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SCATTERPLATE INTERFEROMETRY

The University of Arizona

PH.D.

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SCATTERPLATE INTERFEROMETRY

by

Lawrence Francis Rubin

A Dissertation Submitted to the Faculty of the

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In the Graduate College

THE UNIVERSITY OF ARIZONA

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ABSTRACT

This study provides a description of scatterplate interferometry from both a theoretical and experimental viewpoint. Although this interferometer is a relatively simple and very effective device for use in the optical shop, very little beyond a qualitative description has been previously presented. Through the use of Fourier optics and stochastic processes techniques, a more formalized theory of scatterplate interferometry is shown particularly in how the flip or inversion symmetry of the scatterplate itself affects the flip symmetry. Also described are the effects on the fringe quality produced by non laser sources as well as aberrations in the interferometer optics. Experimental verification of these aspects is shown to be in good agreement with the theory.

Of interest to the shop user of this interferometer is the material presented on scatterplate fabrication. Techniques for making the conventional, "double-exposure" or "180°-type" scatterplate, are very simple to those with access to a holography lab. Additional refinements including null testing with the scatterplate and an infrared scatterplate interferometer are also described.

CHAPTER 1

INTRODUCTION

The first observation of fringes in scattered light was by Newton. In his experiment, sunlight was passed through a pinhole positioned at the center of curvature of a back silvered spherical mirror. What he saw centered about the pinhole was a series of colored rings which were greatly enhanced when the front surface of the mirror was dusted with a fine powder. As described by DeWitte (1967), the principles used to explain the rings (first outlined by Young, later detailed by Stokes) are very similar to those describing the scatterplate interferometer. The actual interferometer in use today was invented by J. M. Burch as reported in a short communication that appeared in Nature in 1953.

Most of what Burch has written on the subject appeared in an International Commission on Optics (ICO) proceedings of 1970. Some Fourier techniques are used to outline the basic theory of the interferometer although most of what was discussed was confined to generalities. Very little was mentioned about making the scatterplate itself. Later articles by R. M. Scott (1969) on scatterplate interferometry and J. B. Houston (1970) on scatterplate fabrication provided more information in these areas. Part of the purpose of this study is to review and perhaps clarify what has been written on scatterplate interferometry

as well as addressing problems that were not previously mentioned. One point in particular that was found to be lacking in previous literature is an analysis of the flip symmetry of the scatterplate which turns out to have a decisive effect on the quality of the fringe pattern.

The next chapter will provide a review of previous work on scatterplate interferometry as well as discussing techniques of testing and fabrication from a "user-oriented" viewpoint. The material presented in this chapter is not meant to be supported by firm mathematical proof. Some of the more theoretical aspects are either left for later chapters or listed in the references. Chapter 3 presents the formalized theory of the interferometer operation using Fourier analysis and develops to a further extent the work started by Burch. Chapter 4 uses results derived in the previous chapter to study the importance of flip symmetry. The idea of a phase transfer function of the scatterplate is found to be very useful here. Chapter 5 is a further development of the interferometer operation which discusses scatter intensity profiles and the affect of aberrations in the auxiliary optics and finite source size on the fringe contrast. Also included is a discussion of how energy is conserved in a scatterplate. Chapter 6 shows how the scatterplate can be modified to perform null testing and the last chapter deals with infrared scatterplate interferometry.

CHAPTER 2

A USERS VIEWPOINT OF SCATTERPLATE INTERFEROMETRY

The scatterplate belongs to a class of interferometers known as common-path interferometers. This is a consequence of the fact that the division of the reference and test beams is accomplished through diffraction. In this case, the diffracting element is a weak diffuser that possesses a flip or inversion symmetry about a unique point. That is,

$$S(x,y) = S(-x,-y) \quad (2-1)$$

where S = scatterplate amplitude transmission

$S(0,0)$ = center of flip symmetry.

In other common-path interferometers such as the Smartt (Smartt and Strong 1972) or the Fresnel zone plate (Murty 1963), the diffracting elements are a single pinhole and a pair of zone plates, respectively.

The benefits of this class of interferometers are well known:

1. Less sensitivity to vibrations and turbulence.
2. No precision auxiliary optics.
3. White light sources may be used.

Since there is no piston error, relative longitudinal motion between the interferometer and test optics introduces defocus only which is

usually imperceptible if the vibration movements are small in amplitude. Determination of the sign of the wavefront error by "pushing" on the interferometer as is done with a Twyman-Green is not very sensitive a method when applied to common-path interferometers. Large longitudinal movements with the micrometer controls are needed to determine the wavefront error sign.

The aberrations present in the illumination and imaging optics will for the most part affect the reference and test beams equivalently so that optics of reasonable quality may be used.

Common-path also implies equal path so that white light may be used as a source illumination. The first scatterplate interferometers in fact used such sources. There are however limitations imposed on the spectral distribution and source size as well as brightness requirements in the interferogram which are discussed further on.

Qualitative Description of Scatterplate Interferometry

Although the principles of operation are virtually the same, several versions of the scatterplate interferometer have been developed. A schematic of the one used in this study is shown in Figure 1. The main feature of this system is that the flip symmetry is built into a single scatterplate which is achieved by double exposing a laser speckle pattern onto a photographic emulsion and rotating the emulsion precisely 180° between exposures. Being autostigmatic, this configuration is useful for testing mirror systems. Figure 2 shows a schematic of the

Fig. 1. Schematic of a scatterplate interferometer.

Fig. 2. Burch's configuration of the scatterplate interferometer using two unsymmetrical scatterplates.

interferometer as it was originally conceived by Burch. Instead of a single "180 degree type" scatterplate, two identical weak scatterers (neither of which possesses flip symmetry) and a pellicle beamsplitter are used. Although there is an added difficulty in aligning the image of the first plate on the second, this scheme lends itself well to single pass lens testing as shown in Figure 3. As Burch mentions, if the separate plates are made with the proper magnification of identical structure, there is the possibility of testing at conjugates other than one to one. By replacing the second scatterplate with a precision flat mirror, a double pass test with a single unsymmetrical scatterplate becomes possible (Schoemaker and Murty 1966). However since all odd order aberrations are cancelled, tilt has to be added by means of a plane parallel plate placed halfway through the cone of scattered light. As shown in Figure 1, tilt is ordinarily added by lateral movements of the scatterplate while defocus is added by longitudinal movements.

It is convenient for means of understanding to separate the beam amplitudes in the fringe plane into four distinct types:

DD (direct-direct) - light unscattered after two passages through the scatterplate (or through both scatterplates if they are separated).

DS (direct-scattered) - light unscattered the first passage (plate) and scattered the second passage (plate).

Fig. 3. Single pass lens testing using two unsymmetrical scatterplates.

SD (scattered-direct) - light scattered the first passage (plate) and unscattered the second passage (plate).

SS (scattered-scattered) - light scattered at both passages (plates).

Figure 4 shows an interferogram obtained when testing a parabolic mirror.

The fringe contours are a result of the interference of the DS (reference beam) and the SD (test beam). The degree to which these two beams are "phased" properly depends on how well the flip symmetry is achieved. The DD light forms the characteristic hotspot seen in scatterplate interferograms. Note that since the interferometer is common-path, the hotspot always lies within a bright fringe. Its position on the mirror surface defines the zero phase point. The SS light is not phased with any of the other beams and adds essentially a background intensity which reduces the fringe contrast, given by

$$\gamma_B = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{1}{1 + \frac{I_{SS}}{2I_{SD}}} \quad (2-2)$$

where I_{SS} = intensity of the SS light

$I_{SD} = I_{DS}$ = intensity of the SD light

$I_{\max} = 4I_{SD} + I_{SS}$

$I_{\min} = I_{SS}$

As can be seen in Figure 4, the distribution of scattered light is not constant over the test aperture, the reasons for which will

Fig. 4. Scatterplate interferogram of a 15cm diameter, $f/4$ parabolic mirror.

become clear later on. A consequence is that the contrast reduction, γ_B , is also not constant across the aperture.

The relative intensities between these beams is determined by the diffraction efficiency of the scatterplate. Although the DS and SD intensities are equal regardless of the efficiency, there is a tradeoff between bright interferograms with the high efficiencies and high fringe contrast with the low efficiencies. 50% has been found to be a suitable diffraction efficiency for the general purpose scatterplate.

It is interesting to note the way in which energy is conserved in a scatterplate interferometer when a null fringe appears across the test aperture (Rubin and Wyant 1979). Interferometers such as the Twyman-Green and Mach-Zehnder are two-port interferometers so that when a null fringe appears in one port, a bright one appears in the other port and the total energy is conserved. The scatterplate has no complimentary fringe pattern and should the hotspot be focused on a small "bump" or "hole" on a mirror surface a null fringe could certainly be possible. Chapter 5 will show how the hotspot varies in intensity to conserve the overall energy.

Fabrication

A simple schematic for generating a speckle pattern is shown in Figure 5. The speckle size is determined by the ground glass aperture, d , and the ground glass to emulsion distance, D . Since almost no direct beam passes through the ground glass, the full angle, θ_F , given by

$$\theta_F = 2 \tan^{-1} [d/2D] \quad (2-3)$$

becomes the half angle of the scattered light after the emulsion is processed and used in the interferometer. In terms of holography, this fabrication method can be thought of as an on-axis hologram of a ground glass in which the self interference is dominant. Unfortunately the distribution of scattered light from a scatterplate made using this configuration is not uniform out to the angle, θ . The frequency content of exposing intensity (power spectrum) goes as the autocorrelation of the ground glass aperture.

Stated in more physical terms, each spatial frequency in the power spectrum can be thought of as a superposition of fringe patterns due to pairs of points of a given spacing in the ground glass. Since there are fewer pairs of points with a wide spacing in the ground glass aperture, the higher spatial frequencies are modulated less. In some cases, improvement in illumination of low f/no. test systems can be accomplished by apodizing the ground glass aperture to enhance the higher spatial frequencies in the scatterplate (i.e. central obscurations). However this technique is limited in usefulness since the low spatial frequencies are also enhanced which causes a larger area of fringe "burnout" in the interferogram.

An interesting attempt at alleviating the uneven scatter profile is shown in a fabrication technique in Figure 6. Each point in the ground glass produces a plane wave which interferes with the reference beam so the average speckle size at the film emulsion has approximately doubled from that due to interference between different points on the

Fig. 5. Fabrication schematic for scatterplates.--The source is the $.5145\mu\text{m}$ line of the Argon ion laser.

Fig. 6. Strong reference beam method of producing scatterplates.

ground glass. If the reference beam is strong enough, the self interference term which contains the autocorrelation is negligible compared to the ground glass and reference interference term. Since a hologram of the ground glass is being taken, the light scattered from the beam-splitter also appeared in the reconstruction step (when the plate is used in the interferometer) which produced some annoying side effects.

Bleaching the emulsion greatly diminishes the burning out of fringes by the hotspot as shown in Figures 7a & 7b. Too bright a hotspot can burn out a significant portion of the interferogram in large $f/\text{no.}$ systems. Figures 8a & 8b show the effect a 20% and a 60% diffraction efficiency scatterplate have on an $f/30$ mirror. As a general rule with large $f/\text{no.}$ systems, it is best to sacrifice fringe contrast in favor of a less intense hotspot.

Figure 9 shows kinematic method of obtaining a quick, precise 180° rotation of the film emulsion. One pair of opposite sides of a film holder are accurately machined parallel to one another so that contact against the three defining pins ensures an accurate 180° rotation. In most cases, 30 sec. of rotational error (corresponding to about a few ten thousandths an inch/inch antiparallelism) has a negligible affect on the interferometer performance. A qualitative discussion of the effect of flip symmetry on the fringe contrast is discussed in the next section while the formalized theory is left for Chapter 4.

It is also convenient if the other pair of opposite sides on the film holder are machined reasonably parallel (a few thousandths of

(a)

Fig. 7. Scatterplate interferograms.

(a) With unbleached emulsion.

(b)

Fig. 7. Continued.

(b) With bleached emulsion.

(a)

(b)

Fig. 8. Scatterplate interferograms of a 25 cm diameter, $f/30$ sphere.

(a) With 20% and (b) 60% diffraction efficiency scatterplates.

Fig. 9. Kinematic mounting for a precise 180° rotation of the emulsion in the double exposure method.

an inch/inch antiparallelism). The center of the rotator will then be the center of symmetry of the scatterplate. Assuming the hole for exposing the emulsion is centered in the rotator, a small black dot can be placed in the center of the exposed portion on the film after it is processed to mark the center of flip symmetry. Cross hairs and other shadow casting markers used in front of the emulsion in the fabrication stage are not as useful since bleaching removes all trace of them, along with other film structure.

It is of some interest to mention a single exposure technique for making scatterplates suggested by R. V. Shack. Although it has not proven as successful as the double exposure method, this technique illustrates an interesting application of Fourier theory. In the modified setup depicted in Figure 10, the ground glass scatterer has been replaced by a random pinhole array which is placed in focused rather than collimated light. Since the Fourier transform intensity of any real valued amplitude mask already has flip symmetry, the emulsion can be exposed once by placing it at the focus of the incoming light beam. The quality of the scatterplate is limited by the amount of defocus and spherical aberration in the beam since any phase distribution across the pinhole mask destroys the symmetry in the Fourier plane. This becomes increasingly more difficult to achieve for low $f/\text{no.}$ exposure systems since they have to be nearly diffraction limited.

The pinhole mask was made by aluminizing a finely dusted optical flat and then blowing the dust away with an air jet after deposition of

Fig. 10. Schematic of the single exposure method of producing scatterplates.--The focus of the undiffracted beam marks the center of the flip symmetry.

the aluminum. The phase variations in even its unbleached state precluded the use of photographic emulsion as a scattering element. Good quality scatterplates were made down to approximately $f/6$ scattering angle using this method.

Qualitative Discussion of Flip Symmetry

Figure 11 shows a photograph through a microscope of the flip symmetry of a scatterplate made with the double exposure process. In any interferometer the phase errors introduced by the interferometer itself should be negligible compared to the test errors. With the Twyman-Green interferometer, this is achieved by using precision made reference flats, divergers and beamsplitters. In an analogous sense, the flip symmetry in the scatterplate interferometer can be considered as the "precision component." As mentioned earlier, once the proper symmetry is achieved (not particularly difficult in any holography lab), aberrations in the auxiliary optics have little effect on the fringe quality.

A few results from Fourier theory provide an understanding of why flip symmetry is necessary in the scatterplate. An equivalent and more useful definition of eq. (2-1) can be arrived at by modeling the scatterplate as a superposition of sinusoidal gratings of many different spatial frequencies. In this context, perfect flip symmetry means a "peak" or a "trough" of each of these gratings must be located at the center of symmetry, $S(0,0)$. Stated in the more formal language of Fourier optics, the scatterplate phase transfer function (PTF) assumes

Fig. 11. Photograph of an unbleached scatterplate through a microscope.--The cross indicates the center of the flip symmetry of the plate.

values of 0 and $\pm\pi$ only. As more and more errors creep into the fabrication, one would expect a progression from the perfect flip symmetry to an ordinary random scatterer where the PTF is uniformly distributed between $+\pi$ and $-\pi$.

As mentioned previously, the fringe contours are the result of interference of the SD and DS amplitudes. The scatterplate PTF imparts a $+\phi$ phase shift, say, to the test beam in the first passage and $-\phi$ to the reference beam the second passage for a total of 2ϕ shift in addition to the test aberrations in the interferogram. Perfect flip symmetry implies only the test aberrations are registered.

Contour fringes represent an average value of the wavefront over a given fringe width. The contrast is best when the wavefront profiles have broad, continuous features as in smooth aspherics rather than rough, jagged structure as in a ground surface. PTF's of imperfect flip symmetries will then affect the fringe pattern in one of two ways. For PTF's of a "slowly varying" dependence on spatial frequency, the bending of the fringes registers the sum of the PTF and the test aberrations. This result can be used to construct scatterplate PTF's which cancel aberrations so that with some modification of the interferometer, a null test is possible (Rubin in press). Burch mentioned this possibility in his later work where he described the scatterplate as a special case of a holographic interferometer. For "rapidly varying" PTF's, the fringe contrast is reduced because of this averaging of large phase variations of small spatial extent.

Fabrication errors in the double exposure method (i.e. exposure mismatch and an imprecise 180° rotation) result in PTF's of the rapidly varying type. The expression for the fringe contrast for these two errors are

$$\gamma_e = \frac{2B}{1+B^2} \quad (2-4)$$

$$\gamma_r = \text{besinc} [\theta_E / (f\#\lambda/s_o)] \quad (2-5)$$

where B = ratio of the diffraction efficiencies of the two separate exposures

θ_E = deviation from 180° rotation

$f\#$ = scattering f/no.

s_o = scatterplate aperture radius.

Both expressions are graphed in Figures 12 and 13.

Unequal diffraction efficiencies of the separate exposures can be thought of as equivalent error in the DS and SD amplitudes since

$$\gamma = \frac{2 \left(\frac{I_{SD}}{I_{DS}} \right)^{1/2}}{1 + \frac{I_{SD}}{I_{DS}}} \quad (2-6)$$

where $\frac{I_{SD}}{I_{DS}} = B^2$

which is the same result as in eq. (2-4). If there is a linear

Fig. 12. Fringe contrast, γ_{rot} , in the presence of rotational error between the two exposures in the scatterplate fabrication.

Fig. 13. Fringe contrast, γ_E , in the presence of unequal exposures during the scatter-plate fabrication.

relationship between exposure and diffraction efficiency, then B will also be the ratio of separate exposures. The resulting fringe contrast then can be reduced if either the laser intensity or the shutter speed is not constant between the two exposures.

An effect related to unequal exposures is the fact that the sensitivity of photographic film to each exposure in a multiple exposure sequence is not the same (Bestenreiner, Dem, and Friess 1970). The first exposure can alter the recorded structure of an otherwise identical second one. Although more experimentation is needed, the author feels this affect causes a slight but measurable reduction in fringe contrast. In low efficiency scatterplates, the fringe contrast near the hotspot should be nearly perfect. However, measurement with a silicon diode array in the image plane could show no scatterplates with contrasts better than about .9.

Unlike unequal exposures, rotational error results in a contrast reduction that depends on the distance, r_1 from the hotspot, in the test aperture for a given focal distance of the test optics. The scattering $f/\text{no.}$ is the ratio of the focal distance, R , to $2r_1$. As seen in Figure 14, the contrast drops further away from the hotspot. The contrast can also be improved by reducing the scatterplate aperture but at the cost of increased speckle size (for laser sources) and reduced image brightness. Figure 13 shows very small contrast reductions for θ_E out to $f\lambda/5s_0$ which is approximately 30-40 sec of arc at $f/8$, $\lambda=.5145$, and $s_0 = 5\text{mm}$. There are rapidly diminishing returns if the error is reduced

Fig. 14. Interferogram using a scatterplate with rotational error in it.

below this limit even for faster $f/\text{no.}$ systems because the improvement is negated by other errors such as non-isoplanatism, described in the next section.

Figure 15 illustrates the appearance of rotational error in an interferogram to an observer. When constricted to a small point, the eye views the fringe pattern through a small portion of the scatterplate instead of averaging over all points on the scatterplate as a camera lens. A semi-quantitative explanation of the contrast reversal bands can be arrived at through a ray analysis.

In the presence of rotational error, identical scatterpoints are no longer exactly at 180° opposite points from the center of symmetry but rather at $180^\circ - \theta_E$. The entrance and exit points of ray paths through a scatterplate with error θ_E is shown in Figure 16. If the eye pupil is centered on a small region $D/2$ from the center of symmetry, the rays entering the plate in the first passage are centered about a point 180° opposite $S(0,0)$. Since the pairs of points A and C' and Band A' are identical scatterpoints, the fringe pattern will be a superposition of two different scattering schemes. The first scheme has light scattered from A the first passage and direct through A' the second passage (SD) interfering with the direct light through C and scattered from A' (DS). On the other hand, the second scheme has light unscattered the first passage through A and scattered the second passage through A' (DS) interfering with light scattered at B and exiting at B' (SD). An expansion of OPD of these two interference schemes shows the

Fig. 15. Interferogram, as it appears to the observer, using a scatterplate with rotational error present.

Fig. 16. Entrance and exit points of ray paths through a scatterplate with rotational error present.

primary term to be tilt whose sign is opposite for each case. So the eye sees a superposition of two fringe patterns one which has slightly more tilt than the other due to a small rotational error in the scatterplate. If the illuminating beam is masked off properly to simulate viewing by the eye, the angular error can be estimated by counting the number contrast reversal bands, n , from a photograph as in Figure 15.

$$\theta_E = \frac{nf\#\lambda}{D_r} \quad (2-7)$$

If the entrance and exiting light cones through the scatterplate are separated horizontally, the bands seen in the fringe plane will also be horizontal since the relative tilt generated between the two scattering schemes is in the vertical direction.

Contrast Reduction in the Interferometer:
Non Isoplanatism and Background Intensity

In spite of the fact that a scatterplate might have perfect flip symmetry, other errors can degrade the performance of this interferometer, the most fundamental of which is non-isoplanatism.

The discussion using Fourier theory assumes isoplanatism, that is, each pair of symmetrical scatterpoints produces an identical fringe pattern in the image plane. In this respect, the scatterplate might be termed as a multiple beam interferometer except in this case, the superposition is on an intensity basis. Depending on the system under test

however, the various pairs of SD and DS rays can have a distribution of phase shifts between them due to different off-axis path lengths. Figure 17 depicts these ray paths which are used to calculate the OPD. As the derivation in Chapter 5 will show, the primary off-axis aberration to be considered is third order astigmatism. The superposition of these individually phase shifted patterns results in a contrast reduction and phase shift in the observed fringe pattern. Figure 18 shows the contrast and phase graphs as a function of $\rho^2 s_o^2 / \lambda R^3$, the maximum waves of astigmatism present.

Non-isoplanatism is apparent when viewing the fringes of a fast f/no. system under test. As one moves his eye from side to side behind the scatterplate, the fringes, especially near the edge of the aperture, appear to bend. The eye again is constricted to a small region and is essentially viewing an individually phase shifted fringe pattern produced by a single pair of symmetrical scatterpoints.

As in the case of rotational error, the contrast drops further away from the hotspot and can also be improved by reducing the scatterplate aperture with the same tradeoffs in speckle size and image brightness.

A consequence of a non-constant scattering profile is that the contrast reduction due to the background intensity, I_{SS} , is also not constant. Since I_{SS} has twice the spatial frequency range that of the SD or DS (signal) light, the ratio of the background to the signal, I_{SS}/I_{SD} , becomes very large near the edge of the scattering cone. Some

Fig. 17. Ray paths of the SD and DS beams for off-axis scatterpoints.

Fig. 18. Contrast and phase graphs for various amounts of non-isoplanatism.

measured values of this ratio appear in Figure 19 for two different diffraction efficiencies. These profiles are measured by using an array of silicon diodes in the image plane of the interferometer. A piece of black tape is placed over the hotspot on the mirror surface to prevent the return of I_{DS} and I_{DD} . The oscilloscope trace from the diode array is similar to the graph in Figure 20. The only intensity returning to the image plane at the small obscuration is I_{SS} . By placing small obscurations at other points on the mirror, I_{SS}/I_{SD} can be measured at points other than at the hotspot. Since this ratio is larger further away from the hotspot, overfilling the test aperture with scattered light not only produces a more even illumination but also a more even fringe contrast.

Source Requirements

Although the first scatterplate interferometers used non-laser sources, it is important to realize the limitations of their use. Both the spatial and temporal coherence of the source should be considered.

In order to have reasonable brightness in the image plane, the "pinhole" in Figure 1 has now become an extended source. The fringe pattern again is the result of a superposition of phase shifted fringe patterns from each point on the source. The high spatial frequency in the image intensity (i.e. regions of high wavefront slope and the background speckle) is first susceptible to contrast reduction as the source size is increased since the final image intensity is a convolution of

Fig. 19. Plots of the ratio of the background to scattered-direct intensities, I_{SS}/I_{SD} , across a test aperture for two different scatter efficiencies, α .

Fig. 20. Plots of oscilloscope traces of a detector array in the image plane.

the source distribution with the fringe pattern with a point source. As a rule of thumb, the fringe contrast is zero when the imaged source size approaches the fringe spacing. Alignment of the interferometer is somewhat more difficult because the very fine fringes present in rough alignment (with large amounts of tilt and defocus) are not seen as well as with an extended source. However background speckle is eliminated.

The source spectral distribution can also reduce fringe contrast. The resulting fringe pattern is a superposition of fringe patterns of a different spatial frequency so that the range of OPD measurable is limited by the source's spectral distribution. In this case, the spectral distribution causes the fringe contrast to drop further away from the hotspot.

Quality of Interferometer Optics

The beam should "see" as little aberration from the test optics as possible. Unlike the Twyman-Green where errors in the reference flat are directly registered in the fringe pattern, the scatterplate because of its multiple-beam characteristic averages over phase errors introduced through the reference beam. The effect of an enlarged hotspot can be seen by defocusing the projection lens. As seen in Figures 21a and 21b, the contrast remains fairly good until the amount of aberration (in this case tilt) across the enlarged spot becomes significant (in b). Spherical aberration in the projection lens and deviations in flatness of the beamsplitter can also increase the hotspot size, but for most test systems these errors would have to be extremely large.

(a)

Fig. 21. Scatterplate interferogram with the hotspot not focused on the mirror.

(a) No tilt present in the aperture.

(b)

Fig. 21. Continued.

(b) Tilt present in the
aperture.

Beamsplitters and doublets with a few wavelengths of error that can be purchased as off-the-shelf items are adequate for use in a scatterplate interferometer.

Much of what was just said about the interferometer illuminating optics applies as well for the imaging optics. In the presence of test aberration, the reference and test beams do not travel along the exact same paths after the second passage through the scatterplate. Instead they are laterally shifted with respect to one another by a few thousandths of an inch depending on the amount of aberration present. Some care, then, should be taken to minimize any relative phase shifts that might occur between these two beams from poorly made beamsplitters and imaging lenses. Variations in thickness of the film emulsion and the glass substrate can also introduce phase shifts between the reference and test beam but usually the separation between these two beams in the plane of the scatterplate is so small this effect can be neglected for most practical purposes.

The effects of defocus in the imaging lens are significant. The higher spatial frequency portions of the fringe pattern are washed out first since they correspond to larger separations in the reference and test beams. It is interesting to note that for further amounts of image lens defocus, there appears a fringe contrast reversal which suggests a besinc-type dropoff in contrast with imaging defocus. Another way of stating this in terms of interferometry is that the fringes in a scatterplate interferometer are localized in the plane of the hotspot focus.

Multiple Surface Test Systems

Up to now, only single surface test configurations have been considered. When multiple surface systems are under test, care should be taken to minimize the hotspot diameter through any surface. For a single lens shown in autocollimation with a flat in Figure 22, the projection lens should focus the direct beam on the flat. In this way, the size of the hotspot through the lens in both passages is the smallest. Obviously, the closer the flat is to the lens the smaller the hotspot is through the lens. However, with a parabola in the null configuration, the flat and the mirror must be separated and a certain amount of contrast reduction is inevitable.

Since the null configuration for the parabola is encountered often, an outline of its alignment procedure is appropriate. Fringes in a scatterplate can be viewed easily by first superimposing the two blurred hotspots on the parabola (the forward and return passages of the reference beam). This assumes the flat, parabola, and scatterplate are placed at roughly the right spacings between each other and the correct height off the test bench. The difficulty often arises when trying to remove the off-axis aberrations present (usually third order coma). In the Twyman-Green, the star image is of some advantage here. By moving the star image in the direction of the "tail" of the coma pattern with movements on the interferometer and bringing the image back with tilt adjustments on the autocollimating flat, the asymmetry in the star pattern can be removed. The scatterplate has only an interferogram from

Fig. 22. Autostigmatic lens testing schematic.

which to read and make adjustments. By removing all the tilt and defocus in the interferogram, the direction of image movement is reduced to one of two ways shown in Figure 23.

Should the parabola be uncoated, an additional problem is encountered. The backscatter from the scatterplate itself is now much brighter than the signal returning from the test system. Tilting the scatterplate with respect to the incoming beam from the projection lens can relieve this problem somewhat. A better solution is achieved by rotating the signal polarization 90° with respect to the backscatter polarization. The standard technique is to place a $1/4\lambda$ plate immediately after the scatterplate and a polarizer near the image lens. If the laser is polarized, proper orientation of the $1/4\lambda$ plate and the polarizer removes the backscatter. The $1/4\lambda$ plate has to be tilted slightly to prevent its specular back reflection from reaching the image plane. For those with thin film facilities available, anti-reflection coating can be applied to both sides of the scatterplate.

Fig. 23. Drawing a coma interferogram without tilt or defocus.--The arrow indicates the direction of beam tilt.

CHAPTER 3

FOURIER ANALYSIS OF THE SCATTERPLATE

This chapter lays the theoretical groundwork for a more formal analysis of the scatterplate interferometer. Fourier theory along with some results from stochastic processes provides the mathematical methods for deriving an expression for the fringe contrast as a function of the scatterplate PTF distribution.

Fourier Analysis

The first step is to derive an expression for the fringe contours in the image plane. By unfolding the schematic in Figure 1 as shown in Figure 24, the scatterplate appears very similar to a coherent optical processor. The usual step by step Fourier techniques of propagation and multiplication of phase masks are used to determine the image plane intensity. Starting just after the first scatterplate, the amplitude is written as

$$S^+ e^{-i \frac{k}{2R} (x_1^2 + y_1^2)} \quad (3-1)$$

where S = scatterplate amplitude transmission

R = nominal mirror radius

$k = 2\pi/\lambda$

The $+$ and $-$ superscripts denote an inversion of the coordinates of S referenced to the orientation of the first scatterplate.

Fig. 24. Unfolded schematic of a scatterplate interferometer.

By convolving this amplitude with the free space propagator, the amplitude just before the test lens is

$$\begin{aligned} & \int S^+ e^{-\frac{ik}{2R} (x_1^2 + y_1^2)} \exp \left\{ i \frac{\pi}{R\lambda} [(x_1 - x_2)^2 + (y_1 - y_2)^2] \right\} dx_1 dy_1 \\ &= \exp \left\{ \frac{ik}{2R} (x_2^2 + y_2^2) \right\} \widetilde{S^+} \end{aligned} \quad (3-2)$$

where $\widetilde{S^+}$ is the Fourier transform of S^+ . The mathematics is simplified if the scattered amplitude distribution just fills the test lens aperture. In this case, the amplitude just after the test lens is

$$\exp \left\{ -i \frac{k}{2R} (x_2^2 + y_2^2) \right\} e^{ikW} \widetilde{S^+} \quad (3-3)$$

where W is the wavefront aberration in the test lens. After another propagation to the second scatterplate, the amplitude just after the image lens is

$$\begin{aligned} & \int \exp \left\{ \frac{-ik}{2R} (x_2^2 + y_2^2) \right\} \widetilde{S^+} e^{ikW} \exp \left\{ \frac{ik}{2R} [(x_3 - x_2)^2 + (y_3 - y_2)^2] \right\} \\ & dx_2 dy_2 S^+ \exp \left\{ \frac{-ik}{2f_I} (x_3^2 + y_3^2) \right\} \\ &= \exp \left\{ i \frac{k}{2} (x_3^2 + y_3^2) \left(\frac{1}{R} - \frac{1}{f_I} \right) \right\} \widetilde{S^+ e^{ikW}} S^+ \end{aligned} \quad (3-4)$$

where f_I is the focal length of the image lens. The final amplitude in the image plane is

$$\int \exp \left\{ \frac{ik}{2} (x_3^2 + y_3^2) \left(\frac{1}{R} - \frac{1}{f_I} \right) \right\} \widetilde{S^+} e^{ikW} .$$

$$S^+ \exp \left\{ \frac{ik}{2Z_I} [(x_4 - x_3)^2 + (y_4 - y_3)^2] \right\} dx_3 dy_3$$

$$= \exp \left\{ \frac{ik}{2Z_I} (x_4^2 + y_4^2) \right\} \widetilde{S^+} e^{ikW} S^+ \quad (3-5)$$

where Z_I is the distance between image lens and image plane and the relation $\frac{1}{R} + \frac{1}{Z_I} - \frac{1}{f_I} = 0$ is used. Eq. (3-5) can be squared and rearranged,

$$I_f = \left| \left[e^{ikW} \widetilde{S^+} \right] ** \widetilde{S^-} \right|^2 \Big|_{\text{Image inverted}} \quad (3-6)$$

where $**$ denotes a two dimensional convolution. This is the expression that will be used throughout the rest of this study. For simplicity, the image inverted will be dropped from here on in. It is important to realize that since eq. (3-6) uses linear systems theory, the effects of non-isoplanatism discussed in the previous chapter are not considered. However, its application in other aspects such as flip symmetry and scattered intensity profiles is very useful.

Statistical Analysis

It is convenient to use the following form for S in eq. (3-6) as first done by Burch

$$S = A[S_0 + S_1] \quad (3-7)$$

where A = scatterplate aperture function

S_0 = average scatterplate transmission

S_1 = variations in the scatterplate transmission about the mean.

$$\begin{aligned}
 I_f &= \left| \left\{ e^{ikW} [\widetilde{A} ** (S_0\delta + \widetilde{S_1^+})] \right\} ** [\widetilde{A} * (S_0\delta + \widetilde{S_1^-})] \right|^2 \\
 &= \left| S_0^2 \widetilde{A} + e^{ikW} S_0 \widetilde{AS_1^+} + S_0 \widetilde{AS_1^-} + (e^{ikW} \widetilde{AS_1^+}) ** \widetilde{AS_1^-} \right|^2
 \end{aligned}
 \tag{3-8}$$

where $\widetilde{A} ** \widetilde{S_1^+} = \widetilde{AS_1^+}$. The approximation used in deriving eq. (3-8) is that W is assumed to be slowly varying over A , the Airy pattern of the scatterplate aperture. In other words, the reference beam should "see" a negligible amount of test error. It is useful to point out that the four terms inside the brackets are the DD, SD, DS, and SS amplitudes respectively. The fringe contours (signal term of I_f) then are given by the squared magnitude of the sum of the DS and SD terms,

$$\begin{aligned}
 I_{sig} &= S_0^2 \left| \widetilde{A} ** \widetilde{S_1^+} \right|^2 + S_0^2 \left| \widetilde{A} ** \widetilde{S_1^-} \right|^2 + 2S_0^2 \operatorname{Re} \left\{ e^{ikW} (\widetilde{A} ** \widetilde{S_1^+}) \cdot \right. \\
 &\quad \left. (\widetilde{A} ** \widetilde{S_1^-})^* \right\}
 \end{aligned}
 \tag{3-9}$$

Although the result will be the same, eq. (3-9) is handled slightly different for pure amplitude and pure phase scatterplates.

In the first case eq. (3-9) reduces to

$$I_{\text{sig}} = 2S_0^2 \left[|\tilde{A}^{**}|^2 |\tilde{S}_1| e^{i\psi} + \text{Re} \left\{ e^{ikW} (\tilde{A}^{**} |\tilde{S}_1| e^{i\psi})^2 \right\} \right] \quad (3-10)$$

where

$$\tilde{S}_1^+ = |\tilde{S}_1| e^{i\psi} \quad (3-11a)$$

$$\tilde{S}_1^- = |\tilde{S}_1| e^{-i\psi} \quad (3-11b)$$

Eq. (3-11b) is a statement of the hermitian property of the Fourier transforms of real-valued functions.

For the pure phase scatterplate, an approximation is necessary in order to prevent the mathematics from becoming too cumbersome. The exact form for S is written as

$$S_{\text{phase}} = e^{i \frac{2\pi}{\lambda} (n-1)h} = 1 + \sum_{q=1}^{\infty} \frac{\left(i \frac{2\pi}{\lambda} (n-1)h \right)^q}{q!} = S_0 + S_1 \quad (3-12)$$

where it is assumed for simplicity that the phase variations are due to surface height variations, h . Theoretically phase gratings are not band limited but for this analysis, all orders of $q > 1$ are assumed to be negligible compared to the first order. The equations corresponding to (3-11a) and (3-11b) for the phase scatterplate become

$$\tilde{S}_1^+ = |\tilde{S}_1| e^{i(\psi - \frac{\pi}{2})} \quad (3-13a)$$

$$\tilde{S}_1^- = |\tilde{S}_1| e^{-i(\psi - \frac{\pi}{2})} \quad (3-13b)$$

or the anti-hermitian property for the Fourier transforms of pure imaginary functions. In either the amplitude or phase case, the result of eq. (3-10), is the same. The section at the end of this chapter will discuss how these higher orders might be incorporated into eq. (3-9).

Although ψ represents the phase shift of the sinusoidal gratings in S_1 , strictly speaking, it is not the PTF. It should be remembered that S_1 is of infinite extent so that the gratings within it are truncated by the finite aperture, A . Actually, the phase of the "smoothed over" version of ψ

$$\widetilde{A^{**}}|\widetilde{S_1}|e^{i\psi} = Me^{i\phi} \quad (3-14)$$

is the PTF in the correct sense. Using eq. (3-10), it is seen that 2ϕ is the phase directly registered into the fringe pattern

$$I_{\text{sig}} = 2S_0^2 M^2 [1 + \cos(kW + 2\phi)] \quad (3-15)$$

As discussed in the previous chapter, the size of the correlation width of ϕ in comparison to changes in W determine if ϕ is either "slowly" or "rapidly" varying. This correlation width is of course closely related to how ψ changes with spatial frequency. Should ψ vary negligibly over A , it can be factored out of the convolution in eq. (3-14) in which case ψ will be equal to ϕ . Chapter 6 discusses a holographic process where slowly varying phase shifts in ψ are encoded into the scatterplate to cancel the test aberration.

At the other extreme, for ψ rapidly changing over A , eq. (3-14) becomes a statistical quantity. The correlation width of ϕ in this case will be about the size of A . When observing a fringe pattern what is of interest are gross characteristics (i.e. the fringes), not the small rapidly varying structure of ψ within it. I_{sig} then, is averaged over an area small compared to the fringe spacing but large enough to include many variations of M and ψ . Physically, this can be thought of as averaging due to a finite detector area of the impulse response of the imaging lens. In either case, the spatial average can be equated to a statistical average by invoking the ergodic theorem (Beckmann and Spizzichino 1963). The observed signal then is

$$\langle I_{\text{sig}} \rangle = 2S_o^2 \langle M^2 [1 - \cos(kW + 2\phi)] \rangle \quad (3-16)$$

where $\langle \rangle$ denotes "statistical average of." The task of the next chapter is to determine the probability density function of M and ϕ , $f_{M\phi}$, in the presence of fabrication errors in the double exposure methods. Once this is achieved, it remains then to substitute $f_{M\phi}$ into the integrals of eq. (3-16) to determine the fringe contrast.

Discussion of Phase Scatterplate PTF's

When the scatterplate is bleached, the superposition of sinusoidal gratings becomes a superposition of sinusoidal phase gratings, each of which now scatters light into an infinite series of orders. A phase grating shifted by ψ with respect to the center of flip

symmetry, shifts each of its orders by $q\psi$, where q is the order number. The sum in eq. (3-14) at any spatial frequency f , now becomes a sum of dephased besinc patterns from the 1st order of the phase gratings near the frequency f plus those of the 2nd order of the phase gratings near the frequency $f/2$ and so on. Using eq. (3-12), the exact expression for the phase scatterplate in eq. (3-14) is

$$Me^{i\phi} = \widetilde{A} \sum_{q=1}^{\infty} \frac{\left(i \frac{2\pi}{\lambda} (n-1)h\right)^q}{q!} = \sum_{q=1}^{\infty} M_q e^{i\psi_q} \quad (3-17)$$

where each partial phasor sum $M_q e^{i\psi_q}$ (which are statistically independent from one another) is the sum of dephased besincs due to order q . Unfortunately, eq. (3-17) is a sum of non-stationary phasors since the spread in random phases ψ_q doubles from each order to the next. Although, the resultant ϕ is still 0 and $\pm\pi$ for perfect flip symmetry, it seems its variance for the imperfect symmetry case will be increased slightly due to the higher orders. This means for higher diffraction efficiencies where the higher order become prominent, the contrast reduction observed will be slightly less than that predicted by the first order alone.

CHAPTER 4

THE EFFECT OF FLIP SYMMETRY ON FRINGE CONTRAST

Before determining the statistics of the double exposure method of making scatterplates, it is instructive to first consider the single exposed speckle pattern. For those familiar with the references on laser speckle and stochastic processes (Goodman 1975, Papoulis 1965), much of what is presented in the next section will be review. However, once this case is understood, the application of the same technique to the double exposure case is straightforward.

Calculation of PTF Probability Densities

The spectrum of S_1 in a single exposure scatterplate will be modeled as a random array of delta functions with the following assumptions.

1. The amplitude and phase, $|\widetilde{S}_1|$ and ψ , for any given delta function are statistically independent.
2. In a completely random speckle pattern, the ψ 's are distributed uniformly between π and $-\pi$.
3. The statistics of each delta function member of S_1 are independent of one another.

The basis for these assumptions is very similar to that found in laser speckle theory.

What appears in the Fourier plane of S_1 then is the convolution of this delta function array with the transform of the scatterplate aperture A. At any coordinate in the Fourier plane, an amplitude sum of dephased besinc functions occur as shown in Figure 25. It is assumed that the delta functions of S_1 are "closely spaced" enough to result in an overlap of several different besinc functions. The phasor sum, then, can be written as

$$Me^{i\phi} = \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i}{r_0}\right) e^{i\psi_i} \quad (4-1)$$

where

$$r_0 = \frac{\lambda R}{2s_0}$$

ρ_i = distances in the Fourier plane of the besinc functions away from the particular point of summation.

Since the amplitudes, $|\widetilde{S}_1|_i$, are given by the autocorrelation of the scattering aperture used to produce the speckle pattern in the fabrication process, $|\widetilde{S}_1|_i$ is distributed uniformly between zero and some constant given by the envelope of this autocorrelation at a particular spatial frequency. Strictly speaking, the statistics of $|\widetilde{S}_1|_i$ are non-stationary since this autocorrelation is not uniform over the Fourier plane. However, it will be assumed that locally (over an area comparable to $\widetilde{A}$), $|\widetilde{S}_1|_i$ is stationary.

Fig. 25. Superposition of dephased besinc patterns in the Fourier plane of a single (unsymmetrical) exposed scatterplate.

The theory of random phasor sums (Beckmann and Spizzichino 1963) provides a method of determining the statistics of M and ϕ in eq. (4-1) given the known statistics of $|\widetilde{S}_1|_i$ and ψ_i . It is convenient first to determine the statistics of the real and imaginary parts of eq. (4-1) separately and then make a transformation of variables back to M and ϕ .

$$X = \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc} \left(\frac{\rho_i}{r_0} \right) \cos \psi_i \quad (4-2)$$

$$Y = \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc} \left(\frac{\rho_i}{r_0} \right) \sin \psi_i \quad (4-3)$$

Using the assumptions outlined previously

$$\langle X \rangle = \langle Y \rangle = \langle XY \rangle = 0 \quad (4-4)$$

By invoking the Central Limit Theorem, the probability density of X and Y is Gaussian,

$$f_{XY} = \frac{1}{2\pi\sigma_X\sigma_Y} \exp \left\{ -\frac{X^2}{2\sigma_X^2} - \frac{Y^2}{2\sigma_Y^2} \right\} \quad (4-5)$$

where

$$\begin{aligned} \sigma_X^2 = \langle X^2 \rangle &= \sum_{i=1}^N \langle (|\widetilde{S}_1|_i)^2 \rangle \text{besinc}^2 \left(\frac{\rho_i}{r_0} \right) \langle \cos^2 \psi_i \rangle \\ &= \frac{1}{2} \langle (|\widetilde{S}_1|_i)^2 \rangle \sum_{i=1}^N \text{besinc}^2 \left(\frac{\rho_i}{r_0} \right) \end{aligned} \quad (4-6)$$

$$\begin{aligned}
\sigma_y^2 &= \langle Y^2 \rangle = \sum_{i=1}^N \langle (|\widetilde{S}_1|_i)^2 \rangle \text{besinc}^2\left(\frac{\rho_i}{r_o}\right) \langle \sin^2 \psi_i \rangle \\
&= \frac{1}{2} \langle (|\widetilde{S}_1|_i)^2 \rangle \sum_{i=1}^N \text{besinc}^2\left(\frac{\rho_i}{r_o}\right)
\end{aligned} \tag{4-7}$$

Transforming f_{XY} into polar coordinates

$$\begin{aligned}
M^2 &= X^2 + Y^2 \quad \phi = \tan^{-1} \frac{Y}{X} \\
f_{M\phi} &= \frac{M}{2\pi\sigma^2} e^{-\frac{M^2}{2\sigma^2}}, \quad \phi \text{ uniform between } \pi \text{ and } -\pi
\end{aligned} \tag{4-8}$$

The phase, ϕ , is uniformly distributed because of the equal variances σ_x^2 and σ_y^2

$$\sigma_x = \sigma_y = \sigma \tag{4-9}$$

Eq. (4-8) leads to the well known Rayleigh distribution.

Phasor Sums of Double Exposure Speckle Patterns

If a second exposure of the same speckle pattern precisely 180° is present, a complex conjugate sum is added to eq. (4-1)

$$\begin{aligned}
Me^i &= \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i}{r_o}\right) e^{i\psi_i} + |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i}{r_o}\right) e^{-i\psi_i} \\
&= 2 \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i}{r_o}\right) \cos \psi_i
\end{aligned} \tag{4-10}$$

The imaginary part of this expression is zero so that ϕ assumes values of 0 and $\pm\pi$. For the phase scatterplate the corresponding expression for $Me^{i\phi}$ becomes

$$\begin{aligned} Me^{i\phi} &= \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i}{r_o}\right) e^{i(\psi - \frac{\pi}{2})} + |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i}{r_o}\right) e^{-i(\psi - \frac{\pi}{2})} \\ &= 2 \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i}{r_o}\right) \sin\psi_i \end{aligned} \quad (4-11)$$

where eq. (3-13a) and (3-13b) have been used. In these expressions and those which follow, the only difference between the pure amplitude and pure phase scatterplates, as far as the mathematics is concerned, is switching of a $\cos\psi_i$ to a $\sin\psi_i$ and vice versa.

The inclusion of a small amount of rotational error, θ_E , between exposures implies the pairs of conjugate besinc functions in the Fourier plane have an angular shear between them instead of being directly superimposed. Figure 26 illustrates a simplified view of what takes place in the Fourier plane with and without rotational error present. Eq. (4-10) is rewritten as

$$Me^{i\phi} = \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i}{r_o}\right) e^{i\psi_i} + |\widetilde{S}_1|_i \text{besinc}\left(\frac{\rho_i'}{r_o}\right) e^{-i\psi_i} \quad (4-12)$$

ρ and ρ' , more clearly seen in a top view of Figure 26b, are the different distances of the besinc pairs away from the summation point A. If the point A is located, for simplicity, on the x axis, a small

Fig. 26. Besinc pattern in the Fourier plane of a double exposed scatterplate.

(a) With an exact 180° and (b) an imperfect 180° rotation between exposures.

Fig. 26. Continued.

(c) A top view of Figure 26b showing the coordinates relative to the summation point A.

amount of rotation implies ρ' can be approximated by the vector sum of ρ and a small y direction vector of length $r_1\theta_E$. This simplifies the mathematics when integrations of these besincs are performed.

In integral notation

$$X = \frac{\pi}{4r_o^2} \int |\widetilde{S}_1| \left[\text{besinc} \left(\frac{\rho}{r_o} \right) + \text{besinc} \left(\frac{\rho'}{r_o} \right) \right] \cos\psi \, d\rho d\omega \quad (4-13)$$

$$Y = \frac{\pi}{4r_o^2} \int |\widetilde{S}_1| \left[\text{besinc} \left(\frac{\rho}{r_o} \right) - \text{besinc} \left(\frac{\rho'}{r_o} \right) \right] \sin\psi \, d\rho d\omega \quad (4-14)$$

where ρ and ω are polar coordinates referenced to an origin at point A. From hereon in, the mathematics follows the same procedure as in the previous section using the following variances in X and Y

$$\begin{aligned} \sigma_X^2 &= \langle X^2 \rangle = \frac{\pi}{4r_o^2} \left\langle \int |\widetilde{S}_1|^2 \left[\text{besinc} \left(\frac{\rho}{r_o} \right) + \text{besinc} \left(\frac{\rho'}{r_o} \right) \right]^2 \cos^2\psi \, d\rho d\omega \right\rangle \\ &= \langle |\widetilde{S}_1|^2 \rangle \left[1 + \text{besinc} \left(\frac{r_1\theta_E}{r_o} \right) \right] \end{aligned} \quad (4-15)$$

where

$$\frac{\pi}{4r_o^2} \int \text{besinc}^2 \left(\frac{\rho}{r_o} \right) d\rho d\omega = \frac{\pi}{4r_o^2} \int \text{besinc}^2 \left(\frac{\rho'}{r_o} \right) d\rho d\omega = 1$$

and

$$\frac{\pi}{4r_o^2} \int \text{besinc} \left(\frac{\rho}{r_o} \right) \text{besinc} \left(\frac{\rho'}{r_o} \right) d\rho d\omega = \text{besinc} \frac{r_1\theta_E}{r_o}$$

Similarly for Y

$$\sigma_y^2 = \langle |\widehat{S}_1|^2 \rangle \left(1 - \text{besinc} \frac{r_1 \theta_E}{r_o} \right) \quad (4-16)$$

Since

$$\langle X \rangle = \langle Y \rangle = \langle XY \rangle = 0$$

the same form for f_{XY} as in eq. (4-5) may be used here except now the variances in X and Y are unequal. Such distributions are known as Hoyt (Hoyt 1947) distributions and were first solved in 1947.

When transformed into polar coordinates, the probability density function becomes

$$f_{M\phi} = \frac{M e^{-cM^2}}{2\pi\sigma_x\sigma_y} \quad (4-17)$$

where

$$c = \frac{\sin^2\phi}{2\sigma_y^2} + \frac{\cos^2\phi}{2\sigma_x^2}$$

Using eq. (3-16), the observed signal intensity can now be calculated

$$\begin{aligned} \langle I_{\text{sig}} \rangle &= 2S_o^2 \left(\langle M^2 \rangle + \langle M^2 \cos 2\phi \rangle \cos kW + \langle M^2 \sin 2\phi \rangle \sin kW \right) \\ &= 2S_o^2 (\sigma_x^2 + \sigma_y^2) \left(1 + \frac{\sigma_x^2 - \sigma_y^2}{\sigma_y^2 + \sigma_x^2} \cos kW \right) \end{aligned} \quad (4-18)$$

where

$$\langle M^2 \rangle = \sigma_x^2 + \sigma_y^2$$

$$\langle M^2 \cos 2\phi \rangle = \sigma_x^2 - \sigma_y^2$$

$$\langle M^2 \sin 2\phi \rangle = 0$$

Substituting eq. (4-15) and eq. (4-16), the fringe contrast is then

$$\gamma_r = \frac{\sigma_x^2 - \sigma_y^2}{\sigma_y^2 + \sigma_x^2} = \text{besinc} \left(\frac{\theta_E}{\frac{f\#\lambda}{s_0}} \right) \quad (4-19)$$

where

$$r_0 = \frac{\lambda R}{2s_0} \quad \text{and} \quad f\# = \frac{R}{2r_1}$$

Note that there is no net fringe shift due to rotational error, only contrast reduction. Also the switching of the $\cos\psi$ and $\sin\psi$ for the pure phase scatterplate leaves eq. (4-19) the same since the variances of X and Y are unchanged.

For unmatched exposures during fabrication, the phasor sum corresponding to eq. (4-10) is

$$M e^{i\phi} = \sum_{i=1}^N |\widetilde{S}_1|_i \text{besinc} \left(\frac{\rho_i}{r_0} \right) e^{i\psi_i} + B |\widetilde{S}_1|_i \text{besinc} \left(\frac{\rho_i}{r_0} \right) e^{-i\psi_i} \quad (4-20)$$

The amplitudes of the conjugate sum are changed uniformly by the factor B which equals the ratio of the two exposures if the film makes a linear recording. No generality will be lost if B is assumed to be less than or equal to one.

The same form for $f_{M\phi}$ can be used again with the following variances.

$$\begin{aligned} \sigma_x^2 &= \langle X^2 \rangle = \frac{1}{\pi r_0^2} \int \langle |\widetilde{S}_1|^2 \rangle \text{besinc}^2 \frac{\rho}{r_0} (1+B)^2 \langle \cos^2 \psi \rangle d\rho d\omega \\ &= \langle |\widetilde{S}_1|^2 \rangle (1+B)^2 \end{aligned} \quad (4-21)$$

$$\sigma_y^2 = \langle |\widetilde{S}_1|^2 \rangle (1-B)^2 \quad (4-22)$$

Using eq. (4-18), the contrast becomes

$$\gamma_e = \frac{2B}{1+B^2} \quad (4-23)$$

Experimental Verification

Verification of eq. (4-19) was carried out by placing a linear array of silicon detectors in the image plane of the scatterplate interferometer and measuring the contrast directly off the oscilloscope. The results are plotted in Figures 27a and 27b along with the theoretically predicted values.

Several schemes were considered for introducing the desired amounts of rotational error into the interferometer. The most useful method was to manufacture the error into a single plate. Figure 28 shows how desired amounts of error are introduced during fabrication. A small plane parallel plate, aluminized on one side is mounted vertically on the rotating film holder. Autocollimating off the front and back of this mirror surface insures a precise 180° rotation (~ 1 second of error). By replacing one of the defining pins by a rotating eccentric pin, the right amount of angular error can be added and then measured by the autocollimator. Small amounts of wedge (< 1 min. arc) in the plane parallel plate are permissible as long as the direction of the wedge is oriented parallel to the axis of the film holder rotation.

Fig. 27. Experimentally measured values of (a) γ_r and (b) γ_E .

Fig. 28. Modified film emulsion rotator to introduce rotational error, θ_E .

In this way, any wedge in the plate only causes a lateral shift in the return image of the crosshairs between 180° rotations of the film holder.

The exposure mismatch was also incorporated into a single plate, in this case by changing the exposure time on the shutter. The accuracy of this method is limited by the repeatability of the mechanical shutter speeds which were checked with a detector and an oscilloscope and found to vary within about 10% of the dial reading on the shutter. Also, the position of the laser beam within the shutter aperture should be the same between exposures since the shutters show as much as 30% variability in exposure time over the aperture.

The direct reading of the contrast off the oscilloscope has to be corrected to account for the effects of rotational error or exposure mismatch only. To minimize the effects of non-isoplanatism, a large enough scattering f/no. must be found through the use of Figure 18. The measurements in Figure 27a were taken at f/8 with a 1 cm diameter scatterplate aperture. By using eq. (2-2), the effects of the background intensity on contrast can be corrected for. This necessitates an additional measurement of the ratio I_{SS}/I_{SD} (using the techniques described in Chapter 1) at the chosen scattering f/no.

The results in Figures 27a and 27b show a good agreement with the theory except where the errors were small. In spite of all these corrections a near perfect contrast was never measured. It was mentioned in Chapter 2 that the way in which photographic emulsion records

multiple exposures might explain this effect. Although further investigation is needed, the author feels the first exposure is somehow altering the sensitivity of the film in recording the second exposure.

CHAPTER 5

ANALYSIS OF INTERFEROMETER OPERATION

This chapter continues the analytic treatment of the principles behind the scatterplate interferometer. Using the signal portion of the image plane intensity derived in Chapter 3, eq. (3-8), the effects of both aberration in the auxiliary optics and extended source size on the fringe contrast are discussed. An analysis of the entire eq. (3-8), including the hotspot and background intensities, shows how energy is conserved when a null fringe appears and how fringe contrast is reduced by the background intensity. Finally, the mathematical expressions for the contrast and phase graphs in Figure 18 for non-isoplanatism are derived.

Tilt and Defocus

The addition of tilt and defocus into a scatterplate is well known and is included here for completeness. Assuming perfect flip symmetry, the expression for I_{sig} in eq. (3-9) reduces to

$$\begin{aligned} I_{\text{sig}} &= 2S_0^2 [|\widetilde{AS_1}|^2 + \text{Re}\{e^{ikW}(\widetilde{AS_1})^2\}] \\ &= 2S_0^2 |\widetilde{AS_1}|^2 (1 + \cos kW) \end{aligned} \quad (5-1)$$

where $S_1^+ = S_1^- = S_1$. The fringe contour term is an envelope for the

rapidly varying speckle term, $|\widetilde{AS_1}|^2$. As long as the scatterplate aperture is large enough, the speckle will not make fringe observation difficult.

Movement of the center of symmetry of the optical axis in the x direction, say, a distance x_0

$$\widetilde{AS_1(x-x_0)} = \widetilde{AS_1} \exp \left\{ iA_{111}x_2/d_0 \right\} \quad (5-2)$$

where

$$A_{111} = \frac{2\pi x_0}{\lambda f\#}, \text{ amount of tilt phase added across test aperture}$$

x_2 = test aperture coordinate

$f\#$ = test system f/no.

d_0 = test aperture radius.

Substituting eq. (5-2) into (5-1)

$$I_{sig} = 2S_0^2 |\widetilde{AS_1}|^2 (1 + \cos kW + 2A_{111}x_4/d_I) \quad (5-3)$$

where d_I is the radius of test aperture in the image plane.

Longitudinal movement of the scatterplate away from the paraxial focus introduces a quadratic phase factor outside of AS_1 so that I_{sig} becomes

$$I_{sig} = 2S_0^2 |\widetilde{AS_1}|^2 \{1 + \cos[kW + A_{020}(x_4^2 + y_4^2)/d_I^2]\} \quad (5-4)$$

where

$\Delta \ell$ = longitudinal movement of the scatterplate

$A_{020} = \frac{2\pi\Delta\ell d_0^2}{\lambda R^2}$, amount of defocus phase added across test aperture.

Aberrations in the Auxiliary Optics

As stated in the first chapter, the aberrations in the auxiliary optics have to be fairly severe before their effect on the fringe pattern are observed. In most cases because of the averaging property of the scatterplate, net fringe shifts are negligible with the primary effect being contrast reduction. The simplest aberration in the auxiliary optics to handle analytically is defocus.

Projection Lens Defocus

If the hotspot is not focused on the test optics, the new form for S^+ is

$$\begin{aligned} S^+ &= S_0 A \exp\{i a_{020}(x_1^2 + y_1^2)/s_0^2\} \\ &+ A \exp\{i a_{020}(x_1^2 + y_1^2)/s_0^2\} S_1 \end{aligned} \quad (5-5)$$

where

$$\begin{aligned} a_{020} &= \text{amount of defocus phase across the scatterplate aperture} \\ &= \frac{\pi \Delta Z}{R(R + \Delta Z)} \end{aligned}$$

ΔZ = distance between hotspot focus and test surface.

For large a_{020} (i.e. many cycles in phase), the geometric approximation of its Fourier transform may be used

$$\overbrace{A \exp\{i a_{020}(x_1^2+y_1^2)/s_o^2\}} \approx \exp\{i a_{020}(x_2^2+y_2^2)/P_o^2\} \frac{\text{cyl}\left(\frac{P}{P_o}\right)}{\pi P_o^2} \quad (5-6)$$

where

P_o = defocused hotspot radius measured in the test aperture.

The signal terms of I_f become

$$\begin{aligned} I_{\text{sig}} = & \frac{1}{\pi m^2 P_o^2} \left| \left\{ \exp[i a_{020}(x_4^2+y_4^2)/m^2 P_o^2] \text{cyl}\left(\frac{P}{2mP_o}\right) \right\} ** \widetilde{S}_1 \right|^2 \\ & + \frac{1}{\pi m^2 P_o^2} \left| \left\{ e^{ikW} \exp[i a_{020}(x_4^2+y_4^2)/m^2 P_o^2] \right. \right. \\ & \quad \left. \left. \text{cyl}\left(\frac{P}{2mP_o}\right) \right\} ** \widetilde{S}_1 \right|^2 \\ & + \frac{2}{\pi m^2 P_o^2} \text{Re} \left\{ \left(\left\{ e^{ikW} \exp[i a_{020}(x_4^2+y_4^2)/m^2 P_o^2] \right\} ** \widetilde{S}_1 \right) \right. \\ & \quad \left. e^{-ikW} \left(\left\{ \exp[i a_{020}(x_4^2+y_4^2)/m^2 P_o^2] \right. \right. \right. \\ & \quad \left. \left. \text{cyl}\left(\frac{P}{2mP_o}\right) \right\} ** \widetilde{S}_1 \right)^* \right\} \quad (5-7) \end{aligned}$$

where m is the magnification of the test aperture in the image plane. W is again assumed to be slowly varying compared to A . As unwieldy as eq. (5-7) appears, it is virtually the same equation as (5-1) except that

the exact Fourier transform of A has been replaced by its Fresnel transform, a cylinder function with a quadratic phase across it.

Both

$$\exp \left\{ i a_{020} (x_4^2 + y_4^2) / (mP_0)^2 \right\} \text{cyl} \left(\frac{P}{mP_0^2} \right) \}^{**} \widetilde{S}_1 = \int \hat{a} e^{i\hat{\beta}} dA_I \quad (5-8)$$

and

$$\begin{aligned} & \left[e^{ikW} \exp \left\{ i a_{020} (x_4^2 + y_4^2) / (mP_0)^2 \right\} \text{cyl} \left(\frac{P}{mP_0^2} \right) \right]^{**} \widetilde{S}_1 \\ & = \int \hat{a} e^{i\hat{\beta}} e^{i\hat{W}} dA_I \end{aligned} \quad (5-9)$$

are statistical quantities and can be written in phasor sum form where dA_I is the area increment in the image plane. The convolutions in eqs. (5-8) and (5-9) can be pictured as a superposition of cylinder functions pictured in Figure 29, each being scaled by the random variable $\hat{a}$ which is a particular delta function height in the random array of delta functions of S_1 . The random variables $\hat{W}$ and $\hat{\beta}$ arise from phase distributions associated with displaced versions of e^{ikW} and $\exp\{ia_{020}(x_4^2+y_4^2)/(mP_0)^2\}$ respectively. Since the amount of defocus, a_{020} , is several wavelengths, $\hat{\beta}$ is assumed to be uniformly distributed between $-\pi$ and π . W , however, is more slowly varying across the width of the $\text{cyl} \left(\frac{P}{2mP_0} \right)$ and hence $\hat{W}$ does not have as broad a distribution. It can be seen that the fringe contrast is reduced as the amount of aberration across the defocused hotspot increases.

It might appear at first glance that the correlation width of the statistical variations in eqs. (5-8) and (5-9) are on the order of width of the cylinder just as the correlation widths discussed in Chapter 3, eq. (3-14), were on the order of the Airy pattern diameter. However, the presence of the strong quadratic phase variation represented by the parabolas above each cylinder in Figure 29 limits the correlation width still comparable to A . It is assumed that this width is much smaller than a fringe spacing so that the resulting fringe pattern is again a statistical average of eq. (5-7)

$$\begin{aligned}
 \langle I_{\text{sig}} \rangle &= \frac{1}{\pi(mP_0)^2} \left\langle \left| \int \hat{a} e^{i\hat{W}} e^{i\hat{\beta}} dA_I \right|^2 + \left| \int \hat{a} e^{i\hat{\beta}} dA_I \right|^2 \right. \\
 &\quad \left. + 2\text{Re}\{e^{ikW} \int \hat{a} e^{i\hat{W}} e^{i\hat{\beta}} dA_I (\int \hat{a} e^{i\hat{\beta}} dA_I)^* \} \right\rangle \\
 &= 2\langle \hat{a}^2 \rangle (1 + \text{Re}\{e^{ikW} \langle e^{i\hat{W}} \rangle\})
 \end{aligned} \tag{5-10}$$

Mutual independence of a , $\hat{\beta}$, and $\hat{W}$ assures that the statistical average of the cross terms of the square of the sums

$$\left\langle \left| \int \hat{a} e^{i\hat{W}} e^{i\hat{\beta}} dA_I \right|^2 \right\rangle \text{ and } \left\langle \left| \int \hat{a} e^{i\hat{\beta}} dA_I \right|^2 \right\rangle \tag{5-11}$$

are zero.

For only tilt present in the test system, $\hat{W}$ will be symmetric and uniform so that the fringe contrast is

Fig. 29. Superposition of defocused Airy patterns in the Fresnel region of a scatterplate.

$$\gamma_{\text{proj}} = \langle e^{i\hat{W}} \rangle = \text{besinc}(N_n) \quad (5-12)$$

where N_n is the maximum number of waves of tilt across the defocused hotspot.

Figure 30 shows some experimental contrast measurements of eq. (5-12) with a 3" diameter, 15" radius of curvature sphere. The measurements compare well with the theory except near the focused hotspot. This is again believed to be due to the same film effects described in the rotational error measurements.

Image Lens Defocus

Defocus in the image lens is handled very similar to the projection lens, only now the quadratic phase term appears across the S^- term in eq. (3-6)

$$S^- = A \exp\{i a_{020}(x_3^2 + y_3^2)/s_0^2\} (S_0 + S_1) \quad (5-13)$$

$$I_f = \left| [e^{ikW} (S_0 \widetilde{A} + \widetilde{A} S_1)] ** \overbrace{(A \exp\{i a_{020}(x_3^2 + y_3^2)/s_0^2\} + \overbrace{A S_1 \exp\{i a_{020}(x_3^2 + y_3^2)/s_0^2\}}^2)} \right|^2 \quad (5-14)$$

Fig. 30. Measured values of the fringe contrast for two different defocused hotspot sizes.--The X axis is the amount of tilt across the test aperture.

Squaring only the signal terms and using a similar approximation to eq. (5-6)

$$\begin{aligned}
 I_{\text{sig}} = \frac{1}{\pi q_0^2} & \left[\left| \left(\exp\{i a_{020}(x_4^2 + y_4^2)/q_0^2\} \text{cy1}\left(\frac{q}{2q_0}\right) \right) ** \widetilde{S}_1 \right|^2 \right. \\
 & + \left| \left(e^{ikW} \widetilde{S}_1 \right) ** \left(\exp\{i a_{020}(x_4^2 + y_4^2)/q_0^2\} \text{cy1}\left(\frac{q}{2q_0}\right) \right) \right|^2 \\
 & + 2\text{Re} \left\{ \left(\exp\{i a_{020}(x_4^2 + y_4^2)/q_0^2\} \text{cy1}\left(\frac{q}{2q_0}\right) \right) ** \widetilde{S}_1 \right. \\
 & \quad \left. \left(\left(\exp\{i a_{020}(x_4^2 + y_4^2)/q_0^2\} \text{cy1}\left(\frac{q}{2q_0}\right) \right) ** \right. \right. \\
 & \quad \left. \left. \left(\widetilde{S}_1 e^{ikW} \right)^* \right) \right] \quad (5-15)
 \end{aligned}$$

where q_0 is the radius of the defocused image of the hotspot in the image plane.

Statistical expressions similar to eqs. (5-8) and (5-9) appear in the cross term of the previous equation.

$$\left[\exp\{i a_{020}(x_4^2 + y_4^2)/q_0^2\} \text{cy1}\left(\frac{q}{2q_0}\right) \right] ** \widetilde{S}_1 = \int \hat{a} e^{i\hat{\beta}} dA_I \quad (5-16)$$

and

$$\left[\exp\{i a_{020}(x_4^2 + y_4^2)/q_0^2\} \text{cy1}\left(\frac{q}{2q_0}\right) \right] ** (\widetilde{S}_1 e^{ikW}) = \int \hat{a} e^{i\hat{\beta}} e^{i\hat{W}} dA_I \quad (5-17)$$

Now the random array of δ functions, $\widetilde{S}_1$, is multiplied by a phase envelope, e^{ikW} . The distribution of phases, $\hat{W}$, due to W now depends on the position in the image plane. The areas of rapid change result in a wider distribution in $\hat{W}$ and hence greater contrast reduction. An expression for this contrast reduction for an arbitrary W would be difficult. More important is understanding that unlike projection lens defocus where the contrast is reduced evenly over the entire fringe plane, with image lens defocus, the high spatial frequency regions are the most sensitive to contrast reduction. However, if the only aberration present is tilt, the contrast reduction will behave the same in both cases.

Extended Non-Coherent Sources

The fringe pattern in this case can be thought of as an intensity superposition of fringe patterns from each point in the source. For a quasi-monochromatic source, the superposition is written as

$$I_{\text{EXT}} = \text{cyl} \left(\frac{e}{e_0} \right) ** I_{\text{sig}} \quad (5-18)$$

where e_0 is the diameter of the source in the image plane.

I_{sig} is the fringe pattern due to a point source, eq. (5-1).

$$I_{\text{EXT}} = \text{cyl} \frac{e}{e_0} ** \{ |AS_1|^2 (1 + \cos kW) \}$$

$$\approx \langle |AS_1|^2 \rangle [1 + \text{cyl} \left(\frac{e}{e_0} \right) ** \cos kW] \quad (5-19)$$

where $\langle |\widetilde{AS_1}|^2 \rangle$ is the average value of $|\widetilde{AS_1}|^2$ across the image source size. The last step is a valid approximation since the average intensity of the speckle pattern given by $|AS_1|^2$ is constant across the image source size. For the simplest case of tilt only

$$\text{cyl } \frac{e}{e_0} ** \cos \left(\frac{2\pi A_{111}}{L_0} x_4 \right) = \text{besinc} \left(\frac{\frac{e_0}{A_{111}}}{L_0} \right) \quad (5-20)$$

where L_0 is the radius of the imaged test aperture.

An experimental plot of the contrast versus tilt for two source sizes is shown in Figure 31 and compares well with the theory. The quasi-monochromatic source in the lab was simulated by placing a rotating ground glass just after the pinhole in the interferometer.

Conservation of Energy in a Scatterplate

Interferometers such as the Twyman-Green and the Mach-Zehnder are two-port interferometers. When a dark fringe appears in one port a bright fringe appears in the other so that the total energy is conserved. In a scatterplate, no such complimentary fringe pattern exists and the question then arises as to its mechanism for conservation of energy.

The conventional scatterplate interferometer can not have a null fringe across the entire aperture unless the hotspot should be focused on a small bump or hole. To investigate energy conservation then, the hotspot is reflected off a separate mirror that is mounted on a piezoelectric crystal as shown in Figure 32. In this way, a detector

Fig. 31. Measured values of the fringe contrast vs. tilt across the aperture for two different source sizes.

Fig. 32. Schematic of a scatterplate interferometer used in a non-common path mode.

placed on the hotspot can measure intensity variations as piston error is added by the PZT. Since the test mirror is spherical, piston error is the only aberration present in the system. This error, W_0 , is introduced into the formalism by the following expression for S^+

$$S^+ = [S_0 e^{ikW_0} + S_1] \quad (5-21)$$

On the first passage through the scatterplate, the dc or average component is phase shifted with respect to the scattered light by the PZT. S_1 in both S^+ (the first passage) and S^- (the second passage) is assumed to have perfect flip symmetry.

With $W=0$, the entire expression for I_f using (5-21) becomes

$$\begin{aligned} I_f &= |(S_0 e^{ikW_0} \widetilde{A + AS_1})^{**} (S_0 \widetilde{A + AS_1})|^2 \\ &= S_0^4 \widetilde{A^2} + 2S_0^3 \widetilde{A} \operatorname{Re}\{\widetilde{AS_1} (1 + e^{ikW_0})\} + 2S_0^2 \widetilde{A} \operatorname{Re}\{e^{ikW_0} \widetilde{AS_1^2}^*\} \\ &\quad + 2S_0^2 |\widetilde{AS_1}|^2 (1 + \cos kW_0) \\ &\quad + |\widetilde{AS_1^2}|^2 + 2S_0 \operatorname{Re}\{\widetilde{AS_1} \widetilde{AS_1^2}^* (1 + e^{ikW_0})\} \end{aligned} \quad (5-22)$$

Because of the common A factor, the first three terms are the hotspot intensity. They are combinations of the DD amplitude and the SS amplitude scattered in the direction of the hotspot. The next two terms form the signal while the last two are the background.

Unexpectedly, the behavior of I_f with the "phase-type" (bleached emulsion) and the "amplitude-type" (unbleached emulsion) differed. In the former case, the energy exchange occurred between the

signal and the hotspot while in the latter the exchange is between the signal and absorption in the scatterplate itself.

An analogous effect occurs in the Twyman-Green interferometer when using either a dielectric or metallic beamsplitter. For a dielectric, the exchange in energy is between the two ports of the interferometer, which corresponds to the exchange between the signal and hotspot for the phase scatterplate. However, with the metallic beamsplitter, it is possible to have a null fringe in both ports. As yet this effect is not fully understood but the energy is believed to be absorbed in by the beamsplitter.

For the phase scatterplate, the amplitude transmission is

$$S_{\text{phase}} = A[1+ik(n-1)h] \quad (5-23)$$

where h is the surface height variations in the film emulsion, n is the emulsion index of refraction, and k is equal to $2\pi/\lambda$. This is an expansion of the term $\exp ik(n-1)h$ for small variations h . The exact form is again difficult to handle analytically and the results are not as easy to interpret. Unfortunately, eq. (5-23) has a modulus greater than one. Its use will be justified since it correctly describes the experimental results.

Substituting eq. (5-23) into (5-22)

$$\begin{aligned} I_f = \widetilde{A} \left[\widetilde{A} + 2k(n-1)\widetilde{A}h \sin\phi - 2\widetilde{A}h^2k^2(n-1)^2\cos kW_0 \right] & \text{ } \text{hotspot} \\ + 2k^2(n-1)(\widetilde{A}h)^2(1+\cos kW_0) & \text{ } \text{signal} \\ + k^4(n-1)^4(\widetilde{A}h^2)^2 + 2k^3(n-1)^3\widetilde{A}h \widetilde{A}h^2\sin kW_0 & \text{ } \text{background} \end{aligned} \quad (5-24)$$

Integrating I_f over the image plane gives the power transfer among the various terms

$$\begin{aligned} \int I_f dA_I &= A_S - 2k^2(n-1)^2 A_S \langle h^2 \rangle \cos kW_0 && \text{hotspot} \\ &+ 2k^2(n-1)^2 A_S \langle h^2 \rangle (1 + \cos kW_0) && \text{signal} \\ &+ k^4(n-1)^4 A_S \langle h^4 \rangle && \text{background} \end{aligned} \quad (5-25)$$

$$\begin{aligned} &= A_S + 2k^2(n-1)^2 A_S \langle h^2 \rangle + k^4(n-1)^4 A_S \langle h^4 \rangle, \\ &\text{independent of } W_0 \end{aligned} \quad (5-26)$$

where dA_S is the area increment in the scatterplate plane and Rayleigh's theorem (Gaskill 1976)

$$\int \widetilde{A} \widetilde{Ah^2} dA_I = \int A Ah^2 dA_S = A_S \langle h^2 \rangle \quad (5-27)$$

Since I_f is a constant with time, eq. (5-25) and eq. (5-26) differ from the energy transfer by a constant factor, the integration time. These equations show the energy transfer occurs between the signal and the hotspot. No net energy is transferred to the background. Instead it appears to scintillate with the addition of piston error.

The amplitude version of the scatterplate uses the form

$$S_A = A(t_0 + t_1) \quad (5-28)$$

where t_0 is the average amplitude transmission of the scatterplate and t_1 is the variations in amplitude transmission about the mean.

Substituted into I_f eq. (5-22) yields

$$\begin{aligned}
 I_f = & \widetilde{A}[t_o^4 \widetilde{A} + 2t_o^3 \widetilde{A}t_1(1+\cos kW_o) + 2t_o^2 \widetilde{A}t_1^2 \cos kW_o] \quad \text{]hotspot} \\
 & + 2t_o^2 (\widetilde{A}t_1)^2 (1+\cos kW_o) \quad \text{]signal} \\
 & + (\widetilde{A}t_1^2)^2 + 2t_o \widetilde{A}t_1 \widetilde{A}t_1^2 (1+\cos kW_o) \quad \text{]background}
 \end{aligned}
 \tag{5-29}$$

Integrating I_f over A_I again gives the power transfer

$$\begin{aligned}
 \int I_f dA_I = & t_o^4 A_S + 2t_o^2 A_S \langle t_1^2 \rangle \cos kW_o \quad \text{]hotspot} \\
 & + 2t_o^2 A_S \langle t_1^2 \rangle (1+\cos kW_o) \quad \text{]signal} \\
 & + \langle t_1^4 \rangle \quad \text{]background}
 \end{aligned}
 \tag{5-30}$$

There is still no net energy transfer to the background, however, now the hotspot and signal terms are oscillating in phase with kW_o rather than 180° out of phase as with the phase scatterplate. The difference between these two cases is that the SS component of the hotspot term receives a 180° phase shift with respect to the DD component. This is because cosinusoidal phase gratings shift the ± 1 diffracted orders by 90° with respect to the undiffracted light and the SS beam (twice scattered) receives a 180° total phase shift. The signal and hotspot intensities will oscillate 180° out of phase. The amplitude scatterplate does not impart these phase shifts to the diffracted light

so both the signal and hotspot intensities will be in phase as kW_0 is varied. The additional energy in this case must be absorbed by the scatterplate.

Figure 33 shows qualitatively how this occurs. For simplicity the scatterplate amplitude transmission shown in A has a bar target profile so the amplitude and intensity will have the same form. B shows the amplitude just before the second scatterplate passage when $kW_0 = \pi$. The C component has been shifted by 180° by the PZT mirror and the squared modulus (I_{S+}) will be contrast reversed as shown in C (the dotted line). When $kW_0 = \pi$ additional energy is being absorbed in the second passage since light areas of I_{S+} are falling on dark areas of $|S^-|^2$. Mathematically, the analysis goes as the following.

The intensity just before the second plate is

$$I_3 = |A(e^{ikW_0}t_0 + t_1)|^2 = A^2(t_0^2 + 2t_1t_0\cos kW_0 + t_1^2) \quad (5-31)$$

The intensity transmission of the second scatterplate

$$T_3 = A^2(t_0^2 + 2t_1t_0 + t_1^2) \quad (5-32)$$

so that the energy being absorbed by the scatterplate in both passages is

$$E_S = \int_{A_S} (1 - I_3 T_3) dA = A_S - (A_S t_0^4 + \langle t_1^4 \rangle + 2t_0^2 \langle t_1^2 \rangle + 4t_0^2 \langle t_1^2 \rangle \cos kW_0) \quad (5-33)$$

Fig. 33. Simplified view of how a pure amplitude scatter-plate absorbs energy.

and the sum of E_s and the energy in the image plane

$$E_s + \int_{A_s} I_f dA = A_s \quad (5-34)$$

a constant with respect to W_0 .

Contrast Reduction Due to Non-Isoplanatism

The effect of non-isoplanatism on the fringe pattern (as discussed in Chapter 1) is found by an intensity superposition of individual, phase shifted fringe patterns from each pair of symmetrical points on the scatterplate. From Figure 17, the OPD between the reference and test rays for any pair of these points is

$$\begin{aligned} \text{OPD} &= \{[\rho_x - s \cos \eta]^2 + [\rho_y - s(\sin \eta)]^2 + R^2 - \rho^2\}^{\frac{1}{2}} \\ &\quad + \{[\rho_x + s \cos \eta]^2 + [\rho_y + s(\sin \eta)]^2 + R^2 - \rho^2\}^{\frac{1}{2}} - 2\sqrt{R^2 + s^2} \\ &= \frac{[\rho_x s \cos \eta + \rho_y s(\sin \eta)]^2}{R^3} + \frac{3s^2}{2R^5} [\rho_x s \cos \eta + \rho_y s(\sin \eta)]^2 + \dots \end{aligned} \quad (5-35)$$

The dominant term is third order astigmatism. For most cases, the second term in the above equation is negligible and the intensity in the fringe plane (signal terms only) is

$$I_{\text{sig}} = \frac{1}{\pi s_0^2} \int_{A_s} \left[1 + \cos \frac{2\pi}{\lambda} \left\{ W - \frac{[\rho_x s \cos \eta + \rho_y s(\sin \eta)]^2}{R^3} \right\} \right] dA \quad (5-36)$$

where W is the test aberrations and A_s the area of the scatterplate.

The result is a series solution which appears in Figure 18. There is both a contrast reduction and more seriously, a resultant fringe shift. The expansion for the phase

$$\text{phase} = -\frac{1}{4} \left(\frac{2\pi s_0^2 \rho^2}{\lambda R^3} \right) + \frac{1}{384} \left(\frac{2\pi s_0^2 \rho^2}{\lambda R^3} \right)^3 + \dots \quad (5-37)$$

shows the dominant term in most cases of interest is defocus.

Effect of Background Scatter on Fringe Contrast

In Chapter 1, the contrast reduction due to the background intensity was given as

$$\gamma_B = \frac{1}{1 + \frac{I_{SS}}{2I_{SD}}}$$

Since the scattering intensity profile is not constant across the test surface, γ_B will also not be constant. The determination of I_{SS}/I_{SD} is carried out by an examination of eq. (5-22). I_{SD} is the envelope of the signal term which is

$$(\widetilde{AS_1})^2 \quad (5-38)$$

the square of the spatial frequency content of the scatterplate amplitude transmission. As explained earlier, the frequency content is determined by the autocorrelation of the ground glass aperture in the fabrication schematic in Figure 5. The envelope of I_{SD} then is the square of the standard MTF for an unaberrated circular aperture as

shown in Figure 34. The constant M_0 , the zero frequency intensity of the signal, is determined by calculating the power in the SD beam

$$\int I_{DS} dA_I = M_0 \int |MTF|^2 dA_I = (1-\alpha)\alpha \quad (5-39)$$

$$M_0 = \frac{2.77\alpha(1-\alpha)}{f_0^2} \quad (5-40)$$

where $\int |MTF|^2 dA_I = \left(\frac{\pi f_0^2}{4}\right)^2 \int r \text{besinc}^4(f_0 r) dr d\theta$.

The profile for I_{SS} is the envelope of the term

$$(AS_1^2)^2 = \frac{N_0}{n_0} (MTF**MTF)^2 \quad (5-41)$$

where N_0 is the value of I_{SS} at the zero frequency, n_0 is the normalization constant equal to $\frac{\pi^4}{64} f_0^4 (.585)^2$, and f_0 is the cutoff frequency of MTF which is graphed in Figure 35. The power in I_{SS} is

$$\frac{N_0}{n_0} \int (MTF**MTF) dA_I = \alpha^2 \quad (5-42)$$

$$N_0 = \frac{\alpha^2 (.585)^2}{f_0^2 (.307)} \quad (5-43)$$

The ratio I_{SS}/I_{SD} can now be written as

$$\frac{I_{SS}}{I_{SD}} = .4 \frac{\alpha}{(1-\alpha)} \left(\frac{MTF**MTF}{MTF} \right)^2 \frac{1}{n_0} \quad (5-44)$$

Figure 36 is a graph of the contrast reduction when eq. (5-43) is substituted into eq. (5-36).

Fig. 34. Normalized signal intensity profile.

Fig. 35. Normalized background intensity profile.

Fig. 36. Contrast reduction due to background scatter.

The x-axis is the relative distance from the hotspot while α is the various diffraction efficiencies.

The results from Figure 36 are valid only as long as the scattering into higher orders is negligible compared to the first. Higher order scattering has not been considered here since it would worsen the already cumbersome mathematics. For bleached photographic emulsion, scatter efficiencies of up to 65% are possible with scattering into the first order alone.

CHAPTER 6

NULL TESTING IN A MODIFIED SCATTERPLATE

In Chapters 1 and 2, the "slowly varying" PTF was shown to be directly registered into the fringe pattern as seen from eq. (3-15).

$$I_{\text{sig}} = 2S_o^2 M^2 [1 + \cos(kW + 2\phi)] \quad (3-15)$$

A natural progression from this result is to investigate the possibility of producing a PTF in the scatterplate which cancels the test aberration, W . Since a sinusoidal grating that is phase shifted by ϕ imparts a $+\phi$ to the $+1$ order and a $-\phi$ to the -1 order, PTF's are intrinsically anti-symmetric. However, most aberrations such as 4th order spherical are symmetric. This problem is circumvented by placing the hotspot on a separate reference mirror so that only positive spatial frequencies in the scatterplate are used for testing. This chapter describes techniques similar to those found in holographic interferometry, to perform a null test on a 20 cm diameter $f/3$ parabola at its nominal radius of curvature.

Fabrication Setup

Figure 37 illustrates an optical method for producing the desired amount of phase to cancel the spherical aberration introduced by the parabola. The setup is basically a Fourier transform system.

Fig. 37. Optical method for producing null scatterplates.

Fig. 38. Testing schematic for the modified scatterplate interferometer.

The amount of spherical aberration introduced into the system is controlled by changing the conjugate ratio of the first lens which in this case was a short focal length corrected doublet put in "backwards." The amplitude and phase across the pinhole mask is Fourier transformed and interfered with a collimated reference beam at the emulsion. The mask is identical to the one used in the single exposure technique discussed in Chapter 2.

The intensity in the film plate is given by

$$I_F = \left| b e^{i2\pi F_O X_F} + \widetilde{P e^{ikW}} \right|^2 \quad (6-1)$$

where

b = constant amplitude of the reference wave

F_O = carrier frequency of the reference wave

W = induced spherical aberration

P = pinhole mask transmission amplitude

$$\begin{aligned} &= b^2 + b e^{i2\pi F_O X_F} \left(\widetilde{P e^{ikW}} \right)^* + b e^{-i2\pi F_O X_F} \left(\widetilde{P e^{ikW}} \right) \\ &+ \left| \widetilde{P e^{ikW}} \right|^2 \end{aligned} \quad (6-2)$$

Assuming the film makes a linear recording of the above intensity, its transmission amplitude will be proportional to

$$\begin{aligned} S \propto & b^2 + b \left[\widetilde{e^{-ikW(x+x_O)} P(x+x_O)} \right] + b \left[\widetilde{e^{ikW(x-x_O)} P(x-x_O)} \right] \\ &+ \left| \widetilde{P e^{ikW}} \right|^2 \end{aligned} \quad (6-3)$$

where x_0 is the coordinate shift in the plane of P due to the carrier frequency. If W generated in Figure 37 is one half that of the test aberration and of opposite sign, the third term in eq. (6-3) possesses the correct PTF to cancel that test aberration.

As suggested by Burch, the scatterplate can be considered as a special case of a holographic interferometer where a "stored" wavefront is interfered with a conjugate test wavefront in the "playback" stage. The playback schematic as seen in Figure 38 is another Fourier transforming system except that due to the double passage of light through S, a Fourier transform is performed on both the test and reference beams.

Figures 39a and 39b are interferograms of the parabola tested at its nominal radius with and without cancellation. Note the absence of the hotspot in Figure 39b since it is no longer being tested common path. Also if F_0 is large enough, an added benefit of this off-axis configuration is that the background intensity does not reduce the contrast of the signal since it is biased at twice the carrier frequency.

The limiting f/no. of the scattered light is given by the focal length of the second transforming lens and the size of the pinhole mask.

$$f/\text{no.} = \frac{f_T}{D_p} \quad (6-4)$$

where f_T is the focal length of the second transform lens and D_p is the diameter of the pinhole mask.

(a)

Fig. 39. Interferogram of a parabolic mirror at its nominal radius of curvature.

(a) Without null correction.

(b)

Fig. 39. Continued.

(b) With null correction.

Alignment of the Fabrication System

Although only a single exposure is necessary in the fabrication, the difficulties in its alignment as compared to the double exposure method have increased significantly. Examination of eq. (6-3) shows the non-linear phase errors introduced in misalignment can be of two distinct types: those that appear "inside" and "outside" of the Fourier transform in the third term. A detailed mathematical analysis will not be given here, but it will suffice to say that in general non-linear aberrations outside the brackets reduce the fringe contrast in the interferometer while those within the brackets bend the fringes.

Sources of the first type of phase error are:

1. Aberrations present in the reference beam.
2. Misalignment of the second transform lens.
3. Pinhole mask and film emulsion not placed precisely at the front and rear focal points (respectively) of the second transform lens.

The first misalignment is not difficult to remove if the standard 50 mm corrected doublets are used as a collimating lens. The reference beam need only be 20 mm in diameter (the size of the film emulsion) so that the stopped down collimating lens can be aligned adequately with a lateral shear plate. The next alignment error proves to be a little more involved and will be discussed shortly. The pinhole mask is aligned reasonably accurately by reflecting the reference beam

back through the second transform lens. By centering this focused light onto a single pinhole in the mask, the correct longitudinal position can be determined just as in aligning a spatial filter assembly. The emulsion position is obtained by placing a weak scatterer in the film holder and observing the speckle pattern generated by the focused illumination leaving the transforming system. A weak scatterer is used because it generates a speckled interference pattern as seen in Figures 40a and 40b. Since the beam has spherical aberration, the paraxial focus position is determined when the speckle in the center of the pattern is the largest as in Figure 40a. Those familiar with the Smartt interferometer will recognize some similarities here since Figures 40a and 40b are a coherent superposition of point diffraction patterns.

Alignment of the total system has to be done in the presence of many waves of spherical aberration. The required amount of correction for the parabola in Figures 39a and 39b is approximately 15 waves at $.5145\mu\text{m}$. Any off-axis aberrations introduced by the aberration generator and first transform lens are directly registered in the fringe pattern when the plate is used in the interferometer.

The star test was found to be very sensitive in detecting the presence of small amounts of off-axis aberration if large amounts of defocus are added to the pattern. The "Maltese Cross" pattern appears in the central portion of a star image with small amounts of astigmatism as shown in Figures 41a and 41b. The order of the lens alignment goes

(a)

Fig. 40. Interference pattern using a weak diffuser in the film emulsion plane in Fig. 37.

(a) The diffuser is placed at the paraxial focus of the undiffracted beam.

(b)

Fig. 40. Continued.

(b) The diffuser is placed at mid focus of the undiffracted beam.

(a)

Fig. 41. Star test patterns in the presence of large amounts of defocus and spherical aberration.

(a) No astigmatism is present.

(b)

Fig. 41. Continued.

(b) The Maltese Cross pattern in the center indicates a small amount of astigmatism present.

as follows:

1. The first lens (aberration generator) is aligned with the star test.
2. The second transform lens is added and the two lenses are star tested together.
3. The final star test is done at the rear focal point of the second transform lens after the first transform lens is added.

A lateral shearing plate is used only to check the collimation of the first transform lens.

Interferometer Alignment

Not surprisingly, the alignment of the scatterplate in Figure 38 also proves itself more involved than the conventional interferometer of Figure 1. The scattering field of these plates instead of being a single cone of light centered about the hotspot, now consists of two distinct cones of light separated by an angular distance of $F_0\lambda$ on either side of the hotspot as can be deduced from eq. (6-3). As it might be expected, the phases across each of these cones are conjugate to one another so that the "right" one (after some trial and error) has to be chosen for cancellation.

The holographic and test wavefronts should be centered with respect to one another to eliminate the coma due to the lateral shearing of the two spherically aberrated wavefronts. Micrometer tilt controls on the beamsplitter mounting are a useful aid to centering the

scattered cone on the mirror. The direction in which to tilt the beam-splitter can be found by removing the tilt and defocus in the interferograms as done in Figure 23. The direction of beam tilt is then reduced to one of two ways indicated in the Figure.

The contrast of the fringes can also be reduced because of non-isoplanatism as found in the conventional scatterplate interferometer.

The testing scheme shown in Figure 38 can be thought of as a section of a large very fast $f/\text{no.}$ mirror. The phase shifts in eq. (5-35) can become very large. If the test mirror and hotspot are separated horizontally, the contrast can be improved by masking the scatterplate with a vertical slit centered on the center of flip symmetry. This eliminates those points in the scatterplate which create the largest phase errors.

Scaling of the Correction with Wavelength

The manner in which the modified scatterplate cancels test aberrations differs from the holographic null corrector used in conjunction with a Twyman-Green interferometer (MacGovern and Wyant 1971). The holographic element in this scheme is essentially a fringe pattern of the desired wavefront that is Moiré interfered with the test fringe pattern. As such, the number of waves of correction is independent of the wavelength used in the interferometer.

In the modified scatterplate, the number of waves of correction across the cone of scattered light is also independent of wavelength

but the diameter of the cone is directly proportional to the wavelength. The correction in this case scales according to the type of aberration. For example, the null test depicted in Figures 39a and 39b were done at $.5145\mu\text{m}$ with approximately 30 waves of 4th order spherical aberration correction. Assuming the cone of scattered light just fills the mirror diameter for $\lambda = .5145\mu\text{m}$, at $\lambda = .6328\mu\text{m}$ the cone diameter will be scaled up by the factor

$$C_o = \frac{.6328}{.5145} = 1.23 \quad (6-5)$$

The amount of correction across the same mirror at $.6328$ is $30/C_o^4 \approx 13$ waves. Meanwhile the mirror aberration is directly scaled with wavelength to approximately 24 waves. The "null" test at this wavelength will show 11 waves of under correction. The Twyman-Green holographic test will result in $30-30/C_o \approx 6$ waves of over-correction with the same wavelength range.

CHAPTER 7

AN INFRARED SCATTERPLATE INTERFEROMETER

The advantages of testing at infrared wavelengths ($\lambda = 10.6\mu\text{m}$) are similar to those found in other long wavelength interferometers (Kwon 1980). Both large aspherics as well as unpolished surfaces can be tested directly. In addition to the description of the fabrication and testing of the IR scatterplate, this paper enumerates some interesting effects a pyroelectric vidicon camera (PEV) has on detecting the fringe pattern of this interferometer.

Fabrication

The scatterplates for the infrared are chemically etched silicon wafers. Although silicon has a slightly smaller index of refraction (~ 3.5 at $\lambda = 10.6\mu\text{m}$ than germanium, ~ 4.0) it etches with acids that do not attack the resist mask as readily. Other IR materials such as barium fluoride have such a low index of refraction that the etch depths become large enough to make undercutting of the resist mask a problem.

The standard procedure for etching crystalline silicon is outlined in the following steps:

1. Place the polished samples of silicon into an oxygen atmosphere overnight.
2. Spin coat a layer of photoresist a few microns thick.

3. Expose the resist through a photographic film mask and process with the photoresist developer. The exposure and developing times should such that the structure "bottoms out" on the SiO_2 substrate.

4. Etch through the SiO_2 layer with a 1:5 mixture of Hydrofluoric (HF) acid (48% concentration) to H_2O .

5. Remove the photoresist and etch through the SiO_2 mask with a 20:1 mixture of concentrated acid to 48% concentrated HF acid. The etch rates in this case are usually in the neighborhood of $1\mu\text{m}/\text{min}$.

The SiO_2 layer is used as a secondary mask because nitric acid attacks photoresist. In this case however, it is possible to eliminate the use of the SiO_2 mask by etching directly through the photoresist mask with a 4:1 mixture of nitric to HF acid for 15-20 seconds. In most cases the required etch depths (about $2.5\mu\text{m}$) were achieved before the resist was completely removed by the acids. Kodak 747 negative resist was found to be more durable for this use than the Shipley AZ-1350 positive resist.

The surface height profiles after etching are approximately binary as can be seen in the Talystep scans in Figure 42. The Talystep measures surface profiles directly through a very fine tipped mechanical probe. Since a phase step of multiples of 2π diffracts no light, the scatter efficiency of a binary profile is periodic with the peak to peak etch depth (Brooks 1976),

Fig. 42. Talystep scan of acid-etched silicon scatterplates for the infrared.--Each vertical division equals $.5\mu\text{m}$.

$$\alpha = \sin^2 \sigma_p \quad (7-1)$$

where σ_p is the rms phase variations of the binary profile. The relationship between σ_p and h depends on the duty cycle of the surface height profile. If there are as many "peaks as valleys" (i.e. a full duty cycle)

$$\sigma_p = \frac{\pi}{\lambda} (n-1)h \quad (7-2)$$

It was found through numerous Talystep scans, that something less than a full duty cycle is what actually occurs. This means σ_p is some fraction of the value given in the above equation depending on the degree to which the photoresist "bottoms out" on the silicon substrate after development. In most cases an etch depth of 1.5 μm gives a scatter efficiency in the range of 75%.

It is important to have a fairly high scatter efficiency since an intense hotspot can easily damage the pyroelectric vidicon (PEV), the infrared detector used in this experiment. As the next section describes, if only the signal intensity is modulated, the reduction in fringe contrast due to high background intensities is negligible since the PEV is an AC detector.

Testing

Assuming a familiarity with the techniques of alignment in the infrared which are enumerated in the previous references, the alignment of the test setup follows closely with the visible wavelength

version. Care should be taken to prevent the specular back reflection off the scatterplate from reaching the PEV by tilting the scatterplate slightly with respect to the incoming beam. The 30% Fresnel reflection coefficient of silicon is enough to damage the detector at the laser power levels (~ 3 W) needed to observe the signal.

The alignment of the scatterplate is done through trial and error. This process is less time consuming if the following steps are taken beforehand.

1. With the scatterplate removed, the incoming beam is focused on the mirror surface.

2. The test mirror is then tilted so the reflected and incoming beams are superimposed.

3. Placing the scatterplate slightly away from paraxial focus and centered on the incoming beam and with a usually minimal amount of searching with the tilt controls, fringes can be found. The scatterplate should be made so that the center of symmetry (which can be found with unaided eye since the structure is large enough) is in the middle of the plate.

Modulation of the Fringe Pattern Intensity

In other IR interferometers, signal modulation is achieved by chopping the incoming beam or panning the PEV camera. Unfortunately, these methods do not work as well with the scatterplate since the hot-spot and background intensities are also being modulated. In this

case, a modulated hotspot causes a significant portion of the interferogram to be blacked out due to the finite cooling time necessary in the PEV retina. A modulated background, on the other hand, reduces the fringe contrast in the same manner as discussed in the visible wavelength scatterplate. With the high scatter efficiencies necessary here, this contrast reduction is significant. It is interesting to note a subtle difference in the fringe positions on the TV monitor compared with those on the PEV when the signal is chopped or panned. With the first method, the TV fringe pattern is either in phase or 180° out of phase with the PEV fringes depending on what part of the chopping cycle is taking place. The maximum temperature change on the retina occurs on the peaks and troughs of the fringe intensity incident on it. In the panned mode however, bright fringes on the TV monitor correspond to the maximum slope of the intensity on the PEV. The TV fringes are shifted a quarter cycle ahead of the PEV fringes in the direction of panning motion since they are the derivative of the intensity pattern of the PEV camera.

The only alternative method of signal modulation is through x-y tilt motions on the interferometer. Although there is a perfect fringe contrast since only the signal intensity is modulated, the quarter cycle shift explained earlier produces a somewhat annoying artifact in the interferograms as shown in Figure 43. In this figure, a set of fringes vertically oriented is being modulated by tilt motions in the vertical direction. The abrupt contrast reversal can

Fig. 43. IR interferogram using a pyroelectric vidicon camera (PEV).--The modulation of the signal is achieved by adding vertical tilt. This is the same mirror tested in the visible in Fig. 4.

be understood by illustrating how the fringes on the PEV are moving as shown in Figure 44. The solid and dashed sets of lines show two separate positions of these fringes under vertical tilt motions. They appear to pivot about a horizontal line passing through the hotspot. The dotted lines indicate the maximum change in intensity for a single fringe. The contrast reversal appears because a given fringe moves in opposite directions about the pivot point. At the pivot points, no signal is modulated hence the zero contrast band modulating the fringes with a horizontal tilt does not alleviate the problem as shown in Figure 45. With this motion the fringes appear to expand and contract away from a vertical line passing through the hotspot. The TV fringes will show a half cycle to be missing since the maximum intensity slopes are a quarter cycle on either side of this vertical line.

In principle it might seem possible to eliminate the contrast reversal band by defocus movements on the interferometer. However, excessively large movements are necessary before any kind of modulation is achieved.

It was found that simultaneous x-y tilt motions seemed to eliminate the band in some of the TV frames as shown in Figure 46. This result appears to contradict what was discussed about the fringe motion in a scatterplate. The center bright fringe through the hotspot always pivots about the hotspot no matter how tilt is added so there should always be a contrast reversal along it. With rapid and somewhat jerky tilt movement, the PEV is somehow averaging to produce what

Fig. 44. Drawing showing the movement of vertically oriented fringes with vertical tilt being added.--The solid and dashed lines show the fringe positions for two different amounts of vertical tilt.

Fig. 45. IR interferogram with signal modulation achieved by horizontal tilt movements.

Fig. 46. IR interferogram with signal modulation achieved by simultaneous X-Y tilt movements.

appears to be a continuous fringe. The effect is not fully understood at this point and would require a further investigation of the PEV operation. As the next section will discuss this contrast reversal band is not as troublesome as it might seem if the interferogram is analyzed properly.

Interferogram Reduction

For a computer analysis which requires manual digitization of the interferogram, the fringe pattern in Figure 43 can be handled easily as long as fringe centers are plotted along bright fringes in one half and corresponding dark fringes in the other. Such reductions were performed and compared favorably with an interferogram analysis of the same mirror in the visible to within $\lambda/5$ ($\lambda = 10.6\mu\text{m}$). Other than the digitization process itself, one source of error arises from a characteristic of the PEV which is a distortion of the signal in the neighborhood of an intensity hotspot (Rohler and Weiss 1978).

Assuming the hotspot is in the center of the test aperture, the interferogram in Figure 43 might be thought of as one due to a parabola with a $\lambda/2$ thin film over half the aperture. A polynomial fit with fringe centers plotted along dark fringe everywhere might be expected to affect only the odd orders of aberration. This was found to be the case with the FRINGE analysis program used at the Optical Sciences Center. The spherical aberration found with this method of digitization compared favorably with the same mirror tested in the visible.

APPENDIX A

LIST OF VARIABLES

<u>Variable</u>	<u>Eq.</u>	<u>Page</u>
S - scatterplate amplitude transmission	2-1	3
$S^+ - S(x_1, y_1)$	3-1	47
$S^- - S(-x, -y) \ x_1, y_1$. Coordinates in the plane of the scatterplate.	3-1	47
$S(0,0)$ - center of flip symmetry	2-1	3
S_0 - average value of S (general form)	3-7	51
S_1 - variations of S about mean (general form)	3-7	51
t_0 - average transmission for "density-type" scatterplate	5-28	88
t_1 - form for S, density type scatterplate	5-28	88
$i \frac{2\pi}{\lambda} (n-1)h$ - form for S, phase type scatterplate	3-12	52
A - scatterplate aperture function (binary)	3-7	50
α - diffraction efficiency of scatterplate	5-39	94
B - ratio of diffraction efficiencies of the separate exposures during fabrication	2-4	25
a_{020} - defocus present across scatterplate when hotspot is not focused on the mirror	5-5	74
$\hat{\beta}$ - random variable representing the distribution in phase due to shift version of the defocused hotspot in the image plane	5-8	76

<u>Variable</u>	<u>Eq.</u>	<u>Page</u>
D_r - separation of entrance and exit cones of light across the scatterplate for measurement of rotational error present	2-7	33
γ_B - fringe contrast in the presence of the background intensity	2-2	9
γ_e - fringe contrast in the presence of exposure error between the two exposures in the fabrication	2-4	25
γ_r - fringe contrast in the presence of rotational error between the two exposures during fabrication	2-5	25
A_I - area in the image plane	5-9	76
A_S - area in the scatterplate plane	5-25	88
e_o - size of the extended source in image plane	5-18	82
f_I - focal length of image lens	3-4	49
F_o - carrier frequency in the null fabrication setup	6-1	101
$f\#$ - scattering f/no.	2-5	25
f_T - focal length of second transform lens in null fabrication setup	6-4	102
D_p - diameter of pinhole mask in null fabrication setup	6-4	102
δ - Dirac delta function, $\delta(x,y)$	3-8	51

<u>Variable</u>	<u>Eq.</u>	<u>Page</u>
M_0 - zero frequency intensity of the signal	5-40	94
N_0 - zero frequency intensity of the background	5-41	94
M, ϕ - resultant amplitude and phase of a random phasor sum	3-14	53
$ S_1 , \psi$ - component amplitude and phase contributions in a random phasor sum	3-11	52
I_{SD} - intensity of scattered direct beam	2-2	9
I_{SS} - intensity of scattered, scattered beam	2-2	9
I_f - intensity in the fringe plane	3-6	50
I_{sig} - signal intensity	3-9	51
θ_E - rotational error in 180° angle	2-5	25
θ_F - fabrication scattering angle	2-3	11
ϕ - scatterplate phase transfer function	3-14	53
N_H - no. of fringes across the defocus hotspot	5-12	79
r_O - radius of scatterplate aperture Airy pattern	4-1	57
R - effective focal length of test optics	-	28
p_x, p_y - mirror aperture coordinates	Fig. 17	35
s_O - radius of scatterplate aperture	2-5	25
W - aberration function	3-3	49
$\hat{W}$ - random variable which represents the distribution in phase due to shift versions of W	5-9	76
X, Y - real and imaginary parts of a phasor sum	4-2, 4-3	59
σ_x, σ_y - variances of the real and imaginary parts of phasor sum	4-6, 4-7	60

<u>Variable</u>	<u>Eq.</u>	<u>Page</u>
Z_I - distance between image lens and image plane	3-5	50

REFERENCES

- Beckmann, P. and A. Spizzichino, The Scattering of Electromagnetic Waves from Rough Surfaces, Pergamon Press, Oxford, England, 1963, pp 119-151.
- Bestenreiner, F., R. Dem, and V. Friess, Photo. Science Eng. 14, 1 (1970).
- Brooks, L. D., "Construction and Calibration of Phase Perturbation Plates in Photoresist," M.S. Thesis, Optical Sciences Center, University of Arizona, 1976.
- Burch, J. M., "Scatter Fringes of Equal Thickness," Nature (Lond.), Vol. 171, p 889, (1953).
- Burch, J. M., "Interferometry in Scattered Light," in Optical Instruments and Techniques, J.H. Dickson (ed.), Oriel Press, England, 1970, p 220.
- DeWitte, A. J., "Interference in Scattered Light," Am. J. Phys., Vol. 35, p 301, (1967).
- Gaskill, J. D., Linear Systems, Fourier Transforms, and Optics, Wiley-Interscience, New York, 1978, p 216.
- Goodman, J. W., "Statistical Properties of Laser Speckle Patterns," in Laser Speckle, J.C. Dainty (ed.), Springer Verlag, New York, (1975).
- Houston, J. B., "How to Make and Use a Scatterplate Interferometer," Opt. Spectra 4(6), 32, (1970).
- Hoyt, R. S., "Probability Functions for the Modulus and Angle of the Normal Complex Variate," Bell System Tech. J. 26, pp 318-359, (1947).
- Kwon, O., et al., "Rough Surface Interferometry at 10.6 μ m," Appl. Opt. 19, (1980).
- MacGovern, A. J. and J. C. Wyant, "Computer Generated Holograms for Testing Optical Elements," Appl. Opt. 619, (1971).

- Murty, M. V. R. K., "Common Path Interferometer Using Fresnel Zone Plates," JOSA, 53, 568, (1963).
- Papoulis, A., Probability, Random Variables, and Stochastic Processes, McGraw-Hill, New York, 1965, p 266.
- Rohler, D. R. and P. B. Weiss, "Use of Pyroelectric Vidicons for Measurements on CO₂ Laser Systems," presented at the Inertial Confinement Fusion Conference at San Diego, California, February 1978.
- Rubin, L. F., "Null Testing in a Modified Scatterplate Interferometer," in press in Applied Optics.
- Rubin, L. F. and J. C. Wyant, "Energy Distribution in a Scatterplate Interferometer," J. Opt. Soc. Am., Vol. 69, No. 9, 1305, (1979).
- Schoemaker, A. H. and M. V. R. K. Murty, "Some Further Aspects of Scatter Fringe Interferometry," Appl. Opt. 5, 603, (1966).
- Scott, R. M., "Scatter Plate Interferometry," Appl. Opt. 8, 531, (1969).
- Smartt, R. N. and J. Strong, "Point diffraction interferometer," JOSA, 62, 737, (1972).