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**PHASE-SHIFTING BIREFRINGENT SCATTERPLATE
INTERFEROMETER**

by

Michael Brenton North Morris

**A Dissertation Submitted to the Faculty of the
COMMITTEE ON OPTICAL SCIENCES (GRADUATE)**

**In Partial Fulfillment of the Requirements
For the Degree of**

DOCTOR OF PHILOSOPHY

**In the Graduate College
THE UNIVERSITY OF ARIZONA**

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DEDICATION

**To my beautiful wife whose endless patients and selfless
encouragement enabled me to pursue my dream.**

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ABSTRACT

A new phase-shifting scatterplate interferometer is realized by exploiting the polarization characteristics of a birefringent scatterplate. Controlling the component of polarization that is scattered allows the birefringent scatterplate to separate the test and reference beams. The advantages of this design are that it does not require auxiliary optics to be placed near the surface under test and the “hot spot” and background irradiance, which are inherent to scatterplate interferometers, can be eliminated.

This study provides a description of the phase-shifting birefringent scatterplate interferometer, expands the theoretical model of the scatterplate interferometer to include polarization and phase shifting, analyzes the performance of the new interferometer and discusses possible sources of error induced by the design. In addition, a few component specific topics are addressed. Two methods for generating the birefringent scatterplate are presented and the role the scatterplate plays in removing the “hot spot” is explored. Furthermore, the practicality of using a liquid crystal retarder for phase shifting is analyzed in the process of determining the performance of the interferometer.

INTRODUCTION

The most significant advancement in interferometry and other types of instrumentation over the past thirty years has been the development of the computer. The prime manifestation of this occurrence has been the evolution of phase-shifting interferometry. Phase-shifting interferometry records a series of interferograms while the reference phase is changed. The wavefront is encoded in the irradiance distribution of the fringe pattern and a simple point-by-point calculation in the computer recovers the phase. Some of the advantages of phase-shifting interferometry over other measurement techniques are high measurement accuracy, rapid measurement capability, and good results with low contrast fringes. Accuracies on the order of $1/1000$ of a wave are achievable. Phase-shifting interferometry was first conceived by Carré in 1966 and was more fully developed later by Crane, Bruning, and Wyant to name a few (Crane 1969, Bruning 1974, Wyant 1975). Unfortunately, there is a limit imposed on every measurement technique. For phase-shifting interferometry the limiting error is usually fringe movement caused by vibrations and index variations in the optical path.

Common-path interferometers are a “dream come true” and a nightmare for optical engineers interested in phase-shifting interferometry. They are a “dream come true” because their common-path design makes them largely insensitive to vibrations and index variation in the optical path, and a nightmare because it is difficult to separate the test and reference beams for phase shifting. The scatterplate, point diffraction, and Fresnel zone plate interferometers are common-path interferometers that use diffraction to create the reference beam. There is also a subset of common-path interferometers that

shear the test beam by separating it into two orthogonal polarization components. Regardless of how the reference beam is created, the test and reference beams traverse the same optical path. Since it is no longer feasible to place a phase shift device in just one beam, phase shifting becomes difficult.

A few clever groups have phase shifted the point diffraction and scatterplate interferometers with varying success. Osuk Kwon, who fabricated the interferometer onto a sinusoidal grating, was the first to phase shift the point diffraction interferometer (Kwon 1984). The diffraction orders contained the desired phase shift. Later Carolyn Mercer et. al. phase shifted a point diffraction interferometer by embedding a microsphere into a thin liquid crystal layer (Mercer 1995). The microsphere created the reference beam and the liquid crystals produced a variable phase shift. Most recently the point diffraction interferometer was phase shifted by H. Medeck et. al. (Medeck 1996). In this setup the beam coming from the test optic is divided into two beams with a small angular separation using a grating. The zero order beam focuses onto a small pinhole creating the reference and the first order beam passes directly through a much larger hole that is slightly offset from the pinhole. The phase shift is created by shifting the grating laterally. The scatterplate interferometer has also been successfully phase shifted by two research groups: J. Huang et. al. and D. Su et. al (Huang 1988, Su 1991). Both methods exploit polarization to separate the test and reference beams by placing an auxiliary optic near the test mirror. Huang et. al. placed a small quarter-wave plate near the test surface which rotated only the incident linear polarization of the reference beam by 90° . With orthogonal polarizations in the test and reference beams the interferometer is phase

shifted using an electro-optic light modulator. Su et. al. playing on the same theme, placed a large polarizer with a small hole in the center near the test surface. In combination with additional polarization manipulating optics, the polarizer rotates the polarization of the test beam 90° .

The phase-shifting scatterplate interferometer presented in this dissertation also uses polarization to separate the test and reference beams. However, it does not require optics to be placed near the test surface. The secret to this more practical approach is the use of a birefringent scatterplate to separate the test and reference beams. This work will begin by describing the operation of the conventional scatterplate interferometer, both qualitatively and mathematically. After a brief review of polarization and birefringence, the operation of the phase-shifting birefringent scatterplate interferometer will be presented. Moreover, a new mathematical description will be developed which takes into account polarization and phase-shifting. Next, the manufacturing process used to generate the birefringent scatterplate will be outlined and the role etching plays in reducing the “hot spot” amplitude explored. Additionally, the calibration technique used to characterize the liquid crystal retarder phase shifter will be examined. Finally, the performance of the interferometer will be determined and improvements suggested.

CONVENTIONAL SCATTERPLATE INTERFEROMETER

Due to its common-path design and unique ability to average many measurements at one time the scatterplate interferometer is the ideal choice for a phase-shifting common-path interferometer. The scatterplate interferometer belongs to a subset of interferometers known as common-path. As the name implies, a common-path interferometer is one in which the test and reference beams traverse nearly the same optical path. The advantages of common-path interferometers are well documented (Burch 1953, Burch 1970, Scott 1970, Rubin 1980).

1. Reduced sensitivity to vibrations and air turbulence
2. No precision auxiliary optics required
3. White light source may be used

Since both the test and reference beams travel the same path it is easy to imagine that, for the most part, both beams experience the same phase variations due to optical path perturbations brought on by vibrations and air turbulence. Less obvious is the absence of piston error, relative longitudinal motion between the interferometer and test optic introduces defocus only, which is usually undetectable if the vibration movements are small (Rubin 1980). Another advantage of the common-path design is ability to use moderate quality optics in the interferometer. The aberrations present in the illumination and imaging optics will primarily affect the test and reference beams equivalently. In addition, common-path also implies equal path, accordingly white light sources can be used. There are, however, limitations imposed on the source size and spectral distribution (Rubin 1980).

Often in a common-path interferometer the division of the test and reference beams is accomplished through diffraction, as is the case for the scatterplate interferometer. The diffracting element in the scatterplate interferometer is a weak diffuser that possesses inversion symmetry about a unique point. Mathematically,

$$S(x, y) = S(-x, -y), \quad (2-1)$$

where S is the scatterplate amplitude transmission and $S(0,0)$ is the center of symmetry. In other common-path interferometers, such as the Smartt or Fresnel Zone plate, the diffracting elements are a single pinhole and a zone plate respectively.

In addition to being common-path, the scatterplate interferometer has the unique property of averaging many measurements at one time. Since each scatter-point on the scatterplate scatters the light in a unique manner, each scatter point makes a statistically independent measurement of the surface under test and the resulting interferogram is an average of all the measurements. Any small spatial phase variations present in the beam incident on the scatterplate will be averaged, significantly reducing the errors associated with them. Furthermore, errors associated with scatterplate asymmetry and non-isoplanatism manifest themselves as reductions in fringe visibility rather than direct phase measurement errors.

QUALITATIVE DESCRIPTION OF SCATTERPLATE INTERFEROMETER:

The scatterplate interferometer is schematically a simple design. The configuration for testing spherical mirrors is shown in figure 2-1. A small circular aperture is illuminated with a laser or broadband source producing a source of limited

extent for the interferometer. A focusing lens is used to image the source onto the test mirror by way of a beam splitter that removes the source from the path of the return beam and a scatterplate. The scatterplate is placed at the center of curvature of the test mirror. Part of the light incident on the scatterplate scatters illuminating the entire mirror surface and part of it passes directly through imaging to a point on the test mirror. After reflecting off the test surface the light again propagates through the scatterplate scattering a portion of the beam. Finally a focusing lens is used to image the interference fringes onto a screen or detector.

Figure 2-1: Scatterplate Interferometer for Testing Concave Mirrors

The question now arises, if all the light travels the same path, what is interfering to produce fringes. To answer the question it is necessary to look more closely at the effect of the scatterplate. Each time the light encounters the scatterplate some of it is scattered and a portion passes directly through the plate. Since the scatterplate is

traversed twice there are four permutations of the beam that arrives in the image plane: 1) Scattered-Scattered, 2) Scattered-Direct, 3) Direct-Scattered, and 4) Direct-Direct. An examination of each of these combinations will uncover their role in producing fringes. The direct-direct beam passes directly through the scatterplate both times it is encountered forming an image of the source called the “hot spot” in the image plane. Since the “hot spot” is never scattered, it does not contribute to the production of interference fringes. Similarly, the scattered-scattered beam does not play a role in the formation of interference fringes. The light is scattered both times it passes through the scatterplate producing background irradiance in the image plane. If a laser source is used the background irradiance as well as the interference fringes will contain a speckle pattern. The direct-scattered beam is the reference beam of the interferometer. The light passes directly through the scatterplate on the first pass and forms an image of the source on the test surface. If the image of the source is small enough the phase variations introduced into the beam on reflection are negligible. On the return leg the light is scattered. The scattered-direct beam serves as the test beam of the interferometer. The light is scattered on its initial pass through the scatterplate illuminating the entire test mirror. Any departures from a sphere will introduce phase variations in the beam. The light then passes directly through the scatterplate producing fringes when it interferes with the reference beam.

Figure 2-2: Inversion Symmetry Designed into Binary Scatterplate

It seems unlikely that two beams of light that have been scattered at different positions in the interferometer could possibly produce interference fringes. In general, the process described above would not produce interference fringes, however Jim Burch being a clever man came up with the solution. The scatterplate must have inversion symmetry (Burch 1953). Inversion symmetry means that each scatter point has an exact twin located directly opposite the center point of the scatterplate at an equal distance from it as described by equation (2-1). Figure 2-2 shows an example of inversion symmetry designed into a binary photomask used to expose scatterplates. The reason inversion symmetry works is revealed by examining individual scatter points and their effect on the reference and test beams. Keep in mind that examining individual points is not the complete story. Summing the wavefronts from all of the scatter points forms the contour

fringes. Figure 2-3a summarizes the effect of inversion symmetry for a perfectly aligned system. When the incident light interacts with the scatterplate, the light scattered at scatter point $S(x,y)$ acts like a point source. Since the scatterplate is located at the center of curvature of the test mirror, point $S(x,y)$ is imaged back into the plane of the scatterplate at the conjugate point $S'(x,y)$. The reference beam passes directly through the scatterplate reflects off the test mirror and scatters at point $S(-x,-y)$. With inversion symmetry and proper alignment point $S(x,y)$ and $S(-x,-y)$ are scattered in exactly the same manner and the test and reference beams both appear as point sources located at point $S(-x,-y)$. As a result, the phase change due to scattering is the same for both beams.

Figure 2-3: Point-To-Point Analysis of Inversion Symmetry:a) Perfect Alignment, b) Lateral misalignment of Scatterplate, c) Longitudinal Misalignment of Scatterplate

The above discussion of inversion symmetry was for a system with perfect alignment, but this is not always the case. The effect of misalignment of the scatterplate will now be discussed. Lateral movement of the scatterplate produces tilt in the contour fringes. Figure 2-3b demonstrates the consequence of not aligning the center point of the scatterplate with the axis of the interferometer. The image of $S(x,y)$, although still in the plane of the scatterplate, no longer coincides with the symmetric point $S(-x,-y)$. The result is tilt in the contour fringes produced by the interference of two laterally shifted point sources. Similarly, longitudinal misalignment of the scatterplate adds defocus to the contour fringe pattern. Figure 2-3c shows the effect of not placing the scatterplate at the center of curvature of the test mirror. The image of point $S(x,y)$ is still at the same lateral position as the symmetric point $S(-x,-y)$, however it no longer lies in the plane of the scatterplate. The interference of the two longitudinally shifted point sources produces defocus in the contour fringes. Adjusting the scatterplate position adds an important flexibility for minimizing the number of contour fringes across the image plane. Figure 2-4 contains typical fringe patterns obtained using the scatterplate interferometer. The fringe patterns contain varying amounts of tilt and defocus. Notice that the fringe patterns contain speckle as is consistent with the use of a coherent source.

Figure 2-4: Typical Scatterplate Interferograms

SCALAR DIFFRACTION DESCRIPTION OF SCATTERPLATE INTERFEROMETER:

A theoretical model of the scatterplate interferometer can be easily obtained using scalar diffraction theory (Johnson 1979). The model presented here will serve as a starting point for modeling the phase-shifting scatterplate interferometer discussed in chapter four. In addition, the model will provide insight into the source of the four components of the electric field incident on the image plane. It is important to realize that the linear system theory presented here does not consider non-isoplanatism. The contrast reduction due to off axis imaging will be discussed in the next section.

Figure 2-5: Unfolded Scatterplate Interferometer

The first step is to derive an expression for the irradiance in the image plane. By unfolding figure 2-1 as shown in figure 2-5, the layout of the interferometer becomes easy to visualize. The usual step-by-step techniques of beam propagation and multiplication by transmission functions are used to determine the image plane irradiance (Goodman pp.33-120, Gaskill pp.449-512). Starting just after the first scatterplate, the electric field is described by

$$u'_1 = S^+ e^{-i\frac{k}{2R}(x_1^2 + y_1^2)}, \quad (2-2)$$

where R is the radius of curvature of the test mirror, $k=2\pi/\lambda$, and S^+ is the scatterplate transmission function. Note that a parabolic approximation is used and the unimportant constant phase factors are ignored. The $+$ and $-$ superscripts denote the inversion of the scatterplate transmission function between passes. The electric field just before the test mirror is obtained by convolving equation (2-2) with the free space impulse response

$$\begin{aligned}
 u_2 &= \iint u'_1 e^{i\frac{k}{2R}\left\{(x_1-x_2)^2+(y_1-y_2)^2\right\}} dx_1 dy_1, \\
 &= \iint \{S^+\} e^{-i\frac{k}{2R}(x_2^2+y_2^2)}
 \end{aligned} \tag{2-3}$$

where $\iint\{\}$ denotes a two dimensional Fourier Transform. Multiplying equation (2-3) by the quadratic phase, surface errors, and transmission associated with the test mirror results in the electric field just after the test mirror

$$u'_2 = \iint \{S^+\} T_m e^{ikW} e^{-i\frac{k}{2R}(x_2^2+y_2^2)}, \tag{2-4}$$

where W is the wavefront aberration of the test mirror. Transmission function T_m limits the clear aperture of the test mirror. After another propagation to the second scatterplate and multiplication by the transmission functions of the lens and scatterplate, the electric field to the right of the imaging lens becomes

$$u'_3 = S^- \iint \left\{ \iint \{S^+\} T_m e^{ikW} \right\} e^{-i\frac{k}{2Z_l}(x_3^2+y_3^2)}, \tag{2-5}$$

where Z_l is the distance from the imaging lens to the image plane. Finally, propagating to the image plane produces the electric field of the image

$$u_4 = \iint \left\{ S^- \iint \left\{ \iint \{S^+\} T_m e^{ikW} \right\} \right\} e^{i\frac{k}{2Z_l}(x_4^2+y_4^2)}. \tag{2-6}$$

The irradiance in the image plane is obtained by multiplying (2-6) by its complex conjugate. Rearranging equation (2-6) and squaring the modulus gives the irradiance in the image plane

$$E_4 = \left| \left(\iint \{S^+\} T_m e^{ikW} \right) ** \iint \{S^-\} \right|^2. \tag{2-7}$$

Here $**$ is used to denote a two dimensional convolution.

Burch suggested that the transmission variation in the scatterplate be represented by

$$S = A(S_0 + S_1), \quad (2-8)$$

where A is the aperture function, S_0 is the average scatterplate transmission and S_1 is the variations in the scatterplate transmission about the mean (Burch 1970). The image plane irradiance becomes

$$E_4 = \left| \begin{aligned} & (ff\{AS_0\}T_m e^{ikW}) ** ff\{AS_0\} + (ff\{AS_1\}T_m e^{ikW}) ** ff\{AS_0\} \\ & + (ff\{AS_0\}T_m e^{ikW}) ** ff\{AS_1\} + (ff\{AS_1\}T_m e^{ikW}) ** ff\{AS_1\} \end{aligned} \right|^2. \quad (2-9)$$

The approximation used in deriving equation (2-9) is that the wavefront aberration W is assumed to be slowly varying, such that the change across the Airy disk is negligible. It is interesting to note that the four terms in equation (2-9) are the direct-direct, direct-scattered, scattered-direct, and scattered-scattered contributions respectively.

FUNDAMENTAL LIMIT ON SCATTERPLATE SIZE:

The scatterplate model introduced in the previous section does not take into account the change in imaging properties for a scatter point that is located off-axis. Fortunately, a simple geometrical model can describe the limitation that the so-called “non-isoplanatism” places on the size of the scatterplate (Su 1984). The geometrical treatment presented in this section will consider the optical path difference for a ray that reflects off the edge of the test mirror and one that reflects off the center of the test mirror

for an off-axis scatter point. A diagram depicting the two optical paths is shown in figure 2-6.

Figure 2-6: Schematic Diagram for Non-Isoplanatism Calculation

Line segments S^+P , PS^- , and S^+V describe the optical path difference between the test and reference beams exactly as follows.

$$\begin{aligned} OPD &= \overline{S^+P} + \overline{PS^-} - 2\overline{S^+V} \\ &= \sqrt{R^2 + S^2 - 2\rho S} + \sqrt{R^2 + S^2 + 2\rho S} - 2\sqrt{R^2 + S^2} \end{aligned} \quad (2-10)$$

In order to simplify the expression further it is necessary to use the binomial expansion on the first two expressions of equation (2-10). If $2\rho S/(R^2 + S^2)$ is less than 0.18 then the quadratic term is dominant and higher order terms can be neglected. Substituting with the expansion equation (2-10) becomes

$$\begin{aligned}
 OPD &= \sqrt{R^2 + S^2} \left(1 - \frac{\rho S}{2(R^2 + S^2)} - \frac{\rho^2 S^2}{2(R^2 + S^2)^2} \right) \\
 &\quad + \sqrt{R^2 + S^2} \left(1 + \frac{\rho S}{2(R^2 + S^2)} - \frac{\rho^2 S^2}{2(R^2 + S^2)^2} \right) - 2\sqrt{R^2 + S^2} \\
 &= -\frac{\rho^2 S^2}{2(R^2 + S^2)^{\frac{3}{2}}}
 \end{aligned} \tag{2-11}$$

Subsequently noting that R is much greater than S gives the final expression for the extent of the non-isoplanatism

$$OPD = -\frac{\rho^2 S^2}{R^3}. \tag{2-12}$$

The question is, how does the optical path difference associated with off-axis scatter points affect the fringe pattern. Intuition might point to an error in the fringe pattern itself. Fortunately, the phase error induced is focus, which is generally subtracted from most interference measurements. The optical path difference turns out to be scatterplate aperture dependent focus. The summation of the wavefronts from each scatter point results in a somewhat broadened set of focus fringes. The more significant problem is that non-isoplanatism leads to a reduction in fringe contrast. Think of it this way. Light scattered off each point in the scatterplate will experience peak wavefront errors ranging from zero to some maximum value depending on its radial distance from the optical axis. Combining the wavefronts from all of the scatter points affectively averages the wavefront reducing the contrast of the fringe pattern. The reduction in fringe contrast places a fundamental limit on the size of the scatterplate that can be used. The limiting

case is when the peak wavefront error equals one half of the wavelength of the light source. At this limit the fringe contrast drops to 0.64 at the edge of the test mirror. This can be expressed mathematically as follows

$$S_{\text{limit}} = \sqrt{\frac{R^3 \lambda}{2 \rho^2}}. \quad (2-13)$$

The radius of the “used” scatterplate aperture is limited to a size dictated by equation (2-13) that is dependent on the radius of curvature of the test mirror and the source wavelength.

POLARIZATION REVIEW

Polarization is an important concept that is used extensively in the phase-shifting birefringent scatterplate interferometer. As a matter of fact, it is the physical property of electromagnetic waves that makes phase-shifting a scatterplate interferometer possible. Understanding polarization and its manipulation is imperative to appreciate how the phase-shifting scatterplate interferometer operates. As a result, this chapter is dedicated to the understanding of polarization, uniaxial crystals, wave plates, and Jones calculus. First, a physical overview of polarization will be presented, including the special cases of linear and circular polarization (Hecht pp. 270-274). The remainder of the chapter will be dedicated to the manipulation of polarization. A study of uniaxial crystals will provide insight into the operation of wave plates and the birefringent scatterplate, which are both made with uniaxial crystals (Born pp.790-800, Yariv pp.11-16). Next wave plates will be examined as a tool for manipulating polarization. Finally, Jones calculus, a mathematical description for completely polarized coherent systems, will be explored (Hecht pp.321-326, Yariv pp.16-29).

POLARIZATION:

Simply stated, polarization is a description of the transverse vibrations of the electric field and how they change in time and space. For instance, if the electric field oscillates in a plane it is said to have linear polarization because when viewed along the axis of propagation, the oscillation traces out a line. Furthermore, an electric field for which the plane of oscillation changes randomly in time is considered unpolarized. The

descriptions of polarization that follow will be for completely polarized light, that is the plane of oscillation of the electric field is predictable in time and space.

A harmonic plane wave can be represented mathematically by

$$\mathbf{u}(\mathbf{r}, t) = a \cos(\tau + \delta) \quad (3-1)$$

$$\tau = \omega t$$

$$\delta = n(\mathbf{k} \cdot \mathbf{r}) + \delta_0$$

where τ is the time varying term, δ is the space varying term and δ_0 is the initial phase of the oscillation. Providing the z-axis is the direction of propagation, the plane wave can be expressed as a superposition of orthogonal optical disturbances as follows

$$\begin{aligned} \mathbf{u}(z, t) &= u_x(z, t) + u_y(z, t) \\ &= \hat{i} a_x \cos(\tau + \delta_x) + \hat{j} a_y \cos(\tau + \delta_y) \end{aligned} \quad (3-2)$$

The relative amplitudes a_x and a_y , and the phase difference $\delta_y - \delta_x$ determine the amplitude and direction of the electric field oscillations in time and space. There are two cases of particular interest. The first is when the phase difference between the two orthogonal oscillations is zero (or $m2\pi$). It is apparent from equation (3-2) that the x and y components oscillate in phase and their resulting vector sum traces out a line in the x-y plane. Since the oscillation of the electric field is in a constant plane, the light is classified as linearly polarized. The optical disturbance will also oscillate in a plane if the two orthogonal components are π radians out of phase. However, the plane of oscillation will be perpendicular to the previous case. The relative magnitudes of the x and y components determine the orientation of the plane of polarization.

Figure 3-1: States of Polarization: a) linear, b) circular and c) elliptical

The other important state of polarization is one where the orientation and amplitude of the electric field traces a circle in the x-y plane as a function of time and space. In other words, the magnitude of the vector sum of the two orthogonal

components is constant. This so-called circular polarization is achieved when the relative magnitudes a_x and a_y are equal and the phase difference between the orthogonal components is $\pm\pi/2+m2\pi$. Substituting into equation (3-2) gives

$$\mathbf{u}(z,t) = a(\hat{i}\cos(\tau) \pm \hat{j}\sin(\tau)). \quad (3-3)$$

Regardless of the value of τ , the magnitude of the electric field is a . If the sign of the y component is negative the polarization is right circular because the plane of polarization rotates counter clockwise when viewed from “down stream.” Similarly, the polarization is termed left circular if the sign of the y component is positive.

In general, except for the two special cases discussed above the polarization is elliptical. Manipulating the x and y components and squaring produces the following equation of an ellipse.

$$\left(\frac{u_x}{a_x}\right)^2 + \left(\frac{u_y}{a_y}\right)^2 - 2\frac{u_x}{a_x}\frac{u_y}{a_y}\cos(\delta) = \sin^2(\delta) \quad (3-4)$$

$$\delta = \delta_y - \delta_x$$

The relative magnitudes of the x and y components and the phase difference δ determine the shape and orientation of the ellipse. Figure 3-1 shows the vector sum for the three cases of polarization discussed above.

UNIAXIAL CRYSTAL:

Uniaxial crystals are the most commonly used birefringent materials. They are often used to make wave plates for manipulation of polarization. In addition, the

birefringent scatterplate used to phase shift the scatterplate interferometer discussed in the next chapter is made of calcite, a uniaxial crystal. Therefore, it is worthwhile to discuss the physical properties of the crystal.

In general, crystals have unique physical properties because they are electrically anisotropic. In other words, the electric and displacement vectors within a crystal are no longer parallel. Each component of the displacement vector is linearly related to components of the electric vector as follows

$$\begin{aligned}\frac{D_x}{\epsilon_0} &= \epsilon_{xx}u_x + \epsilon_{xy}u_y + \epsilon_{xz}u_z \\ \frac{D_y}{\epsilon_0} &= \epsilon_{yx}u_x + \epsilon_{yy}u_y + \epsilon_{yz}u_z \\ \frac{D_z}{\epsilon_0} &= \epsilon_{zx}u_x + \epsilon_{zy}u_y + \epsilon_{zz}u_z\end{aligned}\quad (3-5)$$

ϵ_{nn} make up the electric tensor. A coordinate transformation to the principal dielectric axes reduces (3-5) to

$$\frac{D_x}{\epsilon_0} = \epsilon_x u_x, \quad \frac{D_y}{\epsilon_0} = \epsilon_y u_y, \quad \frac{D_z}{\epsilon_0} = \epsilon_z u_z. \quad (3-6)$$

Examining the electric energy density results in a geometrical description of the consequence of this anisotropy called an index ellipsoid. As the name implies, the refractive index along each principal axis of the crystal can be different. In general, the index along any line through the crystal is described using the index ellipsoid. The electric energy density is given by,

$$w_e = \frac{1}{8\pi} \mathbf{E} \cdot \mathbf{D}. \quad (3-7)$$

Substituting equation (3-6) into equation (3-7) and rearranging the terms produces an expression that begins to resemble an ellipse

$$\frac{8\pi w_e}{\epsilon_0} = \frac{D_x^2}{\epsilon_x} + \frac{D_y^2}{\epsilon_y} + \frac{D_z^2}{\epsilon_z}. \quad (3-8)$$

The expression can be further simplified by making the following substitutions

$$x = \frac{D_x}{\sqrt{8\pi w_e}}, \quad y = \frac{D_y}{\sqrt{8\pi w_e}}, \quad z = \frac{D_z}{\sqrt{8\pi w_e}} \quad (3-9)$$

and

$$n_x^2 = \frac{\epsilon_x}{\epsilon_0}, \quad n_y^2 = \frac{\epsilon_y}{\epsilon_0}, \quad n_z^2 = \frac{\epsilon_z}{\epsilon_0}, \quad (3-10)$$

where n is the index of refraction. The substitution with the index of refraction is only valid for dielectric materials. The result is the so-called index ellipsoid.

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (3-11)$$

This equation represents an ellipsoid, the semiaxes of which are equal to the indices of refraction along the principal axes of the crystal as shown in figure 3-2. With the aid of the index ellipsoid, the index of refraction along any line through the crystal can be determined. The index is equal to the magnitude of a vector drawn from the center of the

ellipsoid to its surface along the displacement vector. The magnitude of the vector is given by the vector sum of the x, y and z components.

Figure 3-2: Index Ellipsoid

A uniaxial crystal is a crystal for which the index of refraction along the y and z axes is the same,

$$n_0 = n_y = n_z. \quad (3-12)$$

Consequently, when viewed along the x-axis the ellipsoid appears as a circle of radius n_0 .

In other words, if the Poynting vector is parallel with the x-axis, the crystal acts

electrically isotropic. On the other hand, when viewed along any axis in the y-z plane the ellipsoid appears as an ellipse with principal radii equal to n_o and n_x . n_x is often referred to as the extraordinary index, n_e . Figure 3-3 contains examples of these two views.

Figure 3-3: Index Ellipse for Uniaxial Crystal: a) x-y Plane and b) x-z Plane

Uniaxial crystals oriented with the Poynting vector in the y-z plane are commonly used to make wave plates for manipulating polarization. Assuming there is no absorption in the crystal, the only way to change the state of polarization is to manipulate the relative phase difference between the x and y components. In a uniaxial crystal this is accomplished by altering the relative optical path of the two components of polarization. The incident beam is divided into two orthogonal components of polarization along the principal axes of the crystal. Since the axes have different indices of refraction, the

optical paths for the two polarization components differ. The optical path difference is given by

$$\begin{aligned}\delta &= \delta_y - \delta_x = (\Delta n) k \Delta z \\ \Delta n &= n_e - n_o\end{aligned}\tag{3-13}$$

where Δz is the thickness of the crystal. For a given material, Δz is the only effective variable for creating wave plates. The two most widely used wave plates are the quarter-wave plate and the half-wave plate. The relative phase difference, δ for each is $\pi/2$ and π respectively.

JONES CALCULUS:

Although the effect of a wave plate on the polarization state of the incident light is easy to visualize for one component, in general, it is much more complicated for systems of components. As a result, a formalism for tracing the state of polarization through a system is needed. For completely polarized light Jones calculus can be used. Jones calculus is a matrix approach in which the two orthogonal components of polarization are placed in a matrix as follows

$$\mathbf{u}(z, t) = e^{i\tau} \begin{pmatrix} a_x e^{i\delta_x} \\ a_y e^{i\delta_y} \end{pmatrix}\tag{3-14}$$

For the most part, the time varying term is neglected. Furthermore, in many applications it is not necessary to know the exact amplitudes and phases (Hecht pp.324). Only the relative values are important. For example, light that is linearly polarized at 45° with respect to the z-axis can be expressed in a normalized manner

$$\mathbf{u}_{45} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (3-15)$$

Similarly, in normalized form vertical and horizontal polarization are expressed as

$$\mathbf{u}_v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathbf{u}_h = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (3-16)$$

Right and left circular polarization can also be represented in a normalized fashion

$$\mathbf{u}_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \mathbf{u}_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}. \quad (3-17)$$

While the state of polarization is expressed as a two-dimensional vector, the optical elements are expressed as 2x2 matrices. Table 3-1 contains the Jones matrix representation of four common optical elements.

Optical Element	Jones Matrix
Horizontal Linear Polarizer	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
Vertical Linear Polarizer	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
Quarter-Wave Plate Fast Axis Vertical	$e^{i\frac{\pi}{4}} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$
Half-Wave Plate Fast Axis Vertical	$e^{i\frac{\pi}{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Table 3-1: Index Ellipse for Uniaxial Crystal: a) x-y Plane and b) x-z Plane

The output state of polarization is obtained through simple matrix multiplication. For example, the emerging polarization state from a quarter-wave plate oriented with its fast axis vertical for a linearly polarized input oriented at 45° is given by

$$\mathbf{u}_{\text{out}} = \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} . \quad (3-18)$$

= Right Circular Polarization

The additional phase factor in front of the vector cancels when the modulus of the electric field is squared to obtain the irradiance distribution.

A wave plate of arbitrary retardance with its fast axis vertical can be expressed as follows

$$e^{i\frac{\delta}{2}} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\delta} \end{pmatrix}, \quad (3-19)$$

where δ is the relative phase difference induced between the two polarization components. In addition, any of the matrices can be rotated to an arbitrary orientation by multiplying by a rotation matrix thusly

$$\mathbf{M}(\theta) = \mathbf{R}(\theta) \mathbf{M}(0) \mathbf{R}(-\theta), \quad (3-20)$$

where

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (3-21)$$

EFFECT OF SCATTERING ON POLARIZATION:

As the phase-shifting scatterplate interferometer makes extensive use of polarization before and after the light has been scattered, it is important to know how the polarization state is altered after scattering. Of particular interest is how elliptical a linearly polarized beam becomes after scattering. Unfortunately, mathematically describing the polarization change due to scattering from rough surfaces is extremely difficult (Stover pp.111-132). Consequently, experimental results will be used to examine the change in polarization.

Figure 3-4: Setup for Measuring Polarization Change due to Scattering

The experimental setup used to measure the degree of ellipticity in a scattered linearly polarized beam is shown in figure 3-4. A Helium-Neon laser beam passes through a linear polarizer and scatters off a ground glass plate. The ground glass plate is a fused silica slide that was ground with 5 μ m grit. The scattered light passes through a second polarizer and is measured through a 1mm diameter aperture with a power meter. The second polarizer and iris, as well as the detector, are mounted on a translation stage.

A micrometer is used to translate the detection optics in a transverse direction. At each measurement location the second polarizer is rotated and the minimum and maximum power readings are used to determine the extinction ratio. The higher the extinction ratio the closer the polarization is to linear. Figure 3-5 contains a plot of the extinction ratio as a function of the angle off the optical axis for three different orientations of the linearly polarized input.

Figure 3-5: Polarization Change due to Scattering

The extinction ratio remains over 200 out to 6°, which corresponds to measuring an F/2.4 mirror with the scatterplate interferometer.

PHASE-SHIFTING SCATTERPLATE INTERFEROMETER

Interferometers can have fantastic surface height measurement accuracy. Generally, in a phase-shifting interferometer environmental effects such as vibrations and air turbulence limit the measurement accuracy and repeatability. One way of reducing the sensitivity to these effects is to use a common-path interferometer. As the name implies, in a common-path interferometer both the test and reference beams traverse nearly the same optical path. For the most part, both beams experience similar phase variations due to environmental effects. Unfortunately, the same feature that makes common-path interferometers attractive also makes them difficult to phase shift. Since the test and reference beams travel along the same path they are difficult to separate. The challenge is to produce a variable phase difference between the two beams. In the case of the scatterplate interferometer two research groups have met this challenge: Huang et al and Su et al (Huang 1988, Su 1991). Both methods exploit polarization to separate the test and reference beams by placing an auxiliary optic near the surface under test. The interferometer presented here also uses polarization to achieve the goal of phase shifting. However, it does not require optics to be placed near the test surface. In addition, the “hot spot” produced by the beam that travels directly through the scatterplate on both passes can be eliminated. This chapter will present a qualitative and mathematical description of the interferometer. The topic of removing the “hot spot” will be covered in a later chapter.

BIREFRINGENT SCATTERPLATE:

Before the operation of the interferometer as a whole can be understood it is necessary to independently examine the workings of the birefringent scatterplate. In the phase-shifting scatterplate interferometer, the goal is to control when the test and reference beams are scattered. Here the birefringent scatterplate shown in figure 4-1 provides the desired control. The appropriate aperiodic pattern with inversion symmetry is etched into a calcite retarder using a chemical etching process. An index matching oil chosen to match the ordinary index of the crystal is then pressed between the calcite and a glass slide. The end result is that for light polarized along the ordinary axis of the crystal, the index of the oil and the index of the crystal appear the same and the light passes directly through the scatterplate, whereas light polarized along the extraordinary axis of the crystal sees an index difference and is scattered by the rough surface. This is the property that supplies the control over scattering necessary to phase shift the scatterplate interferometer.

Figure 4-1: Birefringent Scatterplate

Matching the index of the oil with the ordinary index of the calcite is fairly straightforward. However, it is important to consider the dispersion of both materials. Literature usually quotes the indices of refraction at the sodium D line. Unfortunately, matching the indices at 589nm does not guarantee that the indices will match at a different wavelength. The dispersion of calcite is described by the following equations (Tropf pp. 33.62).

$$n_o(\lambda) = 1 + \frac{0.8559\lambda^2}{\lambda^2 - (0.0588)^2} + \frac{0.8391\lambda^2}{\lambda^2 - (0.141)^2} + \frac{0.0009\lambda^2}{\lambda^2 - (0.197)^2} + \frac{0.6845\lambda^2}{\lambda^2 - (7.005)^2} \quad (4-1)$$

and

$$n_e(\lambda) = 1 + \frac{1.0856\lambda^2}{\lambda^2 - (0.07897)^2} + \frac{0.0988\lambda^2}{\lambda^2 - (0.142)^2} + \frac{0.317\lambda^2}{\lambda^2 - (11.468)^2} \quad (4-2)$$

where λ is the wavelength of the source in microns. The manufacturer usually supplies the wavelength dependence of the index matching oil. The dispersion equation for Cargille's $n_d=1.662$ oil is

$$n_{1.662} = 1.621298 + \frac{1192615}{(\lambda \cdot 10^4)^2} + \frac{7.658881 \cdot 10^{12}}{(\lambda \cdot 10^4)^4}. \quad (4-3)$$

Figure 4-2 shows a plot of the dispersion curves for the ordinary index of calcite and the index matching oil. At the operating wavelength of 633nm the indices differ by only 0.00005.

Figure 4-2: Dispersion Analysis of Birefringent Scatterplate

Since the purpose of the scatterplate is to control scattering, it is of interest to know the amount of light scattered as a function of polarization. Intuition says that the percentage of light scattered will vary with polarization angle. A formal description is obtained using the index ellipse discussed in the previous chapter. From figure 3-3b it is easy to see that the index of the calcite will oscillate from n_o to n_e and back when the incident linear polarization is rotated 180° . Starting with the equation of the ellipse shown in figure 3-3b,

$$\frac{x^2}{n_e^2} + \frac{y^2}{n_o^2} = 1 \quad (4-4)$$

and replacing x and y with

$$\begin{aligned} x &= n(\theta) \sin(\theta) \\ y &= n(\theta) \cos(\theta) \end{aligned} \quad (4-5)$$

results in an expression for the index as a function of polarization angle

$$n(\theta) = \sqrt{\frac{n_e n_o}{n_o^2 \sin^2(\theta) + n_e^2 \cos^2(\theta)}} \quad (4-6)$$

The validity of this equation is easily confirmed by noting that the value of $n[\theta]$ at 0° and 90° is n_o and n_e respectively. The question is how does the oscillating index correspond to a change in the percentage of scattered light. One model for the amount of light scattered from a rough surface uses the Strehl ratio (Wyant 1992). This model is only valid if the height distribution of the surface roughness is Gaussian and $\Delta n \cdot \sigma$ is greater than 0.1λ . Using the Strehl ratio the fraction of light scattered is given by

$$\text{Scatter Ratio} \approx 1 - e^{-\left(\frac{2\pi n \sigma}{\lambda}\right)^2}, \quad (4-7)$$

where n is the index of refraction, σ is the RMS surface height, and λ is the wavelength of the source. Substituting equation (4-6) into equation (4-7) and plotting produces the oscillating scatter ratio shown in figure 4-3. Notice that since the curve varies slowly near the local minima and maxima, which are the points of interest on the plot, small misalignments in the orientation of the scatterplate will not significantly affect the interferometer performance.

Figure 4-3: % Scatter from scatterplate as a function of polarization angle.
($\sigma = 1\mu\text{m}$, $\lambda = 0.633\mu\text{m}$)

QUALITATIVE DESCRIPTION OF PHASE-SHIFTING SCATTERPLATE INTERFEROMETER:

It is instructive to examine the operation of the phase-shifting scatterplate interferometer qualitatively. The schematic diagram of the interferometer is shown in figure 4-4. If the polarization elements such as the wave plates and polarizers are removed, it is a conventional scatterplate interferometer discussed at length in chapter 2. The polarizer passes linearly polarized light oriented at 45° with respect to the optic axis of the calcite scatterplate, providing equal amplitudes for the component of the beam polarized along the optic axis and the component polarized orthogonal to the optic axis. The component parallel to the optic axis will see the extraordinary index of the crystal and the perpendicular component will see the ordinary index. A liquid crystal retarder produces a variable phase shift between the two orthogonal components of polarization, and the rotating ground glass plate reduces the speckle in the interferogram by effectively enlarging the apparent source size. The source created at the ground glass plate is imaged onto the test mirror by way of the scatterplate and a quarter-wave plate oriented with its fast axis at 45° with respect to the optic axis of the crystal. The scatterplate is located at the center of curvature of the test mirror and scatters only the component of the beam that is polarized parallel to the optic axis of the crystal. The perpendicular component passes through the scatterplate and forms an image of the source on the test mirror. Both the scattered and direct beams pass through a quarter-wave plate twice producing a 90° rotation in the polarization of each. As a result, on the second pass through the scatterplate the beams change roles and the one that traveled directly through on the first pass is now scattered and the one that was scattered now passes directly through. The

outcome is a scattered-direct and direct-scattered beam with orthogonal polarization. Notice that if there is no unwanted scattering when the beams pass directly through the scatterplate, there is no background irradiance in the interference pattern. Similarly, if all of the light is scattered by the scatterplate on the passes when scattering is wanted, the “hot spot” will not exist. The topic of removing the “hot spot” will be treated in a later chapter. Finally, the test mirror is imaged onto a CCD array through an analyzer, which serves to combine the test and reference beams for observation of interference fringes. The end result is that the interference fringes can be phase shifted by applying a voltage to the liquid crystal retarder. Figure 4-5 contains a series of four shifted interferograms.

Figure 4-4: Phase-shifting Scatterplate Interferometer

Figure 4-5: Phase-shifted Fringe Patterns

DIFFRACTION DESCRIPTION OF PHASE-SHIFTING SCATTERPLATE INTERFEROMETER:

In the spirit of Jones Calculus the model for the conventional scatterplate interferometer can be extended to describe the phase-shifting birefringent scatterplate interferometer by separating two orthogonal components of polarization into a vector. For simplicity, the components are chosen such that they are parallel with the ordinary and extraordinary axes of the birefringent scatterplate. For the geometry discussed previously the components are along the x and y-axes. Again the resulting irradiance pattern is obtained by a combination of beam propagation and multiplication by transmission functions, only now a matrix describes each step.

Before stepping through the system, a matrix description for each component must be defined. The quarter-wave plate, liquid crystal retarder, and analyzer are common optical components that do not require any special consideration. The quarter-wave plate is rotated so that its fast axis is oriented at 45° with respect to vertical by multiplying by the rotation matrix

$$\begin{aligned}
 \mathbf{QW}(45^\circ) &= \mathbf{R}(45^\circ) \mathbf{QW}(0^\circ) \mathbf{R}(-45^\circ) \\
 &= \frac{e^{i\frac{\pi}{4}}}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \\
 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}
 \end{aligned} \tag{4-8}$$

In a like manner, the analyzer is simply a rotated linear polarizer oriented at 45°,

$$\mathbf{AN}(45^\circ) = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{4-9}$$

The liquid crystal retarder, a wave plate with arbitrary retardance, is described by equation (3-19). Here δ is the induced phase shift between the test and reference beams. The scatterplate and test mirror are not commonly described by Jones Matrices. However, examining how they act for each component of polarization will point to a matrix representation. The test mirror acts the same for both components of polarization. Therefore, it must operate on both components equivalently. Mathematically, the reflectance function of the mirror is represented as follows

$$\mathbf{M} e^{-i\frac{k}{R}(x_1^2+y_1^2)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} T_m e^{ikW} e^{-i\frac{k}{R}(x_1^2+y_1^2)}. \quad (4-10)$$

where R is the radius of curvature of the mirror, W represents the aberrations in the mirror and T_m is the reflectance function for the mirror. The quadratic phase factor associated with the focusing properties of the lens is removed from the matrix in order to simplify the analysis. The scatterplate, on the other hand, acts differently for each component of polarization. For light polarized along the extraordinary axis, the birefringent scatterplate acts like a conventional scatterplate. Whereas, light polarized along the ordinary axis of the crystal sees the birefringent scatterplate as a clear aperture. Using equation (2-8) the scatterplate is described by

$$\mathbf{S}^* = \begin{pmatrix} AS_0 & 0 \\ 0 & A(S_0 + S_1^*) \end{pmatrix}. \quad (4-11)$$

where A is the scatterplate aperture, S_0 is the average phase, and S_1^* represents variations in phase about the average. The superscript on S_1 denotes the inversion of the scatterplate transmission function between passes.

Figure 4-6: Unfolded Phase-shifting Scatterplate Interferometer

Now that all of the components are accounted for, the system can be traced step-by-step with the usual techniques. Figure 4-6 shows an unfolded schematic diagram of the interferometer. Starting just after the first quarter wave plate, the electric field is represented as

$$\mathbf{u}'_1 = (\mathbf{QW} \mathbf{S}^- \mathbf{LQ} \mathbf{u}_{in}) e^{-i \frac{k}{2R} (x_1^2 + y_1^2)}. \quad (4-12)$$

where $\mathbf{LQ}$ and $\mathbf{u}_{in}$ are the Jones matrices for the liquid crystal retarder and the relative amplitude of the two polarization components present in the incident radiation. $\mathbf{u}_{in}$ is given by

$$\mathbf{u}_{in} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (4-13)$$

Note that the parabolic approximation is used and the unimportant constant phase factors are ignored. The electric field just before the test mirror is obtained by convolving equation (4-12) with the free space impulse response

$$\begin{aligned} \mathbf{u}_2 &= \iint \mathbf{u}'_1 e^{i\frac{k}{2R}\{(x_1-x_2)^2+(y_1-y_2)^2\}} dx_1 dy_1 \\ &= \mathcal{F}\{\mathbf{QW S}^* \mathbf{LQ u}_{in}\} e^{i\frac{k}{2R}(x_2^2+y_2^2)} \end{aligned} \quad (4-14)$$

where $\mathcal{F}\{\}$ denotes a two dimensional Fourier Transform. Multiplying equation (4-14) by the quadratic phase factor and the test mirror matrix results in the electric field just after the test mirror.

$$\mathbf{u}'_2 = \left(\mathbf{M} \mathcal{F}\{\mathbf{QW S}^* \mathbf{LQ u}_{in}\} \right) e^{-i\frac{k}{2R}(x_2^2+y_2^2)}. \quad (4-15)$$

After another propagation to the second quarter-wave plate and multiplication by the transmission functions of the quarter-wave plate, scatterplate, and lens, the electric field to the right of the imaging lens becomes

$$\mathbf{u}'_3 = \left(\mathbf{S}^* \mathbf{QW} \mathcal{F}\{\mathbf{M} \mathcal{F}\{\mathbf{QW S}^* \mathbf{LQ u}_{in}\}\} \right) e^{-i\frac{k}{2z_l}(x_3^2+y_3^2)}, \quad (4-16)$$

where Z_l is the distance from the imaging lens to the image plane. Finally, propagating to the image plane and multiplying by the analyzer produces the electric field in the image plane

$$\mathbf{u}_4 = \left(\mathbf{AN} \mathcal{F}\{\mathbf{S}^* \mathbf{QW} \mathcal{F}\{\mathbf{M} \mathcal{F}\{\mathbf{QW S}^* \mathbf{LQ u}_{in}\}\}\} \right) e^{i\frac{k}{2z_l}(x_4^2+y_4^2)}. \quad (4-17)$$

The irradiance in the image plane is obtained by multiplying (4-17) by its complex conjugate

$$E_4 = \left| \mathbf{AN} \, \text{ff} \left\{ \mathbf{S}^- \mathbf{QW} \, \text{ff} \left\{ \mathbf{M} \, \text{ff} \left\{ \mathbf{QW} \mathbf{S}^+ \mathbf{LQ} \mathbf{u}_{in} \right\} \right\} \right\} \right|^2. \quad (4-18)$$

Performing the matrix multiplication and rearranging equation (4-18) results in an expression for the image plane irradiance, similar to the one obtained in the conventional scatterplate interferometer

$$E_4 = \left| \begin{aligned} & 2\cos\left(\frac{\delta}{2}\right) \left(\left(\text{ff} \{AS_0\} T_m e^{ik''} \right) ** \text{ff} \{AS_0\} \right) \\ & + \left(\left(\text{ff} \{AS_0\} T_m e^{ik''} \right) ** \text{ff} \{AS_1^+\} \right) e^{-i\frac{\delta}{2}} \\ & + \left(\left(\text{ff} \{AS_1^-\} T_m e^{ik''} \right) ** \text{ff} \{AS_0\} \right) e^{i\frac{\delta}{2}} \end{aligned} \right|^2. \quad (4-19)$$

The three terms in equation (4-19) are the direct-direct, scattered-direct, and direct-scattered beams respectively. There are three interesting features to equation (4-19). There is no scattered-scattered beam associated with the phase-shifting birefringent scatterplate interferometer. In addition, the amplitude of the direct-direct beam oscillates with the phase shift δ . Most importantly, there is a controllable relative phase shift between the test and reference beams.

SCATTERPLATE MANUFACTURE

The birefringent scatterplate is the key component of the phase-shifting scatterplate interferometer presented in this dissertation. This chapter will broach the subject of its manufacture. There are many issues to be considered, including pattern generation, average feature size, inversion symmetry, and etching rates. Each of these issues will be addressed in the context of describing the entire manufacturing process. First, an overview of the entire process will be presented in general terms. Then, a detailed presentation on exposing the scatterplate pattern using two different methods will follow. Finally, the role that etching plays in removing the “hot spot” will be investigated.

MANUFACTURING PROCESS:

Since the operation of the interferometer depends on the birefringent properties of the scatterplate, it is necessary to etch the scatterplate pattern directly into the birefringent material. The process presented here will work for most birefringent materials, however the chemicals used to clean and etch the scatterplate may vary. Etching precise patterns into birefringent materials is a new concept and there is no literature to speak of that outlines a process. The “recipes” presented here are specific to calcite and were obtained through trial and error.

Figure 5-1: Scatterplate Manufacturing Process

The birefringent scatterplate is made using a six step etching process outlined in figure 5-1. First, a good quality wave plate made with calcite is cleaned in four stages using acetone, isopropanol, deionized water, and a plasma chamber. The calcite is placed in each of the chemicals in the order listed above for 30 seconds and agitated gently. Upon removal from the deionized water, the calcite is dried and placed in a plasma chamber for 30 seconds to remove any remaining impurities on the surface. If the substrate is not clean the photoresist will not adhere to the surface. The next step in the

manufacturing process is to spin coat the sample with photoresist. There are many photoresists available with different viscosities and sensitivities. Shiply's 1813 photoresist was used for its availability. The scatterplate was coated with photoresist by spinning it at 500rpm for 5 seconds and increasing the speed to 5000rpm for 30 seconds. The speed and duration of the spin coating, as well as the viscosity, determine the thickness of the deposited photoresist. For this application the thickness of the photoresist is not critical providing it stands up to the chemical etching. After coating, the sample is "soft baked" on a hot plate for 1.5 minutes at 100C to solidify the photoresist. The scatterplate pattern is then exposed into the photoresist using either a speckle pattern or a photomask. Developing removes the photoresist from the calcite only in the regions that are exposed with flux levels above 150 mJ/cm^2 . The pattern is developed by placing it in Shiply's 352 developer for 45 seconds and rinsing in deionized water for 1 minute. After developing the sample is "hard baked" for another 1.5 minutes on the hot plate. The birefringent scatterplate is then chemically etched using an extremely weak solution of hydrochloric acid. A 37% solution of HCl is diluted 5000 to 1 in deionized water and the scatterplate is gently agitated in the solution for 3-5 minutes depending on the etch depth desired. Figure 5-2 shows a plot of the etch depth as a function of etching time for calcite. Rinsing the calcite in deionized water for 1 minute terminates the chemical etching. Finally, the photoresist is removed using the same cleaning process used to clean the blank substrate. Figure 5-3 shows a surface plot of an etched scatterplate.

Figure 5-2: Etch Depth Vs. Etch Time for Calcite in Diluted HCl

Figure 5-3: Surface Plot of Birefringent Scatterplate Manufactured using a Photomask

HOLOGRAPHIC EXPOSURE:

A common method for exposing scatterplates is to double expose a speckle pattern, rotating the scatterplate 180° between each exposures (Rubin 1980). Figure 5-4 shows the holographic setup for exposing scatterplates. An Argon-Ion laser beam tuned to a wavelength of 458nm is expanded using a beam expander. The beam is then scattered by a ground glass plate and a speckle pattern is created in the film plane. The size of the smallest features in the speckle pattern is determined by the diameter of the beam incident on the ground glass plate and the distance from the ground glass to the film plane. In turn, the size of the speckle pattern features determines the $F/\#$ of the mirror

that can be measured. The photoresist-coated sample is placed in the film plane and rotated 180° between two exposures. The rotation is controlled via a kinematic mount, shown in figure 5-3, to $\pm 0.05^\circ$. The end result is a random pattern with inversion symmetry.

Figure 5-4: Holographic Exposure of Scatterplate

A simple geometric analysis will reveal the relationship between the holographic setup and the $F/\#$ of the mirror that can be measured. There are two geometries to consider in this analysis. First, the exposing geometry will determine the smallest feature size on the scatterplate and then the geometry of the scatterplate interferometer will

determine the fastest mirror that can be tested. Figure 5-5 shows both geometries. When exposing the speckle pattern, interference between the two most widely separated points on the ground glass produces the smallest features. The radius of the smallest features is determined by finding the first interference minimum off the optical axis. For the geometry shown the radius is given by

$$r = \frac{z \lambda}{D}. \quad (5-1)$$

where z is the distance from the ground glass plate to the film plane, λ is the wavelength of the source, and D is the diameter of the beam incident on the ground glass plate. The analysis for the scattered light is similar except that the smallest features on the scatterplate are used to determine the radius at which the irradiance at the mirror falls off to zero. Repeating the analysis for the interferometer geometry and substituting for r produces the desired relation,

$$F/\# = \frac{z}{2D} \quad (5-2)$$

where

$$F/\# := \frac{f}{2\rho}. \quad (5-3)$$

ρ is the radius of the mirror aperture and f is the focal length of the mirror. The ratio of z over D determines the $F/\#$ of the mirrors that can be measured using the exposed scatterplate. In a practical measurement, it is not desirable to have the irradiance drop

off significantly at the edge of the mirror. If the minimum $F/\#$ is half the $F/\#$ of the test mirror, the irradiance will drop by 50% at the edge of the test mirror.

a)

b)

Figure 5-5: F/# Analysis for Holographic Exposure; a) Recording Geometry b) Interferometer Geometry

The advantage of using the holographic exposure is that low $F/\#$ scatterplates are easily created. The disadvantage is the elaborate setup required for each exposure and the possible errors in symmetry due to rotating the scatterplate between exposures.

PHOTOMASK:

A better method for exposing the scatterplates is to use a photomask. A photomask is a glass substrate on which the desired pattern is written into chrome using an electron beam. Once the pattern is written, the photomask can be used repeatedly to manufacture many scatterplates. The photomask is placed in close contact with the photoresist-coated sample and illuminating the sample with a UV source exposes the pattern. The exposing is usually done using an apparatus known as a mask-aligner and only takes a few minutes. With one photomask many scatterplates can be rapidly exposed. Most of the work associated with using a photomask is in its design.

Normally, designing a photomask is not a labor-intensive activity. There are many programs such as DW2000 and Autocad dedicated to simplifying the design process. Built in modules allow the user to easily create and place the required shapes. Unfortunately, the unusually large number of features and the required randomness of the scatterplate pattern, made it difficult to design. The large number of features in the mask made the file sizes enormous and consequently, difficult to work with. The file size for the $F/6$ mask design was 70MB and contained over 100,000 features. Files of this size were problematic for the mask manufacturer and special concessions had to be made. In addition to designing the mask, it was necessary to write programs to convert the data

into a file format that the mask designer could more easily handle. The files were converted from a pixilated bitmap format to the object-oriented format of Autocad. Another stumbling block was the required randomness of the scatterplate pattern. The random number generator and programming language included with the available mask design program had limitations that made its use impractical. As a result, a program was written in IDL to generate the scatterplate pattern from scratch. The program used the built in random number generator to randomly fill pixels in an array until it was approximately 25% full. Then a copy of the array was created, rotated 180°, and summed with the original to create inversion symmetry. Pixels with values greater than one were set equal to one. The final pattern was an array of ones and zeros that were pseudorandom with inversion symmetry. A subsection of the mask design is shown in figure 2-2. Taking an autocorrelation checked the randomness of the pattern and rotating a copy of the scatterplate pattern 180° and subtracting it from the original checked symmetry. With perfect symmetry the difference should result in an array of zeros. The autocorrelation of a completely random pattern with inversion symmetry should look like a $\text{TRI}\{\}$ function with a sharp spike in the center (Gaskill pp.150-176). Figure 5-6 shows the autocorrelation for the final mask design and a faulty pattern created using the mask design software. The peaks in the autocorrelation of the faulty mask indicate a reoccurring pattern that will result in undesirable diffraction orders distributed throughout the irradiance pattern on the test mirror. The code used to create and test the mask as well as a brief description can be found in appendix A. The same pattern was scaled and used to create four scatterplate masks on the same substrate: F/5, F/6, F/8, F/12.

a)

b)

Figure 5-6: 1-D Autocorrelation of the scatterplate design; a) Final Design, b) Faulty Pattern

As with the holographic exposure, the feature sizes in the mask determine the $F/\#$ of the mirror that can be measured using that particular scatterplate. It turns out that for a randomly generated pattern the average feature size in one dimension is two pixels and

the minimum feature size is one pixel. Since the irradiance pattern at the mirror is the squared modulus of the Fraunhofer diffraction pattern of the scatterplate and the shape of the smallest feature is known, the F/# can be determined by taking the Fourier transform of the smallest feature in the mask. The fastest mirror that can be completely illuminated is one for which the envelope of the irradiance pattern drops to zero at the edge of the mirror. The envelope function is given by

$$\begin{aligned}
 E_{\text{envelope}} &= \left| \iint \left\{ \text{Rect}\left(\frac{x}{a}\right) \text{Rect}\left(\frac{y}{a}\right) \right\} \right|^2 \\
 &= \left| a^2 \text{Sinc}(a\xi) \text{Sinc}(a\eta) \right|^2 \\
 &= a^4 \text{Sinc}^2(a\xi) \text{Sinc}^2(a\eta)
 \end{aligned} \tag{5-4}$$

where

$$\xi = \frac{x}{\lambda f} \tag{5-5}$$

and

$$\eta = \frac{y}{\lambda f}. \tag{5-6}$$

f is the focal length of the test mirror, a is the width of the smallest feature, and λ is the wavelength of the source. Figure 5-7 shows a one-dimensional plot of the resulting envelope function. The irradiance pattern drops to zero at

$$x = \frac{\lambda f}{a}. \tag{5-7}$$

which gives the following relation for the minimum F/#

$$\left(\frac{F}{\#}\right)_{\min} = \frac{a}{2\lambda}. \quad (5-8)$$

In a practical measurement, it is not desirable to have the irradiance drop off significantly at the edge of the test mirror. As a rule of thumb, a scatterplate should be designed such that the minimum $F/\#$ is one-half the $F/\#$ of the test mirror.

Figure 5-7: Irradiance Envelope at Test Mirror

HOT SPOT REDUCTION:

The binary nature of the birefringent scatterplate and the separation of the test and reference beams in polarization space, facilitate the reduction of the amplitude of the “hot

spot.” With proper etching it is possible to generate a scatterplate that will scatter the majority of light polarized along the optic axis of the crystal. As mentioned in the description of the phase-shifting scatterplate interferometer, if all of the light polarized along the optic axis of the crystal is scattered, the “hot spot” will not exist. A review of the operation of the birefringent scatterplate will emphasize this point. On the first pass through the scatterplate a portion of the light polarized along the optic axis of the crystal is scattered, while the majority of the light polarized along the ordinary axis passes directly through. Neglecting the small component of light scattered by the later polarization, there are three beams present at the test mirror: a scattered beam polarized along the optic axis, a direct beam polarized along the optic axis, and a direct beam polarized orthogonal to the optic axis. However, if the scatterplate is designed such that the majority of the light polarized along the optic axis is scattered, then the direct beam for this polarization can be neglected leaving only two beams at the test mirror. Both polarization components are rotated 90° by the quarter-wave plate, changing their roles on the second pass through the scatterplate. Again, if the light is strongly scattered, there are only two beams that emerge: a direct-scattered beam and a scattered-direct beam. The direct-direct beam that is associated with the “hot spot” is noticeably missing.

The question is, under what conditions does the binary scatterplate scatter all of the light polarized along the optic axis of the crystal. Fortunately, a complete mathematical analysis is not necessary. The central ordinate theorem is sufficient to determine the appropriate etch depth. The central ordinate theorem states, “The area of a function is equal to the central ordinate of its Fourier Transform (Gaskill pp.194).” Thus,

the central ordinate of the irradiance distribution at the test mirror is zero, if the area under the scatterplate function is zero. In other words, there is no direct beam. For a binary phase scatterplate function with an equal number of high and low points, the area of the function is equal to zero when the step height is given by

$$\Delta z = \left(\frac{\lambda}{2\pi} \right) \frac{\pi + 2m\pi}{\Delta n}, \quad (5-9)$$

where Δz is the step height, m is an integer, and Δn is the index difference between the oil and the crystal. Figure 5-8 shows the percentage of light scattered by a binary phase scatterplate as a function of etch depth. Theoretically, with proper etching all of the light polarized along the optic axis of the crystal can be scattered. However, non-uniformities in the etch depth and inconsistent etch rates prevent the manufacture of an ideal pattern.

Figure 5-8: Percentage of light scattered by a perfect binary phase scatterplate as a function of etch depth

Unfortunately, a significant reduction in the “hot spot” amplitude was not demonstrated. The acid etch used to generate the scatterplates did not faithfully reproduce the mask pattern or etch to a uniform depth. Consequently, the area under the scatterplate function was not zero. Figure 5-9 contains a subsection of the scatterplate measured on the WYKO NT2000 vertical scanning microscope. The plot demonstrates

the shortcomings of acid etching. The most obvious is the etch depth non-uniformity. The etch depth varies as much as one-third the average etch depth across a given feature. As a result, the pattern placed in the calcite is no longer strictly binary. In addition, the etched features are smaller than the ones exposed into the photoresist. Although the surface of each feature is completely exposed the acid tends not to etch at the boundaries of the photoresist. Perhaps surface tension prevents the acid from interacting with the calcite near the edge of the features. The effect is to round the edges of the features and narrow their width. The implication for "hot spot" removal is that the pattern no longer has a fifty-fifty ratio of area between the high and low steps. It should be noted that even with the pattern defects brought on by etching the scatterplates produce high contrast fringes.

Title:

Figure 5-9: Subsection of Etched Scatterplate Pattern Measured with Vertical Scanning Interference Microscope

Another option is to etch the binary pattern many waves deep. At these depths the diffraction analysis breaks down and the scattering is better described by the strehl ratio. Accordingly, most of the light is scattered regardless of the exact etch depth. In spite of the simplicity of this approach, it too has drawbacks. Deep etches severely reduced the fringe contrast and increased the background irradiance. Useful fringe contrast was unattainable for etch depths one or more waves deep. The increase in background irradiance indicates that the scatterplate is scattering light even when the indices of

refraction are matched. The viscosity of the oil may be preventing it from completely filling all of the features on the scatterplate. Moreover, distortion of the scatterplate pattern increases with etch depth.

LIQUID CRYSTAL RETARDER CALIBRATION

Liquid crystal retarders are not commonly used for phase shifting interferometers. In most cases, an electro-optic light modulator is used when the test and reference beams are separated in polarization space. This chapter addresses the issues associated with using a liquid crystal retarder as a phase-shifting device as well as its calibration. First a brief discussion of the operating characteristics of the liquid crystal retarder will be presented. Then the four-pass technique used to calibrate the liquid crystal retarder will be discussed. Next, the curve fitting necessary for interfacing with Intellwave will be outlined. Finally, the errors associated with the calibration will be analyzed.

LIQUID CRYSTAL RETARDER OPERATING CHARACTERISTICS:

There are a few important characteristics of the liquid crystal retarder that should be considered before using it for phase shifting. First, the response of the liquid crystal retarder is extremely nonlinear with voltage. The optical path difference induced between the fast and slow axes of the device is a complicated exponential function of voltage. Figure 6-1 shows a response curve for the liquid crystal retarder. Secondly, the response time varies with voltage and direction. The same shift in retardance will take longer at lower voltages than at higher voltages. In addition, the same step will take longer when moving toward lower voltages than it would stepping toward higher voltages. In other words, the response time of the liquid crystal retarder has hysteresis. These first two properties make the integrating bucket approach to phase shifting impractical. The liquid crystal retarder is better suited for phase stepping. Lastly,

retardance variations up to two percent of the average retardance across the full aperture of the liquid crystal retarder are possible. Fortunately, the scatterplate interferometer averages many measurements at one time. Each scatter point on the scatterplate performs a statistically independent measurement of the test mirror. The summation of the wavefronts from each scatter point forms the interference fringes. For the most part, the phase variations across the aperture will manifest themselves as a broadening of the fringe pattern and reduction in fringe contrast. For reliable phase shifting it is advisable to limit the usable area of the liquid crystal retarder to a small fraction of the full aperture. Currently, only about 1% of the aperture area is in use.

Figure 6-1: Response of Liquid Crystal Retarder

LIQUID CRYSTAL RETARDER CALIBRATION:

The liquid crystal retarder was calibrated using the four-pass setup shown in figure 6-2. Twenty-three data points separated by an eighth-wave of retardance were taken across the full range of the liquid crystal retarder. The four-pass calibration setup is equivalent to placing four aligned and correlated variable retarders oriented at 45° between two parallel polarizers oriented vertically. If the total induced retardance is a half-wave, the linear polarization leaving the first polarizer is rotated 90° and blocked by the second polarizer. Since the total induced retardance is four times the retardance of the liquid crystal retarder, a minimum in detector irradiance is observed at

$$\delta = \frac{\lambda}{8} + \frac{m\lambda}{4}, \quad (6-1)$$

while peaks in the detector signal occur when

$$\delta = \frac{m\lambda}{4}. \quad (6-2)$$

m is an integer. The four-pass configuration not only produces measurements at eighth-wave steps, it has four times the sensitivity of a single pass measurement. The voltages associated with each extreme in the detector signal were recorded. Due to drift in the source power and stray light the calibration technique was limited to an accuracy of ± 0.02 Volts.

Unfortunately, the errors in retardance are not linearly proportional to errors in the voltage measurements. Examining figure 6-1, it becomes clear that the same size voltage interval corresponds to a much larger retardance range at lower voltages than it does for higher voltages. The implication is that the error estimate for the retardance will be a function of voltage. To represent the statistical nature of the measurement errors, the peak-to-valley error approximation is converted to standard deviation by assuming that it is two standard deviations. The resulting standard deviation in voltage is 0.01 volts. Using the usual technique for propagation of errors, the standard deviation in retardance relates to errors in voltage as follows

$$\sigma_R = \left(\frac{\partial R(V)}{\partial V} \right) \sigma_V, \quad (6-3)$$

where σ_R and σ_V are the standard deviations in retardance and voltage respectively, and $R[V]$ is the function representing the curve in figure 6-1 (Squires pp.33-46). Fitting a curve to the measured calibration data results in the following expression for $R[V]$.

$$R(V) = 0.560463 - 0.0067624V + 0.972123e^{2.674 - 1.58569V + 0.213627V^2 - 0.0122023V^3}. \quad (6-4)$$

Substituting into equation (6-3), the standard deviation in retardance becomes

$$\sigma_R = \left(\frac{-0.0067624 + 0.972123 * (-1.58569 + 0.427253V - 0.036607V^2)}{e^{2.674 - 1.58569V + 0.213627V^2 - 0.0122023V^3}} \right) \sigma_V. \quad (6-5)$$

The standard deviation is a function of voltage. At 1.25 volts the standard deviation in retardance is 0.0290 waves and at 5.0 volts it becomes 0.00106 waves.

CALIBRATION DATA AND INTELLIWAVE:

Depending on the software used to control the interferometer, the voltage values obtained from the calibration may be used directly for phase shifting. On the other hand, some software packages require an analytical expression that describes the nonlinearity. Intellwave, the software package used to control the phase-shifting scatterplate interferometer, requires an expression that translates retardance values into the appropriate output voltage. This necessitates that a curve similar to figure 6-1 but inverted, be fit to the calibration data. Through trial and error it was determined that the response of the liquid crystal retarder is a complicated exponential of the form

$$Voltage = a + b\delta + ce^{l+m\delta+n\delta^2+p\delta^3+q\delta^4+r\delta^5}, \quad (6-6)$$

where a, b, c, l, m, n, p, q, and r are coefficients to be determined. A curve was fit to the calibration data using a three-step process in Mathematica. First, a curve was fit to the natural log of the voltages using the built in linear fit routine to obtain values for coefficients l-n and p-r. These values were then inserted into equation (6-6) and a linear fit performed on the original data to obtain coefficients a-c. The last step was to use the values of the coefficients as a starting point for a nonlinear fit. The adjusted values of the coefficients are shown in table 6-1 and figure 6-3 shows the resulting fit. The root mean square error in phase shift due to the fit was 0.002 waves.

Coefficient	Value	Coefficient	Value
a	2.72225	n	8.57746
b	-0.59205	p	-9.47005
c	0.86083	q	4.56623
l	2.34436	r	-0.72785
m	-7.03689		

Table 6-1: Coefficients for Liquid Crystal Retarder Calibration

Figure 6-3: Liquid Crystal Retarder Calibration Fit

PHASE SHIFT ERRORS:

Combining the error in the calibration measurement with that of the curve fit gives an estimate of the quality of the phase shift calibration. Remembering that variances add, the standard deviation in the phase steps due to the entire calibration process is given by

$$\sigma_{total} = \sqrt{\sigma_R^2 + \sigma_F^2} . \quad (6-7)$$

where σ_F is the standard deviation in retardance due to the curve fit (Frieden 72-98). On average, the root-mean-square phase step error across the aperture measured by Intellwave was 0.005 waves. The consequence for the unwrapped surface data was determined by choosing the approximate voltage range used by the interferometer (1.56V-3.31V), calculating the standard deviation at each phase step, and inserting the corresponding normally distributed error into the phase-shifting algorithm. The 5-B algorithm outlined by Joanna Schmit and Katherine Creath was used (Schmit 1995). The 5-B algorithm was chosen for its error compensation and the small number of steps required. The response time of the liquid crystal retarder is relatively slow, about 0.3 seconds per step. Any more than five steps and the environmental errors outweigh the benefits of more steps. The 5-B algorithm is given by

$$-Tan(\phi) = \frac{-E_1 + 4E_2 - 4E_4 + E_5}{E_1 + 2E_2 - 6E_3 + 2E_4 + E_5} , \quad (6-8)$$

where ϕ is the relative phase associated with the departure of the surface from a sphere and E1-E5 are fringe patterns separated by quarter-wave phase shifts. Figure 6-4 shows the typical double frequency phase error for three intensity fringes caused by the phase

shift miscalibration. The analysis was repeated fifty times producing an average peak-to-valley error of 0.0084 waves with a standard deviation of 0.0051 waves.

Figure 6-4: Typical Double Frequency Phase Error Caused by Phase shift Miscalibration

PERFORMANCE AND ERROR ANALYSIS

This chapter gets to the heart of the matter. How well does the interferometer work? Traditionally, performance is analyzed by examining both accuracy and repeatability. This analysis will follow suit. The accuracy will be examined in three ways. First a measurement taken with the phase-shifting birefringent scatterplate interferometer will be compared with one taken using a commercial interferometer, then two measurements with different amounts of tilt will be subtracted, and finally a few additional sources of systematic error will be examined. Comparing measurements will confirm the general validity of the interferometer output, the subtraction will estimate the errors due to phase shift miscalibration, which usually limits the accuracy of a phase-shifting interferometer, and the exploration of the additional systematic errors will determine if they are of concern. The precision will be studied in two ways. First, two measurements with tilt and focus minimized will be subtracted and then, three tilt fringes will be introduced into the interferogram and the subtraction repeated. The first subtraction is the ultimate indicator of the interferometer's potential. With the fringe pattern "nulled", measurement errors due to fringe movement are minimized allowing the fundamental repeatability of the interferometer to be observed. Adding tilt into the fringe pattern before subtracting enhances the phase-shift errors associated with vibrations and index variation in the optical path enabling their effect on repeatability to be determined.

INTERFEROMETER ACCURACY:

The best way to estimate the accuracy of an interferometer is to measure a “known” surface or equivalently, compare measurements with a calibrated instrument. Figure 7-1 contains a surface plot of the test mirror obtained using the phase-shifting scatterplate interferometer and figure 7-2 shows a surface measurement of the same mirror acquired with a commercial phase-shifting Fizeau interferometer. Due to the long focal length of the test mirror and the limited size of the isolation table, a folding flat was used in the Fizeau measurement. In double pass, the possible peak-to-valley error in the measurement created by the flat was 0.073 waves. Despite the measurement uncertainty due to the folding flat, the scatterplate and Fizeau measurements compare well. The peak-to-valley difference is less than 0.035 waves and the root-mean-square surface departures agree to within 0.000015 waves.

Figure 7-1: Surface Measurement Taken With Phase-shifting Scatterplate Interferometer

Figure 7-2: Surface Measurement Taken with Wyko 6000 Phase-shifting Fizeau Interferometer

The most common systematic error associated with phase-shifting interferometry is due to phase shift miscalibration. A good estimate of the error is obtained by subtracting two measurements with different amounts of tilt in the fringe pattern. Since phase shift miscalibration results in a phase error that has twice the spatial frequency of the fringe pattern, subtracting measurements will not cancel the error. Figure 7-3 shows the subtraction of two measurements, one with no tilt and one with three fringes of tilt. The peak-to-valley phase-shift error is 0.035 waves and the root-mean-square error is 0.0063 waves.

Figure 7-3: Subtraction of Two Consecutive Measurements, One with Fringes “Nulled” and One with 3 Tilt Fringes Added

There are two additional sources of systematic error that need to be examined. Both polarization component misalignment and the introduction of optics between the scatterplate and test mirror can adversely affect the interferometer performance. The introduction of polarization components into the optical path of the interferometer not only facilitates phase shifting, it creates new sources of error. Of particular interest is the consequence of errors in the rotational orientation of each component. Rotating the polarizers is an effective control for maximizing the fringe contrast, which does not introduce errors into the measurement. The orientations of the liquid crystal retarder, quarter-wave plate, and birefringent scatterplate are possible sources of error. The error associated with each component will be explored by tracing the optical path through the system.

The simplest case to analyze is rotation errors in the quarter-wave plate. For the ideal case where the scatterplate scatters one hundred percent of the light polarized along the extraordinary axis of the crystal, the electric field after the first pass through the scatterplate has two components, a scattered beam and a direct beam. A relative phase shift is introduced between the two components by the liquid crystal retarder. The two beams can be expressed in a Jones matrix as follows

$$\begin{pmatrix} S e^{i\delta} \\ D \end{pmatrix}, \quad (7-1)$$

where S and D represent the percentage of incident light divided into the scattered and direct beams respectively. In double pass, the quarter-wave plate acts like a half-wave plate, which rotates the linearly polarized components by twice the angle between the fast

axis of the quarter-wave plate and the axis of the polarization component. With perfect alignment, the two polarization components are rotated 90° and the polarization of the scattered beam lines up with the ordinary axis of the scatterplate. Similarly, the direct beam aligns with the extraordinary axis. Directly after the second pass through the scatterplate there are again two beams, a scattered-direct and a direct-scattered. However, if the quarter-wave plate is rotated by an angle θ , the scattered and direct beams from the first pass through the scatterplate no longer align with the axes of the crystal on the second pass. Hence, the scattered beam from the second pass contains components from both the direct and scattered beams of the first pass. Similarly, the direct beam will contain both components from the first pass. The resulting Jones vector is

$$\begin{pmatrix} DS(\cos^2(2\theta) - \sin(2\theta)\cos(2\theta)) + SS e'' e' (\sin^2(2\theta) + \sin(2\theta)\cos(2\theta)) \\ SD e'' e' (\cos^2(2\theta) + \sin(2\theta)\cos(2\theta)) + DD(\sin^2(2\theta) - \sin(2\theta)\cos(2\theta)) \end{pmatrix} \quad (7-2)$$

where θ is the rotation error in the quarter-wave plate. Instead of having two beams present at the CCD there are now four. The additional beams are direct-direct and scattered-scattered. Misalignment of the quarter-wave plate leads to an increase in the hot spot irradiance, the introduction of background irradiance, and a reduction in the fringe modulation. Small rotation errors will not degrade the performance of the system.

The errors associated with misalignment of the liquid crystal retarder are found in a similar manner. Rotating the liquid crystal retarder produces a mixing of the phase shift in the test and reference beams. Both the test and reference beam will have a shifted and stationary component. With mixing, equation (7-1) becomes

$$\begin{pmatrix} S\{e^{i\delta}L(\theta) + M(\theta)\} \\ D\{P(\theta) + e^{i\delta}Q(\theta)\} \end{pmatrix}, \quad (7-3)$$

where

$$L(\theta) = \cos^2(2\theta) - \sin(2\theta)\cos(2\theta), \quad (7-4)$$

$$M(\theta) = \sin^2(2\theta) + \sin(2\theta)\cos(2\theta), \quad (7-5)$$

$$P(\theta) = \cos^2(2\theta) + \sin(2\theta)\cos(2\theta), \quad (7-6)$$

and

$$Q(\theta) = \sin^2(2\theta) - \sin(2\theta)\cos(2\theta). \quad (7-7)$$

Instead of two beams leaving the scatterplate on the first pass there are now four. The quarter-wave plate rotates the polarization components 90° and the four beams interact with the scatterplate. Immediately after the scatterplate the Jones vector becomes

$$\begin{pmatrix} DS(P(\theta) + e^{i\delta}Q(\theta)) \\ SD e^{ikw} (e^{i\delta}L(\theta) + M(\theta)) \end{pmatrix}. \quad (7-8)$$

Again there are four beams present at the CCD. However, this time it is not clear how the additional components in the test and reference beams affect the measurement. Combining the polarization components and multiplying by the complex conjugate results in the following expression for the irradiance at the detector

$$\begin{aligned}
E_{out} = (DS)^2 \{ & L(\theta)^2 + M(\theta)^2 + P(\theta)^2 + Q(\theta)^2 \} \\
& + 2(DS)^2 \{ L(\theta)M(\theta) + P(\theta)Q(\theta) \} \cos(\delta) \\
& + 2(DS)^2 \{ M(\theta)P(\theta) + L(\theta)Q(\theta) \} \cos(kW) . \\
& + 2(DS)^2 M(\theta)Q(\theta) \cos(kW - \delta) \\
& + 2(DS)^2 L(\theta)P(\theta) \cos(kW + \delta)
\end{aligned} \tag{7-9}$$

There are four important changes in the irradiance pattern in the image plane.

1. The average irradiance experiences a small modulation.
2. The modulation of the original set of interference fringes is reduced slightly.
3. Background fringes that do not shift with δ are introduced.
4. Background fringes that shift in the opposite direction of the original set are present.

For small orientation errors coefficients M and Q are much smaller than L and P . Accordingly, the modulation of the background fringes and the change in average irradiance is extremely small. In order to determine the error associated with the misalignment of the liquid crystal retarder, the irradiance distribution in equation (7-9) was analyzed using the 5-B phase-shifting algorithm. A linear surface height change of 3 fringes and an orientation error of 1° were used. The unwrapped phase was subtracted from the ideal surface. The results are plotted in figure 7-4. The peak-to-valley error was 0.0001 waves.

Figure 7-4: Measurement Error for 1° Misalignment of Liquid Crystal Retarder

The most complicated analysis is for errors in the orientation of the birefringent scatterplate. Rotating the birefringent scatterplate is equivalent to rotating both the liquid crystal retarder and the quarter-wave plate. Simultaneously, inserting the orientation errors for the first two cases and tracing the optical path produces the following Jones vector directly after the second pass through the scatterplate

$$\begin{pmatrix} DS\{P(\theta) + e^{i\delta}Q(\theta)\}L(2\theta) + SS e^{ikw}\{e^{i\delta}L(\theta) + M(\theta)\}M(2\theta) \\ SD e^{ikw}\{e^{i\delta}L(\theta) + M(\theta)\}P(2\theta) + DD\{P(\theta) + e^{i\delta}Q(\theta)\}Q(2\theta) \end{pmatrix} \quad (7-10)$$

There are now eight beams present at the detector array. Notice that two of the beams are direct-direct and two are scattered-scattered which do not interfere to produce fringes. The four remaining beams are the same as the ones in equation (7-8) with a reduction in

amplitude. Misaligning the orientation of the scatterplate not only leads to a slight measurement error, it also increases the hot spot irradiance and introduces background irradiance. However, the induced measurement error is negligible for a 1° misalignment.

Ideally, it is preferable not to have any optics between the scatterplate and the test mirror. Unfortunately, the current design of the phase-shifting scatterplate interferometer requires a cover slide and quarter-wave plate to be placed in this region. The following analysis will describe the errors induced by the presence of these components by treating them as plane parallel plates. Figure 7-5 schematically represents the test and reference beams passing through a plane parallel plate. Since the “hot spot” is small in extent the reference beam can be represented by one cone of light focusing to a point on the optical axis at the mirror. The F/# of the reference beam is determined by the scatterplate aperture and the focal length of the test mirror. The test beam, on the other hand is made up of many cones of light emanating from each scatter point on the scatterplate. The F/# of the test beam is determined by the F/# of the mirror. The aberrations induced by a plane parallel plate are well known (Wyant 1992). Spherical Aberration is given by

$$\Delta W_{sph} = \frac{-T}{\left(F/\#\right)^4} \left(\frac{n^2 - 1}{128n^3} \right), \quad (7-11)$$

coma is equal to

$$\Delta W_{coma} = \frac{-T\bar{U}}{\left(F/\#\right)^3} \left(\frac{n^2 - 1}{16n^3} \right) \cos(\theta), \quad (7-12)$$

and astigmatism is quantified by

$$\Delta W_{astig} = \frac{-T\bar{U}^2}{\left(F/\#\right)^2} \left(\frac{n^2 - 1}{8n^3} \right) \cos^2(\theta), \quad (7-13)$$

where T is the thickness of the plate, $\bar{U}$ is the angle between the surface normal of the plate and the axis of the cone of light passing through the plate, n is the index of refraction in the plate, and θ is the orientation in the pupil plane relative to $\bar{U}$. There is also defocus present, however it is unimportant for this analysis. Spherical aberration is present in both the test and reference beams. The magnitude of the spherical aberration for each beam is determined by the $F/\#$ associated with its cone of light. Since the cone angles differ for the test and reference beams the spherical aberration induced in each will be different. Assuming a plate thickness of 4mm, an $F/6$ test mirror, a focal length of 100cm and a scatterplate diameter of 1cm (the approximate values for setup used to characterize the system), the spherical aberration added to the test and reference beams by the plane parallel plate is -0.0143 waves and -1.15×10^{-8} waves respectively. The difference in magnitude of the spherical aberration present in each beam constitutes a measurement error. The peak-to-valley measurement error due to spherical aberration induced by the optics is 0.0143 waves. Spherical aberration is a potential problem that must be accounted for. Conversely, coma and astigmatism are not significant. The coma added to the test beam for an off axis scatter point is equal and opposite for each pass through the plane parallel plate and tends to cancel. The astigmatism for each pass through the scatterplate adds, however the dependence on $\bar{U}^2$ drastically reduces its magnitude. Using the same system values utilized to determine the spherical aberration,

the astigmatism introduced into a test beam emanating from the edge of the scatterplate is 5.14×10^{-5} waves.

Building on the plane parallel plate analysis, it is easy to visualize that the shape of the optical element adds a new dimension to the error calculation. If the plane parallel plate is replaced by an element with power, the position, shape, and thickness of an optical element all become important factors for determining the systematic errors introduced into the measurement. A striking example of the errors induced by an element with power was observed in the phase-shifting scatterplate interferometer. When the scatterplate was placed in the interferometer such that the etched surface faced away from the test mirror, 0.052 waves of astigmatism were introduced into the measurement. The error was due to the scatterplate itself. Since the test and reference beams were created before they passed through the calcite, the shape and thickness of the scatterplate induced astigmatism in the measurement. Flipping the scatterplate eliminated the astigmatism.

The errors associated with local variations in the thickness are less defined and depend on the position of the plane parallel plate. If the plate is thin and placed in contact with the scatterplate, local variations in thickness will introduce little or no error into the beam. In close proximity to the scatterplate both the test and reference beams travel through the same localized volume within the plate and “see” the same thickness. In contrast, if the plane parallel plate is placed near the test mirror, the test and reference beams travel through completely different parts of the plate and local variations introduce errors in the measurement. The magnitude of the errors can be determined by tracing the optical path of each beam. In reality, the induced errors are somewhere between the two cases, favoring the first. For accurate measurements it is wise to place the quarter-wave plate as close to the scatterplate as possible.

From the measurements and analysis above it is evident that phase shift miscalibration limits the accuracy of the interferometer.

INTERFEROMETER REPEATABILITY:

The repeatability of an interferometer is the most important figure of merit for determining the performance. Repeatability tells the user whether or not they can trust the output of an interferometer from one measurement to the next. The repeatability is analyzed by subtracting two sets of measurements. First, two consecutive measurements with tilt and focus adjusted to minimize the number of fringes across the test surface are subtracted. With the fringes “nulled” the measurement errors due to fringe movement are minimized, allowing the fundamental performance of the interferometer to be examined.

Figure 7-6 shows the subtraction. The resulting root-mean-square noise is 0.0025 waves. Next, two consecutive measurements with three fringes of tilt in the interferogram are subtracted. Adding tilt into the fringe pattern before subtracting enhances the phase-shift errors associated with vibrations and index variation in the optical path, enabling their effect on repeatability to be determined. Figure 7-7 contains the subtraction. The environmental factors in the lab do not significantly reduce the repeatability. The root-mean-square repeatability is limited to 0.003 waves.

Figure 7-6: Subtraction of Two Measurements with Fringes “Nulled”

Figure 7-7: Subtraction of Two Consecutive Measurements with 3 Tilt Fringes Added to the Interferogram

LIMITATIONS AND IMPROVEMENTS

This final chapter will address limitations on the performance of the phase-shifting birefringent scatterplate interferometer and suggest improvements that will reduce the shortcomings. While there are always countless improvements that can be considered, only the most prominent will be addressed. First, the performance degradation due to the liquid crystal retarder will be explored, then the problems associated with the “hot spot” will be outlined, and finally the need to reduce the magnification of the source image on the test mirror will be discussed.

Although satisfactory performance was obtained using the liquid crystal retarder, a significant improvement can be realized by its replacement. The nonlinear response of the liquid crystal retarder makes calibrating the phase steps difficult, the slow response time increases the vulnerability to environmental effects, and the phase variations across the aperture are a potential source of error. The nonlinear relationship between retardance and voltage rules out the use of common statistical methods to calibrate the phase steps. Typically, commercial interferometers calibrate the phase steps by recording a series of interferograms with equal phase shifts between the frames and use an algorithm to extract the linear phase shift between the steps. One such algorithm uses five frames:

$$\alpha(x, y) = \text{ArcCos} \left(\frac{1}{2} \left(\frac{E_5 - E_1}{E_4 - E_2} \right) \right), \quad (8-1)$$

where α is the linear phase shift between steps and E_1 - E_5 are fringe patterns separated by a phase shift of α (Schweider 1993, Cheng 1985). The linear phase shift is calculated for each point in the interferogram and a histogram is used to determine the average phase shift. Unfortunately, this method assumes a linear phase shift between steps, which precludes its use for calibrating the liquid crystal retarder. Instead a more cumbersome method such as the one described in chapter 6 must be used increasing the likelihood of introducing errors into the calibration.

Taking rapid measurements can improve the performance of the phase-shifting scatterplate interferometer. While the scatterplate interferometer is relatively insensitive to piston motions in the test mirror, tilt motions and index variations in the optical path near the test mirror result in fringe movement. Rapid measurements reduce the errors associated with the fringe movement by capturing all of the necessary frames before the fringes move significantly. Generally, there is a performance trade off between the number of phase steps used in a measurement and the errors brought on by environmental factors. Quick measurements reduce the errors due to local fringe movement, while using more steps reduces the sensitivity to phase shift miscalibration. Fast response times facilitate the implementation of 6 and 7 step phase-shifting algorithms. The slow response time and nonlinear output of the liquid crystal retarder can be overcome by replacing it with an electro-optic light modulator that has a linear response.

Another consideration is the phase variations across the aperture of the liquid crystal retarder. The literature for the variable retarder estimates the phase variations across the aperture to be 2% of the average phase. Although the scatterplate

interferometer averages many measurements at one time, the phase variations can limit the performance of the interferometer by broadening the fringes and reducing the contrast. Phase-shifting interferometers generally produce good results with low contrast fringes. However, coupled with the extreme irradiance in the “hot spot”, the contrast reduction can further limit the quantization levels available for digitizing the interference pattern.

The remaining performance limitations are directly related to the “hot spot”. The amplitude of the direct-direct beam is, for the most part, orders of magnitude greater than the modulation of the fringe pattern, severely limiting the quantization levels available for digitizing the interference pattern. Phase errors due to quantization are given by

$$\sigma_{\phi} \approx \frac{2}{Q\sqrt{3N}}, \quad (8-2)$$

where σ_{ϕ} is the standard deviation in the phase in units of radians, N is the number of steps in the algorithm, and Q is the number of quantization levels (Brophy 1990). Reducing the quantization levels from 256 to 128 increases the root mean square error in phase by a factor of two. One simple solution is to increase the gain of the camera allowing the “hot spot” to saturate. Unfortunately, the electrons stored in the overly saturated pixels tend to leak into neighboring pixels, causing the “hot spot” to bloom out and streaks to form across the image. A better solution is to reduce the “hot spot” amplitude. As described in chapter 5, proper etching of the scatterplate can significantly reduce the amplitude of the direct-direct beam. The acid etching used to generate the scatterplates for this research failed to faithfully reproduce the scatterplate pattern.

Perhaps another etching method such as an ion mill will facilitate the “hot spot” reduction.

Finally, the magnification of the source image on the test mirror is a potential source of error that should be addressed. Even with an inverted-telephoto imaging lens, the disproportionate object and image distances resulted in magnifications as large as 50x. The accuracy of the measurements depends on the quality of the reference. If the source image is too large, the assumption of a perfect reference is no longer valid. The risk of error can be reduced by carefully designing a multiple-element imaging lens for which the principal planes are located a considerable distance beyond the lens.

SUMMARY

This dissertation describes and analyzes a new phase-shifting scatterplate interferometer. The interferometer separates the test and reference beams by exploiting the polarization characteristics of a birefringent scatterplate. The scatterplate scatters only the component of polarization oriented along the extraordinary axis of the crystal. Together with two polarizers and a quarter-wave plate, the birefringent scatterplate produces test and reference beams with orthogonal polarizations. A variable phase shift is induced between the beams using a liquid crystal retarder.

The birefringent scatterplate was manufactured by etching the pattern into a good-quality calcite wave plate. The scatterplate pattern was exposed into photoresist using two distinct methods and an extremely weak solution of hydrochloric acid etched the pattern into the crystal. Exposing the pattern holographically, although labor intensive, was the best method for creating low $F/\#$ scatterplates, while using a photomask for the exposures facilitated rapid manufacturing of high quality scatterplates. With proper design and etching it is possible to reduce the “hot spot” amplitude. Unfortunately, the acid etching did not faithfully reproduce the scatterplate pattern and a significant reduction was not demonstrated.

The liquid crystal retarder was difficult to calibrate and its slow response time increased the sensitivity to environmental effects. The nonlinear response of the liquid crystal retarder ruled out the use of common statistical methods for phase-shift calibration. As a result, a separate experimental setup was used, increasing the likelihood of errors. The estimated surface height error due to the limitations of the calibration was

0.0084 waves peak-to-valley. Subtracting consecutive measurements with differing amounts of tilt set the peak-to-valley error three times higher. A reduction in the surface height error was possible if more steps were used to calculate the phase. However, the slow response of the liquid crystal retarder offset the benefits by increasing the sensitivity to environmental effects. The liquid crystal retarder limited the performance of the interferometer.

Despite the limitations imposed by the liquid crystal retarder, the performance of the interferometer was admirable. Subtracting two consecutive measurements demonstrated a repeatability of 0.003 waves and comparing measurements with a commercial interferometer established a peak-to-valley accuracy of 0.03 waves, which was well within the uncertainty of the commercial interferometer measurement.

APPENDIX A

MASK_GENII:

```

*****
.*
.*
.* Mask_genII
.*
.*
.* Written by Michael North Morris 11/12/99
.* Last updated: 12/04/99
.*
.* Mask_genII is an IDL program that generates and tests a photomask
.* pattern used to expose scatterplates for a scatterplate interferometer.
.* The program calls four subroutines:
.*
.*   ave_features - Determines the average 1-D feature size
.*
.*   mask_acorr - Takes the autocorrelation of the center 101x101 pixels
.*
.*   dxfwrtvii - Converts data to autocad DXF format
.*
.*   remove_enclosed - frees enclosed features that may be missed by dxfwrtvii.
.*
.* The program performs the following tasks in order to create the mask pattern.
.*   1. Generates a psuedorandom pattern with inversion symmetry.
.*   2. Analyzes the randomness by taking the autocorrelation and the
.*      and the symmetry by subtraction.
.*   3. Removes enclosed features that may be lost in data conversion.
.*   4. Analyzes randomness and symmetry again
.*   5. Creates 4 different masks by scaling the same pattern.
.*   6. Converts data to DXF format.
.*
*****

pro mask_genII,ext

    n=2001 ;Size of array

    cd,'d:\michael\mask';Set working directory

    ;Resolve Subroutines
    resolve_routine,'ave_features'
    resolve_routine,'mask_acorr'
    resolve_routine,'dxfwrtvii'
    resolve_routine,'remove_enclosedII'

    journal,ext+'.jrn'

    print,'Beginning mask generation'
    print,'Start Generation: '+systemtime(0)

```

```

*****
;* Create random pattern with inversion symmetry
*****

:Generate partially filled array
mask=randomu(seed,n+2,n+2)
mask(where(mask ge 0.705))=1
mask(where(mask lt 0.705))=0

:Sum array with rotated version of itself.
mask=mask(1:n,1:n)+rotate(mask(1:n,1:n),2)
mask(where(mask gt 0))=1

window,1,xsize=4*n,ysize=4*n
tvscf,congrid(mask,4*n,4*n)

*****
;* Analyze randomness and symmetry
*****
:analysis before moving pixels
print,'fill=',total(mask)/float(n)^2           ; What is fill ratio?
symmetry=where(mask-rotate(mask,2) ne 0)         ; Check symmetry
print,'Symmetry Check',size(symmetry)
ave_features,mask,ext                           ; Average feature size

:Autocorrelation of center 101x101 array
mask_acorr,mask((float(n)-1.0)/2.0-50.0:(float(n)-1.0)/2.0+50.0, $
               (float(n)-1.0)/2.0-50.0:(float(n)-1.0)/2.0+50.0),before_`+ext

*****
;* Remove enclosed features
*****
:free enclosed features
remove_enclosedII,mask

window,2,xsize=4*n,ysize=4*n
tvscf,congrid(mask,4*n,4*n)

*****
;* Recheck pattern for randomness and symmetry
*****
:analysis after moving pixels
print,'fill=',total(mask)/float(n)^2           ; What is fill ratio?
symmetry=where(mask-rotate(mask,2) ne 0)         ; Check symmetry
print,'Symmetry Check',size(symmetry)
ave_features,mask,ex                           ; Average feature size

:Autocorrelation of center 101x101 array
mask_acorr,mask((float(n)-1.0)/2.0-50.0:(float(n)-1.0)/2.0+50.0, $

```



```

(float(n)-1.0)/2.0-50.0:(float(n)-1.0)/2.0+50.0), before_ '+ext

:Save mask to file
save,filename='mask_'+ext+'.dat',mask

:*****
:* Create four different masks by scaling features and convert
:* resulting dat to DXF format.
:*****
;write f/4 mask
mask2=mask
mask2(75:124,*)=0
mask2(*,75:124)=0
mask2(1876:1925,*)=0
mask2(*,1876:1925)=0
dxfwrtvii,mask2,4,'f4':Convert to DXF

;write f/6 mask
mask2=mask(1000-666:1000+666,1000-666:1000+666)
mask2(50:82,*)=0
mask2(*,50:82)=0
mask2(1250:1282,*)=0
mask2(*,1250:1282)=0
dxfwrtvii,mask2,6,'f6':Convert to DXF

;write f/8 mask
mask2=mask(1000-500:1000+500,1000-500:1000+500)
mask2(37:61,*)=0
mask2(*,37:61)=0
mask2(939:963,*)=0
mask2(*,939:963)=0
dxfwrtvii,mask2,8,'f8':Convert to DXF

;write f/12 mask
mask2=mask(1000-333:1000+333,1000-333:1000+333)
mask2(25:41,*)=0
mask2(*,25:41)=0
mask2(625:641,*)=0
mask2(*,625:641)=0
dxfwrtvii,mask2,12,'f12':Convert to DXF

print,End Generation: '+systemtime(0)
journal
end

```

AVE_FEATURES:

```

*****
:* Ave_features.pro
:*
:* Written by Michael North Morris 11/13/99
:*
:* Calculates the arithmetic mean for the 1-D feature
:* sizes in mask for both the x and y directions
*****
pro ave_features,mask,ext

    print,'Calculating Average Feature Size.'

    :Initialize variables
    maxx=fix(sqrt(n_elements(mask)))
    flag=0
    n=0
    index=0.0
    features=fltarr(n_elements(mask))

    *****
    :*Calculate average x feature size
    *****
    :Loop makes a histogram of feature sizes
    for j=0,maxx-1 do begin
        for i=0,maxx-1 do begin
            case 1 of
                (mask(i,j) eq 1) and (flag eq 0): begin
                    flag=1
                    n=1
                    index=index+1
                end
                (mask(i,j) eq 1) and (flag eq 1): begin
                    n=n+1
                end
                (mask(i,j) eq 0) and (flag eq 1): begin
                    flag=0
                    features(index-1)=n
                    n=0
                end
            else: begin
                flag=0
                n=0
            end
        endcase
    endfor
    if flag eq 1 then features(index-1)=n
    flag=0
    n=0
endfor
momentx=moment(features(0:index-1))

```

```

ave=momentx(0)                                ;Determine ave. feature size
print,'average x size =',ave
var=momentx(1)
print,'variance in x size =',var

:*****
;Calculate average y feature size
:*****
flag=0
n=0
index=0.0
features=flarr(n_elements(mask))

:loop makes a histogram of feature sizes
for i=0,maxx-1 do begin
    for j=0,maxx-1 do begin
        case 1 of
            (mask(i,j) eq 1) and (flag eq 0): begin
                flag=1
                n=1
                index=index+1
            end
            (mask(i,j) eq 1) and (flag eq 1): begin
                n=n+1
            end
            (mask(i,j) eq 0) and (flag eq 1): begin
                flag=0
                features(index-1)=n
                n=0
            end
            else: begin
                flag=0
                n=0
            end
        endcase
    endfor
    if flag eq 1 then features(index-1)=n
    flag=0
    n=0
endfor
momenty=moment(features(0:index-1))
ave=momenty(0)                                ;Determine ave feature size
print,'average y size =',ave
var=momenty(1)
print,'variance in y size =',var

save,filename='mask_moment_'+ext+'.dat',momentx,momenty
end

```

MASK_ACORR:

```

*****
;* Mask_Acorr.pro
;*
;* Written by Michael North Morris 11/15/99
;*
;* Mask_Acorr takes the autocorrelation of a two dimensional function by sliding an inverted
;* copy of the function across itself.
;*
*****

pro mask_acorr,array,ext

  cd,'d:\michael\mask';Set working directory
  print,Calculating the Auto-Correlation. Please be Patient....'
  print,Started: '+systemtime(0)

  n_sqr=n_elements(array)
  n=sqrt(n_sqr)                                ;Determine number of elements in array
  print,Length of Array is',fix(n)

  *****
  ;* Create array for sliding image across itself
  *****
  print,'creating arrays'
  a1=fltarr(3*n-2,3*n-2)
  a1(0:n-1,0:n-1)=array      ;original function
  a2=fltarr(3*n-2,3*n-2)
  a2(0:n-1,0:n-1)=rotate(array,2)      ;inverted copy of original function
  a2=shift(a2,n-1,n-1)

  *****
  ;* creaate array to hold autocorrellation
  *****
  print,'creating acorr'
  acorr=fltarr(2*n-1,2*n-1)

  *****
  ;* Take autocorrelation
  *****
  ;Loop for sliding arrays across each other
  print,'starting loop'
  for i=0,2*n-2 do begin
    ;print,i
    for j=0,2*n-2 do begin
      acorr(i,j)=total(a2*shift(a1,i,j))
    endfor
  endfor
  acorr=acorr/n^2

  save,filename='acorr.dat',acorr

```

```

:*****
:* Plot results
:*****
window,0,xsize=2*n-1,ysize=2*n-1
tvscl,acorr
write_bmp,'acorr_'+ext+'.bmp',tvrdr()
window,2
plot,acorr(*,N-1),background=255,color=0
write_bmp,'acorr_1d_'+ext+'.bmp',tvrdr()

print,Completed: '+sytime(0)
end

```

REMOVE_ENCLOSED:

```

:*****
:* Remove_enclosed.pro
:*
:*   Written by Michael North Morris 12/02/99
:*
:* remove_enclosed searches out pixels that contain a value of zero and are not
:* directly tracable to the edge of the array with a path of zeros and
:* randomly moves the by one pixel. The process is repeated until there are no
:* enclosed features.
:*
:* This process is necessary becuase DXFwritevii fails to find these features in
:* the conversion of the data.
:*****

```

pro remove_enclosedII,mask

```

cd,'d:\michael\mask' ;set working directory
print,Removing enclosed features. Please be Patient'
print,Started: '+sytime(0)

n=fix(sqrt(n_elements(mask))) ; Determine number of elements in array
count=0

flg0=0

:*****
:* Loop repeats process until no enclosed features are left.
:*****
while flg0 eq 0 do begin
    flg0=1

```

```

*****
:* Create maps for tracking enclosed features
*****
map=mask
map2=ftarr(n,n)

window,0,xsize=2*n,ysize=2*n
tvsc1.congrid(map,2*n,2*n)

*****
:* Begin search for enclosed features
*****
:mark open pixels at edges and set to 2
for i=0,n-1 do begin
    if map(i,0) eq 0 then map(i,0)=2
    if map(i,n-1) eq 0 then map(i,n-1)=2
endfor
for j=0,n-1 do begin
    if map(0,j) eq 0 then map(0,j)=2
    if map(n-1,j) eq 0 then map(n-1,j)=2
endfor

window,0,xsize=2*n,ysize=2*n
tvsc1.congrid(map,2*n,2*n)

:propagate from edges
flg2=0
while flg2 eq 0 do begin
    flg2=1
    a=where((map eq 0) and (shift(map,-1,0) eq 2))
    if size(a,/n_dimensions) ne 0 then begin
        flg2=0
        map(a)=2
    endif
    a=where((map eq 0) and (shift(map,0,-1) eq 2))
    if size(a,/n_dimensions) ne 0 then begin
        flg2=0
        map(a)=2
    endif
    a=where((map eq 0) and (shift(map,1,0) eq 2))
    if size(a,/n_dimensions) ne 0 then begin
        flg2=0
        map(a)=2
    endif
    a=where((map eq 0) and (shift(map,0,1) eq 2))
    if size(a,/n_dimensions) ne 0 then begin
        flg2=0
        map(a)=2
    endif
    a=where((map eq 0) and (shift(map,-1,-1) eq 2))
    if size(a,/n_dimensions) ne 0 then begin
        flg2=0
    endif
endwhile

```

```

        map(a)=2
    endif
    a=where((map eq 0) and (shift(map,1,1) eq 2))
    if size(a,/n_dimensions) ne 0 then begin
        flg2=0
        map(a)=2
    endif
    a=where((map eq 0) and (shift(map,1,-1) eq 2))
    if size(a,/n_dimensions) ne 0 then begin
        flg2=0
        map(a)=2
    endif
    a=where((map eq 0) and (shift(map,-1,1) eq 2))
    if size(a,/n_dimensions) ne 0 then begin
        flg2=0
        map(a)=2
    endif
    endif

window,0,xsize=4*n,ysize=4*n
tvsc1,congrid(map,2*n,2*n)
endwhile

: After 5 cycles fill enclosed features to end loop
a=where(map eq 0)
if count eq 5 then begin
    mask(a)=1
endif else begin
:*****
:
:* randomly kick enclosed features one pixel.
:*****
sz=size(a)
if size(a,/n_dimensions) ne 0 then begin
    flg0=0
    for i=0,sz(1)-1 do begin
        if a(i) le (float(n_elements(mask))-1.0)/2.0 then begin
            u=a(i) mod n
            v=floor(float(a(i))/float(n))
            rnd=randomu(seed)
            case 1 of
                rnd le 0.125:begin
                    mask(u,v)=1
                    mask(u-1,v)=0
                    mask(n-1-u,n-1-v)=1
                    mask(n-1-(u-1),n-1-v)=0
                end
                (rnd le 0.250) and (rnd gt 0.125):begin
                    mask(u,v)=1
                    mask(u+1,v)=0
                    mask(n-1-u,n-1-v)=1
                    mask(n-1-(u+1),n-1-v)=0
                end
                (rnd le 0.375) and (rnd gt 0.250):begin
                    mask(u,v)=1

```

```

        mask(u,v-1)=0
        mask(n-1-u,n-1-v)=1
        mask(n-1-u,n-1-(v-1))=0
    end
    (rnd le 0.500) and (rnd gt 0.375):begin
        mask(u,v)=1
        mask(u,v+1)=0
        mask(n-1-u,n-1-v)=1
        mask(n-1-u,n-1-(v+1))=0
    end
    (rnd le 0.625) and (rnd gt 0.500):begin
        mask(u,v)=1
        mask(u-1,v-1)=0
        mask(n-1-u,n-1-v)=1
        mask(n-1-(u-1),n-1-(v-1))=0
    end
    (rnd le 0.750) and (rnd gt 0.625):begin
        mask(u,v)=1
        mask(u-1,v+1)=0
        mask(n-1-u,n-1-v)=1
        mask(n-1-(u-1),n-1-(v+1))=0
    end
    (rnd le 0.875) and (rnd gt 0.750):begin
        mask(u,v)=1
        mask(u+1,v-1)=0
        mask(n-1-u,n-1-v)=1
        mask(n-1-(u+1),n-1-(v-1))=0
    end
    (rnd le 1.000) and (rnd gt 0.875):begin
        mask(u,v)=1
        mask(u+1,v+1)=0
        mask(n-1-u,n-1-v)=1
        mask(n-1-(u+1),n-1-(v+1))=0
    end
endcase
endif
endfor
endif
endelse
count=count+1
print,count
endwhile      ;repeat until no enclosed features exist or for 5 loops

window,1,xsize=2*n,ysize=2*n
tvsc1,congrid(mask,2*n,2*n)

Print,'Completed: '+systemtime(0)
End

```


DXFWRTVII:

```

*****
;* dxfwrtvii.pro
;*
;* Written by Michael North Morris 10/20/99
;*
;* dxfwrtvii converts a scatterplate pattern into a dxf file which can be
;* read by autocad.
;*
;*      sz determines the side dimension of one pixel. This determines the F/#
;*      of the scatterplate.
*****

pro dxfwrtvii,mask,sz,ext

  cd,'d:\michael\mask' ; Set working Directory
  maxx=sqrt(n_elements(mask)) ;Determine size of array

  ;Map is an array used to keep track of which pixels have already
  ;been accounted for
  map=fltarr(maxx,maxx)

  ;open file for writing
  print,'writing DXF file'
  u=1
  openw,u,'mask_'+ext+'.dxf'

  *****
  ;*Write header information at beginning of file
  *****
  printf,u,0
  printf,u,'SECTION'
  printf,u,2
  printf,u,'ENTITIES'
  printf,u,0

  *****
  ;Trace around features (filled with ones) in a clockwise direction
  *****
  for j=0,maxx-1 do begin
    for i=0,maxx-1 do begin
      dir=1
      if (mask(i,j) EQ 1) and map(i,j) EQ 0 then begin
        map(i,j)=1
        flag=0
        ;function duplicate checks to see if a pixel has already been accounted
        ;for. If it has it returns 0. Otherwise it returns 1.
        flag=duplicate(i,j,maxx,dir,mask,map)

        x=i
        y=j
      end
    end
  end

```

```

if flag EQ 1 then begin
  dir=0
endif else begin
  dir=1
  :Procedure polygon writes the appropriate polygone header to the file.
  polygon,x,y,sz,u
; The following case statements trace around a feature in a clockwise direction.
while (dir GT 0) do begin
  case dir of
    1: begin
      case 1 of
        (y GT 0) and (x LT maxx-1):begin
          case 1 of
            (mask(x+1,y) EQ 1) and (mask(x+1,y-1) EQ 1):begin
              x=x+1
              y=y-1
              dir=4
              map(x,y)=1
              :procedure line writes the coordinates to which a line
              :is drawn in the next step.
              line,x,y,sz,u,dir
              if (x EQ i) and(y EQ j) then dir=0
            end
            (mask(x+1,y) EQ 1):begin
              x=x+1
              map(x,y)=1
              line,x,y,sz,u,dir
            end
          else:begin
            dir=2
            line,x,y,sz,u,dir
          end
        endcase
      end
      (x LT maxx-1): begin
        case 1 of
          (mask(x+1,y) EQ 1):begin
            x=x+1
            map(x,y)=1
            line,x,y,sz,u,dir
          end
          else:begin
            dir=2
            line,x,y,sz,u,dir
          end
        endcase
      end
      else:begin
        dir=2
        line,x,y,sz,u,dir
      end
    endcase
  end
endcase

```

```

end
2: begin
  case 1 of
    (y LT maxx-1) and (x LT maxx-1):begin
      case 1 of
        (mask(x,y+1) EQ 1) and (mask(x+1,y+1) EQ 1):begin
          x=x+1
          y=y+1
          dir=1
          map(x,y)=1
          line.x,y,sz,u,dir
        end
        (mask(x,y+1) EQ 1):begin
          y=y+1
          map(x,y)=1
          line.x,y,sz,u,dir
        end
        else:begin
          dir=3
          line.x,y,sz,u,dir
        end
      endcase
    end
    (y LT maxx-1): begin
      case 1 of
        (mask(x,y+1) EQ 1):begin
          y=y+1
          map(x,y)=1
          line.x,y,sz,u,dir
        end
        else:begin
          dir=3
          line.x,y,sz,u,dir
        end
      endcase
    end
    else:begin
      dir=3
      line.x,y,sz,u,dir
    end
  endcase
end
3: begin
  case 1 of
    (y LT maxx-1) and (x GT 0):begin
      case 1 of
        (mask(x-1,y) EQ 1) and (mask(x-1,y+1) EQ 1):begin
          x=x-1
          y=y+1
          dir=2
          map(x,y)=1
          line.x,y,sz,u,dir
        end
      end
    end
  end
end

```

```

        (mask(x-1,y) EQ 1):begin
            x=x-1
            map(x,y)=1
            line,x,y,sz,u,dir
        end
        else:begin
            dir=4
            line,x,y,sz,u,dir
            if (x EQ i) and (y EQ j) then dir=0
        end
    endcase
end
(x GT 0): begin
    case 1 of
        (mask(x-1,y) EQ 1):begin
            x=x-1
            map(x,y)=1
            line,x,y,sz,u,dir
        end
        else:begin
            dir=4
            line,x,y,sz,u,dir
            if (x EQ i) and (y EQ j) then dir=0
        end
    endcase
end
else:begin
    dir=4
    line,x,y,sz,u,dir
    if (x EQ i) and (y EQ j) then dir=0
end
endcase
end
4: begin
    case 1 of
        (y GT 0) and (x GT 0):begin
            case 1 of
                (mask(x,y-1) EQ 1) and (mask(x-1,y-1) EQ 1):begin
                    x=x-1
                    y=y-1
                    dir=3
                    map(x,y)=1
                    line,x,y,sz,u,dir
                end
                (mask(x,y-1) EQ 1):begin
                    y=y-1
                    map(x,y)=1
                    line,x,y,sz,u,dir
                    if (x EQ i) and (y EQ j) then dir=0
                end
            end
        else:begin
            dir=1
            line,x,y,sz,u,dir
        end
    end
end

```

```

        end
      endcase
    end
    (y GT 0): begin
      case 1 of
        (mask(x,y-1) EQ 1):begin
          y=y-1
          map(x,y)=1
          line,x,y,sz,u,dir
          if (x EQ i) and (y EQ j) then dir=0
        end
        else:begin
          dir=1
          line,x,y,sz,u,dir
        end
      endcase
    end
    else:begin
      dir=1
      line,x,y,sz,u,dir
    end
  endcase
end
endcase
endwhile
;seqend writes the closing footer for a particular polygon
seqend,u
endelse
endif
endfor
endfor

; Writes of of file
printf,u,ENDSEC'
printf,u,0
printf,u,EOF'

close,u      ;close file
end

;*****
;* Pocedure polygon writes a header for a new feature
;*****
pro polygon,x,y,sz,u

  printf,u,POLYLINE'
  printf,u,100
  printf,u,'AcDbEntity'
  printf,u,8
  printf,u,'1'
  printf,u,100

```

```

printf,u,'AcDbPolyline'
printf,u,66
printf,u,1
printf,u,10
printf,u,x*sz
printf,u,20
printf,u,y*sz
printf,u,30
printf,u,0.0
printf,u,70
printf,u,1
printf,u,0
printf,u,'VERTEX'
printf,u,100
printf,u,'AcDbEntity'
printf,u,8
printf,u,'l'
printf,u,100
printf,u,'AcDbVertex'
printf,u,100
printf,u,'AcDb2dVertex'
printf,u,10
printf,u,(x+1)*sz
printf,u,20
printf,u,y*sz
printf,u,30
printf,u,0.0
printf,u,0
end

```

```

:*****
:*Procedure line writes the coordinates of the next vertex
:*in the polygon
:*****
pro line,x,y,sz,u,dir

```

```

case dir of
  1:begin
    printf,u,'VERTEX'
    printf,u,100
    printf,u,'AcDbEntity'
    printf,u,8
    printf,u,'l'
    printf,u,100
    printf,u,'AcDbVertex'
    printf,u,100
    printf,u,'AcDb2dVertex'
    printf,u,10
    printf,u,(x+1)*sz
    printf,u,20
    printf,u,y*sz
    printf,u,30

```

```

        printf,u,0.0
        printf,u,0
    end
    2:begin
        printf,u, VERTEX'
        printf,u,100
        printf,u,'AcDbEntity'
        printf,u,8
        printf,u,'1'
        printf,u,100
        printf,u,'AcDbVertex'
        printf,u,100
        printf,u,'AcDb2dVertex'
        printf,u,10
        printf,u,(x+1)*sz
        printf,u,20
        printf,u,(y+1)*sz
        printf,u,30
        printf,u,0.0
        printf,u,0
    end
    3:begin
        printf,u, VERTEX'
        printf,u,100
        printf,u,'AcDbEntity'
        printf,u,8
        printf,u,'1'
        printf,u,100
        printf,u,'AcDbVertex'
        printf,u,100
        printf,u,'AcDb2dVertex'
        printf,u,10
        printf,u,(x)*sz
        printf,u,20
        printf,u,(y+1)*sz
        printf,u,30
        printf,u,0.0
        printf,u,0
    end
    4:begin
        printf,u, VERTEX'
        printf,u,100
        printf,u,'AcDbEntity'
        printf,u,8
        printf,u,'1'
        printf,u,100
        printf,u,'AcDbVertex'
        printf,u,100
        printf,u,'AcDb2dVertex'
        printf,u,10
        printf,u,x*sz
        printf,u,20
        printf,u,y*sz
    end

```

```

        printf,u,30
        printf,u,0.0
        printf,u,0
    end
endcase
end

.*****
:*Procedure seqend writes the closing footer which denotes that
:*the polygon is finished
.*****
pro seqend,u
    printf,u,'SEQEND'
    printf,u,100
    printf,u,'AcDbEntity'
    printf,u,8
    printf,u,'1'
    printf,u,0
end

.*****
:*Function duplicate checks to see if a pixel is internal to a
:*feature that has already been traced. If it has it returns 1 onethwise
:*it returns zero
.*****
function duplicate,i,j,maxx,dir,mask,map
    x=i
    y=j
    flag=0
    count=0

    while (dir GT 0) do begin
        count=count+1
        :print,count
        case dir of
            1: begin
                case 1 of
                    (y GT 0) and (x LT maxx-1):begin
                        case 1 of
                            (mask(x+1,y) EQ 1) and (mask(x+1,y-1) EQ 1):begin
                                x=x+1
                                y=y-1
                                dir=4
                                if (x EQ i) and (y EQ j) then dir=0
                                if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
                                end
                                (mask(x+1,y) EQ 1):begin
                                    x=x+1
                                    if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
                                    end
                                end
                            else:begin

```



```

        dir=2
    end
endcase
end
(x LT maxx-1): begin
    case 1 of
        (mask(x+1,y) EQ 1):begin
            x=x+1
            if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
            end
        else:begin
            dir=2
        end
    endcase
end
else:begin
    dir=2
end
endcase
end
2: begin
    case 1 of
        (y LT maxx-1) and (x LT maxx-1):begin
            case 1 of
                (mask(x,y+1) EQ 1) and (mask(x+1,y+1) EQ 1):begin
                    x=x+1
                    y=y+1
                    dir=1
                    if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
                    end
                (mask(x,y+1) EQ 1):begin
                    y=y+1
                    if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
                    end
                else:begin
                    dir=3
                end
            endcase
        end
        (y LT maxx-1): begin
            case 1 of
                (mask(x,y+1) EQ 1):begin
                    y=y+1
                    if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
                    end
                else:begin
                    dir=3
                end
            endcase
        end
    else:begin
        dir=3
    end
end

```

```

        endcase
    end
3: begin
    case 1 of
        (y LT maxx-1) and (x GT 0):begin
            case 1 of
                (mask(x-1,y) EQ 1) and (mask(x-1,y+1) EQ 1):begin
                    x=x-1
                    y=y+1
                    dir=2
                    if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
                end
                (mask(x-1,y) EQ 1):begin
                    x=x-1
                    if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
                end
            else:begin
                dir=4
                if (x EQ i) and (y EQ j) then dir=0
            end
        endcase
    end
    (x GT 0): begin
        case 1 of
            (mask(x-1,y) EQ 1):begin
                x=x-1
                if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
            end
        else:begin
            dir=4
            if (x EQ i) and (y EQ j) then dir=0
        end
    endcase
end
else:begin
    dir=4
    if (x EQ i) and (y EQ j) then dir=0
end
endcase
end
4: begin
    case 1 of
        (y GT 0) and (x GT 0):begin
            case 1 of
                (mask(x,y-1) EQ 1) and (mask(x-1,y-1) EQ 1):begin
                    x=x-1
                    y=y-1
                    dir=3
                    if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
                end
            (mask(x,y-1) EQ 1):begin
                y=y-1
                if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
            end
        end
    end
end

```

```

        if (x EQ i) and (y EQ j) then dir=0
      end
    else:begin
      dir=1
    end
  endcase
end
(y GT 0): begin
  case 1 of
    (mask(x,y-1) EQ 1):begin
      y=y-1
      if (map(x,y) EQ 1) and (x NE i) and (y NE j) then flag=1
      if (x EQ i) and (y EQ j) then dir=0
      end
    else:begin
      dir=1
    end
  endcase
end
else:begin
  dir=1
end
endcase
end
endcase
if count GE 500 then flag=1
if flag EQ 1 then dir=0
endwhile

return.flag
end

```

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