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MAXIMIZING CUSTOMER SATISFACTION BY OPTIMAL
SPECIFICATION OF ENGINEERING CHARACTERISTICS

by

Donald Wight Dawson

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF SYSTEMS AND INDUSTRIAL ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

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of Engineering Characteristics

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SIGNED: *Donald W. Dawson*

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ABSTRACT

The House of Quality (HOQ) has been widely discussed as a mechanism for capturing the Voice of the Customer and guiding the process of converting the customer's voice into internal engineering specifications. However, the discussion of this process to date has tended to be qualitative rather than quantitative. Various heuristic practices have evolved with little substantiation of their value or evaluation of their impact other than illustration in a few case studies. Examples of practices promoted by several authors and practitioners include the inclusion of the relative magnitude of effects, but not direction, in the body of the matrix; the restriction of relationships to an integer scale of subjective, ratio values; and allocating available engineering resources to the hard-to-improve characteristics (because the easy ones will presumably take care of themselves).

The process for setting target specifications given the HOQ data has not been well defined either. Design teams are required to develop their own ad hoc rules. When multiple engineering characteristics affect a single customer attribute or when an engineering characteristic impacts multiple customer attributes, the design problem becomes difficult and current procedures are of little use to the design team. This is particularly true when some interactions are negative. The result may be lost opportunity from the selection of non-optimal designs.

This research takes a scientific approach to optimally specifying the target values for engineering characteristics in a competitive marketplace. A methodology for building linear or nonlinear statistical models of customer preference (value) for each customer attribute followed by an optimization routine that will obtain the optimal specification of

engineering characteristics is proposed. The optimization model considers economic and technological constraints as well as customer preferences. The process results in optimal, feasible design specifications. In addition the effect of variability in parameter estimates on the ability to correctly identify the key engineering characteristics for optimizing customer value is investigated. Model performance is also analyzed through random generation of 320 problem instances of varying linearity, size, density, monotonicity and correlation. Finally, the methodology is applied to a problem from industry and the results are compared with an actual, subjectively-derived design.

Chapter 1

Introduction

It is better never to begin a good work than, having begun it, to stop [5].

1.1 Background

Positive revenue flow is the principal motivation for most organizations. All other organizational goals – survival, security or expansion – are dependent upon the organization's profits. Most organizations also have competition, which is the primary threat to revenues. Considerations such as decreasing costs and reducing cycle time are important to market competitiveness. Successful organizations continually change to cope with this threat. Every decision an organization makes (or does not make) in support of evolution usually affects its competitiveness, for better or worse.

Where product planning and development decisions are concerned, the use of Quality Function Deployment (QFD) has gained extensive international support [8]. QFD is a structured methodology for specifying the customer's desires and evaluating the impact of alternative products on those desires. The fundamental tenet of QFD is that increasing customer satisfaction should be *THE* goal. When organizations direct their efforts toward meeting the customer's needs, conflict between competing objectives (increasing revenues, decreasing costs and reducing cycle time) is minimized. By reducing midcourse changes and implementation errors, QFD can decrease both cycle time and costs. Furthermore, its contribution to the production of goods or services which are more attractive to the customer helps to increase revenues [9].

1.2 Problem Statement

To date, practitioners have primarily used QFD to focus on the customer subjectively. However, QFD's objective is to achieve the highest level of customer contentment subject to organizational constraints on cost, time and other resources. Therefore, the problem is to (i) empirically derive and analyze the customer's preferences and (ii) determine the mix and levels of engineering characteristics (ECs) which maximize customer satisfaction. These EC values must also balance development and production constraints with market competitiveness.

1.3 Research Contributions

Quantitative modeling procedures will be presented for choosing optimal engineering characteristic values. Because tradeoffs among market competitiveness (functionality), development costs and time ultimately determine a product's viability, customer satisfaction must be considered over the product's entire life-cycle. Figure 1.1 details the various facets of a proposed methodology for efficiently maximizing customer satisfaction in a constrained environment. Specifically, this dissertation addresses the following issues:

a. Voice of the Customer.

(1) Determination of the relative importance and preferential independence for the Customer Attributes (CAs).

(2) Collection of the Customer's Perceptions (or Assessments) by way of tailored experimental designs.

(3) Quantification of the significant CA/EC relationships into suitable value functions (regression analysis).

Figure 1.1 Research Objectives

b. Interaction among Engineering Characteristics

(1) Transformation of contradictory and complimentary CA/EC relationships to a practicable design space.

(2) Translation of dependent cost and time functions into applicable constraints.

c. Maximizing Customer Satisfaction.

- (1) Optimization of customer value via a linear mathematical program.
- (2) Enhancement of the model to account for quadratic terms.
- (3) Sensitivity of the model to linearization of non-linear constraints.
- (4) Validation of the objective function (with regard to consistency of matrix values).
- (5) Validation of the methodology by application to industrial data.

1.4 Organization of Dissertation

This chapter has provided a brief description of the problem under study along with the motivation for investigating these issues further. Chapter 2 describes the mechanics of QFD and current research efforts in the area of satisfying the customer. Chapter 3 presents a basic methodology with mathematical program for quantifying the voice of the customer and maximizing satisfaction. Chapter 4 expands this methodology to accommodate nonlinear functions and dependency among the engineering characteristics. Chapter 5 evaluates the performance of the mathematical program and compares its performance to that of an accepted heuristic. Chapter 6 validates the methodology through application to an actual problem from industry and Chapter 7 suggests potential directions for future research.

Chapter 2

Literature Review

Of a good beginning cometh a good end [21].

2.1 Introduction

The overwhelming majority of QFD-related research reflects the subjective mindset of most practitioners. This emphasis on qualitative (rather than quantitative) issues is reflected by Clausing and Hauser in their justification of QFD:

A set of planning and communication routines, quality function deployment focuses and coordinates skills within an organization (QFD) is a kind of conceptual map that provides the means for interfunctional planning and communications. People with different problems and responsibilities can thrash out design priorities while referring to patterns of evidence [8].

Recently, certain objective measures have been applied to QFD in an attempt to introduce quantitative rigor to the methodology. While such attempts are in the minority of research efforts, they appear to offer valid principles for successful implementation of QFD.

2.2 Decision Analysis

Important decisions often involve conflicting objectives. Seldom does an option exist that is best for all interests. Hard decisions therefore represent tradeoffs by the decision maker. These tradeoffs represent appraisals of both the relative desirability of the available options for each dimension of the decision and the relative importance of those dimensions [28]. Most tradeoffs are further complicated by the dichotomy of

decisions (Figure 2.1). Can the outcome of a decision be described by one attribute? The presence of multiple factors is the norm with hard decisions. Is uncertainty involved? If so, does a definitive distribution of the possible outcomes exist?

<i>Single Attribute w / Certainty (x)</i>	<i>Multiple Attributes w / Certainty (x)</i>
<i>Single Attribute w / Uncertainty ($\tilde{x}$)</i>	<i>Multiple Attributes w / Uncertainty ($\tilde{x}$)</i>

Figure 2.1 Dichotomy of Decisions (Single Decision Maker) [28]

The purpose of decision analysis is to structure and simplify the task of making hard decisions. The paradigm of decision analysis can be summarized as a sequence of five subanalyses:

Preanalysis $\Rightarrow$ Structural $\Rightarrow$ Uncertainty $\Rightarrow$ Utility/Value $\Rightarrow$ Optimization

Various techniques from multiattribute utility theory (MAUT) exist to aid the decision maker at each step of the process. Almost all are based upon the maximization of expected utility. While the topic of utility theory is not the focus of this dissertation, it is an integral component of the research effort. Therefore, a brief discussion of the decision paradigm and the basic fundamentals of utility theory follows.

2.2.1 The Decision Paradigm

Preanalysis: Given indecision about which course of action to take, the decision

maker identifies viable alternatives. Each alternative usually involves decisions that can be made now, decisions that can be deferred and decisions that require additional information. *Structural*: Each alternative is divided into those choices over which the decision maker has control (decision nodes) and events subject to external influence (chance nodes). Decision trees typically model these nodes as a sequence of “If” → “Then” → “If” → ... → “Then” decisions. *Uncertainty*: The decision maker assigns probabilities to the possible outcomes for each chance event. *Utility/Value*: The decision maker assigns relative worth to the chance events’ possible outcomes. This is normally accomplished through the use of a utility or value function. Utility functions model the cardinal value a decision maker places on an event’s various outcomes. Value functions model the ordinal values of the decision maker. The next section further elaborates on the differences between the two types of functions. *Optimization*: The decision maker calculates the optimal course of action by choosing that sequence of events which maximizes expected (or some other relevant function of) utility. Dynamic programming algorithms are frequently used to avoid enumeration of all possible sequences.

2.2.2 Fundamentals of Utility Theory

Utility theory is concerned with the problem of value tradeoffs. The decision maker is faced with the dilemma of trading off the achievement of one objective against one or more other objectives. If there is no uncertainty involved and one knows the outcome of each alternative, then the problem reduces to how much achievement of one

objective the decision maker is willing to give up to improve achievement on the other objectives. If uncertainty is involved, the tradeoff dilemma remains, but the problem is more complex because it is unclear what the outcome of each alternative is.

Let $(x_1, x_2, \dots, x_n)$ & $(x'_1, x'_2, \dots, x'_n)$ represent different outcomes of an event with n attributes. Assume more (larger x_i) is better for each attribute i . Dominance is defined as

$$x_i \geq x'_i \quad \forall i \quad (2.1a)$$

and

$$x_i > x'_i \quad \text{for at least one } i \quad (2.1b)$$

The region of dominance is called the efficient frontier or Pareto optimal set. The concept of dominance is only concerned with the ordinal dimension of the outcome $(x_1, x_2, \dots, x_n)$. For example, let $x_i = 8$ and $x'_i = 4$. Then $x_i > x'_i$. Conversely, the cardinal dimension of the outcome accounts for the fact that x_i is twice as large as x'_i or the difference between $(x_i = 8, x'_i = 4)$ is greater than the difference between $(x_i = 7, x'_i = 5)$.

Let v represent a function which associates a real number $v(x)$ to each x in the outcome space. v is defined as a value function (or an ordinal utility function) representing the decision maker's preference structure if and only if

$$x \cong x' \Leftrightarrow v(x) = v(x') \quad (2.2a)$$

and

$$x \succ x' \Leftrightarrow v(x) > v(x') \quad (2.2b)$$

where the symbols $\cong$ and $\succ$ mean *indifferent to* and *preferred to*, respectively.

Consequently, knowledge of the decision maker's value function uniquely specifies his entire preference structure.

The decision maker's problem is to choose $(x_1, x_2, \dots, x_n)$ such that

$$v(x_1, x_2, \dots, x_n) \geq v(x'_1, x'_2, \dots, x'_n) \quad \text{for all } (x'_1, x'_2, \dots, x'_n) \quad (2.3)$$

where v is a value function. If there are multiple outcomes for an event, the decision maker's problem is to choose $(x_1, x_2, \dots, x_n)$ so that v is maximized. Assuming v is a continuous function and the set of all possible outcomes is compact, then a maximal outcome exists.

Theoretically, value functions assume riskless events; there is certainty about the probability of each outcome ($p = 1$). Utility functions address the case where uncertainty exists for some or each outcome ($p < 1$). The fundamental assertion of utility theory is that the expected value of the probability distribution for a set of outcomes may be used to numerically transform $(x_1, x_2, \dots, x_n)$ into cardinal values. The resulting utility function $u(x)$ accounts for both the decision maker's preference structure and his aversion to risk. Many utility theorists take the distinction between value and utility functions seriously. Others feel that gamble-based elicitation provide nothing more than a consistency check of the value function.

Our conclusion . . . is that for theoretical, psychological, and practical reasons the distinctions between utility and value are spurious because (1) there are no sure things, and therefore values that are attached to presumably 'riskless' outcomes are in fact attached to gambles; (2) risk aversion can frequently be explained by marginally decreasing value functions and/or by regret attributes of a value function; (3) repetitive elicitation tend to eliminate risk aversion; (4) error and method variance

within value and utility measurement procedures overshadow to a great extent the subtle distinctions that one may extract from the theoretical differences [53].

The issue is largely irrelevant to this research. Keeney and Raiffa [28] show that a value function represents the degenerative case of a utility function when $u(x) \geq u(x')$. Without loss of generality, one can assume such dominance in the application of utility theory to QFD.

Numerous studies have examined the differences between subjective (value elicitation) and objective (policy capturing) approaches to determining weights in additive utility models. Some have argued that it matters little which procedure is used, whereas others strongly favor one over the other. Objective approaches tend to be statistical in nature and often suffer from small sample size and correlation among the predictor variables. Subjective techniques are usually based upon decomposed judgments, which while simpler are often unrepresentative. Edwards and Barron [16] favor subjective processes only if the subject's value function is strictly monotonic. Srivastava et al. [47] and Shoemaker and Waid [46] conducted separate experiments to determine the most accurate method of assessing the weights an individual implicitly applies to multiple attributes. Taken together, the results create as many questions as they answer. Shoemaker and Waid found that, on average, subjective and objective approaches predicted about equally well. Furthermore, nonlinear models were found to be inferior to linear models. Srivastava et al. tentatively concluded that a subjective technique such as SMART offers the best combination of predictive performance and ease of elicitation.

They speculate that humans are capable of discriminating between the importance of different attributes only up to rank order. Measurement beyond that level adds nothing meaningful to the quantification of preference. Edwards and Barron offer the most relevant observation, however.

The most important goal of decision analysis is insight, not numerical treatment . . . nothing that can be done with multiattribute utilities after they have been elicited is nearly as valuable as the insights that sometimes emerge during the elicitation [16].

2.3 Quality Function Deployment

While the United States was preoccupied with the Vietnam conflict in the 1960s, Japan quietly became the world's most efficient producer of steel. How an island with virtually no natural resources was able to import materials and convert them into quality products more economically than any other industrialized nation was the genesis for what is today called Quality Function Deployment (QFD).

Japan's strategic industrialization specifically began with the production of supertanker cargo ships [20]. Most were originally built by Mitsubishi in the shipyards of Kobe and were a logistical nightmare. Unlike automobiles, each order was for a single ship with unique and specific requirements by the customer. Mitsubishi turned to the Japanese government for assistance in developing a process whereby each stage of construction would be linked to fulfilling the customer's requirements. The rest, as "they" say, is history.

Toyota adopted Mitsubishi's approach in the early 1970s in response to public

perception of their automobiles. Customers thought the vehicles were cheap and rusted more quickly than Volkswagens. Toyota initially utilized QFD to redesign pickup truck doors. Ultimately, the process was successfully extended to all Toyota designs (Figure 2.2) and other Japanese companies quickly followed suit.

Figure 2.2 Impact of QFD at Toyota [8]

The actual name for the methodology developed is *hin shitsu* (features, attributes or qualities) *ki nou* (function or mechanization) *ten kai* (evolution, diffusion or deployment). QFD could just as easily be called Feature Mechanization Diffusion; regardless of the acronym, much is lost in the translation [20]. The Japanese developed a procedure for determining exactly what their customers wanted and a rational system for deciding how to satisfy those needs with constrained resources. In other words, QFD determines what qualities the customer desires, what function(s) the product serves and how best to produce what the customer wants with the available resources.

American companies first became interested in QFD during the quality movement of the 1980s. Unfortunately, the process was often adopted for the sole purpose of producing goods at reduced cost – not to satisfy customers' desires (Figure 2.3).

Figure 2.3 QFD Paradigm Shift [8]

As U.S. industry gradually came to the realization that customer satisfaction defines success, QFD has gained extensive support with a wide range of applications. The accompanying research can generally be separated into four broad categories: these categories also accurately describe the phased activities of QFD. Specifically, they are (1) planning for QFD implementation, (2) gathering/translating the voice of the customer, (3) building the House of Quality and (4) analyzing/interpreting the results.

2.3.1 Planning for Implementation

Most of the literature in this area deals with approaches for deciding on the objectives, customer, time horizon, product concept and/or team. Fowler [18] discusses

the problems associated with implementing QFD for the first time. Specifically, he describes what projects have the highest propensity for failure: those which are sacred cows (the CEO's pet), have high esteem value or represent inordinate sunk costs. Fowler recommends the initial application of QFD be in concert with a heart-of-the-line product possessing maximum potential for spin off to other models.

Lyman and Richter [37] offer the Myers Briggs Type Indicator (MBTI) as an aid to improve the effectiveness of QFD teams. Based on Jung's theory of personality types, the MBTI is a forced-choice questionnaire which classifies the subject as one of sixteen possible characterizations. Given the premise that differences are vital to the successful QFD team, the MBTI helps ensure that the team is not only cross-functional, but cross-type as well.

Shillito [44] recommends customer-oriented product concepting (COPC) to derive recommendations for product design. Such an approach assumes that both management's business strategy and the market segment are clearly defined. COPC is a 14-step, interdisciplinary team process that yields a series of technology paths representing different product concepts. Typically, two distinct technology paths emerge as final recommendations: one for the present and one for the future. The former will get the product to market on schedule, the latter is clearly superior but requires additional research and development. This future path ultimately brings the most added value to customer and company alike.

Cohen [9] emphasizes the efficiency of addressing product robustness during the planning phase. Any product (or service) development project is eventually subjected to

reliability analysis. A firm may plan and design for reliability beforehand or react to dissatisfied customers and address reliability as a last resort. Experience has shown that it is usually far more expensive, if not impossible, to build in reliability after the fact. The most accepted reliability planning tools are the Process Decision Chart (PDPC), Fault Tree Analysis (FTA), Failure Mode and Effects Analysis (FMEA) and Robust Engineering. With PDPC, the team draws a flow diagram that details how the customer uses the product. At each interaction, the team lists all probable mishaps and associated countermeasures. The primary focus is on those countermeasures which prevent the mishap and can be incorporated into the design process.

Clausing [7] describes Fault Tree Analysis as the systematic decomposition of possible product failures into component causes relative to a particular function of the product. Analysis begins with a list of the product's functions, followed by generation of the causes for each functional failure. The failure causes in turn each have a set of component causes and so on. FTA provides a hierarchical tree which the design team can use to minimize product failure.

Juran and Gryna [26] are among the many proponents of Failure Mode and Effects Analysis. FMEA is based on the design of the product and begins with analysis of its major components. For each component, the possible types of failure are identified. This identification process is extended iteratively down to individual parts. Additionally, the probability of occurrence and severity of impact for each type of failure is estimated at every level. Consequently, concepts with poor reliability potential are quickly identified and discarded.

Robust Engineering is an outgrowth of statistical experimental design. Developed by Taguchi [17], engineers identify those design factors that are the principal determinants of system performance. Target values are then set for these factors to promote consistent operation in spite of environmental variation: product degradation over time and variation in customer usage.

Figure 2.4 Number of Decisions for 3 Factors [9]

Perhaps the most critical facet of all planning activities is identification of the customer. Cohen [9] proposes a two-step process: identify all possible customers and then identify the key customers. Use of an affinity diagram helps to group customers into clearly defined groups; a prioritization matrix or Saaty's [42] Analytic Hierarchy Process (AHP) are accepted tools for selecting the key customer group. Both approaches utilize weighted criteria to rate all of the customer groups. AHP requires pairwise comparisons

between groups over all criteria, a prioritization matrix simply assigns a numerical value (high/medium/low) to each group for every criterion. Occasionally, multiple key groups are identified (e.g., distributors, retailers and users). When this occurs, customer satisfaction may be defined very differently for each group but the product must still be designed to meet the diverse needs. Figure 2.4 compares the two methods for 3 customer groups (or factors).

Successful implementation of the QFD methodology requires a realistic time horizon. Care must be taken, however, to avoid quickly ruling out objectives that initially appear to be overly expensive or time-consuming. Often, such alternatives represent innovative solutions. The Pugh Concept Selection Process (PCSP) is a 7-step procedure that helps to identify these “breakthrough” opportunities [41]. Pugh’s major tenets are: (1) Aim for a world-class concept, settle for nothing less; (2) Start with the best current concept, even if it is the competition’s; and (3) Either beat (2) or use it. PCSP increases the likelihood that the best aspects of each alternative will be present in the team’s final choice and that the entire team will support its implementation.

2.3.2 The Voice of the Customer

Capturing the voice of the customer (VOC) for QFD consists of two distinct activities: gathering qualitative data and then quantifying same. Qualitative data are usually obtained by talking to and observing customers. Quantitative data are generally acquired through surveys or polls. Various approaches exist for gathering qualitative data (or the customer’s stated importance). The three most popular vehicles are interviews

(single customer), focus groups (8-12 customers) and observation of the product in use.

Griffin and Hauser [19] have found that one 2-hour focus group reveals roughly the same number of needs as two 1-hour interviews (Figure 2.5). Because interviews are less

Figure 2.5 Identification of Customer Needs [48]

expensive than focus groups, they recommend the former as the primary data collection method. Griffin and Hauser also have set bounds on the number of interviews necessary for revealing the majority of customer needs. For most products, they predict that less than 10 and more than 50 interviews is probably inadequate. In contrast, von Hippel [24] suggests that *lead users* are the most efficient source of customer needs. Lead users are customers with needs in advance of the rest of the market and who will benefit the most from product innovations. Specifically, lead users are better able to articulate desired latent attributes ('delighters') and reveal breakthrough opportunities.

The techniques for conducting successful interviews are a subject unto themselves. To summarize, the following guidelines promote effective interaction between interviewers and customers: use open-ended questions (more than yes/no answer required), integrate visual stimuli and props, suppress preconceived hypotheses about the product technology, have the customer use the product, investigate expressions of latent needs (flexible focus) and collect nonverbal information. Wixon et al. [54] have expanded the scope of the standard interview via Contextual Inquiry (CI). The primary enhancements are an emphasis on observation of the product in the context of the customer's activities and interaction between interviewer and customer – a joint search for information.

Kano [27] developed a model of customer satisfaction relative to product characteristics. Kano's model divides these characteristics into three separate categories: *Dissatisfiers*, *Satisfiers* and *Delighters*. Dissatisfiers are the absence of expected quality – a product characteristic which the customer takes for granted but causes dissatisfaction when missing. Satisfiers are desired quality – something customers want in the product and request. Delighters are unexpected quality – latent product attributes that are pleasant surprises to customers when initially encountered (Figure 2.6). Kano's classifications reveal two critical issues: (i) all product characteristics are not equal and (ii) quality is not synonymous with simply removing dissatisfiers. Any competitive strategy for product development will address the current level of satisfaction within each category along with the proportion of resources to allocate among them.

Figure 2.6 Kano's Model [27]

Figure 2.7 Low Revealed Importance [9]

Figure 2.8 High Revealed Importance [9]

Unfortunately, product characteristics based on customer need is not the same as the needs of the customer. Klein [30] has developed a model which measures the

significance of customer needs utilizing both stated and revealed importance (Figures 2.7 and 2.8). Stated importance is self-explanatory; the customer is asked directly how important a specific attribute is. Revealed importance is obtained by examining the correlation between attribute satisfaction and overall product satisfaction. An attribute whose satisfaction level is statistically linked to overall satisfaction has revealed importance (Figure 2.9). Taking both revealed and stated importance into account, customer needs may be classified as: *Expected* (high stated, low revealed), *Low-Impact* (low stated, low revealed), *High-Impact* (high stated, high revealed) and *Hidden* (low stated, high revealed). Efficient design teams concentrate on the expected and high-impact needs while subjecting the hidden attributes to additional analysis.

Stated Importance	Very Important	<i>Expected</i> <ul style="list-style-type: none"> - Must be met - Dissatisfaction if not met - Minimal satisfaction gain if met 	<i>High Impact</i> <ul style="list-style-type: none"> - High satisfaction if met - Dissatisfaction if not met
	Not Important	<i>Low Impact</i> <ul style="list-style-type: none"> - Minimal influence on overall satisfaction whether met or not 	<i>Hidden</i> <ul style="list-style-type: none"> - Opportunity for competitive advantage - More important satisfier than customer admits
		Weak Link	Strong Link
		Revealed Importance	

Figure 2.9 Klein's Grid of Customer Needs [30]

Perhaps the most neglected VOC source is negative customer data (or complaints) [9]. Companies usually compile fairly extensive customer complaint databases but

seldom analyze them from a QFD perspective. This is due in large part to the fact that companies, not customers, classify complaints. If customers were able to dictate the urgency or priority of the complaint, these databases could provide extensive information where expected quality and 'dissatisfiers' are concerned. Even random sampling would produce meaningful data economically.

Before the QFD team can focus on quantitative data, the many needs of the customer must be organized. Ulrich and Eppinger [48] recommend the use of an affinity diagram or hierarchy. The goal is to group the data into sets of primary needs which are further characterized by secondary and tertiary needs. The most important aspect of this exercise is to eliminate redundant needs. Once the hierarchy is established, quantitative data can be gathered, usually via survey. Three types of quantitative data are typically sought for QFD: relative importance of needs, the customer's satisfaction level per need and the competition's satisfaction level per need. Because surveys are often difficult, costly and time-consuming (e.g., appropriate audience, sample size, adequate response, etc.), many QFD teams simply assign the quantitative data by consensus. This is invariably a mistake of convenience.

Without question, the most popular method for quantifying the relative importance of customer needs is the previously mentioned AHP. Sometimes called ratio-scale importance, AHP asks the customer to make pairwise comparisons of all qualitative needs *at each hierarchical level*. Comparisons are made using an integer scale of

$\{1 \leftrightarrow 9\}$, where 1 represents equal importance and 9 signifies absolute importance (Figure 2.10). A matrix is then constructed listing the pairwise comparisons $i \rightarrow j$ with reciprocals assigned to comparisons $j \rightarrow i$. AHP then determines the principle eigenvector of this matrix which represents the relative importance of each element at that level of the hierarchy. By sequentially multiplying the matrices of each hierarchical level, the composite relative importance of each customer need is quantified [42].

<u>Intensity</u>	<u>Definition</u>	<u>Explanation</u>
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience/judgment slightly favor activity over other
5	Essential or strong importance	Experience/judgment strongly favor activity over other
7	Very strong or demonstrated importance	Activity is favored very strongly; dominance demonstrated in practice
9	Absolute importance	Evidence favoring activity over other is of highest order
2,4,6,8	Intermediate values	When compromise is needed

Figure 2.10 AHP Comparison Scale [42]

For example, let A, B, C and D represent the four elements of a hierarchical level; together, they comprise a $[4 \times 4]$ pairwise comparison matrix (Figure 2.11). By convention, the comparison of strength is always of an activity appearing in the

	A	B	C	D
A	1	5	6	7
B	$\frac{1}{5}$	1	4	6
C	$\frac{1}{6}$	$\frac{1}{4}$	1	4
D	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{4}$	1

Figure 2.11 Sample Pairwise Comparison Matrix

column on the left against an activity appearing in the row on top. There are 16 spaces in the matrix. Of these, the four diagonal entries have the value 1 because we are comparing elements to themselves. Of the remaining twelve, only six values are generated; the other six are reverse comparisons and should rationally be reciprocals of the first six.

A	3.81		A	0.61
B	1.48		B	0.24
C	0.64		C	0.10
D	0.28		D	0.05

$\xrightarrow{\text{Normalized}}$

Figure 2.12 Vector of Priorities

A vector of priorities is then derived from the matrix (Figure 2.12). In mathematical terms the principal eigenvalue and associated eigenvector are computed, and after normalization, the eigenvector becomes the priority vector. The principal eigenvector represents the singular values of the matrix and define its spectrum [12]. In other words, this eigenvector measures the amount each element stretches or shrinks the domain [35] of that particular hierarchical level. Saaty [42] states that multiplying the n elements in each row and taking the n th root provides a good approximation (accuracy to within 0.01). His claim is not verified mathematically however, and in certain cases this approximation can lead to large errors.

Customer and competitive satisfaction performance data are usually collected and quantified with less rigor. The preferred method is to simply assign numerical values to the various customer responses and then compute weighted averages. Satisfaction levels are typically expressed on four- to six-point scales. The problem with such an approach is that the weighted average value for a particular need may not be representative of the customers' actual responses. These data should be plotted to determine if the customer base is segmented into extremes. Connolly and Srivastava [10] have indirectly examined this issue in terms of multiattribute evaluation. Their findings indicate that customers may view satisfaction from two different perspectives. Needs implicitly represent either substantive value elements (components) or diagnostic indicators (cues) to a particular customer. The distinction is critical to quantifying the VOC. If a customer is component-oriented, the satisfaction threshold is likely much lower relative to another customer using cues. The former will likely express satisfaction in terms of a continuum while the latter may tend to exhibit an all-or-nothing scale of appraisal.

Beach's Image Theory [4] supports the presence of customer satisfaction thresholds within QFD. Specifically, the customer employs a compatibility test for a product in terms of its CAs (Figure 2.13). If the number of EC_j violations exceeds the rejection threshold for any CA_j , the customer will not accept the product. The compatibility test is not compensatory . . . non-violations do not compensate for violations because the compatibility test is entirely dependent on the latter. With regard to any attribute, a violation is all-or-nothing (1 or 0); the overall non-compatibility (C_{nc})

Perceived Compatibility: $C_{nc} = \sum_i \beta_{ij} \cdot x_{ij} \quad \forall j$ <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <div style="text-align: center;"> where $x_{ij} = 0 \text{ or } 1$ $0 \leq \beta_{ij} \leq 1$ </div> <div style="text-align: right;"> $\forall i, j$ $\forall i, j$ </div> </div>

Figure 2.13 Non-Compensatory Model [4]

of the product is the sum of the weights of all its violations and is ≥ 0 . Conversely, the product is accepted once the information for its attributes is exhausted and C_{nc} 's value is above all the rejection thresholds. Therefore, C_{nc} behaves as a step-function (x_{ij} = status of EC_i for CA_j and β_{ij} = relative importance of EC_i to CA_j).

QFD practitioners have historically employed only a compensatory approach: x_{ij} is not an all-or-nothing proposition. Non-violations compensate for violations (Figure 2.14). In other words, x_{ij} can take any value in the interval $[0, 1]$. Therefore, the compensatory counterpart would take the form:

Perceived Compatibility: $C_c = \sum_i \beta_{ij} \cdot x_{ij} \quad \forall j$ <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <div style="text-align: center;"> where $0 \leq x_{ij} \leq 1$ $0 \leq \beta_{ij} \leq 1$ </div> <div style="text-align: right;"> $\forall i, j$ $\forall i, j$ </div> </div>
--

Figure 2.14 Compensatory Model

Because $0 \leq x_{ij} \leq 1$, C_c is not a step-function. Therefore, the fundamental difference between the two approaches is that the expected value of C_c will always be greater than (or at worst equal to) that of C_{nc} .

The practical implication of choosing one approach over another is one of cues versus components. Connolly and Srivastava [10] defined the former as diagnostic indicators and the latter as substantive value elements. If the CAs are treated as cues for incompatibility, then a non-compensatory function with its rejection threshold is appropriate. Conversely, if the CAs are viewed as components of incompatibility, then a compensatory function should be used. Thus, implementation of a compensatory model in a non-compensatory context could mask the true level of incompatibility.

In reality, customer behavior often reflects a combination of the two models. Customer acceptance of the product might depend upon a unique rejection threshold for each CA_j and is therefore non-compensatory for certain values of x_{ij} . Once these thresholds are met, the compensatory model more accurately describes consumer behavior with its associated tradeoffs. Regardless, the issue of rejection thresholds must be addressed in order to accurately quantify the relationship between a CA and its ECs.

2.3.3 The House of Quality

Once the VOC has been quantified, the design team must concentrate on establishing product specifications, generating concepts and selecting a concept. QFD's ubiquitous tool for accomplishing these steps is the House of Quality (HOQ). Whether modifying an existing product or developing a new design, the HOQ relates the customer's needs with technical responses for meeting same (Figure 2.15). As such, the VOC represents several components of the HOQ: the *customer attributes* (CAs), their

Figure 2.15 Generic House of Quality

relative importance and *competitive assessment*. Once the VOC has been quantified (the “whats”), the QFD team must provide the “hows” in the form of *engineering characteristics* (ECs), determine the relationship between each CA and EC and assess the correlation between ECs. The relationship between a specific technical response and customer need is entered in the Planning Matrix (PM) of the HOQ; the interrelationship between technical responses is entered in the Correlation Matrix (CM) – the HOQ’s roof. Many techniques have been proposed to construct the PM and CM in an effort to meet the customer’s needs.

Before addressing the subjective nature of these matrices, it should be emphasized that QFD teams only make their job more difficult when technical responses are expressed ambiguously. Clausing [7] provides guidelines for effectively characterizing ECs: (1) ensure the technical response directly impacts upon customer perception; (2) define the unit of measurement; (3) define the direction of goodness (the more/less the better); (4) describe how each measurement will be taken; and (5) record all assumptions for each measurement.

Figure 2.16 HOQ with Positive and Negative Relationships

Once the ECs are defined, the QFD team assesses the impact of each on the various CAs. Clausing and Hauser [8] recommend the use of numbers or symbols to indicate the strength of these relationships. This typically translates to positive scales such as [1-3-9] or [1-3-5] denoting weak, medium and strong relationships.

Unfortunately, the issue of negative relationships between CAs and ECs is either ignored or discouraged. Cohen [9] admits such situations arise but complicate the process. He advocates the conversion of any negative linkage (to positive) by redefining the EC. This is not always practical, however (Figure 2.16). If an EC is related to two CAs, one positively and one negatively, then redefinition will only reverse the two relationships (not eliminate negativity).

Similar restrictions surround the CM, where the QFD team maps interdependency between ECs. Cohen [9] recognizes the reality of positive and negative correlations between ECs but fails to discuss the impact of negative values on the prioritization of ECs. It is not coincidental that Wasserman [49], Lyman [36] and Xing et al. [55] offer normalization procedures for the PM in support of prioritizing the ECs which appear to stress the utilization of strictly positive values. These are shown in Equations (2.4) through (2.7), where R_{ij} represents the quantified relationship between EC_i and CA_j and γ_{ik} signifies the correlation between EC_i and EC_k .

$$R_{ij}^{Normalized} = \frac{R_{ij}}{\sum_i R_{ij}} \quad \forall j \quad (\text{Lyman}) \quad (2.4)$$

$$R_{ij}^{Normalized} = \frac{\sum_k R_{kj} \cdot \gamma_{ik}}{\sum_i \sum_k R_{ij} \cdot \gamma_{ik}} \quad \forall j \quad (\text{Wasserman}) \quad (2.5)$$

$$\left\{ \begin{array}{ll} \text{Minimize} & \varepsilon \\ \text{subject to} & \sum_k R_{kj}^{Normalized} \cdot \gamma_{ik} - R_{ij} \leq \varepsilon \quad \forall i, j \\ & R_{ij} - \sum_k R_{kj}^{Normalized} \cdot \gamma_{ik} \leq \varepsilon \quad \forall i, j \\ & \varepsilon \geq 0 \end{array} \right\} \quad (\text{Xing et al.}) \quad (2.6)$$

Let $[PM] = (R_{ij})$, $[CM^{Normalized}] = (\gamma_{ik})$ and $[PM^{Normalized}] = (R_{ij}^{Normalized})$. If $[CM^{Normalized}]$ is invertible and $\varepsilon = 0$, then an exact fit is possible and Equation (2.6) can be reduced to

$$[PM^{Normalized}] = [PM] \times [CM^{Normalized}]^{-1} \quad (\text{Xing et al.}) \quad (2.7)$$

If $[CM^{Normalized}]$ is not invertible, then the best fit has to be determined. The equations in (2.6) seek to minimize the maximum error. An alternative approach for finding the best fit might be developed using the method of least squares (where the sum of squared errors is minimized).

It is not clear how negative entries in the PM (R_{ij}) and CM (γ_{ik}) affect the matrix transformations of Equations (2.4) through (2.7). The QFD community seems content to simply label negative relationships as “breakthrough” opportunities or resort to psychology: Humans understand properties best if they are stated in a positive manner and if properties are chosen so that ‘more is better’ [2]. More is not always better, however; negative relationships are often simply unavoidable (e.g., CA_j = fuel efficiency, EC_i = vehicle weight).

2.3.4 Analysis

Beyond the presence of negative activities, Yoder and Mason feel there are several limitations to the traditional approach of relating ECs to CAs subjectively.

Firstly, it is not reasonable to expect that the strength of the relationship is consistent over all levels of output performance. Secondly, the expert opinion on the strength or direction of improvement may not be correct within the context of the QFD analysis. Thirdly, when setting targets for the outputs, relationship strength alone does not provide a clear indication for what level of output performance is optimal [57].

They offer linear regression as a vehicle for systematically evaluating these relationships. Based upon the customer's perceptions, the competitive assessment of an attribute is treated as the dependent variable; the benchmark values for the engineering characteristics become the independent variables. Regression then fits a line (or curve) for each customer attribute (or row of the PM). Thus, an equation is obtained to predict the value of a CA, given specified levels for the significant ECs. Descriptive statistics may then be used to determine how well the regression equation describes the relationship.

Ultimately . . . (these) relationships are best described as mathematical functions which can be linear, logarithmic, or parabolic based on the customers' perceptions and expectations for that characteristic. These functions describe the change in product utility the customer experiences as the product characteristic is changed. The shapes of these functions vary for every related customer requirement and product characteristic [57].

The methodology of Yoder and Mason is theoretically sound and has excellent potential. Some intermediate EC value may satisfy the customer – *more* then becomes either redundant or detrimental where customer utility is concerned (Figures 2.17 and

2.18). Unfortunately, regression analysis can require large amounts of data to provide meaningful results for even a 1st order model. Capturing the VOC in this manner becomes even more difficult for new technologies or products with minimal competition.

Figure 2.17 More $\equiv$ Redundant

Figure 2.18 More $\equiv$ Detrimental

Cristiano, White and Liker [11] offer a target-setting procedure which integrates multiattribute utility theory with QFD. This process selects values for the engineering characteristics which meet or exceed the customer's demanded quality along each dimension. The result is a set of minimally acceptable EC values which quantify that portion of the design space satisfying the customer's preferences. Crisitano et al. admit that their approach is undermined if the engineering characteristics are correlated.

Wasserman [49] likens the maximization of customer satisfaction to an integer relaxation of the *Knapsack* problem. Satisfying the customer is equivalent to filling the knapsack with targeted values for the ECs. The ECs are prioritized by their *technical importance to cost* ratio. Design resources are then allocated incrementally so that those engineering characteristics with the highest priority are elevated to "world-class" status.

Chapter 3

Deterministic Model Formulation

In this world there are only two tragedies. One is not getting what one wants, and the other is getting it [51].

3.1 Introduction

The work of Cristiano et al. [11] offers a set of minimally acceptable engineering characteristic (EC) values which quantify that portion of the design space satisfying the customer's preferences. However, QFD's objective is to achieve the highest level of customer contentment subject to organizational constraints on cost, time and other resources. Therefore, a mathematical programming model is proposed (Section 3.2) to assist in selecting the mix and levels of engineering characteristics that maximize customer satisfaction. This linear model chooses EC values which balance development and production constraints with market competitiveness. Section 3.3 applies the model to an actual design problem from the literature. In Section 3.4 a procedure for objectively deriving and analyzing the Voice of the Customer is presented. Finally, Section 3.5 examines the robustness of the optimal solution for both the subjective and objective approaches.

3.2 Basic Mathematical Program

If the ultimate goal of QFD is to produce what the customer wants – contingent upon the available resources – then an understanding of the relationships between the customer's desired attributes (CAs) and a product's engineering characteristics is

necessary for success. While this goal translates to maximizing customer satisfaction, determining the strength of these relationships is another matter. As previously described, the relationship between a CA and an EC is typically categorized as strong, medium, weak or non-existent in the planning matrix. Target values for each EC are then set to accentuate the strongest relationships. Implicit to such an approach is the assumption that the product's worth to the customer is a linear function of each EC value. Thus, the relationship between a CA and an EC is simply the estimated rate of change for customer value (Figure 3.1).

Figure 3.1 Characterization of CA/EC Relationships [57]

Consider a House of Quality where (1) the engineering characteristics are uncorrelated, (2) the relative technical difficulty of each EC_i is expressed in terms of the cost/time to develop and produce and (3) the relationship between each CA_j and the ECs

can be described via a linear function with coefficients β_{ij} , and (4) preferential independence exists among the customer attributes. Let $\underline{V}_k$ be the vector of customer attribute values for some alternative design k . For example, the value of the j^{th} customer attribute for design k is

$$V_{jk} = \beta_{1j} \cdot EC_{1j} + \beta_{2j} \cdot EC_{2j} + \cdots + \beta_{1j} \cdot EC_{1j} + \varepsilon \quad (3.1)$$

where $\beta_{ij} \in \{\square, \Delta, O, \blacksquare, \blacktriangle, \bullet\}$ and ε is an error term representing randomness in customer attitude and model specification. Define $\underline{V}_k^-$ as the vector of V_{jk} values for $j \neq m$ or n . A pair of customer attributes is preferentially independent of $\underline{V}_k^-$ if the preference for $\{V_{mk}, V_{nk}, V_k^-\}$ to $\{V'_{mk}, V'_{nk}, V_k^-\}$ does not depend on $\underline{V}_k^-$. If and only if preferential independence exists, then customer satisfaction can be measured as an additive function across attributes [28]. In this case one can assign a value (S_k) to a weighted combination of the CA values by

$$S_k = \sum_j \lambda_j \cdot V_{jk} \quad (3.2)$$

where $0 \leq V_{jk} \leq 1$ and the λ_j are the relative importance weights for the customer attributes normalized such that $\sum_j \lambda_j = 1$. The importance weights can be found using the Analytic Hierarchy Process or a similar procedure. To allow direct comparison, assume the V_{jk} values of Equation (3.1) are normalized such that $V_j = 1$ if all EC_i are set to their most favorable value in design k . The objective is to maximize total customer satisfaction subject to technological and economic feasibility. In dealing with the entire set of possible design specifications, the subscript k is dropped. This leads to the

following mathematical program:

Let S = the overall level of customer satisfaction

x_i = the optimal value of EC_i

x_i^{cc} = the standardized value of x_i

x_i^0 = the initial value of EC_i

L_i & U_i = the technically feasible lower & upper bounds of EC_i

V_j = the satisfaction level of CA_j

π_j = the minimal acceptable satisfaction level of CA_j

λ_j = the relative importance of CA_j

Z_i = the change from x_i^0 in EC_i

V_j^{min} & V_j^{max} = the minimum & maximum values of V_j ($\forall [L_i, U_i]$)

C_p = the production budget (c_{p_i} = the production cost for EC_i)

C_D = the research and development budget (c_{D_i} = the R&D cost for EC_i)

T = the allowable development time (t_i = development time for EC_i)

$$\text{then} \quad \text{Maximize} \quad S = \sum_j \lambda_j \cdot V_j \quad (3.3)$$

$$\text{subject to} \quad \pi_j \leq V_j \leq 1 \quad (\pi_j \geq 0) \quad \forall j \quad (3.4)$$

$$V_j = \left(\sum_i \beta_{ij} x_i^{cc} - V_j^{min} \right) \div (V_j^{max} - V_j^{min}) \quad \forall j \quad (3.5)$$

$$L_i \leq x_i \leq U_i \quad \forall i \quad (3.6)$$

$$x_i^{\alpha} = \frac{\left\{ x_i - \left(\frac{U_i + L_i}{2} \right) \right\}}{\frac{U_i - L_i}{2}} \quad \forall i \quad (3.7)$$

$$Z_i = |x_i - x_i^0| \quad \forall i \quad (3.8)$$

$$\sum_i c_{p_i} \cdot Z_i \leq C_p \quad (3.9)$$

$$\sum_i c_{D_i} \cdot Z_i \leq C_D \quad (3.10)$$

$$\sum_i t_i \cdot Z_i \leq T \quad (3.11)$$

$$\text{or} \quad \max(t_i \cdot Z_i) \leq T \quad \forall i \quad (3.12)$$

The objective function of Equation (3.3) chooses that vector of engineering characteristic values ($\underline{x}$) which generates the maximum amount of satisfaction relative to the customer's stated importance (λ_j) of the attributes. L_i and U_i in Equation (3.6) are the lower (*minimum*) and upper (*maximum*) feasible bounds of EC_i or its technically achievable range. Given a feasible set of x_i values, V_j in Equation (3.5) is the resulting percentage of customer satisfaction achieved at or above a minimum threshold value (π_j) for each CA_j .

Equation (3.5) describes the standardized numerical relationship between each CA_j and the ECs of the HOQ matrix. These relationships can be subjectively assigned or derived objectively. Typically, symbols $\{\square, \Delta, \circ, \blacksquare, \blacktriangle, \bullet\}$ are initially utilized by design teams and then converted to a logarithmic scale: $\beta_{ij} \in \{-9, -3, -1, 0, 1, 3, 9\}$. Conversely, data-driven, empirical models can be used to quantify the customer's

assessments. This will be discussed more extensively later in the chapter (Section 3.4).

Regardless of the coefficients' origin, V_j falls somewhere between the minimum (V_j^{min}) and maximum (V_j^{max}) values of CA_j for the design space defined by all $[L_i, U_i]$.

The x_i are coded (or normalized) to ensure uniform marginal rates of improvement among the ECs in Equation (3.7). The coded values satisfy $-1 \leq x_i^{cc} \leq +1$.

To illustrate, suppose $CA_1 = (3 \cdot EC_1) - (9 \cdot EC_2)$ where EC_1 is fuel efficiency ($10 \text{ mpg} \leq x_1 \leq 50 \text{ mpg}$) and EC_2 is engine emissions ($1 \text{ ppm} \leq x_2 \leq 9 \text{ ppm}$). Let $\underline{x}^* = [30, 3]$. The minimum performance is the point of reference $\underline{x}^0 = [10, 9]$. Then $(V_j^{min}, V_j^{max}) = (-51, 141) = (\{3 \cdot 10 - 9 \cdot 9\}, \{3 \cdot 50 - 9 \cdot 1\})$ and $\underline{x}^*$ represents an improvement of 50% for EC_1 and 75% for EC_2 (relative to x_i^0). Since the contribution of EC_2 is weighted three times as heavily as that of EC_1 , V_1 should reflect an achieved level of $\left(\frac{3 \cdot 0.50 + 9 \cdot 0.75}{12}\right)$

$= 68.75\%$ customer satisfaction within the range $\{-51 \leftrightarrow 141\}$. With uncoded x_i , $V_1 =$

$$\left(\frac{3 \cdot 30 - 9 \cdot 3 - (-51)}{141 - (-51)}\right) = 59.38\% \text{ because the marginal rates of unit improvement for } x_1$$

and x_2 are 2.5% $\left(\frac{1}{30-10}\right)$ and 12.5% $\left(\frac{1}{9-1}\right)$, respectively. Coding of the x_i produces a

consistent scaling of the design space with appropriate values for V_1 : $\underline{x}^{cc} = [0, -0.5]$,

$$(V_1^{min}, V_1^{max}) = (-12, 12) = (\{3 \cdot (-1) - 9 \cdot 1\}, \{3 \cdot 1 - 9 \cdot (-1)\}) \text{ and } V_1 =$$

$$\left(\frac{3 \cdot 0 - 9 \cdot (-0.5) - (-12)}{12 - (-12)}\right) = 68.75\%.$$

Z_i in Equation (3.8) is the improvement of EC_i relative to its default value x_i^0 .

Normally, x_i^0 is set at either L_i or U_i depending on the direction of technical improvement. In the example above, $x_1^0 = 10 = L_1$ and $x_2^0 = 9 = U_2$. C_P is the total production budget and c_{pi} is the production cost per unit of improvement of EC_i in Equation (3.9). C_D is the total research and development budget and c_{Di} is the individual R&D cost per unit of improvement of EC_i in Equation (3.10). T is the total allowable development time and t_i is the development time per unit of improvement of EC_i in Equations (3.11) and (3.12). The two constraints differ in that Equation (3.12) addresses concurrent development of the ECs. Application of the basic mathematical program to sample data is the subject of the next section.

3.3 Example Problem: Designing a Pencil

The original pencil design problem (discussed in [11] and [49]) is depicted in Figure 3.2. The ECs are correlated as shown by the $\blacktriangle$ symbol in the roof of the matrix. Conventional symbols describe the EC $\leftrightarrow$ CA relationships and preferential independence of the CAs is assumed. The relative technical difficulty of each EC is expressed in terms of a cost coefficient (c_i) which denotes the incremental change in cost associated with a unit variation in EC_i . Ignoring the matrix roof and the issue of correlation, EC_3 is incorrectly symbolized for both CA_2 and CA_3 . The depiction is positive: the more lead dust generated, the better. Figure 3.3 corrects this error and will be referred to as the modified pencil design (MPD) problem. Figure 3.4 transforms the modified matrix to the traditional logarithmic scale; these values become the β_{ij} for each CA in Equation (3.5) of the basic model.

Figure 3.2 Original Pencil Design Matrix [11]

Figure 3.3 Modified Pencil Design Matrix

Figure 3.4 Transformation of Modified *Pencil Design* Problem

The mathematical program was solved (Microsoft Excel 7.0) for three cases whose conditions are summarized in Table 3.1. Each optimal design vector ($\underline{x}_k^*$) represents the maximum amount of customer value attainable for instance k . A brief description of each instance follows.

Instance #1. If cost and time are unconstrained, an overall level of 100% customer satisfaction is obtained for $\underline{x}_1^* = [5.5, 6.0, 2.0]$. Due to the inaccurate EC_3 relationships of Figure 3.2, this vector contradicts the preferred region of the parameter space as specified by Cristiano et al. [11] for the original pencil design problem.

Instance #2. Activation of a general budget constraint $\left(\sum_i c_i \cdot Z_i \leq C_{\text{Budget}} = 2.0 \right)$, with no threshold for V_j ($0 \leq V_j \leq 1, \forall j$), significantly reduces overall customer

satisfaction to 61% for $\mathbf{x}_2^* = [5.0, 2.5, 2.0]$.

Instance #3. In conjunction with the budget constraint, these are the EC values which maximize the minimum *feasible* threshold value for V_j . This occurs at $\pi_j = 0.43$ ($\forall j$). Customer satisfaction decreases to 45% for $\mathbf{x}_3^* = [4.65, 4.02, 3.12]$.

Modified Pencil Design (MPD) Problem			
	<i>Instance #1</i> (Unconstrained)	<i>Instance #2</i> (w/ Budget)	<i>Instance #3</i> (w/ Budget & Thresholds)
$\sum \lambda_j V_j$	1.00	0.61	0.44
λ_1	0.33	0.33	0.33
λ_2	0.42	0.42	0.42
λ_3	0.25	0.25	0.25
V_1	1.00	0.67	0.43
V_2	1.00	0.75	0.44
V_3	1.00	0.28	0.43
x_1^{ec}	1.00	0.33	-0.13
x_2^{ec}	1.00	-1.00	-0.13
x_3^{ec}	-1.00	-1.00	0.12
π_1	0	0	0.43
π_2	0	0	0.43
π_3	0	0	0.43
Z_1	1.50	1.00	0.65
Z_2	3.50	0.00	1.52
Z_3	2.00	2.00	0.88
$4 \leq x_1 \leq 5.5$	5.50	5.00	4.65
$2.5 \leq x_2 \leq 6$	6.00	2.50	4.02
$2 \leq x_3 \leq 4$	2.00	2.00	3.12
$\sum c_i Z_i$	4.60	2.00	2.00
C_{Budget}	N/A	2.00	2.00

Table 3.1 MPD Problem Results

Figure 3.5 π_j vs. $\sum \lambda_j V_j$ (MPD Problem)

Figure 3.5 shows the impact of the budget constraint and π_j values ≥ 0 on the linear program. Within the *MPD* problem's feasible region, the budget constraint appears to be the primary determinant of customer satisfaction. These results emphasize the necessity for analyzing costs in conjunction with the QFD effort. Otherwise, any solution may be sub-optimal relative to market competitiveness, particularly if cost is not modeled as a CA. In the case of a product's subsystems or components, value engineering methods can effectively determine the hierarchical costs of the ECs under consideration. Thus far, it has been assumed that subjective symbols $\{\square, \Delta, \bigcirc, \blacksquare, \blacktriangle, \bullet\}$ and their

transformed values accurately describe significant relationships for the CAs and their associated ECs. In the basic mathematical programming model, a product's overall desirability depends upon the value functions of Equation (3.5). Therefore, each V_j implicitly models some combination of the customer's preferences. Objectively revealing these desires is the subject of the next section.

3.4 Empirical Modeling Approach

Relative Importance			EC ₁	EC ₂	EC ₃
Easy to Hold	CA ₁	4	▲		
Does not Smear	CA ₂	5		▲	□
Point Lasts	CA ₃	3	●	■	△
Cost _i			1.0	0.6	0.5
			Length of Pencil (inches)	Time between Sharpening (pages)	Lead Dust Generated (mg/cm ²)

■ / □ = STRONG Pos/Neg

▲ / △ = MEDIUM Pos/Neg

● / ○ = WEAK Pos/Neg

Figure 3.6 Modified *Pencil Design Matrix* without Roof

Consider again the modified HOQ (Figure 3.6) and work of Yoder and Mason [57]. In order to build an empirical model for each CA, the target customer's assessment of the competing products is treated as the dependent variable and the engineering characteristic(s) for that attribute are the independent variable(s). Utilizing regression analysis, a 1st order value function of the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \varepsilon \quad (3.13)$$

can be derived by the method of least squares for each CA. Equation (3.13) can be written in matrix notation as

$$y = X\beta + \varepsilon$$

where ε represents model inadequacy and the randomness of human behavior. The least squares estimator of β which minimizes the sum of the squares of the ε_i is

$$\hat{\beta} = [X^T X]^{-1} X^T y \quad (3.14)$$

$$\text{where } y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Because solving linear equations is computationally more efficient, Equation (3.14) is equivalent to $[X^T X] \hat{\beta} = X^T y$. Also note that the least squares method requires the inverse of $[X^T X]$. Because the data for the regression calculations may differ dramatically in order (e.g., $1.43\text{E}-06$, $8.77\text{E}+03$ and 6.32), $[X^T X]$ can be badly conditioned. The x_{it} are therefore coded to prevent an ill-conditioned $[X^T X]$ by standardizing the design space [13]. An alternative approach to the problem of matrix conditioning is offered by orthogonalization [56]. The following coding scheme [38] produces values between +1 and -1 for all x_{it} . This is Equation (3.7) of the basic model:

$$x_{it}^{\text{coded}} = \frac{\left\{ x_{it} - \left(\frac{\max x_{it} + \min x_{it}}{2} \right) \right\}}{\frac{\max x_{it} - \min x_{it}}{2}}$$

The intercept (β_0) in Equation (3.13) is of little interest when deriving value functions. Centering the data eliminates β_0 from consideration and yields an equivalent model of the form

$$y - \bar{y} = \beta_1(x_1 - \bar{x}_1) + \beta_2(x_2 - \bar{x}_2) + \cdots + \beta_k(x_k - \bar{x}_k) + \varepsilon \quad (3.15)$$

where the degree of freedom lost to the restriction $\sum_{i=1}^n (y_i - \bar{y}) = 0$ is compensated for by fitting one less parameter [13]. The $x_{it}^{centered} = x_{it} - \bar{x}_{it}$ are then coded as shown above such that

$$x_{it}^{coded \& centered} = x_{it}^{cc} = \frac{\left\{ x_{it}^{centered} - \left(\frac{\max x_{it}^{centered} + \min x_{it}^{centered}}{2} \right) \right\}}{\frac{\max x_{it}^{centered} - \min x_{it}^{centered}}{2}}$$

In addition to modeling the relationship between a CA and its ECs, the value function can be analyzed statistically to determine its legitimacy. Analysis of residuals and the amount of curvature will reveal the *validity* of a value function; the coefficient of determination (adjusted R^2) helps to judge its *completeness*. Such analysis can expose an imprecise value function . . . nonlinear relationships might exist or additional ECs may be needed to explain the customer's preferences.

<u>Source of Variation</u>	<u>Sum of Squares</u>	<u>Degrees of Freedom</u>	<u>Mean Square</u>	<u>F</u>
Regression	SS_R	k	MS_R	MS_R/MS_E
Error (Residual)	SS_E	n - k - 1	MS_E	
Total	SS_{yy}	n - 1		

Table 3.2 Analysis of Variance

In Figure 3.6, each response variable ($CA_j, j = 1, 2 \text{ or } 3$) could be related to as many as three regressor variables ($EC_i, i = 1, 2 \text{ or } 3$). A minimum of four ($k + 1$) observations are therefore necessary to account for the unknown regression coefficients $\beta_{0j}, \beta_{1j}, \beta_{2j}$ and β_{3j} . However, in order to measure the significance of each value function, at least five ($k + 2$) observations are needed (Table 3.2). In this case there are not enough data to objectively determine how the ECs define a particular CA_j ($k = 3$ and $n = 4$). The issue of robust statistical models (and techniques for capturing the requisite data to support them) follows.

3.4.1 Data Acquisition through Design of Experiments

Experimental design methods can play an important role in the optimization of customer satisfaction. Systematically and accurately capturing the voice of the customer establishes a value function's legitimacy. The statistical design of experiments (DOE) is a planning process whereby appropriate data are collected and then analyzed by statistical methods. Where the HOQ is concerned, such an approach yields valid, objective conclusions by choosing (1) the ECs to be varied in an experiment, (2) the ranges over which the ECs will be varied and (3) the levels at which observations will be made.

Factorial designs are especially efficient in determining the effect of several factors (ECs) on a response (CA). While a 2^k design with k factors each at two levels is most preferred, the data requirements for a complete replicate rapidly become unrealistic as k increases. Assuming higher-order interactions between factors are negligible, the

pertinent information on main effects and lower-order interactions may be obtained with a *fractional* factorial design. A 2^{k-p} design is simply a $(1/2)^p$ fraction of the 2^k design.

The value of p determines the fractional design's resolution (R). A design is of resolution R if no p -factor effect is aliased with another effect containing less than $R - p$ factors.

Two (or more) factors are aliases if their individual effects are represented by a single, composite effect. The higher the resolution, the lower the impact of the design's alias structure. The goal is to utilize fractional designs with the highest possible resolution subject to constraints on data collection [38]. Because there are only two levels for every factor, the 2^k family of designs assume the response is approximately linear over the specified range of factor levels. The addition of a center point to these designs provides a means for assessing the amount of response curvature.

Once the industrial design process has identified the customer's needs (CA_j) and weighted them accordingly (λ_j), customer receptivity is evaluated – usually through the use of prototypes. Prototypes are approximations of actual products and come in various

forms: $\begin{matrix} \text{Physical} \\ \updownarrow \\ \text{Analytical} \end{matrix}$ versus *Focused* $\leftrightarrow$ *Comprehensive* [48]. Focused, physical prototypes

accentuate a product's features and its intended competitive niche. They are ideally suited for capturing the voice of the customer as it relates to the observations of an experimental design. The more unique the product, the larger the role of prototypes. With revolutionary products or new technologies, prototypes reduce the risk of uncertainty. However, such benefits must be weighed against the time and costs

associated with producing prototypes. When failure costs are low or the product's technologies are well established, 3-dimensional computer-aided design (CAD) images may be a more efficient approach to soliciting customer assessments. Regardless of the approach taken, gathering customer data for the design points can be arduous. This investment of time is nevertheless essential to the DOE methodology as demonstrated in its application to the *pencil design* problem.

3.4.2 An Empirically-Designed Pencil

Using the framework of the *pencil design* problem, an experiment utilizing a 2^{3-1} design with single center point was chosen to elicit customer responses. Combinations of three sizes (4", 6" and 8") of prototype pencils with different styles of lead (0.5mm, 0.7mm and 1.0mm) generated the design points. The thicker the lead, (1) the shorter the time between sharpenings and (2) the more residue produced. The mean responses of three University of Arizona graduate students were then used to derive the customer's value functions for CA_1 , CA_2 and CA_3 . The data in Figure 3.7 were obtained.

Properly fitting a regression model requires several assumptions: (1) errors are uncorrelated random variables with a mean of zero and constant variance; (2) errors are normally distributed (to permit standard statistical tests); and (3) the form of the model is correct. The 2^{3-1} design is a resolution III design and aliases all main effects with two-factor interactions. After examining the products available, the more restricted design space of the *MPD* problem (Table 3.3) was expanded. The two levels chosen for the factors represent a more realistic $[L_i, U_i]$ for each EC_i .

Responses					
Natural			Centered		
CA_1	CA_2	CA_3	CA_1	CA_2	CA_3
1.5	1	1	-1.8	-2	-1.9
4	3.5	1.5	+0.7	+0.5	-1.4
2	5	5	-1.3	+2	+2.1
5	3	3.5	+1.7	0	+0.6
4	2.5	3.5	+0.7	-0.5	+0.6

Design Points								
Natural			Centered			Coded & Centered		
EC_1	EC_2	EC_3	EC_1	EC_2	EC_3	EC_1	EC_2	EC_3
4	2	4	-2	-2	+1	-1	-1	+1
8	2	2	+2	-2	-1	+1	-1	-1
4	6	2	-2	+2	-1	-1	+1	-1
8	6	4	+2	+2	+1	+1	+1	+1
6	4	3	0	0	0	0	0	0

Matrices									
$y =$			$X =$						
-1.8			-1	-1	+1				
+0.7			+1	-1	-1				
-1.3			-1	+1	-1				
+1.7			+1	+1	+1				
+0.7			0	0	0				
-2						-1	-1	+1	
+0.5						+1	-1	-1	
+2						-1	+1	-1	
0						+1	+1	+1	
-0.5						0	0	0	
-1.9							-1	-1	+1
-1.4							+1	-1	-1
+2.1							-1	+1	-1
+0.6							+1	+1	+1
+0.6							0	0	0

Figure 3.7 2^{3-1} w/ Center Point Design

<u>MPD Problem</u>	<u>PD^{k-p} Problem</u>
$4 \leq EC_1 \leq 5.5$	$4 \leq EC_1 \leq 8$
$2.5 \leq EC_2 \leq 6$	$2 \leq EC_2 \leq 6$
$2 \leq EC_3 \leq 4$	$2 \leq EC_3 \leq 4$

Table 3.3 Feasible Design Space

Henceforth, data from the 2^{3-1} design with its extended upper/lower bounds will be referred to as the PD^{k-p} problem. Stepwise regression analysis produces the value functions of Table 3.4 for the PD^{k-p} problem. The linearity assumption is not necessarily warranted (Figures A.1a and A.1b).

$CA_1 = (1.375)EC_1$	
$CA_2 =$	$+ (0.875)EC_2 - (1.125)EC_3$
$CA_3 =$	$+ (1.500)EC_2 - (0.500)EC_3$

Table 3.4 Value Functions for PD^{k-p} Problem

While the value functions are still statistically significant after reducing the degrees of freedom for elimination of β_{0j} ($F_{sig} \approx 0.001$), 1st order models account for only $\approx 86\%$ (adjusted R^2) of the customer's assessments. However, despite the small number of observations the residuals appear to support the assumption $\varepsilon_i \sim NID(0, \sigma^2)$.

The mathematical program was again solved for three instances of the PD^{k-p} problem. The results are summarized in Table 3.5. A level of 100% customer satisfaction was obtained for $\underline{x}_1^* = [8.0, 6.0, 2.0]$. Adding the budget constraint decreases

the customer satisfaction level to 45% for $\underline{x}_2^* = [4.0, 3.67, 2.0]$. The maximum minimum *feasible* threshold value for V_j occurs at $\pi_j = 0.27$ ($\forall j$) and $\underline{x}_3^* = [5.08, 3.07, 3.44]$; overall customer satisfaction is lowered further to 27%.

Factorial Pencil Design (PD^{k-P}) Problem			
	<i>Instance #1</i> (Unconstrained)	<i>Instance #2</i> (w/ Budget)	<i>Instance #3</i> (w/ Budget & Thresholds)
$\sum \lambda_j V_j$	1.00	0.45	0.27
λ_1	0.33	0.33	0.33
λ_2	0.42	0.42	0.42
λ_3	0.25	0.25	0.25
V_1	1.00	0.00	0.27
V_2	1.00	0.74	0.27
V_3	1.00	0.56	0.27
x_1^{cc}	1.00	-1.00	-0.46
x_2^{cc}	1.00	-0.17	-0.47
x_3^{cc}	-1.00	-1.00	0.44
π_1	0	0	0.27
π_2	0	0	0.27
π_3	0	0	0.27
Z_1	4.00	0.00	1.08
Z_2	4.00	1.67	1.07
Z_3	2.00	2.00	0.56
$4 \leq x_1 \leq 8$	8.00	4.00	5.08
$2 \leq x_2 \leq 6$	6.00	3.67	3.07
$2 \leq x_3 \leq 4$	2.00	2.00	3.44
$\sum c_i Z_i$	7.40	2.00	2.00
C_{Budget}	N/A	2.00	2.00

Table 3.5 PD^{k-P} Problem Results

Relative to optimization of the *MPD* problem (Table 3.1), the reduction in customer satisfaction for Instances #2 and #3 is a result of the PD^{k-P} problem's expanded ranges for EC_1 and EC_2 . If the value functions of the PD^{k-P} problem are replaced with the subjective

Expanded Modified Pencil Design (MPD_E) Problem			
	<i>Instance #1</i> (Unconstrained)	<i>Instance #2</i> (w/ Budget)	<i>Instance #3</i> (w/ Budget & Thresholds)
$\sum \lambda_j V_j$	1.00	0.49	0.27
λ_1	0.33	0.33	0.33
λ_2	0.42	0.42	0.42
λ_3	0.25	0.25	0.25
V_1	1.00	0.00	0.27
V_2	1.00	0.85	0.28
V_3	1.00	0.52	0.27
x_1^{cc}	1.00	-1.00	-0.46
x_2^{cc}	1.00	-0.17	-0.47
x_3^{cc}	-1.00	-1.00	0.44
π_1	0	0	0.27
π_2	0	0	0.27
π_3	0	0	0.27
Z_1	4.00	0.00	1.08
Z_2	4.00	1.67	1.07
Z_3	2.00	2.00	0.56
$4 \leq x_1 \leq 8$	8.00	4.00	5.08
$2 \leq x_2 \leq 6$	6.00	3.67	3.07
$2 \leq x_3 \leq 4$	2.00	2.00	3.44
$\sum c_i Z_i$	7.40	2.00	2.00
C_{Budget}	N/A	2.00	2.00

Table 3.6 MPD_E Problem Results

relationships $\{\square, \Delta, \bigcirc, \blacksquare, \blacktriangle, \bullet\}$ of the *MPD* problem, the customer satisfaction levels of Table 3.6 are achieved over the extended design space. This revision to the *MPD* problem will be referred to as the *MPD_E* problem. Note that the different sets of coefficients produce the same optimal design vector ($\underline{x}_k^*$) for all instances of the *PD^{k-p}* and *MPD_E* problems.

This is not coincidental and is the subject of the next section.

3.5 Sensitivity Analysis

Due to the estimation of the objective function, either through subjective assessment or empirical model building, it is of interest to explore the sensitivity of the solution with respect to the objective function. As long as the current basis is correct, the solution is optimal. A graphical comparison of the value functions versus the subjective relationships of CA_1 , CA_2 and CA_3 reveals similar normalized design spaces (Figures A.2a through A.2h). Sensitivity analyses of the cost coefficients for the *PD^{k-p}* and *MPD_E* problems are detailed in Figure A.3. Assuming β_{kj} is fixed ($\forall k \neq i$), this is the range of values for each β_{ij} over which the current basis remains optimal [39]. In most cases, the permissible range of values for each *PD^{k-p}* coefficient includes its *MPD_E* counterpart. However, the subjective coefficients' ranges do not generally include the corresponding β_{ij} of the *PD^{k-p}* problem. A more comprehensive test of each $\underline{x}^*$ is therefore necessary before drawing any conclusions as to robustness.

The current basic feasible solution is optimal for each (maximization) problem instance as long as the vector $z_j - c_j \geq 0$ ($\forall j$). Here $z_j = c_B B^{-1} a_j$, where c_B = the vector

of basic variable cost coefficients, B = the current basis, a_j = the column of the constraint (activity) matrix for the non-basic variable j and c_j = the non-basic variable's cost coefficient. Even though the objective function value may change, as long as $z_j - c_j$ is nonnegative ($\forall j$) the current basis remains optimal [3].

The *100% Rule* [6] and [52] can often determine whether or not the current basis remains optimal for simultaneous changes in the cost coefficients of both basic and non-basic variables. Let c_k = the original cost coefficient for variable k , Δc_k = the change in c_k , I_k = the maximum allowable increase in c_k and D_k = the maximum allowable decrease in c_k (where I_k and D_k are obtained from sensitivity analysis of $\underline{x}^*$). Then, define the ratio r_k as

$$r_k^+ = \frac{\Delta c_k}{I_k} \quad \text{if} \quad \Delta c_k \geq 0 \quad (3.16a)$$

$$r_k^- = \frac{-\Delta c_k}{D_k} \quad \text{if} \quad \Delta c_k \leq 0 \quad (3.16b)$$

Obviously, either r_k^+ or $r_k^- = 0$ ($\forall k$). If $\sum_K r_k \leq 1$ (or equivalently, if $\sum_K r_k$ expressed as a percentage is $\leq 100\%$), the current basis remains optimal for the set of all variables (K).

If $\sum_K r_k > 1$, the current basis may or may not be optimal and the vector $z_j - c_j$ must be calculated. A proof of the *100% Rule* is shown in Appendix A.

The goal is to find the probability that the current basis is optimal. This is a function of the random parameter estimates (β_{ij}) in the value functions. With regard to objectively derived value functions, the least squares estimator $[X^T X]^{-1} X^T y$ is a linear

combination of the responses, normally distributed with mean vector μ and covariance matrix $\sigma^2 [X^T X]^{-1}$. The test statistic

$$\frac{[\beta - \mu]^T [X^T X] [\beta - \mu]}{p \cdot MS_E} = \frac{[\Delta\beta]^T [X^T X] [\Delta\beta]}{p \cdot MS_E}$$

specifies a joint confidence region for the regression parameters (β) and has an F distribution with p numerator and $n-p$ denominator degrees of freedom. Consequently, Equation (3.17) describes an elliptical-shaped region [23]. If used in conjunction with the *100% Rule*, the test statistic provides a lower bound on the probability that the design vector ($\underline{x}^*$) is optimal given the value functions' β_{ij} are random variables.

$$\text{Prob} \left(\frac{[\Delta\beta]^T [X^T X] [\Delta\beta]}{p \cdot MS_E} \leq F_{\alpha, p, n-p} \right) = 1 - \alpha. \quad (3.17)$$

Figure 3.8 graphically portrays some $100(1-\alpha)\%$ elliptical confidence regions for CA_2 .

Note that $[X^T X]$ is a diagonal matrix (no covariance between β_{22} and β_{32}). The diamond-shaped region depicts those values of β_{22} and β_{32} such that $\sum r_{ij} = 1$. The following mathematical program determines that joint confidence region for all coefficients where $\sum r_{ij} \leq 1$. In other words, the maximum joint confidence region whose ellipse satisfies $\sum r_{ij} \leq 1$ for all combinations of r_{ij}^+ and r_{ij}^- . The desired probability $(1 - \alpha)$ is then given by F_0 (where $F_0 = F_{\alpha, p, n-p}$). The 76% confidence region in Figure 3.8 meets this criterion.

Figure 3.8 CA_2 Elliptical Joint Confidence Regions for PD^{k-P} Problem (Instance #2)

$$\text{Minimize } F_0 \quad (3.18)$$

$$\text{subject to } \sum_i \sum_j (r_{ij}^+ + r_{ij}^-) = 1 \quad (3.19)$$

$$r_{ij}^+ \cdot r_{kj}^- = 0 \quad \forall j \text{ \& } i = k \quad (3.20)$$

$$0 \leq r_{ij}^+ \leq 1 \quad \forall i, j \quad (3.21a)$$

$$0 \leq r_{ij}^- \leq 1 \quad \forall i, j \quad (3.21b)$$

$$\text{where } F_0 = \frac{[Ir^+ - Dr^-]^T [X^T X] [Ir^+ - Dr^-]}{p \cdot MS_E} \quad (3.22)$$

$$r_{ij}^+ = \frac{\beta_{ij}^* - \hat{\mu}_{ij}}{I_{ij}} \quad \forall i, j \quad (3.23a)$$

$$r_{ij}^- = \frac{\hat{\mu}_{ij} - \beta_{ij}^-}{D_{ij}} \quad \forall i, j \quad (3.23b)$$

$$I_{ij} = U_{ij} - \hat{\mu}_{ij} \quad \& \quad D_{ij} = \hat{\mu}_{ij} - L_{ij} \quad \forall i, j \quad (3.24)$$

Confidence intervals may also be used in conjunction with the *100% Rule* to obtain a lower bound on the probability that $\underline{x}^*$ is optimal. Let $\beta_{ij} = \mu_{ij} \pm (\Delta \cdot \sigma_{ij})$ where σ_{ij} = standard error ($\hat{\beta}_{ij}$). Several methods are available for choosing Δ [38]. The Scheffe' S-method is one approach and sets

$$\Delta = (p \cdot F_{\alpha, p, n-p})^{\frac{1}{2}}. \quad (3.25)$$

The following mathematical program determines the joint confidence region which satisfies the *100% Rule* ($\sum r_{ij} = 1$) given variation in all coefficients. The desired probability $(1 - \alpha)$ is then given by S_0 (where $S_0 = F_{\alpha, p, n-p}$).

$$\text{Minimize } \Delta \quad (3.26)$$

$$\text{subject to } \sum_i \sum_j r_{ij} = 1 \quad (3.27)$$

$$0 \leq r_{ij} \leq 1 \quad \forall i, j \quad (3.28)$$

where $\Delta = (p \cdot S_0)^{1/2}$ (3.29)

$$r_{ij} = \left(\frac{\Delta \cdot \sigma_{ij}}{I_{ij}}, \frac{\Delta \cdot \sigma_{ij}}{D_{ij}} \right) \quad \forall i, j \quad (3.30)$$

$$I_{ij} = U_{ij} - \hat{\mu}_{ij} \quad \& \quad D_{ij} = \hat{\mu}_{ij} - L_{ij} \quad \forall i, j \quad (3.31)$$

Figure 3.9 CA_2 Rectangular Joint Confidence Regions for PD^{k-p} Problem (Instance #2)

Equation (3.25) describes a rectangular region for $\hat{\beta}$. Figure 3.9 graphically portrays some $100(1-\alpha)\%$ rectangular confidence regions for CA_2 . As shown, F_0 is a

more efficient statistic than S_0 . The volume of the ellipse is always less than the volume of the rectangle for the same α . However, the rectangular region is easier to construct as the number of parameters increases [38]. Unfortunately, the joint confidence regions (F_0 and S_0) provide weak lower bounds on the probability that $\underline{x}^*$ is optimal. As the number of parameters (p) increases, $F_0, S_0 \rightarrow 0$. Table 3.7 compares the values for the F_0 and S_0 joint confidence regions with two Monte Carlo simulations of $\underline{x}^*$ for the PD^{k-p} problem (Instance #2 only). Both simulations consisted of $n = 2500$ randomly generated multi-variate normal [32] vectors where $n = \frac{p(1-p)}{\sigma_{\text{Binomial}}}$. The value p is chosen so as to maximize the numerator given the desired degree of precision reflected by σ : $n = \frac{0.5^2}{0.01}$. The first simulation determined the percentage of random vectors violating the *100% Rule*. The second determined the percentage of vectors which forced a change in the current basis ($z_j - c_j < 0$). Of the 2500 randomly generated multi-variate normal vectors, $\approx 21.48\%$ failed to satisfy the constraint $\sum r_{ij} = 1$ and $\approx 4.28\%$ violated $z_j - c_j \geq 0$. As predicted, the F_0 joint confidence region provided a tighter lower bound than its S_0 counterpart (Figures A.4a and A.4b). In fact, with only five parameters the rectangular (S_0) joint confidence interval is so large that it is meaningless.

In contrast, generation of 2500 random vectors for the subjective coefficients produced a significantly lower probability that $\underline{x}^*$ is optimal for both $z_j - c_j$ and the *100% Rule*. Assuming the coefficients' values are distributed as depicted in Figure A.5, $\underline{x}^*$ of the $MPDE$ problem is much more sensitive (Table 3.7) to variation. This is to be

Lower Bounds on Prob ($\underline{x}^*$) = Optimum		
	PD^{k-p} Problem	MPD_E Problem
F_0 Statistic	18.25%	N/A
S_0 Statistic	0.90%	N/A

Simulation Results ($n = 2500$)		
	PD^{k-p} Problem	MPD_E Problem
	-Infinity = $\mu - 100\sigma$	-Infinity = -1.0E+10
$\sum r_{ij} \leq 1$ (100% Rule)	78.52%	27.72%
$z_j - c_j \geq 0$	95.72%	45.56%

Table 3.7 PD^{k-p} Problem vs. MPD_E Problem (Instance #2)

expected given the discrete nature of the subjective coefficients. For example, suppose $\beta_{ij} = 3$ and a positive relationship (more is better) exists between CA_j and EC_i . Then β_{ij} can realistically only take the values $\{1, 3 \text{ or } 9\}$ and I_{ij} could certainly be less than $6 = |9 - 3|$ or D_{ij} less than $2 = |1 - 3|$ (implying r_{ij}^+ or r_{ij}^- may be greater than 1). Considering the entire set of subjective coefficients (β), the probabilities that $\sum r_{ij} \leq 1$ and $z_j - c_j \geq 0$ can only decrease for discretely distributed parameters.

At first glance, it appears the only difference between the use of value functions (PD^{k-p} problem) versus subjective relationships (MPD_E problem) is the absence of EC_i in the derived value function for CA_j . But given larger problem instances and similar subjective errors, the likelihood of sub-optimal EC values can only increase.

Furthermore, $\underline{x}^*$ is considerably more robust when continuously distributed coefficients are employed. As such, the objective derivation of the relationship between each CA/EC appears to be a worthwhile endeavor with regard to maximizing customer satisfaction.

3.6 Summary

The use of mathematical programming effectively maximizes customer satisfaction while accounting for the impact of cost considerations. Such an approach is all the more powerful in the presence of linear value functions – which more accurately capture the customer's preferences – and independent engineering characteristics. Unfortunately, the voice of the customer is not always linear and the ECs are often related. The issues of non-linearity and correlation within the HOQ are the focal points of the next chapter.

Chapter 4

Non-Linearity & Correlation

That which is crooked cannot be made straight [43].

4.1 Introduction

Thus far, the House of Quality has been presented in fairly simplistic terms. The engineering characteristics were assumed to be uncorrelated and the relationship between each customer attribute and its supporting ECs could be accurately described by a linear function. Unfortunately, most applications of QFD are not this accommodating. Although Yoder and Mason [57] feel that value functions generally do not benefit from the addition of nonlinear terms, instances of parabolic relationships between the customer's competitive assessments and a product's characteristics occur frequently. Additionally, correlation among the ECs obfuscates the quest for optimal customer satisfaction. To this end, the mathematical programming model proposed earlier is modified to accommodate these complications.

4.2 Non-Linearity

Consider again the PD^{k-p} problem of Chapter 3. While a 1st order model (with or without interaction terms) will often suffice, the reality of numerous parameters (ECs) and/or nonlinear relationships can limit the utility of regression-derived value functions. While perfect linearity is unnecessary, the graphs of Figure 4.1 show that significant non-linearity may exist (particularly for EC_1). Unlike the value functions produced when

Figure 4.1 CA/EC Relationships for PD^{k-P} Problem

multiple engineering characteristics support a CA, these comparisons are only one-dimensional. Nevertheless, the assumption of linearity appears to have generated inaccurate 1st order polynomials for all of the CAs. This suspicion can be verified by the

addition of a center point to the 2^{3-1} design which does not influence the effect estimates but does offer protection against curvature. If the difference between the average of the responses at the factorial points and the response at the center point is large, then curvature is present [38]. Specifically, comparison of the error mean square to the sum of squares for curvature (SS_C) tests the validity of each linear value function (where $\bar{y}_F$ is the average response at the factorial design points, y_C is the response at the center point, n_F is the number of factorial points and $n_C = 1$).

$$SS_C = \frac{n_F \cdot n_C \cdot (\bar{y}_F - y_C)^2}{n_F + n_C} \quad (4.1)$$

The computations of Figure 4.2 verify the presence of response curvature and suggest that a 2nd order model might provide a better fit for the data. Note that the

$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \beta_{11} x_1^2 + \cdots + \beta_{kk} x_k^2 + \beta_{12} x_1 x_2 + \cdots + \beta_{k-1,k} x_{k-1} x_k + \varepsilon \quad (4.2)$$

model of Equation (4.2) contains $p = 1 + 2k + [(k^2 - k) \div 2]$ distinct parameters.

Additionally, each variable must be observed at three or more levels in order to estimate the quadratic terms. One common approach uses a minimum of 2^k factorial points + $2k$ axial points + 1 center point to construct a Central Composite Design (CCD) and statistically verify a 2nd order value function. In terms of the HOQ for the *pencil design* problem, this translates to greater than 15 observations when only four are available (Figure 4.3). Fortunately, designs other than the CCD are effective. These saturated (or near-saturated) 2nd order designs require less data (approximately p design points) but sacrifice efficiency in the estimation of linear and interaction coefficients. For example,

Figure 4.2 Tests for Curvature in PD^{k-p} Problem

a Small Composite Design (SCD) for $k = 3$ needs a sample size of 10 (center point not included) [40]. Structured input from the customer must still be obtained while

constructing the HOQ, but surveys and prototypes created in accordance with the design methodology suggested above are a small price to pay for objective accuracy.

Figure 4.3 Modified Pencil Design Problem

Care should be taken when selecting design points for the ECs. Extrapolation with regression equations can produce inaccurate predictions, particularly with nonlinear models. Customer perceptions from the region of optimality for each EC are crucial to model validity. Furthermore, quantification of the VOC can become operationally impractical for higher-order polynomials. The analytic effort should focus primarily on the latent attributes of lead users (customers with needs in advance of the rest of the market). In particular, lead users benefit the most from product innovations and are better able to reveal breakthrough opportunities (*exciters*).

Again using the framework of the *pencil design* problem (Figure 4.4), an experiment utilizing a SCD design ($\alpha = 1.0$) was chosen to elicit customer responses (Figure 4.5). As with the 2^{3-1} design of Chapter 3, combinations of three sizes (4", 6" and 8") of prototype pencils with different styles of lead (0.5mm, 0.7mm and 1.0mm)

Relative Importance			EC ₁	EC ₂	EC ₃
Easy to Hold	CA ₁	4
Does not Smear	CA ₂	5
Point Lasts	CA ₃	3
Cost _i			1.0	0.6	0.5
			Length of Pencil (inches)	Time between Sharpening (pages)	Lead Dust Generated (mg/cm ²)

■ / □ - STRONG Pref/ing

▲ / △ - MEDIUM Pref/ing

● / ○ - WEAK Pref/ing

Figure 4.4 Modified Pencil Design Matrix without Roof

EC ₁	EC ₂	EC ₃
-1	-1	-1
+1	+1	-1
+1	-1	+1
-1	+1	+1
- α	0	0
+ α	0	0
0	- α	0
0	+ α	0
0	0	- α
0	0	+ α
0	0	0

Figure 4.5 Small Composite Design ($k = 3$)

generated the design points. The thicker the lead, (1) the shorter the time between sharpenings and (2) the more residue produced. The mean responses (Figure 4.6) of three University of Arizona graduate students were then used to derive the customer's value functions for CA_1 , CA_2 and CA_3 . This will be referred to as the PD^{SCD} problem.

CA_1	CA_2	CA_3
1	2.5	1
5	5	5
4.5	1	1
1	3	3
2	3	2
4.5	3	2
5	2	1
5	4.5	4
5	4	2
4.5	2	1
5	3.5	2

Figure 4.6 Responses for PD^{SCD} Problem

The intercept (β_0) in Equation (4.2) is still of little interest. However, centering the data will not eliminate β_0 from consideration due to the 2nd order terms. The design points were therefore coded (but not centered) to produce values between +1 and -1 and standardize the design space (Figure 4.7). Stepwise regression analysis eliminated insignificant terms and produced 2nd order value functions (Figure B.1a through B.1c) which are statistically significant ($1.98E-06 \leq F_{sig} \leq 4.08E-05$). The engineering

Natural		
EC_1	EC_2	EC_3
4	2	2
8	6	2
8	2	4
4	6	4
4	4	4
8	4	3
6	2	3
6	6	3
6	4	2
6	4	4
6	4	3

Coded								
EC_1	EC_2	EC_3	EC_1^2	EC_2^2	EC_3^2	EC_1EC_2	EC_1EC_3	EC_2EC_3
-1	-1	-1	+1	+1	+1	+1	+1	+1
+1	+1	-1	+1	+1	+1	+1	-1	-1
+1	-1	+1	+1	+1	+1	-1	+1	-1
-1	+1	+1	+1	+1	+1	-1	-1	+1
-1	0	0	+1	0	0	0	0	0
+1	0	0	+1	0	0	0	0	0
0	-1	0	0	+1	0	0	0	0
0	+1	0	0	+1	0	0	0	0
0	0	-1	0	0	+1	0	0	0
0	0	+1	0	0	+1	0	0	0
0	0	0	0	0	0	0	0	0

Figure 4.7 Design Points for PD^{SCD} Problem

characteristics modeled adequately account for the customer's assessments ($95\% \leq$ adjusted $R^2 \leq 98\%$) too. Graphic illustrations of the design space and the quadratic

polynomials of Table 4.1 are shown in Figures B.2a through B.2c. Contrast these results with the 1st order value functions of Chapter 3 (Table 4.2).

$$CA_1 = 4.9 + (1.667)EC_1 - (1.9)EC_1^2$$

$$CA_2 = 3.2 + (1.167)EC_2 - (0.9167)EC_3 - (0.283)EC_3^2$$

$$CA_3 = 1.895 + (1.5)EC_2 + (0.7631)EC_2^2 - (0.5)EC_3 - (0.2368)EC_3^2 - (0.5)EC_2EC_3$$

Table 4.1 2nd Order Value Functions for PD^{SCD} Problem

$$CA_1 = (1.375)EC_1$$

$$CA_2 = + (0.875)EC_2 - (1.125)EC_3$$

$$CA_3 = + (1.500)EC_2 - (0.500)EC_3$$

Table 4.2 1st Order Value Functions for PD^{k-P} Problem of Chapter 3

4.3 Design Optimality

A manufacturer of industrial control equipment (Siemens Industrial Automation) recently used QFD with its product development teams [25]. Their original HOQ matrix was comprised of 40 customer attributes (rows) supported by 103 engineering characteristics (columns). Although the matrix was later reduced to 19 CAs and 63 ECs, this example emphasizes the ponderous data requirements associated with the design of experiments – a saturated design would require a minimum of $1 + 2k + [(k^2 - k) \div 2] = 2080$ observations.

By focusing on the latent attributes of lead users and subjectively eliminating CA/EC relationships, Siemens was ultimately able to define the customer's desires in terms of 3 CAs and 14 ECs. Yet the realities of cost/time constraints and nonlinear customer responses will invariably necessitate the use of saturated 2nd order designs. It is inevitable that such designs will be less efficient than standard schemes (i.e.; Central Composite and Box-Behnken designs) with regard to either the moment matrix $[X^T X]$ or scaled prediction variance $\frac{N \cdot \text{Var } \hat{y}(x)}{\sigma^2}$. However, economic alternatives such as the SCD, Koshal and Plackett-Burman designs possess many desirable properties and will produce satisfactory results under different conditions. Computer-generated designs also fall into this category and are particularly applicable to QFD.

Based upon the user's input of model, variable ranges, required sample size and other restrictions such as fixed design points, computer algorithms seek to generate the best designs in terms of alphabetic optimality/efficiency criteria. Such criteria tend to minimize the variance and correlation between parameters [40]. The most popular measures are D-optimality and G-optimality (along with their associated efficiencies). A

D-optimal design is one which maximizes $\frac{\text{Determinant}[X^T X]}{N^p}$; a small $\text{Det}[X^T X]$

indicates poor estimation of the model's coefficients (β_{ij}) since $\text{Cov}(\hat{\beta}) = \sigma^2 [X^T X]^{-1}$.

A *G-optimal* design is one for which $\text{Max}_{x \in R} \left\{ \frac{N \cdot \text{Var } \hat{y}(x)}{\sigma^2} \right\}$ is minimized. Of these two

criteria, D-optimality is by far the more widely used. The D-efficiency of a design is

defined as $\left(\frac{\text{Det}[X^T X]}{\text{Max}_{\zeta} \text{Det}[X^T X]} \right)^{1/p}$ where Max_{ζ} signifies the maximum over all designs in the class ζ .

QFD with its associated cubodial constraints lends itself nicely to the use of alphabetic optimality and computer-generated designs. Using the original *pencil design* problem as an example, four design points already exist:

	EC_1	EC_2	EC_3
Product A	5	3	3
Product B	5	2.5	2
Product C	4	5	4
Product D	5.5	6	2

EC_1	EC_2	EC_3	
-0.5	-0.5	0	
-0.5	-0.75	-1	
0	+0.5	+1	
-0.25	+1	-1	$4 \leq EC_1 \leq 8$
+1	-1	+1	$\Leftrightarrow 2 \leq EC_2 \leq 6$
-1	+1	+1	
-1	-1	+1	$2 \leq EC_3 \leq 4$
+1	+1	+1	
-1	+0.33	-1	
0	-1	0	
+1	+0.33	-1	

Figure 4-8 PD^{D-Opt} Computer-Generated Design

A design team could augment these original observations and construct a *D-optimal* design (PD^{D-Opt}) with any number of software packages. When $k = 3$ and $n = 11$, Design-Expert 5.0.4 produces the augmented design shown (coded) in Figure 4.8 for a 2nd order model. A word of caution is in order, however. Computer-generated *optimal* designs for higher-order models are usually inferior to those previously discussed (CCD, BBD, SCD, etc.). The latter produce reliable results under a wide variety of conditions; the former are optimal with respect to a single criterion. *D-optimal* designs react negatively in terms of efficiency to the use of center runs (which decrease the value of $\text{Det}[X^T X]$). For example, consider the model

$$y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{23} x_2 x_3 \quad (4.3)$$

This is the 2nd order value function for CA_3 (Table 4.1). Figure 4.9 graphically portrays the stability of $\sqrt{\text{Var } \hat{y}(x)}$ for Equation (4.3) and the Small Composite Design (PD^{SCD}). Figure 4.10 plots $\sqrt{\text{Var } \hat{y}(x)}$ for the same 2nd order model and the computer-generated *D-optimal* design (PD^{D-Opt}). As expected, the variance of the estimates for responses at the center of the *D-optimal* design is significant (Figures B.3a through B.3l show similar comparisons for CA_1 and CA_2). The addition of a center point and regeneration of the *D-optimal* design reduces this instability as shown in Figure 4.11. The design points for the *D-optimal* design with center point are shown in Figure 4.12.

Figure 4.9 Small Composite Design with Center Point

Figure 4.10 D-Optimal Design without Center Point

Figure 4.11 *D-Optimal Design with Center Point*

EC_1	EC_2	EC_3	
-0.5	-0.5	0	
-0.5	-0.75	-1	
0	+0.5	+1	
-0.25	+1	-1	$4 \leq EC_1 \leq 8$
+1	-1	+1	$\Leftrightarrow 2 \leq EC_2 \leq 6$
+1	+1	-1	
-1	0	-1	
-0.33	-1	+1	$2 \leq EC_3 \leq 4$
0	+1	0	
+1	-1	-1	
0	0	0	

Figure 4.12 *PD^{D-Opt} Computer-Generated Design with Center Point*

Comparison of the SCD with either version of the *D-optimal* design (Table 4.3) confirms that design efficiencies decrease as $\frac{\text{Var } \hat{y}(x)}{\sigma^2}$ stabilizes. Design efficiencies therefore represent only one dimension of optimality when searching for the “best” design. Nevertheless, while computer-generated designs may possess some undesirable qualities, they can be a valuable tool for developing product prototypes more costly than a pencil.

<i>Design</i>	PD^{SCD}	PD^{D-Opt}	PD^{D-Opt} w/ Center Point	Max_{ζ}
$\text{Det}[X^T X]$	9.42E + 04	1.12E + 06	5.28E + 04	6.32E + 06
$D_{\text{efficiency}}$	65.66%	84.09%	61.96%	100%

Table 4.3 Design Efficiencies for 2nd Order Model ($k = 3$, $n = 11$)

4.4 Correlation

The presence of coupling relationships (i.e.; correlation) among the engineering characteristics has a dramatic impact on customer satisfaction. If not addressed, dependence results in a duplication of effort and sub-optimal EC values. Several techniques ([36], [49] and [55]) have been offered to circumvent such dependencies. All are variations of an attempt to orthogonally transform the planning matrix [PM] – that portion of the HOQ containing β_{ij} – and thereby minimize the effects of coupled ECs. Representing the HOQ’s roof, the correlation matrix is typically coded in accordance with the subjective measures $\{\square, \Delta, \circ, \bullet, \blacktriangle, \blacksquare\} \equiv \{-9, -3, -1, 1, 3, 9\}$. After

normalization, the inner (or dot) products of these unit vectors $[CM^{Normalized}]$ are then assumed to span the design space and quantify the correlation between ECs. Finally, the impact of dependent ECs on the β_{ij} is presumably reduced via the transformation $[PM^{Normalized}] = [PM] \times [CM^{Normalized}]^{-1}$.

Such procedures are neither practical nor effective. The utilization of a correlation matrix is restricted solely to 1st order terms without interaction. Non-linearity in the customer's response frustrates any attempt to subjectively assess dependence among higher-order terms. Furthermore, while the transformation $[PM^{Normalized}] = [PM] \times [CM^{Normalized}]^{-1}$ appears to utilize the principal components of the design space, $[CM^{Normalized}]$ does not comprise the eigenvectors of the covariance matrix $[PM]^T [PM]$. In fact, $[CM^{Normalized}]$ does not even represent the correlation matrix associated with $[PM]^T [PM]$. The matrix $[PM^{Normalized}]$ therefore does not constitute an orthogonal change of the basis and fails to suppress redundant information [33].

Figure 4.13 Subjective Pencil Design HQQ

Figure 4.14 Coded Pencil Design HQQ

$$\begin{array}{cc}
 \mathbf{PM} = \begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & -9 \\ 1 & 9 & -3 \end{vmatrix} & \mathbf{CM} = \begin{vmatrix} 9 & 0 & 0 \\ 0 & 9 & -3 \\ 0 & -3 & 9 \end{vmatrix} \\
 \\
 \mathbf{CM}^{Norm} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.949 & -0.316 \\ 0 & -0.316 & 0.949 \end{vmatrix} & \mathbf{PM}^{Norm} = \begin{vmatrix} 3 & 0 & 0 \\ 0 & 0 & -9.487 \\ 1 & 9.487 & 0 \end{vmatrix}
 \end{array}$$

Figure 4.15 $[\mathbf{PM}^{Normalized}] = [\mathbf{PM}] \times [\mathbf{CM}^{Normalized}]^{-1}$

This point can more effectively be illustrated by the *pencil design* problem.

Figures 4.13 and 4.14 show the subjective and coded HOQ with correlation matrices.

Figure 4.15 displays the matrix operations connected with the transformation of $[\mathbf{PM}]$,

henceforth referred to as the transformed modified *pencil design* (MPD^T) problem. The

mathematical program was solved for three instances of the MPD^T problem where $\beta_{ij} \in$

$[\mathbf{PM}^{Normalized}]$. The results (Microsoft Excel 7.0) are summarized in Table 4.4. A

level of 100% customer satisfaction was obtained for $x_1^* = [5.5, 6.0, 2.0]$. Adding the

budget constraint decreases the customer satisfaction level to 65% for $x_2^* = [5.0, 2.5, 2.0]$.

The maximum minimum *feasible* threshold value for V_j occurs at $\pi_j = 0.43$ ($\forall j$) and x_3^*

$= [4.65, 4.02, 3.13]$; overall customer satisfaction is lowered further to 44%. Compare

these results to those obtained in Chapter 3 for the MPD problem where the issue of

correlation was ignored (Table 3.1). The EC values are virtually identical because the

transformation $[\mathbf{PM}^{Normalized}] = [\mathbf{PM}] \times [\mathbf{CM}^{Normalized}]^{-1}$ fails to address the impact

of dependent ECs. In fact, $[PM^{Normalized}]$ is not even an orthogonal basis for the design space and represents nothing more than linear combinations of linear combinations.

Transformed Modified Pencil Design (MPD^T) Problem			
	Instance #1 (Unconstrained)	Instance #2 (w/ Budget)	Instance #3 (w/ Budget & Thresholds)
$\sum \lambda_j V_j$	1.00	0.65	0.44
λ_1	0.33	0.33	0.33
λ_2	0.42	0.42	0.42
λ_3	0.25	0.25	0.25
V_1	1.00	0.67	0.43
V_2	1.00	1.00	0.44
V_3	1.00	0.06	0.43
x_1^{ec}	1.00	0.33	-0.13
x_2^{ec}	1.00	-1.00	-0.13
x_3^{ec}	-1.00	-1.00	0.13
π_1	0	0	0.43
π_2	0	0	0.43
π_3	0	0	0.43
Z_1	1.50	1.00	0.65
Z_2	3.50	0.00	1.52
Z_3	2.00	2.00	0.87
$4 \leq x_1 \leq 5.5$	5.50	5.00	4.65
$2.5 \leq x_2 \leq 6$	6.00	2.50	4.02
$2 \leq x_3 \leq 4$	2.00	2.00	3.13
$\sum c_i Z_i$	4.60	2.00	2.00
C_{Budget}	N/A	2.00	2.00

Table 4.4 MPD^T Problem Results

4.5 Revised Model

As previously stated, the relationships between the desired attributes and a product's engineering characteristics explicitly determine the level of customer satisfaction. The relationships *among* ECs implicitly determine same. While the CAs may legitimately be viewed as independent, dependent ECs must be accounted for during the process of setting optimal values. Unfortunately, transforming the correlation matrix does not address this issue. Therefore, a revision to the basic mathematical program of Chapter 3 (Section 3.2) is suggested in the form of two modifications.

The first is the incorporation of correlation coefficients (θ_{ik}) which quantitatively set the amount of change in EC_k as a result of a *unit* increase (or decrease) in EC_i . Such relationships are assumed to be strictly linear. In the *pencil design* problem for example, if EC_3 were to increase 1 mg/cm^2 then EC_2 might objectively be found to decrease by 2 pages. The corresponding correlation coefficients could then be set at $\theta_{32} = \frac{+1}{-2} = -0.5$ and $\theta_{23} = \frac{+1}{-0.5} = -2$.

The second modification acknowledges the likelihood of nonlinear value functions. In the basic model, the value functions' intercepts (β_{0j}) were eliminated by centering the data. With the addition of 2nd order terms, this is no longer possible. In the revised model, design points are simply scaled (coded) to produce values between +1 and -1 which normalizes the design space. These changes lead to the following mathematical program:

Let S = the overall level of customer satisfaction

x_i = the optimal value of EC_i

x'_i = the correlated realization of x_i

x_i^c = the normalized value of x'_i

x_i^0 = the initial value of EC_i

L_i & U_i = the technically feasible lower & upper bounds of EC_i

V_j = the satisfaction level of CA_j

π_j = the minimal acceptable satisfaction level of CA_j

λ_j = the relative importance of CA_j

Z_i = the energy (effort) expended to directly improve x_i

θ_{ik} = the change in EC_k per unit change in EC_i

V_j^{min} & V_j^{max} = the minimum & maximum values of V_j ($\forall [L_i, U_i]$)

C_p = the production budget (c_{p_i} = the production cost for EC_i)

C_D = the research and development budget (c_{D_i} = the R&D cost for EC_i)

T = the allowable development time (t_i = development time for EC_i)

$$\text{then Maximize } S = \sum_j \lambda_j \cdot V_j \quad (4.4)$$

$$\text{subject to } \pi_j \leq V_j \leq 1 \quad (\pi_j \geq 0) \quad \forall j \quad (4.5)$$

$$V_j = \frac{\beta_{0j} + \sum_i \beta_{ij} x_i^c + \sum_i \beta_{ij} (x_i^c)^2 + \sum_i \beta_{i-1,j} x_{i-1}^c x_i^c - V_j^{min}}{V_j^{max} - V_j^{min}} \quad \forall j \quad (4.6)$$

$$x'_i = x_i^0 + Z_i + \sum_{k \neq i} \theta_{ik} Z_k \quad \forall i \quad (4.7)$$

$$L_i \leq x_i \leq U_i \quad \forall i \quad (4.8)$$

$$x_i^c = \frac{x_i' - \frac{U_i + L_i}{2}}{\frac{U_i - L_i}{2}} \quad \forall i \quad (4.9)$$

$$Z_i = |x_i - x_i^0| \quad \forall i \quad (4.10)$$

$$\theta_k = \frac{\partial x_i}{\partial x_k} \quad \forall i, k \quad (4.11)$$

$$\sum_i c_{p_i} \cdot Z_i \leq C_p \quad (4.12)$$

$$\sum_i c_{D_i} \cdot Z_i \leq C_D \quad (4.13)$$

$$\sum_i t_i \cdot Z_i \leq T \quad (4.14)$$

$$\text{or} \quad \max(t_i \cdot Z_i) \leq T \quad \forall i \quad (4.15)$$

In the revised model, the Z_i are the decision variables. EC values are modified by imparting effort Z_i to EC_i . The resultant EC values (x_i') are found by solving the interdependent set in Equation (4.7). After being coded in Equation (4.9), the x_i' then determine the value functions of Equation (4.6).

4.6 Application

Consider the modified *pencil design* problem a final time. Using the 2nd order value functions (Table 4.1) with the revised model, the mathematical program was again solved for three instances of the PD^{SCD} problem where EC_2 and EC_3 are correlated. The results are summarized in Table 4.5. A level of 100% customer satisfaction was obtained for $\underline{x}_1^* = [6.88, 2.0, 2.0]$. Adding the budget constraint decreases the customer

Revised Model w/ SCD (2nd Order) Value Functions			
	<i>Instance #1</i> (Unconstrained)	<i>Instance #2</i> (w/ Budget)	<i>Instance #3</i> (w/ Budget & Thresholds)
$\sum \lambda_j V_j$	1.00	0.86	0.74
λ_1	0.33	0.33	0.33
λ_2	0.42	0.42	0.42
λ_3	0.25	0.25	0.25
V_1	1.00	0.57	0.66
V_2	1.00	1.00	0.85
V_3	1.00	1.00	0.66
x_1^c	0.44	-0.50	-0.41
x_2^c	1.00	1.00	0.62
x_3^c	-1.00	-1.00	-0.62
π_1	0	0	0.66
π_2	0	0	0.66
π_3	0	0	0.66
Z_1	2.88	1.00	1.19
Z_2	0.00	0.00	0.00
Z_3	-2.00	-2.00	-1.62
x_1^*	6.88	5.00	5.19
x_2^*	6.00	6.00	5.24
x_3^*	2.00	2.00	2.38
θ_{23}	-2	-2	-2
θ_{32}	-0.5	-0.5	-0.5
$4 \leq x_1 \leq 8$	6.88	5.00	5.19
$2 \leq x_2 \leq 6$	2.00	2.00	2.00
$2 \leq x_3 \leq 4$	2.00	2.00	2.38
$\sum c_i Z_i$	3.88	2.00	2.00
C_{Budget}	N/A	2.00	2.00

Table 4.5 PD^{SCD} Problem Results

satisfaction level to 86% for $x_2^* = [5.0, 2.0, 2.0]$. The maximum minimum *feasible*

threshold value for V_j occurs at $\pi_j = 0.66$ ($\forall j$) and $x_3^* = [5.19, 2.00, 2.38]$; overall customer satisfaction is lowered further to 74%. Compare these results with those of the PD^{k-P} problem which utilized a factorial design and ignored the issue of correlation (Table 3.5). The increase in customer satisfaction is due to a more precise modeling of the customer via nonlinear value functions and the use of correlation coefficients. Because of the relationship between EC_2 and EC_3 , the revised model was able to improve both by only changing EC_3 . The decrease in lead dust equated directly to an increase in the time between sharpenings. Therefore, no resources were directed towards improving EC_2 and could be allocated elsewhere.

4.7 Summary

The pragmatic certainties of nonlinear product assessments and dependence among the engineering characteristics serve to complicate the maximization of customer satisfaction. To ignore either defeats the purpose of QFD, however. Non-linearity in the voice of the customer can be addressed by higher-order value functions derived through the prudent use of experimental design methodology. Such a process requires extensive customer data – the price of objectivity. Correlation represents either positive or negative duplication of effort and must be harnessed to further increase customer satisfaction. Modifications to the basic mathematical program suggest the impact of both factors can be accounted for. Whether or not the revised model represents a viable tool is contingent upon its successful application to larger problem instances.

Chapter 5

Evaluation & Validation

The opposite is beneficial; from things that differ comes the fairest attunement [31].

5.1 Introduction

The preceding chapters offer a theoretical methodology for maximizing customer satisfaction with Quality Function Deployment. The purpose of this chapter is to apply the methodology to larger problem instances and evaluate its performance. Specifically, the issues of linearity, correlation, monotonicity, matrix size and density will be examined relative to the House of Quality. Sensitivity to variation in these parameters is an indicator of how efficiently the mathematical model maximizes the objective function in a constrained environment. To determine the objective function's validity, 320 unique problem instances were generated and solved for the optimal setting of each engineering characteristic (EC). Each parameter is then evaluated for significance in terms of its impact on the model's behavior and contribution to increasing the customer's satisfaction. In addition the optimal solution is compared to that of a greedy heuristic.

5.2 Test of Computational Feasibility

5.2.1 Test Design

As previously stated, this chapter considers the impact of variation in five parameters on the objective function. These parameters and their two levels are as follows:

(1) Value Functions:

- a) Nonlinear (2nd order)
- b) Linear approximation (1st order)

(2) HOQ Size:

- a) 5 CAs with 5 ECs $\Rightarrow 5 \times 5$ matrix
- b) 15 CAs with 15 ECs $\Rightarrow 15 \times 15$ matrix

(3) HOQ Density (active cells in the planning matrix):

- a) 50% & 75% $\Rightarrow 5 \times 5$ matrix
- b) 25% & 50% $\Rightarrow 15 \times 15$ matrix

(4) System Coherency:

- a) Strictly increasing value functions (monotonic)
- b) Non-monotonic value functions

5) Correlation:

- a) Positive/negative dependency among ECs (correlated)
- b) Independent ECs (uncorrelated)

Construction of a problem instance began with the generation of a distinct HOQ consisting of either a 5×5 or 15×15 planning matrix. Once matrix size was determined, the HOQ's active cells were randomly designated as active or inactive for each problem instance. The density of active cells for 5×5 matrices was either 50% or 75%; for 15×15 matrices either 25% or 50%. The higher minimum (50%) for 5×5 matrices was helpful to insure each row had at least one active cell. For each of the four combinations

of problem size and density, the ECs were randomly designated as either “+” (more is better) or “-” (less is better). This designation remained constant for all problem instances of a given size and density (e.g., 5×5 HOQ matrix with 75% active cells).

Given the particular direction of technical improvement, each CA/EC relationship was selected from the models of Figure 5.1. Note that models 4 and 5 do not increase monotonically. For non-monotonic problem instances, CA/EC(+) relationships were randomly chosen from models 1-5 and CA/EC(-) relationships from models 4-8. In the case of monotonic problem instances, CA/EC(+) relationships were limited (randomly) to models 1-3 and CA/EC(-) relationships to models 6-8. The model selected for each CA/EC relationship is shown for the eight problem templates of Figures C.1.a through C.1h. The CA/EC relationship for a particular matrix cell remained constant for all instances of a specific template (or combination of problem size, density and monotonicity). In other words, the value in the template indicates the CA/EC model used to generate β_{ij} for the corresponding cell_{ij}.

Value functions comprised of all factors (ECs) were then derived from least-squares regression in the following manner. Consider a single row (or CA) from the HOQ matrix. A random value was first generated for each EC such that with equal probability $EC_i = \{-1, -0.5, -0.1, 0, 0.1, 0.5, 1\}$; the appropriate response as a function of the assigned model (Figure 5.1) was then determined. The average response

$\left(\bar{y}_j = \sum_i^n y_{EC_i} \div n \right)$ across all ECs became the dependent variable for the design point (or

Figure 5.1 CA/EC Relationship Models

vector of random, independent values). Such an approach was utilized simply to ensure each response fell in the conventional interval [1, 5] and to avoid arbitrarily weighting one or more of the ECs. Based on the number of ECs ($n = 5$ or 15), this process was repeated to generate the requisite number of design points for a fractional Central Composite Design (CCD). In order to fit a quadratic model, Design Expert (version 5.0.4) required 160 observations for 15 factors and 32 observations for 5 factors. Once a CCD had been constructed for each matrix row, 2nd order regression equations with all terms were fitted to produce $(5 \text{ rows} \times 4 \text{ templates})_{5 \times 5} + (15 \text{ rows} \times 4 \text{ templates})_{15 \times 15} = 80$ total quadratic value functions. Eighty linear counterparts were then obtained by simply fitting a 1st order model to the same set of responses. The eight HOQs of Figures C.1a through C.1h with either quadratic or linear value functions for each CA now represented 16 unique problem templates (Figure 5.2).

Figure 5.2 Problem Templates

Assuming $\hat{\beta}_{ij} \sim N(\beta_{ij}, \hat{\sigma}^2 [X^T X]^{-1})$ where $\hat{\sigma}^2 = \text{mean square error } (MS_E)$,

random coefficients ($\hat{\beta}_{ij}$) were then generated for all value functions to create 10 distinct problem instances for every template. Based on the template's density, the cells of the

Figure 5.3 Design Cost Models

Figure 5.4 Production Cost Models

HOQ for each of these 160 problems (16 templates $\times$ 10 instances) were also randomly designated as active or inactive. If a cell's status was inactive for a specific instance, the random coefficient ($\hat{\beta}_{ij}$) was deleted from the value function. Additionally, correlation matrices (θ_{ik}) of 25% density were randomly generated for the 160 problem instances:

- (i) Instances 1-8 of each template, $\theta_{ik} \sim U[-1, +1]$, $\theta_{ii} = +1$;
- (ii) Instance 9 (strictly positive) of each template, $\theta_{ik} \sim U[0, +1]$, $\theta_{ii} = +1$;
- (iii) Instance 10 (strictly negative) of each template, $\theta_{ik} \sim U[-1, 0]$, $\theta_{ii} = +1$.

Every instance of each template was then augmented with an uncorrelated version such that $\theta_{ik} = 0$ and $\theta_{ii} = +1$. Thus, a total of 320 distinct problems ultimately existed for evaluation and validation of the mathematical model.

Finally, relative importance scalars (λ_j) along with nonlinear design and production cost functions were randomly generated for each family of problem instances. For any matrix row, $\lambda_j = \{1, 3, 5, 7, 9\}$ with equal probability. Based on the direction of technical improvement for an EC, its design and production cost functions were randomly chosen from those shown in Figures 5.3 and 5.4 (i.e.; model 1 or 2 when "more" was better, model 3 or 4 when "less" was better).

5.2.2 Analysis of Results

GAMS (Version 2.25) was employed to code and solve each problem instance. First, the maximum and minimum of each value function over the design space ($-1 \leq EC_i \leq +1$) was determined. After normalization of the value functions, the

mathematical program was then optimized subject to the constraints outlined in Table 5.1. Because of the presence of nonlinear design and production cost functions, all problem instances were optimized using CONOPT2, the NLP solver within GAMS. While arbitrary, the thresholds for the total design and production costs were chosen to avoid unconstrained problem instances.

<u>Matrix Size</u>	<u>5 × 5</u>	<u>15 × 15</u>
$\sum_i \text{Design Cost}_i$	≤ 37.50	≤ 112.50
$\sum_i \text{Production Cost}_i$	≤ 187.50	≤ 562.50

Table 5.1 Design and Production Cost Constraints

CONOPT2 is a *Generalized Reduced Gradient* (GRG) algorithm specifically designed for large nonlinear problems expressed in the form:

$$\begin{aligned}
 &\text{Min/Max} && f(\mathbf{x}) \\
 &\text{subject to} && g(\mathbf{x}) = \mathbf{b} \\
 &&& \text{Lower Bound} \leq \mathbf{x} \leq \text{Upper Bound}
 \end{aligned}$$

where f and g are differentiable nonlinear functions. CONOPT2 is based on the algorithm of Abadie and Carpenter [1]. The actual implementation has many modifications to make it more efficient for large formulations written in the GAMS language. The key steps in the GRG-based algorithm of CONOPT2 are:

- (1) Initialize and find a feasible solution.

- (2) Compute the Jacobian ($\mathbf{J}$) of the constraints.
- (3) Select a set of n basic variables ($\mathbf{x}_b$) such that $\mathbf{B}$ (the sub-matrix of basic columns from $\mathbf{J}$) is nonsingular. Factorize $\mathbf{B}$. The remaining variables ($\mathbf{x}_n$) are nonbasic.
- (4) Solve $\mathbf{B}^T \mathbf{u} = \partial f / \partial \mathbf{x}_b$ for the multipliers $\mathbf{u}$.
- (5) Compute the reduced gradient, $\mathbf{r} = \partial f / \partial \mathbf{x} - \mathbf{J}^T \mathbf{u}$. By definition, $\mathbf{r}$ will be zero for all basic variables.
- (6) If $\mathbf{r} = 0$ (or is close to zero), then stop. $\mathbf{x}$ is a Kuhn-Tucker point and is optimal.
- (7) Select the subset of nonbasic variables ($\mathbf{x}_s$) that will most profitably change in the search direction $\mathbf{d}_s$ relative to $\mathbf{r}_s$.
- (8) Perform a line search in the direction $\mathbf{d}$ such that $\mathbf{x}_s$ changes according to $\mathbf{d}_s$ and $\mathbf{x}_b$ satisfies $g(\mathbf{x}_b, \mathbf{x}_s) = \mathbf{b}$.
- (9) Return to step (2).

Depending on the specific problem to be solved, these individual steps can be more elaborate. Step (1) consists of several pre-processing activities to include some scaling procedures. The optimization steps are tailored to whether or not the problem appears to be “almost” linear. For “almost” linear problems, (i) much of the linear algebra associated with matrices $\mathbf{J}$ and $\mathbf{B}$ can be avoided; (ii) second-order information is irrelevant to step (7); and (iii) step (8) can be simplified because the optimal step is usually determined by the first variable that reaches a bound. When the problem is predominantly nonlinear, the set of basic variables will usually remain constant over

several iterations – sparse matrix algebra takes advantage of this during factorization of the basis. Furthermore, CONOPT2 improves the rate of convergence by employing a reduced Hessian matrix to utilize second-order information. Additional details about the algorithm and GAMS/CONOPT2 can be found in [14] and [15].

<i>Linearity</i>	<i>Size</i>	<i>Density</i>	<i>Monotonicity</i>	<i>Table</i>	<i>Correlation</i>	<i>Customer Satisfaction</i>	<i>EC Fulfill</i>	<i>Model Time</i>	<i>Solution Time</i>
Non-Linear	5 x 5	75%	M	D.1	C	70.5%	59.9%	5.5	17.1
					U	77.9%	60.7%	4.1	13.5
			N-M	D.2	C	79.2%	64.6%	4.0	13.3
		50%			U	83.6%	63.3%	4.2	14.2
			M	D.3	C	82.5%	70.7%	4.3	14.2
					U	85.2%	64.0%	4.0	13.7
	15 x 15	25%	N-M	D.4	C	81.8%	61.5%	5.0	15.6
					U	83.9%	60.6%	4.1	13.0
			M	D.5	C	80.9%	81.2%	25.4	82.6
		50%			U	84.2%	66.2%	25.2	90.7
			N-M	D.6	C	78.7%	69.2%	26.5	85.9
					U	81.1%	63.8%	25.4	91.4
			M	D.7	C	70.7%	79.5%	28.9	96.2
					U	71.9%	67.2%	27.9	102.7
			N-M	D.8	C	67.1%	65.4%	28.0	112.7
					U	69.6%	60.9%	28.0	119.5
Linear	5 x 5	75%	M	D.9	C	81.0%	73.3%	4.9	16.9
					U	81.2%	69.7%	3.4	11.0
			N-M	D.10	C	82.9%	65.2%	3.5	11.1
		50%			U	83.3%	63.6%	3.4	10.7
			M	D.11	C	87.5%	71.6%	4.8	16.6
					U	87.8%	65.5%	3.3	10.2
	15 x 15	25%	N-M	D.12	C	82.5%	51.4%	3.4	10.4
					U	85.0%	48.9%	3.3	9.9
			M	D.13	C	91.2%	87.7%	9.9	30.2
		50%			U	84.4%	69.5%	9.6	29.6
			N-M	D.14	C	86.5%	73.1%	11.1	34.9
					U	84.3%	62.0%	9.8	30.3
			M	D.15	C	85.3%	85.5%	11.5	37.1
					U	78.7%	73.0%	10.1	32.3
			N-M	D.16	C	79.9%	69.4%	11.4	36.2
					U	76.9%	62.6%	10.4	33.7

Table 5.2 Summary of Results (Averages for 10 Problem Instances)

The results for each problem instance are presented in Appendix D and summarized above in Table 5.2. Figures 5.5 and 5.6 show the organization of the tables in Appendix D. Each table reports the objective function (*Customer Satisfaction*) and

Figure 5.5 Organization of Nonlinear Problem Instances (Appendix D)

Figure 5.6 Organization of Linear Problem Instances (Appendix D)

associated fulfillment of the EC_i . Fulfillment is a measure of how closely the realized value of an EC approaches its preferred setting (-1, 0, or +1). For correlated problem

instances, the realized EC_i values reflect the impact of dependent engineering characteristics. Two execution times are also reported. *Modeling* time is the period needed by GAMS for problem setup, to include determining the maximum and minimum of all value functions. *Elapsed* time is the interval CONOPT2 required to optimize the mathematical model. These times are reported in actual seconds (*Pentium 75*). For economy of presentation, the random designations (response, design and production cost models plus active cells), coefficients ($\hat{\beta}_{ij}$), vectors of relative importance (λ_j), and the correlation matrix for each problem instance are not included in Appendix D. However, all data are readily available upon request.

The first column (*Customer Satisfaction*) of Table 5.2 is graphically portrayed in Figure 5.7. Note this is the average value ± 2.24 standard deviations ($\alpha = 0.025$) of the objective function for 20 problem instances (correlated & uncorrelated) and is indexed to the relevant table in Appendix D. While inconclusive, it appears the mathematical program returns a higher level of customer satisfaction for certain problem templates. To investigate this issue further, each parameter was tested for significance via an analysis of variance (ANOVA). The results are shown in Table 5.3. Of the five parameters, three are significant – linearity, matrix size and density. Conversely, monotonicity and correlation did not singularly effect the objective function. Interaction among factor pairs is fairly extensive too.

Since the mathematical program was specifically formulated to account for dependency among the ECs, the insignificance of correlation is disturbing. Fortunately,

Figure 5.7 Average Customer Satisfaction Achieved by Mathematical Program

<i>Factor</i>	<i>Name</i>	<i>-1 Level</i>	<i>+1 Level</i>
A	Linearity	Linear	Non-Linear
B	Size	5 x 5	15 x 15
C	Density	Sparse	Dense
D	Monotonicity	Monotonic	Non-Monotonic
E	Correlation	Uncorrelated	Correlated

<i>Source</i>	<i>Sum of Squares</i>	<i>DF</i>	<i>Mean Square</i>	<i>F</i>	<i>Prob > F</i>
Model	0.9994	31	0.0322	7.0553	<0.0001
Residual	1.3161	288	0.0046		
Total	2.3155	319			

<i>Factor</i>	<i>Coefficient Est.</i>	<i>Stand. Error</i>	<i>t for H₀</i>	<i>Prob > t </i>
Intercept	0.8084	0.0038		
A-Linearity	-0.0280	0.0038	-7.3972	<0.0001
B-Size	-0.0139	0.0038	-3.6874	0.0003
C-Density	-0.0337	0.0038	-8.9089	<0.0001
D-Monotonicity	-0.0045	0.0038	-1.1784	0.2396
E-Correlation	-0.0034	0.0038	-0.9055	0.3659
AB	-0.0114	0.0038	-3.0126	0.0028
AC	-0.0087	0.0038	-2.2965	0.0224
AD	0.0054	0.0038	1.4315	0.1534
AE	-0.0129	0.0038	-3.4030	0.0008
BC	-0.0108	0.0038	-2.8638	0.0045
BD	-0.0100	0.0038	-2.6455	0.0086
BE	0.0091	0.0038	2.4040	0.0168
CD	0.0078	0.0038	2.0732	0.0390
CE	-0.0006	0.0038	-0.1712	0.8642
DE	-0.0024	0.0038	-0.6277	0.5307
ABC	-0.0037	0.0038	-0.9783	0.3287
ABD	-0.0048	0.0038	-1.2760	0.2030
ABE	-0.0047	0.0038	-1.2413	0.2155
ACD	0.0020	0.0038	0.5350	0.5930
ACE	-0.0025	0.0038	-0.6574	0.5114
ADE	0.0042	0.0038	1.1222	0.2627
BCD	-0.0098	0.0038	-2.5859	0.0102
BCE	0.0023	0.0038	0.6210	0.5351
BDE	-0.0031	0.0038	-0.8179	0.4141
CDE	0.0010	0.0038	0.2688	0.7883
ABCD	-0.0009	0.0038	-0.2440	0.8074
ABCE	0.0033	0.0038	0.8791	0.3801
ABDE	0.0005	0.0038	0.1348	0.8929
ACDE	-0.0009	0.0038	-0.2307	0.8177
BCDE	-0.0018	0.0038	-0.4871	0.6266
ABCDE	-0.0011	0.0038	-0.2886	0.7731

Table 5.3 Average Customer Satisfaction ANOVA

this seemingly counterintuitive behavior by the model can be traced to the manner in which problem instances were generated. Once the value functions' coefficients ($\hat{\beta}_{ij}$) for each problem template had been derived, a unique set of coefficients ($\hat{\beta}'_{ij}$) was randomly generated for each instance within that template such that $\hat{\beta}'_{ij} \sim N(\hat{\beta}_{ij}, \hat{\sigma}^2)$. The active cells for each problem instance were separately and randomly designated based on the template's density. Finally, the actual value functions for a problem instance were constructed using only those coefficients associated with active cells. The consequence of sequentially inducing coefficient variation and eliminating some $\hat{\beta}'_{ij}$ was a set of value functions that did not necessarily span the design space for all ECs – an outcome not normally associated with actual data (as shown in the next chapter). The following example should illustrate.

<i>Fitted</i> β	(+)	+	+	-	+	-	Max V_j	Min V_j
	x_0	x_1	x_2	x_3	x_4	x_5		
CA_1	3.37	0.38	0.54	-0.38	0.42	-0.04	5.13	1.61
CA_2	3.14	0.46	0.28	-0.48	0.14	-0.14	4.64	1.63
CA_3	2.75	0.49	0.50	-0.37	0.25	-0.31	4.68	0.83
CA_4	3.55	0.27	0.52	-0.33	0.40	-0.26	5.33	1.77
CA_5	3.07	0.27	0.41	-0.50	0.37	-0.35	4.98	1.17

Table 5.4 $\hat{\beta}_{ij}$ for L-5-50-M Problem Instances

Table 5.4 shows the derived coefficients ($\hat{\beta}_{ij}$) for the problem template L-5-50-M: linear (or 1st order), 5×5 , 50% density and monotonic. Table 5.5 shows two hypothetical problem instances where the ECs are uncorrelated ($\theta_{ik} = 0$ and $\theta_{ii} = 1$).

Active	x_0	x_1	x_2	x_3	x_4	x_5	λ_j
CA ₁	*					*	7
CA ₂	*			*			5
CA ₃	*	*	*	*			7
CA ₄	*	*	*		*	*	1
CA ₅	*	*	*	*		*	1

Active	x_0	x_1	x_2	x_3	x_4	x_5	λ_j
CA ₁	*					*	1
CA ₂	*			*			1
CA ₃	*	*	*		*		1
CA ₄	*	*	*			*	9
CA ₅	*	*	*		*	*	9

Random	(+)	+	+	-	+	-	Max	Min
β	x_0	x_1	x_2	x_3	x_4	x_5	V_j	V_j
CA ₁	3.34	0.51	0.51	-0.35	0.37	0.05	5.04	1.64
CA ₂	3.12	0.69	0.31	-0.30	-0.03	0.03	4.36	1.89
CA ₃	2.85	0.55	0.45	-0.37	0.09	-0.30	4.60	1.10
CA ₄	3.52	0.18	0.73	-0.15	0.38	-0.54	5.50	1.55
CA ₅	3.19	0.07	0.46	-0.20	0.35	-0.32	4.59	1.79

Random	(+)	+	+	-	+	-	Max	Min
β	x_0	x_1	x_2	x_3	x_4	x_5	V_j	V_j
CA ₁	3.34	0.51	0.51	-0.35	0.37	0.05	5.04	1.64
CA ₂	3.12	0.69	0.31	-0.30	-0.03	0.03	4.36	1.89
CA ₃	2.85	0.55	0.45	-0.37	0.09	-0.30	4.60	1.10
CA ₄	3.52	0.18	0.73	-0.15	0.38	-0.54	5.50	1.55
CA ₅	3.19	0.07	0.46	-0.20	0.35	-0.32	4.59	1.79

Active	(+)	+	+	-	+	-	Max	Min
β	x_0	x_1	x_2	x_3	x_4	x_5	V_j	V_j
CA ₁	3.34					0.05	3.39	3.29
CA ₂	3.12			-0.30			3.42	2.82
CA ₃	2.85	0.55	0.45	-0.37			4.22	1.48
CA ₄	3.52	0.18	0.73		0.38	-0.54	5.35	1.69
CA ₅	3.19	0.07	0.46	-0.20		-0.32	4.24	2.14

Active	(+)	+	+	-	+	-	Max	Min
β	x_0	x_1	x_2	x_3	x_4	x_5	V_j	V_j
CA ₁	3.34					0.05	3.39	3.29
CA ₂	3.12			-0.30			3.42	2.82
CA ₃	2.85	0.55	0.45		0.09		3.94	1.76
CA ₄	3.52	0.18	0.73			-0.54	4.97	2.07
CA ₅	3.19	0.07	0.46		0.35	-0.32	4.38	1.99

Customer Satisfaction > EC Fulfillment							Overall	Avg
Realized	x_0	x_1	x_2	x_3	x_4	x_5	Satisfy	Fulfill
x_1	(1)	1	1	-1	1	1	97.1%	80.0%

Customer Satisfaction < EC Fulfillment							Overall	Avg
Realized	x_0	x_1	x_2	x_3	x_4	x_5	Satisfy	Fulfill
x_1	(1)	1	1	-1	1	1	72.6%	80.0%

Table 5.5 Random Variance in Achieved Customer Satisfaction (ECs Uncorrelated)

The average EC fulfillment (*Avg Fulfill*) for the optimal vector $\underline{x}^* = [+1, +1, -1, +1, +1]$ is $80\% \left(\frac{100_{EC_1\%} + 100_{EC_2\%} + 100_{EC_3\%} + 100_{EC_4\%} + 0_{EC_5\%}}{5} = 80 \right)$. Note that the same set of revised value functions ($\hat{\beta}'_{ij}$) are used for both instances and the cost constraints have been omitted. Yet despite varying only the active cells and the vector of relative importance (λ_j), the fluctuation of the objective function (*Overall Satisfy*) for the same optimal vector ($\underline{x}^*$) is extraordinary. Consider that within any problem template, the introduction of correlation among the ECs, cost constraints and different $\hat{\beta}'_{ij}$ further nullifies the relationship between *Customer Satisfaction* and *EC Fulfillment*. While the objective function for each instance does not represent an affine combination of *EC Fulfillment*, testing the model's computational feasibility and examining its performance was the purpose of this problem set (as opposed to verifying the tenets of linear algebra and regression). In order to isolate the model's treatment of correlation, *EC Fulfillment* provides an alternative measure of the mathematical program's performance. As described previously, fulfillment is the average percentage the ECs realize relative to their technical goal ($-1, 0$, or $+1$) in the optimal solution for each instance. The second column (*EC Fulfill*) of Table 5.2 is graphically portrayed in Figure 5.8. This is the average value ± 2.24 standard deviations ($\alpha = 0.025$) for 10 problem instances and is indexed to the relevant table in Appendix D. The results of the ANOVA are shown in Table 5.6. Of the five parameters, four are significant – linearity, matrix size, monotonicity and correlation. Matrix density did not singularly affect *EC Fulfillment*.

Figure 5.8 Average EC Fulfillment Achieved by Mathematical Program

<i>Factor</i>	<i>Name</i>	<i>-1 Level</i>	<i>+1 Level</i>
A	Linearity	Linear	Non-Linear
B	Size	5 x 5	15 x 15
C	Density	Sparse	Dense
D	Monotonicity	Monotonic	Non-Monotonic
E	Correlation	Uncorrelated	Correlated

<i>Source</i>	<i>Sum of Squares</i>	<i>DF</i>	<i>Mean Square</i>	<i>F</i>	<i>Prob > F</i>
Model	2.17	31	0.0700	19.56	< 0.0001
Residual	1.03	288	0.0036		
Total	3.21	319			

<i>Factor</i>	<i>Coefficient Est.</i>	<i>Stand. Error</i>	<i>t for H₀</i>	<i>Prob > t </i>
Intercept	0.6700	0.0033		
A-Linearity	-0.0100	0.0033	-3.120	0.0020
B-Size	0.0380	0.0033	11.390	< 0.0001
C-Density	0.0052	0.0033	1.550	0.1214
D-Monotonicity	-0.0440	0.0033	-13.030	< 0.0001
E-Correlation	0.0340	0.0033	10.060	< 0.0001
AB	-0.0079	0.0033	-2.370	0.0183
AC	-0.0150	0.0033	-4.470	< 0.0001
AD	0.0190	0.0033	5.580	< 0.0001
AE	-0.0053	0.0033	-1.580	0.115
BC	-0.0110	0.0033	-3.290	0.0011
BD	-0.0085	0.0033	-2.540	0.0116
BE	0.0200	0.0033	5.980	< 0.0001
CD	0.0100	0.0033	3.010	0.0029
CE	-0.0076	0.0033	-2.260	0.0247
DE	-0.0120	0.0033	-3.680	0.0003
ABC	0.0120	0.0033	3.490	0.0006
ABD	-0.0100	0.0033	-2.980	0.0032
ABE	-0.0018	0.0033	-0.520	0.6009
ACD	-0.0013	0.0033	-0.380	0.7032
ACE	0.0008	0.0033	0.230	0.8154
ADE	-0.0009	0.0033	-0.260	0.7913
BCD	-0.0160	0.0033	-4.930	< 0.0001
BCE	-0.0009	0.0033	-0.280	0.7799
BDE	-0.0065	0.0033	-1.950	0.0519
CDE	0.0039	0.0033	1.180	0.2408
ABCD	0.0004	0.0033	0.130	0.8997
ABCE	0.0032	0.0033	0.960	0.3397
ABDE	-0.0021	0.0033	-0.610	0.5394
ACDE	0.0020	0.0033	0.590	0.5551
BCDE	-0.0020	0.0033	-0.590	0.5565
ABCDE	-0.0018	0.0033	-0.540	0.5911

Table 5.6 Average EC Fulfillment ANOVA

SUMMARY				
<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>EC Average</i>	<i>Variance</i>
<i>Correlated</i>	160	112.918	70.6%	0.013
<i>Uncorrelated</i>	160	102.140	63.8%	0.005

ANOVA					
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>
Between Groups	0.363	1	0.363	40.582	6.6E-10
Within Groups	2.844	318	0.009		
Total	3.207	319			

Table 5.7 Average EC Fulfillment ANOVA for Correlation

SUMMARY				
<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>EC Average</i>	<i>Variance</i>
<i>5 x 5 (C only)</i>	80	53.613	67.0%	0.006
<i>15 x 15 (C only)</i>	80	59.304	74.1%	0.017

ANOVA					
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>
Between Groups	0.202	1	0.202	17.599	4.5E-05
Within Groups	1.817	158	0.012		
Total	2.020	159			

Table 5.8 Average EC Fulfillment ANOVA for Size (Correlated Instances only)

SUMMARY				
<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>EC Average</i>	<i>Variance</i>
<i>5 x 5 (U only)</i>	80	50.844	63.6%	0.004
<i>15 x 15 (U only)</i>	80	51.296	64.1%	0.006

ANOVA					
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>
Between Groups	0.001	1	0.001	0.245	0.6210
Within Groups	0.823	158	0.005		
Total	0.824	159			

Table 5.9 Average EC Fulfillment ANOVA for Size (Uncorrelated Instances only)

First consider the issue of correlation. Tables 5.7 through 5.9 show that for this problem set the model accentuates the presence of positive correlation and de-emphasizes negative correlation with regard to average EC fulfillment (*EC Average*: 67.0% $\rightarrow$ 63.6% & 74.1% $\rightarrow$ 64.1%). This is accomplished more easily in larger problem instances (Δ *EC Average*: 3.4% vs. 10.0%). All correlation matrices had a density of 25%; 15×15 matrices therefore provided the model with more degrees of freedom for circumventing negative correlation.

The significance of monotonicity is puzzling and can be traced to the fact that quadratic value functions generally provided the best fit of non-monotonic data while linear value functions predicted monotonic instances better. Non-monotonic problem instances reflect the presence of saddle points in the design space. Consider the HOQ templates of Figures C.1g and C.1h. A comparison of EC_1 and EC_3 from CA_5 is shown below (Figure 5.9) for the fitted quadratic regression models. The response surface for

Figure 5.9 2nd Order Models for 5×5 HOQ (@ 75% Density)

CA_5 (monotonic) is best described by a linear model; a quadratic model predicts CA_5 (non-monotonic) better. This is the case for the majority of CAs in the HOQ templates. Furthermore, the ANOVA of Table 5.10 reveals that the addition of quadratic terms slightly decreases average EC fulfillment (*EC Average*: 74.5% $\rightarrow$ 68.7%) for monotonic value functions.

SUMMARY				
<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>EC Average</i>	<i>Variance</i>
<i>Non-Linear (M only)</i>	80	54.925	68.7%	0.011
<i>Linear (M only)</i>	80	59.583	74.5%	0.008

ANOVA					
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>
Between Groups	0.136	1	0.136	14.239	2.3E-04
Within Groups	1.505	158	0.010		
Total	1.641	159			

Table 5.10 Average *EC Fulfillment* ANOVA for Linearity (Monotonic Instances only)

In fact, the use of quadratic value functions within this problem set actually decreases the mathematical model's performance where maximizing *EC Fulfillment* is concerned. Based on the best regression fit (Figures C.1a through C.1h), a quadratic model was essentially valid for non-monotonic instances only. However, a comparison of these problems with their linear approximations shows 1st order models to be more effective. The ANOVA in Table 5.11 indicates linear models provide a robust estimation of the nonlinear response surfaces within this design space (*EC Average*: 63.7% $\rightarrow$ 62.0%). Figure 5.10 shows that in all but one case the 1st order models meet or exceed the average realization of their 2nd order counterpart. Finally, linearity as a parameter

ceases to be of significance ($P\text{-value} = 0.1801$) in terms of model consistency. This suggests that comparable results might be obtained beyond these data through the exclusive use of 1st order models.

SUMMARY				
Groups	Count	Sum	EC Average	Variance
Non-Linear (NM only)	80	50.934	63.7%	0.003
Linear (NM only)	80	49.616	62.0%	0.009

ANOVA					
Source of Variation	SS	df	MS	F	P-value
Between Groups	0.011	1	0.011	1.812	0.1801
Within Groups	0.947	158	0.006		
Total	0.958	159			

Table 5.11 Average EC Fulfillment ANOVA for Linearity (Non-Monotonic Instances only)

Figure 5.10 Average EC Fulfillment (Non-Monotonic Instances only)

5.3 Comparison of Mathematical Program and Greedy Heuristic

As cited in Chapter 2, numerous techniques for optimizing customer satisfaction have been suggested for use in conjunction with QFD. Most of these processes are qualitative in nature. Those quantitative algorithms which are documented typically treat the subjective relationships of the HOQ as linear functions and then combine a normalization of the matrix with various heuristics to systematically allocate resources. Transformation of the HOQ is undertaken to address the effects of correlation and rank the ECs in terms of their technical importance.

$$[\nu] = [\theta]^{Normalized} \quad (5.1)$$

$$[\gamma] = [\nu]^T [\nu] \quad (5.2)$$

$$R_{ij}^{Normalized} = \frac{\sum_k R_{kj} \cdot \gamma_{ik}}{\sum_i \sum_k R_{ij} \cdot \gamma_{ik}} \quad \forall j \quad (5.3)$$

$$w_i = \sum_j \lambda_j \cdot |R_{ij}^{Normalized}| \quad \forall i \quad (5.4)$$

$$Index_i = \frac{w_i}{c_i} \quad (5.5)$$

where θ_{ik} = correlation between EC_i and EC_k
 ν_i = unit vectors of θ_i
 γ_{ik} = orthogonal transformation of θ_{ik}
 R_{ij} = quantified relationship between EC_i and CA_j
 w_i = relative technical importance of EC_i
 c_i = cost per unit improvement of EC_i

Equations (5.1) through (5.4) list the sequential computations which comprise the normalization procedure of Wasserman [49]. After orthogonal transformation of the HOQ, Wasserman suggests the allocation of resources for optimally setting the final EC values be based on the index $\frac{w_i}{c_i}$ of Equation (5.5) and a relaxed version of the *Knapsack* problem. This greedy heuristic attempts to maximize the contents of a container (knapsack), given an assortment of items (each of varying size and inherent value) and an overall capacity bound. The *Knapsack* problem normally is restricted to integer values; in this case, that restriction is relaxed for the final item chosen such that the capacity (resource) constraint is tight.

θ_{ik}	0	1	2	3	4	5
0	1					
1		1	0.185			
2			1			0.622
3		0.152		1		
4			-0.967		1	
5				-0.085		1

v_{ik}	0	1	2	3	4	5
0	1					
1		0.989	0.132			
2			0.712			0.528
3		0.150		0.996		
4			-0.689		1	
5				-0.084		0.849

γ_{ik}	0	1	2	3	4	5
0	1					
1		1	0.130	0.149		
2		0.130	1		-0.689	0.376
3		0.149		1		-0.072
4			-0.689		1	
5			0.376	-0.072		1

$\beta_{ij} =$						
R_{ij}	x_0	x_1	x_2	x_3	x_4	x_5
CA_1	3.384		0.698		0.486	0.014
CA_2	3.137		0.287	-0.562		-0.168
CA_3	2.698	0.515	0.453		0.276	
CA_4	3.517	0.374				
CA_5	2.974	0.406			0.434	-0.347

Table 5.12 Heuristic Transformation Matrices (L-5-50-M-1C)

R_{ij}^{Norm}	λ_j	x_0	x_1	x_2	x_3	x_4	x_5
CA ₁	7	0.821	0.022	0.089	-0.0002	0.001	0.067
CA ₂	9	1.232	-0.018	0.088	-0.216	-0.078	-0.008
CA ₃	5	0.708	0.150	0.087	0.020	-0.010	0.045
CA ₄	3	0.880	0.094	0.012	0.014		
CA ₅	9	0.937	0.128	-0.119	0.027	0.137	-0.109
w_i		31.44	2.50	2.95	2.33	1.98	1.75
c_i		-	47.5	47.5	47.5	47.5	47.5
w_i/c_i		-	0.053	0.062	0.049	0.042	0.037
Rank _i		-	2	1	3	4	5

L-5-50-M-1C	+	+	-	+	-	Overall	EC	$\Sigma \$$	
Solution	x_1	x_2	x_3	x_4	x_5	Satisfy	Fulfill	Design	Production
Heuristic	1	1	-1	-0.155	1	74.8%	68.5%	29.17	187.50
Optimal	0.459	1	-1	-0.033	-0.915	81.3%	83.4%	37.50	186.32

Table 5.13 Normalized HOQ and Optimal EC Settings (L-5-50-M-1C)

For example, applying these steps to the randomly generated data (θ_{ik} and $\hat{\beta}'_{ij}$ matrices) from the first instance of problem template L-5-50-M (correlated) gives the normalized results of Table 5.12. Application of the greedy heuristic is depicted in Table 5.13. Note the presence of the intercept term for each value function ($\hat{\beta}'_{0j}$) and that the cost factor for the generated problem set is a constant. Both can be ignored when ordinally ranking the ECs and setting their final, optimal value.

In order to better measure the mathematical program's efficiency, the results of the 160 linear problem instances will be contrasted to those obtained by transforming the HOQ and setting EC levels via this greedy heuristic. The two methodologies will be referred to simply as "optimal" and "heuristic" for comparison purposes. The nonlinear

problem instances are excluded because none of the cited transformation procedures will admit interaction or 2nd order terms. That cost constraint (Table 5.1) which was met first by the heuristic represented the overall capacity bound for the knapsack. In almost all

cases, this was the cost of design $\left(\sum_i \text{Design Cost}_i \leq \begin{Bmatrix} \$37.50 \\ \$112.50 \end{Bmatrix} \right)$. Table 5.14

summarizes the results of applying the normalization procedure and heuristic to the linear problem instances.

Size	Density	Monotonicity	Table	Correlation	Customer	EC
					Satisfaction	Fulfill
5 x 5	75%	M	D.9	C	76.5%	36.7%
				U	74.8%	42.7%
		N-M	D.10	C	74.2%	60.3%
				U	74.5%	60.8%
	50%	M	D.11	C	81.6%	62.3%
				U	87.8%	61.8%
		N-M	D.12	C	69.9%	48.9%
				U	74.2%	46.6%
15 x 15	25%	M	D.13	C	71.8%	62.4%
				U	78.6%	61.8%
		N-M	D.14	C	72.1%	58.1%
				U	76.8%	59.7%
	50%	M	D.15	C	66.4%	55.0%
				U	72.9%	49.4%
		N-M	D.16	C	66.1%	56.2%
				U	71.4%	57.7%

Table 5.14 Summary of Heuristic Results (Averages for 10 Problem Instances)

Figure 5.11 graphically compares the heuristic and model results in terms of *EC Fulfillment* (average value $\pm 2.24\sigma$). For this problem set, the model unilaterally outperforms the heuristic. The difference is especially large for correlated problem instances. As will be discussed shortly, this is because the heuristic performs similarly for both correlated and uncorrelated instances within each template. An ANOVA for the

Figure 5.11 Average EC Fulfillment Achieved by Heuristic & Model (Linear Instances Only)

<i>Factor</i>	<i>Name</i>	<i>-1 Level</i>	<i>+1 Level</i>
A	Methodology	Heuristic	Model
B	Size	5 x 5	15 x 15
C	Density	Sparse	Dense
D	Monotonicity	Monotonic	Non-Monotonic
E	Correlation	Uncorrelated	Correlated

<i>Source</i>	<i>Sum of Squares</i>	<i>DF</i>	<i>Mean Square</i>	<i>F</i>	<i>Prob > F</i>
Model	3.86	31	0.1200	24.86	< 0.0001
Residual	1.44	288	0.0050		
Total	5.30	319			

<i>Factor</i>	<i>Coefficient Est.</i>	<i>Stand. Error</i>	<i>t for H₀</i>	<i>Prob > t </i>
Intercept	0.6200	0.0040		
A-Methodology	0.0660	0.0040	16.710	< 0.0001
B-Size	0.0360	0.0040	9.000	< 0.0001
C-Density	-0.0033	0.0040	-0.840	0.3997
D-Monotonicity	-0.0260	0.0040	-6.580	< 0.0001
E-Correlation	0.0190	0.0040	4.880	< 0.0001
AB	0.0100	0.0040	2.650	0.0085
AC	0.0240	0.0040	5.940	< 0.0001
AD	-0.0360	0.0040	-9.160	< 0.0001
AE	0.0200	0.0040	4.970	< 0.0001
BC	-0.0130	0.0040	-3.220	0.0014
BD	-0.0024	0.0040	-0.610	0.5391
BE	0.0130	0.0040	3.290	0.0011
CD	0.0330	0.0040	8.240	< 0.0001
CE	-0.0055	0.0040	-1.380	0.1690
DE	-0.0063	0.0040	-1.600	0.1113
ABC	-0.0100	0.0040	-2.520	0.0122
ABD	0.0039	0.0040	0.990	0.3252
ABE	0.0088	0.0040	2.210	0.0276
ACD	-0.0210	0.0040	-5.370	< 0.0001
ACE	-0.0029	0.0040	-0.730	0.4661
ADE	-0.0051	0.0040	-1.300	0.1962
BCD	-0.0250	0.0040	-6.440	< 0.0001
BCE	0.0025	0.0040	0.620	0.5359
BDE	-0.0074	0.0040	-1.880	0.0613
CDE	0.0006	0.0040	0.160	0.8726
ABCD	0.0086	0.0040	2.170	0.0312
ABCE	-0.0066	0.0040	-1.670	0.0969
ABDE	0.0030	0.0040	0.750	0.4562
ACDE	0.0013	0.0040	0.330	0.7385
BCDE	-0.0028	0.0040	-0.700	0.4853
ABCDE	0.0026	0.0040	0.660	0.5125

Table 5.15 Average EC Fulfillment ANOVA (Heuristic vs. Model)

two approaches is shown in Table 5.15. Of the five parameters, four are significant – methodology, correlation, monotonicity and matrix size. Again, matrix density did not singularly effect the results.

Table 5.16 summarizes the disparity between the model and heuristic for this problem set. The heuristic with its associated transformation procedure returns significantly lower levels of *EC Fulfillment* with the same variance. Admittedly, it is difficult to separate the impact of matrix normalization from that of the heuristic on *EC Fulfillment*. Allocation schemes other than the greedy approach might obtain results which more closely parallel the mathematical program.

SUMMARY					
<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>EC Average</i>	<i>Variance</i>	
<i>Heuristic</i>	160	88.049	55.0%	0.012	
<i>Model</i>	160	109.199	68.2%	0.012	

ANOVA					
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>
Between Groups	1.398	1	1.398	113.888	6.3E-23
Within Groups	3.903	318	0.012		
Total	5.301	319			

Table 5.16 Average *EC Fulfillment* ANOVA for Methodology

However, Table 5.17 suggests that matrix normalization fails to address the issue of dependency among the ECs . . . there is no significant difference between the correlated and uncorrelated results for heuristic problem instances. While there is no benefit gained here by denouncing matrix normalization, these results suggest that the mathematical program may consider the issue of correlation more efficiently.

SUMMARY				
<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>EC Average</i>	<i>Variance</i>
<i>Correlated (Heuristic)</i>	80	43.995	55.0%	0.013
<i>Uncorrelated (Heuristic)</i>	80	44.054	55.1%	0.011

ANOVA					
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>
Between Groups	2.2E-05	1	2.2E-05	0.002	0.9663
Within Groups	1.943	158	0.012		
Total	1.943	159			

Table 5.17 Average *EC Fulfillment* ANOVA for Correlation (Heuristic Only)

5.4 Summary

The author does not consider the problem set and associated results of this chapter to be an exhaustive test of the proposed mathematical model. Instead, two issues emerge as candidates for additional research. The first is the mathematical model's ability to account for dependency among the ECs. Relative to the problem set presented here, the results are most encouraging: the mathematical model appears to optimize positively and negatively correlated ECs in an efficient manner. The second is the robustness of linearly approximated value functions. If such approximations consistently return the levels of *EC Fulfillment* associated with these problem instances, this methodology becomes all the more feasible. The practitioner need not be concerned with the expensive and time consuming process of constructing higher-order response surfaces to derive the customer's value functions. Nevertheless, determining when linear models fail to sufficiently approximate higher-order value functions and therefore degrade the mathematical program's performance is a topic requiring further investigation.

Chapter 6

An Industrial Case Study

Proverbs contradict each other [34].

6.1 Introduction

By maximizing customer satisfaction, the proposed mathematical model explicitly seeks to address an organization's market competitiveness. Goods or services which are more attractive to the customer ultimately help to increase revenues. This chapter presents an application of the methodology to actual industrial data. The results are then compared to those product planning and development decisions made in conjunction with a strictly subjective utilization of Quality Function Deployment (QFD).

This example was obtained from a large defense contractor in Tucson, Arizona – henceforth referred to as *the company*. Due to proprietary considerations, all data have been sanitized and do not represent current technology. The product design is prototypical in nature and is presented solely for illustrative purposes. Nevertheless, the conclusions drawn are based entirely on real analysis conducted by the company in support of achieving the highest level of customer contentment subject to organizational and market constraints.

6.2 Background

As part of the defense industry's efforts to plan strategically for the next century, the company employed QFD to help design a generic guidance system for future

generations of missiles. The guidance system of a missile is the primary determinant of both its effectiveness and cost. As such, the company focused on this dominant component in its application of QFD. Figure 6-1 shows the subjective House of Quality (HOQ) used in their design process, to include the technically feasible range (*High/Low*) and direction of improvement ($\uparrow/\downarrow$) for each Engineering Characteristic (EC).

Figure 6.1 Sanitized HOQ from Industry

In order to apply the mathematical model to this data, the customer's preferences had to be derived empirically. Sixteen model systems already existed as part of the company's research and development efforts. These 16 prototypes had been constructed

θ_{ik}	1	2	3	4	5	6	7	8	9	10
1	1	0.800	-0.250	-0.200			-0.408			
2	0.400	1								
3	-0.400		1	0.200		-0.080				-0.202
4	-0.400		0.250	1		-0.080				-0.202
5					1					
6			-0.250	-0.200		1			-2.220	
7	-0.400						1			-0.404
8								1	-2.220	
9						-0.080		-0.001	1	-0.404
10			-0.250	-0.200			-0.816		-4.440	1

	(+)	-	-	+	+	+	+	-	+	-	-
β_{ij}	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
CA_1	2.252	-0.821	-0.323	0.350	0.323						
CA_2	2.486	-0.102	-1.084	0.392	0.361						
CA_3	1.804		-0.218	0.711	0.655		0.246	-0.218			
CA_4	2.401							-0.475	0.476	-0.476	-0.462
CA_5	1.206						1.587			-0.469	
CA_6	1.756										-1.766
CA_7	2.792					1.233	0.155		0.137	-0.411	
CA_8	-400.332	9.779	2.289	-22.550	-8.242	-2.786	-79.952	26.739	-32.967	404.595	155.685
CA_9	-439.237	6.793	1.590	-46.994	-5.725	-1.935	-499.855	167.167	-2.290	93.685	3.605
CA_{10}	1.830	-0.026	-0.030	0.033	0.030	0.030	0.342	-0.302	0.030	-0.303	-0.883

β_{ij}	x_1^2	x_2^2	x_3^2	x_4^2	x_5^2	x_6^2	x_7^2	x_8^2	x_9^2	x_{10}^2
CA_1	0.370		0.348							
CA_2	0.046		0.390							
CA_3			0.707			0.241	0.106			
CA_4							0.230			0.308
CA_5						1.551				
CA_6										1.176
CA_7						0.151				
CA_8	-5.232	-1.635	-3.578			-56.279	-21.879		-331.033	-131.514
CA_9	-3.634	-1.136	-7.455			-351.851	-136.784		-76.651	-3.045
CA_{10}			0.033			0.334	0.147			0.588

Table 6.1 Empirical Data for Industrial Example

Figure 6.2. Industrial Cost Functions

to represent a 2^{10-6} (or $1/64^{\text{th}}$) factorial design. Assuming the customer's value functions were not strictly linear, a saturated composite design was constructed using two blocks. Block 1 was comprised of the original 16 prototypes with their associated design points; Block 2 added an additional 22 observations via computer-aided design (Figures E.1a and E.1b). Company representatives participated in evaluating these designs. The 38 responses for each Customer Attribute (CA) represent the average elicited satisfaction of three subject matter experts from the company (Figure E.2). The value function for each CA was then obtained from stepwise regression. Due to the design's resolution, most of the interaction terms for a 2nd order model were aliased and therefore all were omitted. Table 6.1 shows these empirically derived value functions ($\hat{\beta}_{ij}$) as well as the relational dependence among ECs (θ_{ik}). Composite (R&D plus production) cost functions were utilized for the ECs and are shown in Figure 6.2. Linear approximations of same were also generated to analyze design sensitivity to cost variation.

6.3 Analysis of Results

Subject to the constraints $\sum_i Cost_i \leq \$800K$ and $-1 \leq EC_i \leq +1$, GAMS (Version 2.25) was again employed to optimize the mathematical program. The EC values of Table 6.2 achieved levels of 54.3% *Customer Satisfaction* and 68.6% average *EC Fulfillment* at a total cost of \$712K. Note that the cost constraint was *not* binding; this is due to CAs 8 and 9 which represent cost-related preferences of the customer. The coefficients ($\hat{\beta}_{ij}$) of Table 6.1 show the NLP was faced with the quandary of maximizing

CAs 1-7 and 10 and minimizing CAs 8 and 9. In other words, the problem contained dual objective functions.

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀
<i>Optimal (Coded) Setting</i>	-1.0	-0.71	-1.0	1.0	1.0	0.60	0.50	-0.82	0.32	-0.21
<i>Realized (Correlated) Value</i>	-0.94	-1.0	-0.52	0.57	1.0	-1.0	0.10	1.0	0.17	1.0
<i>Optimal (Natural) Setting</i>	5.0	0.86	17.0	30.0	2.0	25.0	3.76	1361	0.69	4.0

Table 6.2 Optimal *Objective* Solution for Industrial Example

This conflict produced a technologically sub-optimal solution. To address this dilemma, CAs 8 and 9 were deleted and the problem reoptimized. The EC values of Table 6.3 achieved levels of 81.2% *Customer Satisfaction* and 83.5% average *EC Fulfillment* at a total cost of \$800K for the modified problem.

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀
<i>Optimal (Coded) Setting</i>	-1.0	-0.71	-1.0	1.0	1.0	-0.60	-0.97	-0.82	-1.0	0.21
<i>Realized (Correlated) Value</i>	-0.94	-1.0	-0.52	0.57	1.0	1.0	-0.19	1.0	-0.52	-1.0
<i>Optimal (Natural) Setting</i>	5.0	0.86	17.0	30.0	2.0	10.0	0.18	1361	0.10	6.1

Table 6.3 Optimal *Objective* Solution for Industrial Example (*Modified*)

Given the time and effort required to capture the voice of the customer and the uncertainty of forecasting costs, it was important to investigate the solution's sensitivity to the substitution of linear value and cost functions. Stepwise regression analysis produced the 1st order value functions ($\hat{\beta}_{ij}$) of Table 6.4; the linearly approximated cost functions are shown in Figure 6.2. Three permutations of the modified problem were then optimized by GAMS. In all cases, the cost constraint remained tight at \$800K.

	(+)	-	-	+	+	+	+	-	+	-	-
p_{ij}	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
CA_1	2.657	-0.821	-0.323	0.350	0.323						
CA_2	2.739	-0.102	-1.084	0.392	0.361						
CA_3	2.382		-0.218	0.711	0.655		0.246	-0.218			
CA_4	2.717							-0.475	0.476	-0.476	-0.462
CA_5	2.074						1.587			-0.469	
CA_6	2.431										-1.766
CA_7	2.894					1.233	0.155		0.137	-0.411	
CA_{10}	2.437						0.342	-0.302		-0.303	-0.883

Table 6.4 1st Order Value Functions for Industrial Example (Modified)

Figures 6.3 through 6.5 compare the realized (correlated) value for each EC where:

- (1) *NL* refers to the original quadratic solution (Table 6.3).
- (2) *L-V* represents the case of linear value functions (Table 6.4) with quadratic cost functions.
- (3) *L-C* signifies linearly approximated cost functions (Figure 6.2) with 2nd order value functions.
- (4) *L* denotes the application of strictly linear functions.

The substitution of linear functions results in different but very similar designs. Table 6.5 summarizes the results achieved by these permutations. Within this design space, the use of 1st order value functions had the most positive impact. The linearly approximated cost functions slightly degraded the model's performance, however.

	Overall Satisfy	EC Fulfill
<i>NL</i>	81.2%	83.5%
<i>L-V</i>	90.1%	87.9%
<i>L-C</i>	74.4%	72.0%
<i>L</i>	85.2%	82.9%

Table 6.5 Summary of Permutations

Figure 6.3 1st Order Value and Quadratic Cost Functions

Figure 6.4 2nd Order Value with Linearly Approximated Cost Functions

Figure 6.5 1st Order Value with Linearly Approximated Cost Functions

Contrast these results with those produced by the subjective HOQ of Figure 6.1 (CAs 8 and 9 excluded). Normalizing the HOQ in accordance with the procedure $[PM^{Normalized}] = [PM] \times [CM^{Normalized}]^{-1}$ discussed in Chapters 2 and 4 [55], the company's design team generated the matrix of Figure 6.6. The absolute relative technical importance measures (w_i), cost coefficients (c_i), value indices (w_i / c_i) and corresponding fulfillment priorities of the ECs are shown too.

Based upon the fulfillment priority, available funds (\$800K) were then distributed sequentially in descending order to set each EC at its optimal technical limit. As discussed previously, such an allocation procedure is simply an integer relaxation of the *Knapsack* problem. Subjective input in conjunction with this procedure advocates the design of Table 6.6 for achievement of the highest level of customer satisfaction. Average *EC Fulfillment* is 48.7% with ECs 8, 4, 6 and 3 consuming the entire budget.

	Weight	EC ₁ ↓	EC ₂ ↓	EC ₃ ↑	EC ₄ ↑	EC ₅ ↑	EC ₆ ↑	EC ₇ ↓	EC ₈ ↑	EC ₉ ↓	EC ₁₀ ↓
CA ₁	4	0.579	0.627	-0.038	-0.038		-0.130	-0.208		0.163	0.046
CA ₂	3	0.197	0.583	0.240	0.240		-0.251	0.021		0.135	-0.165
CA ₃	5	-0.570	-0.017	0.884	0.884		-0.516	0.510	0.029	0.335	-0.539
CA ₄	5	-0.055	-0.088	0.128	0.128		0.056	0.418	0.397	0.106	-0.088
CA ₅	7	1.780		-3.706	-3.706		5.599	0.936	-0.130	-0.964	1.190
CA ₆	5	-1.209		1.511	1.511		-1.292	2.279	-0.765	2.162	-3.198
CA ₇	2	0.024		0.031	0.031	0.819	-0.024	0.115	-0.048	0.178	-0.126
CA ₁₀	2	3.630	-0.582	-5.453	-5.453		6.771	-2.798	1.251	-4.236	7.869
		31.85 72.64	5.94 342.3	58.39 102.6	58.39 30.0	1.64 780.0	63.38 116.4	29.31 198.6	9.46 0.30	29.65 1818.8	44.12 190.8
		0.442 5	0.017 8	0.491 4	1.68 2	0.002 10	0.544 3	0.148 7	31.53 1	0.016 9	0.231 6

w_i
 c_i
 w_i / c_i
Priority_i

Figure 6.6 Normalized Subjective Industrial Example (Modified)

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀
Optimal (Coded) Setting	1.0	1.0	0.75	1.0	-1.0	1.0	1.0	1.0	1.0	1.0
Optimal (Natural) Setting	10.0	3.0	23.99	30	0.7	30.0	5.0	5000	1.0	10.0

Table 6.6 Optimal Subjective Solution for Industrial Example (Modified)

Figure 6.7 graphically portrays the discrepancy between the company's results and those from the mathematical program. Not surprisingly, the "optimal" subjective vector of Table 6.6 only achieves levels of 42.6% *Customer Satisfaction* with the 2nd order value functions of Table 6.1 and 41.0% *Customer Satisfaction* with the 1st order value functions of Table 6.4.

Figure 6.7 Subjective vs. Objective Interpretation of Industrial Example

Weight	EC ₁ ↓	EC ₂ ↓	EC ₃ ↑	EC ₄ ↑	EC ₅ ↑	EC ₆ ↑	EC ₇ ↓	EC ₈ ↑	EC ₉ ↓	EC ₁₀ ↓	
CA ₁	4	-9	-3	3	3						
CA ₂	3	-1	-3	3	3						
CA ₃	5		-3	9	9		3	-3			
CA ₄	5						-3	3	-3	-3	
CA ₅	7						9		3		
CA ₆	5										-9
CA ₇	2					9	1		1	-3	
CA ₈	7	-1		1	1		1	-1	1	-9	-3
CA ₉	9	-1		3	1		9	-9		-3	
CA ₁₀	2						1	-1		-1	-3
	10 5	3 0.5	25 17	30 20	2 0.7	30 5	5 0.1	5000 3000	1 0.1	10 0.1	High Low

Figure 6.8 Sanitized HOQ from Industry (Revised)

In an attempt to isolate the cause of the enormous disparity between the two approaches, it was suggested that the original subjective HOQ (Figure 6.1) be revised to admit negative values in the planning matrix. The use of a strictly positive scale of comparison suggested that “more” was always better and contradicted the cost functions. Figure 6.8 shows the revised subjective HOQ (correlation matrix unchanged). Figure 6.9 summarizes the normalization of same (again omitting CAs 8 and 9). The suggested optimal design is identical to that of Table 6.6. While the sequence of priority changes, it alters only the order of those ECs below the total cost threshold (\$800K). One must conclude that for this problem the subjective approach and normalization procedure neither adequately addresses EC dependence nor provides a design which optimizes customer satisfaction.

	Weight	<div> <div>EC₁↓</div> <div>EC₂↓</div> <div>EC₃↑</div> <div>EC₄↑</div> <div>EC₅↑</div> <div>EC₆↑</div> <div>EC₇↓</div> <div>EC₈↑</div> <div>EC₉↓</div> <div>EC₁₀↓</div> </div>									
		EC ₁ ↓	EC ₂ ↓	EC ₃ ↑	EC ₄ ↑	EC ₅ ↑	EC ₆ ↑	EC ₇ ↓	EC ₈ ↑	EC ₉ ↓	EC ₁₀ ↓
CA ₁	4	3.139	2.066	-2.512	-2.512		1.375	-1.460		-0.431	1.336
CA ₂	3	1.446	1.466	-1.063	-1.063		0.592	-0.545		-0.279	0.445
CA ₃	5	-1.913	-0.945	2.500	2.500		-1.580	0.667	0.089	0.584	-0.902
CA ₄	5	-7.145	-11.4	16.56	16.56		-18.59	54.21	-101.4	100.26	-48.01
CA ₅	7	-2.721		7.940	7.940		-11.80	1.431	-3.640	8.260	-6.408
CA ₆	5	-1.209		1.511	1.511		-1.292	2.279	-0.765	2.162	-3.198
CA ₇	2	0.031		-0.191	-0.191	1.028	0.297	-0.145	0.328	-0.462	0.305
CA ₁₀	2	-0.305	-0.249	0.360	0.360		-0.148	0.715	-0.010	0.398	-0.122
		87.95	74.88	172.8	172.8	2.06	198.1	395.0	537.5	577.2	312.9
		72.04	342.3	102.6	30.0	788.0	116.4	198.6	0.30	1018.4	190.9
		1.284	0.216	1.670	5.71	0.003	1.696	1.521	1.792	0.317	1.638
		7	9	4	2	10	3	6	1	8	5

w_i
 c_i
 w_i / c_i
Priority_i

Figure 6.9 Normalized Subjective Industrial Example (Revised & Modified)

6.4 Summary

Application of the mathematical program to industrial data corroborated many of the conclusions drawn in previous chapters. The model again outperformed the conventional heuristic of coupling subjective assessments with a transformation of the design space. Dependence among the ECs remained a powerful tool for enhancing customer satisfaction in the presence of economic and technological constraints. 1st order value functions continued to offer a fairly robust alternative to the more expensive and time-consuming higher-order customer elicitations. However, linear approximation of cost functions showed less promise.

Chapter 7

Extensions

We should all be concerned about the future because we will have to spend the rest of our lives there [29].

7.1 Synopsis of Contributions

The House of Quality has been widely discussed as a mechanism for converting customer attributes into engineering characteristics to ensure the design quality of new products and processes. In the past, this process has been subjective and heuristic. This research effort presented a mathematical programming model for determining the optimal settings for engineering characteristics based upon value functions constructed to capture customer preferences. Additionally, the math program accounts for the effects of both positive and negative dependency among engineering characteristics. The model can be used with either traditional subjective measures of customer preference or incorporate empirical models derived from quantitative data. The robustness of the optimal solution to randomness in parameter estimates was investigated and the model's performance was compared to that of an accepted heuristic. In both cases, the results encourage further study.

The major findings of this research include:

- (1) Current matrix transformation procedures fail to account for the impact of dependency among engineering characteristics.
- (2) The optimization model returns greater levels of *EC Fulfillment* than a conventional heuristic.

(3) When determining optimal target values for ECs, the impact of positive and negative correlation can be respectively accentuated and minimized in the presence of resource constraints.

(4) Optimization offers an efficient and feasible approach to maximizing *Customer Satisfaction* for multivariate, nonlinear problems.

(5) The data requirements for empirically deriving the voice of the customer are significant for all but the most trivial applications of the methodology.

7.2 Suggestions for Future Research

As stated previously, the author believes additional testing of the mathematical program could be beneficial. The set of 320 problem instances were randomly generated primarily to test the model's computational efficiency. Such testing should focus on the model's performance with more precise value functions and correlation coefficients. For instance, the problem set's correlation coefficients were all of the same order of magnitude ($-1 \leq \theta_{ik} \leq +1$). That is undoubtedly an overly conservative replication of what a practitioner (or the model) might encounter. Furthermore, the value functions usually did not span the entire design space. Again, this was not the goal of the problem set but future research efforts could examine the synergy between the objective function (*Customer Satisfaction*) and *EC Fulfillment* in the presence of rational value functions.

The model's sensitivity to parameter variation should be more extensively investigated too. The robustness of discrete coefficients (β_{ij}), either objectively derived

or subjectively assigned is open to debate. The example given in Chapter 3 is very limited and based largely upon intuition. It is no substitute for comprehensive testing of randomly generated discrete coefficients from authentic distributions. The penalties associated with subjective evaluations and forced discretization of same may or may not be significant. Problems could be devised where model deviations for exact relationships are tightly controlled and their effect evaluated. For instance, if $\beta_{ij} = 5.25$ but a $[1, 3, 9]$ scale must be used, the error in the value function might be ξ . As a consequence ξ translates to an error of ψ in the objective function.

Perhaps the most intriguing result of Chapters 5 and 6 was the performance of linearly approximated value and cost functions. The former achieved results as good or better than their nonlinear counterparts; the latter were not so efficient. Obviously the ramifications of dispensing with higher-order functions make it worth exploring this issue.

Other areas related to model behavior that are candidates for evaluation include:

- (1) Integer constraints for the ECs (mixed integer programming).
- (2) *Piecewise* linearization of nonlinear functions (as opposed to strict linear approximations).
- (3) Assessment of optimization heuristics beyond a relaxation of the *Knapsack* problem.
- (4) Disjoint feasible ranges for x_i (e.g., $[2 \leq x_i \leq 4]$ & $[6 \leq x_i \leq 8]$ vs. $[2 \leq x_i \leq 8]$).
- (5) Impact of nonlinear correlation (θ_{ik}) functions.

To expand further on topic (3), fuzzy logic [50] and factor analysis [45] could be considered for incorporation into the methodology. Both of these techniques have been offered as improvements to the subjective HOQ and might possibly enhance the employment of the mathematical program.

APPENDIX A

100% RULE PROOF

Theorem: Let U_k and L_k represent the maximum and minimum values of objective function coefficient c_k for which the current basis remains optimal given all other coefficients are held constant. The *100% Rule* holds that the current basis remains optimal (for a maximization problem) if $\sum_K r_k^+ + r_k^- \leq 1$ where K is the set of all variables and

$$r_k^+ = \frac{\Delta c_k}{U_k - c_k} \quad \text{if} \quad \Delta c_k \geq 0 \quad (\text{A-1a})$$

$$r_k^- = \frac{-\Delta c_k}{c_k - L_k} \quad \text{if} \quad \Delta c_k < 0 \quad (\text{A-1b})$$

Proof: For any basic or non-basic variable k , define its changed objective function coefficient as c_k' where

$$c_k' = c_k + \Delta c_k \quad (\text{A-2})$$

$$= c_k + (U_k - c_k)r_k^+ \quad \text{or} \quad c_k - (c_k - L_k)r_k^- \quad (\text{A-3})$$

$$= U_k r_k^+ + (1 - r_k^+)c_k \quad \text{or} \quad L_k r_k^- + (1 - r_k^-)c_k \quad (\text{A-4})$$

$$= U_k r_k^+ + \sum_{i \neq k} r_i c_k + \left(1 - \sum_K r_i\right) c_k \quad \text{or} \quad L_k r_k^- + \sum_{i \neq k} r_i c_k + \left(1 - \sum_K r_i\right) c_k \quad (\text{A-5})$$

Since it is apparent the argument is equivalent for r_k^+ or r_k^- , without loss of generality

$$c_k' = M_k r_k + \sum_{i \neq k} r_i c_k + \left(1 - \sum_K r_i\right) c_k \quad (\text{A-6})$$

where $M_k = U_k$ or L_k and $r_k = r_k^+$ or r_k^- based upon the direction of change for c_k .

Expanding Equation (A-6) to include all of K

$$\begin{aligned} [c'_1, c'_2, \dots, c'_K] = & r_1 [M_1, c_2, \dots, c_K] + r_2 [c_1, M_2, \dots, c_K] + \dots + r_K [c_1, c_2, \dots, M_K] \\ & + (1 - r_1 - r_2 - \dots - r_K) [c_1, c_2, \dots, c_K] \end{aligned} \quad (\text{A-7})$$

Let the set B contain all current basic variables and c_j represent any non-basic variable.

The current basis remains optimal [22] as long as $z'_j - c'_j \geq 0$ ($\forall j$), where $z'_j = c_B B^{-1} a_j$.

For $[c'_1, c'_2, \dots, c'_B]$ and all c'_j

$$\begin{aligned} c'_B B^{-1} a_j - c'_j = & r_1 \{ [M_1, c_2, \dots, c_B] B^{-1} a_j - c_j \} + r_2 \{ [c_1, M_2, \dots, c_B] B^{-1} a_j - c_j \} + \\ & + \dots + r_B \{ [c_1, c_2, \dots, M_B] B^{-1} a_j - c_j \} + r_j \{ [c_1, c_2, \dots, c_B] B^{-1} a_j - M_j \} \\ & + (1 - r_1 - r_2 - \dots - r_K) \{ [c_1, c_2, \dots, c_B] B^{-1} a_j - c_j \} \end{aligned} \quad (\text{A-8})$$

First consider only the quantities inside the $\{ \}$. Only one term is changed in each and it is always within its allowable range ($M_k = U_k$ or L_k); all other terms are held constant.

As such, each $\{ \}$ must be nonnegative as defined by sensitivity analysis. Subsequent multiplication of each $\{ \}$ by a constant ($r_k \geq 0$) produces a nonnegative linear combination of the $\{ \}$.

$$\therefore z'_j - c'_j \geq 0 (\forall j) \text{ if } \sum_K r_k \leq 1.$$

<i>Regression Statistics</i>	
Multiple R	0.958
R ²	0.917
Adjusted R ²	0.872
Std. Error	0.507
Observations	15

<i>Parameter</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	0	0.131	0	1.00	-0.296	0.296
β_{11}	1.375	0.253	5.425	4.19E-04	0.802	1.948
β_{22}	0.875	0.253	3.452	7.25E-03	0.302	1.448
β_{33}	-1.125	0.253	-4.439	1.63E-03	-1.698	-0.552
β_{21}	1.500	0.253	5.918	2.24E-04	0.927	2.073
β_{33}	-0.500	0.253	-1.973	8.00E-02	-1.073	0.073

<i>ANOVA</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	5	25.69	5.138	19.995	1.25E-04
Residual	9	2.31	0.257		
Total	14	28.00			

<i>Observation</i>	<i>Residual</i>
1	-0.425
2	-0.675
3	0.075
4	0.325
5	0.7
6	0
7	0.25
8	0
9	0.25
10	-0.5
11	0.1
12	-0.4
13	0.1
14	-0.4
15	0.6
Σ Residuals	0.00

Table A.1a $PD^{k,r}$ Regression Analysis w/ df Uncorrected for $\beta_{0j} = 0$

<i>Regression Statistics</i>	
Multiple R	0.958
R ²	0.917
Adjusted R ²	0.858
Std. Error	0.575
Observations	15

<i>Parameter</i>	<i>Coefficient</i>	<i>Std. Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
β_{11}	1.375	0.287	4.785	2.00E-03	0.695	2.055
β_{21}	0.875	0.287	3.045	1.87E-02	0.195	1.555
β_{31}	-1.125	0.287	-3.915	5.79E-03	-1.805	-0.445
β_{41}	1.500	0.287	5.220	1.23E-03	0.820	2.180
β_{51}	-0.500	0.287	-1.740	1.25E-01	-1.180	0.180

<i>ANOVA</i>	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	5	25.69	5.138	15.551	1.14E-03
Residual	7	2.31	0.330		
Total	12	28.00			

<i>Observation</i>	<i>Residual</i>
1	-0.425
2	-0.675
3	0.075
4	0.325
5	0.7
6	0
7	0.25
8	0
9	0.25
10	-0.5
11	0.1
12	-0.4
13	0.1
14	-0.4
15	0.6
Σ Residuals	0.00

Table A.1b $PD^{k,p}$ Regression Analysis w/ df Corrected for $\beta_{0j} = 0$

Instance #1 (Unconstrained)						
	PD ^{k-p} Cost Coefficients			MPD _E Cost Coefficients		
	Value μ_{ij}	Allowable Minimum	Allowable Maximum	Value μ_{ij}	Allowable Minimum	Allowable Maximum
β_{11}	1.375	0	Infinity	3	0.104	Infinity
β_{22}	0.875	-0.900	Infinity	3	2.88E-05	26.997
β_{33}	-1.125	-Infinity	0.300	-9	-Infinity	-1.002
β_{13}	0	N/A	N/A	1	-15.735	Infinity
β_{23}	1.5	-1.458	Infinity	9	3.588	52.333
β_{33}	-0.5	-Infinity	1.875	-3	-Infinity	11.440

Instance #2 (Budget Constraint)						
	PD ^{k-p} Cost Coefficients			MPD _E Cost Coefficients		
	Value μ_{ij}	Allowable Minimum	Allowable Maximum	Value μ_{ij}	Allowable Minimum	Allowable Maximum
β_{11}	1.375	-Infinity	2.543	3	-Infinity	3.985
β_{22}	0.875	0.060	2.520	3	1.106	20.604
β_{33}	-1.125	-Infinity	-0.440	-9	-Infinity	-1.665
β_{13}	0	N/A	N/A	1	-Infinity	6.698
β_{23}	1.5	0.142	4.242	9	5.585	40.778
β_{33}	-0.5	-Infinity	0.642	-3	-Infinity	10.244

Instance #3 (Budget and Threshold Constraints)						
	PD ^{k-p} Cost Coefficients			MPD _E Cost Coefficients		
	Value μ_{ij}	Allowable Minimum	Allowable Maximum	Value μ_{ij}	Allowable Minimum	Allowable Maximum
β_{11}	1.375	-Infinity	14.323	3	-Infinity	46.771
β_{22}	0.875	-Infinity	2.520	3	-Infinity	20.604
β_{33}	-1.125	-Infinity	-0.440	-9	-Infinity	-1.665
β_{13}	0	N/A	N/A	1	-Infinity	253.906
β_{23}	1.5	-Infinity	4.242	9	-Infinity	40.778
β_{33}	-0.5	-Infinity	0.642	-3	-Infinity	10.244

Table A.2 Sensitivity Analyses

$$\begin{aligned}
 F_0 &= 0.426 = 18.25\% (F_{0.0175, 5, 7}) \\
 I_{11} &= 1.168 & [I_{ij}r_{ij}^+ - D_{ij}r_{ij}^-]^T &= 0.151 \quad 0.216 \quad 0.257 \quad 0.129 \quad 0.154 = [\Delta]^T \\
 I_{22} &= 1.645 \\
 I_{32} &= 0.685 & & 4 \quad 0 \quad 0 \quad 0 \quad 0 \\
 I_{23} &= 2.742 & & 0 \quad 4 \quad 0 \quad 0 \quad 0 \\
 I_{33} &= 1.140 & [X^T X] &= 0 \quad 0 \quad 4 \quad 0 \quad 0 \\
 D_{11} &= 28.738 & & 0 \quad 0 \quad 0 \quad 4 \quad 0 \\
 D_{22} &= 0.815 & & 0 \quad 0 \quad 0 \quad 0 \quad 4 \\
 D_{32} &= 28.738 \\
 D_{23} &= 1.360 & & 0.151 \\
 D_{33} &= 28.738 & & 0.216 \\
 \mu_{11} &= 1.375 & [I_{ij}r_{ij}^+ - D_{ij}r_{ij}^-] &= 0.257 = [\Delta] \\
 \mu_{22} &= 0.875 & & 0.129 \\
 \mu_{32} &= -1.125 & & 0.154 \\
 \mu_{23} &= 1.500 \\
 \mu_{33} &= -0.500 & [D]^T [X^T X] [D] &= 0.603 \quad 0.864 \quad 1.028 \quad 0.518 \quad 0.618 \\
 & & [D]^T [X^T X] [D] &= 0.704 \\
 (r_{11}^+) \cdot (r_{11}^-) &= 0 \\
 (r_{22}^+) \cdot (r_{22}^-) &= 0 & s^2 &= 0.330 \\
 (r_{32}^+) \cdot (r_{32}^-) &= 0 & ps^2 &= 1.652 \\
 (r_{23}^+) \cdot (r_{23}^-) &= 0 & \sigma_{ij} &= 0.287 \\
 (r_{33}^+) \cdot (r_{33}^-) &= 0 \\
 \beta_{11}^+ &= 1.526 & r_{11}^+ &= 0.129 \\
 \beta_{22}^+ &= 0.875 & r_{22}^+ &= 0 \\
 \beta_{32}^+ &= -0.868 & r_{32}^+ &= 0.375 \\
 \beta_{23}^+ &= 1.500 & r_{23}^+ &= 0 \\
 \beta_{33}^+ &= -0.346 & r_{33}^+ &= 0.135 \\
 \beta_{11}^- &= 1.375 & r_{11}^- &= 0 \\
 \beta_{22}^- &= 0.659 & r_{22}^- &= 0.265 \\
 \beta_{32}^- &= -1.125 & r_{32}^- &= 0 \\
 \beta_{23}^- &= 1.371 & r_{23}^- &= 0.095 \\
 \beta_{33}^- &= -0.500 & r_{33}^- &= 0 \\
 & & \Sigma r_{ij} &= 1.00
 \end{aligned}$$

Table A.3a F_0 Statistic for PD^k -P Problem (Instance #2)

$\Delta^* = 0.675 = 0.90\% = (F_{0.9910, 5, 7})$		
$I_{11} = 1.168$		$r_{11} = 0.166$
$I_{22} = 1.645$		$r_{22} = 0.238$
$I_{32} = 0.685$		$r_{32} = 0.283$
$I_{23} = 2.742$		$r_{23} = 0.143$
$I_{33} = 1.140$		$r_{33} = 0.170$
$D_{11} = 28.738$		$\Sigma r_{ij} = 1.00$
$D_{22} = 0.815$		
$D_{32} = 28.738$		$\beta_{11} = 1.569$
$D_{23} = 1.360$		$\beta_{22} = 0.681$
$D_{33} = 28.738$		$\beta_{32} = -0.931$
		$\beta_{23} = 1.306$
		$\beta_{33} = -0.306$
$\mu_{11} = 1.375$		
$\mu_{22} = 0.875$		
$\mu_{32} = -1.125$		$\sigma_{ij} = 0.287$
$\mu_{23} = 1.5$		
$\mu_{33} = -0.5$		$\Delta \times \sigma_{ij} = 0.194$

Table A.3b S_0 Statistic for PD^k-P Problem (Instance #2)

Figure A.1a $V_1 \Rightarrow MPDE \{CA_1 = 3 \times EC_1\}$

Figure A.1b $V_1 \Rightarrow PD^{k-p} \{CA_1 = 1.375 \times EC_1\}$

Figure A.1c $V_2 \Rightarrow MPDE \{CA_2 = 3 \times EC_2 - 9 \times EC_3\}$

Figure A.1d $V_2 \Rightarrow PD^{k-p} \{CA_2 = 0.875 \times EC_2 - 1.125 \times EC_3\}$

Figure A.1e $V_3 \Rightarrow MPDE \{CA_3 = EC_1 + 9 \times EC_2 - 3 \times EC_3\}$

Figure A.1f $V_3 \Rightarrow PD^{k-p} \{CA_3 = 1.5 \times EC_2 - 0.5 \times EC_3\}$

Figure A.1g $V_3 \Rightarrow MPDE \{CA_3 = EC_1 + 9 \times EC_2 - 3 \times EC_3\}$

Figure A.1h $V_3 \Rightarrow MPDE \{CA_3 = EC_1 + 9 \times EC_2 - 3 \times EC_3\}$

Figure A.2 Hypothetical Distributions for Subjective β_{ij} (MPDE)

APPENDIX B

<i>Regression Statistics: CA₁</i>	
Multiple R	0.981
R Square	0.962
Adjusted R Square	0.953
Standard Error	0.359
Observations	11

ANOVA	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance</i>
Regression	2	26.512	13.256	102.628	1.98E-06
Residual	8	1.033	0.129		
Total	10	27.545			

<i>Parameter</i>	<i>Coefficients</i>	<i>tandard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
β_0	4.900	0.161	30.486	1.46E-09	4.529	5.271
β_1	1.667	0.147	11.359	3.25E-06	1.328	2.005
β_{11}	-1.900	0.218	-8.731	2.32E-05	-2.402	-1.398

<i>Observation</i>	<i>Residuals</i>
1	-0.333
2	0.333
3	-0.167
4	-0.333
5	0.667
6	-0.167
7	0.100
8	0.100
9	0.100
10	-0.400
11	0.100
Σ Residuals	0.00

Table B.1a *PDSCD* Regression Analysis for CA_1

<i>Regression Statistics: CA₂</i>	
Multiple R	0.989
R Square	0.978
Adjusted R Square	0.969
Standard Error	0.207
Observations	11

ANOVA	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance</i>
Regression	3	13.427	4.476	104.434	3.56E-06
Residual	7	0.300	0.043		
Total	10	13.727			

<i>Parameter</i>	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
β_0	3.200	0.093	34.564	4.40E-09	2.981	3.419
β_2	1.167	0.085	13.804	2.47E-06	0.967	1.367
β_3	-0.917	0.085	-10.846	1.25E-05	-1.117	-0.717
β_{33}	-0.283	0.125	-2.260	5.83E-02	-0.580	0.013

<i>Observation</i>	<i>Residuals</i>
1	-0.167
2	0.000
3	0.167
4	-0.167
5	-0.200
6	-0.200
7	-0.033
8	0.133
9	0.167
10	0.000
11	0.300
Σ Residuals	0.00

Table B.1b *PD^{SCD}* Regression Analysis for CA₂

Regression Statistics: CA₃	
Multiple R	0.996
R Square	0.991
Adjusted R Square	0.982
Standard Error	0.178
Observations	11

ANOVA	df	SS	MS	F	Significance
Regression	5	17.478	3.496	110.697	4.08E-05
Residual	5	0.158	0.032		
Total	10	17.636			

Parameter	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
β_0	1.895	0.091	20.785	4.77E-06	1.660	2.129
β_2	1.500	0.073	20.676	4.90E-06	1.314	1.686
β_{22}	0.763	0.112	6.835	1.02E-03	0.476	1.050
β_3	-0.500	0.073	-6.892	9.85E-04	-0.686	-0.314
β_{33}	-0.237	0.112	-2.121	8.74E-02	-0.524	0.050
β_{23}	-0.500	0.089	-5.627	2.45E-03	-0.728	-0.272

Observation	Residuals
1	0.079
2	0.079
3	0.079
4	0.079
5	0.105
6	0.105
7	-0.158
8	-0.158
9	-0.158
10	-0.158
11	0.105
Σ Residuals	0.00

Table B.1c PD^{SCD} Regression Analysis for CA₃

Figure B.1a $CA_1 = 4.9 + (1.667)EC_1 - (1.9)EC_1^2$

Figure B.1b $CA_2 = 3.2 + (1.167)EC_2 - (0.9167)EC_3 - (0.283)EC_3^2$

Figure B.1c $CA_1 = 1.895 + (1.5)EC_2 + (0.763)EC_2^2 - (0.5)EC_3 - (0.237)EC_3^2 - (0.5)EC_2EC_3$.

Figure B.2a $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_1 x_1 + \beta_{11} x_1^2$

Figure B.2b $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_1 x_1 + \beta_{11} x_1^2$

Figure B.2c $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_1 x_1 + \beta_{11} x_1^2$

Figure B.2d $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_1 x_1 + \beta_{11} x_1^2$

Figure B.2e $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{33} x_3^2$

Figure B.2f $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{33} x_3^2$

Figure B.2g $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{33} x_3^2$

Figure B.2h $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{33} x_3^2$

Figure B.2i $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{23} x_2 x_3$

Figure B.2j $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{23} x_2 x_3$

Figure B.2k $\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{23} x_2 x_3$

$$\sqrt{\text{Var}[\hat{y}(x)]} \rightarrow y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{23} x_2 x_3$$

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀	EC ₁₁	EC ₁₂	EC ₁₃	EC ₁₄	EC ₁₅	Best Regression
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	
CA ₁	1	1	3	8	1	8	3	8	3	6	3	2	8	7	1	Linear
CA ₂	3	1	2	8	1	6	3	7	1	6	1	3	7	6	2	Linear
CA ₃	2	1	1	7	3	7	3	7	3	7	1	2	8	6	2	Linear
CA ₄	2	3	2	7	3	8	3	8	3	7	3	1	7	6	1	Linear
CA ₅	2	2	1	8	1	8	2	6	1	6	1	2	7	8	3	Linear
CA ₆	2	2	2	8	3	8	2	8	2	6	1	3	6	8	2	Linear
CA ₇	1	1	2	7	2	8	2	8	1	7	2	3	7	8	2	Linear
CA ₈	1	3	3	6	2	8	1	8	2	8	3	2	6	7	1	Linear
CA ₉	1	2	1	6	3	6	2	8	3	8	3	2	7	6	1	Linear
CA ₁₀	2	2	2	7	2	8	2	7	2	7	1	2	6	8	1	Linear
CA ₁₁	1	1	2	6	3	6	2	7	3	8	1	1	8	7	1	Linear
CA ₁₂	3	1	2	8	1	7	3	6	1	6	3	3	7	6	2	Linear
CA ₁₃	1	1	3	8	3	8	3	7	1	7	1	1	6	7	2	Linear
CA ₁₄	2	3	2	7	1	8	2	6	2	6	3	2	7	6	2	Linear
CA ₁₅	1	1	1	7	2	8	2	7	3	6	1	1	8	7	2	Linear

Table C.1a Monotonic 15 x 15 HOQ @ 25% Density

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀	EC ₁₁	EC ₁₂	EC ₁₃	EC ₁₄	EC ₁₅	<i>Best Regression</i>
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	
CA ₁	5	5	2	7	1	6	3	5	2	4	5	3	6	4	1	<i>Quadratic</i>
CA ₂	1	3	3	4	4	5	3	7	4	5	2	5	6	4	2	<i>Quadratic</i>
CA ₃	4	4	3	6	1	6	4	8	3	8	4	4	4	8	3	<i>Quadratic</i>
CA ₄	3	1	3	7	2	8	4	8	4	6	3	1	7	4	3	<i>Linear</i>
CA ₅	1	5	5	4	4	4	3	4	1	7	2	1	7	5	1	<i>Quadratic</i>
CA ₆	3	5	2	6	1	5	5	6	2	8	2	5	6	7	3	<i>Linear</i>
CA ₇	2	4	1	7	3	5	2	8	4	5	3	3	8	6	1	<i>Linear</i>
CA ₈	1	3	5	5	1	7	4	8	4	4	3	4	8	8	1	<i>Quadratic</i>
CA ₉	2	5	5	6	5	7	5	7	3	6	2	2	8	4	3	<i>Linear</i>
CA ₁₀	1	3	5	7	1	7	1	7	4	8	2	2	8	8	5	<i>Linear</i>
CA ₁₁	2	1	3	7	3	5	1	4	1	4	5	5	7	7	4	<i>Quadratic</i>
CA ₁₂	5	5	3	4	4	6	2	7	3	8	2	3	4	4	1	<i>Quadratic</i>
CA ₁₃	5	3	5	8	1	4	5	4	3	4	2	1	5	8	5	<i>Quadratic</i>
CA ₁₄	2	5	5	7	4	6	3	7	3	6	5	2	7	7	1	<i>Quadratic</i>
CA ₁₅	2	5	4	8	4	8	2	4	2	5	1	2	6	4	3	<i>Quadratic</i>

Table C.1b Non-Monotonic 15 x 15 HOQ @ 25% Density

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀	EC ₁₁	EC ₁₂	EC ₁₃	EC ₁₄	EC ₁₅	<i>Best Regression</i>
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	
CA ₁	6	3	3	2	2	3	6	3	7	3	7	3	7	7	1	<i>Linear</i>
CA ₂	8	2	1	1	2	1	6	2	6	1	6	2	6	7	1	<i>Quadratic</i>
CA ₃	7	2	3	3	1	2	8	2	7	1	6	1	6	7	2	<i>Linear</i>
CA ₄	6	2	2	3	2	2	8	3	6	1	7	2	6	6	2	<i>Linear</i>
CA ₅	8	1	1	3	1	3	8	2	7	3	6	3	6	8	1	<i>Linear</i>
CA ₆	6	2	3	1	3	1	6	1	6	3	7	3	7	8	2	<i>Linear</i>
CA ₇	6	1	3	2	3	3	7	2	7	2	8	3	6	7	3	<i>Linear</i>
CA ₈	6	2	1	1	3	1	8	3	8	1	8	3	6	8	1	<i>Linear</i>
CA ₉	8	3	1	1	1	3	7	3	6	2	7	3	6	8	3	<i>Linear</i>
CA ₁₀	8	3	2	3	3	1	8	1	6	1	8	3	6	8	2	<i>Linear</i>
CA ₁₁	7	1	3	1	3	3	7	1	8	3	7	3	6	7	3	<i>Linear</i>
CA ₁₂	8	3	2	3	2	2	7	1	8	1	8	3	7	6	2	<i>Linear</i>
CA ₁₃	7	1	1	2	3	1	7	3	7	1	8	1	7	8	1	<i>Linear</i>
CA ₁₄	6	2	2	3	2	3	6	3	7	1	6	1	8	8	1	<i>Linear</i>
CA ₁₅	6	2	3	3	2	3	8	2	7	2	6	2	8	7	1	<i>Linear</i>

Table C.1c Monotonic 15 x 15 HOQ @ 50% Density

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀	EC ₁₁	EC ₁₂	EC ₁₃	EC ₁₄	EC ₁₅	<i>Best Regression</i>
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	
CA ₁	6	4	1	2	3	1	5	2	6	4	8	5	4	5	5	<i>Quadratic</i>
CA ₂	5	2	3	1	3	4	5	5	5	2	6	1	5	4	5	<i>Quadratic</i>
CA ₃	7	2	1	1	5	2	6	1	7	1	4	2	7	6	3	<i>Quadratic</i>
CA ₄	8	3	5	2	4	2	5	2	8	3	7	2	4	7	5	<i>Quadratic</i>
CA ₅	8	5	2	3	3	3	4	2	6	1	8	2	7	7	4	<i>Linear</i>
CA ₆	6	4	5	3	4	2	6	2	5	5	8	4	6	4	3	<i>Quadratic</i>
CA ₇	5	3	3	3	2	5	5	5	6	5	4	4	7	5	1	<i>Linear</i>
CA ₈	4	3	3	1	1	2	5	2	6	5	6	3	5	4	4	<i>Quadratic</i>
CA ₉	7	3	4	3	2	4	4	1	5	4	6	5	8	4	3	<i>Linear</i>
CA ₁₀	4	2	3	2	2	2	7	2	4	1	4	4	7	7	1	<i>Quadratic</i>
CA ₁₁	5	3	5	2	2	1	6	3	8	1	7	1	8	4	2	<i>Linear</i>
CA ₁₂	6	4	2	5	4	1	7	4	6	5	5	5	4	5	1	<i>Quadratic</i>
CA ₁₃	7	5	5	1	3	1	4	3	7	5	5	2	8	7	4	<i>Quadratic</i>
CA ₁₄	7	2	5	4	5	4	5	1	7	3	5	1	4	4	3	<i>Quadratic</i>
CA ₁₅	4	3	5	4	2	1	6	1	4	5	5	2	8	8	3	<i>Linear</i>

Table C.1d Non-Monotonic 15 x 15 HOQ @ 50% Density

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	Best Regression
	+	+	-	+	-	
CA ₁	1	3	7	3	7	Linear
CA ₂	2	3	8	1	7	Linear
CA ₃	3	3	6	2	6	Linear
CA ₄	2	3	7	3	7	Linear
CA ₅	1	2	8	3	7	Linear

Table C.1e Monotonic 5 x 5 HOQ @ 50% Density

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	Best Regression
	+	+	-	+	-	
CA ₁	2	3	7	4	7	Quadratic
CA ₂	3	3	8	3	6	Linear
CA ₃	5	5	5	1	7	Quadratic
CA ₄	5	5	5	5	7	Quadratic
CA ₅	2	4	7	3	5	Quadratic

Table C.1f Non-Monotonic 5 x 5 HOQ @ 50% Density

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	Best Regression
	-	-	+	+	+	
CA ₁	8	6	2	2	3	Linear
CA ₂	7	6	1	2	3	Linear
CA ₃	6	6	2	2	1	Quadratic
CA ₄	8	7	1	2	3	Linear
CA ₅	8	8	1	1	3	Linear

Table C.1g Monotonic 5 x 5 HOQ @ 75% Density

	EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	Best Regression
	-	-	+	+	+	
CA ₁	5	8	1	4	5	Quadratic
CA ₂	7	4	4	3	1	Quadratic
CA ₃	7	8	1	1	2	Linear
CA ₄	5	5	4	2	1	Quadratic
CA ₅	4	8	5	1	1	Quadratic

Table C.1h Non-Monotonic 5 x 5 HOQ @ 75% Density

Instance	Optimal (Coded) Setting					Realized (Coded) Value					(Secs)			
	-	-	+	+	+	-	-	+	+	+	Overall Satisfy	Avg x_i Fulfill	Model Time	Soln Time
1-C	-0.171	-0.379	-0.524	0.219	0.958	-0.171	-0.371	-1	0.322	0.958	69.7%	58.2%	5.57	18.0
1-U	-0.54	-0.606	-1	0.177	1	-0.54	-0.606	-1	0.177	1	72.1%	63.2%	4.06	13.0
2-C	-1	-0.429	1	0.272	-0.87	-1	-0.429	0.779	0.272	-0.124	79.3%	73.6%	5.48	16.0
2-U	-1	-0.796	-1	0.153	1	-1	-0.796	-1	0.153	1	85.8%	69.5%	4.05	13.0
3-C	1	0.596	-1	1	1	0.267	1	-1	1	0.025	61.2%	37.6%	5.52	17.0
3-U	1	1	-1	1	1	1	1	-1	1	1	79.7%	40.0%	3.97	12.0
4-C	-0.589	1.000	-0.585	1	0.109	-1	1.000	-1	0.948	0.109	81.3%	50.6%	5.93	19.0
4-U	-1	1	-1	1	0.108	-1	1	-1	1	0.108	81.9%	51.1%	4.47	15.0
5-C	-0.725	0.017	1	-1	1	-0.486	0.017	0.414	-1	0.473	53.1%	53.6%	5.21	16.0
5-U	0.624	1	1	1	1	0.624	1	1	1	1	77.6%	63.8%	4.17	13.0
6-C	-0.569	0.979	0.786	0.533	-1	-1	1	1	1	-0.999	69.2%	60.0%	5.54	17.0
6-U	-0.551	1	1	0.192	0.966	-0.551	1	1	0.192	0.966	61.7%	67.1%	4.02	13.0
7-C	0.091	-0.608	-0.109	0.274	0.646	0.091	-1	-0.172	0.425	1	76.3%	71.6%	5.38	17.0
7-U	-0.014	-1	0.229	0.289	-1	-0.014	-1	0.229	0.289	-1	82.3%	55.3%	4.02	13.0
8-C	-0.794	1	1	1	-0.203	-0.794	1	0.641	0.429	-0.203	63.5%	56.6%	5.30	17.0
8-U	-0.902	1	1	1	-0.852	-0.902	1	1	1	-0.852	84.1%	60.5%	3.93	13.0
9-C (+)	0.324	0.482	0.812	0.683	1	0.636	0.482	1	1	1	76.9%	68.8%	5.46	17.0
9-U	0.735	0.458	1.000	1	1	0.735	0.458	1.000	1	1	76.7%	68.1%	4.21	15.0
10-C (-)	0.755	0.386	1	1	1	0.636	0.386	0.984	0.846	1	74.1%	68.1%	5.48	17.0
10-U	0.735	0.458	1.000	1	1	0.735	0.458	1.000	1	1	76.7%	68.1%	4.21	15.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 74.2%

Avg Satisfaction (C) : 70.5%

Avg Satisfaction (U) : 77.9%

Avg Fulfillment (C & U) : 60.3%

Avg Fulfillment (C) : 59.9%

Avg Fulfillment (U) : 60.7%

Table D.1 NL-5-75-M (Mathematical Program)

Instance	Optimal (Coded) Setting					Realized (Coded) Value					(Secs)			
	-	-	+	+	+	-	-	+	+	+	Overall	Avg x_i	Model	Soln
	x_1	x_2	x_3	x_4	x_5	x_1	x_2	x_3	x_4	x_5	Satisfy	Fulfill	Time	Time
1-C	0.240	-0.523	-0.524	0.230	1	0.240	-0.513	-1	0.338	1	84.4%	59.2%	3.90	13.0
1-U	0.210	-0.979	-1	0.155	1	0.210	-0.979	-1	0.155	1	89.0%	60.1%	4.05	14.0
2-C	0.235	-1	-1	1	1	0.235	-1	-0.779	1	0.143	72.4%	66.7%	4.02	13.0
2-U	0.281	-1	-1	1.000	1	0.281	-1	-1	1.000	1	89.7%	69.8%	4.11	12.0
3-C	1	-0.596	1	0.987	-1	0.267	-1	1	0.987	-0.025	72.7%	73.4%	4.07	13.0
3-U	-0.417	-1	1	-1	-1	-0.417	-1	1	-1	-1	75.9%	48.0%	4.11	14.0
4-C	0.589	-0.386	0.585	-1	1	1	-0.386	1	-0.948	1	74.4%	52.5%	4.45	15.0
4-U	1	-0.908	-1	0.554	1	1	-0.908	-1	0.554	1	73.9%	58.1%	4.46	16.0
5-C	-0.028	0.025	1	1	0.320	-0.019	0.025	0.414	1	0.151	61.9%	65.8%	4.03	14.0
5-U	0.084	0.241	1	1	0.512	0.084	0.241	1	1	0.512	81.7%	69.9%	3.94	13.0
6-C	0.042	-0.948	-0.786	0.533	1	0.074	-0.968	-1	1	0.999	88.5%	71.2%	3.90	13.0
6-U	0.144	-0.864	-1	0.899	1	0.144	-0.864	-1	0.899	1	83.4%	68.8%	3.99	14.0
7-C	0.550	-0.608	-0.066	0.645	0.646	0.550	-1	-0.104	1	1	92.4%	67.6%	3.91	13.0
7-U	1	-1	-1	1	0.941	1	-1	-1	1	0.941	89.9%	63.8%	4.05	15.0
8-C	1	-0.297	0.244	0.751	1	1	-0.297	0.144	0.322	1	75.0%	52.8%	3.98	13.0
8-U	1	-0.048	0.227	0.845	1	1	-0.048	0.227	0.845	1	77.3%	58.2%	3.98	14.0
9-C (+)	-0.003	-0.25	0.242	0.683	1	-0.007	-0.25	0.298	1	1	88.0%	69.9%	3.96	13.0
9-U	0.088	-0.251	0.187	1	1	0.088	-0.251	0.187	1	1	87.7%	68.3%	4.45	15.0
10-C (-)	0.125	-0.308	0.160	1	1	0.105	-0.308	0.158	0.846	1	82.6%	66.4%	3.92	13.0
10-U	0.088	-0.251	0.187	1	1	0.088	-0.251	0.187	1	1	87.7%	68.3%	4.45	15.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 81.4%

Avg Satisfaction (C) : 79.2%

Avg Satisfaction (U) : 83.6%

Avg Fulfillment (C & U) : 63.9%

Avg Fulfillment (C) : 64.6%

Avg Fulfillment (U) : 63.3%

Table D.2 NL-5-75-NM (Mathematical Program)

Instance	Optimal (Coded) Setting					Realized (Coded) Value					(Secs)			
	+	+	-	+	-	+	+	-	+	-	Overall Satisfy	Avg x_i Fulfill	Model Time	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_1	x_2	x_3	x_4	x_5				
1-C	0.589	0.617	-0.868	-1	-0.428	0.698	1	-1	-0.033	-0.391	84.1%	80.6%	3.84	12.0
1-U	0.896	1	1	0.638	-0.455	0.896	1	1	0.638	-0.455	77.8%	69.9%	3.76	13.0
2-C	0.127	1	-0.588	-1	-0.733	0.127	1	-0.747	-0.025	-0.733	91.5%	75.8%	3.91	13.0
2-U	-0.036	1	-0.743	-1	-0.765	-0.036	1	-0.743	-1	-0.765	93.5%	64.7%	3.96	14.0
3-C	-1	1	-0.541	0.826	-0.305	-0.534	1	-0.618	1	-0.252	85.5%	73.4%	3.91	13.0
3-U	-1	1	-0.437	1	-0.416	-1	1	-0.437	1	-0.416	84.3%	68.5%	4.01	13.0
4-C	0.333	0.674	-0.496	0.748	0.550	0.333	0.571	-0.690	0.768	0.550	83.5%	68.1%	3.88	13.0
4-U	0.332	0.828	-0.593	0.784	0.829	0.332	0.828	-0.593	0.784	0.829	83.4%	67.1%	3.94	13.0
5-C	0.588	-1	-0.946	1	-0.232	0.297	-0.450	-0.724	0.884	-0.357	68.6%	68.1%	4.03	15.0
5-U	0.527	0.917	1	1	-0.382	0.527	0.917	1	1	-0.382	77.2%	68.3%	3.95	15.0
6-C	0.475	-0.754	-1	1	-0.014	1	-1	-0.689	1	-0.015	90.2%	67.0%	3.94	12.0
6-U	1	-1	-1	1	0.164	1	-1	-1	1	0.164	97.8%	68.4%	3.91	13.0
7-C	0.973	-0.183	-0.569	-1	-0.034	1	-0.200	-1	-1	-0.063	95.1%	58.6%	4.31	16.0
7-U	1	-1	-1	-1	0.515	1	-1	-1	-1	0.515	92.6%	44.9%	3.97	12.0
8-C	0.456	1	-0.139	0.249	-0.253	1	1	-0.269	0.249	-0.296	76.5%	78.1%	5.50	17.0
8-U	1	-0.923	-0.796	0.983	-0.223	1	-0.923	-0.796	0.983	-0.223	73.4%	70.8%	4.02	14.0
9-C (+)	0.509	0.565	-0.472	0.023	0.252	1	1	-1	0.036	0.252	88.9%	77.8%	5.71	16.0
9-U	0.370	0.936	-1	-1	0.444	0.370	0.936	-1	-1	0.444	85.8%	58.6%	4.44	15.0
10-C (-)	0.604	0.623	-1	-1	-0.036	0.450	0.080	-0.500	-0.165	-0.036	61.2%	59.0%	4.44	15.0
10-U	0.370	0.936	-1	-1	0.444	0.370	0.936	-1	-1	0.444	85.8%	58.6%	4.44	15.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 83.8%

Avg Satisfaction (C) : 82.5%

Avg Satisfaction (U) : 85.2%

Avg Fulfillment (C & U) : 67.3%

Avg Fulfillment (C) : 70.7%

Avg Fulfillment (U) : 64.0%

Table D.3 NL-5-50-M (Mathematical Program)

Instance	Optimal (Coded) Setting					Realized (Coded) Value					(Secs)			
	+	+	-	+	-	+	+	-	+	-	Overall Satisfy	Avg x_1 Fulfill	Model Time	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_1	x_2	x_3	x_4	x_5				
1-C	0.134	0.617	-0.868	-0.682	-0.856	0.159	1	-1	-0.023	-0.784	74.7%	62.7%	5.50	17.0
1-U	-0.079	1	0.516	1	-0.059	-0.079	1	0.516	1	-0.059	70.4%	59.9%	3.95	12.0
2-C	0.009	0.088	-0.080	1	-0.057	0.009	0.088	-0.102	0.025	-0.057	92.3%	62.2%	5.51	17.0
2-U	0.008	0.050	-0.100	1	-0.052	0.008	0.050	-0.100	1	-0.052	95.2%	68.0%	3.95	13.0
3-C	-0.052	0.928	0.099	0.826	-0.799	-0.028	0.928	0.113	1	-0.660	76.6%	68.7%	5.54	18.0
3-U	0.583	1	0.180	1	0.223	0.583	1	0.180	1	0.223	76.9%	61.6%	4.06	13.0
4-C	0.409	-0.040	-0.140	0.975	-0.098	0.409	-0.034	-0.194	1	-0.098	92.2%	66.9%	4.14	12.0
4-U	0.397	-0.034	-0.189	1	-0.053	0.397	-0.034	-0.189	1	-0.053	92.1%	66.8%	3.97	12.0
5-C	0.356	0.220	-0.026	-1	-0.268	0.180	0.099	-0.020	-0.884	-0.413	82.3%	57.9%	5.47	18.0
5-U	0.187	0.099	-0.020	-1	-0.419	0.187	0.099	-0.020	-1	-0.419	84.0%	57.3%	3.90	13.0
6-C	0.007	-0.754	-1	1	-0.281	0.014	-1	-0.689	1	-0.299	75.6%	59.8%	5.44	17.0
6-U	0.014	-0.682	-1	1	-0.255	0.014	-0.682	-1	1	-0.255	79.2%	61.5%	3.96	14.0
7-C	0.067	-0.541	-0.280	1	0.109	0.069	-0.591	-0.667	1	0.204	93.8%	59.4%	3.91	13.0
7-U	0.068	-0.489	-0.457	1	0.227	0.068	-0.489	-0.457	1	0.227	93.7%	60.3%	3.93	12.0
8-C	0.456	1	0.121	-1	-0.523	1	1	0.234	-1	-0.612	86.0%	53.2%	4.03	13.0
8-U	1	1	0.234	-1	-0.612	1	1	0.234	-1	-0.612	86.0%	53.2%	3.96	13.0
9-C (+)	0.030	-0.012	-0.472	0.000	-0.595	0.059	-0.021	-1	0	-0.595	80.9%	62.1%	3.90	12.0
9-U	0.059	-0.021	-1	-0.591	-0.595	0.059	-0.021	-1	-0.591	-0.595	80.9%	58.5%	4.41	14.0
10-C (-)	0.065	0.168	-1	-0.876	-0.641	0.048	0.022	-0.500	-0.144	-0.641	63.9%	62.6%	6.15	19.0
10-U	0.059	-0.021	-1	-0.591	-0.595	0.059	-0.021	-1	-0.591	-0.595	80.9%	58.5%	4.41	14.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 82.9%

Avg Satisfaction (C) : 81.8%

Avg Satisfaction (U) : 83.9%

Avg Fulfillment (C & U) : 61.1%

Avg Fulfillment (C) : 61.5%

Avg Fulfillment (U) : 60.6%

Table D.4 NL-5-50-NM (Mathematical Program)

Instance	Optimal (Coded) Setting															(Secs)	
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	Overall Satisfy	Model Time
1-C	0.635	0.749	0.454	-1	0.566	0.770	-1	-0.879	0.337	-0.484	0.729	0.911	1	-0.554	0.458	84.3%	26.53
1-U	1	-1	-0.358	-1	1	1	0.563	-1	0.411	-1	1	0.652	1	0.664	1	85.8%	24.80
2-C	0.794	0.943	0.511	1	-0.322	-0.495	-1	1	1	-0.210	0.439	0.551	-0.856	0.894	0.904	74.7%	26.36
2-U	-1	-0.348	0.896	-1	0.570	-1	0.264	1	-1	-0.867	1	1	-0.813	0.183	-1	80.8%	25.08
3-C	-1	0.448	-1	-0.592	-1	-0.495	0.679	0.907	0.907	1	1	-0.685	-0.376	1	0.722	59.7%	24.94
3-U	1	-1	1	1	1	-1	0.539	-1	1	0.223	1	1	-1	1	-1	88.9%	24.80
4-C	-0.861	0.068	0.507	-0.570	0.973	-0.661	-1	-0.620	0.417	-0.644	1	0.358	1	-0.491	0.722	87.0%	25.44
4-U	1	-0.070	0.573	-1	0.746	-0.322	1	-0.943	-1	0.030	1	-0.189	0.462	0.078	-0.163	79.4%	25.74
5-C	0.540	1	1	-0.076	0.256	1	0.377	-0.281	0.678	-0.159	0.627	0.641	-0.622	1	1	83.1%	25.41
5-U	0.275	1.000	-1	-0.115	-0.069	1	1	-1	0.900	1	1	1	-1	-0.894	1	82.9%	25.62
6-C	0.310	0.406	0.520	0.445	0.402	-0.949	1	-0.419	0.524	0.569	-0.941	0.630	-0.400	-0.151	0.324	94.7%	25.15
6-U	1	1	0.274	1	1	-0.859	0.907	-0.361	-1	-0.385	1	1	-1	1	1	88.4%	25.11
7-C	0.561	-1	1	-1	-0.919	1	0.303	-1	1	-0.549	0.969	-0.513	-0.491	-1	0.328	86.9%	24.10
7-U	-0.271	1	1	1	1	0.906	0.383	-1	0.627	-1	1	-1	-1	-1	-1	88.6%	24.39
8-C	0.170	1	0.498	-0.373	-1	1	1	-0.061	-1	-1	0.543	0.032	-0.254	-0.633	0.499	82.7%	25.02
8-U	0.183	-1	0.812	-0.586	0.981	1	1	0.082	1	-1	1	0.001	-1	1	1	87.3%	25.27
9-C (+)	-0.020	-0.033	0.376	-0.319	0.281	-0.872	0.445	-0.337	0.243	0.204	0.488	0.477	-0.442	-0.080	0.352	84.3%	25.46
9-U	-0.746	-0.161	0.995	-1	-1	-0.662	-0.411	-1	1	0.737	-0.329	1	-1	-0.122	0.593	79.7%	25.48
10-C (-)	-1	0.058	-1	1	-0.692	-1	-1	1	-1	0.695	1	-1	1	0.171	1	71.1%	25.33
10-U	-0.746	-0.161	0.995	-1	-1	-0.662	-0.411	-1	1	0.737	-0.329	1	-1	-0.122	0.593	79.7%	25.48

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 82.5%

Avg Satisfaction (C) : 80.9%

Avg Satisfaction (U) : 84.2%

Table D.5a NL-15-25-M (Mathematical Program)

Instance	Realized (Coded) Value															(Secs)	
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	Avg x_i	Soln
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	Fulfill	Time
1-C	1	1	0.402	-0.669	1	1	0.284	-1	0.026	-1	1	0.802	-0.381	-1	1	81.9%	81.0
1-U	1	-1	-0.358	-1	1	1	0.563	-1	0.411	-1	1	0.652	1	0.664	1	65.3%	79.0
2-C	1	1	0.711	-0.530	-0.162	-1	0.057	0.562	0.372	-1	1	1	-0.856	0.963	1	76.1%	84.0
2-U	-1	-0.348	0.896	-1	0.570	-1	0.264	1	-1	-0.867	1	1	-0.813	0.183	-1	59.6%	94.0
3-C	-0.162	1	0.514	-1	0.344	-1	1	-1	1	0.999	-0.905	-1	-1	-0.507	1	71.0%	74.0
3-U	1	-1	1	1	1	-1	0.539	-1	1	0.223	1	1	-1	1	-1	67.7%	76.0
4-C	1	0.068	1	-1	1	-1	0.313	-1	1	-1	0.923	0.671	0.386	-1	0.593	87.3%	85.0
4-U	1	-0.070	0.573	-1	0.746	-0.322	1	-0.943	-1	0.030	1	-0.189	0.462	0.078	-0.163	65.3%	110.0
5-C	0.540	0.849	0.948	-0.306	0.222	-0.026	1	-1	1	-0.286	1	1	-1	-0.082	0.580	82.8%	82.0
5-U	0.275	1.000	-1	-0.115	-0.069	1	1	-1	0.900	1	1	1	-1	-0.894	1	70.4%	90.0
6-C	1	1	0.724	-1	1	-1	0.474	-1	0.461	-1	1	1	-1	-0.301	1	93.2%	76.0
6-U	1	1	0.274	1	1	-0.859	0.907	-0.361	-1	-0.385	1	1	-1	1	1	72.6%	81.0
7-C	1	0.063	0.974	-0.815	1	-0.565	0.722	-0.665	0.587	-1	1	-1	-1	-0.617	0.017	80.1%	81.0
7-U	-0.271	1	1	1	1	0.906	0.383	-1	0.627	-1	1	-1	-1	-1	-1	66.1%	82.0
8-C	0.266	0.779	0.968	-1	0.483	0.231	0.910	-0.201	0.831	-0.745	1	0.052	-1	-1	1	83.3%	81.0
8-U	0.183	-1	0.812	-0.586	0.981	1	1	0.082	1	-1	1	7E-04	-1	1	1	68.3%	93.0
9-C (+)	-0.041	-0.088	1	-1	1	-1	0.825	-1	1	0.713	1	1	-1	-0.450	0.657	83.6%	83.0
9-U	-0.746	-0.161	0.995	-1	-1	-0.662	-0.411	-1	1	0.737	-0.329	1	-1	-0.122	0.593	63.3%	101.0
10-C (-)	0.143	-0.073	0.986	-0.451	1	-0.924	0.400	-0.385	0.713	-0.681	0.055	0.038	-0.170	-0.773	0.206	72.8%	99.0
10-U	-0.746	-0.161	0.995	-1	-1	-0.662	-0.411	-1	1	0.737	-0.329	1	-1	-0.122	0.593	63.3%	101.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Fulfillment (C & U) : 73.7%

Avg Fulfillment (C) : 81.2%

Avg Fulfillment (U) : 66.2%

Table D.5b NL-15-25-M (Mathematical Program)

Instance	Optimal (Coded) Setting															(Secs)	
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	Overall Satisfy	Model Time
1-C	0.635	0.749	0.144	-1	0.566	0.770	-1	0.879	1.000	-0.484	0.030	1	1	-0.554	0.458	70.3%	26.61
1-U	1	-1	-0.568	-1	1	1	1	1	0.725	-1	-0.046	0.203	-0.974	-1	1	75.6%	25.78
2-C	0.015	-0.527	0.324	0.940	1	-0.495	-1	1	1	-0.152	0.035	0.551	-1	-0.929	0.904	70.3%	26.50
2-U	-0.112	-0.694	-0.439	-0.208	1	-1	-0.168	1	1	-0.183	-0.484	1	-0.775	1	1	73.5%	25.03
3-C	-0.095	0.448	-1	-0.592	-1	0.387	0.679	0.907	0.907	0.757	-0.654	0.685	-0.376	-1	0.722	77.9%	25.12
3-U	-0.04	0.361	-1	-1	1	0.789	1	-1	1	0.797	0.581	1	-1	1	1	79.4%	24.81
4-C	-0.861	-0.043	0.507	-0.570	0.973	0.661	-1	-0.355	0.417	0.644	0.357	0.534	-1	-0.037	1	80.2%	25.61
4-U	1	-0.139	0.225	-1	1	1	0.478	-0.205	-1	1	-0.1	0.788	-0.317	0.242	1	77.0%	25.58
5-C	0.632	1	-0.255	-0.206	1	1	0.377	-0.216	0.678	0.344	0.627	0.641	-0.182	1	1	81.3%	27.23
5-U	0.327	-1	-0.467	-1	1	-1	1	-0.83	1	0.442	1	-1	-0.125	1	1	87.1%	25.25
6-C	0.134	0.406	0.718	0.198	0.402	-0.515	1	-0.067	-0.378	0.569	-0.941	0.630	-0.400	-0.501	0.324	84.9%	29.64
6-U	0.327	1	0.679	-0.119	1	-0.278	1	0.231	-1	-0.775	1	1	-1	1	1	83.7%	28.21
7-C	0.561	-1	1	-1	-0.919	1	0.306	-1	1	-0.549	0.969	-0.513	-0.491	-1	0	81.0%	25.90
7-U	1	1	1	0.441	1	1	0.677	-1	0.788	-1	1	-1	-0.906	1	-1	86.1%	24.45
8-C	0.138	-0.060	0.382	-0.373	-1	1	1	-0.205	-1	-1	0.534	0.617	-0.254	-0.633	0.475	72.8%	26.30
8-U	0.144	-0.027	0.113	-0.935	1	1	1	-0.062	1	-1	0.689	-0.080	-1	1	0.979	78.3%	24.88
9-C (+)	0.493	-0.010	0.364	-0.319	0.281	-0.872	0.539	0.337	0.243	-0.286	0.488	0.252	-0.014	-0.179	0.407	89.0%	26.79
9-U	1	-0.103	0.454	-1	1	-0.521	0.563	1	1	-1.000	-1	0.140	0.661	-1	0.124	85.3%	25.16
10-C (-)	-1	0.015	-0.836	1	-0.692	-1	-1	1	-1	1	1	-1	-0.696	0.221	1	78.9%	25.47
10-U	1	-0.103	0.454	-1	1	-0.521	0.563	1	1	-1.000	-1	0.140	0.661	-1	0.124	85.3%	25.16

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 79.9%

Avg Satisfaction (C) : 78.7%

Avg Satisfaction (U) : 81.1%

Table D.6a NL-15-25-NM (Mathematical Program)

Instance	Realized (Coded) Value															(Secs)	
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	Avg x_i	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}		
1-C	1	1	0.127	-0.669	1	1	0.284	1	0.076	-1	0.041	0.880	-0.381	-1	1	64.3%	83.0
1-U	1	-1	-0.568	-1	1	1	1	1	0.725	-1	-0.046	0.203	-0.974	-1	1	65.3%	78.0
2-C	0.019	-0.559	0.451	-0.498	0.502	-1	0.057	0.562	0.372	-0.722	0.08	1	-1	-1	1	66.0%	94.0
2-U	-0.112	-0.694	-0.439	-0.208	1	-1	-0.168	1	1	-0.183	-0.484	1	-0.775	1	1	59.2%	94.0
3-C	-0.015	1	0.514	-1	0.344	0.782	1	-1	1	0.756	0.592	1	-1	0.507	1	67.8%	84.0
3-U	-0.04	0.361	-1	-1	1	0.789	1	-1	1	0.797	0.581	1	-1	1	1	67.1%	85.0
4-C	1	-0.043	1	-1	1	1	0.313	-0.572	1	1	0.329	1	-0.386	-0.075	0.768	65.8%	91.0
4-U	1	-0.139	0.225	-1	1	1	0.478	-0.205	-1	1	-0.1	0.788	-0.317	0.242	1	59.3%	97.0
5-C	0.632	0.849	-0.242	-0.833	0.869	-0.026	1	-0.769	1	0.619	1	1	-0.293	-0.082	0.580	68.9%	88.0
5-U	0.327	-1	-0.467	-1	1	-1	1	-0.83	1	0.442	1	-1	-0.125	1	1	63.4%	93.0
6-C	0.433	1	1	-0.445	1	-0.543	0.474	-0.161	-0.332	-1	1	1	-1	-1	1	70.9%	92.0
6-U	0.327	1	0.679	-0.119	1	-0.278	1	0.231	-1	-0.775	1	1	-1	1	1	65.8%	99.0
7-C	1	0.063	0.974	-0.815	1	-0.565	0.729	-0.665	0.587	-1	1	-1	-1	-0.617	0	70.6%	73.0
7-U	1	1	1	0.441	1	1	0.677	-1	0.788	-1	1	-1	-0.906	1	-1	59.4%	79.0
8-C	0.215	-0.047	0.743	-1	0.483	0.231	0.910	-0.673	0.831	-0.745	0.985	1	-1	-1	0.952	75.8%	85.0
8-U	0.144	-0.027	0.113	-0.935	1	1	1	-0.062	1	-1	0.689	-0.08	-1	1	0.979	68.9%	99.0
9-C (+)	1	-0.027	0.968	-1	1	-1.000	1.000	1	1	-1	1	0.528	-0.031	-1	0.761	74.6%	84.0
9-U	1	-0.103	0.454	-1	1	-0.521	0.563	1	1	-1.000	-1	0.140	0.661	-1	0.124	64.6%	95.0
10-C (-)	0.143	-0.019	0.825	-0.451	1	-0.924	0.400	-0.385	0.713	-0.979	0.055	0.038	0.118	-1	0.206	67.3%	85.0
10-U	1	-0.103	0.454	-1	1	-0.521	0.563	1	1	-1.000	-1	0.140	0.661	-1	0.124	64.6%	95.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Fulfillment (C & U) : 66.5%

Avg Fulfillment (C) : 69.2%

Avg Fulfillment (U) : 63.8%

Table D.6b NL-15-25-NM (Mathematical Program)

Instance	Optimal (Coded) Setting															(Secs)	
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	Overall Satisfy	Model Time
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}		
1-C	-0.267	1	0.637	0.623	1	-1	-1	0.416	-1	1	1	0.388	1	-1	0.615	65.6%	28.64
1-U	-0.278	0.004	-1	1	1	0.264	-0.980	-0.250	1	1	-0.741	0.784	-1	-1	1	73.4%	27.69
2-C	-0.476	1	0.301	1	0.861	0.487	-0.369	0.355	-0.466	-1	-1	-0.019	-0.463	1	1	71.0%	29.06
2-U	-0.059	1	-1	1	1	-1	-0.071	1	-0.583	-1	-1	0.041	-0.159	-1	1	68.2%	27.71
3-C	-0.362	-0.832	0.542	-1	-1	1	-0.735	0.909	-0.595	0.652	0.121	-1	0.427	-0.805	1	69.4%	29.07
3-U	-1	-1	1.000	-1	0.855	0.232	-1	0.362	1	1	-0.767	0.383	1	1	0.864	65.5%	27.82
4-C	-0.378	0.428	-0.856	1	-1	0.386	-0.784	0.499	-0.502	1	0.748	1	-1	-0.306	0.366	66.3%	28.80
4-U	-0.988	1	1	1	-1	-1	-1	1	-0.525	1	1	1	-1	1	-0.959	71.4%	27.30
5-C	-0.835	0.847	1	1	0.592	-1	-0.769	-1	1	1	1	1	-0.962	1	0.448	74.8%	28.62
5-U	1	0.009	0.958	-1	1	0.030	-1	0.749	-1	1	-1	0.925	-1	1	1	77.3%	27.47
6-C	-1	0.612	0.471	0.416	0.483	0.401	-1	0.147	-0.137	0.481	1	-1	-0.379	-0.917	1	61.5%	28.58
6-U	-1	0.703	0.530	1	1	1	1	0.305	-0.926	1	1	-1	1	-0.225	0.877	67.3%	26.95
7-C	0.956	0.599	0.428	-0.501	0.504	0.674	-0.695	0.791	-1	0.752	-0.555	0.264	1	1	0.708	75.7%	29.11
7-U	1	1	-1	0.722	0.404	1	-1	-1	-0.732	1	0.457	0.138	-1	1	1	71.0%	27.30
8-C	-0.398	0.945	-1	0.614	-1	0.686	-1	-1	-0.467	1	1	0.269	-0.64	-0.563	0.495	80.5%	29.73
8-U	-1	0.962	1	-0.013	0.377	1	-1	1	1	0.289	-1	-1	1	-1	1	80.3%	28.30
9-C (+)	-0.392	0.356	0.249	0.529	0.308	0.518	-0.374	-0.183	-0.532	0.427	-0.422	-1	-0.476	-0.136	0.568	79.1%	27.89
9-U	-0.947	0.650	1	-0.353	-0.370	1	-1	-1	-1	1	-1	-1	-1	1	0.798	72.2%	29.28
10-C (-)	0.951	-1	-0.429	1	-0.167	-1	1	-0.365	1	-0.583	0.980	-0.961	1	0.243	-1	63.2%	29.28
10-U	-0.947	0.650	1	-0.353	-0.370	1	-1	-1	-1	1	-1	-1	-1	1	0.798	72.2%	29.28

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 71.3%

Avg Satisfaction (C) : 70.7%

Avg Satisfaction (U) : 71.9%

Table D.7a NL-15-50-M (Mathematical Program)

Instance	Realized (Coded) Value															(Secs)	
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	Avg x_i	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	Fulfiln	
1-C	-0.276	0.133	1	1	0.053	0.710	-0.201	0.993	-0.488	0.670	0.299	0.032	-0.055	-0.822	1	73.8%	90.0
1-U	-0.278	0.004	-1	1	1	0.264	-0.98	-0.250	1	1	-0.741	0.784	-1	-1	1	72.7%	101.0
2-C	-1	0.480	-0.042	1	1	1	-1	1	-1	0.693	-0.811	-0.032	-0.727	0.276	0.211	81.9%	102.0
2-U	-0.059	1	-1	1	1	-1	-0.071	1	-0.583	-1	-1	0.041	-0.159	-1	1	66.4%	95.0
3-C	-1	0.063	1	0.057	0.18	0.247	-1	1	-1	1	0.176	-0.607	1	-1	0.202	69.9%	94.0
3-U	-1	-1	1	-1	0.855	0.232	-1	0.362	1	1	-0.767	0.383	1	1	0.864	58.2%	104.0
4-C	-1	1	1	0.209	0.481	1	-1	1	-0.888	0.055	1	1	-0.198	-1	1	82.8%	90.0
4-U	-0.988	1	1	1	-1	-1	-1	1	-0.525	1	1	1	-1	1	-0.959	65.2%	90.0
5-C	-1	1	0.512	0.875	1	0.513	-1	0.469	-0.541	0.657	-0.269	0.663	-1	0.203	1	84.3%	96.0
5-U	1	0.009	0.958	-1	1	0.030	-1	0.749	-1	1	-1	0.925	-1	1	1	72.2%	120.0
6-C	-0.650	0.797	1	1	1	1	-0.636	0.220	-0.039	-1	0.975	-0.398	-1	-0.615	0.674	70.9%	101.0
6-U	-1	0.703	0.53	1	1	1	1	0.305	-0.926	1	1	-1	1	-0.225	0.877	65.2%	99.0
7-C	1	1	1	-0.079	1	1	-1	1	-1	1	-0.493	0.264	0.018	-0.043	1	79.0%	97.0
7-U	1	1	-1	0.722	0.404	1	-1	-1	-0.732	1	0.457	0.138	-1	1	1	61.8%	103.0
8-C	-1	1	0.327	1	0.804	1	-0.846	0.677	-1	0.408	-0.692	1	-1	-1	1	92.5%	96.0
8-U	-1	0.962	1	-0.013	0.377	1	-1	1	1	0.289	-1	-1	1	-1	1	72.1%	101.0
9-C (+)	-1	1	1	1	0.556	1	-1	-1	-1	1	-1	-1	-1	-0.640	1	84.0%	89.0
9-U	-0.947	0.650	1	-0.353	-0.370	1	-1	-1	-1	1	-1	-1	-1	1	0.798	68.9%	107.0
10-C (-)	-1	0.579	1	0.498	0.216	0.498	-0.654	1	-0.064	1	-1	-0.961	-0.129	-1	0.168	76.2%	107.0
10-U	-0.947	0.650	1	-0.353	-0.370	1	-1	-1	-1	1	-1	-1	-1	1	0.798	68.9%	107.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Fulfillment (C & U) : 73.3%

Avg Fulfillment (C) : 79.5%

Avg Fulfillment (U) : 67.2%

Table D.7b NL-15-50-M (Mathematical Program)

Instance	Optimal (Coded) Setting															(Secs)	
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	Overall Satisfy	Model Time
	x ₁	x ₂	x ₃	x ₄	x ₅	x ₆	x ₇	x ₈	x ₉	x ₁₀	x ₁₁	x ₁₂	x ₁₃	x ₁₄	x ₁₅		
1-C	-0.968	1	0.637	0.623	-1	-1	-1	0.419	-1	0.749	1	1	1	1	0.615	57.9%	28.04
1-U	1	0.979	1	1	0.045	1	-1	1	1	0.483	1	0.840	-1	1	1	66.2%	26.51
2-C	-0.378	1	1	1	0.861	-0.487	-0.369	0.355	-0.466	0.346	1	0.085	-0.637	-1	0.038	65.6%	26.54
2-U	-0.73	1	-1	1	1	-1	-1	1	-1	-0.140	1	0.042	-1	-1	-1	72.3%	27.39
3-C	0.362	-1	-0.176	-1	-1	0.821	-0.735	0.909	-0.595	0.652	-0.691	0.851	-0.427	-0.805	0.407	60.3%	28.36
3-U	1	1	0.275	-1	-1	0.604	-1	1	-1	-1	-0.445	0.319	0.066	-1	0.744	63.4%	27.09
4-C	-0.378	0.428	-0.856	1	-1	0.386	-0.784	0.499	-0.252	1	0.748	-0.027	-1	-0.306	0.282	67.7%	27.85
4-U	-1	1	1	1	1	1	1	0.175	-0.068	-0.243	1	-0.438	-0.508	1	1	69.1%	26.53
5-C	-0.835	0.847	1	1	0.592	-1	-0.769	-1	1	0.744	1	1	-0.962	1	0.448	71.5%	27.89
5-U	-1	1	0.429	1	1	0.052	0.525	0.454	1	-0.710	0.237	0.581	-0.963	-1	1	66.5%	26.84
6-C	-1	0.769	0.209	0.416	0.483	0.401	-1	0.607	-0.512	0.481	1	-0.574	-0.186	0.342	1	68.5%	28.19
6-U	-1	0.388	0.096	1	1	1	1	0.739	-0.653	0.874	1	-0.261	0.707	0.122	0.977	71.9%	26.67
7-C	-0.626	0.599	0.428	-0.139	0.504	0.674	-0.695	-0.791	1	0.739	-1	1	1	1	0.708	69.6%	27.53
7-U	-0.258	1	-1	0.056	-0.353	1	-1	-1	-0.855	0.537	-0.441	1	0.663	-1	0.598	68.1%	26.60
8-C	-0.281	0.945	-1	0.614	-1	0.686	-1	-0.898	-0.467	1	1	0.269	-0.640	-0.563	0.495	78.3%	28.64
8-U	-0.11	1	0.355	1	-0.799	1	-1	0.832	-1	0.146	-0.998	-1	1	-1	1	77.3%	27.69
9-C (+)	-0.392	0.356	0.249	0.529	0.367	0.518	-0.347	-0.183	-0.532	0.427	-0.303	-1	-0.476	-0.212	0.568	73.6%	29.29
9-U	-0.545	1	0.854	1	0.773	1	-0.575	-1	-1	0.649	-0.315	-1	1	1	1	70.8%	32.20
10-C (-)	0.316	-1	-0.110	1	-0.243	1	1	-0.365	1	-0.583	0.452	0.197	1	0.243	-1	57.7%	27.68
10-U	-0.545	1	0.854	1	0.773	1	-0.575	-1	-1	0.649	-0.315	-1	1	1	1	70.8%	32.20

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 68.4%

Avg Satisfaction (C) : 67.1%

Avg Satisfaction (U) : 69.6%

Table D.8a NL-15-50-NM (Mathematical Program)

Instance	Realized (Coded) Value															(Secs)	
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	Avg x_i	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	Fulfill	
1-C	-1	0.133	1	1	-0.053	0.710	-0.201	1	-0.488	0.502	0.299	0.082	-0.055	0.822	1	63.0%	100.0
1-U	1	0.979	1	1	0.045	1	-1	1	1	0.483	1	0.840	-1	1	1	61.6%	105.0
2-C	-0.795	0.480	-0.139	1	1	-1	-1	1	-1	-0.240	0.811	0.141	-1	-0.276	0.008	63.0%	106.0
2-U	-0.730	1	-1	1	1	-1	-1	1	-1	-0.140	1	0.042	-1	-1	-1	60.4%	117.0
3-C	1	0.076	-0.325	0.057	0.180	0.203	-1	1	-1	1	-1	0.517	-1	-1	0.082	61.2%	118.0
3-U	1	1	0.275	-1	-1	0.604	-1	1	-1	-1	-0.445	0.319	0.066	-1	0.744	55.6%	124.0
4-C	-1	1	1	0.209	0.481	1	-1	1	-0.445	0.055	1	-0.027	-0.198	-1	0.769	66.1%	109.0
4-U	-1	1	1	1	1	1	1	0.175	-0.068	-0.243	1	-0.438	-0.508	1	1	61.8%	110.0
5-C	-1	1	0.512	0.875	1	0.513	-1	0.469	-0.541	0.489	-0.269	0.663	-1	0.203	1	71.1%	116.0
5-U	-1	1	0.429	1	1	0.052	0.525	0.454	1	-0.710	0.237	0.581	-0.963	-1	1	64.2%	130.0
6-C	-0.650	1	0.443	1	1	1	-0.636	0.912	-0.145	-1	0.975	-0.228	-0.49	0.230	0.674	63.1%	110.0
6-U	-1	0.388	0.096	1	1	1	1	0.739	-0.653	0.874	1	-0.261	0.707	0.122	0.977	61.6%	108.0
7-C	-0.655	1	1	-0.022	1	1	-1	-1	1	0.983	-0.889	1	0.018	-0.043	1	59.9%	116.0
7-U	-0.258	1	-1	0.056	-0.353	1	-1	-1	-0.855	0.537	-0.441	1	0.663	-1	0.598	57.2%	115.0
8-C	-0.705	1	0.327	1	0.804	1	-0.846	0.608	-1	0.408	-0.692	1	-1	-1	1	77.1%	112.0
8-U	-0.11	1	0.355	1	-0.799	1	-1	0.832	-1	0.146	-0.998	-1	1	-1	1	65.3%	116.0
9-C (+)	-1	1	1	1	0.662	1	-0.927	-1	-1	1	-0.717	-1	-1	-1	1	68.5%	110.0
9-U	-0.545	1	0.854	1	0.773	1	-0.575	-1	-1	0.649	-0.315	-1	1	1	1	60.8%	135.0
10-C (-)	-0.379	0.579	0.257	0.498	0.314	-0.498	-0.654	1	-0.064	1	-0.461	0.197	-0.129	-1	0.168	61.4%	130.0
10-U	-0.545	1	0.854	1	0.773	1	-0.575	-1	-1	0.649	-0.315	-1	1	1	1	60.8%	135.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Fulfillment (C & U) : 63.2%

Avg Fulfillment (C) : 65.4%

Avg Fulfillment (U) : 60.9%

Table D.8b NL-15-50-NM (Mathematical Program)

Instance	Optimal (Coded) Setting					Realized (Coded) Value					(Secs)			
	-	-	+	+	+	-	-	+	+	+	Overall	Avg x_i	Model	Soln
	x_1	x_2	x_3	x_4	x_5	x_1	x_2	x_3	x_4	x_5	Satisfy	Fulfill	Time	Time
1-C	0.280	-0.276	0.524	0.356	0.624	0.280	-0.271	1	0.523	0.624	75.8%	71.4%	4.61	16.0
1-U	0.262	-0.31	1	0.054	0.642	0.262	-0.310	1	0.054	0.642	73.0%	67.4%	3.18	10.0
2-C	-1	-1	-0.577	1	-1	-1	-1	-0.449	1	-0.143	82.3%	74.1%	4.76	16.0
2-U	-0.673	-1	-1	0.739	1	-0.673	-1	-1	0.739	1	81.0%	74.1%	3.33	10.0
3-C	1	-0.596	1	0.987	-1	0.267	-1	1	0.987	-0.025	75.5%	77.0%	4.96	17.0
3-U	-1	1	0.885	0.558	0.471	-1	1	0.885	0.558	0.471	73.1%	69.1%	3.81	15.0
4-C	0.370	-0.330	0.585	-0.317	0.885	0.627	-0.330	1	-0.300	0.885	78.8%	62.9%	4.89	17.0
4-U	0.821	-0.319	1	-0.283	0.875	0.821	-0.319	1	-0.283	0.875	78.0%	60.9%	3.35	10.0
5-C	-1	-0.180	1	1	-1	-0.67	-0.180	0.414	1	-0.473	71.3%	67.9%	4.89	16.0
5-U	-1	-0.113	1	1	-0.769	-1	-0.113	1	1	-0.769	82.5%	73.4%	3.31	11.0
6-C	-0.569	0.055	0.786	0.533	-0.175	-1	0.057	1	1	-0.175	79.1%	77.7%	4.90	17.0
6-U	-1	-0.361	-1	0.463	1	-1	-0.361	-1	0.463	1	81.8%	68.2%	3.34	11.0
7-C	0.854	-0.608	0.633	0.645	0.646	0.854	-1	1	1	1	93.9%	81.5%	4.92	18.0
7-U	1	-0.242	0.931	0.634	0.961	1	-0.242	0.931	0.634	0.961	80.8%	67.7%	3.44	11.0
8-C	1	-1	1	-0.165	1	1	-1	0.641	-0.071	1	89.4%	65.7%	4.84	17.0
8-U	1	-1	1	-0.165	1	1	-1	1	-0.165	1	93.8%	68.4%	3.39	10.0
9-C (+)	-0.509	-0.119	-0.03	0.683	1	-1	-0.119	-0.037	1	1	81.2%	80.8%	4.77	17.0
9-U	1	-0.853	0.530	1	1	1	-0.853	0.530	1	1	84.1%	73.8%	3.44	11.0
10-C (-)	1	-0.858	0.522	1	1	0.842	-0.858	0.514	0.846	1	82.6%	73.8%	5.52	18.0
10-U	1	-0.853	0.530	1	1	1	-0.853	0.530	1	1	84.1%	73.8%	3.44	11.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 81.1%

Avg Satisfaction (C) : 81.0%

Avg Satisfaction (U) : 81.2%

Avg Fulfillment (C & U) : 71.5%

Avg Fulfillment (C) : 73.3%

Avg Fulfillment (U) : 69.7%

Table D.9a L-5-75-M (Mathematical Program)

Instance	Optimal (Coded) Setting					Overall Satisfy	Avg x_i Fulfill
	-	-	+	+	+		
	x_1	x_2	x_3	x_4	x_5		
1-C	1	-0.331	1	-1	1	66.7%	26.7%
1-U	1	-1	-0.515	-1	1	57.8%	35.2%
2-C	-1	-1	-1	0.251	1	79.0%	32.5%
2-U	-1	-1	-1	1	-0.576	75.6%	55.8%
3-C	-0.536	1	1	-1	1	59.5%	24.6%
3-U	-1	-0.418	1	-1	-1	63.4%	25.8%
4-C	1	-1	1	-1	0.331	69.5%	26.7%
4-U	1	-1	1	-1	0.331	69.5%	26.7%
5-C	-1	-1	1	-1	0.251	84.0%	7.5%
5-U	-1	-1	1	-0.624	-1	81.4%	23.8%
6-C	-1	1	-1	1	0.536	70.9%	64.6%
6-U	-1	1	-1	0.911	1	75.3%	59.1%
7-C	1	-0.941	-1	1	1	84.7%	60.6%
7-U	1	-0.941	-1	1	1	84.7%	60.6%
8-C	1	-1	1	-0.165	1	93.8%	28.4%
8-U	1	-1	1	-0.165	1	93.8%	28.4%
9-C (+)	-1	-1	0.251	1	-1	73.3%	47.5%
9-U	1	-1	1	1	-0.577	73.3%	55.8%
10-C (-)	1	-1	0.251	1	1	83.6%	47.5%
10-U	1	-1	1	1	-0.577	73.3%	55.8%

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 75.6%

Avg Satisfaction (C) : 76.5%

Avg Satisfaction (U) : 74.8%

Avg Fulfillment (C & U) : 39.7%

Avg Fulfillment (C) : 36.7%

Avg Fulfillment (U) : 42.7%

Table D.9b L-5-75-M (Greedy Heuristic)

Instance	Optimal (Coded) Setting					Realized (Coded) Value					(Secs)			
	-	-	+	+	+	-	-	+	+	+	Overall	Avg x_i	Model	Soln
	x_1	x_2	x_3	x_4	x_5	x_1	x_2	x_3	x_4	x_5	Satisfy	Fulfill	Time	Time
1-C	1	-0.072	0.524	0.569	0.725	1	-0.071	1	0.836	0.725	84.9%	63.4%	3.15	10.0
1-U	1	-0.132	1	0.292	0.769	1	-0.132	1	0.292	0.769	80.9%	57.4%	3.17	10.0
2-C	1	-1	1	1	-0.624	1	-1	0.779	1	-0.089	79.9%	65.2%	3.72	11.0
2-U	1	-1	1	0.180	1	1	-1	1	0.180	1	86.1%	62.2%	3.42	10.0
3-C	1	-0.596	1	0.987	-1	0.267	-1	1	0.987	-0.025	80.8%	73.4%	3.68	15.0
3-U	1	-1	0.927	0.536	0.422	1	-1	0.927	0.536	0.422	76.3%	63.5%	3.68	15.0
4-C	0.589	0.105	0.585	0.192	0.935	1	0.105	1	0.182	0.935	79.9%	55.3%	3.68	10.0
4-U	1	-0.561	-1	0.903	1	1	-0.561	-1	0.903	1	79.4%	60.2%	3.46	10.0
5-C	1	-1	1	1	0.251	0.670	-1	0.414	1	0.119	74.0%	66.4%	3.65	11.0
5-U	1	-1	1	1	0.251	1	-1	1	1	0.251	82.8%	69.0%	3.43	11.0
6-C	0.155	-0.979	-0.786	0.533	1	0.272	-1	-1	1	0.999	78.5%	69.8%	3.69	11.0
6-U	1	-1	-1	0.911	1	1	-1	-1	0.911	1	77.7%	62.9%	3.33	10.0
7-C	0.854	-0.608	0.633	0.645	0.646	0.854	-1	1	1	1	93.2%	73.2%	3.36	11.0
7-U	1	-0.941	-1	1	1	1	-0.941	-1	1	1	81.1%	63.6%	3.25	11.0
8-C	1	-1	-0.004	0.650	1	1	-1	-0.002	0.279	1	85.8%	55.3%	3.30	11.0
8-U	1	-1	-0.624	1	1	1	-1	-0.624	1	1	89.8%	64.0%	3.29	10.0
9-C (+)	0.509	-0.689	0.214	0.683	1	1	-0.689	0.263	1	1	84.8%	64.2%	3.55	11.0
9-U	1	-0.970	0.321	1	1	1	-0.97	0.321	1	1	89.4%	66.4%	3.31	10.0
10-C (-)	1	-0.974	0.311	1	1	0.842	-0.974	0.306	0.846	1	87.3%	65.7%	3.42	10.0
10-U	1	-0.970	0.321	1	1	1	-0.970	0.321	1	1	89.4%	66.4%	3.31	10.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 83.1%

Avg Satisfaction (C) : 82.9%

Avg Satisfaction (U) : 83.3%

Avg Fulfillment (C & U) : 64.4%

Avg Fulfillment (C) : 65.2%

Avg Fulfillment (U) : 63.6%

Table D.10a L-5-75-NM (Mathematical Program)

Instance	Optimal (Coded) Setting					Overall Satisfy	Avg x_i Fulfill
	x_1	x_2	x_3	x_4	x_5		
1-C	1	1	1	0.331	1	76.3%	52.0%
1-U	1	1	-0.515	1	1	65.6%	52.0%
2-C	-0.251	-1	-1	1	1	70.1%	72.0%
2-U	-0.251	-1	-1	1	1	70.1%	72.0%
3-C	1	-1	1	-1	0.911	68.2%	55.6%
3-U	1	-1	1	0.911	-1	70.2%	54.9%
4-C	1	1	1	1	0.331	68.1%	57.3%
4-U	1	-1	-0.515	1	-1	40.3%	48.0%
5-C	-1	-1	1	-0.624	-1	68.5%	49.5%
5-U	0.624	-1	1	1	-1	78.1%	59.0%
6-C	-0.941	-1	-1	1	-1	49.1%	56.0%
6-U	1	-1	-1	1	0.536	71.1%	62.1%
7-C	1	-0.941	-1	1	1	81.1%	63.6%
7-U	1	-1	-1	1	0.941	80.8%	63.8%
8-C	1	-1	-0.624	1	1	89.8%	64.0%
8-U	1	-1	-0.624	1	1	89.8%	64.0%
9-C (+)	0.624	-1	-1	1	1	81.6%	67.0%
9-U	1	-1	0.251	1	1	89.4%	66.0%
10-C (-)	1	-1	0.251	1	1	89.4%	66.0%
10-U	1	-1	0.251	1	1	89.4%	66.0%

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 74.3%

Avg Satisfaction (C) : 74.2%

Avg Satisfaction (U) : 74.5%

Avg Fulfillment (C & U) : 60.5%

Avg Fulfillment (C) : 60.3%

Avg Fulfillment (U) : 60.8%

Table D.10b L-5-75-NM (Greedy Heuristic)

Instance	Optimal (Coded) Setting					Realized (Coded) Value					(Secs)			
	+	+	-	+	-	+	+	-	+	-	Overall Satisfy	Avg x_1 Fulfill	Model Time	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_1	x_2	x_3	x_4	x_5				
1-C	0.388	0.617	-0.868	-1	-1	0.459	1	-1	-0.033	-0.915	81.3%	83.4%	4.88	17.0
1-U	0.787	1	1	0.674	-0.779	0.787	1	1	0.674	-0.779	77.2%	72.4%	3.23	10.0
2-C	-0.515	1	-0.787	-1	-1	-0.515	1	-1	-0.025	-1	94.7%	74.6%	4.64	16.0
2-U	-1	1	-1	-0.577	-1	-1	1	-1	-0.577	-1	92.7%	64.2%	3.04	10.0
3-C	0.545	1	-0.876	0.826	1	0.291	1	-1	1	0.826	93.9%	74.7%	5.48	19.0
3-U	0.497	1	-0.875	1	1	0.497	1	-0.875	1	1	94.1%	73.7%	3.61	11.0
4-C	0.962	0.061	-0.719	0.975	1	0.962	0.052	-1	1	1	93.9%	70.1%	4.76	16.0
4-U	0.710	-0.052	-1	1	1	0.710	-0.052	-1	1	1	91.4%	66.6%	3.27	10.0
5-C	0.668	0.994	1	0.824	-0.649	0.337	0.447	0.765	0.728	-1	70.5%	67.5%	4.86	17.0
5-U	1	1	0.577	1	1	1	1	0.577	1	1	76.0%	64.2%	3.38	10.0
6-C	0.475	-0.754	-1	1	-0.014	1	-1	-0.689	1	-0.015	93.0%	67.0%	5.06	17.0
6-U	1	-1	-1	1	0.164	1	-1	-1	1	0.164	98.9%	68.4%	3.33	10.0
7-C	0.818	-0.916	-0.569	-0.225	-0.537	0.841	-1	-1	-0.225	-1	88.5%	66.2%	4.90	17.0
7-U	0.725	-1	-0.586	-0.295	-1	0.725	-1	-0.586	-0.295	-1	78.7%	60.2%	3.40	11.0
8-C	0.456	-0.295	-0.517	-0.071	-0.854	1	-0.295	-1	-0.071	-1	91.5%	76.3%	5.18	17.0
8-U	0.837	-1	-1	0.312	-1	0.837	-1	-1	0.312	-1	89.0%	71.5%	3.27	10.0
9-C (+)	0.509	0.565	-0.472	-0.630	-0.547	1	1	-1	-1	-0.547	99.0%	75.5%	4.76	17.0
9-U	0.512	0.901	-1	-1	0.718	0.512	0.901	-1	-1	0.718	89.8%	57.0%	3.23	10.0
10-C (-)	0.995	-0.084	-1	-1	-0.011	0.741	-0.011	-0.500	-0.165	-0.011	68.3%	60.8%	3.71	13.0
10-U	0.512	0.901	-1	-1	0.718	0.512	0.901	-1	-1	0.718	89.8%	57.0%	3.23	10.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 87.6%

Avg Satisfaction (C) : 87.5%

Avg Satisfaction (U) : 87.8%

Avg Fulfillment (C & U) : 68.6%

Avg Fulfillment (C) : 71.6%

Avg Fulfillment (U) : 65.5%

Table D.11a L-5-50-M (Mathematical Program)

Instance	Optimal (Coded) Setting					Overall Satisfy	Avg x_i Fulfill
	x_1	x_2	x_3	x_4	x_5		
1-C	1	1	-1	-0.155	1	74.8%	68.5%
1-U	1	1	0.575	1	1	75.6%	64.3%
2-C	-1	1	-1	-0.576	-1	92.9%	64.2%
2-U	-1	1	-1	-0.576	-1	92.9%	64.2%
3-C	1	1	-1	-0.576	1	85.9%	64.2%
3-U	0.250	1	-1	1	1	93.7%	72.5%
4-C	0.911	-1	-1	-1	-1	49.5%	59.1%
4-U	0.911	-1	-1	1	1	90.0%	59.1%
5-C	-0.146	1	1	1	-1	71.4%	68.5%
5-U	1	1	0.576	1	1	76.0%	64.2%
6-C	1	-1	-1	1	0.164	99.1%	68.4%
6-U	1	-1	-1	1	0.164	99.1%	68.4%
7-C	1	0.390	-1	-1	1	74.8%	53.9%
7-U	1	-1	-1	0.390	1	82.3%	53.9%
8-C	1	-1	-1	-0.156	-1	88.4%	68.4%
8-U	-0.624	1	-1	-1	-1	89.4%	63.8%
9-C (+)	0.390	1	-1	-1	1	89.5%	53.9%
9-U	0.390	1	-1	-1	1	89.5%	53.9%
10-C (-)	0.390	1	-1	-1	1	89.5%	53.9%
10-U	0.390	1	-1	-1	1	89.5%	53.9%

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 84.7%

Avg Satisfaction (C) : 81.6%

Avg Satisfaction (U) : 87.8%

Avg Fulfillment (C & U) : 62.1%

Avg Fulfillment (C) : 62.3%

Avg Fulfillment (U) : 61.8%

Table D.11b L-5-50-M (Greedy Heuristic)

Instance	Optimal (Coded) Setting					Realized (Coded) Value					(Secs)			
	+	+	-	+	-	+	+	-	+	-	Overall Satisfy	Avg x_1 Fulfill	Model Time	Soln Time
1-C	0.388	0.617	-0.868	-1	-1	0.459	1	-1	-0.033	-0.915	76.9%	62.5%	3.23	9.0
1-U	0.312	1	-0.837	1	1	0.312	1	-0.837	1	1	81.7%	53.7%	3.26	9.0
2-C	-1	1	0.787	1	-1	-1	1	1	0.025	-1	85.8%	38.2%	3.33	10.0
2-U	-1	1	1	1	-1	-1	1	1	1	-1	88.0%	44.0%	3.17	10.0
3-C	-0.684	1	-0.161	0.826	-1	-0.365	1	-0.184	1	-0.826	84.1%	65.8%	3.63	11.0
3-U	-0.329	1	0.117	1	-1	-0.329	1	0.117	1	-1	84.0%	65.8%	3.49	11.0
4-C	0.981	0.005	-0.719	0.975	1	0.981	0.004	-1	1	1	92.8%	52.0%	3.20	10.0
4-U	0.731	-0.106	-1	1	1	0.731	-0.106	-1	1	1	90.4%	51.7%	3.19	9.0
5-C	1	1	1	-1	-0.649	0.505	0.450	0.765	-0.884	-1	72.1%	49.0%	3.37	9.0
5-U	1	1	1	-1	-1	1	1	1	-1	-1	80.4%	44.0%	3.31	10.0
6-C	-0.475	-0.754	-1	1	-0.720	-1	-1	-0.689	1	-0.765	86.1%	47.7%	3.30	10.0
6-U	-1	-0.577	-1	1	-1	-1	-0.577	-1	1	-1	89.4%	51.4%	3.35	10.0
7-C	0.421	-0.916	-0.569	1	0.537	0.433	-1	-1	1	1	91.3%	45.1%	3.34	10.0
7-U	0.674	-1	-0.819	1	1	0.674	-1	-0.819	1	1	91.5%	45.0%	3.34	10.0
8-C	0.456	0.114	-0.517	-1	-0.854	1	0.114	-1	-1	-1	89.3%	56.0%	3.29	11.0
8-U	-0.624	1	-1	-1	-1	-0.624	1	-1	-1	-1	88.0%	49.3%	3.34	10.0
9-C (+)	-0.509	0.565	-0.472	-0.630	-0.976	-1	1	-1	-1	-0.976	80.6%	43.9%	3.62	15.0
9-U	-1	0.909	-1	-1	-0.552	-1	0.909	-1	-1	-0.552	78.4%	42.2%	3.32	10.0
10-C (-)	-1	0.390	-1	-1	-1	-0.745	0.050	-0.500	-0.165	-1	65.6%	53.6%	3.26	9.0
10-U	-1	0.909	-1	-1	-0.552	-1	0.909	-1	-1	-0.552	78.4%	42.2%	3.32	10.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 83.7%

Avg Satisfaction (C) : 82.5%

Avg Satisfaction (U) : 85.0%

Avg Fulfillment (C & U) : 50.2%

Avg Fulfillment (C) : 51.4%

Avg Fulfillment (U) : 48.9%

Table D.12a L-5-50-NM (Mathematical Program)

Instance	Optimal (Coded) Setting					Overall Satisfy	Avg x_i Fulfill
	x_1	x_2	x_3	x_4	x_5		
1-C	-1	1	0.624	1	-1	78.2%	49.3%
1-U	-1	1	0.624	1	-1	78.2%	49.3%
2-C	-1	1	-1	-0.576	-1	82.1%	46.5%
2-U	-1	1	-1	-0.576	-1	82.1%	46.5%
3-C	-1	1	-0.180	1	-1	83.0%	57.6%
3-U	-1	1	-1	1	0.624	76.4%	44.5%
4-C	-1	-1	-1	1	-0.417	76.5%	45.7%
4-U	0.911	-1	-1	1	1	89.3%	44.2%
5-C	-0.146	1	-1	-1	-1	69.4%	56.0%
5-U	1	-0.624	-1	-1	-1	59.4%	51.0%
6-C	1	-1	-1	1	0.164	43.1%	54.0%
6-U	1	-1	-1	1	0.164	43.1%	54.0%
7-C	-1	-1	-0.331	1	-1	58.4%	49.3%
7-U	0.390	-1	-1	1	1	91.1%	45.2%
8-C	1	-0.624	-1	-1	-1	85.6%	51.0%
8-U	-0.624	1	-1	-1	-1	88.1%	49.3%
9-C (+)	1	0.390	-1	-1	1	61.1%	40.0%
9-U	0.390	1	-1	-1	1	67.2%	41.2%
10-C (-)	1	0.390	-1	-1	1	61.1%	40.0%
10-U	0.390	1	-1	-1	1	67.2%	41.2%

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 72.0%

Avg Satisfaction (C) : 69.9%

Avg Satisfaction (U) : 74.2%

Avg Fulfillment (C & U) : 47.8%

Avg Fulfillment (C) : 48.9%

Avg Fulfillment (U) : 46.6%

Table D.12b L-5-50-NM (Greedy Heuristic)

Instance	Optimal (Coded) Setting															(Secs)	
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	Overall Satisfy	Model Time
1-C	0.635	0.749	1	-1	0.566	0.287	-1	-0.879	-0.683	-0.484	0.729	1	1	-0.554	0.458	91.1%	11.16
1-U	1	-1	1	-1	-1	1	1	-1	0.146	-1	1	0.876	0.396	-0.155	1	90.5%	10.26
2-C	0.247	0.541	0.718	1	1	-0.495	-1	-0.271	0.385	-0.210	0.439	0.551	-1	-0.204	0.904	85.2%	11.12
2-U	-0.432	0.005	1	-1	1	-1	-0.125	-0.265	0.722	0.059	1	0.284	-0.904	0.344	-1	78.2%	9.32
3-C	1	0.448	-1	-0.592	-1	-0.495	0.679	0.907	0.907	-1	-1	0.685	-0.376	1	0.722	92.6%	9.53
3-U	0.441	-1	1	1	1	-0.312	1	-1	1	0.053	1	0.512	-1	-0.604	0.538	85.7%	10.00
4-C	-0.861	0.862	0.507	-0.57	0.895	-0.661	-1	-0.620	0.081	-0.494	1	0.255	0.157	-0.491	0.411	89.3%	9.84
4-U	1	0.487	0.509	-1	0.498	-0.546	1	-0.126	-1	0.247	1	-0.643	-0.261	-0.348	0.257	74.7%	9.52
5-C	0.939	-0.112	1	-0.247	1	1	0.377	-0.281	0.678	-0.556	0.627	0.641	-0.622	1	1	91.6%	9.68
5-U	0.778	0.360	1	-0.885	-1	-1	1	-1	1	1	1	-1	-1	0.770	1	84.9%	9.96
6-C	0.310	0.406	0.718	0.445	0.402	-0.949	1	-0.419	1	0.569	-0.941	0.630	-0.400	-0.501	0.324	97.9%	9.69
6-U	1	1	0.717	-0.563	0.488	-0.881	1	-0.389	0.972	1	1	1	-1	1	-1	85.0%	9.44
7-C	0.561	-1	1	-1	-0.919	1	0.420	-1	1	-0.549	0.969	0.513	-0.491	-1	-1	90.6%	9.40
7-U	-1	1	1	1	0.796	0.771	1	-1	0.872	-1	0.937	0.477	-1	-1	-1	90.9%	9.29
8-C	0.641	1	0.514	-0.373	-1	0.102	1	-0.305	-1	-1	0.543	0.617	-0.254	-0.633	0.499	91.6%	9.59
8-U	0.609	0.731	0.335	-0.487	1	-1	1	-0.631	1	-1	1	-1	-0.836	1	-1	81.7%	9.51
9-C (+)	0.493	0.373	0.376	-0.319	0.281	-0.616	0.270	-0.337	0.243	-0.286	-0.488	0.477	-0.442	-0.179	0.535	97.9%	9.64
9-U	1	1	0.876	-1	-1	-0.601	-0.146	1	1	-0.758	-1	0.841	1	-1	0.530	86.3%	9.54
10-C (-)	-1	-0.797	-1	1	-0.692	-1	-1	1	-1	1	1	-1	1	0.221	1	83.8%	9.43
10-U	1	1	0.876	-1	-1	-0.601	-0.146	1	1	-0.758	-1	0.841	1	-1	0.530	86.3%	9.54

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 87.8%

Avg Satisfaction (C) : 91.2%

Avg Satisfaction (U) : 84.4%

Table D.13a L-15-25-M (Mathematical Program)

Instance	Realized (Coded) Value															(Secs)	
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	Avg x_i	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	Fulfil	
1-C	1	1	0.884	-0.669	1	0.372	0.284	-1	-0.052	-1	1	0.880	-0.381	-1	1	85.6%	35.0
1-U	1	-1	1	-1	-1	1	1	-1	0.146	-1	1	0.876	0.396	-0.155	1	69.3%	31.0
2-C	0.311	0.574	1	-0.530	0.502	-1	0.057	-0.152	0.143	-1	1	1	-1	0.220	1	80.2%	36.0
2-U	-0.432	0.005	1	-1	1	-1	-0.125	-0.265	0.722	0.059	1	0.284	-0.904	0.344	-1	67.4%	29.0
3-C	0.162	1	0.514	-1	0.344	-1	1	-1	1	-0.999	0.905	1	-1	-0.507	1	91.4%	29.0
3-U	0.441	-1	1	1	1	-0.312	1	-1	1	0.053	1	0.512	-1	-0.604	0.538	74.5%	29.0
4-C	1	0.862	1	-1	0.920	-1	0.313	-1	0.193	-0.767	0.923	0.478	0.061	-1	0.316	85.7%	29.0
4-U	1	0.487	0.509	-1	0.498	-0.546	1	-0.126	-1	0.247	1	-0.643	-0.261	-0.348	0.257	67.1%	29.0
5-C	0.939	-0.095	0.948	-1	0.869	-0.026	1	-1	1	-1	1	1	-1	-0.082	0.580	87.8%	29.0
5-U	0.778	0.360	1	-0.885	-1	-1	1	-1	1	1	1	-1	-1	0.770	1	70.8%	30.0
6-C	1	1	1	-1	1	-1	0.474	-1	0.879	-1	1	1	-1	-1	1	97.8%	28.0
6-U	1	1	0.717	-0.563	0.488	-0.881	1	-0.389	0.972	1	1	1	-1	1	-1	73.4%	29.0
7-C	1	0.063	0.974	-0.815	1	-0.565	1	-0.665	0.587	-1	1	1	-1	-0.617	-0.051	87.5%	28.0
7-U	-1	1	1	1	0.796	0.771	1	-1	0.872	-1	0.937	0.477	-1	-1	-1	71.0%	29.0
8-C	1	0.779	1	-1	0.483	0.024	0.910	-1	0.831	-0.745	1	1	-1	-1	1	92.4%	29.0
8-U	0.609	0.731	0.335	-0.487	1	-1	1	-0.631	1	-1	1	-1	-0.836	1	-1	72.1%	30.0
9-C (+)	1	1	1	-1	1	-0.706	0.500	-1	1	-1	-1	1	-1	-1	1	90.7%	29.0
9-U	1	1	0.876	-1	-1	-0.601	-0.146	1	1	-0.758	-1	0.841	1	-1	0.530	64.9%	30.0
10-C (-)	0.143	1	0.986	-0.451	1	-0.924	0.400	-0.385	0.713	-0.979	0.055	0.038	-0.170	-1	0.206	78.2%	30.0
10-U	1	1	0.876	-1	-1	-0.601	-0.146	1	1	-0.758	-1	0.841	1	-1	0.530	64.9%	30.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Fulfilment (C & U) : 78.6%

Avg Fulfilment (C) : 87.7%

Avg Fulfilment (U) : 69.5%

Table D.13b L-15-25-M (Mathematical Program)

Instance	Optimal (Coded) Setting															Overall Satisfy	Avg x_i Fulfill
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}		
1-C	-1	-1	1	-1	1	-0.604	1	-1	-1	-1	1	1	1	-1	-1	86.0%	65.3%
1-U	-1	-1	1	-1	-0.276	1	1	-1	-1	-1	1	1	1	-1	1	89.5%	62.4%
2-C	1	-1	1	-1	-1	-1	1	1	-0.171	1	1	1	-1	1	-1	64.5%	56.1%
2-U	-1	-1	1	-1	1	-1	-1	-1	1	1	-1	-0.291	-1	1	1	73.5%	55.7%
3-C	-1	1	1	-1	-1	-1	1	-1	-0.944	-1	1	-1	-1	1	1	71.7%	66.9%
3-U	1	-1	1	-1	-1	-1	1	-1	1	0.699	1	-1	-1	1	1	82.3%	67.7%
4-C	-1	-1	1	-0.044	-1	-1	-1	-1	-1	-1	1	-1	-1	-1	-1	55.2%	50.1%
4-U	-1	-1	1	-0.044	1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	60.7%	50.1%
5-C	-1	-0.700	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	1	81.5%	67.7%
5-U	1	-0.945	1	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	82.8%	66.9%
6-C	1	-1	1	0.7	1	-1	1	1	-1	-1	1	1	-1	1	1	78.5%	67.7%
6-U	1	1	1	0.7	1	-1	1	-1	-1	1	-1	1	-1	1	1	79.9%	67.7%
7-C	-1	1	1	-1	1	-1	-1	1	1	0.888	1	1	-1	-1	-1	75.1%	67.0%
7-U	-1	1	1	1	1	1	-0.924	-1	1	-1	1	1	-1	-1	-1	88.1%	66.9%
8-C	1	-1	-1	-1	-1	1	1	-0.604	1	-1	1	-1	-1	-1	1	72.2%	65.3%
8-U	-1	1	-1	-1	-0.211	-1	1	-1	1	-1	-1	-1	-1	1	1	71.5%	62.6%
9-C (+)	1	-1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	-1	0.615	52.8%	58.7%
9-U	1	-1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	0.698	78.7%	59.0%
10-C (-)	1	1	1	-1	-1	1	0.698	1	1	-1	-1	1	1	-1	-1	80.3%	59.0%
10-U	1	-1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	0.698	78.7%	59.0%

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 75.2%

Avg Satisfaction (C) : 71.8%

Avg Satisfaction (U) : 78.6%

Avg Fulfillment (C & U) : 62.1%

Avg Fulfillment (C) : 62.4%

Avg Fulfillment (U) : 61.8%

Table D.13c L-15-25-M (Greedy Heuristic)

Instance	Optimal (Coded) Setting															(Secs)	
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	Overall Satisfy	Model Time
1-C	0.635	-0.640	1	-1	0.566	-0.770	-1	-0.879	-0.450	-0.484	0.729	1	1	-0.554	0.458	85.4%	11.00
1-U	1	-1	1	-1	-1	1	1	-1	-0.626	-1	0.974	1	0.083	-0.207	1	87.1%	9.54
2-C	-0.794	-0.943	0.718	1	1	-0.495	-1	-0.724	1	-0.210	0.439	0.551	-1	-0.077	0.904	83.9%	11.03
2-U	-1	-1	1	-1	1	-1	0.240	-0.053	0.741	-0.128	-1	0.555	-1	1	1	81.8%	9.75
3-C	1	0.448	-1	-0.592	-1	-0.495	0.679	0.907	0.907	-1	-1	0.685	-0.376	1	0.722	90.4%	11.38
3-U	0.769	-1	-1	-1	1	-0.607	1	-1	0.952	0.200	1	0.799	-1	1	0.887	90.3%	9.72
4-C	-0.861	1	0.507	-0.570	0.483	-0.661	-1	-0.620	0.366	-0.366	-1	0.534	-0.65	-0.491	1	83.4%	11.80
4-U	1	0.889	1	-1	-0.237	0.109	1	-0.404	-1	0.697	-1	-0.647	-0.747	-0.581	0.691	76.6%	9.77
5-C	1	-1	1	-0.247	1	0.892	0.377	-0.281	0.678	-0.556	0.627	0.641	-0.622	1	1	86.9%	10.97
5-U	0.953	-1	-1	-1	1	1	1	-1	0.751	-1	1	-1	-1	-0.322	1	88.3%	10.16
6-C	0.310	0.406	0.718	0.445	0.402	-0.949	1	-0.419	1	0.569	-0.941	0.630	-0.400	-0.501	0.324	96.6%	11.34
6-U	1	-1	0.867	-0.582	0.501	-1	-1	-0.795	1	-0.351	1	1	-0.853	1	1	85.0%	9.99
7-C	0.561	-1	1	-1	-0.919	-0.318	0.420	-1	1	-0.549	0.969	0.513	-0.491	-1	-1	87.9%	11.00
7-U	-1	1	1	-1	0.607	1	1	-1	0.904	-1	0.509	-0.786	-1	-1	-1	89.4%	9.87
8-C	0.641	1	0.514	-0.373	-1	-1	-1	-0.305	-1	-1	0.534	0.617	-0.254	-0.633	0.499	80.5%	11.05
8-U	1	-0.525	-1	-1	1	1	-1	-0.505	1	-1	1	1	-1	1	1	80.6%	9.75
9-C (+)	0.493	0.373	0.376	-0.319	0.281	-0.445	0.237	-0.337	0.243	-0.286	-0.002	0.477	-0.442	-0.179	0.535	91.5%	11.00
9-U	1	1	0.951	-1	-1	-0.130	-0.479	1	1	-0.920	-0.801	0.883	1	-1	0.588	81.9%	9.76
10-C (-)	-1	-0.797	-1	1	-0.692	-1	-1	1	-1	1	1	-1	1	0.221	1	78.4%	10.02
10-U	1	1	0.951	-1	-1	-0.130	-0.479	1	1	-0.920	-0.801	0.883	1	-1	0.588	81.9%	9.76

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 85.4%

Avg Satisfaction (C) : 86.5%

Avg Satisfaction (U) : 84.3%

Table D.14a L-15-25-NM (Mathematical Program)

Instance	Realized (Coded) Value															(Secs)	
	+	+	+	-	+	-	+	-	+	-	+	+	-	-	+	Avg x_i	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	Fulfil	
1-C	1	-0.856	0.884	-0.669	1	-1	0.284	-1	-0.034	-1	1	0.880	-0.381	-1	1	71.4%	35.0
1-U	1	-1	1	-1	-1	1	1	-1	-0.626	-1	0.974	1	0.083	-0.207	1	61.0%	31.0
2-C	-1	-1	1	-0.530	0.502	-1	0.057	-0.407	0.372	-1	1	1	-1	-0.083	1	61.2%	36.0
2-U	-1	-1	1	-1	1	-1	0.240	-0.053	0.741	-0.128	-1	0.555	-1	1	1	57.9%	29.0
3-C	0.162	1	0.514	-1	0.344	-1	1	-1	1	-0.999	0.905	1	-1	-0.507	1	77.0%	35.0
3-U	0.769	-1	-1	-1	1	-0.607	1	-1	0.952	0.200	1	0.799	-1	1	0.887	68.8%	30.0
4-C	1	1	1	-1	0.496	-1	0.313	-1	0.878	-0.568	-0.923	1	-0.251	-1	0.768	70.4%	37.0
4-U	1	0.889	1	-1	-0.237	0.109	1	-0.404	-1	0.697	-1	-0.647	-0.747	-0.581	0.691	56.3%	31.0
5-C	1	-0.849	0.948	-1	0.869	-0.023	1	-1	1	-1	1	1	-1	-0.082	0.580	74.4%	35.0
5-U	0.953	-1	-1	-1	1	1	1	-1	0.751	-1	1	-1	-1	-0.322	1	64.1%	31.0
6-C	1	1	1	-1	1	-1	0.474	-1	0.879	-1	1	1	-1	-1	1	81.1%	35.0
6-U	1	-1	0.867	-0.582	0.501	-1	-1	-0.795	1	-0.351	1	1	-0.853	1	1	66.6%	31.0
7-C	1	0.063	0.974	-0.815	1	0.180	1	-0.665	0.587	-1	1	1	-1	-0.617	-0.051	74.3%	35.0
7-U	-1	1	1	-1	0.607	1	1	-1	0.904	-1	0.509	-0.786	-1	-1	-1	61.9%	30.0
8-C	1	0.779	1	-1	0.483	-0.231	-0.910	-1	0.831	-0.745	1	1	-1	-1	1	74.9%	36.0
8-U	1	-0.525	-1	-1	1	1	-1	-0.505	1	-1	1	1	-1	1	1	64.8%	30.0
9-C (+)	1	1	1	-1	1	-0.510	0.440	-1	1	-1	-0.003	1	-1	-1	1	79.4%	35.0
9-U	1	1	0.951	-1	-1	-0.13	-0.479	1	1	-0.920	-0.801	0.883	1	-1	0.588	59.5%	30.0
10-C (-)	0.143	1	0.986	-0.451	1	-0.924	0.400	-0.385	0.713	-0.979	0.055	0.038	-0.170	-1	0.206	67.2%	30.0
10-U	1	1	0.951	-1	-1	-0.130	-0.479	1	1	-0.920	-0.801	0.883	1	-1	0.588	59.5%	30.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Fulfilment (C & U) : 67.6%

Avg Fulfilment (C) : 73.1%

Avg Fulfilment (U) : 62.0%

Table D.14b L-15-25-NM (Mathematical Program)

Instance	Optimal (Coded) Setting															Overall Satisfy	Avg x_i Fulfill
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}		
1-C	-1	-1	1	-1	1	1	1	1	1	-1	1	1	1	-1	-0.212	75.3%	58.5%
1-U	-1	-1	1	-1	-1	1	1	-1	-1	-1	1	1	-0.604	-1	1	87.0%	61.4%
2-C	1	-1	1	-1	-1	-1	1	1	1	0.066	-1	1	-1	1	-1	72.6%	55.5%
2-U	-1	-1	1	-1	-1	-1	-1	-1	1	0.171	-1	1	-1	1	1	76.2%	56.2%
3-C	1	1	1	-1	-1	-1	1	1	-0.700	-1	1	-1	-1	1	1	79.6%	61.8%
3-U	1	-1	-1	-1	-1	-1	1	-1	1	-1	1	-0.511	-1	1	1	87.3%	65.7%
4-C	1	-1	-1	-1	1	-1	-1	-1	-1	1	-1	-1	-1	-1	0.746	58.1%	54.5%
4-U	-1	-1	1	-1	1	-1	-1	-1	-1	1	-1	-1	-1	-1	0.782	63.7%	53.3%
5-C	-1	1	1	-1	-1	-1	1	-1	1	0.959	1	-1	-1	1	1	79.4%	62.3%
5-U	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	0.943	1	82.1%	64.4%
6-C	-1	1	1	0.512	1	-1	1	-1	-1	-1	1	1	1	1	1	68.4%	60.3%
6-U	-1	1	1	0.512	1	-1	-1	-1	-1	-1	1	1	-1	1	1	70.2%	61.6%
7-C	-0.959	1	1	-1	1	1	-1	-1	1	-1	1	-1	-1	-1	-1	84.7%	60.1%
7-U	-1	1	1	-1	1	1	-0.959	-1	1	-1	1	-1	-1	-1	-1	84.8%	60.1%
8-C	1	-1	-1	-1	-1	-1	1	-1	0.604	1	-1	1	-1	-1	1	57.9%	62.2%
8-U	-1	-1	-1	-1	1	-1	1	-1	1	-1	-0.275	-1	-1	1	1	66.4%	61.1%
9-C (+)	1	-1	1	-1	-1	1	-1	0.126	1	-1	1	-1	1	-1	1	68.6%	54.6%
9-U	1	-1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	0.698	75.1%	56.7%
10-C (-)	1	1	1	0.053	-1	-1	-1	1	1	-1	-1	1	1	-1	-1	76.3%	51.3%
10-U	1	-1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	0.698	75.1%	56.7%

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 74.4%

Avg Satisfaction (C) : 72.1%

Avg Satisfaction (U) : 76.8%

Avg Fulfillment (C & U) : 58.9%

Avg Fulfillment (C) : 58.1%

Avg Fulfillment (U) : 59.7%

Table D.14c L-15-25-NM (Greedy Heuristic)

Instance	Optimal (Coded) Setting															(Secs)	
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	Overall Satisfy	Model Time
1-C	-0.968	1	0.637	0.623	-1	-1	-1	0.419	-1	1	-0.614	0.086	1	-1	0.615	74.6%	11.30
1-U	-0.036	1	-1	1	1	0.157	-0.788	-0.097	-0.984	0.271	0.036	0.743	-1	-1	0.965	78.6%	9.83
2-C	-0.476	1	-1	1	0.861	0.487	-0.369	0.355	-0.466	-1	-1	0.601	-0.637	-1	-0.612	88.4%	11.76
2-U	0.091	1	1	1	0.539	0.346	-0.820	1	-0.401	-0.063	-1	0.111	0.052	-1	-1	79.0%	10.02
3-C	-0.362	-1	0.542	-1	-1	0.053	-0.735	0.909	-0.595	0.652	-0.691	0.957	-0.427	-0.805	-0.255	82.4%	12.12
3-U	-1	-1	0.900	0.697	-1	0.386	-0.501	0.365	-1	0.465	-0.576	0.540	0.098	-0.747	0.245	70.1%	10.61
4-C	-0.378	0.428	-0.856	1	-1	0.386	-0.784	0.499	-0.566	-0.290	-0.748	1	-1	-0.306	0.366	87.1%	11.78
4-U	-0.418	1	1	1	0.422	0.026	-1	1	-0.153	1	1	1	-0.627	0.040	0.056	79.4%	9.96
5-C	-0.835	0.847	1	1	0.592	-1	-0.769	-1	1	1	1	1	-0.962	0.386	0.448	85.0%	11.48
5-U	1	0.605	1	-1	0.717	0.322	-1	0.663	-0.017	0.875	-1	0.220	-0.645	-1	1	80.6%	9.92
6-C	-1	0.769	0.471	0.416	0.483	0.401	1	0.665	0.322	-0.481	-1	1	-0.379	-0.699	0.112	83.9%	11.35
6-U	1	0.583	0.719	-1	1	0.672	-1	0.650	-0.314	0.557	-1	1	-0.244	-0.619	0.027	76.7%	10.18
7-C	-0.522	0.599	0.428	-0.381	0.504	0.674	-0.695	-0.791	-0.802	0.752	-0.256	1	1	1	0.708	82.4%	11.40
7-U	-0.058	1	-1	0.699	-0.016	1	-1	-1	-0.345	0.616	0.103	0.771	-0.698	-1	0.396	76.0%	10.04
8-C	-0.398	0.945	-1	0.614	-1	0.686	-1	-1	-0.467	1	1	0.269	-0.64	-0.563	0.495	89.5%	11.78
8-U	-1	1	1	1	-0.179	1	-1	1	-1	0.841	-0.406	-0.659	1	-1	-1	83.2%	10.11
9-C (+)	-0.392	0.356	0.249	0.508	0.426	0.518	-0.374	0.183	-0.532	0.427	-0.422	-0.044	-0.476	-0.212	0.219	93.1%	11.74
9-U	-0.568	0.384	1	0.688	0.503	1	-1	1	1	0.322	-1	0.663	-1	1	0.316	81.5%	10.41
10-C (-)	0.951	-1	-0.429	1	-0.772	-1	1	-0.365	1	-0.583	0.980	1	1	0.243	-1	86.5%	10.25
10-U	-0.568	0.384	1	0.688	0.503	1	-1	1	1	0.322	-1	0.663	-1	1	0.316	81.5%	10.41

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 82.0%

Avg Satisfaction (C) : 85.3%

Avg Satisfaction (U) : 78.7%

Table D.15a L-15-50-M (Mathematical Program)

Instance	Realized (Coded) Value															(Secs)	
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	Avg x_i	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	Fulfill	
1-C	-1	0.133	1	1	-0.053	0.710	-0.201	1	-0.488	0.670	-0.184	0.007	-0.055	-0.822	1	77.4%	37.0
1-U	-0.036	1	-1	1	1	0.157	-0.788	-0.097	-0.984	0.271	0.036	0.743	-1	-1	0.965	76.0%	31.0
2-C	-1	0.480	0.139	1	1	1	-1	1	-1	0.693	-0.811	1	-1	-0.276	-0.129	87.6%	37.0
2-U	0.091	1	1	1	0.539	0.346	-0.820	1	-0.401	-0.063	-1	0.111	0.052	-1	-1	73.4%	31.0
3-C	-1	0.076	1	0.057	0.18	0.013	-1	1	-1	1	-1	0.581	-1	-1	-0.051	82.9%	38.0
3-U	-1	-1	0.900	0.697	-1	0.386	-0.501	0.365	-1	0.465	-0.576	0.540	0.098	-0.747	0.245	67.7%	33.0
4-C	-1	1	1	0.209	0.481	1	-1	1	-1	-0.016	-1	1	-0.198	-1	1	89.6%	37.0
4-U	-0.418	1	1	1	0.422	0.026	-1	1	-0.153	1	1	1	-0.627	0.040	0.056	75.5%	32.0
5-C	-1	1	0.512	0.875	1	0.513	-1	0.469	-0.541	0.657	-0.269	0.663	-1	0.078	1	84.7%	38.0
5-U	1	0.605	1	-1	0.717	0.322	-1	0.663	-0.017	0.875	-1	0.220	-0.645	-1	1	73.5%	31.0
6-C	-0.650	1	1	1	1	1	0.636	1	0.091	1	-0.975	0.398	-1	-0.469	0.075	82.8%	38.0
6-U	1	0.583	0.719	-1	1	0.672	-1	0.650	-0.314	0.557	-1	1	-0.244	-0.619	0.027	71.3%	34.0
7-C	-0.578	1	1	-0.060	1	1	-1	-1	-0.802	1	-0.227	1	0.018	-0.043	1	78.6%	37.0
7-U	-0.058	1	-1	0.699	-0.016	1	-1	-1	-0.345	0.616	0.103	0.771	-0.698	-1	0.396	68.2%	32.0
8-C	-1	1	0.327	1	0.804	1	-0.846	0.677	-1	0.408	-0.692	1	-1	-1	1	92.5%	37.0
8-U	-1	1	1	1	-0.179	1	-1	1	-1	0.841	-0.406	-0.659	1	-1	-1	74.7%	33.0
9-C (+)	-1	1	1	0.96	0.768	1	-1	1	-1	1	-1	-0.044	-1	-1	0.386	93.6%	38.0
9-U	-0.568	0.384	1	0.688	0.503	1	-1	1	1	0.322	-1	0.663	-1	1	0.316	74.8%	33.0
10-C (-)	-1	0.579	1	0.498	1	0.498	-0.654	1	-0.064	1	-1	1	-0.129	-1	0.168	85.3%	34.0
10-U	-0.568	0.384	1	0.688	0.503	1	-1	1	1	0.322	-1	0.663	-1	1	0.316	74.8%	33.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Fulfillment (C & U) : 79.2%

Avg Fulfillment (C) : 85.5%

Avg Fulfillment (U) : 73.0%

Table D.15b L-15-50-M (Mathematical Program)

Instance	Optimal (Coded) Setting															Overall Satisfy	Avg x_i Fulfill
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}		
1-C	1	1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	-0.859	1	66.4%	52.9%
1-U	1	1	-0.994	1	1	1	-1	-1	-1	1	1	1	-1	1	1	72.7%	53.4%
2-C	0.471	1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	71.1%	58.2%
2-U	1	1	1	-0.915	1	1	-1	1	-1	-1	-1	-1	1	-1	-1	71.4%	46.4%
3-C	1	1	1	1	-1	-1	-0.782	-1	1	-1	-1	1	1	-1	1	65.3%	67.4%
3-U	1	-1	1	1	-1	-1	-1	-1	1	0.818	-1	1	1	-1	1	65.0%	53.9%
4-C	-1	-0.211	1	-1	1	-1	1	1	-1	1	-1	-1	-1	1	1	64.9%	49.3%
4-U	-1	1	1	1	1	-1	1	1	-0.559	1	1	1	-1	1	-1	74.0%	54.8%
5-C	0.888	1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	79.5%	39.6%
5-U	1	1	1	-1	1	1	-1	1	-1	1	-1	-0.511	-1	1	-1	81.5%	41.6%
6-C	1	-1	1	1	1	0.615	-1	-1	1	1	-1	1	1	-1	-1	62.4%	48.0%
6-U	1	1	1	-1	-1	1	-1	1	-0.736	1	-1	-1	1	-1	-1	68.6%	34.2%
7-C	1	1	1	1	-1	-1	1	-0.792	-1	1	-1	1	1	-1	-1	61.3%	52.6%
7-U	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	0.792	1	68.0%	67.4%
8-C	-1	1	1	1	1	-1	1	-0.351	-1	-1	-1	1	-1	-1	-1	63.7%	64.5%
8-U	-1	1	1	1	-1	1	-1	1	0.462	-1	-1	-1	1	-1	1	77.9%	38.2%
9-C (+)	-0.559	1	1	-1	1	-1	-1	1	1	-1	-1	-1	-1	-1	1	65.2%	68.1%
9-U	-1	1	1	1	0.647	-1	-1	-1	1	1	-1	1	-1	1	-1	74.9%	52.2%
10-C (-)	-1	1	-0.144	-1	1	-1	-1	-1	-1	1	-1	1	1	1	1	64.7%	49.5%
10-U	-1	1	1	1	0.647	-1	-1	-1	1	1	-1	1	-1	1	-1	74.9%	52.2%

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 69.7%

Avg Satisfaction (C) : 66.4%

Avg Satisfaction (U) : 72.9%

Avg Fulfillment (C & U) : 52.2%

Avg Fulfillment (C) : 55.0%

Avg Fulfillment (U) : 49.4%

Table D.15c L-15-50-M (Greedy Heuristic)

Instance	Optimal (Coded) Setting															(Secs)	
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	Overall Satisfy	Model Time
1-C	-0.968	1	0.637	0.623	-1	-1	-1	0.419	-1	1	-0.685	-0.067	1	-1	0.615	70.4%	10.15
1-U	-0.359	1	0.981	1	1	0.500	1	0.300	-1	0.227	-0.478	0.919	-1	1	1	76.3%	10.06
2-C	-0.476	1	-1	1	0.861	0.487	-0.369	0.355	-0.466	-1	-1	0.601	-0.637	-1	-0.612	81.7%	11.51
2-U	-0.605	1	-1	1	1	0.553	-0.781	1	-1	0.494	1	0.045	-0.496	-1	-1	76.2%	10.04
3-C	-0.362	-1	-0.542	-1	-1	0.708	-0.735	0.909	-0.595	0.652	-0.691	0.357	-0.427	-0.805	0.756	75.9%	11.73
3-U	1	1	-1	0.653	-1	0.583	-0.808	0.928	-1	0.153	-0.928	-0.517	-0.462	-0.173	0.799	74.9%	10.13
4-C	-0.378	0.428	-0.856	1	-1	0.386	-0.116	0.499	-0.566	1	-0.748	1	-1	0.306	0.366	78.7%	11.71
4-U	-0.896	1	-1	1	1	0.567	1	1	-0.903	1	1	1	-0.766	1	0.560	77.7%	10.53
5-C	-0.835	0.847	1	1	0.592	-1	-0.769	-1	1	-1	1	1	-0.962	-1	0.448	78.7%	11.78
5-U	1	0.416	0.949	1	1	0.045	-1	0.789	1	-1	-1	0.669	-0.915	-1	1	78.8%	10.90
6-C	-1	0.769	0.471	0.416	0.483	0.401	1	0.665	-0.726	-0.481	1	1	-0.379	-0.926	0.507	78.5%	11.59
6-U	-1	0.922	0.278	-1	1	0.958	1	1	-0.803	0.114	1	1	-0.885	-0.285	-0.137	74.2%	11.04
7-C	-0.226	0.599	0.428	-0.443	0.504	0.674	-0.695	0.791	-0.194	0.752	-0.134	0.961	1	1	0.708	80.3%	11.67
7-U	-0.040	1	-1	0.946	0.258	1	-1	1	-0.046	0.427	-0.084	0.812	-0.159	1	0.611	72.4%	10.82
8-C	-0.398	0.945	-1	0.614	-1	0.686	-1	-1	-0.467	1	1	0.269	0.640	-0.563	0.495	83.7%	11.68
8-U	-1	1	1	1	-0.366	1	-1	1	-1	0.401	-0.934	-1	1	-1	-1	81.5%	10.45
9-C (+)	-0.392	0.356	0.249	0.529	0.422	0.518	-0.374	0.183	-0.532	0.427	-0.422	-0.161	-0.476	0.212	0.568	90.0%	11.68
9-U	-0.791	0.829	-1	0.813	0.738	1	-1	1	1	0.975	-1	0.646	1	1	1	78.3%	10.17
10-C (-)	0.951	-1	-0.429	1	-0.772	-1	1	-0.365	1	-0.583	0.980	1	1	-0.243	-1	80.7%	10.71
10-U	-0.791	0.829	-1	0.813	0.738	1	-1	1	1	0.975	-1	0.646	1	1	1	78.3%	10.17

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 78.4%

Avg Satisfaction (C) : 79.9%

Avg Satisfaction (U) : 76.9%

Table D.16a L-15-50-NM (Mathematical Program)

Instance	Realized (Coded) Value															(Secs)	
	-	+	+	+	+	+	-	+	-	+	-	+	-	-	+	Avg x_i	Soln Time
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	Fulfiln	
1-C	-1	0.133	1	1	-0.053	0.710	-0.201	1	-0.488	0.670	-0.205	-0.005	-0.055	-0.822	1	66.0%	31.0
1-U	-0.359	1	0.981	1	1	0.5	1	0.300	-1	0.227	-0.478	0.919	-1	1	1	68.6%	32.0
2-C	-1	0.480	0.139	1	1	1	-1	1	-1	0.693	-0.811	1	-1	-0.276	-0.129	73.7%	36.0
2-U	-0.605	1	-1	1	1	0.553	-0.781	1	-1	0.494	1	0.045	-0.496	-1	-1	62.5%	32.0
3-C	-1	0.076	-1	0.057	0.180	0.175	-1	1	-1	1	-1	0.217	-1	-1	0.153	62.0%	38.0
3-U	1	1	-1	0.653	-1	0.583	-0.808	0.928	-1	0.153	-0.928	-0.517	-0.462	-0.173	0.799	57.4%	31.0
4-C	-1	1	1	0.209	0.481	1	-0.148	1	-1	0.055	-1	1	-0.198	1	1	70.5%	37.0
4-U	-0.896	1	-1	1	1	0.567	1	1	-0.903	1	1	1	-0.766	1	0.560	63.3%	35.0
5-C	-1	1	0.512	0.875	1	0.513	-1	0.469	-0.541	-0.657	-0.269	0.663	-1	-0.203	1	69.6%	37.0
5-U	1	0.416	0.949	1	1	0.045	-1	0.789	1	-1	-1	0.669	-0.915	-1	1	61.5%	37.0
6-C	-0.650	1	1	1	1	1	0.636	1	-0.206	1	0.975	0.398	-1	-0.621	0.342	68.3%	37.0
6-U	-1	0.922	0.278	-1	1	0.958	1	1	-0.803	0.114	1	1	-0.885	-0.285	-0.137	62.9%	36.0
7-C	-0.236	1	1	-0.070	1	1	-1	1	-0.194	1	-0.120	0.961	0.018	-0.043	1	66.2%	38.0
7-U	-0.040	1	-1	0.946	0.258	1	-1	1	-0.046	0.427	-0.084	0.812	-0.159	1	0.611	62.4%	32.0
8-C	-1	1	0.327	1	0.804	1	-0.846	0.677	-1	0.408	-0.692	1	-1	-1	1	77.9%	37.0
8-U	-1	1	1	1	-0.366	1	-1	1	-1	0.401	-0.934	-1	1	-1	-1	63.8%	34.0
9-C (+)	-1	1	1	1	0.762	1	-1	1	-1	1	-1	-0.161	-1	1	1	73.5%	38.0
9-U	-0.791	0.829	-1	0.813	0.738	1	-1	1	1	0.975	-1	0.646	1	1	1	61.6%	34.0
10-C (-)	-1	0.579	1	0.498	1	0.498	-0.654	1	-0.064	1	-1	1	-0.129	1	0.168	66.2%	33.0
10-U	-0.791	0.829	-1	0.813	0.738	1	-1	1	1	0.975	-1	0.646	1	1	1	61.6%	34.0

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Fulfillment (C & U) : 66.0%

Avg Fulfillment (C) : 69.4%

Avg Fulfillment (U) : 62.6%

Table D.16b L-15-50-NM (Mathematical Program)

Instance	Optimal (Coded) Setting															Overall Satisfy	Avg x_i Fulfill
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}		
1-C	1	1	1	1	1	-1	1	1	-1	1	1	1	0.993	-1	1	65.8%	60.0%
1-U	1	1	1	1	1	-1	-0.859	-1	-1	1	1	1	-1	1	1	68.5%	58.7%
2-C	-1	1	1	-1	1	1	1	1	-1	-1	1	-1	-1	-1	-0.878	70.2%	56.7%
2-U	1	1	1	1	1	1	-1	1	-1	-1	1	-1	0.471	-1	-1	71.4%	57.0%
3-C	1	1	-1	1	-1	-1	-1	1	1	-1	-1	0.782	1	-1	1	66.6%	50.2%
3-U	1	-1	1	1	-1	0.818	-1	1	1	-1	-1	1	1	1	1	62.4%	51.4%
4-C	-0.647	-1	1	-1	1	1	1	1	-1	1	-1	-1	-1	1	1	71.5%	59.3%
4-U	-0.647	1	1	1	1	1	1	1	-1	-1	1	-1	-1	1	1	74.4%	62.4%
5-C	1	1	1	-1	-0.888	1	-1	1	-1	-1	-1	1	-1	-1	-1	68.7%	58.4%
5-U	1	1	1	1	1	-0.699	-1	1	1	-1	-1	1	-1	1	1	78.8%	60.5%
6-C	1	1	1	1	-1	1	-1	-1	1	1	-1	-1	-1	-0.698	-1	56.3%	52.1%
6-U	1	1	1	-1	-1	1	-1	1	-1	-1	-1	-1	-1	-0.736	-1	68.5%	53.5%
7-C	1	1	-1	1	-1	-1	1	-0.751	-1	1	-1	1	-1	-1	-1	58.9%	51.0%
7-U	1	1	1	1	-1	1	1	-1	1	1	1	1	-1	0.792	1	68.2%	52.0%
8-C	-1	1	1	1	-1	-0.408	-1	1	-1	-1	-1	1	-1	1	-1	74.9%	60.9%
8-U	0.462	1	1	1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	78.5%	60.6%
9-C (+)	1	1	1	1	1	-0.212	-1	1	1	1	1	-1	-1	1	1	68.3%	56.8%
9-U	-1	1	-0.144	1	1	-1	1	1	1	1	-1	-1	-1	1	1	71.6%	60.3%
10-C (-)	-0.511	-1	1	-1	1	1	-1	1	1	-1	-1	-1	-1	-1	1	59.3%	56.8%
10-U	-1	1	-0.144	1	1	-1	1	1	1	1	-1	-1	-1	1	1	71.6%	60.3%

"+" = More is better (+1 = Best)

"-" = Less is better (-1 = Best)

Avg Satisfaction (C & U) : 68.7%

Avg Satisfaction (C) : 66.1%

Avg Satisfaction (U) : 71.4%

Avg Fulfillment (C & U) : 56.9%

Avg Fulfillment (C) : 56.2%

Avg Fulfillment (U) : 57.7%

Table D.16c L-15-50-NM (Greedy Heuristic)

APPENDIX E

Observation	CA ₁	CA ₂	CA ₃	CA ₄	CA ₅	CA ₆	CA ₇	CA ₈	CA ₉	CA ₁₀
1	3.50	1.97	2.24	2.93	0.74	4.70	1.32	-647.86	-309.52	2.88
2	4.17	4.17	3.17	2.11	4.85	1.29	4.64	-1249.80	-1491.86	2.46
3	1.82	1.73	2.56	3.88	1.65	1.21	2.11	-1150.98	-727.89	2.27
4	4.19	2.75	4.48	3.95	4.88	4.82	2.23	-1620.98	-1837.80	4.77
5	1.84	3.20	1.28	3.89	1.68	4.74	4.58	-1402.54	-396.37	3.49
6	1.88	1.79	2.67	2.96	1.65	1.21	4.31	-1119.24	-809.72	2.27
7	2.49	3.93	3.48	3.00	3.95	4.78	3.85	-757.28	-1549.91	4.17
8	4.81	4.89	4.43	3.05	0.83	1.33	3.84	-416.60	-655.14	1.90
9	3.15	4.66	3.98	2.97	1.68	4.74	1.90	-1392.62	-493.35	3.49
10	2.52	2.52	4.03	2.08	3.92	1.26	4.07	-498.89	-1303.75	1.86
11	2.89	1.29	1.78	4.87	4.88	4.82	4.91	-1630.90	-1740.81	4.77
12	4.11	4.11	3.06	3.02	4.85	1.29	2.45	-1281.55	-1410.03	2.46
13	1.14	0.96	1.27	1.03	3.79	1.09	1.27	-365.80	-1189.86	1.62
14	3.44	1.91	2.13	2.01	0.74	4.69	3.51	-558.88	-226.27	2.87
15	3.42	3.33	1.67	2.00	0.71	1.16	1.04	-283.51	-541.26	1.65
16	2.55	3.99	3.60	3.92	3.95	4.79	1.66	-846.26	-1633.16	4.17
17	1.93	1.40	1.59	2.38	1.19	1.74	2.78	-399.68	-438.78	1.80
18	2.95	3.27	3.22	2.44	1.24	1.80	2.82	-426.46	-493.69	1.90
19	2.57	3.57	2.02	2.42	1.22	1.78	2.81	-404.26	-441.96	1.86
20	3.44	2.63	1.81	2.42	1.22	1.78	2.81	-415.34	-449.66	1.87
21	2.25	2.48	1.80	2.40	1.21	1.76	2.79	-381.36	-399.70	1.83
22	2.27	2.50	2.13	3.11	1.23	1.79	2.81	-448.95	-743.19	2.28
23	2.24	2.47	1.80	2.38	1.19	1.74	1.56	-397.55	-437.30	1.80
24	2.25	2.49	1.80	2.40	1.21	1.76	2.79	-400.33	-439.24	1.83
25	2.28	2.51	2.29	2.44	4.34	1.80	3.10	-536.56	-1290.94	2.51
26	2.25	2.49	1.80	2.40	1.21	1.76	2.79	-400.33	-439.24	1.83
27	2.27	2.51	1.82	3.17	1.23	4.70	2.81	-687.53	-445.89	3.30
28	2.26	2.50	1.81	2.88	1.67	1.78	3.20	-1135.96	-609.57	2.13
29	1.93	2.12	1.15	2.38	1.19	1.74	2.78	-392.09	-433.51	1.80
30	1.80	2.43	1.80	2.39	1.20	1.75	2.79	-395.78	-436.08	1.82
31	2.26	2.50	1.81	2.88	1.22	1.78	2.93	-433.30	-441.53	1.86
32	2.25	2.48	1.80	2.25	1.20	1.17	2.79	-376.16	-438.68	1.54
33	2.25	2.48	1.69	2.16	1.20	1.75	2.78	-395.47	-408.85	1.67
34	2.24	2.47	1.80	1.92	1.19	1.74	2.65	-367.36	-436.95	1.80
35	2.26	2.50	1.81	2.42	1.22	1.78	4.02	-403.12	-441.17	1.86
36	2.24	2.47	1.80	1.92	0.74	1.74	2.38	-326.77	-422.20	1.53
37	2.57	2.85	2.46	2.42	1.22	1.78	2.81	-408.57	-444.96	1.86
38	2.25	2.49	1.80	2.40	1.17	1.76	2.79	-376.66	-291.23	1.82

Table E.1 Average Responses for Augmented 2¹⁰⁻⁶ Design

EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀
Block 1									
-1	-1	-1	-1	-1	-1	-1	-1	+1	+1
+1	-1	-1	-1	+1	-1	+1	+1	-1	-1
-1	+1	-1	-1	+1	+1	-1	+1	-1	-1
+1	+1	-1	-1	-1	+1	+1	-1	+1	+1
-1	-1	+1	-1	+1	+1	+1	-1	-1	+1
+1	-1	+1	-1	-1	+1	-1	+1	+1	-1
-1	+1	+1	-1	-1	-1	+1	+1	+1	-1
+1	+1	+1	-1	+1	-1	-1	-1	-1	+1
-1	-1	-1	+1	-1	+1	+1	+1	-1	+1
+1	-1	-1	+1	+1	+1	-1	-1	+1	-1
-1	+1	-1	+1	+1	-1	+1	-1	+1	-1
+1	+1	-1	+1	-1	-1	-1	+1	-1	+1
-1	-1	+1	+1	+1	-1	-1	+1	+1	+1
+1	-1	+1	+1	-1	-1	+1	-1	-1	-1
-1	+1	+1	+1	-1	+1	-1	-1	-1	-1
+1	+1	+1	+1	+1	+1	+1	+1	+1	+1

Figure E.1a Block 1 of Augmented Design

EC ₁	EC ₂	EC ₃	EC ₄	EC ₅	EC ₆	EC ₇	EC ₈	EC ₉	EC ₁₀
Block 2									
-1	0	0	0	0	0	0	0	0	0
+1	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0
0	+1	0	0	0	0	0	0	0	0
0	0	-1	0	0	0	0	0	0	0
0	0	+1	0	0	0	0	0	0	0
0	0	0	-1	0	0	0	0	0	0
0	0	0	+1	0	0	0	0	0	0
0	0	0	0	-1	0	0	0	0	0
0	0	0	0	+1	0	0	0	0	0
0	0	0	0	0	-1	0	0	0	0
0	0	0	0	0	+1	0	0	0	0
0	0	0	0	0	0	-1	0	0	0
0	0	0	0	0	0	+1	0	0	0
0	0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	0	+1	0	0
0	0	0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	0	+1	0
0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	0	+1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Figure E.1b Block 2 of Augmented Design

GLOSSARY

AHP		Analytic Hierarchy Process
ANOVA		Analysis of Variance
a_k		k^{th} column of constraint matrix
BBD		Box-Behnken Design
β_{ij}		Empirical relationship between EC_i & CA_j
B		Current basis matrix
CA		Customer Attribute
CAD		Computer-aided Design
CCD		Central Composite Design
CI		Contextual Inquiry
COPC		Customer-Oriented Product Concepting
CM		Correlation Matrix (γ_{ik} or θ_{ik})
C_D		Design budget
C_P		Production budget
c_{D_i}		Design cost for EC_i
c_{P_i}		Production cost for EC_i
c_k		k^{th} variable's cost coefficient
DOE		Design of Experiments
D_k		Maximum allowable decrease in c_k

EC		Engineering Characteristic
FEMA		Failure Mode and Effects Analysis
FTA		Fault Tree Analysis
γ_{ik}		Correlation of EC_i & EC_k ($\gamma_{ik} = v_i \cdot v_k$)
GRG		Generalized Reduced Gradient
HOQ		House of Quality
I_k		Maximum allowable increase in c_k
k		Number of factors or alternative index
λ_j		Relative importance of CA_j
L_i		Technically feasible lower bound of EC_i
MAUT		Multiattribute Utility Theory
MBTI		Myers Briggs Type Indicator
MPD		Modified Pencil Design problem
MPD_E		MPD problem w/ extended design space
MPD^T		Transformed MPD problem
MS_E		Mean square error
v_{ik}		Normalized value of θ_{ik}
N		Total number of observations
n_C		Number of center points
n_F		Number of factorial design points

PCSP		Pugh Concept Selection Process
PDPC		Process Decision Chart
PD^{D-Opt}		<i>D-optimal</i> Pencil Design problem
PD^{k-p}		2^{3-1} Pencil Design problem
PD^{SCD}		Small Composite (Pencil) Design problem
π_j		Minimum acceptable satisfaction for CA_j
PM		Planning Matrix (R_{ij} or β_{ij})
p		Number of parameters or probability
QFD		Quality Function Deployment
R		Design Resolution
R_{ij}		Quantified relationship of EC_i & CA_j
r_k^+		$\frac{\Delta c_k}{I_k}$
r_k^-		$\frac{-\Delta c_k}{D_k}$
SCD		Small Composite Design
SS_C		Sum of Squares for Curvature
θ_{ik}		Change in EC_k per unit change in EC_i
T		Total allowable development time
t_i		Development time for EC_i
U_i		Technically feasible upper bound of EC_i

VOC		Voice of the Customer
V_j		Satisfaction level of CA_j
$V_j^{\max}$		Maximum value of V_j over all $[L_i, U_i]$
$V_j^{\min}$		Minimum value of V_j over all $[L_i, U_i]$
w_i		Relative technical importance of EC_i
x_i		Value of EC_i
x_i^c		Coded (standardized) value of x_i
x_i^{cc}		Coded & centered (normalized) value of x_i
x_i^0		Initial value of EC_i
x_i'		Correlated realization of x_i
x_i^*		Optimal value of EC_i
y_C		Response at center point
$\bar{y}_F$		Average response at factorial design points
Z_i		Change in EC_i from x_i^0
z_k		$c_B B^{-1} a_k$

REFERENCES

- [1] Abadie, J., and Carpenter, J., 1969. *Generalization of the Wolfe Reduced Gradient Method for the Case of Nonlinear Constraints*, Optimization, pp. 37-47.
- [2] Bahill, T., and Chapman, W., 1993. *A Tutorial on Quality Function Deployment*, Engineering Management Journal, Vol 5, No 3, pp. 24-35.
- [3] Bazaraa, M., Jarvis, J. and Sherali, H., 1990. Linear Programming and Network Flows, Wiley, New York.
- [4] Beach, L., 1990. Image Theory: Decision Making in Personal and Organizational Contexts, Wiley, New York.
- [5] Bede, V., 1994. The Ecclesiastical History of the English People, Oxford University Press, New York.
- [6] Bradley, S., Hax, A. and Magnanti, T., 1977. Applied Mathematical Programming, Addison-Wesley, Reading, Massachusetts.
- [7] Clausing, D., 1994. Total Quality Development, ASME Press, New York.
- [8] Clausing, D., and Hauser, J., 1988 (May-June). *The House of Quality*, Harvard Business Review, , pp. 63-73.
- [9] Cohen, L., 1995. Quality Function Deployment, Addison-Wesley, Reading, Massachusetts.
- [10] Connolly, T., and Srivastava, J., 1995 (November). *Cues and Components in Multiattribute Evaluation*, Organizational Behavior and Human Decision Processes, Vol 64, No 2, pp. 219-228.
- [11] Cristiano, J., White, C. and Liker, J., 1994. *Set-Based Target Setting with Precise Rate of Importance Weights in Quality Function Deployment*, Sixth Symposium on Quality Function Deployment, ASI & GOAL/QPC, pp. 239-249.
- [12] Datta, B., 1995. Numerical Linear Algebra, Brooks/Cole, Pacific Grove, California.
- [13] Draper, N. and Smith, H., 1981. Applied Regression Analysis, Wiley, New York.
- [14] Drud, A., 1996. CONOPT: A System for Large Scale Nonlinear Optimization, ARKI Consulting and Development, Bagsvaerd, Denmark.

- [15] Drud, A., 1992. *CONOPT – A Large Scale GRG Code*, ORSA Journal on Computing, Vol 6, pp. 207-216.
- [16] Edwards, W., and Barron, F., 1994. *SMARTS and SMARTER: Improved Simple Methods for Multiattribute Utility Measurement*, Organizational Behavior and Human Decision Processes, Vol 60, pp. 306-325.
- [17] Eureka, W. and Ryan, N., 1995. *Quality Up, Costs Down: Taguchi Methods and QFD*, ASI Press, Dearborn, Michigan.
- [18] Fowler, T., 1990. *Value Analysis in Design*, Van Nostrand Reinhold, New York.
- [19] Griffin, A. and Hauser, J., 1993. *The Voice of the Customer*, Marketing Science, Vol 12, No 1, pp. 1-27.
- [20] Guinta, L. and Praizler, N., 1993. *The QFD Book*, American Management Association, New York.
- [21] Heywood, J., 1963. *A Dialogue of Proverbs*, University of California Press, Berkeley.
- [22] Hillier, F. and Lieberman, G., 1986. *Introduction to Operations Research*, Holden-Day, Oakland.
- [23] Hines, W. and Montgomery, D., 1990. *Probability and Statistics in Engineering and Management Science*, Wiley, New York.
- [24] Hippel, E. von, 1988. *The Sources of Innovation*, Oxford University Press, New York.
- [25] Hunter, M. and Van Landingham, R., 1994. *Listening to the Customer Using QFD*, Quality Progress, April, pp. 55-59.
- [26] Juran, J. and Gryna, F., 1993. *Quality Planning and Analysis*, McGraw-Hill, New York.
- [27] Kano, N., et al., 1984. *Attractive Quality and Must-Be Quality*, Japan Society for Quality Control, Vol 12, No 2, translated by Glenn Mazur.
- [28] Keeney, R. and Raiffa, H., 1976. *Decisions with Multiple Objectives*, Wiley, New York.
- [29] Kettinger, C., 1961. *Prophet of Progress*, Dutton, New York.

- [30] Klein, R., 1990. *New Techniques for Listening to the Voice of the Customer*, Second Symposium on Quality Function Deployment, ASI & GOAL/QPC, p. 197.
- [31] Laertius, D., 1925. Lives of Eminent Philosophers, Heinemann, London.
- [32] Law, A. and Kelton, W., 1982. Simulation Modeling and Analysis, McGraw-Hill, New York.
- [33] Lay, D., 1994. Linear Algebra and Its Applications, Addison-Wesley, Reading, Massachusetts.
- [34] Lec, S., 1962. Unkempt Thoughts, St. Martin's Press, New York.
- [35] Luenberger, D., 1979. Introduction to Dynamic Systems, Wiley, New York.
- [36] Lyman, D., 1990. *Deployment Normalization*, Second Symposium on Quality Function Deployment, ASI & GOAL/QPC, pp. 307-315.
- [37] Lyman, D. and Richter, K., 1993. *QFD and Personality Type*, Fifth Symposium on Quality Function Deployment, ASI & GOAL/QPC, pp. 197-205.
- [38] Montgomery, D. and Peck, E., 1992. Introduction to Linear Regression Analysis, Wiley, New York.
- [39] Murty, K., 1983. Linear Programming, Wiley, New York.
- [40] Myers, R. and Montgomery, D., 1995. Response Surface Methodology, Wiley, New York.
- [41] Pugh, S., 1991. Total Design, Addison-Wesley, Reading, Massachusetts.
- [42] Saaty, T., 1980. The Analytic Hierarchy Process, McGraw-Hill, New York.
- [43] Scofield Reference Bible, King James Version, Ecclesiastes, 1: 15.
- [44] Shillito, M., 1994. Advanced QFD, Wiley, New York.
- [45] Shin, J., and Kim, K., 1997. *Restructuring a House of Quality Using Factor Analysis*, Quality Engineering, Vol 9, No 4, pp. 739-746.
- [46] Shoemaker, P., and Waid, C., 1982. *An Experimental Comparison of Different Approaches to Determining Weights in Additive Utility Models*, Management Science, Vol 28, NO 2, pp. 182-196.

- [47] Srivastava, J., Connolly, T., and Beach, L., 1995 (July). *Do Ranks Suffice? A Comparison of Alternative Weighting in Value Elicitation*, Organizational Behavior and Decision Processes, Vol 63, No 1, pp. 112-116.
- [48] Ulrich, K. and Eppinger, S., 1995. Product Design and Development, McGraw-Hill, New York.
- [49] Wasserman, G., 1993. *On How to Prioritize Design Requirements During the QFD Process*, IIE Transactions, Vol 25, No 3, pp. 59-65.
- [50] Wasserman, G., et al., 1993. *Using Fuzzy Set Theory to Derive an Overall Customer Satisfaction Index*, Fifth Symposium on Quality Function Deployment ASI & GOAL/QPC, pp. 36-54.
- [51] Wilde, O., 1978. Lady Windermere's Fan, Act III, Methuen, London.
- [52] Winston, W., 1995. Introduction to Mathematical Programming, Duxbury Press, Belmont, California.
- [53] Winterfeldt, D. von and Edwards, W., 1992. Decision Analysis and Behavioral Research, Cambridge University Press, New York.
- [54] Wixon, D., Holtzblatt, K. and Knox, S., 1990. *Contextual Design: An Emergent View of System Design*, Human Factors in Computing Systems, ACM, New York.
- [55] Xing, W., Jiang, Q., and Xie, J., submitted 1996. *Normalizing QFD Planning Matrix Using Linear Programming*, Tsinghua University, Beijing, China.
- [56] Yakowitz, S., and Szidarovszky, F., 1989. An Introduction to Numerical Computations, Macmillan, New York.
- [57] Yoder, B. and Mason, D., 1995. *Evaluating QFD Relationships Through the Use of Regression Analysis*, Seventh Symposium on Quality Function Deployment ASI & GOAL/QPC, pp. 35-59.