64/
      NAMELIST/ERR/IFFER
50     ABSQ(C)=REAL(C)**2+AIMAG(C)**2
      REXP(X)=EXP(AMAX1(-600.,AMIN1(600.,X)))
      ENTRY CIRCLE
      APSQ = APRAD**2
      DO 1450 I=1,NSIZE
80     DO 1440 J=1,NSIZE
      DUM = ((J-MID)**2 + (I-MID)**2)*DLX**2
      IF (DUM.GT.APSQ) C(I,J) = 1.E-30

```

SUBROUTINE BSUB

74/175 OPT=2

FTN 4.8+528

```

1440 CONTINUE
1450 CONTINUE
60      RETURN
        ENTRY UNITY
        DO 1550 I=1,NSIZE
        DO 1540 J=1,NSIZE
1540    C(I,J) = 1.
65      1550 CONTINUE
        RETURN
        ENTRY PROPST
        CALL FFT2D(C,MSIZE,0,KDIM)
        KAY = 2.*PI/WVL
70      DLALF = WVL/FLDSZ
        DO 2000 I=1,MID
        ALF2 = (DLALF*(I-1))**2
2000    FAC(I) = -(PI/WVL)*ALF2*(1.+.25*ALF2*(1.+.5*ALF2))
        RETURN
75      ENTRY FRMSPC
        INV = 1
        GO TO 2200
        ENTRY FRMFLO
        INV = 2
80      2200 CONTINUE
        CALL FFT2D(C,MSIZE,INV,KDIM)
        RETURN
        ENTRY LENS
        APSQ = APRAD**2
85      PHAS = PI*APSQ/WVL/FOCLLN
        DO 2320 I=1,NSIZE
        DO 2318 J=1,NSIZE
        DUM = ((J-MID)**2 + (I-MID)**2)*DLX**2
        PHASE = PI*DUM/WVL/FOCLLN
90      IF (DUM.GT.APSQ) C(I,J) = 1.E-30
        IF (DUM.GT.APSQ) GO TO 2318
        C(I,J) = C(I,J)*CEXP(-EYE*(PHASE-PHAS))
2318 CONTINUE
95      2320 CONTINUE
        RETURN
        ENTRY PROP
        DLZ = ZPOS(NSCRN+1) - ZPOS(NSCRN)
        IF (NSTEP.EQ.-1) DLZ = ZPOS(NSCRN)-ZPOS(NSCRN-1)
        FCUT = ACUT*FLDSZ**2/(WVL*RANGE)
100     DO 2410 N=1,MID
        F = FLOAT(N)
        RDUM(N) = REXP(-((F-1.)/FCUT)**NAPDPR)
        CDUM(N) = CEXP(EYE*DLZ*FAC(N))*RDUM(N)
105     2410 CONTINUE
        DO 2420 I=1,NSIZE
        DO 2418 J=1,NHI
        C(I,J) = C(I,J)*CDUM(J)
        C(I,J+NHI) = C(I,J+NHI)*CDUM(MID+1-J)
2418 CONTINUE
110    2420 CONTINUE
        DO 2440 J=1,NSIZE
        DO 2438 I=1,NHI
        C(I,J) = C(I,J)*CDUM(I)
        C(I+NHI,J) = C(I+NHI,J)*CDUM(MID+1-I)

```


SUBROUTINE BSUB

74/175 OPT=2

FTN 4.8+528

```
115      2438 CONTINUE
      2440 CONTINUE
          NSCRN = NSCRN + NSTEP
          RETURN
          ENTRY PRPFWD
120      NSTEP = 1
          NSCRN = 1
          RETURN
          END
```

SUBROUTINE CSUB

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE CSUB
      REAL      INCHES      ,KM      ,METERS      ,KW      ,MW
      COMPLEX    C          ,CDUM      ,EYE      ,TRAMP      ,CS
      LOGICAL    LTRID      ,LGAMP      ,LPLOTI      ,LCLEAN      ,LCO2
5      B          ,LMVGTR      ,LJANRY      ,LPFAST      ,LVERT      ,LREFI
      C          ,LCOPI0      ,LCOPI1      ,LCO2SW      ,LPLOTP      ,LTURB
      D          ,LPRINT
      COMMON/REAL/ DLX      ,DLZ      ,ELRAD      ,ELSEP      ,FLDSZ
10     B          ,FOCLLN      ,PI      ,WVL      ,RANGE      ,KAY
      C          ,INCHES      ,KM      ,METERS      ,CM
      D          ,POWER      ,DISTNO      ,SLEWNO      ,VORBIT      ,STREHL
      E          ,FREQ      ,CAVE      ,CRMS      ,PHVAR      ,EO
      F          ,APRAD      ,SRUFF      ,HTRANS      ,VMAX      ,VTRANS
15     G          ,VTARG      ,VWIND      ,WVPR      ,RELHUM      ,ELANG
      H          ,WATTS      ,KW      ,MW      ,OMEGA      ,ACUT
      I          ,RBUKIT      ,FLDMG      ,WAVEL      ,TRANS      ,TRPOW
      J          ,TRBI      ,XAVE      ,YAVE      ,SPOTSZ      ,HTARGET
      K          ,TURBJTR      ,TRAKJTR      ,BMJTR      ,RO      ,TURBTST
20     COMMON/INT / INLOOP      ,INUM      ,INTVL      ,MSIZE      ,NEL
      B          ,NRINGS      ,NSCRN      ,NSIZE      ,NSTEP      ,NHOP
      C          ,IFFER      ,MID      ,NHI      ,NAPOPR      ,NROSPT
      D          ,NPTS      ,ITAPE      ,NPLOTS      ,NSEED      ,NDZ
      E          ,NTIME      ,NSIZ2      ,NPLOT      ,NSCALE
25     F          ,ICD      ,NCS      ,NCF      ,IPRINT      ,ITR
      G          ,MA      ,I      ,J      ,NIT
      COMMON/CMPLX/EYE
      COMMON/LOG /LTRID      ,LGAMP      ,LPLOTI      ,LCLEAN      ,LCO2
      B          ,LMVGTR      ,LJANRY      ,LPFAST      ,LVERT      ,LREFI
      C          ,LCOPI0      ,LCOPI1      ,LCO2SW      ,LPLOTP      ,LTURB
30     D          ,LPRINT
      COMMON/ARRAY/ELXPOS(220)      ,ELYPOS(220)      ,NPOL(220)
      B          ,TRAMP(220)      ,TRPHAS(220)      ,TRPWR(220)
      C          ,C(64,64)      ,CDUM(64)      ,RDUM(64)
35     D          ,IDUM(64)      ,SFFT(16)      ,FAC(33)
      E          ,ZPOS(21)      ,XSPACE(6)      ,YSPACE(6)
      F          ,DELZ(21)      ,ROSPMG(10)      ,PMAX(10)
      G          ,ROXPDS(10)      ,ROYPOS(10)      ,ROSPWD(10)
      H          ,ACDOL(21)      ,STORE(64,64)      ,CS(64,64)
40     I          ,TEMP(21)      ,HEIGHT(21)      ,TAU(21)
      J          ,ALPHA(21)      ,AH20(21)      ,ACQ2(21)
      K          ,PX(220)      ,PY(220)      ,PB(220)
      L          ,AL(500)      ,H(500)      ,THP(500)
      M          ,RAD(500)      ,VW(500)      ,Z(500)
45     N          ,DIST(21)      ,DZ(500)      ,ARATIO(500)
      O          ,X(64)      ,Y(64)      ,ZK(22)      ,ACDEF(22)      ,SOME(22)
      P          ,ZZ(22)      ,V(64,64)      ,SZ(22)      ,BC(22)      ,CP(22,22)
      Q          ,S(64,64)
      DATA KDIM/64/
      NAMELIST/ERR/IFFER
50     ABSQ(C)=REAL(C)**2+AIMAG(C)**2
      REXP(X)=EXP(AMAX1(-600.,AMIN1(600.,X)))
      ENTRY STRPHS
      DO 2570 JJ=1,NSIZE
      J = MID + JJ - 1
55     IF (J.GT.NSIZE) J=NSIZE-JJ+1
      DO 2568 II=1,NSIZE
      I = MID+II-1

```

SUBROUTINE CSUB

74/175 OPT=2

FTN 4.8+528

```

        IF (I.GT.NSIZE) I=NSIZE-II+1
        RC = REAL (C(I,J))
60      AIC = AIMAG(C(I,J))
        AMPS = ABSQ(C(I,J))
        AMPL = SQRT(AMPS)
        IF ((AIC.EQ.0.) .AND. (RC.EQ.0.)) PHA = 0.
        IF ((AIC.EQ.0.) .AND. (RC.EQ.0.)) GO TO 2558
65      PHA = ATAN2(AIC,RC)
2558   CONTINUE
2560   IF (II.EQ.1) GO TO 2564
        IF (I.GT.MID) PHADIF=STORE(I-1,J)-PHA
        IF (I.LT.MID) PHADIF=STORE(I+1,J)-PHA
70      2561 PRV = PHADIF + PHA
2562   IF (ABS(PHADIF).GT.PI) PHA=PHA+2.*SIGN(PI,PHADIF)
        PHADIF = PRV - PHA
        IF (ABS(PHADIF).GT.PI) GO TO 2562
        GO TO 2566
75      2564 IF (J.EQ.MID) GO TO 2566
        PHADIF = STORE(I,J-1) - PHA
        IF (J.LT.MID) PHADIF=STORE(I,J+1) - PHA
        PRV = PHADIF + PHA
2565   IF (ABS(PHADIF).GT.PI) PHA=PHA+2.*SIGN(PI,PHADIF)
        PHADIF = PRV - PHA
        IF (ABS(PHADIF).GT.PI) GO TO 2565
80      2566 STORE(I,J) = PHA
        IF (CABS(C(I,J)) .LT. 1.E-25) STORE(I,J)=0.
        CS(I,J) = CMPLX(AMPS,PHA)
85      2568 CONTINUE
2570   CONTINUE
        RETURN
        ENTRY PRPBAK
        NSTEP = -1
90      RETURN
        ENTRY PRPSCN
        INUM = 20 + NSCRN
        REWIND INUM
        CALL READA(INUM,STORE,NSIZ2)
95      DUM = EXP(-DELZ(NSCRN)*(ALPHA(NSCRN)/2.))
        DO 3320 J=1,NSIZE
        DO 3310 I=1,NSIZE
        IJ = NSIZE*(I-1) + J
100     3310 C(I,J) = C(I,J)*DUM*CEXP(EYE*STORE(IJ))
        3320 CONTINUE
        RETURN
        ENTRY VACSCN
        DUM = EXP(-DELZ(NSCRN)*(ALPHA(NSCRN)/2.))
        DO 3370 J=1,NSIZE
        DO 3365 I=1,NSIZE
105     3365 C(I,J) = C(I,J)*DUM
        3370 CONTINUE
        RETURN
        ENTRY BLMSET
110     NMAX = 500
        KMAX = 499
        P1 = 2.899E-5
        P2 = .2030
        P3 = 3.146E-5

```

SUBROUTINE CSUB

74/175 OPT=2

FTN 4.8+528

```

115      P4 = .7322
        A1 = 2.E-6
        B1 = 3.85E-5
        A2 = 7.E-5
        B2 = .2425E-3
120      Q2 = 12.
        Q3 = 11.7
        Q4 = .1
        Q5 = .01
        Z(1) = 0.
125      DZ(1) = 1.
        BI = ALOG(VTARG/VWIND)/NMAX
        BIL=0.
        DO 3402 N=1,KMAX
          DZ(N+1) = (1+BIL)*DZ(N)
130      3402 Z(N+1) = Z(N) + DZ(N)
          ZMAX = Z(NMAX)
          DO 3404 N=1,KMAX
            DZ(N) = DZ(N)*RANGE/ZMAX
          3404 Z(N+1) = Z(N) + DZ(N)
135      SEA = SIN(ELANG)
          DO 3410 N=1,NMAX
            H(N) = Z(N)*SEA+HTRANS
            HK = H(N)/KM
            IF(LCOPI0) GO TO 3470
            IF(LCOPI1) GO TO 3480
            IF(LCO2) GO TO 3405
            IF(LCO2SW) GO TO 3405
            AL(N) = P1*EXP(-HK*P2) + P3*EXP(-HK*P4)
            GO TO 3406
145      3405 CONTINUE
            IF(HK.LT.16.) ALC = 2.389E-3/(30.25+HK**2)
            IF((HK.GE.16.).AND.(HK.LE.55.)) ALC=1.224E-3/(144.+(HK-29.)**2)
            IF(HK.GE.55.) ALC = 2.205*EXP(-.25945*HK)
            IF (LJANRY)
150      B ALH = .0334E-3*EXP(-.705*HK) + .101E-3*EXP(-1.15*HK)
            IF (.NOT.LJANRY)
            B ALH = .075E-3*EXP(-.635*HK) + .433E-3*EXP(-1.01*HK)
            IF(LCO2SW) ALC=0.
            AL(N) = ALC + ALH
155      ARATIO(N) = ALC/AL(N)
            GO TO 3406
          3470 IF (HK.LT.15.) 3472,3474
          3472 ACO = 3.4E-4
            BCO = -.733
            GO TO 3490
          3474 ACO = 2.3E-7
            BCO = -2.47E-1
            GO TO 3490
          3480 IF (HK.LT.9.) 3482,3484
165      3482 ACO = 2.5E-3
            BCO = -.7
            GO TO 3490
          3484 IF (HK.LT.15.) 3486,3488
170      3486 ACO = .1054
            BCO = -1.115
            GO TO 3490

```

SUBROUTINE CSUB

74/175 OPT=2

FTN 4.0+528

```

3488 ACO = 3.8E-8
BCD = -.126
175 3490 AL(N) = ACO*EXP(BCD*HK)
3406 CONTINUE
      IF (H(N).LE.11.5E3) TMP(N)=288.-6.4*H(N)/KM
      IF (H(N).GT.11.5E3) TMP(N)=217.
      IF (H(N).GT.25.5E3) TMP(N)=140.+3.*H(N)/KM
      IF (HK.GT.47.5) TMP(N)=407.3-2.68*HK
180  RAD(N) = APRAD+(Z(N)/RANGE)*(0.5*WVL*RANGE/APRAD-APRAD)
      RAD(N) = APRAD+(Z(N)/RANGE)*(0.5*WVL*RANGE/APRAD)
      RAD(N) = ELRAD
      IF (.NOT.LMVGTR)
        *VW(N)=VMAX*Q2/(Q2+(HK-Q3)**2+Q4*(HK-Q3)**3+Q5*(HK-Q3)**4)
185  IF (LMVGTR) VW(N) = VTRANS
3410 CONTINUE
      SUM = 0.
      DO 3420 N=1,NMAX
190 3420 SUM = SUM+(AL(N)/((VW(N)+OMEGA*Z(N))*RAD(N)*TMP(N)))*DZ(N)
      PSUM = SUM/(NHOP-.8)
      M = 2
      ZPOS(1) = 0.
      HEIGHT(1) = HTRANS
      NN = 0
195  TPE = 0.
      TOW = 0.
      ALPH = 0.
      SUM = 0.
      DST = 0.
200  DO 3440 N=1,NMAX
      NN = NN+1
      G1 = (RELHUM/3.48E-3)*EXP(-H(N)*5.75E-4)
      G2 = EXP(-H(N)*1.64E-4)/4.08E-2
      G3 = 8.5*WVPR/(1.+1.E-3*H(N))+38.6*EXP(-1.7E-4*H(N))
205  TAW = (1./(G1+G2+G3))*(1.-5.*WVPR/(6.*17.5))
      TOW = TOW + TAW*DZ(N)
      DUM = TAW * (VW(N)+OMEGA*Z(N))
      COOL = 1.
      IF (LCO2) COOL=1.-2.441*ARATIO(N)*EXP(-2*APRAD/DUM)
210  SUM = SUM+(AL(N)/((VW(N)+OMEGA*Z(N))*RAD(N)*TMP(N)))*DZ(N)
      DST = DST+(AL(N)/((VW(N)+OMEGA*Z(N))*TMP(N)))*DZ(N)*COOL
      ALPH = ALPH + AL(N)*DZ(N)
      TPE = TPE + TMP(N)*DZ(N)
      ZMAX = RANGE
215  IF (Z(N).GT.ZMAX) GO TO 3435
      IF (N.EQ.NMAX) GO TO 3435
      IF (SUM.GE.PSUM) 3435,3440
3435 ZPOS(M) = Z(N)
      IF (N.EQ.NMAX) ZPOS(M) = RANGE
220  DELZ(M-1) = ZPOS(M) - ZPOS(M-1)
      ALPHA(M-1) = ALPH/DELZ(M-1)
      DIST(M-1) = DST
      HEIGHT(M) = H(N)
      TAU(M-1) = TOW/DELZ(M-1)
225  TEMP(M-1) = TPE/DELZ(M-1)
      IDUM(M-1) = N
      M = M+1
      NN = 0

```

SUBROUTINE CSUB

74/175 OPT=2

FTN 4.8+528

```

230      DST = 0.
      SUM = 0.
      ALPH = 0.
      TQW = 0.
      TPE = 0.
344C CONTINUE
235      TRANS = 1.
      DO 3450 N=1,NHOP
3450 TRANS = TRANS*EXP(-ALPHA(N)*DELZ(N))
      WRITE (6,3499) TRANS
      MHOP = NHOP+1
240      WRITE (6,3491) (ZPOS(M),M=1,MHOP)
      WRITE (6,3492) (DELZ(M),M=1,MHOP)
      WRITE (6,3493) (HEIGHT(M),M=1,MHOP)
      WRITE (6,3494) (ALPHA(M),M=1,MHOP)
      WRITE (6,3495) (DIST(M),M=1,MHOP)
245      WRITE (6,3497) (TAU(M),M=1,MHOP)
      WRITE (6,3498) (TEMP(M),M=1,MHOP)
      WRITE (6,3496) (IDUM(M),M=1,MHOP)
3491 FORMAT(1H0,2X,10HZPOSITION=,8X,(9E12.4))
3492 FORMAT(1H ,2X,10HDELZ= ,8X,(9E12.4))
250 3493 FORMAT(1H ,2X,10HHEIGHT= ,8X,(9E12.4))
3494 FORMAT(1H ,2X,10HALPHA= ,8X,(9E12.4))
3495 FORMAT(1H ,2X,11HDISTORTION=,7X,(9E12.4))
3496 FORMAT(1H ,2X,13HSTEP NUMBER= ,9I12))
3497 FORMAT(1H ,2X,4HTAU=,14X,(9E12.4))
255 3498 FORMAT(1H ,2X,12HTEMPERATURE=,6X,(9E12.4))
3499 FORMAT(///,2X,13HTRANSMISSION=,5X,F8.4)
      RETURN
      ENTRY SCNBK
      INUM=20+NSCRN
260 REWIND INUM
      CALL READA(INUM,STORE,NSIZ2)
      DUM=EXP(-DELZ(NSCRN)*(ALPHA(NSCRN)/2.))
      WAVEBK=10.E-6
      DO 1814 J=1,NSIZE
265 DO 1815 I=1,NSIZE
      IJ=NSIZE*(I-1) + J
1815 C(I,J)=C(I,J)*DUM*CEXP(EYE*STORE(IJ)*(WAVEL/WAVEBK))
1814 CONTINUE
      RETURN
270 ENTRY BAKLENS
      APSQ=APRAD**2
      PHAS=PI*APSQ/WVL/(RANGE/2.)
      DO 1909 I=1,NSIZE
      DO 1910 J=1,NSIZE
275 DUM=((J-MID)**2+(I-MID)**2)*DLX**2
      PHASE=PI*DUM/WVL/(RANGE/2.)
      IF(DUM.GT.APSQ) C(I,J)=1.E-30
      IF(DUM.GT.APSQ) GO TO 879
      C(I,J)=C(I,J)*CEXP(-EYE*(PHASE-PHAS))
280 879 CONTINUE
      1910 CONTINUE
      1909 CONTINUE
      RETURN
      END

```

SUBROUTINE DSUB

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE DSUB
      REAL      INCHES ,KM      ,METERS ,KW      ,MW
      COMPLEX   C      ,COUM    ,EYE     ,TRAMP   ,CS
      LOGICAL   LTRID   ,LGAMP   ,LPLOTI  ,LCLEAN  ,LCO2
5      B        ,LMVGTR ,LJANRY ,LPFAST  ,LVERT   ,LREFI
      C        ,LCOP10 ,LCOP11 ,LCO2SW  ,LPLOTP  ,LTURB
      D        ,LPRINT
      COMMON/REAL/ DLX ,DLZ ,ELRAD ,ELSEP ,FLDSZ
10     B        ,FOCLLN ,PI      ,WVL    ,RANGE ,KAY
      C        ,INCHES ,KM      ,METERS ,CM
      D        ,POWER  ,DISTNO ,SLEWNO ,VORBIT ,STREHL
      E        ,FREQ   ,CAVE   ,CRMS   ,PHVAR  ,EO
      F        ,APRAD  ,SRUFF  ,HTRANS ,VMAX   ,VTRANS
15     G        ,VTARG ,VWIND ,WVPR   ,RELHUM ,ELANG
      H        ,WATTS  ,KW      ,MW      ,OMEGA ,ACUT
      I        ,RBUKIT ,FLDMG  ,WAVEL  ,TRANS  ,TRPOW
      J        ,TRBI   ,XAVE   ,YAVE   ,SPOTSZ ,HTARGET
      K        ,TURBJTR ,TRAKJTR ,BMJTR ,RO      ,TURBTST
20     COMMON/INT / IMLOOP ,INUM  ,INTVL ,MSIZE  ,NEL
      B        ,NRINGS ,NSCRN  ,NSIZE  ,NSTEP  ,NHOP
      C        ,IFFER  ,MID    ,NHI    ,NAPDPR ,NROSPT
      D        ,NPTS   ,ITAPE  ,NPLOTS ,NSEED  ,NDZ
      E        ,NTIME  ,NSIZ2  ,NPLOT  ,NSCALE
25     F        ,ICD   ,NCS    ,NCF    ,IPRINT ,ITR
      G        ,HA     ,I      ,J      ,NIT
      COMMON/CMPLX/EYE
      COMMON/LOG /LTRID ,LGAMP ,LPLOTI ,LCLEAN ,LCO2
30     B        ,LMVGTR ,LJANRY ,LPFAST  ,LVERT   ,LREFI
      C        ,LCOP10 ,LCOP11 ,LCO2SW  ,LPLOTP  ,LTURB
      D        ,LPRINT
      COMMON/ARRAY/ELXPOS(220) ,ELYPPOS(220) ,NPOL(220)
      B        ,TRAMP(220) ,TRPHAS(220) ,TRPWR(220)
      C        ,C(64,64) ,CDUM(64) ,RDUM(64)
35     D        ,IDUM(64) ,SFFT(16) ,FAC(33)
      E        ,ZPOS(21) ,XSPACE(6) ,YSPACE(6)
      F        ,DELZ(21) ,ROSPMG(10) ,PHAX(10)
      G        ,ROXPOS(10) ,ROYPOS(10) ,ROSPWD(10)
      H        ,ACDOL(21) ,STORE(64,64) ,CS(64,64)
40     I        ,TEMP(21) ,HEIGHT(21) ,TAU(21)
      J        ,ALPHA(21) ,AH20(21) ,ACQ2(21)
      K        ,PX(220) ,PY(220) ,PB(220)
      L        ,AL(500) ,H(500) ,THP(500)
      M        ,RAD(500) ,VW(500) ,Z(500)
      N        ,DIST(21) ,DZ(500) ,ARATIO(500)
45     O        ,X(64) ,Y(64) ,ZK(22) ,ACDEF(22) ,SOME(22)
      P        ,ZZ(22) ,V(64,64) ,SZ(22) ,BC(22) ,CP(22,22)
      Q        ,S(64,64)
      DATA KDIM/64/
      NAMELIST/ERR/IFFER
50     ABSQ(C)=REAL(C)**2+AIMAG(C)**2
      REXP(X)=EXP(AMAX1(-600.,AMIN1(600.,X)))
      ENTRY VBLOOM
      INUM = 20 + NSCRN
      REWIND INUM
55     FACTOR = 1.
      IF(LGAMP) FACTOR = 1./(1.-EXP(-(APRAD/ELRAD)**2))
      DISTNO = .234E-6*RANGE*POWER*KAY*DIST(NSCRN)/ELRAD

```

SUBROUTINE DSUB

74/175 OPT=2

FTN 4.8+528

```

        PHMULT = DISTNO/(RANGE*ELRAD*PI*NEL)
        DO 3660 J=1,NSIZE
60      RISUM = 0.
        DO 3650 I=1,NSIZE
3644      BINT = REAL(C(I,J)*CONJG(C(I,J)))
3646      RISUM = RISUM + BINT*DLX
        IJ = NSIZE*(I-1) + J
65      3650 STORE(J,I) = -PHMULT * RISUM * FACTOR
3660      CONTINUE
        CALL WRITEA(INUM,STORE,NSIZ2)
        END FILE INUM
        RETURN
70      ENTRY REFLCT
        WO=1.414214*WAVEL*RANGE/(PI*2.*APRAD)
        ANGX=ROXPOS(1)/RANGE
        ANGY=ROYPOS(1)/RANGE
75      THETX=ATAN(ANGX)
        THETY=ATAN(ANGY)
        ALFAX=SIN(THETX)
        ALFAY=SIN(THETY)
        DO 4120 I=1,NSIZE
        DO 4118 J=1,NSIZE
80      DUM = 0.
        DELX=DLX*(I-MID)-ROXPOS(1)
        DELY=DLX*(J-MID)-ROYPOS(1)
        DUM=1.0*EXP(AMAX1(-100.,-1.*(DELX/WO)**2-1.0*(DELY/WO)**2))
        C(I,J)=DUM
85      PHASDF=KAY*(ALFAX*DELX+ALFAY*DELY)
        C(I,J)=C(I,J)*CEXP(-EYE*PHASDF)
4118      CONTINUE
4120      CONTINUE
        WRITE(6,435) XAVE,YAVE,ROXPOS(1),ROYPOS(1),WO
90      435 FORMAT(/5X* XAVE=*,E10.3,*, YAVE=*,E10.3,*, ROXPOS=*,
        BE10.3,*, ROYPOS=*,E10.3,*, WO=*,E10.3)
        RETURN
        ENTRY BUCKET
        BEST = (RBUKIT/DLX)**2
95      KIT = 0
        BUKIT = 0.
        DO 4290 I=1,NSIZE
        DO 4290 J=1,NSIZE
        IF (((I-MID)**2+(J-MID)**2).GT.BEST) GO TO 4290
100     KIT = KIT + 1
        BUKIT = BUKIT + ABSQ(C(I,J))
4290      CONTINUE
        TRNS = TRANS
        IF(NSCRN.EQ.1) TRNS=1.
105     BUKIT = BUKIT*DLX*DLX*POWER/(TRPOW*TRNS*MW)
        RELPWR = BUKIT*MW/POWER
        WRITE (6,4291) KIT,BUKIT,RELPWR
4291     FORMAT(/10X,I4,3X*ON-AXIS POWER IN BUCKET =*,F10.5,4H MW,,10X,
        B*RELATIVE POWER =*,F10.5)
110     TMAX = -1.E9
        DO 4292 I=1,NSIZE
        DO 4292 J=1,NSIZE
        TESA = ABSQ(C(I,J))
        IF(TESA.LE.TMAX) GO TO 4292

```


SUBROUTINE DSUB

74/175 OPT=2

FTN 4.8+528

```

115          THAX = TESA
            MIDI = I
            MIDJ = J
            ITEST = 1
120          4292 CONTINUE
            4295 CONTINUE
            KIT = 0
            BUKIT = 0.
            DO 4293 I=1,NSIZE
            DO 4293 J=1,NSIZE
125          IF(((I-MIDI)**2+(J-MIDJ)**2).GT.BEST) GO TO 4293
            KIT = KIT + 1
            BUKIT = BUKIT + ABSQ(C(I,J))
            4293 CONTINUE
            BUKIT = BUKIT*DLX*DLX*POWER/(TRPOW*TRNS*MW)
130          RELPWR = BUKIT*MW/POWER
            WRITE (6,4299) KIT,BUKIT,RELPWR
            4299 FORMAT(/10X,I4,3X*PEAK POWER IN BUCKET =    *,F10.5,4H MW,,10X,
            B*RELATIVE POWER =*,F10.5)
            MIDJ = NSIZE/2 + 1
135          ITEST = ITEST + 1
            IF(ITEST.EQ.2) GO TO 4295
            RETURN
            ENTRY CONJC
            DO 5320 I=1,NSIZE
            DO 5310 J=1,NSIZE
140          5310 C(I,J) = CONJG(C(I,J))/CABS(C(I,J))
            5320 CONTINUE
            RETURN
            END

```

SUBROUTINE ESUB

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE ESUB
      REAL      INCHES ,KM      ,METERS ,KW      ,MW
      COMPLEX   C      ,CDUM    ,EYE     ,TRAMP   ,CS
      LOGICAL   LTRID  ,LGAMP   ,LPLOTI  ,LCLEAN  ,LCO2
5      B        ,LMVGTR ,LJANRY ,LPFAST  ,LVERT  ,LREFI
      C        ,LCOP10 ,LCOP11 ,LCO2SW  ,LPLOTP ,LTURB
      D        ,LPRINT
      COMMON/REAL/ DLX ,DLZ ,ELRAD ,ELSEP ,FLDSZ
10     B        ,FOCLLN ,PI     ,WVL    ,RANGE ,KAY
      C        ,INCHES ,KM     ,METERS ,CM     ,
      D        ,POWER  ,DISTNO ,SLEWNO ,VORBIT ,STREHL
      E        ,FREQ   ,CAVE   ,CRMS   ,PHVAR  ,EO
      F        ,APRAD  ,SRUFF  ,HTRANS ,VMAX   ,VTRANS
15     G        ,VTARG ,VWIND ,WVPR   ,RELHUM ,ELANG
      H        ,WATTS  ,KW     ,MW     ,OMEGA  ,ACUT
      I        ,RBUKIT ,FLDMG  ,WAVEL  ,TRANS  ,TRPOW
      J        ,TRBI   ,XAVE   ,YAVE   ,SPOTSZ ,HTARGT
      K        ,TURBJTR ,TRAKJTR ,BMJTR ,RO     ,TURBTST
20     COMMON/INT / IMLOOP ,INUM  ,INTVL ,MSIZE ,NEL
      B        ,NRINGS ,NSCRN  ,NSIZE  ,NSTEP ,NHOP
      C        ,IFFER  ,MID    ,NHI    ,NAPDPR ,NROSPT
      D        ,NPTS   ,ITAPE  ,NPLOTS ,NSEED ,NDZ
      E        ,NTIME  ,NSIZ2  ,NPLOT  ,NSCALE
25     F        ,ICD   ,NCS    ,NCF    ,IPRINT ,ITR
      G        ,MA     ,I      ,J      ,NIT
      COMMON/CMPLX/EYE
      COMMON/LOG /LTRID ,LGAMP ,LPLOTI ,LCLEAN ,LCO2
30     B        ,LMVGTR ,LJANRY ,LPFAST  ,LVERT  ,LREFI
      C        ,LCOP10 ,LCOP11 ,LCO2SW  ,LPLOTP ,LTURB
      D        ,LPRINT
      COMMON/ARRAY/ELXPOS(220) ,ELYP0S(220) ,NP0L(220)
      B        ,TRAMP(220) ,TRPHAS(220) ,TRPWR(220)
      C(64,64) ,CDUM(64) ,RDUM(64)
35     D        ,IDUM(64) ,SFFT(16) ,FAC(33)
      E        ,ZPOS(21) ,XSPACE(6) ,YSPACE(6)
      F        ,DELZ(21) ,ROSPMG(10) ,PHAX(10)
      G        ,ROXPOS(10) ,ROYPOS(10) ,ROSPWD(10)
      H        ,ACDOL(21) ,STORE(64,64) ,CS(64,64)
40     I        ,TEMP(21) ,HEIGHT(21) ,TAU(21)
      J        ,ALPHA(21) ,AH2D(21) ,AC02(21)
      K        ,PX(220) ,PY(220) ,PB(220)
      L        ,AL(500) ,H(500) ,TMP(500)
      M        ,RAD(500) ,VW(500) ,Z(500)
      N        ,DIST(21) ,DZ(500) ,ARATIO(500)
45     O        ,X(64) ,Y(64) ,ZK(22) ,AC0EF(22) ,SOME(22)
      P        ,ZZ(22) ,V(64,64) ,SZ(22) ,BC(22) ,CP(22,22)
      Q        ,S(64,64)
      DATA KDIM/64/
      NAMELIST/ERR/IFFER
50     ABSQ(C)=REAL(C)**2+AIMAG(C)**2
      REXP(X)=EXP(AMAX1(-600.,AMIN1(600.,X)))
      ENTRY TOTPWR
      SUM = 0.
      DO 6720 J=1,NSIZE
85     DO 6710 I=1,NSIZE
6710 SUM = SUM+ABSQ(C(I,J))
6720 CONTINUE

```

SUBROUTINE ESUB

74/175 OPT=2

FTN 4.8+528

```

        SUM = SUM*DLX**2
        TRNS = TRANS
60      IF (NSCRN.EQ.1) TRNS = 1.
        RPWR = SUM*POWER/(TRPOW*TRNS)/MW
        RELPWR = RPWR*MW/POWER
        WRITE (6,6790) RPWR,RELPWR,NSCRN
6790 C FORMAT(1H0,4X,13HLASER POWER =,F10.5,1X,3HMMW,,10X,
65      B*RELATIVE POWER = *,F10.5,1H,,10X,6HNSCRN=,I3)
        RETURN
        ENTRY TRAPWR
        SUM = 0.
        DO 6820 J=1,NSIZE
        DO 6810 I=1,NSIZE
70      6810 SUM = SUM+ABSQ(C(I,J))
        6820 CONTINUE
        TRPOW = SUM*DLX**2
        DUM = POWER/MW
75      WRITE (6,6890) DUM
        TRBI = POWER*(CM*CM/KW)/(PI*NEL*ELRAD**2)
        WRITE (6,6891) TRBI
6890 C FORMAT(1H0,4X,*TRANSMITTER BEAM POWER =*,F10.5,1X,2HMMW)
6891 C FORMAT(1H ,4X,*TRANSMITTER BEAM INTENSITY =*,F10.5,1X,8HkW/CM**2)
80      RETURN
        ENTRY BEAMSZ
        XB=YB=BSUM=XXB=YYB=0.
        DO 7010 I=2,NSIZE
        DO 7010 J=2,NSIZE
85      BINT = STORE(I,J)
        X = DLX*(I-MID)
        Y=(J-MID)*DLX
        XB = XB+BINT*X
        YB = YB+BINT*Y
90      BSUM = BSUM+BINT
        XXB = XXB+BINT*X*X
        YYB = YYB+BINT*Y*Y
7010 CONTINUE
        XAVE = XB/BSUM
        YAVE = YB/BSUM
95      SPOTX=SQRT(XXB/BSUM-XAVE*XAVE)
        SPOTY=SQRT(YYB/BSUM-YAVE*YAVE)
        SPOTSZ=SQRT(SPOTX*SPOTX+SPOTY*SPOTY)
        WRITE(6,7049) XAVE,YAVE,SPOTX,SPOTY,SPOTSZ
100     7049 C FORMAT(1H0,* XAVE = *,E10.3,*, YAVE = *,E10.3,
        A//,* SPOTX = *,E10.3,*, SPOTY = *,E10.3,
        B*, SPOTSZ = *,E10.3)
        RETURN
        ENTRY STRCSQ
105     DO 9000 I=1,NSIZE
        DO 9000 J=1,NSIZE
        9000 STORE(I,J) = ABSQ(C(I,J))
        RETURN
        ENTRY PRNTI
110     N2 = 1
        IF (NSIZE.EQ.64) N2 = 2
        IF (LPFAST) N2 = 1
        DO 9250 NN=1,N2
        I1 = 1+(NN-1)*NHI

```

SUBROUTINE ESUB

74/175 OPT=2

FTN 4.8+928

```

115      I2 = NSIZE/2 + (NN-1)*NHI
          IF (N2.EQ.1) I2 = NSIZE
          IF (LPFAST) I2 = (40./64.)*NSIZE
          WRITE (6,9201)
          RMAX = 0.
120      DO 9220 J=1,NSIZE
          DO 9215 I=1,NSIZE
          RDUM(I) = STORE(I,J)
          9215 RMAX = AMAX1(RMAX,RDUM(I))
          9220 CONTINUE
125      AMX = RMAX
          IF (RMAX.EQ.0.) AMX = 1.
          J2 = NSIZE
          IF (LPFAST) J2 = (40./64.)*NSIZE
          PRINT 9241,1H
130      9241 FORMAT(A1)
          DO 9240 I=1,64
          K=0
          DO 9235 J=1,64
          K = K+1
135      IDUM(K)=STORE(I,J)*100/AMX + 0.5
          9235 CONTINUE
          CALL GRAY(IDUM)
          9240 CONTINUE
          9250 CONTINUE
140      RMAX = TRBI*RMAX*KW
          WRITE (6,9203) RMAX,NSCRN
          TRNS = TRANS
          IF (NSCRN.EQ.1) TRNS = 1.
          DUM = RMAX/TRNS
145      WRITE (6,9204) DUM
          9201 FORMAT(1H1)
          9203 FORMAT(*0*,4X,*PEAK INTENSITY =*,G12.5,9H W/CM**2, ,10X,
          B 6HNSCRN=,I3)
          9204 FORMAT(1H0,4X,45HPEAK INTENSITY DIVIDED BY TRANSMISSION LOSS =,
150      B G12.5, 8H W/CM**2 )
          RETURN
          ENTRY PRNTS
          REWIND INUM
          RMAX = 0.
          RMIN = 0.
155      CALL READA(INUM,STORE,NSIZ2)
          DO 9510 I=1,NSIZ2
          RMAX = AMAX1(RMAX,STORE(I))
          RMIN = AMIN1(RMIN,STORE(I))
160      9510 CONTINUE
          WRITE (6,9590)
          RDIFF = RMAX - RMIN
          REWIND INUM
          DO 9520 J=1,NSIZE
          DO 9515 I=1,NSIZE
          J1 = NSIZE*(J-1) + I
165      9515 IDUM(I) = (STORE(J1) - RMIN)*100./RDIFF + .5
          CALL PRINT3(IDUM,NSIZE)
          9520 CONTINUE
170      WRITE (6,9592) RMAX,RMIN
          9592 FORMAT(1H0,10HMAX VALUE=,E12.4,10X,11H,MIN VALUE=,E12.4)

```

SUBROUTINE ESUB

74/175 OPT=2

FTN 4.8+528

```

          959C FORMAT(1H1)
              RETURN
          ENTRY PRTPHS
175         J1 = MID-14
              J2 = MID+14
              DO 9840 M=1,2
                  IF (M.EQ.1) WRITE (6,9891)
                  IF (M.EQ.2) WRITE (6,9892)
180         I1 = MID+(M-2)*16.
              I2 = MID+(M-1)*16.
              DO 9830 J=J1,J2
                  WRITE (6,9890) (STORE(I,J),I=I1,I2)
185         9830 CONTINUE
              984C CONTINUE
              9890 FORMAT(1H ,2X,17F7.2)
              9891 FORMAT(1H1)
              9892 FORMAT(//)
190         RETURN
              END
```

FUNCTION NODD

74/175 OPT=2

FTN 4.8+528

```
1      FUNCTION NODD(I)
      DIMENSION NO(2)
      DATA NO/1,-1/
      J = I/2
5      K = I-J*2+1
      NODD = NO(K)
      RETURN
      END
```

FUNCTION NOOD

74/175 OPT-2

FTN 4.0+528

```
1      FUNCTION NOOD(I)
      DIMENSION NO(2)
      DATA NO/1,-1/
      J = I/2
5      K = I-J*2+1
      NOOD = NO(K)
      RETURN
      END
```

SUBROUTINE PRINT3 74/175 OPT=2

FTN 4.8+528

```
1      SUBROUTINE PRINT3(IDUM,N)
        DIMENSION IDUM(N)
        WRITE (6,3) IDUM
5        3  FORMAT(1H ,4I13)
        RETURN
        ENTRY PRINT2
        WRITE (6,2) IDUM
        2  FORMAT(1H ,64I2)
        RETURN
10     END
```


SUBROUTINE FFT2D

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE FFT2D(A,M,INV,KD)
      COMPLEX A(KD,KD),U,T,W(8,2)
      IF(INV.EQ.0) GO TO 40
      DO 26 J2=1,2
5      J = 1
      DO 7 I=1,NM1
      IF (I.GE.J) GO TO 5
      DO 4 KK=1,N
      T = A(J,KK)
10     A(J,KK) = A(I,KK)
      A(I,KK) = T
      K = NV2
      6     IF (K.GE.J) GO TO 7
      J = J-K
15     K = K/2
      GO TO 6
      7     J = J+K
      LE = 1
      DO 20 L=1,M
20     LE1 = LE
      LE = LE + LE
      U = (1.,0.)
      UR = REAL(U)
      UA = AIMAG(U)
25     DO 20 J=1,LE1
      DO 10 I=J,N,LE
      IP = I + LE1
      DO 10 KK=1,N
      T = A(IP,KK)*U
30     A(IP,KK) = A(I,KK) - T
      A(I,KK) = A(I,KK) + T
20     U = U*W(L,INV)
      DO 25 I=1,N
      DO 25 KK=I,N
35     T = A(I,KK)
      A(I,KK) = A(KK,I)
      A(KK,I) = T
25     A(KK,I) = T
26     CONTINUE
      IF (INV.EQ.2) RETURN
40     DO 30 I=1,N
      DO 30 KK=1,N
30     A(I,KK) = A(I,KK)*FN
      RETURN
40     N = 2**M
45     FN = (1./N)**2
      NV2 = N/2
      NM1 = N - 1
      DO 50 L=1,M
      ANG = -2.*3.14159265358979/2**L
50     W(L,1) = CMPLX(COS( ANG),SIN( ANG))
      W(L,2) = CMPLX(COS(-ANG),SIN(-ANG))
      RETURN
      END

```

SUBROUTINE GRAY

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE GRAY(IDUM)
      DIMENSION IDUM(64),PBUF(64),CHAR(8,4)
      DATA BLANK,PLUS/1H ,1H+/
      DATA CHAR/ 1H ,1H.,1HL,1HU,1HO,1HH,1HW,1HW,
5      +          1H ,1H ,1H ,1H ,1H-,1HI,1HM,1HM,
      +          1H ,1H ,1H ,1H ,1H ,1H ,1HT,1HI,
      +          1H ,1H ,1H ,1H ,1H ,1H ,1H ,1HA/
      NOFF=2
      RM=0.08
10     NP1=65
      DO 100 IX=1,64
      CX=1.0+0.08*IDUM(IX)
      IF (CX.LE.1.0) CX=1.5
      IF (CX.GE.9.0) CX=8.5
15     100 IDUM(IX)=CX
      CC=PLUS
      DO 500 L=1,4
      DO 200 IX=1,64
      M=IDUM(IX)
20     200 PBUF(IX)=CHAR(M,L)
      DO 300 IX=1,64
      M=NP1-IX
      IF (PBUF(M).NE.BLANK) GO TO 400
25     300 CONTINUE
      IF (CC.NE.BLANK) GO TO 500
      M=1
      400 PRINT 1000,CC,NOFF,(PBUF(IX),IX=1,M)
      500 CC=PLUS
1000  FORMAT(A1,T=,64(A1,1X))
30     PRINT 1000,BLANK
      RETURN
      END

```

APPENDIX C

IMAGE RECONSTRUCTION PROGRAM LISTING

PROGRAM IMAGE

74/175 OPT=2

FTN 4.8+528

```

1      PROGRAM IMAGE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
      *TAPE14,TAPE15=5000,TAPE16,TAPE17=5000,TAPE30,TAPE31)
      DIMENSION VPSF(64,64),BPSF(64,64),TARG(64,64),PARTIM(64,64)
      DIMENSION TOTIM(128,128)
5      DIMENSION WORK(128,128)
      DIMENSION TDUM(128)
C      TAPE14 IS THE VAC PSF CENTERS
C      TAPE15 IS THE VAC PSF ARRAY
C      TAPE16 IS THE BLOOMED PSF CENTERS
10     C      TAPE17 IS THE BLOOMED PSF ARRAY
C      TAPE30 IS THE TOTAL BLOOMED IMAGE
C      TAPE31 IS THE TOTAL VACUUM IMAGE
      NISOX=8
      NISOY=8
15     C      IVAC = 0 MEANS BLOOMED IMAGE
C      IVAC = 1 MEANS VACUUM IMAGE
      IVAC=0
      IVAC=1
      FLDSZ=3.0
20     DLX=FLDSZ/63.
      REWIND 14
      REWIND 15
      REWIND 16
      REWIND 17
25     APSQ=(12.*DLX)**2
      REWIND 30
      REWIND 31
      DO 19 I=1,64
      DO 20 J=1,64
30     DUM=((J-33)**2+(I-33)**2)*DLX**2
      IF(DUM.LE.APSQ) TARG(I,J)=1.0
      IF(DUM.LE.APSQ) GO TO 15
      TARG(I,J)=1.E-30
      15 CONTINUE
      20 CONTINUE
      19 CONTINUE
      CALL PARAM(BIG,TARG,64,XSIG,YSIG,XAVE,YAVE,1)
      DO 73 I=1,128
      DO 74 J=1,128
40     IF (I.LE.32.OR.I.GE.96.OR.J.LE.32.OR.J.GE.96)
      *   WORK(I,J)=0.
      IF (I.LE.32.OR.I.GE.96.OR.J.LE.32.OR.J.GE.96)
      *   GO TO 1022
      WORK(I,J)=TARG(I-32,J-32)
45     1022 CONTINUE
      74 CONTINUE
      73 CONTINUE
      CALL PLOT2D(WORK,1)
      DO 50 NX=3,6
      DO 51 NY=3,6
50     IF(IVAC.EQ.1) READ(15) VPSF
      IF(IVAC.EQ.0) READ(17) BPSF
      IF(IVAC.EQ.1) JCPSF=0
      IF(IVAC.EQ.1) ICPSF=0
55     IF(IVAC.EQ.1) GO TO 1931
      DO 37 I,CAS=1,2
      DIF=0.

```

PROGRAM IMAGE

74/175 OPT=2

FTN 4.8+528

```

        READ(14) VPSFCTR
        READ(16) BPSFCTR
60      DIF=(BPSFCTR-VPSFCTR)/DLX
        IF(DIF.LT.0.) DIF=DIF-0.5
        IF(DIF.GE.0.) DIF=DIF+0.5
        IF(IJCAS.EQ.1) JCPSF=IFIX(DIF)
        IF(IJCAS.EQ.2) ICPSF=IFIX(DIF)
65      37 CONTINUE
        1931 CONTINUE
        CALL TRNSFR(TARG,PARTIM,NX,NY,NISOX,NISOY)
        IF(IVAC.EQ.1) CALL CONVOL(VPSF,PARTIM,PARTIM)
        IF(IVAC.EQ.0) CALL CONVOL(BPSF,PARTIM,PARTIM)
70      CALL TOTAL(PARTIM,TOTIM,NX,NY,NISOX,NISOY,JCPSF,ICPSF)
        CALL PLOT2D(TOTIM,1)
        51 CONTINUE
        50 CONTINUE
        CALL PARAM(TOTIM,SMALL,128,XSIG,YSIG,XAVE,YAVE,0)
75      C AFTER CALLING PARAM...
        C FIND MAX FOR NORMALIZATION
        DO 8711 I=1,128
        DO 8712 J=1,128
            TDUM(I)=TOTIM(I,J)
80      RMAX=AMAX1(RMAX,TDUM(I))
            WORK(I,J)=0.
        8712 CONTINUE
        8711 CONTINUE
        AMX=RMAX
85      IF(RMAX.EQ.0.) AMX=1.0
        WRITE(6,672) AMX
        672 FORMAT(/,2X,* MAX VALUE IN TOTAL IMAGE ARRAY IS *,E11.3)
        C XAVE,YAVE ARE NOW TOTIM CENTROID INCREMENTS WRT (65,65)
        IF(XAVE.LT.0.) JCT=IFIX(XAVE+0.5)
90      IF(XAVE.GE.0.) JCT=IFIX(XAVE+0.5)
        IF(YAVE.LT.0.) ICT=IFIX(YAVE+0.5)
        IF(YAVE.GE.0.) ICT=IFIX(YAVE+0.5)
        C MOVE TOTIM TO CENTER OF A 128X128 CALLED WORK
        DO 8715 I=1,128
        DO 8716 J=1,128
95      IF((I+ICT).LT.1.OR.(I+ICT).GT.128.OR.
            * (J+JCT).LT.1.OR.(J+JCT).GT.128) GO TO 8717
            WORK(I,J)=TOTIM(I+ICT,J+JCT)
        8717 CONTINUE
100     8716 CONTINUE
        8715 CONTINUE
        CALL PARAM(WORK,SMALL,128,XSIG,YSIG,XAVE,YAVE,0)
        CALL PLOT2D(WORK,1)
        CALL PLOT2D(WORK,3)
105     C TOTIM IS NOW CENTERED WRT CENTROID OF TOTAL IMAGE
        IF(IVAC.EQ.0) WRITE(30) WORK
        IF(IVAC.EQ.1) WRITE(31) WORK
        REWIND 30
        REWIND 31
110     WRITE(6,601)
        601 FORMAT(*0 END OF PROGRAM*)
        STOP
        END

```

SUBROUTINE TRNSFR 74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE TRNSFR(A,B,NX,NY,NISOX,NISOY)
      C      SUBR TRANSLATES TARGET PATCH TO CENTER OF ARRAY B (64X64)
      C      NISOX,NISOY MUST BE 2,4,8,16,32 OR 64
      DIMENSION A(64,64),B(64,64)
5      JADEL=64/NISOX
      IADEL=64/NISOY
      MIDAJ=JADEL/2+1
      MIDAI=IADEL/2+1
10     IAL=(NY-1)*IADEL+1
      IAH=IAL+IADEL-1
      JAL=(NX-1)*JADEL+1
      JAH=JAL+JADEL-1
      DO 1 I=IAL,IAH
      DO 2 J=JAL,JAH
15     B(I+33-IAL-MIDAI+1,J+33-JAL-MIDAJ+1)=A(I,J)
      2 CONTINUE
      1 CONTINUE
      RETURN
      END

```

SUBROUTINE TOTAL 74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE TOTAL(PARTIM,TOTIM,NX,NY,NISOX,NISOY,JCPSE,ICPSE)
      C      SUBR TAKES PARTIAL IMAGE AND ADDS IT TO TOTAL IMAGE ARRAY
      DIMENSION PARTIM(64,64),TOTIM(128,128)
      JADEL=64/NISOX
5      IADEL=64/NISOY
      MIDAJ=JADEL/2+1
      MIDAI=IADEL/2+1
      C      JCT=X-CENTROID OF TARGET PATCH WRT TOTAL TARGET CENTER
      C      JCPSE=X-CENTROID OF BLOOMED PSF WRT VAC PSF LOCATION
10     JCT=((NX-1)*JADEL+MIDAJ)-33
      ICT=((NY-1)*IADEL+MIDAI)-33
      INLO=33+ICT+ICPSE
      INHI=INLO+63
      JNLO=33+JCT+JCPSE
      JNHI=JNLO+63
15     DO 178 II=1,128
      DO 179 JJ=1,128
      IF(II.LT.INLO.OR.II.GT.INHI.OR.JJ.LT.JNLO.OR.JJ.GT.JNHI)
      *   TOTIM(II,JJ)=TOTIM(II,JJ)
20     IF(II.LT.INLO.OR.II.GT.INHI.OR.JJ.LT.JNLO.OR.JJ.GT.JNHI)
      *   GO TO 1019
      TOTIM(II,JJ)=TOTIM(II,JJ)+PARTIM(II-INLO+1,JJ-JNLO+1)
1019  CONTINUE
179  CONTINUE
25   178  CONTINUE
      RETURN
      END

```

SUBROUTINE CONVOL

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE CONVOL(AR,BR,RESULT)
      C  A=PSF FUNCTION
      C  B=TARGET FUNCTION
      C  ARRAY IS CENTERED ON (33,33)
5      DIMENSION AR(64,64),BR(64,64),AC(64,64),BC(64,64),RESULT(64,64)
      ACUM(4)
      C  COMPLEX AC,BC
      C  CHANGE REAL TO COMPLEX ARRAYS
10     DO 4150 I=1,64
        DO 4150 J=1,64
          AC(I,J)=AR(I,J)
          BC(I,J)=BR(I,J)
4150   CONTINUE
15     CALL FFT2D(AC,6,0,64)
        CALL FFT2D(AC,6,1,64)
        CALL FFT2D(BC,6,1,64)
        DO 10 I=1,64
          DO 10 J=1,64
20     10 AC(I,J)=AC(I,J)*BC(I,J)
        CALL FFT2D(AC,6,2,64)
      C  REARRANGE ARRAY ABOUT CENTER OF GRID
        DO 20 I=1,32
          DO 20 J=1,32
25     IP=I+32
          JP=J+32
          CDUM(1)=AC(I,J)
          CDUM(2)=AC(IP,JP)
          CDUM(3)=AC(I,JP)
          CDUM(4)=AC(IP,J)
30     AC(IP,JP)=CDUM(1)
          AC(I,J)=CDUM(2)
          AC(IP,J)=CDUM(3)
          AC(I,JP)=CDUM(4)
20     CONTINUE
35     DO 23 I=1,64
        DO 23 J=1,64
23     RESULT(I,J)=CABS(AC(I,J))
      RETURN
      END

```


SUBROUTINE FFT2D

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE FFT2D(A,M,INV,KD)
      COMPLEX A(KD,KD),U,T,W(8,2)
      IF(INV.EQ.0)GO TO 40
      DO 26 J2=1,2
5         J = 1
      DO 7 I=1,NM1
      IF(I.GE.J)GO TO 5
      DO 4 KK=1,N
10         T = A(J,KK)
4         A(J,KK) = A(I,KK)
5         A(I,KK) = T
      K = NV2
6         IF(K.GE.J)GO TO 7
      J = J - K
15         K = K/2
      GO TO 6
      J = J + K
7         LE = 1
      DO 20 L=1,M
20         LE1 = LE
      LE = LE + LE
      U = (1.,0.)
      UR = REAL(U)
      UA = AIMAG(U)
25         DO 20 J=1,LE1
      DO 10 I=J,N,LE
      IP = I + LE1
      DO 10 KK=1,N
      T = A(IP,KK)*U
30         A(IP,KK) = A(I,KK) - T
10         A(I,KK) = A(I,KK) + T
20         U = U*W(L,INV)
      DO 25 I=1,N
      DO 25 KK=1,N
35         T = A(I,KK)
      A(I,KK) = A(KK,I)
25         A(KK,I) = T
26         CONTINUE
      IF(INV.EQ.2)RETURN
40         DO 30 I=1,N
      DO 30 KK=1,N
30         A(I,KK) = A(I,KK)*FN
      RETURN
40         N = 2**M
45         FN = (1./N)**2
      NV2 = N/2
      NM1 = N - 1
      DO 50 L=1,M
50         ANG=-2.*3.14159265/2**L
      W(L,1) = CMPLX(COS( ANG),SIN( ANG))
50         W(L,2) = CMPLX(COS(-ANG),SIN(-ANG))
      RETURN
      END

```

SUBROUTINE PLOT2D

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE PLOT2D(A,IPLT)
      C   IPLT=1 CALLS FOR 8-LEVEL GRAY SCALE
      C   IPLT=2 CALLS FOR NUMBERS FROM 0 TO 100 (***)
      C   IPLT=3 CALLS FOR 8-LEVEL CONTOUR PLOT
5      DIMENSION A(128,128),RDUM(128),IDUM(64),STORE(64,64)
      DIMENSION CHAR(8,4),PBUF(64)
      DIMENSION IPLT(9)
      DATA BLANK,PLUS/1H,1H+/
10     DATA CHAR/      1H,1H.,1HL,1HU,1HD,1HH,1HW,1HW,
      +                1H,1H,1H,1H,1H-,1HI,1HM,1HM,
      +                1H,1H,1H,1H,1H,1H,1HT,1HI,
      +                1H,1H,1H,1H,1H,1H,1H,1HA/
      DATA IPLT/" 1"," 2"," 3"," 4"," 5"," 6"," 7"," 8"," " /
15     WRITE(6,999)
      999 FORMAT(1H1)
      RMAX=0.
      DO 191 J=1,128
      DO 192 I=1,128
20     RDUM(I)=A(I,J)
      192 RMAX=AMAX1(RMAX,RDUM(I))
      191 CONTINUE
      AMX=RMAX
      IF(RMAX.EQ.0.) AMX=1.
      C   FOLLOWING DO LOOP CONTROLS PORTION OF TOTIM TO BE PRINTED
25     DO 991 I=33,96
      DO 990 J=33,96
      STORE(I-32,J-32)=A(I,J)
      990 CONTINUE
      991 CONTINUE
30     PRINT 9741,1H
      9741 FORMAT(A1)
      DO 9240 I=1,64
      KK=0
      DO 9235 J=1,64
35     KK=KK+1
      IDUM(KK)=STORE(I,J)*100/AMX+0.5
      9235 CONTINUE
      IF (IPLT-2) 301,302,303
40     301 CONTINUE
      IXH=64
      NDOFF=2
      NP1=65
      DO 100 IX=1,IXH
      CX=1.0+0.08*IDUM(IX)
45     IF (CX.LE.1.0) CX=1.5
      IF (CX.GE.9.0) CX=8.5
      100 IDUM(IX)=CX
      CC=PLUS
      DO 500 L=1,4
50     DO 200 IX=1,IXH
      M=IDUM(IX)
      200 PBUF(IX)=CHAR(M,L)
      DO 300 IX=1,IXH
      M=NP1-IX
55     IF(PBUF(M).NE.BLANK) GO TO 400
      300 CONTINUE
      IF(CC.NE.BLANK) GO TO 500

```

SUBROUTINE PLOT2D

74/175 OPT=2

FTN 4.8+328

```

      M=1
      400 PRINT 1000,CC,NOFF,(PBUF(IX),IX=1,M)
      500 CC=PLUS
60      1000 FORMAT(A1,T=,64(A1,1X))
      PRINT 1000,BLANK
      GO TO 9241
      302 CONTINUE
65      WRITE(6,3) IDUM
      3 FORMAT(1H,64I2)
      GO TO 9241
      303 CONTINUE
      DO 1013 KL=1,64
70      IF(IDUM(KL).EQ.0) IDUM(KL)=IPLT(9)
      IF(IDUM(KL).GE.99.AND.IDUM(KL).LE.100) IDUM(KL)=IPLT(9)
      DO 1013 INC=1,7
      L1=14*(INC-1)+8
      L2=14*(INC-1)+1
75      C CONDITION FOR 1,2,...,8
      IF(IDUM(KL).GE.L2.AND.IDUM(KL).LE.L2+6) IDUM(KL)=IPLT(INC)
      C CONDITION FOR A BLANK
      IF(IDUM(KL).GE.L1.AND.IDUM(KL).LE.L1+6)
      * IDUM(KL)=IPLT(9)
80      1013 CONTINUE
      WRITE(6,27) IDUM
      27 FORMAT(3X,64A2)
      9241 CONTINUE
      9240 CONTINUE
85      RETURN
      END

```

SUBROUTINE NORMAL

74/175 OPT=2

FTN 4.8+528

```
1      SUBROUTINE NORMAL(C,CMAX,CN)
      DIMENSION C(128,128),TDUM(128)
      DIMENSION CN(128,128)
5      C  THIS SUBR NORMALIZES ARRAY C(I,J) WRT 100 AND PRINTS MAX VALUE
      LIMUP=128
      DO 8711 I=1,LIMUP
      DO 8712 J=1,LIMUP
      TDUM(I)=C(I,J)
      RMAX=AMAX1(RMAX,TDUM(I))
10     8712 CONTINUE
      8711 CONTINUE
      CMAX=RMAX
      IF(RMAX.EQ.0.) CMAX=1.
      WRITE(6,8720) CMAX
15     8720 FORMAT(10X,* CMAX IS EQUAL TO *,E11.3)
      DO 8713 I=1,LIMUP
      DO 8714 J=1,LIMUP
      CN(I,J)=C(I,J)*100./CMAX+0.5
20     8714 CONTINUE
      8713 CONTINUE
      RETURN
      END
```

SUBROUTINE DRIV3D

74/175 OPT=2

FTN 4.8+528

```
1      SUBROUTINE DRIV3D(STORE,NSIZE,NSCALE,NPLOT)
      DIMENSION STORE(128,128)
      CALL INVERT(STORE,NSIZE)
      CALL STRI(STORE,NSIZE,NSCALE,NPLOT)
5      CALL PLTST
      CALL FRAME1(RBX1)
      CALL FRAME2(NLBX,NUBX,NLBY,NUBY,PTMESH,PTVERT,RBX1)
      CALL PLOT3D(STORE,NLBX,NUBX,NLBY,NUBY,PTMESH,PTVERT,NPLOT)
10     CALL PLTCLS
      RETURN
      END
```

SUBROUTINE INVERT 74/175 OPT=2

FTN 4.8+528

```
1      SUBROUTINE INVERT(STORE,NSIZE)
      DIMENSION STORE(128,128),RDUM(128)
      C      INVERTS STORE(I,J) MATRIX FOR 3D PLOTTING.
      JHI      = NSIZE-1
5      DO 8020 J=1,JHI
      IHI      = NSIZE-J
      DO 8020 I=1,IHI
      RDUM(I) = STORE(I,J)
      STORE(I,J) = STORE(NSIZE-J+1,NSIZE-I+1)
10     STORE(NSIZE-J+1,NSIZE-I+1) = RDUM(I)
      8020    CONTINUE
      RETURN
      END
```

SUBROUTINE PARAM

74/175 OPT=2

FTN 4.8+528

```

1      SUBROUTINE PARAM(BIG,SMALL,INDBS,XSIG,YSIG,XAVE,YAVE,IVAC)
C      THIS SUBROUTINE CALCULATES THE MEAN AND VARIANCE
C      IN X AND Y FOR EITHER A 128 X 128 OR A 64 X 64.
C      FOLLOWING APPLIES TO PSF ARRAYS ONLY
5      C      IVAC=0 DENOTES BLOOMED PSF
C      IVAC=1 DENOTES VACUUM PSF
C*****
C      DO NOT USE SUBR PARAM AFTER NORMALIZING ARRAY
C      ----INCORRECT SIGMA VALUES-----
10     C*****
        DIMENSION BIG(128,128),SMALL(64,64)
        XB=YB=BSUM=XXB=YYB=0.
        IF(INDBS.EQ.64) MAXI=64
        IF(INDBS.EQ.64) MAXJ=64
15     IF(INDBS.EQ.128) MAXI=128
        IF(INDBS.EQ.128) MAXJ=128
        DO 100 I=2,MAXI
        DO 101 J=2,MAXJ
        IF(INDBS.EQ.128) BINT=BIG(I,J)
20     IF(INDBS.EQ.64) BINT=SMALL(I,J)
        Y=I-(INDBS/2+1)
        X=J-(INDBS/2+1)
        XB=XB+BINT*X
        YB=YB+BINT*Y
25     BSUM=BSUM+BINT
        XXB=XXB+BINT*X*X
        YYB=YYB+BINT*Y*Y
        101 CONTINUE
        100 CONTINUE
30     XAVE=XB/BSUM
        YAVE=YB/BSUM
        XSIG=SQRT(XXB/BSUM-XAVE*XAVE)
        YSIG=SQRT(YYB/BSUM-YAVE*YAVE)
        IF(INDBS.EQ.64.AND.IVAC.EQ.1) WRITE(6,602)
35     IF(INDBS.EQ.64.AND.IVAC.EQ.0) WRITE(6,598)
        IF(INDBS.EQ.128) WRITE(6,599)
598    FORMAT(2X,* FOLLOWING IS FOR THE BLOOMED PSF(64,64) ARRAY*)
602    FORMAT(2X,* FOLLOWING IS FOR THE VAC PSF(64,64) ARRAY*)
599    FORMAT(2X,* FOLLOWING IS FOR THE TOTAL(128,128) ARRAY*)
40     WRITE(6,600) XAVE,YAVE,XSIG,YSIG
600    FORMAT(2X,*XAVE = *,E12.5,/,
        * 2X,*YAVE = *,E10.3,/,
        * 2X,*XSIG = *,E10.3,/,
        * 2X,*YSIG = *,E10.3)
45     RETURN
        END

```

SUBROUTINE PLTST

74/175 OPT=2

FTN 4.8+528

```
1      SUBROUTINE PLTST
      CALL PLTFIL
      CALL NEWPLT(1,-.5,-.5,11.0,9.0,0.0,0.0,0,6H3DPLT)
      CALL PLOT(5.,3.,-3)
5      RETURN
      END
```


SUBROUTINE STRI

74/175 OPT=2

FTN 4.0+520

```

1      SUBROUTINE STRI(STORE,NSIZE,NSCALE,NPLOT)
      DIMENSION STORE(128,128),PMAX(10)
      C      SCALES STORE(I,J) FOR 3D PLOT
      C      A NEW INTENSITY SCALE IS CALCULATED IF NSCALE=0, OTHERWISE
5      C      SCALE FACTOR IS PMAX(NSCALE). IN THAT CASE, NSCALE MUST BE
      IF(NSCALE.EQ.0) 8105,8112
      8105 PMAX(NPLOT) = 0.
      DO 8110 J=1,NSIZE
      DO 8110 I=1,NSIZE
10     PMAX(NPLOT) = AMAX1(PMAX(NPLOT),STORE(I,J))
      8110 CONTINUE
      GO TO 8114
      8112 PMAX(NPLOT) = PMAX(NSCALE)
      8114 DO 8120 J=1,NSIZE
15     DO 8120 I=1,NSIZE
      STORE(I,J)=STORE(I,J)/PMAX(NPLOT)
      8120 CONTINUE
      RETURN
      END

```

SUBROUTINE FRAME1 74/175 OPT=2

FTN 4.8+528

```
1      SUBROUTINE FRAME1(RBX1)
      C      DEFINE LOWER AND UPPER BOUNDARIES OF X AND Y AXES.
      NLBX  = 2
      NUBX=127
5      NLBY  = NLBX
      NUBY  = NUBX
      PTMESH=.035
      PTVERT = 7.5
10     C PTMESH IS THE PLOTTED MESH SIZE IN INCHES.
      C PTVERT IS THE PLOTTED MAXIMUM VERTICAL HEIGHT IN INCHES.
      RBX1  = NUBX-NLBX+1
      CALL FACTOR(0.72)
      RETURN
      END
```

SUBROUTINE FRAME2 74/175 OPT=2

FTN 4.8+528

```
1      SUBROUTINE FRAME2(NLBX,NUBX,NLBY,NUBY,PTMESH,PTVERT,RBX1)
      NLBX=33
      NUBX=96
      NLBY=33
      NUBY=96
5      C      FRAME1 MUST BE CALLED FIRST TO CALCULATE RBX1.
      RBX2 = NUBX-NLBX+1
      FX   = (RBX1/RBX2)*0.72
      PTVERT =5.0*(RBX2/RBX1)
10     PTMESH=.035
      CALL FACTOR(FX)
      RETURN
      END
```

SUBROUTINE PLOT3D

74/175 OPT=2

FTN 4.8+328

```

1      SUBROUTINE PLOT3D(STORE,NLBX,NUBX,NLBY,NUBY,PTMESH,PTVERT,
      *NPLOT)
      DIMENSION STORE(128,128),SCRATCH(128)
      D = PTMESH
5      NSM1 = NUBX - 1
      NS1 = NUBX + 1
      DO 8320 I=1,NS1
8320   SCRATCH(I) = -99.
      XSHIFT = 0.
10     YSHIFT = 0.
      DO 8340 I=NLBY,NUBY
      CALL PLOT(XSHIFT,YSHIFT,3)
      KPEN = 0
      I1 = 1
15     DO 8330 J=NLBX,NUBX
      I1 = I1 + 1
      X = D*(J - NLBX) + XSHIFT
      Y = .577*D*(J - NLBX) + YSHIFT + PTVERT*STORE(I,J)
      IF(Y.GT.SCRATCH(I1)) GO TO 8325
      KPEN = 1
      GO TO 8330
8325   SCRATCH(I1) = Y
      IF(KPEN.EQ.1) GO TO 8327
      CALL PLOT(X,Y,2)
25     GO TO 8330
8327   CALL PLOT(X,Y,3)
      KPEN = 0
8330   CONTINUE
      XSHIFT = XSHIFT - D
      YSHIFT = YSHIFT + .577*D
30     DO 8335 K=1,NSM1
      SCRATCH(NUBX+1-K) = SCRATCH(NUBX-K)
8335   CONTINUE
8340   CONTINUE
      NSM1 = NUBY - 1
      NS1 = NUBY + 1
35     DO 8350 I=1,NS1
8350   SCRATCH(I) = -99.
      XSHIFT = 0.
      YSHIFT = 0.
40     DO 8390 I=NLBX,NUBX
      CALL PLOT(XSHIFT,YSHIFT,3)
      KPEN = 0
      I1 = 1
45     DO 8370 J=NLBY,NUBY
      I1 = I1 + 1
      X = -D*(J - NLBY) + XSHIFT
      Y = .577*D*(J - NLBY) + YSHIFT + PTVERT*STORE(J,I)
      IF(Y.GT.SCRATCH(I1)) GO TO 8360
      KPEN = 1
50     GO TO 8370
8360   SCRATCH(I1) = Y
      IF(KPEN.EQ.1) GO TO 8365
      CALL PLOT(X,Y,2)
      GO TO 8370
55     CALL PLOT(X,Y,3)
      KPEN = 0

```

SUBROUTINE PLOT3D

74/175 OPT=2

FTN 4.8+528

```
      8370  CONTINUE
            XSHIFT = XSHIFT + D
60          YSHIFT = YSHIFT + .577*D
            DO 8380 K=1,NSM1
            8380  SCRTCH(NS1-K) = SCRTCH(NS1-1-K)
            8390  CONTINUE
            CALL NEWPLT (1,-.5,-.5,11.0,9.0,0.0,0.0,0.0,6HPLOT3D)
65          CALL PLOT(10.,0.,-3)
            WRITE(6,8399) NPLOT
            NPLOT = NPLOT+1
            8399  FORMAT(1H0,37X,5H*****,10X,21HPLOT CREATED, NPLOT =,I3,10X,5H
70          .)
            RETURN
            END
```

SUBROUTINE PLTCLS 74/175 OPT=2

FTN 4.8+528

```
1      SUBROUTINE PLTCLS
        CALL PLOT(5.,0.,999)
        RETURN
        END
```

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