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RESERVOIR.

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FLOW NEAR THE OUTLET OF A GEOTHERMAL
ENERGY RESERVOIR

by

Hugh Donald Murphy

A Thesis Submitted to the Faculty of the
DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY
WITH A MAJOR IN MECHANICAL ENGINEERING

In the Graduate College
THE UNIVERSITY OF ARIZONA

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THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction
by Hugh Donald Murphy
entitled Flow Near the Outlet of a Geothermal Energy Reservoir

be accepted as fulfilling the dissertation requirement for the Degree
of Doctor of Philosophy

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ABSTRACT

Steady, incompressible flow converging radially between two stationary, parallel plates was investigated both numerically and experimentally. Flow ranges investigated were laminar, turbulent, and transitional. For laminar flows at dimensionless radii ($2\bar{r}\sqrt{\pi\bar{v}/Q\bar{t}}$) much greater than one the velocity profile becomes parabolic and invariant. At radii less than one a boundary layer character evolves with an approximately uniform core region and the boundary layer thickness decreases from one-half the disk spacing to values proportional to the local radii as the flow accelerates towards the center. At large radii the friction factor approaches the classic value obtained for fully developed rectilinear flow between infinite plates, $6\bar{v}/\bar{V}\bar{t}$, but at small radii it approaches the constant $2.17/\sqrt{R_0}$, where R_0 is an overall Reynolds number based on the volumetric flow rate and the disk spacing and is independent of radius. Tabular and graphical results are provided for the intermediate range of radii, where both viscous and inertial effects are important, and exact analyses were not available.

Laminar flow experiments in a circular disk geometry were conducted over the half-spacing range 0.46 to 25.5 mm (0.018 to 1.0 in.), overall Reynolds number range 4000 to 106,000 and for the local Reynolds number range 200 to 20,000. Measured pressure drops were in good agreement with laminar predictions, indicating that accelerations were large enough to stabilize the flows completely despite high local Reynolds numbers.

Turbulent flows were examined with a pie-shaped apparatus with a fixed spacing of 12 mm (1/2 in.). Convergent flow angles were adjusted to vary the acceleration at a given flow rate. The ranges of parameters examined were local Reynolds numbers from 1600 to 68,000, acceleration parameters $K_1 = 1/R_0 = (\bar{v}/\bar{v}^2)d\bar{v}/d\bar{r}$ from 2.6×10^{-8} to 1.1×10^{-5} , and convergent half-angles from 0 to 16 degrees. For fully turbulent flow, reasonably good agreement with measured pressures and wall shear stresses was obtained with numerical predictions using a turbulence model based on the van Driest mixing length approach. These comparisons are presented graphically and in tabular form.

In this study a reduction of measured pressure gradients and wall shear stresses compared to turbulent predictions is called laminarization, corresponding with the terminology of some previous investigators who examined accelerating turbulent flows caused by heating gases or by converging planes. The measuring station was two-thirds of the way along the duct. Completion of laminarization, defined by agreement of measurements with the laminar predictions, occurred at values of acceleration parameters K_1 of 2×10^{-6} to 5×10^{-6} and $K_2 = (\bar{v}/\bar{p}U_T^3)(d\bar{p}/d\bar{r})$ of 0.02 to 0.05. For laminarizing flows the van Driest turbulence model was empirically modified to agree with the measurements. A simple algebraic model was obtained that provided predictions in reasonable agreement with the turbulent, laminarizing and laminar flows in this apparatus. Predictions in agreement with the laminarizing flows showed a thickening of the non-dimensional viscous layer.

Preliminary, qualitative measurements of the temporal wall shear stress were obtained with a hot film sensor. These limited data generally showed reductions in turbulent fluctuations which corresponded with the laminarizing process as defined above.

CHAPTER I

INTRODUCTION

Geothermal energy reservoirs can be broadly categorized into two types, hydrothermal and dry hot rock reservoirs. Hydrothermal reservoirs occur in those fortunate places where nature has provided not only a source of thermal energy, hot rock, but also the working fluid, either in the form of dry steam or superheated water, which delivers the geothermal energy close to the earth's surface. In such cases it is a simple process to penetrate the reservoir by drilling and subsequently to convey this fluid, either dry steam, or superheated water that flashes to steam, to steam turbines for conversion of the geothermal energy to electrical energy or for other purposes. Hydrothermal reservoirs are relatively rare, and in the future, will be found with decreasing frequency.

Fortunately, potential dry hot rock reservoirs underlie the entire world. The potential of this resource base is limited only by present and projected drilling techniques. Drilling to depths of 3000 to 6000 m is now becoming practical, which means that useful geothermal power could be produced from approximately 0.6 million square kilometers in the American west; and this resource base by itself could satisfy this nation's energy needs for thousands of years (Smith, 1974).

A program designed to demonstrate the feasibility of extracting energy from dry hot rock has been initiated at the Los Alamos Scientific Laboratory. Basically it has been proposed that man-made geothermal energy reservoirs can be created by drilling into relatively impermeable rock to a depth where the temperature is high enough to be useful; creating a large hydraulic fracture; and then completing the circulation loop by drilling a second hole to intercept the hydraulic fracture. The concept is illustrated in Figure 1. The fracture aperture, $2\bar{t}$,¹ is typically of the order of 1 mm (Perkins and Kern, 1961).

Thermal power is extracted from this system by injecting cold water down the first hole, forcing the water to sweep by the freshly exposed hot rock surface in the reservoir/fracture system, and then returning the hot water to the surface where the energy is extracted from the water for generating electrical power, or for other uses. System pressures are maintained such that only one phase, liquid water, is present in the reservoir and the drilled holes.

The mechanics of the energy extraction process have been discussed by Harlow and Pracht (1972), McFarland (1975), and McFarland and Murphy (1976). All these authors have assumed that, irrespective of position, the velocity profile across the very narrow fracture aperture is parabolic, corresponding to a laminar, fully developed flow between two flat, smooth plates. While this assumption may be valid far from the inlet and outlet, where velocities are low, it may not be true near the outlet, where the rather large flow rate required to produce

1. Symbols are defined in the Nomenclature section following the Appendices.

Figure 1. Man-Made Geothermal Energy Reservoir.

significant levels of power extraction is constrained to smaller and smaller flow areas as the outlet is approached. Thus the outlet region is characterized by large velocities, large accelerations, and large pressure losses. These large pressure losses degrade the overall system efficiency since extracted power must necessarily be diverted to provide pumping power. These pressure losses may be even larger than anticipated because the confining walls of the fracture will no doubt be rough; but even for the idealized case of smooth walls the possibility exists that in some cases the flow may become turbulent near the outlet, further aggravating pressure losses.

Problem Description

In the work described here it will be assumed that the fracture walls are hydrodynamically smooth and parallel near the outlet region. This idealization provides a fundamental technical base and the results should prove useful as a starting point for future studies which will incorporate roughness effects when estimates of the absolute roughness in geothermal reservoirs become available. The water flow in the very small aperture created by the fracturing process can be reasonably approximated as an incompressible, radially converging flow with no azimuthal variation of either pressure or velocity as it converges upon the exit wellbore, which has been directionally drilled so as to intersect the fracture plane at 90 degrees. The geometry and some of the nomenclature to be used are indicated in Figure 2. In the present study the range of interest does not include the region in the immediate vicinity of the exit where significant momentum would be directed in

Figure 2. Idealized Flow Geometry.

the transverse direction. It will be shown that this effect is important only in a very small region next to the outlet, with radial extent of only several apertures. Nor does the range of interest include great radial distances from the exit. Typically the acceleration and non-parabolic velocity effects occur within a relatively short radius, of the order of several meters. This distance is so small compared to the overall fracture radius that the variation in fracture aperture can be ignored in the outlet region of interest.

In essence, then, the idealized problem is that an incompressible, radially converging flow between two stationary disks with constant spacing. Such flows are also of interest in connection with such practical applications as double disk valves and thrust bearings.

For the idealized geothermal reservoir the velocity profile would correspond to a fully developed one at a large radius; however, for some other applications the velocity would be uniform at an entry at large radius. In the latter case the flow develops and approaches a fully developed profile as between infinite parallel plates, though the bulk velocity is continuously increasing as it converges towards the exit. At smaller radii acceleration dominates the flow in the center of the channel causing a flattening of the profile. Since the no-slip condition requires zero velocity at the wall, the profile takes the appearance of a boundary layer flow and potential core even though the location would be considered well downstream in a duct flow. This boundary layer becomes successively thinner as acceleration continues. Some of the physical features are comparable to the flow at the base of an idealized tornado except there is no swirl in the present problem.

Previous Studies

Laminar Flows

It appears that the flow of opposite nature, i.e., radially diverging, decelerating flow has received much more attention in the literature, prompted perhaps by an interest in the fluid dynamics of radial diffusers. In some cases the pressure gradient in these diverging flows becomes so adverse that the flow separates (Woolard, 1957; McGinn, 1955; and Jackson and Symmons, 1965a), leading to the phenomenon sometimes called "stall." However, when this condition is avoided, some of the developments discussed in the literature can also be applied in cases of converging flow, and these are therefore discussed here when appropriate.

There appear to be three levels of approximation of the theory of radially accelerating or decelerating flows. These are: (1) ignoring the acceleration altogether, (2) boundary layer momentum integral approximations, assuming invariant velocity profiles, and (3) perturbation, series expansion, and higher order integral approximations. At the first level Licht and Fuller (1954) confined their attention to cases where the flow was laminar and the local radius was so large that inertial or acceleration effects were negligible compared to viscous effects. (For the remainder of this dissertation the term acceleration will generally be used rather than inertial.) They termed this "creeping" flow and concluded that the velocity profile is parabolic as in parallel duct flow and that the pressure varies logarithmically with radius.

For the situation where acceleration forces cannot be completely ignored a number of authors, working at level 2, have proposed approximate laminar solutions based on the assumption that the velocity profile was invariant with radius. Livesey (1960) assumed the profile was parabolic and found that the pressure change had two components, one due to viscous effects which varied logarithmically with radius as in Licht and Fuller (1954), and one due to acceleration which varied inversely with the square of the radius. With minor alteration Moller (1963) obtained similar results that differed only in numerical coefficients. Moller also showed that similarity solutions valid for all radii could not be obtained for this geometry. Woolard (1957) developed an integral boundary layer momentum equation for the case of diverging flow. An invariant velocity profile, suitable for laminar flow, was used. Even though he integrated the boundary layer momentum equation numerically, he chose to neglect the non-linear terms in the equation. To avoid singularities in the boundary layer integral equation for an inlet condition of uniform velocity, he assumed that at the inlet a small boundary layer thickness pre-existed.

Working at the third level Andreeva (1960) and Savage (1964) were the first authors to show that under acceleration the velocity profile was not invariant, and that the results discussed previously must be increasingly in error as the radius decreases (acceleration increases). Savage (1964) assumed that for laminar flow the velocity profile could be represented as a parabolic profile plus the contributions of other profiles, whose relative contributions depended upon the local radius. These profiles were determined from the stream function forms of the

Navier-Stokes equations. The basic parabolic profile was considered as first order theory and the sum of this profile and the first of the radius-dependent profiles was considered second order theory. Savage carried the concept out only to second order, so that it was not valid for large ratios of acceleration to viscous effects. It is possible, but tedious, to derive higher order approximations but the method is inherently limited to low accelerations. Despite this limitation it was shown that, for diverging flows, deceleration results in a Gaussian velocity profile which could eventually yield separation, i.e., the condition that the velocity gradient at the wall was reduced to zero. For converging flows it could be shown that acceleration yields blunter, more uniform profiles. Despite this more sophisticated analysis, entailing varying velocity profiles, Savage (1964) and other authors (Jackson and Symmons, 1965a) who used these level 3 methods derived radial pressure distributions that varied only in their numerical coefficients from the distributions derived with the level 2 methods. Boyack and Rice (1970) have proposed a higher order approximation based upon the boundary layer momentum integral in which the velocity profile at any radius is represented with four terms, i.e., two more than Savage considered. They found that Savage's solution began to differ from theirs when the ratio of acceleration to viscous forces approached one. Since they did consider more terms in the approximation their analysis extended the range of validity to somewhat higher accelerations, but as noted above even this more refined analysis would eventually become invalid at higher accelerations. Higgins (1975) used asymptotic expansion

techniques to identify three regions for radial outflow, which he labeled as the upstream, downstream and outer regions.

Series expansion methods for laminar flow have also been proposed by Ishizawa (1965) and Wilson (1972), but these were focused upon the entry regions of diverging flows. Both authors indicated that the effects of the conditions at the entry decayed exponentially with downstream distance so that eventually the flow field would become independent of inlet conditions. Finally, experimental data for flows in which the ratio of acceleration to viscous forces was significant (approaching one) were reported by Moller (1963), Morgan and Saunders (1960) and Jackson and Symmons (1965b), but these were all diverging flows and separation effects had been evident in some cases.

In summary, the laminar theories presented in the references cited above are either approximate ones, valid only when the effects of acceleration are small, or else concern themselves with the entry region only. A solution for the situation when both acceleration and viscous effects are important has not been presented even though the problem, particularly its geometry, appears to be of fundamental simplicity and various aspects have been under investigation for over thirty years.

Turbulent Flows

"Simple" Turbulent Flows. When turbulent flows are considered the situation becomes further complicated by the additional transport mechanism afforded by the turbulent motion of the fluid. The fluctuating components of the velocity field result in additional mixing and an increase in the effective transport properties (viscosity and thermal

conductivity) of the fluid. These velocity fluctuations also give rise to additional turbulent stresses, often called Reynolds stresses.

Unfortunately, turbulent transport is not readily predictable, even for flows in which accelerations are absent. Historically, the prediction of these flows has been approached by considering a simplified model of the transport process. A well known and useful example is the Prandtl mixing length model (Schlichting, 1968, pp. 545-549). Here the turbulent component of the kinematic viscosity is taken to be a function of the distance from the wall and the local velocity gradient. Application of this model to simple flows results in the well known "logarithmic law of the wall," so that near the wall (but not within the viscous layer) the local velocity is proportional to the logarithm of the distance from the wall. Space does not permit a general discussion of turbulence and its effects (the unfamiliar reader can refer to the texts authored or edited by Hinze, 1976; Frost and Moulden, 1977; Bradshaw, 1976; and Tennekes and Lumley, 1972); instead the focus here is upon the modification of the effects of turbulence caused by the radial acceleration of the fluid.

Acceleration Effects. Acceleration effects have been studied experimentally for other non-radial flow geometries. It is known that the favorable (i.e., a pressure decrease in the downstream direction) pressure gradients that occur in an accelerating flow have a stabilizing effect on the transition from laminar to turbulent flow (Rosenhead, 1963, pp. 540-544 and Schlichting, 1968, pp. 467-473) whereas adverse pressure gradients, decelerating flows, have a destabilizing effect.

A number of authors (Sternberg, 1954 and Sergienko and Gretsov, 1959) have, in fact, found that an initially turbulent flow can be made to revert towards a laminar state by acceleration. This process has been variously called relaminarization or reverse transition. Launder (1964) coined the term "laminarization" to describe those flows affected by acceleration in this manner but in which the reversion to the laminar state need not be complete. Other authors have suggested that such flows could be described as "laminarescent" (Simpson and Wallace, 1975).

Several authors have proposed various nondimensional parameters involving either velocity, pressure or shear stress gradients which purportedly serve as a guide to the possibility of laminarization. Based upon convective heat transfer measurements Moretti and Kays (1965) found that when the parameter $K_1 = (\bar{v}/\bar{U}_\infty^2)(d\bar{U}_\infty/d\bar{r})$, was equal to or greater than 3.5×10^{-6} the production of turbulence was apparently inhibited, so that after the residual turbulence decayed the boundary layer became completely laminar. In supporting research Kline et al. (1967) noticed that the turbulent bursts, i.e., the outward eruption of fluid away from the wall near the edge of the viscous layer, ceased when K_1 reached the critical value 3.7×10^{-6} . Jones and Launder (1972) found that the peak Reynolds stress due to turbulence became a progressively smaller fraction of the wall shear stress as K_1 increased. They indicated that a turbulent flow could no longer exist when K_1 exceeds a critical value somewhere between 2.5×10^{-6} and 3.5×10^{-6} . Blackwelder and Kovasznay (1972) found that when K_1 exceeded the critical values above, the viscous sublayer thickness increased, the skin friction coefficient at the wall decreased and large departures from the turbulent logarithmic law of the wall occurred.

Patel (1965) also noted major departures from the log law, but preferred to correlate his results in terms of the pressure gradient parameter $K_2 = (\bar{v}/\bar{\rho}\bar{U}_\tau^3) (d\bar{P}/d\bar{r})$. Patel and Head (1968) suggested that the critical value of K_2 was 0.018. They also suggested that a more appropriate laminarization parameter might be defined by replacing the pressure gradient with the shear stress gradient at the wall, $\partial\bar{\tau}/\partial\bar{z}_w$; we shall call it K_3 . For transitional pipe flow they found that with the present nomenclature $K_2 = 2K_3 = 0.018$ whereas for transitional flow between parallel plates they found that $K_2 = K_3 = 0.009$. On the basis of consistency, then, they concluded that K_3 was a better guide to the possibility of laminarization and that its critical value was 0.009. They also suggested that the downstream distance required to complete reverse transition must be dependent upon the momentum boundary layer thickness, $R_2 = \bar{U}_\infty \delta_2 / \bar{v}$.

Narasimha and Sreenivasan (1973) challenged the conclusions of Patel and Head (1968) pointing out that due to algebraic errors the critical value of K_3 should have been 0.004. They concluded that laminarization was due to domination of pressure gradients over the Reynolds stresses and therefore the parameter incorporating the pressure gradient, K_2 , was recommended. In addition they pointed out that parameters such as K_1 through K_3 cannot be universally useful for all levels of acceleration. When the acceleration is very low, as in a near fully developed flow (i.e., one in which the downstream distance in a constant area duct is sufficient that velocity profiles and other flow parameters no longer vary), the flow and its state of turbulence

revert to their equilibrium values and thus the local Reynolds number provides the best guide to the possibility of laminarization.

A number of authors have been careful to point out that there is no theoretical basis for assuming that either K_1 , K_2 , or K_3 is the only parameter of significance. Launder (1964) noted that an additional consequence of acceleration is a thinning of the boundary layer, so that a Reynolds number based upon boundary layer thickness, e.g., R_2 , continuously decreases with acceleration. Thus, Launder pointed out the possibility that laminarization might simply be a low Reynolds number effect, and in fact Kays (1966, p. 93) suggested the critical value, $R_2 = 360$. However, Back, Cuffel and Massier (1970) claimed values as large as 8000 were associated with the reverse transition processes they studied in nozzle flows. Another Reynolds number criterion was proposed by Bradshaw (1969), who postulated that a turbulent flow would become directly dependent upon viscosity, i.e., laminarizing, when the energy-containing and the dissipation ranges of eddy size overlap. This led to a critical "eddy" Reynolds number $R_{\text{eddy}} = \bar{U}_\tau \bar{\ell} / \bar{\nu} = 12$. This "local" criterion has not seen use in the literature, however, since not only must the variation with distance from the wall of the mixing length, $\bar{\ell}$, be known or measured, but also the variation of the local shear velocity, $\bar{U}_\tau(y) = \sqrt{\bar{\tau}(y)/\bar{\rho}}$, must be measured. Therefore it is not too useful for prediction purposes.

Badri Narayanan and Ramjee (1969) attempted to meld all these diverse observations into a unified structure by describing reverse transition in three consecutive states: (1) the disappearance of the turbulent bursts near the wall at $K_1 \approx 3 \times 10^{-6}$, (2) breakdown of the

law of the wall at $K_2 = 0.02$, and (3) the decay of turbulence intensity starting at $R_2 = 300$ to 400. While at first glance this description seems to accommodate all the observations reported above, stage (3) strongly implies that the completion of laminarization is controlled by a critical value of R_2 , whereas Launder and Jones (1969) suggested that K_1 was the critical parameter. In support for the K_1 criterion, Simpson and Wallace (1975) stated that extrapolation of their results from sink flow experiments using inclined plates indicated that a "truly" laminar asymptotic condition would have been likely for $K_1 > 4.1 \times 10^{-6}$.

Laminarization of gases in heated tubes has been studied by McEligot (1963), Bankston (1970), Perkins and McEligot (1975) and others. The acceleration and increase in viscosity caused by heating appears to have some consequences similar to those of isothermal acceleration. In the earlier work, McEligot found that heat transfer coefficients were reduced and approached laminar predictions, and essentially defined the laminarization process in terms of a comparison of wall parameters to predictions. Later, by temperature profile measurements, Perkins and McEligot showed these reductions in wall parameters corresponded to evidence of thicker viscous layers as hypothesized earlier. Using hot wires at the tube center, Bankston found that $\sqrt{(\bar{u}')^2}/\bar{u}$ decayed axially under conditions where laminarization was inferred from the trends of the local Nusselt numbers; he also showed that the laminarization parameters K_1 and K_2 provided good criteria for laminarization in heated gas flows.

In summary, there still remain large uncertainties not only in their precise critical values, but also as to which is the "correct" dimensionless parameter to use as a guide to laminarization. Furthermore

the observations reported above are for flow geometries that differ considerably from the one considered here -- there is no assurance that they can be expected to provide reasonable guides in the prediction of radial sink flows.

Previous Turbulent Radial Flow Studies. To the writer's knowledge no measurements of turbulent, radially converging inflow have been reported in the literature. However, turbulent radial outflow has been investigated by several authors (Hofmann, 1931; Brown and Bradshaw, 1947; Brown, 1947; Paivanas, 1955; Moller, 1963; Bakke, Krieder and Kreith, 1973). Considering however, the earlier remarks upon the dissimilar effects of acceleration and deceleration upon turbulence, the turbulence characteristics of these flows are not likely to have significant bearing upon radial inflows. Several writers (Mohn, 1930; Brown, 1947; Woolard, 1954; Welanetz, 1956; Moller, 1963; and Bakke, Kreider and Kreith, 1973) have also attempted to predict turbulent pressure distributions for radial outflow using the "hydraulic method." As described by Woolard (1957) the hydraulic method treats the flow as one-dimensional and utilizes a modified Bernoulli equation which includes an additional term for friction by means of a friction coefficient. In all cases a friction factor -- Reynolds number correlation developed for fully developed flows without acceleration or deceleration was used; this limitation was appreciated as early as 1957 (Woolard, 1957).

Kreith (1965) has summarized experimental results for the laminarization of radial outflows which were turbulent at the entry to the disks. It was found that as the velocities diminished with increasing radius, the flow laminarized. Laminarization was inferred from hot wire measurements and pressure distribution data. Laminarization occurred at

a radius where laminar flow theory would result in a velocity profile without an inflection point. If laminar flow theory predicted a separation-type profile, the flow was turbulent. Kreith provided a formula for the radius $\bar{r}_1$ at laminarization; in terms of the nomenclature developed in the following chapter, the non-dimensional transition radius was $r_1 = 0.76$ for laminarizing outflows.

In summary neither the measurements nor the prediction techniques developed for radial outflow appear to be appropriate or useful for radial inflow. In fact, the predictive techniques developed appear to be very approximate, even for the case of outflow, and their accuracy is very questionable in the case of large decelerations.

Objectives

This study had as its goals the completion of three major tasks: (1) develop a more exact predictive analysis of laminar flow, valid for either small or large acceleration, (2) devise a turbulent transport model which could be recommended for the prediction of accelerated turbulent flows in radial geometries, and (3) perform a series of experimental studies of accelerated laminar, transitional and turbulent flows which could be used to confirm the laminar analysis, guide the selection of a suitable turbulence model, and determine which, if any, of the laminarization criteria discussed earlier pertains to the radial flow geometry.

Item one was approached via the boundary layer theory. The incorporation of strongly nonlinear acceleration terms necessarily required the development of a numerical method of solution. Item two was

approached by means of modifications to the basic Prandtl mixing length hypothesis. Item three actually required the development of two flow experiments -- one for laminar flow and one for turbulent and transitional flows. The measured experimental results consisted primarily of radial pressure distributions and, for transitional turbulent flows only, wall shear stresses. Because of the extremely thin boundary layers encountered in many of the highly accelerated runs (in some cases δ_1 was less than 250 μm , or 0.001 in.), it was decided that the measurement of velocity profiles was impractical, and these were therefore not attempted. Due to this problem and other experimental limitations, some of the turbulence quantities that are of interest, such as breakdown of the law of the wall and thickening of the viscous sublayer could not be directly measured. However, the use of measured wall shear stresses and pressure variations did allow definite conclusions to be drawn regarding laminarization phenomena in radial flow, and, in conjunction with turbulence modeling, led to inferences regarding the turbulence quantities mentioned above.

The reader is cautioned however that unlike some previous work, in which laminarization was defined or interpreted in terms of cessation of wall bursting, thickening of the linear sublayer and decay of turbulent velocity fluctuations, in the present study laminarization was defined as the reduction of measured pressure gradients and wall shear stresses compared to turbulent predictions. The term complete laminarization is used to indicate agreement with predictions based on a laminar analysis.

CHAPTER II

LAMINAR FLOW ANALYSIS

To reiterate, the physical situation, shown in Figure 2, consists of two stationary, parallel, circular disks separated by a distance $2\bar{z}$ with the origin for the transverse coordinate $\bar{z}$ located at the plane midway between the disks.¹ The radial coordinate, and the transverse and radial components of velocity, are denoted as $\bar{r}$, $\bar{w}$, $\bar{u}$, respectively. The fluid density and kinematic viscosity are $\bar{\rho}$ and $\bar{\nu}$, respectively. The analysis to follow is restricted to primarily radial flow. When a strong transverse component of flow occurs, such as might be expected in the immediate vicinity of the exit pipe sketched in Figure 2, the solution becomes invalid.

For steady, incompressible flow of a laminar viscous fluid with no azimuthal component of velocity (swirl), nor azimuthal variation of pressure or velocity, the governing equations consist of the continuity equation and two equations of motion written in cylindrical coordinates:

$$\frac{1}{\bar{r}} \frac{\partial(\bar{r}\bar{u})}{\partial \bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (2-1a)$$

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} = -\frac{1}{\bar{\rho}} \frac{\partial \bar{P}}{\partial \bar{r}} + \bar{\nu} \left(\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) \quad (2-1b)$$

1. For convenience, we adopt the convention that dimensional variables are identified with an overscore while non-dimensional variables have none.

$$\bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} + \bar{u} \frac{\partial \bar{w}}{\partial \bar{r}} = \frac{-1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial \bar{z}} + \bar{v} \left(\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right) \quad (2-1c)$$

The following initial and boundary conditions are to be satisfied by equations (2-1a, b, c):

$$\bar{u}(\bar{z}, \bar{r}_i) = \bar{u}_i(\bar{z}) ; \bar{w}(\bar{z}, \bar{r}_i) = \bar{w}_i(\bar{z})$$

$$\frac{\partial \bar{u}}{\partial \bar{z}}(0, \bar{r}) = 0 ; \bar{w}(0, \bar{r}) = 0 \quad (2-2)$$

$$\bar{u}(\pm \bar{t}, \bar{r}) = 0 ; \bar{w}(\pm \bar{t}, \bar{r}) = 0$$

The first of these conditions simply provides specified profiles at the entry, $\bar{r}_i$, while the second and third require symmetry about the mid-plane, and the no-slip condition at the impermeable confining surfaces.

Nondimensionalization

In order to transform the governing equations into forms which have no free parameters, we define the following dimensionless variables

$$\left. \begin{aligned} u &= \bar{u}/\bar{V}, w = \bar{w}\sqrt{R_0} / \bar{V} \\ r &= \bar{r}/(\bar{t}\sqrt{R_0}), z = \bar{z}/\bar{t} \\ P &= (\bar{P}_i - \bar{P}) / (\bar{\mu} \bar{Q}/4\pi\bar{t}^3) \end{aligned} \right\} \quad (2-3)$$

where $\bar{V}$ is the bulk average velocity at any radius, $\bar{V} = \bar{Q}/(4\pi\bar{r}\bar{t})$, $\bar{Q}$ is volumetric flow rate, R_0 is an overall Reynolds number, $R_0 = \bar{Q}/(4\pi\bar{v}\bar{t})$,

and $\bar{P}$ and $\bar{P}_i$ are the local and inlet values of the pressure, respectively. The nondimensionalization and order-of-magnitude analysis of the governing equations is described in detail in Appendix A. For converging flow the signs of $\bar{Q}$ and $\bar{V}$ are negative but absolute values are used in evaluating R_0 . It should be noted also that the overall Reynolds number is not a function of radius and, since the nondimensional radius r is scaled by $\sqrt{R_0}$, small values of r can correspond to substantial distances from the center of the disks.

As a consequence of the above definitions, of the assumption that $\sqrt{R_0} \gg 1$, and of the order-of-magnitude analysis described in Appendix A, equations (2-1a, b, c) can be reduced to

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (2-4)$$

and

$$u \frac{\partial u}{\partial r} - \frac{u^2}{r} + w \frac{\partial u}{\partial z} = r^2 \frac{dP}{dr} + r \frac{\partial^2 u}{\partial z^2} \quad (2-5)$$

These equations correspond to the usual boundary layer approximations for internal flows. In the boundary layer approximation P is a function of r only. With three dependent variables (u , w , P) appearing, an additional governing equation is necessary; it is provided by the integral continuity equation,

$$\int_0^1 u \, dz = 1 \quad (2-6)$$

Solutions to equations (2-4) through (2-6) require two boundary conditions and an initial condition. As noted in equation (2-2), the boundary conditions are symmetry at the mid-plane and no-slip at the impermeable wall. The inlet pressure $\bar{P}_i$ can conveniently be specified as zero for reference. These conditions introduce no free parameters. The only remaining requirement is the specification of an entry radius r_i and an entry velocity profile. Thus, for a given entry condition the solution will depend on dimensionless r and z only.

The order-of-magnitude analysis reveals additional features of the flow. The acceleration or convective terms become dominant for small radii, $r \ll 1$. However, as suggested by Prandtl (1904), near the wall a thin boundary layer must remain where the viscous term is of equal importance. The displacement thickness of this dimensionless boundary layer ($\delta_1 = \bar{\delta}_1/\bar{t}$) is of the same order as the radius, that is,

$$\delta_1 = O(r) \text{ for } r \ll 1 \quad . \quad (2-7)$$

Since $\delta \rightarrow 0$ as $r \rightarrow 0$, the boundary layer approximation is valid even as $r \rightarrow 0$. In contrast, at larger radii the importance of the acceleration terms relative to the viscous term declines as $1/r^2$ and the viscous term is balanced by the pressure gradient term only as $r \rightarrow \infty$.

Limiting Solutions

Large Radii. In the limit as $r \rightarrow \infty$, the acceleration terms of equation (2-5) can be neglected and by integration of equations (2-5) and (2-6) it is found that

$$u = \frac{3}{2} (1 - z^2) \quad . \quad (2-8)$$

Between any two dimensionless radii the dimensionless pressure difference becomes

$$P_2 - P_1 = -3 \ln(r_1/r_2) \quad . \quad (2-9)$$

The local friction factor, f , is defined in terms of the local bulk velocity,

$$f = \frac{2\bar{v}}{\bar{v}^2} \frac{\partial \bar{u}}{\partial \bar{z}} \bigg|_{\bar{z}=\bar{t}} \quad .$$

Its asymptotic value as $r \rightarrow \infty$, denoted f_∞ , can be shown to be

$$f_\infty = 24/R \quad (2-10)$$

where R is the local Reynolds number, also defined in terms of the local bulk velocity and hydraulic diameter, $R = 4 \bar{V} \bar{t}/\bar{v}$.

These expressions for the velocity profile and friction factor are identical to those derived for fully developed flow between infinitely wide flat plates and are equivalent to the creeping flow solution (Licht and Fuller, 1954).

Small Radii. In the other limit, $r \rightarrow 0$, the pressure distribution approaches

$$P_2 - P_1 = \frac{1}{2} \left[\frac{1}{r_2^2} - \frac{1}{r_1^2} \right] \quad (2-11)$$

which is the dimensionless Bernoulli equation. Since the ratio of viscous to acceleration terms outside the boundary layers is $O(r^2)$ as $r \rightarrow 0$, the component of pressure change due to viscous effects is small

compared to that due to acceleration effects. Nevertheless, this component is important since it represents the nonrecoverable portion of the pressure drop required to accelerate the fluid.

Since $\delta_1 \rightarrow 0$ as $r \rightarrow 0$ the flow eventually consists of a potential core with velocity $\bar{U}_\infty \approx \bar{Q}/4\pi\bar{r}\bar{t}$, flanked at both walls with thin boundary layers. Following Schlichting (1968, p. 152), one may obtain a similarity solution for these boundary layers as follows. Letting the wall distance parameter be $y = \bar{y}\sqrt{\bar{U}_\infty/\bar{r}}\bar{v}$, where $\bar{y} = \bar{t} - \bar{z}$, and using the stream function $\bar{\psi} = -\sqrt{\bar{v}\bar{Q}/4\pi\bar{t}}\psi$, one finds that the boundary layer equation is satisfied by

$$\psi'''' - \psi\psi'' - (\psi')^2 + 1 = 0 \quad . \quad (2-12).$$

Solutions to this equation must satisfy the boundary conditions

$$\psi(0) = 0; \psi'(0) = 0; \psi'(y \rightarrow \infty) = 1 \quad .$$

Solutions were found numerically by Burggraf, Stewartson, and Belcher (1971). At the wall the principal result was

$$\psi''(0) = f\sqrt{R_0/2} = 1.0866 \quad . \quad (2-13)$$

In summary, at large dimensionless radii, viscous effects dominate, the boundary layer thickness is equal to the half spacing, the velocity profile is parabolic and the pressure varies logarithmically with radius. As the fluid flows inward, acceleration effects become important, and when $r \approx 1$, acceleration and viscous effects are roughly equal. With decreasing r the velocity profile diverges from its

parabolic form, and a thinner boundary layer starts to develop. This boundary layer shrinks as r decreases and the velocity profile becomes increasingly more blunt. The friction factor approaches a constant, with magnitude $2.173/\sqrt{R_0}$, which can be many times greater than the local asymptotic value, $24/R$.

Numerical Solutions

For conditions where both viscous and acceleration effects are significant, the governing equations (2-4) through (2-6) were solved numerically. Equation (2-5) is parabolic so an iterative marching technique (Bankston and McEligot, 1970) developed for internal boundary layer flows was applied. A finite control volume analysis is employed to generate a set of implicit, finite difference equations. These algebraic equations are solved successively and are then iterated to handle the nonlinearities in the convective terms in the momentum equation. The momentum equation and the integral continuity equation are solved simultaneously for $u(z)$ and P and then the continuity equation is solved from the wall to the center-plane to obtain $w(z)$.

The solution accuracy was tested by successively refining the mesh to determine converged results for test cases. To conserve computer time, mesh parameters were then chosen to yield wall friction results within about one percent of converged values. Typically, fifty nodes in the z -direction were sufficient. However, at small r the boundary layer thickness becomes quite small so 100 nodes were used and the nodes were distributed according to a power law in z so that the node spacing rapidly diminished as the wall was approached. Even in

the most extreme case, $r = 10^{-3}$, there were at least 6 nodes within the boundary layer. In the radial direction the nodes were initially spaced by a dimensionless radius increment of 10^{-5} . This increment was allowed to increase by 10 percent for each subsequent radial step until the radial spacing became equal to 0.01 times the dimensionless radius. Thereafter it was maintained 0.01 r . Further details are contained in Appendix B.

CHAPTER III

CALCULATED LAMINAR FLOW RESULTS

Results are presented for five entry radii ranging from 0.1 to 10. Except for special occasions discussed in connection with Figure 4, a uniform entry condition was assumed, i.e., $w = 0$ and $u = 1$ at $r = r_i$. This inlet condition was assumed in order to widen the applicability of these results to other situations. It will be shown that for large r_i the initially uniform velocity profiles first develop into fully developed, parabolic profiles, as would be the case for the geothermal reservoir at large radius. Thus the initial conditions for the geothermal reservoir are contained as a sub-set to those assumed here.

Velocity Profiles

Velocity distributions are presented in Figures 3a and 3b for the cases typifying entry first at a relatively large dimensionless radius and then at a small radius, respectively. For large entry radii viscous effects are expected to dominate after the entry effects decay. For the case where the dimensionless entry radius is 10, the velocity profile evolves from its uniform form at the entry to the parabolic shape of equation (2-8) within a distance $\Delta r < 0.02$. Starting near $r = 1$, acceleration effects become important and the profile departs

Figure 3. Velocity Profiles. (a) Entry at $r_i = 10$. (b) Entry at $r_i = 0.3$.

from its parabolic form, tending, eventually, back to a uniform profile. At $r = 0.01$, the profile is so flat that the center line velocity differs from the bulk velocity by only one percent.

For large entry radii, the entry length required for the evolution of a parabolic profile is so small compared to r_i that changes in radius can be ignored. Thus the entry flow region in this situation does not differ substantially from the entry behavior for flow between two parallel plates with unchanging cross section. This problem has been treated by Schade and McEligot (1971) and others, who concluded that the velocity profile and the friction factor approach their fully developed values when the dimensionless downstream distance is approximately $x^+ = \frac{\bar{x}/\bar{t}}{R} = 0.05$. From their definitions, the dimensionless downstream radius, Δr , in the present work is related to x^+ as $\Delta r = 4x^+/r$, so we expect entry effects to decay when $\Delta r \cong 0.2/r_i$. The flow development is quantized in terms of the centerline velocity variation in Figure 4. In addition to uniform inlet profiles (solid curves) Figure 4 also presents some results for parabolic inlet profiles (dashed curves). Flow is from right to left in this diagram (and the succeeding ones); a value of 1.5 corresponds to the parabolic profile and 1.0 represents uniform flow provided no minima occur in the profile. Examining the uniform inlet curves for $r_i = 3$ and 10, one sees the entry behavior indeed diminishes within $\Delta r = 0.2/r$.

For radii less than about 1 the parabolic profile is not attained with uniform inlet profiles. In the immediate entry the boundary layer grows and the central region accelerates as a potential core until acceleration dominates at small r , as suggested by the order-of-magnitude

Figure 4. Dimensionless Mid-Plane Velocity.

analysis. The boundary layer thickness then decreases and the profile again approaches a uniform shape. Figures 3b and 4 show that beyond the immediate entry the velocity distributions approach a common state $u(z,r)$. While this observation is not surprising for large inlet radii, it is interesting for the flows which do not reach the parabolic profile first. As one might expect, equations (2-4) through (2-6) yield a single solution independent of the entry condition provided the distance from the entry is sufficient. With uniform flow hypothesized for the entry, that distance is reduced at small r since the entry profile is a fair approximation of the solution to which it adjusts. If the entry nozzle geometry in an experiment or in a physical application yields a non-uniform profile -- say closer to parabolic -- at small inlet radii, Figure 4 shows that the entry development length becomes greater than that required for uniform inlet profiles.

Pressure Distributions

Figure 5 summarizes the pressure variation predicted by the present numerical solution in comparison to results for the two limiting cases: inviscid flow and purely viscous flow. At large radii the pressure varies logarithmically with radius as predicted when acceleration may be neglected [equation (2-9)]; this observation is more evident ($r > 1$) in the following Figure 6 with semilogarithmic coordinates. At very small r , the pressure approaches the limit predicted by the inviscid Bernoulli equation (2-11). Between these two limits, viscosity effects and acceleration effects are both important and they interact. The pressure drop is greater than the value predicted by either one of

Figure 5. Dimensionless Pressure Variation.

Figure 6. Comparison of Pressure Variations.

the limiting cases alone, as would be expected. The assumption of a uniform velocity profile at the entry promotes the interaction of viscous and acceleration effects at intermediate values of r . For example, if one examines the results for $r_i = 0.1$, which is small enough so that acceleration effects should dominate, one finds that when $r = 0.05$ the purely viscous and purely inviscid approximations, equations (2-9) and (2-11), predict dimensionless pressure drops of 2 and 150 respectively, while the numerical result is 191, some 26 percent greater than the sum of the first two.

Discussion of Pressure Results

Since the purely viscous equation (2-9) approximates the exact solution well at very large radii while inviscid equation (2-11) works well at very small radii, it may be suggested that a first order approximation for intermediate radii could be obtained by simply summing the two. This result, with minor modification, has been recommended by many other investigators. Livesey (1960) assumed that the velocity profile was parabolic at all radii and, using an integral method, concluded that

$$P = 3 \ln(r_i/r) + 0.60 \left(\frac{1}{r^2} - \frac{1}{r_i^2} \right) . \quad (3-1)$$

The increase in the coefficient of the acceleration term from 0.5 to 0.6 reflects the fact that, for the same average velocity, a parabolic velocity profile possesses 20 percent more momentum than a uniform profile. With a comparable approach Moller (1963) obtained equation (3-1),

but with a coefficient of 0.72 rather than 0.60. Using higher order approximations based upon power series expansions, other authors (Savage, 1964; Jackson and Symmons, 1965a) also obtained equation (3-1), but with a coefficient of 0.771. Both these approximations neglect terms of order $1/r^4$, so the validity is questionable for $r < 1$. Since the exact solution must approach the Bernoulli equation (2-11) in the limit $r \rightarrow 0$, in which the coefficient is exactly 0.5, the approximations above will obviously be in error, at very small r , by 20, 44, and 54 percent, respectively.

These approximate predictions are compared to the present numerical results for $r_i = 10$ in Figure 6. Since Moller's (1963) approximation differs only slightly from that of Savage (1964) or Jackson and Symmons (1965a) it was excluded for clarity. It is clear that they agree reasonably well for radii greater than 0.5. Surprisingly, the simpler approximation of Livesey (1960) agrees best, and actually provides reasonable agreement for radii as low as 0.15, even though it is based upon a parabolic velocity profile -- an assumption that Figure 3 shows to be increasingly incorrect at values of r lower than 0.5. As remarked by Wilson (1972), commenting on the comparison of Savage's theory (1964) and Moller's data (1963) for outward flow at small r , the agreement at values of r as low as 0.5 must be regarded as fortuitous. At small r the actual velocity profile is more uniform than that predicted in these approximations. Consequently, the shear stress and viscous contribution to the pressure gradient are greater. On the other hand, since a uniform velocity profile possesses less momentum than parabolic or near-parabolic profiles, the momentum change and

corresponding pressure gradient due to acceleration are less. These two errors tend to offset each other, so that approximations of the form of equation (3-1) predict the total pressure change reasonably well but are inadequate to differentiate between the amount of the pressure change caused by either viscous or acceleration effects individually. While perhaps of minor concern when only the pressure distribution is required, these effects must be separated when considering phenomena which are sensitive to the wall shear stress, such as erosion, convective heat transfer and convective mass transfer. Furthermore, the pressure drop due to acceleration may be recoverable while the viscous pressure drop is irreversibly lost.

Friction Factor

Figure 7 illustrates the friction factor variation for various entry radii. Due to the uniform entry velocity profile assumed, the friction factor is high in the entry region ($f \sim 1/\sqrt{x^+}$) but tends to merge with the results obtained at larger entry radii within a short downstream distance. Excluding the entry region it is seen that at large r , the numerical results agree with equation (2-10), the limiting value for purely viscous flow. At small r ($r \leq 0.05$) the numerical results approach the limit

$$\frac{f \cdot R}{24} = \frac{0.36}{r} \quad (3-2)$$

Equivalently, it can be shown that this result corresponds to a constant friction factor $f = 2.17/\sqrt{R_0}$, in excellent agreement with the result obtained from the similarity solution, equation (2-13). Although this

Figure 7. Local Friction Factor.

apparent limiting behavior for the friction factor occurs only at small r , it should be recalled that small physical radii are not necessarily implied since r is defined by $(\bar{r}/\bar{t}) = r\sqrt{R_0}$ and $\sqrt{R_0}$ can be large.

Discussion of Friction Factor Results

When the velocity profiles obtained by Savage (1964) and Jackson and Symmons (1965a) are differentiated it can be shown that these approximate theories predict

$$\frac{f \cdot R}{24} = 1 + \frac{1}{35r^2} \quad (3-3)$$

These theories contain no information about the entry region so equation (3-3) must be compared (Figure 7) to numerical results which were obtained for entry radii sufficiently larger than the range of interest. It is found that equation (3-3) is in error by 6 percent at $r = 1$ and by 25 percent at $r = 0.3$. If the relation is extended to even smaller radii it is found that the error actually diminishes to zero near $r = 0.1$ but then begins to increase by an order of magnitude at $r = 0.01$. Livesey's integral analysis (1960) is based on a parabolic profile which is assumed invariant. Consequently, it requires that $f \cdot R/24 = 1$ at all radii which is obviously incorrect.

An expression combining the limiting values of friction factor,

$$\frac{f \cdot R}{24} = \sqrt{1 + \left(\frac{0.36}{r}\right)^2} \quad (3-4)$$

approximates the numerical results reasonably well for all values of radius. This expression is also shown in Figure 7. The maximum error is 8 percent occurring at a radius of 0.3.

Summary Results

Figures 4 and 7 have shown that regardless of the entry radius, the velocity profiles and friction factors downstream tend to approach the same profiles and friction factors obtained with larger entry radii. Thus, there appears to be a unique downstream solution $u(r,z)$ which is approached asymptotically away from the entry. Values of the center-plane velocity, friction factors, the ratios of both the dimensionless displacement and momentum boundary layer thickness to the local dimensionless radius, δ_1/r and δ_2/r , and the shape factor, $H = \delta_1/\delta_2$, for these circumstances are tabulated in Table I. At very large r the dimensionless displacement and momentum boundary layer thicknesses are constants, 0.333 and 0.133 respectively, in exact agreement (to three significant figures) with the boundary layer thicknesses corresponding to parabolic velocity profiles. The friction factor is $24/R$ as expected. At small r , both thicknesses vary with radius, and for radii less than 0.1 the displacement layer thickness is closely approximated as $\delta_1 = 1.1 r$, in good agreement with the order-of-magnitude estimate, equation (2-7). As $r \rightarrow 0$ we again note the excellent agreement between the similarity friction factor, equation (2-13), and the numerical results. In fact it is evident that only at radii as small as 10^{-3} , where the boundary layer is so thin that only 6 nodes were incorporated within it, does some numerical inaccuracy begin to appear. The shape factor evolves from 2.5 at large radii to 1.77 at small radii. Rosenthal (1963, p. 543) has summarized results of hydrodynamic stability calculations which provide estimates of the lowest critical Reynolds numbers, below which laminar flows are stable to disturbances of all

TABLE I

VALUES OF INTEGRAL PARAMETERS FOR THE CASE $r_i \rightarrow \infty$

r	$u(o,r)$	$f\sqrt{R_o}$	$f \cdot R/24$	δ_1/r	δ_2/r	H
10	1.50	60.0	1.00	0.033	0.013	2.50
5	1.50	30.0	1.00	0.067	0.027	2.50
2	1.49	12.3	1.03	0.164	0.066	2.49
1	1.46	6.60	1.10	0.314	0.129	2.43
0.5	1.38	4.00	1.34	0.55	0.24	2.29
0.2	1.22	2.73	2.27	0.90	0.44	2.03
0.1	1.12	2.44	4.13	1.07	0.56	1.89
0.05	1.06	2.38	7.93	1.11	0.64	1.75
0.02	1.02	2.23	18.6	1.13	0.66	1.71
0.01	1.01	2.19	36.7	1.12	0.65	1.72
0.005	1.01	2.17	72.6	1.09	0.62	1.75
0.002	1.00	2.17	181	1.08	0.61	1.77
0.001	1.00	2.16	360	1.08	0.61	1.77

wavelengths. Shape factors lower than two were not considered, but a decrease in H from 2.5 to 2.0 resulted in an increase in minimum critical Reynolds number from 1000 to 40,000, where the Reynolds numbers are based upon $\bar{\delta}_1$. These instability calculations suggest that rapid accelerations, resulting in small values of H , may be unusually resistant to laminar-to-turbulent transition. Further discussion will be deferred to Chapters V and VIII.

CHAPTER IV

LAMINAR FLOW EXPERIMENTS

It had been hoped that both laminar and turbulent radial flow experiments could be conducted with a single experimental apparatus. It soon became apparent, though, that radial acceleration led to stable laminar flows even at Reynolds numbers substantially higher than the transition values conventionally accepted for zero pressure gradients, and turbulent flows could not be obtained with the first apparatus constructed. Turbulent and transitional flows were eventually studied in a second flow apparatus constructed specifically for this purpose. These studies are reported in Chapters VII and VIII. In both experimental apparatus air was used as the modeling fluid. This choice simplified piping and pumping problems and, of course, when the results are non-dimensionalized the choice of fluid is immaterial, so long as it is a Newtonian, incompressible fluid with constant properties. All experiments were conducted at room temperature, approximately 25°C, and the pressure drop in the test section was limited to 5 mm of mercury (0.1 psi). The inlet pressure was quite close to atmospheric pressure, which was approximately 620 mm Hg (12 psia) since most of these experiments took place at the Los Alamos Scientific Laboratory where the elevation is 2.1 km (7000 ft). Thus the maximum pressure drop in the test section represents only 0.8 percent of the inlet pressure and, by the

ideal gas theory, only a 0.8 percent decrease in density occurred so the air flow can be considered essentially incompressible.

Experimental Apparatus

A schematic diagram of the apparatus is shown in Figure 8. Viewing from left to right, one sees two circular disks that provide the radially converging flow test section, an exit pipe centered on the axis of rotational symmetry, a flow meter, and an additional length of pipe. Not shown on the diagram are 4.6 m (15 ft) of flexible air ducting and associated fittings which connect this apparatus to a reversed blower. This blower drew air from the room into the space between the circular disks where it then converged upon the outlet, turned 90 degrees to flow into the outlet pipe, then into the venturi flow meter, more piping, the air ducting, through a "honeycomb" flow straightener not indicated in Figure 8, and finally into the blower, where it was exhausted to the room in which the blower was located. Flow rates were controlled by means of a flow control valve located at the blower outlet. To avoid blower-related vibrations in the test section the blower was actually located in an adjoining room. The 5-m (15-ft) long flexible air ducting effectively prevented vibrations from propagating upstream.

Flow Test Section Description

The circular disks are shown in more detail in Figure 9. They were fabricated from 12-mm (1/2-in.) thick Plexiglas which permitted visualization of the flow when smoke was injected. The surfaces of these disks were smooth, but not absolutely flat; waves of 0.25 mm (0.010 in.)

Figure 8. Laminar Flow Apparatus.

Figure 9. Laminar Flow Test Section.

peak-to-peak flatness variations occurred with a wave length of 50 to 100 mm (2 to 4 in.). The effects of this flatness variation are considered in the uncertainty analysis, Appendix D. Both disks were turned on a lathe to a final outer radius of 0.30 m (11-3/4 in.). This outer radius was fully rounded with a radius of 6 mm (1/4 in.) and polished. Thus, the flow inlet to the disks resembled a converging two-dimensional nozzle, allowing the assumption that the velocity profile was uniform at an effective entry located at a radius of 0.29 m (11-1/2 in.), the location where the rounding was tangent to the inner surface. As shown in Figure 9 three stiffeners were glued to each disk to increase the overall rigidity and to facilitate changes in the disk spacing.

A total of 22 pressure tap holes 0.7 mm (0.027 in.) in diameter were drilled in the left disk. The outermost taps were located at a radius of 0.29 m (11-1/2 in.) to coincide with the edge of the flat surface of the disk. The 0.7-mm holes were drilled 3 mm deep, starting on the inside face of the disk, to avoid burrs in the flow section. The small diameter ensured that the pressure tap geometry would not affect measured pressures (Shaw, 1960). The taps were then completed by drilling larger holes from the opposite side. These larger holes were threaded and Swage-Lok fittings were used to attach rigid plastic tubing (Polypenco) of 3 mm (0.125 in.) outer diameter to each tap. These tubes were connected to the pressure measuring instrument described later. The taps were deliberately laid out along non-radial lines so that if the flow was locally disturbed by one tap, this disturbed flow would not pass over the downstream taps. In the remainder of this dissertation these

lines are referred to as rakes. Three rakes, each consisting of seven taps, were separated in azimuth by approximately 120° , so that radial symmetry (independence of azimuthal angle) could be checked. In these studies only the center tap and two rakes were actually instrumented; the seven taps on the remaining rake were sealed to avoid leaks. The radial spacing between taps was reduced near the center of the disk since velocities and pressure gradients increase there. The measured radial spacings are reported in Appendix C.

In addition to these taps, four additional instrumented taps were drilled on the right disk of Figure 9. Two each, spaced roughly 120° azimuthally, were located at the inner radii of 57 and 70 mm (2.25 and 2.75 in.), directly opposite their counterparts on the left disk. A comparison between the pressures at these opposing taps on the left disk and the right disk provided an indication when conditions were such that significant axial momentum transfer was occurring as the outlet pipe was approached (e.g., too wide a disk spacing).

Remaining Flow Path Description

The outlet pipe was also Plexiglas, with dimensions of 75 mm (3 in.) inner diameter, 6 mm (0.25 in.) wall thickness and 1.5 m (5 ft.) long. The corner radius at the intersection with the disk was 0.5 mm or less. The inner diameter chosen was a compromise; making it smaller would have extended the radial extent of the flow test section between the disks, but would have increased the pressure drop in the pipe. (The possibility of constructing a venturi-shaped exit with a small inlet diameter but a larger outlet nozzle was considered, but rejected as being

too complex and expensive to machine.) Thus, the maximum radial extent of the flow test section inside the disks was 250 mm (10 in.). The useful radial extent was further reduced by several disk spacings because of the flow turning at the outlet. As the flow turned 90° and entered the pipe there was a contraction forming a vena contracta. This contraction was, of course, a function of the disk spacing and flow rate. To avoid the problems of calibrating the venturi flow meter for varying inlet conditions, the pipe was made sufficiently long (20 diameters) so that the flow entering the pipe could expand, reattach to the pipe wall, and eventually evolve to a fully developed turbulent flow profile before entering the venturi. The expansion of the flow following its entry in the pipe resulted in an adverse pressure gradient which led to a high level of turbulence and mixing, as evidenced by smoke visualization studies. This turbulence and enhanced mixing resulted in a relatively rapid development of flow. Reynolds numbers in the pipe were always greater than 5000, so the flow remained turbulent throughout the remainder of its passage down the pipe. Five pressure taps were arranged along the length of the pipe. These pressure data as well as smoke visualization, assisted in determining that the flow was indeed fully developed before entering the venturi. Because the pressure data obtained from the pipe taps were used only for ensuring fully-developed flow at the venturi, and were not otherwise used in this research, these data are not tabulated in Appendix E.

The venturi flow meter was manufactured by Barco, a subdivision of Aeroquip Corporation. It had a nominal pipe diameter of 75 mm (3 in.) and a throat diameter of 37.8 mm (1.49 in.). Pressure taps were located

upstream and at the throat. The pipe at the venturi outlet was actually cut from a larger length that was used for the disk outlet/venturi inlet pipe. The venturi outlet pipe length was 0.6 m (2 ft) providing an undisturbed venturi downstream distance of six diameters. In the downstream end of this pipe, a 100 mm (4 in.) length of paper honeycomb material was inserted to straighten the flow; smoke flow visualization described later, showed that it prevented any swirl components of motion induced by the blower from propagating upstream. The honeycomb walls were hexagonal, 5 mm (0.2 in.) across opposing flats. An additional 2 ft of relatively stiff rubber hose, 63 mm (2.5 in.) in diameter led from the outlet pipe to an aluminum diverging nozzle. This nozzle simply provided a transition from the rubber hose to the flexible air duct of diameter 150 mm (6 in.). Since this flexible hose was relatively long its diameter was increased in order to reduce its pressure loss. For most of the measurements reported, the blower used was a high pressure, Dayton centrifugal blower with 63-mm (2.5-in.) wide, 155-mm (6-in.) long radial blades. This blower had a stated flow capacity of $0.41 \text{ m}^3/\text{s}$ ($14 \text{ ft}^3/\text{s}$) at a pressure difference of 11 mm of mercury (0.22 psi) when operating in the forward mode (positive pressurization) and was operated at 3450 rpm with a Dayton 3-phase, 208 V, 2 hp, ac motor. For some of the earlier experiments a lower capacity Westinghouse blower was used. This blower was operated with both a constant speed, 1/2 hp, 3450 rpm ac motor and a variable speed (1800 rpm maximum) 1 hp dc motor. The horsepower ratings for the fan and motor are nominal ratings. The pressure-flow rate characteristics of these various blower and motor combinations in the reversed mode are shown in Figure 10. Also shown is the variation

Figure 10. Pressure-Flow Rate Characteristics of Various Blower/Motor Combinations and System Pressure Drops.

in the pressure drop of the entire experimental apparatus with flow rate for various values of $\bar{t}$ with the blower outlet control valve wide open. For the small $\bar{t}$, 1.59 mm, the pressure drop across the passage between the disks tends to dominate; whereas, at large $\bar{t}$, the pressure drop associated with the remaining flow path dominated. Thus, even for very large values of $\bar{t}$, say 25 mm, where the pressure drop across the disks from inlet to outlet was only 0.038 mm of Hg, the maximum flow rate attainable with this apparatus was $0.1 \text{ m}^3/\text{s}$ ($3.8 \text{ ft}^3/\text{s}$). As will be seen, this flow rate limitation led to restrictions on the maximum value of R_0 and prevented the attainment of turbulent flows.

Measurements and Uncertainties

Pressure measurements were made with an MKS Baratron capacitance manometer system. This system consisted of two pressure heads and an electronics unit with digital output. One pressure head (145BH-1) ranged from 0 to 1 mm Hg (0.02 psi) whereas the other head (145BH-100) ranged from 0 to 100 mm Hg (2 psi). Both heads were calibrated at the manufacturer's facilities at 10 pressures each, evenly spaced over each head's range. These data are reported in Appendix C. The maximum discrepancy was 0.06 percent, based not upon the full scale reading but on the actual pressure being measured. In actuality, the full benefit of the excellent accuracy of this instrument was not achieved because the measurements were performed in a machine shop building in which changing drafts of air were usually present. These drafts led to slight but perceptible fluctuations of the pressure reading amounting to $\pm 0.001 \text{ mm Hg}$. The effects of this variation as well as other uncertainties are treated in Appendix D.

All pressure measurements in the flow section were performed in a differential mode to improve accuracy and precision. Since the velocity at the outermost tap was not exactly zero, the pressure there was not identical to the ambient air pressure. Because the entry location in the numerical calculation was taken as the position of the outermost taps, coinciding with the end of the convergence formed by the rounding of the outer periphery, to be strictly correct the density and viscosity values should be specified for the absolute pressure corresponding to the outermost tap. This pressure was obtained by measuring the difference between the ambient air and the outermost tap pressures and subtracting this difference from the absolute ambient air pressure, i.e., the local atmospheric air pressure. Eventually it was found that the difference between the ambient air and outermost pressures was so insignificantly small that the properties could be evaluated at either pressure with very little difference, less than 0.01 percent, in the computed results. For this reason measurement of this particular pressure difference was discontinued for the turbulent and transitional experiments described in Chapter VIII, and is not reported in Appendix E. The absolute atmospheric pressure was measured in another building 100 m away, at the same elevation, with a mercury barometer manufactured by the Precision Thermometer and Instrument Company. The resolution of this instrument is 0.1 mm Hg.

The inlet air temperature, necessary for determining air viscosity and density, was measured with a Thermo-Schneider 170-mm long Mercury thermometer graduated in 0.2°C increments from 0 to 100°C . Accuracy is estimated to be at least within 0.5°C .

Flow measurements were obtained with the venturi. Pressures were measured in the MKS Baratron and the flow rates were computed from the calibration equation,

$$\bar{Q} = C_D \pi \bar{D}_2^2 \sqrt{\frac{\Delta \bar{P}}{8 \bar{\rho}} \left[1 - \left(\frac{\bar{D}_2}{\bar{D}_1} \right)^4 \right]} \quad (4-1)$$

where $\bar{D}_1$ and $\bar{D}_2$ are the upstream pipe diameter and the throat diameter, respectively, $\Delta \bar{P}$ is the pressure difference between $\bar{D}_1$ and $\bar{D}_2$, and $\bar{\rho}$ is the mean density, which was evaluated as simply the arithmetic average of $\bar{\rho}_1$ and $\bar{\rho}_2$. The discharge coefficient C_D was evaluated by calibrating the venturi, first with a bell prover at the flow calibration facility of the Sandia Laboratories in Albuquerque, New Mexico, and then with a Merriam laminar flow element at The University of Arizona. (A bell prover is a type of volumetric displacement meter in the shape of a bell-jar, in which air flow displaces oil and the subsequent rate of volume change is related to the flow rate.) Details can be found in Appendix C.

For the relatively small variations from standard conditions encountered during these studies the air density was calculated from an ideal gas equation of state

$$\bar{\rho}(\bar{P}, \bar{T}) = \bar{\rho}_{\text{ref}} \frac{\bar{P}}{760} \frac{300}{\bar{T}} \quad (4-2)$$

where $\bar{\rho}_{\text{ref}}$ is 1.177 kg/m^3 ($0.00228 \text{ slugs/ft}^3$), the density at 760 mm Hg and 27°C (Hilsenrath et al., 1955); $\bar{P}$ is expressed in mm of mercury; and $\bar{T}$ is expressed in degrees Kelvin.

Estimated uncertainties in individual measurements are listed in Table II. These uncertainties and the uncertainties they induce in the experimental results were generally estimated in accordance with the procedures of Kline and McClintock (1953), and are discussed in detail in Appendix D. Briefly however, it is repeated here that tabulated uncertainties are equivalent to two standard deviations, so that a measurement has a 95 percent probability of lying within the uncertainty interval. Efforts to reduce one of the most important uncertainties, that of the disk spacing, are discussed in the next section but the final spacing uncertainties are reported here in Table II.

The examination of estimated experimental uncertainties in pressure differences is a convenient and useful method of assessing how individual measurement uncertainties will affect results (Kline and McClintock, 1953). The major goal of the research was to compare experimental and predicted pressure differences, so each individual measurement uncertainty was analyzed to determine its effects on this comparison.

The numerical predictions are based on measured quantities, such as flow rate, temperature, etc. Therefore the uncertainties in these quantities caused associated uncertainties in the loci of the predicted curves. Uncertainties in the comparison result because of uncertainties in the measured pressure differences and in the predictions. The former are referred to as direct measurement uncertainties and the latter are referred to as associated uncertainties. Associated uncertainties in predictions are caused by uncertainties in the run parameters such as flow rates, temperature, average disk spacing, surface waviness, and

TABLE II

UNCERTAINTIES IN PRESSURE DIFFERENCES DUE TO
MEASUREMENT UNCERTAINTIES, LAMINAR EXPERIMENTS

DIRECT UNCERTAINTIES

<u>Parameter</u>	<u>Measurement Uncertainty (+ or -)</u>
Pressure	0.001 mm Hg

ASSOCIATED UNCERTAINTIES

<u>Parameter</u>	<u>Measurement Uncertainty (+ or -)</u>	<u>Associated Uncertainty in Pressure Differences</u>	
		<u>r >> 1</u>	<u>r << 1</u>
Temperature	0.5°C	0.05% ^a	0.02%
Flow Rate	2.5%	2.5%	5%
Pressure tap radii ^b	0.07%	0.04%	0.14%
Average disk spacing	$\delta \bar{t} = 13 \mu\text{m}$ (0.0005 in.)	300 $\delta \bar{t} / \bar{t} \%$	200 $\delta \bar{t} / \bar{t} \%$
Surface waviness	$\bar{a} = 0.18 \text{ mm}$ (0.007 in.)	50 $\bar{a} / \bar{t} \%$	50 $\bar{a} / \bar{t} \%$

^aAll uncertainties expressed as a percentage pertain to the actual value measured, or the reported pressure difference uncertainty, not, for example to a full scale reading.

^bThe uncertainty in pressure tap location actually varies from tap to tap. Shown here is the worst case.

pressure tap locations. To avoid the complications of comparing a measured pressure drop and its uncertainty to a series of curves representing the associated uncertainties in predicted pressure drop, both types of uncertainty were combined as an overall uncertainty. For simplicity the associated uncertainties were evaluated at the extremes where the dimensionless radius was either much greater or much less than one, so that a simple expression could be obtained for expressing the associated uncertainty in the predicted pressure drop due to an uncertainty in, say, flow rate or disk spacing. For example, for $r \gg 1$, the pressure difference at any radius is linearly related to flow rate; according to the analysis if there is a 2.5 percent uncertainty in the flow rate measurement, then one can expect the associated uncertainty in predicted pressure differences to be approximately 2.5 percent or less.

Appendix D describes the calculation of both measurement and associated uncertainties in pressure differences for all significant sources of uncertainties in detail. These uncertainties are summarized in Table II. For intermediate cases, where the associated uncertainty would fall between the extremes $r \gg 1$ or $r \ll 1$, the maximum associated uncertainty was used. It was found that the most important uncertainties were those of flow rate, surface waviness, and the actual measurement of pressure differences.

Due to the excellent accuracy of the Baratron pressure measuring system, the pressure measurement uncertainty was primarily due to the presence of stray air currents in the shop building where these experiments were conducted. These stray air currents were most significant at

wide disk spacings where pressure differences were small. Under these conditions these currents affected every tap nearly equally. In theory the uncertainty caused by surface waviness could be assessed for every individual pressure tap if the precise geometry, i.e., waviness amplitude as a function of radius and azimuth, were known. However, such detailed measurements were not obtained; only the average amplitude and wavelength of the waviness were measured. In Appendix D these data are analyzed statistically and a maximum uncertainty is assigned for each run. This maximum uncertainty was not a function of tap location. Since other uncertainties specific to a given tap, such as radial position, were insignificant compared to uncertainties due to flow rate, stray air currents, waviness, and average disk spacing, total uncertainties effectively vary only from run to run, and not significantly with tap location.

Procedure

First the disk spacing, $2\bar{t}$, was set to the desired value. Quite early in the course of these experiments it was found that, despite the three stiffeners attached to the outside of each disk, the disks were warped such that the disk spacing at the center was 0.7 mm (0.03 in.) greater than that at the periphery. It was also found that near the outer periphery azimuthal spacing variations of 0.5 mm (0.02 in.) often occurred between the stiffeners. For experiments with half spacings, $\bar{t}$, less than 25 mm (1 in.) this much variance could not be tolerated so metal spacers were distributed near the inlet and outlet of the flow section and the disks were clamped against these spacers. The spacers

were machined from brass square rod stock with dimensions across the square equal to the desired disk spacing. As discussed in Appendix D, the spacers had very uniform thickness and the thickness of each spacer, equivalent to $2\bar{t}$, was measured to within $13\text{ }\mu\text{m}$ (0.0005 in.). These spacers were cut three spacings long and then were ground with one end rounded, the other end tapered, like an airfoil without camber so that the flow disturbance in the downstream laminar flow was minimized. These were then positioned between the disks so that the round end was the leading edge, i.e., upstream. Typically six of these spacers were positioned at the outer radius, at azimuthal positions 60° away from entry taps. This positioning also minimized interference with downstream pressure taps. The flow area blocked by these spacers never exceeded 0.5 percent. In addition to being merely a local effect, the uncertainty in pressure difference was less than 1 percent. An additional three spacers were positioned near the center at the point where the flow turns into the exit pipe. The maximum flow area blockage there amounted to 1.3 percent, but it should be recalled that the flow in this region was already seriously perturbed by the turning of the flow into the outlet pipe.

After setting the spacers and checking the spacing uniformity with feeler gauges, the blower was switched on and the control valve adjusted to yield the desired flow rate. Smoke was then injected with a "Tel Tru" smoke gun (E. V. Hill Inc., San Francisco, California) at several azimuthal positions at the periphery to check visually that the flow was truly radial and laminar. A typical photo is presented as Figure 14 in Chapter V, where it is discussed in more detail. This

smoke gun utilized the reaction between zirconium tetrachloride and water vapor in the air to form a dense gray smoke. Large smoke bursts were then injected to ascertain whether the flow completely filled the exit pipe before entering the venturi.

The complete set of pressure differences were then recorded. As described in Appendix C pressure differences were measured with respect to a reference tap located near the center of the disks. The venturi pressure difference was actually recorded before and after each run to ensure that no significant flow rate changes had occurred during the run. Each run was then terminated by measuring the room air temperature and atmospheric pressure. These data are tabulated in Appendix E.

Data Reduction

Before performing detailed data processing the pressures of the two azimuthally separated rakes of taps were compared at each radius to determine if an azimuthal variation occurred. For all the runs of this study there appears to have been a high degree of azimuthal symmetry -- the pressure difference variations of any two taps at the same radius were typically within 2 percent of the overall pressure drop from the outer radius to the reference tap. In the worst case this variation attained 8 percent, but this case occurred at a very wide disk spacing where the overall pressure drop was so low that the uncertainty in its measurement was 31 percent. In fact, most of the larger variations in pressure difference occurred at the lower flow rates, so that they likely were caused by the stray air currents discussed earlier.

Next the pressure differences at the two small radius taps located at 57 and 70 mm (2.25 and 2.75 in.) were compared for both disks to see if the flow turning was affecting the pressures this far upstream of the exit. When the difference between the pressures for both disks caused by flow turning exceeded 5 percent of the overall pressure drop the data for that particular radius were omitted from further consideration. The overall pressure drop was then calculated by subtracting the pressure difference at the lowest radius tap not affected by flow turning from the pressure difference at the outer entry tap. Following the 5 percent criteria discussed above it was found that when the half spacing was 12.7 mm (0.500 in.) only the taps at 57 mm were affected by flow turning. When the half spacing was 25.5 mm (1.005 in.), the taps at both 57 mm and 70 mm appeared to be affected. However, for this run the overall pressure drop was so low that the experimental uncertainty due to pressure difference measurements alone was 31 percent, so that the 5 percent criteria may have been too restrictive in this particular case.

The basic aim of the data reduction process was to present the data in a form where they could be directly compared with the theoretical predictions of Chapter III. This comparison was accomplished in two stages. First, following each run, the predicted pressure variation for the specified disk spacing and flow rate was obtained with the numerical program described in Appendix B. The actual air density and viscosity appropriate for the measured temperature and inlet pressure were used. The densities and viscosities of water vapor and air mixtures were calculated with the ideal gas equation and the Wilkes equation (Reid and

Sherwood, 1966, p. 421), respectively. The relative humidity in Tucson and Los Alamos is normally quite low, so 40 percent was used as a typical value. It was found that these quantities varied only 0.8 and 0.04 percent from their respective dry air values. Dry air properties were taken from the Tables of Thermal Properties of Gases (Hilsenrath et al., 1955). As discussed earlier, it was assumed that the flow entered with a uniform velocity distribution at an inlet radius of 0.29 m (11.5 in.). A dimensional plot of pressure difference in mm Hg versus radius in m was constructed. The experimental pressure differences were then superimposed upon the predicted results so that any discrepancies could be noted. Large discrepancies called for further investigation and remedial action. Following this initial screening procedure, the results were then reduced to their final form. The measured pressures and radii were non-dimensionalized as discussed in Chapter II and a family of dimensionless pressure-dimensionless radii curves was plotted.

Results

Table III summarizes the conditions of the fifteen runs reported. Excluded from consideration are the early runs in which spacers were not used to control the disk spacing properly when low spacings were desired. A large variation of adjustable parameters was strived for and was attained. Flow rates were varied by over an order of magnitude, from 0.0039 to 0.075 m³/s (0.138 to 2.65 ft³/s) and measurements were obtained at half spacings ranging from 0.46 to 25.5 mm (0.018 to 1 in.), almost two orders of magnitude variation. As a result the overall Reynolds number, R_o , ranged from 4500 to 105,000 and the dimensionless inlet radius, r_i , varied from 0.17 to 3.35.

TABLE III
LAMINAR EXPERIMENT RUNS

Run	$\bar{t}$ Half-Spac- ing (mm/in.)	$\bar{Q}$ Flow Rate (m ³ s ⁻¹ /ft ³ s ⁻¹)	R_o Overall Reynolds Number	r_i Inlet Dimen- sionless Radius	r_{min} Minimum Dimen- sionless Radius	R Local Reynolds Number Range	Overall Pressure Drop (mm Hg)	Difference Between Data and Predictions	Estimated Uncertainty (+ or -)
1	12.7/0.500	0.0407/1.44	12,700	0.205	0.048	2,200-11,300	0.038	-3.6%	7%
2	2.41/0.095	0.0206/0.729	33,800	0.660	0.129	1,100-5,700	0.820	+6.3%	4%
3	2.41/0.095	0.0132/0.466	21,600	0.820	0.160	710-3,650	0.261	+7.9%	4%
4	12.7/0.500	0.0197/0.695	6,120	0.292	0.069	1,060-5,440	0.010	-7.0%	11%
6	1.60/0.063	0.0076/0.269	18,900	1.34	0.262	410-2,100	0.236	-1.0%	7%
7	1.60/0.063	0.00391/0.138	9,700	1.87	0.366	210-1,080	0.093	-12.0%	7%
8	1.60/0.063	0.0106/0.375	26,400	1.13	0.221	570-2,930	0.435	-5.0%	7%
9 ^a	25.5/1.005	0.0287/1.014	4,450	0.172	0.041	1,550-7,930	0.0026	-26.0%	39%
13	1.65/0.065	0.0264/0.934	66,000	0.720	0.140	1,430-7,300	2.34	-5.0%	7%
14	0.80/0.032	0.0118/0.415	57,300	1.52	0.297	640-3,250	2.94	+1.0%	12%
16	3.20/0.126	0.0484/1.71	60,000	0.375	0.073	2,950-15,100	1.47	-8.0%	5%
17	0.46/0.018	0.0048/0.171	48,270	3.35	0.655	261-1,340	3.46	-16.0%	20%
19	3.20/0.126	0.0745/2.63	92,500	0.302	0.059	4,030-20,600	3.43	-7.0%	5%
20	1.60/0.063	0.0425/1.50	105,600	0.566	0.110	2,300-11,700	5.23	-5.0%	7%
21 ^b	3.20/0.126	0.075/2.65	93,300	0.301	0.059	4,050-20,700	3.50	-4.0%	5%

^aSpacers were used to control the disk spacing for all runs with the exception of Run 9.

^bRun 21 was performed with turbulence "trips" of rough sandpaper glued to the disk surfaces at the flow inlet.

For most runs the dimensionless radii were less than one from inlet to outlet, so these flows never attained a parabolic velocity profile but instead had continuously changing profiles. Nevertheless the numerical solutions developed in Chapter III are still applicable and predicted pressure distributions were computed for each run for comparison with data.

As a result of the wide variation in flow rate and spacing the overall pressure drop across the disk flow section varied by three orders of magnitude. This range of pressure drop was well within the measuring capability of the Baratron manometer, with the exception of the runs at pressures less than 0.01 mm Hg, where stray air currents became a significant factor. The minimum dimensionless radius and the maximum local Reynolds number that appear in Table III are calculated at the radius of the innermost tap for which flow turning effects were not apparent for the particular run. As discussed above the overall pressure drop across the test section was measured from the inlet tap to this innermost tap. The tabulated percent difference represents the difference between this measured $\Delta\bar{P}$ and the corresponding predicted laminar $\Delta\bar{P}$ that was calculated for each run, formed from $(\Delta\bar{P}_{\text{meas}} - \Delta\bar{P}_{\text{pred}}) / \Delta\bar{P}_{\text{pred}}$. For comparison, Table III also lists the estimated overall uncertainties, based upon the results of Table II.

In some runs the local Reynolds number at all radii was below 2300, the value conventionally accepted for laminar-to-turbulent transition for non-accelerated duct flow (Hartnett, Koh and McComas, 1962); whereas for other runs it was well above 2300, and yet, as will be discussed in Chapter V all these flows were laminar. Run 21 was almost

identical to run 19 except for the addition of a rough sandpaper "trip" at the inlet. It was thought that such a trip would promote turbulence, but the close agreement of pressure drop in the two runs indicates the acceleration was sufficient to overcome the "trip" and to laminarize the flow completely.

The lowest half spacing tabulated, 0.46 mm, was considered the smallest value for which reasonable results would be expected considering the 0.25 mm (0.010 in.) waves in the disks' surface, and in fact a fairly large (16 percent) discrepancy did occur which was slightly smaller than the estimated uncertainty. The largest spacing, run 9, yielded an inlet radius to half-spacing ratio, $\bar{r}_i/\bar{\ell}$, of 11.5. Larger spacings were not attempted since it was felt that even smaller ratios would not be compatible with the boundary layer assumptions which, according to the order of magnitude analysis of Appendix A, require that $(\bar{r}_i/\bar{\ell})^2 \gg 1$. In actuality a large discrepancy, -26 percent, was found with the largest half spacing attempted. However, this discrepancy was within the estimated uncertainty, which was dominated by stray air currents, so it is not clear that the limits of the boundary layer theory were actually encountered. This run was also mentioned above in connection with the discussion regarding the flow turning effect and the worst case of apparent asymmetry of flow.

Figure 11 compares the predicted and measured pressures for run 9, just discussed. In this figure, as well as the two that follow, the measured pressures are presented in terms of the pressure drops from the entry. These are obtained by subtracting the pressure difference at a particular location relative to the reference tap, from the pressure

Figure 11. Results Obtained with Largest Disk Spacing.

difference measured at the outermost tap. Once again the predictions assume a uniform entry velocity profile at the position of the outermost tap and are based upon the measured flow rate and properties consistent with the experimentally measured room temperature and pressure. Also shown at selected points is the estimated overall uncertainty. Qualitatively the data follow the predicted trend, but large absolute differences do occur.

In Table III it can be seen that the discrepancies obtained with the next highest half spacing, 12.7 mm (0.5 in.), were considerably smaller, so it is concluded that at the very least the boundary layer approximations are valid so long as $\bar{r}/\bar{t} > 23$. Overall, Table III indicates that if the largest and smallest spacings are excluded for the reasons given above, discrepancies between the predicted and measured overall pressure drops of less than 8 percent were obtained. Furthermore, these discrepancies are close to, or within the uncertainty estimates and since both positive and negative discrepancies are well represented it appears that these discrepancies do not have an unforeseen, systematic cause. For some of the runs with narrow spacings the discrepancies are considerably less than the estimated experimental uncertainties, possibly because special pains were taken during these runs to minimize spacing variations.

The results of a typical run are shown in Figure 12. This is run 13, in which the flow rate was $0.0264 \text{ m}^3/\text{s}$ ($0.934 \text{ ft}^3/\text{s}$), the half spacing was 1.65 mm (0.065 in.) and the local Reynolds number varied from 1430 to 7300. Very good agreement is evident with the pressures predicted for laminar flow, even at low radii, where the pressure

Figure 12. Comparison of Theory and Data for Typical Run.

gradient, dP/dr , was very large. The worst discrepancy, 7 percent, occurs at a radius of 0.11 m; the discrepancies at the other radii are considerably less.

Figure 13 presents some of the results in their final dimensionless form. To avoid excessive detail only four of the runs are presented. The dimensionless inlet radii range from about 0.3 to almost 2, spanning almost the full range of the experimental investigation. Furthermore, these radii are such that both acceleration and viscous effects were important, so that the asymptotic relationships, equations (2-9) and (2-11), that pertain when $r \gg 1$ or $r \ll 1$ cannot be used. In addition these runs are reasonably well spaced in r_i , avoiding a cluttered effect on the figure. The predictions shown are analogous to those presented earlier in Figure 5, but here the dimensionless inlet radii were chosen at the values corresponding to experimental conditions so that the predictions and measurements could be directly compared. Because the overall pressure difference uncertainty is a constant percentage of each individual pressure difference for each run, it appears on a logarithmic scale as an uncertainty bar of constant height, so that only one such bar need be shown for each run. Once again good agreement between the data and pressures predicted for laminar flow is noted and it is concluded that, barring a remarkably fortuitous coincidence, the validity of the laminar analysis discussed in Chapters II and III has been established.

Figure 13. Overall Comparison of Measured and Predicted Pressures.

CHAPTER V

DISCUSSION OF LAMINAR FLOW EXPERIMENTS

Referring again to Table III, one sees that with a few exceptions the measured pressure differences are predicted with laminar flow calculations to within the estimated overall uncertainties, for Reynolds numbers ranging from 210 to 20,700. This agreement raises two points for discussion. First, agreement at low Reynolds numbers where one would expect the flows to be laminar serves to establish the validity of the experimental procedures and measurements. Secondly, it appears that based upon this agreement, all these flows were laminar up to Reynolds numbers of 20,700. This laminar flow prevailed despite the fact that no attempts were made to defer laminar-to-turbulent transition. Residual turbulence intensity in the air entering from the surrounding room was not eliminated by screens and the flow test section was neither free of vibrations nor perfectly smooth. Thus, the experimental conditions were favorable for laminar-to-turbulent transition if not suppressed by other phenomena.

Secondary evidence that these radial flows were in fact laminar was provided by smoke visualization studies. A typical photograph of the smoke pathlines is presented in Figure 14. The view here is from the side, between the disks, looking along a direction that corresponds to decreasing radius. The outlet pipe is shown in the lower right hand corner, and the smoke gun is visible at the upper center, near the entry

Figure 14. Smoke Pathline Indicating Laminar Flow.

region. This photograph was obtained during the early phases of the studies before the disks were instrumented with pressure taps. The half spacing was 12 mm (0.50 in.) and the flow rate was $0.041 \text{ m}^3/\text{s}$ ($1.5 \text{ ft}^3/\text{s}$), so the flow conditions were similar to run 1. Perhaps as a result of squeezing the rubber bulb containing the smoke mixture too energetically, the smoke apparently formed a turbulent jet as it left the gun nozzle. However, downstream of the jet region the smoke formed a thin filament as in laminar flow. The absence of rapid mixing and diffusion that would have been induced by turbulence can be noted. Photographs taken in the perpendicular direction, which are not presented here, clearly show that the smoke lines are radial, but reflections from the Plexiglas surfaces tend to obscure the filamentary nature of the flow.

Laminar flow at a Reynolds number as high as 20,000 may seem surprising in view of conventional engineering experience where Reynolds numbers of about 2300 are expected for non-accelerated flows confined between closely spaced plates (Hartnett, Koh and McComas, 1962). However, it is not clear that the transition Reynolds number determined for non-accelerated flows is applicable to accelerated flows. There are differences in velocity profiles, Reynolds numbers based upon boundary layer thickness scales, and shape factors. It may be recalled from the conclusion of Chapter III that changes in H resulted in very large increases in the critical Reynolds numbers that resulted from theoretical instability calculations.

This discussion points out that commonly accepted transition Reynolds numbers based upon non-accelerated duct flows may have no bearing on accelerated radial duct flows.

As indicated in the Introduction, favorable pressure gradients are known to have a stabilizing effect on the transition from laminar to turbulent flow (Rosenhead, 1963, pp. 540-544, and Schlichting, 1968, pp. 467-473). Previous experimental investigations have been concerned with the unexpected or premature laminarization of an initially turbulent flow and the terms relaminarization and reverse transition have become popular in describing this process. Chapters VII and VIII describe experiments in which the more usual laminarization process, relative to turbulent flow, was investigated, but here we are concerned with the occurrence of laminar flow at local Reynolds numbers in excess of the transition values for ducts of constant cross section. Transition criteria for radially accelerating flows do not appear to have been developed yet.

As indicated in the introductory literature survey several authors have examined laminarization and have concluded that it is independent of the local Reynolds number based on hydraulic diameter and instead have found that another factor, quantified as a dimensionless acceleration or pressure gradient, provided an indication when an accelerated turbulent boundary layer would begin a reverse transition process. This dimensionless parameter is usually called K . These observations showed that a turbulent boundary layer tends to revert to a laminar one if K exceeds a critical value for a sustained distance. Although this conclusion is derived from experiments in which the flow was still intermittently turbulent, it is reasonable to expect that if

a flow is initially laminar and K exceeds this critical value, then the flow will remain laminar. Several of these parameters have been proposed, such as

$$K_1 = \frac{\bar{v}}{\bar{U}_\infty^2} \frac{d\bar{U}_\infty}{d\bar{r}}, \quad K_2 = \frac{\bar{v}}{\bar{\rho}\bar{U}_\tau^3} \frac{d\bar{P}}{d\bar{r}}, \quad \text{and} \quad K_3 = \frac{\bar{v}}{\bar{\rho}\bar{U}_\tau^3} \left(\frac{\partial \bar{\tau}}{\partial \bar{z}} \right)_w.$$

In these definitions $\bar{U}_\tau$ is the friction velocity, $\sqrt{\bar{\tau}_w/\bar{\rho}}$, and $(\partial \bar{\tau}/\partial \bar{z})_w$ represents the shear stress gradient at the wall.

The acceleration parameter K_1 was proposed for external boundary layers where the imposed flow field was characterized by a free stream velocity $\bar{U}_\infty(\bar{r})$ and the local pressure gradient was $d\bar{P}/d\bar{r} = \bar{\rho}\bar{U}_\infty d\bar{U}_\infty/d\bar{r}$ from Euler's equations. The acceleration parameter K_1 is not directly applicable to the present case of a confined boundary layer except when the dimensionless radius is very small and the flow resembles a potential core having velocity approximately equal to $\bar{V}$. Then K_1 can be shown to be algebraically equivalent to $1/R_0$. However, for laminarization by heating gas flows in ducts, McEligot, Coon and Perkins (1970) showed that by interpreting the velocity scale to be the bulk velocity the critical value of K_1 was of the same order of magnitude as for external flows. Thus, use of $K_1 = 1/R_0$ might be expected to provide a guide even for $r \gtrsim 1$.

If attention is confined to laminar flows where r is either very small or very large, it is a simple calculation using the asymptotic pressure variations, equations (2-9) and (2-11), and the asymptotic friction factors, equations (2-10) and (2-13), to derive algebraic equivalents to

K_2 and K_3 also. For $r \ll 1$, $(\partial \bar{\tau} / \partial \bar{z})_w$ may be approximated as $\bar{\tau}_w / \bar{\delta}_1$, and $\bar{\delta}_1$ may be estimated as $r \bar{t}$. These approximations lead to $K_2 = K_3 = 0.9 R_0^{-1/4}$. In a similar manner approximations can also be obtained for $r \gg 1$.

Following the same procedures, one can also derive algebraic equivalents for the Reynolds number, R_2 , based on the boundary layer momentum thickness. It may be recalled from the Introduction that some authors suggested that R_2 might serve as a guide to laminarization. Unlike K_1 through K_3 , a flow is expected to remain laminar when R_2 is less than the critical value (Kays, 1966, pp. 92-93). These transformations to the present geometry are summarized in Table IV along with the critical values proposed by the references cited in the table. Although K_2 and K_3 are algebraically equivalent for the present case, the critical values reported for pipe flow experiments differ by the factor two as mentioned in the Introduction.

Regardless of their definitions, at small r all the acceleration parameters and R_2 are consistent in imposing an upper bound on the overall Reynolds number for which laminar flow would be maintained. These are 3×10^5 , 6×10^6 , 1×10^8 (2.6×10^9 had the corrected critical value of Narasimha and Sreenivasan, 1973, been employed) and 4×10^5 , respectively. Even the most restrictive of these bounds implies that in many cases of practical importance the flow can be expected to remain laminar, although the local Reynolds number, R , may be extremely large.

For $r \gg 1$ all the appropriate parameters imply an upper limit on the local Reynolds number. K_2 requires that $R < 4000$ while K_3 requires that $R < 16,000$ (82,000 if corrected) and R_2 requires that $R < 2900$.

TABLE IV

LAMINARIZATION PARAMETERS AT THE EXTREMES OF DIMENSIONLESS
RADII AND THEIR CRITICAL VALUES

	$K_1 = \frac{\bar{v}}{\bar{U}_\infty^2} \frac{d\bar{U}_\infty}{d\bar{r}}$	$K_2 = \frac{\bar{v}}{\bar{\rho}\bar{U}_\tau^3} \frac{d\bar{P}}{d\bar{r}}$	$K_3 = \frac{\bar{v}}{\bar{\rho}\bar{U}_\tau^3} \frac{\partial \bar{\tau}}{\partial \bar{z}_w}$	$R_2 = \frac{\bar{U}_\infty \bar{\delta}_2}{\bar{v}}$
$r \ll 1$	$1/R_0$	$0.9/R_0^{1/4}$	$0.9/R_0^{1/4}$	$0.6\sqrt{R_0}$
$r \gg 1$	No Direct Equivalent	$1.15/\sqrt{R}$	$1.15/\sqrt{R}$	$0.125 R$
Critical Value ^a	3.5×10^{-6}	0.018	0.009	360
Investi- gators	Moretti & Kays (1965)	Patel & Head (1968)	Patel & Head (1968)	Kays (1966)

^aA flow might be expected to laminarize when K_1 through K_3 exceed their critical values, or when R_2 is less than its critical value.

It is interesting to note that for $r \gg 1$, when acceleration effects become negligible, all the criteria do in fact result in a limitation on the local R , as would be expected for flows without acceleration. In fact the transition values implied by K_2 and R_2 appear to be close to the values usually accepted. The fact that K_3 results in much larger values, coupled with the observation that even at $r \ll 1$ the critical values of R_0 imposed by K_3 are clearly out of line with the other estimates, leads one to suspect that K_3 is not appropriate as a laminarization criterion. This topic will be reconsidered in Chapter VIII.

Finally, we note that for all the experimental runs possible with this equipment, summarized in Table III, the maximum overall Reynolds number was 105,000, well below any of the bounds on R_0 given above. For those few runs at r_i of 1 or above, where the R_0 limitation might no longer be appropriate, we find that R was 3250 or less. It is apparent then, that judging by any of these criteria, for any range of dimensionless radii, all these runs should have been, and were indeed, laminar. It is concluded then that these dimensionless laminarization criteria may serve as useful guides for laminarization. Admittedly the picture is not complete since it so far views only the laminar side of the transition. However, it has been shown at least that the experimental results are consistent with the laminarization criteria considered. In Chapter VIII laminarization is examined with both turbulent and laminar flows and more definitive recommendations are given regarding the various criteria.

CHAPTER VI

TURBULENT FLOW PREDICTIONS

The concept of turbulence modeling was briefly mentioned in the Introduction. Perkins (1975) has admirably described the concept, use, and selection of turbulence models and he is freely quoted here.

Historically, the problem of predicting turbulent transport has been approached by considering a simplified model of the transport process (e.g., Prandtl's mixing length model). . . Jones and Launder (1972) have argued that the mixing length concept has been extended as far as would be profitable and that only a turbulence model employing at least the complexity of their two-equation model (one partial differential equation for turbulence kinetic energy, and another for turbulence dissipation rate) will permit a universal modeling. . . .However, engineers probably will continue to depend on predictions made with the relatively simple mixing length models for many years. . . .Several authors (Bankston and McEligot, 1969; McEligot and Bankston, 1969; Morse, 1971) have pointed out the general usefulness and applicability of the van Driest (1956) mixing length model to many flow regimes, particularly for internal turbulent flows.

The reader interested in more details is referred to the reviews of turbulence modeling by Bradshaw (1972) and Reynolds (1976).

With this background then, it seemed logical to perform some preliminary calculations with this model first, and then, in Chapter VIII, to explore further modifications to bring it into line with experimental results for laminarizing flows. Since the van Driest model is probably the most generally accepted model in use today, it will be used in Chapter VIII as a "standard turbulence model," i.e., one end point (the other being the purely laminar prediction) to which the experimental results to be described later will be compared.

With this object in mind we first define the model and incorporate it into the momentum equation. We then apply it to turbulent flow between infinitely wide parallel plates - a situation in which the complicating effect of radial acceleration does not occur and where considerable experimental results exist. These experimental results are then used to test the predictive capability of the model.

The Van Driest Turbulence Model

Prandtl's mixing length theory (Schlichting, 1968, p. 548) results in an expression for the turbulent viscosity of the form

$$\bar{\mu}_t = \bar{\rho} \bar{\ell}^2 \left| \frac{\partial \bar{u}}{\partial \bar{y}} \right| \quad (6-1)$$

where $\bar{\ell}$ is the mixing length and $\bar{y}$ is the distance from the wall. The total effective viscosity, the sum of the molecular and turbulent contributions, is $\bar{\mu}_{\text{eff}} = \bar{\mu} + \bar{\mu}_t$.

For boundary layer calculations, the turbulent boundary layer is usually regarded as a composite layer consisting of inner and outer regions, and the distributions of $\bar{\ell}$ and $\bar{\mu}_t$ are described by two separate empirical expressions in each region. In the inner layer, but outside the viscous layer, $\bar{\ell}$ is approximated as being proportional to $\bar{y}$ but in the outer layer it is taken as proportional to $\bar{\delta}$ (Bradshaw, 1976, p. 217). Therefore, $\bar{\ell} = \kappa \bar{y}$ if $\bar{\ell}$ is less than $\alpha \bar{\delta}$, otherwise $\bar{\ell} = \alpha \bar{\delta}$. The empirical parameters κ and α vary slightly according to experimental data but typical values are $\kappa = 0.4$ and $\alpha = 0.09$ when $\bar{\delta}$ is taken as the boundary layer thickness, defined as the location where the velocity is 0.995 times the centerplane velocity (Bradshaw, 1976, p. 217).

Van Driest (1956) suggested that in the immediate vicinity of the wall the effects of turbulent mixing should be damped by setting

$$\bar{\lambda} = \kappa \bar{y} (1 - e^{-y^+/A^+}) \quad (6-2)$$

where y^+ is the dimensionless wall coordinate,

$$y^+ = \bar{y} \sqrt{\bar{\tau}_w / \bar{\rho}} / \bar{v} \quad (6-3)$$

and A^+ is a constant with value 26.

When y^+ is small, say 5 or less, equation (6-2) has the effect that $\bar{\lambda}$ also is small so that $\bar{\mu}_{\text{eff}} \approx \bar{\mu}$. The net effect is a thin layer variously referred to as the laminar or linear layer. If it is assumed that $\bar{\tau} = \bar{\tau}_w$ in this region it can be shown that $u^+ = \bar{u} / \bar{U}_\tau = y^+$. Many authors have observed that, by simply adjusting the value of A^+ , the thickness of the linear layer could be varied, and better agreement with low Reynolds number and laminarizing flow could be attained. McEligot, Ormand and Perkins (1966) demonstrated that by letting the thickness of the laminar sublayer be a suitable function of the local Reynolds number, good agreement could be obtained with convective heat transfer data for flows in circular tubes at low Reynolds numbers (2500-10,000) and low heating rates. Launder and Jones (1969) showed that by letting A^+ be a function of the laminarization parameter K_2 better agreement could be obtained for flows laminarizing between converging plates. A similar modification will be used in Chapter VIII for the calculation of laminarizing flows, but for the present purposes A^+ will be taken as 26.

In the other extreme, $y^+ \gg A^+$, equation (6-2) is approximated as $\bar{\ell} = \kappa \bar{y}$. Substitution in equation (6-1) gives

$$\bar{\mu}_t = \bar{\rho} \kappa^2 \bar{y}^2 \left| \frac{\partial \bar{u}}{\partial \bar{y}} \right| \quad (6-4)$$

so that, by definition,

$$\bar{\tau}_t = \bar{\rho} \kappa^2 \bar{y}^2 \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 \quad (6-5)$$

where for convenience positive velocity gradients only are considered.

If it is assumed that near the wall -- but far enough away that

$\bar{\tau}_t \gg \bar{\tau}_\ell$ -- the stress is constant, then equation (6-5) may be integrated with the result

$$u^+ = \frac{1}{\kappa} \ln y^+ + C \quad (6-6)$$

where C is an integration constant. This constant is usually evaluated by matching data in the linear layer to data in the logarithmic region where equation (6-6) holds. Despite the fact that equation (6-6) is, strictly speaking, valid only when $\bar{\tau} = \bar{\tau}_w$, it is often used for the entire flow field, even close to the center line of pipe flow where $\bar{\tau} = 0$ (Schlichting, 1968, p. 555). In this guise equation (6-6) is referred to as the "universal logarithmic velocity law," and a wide variety of experimental studies support it (see Schlichting, 1968, p. 565). Based upon pipe flow experiments Schlichting recommends $\kappa = 0.4$ and $C = 5.5$ so that equation (6-6) becomes

$$u^+ = 2.5 \ln y^+ + 5.5 \quad (6-7)$$

The picture that has been sketched so far then, is that in the linear layer, say $y^+ \gtrsim 5$, u^+ is proportional to y ($u^+ \cong y^+$) but when $y^+ \gtrsim 30$ a logarithmic law becomes appropriate. One has to expect deviations in special circumstances, e.g., it has already been pointed out that for low Reynolds numbers the linear layer may be thicker than the values given here, and for extreme cases of acceleration -- where $\bar{\tau}(\bar{y})$ is far from being a constant -- one might also expect variations from the logarithmic law. Ultimately each individual case must be evaluated with the aid of a model, such as the van Driest model or a more advanced model, and for complete reliability the latter predictions should be confirmed by experiment.

The governing equations for isothermal turbulent flows are similar to those for laminar flow except that $\bar{v}$ must be replaced with $\bar{v}_{\text{eff}}$; since the latter quantity varies with $\bar{z}$ as well as $\bar{r}$, this variation must be treated in the viscous terms of equations (2-1b) and (2-1c). Thus, for example, instead of $\bar{v} \partial^2 \bar{u} / \partial \bar{z}^2$ one must write $\partial(\bar{v}_{\text{eff}} \partial \bar{u} / \partial \bar{z}) / \partial \bar{z}$. When the scaling variables, equation (2-3), are substituted into the governing equations (2-1a, b, c) one obtains dimensionless boundary layer equations similar to equations (2-4) through (2-6) with the exception that the momentum equation (2-5) must be rewritten as

$$u \frac{\partial u}{\partial r} - \frac{u^2}{r} + w \frac{\partial u}{\partial z} = r^2 \frac{dp}{dr} + r \frac{\partial}{\partial z} \left(\frac{\bar{v}_{\text{eff}}}{\bar{v}} \frac{\partial u}{\partial z} \right) \quad (6-8)$$

Unlike laminar flow, a free parameter, $\bar{v}_{\text{eff}}/\bar{v}$, now appears. Since $\bar{v}_{\text{eff}}/\bar{v}$ depends upon local Reynolds number, wall and integral parameters such as f , P , and boundary layer thicknesses depend not just on the

dimensionless radius r , but also on the Reynolds number. It can be shown from their definitions that the local Reynolds number is $R = 4\sqrt{R_0}/r$, so that, equivalently, these wall and integral parameters depend on dimensionless radius and the overall Reynolds number. Furthermore, since r is based upon $\bar{v}$, not $\bar{v}_{\text{eff}}$, the dimensionless radius, while still a convenient scaling parameter, can no longer be viewed as a direct measure of the importance of viscous effects relative to acceleration effects, as it was for laminar flows. Thus at $r = 1$ the viscous effects associated with turbulent flows will be, depending upon R_0 , more important than acceleration effects, whereas for laminar flows the two effects were approaching equality.

Presentation of numerical predictions for converging turbulent flows is deferred until Chapter VIII where they can be compared to measurements directly.

Rectilinear Turbulent Flows

The van Driest model introduced above was incorporated into the numerical program discussed in Appendix B. The geometry was also converted to a rectilinear one, as also discussed in Appendix B, so that turbulent flows between infinitely wide, parallel flat plates could be calculated. For this flow situation, Hartnett, Koh, and McComas (1962) found that the measured friction factor for fully developed flow was in close agreement with the Blasius friction factor correlation (Schlichting, 1968, p. 561). When the aspect ratio was either 5 or 10 the measured friction factor was 3 to 4 percent greater than the

Blasius result at $R = 10^4$, and 1 to 2 percent greater at $R = 10^5$.

The Blasius correlation is

$$f = 0.0791 R^{-0.25} \quad 6000 < R < 100,000 \quad . \quad (6-9)$$

Here R is based upon the "equivalent" hydraulic diameter, $\bar{D}_h$, which is four times the duct area divided by the duct perimeter. For infinitely wide plates the equivalent hydraulic diameter is two times the plate spacing, or $4\bar{t}$.

A number of turbulent flow predictions were performed and some results are shown in Figure 15. These results are compared with the Blasius correlation and also the Drew, Koo and McAdams (1932) correlation,

$$f = 0.0014 + 0.125 R^{-0.32} \quad 3000 > R < 3 \times 10^6 \quad .$$

In the range of Reynolds numbers shown there is essentially no difference between the Blasius and the Drew, Koo and McAdams correlations. The friction factors predicted for fully developed flow are 3 to 4 percent higher than both correlations except at the lowest Reynolds number calculated, 9000, where the disagreement is 12 percent. The predictions at the higher Reynolds numbers agree with the data of Hartnett, Koh and McComas (1962).

As suggested above, the friction factors measured by Hartnett, Koh and McComas (1962) in ducts with aspect ratios of 5 or 10 were 3 to 4 percent greater than the Blasius correlation at $R = 9000$. For aspect ratios of 28.5 and 30.0, Beavers, Sparrow and Lloyd (1971) measured friction factors 2 percent higher than the Blasius correlation at $R = 9000$;

Figure 15. Comparison of Predicted Friction Factors with Blasius and Drew, Koo and McAdams Correlations for Rectilinear Flows. 84

whereas for an aspect ratio of 48 Patel and Head (1968) obtained results 11 percent higher than the Blasius correlation at $R = 9000$. One interpretation of the close agreement of the data of Hartnett et al., with those of Beavers et al., for aspect ratios varying from 5 to 30, is that the Patel and Head data are anomalously high, and that the Blasius correlation predicts reasonably accurate friction factors which are perhaps 2 to 4 percent lower than the accepted measurements at $R = 9000$. If this interpretation is correct, the van Driest calculations are 8 to 10 percent higher than the accepted measurements at $R = 9000$. It is likely that had the modification suggested by McEligot, Ormand and Perkins (1966) been utilized even this discrepancy could have been avoided. In any event it is concluded that turbulent flow predictions which are reasonably accurate for engineering purposes can be performed with the present turbulence model even at Reynolds numbers as low as 9000.

CHAPTER VII

TURBULENT, TRANSITIONAL, LAMINARIZING AND LAMINAR FLOW EXPERIMENTS -- DESCRIPTION, PROCEDURE AND MEASUREMENTS

The main purpose of these experiments was to examine the turbulent and laminarizing flows that were unattainable with the laminar flow apparatus of Chapter IV. However, some laminar flows were also examined in connection with the calibration of the shear stress probe described later in this chapter as well as some radially converging flows that were completely laminarized. For brevity the new apparatus will be referred to as the turbulent flow test section. Most of the remarks that prefaced the description of the laminar flow experiments are germane to this chapter also. For example, air was again used as the modeling fluid for the reason stated in Chapter IV and once again pressure drops across the test section were so small that the air flow could be considered incompressible.

Description of Turbulent Flow Test Section

Only the test section itself and the exit pipe were changed for turbulent and laminarizing flows. All the downstream equipment, i.e., the venturi meter, flexible ducting, and blower were unchanged from the laminar experiment. Once again room air was drawn into the flow test section then through the exit pipe and venturi, and finally ejected by the reversed blower. Even the same Polypenco pressure tubing and valve arrangements were reused, as was the MKS pressure measuring system.

The new test section is shown schematically in Figure 16 (a photograph and other details appear in Appendix C). The major test section walls are formed by the two Plexiglas plates and the side walls that complete the test section are formed by movable aluminum side rails. These side rails can be positioned at angles, θ , to the vertical so that various degrees of flow convergence and acceleration can be achieved. To aid accurate positioning of the side rails, lines were carefully positioned and scribed on the opposing Plexiglas surfaces every 2° . Unlike the laminar test section, this test section had an abrupt inlet with 90° corners, which served as turbulence "trips." These sharp corners were provided so that the transition to turbulence (for those flows expected to be turbulent) would be as rapid as possible and not be impeded by, say, an unusually smooth inlet.

In the laminar experiment the full 2π radial circular opening was fixed but one could vary the disk spacing; but in this experiment the spacing was fixed at 12.7 mm (0.500 in.) while the circular opening could be changed with the side rails so that various pie-shaped segments of a circle could be achieved. When the side rails were vertical and parallel ($\theta = 0$) they were separated by 152 mm (6 in.) such that a rectangular duct was formed with an aspect ratio of 12. The effective duct length was 1.14 m (45 in.). Each side rail could be moved a maximum of 20° from vertical. In the radial geometry the effective radius at the outlet, measured from a virtual origin where the side rails would have intersected was simply $0.152/(2 \sin \theta)$ meters and the inlet radius was 1.14 m greater.

Figure 16. Turbulent Flow Test Section.

Starting 25 mm (1 in.) below the inlet, pressure taps were arrayed vertically every 76 mm (3.00 in.). In addition at selected vertical positions up to six additional taps were arranged horizontally; these taps provided a check on azimuthal or spanwise symmetry, i.e., whether the flow was truly radial when $\theta > 0$, or longitudinal when $\theta = 0$. A total of 28 taps were located on the test section. The machining of the taps was similar to the laminar experiment, thus avoiding burrs or flow disturbances. Further details about the test section, its fabrication, and its pressure instrumentation can be found in Appendix C.

Shear Stress Measurements

In addition to the measurements described in Chapter IV, some shear stress measurements were made in this test section with TSI (formerly Thermo Systems Inc.) model 1237, flush mounted hot film probes. These probes consist of thin platinum films which are 0.1 mm long in the flow direction and 1 mm wide in the spanwise direction. These films are mounted in cylindrical probes 3 mm (0.124 in.) in diameter. Each probe was inserted in the center of a 76-mm (3-in.) diameter Plexiglas disk which is 12 mm (1/2 in.) thick. More details and a photograph of the probe-disk assemblies are presented in Appendix C. As described in Appendix C a dial indicator was used and each probe was inserted so that the probe surface containing the film was flush with each disk surface within 5 μm (0.0002 in.). This ensured that when the disk surface was made part of the inner surface of the test section, as described below, the surface mismatch as the air flowed across the disk surface and then over the heated film was insignificant. For all the runs described in

the next chapter, the maximum potential mismatch of $5\text{ }\mu\text{m}$ represents a dimensionless projection, y^+ , into the flow stream of only 0.5, considerably smaller than the thickness of the viscous layer. For comparison Bellhouse and Schultz (1966) found that purposely introducing mismatches of plus and minus $75\text{ }\mu\text{m}$ (0.003 in.) did not affect shear stress measurements in turbulent boundary layers.

As described in more detail in Appendix C, the Plexiglas plate furthest from the viewer of Figure 16 was constructed so that a Plexiglas "slider" could be moved in the streamwise direction. The slider was 64 mm (2.5 in.) wide in the spanwise direction and 2.3 m (94 in.) long in the streamwise direction. Its long direction was parallel to the bisector of the angle formed by the side rails. The inside surface of the slider was intended to be flush with the inside surface of the Plexiglas plate. The probe-disk assemblies were installed flush with the inside surface of the slider, and by moving the slider in the streamwise direction it was intended that shear stress measurements could be obtained as a function of radius. However, it was found that because of variations in the thickness of the Plexiglas and machining difficulties it was not possible to keep the surfaces flush within necessary tolerances at all positions. In some positions the slider projected into the flow field by as much as 0.5 mm (0.020 in.); in others it was retracted 0.5 mm so that the local flow spacing at the probe exceeded that desired by 4 percent. In addition these surface mismatches at the slider-to-Plexiglas plate joints apparently caused secondary flows.

Eventually the sliding feature was abandoned; the slider and shear stress probes were fixed in a single position and the whole

apparatus was planed, sanded, and polished to remove all mismatches. One probe was accidentally destroyed before the experimental program was well underway. The remaining probe, which was used to measure the shear stresses reported in the next chapter, was fixed 0.74 m (29 in.) from the entry. The interested reader is referred to Appendix C for a discussion of the many problems faced in this aborted attempt to obtain radial shear stress profiles, which led to a long and tedious fabrication and machining history. One consequence should be pointed out here, however. As an outcome of these difficulties some measurements were made with a separate, plain Plexiglas plate having neither slider nor shear probes. Thus, some of the turbulent flow experiments, particularly those at low angles of convergence, lack shear stress measurements.

The principle of operation of these probes is described in Appendix C. Basically the film is kept at an elevated constant temperature by means of electronic feedback circuitry. For these measurements a TSI 1050 anemometer system provided the necessary circuitry. Bridge voltage was displayed digitally with a Fluke 8300 A voltmeter. As the flow over the surface film changes, the degree of convective cooling changes and the feedback circuitry adjusts the resistive heating in the film as necessary to maintain constant temperature. Since the filament temperature is constant, so is the filament resistance, and thus the power required is proportional to the square of the bridge voltage, $\bar{E}^2$. Appendix C discusses how the wall shear stress is related to this power using the Lévéque (1928) analysis. This analysis is based upon the assumptions that all heat loss is due to convection and that the thermal boundary layer is so thin, $\delta_t^+ \gtrsim 5$, that it can be assumed that within

this layer $u^+ = y^+$ regardless of whether the flow is laminar or turbulent. While this analysis is based upon parallel rather than converging boundary layer flows, the length of the surface film in the flow direction is so small (0.1 mm or 0.004 in.) that the velocity profile is essentially unchanged over the region of the film; consequently, the results of the analysis should pertain in either case. (As an approximation, $\Delta u/u \approx \Delta r/r$; in the worst case, $\Delta r/r \approx 2 \times 10^{-4}$.) In functional form one expects that so long as the assumptions made are actually operative, the predicted relationship will be

$$\bar{E}^2 - \bar{E}_0^2 = \bar{A} \bar{\tau}_w^{1/3} \quad (7-1)$$

Since the assumptions that lead to equation (7-1) are not always perfectly met the constants $\bar{E}_0$, $\bar{A}$, and even the exponent of $\bar{\tau}_w$ are usually determined by actual calibration.

Appendix C also discusses the in situ calibration of the probe. In brief, the side rails were placed at $\theta = 0$ so that fully developed rectilinear duct flow could be obtained. The probe was located downstream a distance 58 times the spacing so that inlet effects could be ignored. For some laminar runs this distance was not entirely satisfactory, however, as will be seen in Chapter VIII and Appendix C. Fully developed flow assures an equilibrium between shear stresses and pressure gradients, and the required relationships between the two are provided in Appendix C. In operation then, one establishes a flow rate, measures the pressure gradient and $\bar{E}$, calculates $\bar{\tau}_w$ from $d\bar{P}/d\bar{x}$ and relates $\bar{E}$ to $\bar{\tau}_w$. Appendix C (Figure 38) presents the calibration obtained. For most of the range of interest it was found that the functional form of

equation (7-1) was applicable, but at very low shear stresses, there were departures which were probably caused by entry length effects and by failure of the boundary layer assumptions, as suggested in Appendix C. The range of calibration spanned 3 decades of shear stress and extends to shear stresses as low as 8×10^{-3} Pa (2×10^{-4} lb/ft² or 1×10^{-6} psi). It should be pointed out, however, that much of the range of calibration is not of practical interest; the extension of the range was motivated by attempts to calibrate the same probe in the same apparatus for both laminar and turbulent rectilinear duct flows, in an effort to see if the state of flow caused any differences in the probe performance. It is concluded, in Appendix C, that it did not cause any observable difference, in agreement with Bellhouse and Schultz (1966) and Badri Narayanan and Ramjee (1969). For practically all turbulent, laminarizing and laminar radial flows discussed in Chapter VIII shear stresses were 0.04 Pa or greater, on the linear $\bar{E}^2 - \tau_w^{1/3}$ region.

Following calibration the probe was not further physically disturbed for the remainder of these experiments, thus adding confidence in its reliability. The probe was recalibrated in the rectangular duct configuration at frequent intervals and differences from one calibration to another were used to assess some measurement uncertainties (Appendix D).

One might question whether the calibration of the hot film sensor is affected by pressure gradient. Liepmann and Skinner (1954) examined this question partially by means of an order-of-magnitude analysis. For a turbulent flow they propose that the solution from a laminar analysis would be valid provided that the thermal boundary layer thickness $\bar{\delta}_t$

remains within the linear layer; they then evaluate this criterion for flow over a flat plate with no pressure gradient. Further, they partially justify the assumption of a linear velocity profile in favorable pressure gradients by an approximate laminar analysis which allows for the induced modification of $\bar{\tau}(\bar{y})$ near the wall. Brown (1967) obtained a more sophisticated solution of the same problem and confirmed that, as far as this consideration is concerned, the effect of favorable pressure gradients would be negligible except near a stagnation point. He also generalized the criterion for the maximum film length in the streamwise direction to the form $h^+ \gtrsim 64 \text{ Pr}$ where h^+ is the length in wall coordinates, $h^+ = h \bar{U}_\tau / \bar{\nu}$, and Pr is the Prandtl number which is 0.7 for air. This criterion restricts $h^+ \gtrsim 45$; for comparison the maximum effective value of h^+ was estimated to be between 12 and 25 in the present study.

Neither Liepmann and Skinner nor Brown address the possibility that the turbulence and, hence, extent of the linear layer are modified by imposed pressure gradients. Fortunately, Blackwelder and Kovasznay (1972) and others have observed that the viscous layer thickens in favorable pressure gradients so the requirement that $\delta_t^+ \gtrsim 5$ or $h^+ \gtrsim 64 \text{ Pr}$ would be conservative in the present experiment. Liepmann and Skinner could not test the effects of pressure gradients with their equipment. In the present study the rectilinear configuration used for calibration has the smallest cross section, so it was possible to test the proposed correlation over almost the entire range of pressure gradients encountered in the later experiments. The linear relation between E^2 and $\bar{\tau}_w^{1/3}$ was verified for all pressure gradients except at the

lowest flow rates where measurement uncertainties inhibited the comparison. The measured pressure gradient was less than the maximum calibrated value for all runs except one.

The calibrations also spanned the same local Reynolds number range encompassed by the experimental runs. No Reynolds number effect upon the calibration was noted except possibly at Reynolds numbers less than 2500. Then the calibration data fell below the $\bar{E}^2 - \bar{\tau}_w^{1/3}$ relationship, but as discussed in Appendix C this discrepancy is probably due to systematic errors at very low flow rates. Even though the calibration data departed from the linear relationship, the trend of the data above and below the expected transition Reynolds number (about 2300 for the non-accelerating, rectangular duct flows used for these calibrations) was continuous; there was no abrupt change associated with differing flow regimes.

Frequency response of these probes was estimated in air with the electronic square wave method which has recently been reviewed in detail by Freymuth (1977). Briefly a square wave is fed into the anemometer circuitry (the TSI 1050 anemometer has a built-in square wave generator for conveniently conducting these tests) and the pulse response of the system is observed. The frequency response was optimized by varying circuit parameters such as coil induction and bridge resistance. The cutoff frequency where the response of the probe fell off 3 db from the ideal, flat response can then be estimated as

$$\bar{f} = 1/1.5 \bar{\tau}_a \quad (7-2)$$

where $\bar{\tau}_a$ is the time from the start of the pulse to when the pulse has decayed to 3 percent of its maximum value. For the measurements reported here $\bar{f}$ was 30 kHz, whereas the predominant frequency of the turbulent fluctuations was only 1 kHz or less. To avoid any degradation of frequency response in data transmission to the anemometer, a TSI 1015 triax cable with two shields was used to connect the probe to the anemometer.

Finally, it might be noted that before general use each probe was operated overnight at a temperature 30°C higher than the usual operating temperature in order to stabilize each probe and prevent long-term drift.

Procedure, Measurements and Uncertainties

For the most part procedures were similar to those already described for the laminar flow experiments so that only new features are discussed here. The side rail angles were set to the desired value using the scribed lines and then rechecked with simple trigonometrical relationships. Unlike the laminar flow experiments these Plexiglas plates were relatively rigid, and the set spacing, 13 mm, was so large that the use of spacers to control spacing variations was not required for this test section. As will be seen, spacing uncertainties were only 1 percent. In some cases spacers were used in an attempt to reduce this uncertainty even further, but the generally larger set spacing, compared to the laminar flow experiments, required spacers which were wider in the spanwise direction and the area blockage sometimes far exceeded acceptable limits. For example, for parallel side-rail runs,

$\theta = 0$, the blockage was as high as 8 percent. For this reason these cases are not reported here.

Flow rates and pressure difference readings were obtained as previously described, and time-mean shear stresses were obtained, for those runs so instrumented, by means of the anemometer and a Fluke 8300A integrating digital voltmeter. Temporal fluctuations of the shear stress probe were recorded with a Tectronix 475 oscilloscope operated in the ac mode. For typical turbulent runs the fluctuations had minimum frequencies of 10 to 100 Hz but for some of the runs where laminarization was nearly complete the minimum frequency was estimated to be dc to 10 Hz. Photographs were obtained with a Polaroid C-30A camera and Polaroid 3000 speed film.

Tables V and VI summarize the measurement uncertainties and associated uncertainties in pressure differences (Table V) and mean shear stresses (Table VI). The distinction between direct measurement uncertainties and associated uncertainties in prediction of pressure drops caused by uncertainties in flow rates, spacings, etc. was introduced in Chapter IV. Analogous uncertainties can also be derived for wall shear stresses. These uncertainties are discussed in detail in Appendix D. The uncertainty in shear stress due to probe voltage uncertainties was derived from repeatability studies. It effectively includes, in addition to voltage measurement uncertainties, the effects of pressure difference uncertainties caused by stray air currents and by instrument drift, and the possible failure to average out turbulent fluctuations on occasion. The flow rate uncertainty and, at low

TABLE V

UNCERTAINTIES IN PRESSURE DIFFERENCES DUE TO MEASUREMENT
UNCERTAINTIES, TURBULENT AND TRANSITIONAL EXPERIMENTS

DIRECT UNCERTAINTIES

<u>Parameter</u>	<u>Measurement Uncertainty (+ or -)</u>
Pressure	0.001 mm Hg

ASSOCIATED UNCERTAINTIES

<u>Parameter</u>	<u>Measurement Uncertainty (+ or -)</u>	<u>Associated Uncertainty in Pressure Difference (+ or -)</u>
Temperature	0.5°C	0.05% ^a
Flow Rate	2.5%	5%
Pressure Tap Locations ^b	0.1 mm (0.005 in.)	0.01%
Spacing	1%	3%

^aAll uncertainties expressed as a percentage pertain to the actual value measured or the reported pressure difference uncertainty not, for example to a full scale reading.

^bThe uncertainty in pressure tap location actually varies from tap to tap. Shown here is the worst case.

TABLE VI

UNCERTAINTIES IN MEAN SHEAR STRESS DUE TO
MEASUREMENT UNCERTAINTIES, TURBULENT AND TRANSITIONAL EXPERIMENTS

<u>Parameter</u>	<u>Measurement Uncertainty</u>	<u>Associated Shear Stress Uncertainty</u>
Temperature	0.5°C	0.05% ^a
Flow Rate	2.5%	5%
Probe Voltage	0.011 V	0.033/ ΔE
Probe Location	0.1 mm (0.005 in.)	0.03%
Spacing	1%	2%

^aAll uncertainties expressed as a percentage pertain to the actual value
a full-scale reading.

overall pressure differences, stray air currents resulted in the largest pressure difference uncertainties while probe voltage and flow rate uncertainties resulted in the largest shear stress uncertainties.

CHAPTER VIII

TURBULENT, TRANSITIONAL, LAMINARIZING AND LAMINAR FLOW EXPERIMENTS -- RESULTS AND DISCUSSION

Results and their discussion are presented here in two sections, first those results for $\theta = 0$, and secondly those for which $\theta > 0$. The first section represents results for rectangular ducts, and apart from the entry region, the flows are fully developed ones for which considerable data are already available, and for which the turbulent predictions of Chapter VI are readily applied for comparison. In effect then the first set of results mainly validates the experimental apparatus as a research tool and also provides a convenient method of calibrating shear stress probes. The second set of results, for radial flows, breaks new ground. There exists the possibility of laminarization due to acceleration so that the turbulent calculations of Chapter VI may not be applicable without modification to account for this possibility.

Rectilinear Flows, $\theta = 0$

At Reynolds numbers above ~ 2000 to 3000 the flow in rectangular ducts is ordinarily considered to be a fully developed turbulent one when the downstream distance exceeds 20 hydraulic diameters or $\bar{x}/\bar{t} \gtrsim 80$, if the entry is an abrupt one as in this study (Hartnett, Koh and McComas, 1962). The hydraulic diameter, $\bar{D}_h$, is defined as four times the duct cross-sectional area divided by the perimeter. In terms of the duct width, $\bar{b}$, and the half spacing, $\bar{t}$, $\bar{D}_h$ is

$$\bar{D}_h = 4\bar{t}/(1 + 2\bar{t}/\bar{b}) \quad . \quad (8-1)$$

In connection with the shear stress probe calibration in Appendix C it has been shown via a force balance that for a fully developed flow the peripherally-averaged wall shear stress is given as

$$\bar{\tau}_w = -\bar{t}(d\bar{P}/d\bar{x})/(1 + 2\bar{t}/\bar{b}) \quad . \quad (C-12)$$

For the present treatment it has been assumed that the shear stress along the center line is approximated by this average. This latter assumption is reasonably good for large aspect ratios, $\bar{b}/2\bar{t}$, but small aspect ratios can result in secondary flows in the corners (Schlichting, 1968, pp. 576-577) so that the assumption would become weak. However, as mentioned in Chapter VI, Hartnett, Koh and McComas (1962) found that when the aspect ratio was either 5 or 10, and when τ_w was evaluated from equation (C-12), then agreement within 4 percent was obtained with the Blasius equation over the Reynolds number range $R = 6000$ to $100,000$, whereas an aspect ratio of 1 resulted in a 4 percent difference at $R = 100,000$ but a 10 percent difference at $R = 6000$. In the present study the aspect ratio was 12. The Reynolds number was evaluated as

$$R = \bar{V}\bar{D}_h/\bar{\nu} \quad . \quad (8-2)$$

Figure 17 presents results for a typical turbulent experiment in the present investigation. These results have been nondimensionalized in terms of $\bar{D}_h$ and the dynamic pressure $\bar{\rho}\bar{V}^2/2$. Beyond $\bar{x}/\bar{D}_h = 10$ all the

Figure 17. Measured Pressure Distribution for a Rectilinear Flow.

pressures lie on a straight line. The departure at the last point was probably caused by the turning of the flow into the outlet pipe transition. The pressure drops are presented relative to the first pressure tap downstream of the entry, located at 25 mm ($\bar{x}/\bar{D}_h = 1$). Normally one expects pressure gradients near the entry to be higher than those that pertain to fully developed flow, but here we see that near the second pressure tap the gradient is actually slightly lower. This result is attributed to a vena contracta at the entry. Because of the abrupt entry of this test section, a vena contracta probably forms in the vicinity of the first tap, resulting in locally low pressures due to the high velocities in the vena contracta. Thereafter, the vena contracta expands as the velocity profile fills the flow channel, so that a partial pressure recovery occurs which offsets some of the viscous pressure loss.

A check of the transverse, spanwise pressure taps revealed no spanwise dependence of pressure at any of the streamwise positions where these additional taps were located. The pressure gradient was computed from the best fit to the data in Figure 17, the shear stress was evaluated from equation (C-12), and finally the friction factor was computed. Figure 18 presents a summary of a number of such runs in which the flow rate was varied. For comparison the Blasius correlation, the laminar result, and the results of the van Driest model described in Chapter VI are shown. Above $R = 6000$ the data differ with the Blasius correlation by a maximum of 9 percent, only slightly greater than the experimental uncertainties indicated. Above $R = 10,000$ the data agree within about 6 percent or less. Later the van Driest prediction is used for comparison;

Figure 18. Comparison of Measured and Predicted Friction Factors for Rectilinear Flows.

Figure 18 shows these data to be consistently lower by 5 to 10 percent above $R = 10,000$.

As discussed in Appendix C, runs were also made at extremely low rates of flow (as low as $2 \times 10^{-3} \text{ m}^3/\text{s}$ or $0.064 \text{ ft}^3/\text{s}$) in an effort to detect any difference in the shear stress probe characteristics at the transition from laminar to turbulent flow. Obtaining these flow rates often required complete closure of the flow control valve beyond the venturi and in some cases special measures were necessary to reduce residual leakage around the throttle plate within the closed valve. The pressure gradients in the lowest Reynolds number run were so low that pressure difference uncertainties of ± 35 percent were introduced by stray air currents. Furthermore, the entry development region for these laminar and near-laminar flows was so long that fewer taps were actually within the constant $d\bar{P}/d\bar{x}$ region. Only taps beyond this entry were used in data reduction. At $R = 3000$, Hartnett, Koh and McComas (1962) suggest that for abrupt entries, $(\bar{x}/\bar{D}_h)_{\text{entry}} \approx 20$ while the present measurements indicate $(\bar{x}/\bar{D}_h)_{\text{entry}} \approx 24$, so that only the downstream two-thirds of the taps were useful. At the lowest R considered here, $R = 1150$, the same authors suggest $(\bar{x}/\bar{D}_h)_{\text{entry}} = 0.033 R$ for laminar flow and the present measurements suggest $(\bar{x}/\bar{D}_h)_{\text{entry}} = 0.030 R$, so that only three taps were useful.

It is perhaps worth pointing out here, as well as repeating it in Appendix C, that for laminar flows the shear stress probe was located just within the fully developed region when $R \leq 1000$. Thus, between $R = 1000$ and $R \approx 3000$, where a shorter turbulent entry would become apparent, the shear stress calibration was probably affected by entry

effects. While entry effects are important for calibration, where one relies upon the validity of the fully developed relationship between $\bar{\tau}_w$ and $d\bar{P}/d\bar{x}$, they present no problem for measurements in either the laminar or turbulent flows, which do not require fully developed flow for comparison to predictions since the predictions are able to handle developing flows. Fortunately shear stresses for the accelerated experiments were always high enough, even for laminarizing and laminar flows, that one could avoid the regions of calibration where insufficient entry length was a problem in the rectilinear duct flow. One might claim that the linear $\bar{E}^2 - \bar{\tau}_w^{1/3}$ relationship was closely adhered to only by the turbulent rectilinear duct flows, so it might be inappropriate to use this relationship for accelerated laminarizing and laminar flows. However the discussion in Chapter VII and Appendix C showed that the calibration should not be affected by the flow regime per se, so long as $\delta_t^+ < 5$. Ironically, most of the departures from the linear calibration relationship were attributable to systematic effects, such as the entry effect that prefaced this discussion, which were caused by the low flow rates required for laminar rectilinear flow.

In addition to uncertainties due to entry effects there is the possibility that small undetectable leaks were entering the flow system downstream of the test section, but upstream of the venturi flow meter. While these leaks would be insignificant for more typical operating conditions, they could have been important under the extremely low flow rate conditions, and could provide an explanation for the observation that the measured friction factors were less than the theoretical values

at low flow rates. An undiscovered systematic error in the flow meter could also cause this trend.

In summary, it appears that when the estimated overall uncertainties are considered, the results obtained with this flow test section are in agreement with both accepted laminar and turbulent results where appropriate. It is granted, however, that the laminar rectilinear flows were obtained under such unfavorable measurement conditions that these were avoided in the work to be described next.

Converging Radial Flows

Several introductory comments are made before the data are considered in detail: (1) In many studies of acceleration-induced laminarization, the investigators placed a constant area duct ahead of the region where the flow was accelerated, thus ensuring that the flow was fully developed and fully turbulent before undergoing acceleration. In this study, which was primarily motivated by the application to geothermal energy reservoirs, this pre-conditioning was not performed. Instead the flow was immediately accelerated upon entry. While the sharp corners at the inlet would have disturbed the entering flow significantly the available instrumentation did not permit a direct determination where the entering flow became fully turbulent, if indeed it ever did. (2) Since the flow was accelerated immediately, a fully developed profile was never attained, instead these flows had continuously changing profiles as did the laminar experiments of Chapter IV. Once again though, the numerical solutions treat the continuous readjustment of the profiles in these flows so that the predictions based upon laminar and van Driest turbulence

models provide reasonable bases for comparison with the data. (3) The reader is reminded that in this dissertation laminarization is defined as the reduction of measured pressure gradients and wall shear stresses compared to turbulent predictions. The term complete laminarization is used to indicate agreement with predictions based on a laminar analysis. The use of the term laminarization to imply the gradual nature of the process, without demanding completion of the process, is consistent with Launder (1964).

Table VII summarizes the turbulent and laminarizing experiments in a format similar to the one used for the laminar experiments, Table III. These runs are not presented in chronological order but rather in order of increasing half angle, and within each angle, in order of decreasing flow rate. Once again a wide range of flow parameters was desired and was attained. Flow rates, overall Reynolds numbers, pressure drops and shear stresses vary by almost three orders of magnitude. Generally speaking as one scans down the list both overall and local Reynolds numbers decrease, as do pressure drops and shear stress signals. As will be seen, extensive use will be made of the pressure drop from the inlet to the tap nearest to the shear stress probe vicinity, so this pressure drop, rather than the overall test section drop, has been tabulated.

For these runs the dimensionless radii were kept in the range roughly 0.1 to 1 so that acceleration would not overwhelm viscous effects. Compromises had to be accepted. Some level of acceleration was required to study acceleration-induced laminarization, but high levels of acceleration sometimes led to conditions where the component of pressure

TABLE VII
TURBULENT AND LAMINARIZING RADIAL EXPERIMENT RUNS

Run	Half-angle θ , (DEG)	Flow Rate ^a (m ³ /s / ft ³ /s)	$R_o \times 10^{-6}$	Range of Dimen. Radius	Local Reynolds Number at Shear Probe	Range of Local Reynolds Number	ΔP , ^b Inlet To Probe Position (mm Hg)	Shear Stress Signal (Volts)	Estimated Shear Stress Uncertainty (%)
T-1	1/4	58.9 / 2080	37.9	0.445-0.475	52400	51000-54600	3.37	-	-
T-2	1/4	31.7 / 1120	20.4	0.606-0.648	28200	27500-29400	1.15	-	-
T-3	1/2	30.3 / 1070	19.3	0.312-0.352	50300	48100-54700	3.44	-	-
T-4	1/2	15.9 / 563	10.3	0.425-0.485	27800	25700-29300	1.13	-	-
T-5	2	8.16 / 288	5.34	0.148-0.225	51000	39700-60500	3.93	9.141	8
T-6	2	5.32 / 188	3.38	0.186-0.283	33300	25900-39500	1.76	9.019	9
T-7	2	3.99 / 141	2.53	0.214-0.327	24900	19400-29500	1.05	8.952	10
T-8	2	2.66 / 94	1.69	0.262-0.400	16600	12700-19700	0.41	8.870	12
T-9	2	1.13 / 40	0.72	0.402-0.612	7060	5500- 8370	0.112	8.725	20
T-10	2	0.0 / 0.0	0.0	-	0	0	0.0	8.557 ^c	-
T-11	4	4.22 / 149.0	2.71	0.103-0.208	45600	30430-60200	3.39	9.148	8
T-12	4	2.66 / 93.9	1.71	0.129-0.262	28900	19300-38100	1.43	9.029	10
T-13	4	1.99 / 70.4	1.29	0.149-0.302	21600	14400-29100	0.84	8.968	11
T-14	4	1.33 / 47.0	0.85	0.183-0.370	14500	9670-19600	0.42	8.893	13
T-15	4	0.566/ 20.0	0.36	0.281-0.568	6150	4100- 8100	0.091	8.749	25
T-16	4	0.0 / 0.0	0.0	-	0	0	0.0	8.616 ^c	-
T-17	8	2.24 / 79.2	1.44	0.071-0.218	38100	21500-68400	2.38	9.115	8
T-18	8	1.42 / 50.0	0.85	0.092-0.284	22600	12800-39400	1.92	8.985	10
T-19	8	1.00 / 35.2	0.64	0.107-0.330	16900	9450-29400	0.54	8.913	12
T-20	8	0.665/ 23.5	0.43	0.131-0.404	11300	6380-19700	0.25	8.824	15
T-21	8	0.283/ 10.0	0.182	0.202-0.624	4830	2730- 8420	0.056	8.720	26
T-22	8	0.0 / 0.0	0.0	-	0	0	0.0	8.588 ^c	-
T-23	12	1.52 / 53.8	0.99	0.057-0.237	32200	16500-67900	1.77	9.033	9
T-24	12	0.886/ 31.3	0.57	0.076-0.313	18500	9470-39000	0.66	8.910	11
T-25	12	0.665/ 23.5	0.42	0.088-0.362	13800	7070-29100	0.38	8.846	13
T-26	12	0.445/ 15.7	0.284	0.107-0.442	9250	4740-19500	0.18	8.768	18
T-27	12	0.189/ 6.67	0.120	0.165-0.678	3950	2020- 8330	0.042	8.693	29
T-28	12	0.0 / 0.0	0.0	-	0	0	0.0	8.576 ^c	-
T-29	16	1.18 / 41.6	0.76	0.050-0.259	27700	13300-68420	1.40	9.020	9
T-30	16	0.665/ 23.5	0.42	0.066-0.339	15700	7550-38800	0.47	8.898	12
T-31	16	0.498/ 17.6	0.321	0.076-0.391	11700	5630-28900	0.29	8.834	15
T-32	16	0.331/ 11.7	0.214	0.093-0.479	7880	3790-19500	0.143	8.776	18
T-33	16	0.142/ 5.00	0.090	0.144-0.739	3300	1590- 8150	0.031	8.698	31
T-34	16	0.0 / 0.0	0.0	-	0	0	0.0	8.589 ^c	-

^aFlow rate corresponding to full 360° opening (equals $360 \bar{Q}_{act}/20$).

^bActual ΔP measured from first tap at 25mm from inlet to tap 25mm downstream of probe.

^cZero flow shear voltages taken 5 minutes after cessation of flow.

drop due to acceleration alone was so high that the remaining pressure drop attributed to either laminar or turbulent viscous effects was small relative to it. In these cases the differences between the pressure drop predictions for the fully-laminar flow and the fully-turbulent flow were so small that it was difficult, after considering overall uncertainties, to ascertain the extent of laminarization by comparing measured pressure drops to the predicted quantities. This difficulty is described in more detail below.

For practically all runs the uncertainty due to stray air currents is small so that the only significant pressure drop uncertainties are due to flow rate and spacing, and the overall pressure drop uncertainty is 6 percent. Even in the worst case, run T-33, where stray air currents amount to 3 percent, the overall uncertainty is only raised to 6.6 percent, so no distinction is made for the remainder of this work; a constant uncertainty of 6 percent is used. The situation is quite different for shear stresses. Because of the generally low values of $\Delta \bar{E} = \bar{E} - \bar{E}_0$ and the sensitive relationship of $\bar{\tau}_w$ to $\Delta \bar{E}$, these shear stress uncertainties are rather high, and they increase with half angle and with decreasing flow rate. Compounding these uncertainties is the fact that at higher angles the pressure drop component due to acceleration increases so that the differences between the viscous pressure drops predicted for laminar and turbulent flows are diminished relative to the total pressure drop. As will be seen, attempts to attribute differences between measured and predicted results to laminarization become insensitive and result in large uncertainties, particularly at large angles and low flow rates.

Examples of this insensitivity are found in Figures 19 through 21. Figure 19 shows the best situation, i.e., lowest angle and highest local Reynolds number, so that the turbulent and laminar pressure distributions lie furthest apart. The local Reynolds number, R_{probe} was evaluated at the probe position. To avoid the immediate effects of the vena contracta at the entry the pressure tap nearest the inlet has been eliminated from consideration, so the pressure drops are measured with respect to the second tap, 100 mm (4 in.), from the entry. The minimum Reynolds number tabulated for each run in Table VII was also evaluated at this tap, whereas R_{entry} which appears in Figures 19 through 21 was evaluated at the actual entry. This accounts for the slight discrepancies between the two Reynolds numbers. The predictions have also been referenced to the same position so that they are directly comparable. These predictions are based upon a uniform entry velocity profile. It was assumed that the turbulent model was applicable immediately at the entry, i.e., no provision was made for the possible existence of a short laminar boundary layer followed by transition to turbulent flow. Because the vena contracta caused by the abrupt entry was followed by a short region of adverse pressure gradient the transition to turbulence, if it occurred, probably occurred close to the entry; however, the appropriate position of the virtual origin of the turbulent boundary layer remains uncertain since profile measurements were not made. In spite of this uncertainty the pressure data agreed with the predicted distribution within about eight hydraulic diameters for the rectilinear flows.

Figure 19. Measured and Predicted Pressure Distributions, $\theta = 1/4^\circ$.

Figure 20. Measured and Predicted Pressure Distributions, $\theta = 2^\circ$.

Figure 21. Measured and Predicted Pressure Distributions, $\theta = 16^\circ$.

For run T-1, Figure 19, it would appear from any of the laminarization criteria of Table IV that no laminarization should be expected, and the measured pressure drops, while differing from the turbulent prediction by slightly more than the estimated uncertainties, are close to the turbulent predictions. Figure 20 shows a more typical case; here the laminar and turbulent predictions are not as far apart and some indication of the insensitivity problem is seen -- the overall uncertainty ranges from 30 to 50 percent of the difference between the laminar and turbulent predictions. These data are significantly lower than the turbulent prediction and, considering the higher value of K_1 , one might reasonably suggest that the turbulent flow was laminarized to a partial degree. In fact the difference between the data and turbulent predictions will be used below as an ad hoc indicator of laminarization. Also shown in this figure are the results of two turbulence models which have been modified to account for laminarization. These will be discussed in more detail later.

A more severe lack of sensitivity occurs in Figure 21. For this run the dimensionless radius ranges from 0.14 to 0.74 so acceleration effects were large. The laminarization parameters K_1 through K_3 for this flow were so large that a completely laminar flow would be expected; however, the predictions are so close together that the measurements could almost be matched by either prediction and the information content regarding the extent of laminarization is effectively lost. In some cases this is an unimportant point. For the engineer interested in only predicting pressure drops the state of the flow is clearly of little importance for highly accelerated flows like this one.

To quantify the laminarization process further, an ad hoc laminarization indicator has been defined as

$$\phi_p = \frac{\Delta \bar{P}_{\text{meas}} - \Delta \bar{P}_l}{\Delta \bar{P}_t - \Delta \bar{P}_l} \quad (8-3)$$

where, for convenience, all $\Delta \bar{P}$'s are evaluated from the first tap from the entry to the tap nearest the shear stress probe. However, as will be discussed, the result is independent of the radius chosen so that taps other than the one closest to the probe could have been used. When ϕ_p is zero the measurements agree with the laminar prediction and when ϕ_p is unity, the measurements agree with the prediction from the van Driest turbulent model. One should note that actual turbulence quantities such as velocity fluctuations or the kinetic energy of these fluctuations are not involved in the definition of the laminarization indicator, only the effects of turbulence are. However this approach is consistent with some of the philosophy underlying turbulence modeling with the simpler models; one does not usually attempt to calculate actual turbulence quantities but instead tries to predict the effect of turbulence on mean flow quantities. Furthermore the approach is useful in a practical sense. Using the graphical correlation of ϕ developed later, one can compute pressures (and shear stresses and friction factors) whereas the typical design engineer would be hardpressed to use true turbulence quantities such as turbulence kinetic energy, etc. effectively.

Several theoretical objections to the present approach can be raised. For example, Narasimha and Sreenivasan (1973) have argued that inferring laminarization from a "standard" turbulence law suffers from

an inherent difficulty because there is no universal turbulence law which can be expected to yield satisfactory results in all situations. However, as discussed in Chapter VI the van Driest turbulence model has proven useful for a wide variety of flows; there is no reason, a priori, to expect that in the absence of laminarization a change of geometry from the usual linear one to the related radial one considered in this investigation would impair the usefulness of the model. In fact we will see that for some radial flows, where the values of the various laminarization parameters were such that laminarization effects would not be expected, the van Driest model provides pressure drop predictions in reasonable agreement with measurements. Finally, considering the alternative, if one denies the existence of valid turbulence models, then one can only examine how close to completely laminar flow a process is while never knowing how far it differs from a turbulent state. Even Narasimha and Sreenivasan were discomforted by this prospect; they eventually compared measurements with "analyses expected a priori to be valid on either side of laminarization."

With the exception of those runs with $\theta = 12^\circ$ and 16° , where the insensitivity discussed above resulted in unacceptably high uncertainties, all the runs of Table VII were examined according to equation (8-3). The laminarization indicators so generated are presented in Figures 22 and 23. The shear velocity $\bar{U}_\tau$ that appears in the definition of K_2 or K_3 was evaluated with the aid of the shear stress probe, which was not in place for the $\theta = 1/4^\circ$ or $1/2^\circ$ runs. Additionally, those runs which incorporated wall shear stress measurements were examined according to an equation analogous to equation (8-3), but replacing

Figure 22. Laminarization in Terms of Pressure Drops and K_1 .

Figure 23. Laminarization in Terms of Pressure Drops and K_2 or K_3 .

pressures with shear stresses so as to form an ad hoc laminarization indicator based upon wall shear stress, ϕ_T . Once again the 12° and 16° results were excluded because of insensitivity difficulties. The results based upon shear stresses are presented in Figures 24 and 25.

In Figures 22 through 25 it will be noted that at low values of K_1 through K_3 , where one would expect no laminarization, $\phi = 100$ percent, the measurements indicate only 80 to 90 percent. Much of this problem, in particular for the $\theta = 2^\circ$ and $\theta = 4^\circ$ data points, is a result of the low local Reynolds numbers ($R_{\text{probe}} = 4830\text{-}6150$) associated with these data. It will be recalled from Chapter VI that the van Driest model tends to yield turbulent predictions for wall shear stresses and pressure gradients that are approximately 10 percent higher than either the Blasius or Drew, Koo and McAdams correlations. This explanation, however, does not pertain to the $\theta = 1/4^\circ$ and $\theta = 1/2^\circ$ runs where the Reynolds numbers were relatively high. While these data do reach 100 percent, if one includes the uncertainty band, the individual points are consistently less than 100 percent, ranging from 82 to 90 percent. However, these data were obtained with a Plexiglas plate having neither shear probe nor slider assembly. Since it was only used temporarily it was not planed, so that these effects may be attributable to greater spacing and waviness uncertainties which can no longer be quantified because this plate was removed from the experimental apparatus as soon as the improved plate became available.

While it is thus true that large experimental uncertainties mar the graphical correlations of Figures 22 to 25, there are clear trends that the ad hoc laminarization indicators decrease with either increasing

Figure 24. Laminarization in Terms of Shear Stress and K_I .

Figure 25. Laminarization in Terms of Shear Stress and K_2 or K_3 .

K_1 or increasing $K_2(K_3)$. K_1 apparently provides the best correlation. There are no unique critical values but it appears that when based on either pressure drop or shear stress the process is approximately complete at $K_1 = 2 \times 10^{-6}$ to 5×10^{-6} or, with considerable scatter apparent, at K_2 and $K_3 = 0.02$ to 0.05 . With the exception of K_3 these values are reasonably close to the values quoted by other authors that are cited in Table IV. The present value for K_3 is two to five times the value listed in Table IV [five to twelve times when corrected by Narasimha and Sreenivasan (1973)] so it is concluded that unlike K_1 or K_2 , there is too much variation in K_3 for it to serve as a laminarization criterion useful for many flow geometries. It may be recalled that this conclusion was also suggested in Chapter V for laminar flow, where at the limits $r \gg 1$ or $r \ll 1$, choice of K_3 led to bounds on R or R_0 that were clearly out of line with other estimates.

Based on the reduced scatter of the data in Figure 22 compared to that of Figure 23, one might be tempted to argue that K_1 is the more definitive guide, but it must be acknowledged that this study has failed to resolve clearly whether K_1 or K_2 is the appropriate parameter that governs laminarization. The other parameter, R_2 , was not investigated in this study since $\bar{\delta}_2$ was not measured. However, at least a better technological base has been established for the use of K_1 or K_2 , since it has now been established that their numerical values provide approximately the same criteria for radial geometries as for pipe and channel flows.

At this point we return to the earlier remark that ϕ_p became independent of radius, suggesting that the partial laminarization began

close to the entry. As related in the Introduction, Badri Narayanan and Ramjee (1969) have proposed that the laminarization process is a sequential one, involving first the cessation of turbulent bursts, then breakdown of the logarithmic law of the wall, and finally, decay of turbulence intensity. They look on the laminarization process as a wall dominated one in which the wall region reflects the effect of laminarization quickly while the outer regions adjust more slowly. On the other hand Narasimha and Sreenivasan (1973) also suggest that laminarization is an asymptotic process, but they assert that reversion starts in the outer layer and that wall parameters lag. The latter view point is opposite to those of (1) Badri Narayanan and Ramjee, (2) Huffman and Bradshaw (1972), who support the thesis that the sublayer reacts (by thickening) to acceleration-induced laminarization before the fully turbulent outer layer reacts, and (3) Blackwelder and Kovasznay (1972), who showed that the absolute magnitudes of the turbulent velocity fluctuations remained nearly constant in the outer portions of a boundary layer undergoing rapid acceleration. Despite this fundamental difference, however, it still appears that the above investigators claim that laminarization, whether complete or incomplete, is a delayed process, whereas the present experiments indicate that primary adjustment occurs almost immediately upon entry in terms of the laminarization indicators defined above.

In all likelihood the disparity is caused by differences of scale. Narasimha and Sreenivasan (1973) have provided a convenient catalog of many of the experimental configurations used by various authors to investigate laminarization. These configurations include

two-dimensional nozzles, inclined plates or wedges, channels with flexible walls, and pipes with center bodies shaped such that the net cross-sectional area changed in the downstream flow direction. In all these cases however, the flow channel spacings were quite large, typically 300 mm (12 in.), so that the downstream distance over which the acceleration was imposed was only several channel spacings. In contrast the present channel spacing was only 12 mm (0.5 in.), and the acceleration extended over 90 channel spacings. Furthermore, the first pressure tap at which the laminarization process could be checked was already 8 channel spacings downstream of the entry or 14 spacings when the first tap could not be used because of the vena contracta at the entry.

Wall Shear Stress Signals

Temporal Observations

To show that laminarization as defined in terms of ϕ_p and ϕ_τ also corresponds to reductions in turbulence quantities such as $\sqrt{(\bar{u}')^2}/\bar{u}$ it would be desirable to have hot wire measurements across the boundary layer. In many of the converging experiments the estimated boundary layer thicknesses became extremely small, decreasing to 1/4 mm, so obtaining meaningful profile measurements with an immersed hot wire would have been impractical in these cases. However, Eckelmann (1974) has shown that the fluctuating shear stress signal from a heated element mounted at the wall correlates with velocity fluctuations up to a distance from the wall of $y^+ \approx 25$. Thus the temporal history of the shear stress signal can be expected to provide a reasonable picture of turbulent events in the viscous layer. Examination of the temporal history

of the wall shear stress signal was begun late in this investigation to study the feasibility of this proposal. Only limited data were obtained. Consequently, the observations of this subsection are primarily introductory and qualitative in nature.

Oscilloscope photographs of the fluctuating shear stress signal are presented in Figures 26 through 29. To facilitate comparisons all figures have the same scale. These voltage fluctuations can be converted to shear stress fluctuations via equation (C-11) and the $\Delta \bar{E}$'s tabulated in Table VII. In order to focus attention on acceleration effects the photographs have been grouped together so that the Reynolds numbers at the probe are approximately the same in any one figure. High frequency noise attributed to the anemometer electronic circuitry is superimposed upon the shear stress fluctuations shown in Figures 26 through 29. This background noise is approximately 10 mV peak-to-peak and about 10 kHz or more in frequency. The bottom photograph of Figure 29, taken without flow, illustrates this noise.

In Figure 26 a rectilinear flow at high Reynolds number ($R \approx 30,000$ and $\theta = 0^\circ$), shows a typical signal for a fully developed turbulent duct flow. There are large shear stress signal fluctuations with amplitudes as high as 30 mV, corresponding to shear stress fluctuations of 20 percent of the mean value. The frequency of the larger fluctuations, which may correspond to wall bursts, is about 300 Hz. With increasing θ and K_1 , the amplitudes of the larger fluctuations are diminished slightly but the flow remains essentially turbulent and the apparent frequency of the large scale fluctuations is approximately the same. Both of the lower subfigures represent flows with some laminarization in terms of ϕ_p .

Figure 26. Fluctuating Shear Stress Signals, High Reynolds Number.

Figure 27. Fluctuating Shear Stress Signals, Intermediate Reynolds Number.

Figure 28. Fluctuating Shear Stress Signals, Moderate Reynolds Number.

Figure 29. Fluctuating Shear Stress Signals, Low Reynolds Number.

Figure 27 shows results at a reduced Reynolds number, $R \approx 20,000$ with the middle two subfigures at slightly higher Reynolds numbers than the rectilinear flow run. Considering the rectilinear flow run first, one sees that both the amplitude and frequency are slightly reduced compared to the comparable trace in Figure 26. Some investigators such as Black (1968) suggest that bursting frequencies should scale on wall variables as $\bar{U}_t^2/\bar{\nu} \bar{f}_b \approx 100$ while Rao, Narasimha and Badri Narayanan (1971) and others recommend outer variables in the form $\bar{U}_\infty/\delta \bar{f}_b \approx 5$, where $\bar{f}_b$ is the bursting frequency. For the conditions of the present rectilinear flow experiments, these suggestions have been converted to estimated bursting times in Table VIII. Since these turbulent flows are believed to be fully developed the relationship for outer variables has been evaluated as $\bar{V}/\bar{t}\bar{f}_b \approx 5$. The trend in Figures 26 to 29 is in agreement with both methods of scaling; the limited data might be interpreted as favoring the wall scaling, but more measurements would be necessary to discriminate between the two.

TABLE VIII

APPROXIMATE PREDICTED TIMES BETWEEN BURSTS FOR
RECTILINEAR DUCT FLOW

<u>R =</u>	<u>30,000</u>	<u>20,000</u>	<u>10,000</u>	<u>7000</u>
Wall Scaling, msec	1.2	2.5	8.2	15.4
Outer Scaling, msec	1.3	2.0	4.0	5.7

Returning to Figure 27, one sees that as θ or K_1 is increased the amplitude and the frequency of the signal decreases. The lower two subfigures correspond to flows where K_1 is sufficiently large that ϕ_p approaches complete laminarization but it is obvious that there are still significant turbulent events in the flow. On these subfigures one can identify segments which have the appearance of the typical bursting event described by Blackwelder and Kaplan (1976). With the increase in acceleration it is possible that the turbulent boundary layers never reached the center plane in these flows; in that case, both estimates of non-dimensional bursting frequencies would predict an increase in bursting frequency rather than the reduction observed. Thus, this figure shows that the apparent laminarization in terms of wall parameters does correspond to a reduction in turbulent activity.

The accelerated flows in Figures 28 and 29 show almost no evidence of fluctuations. It appears that if $K_1 > 2.3 \times 10^{-6}$ the fluctuations are almost reduced to zero. However, this apparent agreement with laminarization indicators ϕ_p and ϕ_τ may be fortuitous. If wall scaling was the appropriate measure of bursting times and acceleration reduces the frequency as observed by Kline et al. (1967), then the period between bursts could be longer than the 10 msec traces. Another possible explanation for runs T-15, T-20 and T-31 was that at their high values of K_1 they remained as laminar flows after entering the apparatus.

The general observation from Figures 26 to 29 is that at a given approximate Reynolds number increasing convergence leads to reductions in the amplitude and frequency of turbulent fluctuations. Likewise, at a given convergence angle reducing the Reynolds number reduces the

frequency, as expected. However, more extensive measurements are necessary to quantify these conclusions or to attempt to resolve the question of the appropriate scaling of burst frequencies; such measurements are considered beyond the scope of the present work.

Mean Shear Stresses

The time-mean shear stresses are summarized in Table IX as local friction factors. The Reynolds number was evaluated at the probe location. Despite the wide range of accelerations and Reynolds numbers, the friction factor is almost a constant. If the individual friction factors are simply averaged the mean value is 0.0056, with 0.00015 as a standard deviation. If one then considers the uncertainties in the individual data, only 6 of the 25 runs have measured shear stresses which fail to straddle the mean value. Furthermore, these six discrepancies are relatively small and are scattered (one occurs at $\theta = 2^\circ$, 2 at 4° , 2 at 8° and 1 at 12°), so it appears that to a first approximation a constant friction factor represents the data fairly.

Interestingly, Coles (1962) has remarked that 0.0048 appears to be the maximum value of friction factor that a turbulent boundary layer could attain in a zero pressure gradient. Although the connection with accelerated flows would appear tenuous, it is notable that for the sink flow experiments of Jones and Launder (1972) f varied only from 0.0042 to 0.0056 for K_1 ranging from 1.5×10^{-6} to 3×10^{-6} . Using a similar geometry, Simpson and Wallace (1975) also found 0.005 as the maximum f attainable in accelerated flows. In addition Narasimha and

TABLE IX

SUMMARY OF MEASURED FRICTION FACTORS

Half Angle (Deg)	Run	R	r	$K_1 \times 10^6$	$f \times 10^3$
2	T-5	51,000	0.148 - 0.225	0.19	4.8 ± 0.4
	T-6	33,300	0.186 - 0.283	0.30	5.2 ± 0.5
	T-7	24,900	0.214 - 0.327	0.40	5.5 ± 0.6
	T-8	16,600	0.262 - 0.400	0.59	6.3 ± 0.8
	T-9	7,060	0.402 - 0.612	1.39	5.2 ± 1.0
4	T-11	45,600	0.103 - 0.208	0.37	3.8 ± 0.2
	T-12	28,900	0.129 - 0.262	0.59	5.1 ± 0.5
	T-13	21,600	0.149 - 0.302	0.78	5.2 ± 0.6
	T-14	14,500	0.183 - 0.370	1.17	5.9 ± 0.8
	T-15	6,150	0.281 - 0.568	2.75	3.5 ± 0.9
8	T-17	38,100	0.071 - 0.218	0.70	5.4 ± 0.4
	T-18	22,600	0.092 - 0.284	1.17	6.9 ± 0.7
	T-19	16,900	0.107 - 0.330	1.57	6.7 ± 0.8
	T-20	11,300	0.131 - 0.404	2.34	6.0 ± 0.9
	T-21	4,830	0.202 - 0.624	5.49	5.5 ± 1.4
12	T-23	32,200	0.057 - 0.237	1.01	5.6 ± 0.5
	T-24	18,500	0.076 - 0.313	1.76	6.2 ± 0.7
	T-25	13,800	0.088 - 0.362	2.36	5.7 ± 0.7
	T-26	9,250	0.107 - 0.442	3.52	4.5 ± 0.8
	T-27	3,950	0.165 - 0.678	8.26	5.6 ± 1.6
16	T-29	27,700	0.050 - 0.259	1.32	6.0 ± 0.5
	T-30	15,700	0.066 - 0.339	2.33	6.6 ± 0.8
	T-31	11,700	0.076 - 0.391	3.11	6.0 ± 0.9
	T-32	7,880	0.093 - 0.479	4.67	6.0 ± 1.1
	T-33	3,300	0.144 - 0.739	11.10	6.4 ± 2.0

Average = 0.0056 ± 0.0003^a ^aBased on two standard deviations.

Sreenivasan (1973) have also suggested that f is roughly constant throughout the laminarization process.

Simple Turbulence Modeling

Both Launder and Jones (1969) and Huffman and Bradshaw (1972) have observed that an effect of acceleration is a thickening of the linear sublayer next to the wall. These investigators suggested that for predictions this thickening could be embodied in the damping factor A^+ , by letting A^+ be a function of the laminarization parameter K_2 or K_3 rather than a constant. Launder and Jones chose to represent the functional dependence in terms of K_2 for their sink flow between inclined plates. Using a van Driest turbulence model similar to the one used here, they systematically varied A^+ until the computed friction factor matched 0.0048 approximately.

Huffman and Bradshaw also used a van Driest model but, rather than matching f , they required that the predicted velocity profiles match measured profiles from the wall to 20 percent of the flow width (typically $y^+ \sim 10$ to 1000) for various flows. These flows included fully developed duct flows (Patel and Head, 1969) axisymmetric wall jets (Starr and Sparrow, 1967) and boundary layers (Julien, Kays and Moffat, 1970).

Both sets of results are presented in Figure 30. Also shown are calculations from the present study in which a constant friction factor of 0.0048 was forced. While the writer would have preferred to fit $f = 0.0056$ to correspond with typical data of the present study, 0.0048 was used instead so as to allow a direct comparison with Launder

Figure. 30. Comparison of $A^+(K_2 \text{ or } K_3)$ Functions.

and Jones' A^+ variation with K_2 . For these calculations θ was set to 2° and the conditions that corresponded to runs T-5 through T-9 were specified. For each run A^+ was varied iteratively until the friction factor calculated for the measuring station was within 1 percent of that desired. A 2° half angle was used because then the radial variations of K_2 and f were less than ± 4 percent after the entry region. These small variations were desired in order to correspond to the conditions of Launder and Jones or Huffman and Bradshaw, whose results were independent of streamwise position.

Launder and Jones presented their A^+ variation in terms of K_2 , Huffman and Bradshaw used K_3 , and the present calculations used K_2 . However, the present calculations can be compared to either of the previous investigations since $K_2 = K_3$ for radial flows. Since the procedure followed was the same, one would expect good agreement between the present results and those of Launder and Jones, so that the disparities between the two $A^+(K_2)$ relations on Figure 30 might be attributed to the differences of geometry of the flows under consideration.

The results of Huffman and Bradshaw imply a more abrupt laminarization; below $K_3 = 0.015$, A^+ is 60 or less, so that predicted results would not differ significantly from fully turbulent predictions whereas above 0.020 A^+ increases sharply. These authors strongly imply that the critical value of K_3 is about 0.02, not 0.009 as claimed by Patel and Head (1968). Thus, Huffman and Bradshaw are in agreement with the present observations in Figures 23 and 25, that should laminarization be interpreted in terms of K_3 , then the process is essentially completed by $K_3 = 0.02$ to 0.05. The present writer also found a sharp increase of

A^+ above $K_2 = 0.02$ and at $K_2 = 0.025$ A^+ approached infinity, implying that the flow becomes completely laminar. As a check, a fully laminar calculation for the same conditions resulted in $f = 0.0048$. A K_2 value of 0.025 is in reasonably good agreement with the value provided by Patel and Head, 0.018, and again with the values of 0.02 to 0.05 implied in Figures 23 and 25. The results of Launder and Jones do not show any indication of the completion of laminarization -- it is not known whether the calculations simply were not carried out to sufficiently large values of K_2 or whether this is an artifact of their calculational method.

With the $A^+(K_2)$ model derived here, velocity profiles were computed for run T-7. This run had such a modest radial variation that K_2 was almost constant at 0.0067 and $A^+ = 55$. The profile at the shear stress probe location is shown in u^+, y^+ coordinates in Figure 31. Also shown for comparison are the results of the unmodified model, the "universal" logarithmic law and the laminar calculation. As expected, the modified model shows a slightly thickened linear sublayer compared to the usual van Driest model. In addition, predicted pressure drops for the modified model presented here and that proposed by Launder and Jones were presented for this run earlier in Figure 20. Both modified models show improved agreement with the data but the one proposed here is somewhat better. Furthermore it is clear that had the measured friction factor of 0.0056 been used rather than 0.0048, the agreement would have been better yet.

The final picture that evolves from this research, while somewhat uncertain numerically, is a remarkably consistent one. Starting

Figure 31. Comparison of Computed Velocity Profiles, Run T-7.

about $K_1 = 0.3 \times 10^{-6}$ ($K_2 = K_3 = 0.006$) a sustained acceleration can be expected to show evidence of laminarization. Below these values the van Driest model should give reasonable agreement (Figure 18) with the data but above these values some modification is required. Increasing laminarization parameters result in (1) reduced turbulent fluctuations, (2) apparently increased linear sublayer thickness and, (3) a regime in which the friction factor is approximately 0.005 or so. At $K_1 = 2 \times 10^{-6}$ to 5×10^{-6} , or $K_2 = K_3 = 0.02$ to 0.05 , the process is complete and flow is apparently completely laminar. It was shown in Chapter III that when laminar at small nondimensional radii f approaches $2.17\sqrt{K_1}$; thus, if laminarization becomes complete the friction factor is expected to become independent of r at a value determined by the acceleration parameter K_1 . For $K_1 = 5 \times 10^{-6}$, this asymptotic friction factor would be 0.0049 for a laminarized flow. Modified turbulence models, in which A^+ is a function of the acceleration, provide reasonably good agreement with measured pressure distributions and also yield results for the completion of laminarization which are consistent with both the ad hoc ϕ_p and ϕ_τ indications and the apparent disappearance of wall shear stress fluctuations.

CHAPTER IX

CONCLUSIONS

Laminar Flows

A significant parameter characterizing laminar flow converging radially inward between two stationary, parallel disks is the overall Reynolds number formed from the volumetric flow rate, kinematic viscosity and disk spacing as $R_0 = \bar{Q}/(4\pi\bar{\nu}t)$. In turn, the dimensionless radius formed from the local radius, disk spacing and the root of the overall Reynolds number, $r = \bar{r}/(\bar{t}\sqrt{R_0})$, gives the local scale of the importance of viscous effects relative to acceleration effects. At large values of the overall Reynolds number the boundary layer approximations are applicable, even as the dimensionless radius approaches zero.

Dimensional analysis showed that, when properly scaled, the velocities become functions of only r and z and the pressure distribution and friction factors become functions of r alone beyond the entry region. At large radii results correspond to well known values for fully developed, parallel flow between infinite plates. As the radius becomes less than about 3, the numerical predictions diverge from the parallel results; consequent values of the integral parameters are listed in Table I. For $r \lesssim 0.2$ the friction factor approaches a value $2.17/\sqrt{R_0}$ which is independent of radius; this numerical result is confirmed by a similarity analysis.

It was determined that the equation of Livesey (1960) for dimensionless pressure

$$P = 3 \ln(r_i/r) + 0.6 \left(\frac{1}{r^2} - \frac{1}{r^2} \right) \quad (3-1)$$

is in good agreement with the present numerical results for $r \geq 0.1$, but that this approximation, and other comparable approximations, provide poor estimates of that portion of the pressure change that is irreversibly lost due to viscous shear. The present results for pressure distributions, Figures 5 and 6, and the present friction factors, either Figure 7 or Table I are recommended. An expression combining the limiting values of the friction factor,

$$\frac{f \cdot R}{24} = \sqrt{1 + \left(\frac{0.36}{r} \right)^2} \quad (3-4)$$

approximates the numerical results to within 8 percent.

Measurements of pressure distributions for flow between two circular disks over the ranges $210 < R < 20,700$, $0.048 < r < 3.35$ and $9.5 \times 10^{-6} < K_1 = 1/R_0 < 2.2 \times 10^{-4}$, agreed with the numerical predictions within the estimated experimental uncertainty. At lower Reynolds numbers the numerical predictions verified the experimental technique. At higher Reynolds numbers this agreement, in conjunction with smoke flow visualization, confirmed the stability of these converging laminar flows for acceleration parameters, $K_1 = (\bar{v}/\bar{v}^2)d\bar{v}/d\bar{r}$, greater than 9.5×10^{-6} .

Turbulent and Laminarizing Flows

Experiments were conducted for flow between parallel plates, with a sharp-cornered entry and convergence due to adjustable edge walls, over the following ranges: $1590 < R < 68,400$, $2.6 \times 10^{-8} < K_1 < 1.1 \times 10^{-5}$ and $0.05 < r < 0.74$. The numerical analysis was extended to include the van Driest (1956) mixing length model; comparison to measured pressure distributions and friction factors showed:

1) agreement with turbulent predictions if

$$K_1 \gtrsim 3 \times 10^{-7}$$

$$K_2 \gtrsim 0.006$$

$$K_3 \gtrsim 0.006$$

2) agreement with laminar predictions when

$$K_1 \gtrsim 2 \times 10^{-6} \text{ to } 5 \times 10^{-6}$$

$$K_2 \gtrsim 0.02 \text{ to } 0.05$$

$$K_3 \gtrsim 0.02 \text{ to } 0.05$$

where the other acceleration parameters are defined as $K_2 = (\bar{v}/\bar{\rho} \bar{U}_\tau^3) d\bar{P}/d\bar{r}$ and $K_3 = (\bar{v}/\bar{\rho} \bar{U}_\tau^3)(\partial\bar{\tau}/\partial\bar{z})_w$. The criteria based on K_1 or K_2 are in approximate agreement with values found by other investigators for flows accelerating in geometries differing from the present experiment. It is recommended that K_3 be rejected as a useful criterion since its critical values appear to vary significantly from one flow situation to another.

The phenomenon of laminarization was defined in this study as the reduction of measured pressure gradients and wall shear stresses

compared to turbulent predictions to coincide with the usage of Launder (1964). Empirical adjustment of the van Driest parameter A^+ to form a function $A^+(K_2)$, as plotted on Figure 30, yielded numerical predictions in essential agreement with the measurements for laminarizing and laminar flows as well as the turbulent experiments. The resulting predictions of laminarizing flows showed a thickened viscous layer when compared to turbulent predictions in terms of wall law coordinates, $u^+(y^+)$.

Qualitative measurements of the temporal wall shear stress were obtained with a hot film sensor located two-thirds of the way along the converging duct. These limited data showed that at an approximately constant Reynolds number increasing acceleration led to reductions in the amplitude and frequency of turbulent fluctuations. Above $K_1 = 2.3 \times 10^{-6}$ the fluctuations appeared to be reduced to below the level of the electronic noise when the flow reached the sensor station. The observations with the film sensor were generally in agreement with the determination of laminarization from the comparisons between the measured integral parameters and the numerical predictions.

APPENDIX A

NONDIMENSIONALIZATION AND ORDER-OF-MAGNITUDE ANALYSIS

The nondimensionalization and order-of-magnitude analyses presented here are primarily concerned with laminar flow. However some results are qualitatively applicable to turbulent flows if one takes account of the higher effective viscosities. For example, in Chapter VI it is shown that the dimensionless boundary layer momentum equation for turbulent flow [equation (6-8)] is similar to that of laminar flow [equation (2-5)] but an additional parameter $\bar{v}_{\text{eff}}/\bar{v}$ appears.¹ In the development of the equations that follow only laminar flows are considered explicitly.

Nondimensionalization of Governing Equations

A characteristic velocity for the radial velocity component which provides a reasonable scaling ($u \approx 0$ to 1) for the nondimensional radial velocity is given by the bulk velocity $\bar{V} = \bar{Q}/4\pi\bar{r}\bar{t}$. Since the axial velocity component is small, it will be shown that a characteristic velocity that properly scales $\bar{w}$ is $\bar{V}/\sqrt{R_0}$. An obvious characteristic length that properly scales the coordinate $\bar{z}$ is given by $\bar{t}$, the half spacing. Characteristic values for the pressure and radial coordinate $\bar{r}$ are not as obvious, but a reasonable choice for the characteristic pressure is $\bar{\mu} \bar{Q}/4\pi\bar{t}^3$, and since $\bar{r}$ is much larger than $\bar{t}$, a reasonable characteristic

1. Overscored variables are dimensional; those without overbars are dimensionless.

radius is $\bar{t}\sqrt{R_0}$. Formally, then, the following nondimensional quantities are defined

$$\left. \begin{aligned} u &= \bar{u}/\bar{V} \\ w &= \bar{w}\sqrt{R_0}/\bar{V} \\ z &= \bar{z}/\bar{t} \\ r &= \bar{r}/\bar{t}\sqrt{R_0} \\ p &= \frac{\bar{p}_i - \bar{p}}{\bar{\mu}\bar{Q}/4\pi\bar{t}^3} \end{aligned} \right\} \quad (A-1)$$

The first three of these quantities have order-of-magnitude of unity since maximum values of u , w , and z are approximately one, but the order of magnitude of r is variable since the maximum value of $\bar{r}$ is unrestricted ($\bar{r} \rightarrow \infty$). The choice of a characteristic pressure involving viscosity, rather than the dynamic pressure $\bar{\rho}\bar{V}^2$, results in a nondimensional P which can be orders-of-magnitude greater than one for small r , and approximately one at large r .

If the dimensionless variables defined by equation (A-1) are substituted in equation (2-1a), the dimensionless continuity equation becomes

$$\left. \begin{aligned} \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} &= 0 \\ \frac{1}{r} &\quad \frac{w}{t} \end{aligned} \right\} \quad (A-2)$$

The orders-of-magnitude of the terms are written directly below each term. Differentiation of the characteristic scaling velocity with respect to r results in an interesting transformation of the continuity

equation - from one suitable to axisymmetric cylindrical coordinates to one suitable for rectangular cartesian coordinates.

Since both terms in equation (A-2) are equally important it is seen that the order of magnitude of w is $1/r$. It will be shown when the r -component of the Navier-Stokes equation is nondimensionalized that if r is less than one, a boundary layer of nondimensional thickness δ develops. This boundary layer thickness has an order of magnitude equal to r , so the gradient $\partial w / \partial z$ has order of magnitude w/r . Thus, with the aid of equation (A-2) the orders of w are

$$\begin{aligned} w &\approx 1/r & (r > 1) \\ w &\approx 1 & (r < 1) \end{aligned} \quad (A-3)$$

Since w is scaled by $\bar{V}/\sqrt{R_0}$ and R_0 is expected to be large, e.g., typically 10^6 , it is seen that the actual magnitudes of $\bar{w}$ are very small compared to $\bar{u}$.

If the dimensionless variables are substituted in equation (2-1b), the dimensionless equivalent of the r -component of the Navier-Stokes equation becomes

$$\begin{aligned} u \frac{\partial u}{\partial r} - \frac{u^2}{r} + w \frac{\partial u}{\partial z} &= r^2 \frac{\partial P}{\partial r} + r \left(\frac{\partial^2 u}{\partial z^2} + \frac{1}{R_0} \frac{\partial^2 u}{\partial r^2} \right) \\ 1 \frac{1}{r} - \frac{1}{r} + w \frac{1}{1} &= r^2 \frac{\partial P}{\partial r} + r \left(\frac{1}{1} + \frac{1}{R_0} \frac{1}{r^2} \right). \end{aligned} \quad (A-4)$$

The addition and deletion of certain terms in equation (A-4), as compared to equation (2-1b), results because of differentiation of the characteristic

velocity, $\bar{V}$, with respect to $\bar{r}$. The orders of magnitude of the terms in equation (A-4) are also written directly below each term. It can be seen that the second viscous term in parentheses may be neglected when $R_0 r^2 \gg 1$, or $\bar{r}^2 \gg \bar{\epsilon}^2$. Using equation (A-3), one can see that the orders of magnitude of all the acceleration terms are $1/r$ or less (specifically, for $r > 1$, $w \partial u / \partial z \approx 1/r$; but for $r < 1$, $w \partial u / \partial z \approx 1$) while the order of magnitude of the remaining viscous term is r . Thus, as anticipated, acceleration effects dominate at small r ; viscous effects dominate at large r and both effects are approximately of equal importance when $r = 1$. The ratio of viscous to acceleration effects is r^2 . No matter which effect is important, it must always be balanced by the static pressure gradient, so for small r the order of magnitude of $\partial P / \partial r$ is $1/r^3$ while for large r it is $1/r$. Thus

$$\begin{aligned} \partial P / \partial r &\approx 1/r^3 & r < 1 \\ \partial P / \partial r &\approx 1/r & r > 1 \end{aligned} \quad (A-5)$$

A question arises when r is very small -- can the viscous terms be neglected altogether? We know that if viscous terms are neglected completely the order of the partial differential equation is reduced by one and the no-slip condition at the walls cannot be satisfied. All real fluids do satisfy this condition, so it must be concluded that, no matter how large the acceleration terms, there is some small region close to the wall where viscous effects are important. Over this small region, of thickness $\bar{\delta}$, the velocity $\bar{u}$ changes from approximately $\bar{V}$ to zero. Thus the proper order of magnitude of the velocity gradient is

not $\bar{V}/\bar{t}$ but $\bar{V}/\bar{\delta}$, and the second derivative is of order $\bar{V}/\bar{\delta}^2$. In non-dimensional form, with $\delta = \bar{\delta}/\bar{t}$, the order of magnitude of the viscous term in equation (A-4) becomes r/δ^2 , and if this viscous term is to be as important as the acceleration terms at small r , it must be concluded that $\delta \approx r$.

The dimensionless boundary layer thickness cannot exceed one so the complete description is

$$\begin{aligned} \delta &\approx 1 \quad \text{for } r > 1 \\ \delta &\approx r \quad \text{for } r < 1 \end{aligned} \quad (\text{A-6})$$

At small r , the order of magnitude of the first viscous term becomes $1/r^2$ while the order of magnitude of the second viscous term is $1/(R_0 r^2)$. Thus, it is seen that the latter term is truly negligible in comparison to the former, even for small r .

If the dimensionless quantities are substituted in equation (2-1c) and $R_0 > 1$, the dimensionless z -component of the Navier-Stokes equation becomes

$$\begin{aligned} w \frac{\partial w}{\partial z} + u \frac{\partial w}{\partial r} - \frac{uw}{r} &= R_0 r^2 \frac{\partial P}{\partial z} + r \left[\frac{\partial^2 w}{\partial z^2} + \frac{1}{R_0} \left(\frac{\partial^2 w}{\partial r^2} - \frac{1}{r} \frac{\partial w}{\partial r} + \frac{w}{r^2} \right) \right] \\ w \frac{w}{z} + \frac{1}{r} \frac{w}{r} - \frac{1}{r} \frac{w}{r} &= R_0 r^2 \frac{\partial P}{\partial z} + r \left[\frac{w}{z^2} + \frac{w}{R_0 r^2} \right] \end{aligned} \quad (\text{A-7})$$

One may recall that when $r > 1$, $w \approx 1/r$. In addition, since $z \approx 1$, the order of magnitude of the acceleration terms is $1/r^2$; that of the first viscous term is one; and the remaining viscous terms have order $1/R_0 r$,

which is negligible for $r > 1$. Thus, when $r > 1$, the first viscous term dominates in equation (A-7) and $\partial P / \partial z \approx 1/r^2 R_0$.

When r is small, it is surprising to note that both viscous and acceleration effects can be equally important. However, since $w \approx 1$ and $z \approx 1/r$, it can be shown that $\partial P / \partial z \approx 1/R_0 r^3$. When these axial pressure gradients are compared to the radial pressure gradients, it is found that

$$\frac{\partial P}{\partial z} \approx \frac{1}{r R_0} \frac{\partial P}{\partial r} \quad (r > 1) \quad (A-8)$$

$$\frac{\partial P}{\partial z} \approx \frac{1}{R_0} \frac{\partial P}{\partial r} \quad (r < 1)$$

Thus, axial pressure gradients are negligible compared to radial pressure gradients for all r and the radial gradient may be written as a total derivative. Furthermore, since the magnitude of the dimensional velocity $\bar{w}$ is always less than $\bar{u}$ by a factor of $\sqrt{R_0}$ or more, the solution of equation (A-7) is not of practical interest and it can be ignored in comparison to equation (A-4). (Though w is still needed to determine u , it is determined via the continuity equation.)

In final form, the simplified dimensionless equations are continuity,

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (2-4)$$

r-momentum,

$$u \frac{\partial u}{\partial r} - \frac{u^2}{r} + w \frac{\partial u}{\partial z} = r^2 \frac{\partial P}{\partial r} + \frac{\partial^2 u}{\partial z^2} \quad (2-5)$$

and overall continuity,

$$\int_0^1 u \, dz = 1 \quad (2-6)$$

As a consequence of the method of nondimensionalization chosen, equation (A-1), adjustable parameters such as a Reynolds number do not appear in equations (2-4) through (2-6). Thus, for a given inlet condition the solution will depend on r and z only. It will be completely independent of Reynolds number, flow rate, viscosity, or plate spacing. Furthermore, some quantities, such as P , τ_w , and f will be functions of r only since P is essentially independent of z as indicated by equation (A-8) and $\bar{\tau}_w$ is always evaluated at a specific value of z ($z = 1$). The above comments pertain to laminar flow with constant properties. For turbulent flow the effective viscosity is a function of z and Reynolds number, so that the form of equation (2-5) changes and P , τ_w , and f will then depend on R as well as r .

Friction Factors

For laminar flow the order of magnitude of the shear stress at the wall can be estimated as

$$\bar{\tau}_w = \bar{\mu} \left. \frac{\partial \bar{u}}{\partial \bar{z}} \right|_{\bar{z}=\bar{t}} \approx \bar{\mu} \bar{V} / \bar{\delta} \quad (A-12)$$

and the friction factor as

$$f = \frac{\bar{\tau}_w}{\bar{\rho}\bar{V}^2} \approx \frac{2\bar{v}}{\bar{V}\delta} \quad .$$

Using the previous definitions of $\bar{V}$, r and δ , one can derive the order of magnitude of the friction factor

$$f \approx \frac{2r}{\delta\sqrt{R_0}} \quad .$$

It was previously shown that

$$\delta \approx 1 \quad \text{for } r > 1$$

(A-6)

$$\delta \approx r \quad \text{for } r < 1$$

so that

$$f \approx 2r/\sqrt{R_0} \quad \text{for } r > 1$$

(A-13)

$$f \approx 2/\sqrt{R_0} = \text{constant} \quad \text{for } r < 1 \quad .$$

Since the local value of the Reynolds number, based upon the equivalent hydraulic diameter, $4\bar{r}$, is related to r as

$$R = 4\sqrt{R_0}/r \quad ,$$

equations (A-13) are equivalent to

$$\begin{array}{ll}
 f \approx 8/R & \text{for } r > 1 \\
 f \approx 8/rR & \text{for } r < 1
 \end{array}
 \quad (A-14)$$

When $r > 1$, where viscous forces dominate and the velocity profile is approximately a fully developed parabolic one, the predicted order of magnitude of f is $8/R$, whereas the exact relationship for fully developed laminar flow between parallel plates is known to be $24/R$. Thus the order of magnitude analysis predicts the correct functional form but the numerical coefficient is one-third the correct value. On the other hand, when $r < 1$ equation (A-13) shows that $f \approx 2/\sqrt{R_0}$, which not only has the correct form, but is within 8 percent of the result given by the similarity solution, equation (2-13).

APPENDIX B

NUMERICAL METHODS

Finite Control Volume Equations

The parabolic equations (2-4) through (2-6) were solved using an iterative marching technique described by Bankston and McEligot (1970). In actuality the equations were solved in dimensional form first, and the results were then also expressed in non-dimensional form.¹ Having the results in both forms allowed convenient comparison of experimental data and predictions. The method has been described in greater detail in a report by Bankston and McEligot (1969) so that only a brief review need be given here.

The flow field is discretized by laying out n nodes across the disk spacing from the wall to the center plane. The nodes do not have to be equally spaced. Symmetry allows restricting the computation field to only half the spacing. Finite control volumes are then positioned at these nodes with their upper and lower surfaces located half-way between nodes, as indicated in Figure 32. In this figure θ now denotes the azimuthal included angle of the control volume computation cell, rather than the convergence half angle of Chapters VII and VIII.

1. To avoid cumbersome notation in this Appendix, the use of overscores to distinguish dimensional quantities will be omitted.

Figure 32. Finite Control Volume Computation Cell.

We wish to advance from radial position i , where all quantities are known, either because they have just been calculated, or because they are the specified inlet conditions, to the downstream radial position $i + 1$. Algebraic equations expressing conservation of mass and r -directed momentum are written for each control volume.

It is assumed that the average velocity for each volume is given by the velocity at its node point. The shear stress acting at the upper surface of a control volume is evaluated as

$$\tau_{j+1/2} = \mu_{j+1/2} \left[(1-\sigma) \frac{u_{i,j+1} - u_{i,j}}{z_{i,j+1} - z_{i,j}} + \sigma \frac{u_{i+1,j+1} - u_{i+1,j}}{z_{i,j+1} - z_{i,j}} \right] \quad (B-1)$$

The subscript $j+1/2$ merely indicates that the quantity is evaluated not at the node point, but in between, at the control volume surface. For turbulent flows $\mu_{j+1/2}$ is the effective viscosity evaluated with the aid of a turbulence model, as in Chapter VI, whereas for laminar flows $\mu_{j+1/2}$ is simply the molecular viscosity. The parameter σ is a weighting factor which centers the mesh with respect to finite differencing. If σ was 0 the differencing would be purely explicit, while $\sigma = 1$ results in a purely implicit scheme (Carnahan, Luther and Wilkes, 1969, pp. 432-440). Ordinarily $\sigma = 0.5$ (Crank-Nicolson differencing) results in best accuracy for the Heat Conduction Equation (Carnahan, Luther and Wilkes, 1969, p. 451). For the laminar flows reported here σ was taken as 0.6 but for turbulent flows it was found necessary to use $\sigma = 1$ for numerical stability with the mixing length model used.

In order to linearize the algebraic momentum equation it is assumed that quadratic terms such as $u_{i+1,j}^2$, can be expressed as

$u_{i+1,j}^*$ $u_{i+1,j}$, where $u_{i+1,j}^*$ is an estimate of $u_{i+1,j}$, obtained either from the last iteration or from $u_{i,j}$ if just starting the iteration at a new step. The resulting equations are then iterated, continuously updating $u_{i+1,j}^*$, until a suitable convergence criterion is satisfied.

In final form the momentum equation can be written as

$$-A_j [u_{i+1,j+1}] + B_j [u_{i+1,j}] - A_{j-1} [u_{i+1,j-1}] + C_j [P_{i+1}] + D_j = 0 \quad (B-2)$$

where the bracketed terms are the dependent variables. The coefficients A through D are

$$A_j = \frac{(r_i + r_{i+1}) \Delta r \mu_{j+1/2} \sigma}{4(z_{j+1} - z_j)}$$

$$A_{j-1} = \frac{(r_i + r_{i+1}) \Delta r \mu_{j-1/2} \sigma}{4(z_j - z_{j-1})}$$

$$B_j = M_{i+1,j} + A_j + A_{j-1}$$

$$C_j = (r_{i+1} + r_i) \Delta z_j / 2$$

$$D_j = -C_j P_i - \frac{1-\sigma}{\sigma} \left[A_j (u_{i,j+1} - u_{i,j}) - A_{j-1} (u_{i,j} - u_{i,j-1}) \right] \\ - M_{i,j} u_{i,j} + M_{j+1/2} (u_{i,j} + u_{i,j+1} + u_{i+1,j}^* + u_{i+1,j+1}^*) / 4 \\ - M_{j-1/2} (u_{i,j} + u_{i,j-1} + u_{i+1,j}^* + u_{i+1,j-1}^*) / 4$$

The mass transfer rate terms $M_{j+1/2}$ and $M_{j-1/2}$ are evaluated from the continuity equation

$$M_{i+1,j} = M_{k,j} + M_{j+1/2} - M_{j-1/2} = 0 \quad (B-3)$$

where

$$M_{i,j} = \rho u_{i,j} r_i \Delta z_j$$

$$M_{i+1,j} = \rho u_{i+1,j} r_{i+1} \Delta z_j$$

$$M_{j+1/2} = \rho (w_{i,j} + w_{i,j+1} + w_{i+1,j} + w_{i+1,j+1}) (r_i + r_{i+1}) \Delta r / 8$$

$$M_{j-1/2} = \rho (w_{i,j} + w_{i,j-1} + w_{i+1,j} + w_{i+1,j-1}) (r_i + r_{i+1}) \Delta r / 8 \quad .$$

When it is desired to calculate flows between infinitely wide plates, r_i and r_{i+1} are set equal to 1 which effectively converts the geometry from a radial, cylindrical one to a rectilinear one. In this case Δr can be thought of as the downstream or axial increment of distance.

Solution Procedure

The momentum equation (B-2) can be written at the n node points. The dependent variables are the n velocities and one pressure, P_{i+1} . The additional equation required to make the system deterministic is the integral continuity equation

$$4\pi r_{i+1} \sum_{j=1}^n u_{i+1,j} \Delta z_j = Q \quad . \quad (B-4)$$

The resulting $n + 1$ equations are solved simultaneously using Gaussian elimination as outlined by Bankston and McEligot (1969). In numerical form this procedure leads to an efficient tridiagonal matrix algorithm in which two sweeps across the mesh results in the updated estimates of $u_{i+1,j}$ and P_{i+1} .

With these latest estimates, the continuity equation (B-3) is successively solved for each control volume, starting at the wall where $M_{i+1,N} = 0$. For any j , with $M_{i,j}$ and $M_{i+1,j}$ known or estimated and $M_{i,j+1/2}$ estimated from the preceeding control volume, $M_{i,j-1/2}$ can be calculated. In this manner one proceeds to the center plane, $z = 0$, where by symmetry M_i should be zero, providing a good check on the numerical procedure. These latest estimates of the mass transfer rates and velocities are then used to update the coefficients A through D and the entire process is then repeated until convergence is obtained. The criterion taken for convergence was that the maximum difference between successive calculations of pressure gradients, $(P_{i+1} - P_i)/\Delta r$, or the successive velocities at a selected node had to be 0.1 percent or less. Once convergence was achieved integral mass and momentum balances were calculated; typically these were conserved to 10^{-13} percent. Output variables were printed, arrays were updated and calculations for the next radial step were begun.

Numerical Accuracy

As mentioned in Chapter II, the severest test of numerical accuracy occurs at small values of the nondimensional radius, where the boundary layers become very thin. To promote better accuracy the node

spacing was always laid out in such a manner that the node spacing was relatively large near the center plane, but became smaller as the wall was approached. After experimenting with various power laws it was found that $z(j)/t = [(j-1)/(n-1)]^4$, where n is the total number of nodes, resulted in a good balance of accuracy and economical computing times.

Table X shows the results obtained with some grid distributions. The parameters r_i and r are the dimensionless inlet and downstream radii, $(\Delta z/t)_w$ is the dimensionless node spacing at the wall, and n' is the number of nodes that were included in the boundary layer, i.e., from the wall to the position where $u(r,z) = 0.9 u(r,0)$. In all cases r_i is large enough that the results shown should be independent of entry effects. In accordance with the similarity solution of Chapter II, the factor $f\sqrt{R_0}$ should approach the constant 2.173 as $r \rightarrow 0$. In all but the first case, where the spacing was more uniformly distributed, it appears that the similarity solution is closely approached at $r = 0.01$. Somewhat surprisingly it appears that as few as 6 nodes within the boundary layer is sufficient to give good accuracy for friction factors. Doubling the number of nodes (case 3 compared to case 2) results in agreement within 1 percent with the similarity solution for $r = 0.01$ to 0.001, a factor of 10 variation in radius. However, as a practical matter, since the similarity solution is closely attained at $r = 0.01$, results at even smaller radii are not required. It appears that the node spacing for case 2, which results in more economical computing, is very satisfactory and this was, in fact, the spacing distribution used for the bulk of the work.

TABLE X

ASSESSMENT OF NUMERICAL ACCURACY

	<u>Case 1</u>	<u>Case 2</u>	<u>Case 3</u>
$r_i =$	4.2	10	0.1
$n =$	50	50	100
$(\Delta z/t)_w =$	10^{-2}	10^{-3}	5×10^{-4}
$\frac{r}{-}$	$\frac{n' f \sqrt{R_0}}{-}$	$\frac{n' f \sqrt{R_0}}{-}$	$\frac{n' f \sqrt{R_0}}{-}$
0.01	4 2.27	20 2.16	39 2.17
0.005	3 2.80	12 2.15	23 2.16
0.003	2 4.02	8 2.15	15 2.16
0.002		6 2.15	9 2.15
0.001		4 2.19	6 2.16
0.0005			4 2.23

Fortran Program

A Fortran listing of the program is attached. Either laminar or turbulent flows can be predicted. In its present form the van Driest modification of the Prandtl mixing length model that was discussed in Chapter VI is incorporated in the program. The use of comments and the definition of Fortran variables has been quite liberal, and it is expected that these aids, in conjunction with the previous discussion, should enable the interested reader to understand and use the code. Typical running times on a Control Data Corporation 6600 computer were about a minute to calculate 800 radial steps, sufficient to cover a dimensionless radius range from 10 to 0.01.

Originally the program was written with the intent to handle radial variations of disk spacing or density. The second variation is restricted to slightly compressible flow since the energy equation is not solved. The reader is cautioned, however, that these additional features were relatively untested so that calculations with these additional variations may not be necessarily correct. In its present version the program is set up to be useful as either a data analysis program or a purely predictive program. In the latter situation, one simply specifies the overall Reynolds number R_0 and dimensionless radius desired at the entry. Using included properties which happen to be suitable for air and a default value of half-spacing, the program then computes all the dimensionless quantities discussed in Chapters II and VI, and prints out results at values of dimensionless radii convenient for plotting on logarithmic coordinates, as in Chapter III. Presently these dimensionless radii are 1^k , 2^{k-1} , 3^{k-1} , and 5^{k-1} where k takes on, successively, values from +2 to -3.

To analyze the data measured with the experimental apparatus described in Chapters VII and VIII one specifies $KDATA = 1$ as well as the values of the half-angle θ in degrees, the plate spacing in inches, the temperature in degrees centigrade, the atmospheric pressure in millimeters of mercury, and the venturi pressure difference in millimeters of mercury. These dimensional units were selected because these were the units in which each of the input quantities were directly measured; other units can be selected by modifying the program slightly. With these input quantities the program then calculates the density and viscosity of the entering air, the flow rate, and the entry and terminal

radii. There then follows the calculation of the radial variation of pressure differences, shear stresses, velocity profiles, mixing lengths, effective viscosities, laminarization parameters, and various diagnostic information. These output quantities are presented in both dimensional and dimensionless form. Pressure differences are printed in units of millimeters of mercury so that a direct comparison of measured and predicted pressure differences could be quickly and easily performed after each experimental run.

Letting $KTURB = 0$ results in laminar calculations; $KTURB = 1$ gives the original van Driest mixing length turbulence model. Where one wants to calculate flows between infinitely wide plates, simply set $NTYPE = 1$.

The program was written in Fortran IV and with the exception of some plotting routines which are convenient to use, but not essential, does not call upon any external library routine. These plotting routines are easily bypassed by setting $KPLOT = 0$. Therefore the program should be compatible with any Control Data Corporation computer without any modification, and with modification it can be made compatible with IBM computers. Upon receipt of a blank tape the author will provide taped versions of the program to any requestor.

```

      PROGRAM RADFLO(INPUT,OUTPUT,FILM)
CC  PROGRAM SOLVES B L APPROX TO NAV STOKES EQNS FOR RADIAL FLOW
CCCCC BETWEEN TWO DISKS WITH VARYING SPACING AND DENSITIES
CC
      REAL MR,MZ,MRNEW,MASFLO,MU,UFLAG,UFLAGN,TAU,TAUD,RO,RE,X,XBT
      1,T,PR,PRNEW,RHO,RHONNEW,W,DTBDR,DELTBT,CPD,CPDNEW,DELTA,PI,VISC,
      2UBULK,SUMF,SUMF,SUMH,SUMMR,ERRDPM,ERRDPR,REMAIN,DPBDR,DPBDRN,ERRU,
      3 DIMR,RBT,RMAX,MS,MSNEW,DPDRDY,DPDRVT,DPDRAT,DPERR,DPDRDA,
      4DPDRVA,MSA,MSNEWA,TAUDA,CFN,CFNAPP,CDEF,PRIN,TIN,RHOIN
      5 ,DYNPR,D99BT,DIMRIN,DIMRMI,DELRDT,RJRT,UFRIC,H,ACCP,ACCU
      6 ,SH,D1BT,D2BT,RE2,FRORT
      DIMENSION U(100),UNEW(100),UOLD(100),MR(100),MRNEW(100),MZ(100),
      1 TC(100),CPD(100),CPU(100),Z(100),7D(100),MU(100),TCP(100),
      2 A(100),B(100),C(100),D(100),BP(100),DP(100),P(100),Q(100),RC(100)
      3 ,E(100),F(100),H(100),UD(100),DELZ(100),WT(100),WTD(100)
      4 ,UPLUS(100),DUNEW(100),YPLUS(100)
      REAL LD(100),MUD1(100),MUD2(100),MUFFF(100)

CCCC R=RADIUS Z=TRANS DIR U=R COMP VEL WT=Z COMP VEL
      X=NET RADIAL DIST MARCHED
CC MR=MASS FLO FOR DELZ IN R DIR MZ IN Z DIR T=HALF SPACE (FT)
CC MS=TOTAL MOMENTA,SUM OF MR(I)*U(I)
CC PR=PRESS(LB-F/SQ FT) RHO=DENS(SLUG/CU FT) MASFLO=TOTAL FLO(BOTH
CC SIDES) IN SLUG/S MU=EFF VISC TC=CDEF SHEAR STRESS TERM
CCPRM IS PRESS IN MM HG
CC CPD=CDEF DOWN STREAM PRESS CPU=UPSTREAM CDEF DTBDR=DELT/DELR
CCCCC SUFFIX-NEW TO ALL TERMS IS NEW DOWNSTREAM VALUE
CCCCC SUFFIX-D TO ALL TERMS IS NON DIMENSIONAL FORM OF VARIABLE
CCC SUFFIX-IN MEANS INLET VALUE
CC SUFFIXA MEANS APPARENT (ON 1D BASIS) VALUE
      CFN=TAUD/TAUD-FUL DEV (24./RE)
C D99BT= BOUND LAYER THICKNESS( U=.99*UMAX )/ T
C DB3BT=NON DIM B. L. THICK(U=.83333*UBULK)
CC D1BT IS NONDIM B L DISP THICK
CC D2BT IS NONDIM B L MOM THICK
CC RE2 IS REYN NO BASED ON U-MIDPLANE AND BL MOM THICK
CC SH IS D1/D2 (SHAPE FACTOR)
CC ACCP ,ACCU ARE ACC PARAMETERS
CC LD IS MIX LEN DIV BY D99
CC MUD1 IS 1+TURB VISC/MOL VISC
CC MUD2 IS EDDY DIFF/(UFRIC*D99)

```

```

      INITIALIZATION
      PI=3.14159
      D99BT=0.
      D99=0.
      NPRINT=1
      NPRINT=6
      CNPRINT=NUM RADIAL STEPS BETWEEN PRINT-OUTS
      KSHT=0
      LPR=0
C KSHT=1 GIVES PRINT AT DIMR OF 1,2,3.5*10**N ONLY

```

```

      NARRAY=0
CC   NARRAY OF ZERO PRINTS NO ARRAYS
      NARR1=0
CC   NARR1 OF 0 GIVES NO ARRAYS OF Z,V,MUEFF,TCP
CCC  OR 1ST RESULTS OF A-F,TC,CPD,CPU,UNEW
      KPLOT=1
CC   KPLOT 1 GIVES PLOTS
      NTWICK=0
      IF(NTHICK.EQ.0) DTBDR=0.
CC   IF NTHICK=0 DISK SPACING DOES NOT VARY
      NTYPE=0
      IF NTYPE=1 CALC IS FOR FLD BETWEEN INF PLATES
      NDIAGN=1
      NDIAGN=0
CC   NDIAGN=1 GIVES DIAGNOSTICS
      N=50
C    N=NUMB NODESACROSS STREAM
      NN=N-1
      M=0
      M COUNTS NUMB OF STEPS TAKEN IN R DIR
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCC INPUT DATACCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

      KTURB=1
C    KTURB=1 GIVES MIX LENGTH TURB MODEL
      KDATA=1
C    KDATA 1 GIVES DATA ANALYSIS,NOT PRFD,VERSION
      IF(KDATA.NE.1)GO TO 3
CCC  INPUT DATA FOR TEST,DATA ANALYSIS
      SPACE=.504
      TEMP=24.9
      VENTDP=5.14
      PATM=605.2
      DEG=2.0
CC   SPACE(IN),TEMP(DEG C),VENTDP,PATM(MM HG),DEG(HALF)
      TIN=SPACE/24.
      PRIN=0.

      VISC=3.582E-7+1.02E-9*TEMP
CCCLB=F*S/SQ FT
      RHOIN=.00228*PATM*300./((760.+(273.+TEMP))
      QACT=(VENTDP/(1158.*RHOIN))**.5
      QINPUT=QACT/((.01786+.004578*DEG)
      QINPUT=180.*QACT/DEG
CC   FOR FULL 2*PI ADMITTANCE
      THET=PI*DEG/180.
      RSTART=3.75+.25/SIN(THET)
      REWD=.25/SIN(THET)
      IF(NTYPE.EQ.1)QINPUT=4.*PI*QACT
CC   FOR THIS CASE,R=1(ALWAYS),SO QINPUT=2*PI*QACT/CHAN WIDTH(.5FT)
      PRINT 600,SPACE,TEMP,VENTDP,PATM,DEG,TIN,
      1 QACT,QINPUT,VISC,RSTART
500  FORMAT(2X,*SPACE=*,E11.3,* TEMP=*,E11.3,* VENTDP=*,
      1 E11.3,* PATM=*,E11.3,/,2X,*DEG=*,E11.3,* TIN=*,
      2 E11.3,* QACT=*,E11.3,* QINPUT=*,E11.3,* VISC=*,E11.3,
      3 *RSTART=*,E11.3,/)
      3 IF(KDATA.EQ.1)GO TO 4
      RHOIN=.0018
      PRIN=0.
CC   SLUGS/FT**3

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      VISC=3.835E-7
CCLB-F*S/FT**2
      TIN=.02083333
CC   HALF-INCH DISC SPACING
      RO=1.E+6
CCCCCCC DESIRED VALUE SPECIFIED HERECCCCCCC
      RORT=(ABS(RO))**.5
      QINPUT=4.*PI*VISC*TIN*RO/RHOIN
      DIMRIN=10.
      DIMRMI=DIMRIN*1.E-4
      DELRDI=(-4.E-5)/DIMRIN
      RSTART=TIN*RORT*DIMRIN
      4 MASFLO=-QINPUT*RHOIN
CTOTAL BOTH SIDES (SLUG/S)
      X=0.
      PR=PRIN
      T=TIN
      RHO=RHOIN
      RMAX=4000.
      RO=MASFLO/(4.*PI*VISC*TIN)
      RORT=(ABS(RO))**.5
      R=RSTART
      DELR=TIN*RORT*DELRDI
      IF(KDATA.EQ.1) DELR=-.0001
      IF(NTYPE.EQ.1) R=1.
      W=1.
      IF(KTURB.EQ.0) W=.6
W   CENTERS THE MESH WITH RESPECT TO DIFFS IF W=0 EXPLICIT IF W=1 IMP
      UBULK=MASFLO/(4.*PI*RHO*R*T)
      RV2=RHOIN*UBULK*UBULK/2.
      RV2MM=RV2*.359
      UBOLD=UBULK
      MSA=RHO*UBULK*UBULK*R*T
      MS =RHO*UBULK*UBULK*R*T
CC   GOOD ONLY FOR UNIF VEL PROFILE
C   DEFINE GRID SPACING
      DO 2 I=1,N
      ZD(I)=(FLOAT(I-1)/FLOAT(N-1))**.5
      IF(I.LE.20) ZD(I)=.90*FLOAT(I-1)/19.
      IF(I.GT.20) ZD(I)=.90+.1*(FLOAT(I)/FLOAT(N))**.5
      ZD(I)=(FLOAT(I-1)/FLOAT(N-1))**.25

      MU(I)=VISC
CC***** LAMINAR FLOW ONLY *****
      U(I)=1.5*UBULK*(1.-ZD(I)**2)
      U(I)=1.1666*UBULK*(1.-ZD(I))**.1666
      U(I)=UBULK

***** UNIFORM INLET VEL PROFILE *****
      MZ(I)=0.
      2 CONTINUE
      UFRIC=12.
      D99BT=.87
      D99=D99BT*T

C   COME BACK HERE (STATE 10) IF T VARIES
      10 DO 12 I=1,N
      Z(I)=ZD(I)*T
      YPLUS(I)=(T-Z(I))*UFRIC*RHO/VISC
      LD(I)=.4*(T-Z(I))*(1.-EXP(-YPLUS(I)/26.1))/999
      IF(LD(I).GT..09) LD(I)=.09

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```

12 CONTINUE
  DO 14 I=2,NN
    DELZ(I)=(Z(I+1)-Z(I-1))/2.
14 CONTINUE
  DELZ(1)=Z(2)/2.
  DELZ(N)=(Z(N)-Z(N-1))/2.
  DO 16 I=1,N
    MR(I)=RHO*U(I)*R*DELZ(I)
16 CONTINUE
17 CONTINUE
CC RETURN HERE IF KTURB=1
CC FORM PRECEFS CCCCC
  DO 18 I=1,NN
    MUEFF(I)=(MU(I)+MU(I+1))*0.5+0.16557*0.423*VTSC
    1*(0.5*(YPLUS(I)+YPLUS(I+1))-11.*TANH(0.5*
    2(YPLUS(I)+YPLUS(I+1))/11.))*(1.+70(I))*
    3(1.+2.*ZD(I)**2)
    MUEFF(I)=(MU(I)+MU(I+1))*0.5+ABS(0.47*0.99*0.99*(U(I+1)-
    1 U(I))/(Z(I+1)-Z(I))*((LD(I+1)+LD(I))/2.))**2)
    IF(KTURB.EQ.0)MUEFF(I)=(MU(I)+MU(I+1))/2.

    TCP(I)=0.5*MUEFF(I)/(Z(I+1)-Z(I))
18 CONTINUE
  TCP(N)=0.
  IF(KSHT.EQ.1 .OR. NARR1.EQ.0) GO TO 25
  IF(M.NE.1 .AND. MOD(M,NPRINT).NE.0)GO TO 25
  PRINT 601
601 FORMAT(1H1,9X,*Z (FT)*,7X,*Z/T*,11X,*VFL.F/S*,9X,*DELZ,FT*,10X,
  1 *MR*,11X,*MZ*,7X,*EFF VISCOSITY*.5X,*C7FF TCP(I)*,/)
  DO 20 I=1,N
    20 PRINT 602,Z(I),ZD(I),U(I),DELZ(I),MR(I),M7(I),MUEFF(I),TCP(I)
602 FORMAT(2(4X,F11.8),6(4X,E11.4),)
*****
*****START MARCHING *****
25 M=M+1
  DELTBT=OTBOR*DELR/T
  X=X+DELR
  RNEW=R+DELR
  IF(NTYPE.EQ.1) RNEW=R
  K=0
C K IS ITERATION COUNTER
  FORM PRE-CJEFS
  DO 30 I=1,N
    TC(I)=TCP(I)*DELR*(R+RNEW)
    CPD(I)=-DELZ(I)*(RNEW*(1.+DELTBT)+R)/2.
    CPU(I)=-CPD(I)
C INITIAL GUESSES
    MRNEW(I)=MR(I)
    UNEW(I)=R*U(I)/(RNEW*(1.+DELTBT))
30 CONTINUE
    RHONEW=RHO
  NOTE THAT MZ(I) NOT CHANGED-USE INLET DATA TO RESULTS(FROM CONTINUITY)
CC) FOR LAST RADIAL STEP
  ***** FOR NO-SLIP CONDITION*****
  MRNEW(N)=0.
  UNEW(N)=0.
  IF(NARR1.EQ.0) GO TO 32
  IF(M.NE.1.OR.K.NE.0.OR.KSHT.EQ.1) GO TO 32
  PRINT 603
603 FORMAT(1H1,6X,*TC*,13X,*CPD*,12X,*CPU*,12X,*MRNEW*,8X,*UNEW*,/)

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      DO 31 I=1,N
31  PRINT 604,TC(I),CPD(I),CPU(I),MRNEW(I),UNEW(I)
604  FORMAT(5(4X,E11.4))
32  CONTINUE
      FORM PARTIAL COEFS--LEAVE OUT TERMS TO BE UPDATED BY ITERATION
      A(1)=TC(1)*W/(1.+DELTBT)
      BP(1)=A(1)
      C(1)=-CPD(1)
      DP(1)=-CPU(1)*PR-TC(1)*(1.-W)*(U(2)-U(1))-MR(1)*U(1)
      DO 120 I=2,NN
      A(I)=TC(I)*W/(1.+DELTBT)
      BP(I)=A(I)+A(I-1)
      C(I)=-CPD(I)
      DP(I)=-CPU(I)*PR-TC(I)*(1.-W)*(U(I+1)-U(I))+TC(I-1)*(1.-W)*
1    (U(I)-U(I-1))-MR(I)*U(I)
120  CONTINUE
      COMPLETE COEFS USING GUESSES(1 ST ITER) OR UPDATED VALUES
      FROM LAST ITER
200  K=K+1
      B(1)=BP(1)+MRNEW(1)
      D(1)=DP(1)+.25*MZ(1)*(U(1)+UNEW(1)+U(2)+UNEW(2))
      P(1)=A(1)/B(1)
      Q(1)=-D(1)/B(1)
      RC(1)=-C(1)/B(1)
      DO 210 I=2,NN
      B(I)=BP(I)+MRNEW(I)
      D(I)=DP(I)+.25*MZ(I)*(U(I)+UNEW(I)+U(I+1)+UNEW(I+1))
1    -.25*MZ(I-1)*(U(I)+UNEW(I)+U(I-1)+UNEW(I-1))
      P(I)=A(I)/(B(I)-A(I-1)*P(I-1))
      RC(I)=(A(I-1)*RC(I-1)-C(I))/(B(I)-A(I-1)*P(I-1))
      Q(I)=(A(I-1)*Q(I-1)-D(I))/(B(I)-A(I-1)*P(I-1))
210  CONTINUE
      E(N)=1.
      F(N)=0.
      H(N)=0.
      DO 220 I=1,NN
      J=N-I
      E(J)=P(J)*E(J+1)
      F(J)=P(J)*F(J+1)+Q(J)
      H(J)=P(J)*H(J+1)+RC(J)
220  CONTINUE
      IF(NDIAGN.EQ.1) GO TO 222
      IF(M.NE.1.OR.KSHT.FQ.1.OR.K.NE.1) GO TO 225
      IF(NARR1.EQ.0) GO TO 225
222  PRINT 606
606  FORMAT(1H1,*          A          B          C          D          *
1,1X,*P          Q          PC          E          F          *
2,1X,*          H*,//)
      DO 224 I=1,N
224  PRINT 607,A(I),B(I),C(I),D(I),P(I),Q(I),RC(I),E(I),F(I),H(I)
607  FORMAT(1X,10E12.3,)
225  CONTINUE
      CALCULATE PRNEW
      SUME=0.
      SUMF=0.
      SUMH=0.
      DO 230 I=1,N
      SUME=SUME+DELZ(I)*E(I)
      SUMF=SUMF+DELZ(I)*F(I)
      SUMH=SUMH+DELZ(I)*H(I)
230  CONTINUE

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      PRNEW=(MASFLO/(4.*PI*RHONNEW*(1.+DELTA)*RNEW)-SUMF-UNEW(N)*SUME)/
      1SUMF
*****NORMALLY UNEW=0.--NO SLIP BUT UNEW(N) CAN BE SPECIFIED IN*****
****INITIAL GUESSES
****PUT IN EQN OF STATE HERE (RHONNEW=G(RNEW))*****
      RHONNEW=RHO
      INCOMP,ISOTHERMAL FLUID
      CALCULATE NEW VELOCITIES AND MASS FLOWS,MRNEW(I)
      SUMMR=.0
      DO 240 I=1,N
      UNEW(I)=UNEW(N)*E(I)+F(I)+H(I)*RNEW
      MRNEW(I)=RHONNEW*UNEW(I)*RNEW*(1.+DELTA)*DELZ(I)
      SUMMR=SUMMR+MRNEW(I)
240  CONTINUE
      ERRORM=1.-4.*PI*SUMMR/MASFLO
      SOLVE CONTINUITY EQN
      MZ(N)=0.
      WT(N)=0.
      DO 250 I=1,NN
      J=N-I
      MZ(J)=MRNEW(J+1)-MR(J+1)+MZ(J+1)
      WT(J)= 4.*MZ(J)/((RHO+RHONNEW)*(R+RNEW)*DELR)
250  CONTINUE
      REMAIN=MRNEW(1)-MR(1)+MZ(1)
      REMAIN=REMAIN/MASFLO
****CONVERGENCE CHECK
      DPBDRN=(PRNEW-PR)/DELR
      UFLAGN=UNEW(N-1)
      IF(K.EQ.1)GO TO 260
      ERDPDR=1.-DPBDR/DPBDRN
      ERRU=1.-UFLAG/UFLAGN
      IF(ABS(ERDPDR).LE..001 .AND. ABS(ERRU).LE..001
1 .AND. ABS(ERRORM).LE..001 ).NDIAGN=0
      IF(ABS(ERDPDR).LE..001 .AND. ABS(ERRU).LE..001
1 .AND. ABS(ERRORM).LE..001 ) GO TO 300
260  DPBDR=DPBDRN
      UFLAG=UFLAGN
      IF(K.GE.200) PRINT 608
      IF(NDIAGN.EQ.1) PRINT 608
608  FORMAT(1H1,*RESULTS DID NOT CONVERGE-NO CONVERGENCE NO CONVERG*,/)
      IF(K.GE.200) GO TO 300
      IF(NDIAGN.EQ.1) GO TO 300
      GO TO 200
      CONTINUE ITERATING
300  CONTINUE
      CALCULATE OUTPUT PARAMETERS OF INTEREST
      MSNEW=0.
      DO 302 I=1,N
      MSNEW=MSNEW+MRNEW(I)*UNEW(I)
302  CONTINUE
      TAU=(TC(N-1)*((1.-W)*(U(N)-U(N-1))+W*(UNEW(N)-UNEW(N-1))
1 (1.+DELTA))-CPD(N)*PRNEW-CPU(N)*PR)*2./(DELR*(R+RNEW))
      UBULK=MASFLO/(4.*PI*RNEW*T*(1.+DELTA)*RHONNEW)
      MSNEWA=RHONNEW*UBULK*UBULK*RNEW*T*(1.+DELTA)
      TAUO=TAU*2./(RHONNEW*UBULK**2)
      RE=MASFLO/(PI*RNEW*VISC)
      RO=RE*RNEW/(4.*T*(1.+DELTA))
      DIMR=RNEW/(T*(1.+DELTA)*(ABS(RO)**.5)
      UFRIC=(TAU/RHONNEW)**.5
      RPLMOD=RE*T*(1.+DELTA)/RNEW
      IF(NTYPE.EQ.1) RPLMOD=RE*T*(1.+DELTA)/X

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RPLMOD=1./RPLMOD
CC RPLMOD=R/T/RE=RPLUS**2/4
DPDRDY=-(MSNEW-MS)/(.5*DELR*(RNEW*(1.+DELBT)+R)*T)
DPDRVI=TAU*(R+RNEW)/(T*(RNEW*(1.+DELBT)+R))
DPDRAT=DPDRDY/DPDRVI
DPERR=(DPDRDY+DPDRVI-DPDRN)/DPDRN
DYNPR =RHONEW*UBULK*UBULK/2.
DPDRDA=-(MSNEWA-MSA)/(.5*DELR*(RNEW*(1.+DELBT)+R)*T)
DPDRVA=DPDRN-DPDRDA
TAUDA=2.*DPDRVA*T*(RNEW*(1.+DELBT)+R)/((RNEW+R)*RHONEW*UBULK**2)
CFN=TAUD*RE/24.
CFNAPP=TAUDA*RE/24.
PRCDEF=(PRIN-PRNEW)/(VISC*MASF7/(R47 *4.*PI*(T*(1.+DELBT))**3))
IF(NTYPE.EQ.1) PRCDEF=(PRIN-PRNEW)/(.5*RHONEW*
1 UBULK*UBULK)
RBT=RNEW/(T*(1.+DELBT))
XBT=X/(T*(1.+DELBT))
D1BT=(UNEW(1)-UBULK)/UNEW(1)
D2BT=0.
DO 270 I=1,N
D2BT=D2BT+UNEW(I)*DELZ(I)*(UNEW(1)-UNEW(I))
270 CONTINUE
RE2=RHONEW*D2BT/(VISC*UNEW(1))
D2BT=D2BT/(T*(UNEW(1))**2)
SH=D1BT/D2BT
FRORT=TAUD*RORT
ACCP=VISC*DPDRN/((RHONEW**2)*(UFIC**3))
ACCU=VISC*(UBULK-UBOLD)/((UBULK**2)*DELR*RHONEW)
*****CONVERT PRESS TO MM OF HG*****
PRMM=.359*PRNEW
PRMMV2=PRMM-RV2MM
DO 290 I=2,N
IF(ABS(UNEW(I)).LE..833333*ABS(UBULK)) GO TO 292
290 CONTINUE
292 D83BT=1.-ZD(I)-(.833333*ABS(UBULK)-ABS(UNEW(I)))*
1 (ZD(I-1)-ZD(I))/(ABS(UNEW(I-1))-ABS(UNEW(I)))
DO 303 I=2,N
IF(ABS(UNEW(I)).LE..99*ABS(UNEW(1))) GO TO 304
303 CONTINUE
304 D99BT=1.-ZD(I)-(.99*ABS(UNEW(1))-ABS(UNEW(I)))*
1 (ZD(I-1)-ZD(I))/(ABS(UNEW(I-1))-ABS(UNEW(I)))
D99=D99BT*T*(1.+DELBT)
IF(K.GE.200) GO TO 305
IF(NDIAGN.EQ.1) GO TO 305
DO 605 J=1,5
IF(DIMR.LT.(100.+1.E-6)/10.**J.AND.DIMR.GT.(100.-1.E-6)/10.**J)
1 LPR=1
IF(DIMR.LT.(50.+0.0001)/10.**J.AND.DIMR.GT.(50.-0.0001)/10.**J)LPR=1
IF(DIMR.LT.(30.+0.0001)/10.**J.AND.DIMR.GT.(30.-0.0001)/10.**J)LPR=1
IF(DIMR.LT.(20.+0.0001)/10.**J.AND.DIMR.GT.(20.-0.0001)/10.**J)LPR=1
605 CONTINUE
IF(NTYPE.EQ.1) GO TO 407

IF(KSHT.EQ.1.AND.LPR.NE.1) GO TO 330
407 IF(M.NE.1 .AND. MOD(M,NPRINT) .NE.0.AND.KSHT.EQ.0) GO TO 330
PRINT 609,K,M,NTYPE,NTHICK,KTURB,KDATA,W
LPR=0
609 FORMAT (1H1,*CONVERGED RESULTS NUMR ITER IS*,I3,*M IS*,I5,
1 * NTYPE IS*, I2,* NTHICK IS*,I2,* KTURB IS*,I2,*KDATA IS*,
2 I2,* W IS*,F5.2//)

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305 PRINT 610,RNEW,DELR,UBULK,RE,RBT,RJ,DIR,TAU,TAUD,ERDPDR,ERRU,
1 ERRORM,REMAIN,MASFLD,RHNEW,T,X,XBT,PRNEW,DYNPR,DPBDRN,DPDRDY,
2 DPDRVI,DPRAT,DPERR,MSNEW,CFN,CFNAPP,RPLM7D,PRCOEF,
3 D99BT,D83BT,VISC
610 FORMAT(2X,*RNEW=*,E11.4,*FT DELR=*,E11.3,*FT UBULK=*,E11.3,*F/S *,
1 *RE=*,E11.3,* RNEW/T=*,E11.3,* RJ=*,E11.3,/,2X,* DIMR=*,E11.3,
2 * TAU=*,E11.3,* TAUD=*,E11.3,* ERDPDR=*,E11.3,* FRRU=*,E11.3,
3 * ERRORM=*,E11.3,/,2X,* REMAIN=*,E11.3,* MASFLD=*,E11.3,
4 *SLUG/S RHNEW=*,E11.3,*SLUG/CUFT TOLD=*,E11.3,*FT X=*,E10.4,
5 *FT X/T=*,E11.3,/,2X,*PRESS=*,E11.3,*LR/SOFT DYNPR=*,E11.3,
6 * TOT PR GRAD=*,E11.3,* MOM PR GRAD=*,E11.3,* VIS PR GRAD=*,
7 E11.3,/,2X,* MOM/VIS GRAD RAT=*,E11.3,* DPERR=*,E11.3,* TD MOM=*,
8 E11.3,* CFN=*,E11.3,* CFNAPP=*,E11.3,* R/T/RE=*,E11.3,/,* PRCOEF=*,
9 E11.3,* DEL99/T=*,E11.3,* D83BT= *,E11.3,* VISC= *,E11.3/)
RINCH=RNEW*12.
PRINT 612,PRMM,PRMMV2,RINCH
612 FORMAT(2X,* PRMM= *,E11.3,* PRMMV2= *,E11.3,
1 * MM HG R= *,E11.3,* INCH *,/)

PRINT 616,UFRIC,FRORT,ACCP,ACCU
616 FORMAT(2X,*UFRIC=*,E11.3,* FRORT=*,E11.3,* ACCP=*,E11.3,
1 * ACCU=*,E11.3/)
PRINT 617,D1BT,D2BT,RE2,SH
617 FORMAT(2X,*D1BT=*,E11.3,* D2BT=*,E11.3,* RE2=*,E11.3,* SH=*,
1 E11.3/)

CALCULATE DIMENSIONLESS VELOCITIES AND OUTPUT
IF(NTHICK.EQ.0) GO TO 309
DO 307 I=1,N
Z(I)=Z(I)*(1.+DELTBT)
307 CONTINUE
309 CONTINUE
DO 310 I=1,N
UD(I)=UNEW(I)/UBULK
WTD(I)=WT(I)*(ABS(RO))**.5/UBULK
IF(NTYPE.EQ. 1) WTD(I)=WT(I)*ABS(RE)/UBULK
UPLUS(I)=ABS(UNEW(I)/UFRIC)
YPLUS(I)=(T*(1.+DELTBT)-Z(I))*UFRIC*RHNEW/VISC
LD(I)=.4*(T*(1.+DELTBT)-Z(I))*(1.-FYP(-YPLUS(I)/26.))
1/D99
IF(LD(I).GT. .09) LD(I)=.09
310 CONTINUE
DO 312 I=2,NN
DUNEW(I)=.5*ABS((UNEW(I+1)-UNEW(I))/(Z(I+1)-Z(I))+(UNEW(I)-
1 UNEW(I-1))/(Z(I)-Z(I-1)))
312 CONTINUE
DUNEW(N)=ABS((UNEW(N)-UNEW(N-1))/(Z(N)-Z(N-1)))
DUNEW(1)=.0
DO 313 I=1,N
MUD1(I)=1.+RHNEW*LD(I)*LD(I)*D99*D99*UNEW(I)/VISC
MUD2(I)=LD(I)*LD(I)*D99*DUNEW(I)/UFRIC
313 CONTINUE
IF(NARRAY.EQ.0) GO TO 325

PRINT 620
620 FORMAT(7X,*Z*,14X,*UNEW*,13X,*MRNEW*,12X,*7/T*,10X,
1 *U/UBULK*,8X,*DIM WT*,10X,*YPLUS*,12X,*UPLUS*,//)
DO 320 I=1,N
320 PRINT 630,Z(I),UNEW(I),MRNEW(I),77(T),77(T),WTD(I),YPLUS(I),
1 UPLUS(I)
630 FORMAT(8(4X,E11.3))

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        PRINT 631,T,DELTBT,D99BT,D99
631  FORMAT(2X,*DIAGS *,4E11.3,/)

        PRINT 632
632  FORMAT(9X,*Z/T*,11X,*L/D99*,8X,*MUTBT/MJ*,8X,
1  *E/D-UF*,10X,*DU/DY*,//)
        DO 322 I=1,N
322  PRINT 635,ZD(I),LD(I),MUD1(I),MUD2(I),MUDNEW(I)
635  FORMAT(5(4X,E11.3))
325  IF(KPLDT.EQ.0)GO TO 330
        ENCODE(10,640,LABEL) DIMR
640  FORMAT(24R=,F8.1)
        CALL PLOJB(UD,ZD,N,1,0,1,0.,1.,1.,LABEL,10,
1  1HU,1,1HZ,1)
330  CONTINUE
        IF(K.GE.200) GO TO 400
        IF(NDIAGN.EQ.1) GO TO 200
C  UPDATE TO MAKE NEXT RADIAL STEP
        DO 340 I=1,N
        UOLD(I)=U(I)
        U(I)=UNEW(I)
        MR(I)=MRNEW(I)
340  CONTINUE
C  NOTE MZ NOT UPDATED
        T=T*(1.+DELTBT)
        R=RNEW
        IF(R.LT.0.) GO TO 400
        IF(NTYPE.EQ. 1 .AND. RPLMOD.GT. 1.) GO TO 400
        PR=PRNEW
        UBOLD=UBULK
        RHQ=RHONEW
        MS=MSNEW
        MSA=MSNEWA
        DELRDI=1.1*DELRDI
        IF(.01*DIMR.LE.ABS(DELRDI)) DELRDI=-.01*DIMR
        RSTD=0.
        DO 360 J=1,5
        RSTD=100./10.**J
        IF(DIMR.GT.RSTD.AND.DIMR+DELRDI.LT.RSTD)DELRDI=RSTD-DIMR
        RSTD=50./10.**J
        IF(DIMR.GT.RSTD.AND.DIMR+DELRDI.LT.RSTD)DELRDI=RSTD-DIMR
        RSTD=30./10.**J
        IF(DIMR.GT.RSTD.AND.DIMR+DELRDI.LT.RSTD)DELRDI=RSTD-DIMR
        RSTD=20./10.**J
        IF(DIMR.GT.RSTD.AND.DIMR+DELRDI.LT.RSTD)DELRDI=RSTD-DIMR
360  CONTINUE
        DELR=DELRDI*T*RORT
        IF(DELR.LE.-.01.AND.KDATA.EQ.1)DELR=-.01
        NPRINT=12
760  FORMAT(2X,2I3,5E11.3)
        IF(M.LE.10)NPRINT=12
        IF(R.LE.REND-1..AND.KDATA.EQ.1)GO TO 400
        IF(DIMR.LT.DIMRMI)GO TO 400

        IF(NTHICK.NE.0 .AND. R.LT.RMAX) GO TO 10
        IF(KTURB.EQ.1.AND. NTHICK.EQ.0) GO TO 17
        IF (R.LT.RMAX .AND. NTHICK.EQ.0) GO TO 25
400  STOP
        END

```

APPENDIX C

EXPERIMENTAL DETAILS

Laminar Flow Test Section

The laminar flow test section was described in Chapter IV. Here attention is restricted to some of the details of test section fabrication and operation. The disks used in this test section were made from rectangular sheets of transparent acrylic resin Plexiglas (Rohm and Haas). The outer radius of each disk was cut in rough form with a band saw and then turned on a lathe of the Physics Department machine shop at The University of Arizona. The outer radii were also rounded and polished to create a convergent inlet. The remaining fabrication was performed within the Aerospace and Mechanical Engineering Department.

Figure 33 shows the disk closest to the viewer (termed the "left" disk in Chapter IV) with its pressure taps and attached Polypenco tubing. The taps running toward the right were not instrumented, whereas the rest were. To identify the other two lines, or "rakes," of taps conveniently, the rake of taps running towards the bottom was called the G-rake while the remaining rake was called the A-rake. All these taps, as well as those on the other disk, the outlet pipe, and the venturi flow meter were plumbed via the tubing shown to the individual valves shown in Figure 34. All pressure measurements were done in the differential mode, in which any selected pressure served as the input to the high side of

Figure 33. Photograph of Instrumented Laminar Flow Test Section.

Figure 34. Valving Arrangement.

the capacitance manometer, and other selected pressures could serve as input to the low side as discussed below. For radial pressure distributions it might appear most straightforward to measure the pressure at any radius with respect to the centermost tap. However, the abrupt turning of the flow associated with the expansion into the outlet pipe resulted in pressure fluctuations at the centermost tap location. In addition the flow turning led to non-radial flow at $r = 0$,¹ and on occasion, at $r = 57$ mm (2.25 in.) as discussed in Chapter IV. In these situations it was convenient to measure a relatively noise-free pressure difference between the desired radius and perhaps one at 70 mm (2.75 in.), i.e., ΔP_{r-70} . If, for some reason, one wanted the difference between the desired pressure and the centermost tap, one could then measure more carefully a time-averaged pressure difference, between 70 mm and the centermost tap, ΔP_{0-70} and from this the desired pressure difference could be formulated as

$$\Delta P_{r-0} = \Delta P_{r-70} - \Delta P_{0-70} \quad . \quad (C-1)$$

For these reasons control valves were configured so that in addition to the venturi throat diameter pressure, either the pressure at the centermost tap of Figure 33, or the A-rake taps at radii of 57 and 70 mm (2-1/4 and 2-3/4 in.) could serve as the low input. The upstream venturi pressure, as well as any other pressure (with the exception of the venturi throat), could serve as the high input.

1. The use of overscores to distinguish dimensional quantities is omitted in this Appendix.

The plumbing and valving system shown in Figure 34 was made leak tight by pressurizing the system with compressed air at 1 MPa (150 psi). The Polypenco tubes were detached from the disks and plugged with Swagelok fittings for this test. Every valve, fitting and joint was soaked with soapy water, the tell-tale presence of bubbles sought out, and the problem was then fixed. Checks were performed by momentarily pressurizing the system, closing valves so that a residual pressure remained in the system, and then using the decay of this residual pressure as an indication of remaining leaks. After several days of these tests the system was made so leak-free that, other than for atmospheric pressure changes, there were no perceptible pressure changes after two days. The Polypenco tubing was then reattached to the disk pressure taps and these final connections were then checked by sealing the peripheral entry to the disks with plastic tape and then pressurizing the entire system to about 0.01 MPa (1 psi). Such a low pressure was necessary to prevent structural damage to the disks. Checks to ensure that these Polypenco-to-disk connections were not leaking were conducted with the smoke gun during the course of the flow experiments.

Pressure tap locations were laid out with a machinist's rule graduated in 0.4 mm (0.016 in.) increments. Table XI shows the desired radial locations and the actual measurements. Measurements were performed with a vernier caliper with an estimated uncertainty of 0.05 mm (0.002 in.). In general, actual radii are within 0.3 mm (0.012 in.) of the desired locations. However, since the average of the two actual radii was used in the comparison of measured and predicted pressures, it is the variation about this average that is the significant quantity,

TABLE XI

LAMINAR TEST SECTION PRESSURE TAP LOCATIONS
(mm)

<u>Desired Radius</u>	<u>Measured A-rake</u>	<u>Measured G-rake</u>	<u>Average Measurement</u>	<u>Variation About Average</u>
0.0	0.0	-	-	0.00 (<u>+0.00%</u>)
57.15	57.33	57.27	57.30	<u>+0.03</u> (<u>+0.05%</u>)
69.85	70.15	70.05	70.10	<u>+0.05</u> (<u>+0.07%</u>)
88.90	88.70	88.67	88.68	<u>+0.02</u> (<u>+0.02%</u>)
114.30	114.35	114.37	114.36	<u>+0.01</u> (<u>+0.008%</u>)
152.40	152.50	153.55	152.52	<u>+0.03</u> (<u>+0.02%</u>)
203.20	203.38	203.40	203.39	<u>+0.01</u> (<u>+0.005%</u>)
292.10	292.48	292.28	292.38	<u>+0.10</u> (<u>+0.03%</u>)

and it is seen that here the absolute variation is 0.10 mm (0.004 in.) or less, and the percentage variation is 0.07 percent or less of the radius in question.

Turbulent Flow Test Section

The turbulent flow test section was fabricated at a branch shop of the Los Alamos Scientific Laboratory. Once again Plexiglas was chosen as the major material in order to enable flow visualization. Figure 35 is a photograph of the completed test section. The rectangular aluminum side rails shown at angles from the vertical are situated between the two Plexiglas plates and form the lateral sides of the test section. These bars are pin-ended at their lower ends and can be placed at angles of 0 to 20° from the vertical. The remaining vertical and horizontal

Figure 35. Photograph of Turbulent Flow Test Section.

bars appearing are outside the Plexiglas plates, and when clamped together, force the Plexiglas into contact with the inner side rails, thus providing control of the spacing. The total length of test section is 1.19 m (47 in.) but the effective length, from inlet to the pinned connections was 1.14 m (45 in.). The 16 vertical pressure taps were situated every 76.2 mm (3.00 in.) on the plate facing the observer in Figure 35. The first tap was located 25 mm (1 in.) down from the entry. In order to check azimuthal symmetry additional taps were positioned horizontally at 0.33, 0.64, and 0.94 m (13, 25, and 37 in.) from the inlet. In order to avoid burrs in the test section the taps were drilled and completed in the same fashion described in Chapter IV for the laminar flow test section. Each tap and shear stress sensor was laid out with the dial indicator, rather than a steel rule, however, so these taps were even more precisely located than their laminar counterparts. Estimated uncertainties in position, measured relative to the entry, are 0.1 mm (0.005 in.). As mentioned in Chapter VII, the flow path was similar to the laminar test section. The air from the surrounding room flowed through the test section, turned 90° into the transition shown at the bottom of Figure 35 and then flowed into the Plexiglas pipe that serves as the inlet to the venturi flow meter. Downstream components, the flow pipe, venturi, flexible ducting, flow straightener and reversed blower, were similar to or identical to those used in the laminar experiments. In addition the valves, manifolds and tubing from the laminar flow experiment were re-used, thus avoiding the complete leak-proofing of a new pressure measuring system. Final checks were made with the pressure method and the smoke gun as described for the laminar test section.

In Figure 35 it may be noticed that a Plexiglas slider plate extends above and below the test section. This slider is mounted in the Plexiglas plate furthest from the observer and contains two removable Plexiglas disk-shaped inserts, in each of which is contained a flush mounted, hot film probe for measuring wall shear stress. These disk-shaped inserts and probes are positioned immediately above the lower of the two horizontal bars visible in Figure 35; the probes are discussed in more detail later in this appendix.

The original intention was to move the slider during a run so that wall shear stress could be measured as a function of position along the test section; however, a number of difficulties prevented this mode of operation. The slider had to be machined in three pieces due to its length of 2.3 m (94 in.). Each piece was fitted with tongue and groove joints and the assembly was completed by fusing the three pieces together at their ends with methyl chloride. The methyl chloride temporarily dissolves the Plexiglas it contacts, so that if two pieces of Plexiglas are brought together and methyl chloride is introduced at the joint the effect is a local dissolving and joining together, followed by fusion and hardening as the methyl chloride evaporates. This same method was used for all Plexiglas-to-Plexiglas joining operations. Unfortunately, there were mismatches of 0.4 mm (0.015 in.) in the slider after joining because of the thickness variations so that at the joints local discontinuities in the surface smoothness occurred. Although these joints were sanded so that the mismatches were reduced to 0.05 mm (0.002 in.) pressure measurements indicated that the pressure drops between two

adjacent taps across these joints, as well as those across the removable inserts, were 40 to 60 percent higher than elsewhere.

The apparatus was dismantled and the joints and inserts were progressively wet-sanded by hand with 200, 420 and 600 grit paper. Then they were hand polished so smoothly that the joints and inserts were absolutely undetectable by a finger run along the surface. Depending upon the person, the finger is sensitive to abrupt surface discontinuities as small as approximately $13\text{ }\mu\text{m}$ (0.0005 in.).

Reassembly showed that the problem was only partially eliminated. Again greater pressure changes were found near the joints and inserts despite the local enlargement of flow area caused by sanding. It should be pointed out, however, that the various reworking operations, which smoothed out the mismatches in the slider direction, resulted in greater mismatches when traversing from the plate to its contained slider in the spanwise direction perpendicular to the flow. At some positions along the slider these spanwise mismatches were as large as 0.8 mm (0.030 in.). Possibly this spanwise mismatch had introduced an increase in secondary flow effects in the corners of the flow channel. To remedy the situation, the entire assembly, plate, slider and inserts, was then planed in a horizontal mill using a fly cutter. Because of the limited cutting radius the work had to be rotated and remounted several times. This operation resulted in a mismatch of 0.8 mm (0.030 in.), but it occurred at one of the plate corners farthest from the inlet, where there is no flow. The region of the slider-plate interface had no mismatch. All machining marks, including the corner mismatch, were removed by sanding with 80, 180, 370, 400, and then 600 grit paper.

The resultant finish was so smooth that optical transparency was almost entirely restored. As expected, the slider transition from disk insert and plate was then very smooth, but only so long as the slider was not moved from its as-machined position. Relative movements of 0.25 mm (10 in.) or more resulted in 0.25 mm (0.010 in.) mismatches and for this reason all subsequent shear stress measurements were made at the same location; the slider was not moved. Subsequent pressure drop measurements indicated that this procedure was successful. Hardly any of the anomalous pressure gradients that were formerly noted at the slider joints and removable inserts were observed.

These reworking operations took about 2 months. In the interim, to continue the research progress another Plexiglas plate, without the slider, inserts or probes, was substituted for the one being re-worked and pressure measurements made at $\theta = 0, 1/4, 1/2, 2,$ and 4° . When the re-worked plate was returned from machining the measurements were repeated at 0 and 2° with the addition of shear stresses and were extended to 8, 12, and 16° , but the runs at $1/4$ and $1/2^\circ$ were never repeated.

Pressure Measurements

As indicated in Chapter IV all pressure measurements were made with an MKS Baratron capacitance manometer system, type 170, which has a sensitivity of 10^{-5} mm of Hg (2.0×10^{-7} psi or 3×10^{-5} lb/ft²). The system consisted of two pressure heads or transducers, one for pressures to 1 mm Hg (145 BH-1) and one for pressures to 100 mm Hg (145 BH-100), an electronics unit 170 M-6B, and a three-channel digital

readout device, 170 M-27B. Each head contains a regulated heater which maintains the head at a constant temperature and prevents drifts due to ambient temperature changes. In conjunction with the electronics unit and the readout, both heads were calibrated at the manufacturer's facility with an air dead-weight tester. Copies of the original calibration records are attached as Tables XII and XIII. The accuracy of both heads is impressive and the errors introduced by pressure measurements with this system are insignificant compared to the other uncertainties discussed in Appendix D. For example, the worst uncertainty in either Table XII or XIII is 0.06 percent, whereas the associated uncertainties in pressure difference due to flow rate uncertainties alone are at least 2.5 percent.

The sampling time of the digital display could be varied in three discrete steps, from a fraction of a second to several seconds. Changes in sampling time of the MKS system were automatically accompanied by changes in electronic circuitry so that the output voltage corresponding to the pressure difference was integrated over a longer period before being displayed. Longer sample times were used when the pressure was fluctuating due to turbulence.

Flow Rate Measurements

Flow rates were measured with a venturi meter, BR-27478-48-1, manufactured by Barco, a subdivision of Aeroquip Corporation. This venturi has a nominal pipe diameter of 75 mm (3 in.) and throat diameter of 37.83 mm (1.489 in.). As mentioned in Chapter IV the venturi was calibrated with an American Instrument Co. bell prover (serial number

TABLE XII

CALIBRATION RECORD SHEET, 1 mm Hg
PRESSURE SENSOR HEAD

MKS INSTRUMENTS, INC.

PRECISION PRESSURE MEASUREMENT

25 ADAMS STREET, BURLINGTON, MASS. 01803 (617) 272-8255

CALIBRATION RECORD SHEET

The following data was measured on the MKS Baratron Head identified by Type and Serial Number. Calibration is performed using an MKS transfer standard that has been calibrated with a CEC Air Dead-Weight Tester. Pressure readings are in units of standard Millimeters of Mercury.

CALIBRATION DATA

PRESSURE STD mmHG	COMPUTED LINEAR D.C. OUTPUT	ACTUAL D.C. OUTPUT	ERROR (± Hg)	ERROR %
.0000	.0000	.0000	0	0
.10145	1.0145	1.0145	0	0
.20291	2.0291	2.0240	-1	.005
.30437	3.0437	3.0440	+1.3	.01
.40582	4.0582	4.0592	+1.0	.02
.50728	5.0728	5.0754	+2.6	.05
.60874	6.0874	6.0888	+1.4	.02
.71020	7.1020	7.1039	+1.9	.03
.81165	8.1165	8.1201	+3.6	.04
.91311	9.1311	9.1365	+5.4	.06
1.01457	10.1457	10.1522	+6.5	.06

Head Type 145BH-1 Data By: R.S.
 Ser. No. 17725 Checked By: R.S.
 Date: 9.27.75
 Indicator Type 170M-LB Sys. Ck.: 9.8
 Ser. No. 17727

NOTE: Unit must be operated on regulated heat approximately
 1 hour before checking or resetting calibration.
 MKS DIGITAL READOUTS READ DIRECTLY IN PRESSURE.
 Bakeable Heads are calibrated at 25° C and Temp. Coeff. not set at 7.0.

TABLE XIII

CALIBRATION RECORD SHEET, 100 mm Hg
PRESSURE SENSOR HEAD

CALIBRATION RECORD SHEET

The following data was measured on the MKS Baratron Head identified by Type and Serial Number. Calibration is performed using an MKS Transfer standard that has been calibrated with a CEC Air Dead-Weight Tester. Pressure readings are units of standard Millimeters of Mercury.

PRESSURE STD mm Hg	COMPUTED LINEAR D.C. OUTPUT	ACTUAL D.C. OUT- PUT	ERROR ($\pm$ MV)	ERROR (%)
.000	.0000	.0000	0	0.00
10.082	1.0082	1.0082	0	0.00
20.164	2.0164	2.0161	-0.3	0.02
30.246	3.0246	3.0236	-1.0	0.03
40.327	4.0327	4.0317	-1.0	0.03
50.409	5.0409	5.0392	-1.7	0.3
60.491	6.0491	6.0461	-3.0	0.5
70.573	7.0573	7.0542	-3.1	0.4
80.655	8.0655	8.0628	-2.7	0.3
90.737	9.0737	9.0727	-1.0	0.1
101.077	10.1077	10.1122	+4.5	0.4

Head Type 145BH-100

Data By : B.S.

Ser. No. 18230

Checked By : REJ

Date : 10/21/76

Indicator Type _____

Sys. Ck. : 9.9

Ser. No. _____

NOTE: Unit must be operated on regulated heat approximately 1 hour before checking or resetting calibration.

MKS DIGITAL READOUTS READ DIRECTLY IN PRESSURE.

Bakeable heads are calibrated at 25°C and Temp. Comp. pot set at 500.

4055) at the Sandia Laboratory in Albuquerque and with a Merriam laminar flow element (model 50 MH 10-2, type 1, serial number A55931) at The University of Arizona. The bell prover calibration is traceable to the National Bureau of Standards (Sandia certification report, file number 3302B, 4-1-76). While the accuracy claimed is 0.2 percent of full scale it will be shortly seen that this accuracy must have been attained under more ideal conditions than those experienced here. Unfortunately, the capacity of the bell prover did not extend to some of the higher flow rates of interest, so additional calibrations were obtained with the laminar flow element. The calibration of the particular flow element used here is not traceable to the National Bureau of Standards, but based on previous experiences with similar flow elements (Reynolds 1968, p. 92) its accuracy might be estimated to be about 1 percent. However, as will be seen, checks on repeatability suggest a more realistic uncertainty of 2 to 3 percent. Figure 36 presents both calibrations, which are quantified in terms of the discharge coefficient C_D , defined in equation (4-1). The expected trends are evident. At low Reynolds numbers C_D is less than 1 due to viscous effects. At high Reynolds numbers C_D approaches a constant. In this particular case the constant is actually slightly greater than 1. As explained by Streeter (1958 p. 317) this apparent anomaly occurs when the integrated kinetic energy, $\rho \int U^2(r) dA$, at the upstream diameter exceeds the estimate based upon the average bulk velocity, $\rho [\int U(r) dA]^2 / A$ which was used in deriving equation (4-1), by a factor greater than that by which the actual kinetic energy at the venturi throat exceeds the estimate based on average throat velocity. This situation is often the case, since for

Figure 36. Venturi Flow Meter Calibrations.

typical fully developed profiles near the venturi inlet, $(U^2)_{\text{avg}} > (U_{\text{avg}})^2$ but at the throat, the rapid acceleration results in a more blunt profile for which $(U^2)_{\text{avg}} \approx (U_{\text{avg}})^2$.

In the range of overlap, reasonably close agreement was obtained with the two methods of calibration. Most disconcerting however is the fact that when the repeatability was checked, a significant difference occurred for both calibrations. For each type of calibration the second run was performed immediately following the original calibration run, so there is little likelihood that the repeatability differences are due to changes in the calibration systems that evolved with time.

Two approaches to estimating the experimental uncertainty of the venturi are presented below. However, first the reader is cautioned that the "best fit" calibration curve was estimated only -- nonlinear regression techniques, which might have been better, were not used. For the first estimate of uncertainty one could assume that all 28 calibration points are statistically independent and have a normal (Gaussian) distribution. The standard deviation could then be calculated and it would be found that the standard deviation of the data from the "best fit" was only 0.33 percent. This is a misleadingly low estimate of the standard deviation, however, in view of the significant, systematic difference between the original calibrations and their repeated runs. Since the source, if any, of these systematic differences could not be isolated and identified, one is forced to attribute these differences to experimental uncertainties. Thus if the standard deviation from the best fit of only the five repeated points is calculated one obtains a much higher, but a more realistic, conservative value of 1.3 percent.

For a confidence level of 95 percent (20 to 1 odds) the uncertainty in flow rate is 2 standard deviations, or about 2-1/2 percent.

In some of the experimental runs conducted in these studies the flow rate was as much as twice that attainable under calibration, even with the laminar flow element. Fortunately the calibrations did at least extend to rates high enough so that C_D was constant, thus providing a sound basis for extrapolation. The reliability of this procedure was further established by some high flow rate, radial, laminar experiments, where it was found that the differences between measured and predicted pressures were within estimated uncertainties and, in some cases, as small or smaller than the differences obtained with lower flow rates.

Shear Stress Measurements

Shear stresses were measured with flush-mounted, Model 1237 hot film probes manufactured by TSI (formerly Thermo Systems Inc.). These are shown in Figure 37 at the centers of the removable inserts mentioned earlier. The probe bodies are 3-mm (1/2-in.) diameter cylinders which appear black in Figure 37. The actual platinum heated films are 1 mm (0.040 in.) wide in the spanwise direction and 0.1 mm (0.004 in.) long in the streamwise direction, and the film near the center of the photograph can barely be detected as a silvery glint in the figure. Mounting each probe in a removable insert facilitated access to the probes for inspection since the easily removable, replaceable inserts were handled rather than the carefully installed probe itself. Each insert was carefully machined so as to slip fit into the slider. After the planing and sanding operation described earlier the

Figure 37. Shear Stress Probes. -- Pressure Taps Appearing in Figure are on Near Wall, Shear Stress Probes are on the Far Wall Surface.

flow surfaces of the insert and slider were flush to within $10\text{ }\mu\text{m}$ (0.0004 in.). The inserts and the slider were then scribed so that the insert would always be reinserted in its correct position, thus avoiding potential mismatches.

Having thus assured that the inserts could always be reinserted so as to avoid significant mismatch at the insert-slider joint one shear stress probe was then finally inserted into each insert. After inserting the shear stress probes the surface flushness of the probe with the insert was checked by running a dial indicator over the insert and probe. The probe position was adjusted until a maximum mismatch of $5\text{ }\mu\text{m}$ (0.0002 in.) was attained. For comparison, Bellhouse and Schultz (1966) found that purposely introducing mismatches of plus and minus $75\text{ }\mu\text{m}$ (0.003 in.) did not affect shear stress measurements in turbulent boundary layers over flat plates with comparable instrumentation. As a final precaution, the sensors were calibrated in place, and once calibrated were not further disturbed for any of the measurements reported here. In operation, the heated probe was kept at a constant elevated temperature by means of electronic feedback circuitry developed for constant temperature anemometers. For these measurements a TSI 1050 anemometer was used.

Theory of Operation

The principle of operation of the hot film sensor can be derived via a Lvque (1928) analysis as follows. One should recall that we have dropped the use of overscores to distinguish dimensional variables. It is first assumed that the length of the heated film in the flow direction

is so small that the thickness δ_t of the thermal boundary layer induced by the film heating is constrained to

$$\delta_t^+ = \delta_t \sqrt{\tau_w / \rho \nu} < 5 \quad . \quad (C-2)$$

With this stipulation the thermal boundary layer would fall within the linear sublayer where $u^+ = y^+$, or in dimensional form,

$$u = \tau_w y / \mu \quad . \quad (C-3)$$

The rate of heat loss caused by the flow over the film is then determined by the laminar boundary layer equation

$$\rho c u \frac{\partial T}{\partial x} = \lambda_f \frac{\partial^2 T}{\partial y^2} \quad . \quad (C-4)$$

With the substitution of the velocity profile, equation (C-3), equation (C-4) becomes

$$y \frac{\partial T}{\partial x} = \frac{\alpha \mu}{\tau_w} \frac{\partial^2 T}{\partial y^2} \quad (C-5)$$

where c is the specific heat capacity at constant pressure, λ_f is the thermal conductivity, α is the thermal diffusivity, $\alpha = \lambda_f / (\rho c)$, x measures the distance downstream from the start of heating and T is temperature. Equation (C-5) is subject to the initial and boundary conditions

$$T(y, x = 0) = 0$$

$$T(y = 0, x) = T_0 \quad (C-6)$$

$$T(y \rightarrow \infty, x) = 0$$

Here it has been assumed that the fluid temperature has the reference value zero, and that the filament temperature is T_0 . A more appropriate surface boundary condition would probably be a constant wall heat flux. Fortunately these two boundary conditions lead to the same functional form (Worsoe-Schmidt, 1967) so the calibration equations differ only by a constant; since the constant is determined by calibration in practice, no difficulty occurs.

The solution to equations (C-5) and (C-6) was obtained via a similarity transformation by Lvque (1928). The principal result was that the heat flux required to maintain the filament at T_0 was, in the nomenclature used here,

$$q'' = 1.12 \lambda_f T_0 \left(\frac{\tau_w}{9\mu\alpha x} \right)^{1/3} \quad (C-7)$$

When equation (C-7) is integrated with respect to x it is found that the total power to maintain the filament at T_0 is

$$P = E^2/R = 1.68 \lambda_f T_0 s \left(\frac{\tau_w h^2}{9\mu\alpha} \right)^{1/3} \quad (C-8)$$

where P is power, E is the filament voltage, R the operating resistance corresponding to the temperature T_0 , s is the spanwise width of the

filament and h is its length in the flow direction. As long as equation (C-2) holds equation (C-8) is applicable for either turbulent or laminar flows.

In deriving equation (C-8) it was assumed that not only is the velocity profile linear and the filament temperature constant, but also that the boundary layer assumptions apply and that convection is the only heat loss mechanism. In actuality many factors disturb these idealized conditions. For one thing $\delta_t^+ \sim (h^+)^{1/3}$, and for some flows with high wall shear stresses h^+ can lead to δ_t^+ greater than 5, so the linear velocity law may not be fully warranted. As noted in Chapter VII, Brown (1967) provides a maximum length criterion which the present experiments satisfy. In addition, h is so small that even the boundary layer assumption might not be warranted (fluid conduction in the x -direction might be almost as important as in the y -direction); Liepmann and Skinner (1954) address this problem. Finally there are other modes of heat transport not accounted for here, such as conduction to the probe body surrounding the filament. Thus, Bellhouse and Schultz (1966) and others express the relationship between heating and shear stress as $E^2/R = A \tau_w^{1/3}$ plus other heat losses. This relation can be expressed in the form

$$(E^2 - E_o^2)/R = A \tau_w^{1/3} \quad (C-9)$$

where A and a verification of the exponent of τ_w are determined by calibration. The quantity E_o^2/R represents the other heat losses, which in principle can be determined by operating the sensor at zero flow

($\tau_w = 0$) if there is no significant natural convection induced. In the present study E was of the order of 9.5 volts and E_0 was approximately 9 volts.

When in operation to measure τ_w in forced convection, the heat losses contributing to E_0 are conduction heat loss via the substrate and mounting and radiation heat loss to the opposite wall. In the present study the no-flow condition was used to calibrate E_0 so there was the potential for heat transfer by natural convection during the determination of E_0 . However, the calibration data shown later agree with equation (C-9) which implies that $(E^2 - E_0^2)/R$ does represent pure forced convection. It can be argued that if significant natural convection was present during the determination of E_0 , then, since it is absent or is not linearly additive during the calibration with flow, a constant error would be introduced; this error would cause the curve to differ from its linear $(E^2 - E_0^2)$ dependence on $\tau_w^{1/3}$.

During the measurement of E_0 in non-flow runs, the voltage rose rapidly to a value of about 9.0 volts when the circuit was energized. Then E_0 gradually decreased to about 8.6 volts over a period of hours, presumably due to the reduction in heat loss via transient conduction through the mounting. About 90 to 95 percent of this decrease occurred during the first 5 minutes so it was decided to evaluate E_0 consistently at 5 minutes after energizing both for calibration and the experimental runs. Though this procedure gave approximately 0.04 volts error in E_0 , it is believed that it would mostly cancel when the calibration was later applied to deduce τ_w in the experiments.

Shear Stress Fluctuations

In accordance with the mixing length theory, turbulent velocity fluctuations diminish to zero as the wall is approached. However, the rate of reduction is no faster than y itself, so that the gradient of the fluctuations at the wall, i.e., the fluctuations of $\partial u / \partial y|_w$ need not be zero. In fact, Eckelmann (1974) showed that these gradient fluctuations, which are related to the wall shear stress fluctuations, are definitely finite, albeit small.

As noted above it was found that E_0 was approximately 9 volts and E was typically 9.5 volts; the fluctuations of E typically were 0.04 volts peak-to-peak. Since these fluctuations are so small compared to E_0 and E , differential calculus can be used to relate the shear stress fluctuations to the voltage fluctuations. If equation (C-9) is first differentiated, and the result then divided by equation (C-9) one obtains

$$\frac{6E}{E^2 - E_0^2} \frac{dE}{2} = \frac{d\tau_w}{\tau_w} \quad (C-10)$$

where dE and $d\tau_w$ represent the turbulent fluctuations. We recall that $E - E_0 = \Delta E = 0.5$ volts, which is small compared to either E or E_0 . Thus, $E^2 = E_0^2 + 2E_0\Delta E + (\Delta E)^2 \approx E_0^2 + 2E_0\Delta E$ and $E dE = E_0 dE + \Delta E dE \approx E_0 dE$. When these approximations are substituted in equation (C-10), it becomes

$$\frac{d\tau_w}{\tau_w} \approx 3 \frac{dE}{\Delta E} \quad (C-11)$$

One first measures the time-averaged value E , subtracts E_0 to form ΔE , and then uses the calibration curves discussed below to estimate

the mean τ_w . The fluctuating dE is then measured as a function of time with an oscilloscope and the magnitude of the shear stress fluctuations finally is estimated with the aid of equation (C-11).

Probe Calibration

Just prior to calibration the upper probe of Figure 37 was accidentally destroyed by overheating. The remaining probe was calibrated in situ by operating the turbulent flow apparatus with parallel side rails (no radial acceleration). With this situation, the flow downstream of the inlet becomes a fully developed one (for most cases) in which the velocity profile, wall shear stress, pressure gradient, and momentum no longer change with downstream position. In the absence of momentum changes a simple force balance gives

$$\tau_w = -t \frac{dP}{dx} \left(\frac{1}{1+2t/b} \right) \quad (C-12)$$

(again the use of overscores is temporarily suspended) where b is the width of the flow section (the distance between the two parallel side rails), and t is the half spacing.

In deriving equation (C-12) it was assumed that the shear stress on the narrow walls formed by the two side rails was the same as that acting on the walls formed by the Plexiglas plates. This idealization is not necessarily true, but the error is small when the aspect ratio $b/(2t)$ is large. For these experiments the aspect ratio was 12.

The calibration procedure consisted of operating the flow apparatus at various flow rates, with fixed probe resistance R , corresponding to fixed average sensor temperatures, and for each flow rate:

(1) measuring the probe voltage and the streamwise pressure distribution, (2) determining the best straight line that fit the downstream pressures, and evaluating its slope, dP/dx , (3) evaluating τ_w from equation (C-12) and finally, (4) plotting $E^2 - E_o^2$ versus τ_w . Figure 38 presents calibration results on log-log coordinates. A straight line relationship was obtained for shear stresses exceeding 0.03 Pa (7×10^{-4} lb/ft²) and the slope was 1/3, as predicted by equation (C-9).

As indicated in Chapter VII, attempts to operate at even lower Reynolds numbers, e.g., below and just above the usually accepted laminar to turbulent transition values of the Reynolds number, resulted in some departures from this straight line relationship. However, the turbulent flow test section was not designed to operate with laminar flow, and in so doing extremely small flow rates (as low as 0.002 m³/s, 0.064 ft³/s) were required, resulting in pressure gradients as low as 6×10^{-6} mm Hg/m, so that systematic errors may have been introduced as described next. Very low flow rates enhanced the role of fluid conduction of sensor heat in the streamwise direction, which was not accounted for in the boundary layer treatment that led to the $E^2 - \tau_w^{1/3}$ relationship. This additional heat conduction would increase heat losses and result in calibration departures opposite to those actually observed. However, this effect was apparently more than offset by the fact that during laminar flow, the probe was not positioned far enough downstream that the flow could be considered fully developed. Flow development is discussed in more detail in Chapter VIII. When not fully developed, part of the pressure gradient that was used in equation (C-12) to calculate the shear stress was actually attributable to momentum gradients,

Figure 38. Calibration of Shear Stress Probe.

so the shear stresses obtained with the calibration would be too high, resulting in deviation from the $1/3$ slope relationship.

Despite these departures there were no abrupt differences between the trend of the calibrations below $R \approx 2000$ and the trend above $R \approx 2000$, so it is concluded that, as expected from the discussion following equation (C-8), the shear sensor calibration is not influenced by transition from laminar to turbulent flow.

APPENDIX D

UNCERTAINTY ANALYSIS

Since a large portion of this investigation was concerned with comparisons of experiments and predictions, and furthermore, since conclusions regarding laminarization processes were deduced from the extent to which an experiment differed from laminar and turbulent predictions, it was necessary to analyze the experimental uncertainties. The analysis was guided by the methods developed by Kline and McClintock (1953), but on occasion was tempered by important considerations such as overall measurement system repeatability. Examples where the latter consideration led to more realistic and conservative estimates of uncertainties were the venturi calibration and the shear stress calibration discussed in Appendix C. In this section the uncertainties that occur in the individual measurements are discussed. The following sections then discuss how these individual measurement uncertainties propagate into the comparisons as either associated pressure difference or shear stress uncertainties.

It may be recalled from Chapter IV that the numerical predictions are based on measured quantities such as flow rate, temperature, etc. Uncertainties in these quantities cause uncertainties in the predictions which then cause uncertainties in the comparison of measured and predicted quantities. Uncertainties in the predictions are referred to as associated uncertainties while uncertainties in the direct

measurements of pressure differences and shear stresses are referred to as direct measurement uncertainties. As discussed in Chapter IV both types of uncertainties were combined to give an overall uncertainty for comparison purposes.

The uncertainties listed throughout this work are based on two standard deviation uncertainties. In other words the measurement or result reported has a 95 percent probability, or 20 to 1 odds, of being within the reported uncertainty. Overall uncertainties were consequences of uncertainties in the measurements of temperature, pressure, flow rate, shear stress probe voltage, pressure tap and shear stress probe locations, and flow gap spacings. As mentioned in Chapter IV, temperature uncertainties were less than 0.5°C . Pressure uncertainties were due to two sources, uncertainties in the measurement itself and variations caused by stray air currents. The Baratron calibration data presented in Appendix C showed that the uncertainties in the direct measurements were insignificant. However stray air currents introduced ± 0.001 mm Hg pressure variations of very long (several seconds), but uncertain periods, so that these variations could not be avoided by simply sampling for longer periods.

Uncertainties in flow rate were caused by uncertainties in air density caused by pressure and temperature uncertainties, pressure difference uncertainties, and by calibration uncertainties. The venturi pressure drops were normally quite large compared to the air current variations and, furthermore, these fluctuations were diminished in magnitude at the venturi location. Thus the uncertainty caused by air currents was quite small and, except for the extremely low flow rates encountered

when calibrating the shear stress probe at very low Reynolds number, the uncertainty induced in flow rates by this effect was less than 0.8 percent. This uncertainty is small compared to the uncertainties of 2.5 percent introduced by the non-repeatability of the venturi calibrations discussed in Appendix C.

Uncertainties in mean shear stress voltages could have been assessed by considering the combined uncertainties of the TSI 1050 constant temperature anemometer and the Fluke 8300 A digital voltmeter used to record the voltage. Instead, a more realistic estimate of the overall uncertainty of the measurement system, and one which also assessed the repeatability of the calibration itself, was obtained by repeating calibration runs at seven different shear stress levels that spanned the range of interest. The differences between the original and repeated voltages did not appear to be voltage or shear stress dependent, so the voltage uncertainty was taken as twice the standard deviation of the seven differences. This uncertainty was 0.011 V. Furthermore, since the estimate of uncertainty was obtained under typical flow conditions, the effects, if any, of not properly averaging out turbulent fluctuations are also included, as are the errors introduced by stray air currents. The additional uncertainty caused by recording turbulent fluctuating shear stress voltages with a Tektronix 475 oscilloscope is estimated to be less than 3 percent of the voltage which, considering the qualitative way in which these data were put to use, is insignificant.

As discussed in Appendix C, the uncertainties in shear stress sensor locations and pressure tap radii for both experiments were less than 0.07 percent of the radius. The remaining uncertainties were

caused by the uncertainty in flow gap spacing. The spacing uncertainty had two components; one was caused by uncertainty in the average spacing, and the other was a result of the local uncertainty caused by the surface waviness in the laminar flow experiment, discussed in Chapter IV. For the laminar experiments, the average spacing was controlled by metal spacers inserted in the flow field. (As indicated in Chapter IV, the flow blockage associated with these spacers was less than 1 percent.) The spacers were cut from very uniform rectangular stock and were measured to an accuracy of $13\text{ }\mu\text{m}$ (0.0005 in.) with a micrometer, so the uncertainty in average spacing was estimated at $13\text{ }\mu\text{m}$ for the laminar flow experiment. For the laminar flow experiments the surface waviness was a more significant effect. In Chapter 4 the waviness was estimated as 0.25 mm peak-to-peak or 0.13 mm about the mean. Assuming that the waviness patterns of the two disks were statistically independent, one may estimate the combined waviness amplitude about the mean as a factor of $\sqrt{2}$ greater, or 0.18 mm (0.007 in.).

In the turbulent and transitional flow experiments one of the Plexiglas plates was planed on a horizontal mill, as discussed in Appendix C, so that its waviness was less than $50\text{ }\mu\text{m}$ (0.002 in.). The other plate had a waviness of amplitude 0.4 mm (0.015 in.) peak-to-peak, and wavelength $100\text{--}200\text{ mm}$. Fortunately, the streamwise length of this flow apparatus (1.14 m) was long compared to the wavelength so this waviness is not expected to affect overall measurements (see the related discussion for laminar flows in this appendix). Because spacers were not used, however, spacing control was not as good for the turbulent experiment, and it is estimated that there was a 0.13 mm uncertainty.

These measurement uncertainties are summarized in Table II for the laminar flow experiments and Tables V and VI for the turbulent and transitional experiments.

Associated Uncertainties, Laminar Experiments

As indicated in Chapter 4, associated uncertainties in predictions were evaluated at the extremes where the dimensionless radii were either much greater or much less than one, so that simple expressions could be obtained for assessing the associated uncertainty due to an uncertainty in a quantity such as a flow rate or spacing. For example, for $r \gg 1$ the laminar pressure difference from the entry to any other radius is linearly related to flow rate, thus if there exists a 2.5 percent uncertainty (of the measured value) in the flow rate measurement, then one can expect a possible 2.5 percent disagreement between measured and predicted pressure differences due to flow rate uncertainties alone.

In dimensional form the laminar pressure differences at the extreme radii are¹

$$P - P_i = \frac{-3\mu Q}{4\pi t} \ln (r/r_i) \quad r_d \gg 1 \quad (D-1)$$

$$P - P_i = \frac{\rho}{2} \frac{Q^2}{(4\pi t)^2} \left[\frac{1}{r_i^2} - \frac{1}{r^2} \right] \quad r_d \ll 1 \quad (D-2)$$

1. The use of overscores for dimensionless quantities is suspended in this appendix. Dimensionless quantities are subscripted by d.

The associated uncertainty in $\Delta P (=P_i - P)$ caused by the uncertainties in the measurement of Q , t , r , ρ , and μ can be simply derived by taking the logarithmic differential. (Uncertainties in ΔP caused by ρ and μ are eventually tied to uncertainties in T and P due to property variation with temperature and pressure.) For example, the associated uncertainty $d\Delta P$, in ΔP , caused by uncertainty in average spacing is

$$\frac{d\Delta P}{\Delta P} = \frac{-3dt}{t} \quad r_d \gg 1 \quad . \quad (D-3)$$

In a similar manner the associated uncertainties due to the other uncertainties can be derived for $r_d \gg 1$ and $r_d \ll 1$. These associated pressure difference uncertainties are listed in Table II of Chapter IV. It was found that the associated pressure uncertainty due to waviness requires a special treatment which is provided below.

Uncertainties Due to Surface Waviness

In many cases we are interested in the overall pressure drop across the laminar test section. For example, in Table III it was the overall pressure drop, measured from the outermost tap to the tap at the lowest radii unaffected by flow turning, that was compared to the corresponding prediction. However, the wavelength of the surface waviness is smaller than the distance over which the overall pressure difference is measured, so to assess the uncertainty associated with the pressure variation caused by waviness, one must integrate the effect of the waviness over the radial interval of interest.

The half spacing t' can be expressed as the sum of a constant uniform spacing, t , plus a sinusoidal variation

$$t' - t = a \sin (2\pi r/\lambda) \quad (D-4)$$

where a is the waviness amplitude and λ is the wavelength. Since $(t/a)^2 \gg 1$, expansion as a power series gives

$$\left[t + a \sin (2\pi r/\lambda) \right]^{-3} \approx \left[1 - 3 \frac{a}{t} \sin (2\pi r/\lambda) \right] / t^3 \quad (D-5)$$

$$\left[t + a \sin (2\pi r/\lambda) \right]^{-2} \approx \left[1 - 2 \frac{a}{t} \sin (2\pi r/\lambda) \right] / t^2 \quad (D-6)$$

to a close approximation.

If the pressure difference equations (D-1) and (D-2) are differentiated with respect to r , and the r -dependent half-spacing equations (D-5) and (D-6) are substituted, and each result is then integrated, it can be shown that the pressure difference uncertainty associated with waviness is

$$\frac{d\Delta P}{\Delta P} = \frac{3a}{t} \frac{[\text{Si}(2\pi r/\lambda) - \text{Si}(2\pi r_i/\lambda)]}{\ln(r/r_i)} \quad r_d \gg 1 \quad (D-7)$$

and

$$\frac{d\Delta P}{\Delta P} = \frac{a}{t} \left(\frac{4\pi r}{\lambda} \right)^2 \left(\frac{r_i^2}{r_i^2 - r^2} \right) \left[\text{I}(2\pi r/\lambda) - \text{I}(2\pi r_i/\lambda) \right] \quad r_d \ll 1 \quad (D-8)$$

where the integral sine function (Gautschi and Cahill, 1964) is

$$\text{Si}(x) = \int_0^x \frac{\sin u}{u} du = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \dots$$

and

$$I(x) = [\sin x/x^2 - \cos x/x - Si(x)]/2 \quad .$$

In these equations it has been assumed that one is most interested in the pressure drop from the outermost entry tap at r_i to some other tap located at r . If the overall pressure drop is required, r would be specialized to the smallest radius unaffected by flow turning.

Both $Si(x)$ and $I(x)$ are graphed in Figure 39. Several features are apparent. For $r/\lambda > 0.5$ both functions are mildly oscillatory. The amplitudes of the oscillations diminish with increasing r/λ . The Si function approaches $\pi/2$ asymptotically while I approaches $-\pi/4$. The pressure drop uncertainty, proportional to the difference of two Si or I functions, should be relatively small unless r/λ is so small that $Si \rightarrow 0$ for one of the radii. If r/λ and r_i/λ are both large, the associated uncertainty approaches zero but there could be maximum uncertainties locally if r/λ is such that Si is at one of its peaks and r_i/λ is such that Si is at one of its minima.

To evaluate the effect of the surface waviness on the overall pressure drop that was reported for most runs, r was chosen as 57 mm (2.25 in.) and r_i was 272 mm (11.5 in.). Table XIV presents the uncertainties for three values of wavelength. When λ is of the order of 200 mm, the localized errors discussed above can indeed be large. Physical measurements, however, indicated that actual wavelengths were 50-100 mm. So it appears that in either case, $r_d \gg 1$ or $r_d \ll 1$, the associated uncertainty in pressure differences can be conservatively estimated as approximately 0.5 a/t or 50 a/t percent.

Figure 39. The $Si(x)$ and $I(x)$ Functions.

TABLE XIV

ASSOCIATED UNCERTAINTIES IN LAMINAR PRESSURE DIFFERENCES
DUE TO SURFACE WAVINESS

$r = 57 \text{ mm (2.25 in.)}$ AND $r_j = 272 \text{ mm (11.5 in.)}$

Wavelength mm (in.)		Associated Pressure Drop Uncertainties $d(\Delta P)/\Delta P$	
		$r_d \gg 1$	$r_d \ll 1$
50	2	0.1 a/t^*	0.05 a/t
100	4	0.46 a/t	0.52 a/t
200	8	0.29 a/t	3.6 a/t

* $a = \text{amplitude, } t = \text{half spacing.}$

Associated Uncertainties, Turbulent and Laminarizing Experiments

For turbulent and transitional experiments the objective was to obtain wall shear stress measurements at a fixed position, as well as radial pressure distributions, so that the associated uncertainties in both shear stress and pressures had to be considered.

Shear Stresses

In Chapter VIII it was shown that as a first approximation, the friction factor was constant so the shear stress can be expressed as

$$\tau_w = f_p (Q/4\pi r t)^2 / 2 \quad . \quad (D-9)$$

The associated shear stress uncertainties can be determined directly from equation (D-9), again using the usual logarithmic differentials. An additional direct measurement uncertainty comes about because of the

uncertainty in probe voltage; this uncertainty was earlier estimated as 0.011 V. The direct measurement uncertainty due to voltages was evaluated in accordance with equation (C-11). Shear stress uncertainties are summarized in Table VI of Chapter VII.

Pressures

In order to estimate associated pressure difference uncertainties systematically, it was assumed that the total pressure change could be expressed as the sum of an acceleration effect and a viscous effect.

$$P_i - P \cong \frac{\rho}{2} \left(\frac{Q}{4\pi t} \right)^2 \left[\frac{1}{r^2} - \frac{1}{r_i^2} \right] + \int_r^{r_i} \frac{\tau}{t} dr \quad . \quad (D-10)$$

The astute reader will recognize this as the "hydraulic method" (Woolard, 1957), an approximation which was criticized in the Introduction. While this method indeed deserves criticism if one is attempting to predict absolute pressure changes, the method should provide reasonably good estimates of the associated pressure difference uncertainties. With the substitution of equation (D-9), equation (D-10) becomes

$$P_i - P \cong \frac{\rho}{2} \left(\frac{Q}{4\pi t} \right)^2 \left\{ \frac{1}{r^2} - \frac{1}{r_i^2} + \frac{f}{t} \left(\frac{1}{r} - \frac{1}{r_i} \right) \right\} \quad . \quad (D-11)$$

With the usual logarithmic differentiation the effects of uncertainties in ρ and Q are rather obvious. However, the effects of uncertainties in t and r are awkward expressions. For example, the associated pressure difference uncertainty due to uncertainty in the spacing is

$$\frac{d\Delta P}{\Delta P} \approx \frac{\frac{-2dt}{t} \left(\frac{1}{r^2} - \frac{1}{r_i^2} \right) - \frac{3fdt}{t^2} \left(\frac{1}{r} - \frac{1}{r_i} \right)}{\left(\frac{1}{r^2} - \frac{1}{r_i^2} \right) + \frac{f}{t} \left(\frac{1}{r} - \frac{1}{r_i} \right)} \quad (D-12)$$

This is an inconvenient expression to evaluate.

A simpler, conservative estimate can be obtained with the following line of reasoning. First, if accelerations are so great that viscous forces can be ignored, then equation (D-12) simplifies to $d\Delta P/\Delta P \approx -2dt/t$. However, for no acceleration at all (parallel side rails) the viscous term yields $d\Delta P/\Delta P \approx -3dt/t$. For either case, a conservative estimate is

$$d\Delta P/\Delta P \approx -3 dt/t \quad (D-13)$$

and in a similar fashion

$$d\Delta P/\Delta P \approx -2 dr/r \quad (D-14)$$

The associated pressure difference uncertainties are summarized in Table V, Chapter VII.

APPENDIX E

EXPERIMENTAL DATA

The experimental data are tabulated on the following pages. The first section contains the laminar experiments discussed in Chapters IV and V, and the second section contains the turbulent and transitional experiments discussed in Chapter VIII.

Laminar Experiments

The data for all the runs summarized in Table III, Chapter 4, follow. Referring to either Figures 8 or 9, one can see that the left outer disk is simply the disk opposite the one attached to the pipe that conveys the air flow, eventually, to the reversed blower, whereas the right inner disk is the one attached to the same pipe. The terminology A-rake and G-rake refers to the non-radial lines upon which the taps were located. These rakes are separated by approximately 120° of azimuth.

As discussed in Chapter IV and Appendix C, all pressures were measured in a differential mode, so that the pressure at one tap was measured with respect to another selected reference tap. As may be seen in the tabulations, the centermost tap was selected for the first two runs, but thereafter, to avoid some of the pressure fluctuations associated with flow turning, taps at greater radii were used as the reference. The radius was measured from the center line of the outlet pipe.

The atmospheric pressure difference at the bottom of each data listing represents the difference between the atmospheric pressure and

the pressure at the reference tap. If one subtracts from this difference the pressure difference between the entry and reference taps one obtains the pressure difference between the atmosphere and the entry tap. If this difference is then subtracted from the atmospheric pressure one has the correct pressure for evaluating the air properties at the entry tap. In practice it was found that the differences between the atmospheric and entry tap pressures were so small that either pressure could be used and only insignificant changes in entry conditions resulted. For this reason, the so-called atmospheric pressure differences were not measured for the turbulent and laminarizing experiments in the next section.

RUN 1 FLOW RATE = $0.0407 \text{ m}^3/\text{s}$ HALF SPACING = 12.7 mm

TEMPERATURE = 26.0 °C ATMOSPHERIC PRESSURE = 608.2 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO CENTER TAP LOCATED AT RADIUS = 0.00 mm)

LEFT (OUTER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
292.48	0.041	292.28	0.041
203.38	0.039	203.40	0.040
152.50	0.036	153.55	0.036
114.35	0.032	114.37	0.032
88.70	0.025	88.67	0.025
70.15	0.013	70.05	0.014
57.33	0.003	57.27	0.004
00.00	0.000	---	---

RIGHT (INNER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
70.10	0.012	70.08	0.012
57.18	-0.005	57.18	-0.005

Atmospheric Press. Diff. = 0.044 mm of Hg

RUN 2 FLOW RATE = $0.0206 \text{ m}^3/\text{s}$ HALF SPACING = 2.41 mm

TEMPERATURE = 26.2 °C ATMOSPHERIC PRESSURE = 609.6 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO CENTER TAP LOCATED AT RADIUS = 0.00 mm)

LEFT (OUTER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
292.48	0.82	292.28	0.81
203.38	0.76	203.40	0.76
152.50	0.72	153.55	0.72
114.35	0.64	114.37	0.64
88.70	0.54	88.67	0.54
70.15	0.38	70.05	0.40
57.33	0.22	57.27	0.23
00.00	0.00	---	---

RIGHT (INNER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
70.10	0.37	70.08	0.40
57.18	0.21	57.18	0.22

Atmospheric Press. Diff. = 0.84 mm of Hg

RUN 3 FLOW RATE = $0.0132 \text{ m}^3/\text{s}$ HALF SPACING = 2.41 mm

TEMPERATURE = 27.9 °C ATMOSPHERIC PRESSURE = 608.4 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 57.33 mm)

LEFT (OUTER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
292.48	0.261	292.28	0.260
203.38	0.235	203.40	0.237
152.50	0.213	153.55	0.213
114.35	0.180	114.37	0.181
88.70	0.135	88.67	0.135
70.15	0.075	70.05	0.083
57.33	0.000	57.27	0.005
00.00	-0.125	---	---

RIGHT (INNER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
70.10	---	70.08	0.079
57.18	0.001	57.18	0.000

Atmospheric Press. Diff. = 0.270 mm of Hg

RUN 4

FLOW RATE = $0.0197 \text{ m}^3/\text{s}$

HALF SPACING = 12.7 mm

TEMPERATURE = 27.8 °C

ATMOSPHERIC PRESSURE = 603.2 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

<u>Radius (mm)</u>	A-Rake <u>Press. Diff. (mm of Hg)</u>	<u>Radius (mm)</u>	G-Rake <u>Press. Diff. (mm of Hg)</u>
292.48	0.010	292.28	0.010
203.38	0.009	203.40	0.009
152.50	0.008	153.55	0.008
114.35	0.006	114.37	0.006
88.70	0.003	88.67	0.003
70.15	0.000	70.05	0.000
57.33	-0.004	57.27	-0.004
00.00	-0.007	---	---

RIGHT (INNER) DISK TAPS

<u>Radius (mm)</u>	A-Rake <u>Press. Diff. (mm of Hg)</u>	<u>Radius (mm)</u>	G-Rake <u>Press. Diff. (mm of Hg)</u>
70.10	---	70.08	0.001
57.18	-0.007	57.18	-0.007

Atmospheric Press. Diff. = 0.010 mm of Hg

RUN 6 FLOW RATE = $0.0076 \text{ m}^3/\text{s}$ HALF SPACING = 1.60 mm

TEMPERATURE = 29.0 °C ATMOSPHERIC PRESSURE = 606.6 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

<u>Radius (mm)</u>	<u>A-Rake Press. Diff. (mm of Hg)</u>	<u>Radius (mm)</u>	<u>G-Rake Press. Diff. (mm of Hg)</u>
292.48	0.175	292.28	0.174
203.38	0.147	203.40	0.147
152.50	0.122	153.55	0.122
114.35	0.088	114.37	0.088
88.70	0.049	88.67	0.048
70.15	0.000	70.05	-0.001
57.33	-0.061	57.27	-0.057
00.00	-0.143	---	---

RIGHT (INNER) DISK TAPS

<u>Radius (mm)</u>	<u>A-Rake Press. Diff. (mm of Hg)</u>	<u>Radius (mm)</u>	<u>G-Rake Press. Diff. (mm of Hg)</u>
70.10	---	70.08	---
57.18	---	57.18	---

Atmospheric Press. Diff. = 0.183 mm of Hg

RUN 7 FLOW RATE = $0.0039 \text{ m}^3/\text{s}$ HALF SPACING = 1.60 mm

TEMPERATURE = 29.8 °C ATMOSPHERIC PRESSURE = 604.4 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
292.48	0.072	292.28	0.073
203.38	0.059	203.40	0.060
152.50	0.047	153.55	0.049
114.35	0.033	114.37	0.034
88.70	0.019	88.67	0.017
70.15	0.001	70.05	0.000
57.33	-0.021	57.27	-0.020
00.00	-0.055	---	---

RIGHT (INNER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
70.10	---	70.08	-0.001
57.18	-0.020	57.18	-0.020

Atmospheric Press. Diff. = 0.076 mm of Hg

RUN 8 FLOW RATE = $0.0106 \text{ m}^3/\text{s}$ HALF SPACING = 1.60 mm

TEMPERATURE = 29.8 °C ATMOSPHERIC PRESSURE = 604.4 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
292.48	0.317	292.28	0.318
203.38	0.271	203.40	0.273
152.50	0.228	153.55	0.230
114.35	0.168	114.37	0.168
88.70	0.095	88.67	0.092
70.15	0.001	70.05	0.000
57.33	-0.118	57.27	-0.112
00.00	-0.250	---	---

RIGHT (INNER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
70.10	---	70.08	-0.003
57.18	-0.117	57.18	-0.117

Atmospheric Press. Diff. = 0.337 mm of Hg

RUN 9 FLOW RATE = $0.0287 \text{ m}^3/\text{s}$ HALF SPACING = 25.5 mm

TEMPERATURE = 29.7 °C ATMOSPHERIC PRESSURE = 607.0 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
292.48	0.0036	292.28	0.0035
203.38	0.0032	203.40	0.0030
152.50	0.0028	153.55	0.0023
114.35	0.0019	114.37	0.0017
88.70	0.0010	88.67	0.0008
70.15	0.0001	70.05	0.0002
57.33	-0.0005	57.27	-0.0007
00.00	-0.0009	---	---

RIGHT (INNER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
70.10	---	70.08	-0.0027
57.18	-0.0088	57.18	-0.0093

Atmospheric Press. Diff. = 0.0038 mm of Hg

RUN 13 FLOW RATE = 0.0264 m³/s HALF SPACING = 1.65 mm

TEMPERATURE = 29.4 °C ATMOSPHERIC PRESSURE = 608.7 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
292.48	1.62	292.28	1.62
203.38	1.39	203.40	1.38
152.50	1.22	153.55	1.20
114.35	0.94	114.37	0.91
88.70	0.55	88.67	0.52
70.15	0.00	70.05	0.00
57.33	-0.78	57.27	-0.70
00.00	-1.38	---	---

RIGHT (INNER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
70.10	---	70.08	-0.04
57.18	-0.78	57.18	-0.75

Atmospheric Press. Diff. = 1.71 mm of Hg

RUN 14 FLOW RATE = $0.0118 \text{ m}^3/\text{s}$ HALF SPACING = 0.80 mm

TEMPERATURE = 28.4 °C ATMOSPHERIC PRESSURE = 606.8 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

Radius (mm)	A-Rake Press. Diff. (mm of Hg)	Radius (mm)	G-Rake Press. Diff. (mm of Hg)
292.48	2.13	292.28	2.11
203.38	1.81	203.40	1.80
152.50	1.50	153.55	1.52
114.35	1.07	114.37	1.12
88.70	0.55	88.67	0.60
70.15	0.00	70.05	0.02
57.33	-0.81	57.27	-0.83
00.00	-1.86	---	---

RIGHT (INNER) DISK TAPS

Radius (mm)	A-Rake Press. Diff. (mm of Hg)	Radius (mm)	G-Rake Press. Diff. (mm of Hg)
70.10	---	70.08	-0.06
57.18	-0.80	57.18	-0.85

Atmospheric Press. Diff. = 2.20 mm of Hg

RUN 16 FLOW RATE = $0.0484 \text{ m}^3/\text{s}$ HALF SPACING = 3.20 mm

TEMPERATURE = 29.7 °C ATMOSPHERIC PRESSURE = 605.9 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

<u>Radius (mm)</u>	A-Rake <u>Press. Diff. (mm of Hg)</u>	<u>Radius (mm)</u>	G-Rake <u>Press. Diff. (mm of Hg)</u>
292.48	0.982	292.28	0.978
203.38	0.901	203.40	0.895
152.50	0.795	153.55	0.795
114.35	0.619	114.37	0.620
88.70	0.368	88.67	0.375
70.15	0.000	70.05	---
57.33	-0.490	57.27	---
00.00	-0.910	---	---

RIGHT (INNER) DISK TAPS

<u>Radius (mm)</u>	A-Rake <u>Press. Diff. (mm of Hg)</u>	<u>Radius (mm)</u>	G-Rake <u>Press. Diff. (mm of Hg)</u>
70.10	---	70.08	-0.027
57.18	-0.452	57.18	-0.461

Atmospheric Press. Diff. = 1.042 mm of Hg

RUN 17 FLOW RATE = 0.0048 m³/s HALF SPACING = 3.20 mm

TEMPERATURE = 28.2 °C ATMOSPHERIC PRESSURE = 609.2 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
292.48	2.72	292.28	2.71
203.38	2.17	203.40	2.16
152.50	1.74	153.55	1.76
114.35	1.21	114.37	1.26
88.70	0.63	88.67	0.69
70.15	0.00	70.05	0.04
57.33	-0.74	57.27	-0.80
00.00	-2.13	---	---

RIGHT (INNER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
70.10	---	70.08	0.02
57.18	-0.73	57.18	-0.84

Atmospheric Press. Diff. = 2.78 mm of Hg

RUN 19 FLOW RATE = $0.0745 \text{ m}^3/\text{s}$ HALF SPACING = 3.20 mm

TEMPERATURE = 31.4 °C ATMOSPHERIC PRESSURE = 607.1 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
292.48	2.33	292.28	2.34
203.38	2.15	203.40	2.14
152.50	1.87	153.55	1.88
114.35	1.44	114.37	1.48
88.70	0.86	88.67	0.90
70.15	0.00	70.05	0.04
57.33	-1.10	57.27	-1.13
00.00	-2.03	---	---

RIGHT (INNER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
70.10	---	70.08	-0.01
57.18	-1.11	57.18	-1.22

Atmospheric Press. Diff. = 2.49 mm of Hg

RUN 20 FLOW RATE = $0.0425 \text{ m}^3/\text{s}$ HALF SPACING = 1.60 mm

TEMPERATURE = 31.8 °C ATMOSPHERIC PRESSURE = 605.9 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
292.48	3.57	292.28	3.58
203.38	3.24	203.40	3.22
152.50	2.84	153.55	2.78
114.35	2.19	114.37	2.14
88.70	1.28	88.67	1.24
70.15	0.00	70.05	0.03
57.33	-1.66	57.27	-1.62
00.00	-3.04	---	---

RIGHT (INNER) DISK TAPS

A-Rake		G-Rake	
Radius (mm)	Press. Diff. (mm of Hg)	Radius (mm)	Press. Diff. (mm of Hg)
70.10	---	70.08	-0.06
57.18	-1.64	57.18	-1.68

Atmospheric Press. Diff. = 3.77 mm of Hg

RUN 21 FLOW RATE = $0.075 \text{ m}^3/\text{s}$ HALF SPACING = 3.20 mm

TEMPERATURE = 31.4 °C ATMOSPHERIC PRESSURE = 605.0 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO A-RAKE TAP LOCATED AT RADIUS = 70.15 mm)

LEFT (OUTER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
292.48	2.31	292.28	2.32
203.38	2.16	203.40	2.20
152.50	1.90	153.55	1.96
114.35	1.44	114.37	1.42
88.70	0.83	88.67	0.87
70.15	0.00	70.05	0.02
57.33	-1.19	57.27	-1.16
00.00	-2.23	---	---

RIGHT (INNER) DISK TAPS

<u>Radius (mm)</u>	A-Rake Press. Diff. (mm of Hg)	<u>Radius (mm)</u>	G-Rake Press. Diff. (mm of Hg)
70.10	---	70.08	---
57.18	---	57.18	---

Atmospheric Press. Diff. = 2.56 mm of Hg

Turbulent and Laminarizing Experiments

The detailed data for the runs summarized in Table VII, Chapter VIII, follow. Because of the insensitivity problem discussed in Chapter VIII, the runs at half angles of 12 and 16° were not used in the laminarization correlations ϕ_p and ϕ_τ , so the data are not presented here.

The tabulated flow rate corresponds to the equivalent 360° peripheral entry. This rate is most useful since it enters in the calculation of R_0 and r . Actual flow rates are obtainable as $2\theta/360$ times the rate tabulated, where θ is the half angle. The hot film wall shear stress probe was located 0.737 m from the inlet. The runs at $\theta = 0.25$ and 0.5° were performed without shear stress measurements.

The distances from the inlet tabulated in the following tables are measured with respect to a horizontal tangent to the top of the flow section (see Figure 35 for the geometry). These distances can be converted to radial distances by subtracting the distance from the inlet tabulated from the sum of 1.14 and $0.152/(2 \sin \theta)$ where θ is the half angle. All dimensions are in meters; 1.14 m is the distance from the inlet at the point of tangency to the outlet, and $0.152/(2 \sin \theta)$ is the virtual radius of the outlet. Most of the pressure taps are located on a vertical line which is perpendicular to, and intersects the point of tangency of a horizontal line and the top of the test section. Additional taps were located left and right of this vertical line; these taps served as checks on azimuthal symmetry of the flow.

RUN T-1 FLOW RATE = 58.9 m³/s HALF ANGLE = 0.25 DEG

TEMPERATURE = 20.8 °C ATMOSPHERIC PRESSURE = 600.5 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	5.44			
0.1017	5.21	0.3301	0.1904 (L)	---
0.1778	4.80	0.3302	0.1271 (L)	---
0.2541	4.38	0.3301	0.0635 (L)	---
0.3302	3.99	0.3303	0.0635 (R)	---
0.4064	3.61	0.3302	0.1270 (R)	---
0.4826	3.24	0.3301	0.1903 (R)	---
0.5587	2.84			
0.6350	2.47	0.6351	0.1270 (L)	---
0.7113	2.07	0.6350	0.0634 (L)	---
0.7874	1.68	0.6349	0.0635 (R)	---
0.8636	1.28	0.6350	0.1270 (R)	---
0.9398	0.84			
1.0161	0.45	0.9398	0.0635 (L)	---
1.0922	0.00	0.9397	0.0635 (R)	---

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = --- V
 VOLTAGE, E_o , = --- V
 SHEAR STRESS, τ_w = --- Pa

RUN T-2 FLOW RATE = 31.7 m³/s HALF ANGLE = 0.25 DEG

TEMPERATURE = 20.8 °C ATMOSPHERIC PRESSURE = 600.5 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	1.85			
0.1017	1.75	0.3301	0.1904 (L)	---
0.1778	1.62	0.3302	0.1271 (L)	---
0.2541	1.47	0.3301	0.0635 (L)	---
0.3302	1.34	0.3303	0.0635 (R)	---
0.4064	1.21	0.3302	0.1270 (R)	---
0.4826	1.09	0.3301	0.1903 (R)	---
0.5587	0.96			
0.6350	0.83	0.6351	0.1270 (L)	---
0.7113	0.70	0.6350	0.0634 (L)	---
0.7874	0.57	0.6349	0.0635 (R)	---
0.8636	0.43	0.6350	0.1270 (R)	---
0.9398	0.29			
1.0161	0.16	0.9398	0.0635 (L)	---
1.0922	0.00	0.9397	0.0635 (R)	---

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = --- V
 VOLTAGE, E_o , = --- V
 SHEAR STRESS, τ_w = --- Pa

RUN T-3 FLOW RATE = 30.3 m^3/s HALF ANGLE = 0.5 DEG

TEMPERATURE = 20.4 °C ATMOSPHERIC PRESSURE = 600.5 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	5.83			
0.1017	5.71	0.3301	0.1904 (L)	---
0.1778	5.30	0.3302	0.1271 (L)	---
0.2541	4.87	0.3301	0.0635 (L)	---
0.3302	4.44	0.3303	0.0635 (R)	---
0.4064	4.04	0.3302	0.1270 (R)	---
0.4826	3.64	0.3301	0.1903 (R)	---
0.5587	3.22			
0.6350	2.82	0.6351	0.1270 (L)	---
0.7113	2.39	0.6350	0.0634 (L)	---
0.7874	1.96	0.6349	0.0635 (R)	---
0.8636	1.51	0.6350	0.1270 (R)	---
0.9398	1.02			
1.0161	0.54	0.9398	0.0635 (L)	---
1.0922	0.00	0.9397	0.0635 (R)	---

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = --- V
 VOLTAGE, E_o , = --- V
 SHEAR STRESS, τ_w = --- Pa

RUN T-4 FLOW RATE = 15.9 m^3/s HALF ANGLE = 0.5 DEG

TEMPERATURE = 21.7°C ATMOSPHERIC PRESSURE = 608.0 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	1.90			
0.1017	1.81	0.3301	0.1904 (L)	---
0.1778	1.68	0.3302	0.1271 (L)	---
0.2541	1.55	0.3301	0.0635 (L)	---
0.3302	1.42	0.3303	0.0635 (R)	---
0.4064	1.29	0.3302	0.1270 (R)	---
0.4826	1.17	0.3301	0.1903 (R)	---
0.5587	1.03			
0.6350	0.91	0.6351	0.1270 (L)	---
0.7113	0.77	0.6350	0.0634 (L)	---
0.7874	0.63	0.6349	0.0635 (R)	---
0.8636	0.48	0.6350	0.1270 (R)	---
0.9398	0.33			
1.0161	0.17	0.9398	0.0635 (L)	---
1.0922	0.00	0.9397	0.0635 (R)	---

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = --- V
 VOLTAGE, E_o , = --- V
 SHEAR STRESS, τ_w = --- Pa

RUN T-5 FLOW RATE = 8.16 m³/s HALF ANGLE = 2.0 DEG

TEMPERATURE = 24.9 °C ATMOSPHERIC PRESSURE = 605.2 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis	Press. Diff. (mm of Hg)
			(m)	
0.0254	7.67			
0.1017	7.60	0.3301	0.1904 (L)	---
0.1778	7.14	0.3302	0.1271 (L)	---
0.2541	6.66	0.3301	0.0635 (L)	6.26
0.3302	6.24	0.3303	0.0635 (R)	6.25
0.4064	5.80	0.3302	0.1270 (R)	---
0.4826	5.29	0.3301	0.1903 (R)	---
0.5587	4.82			
0.6350	4.36	0.6351	0.1270 (L)	---
0.7113	3.74	0.6350	0.0634 (L)	4.34
0.7874	3.14	0.6349	0.0635 (R)	4.34
0.8636	2.47	0.6350	0.1270 (R)	---
0.9398	1.66			
1.0161	0.88	0.9398	0.0635 (L)	1.68
1.0922	0.00	0.9397	0.0635 (R)	1.69

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 9.141 V
VOLTAGE, E_o , = 8.557 V
SHEAR STRESS, τ_w = 3.50 Pa

RUN T-6 FLOW RATE = 5.32 m³/s HALF ANGLE = 2.0 DEG

TEMPERATURE = 24.9°C ATMOSPHERIC PRESSURE = 605.2 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	3.46			
0.1017	3.44	0.3301	0.1904 (L)	---
0.1778	3.23	0.3302	0.1271 (L)	---
0.2541	3.02	0.3301	0.0635 (L)	2.84
0.3302	2.83	0.3303	0.0635 (R)	2.83
0.4064	2.63	0.3302	0.1270 (R)	---
0.4826	2.40	0.3301	0.1903 (R)	---
0.5587	2.19			
0.6350	1.97	0.6351	0.1270 (L)	---
0.7113	1.70	0.6350	0.0634 (L)	1.97
0.7874	1.42	0.6349	0.0635 (R)	1.97
0.8636	1.12	0.6350	0.1270 (R)	---
0.9398	0.75			
1.0161	0.40	0.9398	0.0635 (L)	0.77
1.0922	0.00	0.9397	0.0635 (R)	0.77

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E_1 = 9.019 V

VOLTAGE, E_C = 8.557 V

SHEAR STRESS, τ_w = 1.58 Pa

RUN T-7 FLOW RATE = 3.99 m^3/s HALF ANGLE = 2.0 DEG

TEMPERATURE = 24.9°C ATMOSPHERIC PRESSURE = 605.2 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	2.05			
0.1017	2.02	0.3301	0.1904 (L)	---
0.1778	1.90	0.3302	0.1271 (L)	---
0.2541	1.78	0.3301	0.0635 (L)	1.67
0.3302	1.66	0.3303	0.0635 (R)	1.66
0.4064	1.54	0.3302	0.1270 (R)	---
0.4826	1.41	0.3301	0.1903 (R)	---
0.5587	1.28			
0.6350	1.16	0.6351	0.1270 (L)	---
0.7113	1.00	0.6350	0.0634 (L)	1.16
0.7874	0.84	0.6349	0.0635 (R)	1.16
0.8636	0.66	0.6350	0.1270 (R)	---
0.9398	0.45			
1.0161	0.24	0.9398	0.0635 (L)	0.45
1.0922	0.00	0.9397	0.0635 (R)	0.45

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 8.952 V
 VOLTAGE, E_o , = 8.557 V
 SHEAR STRESS, τ_w = 0.94 Pa

RUN T-8 FLOW RATE = 2.66 m³/s HALF ANGLE = 2.0 DEG

TEMPERATURE = 24.9°C ATMOSPHERIC PRESSURE = 605.2 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

VERTICAL TAPS		ADDITIONAL HORIZONTAL TAPS		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	0.99			
0.1017	0.95	0.3301	0.1904 (L)	---
0.1778	0.90	0.3302	0.1271 (L)	---
0.2541	0.84	0.3301	0.0635 (L)	0.79
0.3302	0.79	0.3303	0.0635 (R)	0.79
0.4064	0.73	0.3302	0.1270 (R)	---
0.4826	0.67	0.3301	0.1903 (R)	---
0.5587	0.61			
0.6350	0.55	0.6351	0.1270 (L)	---
0.7113	0.48	0.6350	0.0634 (L)	0.55
0.7874	0.40	0.6349	0.0635 (R)	0.55
0.8636	0.32	0.6350	0.1270 (R)	---
0.9398	0.22			
1.0161	0.12	0.9398	0.0635 (L)	0.22
1.0922	0.00	0.9397	0.0635 (R)	0.22

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 8.870 V
 VOLTAGE, E_o , = 8.557 V
 SHEAR STRESS, τ_w = 0.48 Pa

RUN T-9 FLOW RATE = $1.13 \text{ m}^3/\text{s}$ HALF ANGLE = 2.0 DEG

TEMPERATURE = 24.9°C ATMOSPHERIC PRESSURE = 605.2 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	0.200			
0.1017	0.185	0.3301	0.1904 (L)	---
0.1778	0.172	0.3302	0.1271 (L)	---
0.2541	0.160	0.3301	0.0635 (L)	0.149
0.3302	0.147	0.3303	0.0635 (R)	0.149
0.4064	0.137	0.3302	0.1270 (R)	---
0.4826	0.124	0.3301	0.1903 (R)	---
0.5587	0.113			
0.6350	0.101	0.6351	0.1270 (L)	---
0.7113	0.088	0.6350	0.0634 (L)	0.102
0.7874	0.074	0.6349	0.0635 (R)	0.102
0.8636	0.058	0.6350	0.1270 (R)	---
0.9398	0.039			
1.0161	0.020	0.9398	0.0635 (L)	0.040
1.0922	0.000	0.9397	0.0635 (R)	0.040

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 8.225 V
 VOLTAGE, E_o , = 8.557 V
 SHEAR STRESS, τ_w^o = 0.072 Pa

RUN T-11 FLOW RATE = 4.22 m^3/s HALF ANGLE = 4.0 DEG

TEMPERATURE = 22.1°C ATMOSPHERIC PRESSURE = 600.4 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP LOCATED 1.0922 m FROM INLET)

VERTICAL TAPS		ADDITIONAL HORIZONTAL TAPS		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tangent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	7.90			
0.1017	7.88	0.3301	0.1904 (L)	---
0.1778	7.53	0.3302	0.1271 (L)	6.88
0.2541	7.20	0.3301	0.0635 (L)	6.85
0.3302	6.85	0.3303	0.0635 (R)	6.85
0.4064	6.48	0.3302	0.1270 (R)	6.88
0.4826	6.04	0.3301	0.1903 (R)	---
0.5587	5.58			
0.6350	5.12	0.6351	0.1270 (L)	---
0.7113	4.51	0.6350	0.0634 (L)	5.10
0.7874	3.85	0.6349	0.0635 (R)	5.10
0.8636	3.11	0.6350	0.1270 (R)	---
0.9398	2.18			
1.0161	1.19	0.9398	0.0635 (L)	2.22
1.0922	0.00	0.9397	0.0635 (R)	2.22

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 9.148 V
 VOLTAGE, E_o , = 8.616 V
 SHEAR STRESS, τ_w^o = 2.20 Pa

RUN T-12

FLOW RATE = 2.66 m³/s

HALF ANGLE = 4.0 DEG

TEMPERATURE = 22.1°C

ATMOSPHERIC PRESSURE = 600.4 mm of Hg

PRESSURE DIFFERENCES(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	3.32			
0.1017	3.30	0.3301	0.1904 (L)	---
0.1778	3.16	0.3302	0.1271 (L)	2.88
0.2541	3.02	0.3301	0.0635 (L)	2.87
0.3302	2.87	0.3303	0.0635 (R)	2.87
0.4064	2.71	0.3302	0.1270 (R)	2.88
0.4826	2.53	0.3301	0.1903 (R)	---
0.5587	2.34			
0.6350	2.14	0.6351	0.1270 (L)	---
0.7113	1.89	0.6350	0.0634 (L)	2.14
0.7874	1.61	0.6349	0.0635 (R)	2.14
0.8636	1.31	0.6350	0.1270 (R)	---
0.9398	0.92			
1.0161	0.50	0.9398	0.0635 (L)	0.93
1.0922	0.00	0.9397	0.0635 (R)	0.93

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E, = 9.029 V

VOLTAGE, E₀, = 8.616 VSHEAR STRESS, τ_w^0 = 1.15 Pa

RUN T-13 FLOW RATE = 1.99 m³/s HALF ANGLE = 4.0 DEG

TEMPERATURE = 22.1°C ATMOSPHERIC PRESSURE = 600.4 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	1.94			
0.1017	1.91	0.3301	0.1904 (L)	---
0.1778	1.83	0.3302	0.1271 (L)	1.67
0.2541	1.75	0.3301	0.0635 (L)	1.67
0.3302	1.66	0.3303	0.0635 (R)	1.67
0.4064	1.57	0.3302	0.1270 (R)	1.67
0.4826	1.46	0.3301	0.1903 (R)	---
0.5587	1.36			
0.6350	1.24	0.6351	0.1270 (L)	---
0.7113	1.10	0.6350	0.0634 (L)	1.24
0.7874	0.94	0.6349	0.0635 (R)	1.24
0.8636	0.76	0.6350	0.1270 (R)	---
0.9398	0.54			
1.0161	0.30	0.9398	0.0635 (L)	0.54
1.0922	0.00	0.9397	0.0635 (R)	0.54

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E, = 8.968 V

VOLTAGE, E₀, = 8.616 V

SHEAR STRESS, τ_w^0 = 0.67 Pa

RUN T-14 FLOW RATE = 1.33 m³/s HALF ANGLE = 4.0 DEG

TEMPERATURE = 22.1°C ATMOSPHERIC PRESSURE = 600.4 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	0.92			
0.1017	0.89	0.3301	0.1904 (L)	---
0.1778	0.85	0.3302	0.1271 (L)	0.78
0.2541	0.81	0.3301	0.0635 (L)	0.78
0.3302	0.76	0.3303	0.0635 (R)	0.78
0.4064	0.72	0.3302	0.1270 (R)	0.78
0.4826	0.67	0.3301	0.1903 (R)	---
0.5587	0.62			
0.6350	0.57	0.6351	0.1270 (L)	---
0.7113	0.50	0.6350	0.0634 (L)	0.58
0.7874	0.43	0.6349	0.0635 (R)	0.58
0.8636	0.35	0.6350	0.1270 (R)	---
0.9398	0.25			
1.0161	0.13	0.9398	0.0635 (L)	0.26
1.0922	0.00	0.9397	0.0635 (R)	0.26

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 8.893 V
VOLTAGE, E_o , = 8.616 V
SHEAR STRESS, τ_w = 0.34 Pa

RUN T-15 FLOW RATE = $0.566 \text{ m}^3/\text{s}$ HALF ANGLE = 4.0 DEG

TEMPERATURE = 22.1°C ATMOSPHERIC PRESSURE = 600.4 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tangent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	0.182			
0.1017	0.173	0.3301	0.1904 (L)	---
0.1778	0.163	0.3302	0.1271 (L)	---
0.2541	0.155	0.3301	0.0635 (L)	---
0.3302	0.147	0.3303	0.0635 (R)	---
0.4064	0.137	0.3302	0.1270 (R)	---
0.4826	0.127	0.3301	0.1903 (R)	---
0.5587	0.116			
0.6350	0.106	0.6351	0.1270 (L)	---
0.7113	0.092	0.6350	0.0634 (L)	---
0.7874	0.079	0.6349	0.0635 (R)	---
0.8636	0.063	0.6350	0.1270 (R)	---
0.9398	0.044			
1.0161	0.024	0.9398	0.0635 (L)	---
1.0922	0.001	0.9397	0.0635 (R)	---

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 8.749 V
 VOLTAGE, E_o , = 8.616 V
 SHEAR STRESS, τ_w = 0.036 Pa

RUN T-17 FLOW RATE = 2.24 m^3/s HALF ANGLE = 8.0 DEG

TEMPERATURE = 22.8°C ATMOSPHERIC PRESSURE = 599.6 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	7.93			
0.1017	7.92	0.3301	0.1904 (L)	7.34
0.1778	7.74	0.3302	0.1271 (L)	7.32
0.2541	7.54	0.3301	0.0635 (L)	7.31
0.3302	7.29	0.3303	0.0635 (R)	7.30
0.4064	7.05	0.3302	0.1270 (R)	7.32
0.4826	6.75	0.3301	0.1903 (R)	---
0.5587	6.43			
0.6350	6.05	0.6351	0.1270 (L)	6.06
0.7113	5.55	0.6350	0.0634 (L)	6.03
0.7874	4.96	0.6349	0.0635 (R)	6.05
0.8636	4.20	0.6350	0.1270 (R)	6.07
0.9398	3.15			
1.0161	1.82	0.9398	0.0635 (L)	3.19
1.0922	0.00	0.9397	0.0635 (R)	3.18

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 9.115 V
VOLTAGE, E_o , = 8.588 V
SHEAR STRESS, τ_w^o = 2.11 Pa

RUN T-18 FLOW RATE = 1.42 m^3/s HALF ANGLE = 8.0 DEG

TEMPERATURE = 22.8°C ATMOSPHERIC PRESSURE = 599.6 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	2.97			
0.1017	2.93	0.3301	0.1904 (L)	2.72
0.1778	2.87	0.3302	0.1271 (L)	2.71
0.2541	2.78	0.3301	0.0635 (L)	2.71
0.3302	2.70	0.3303	0.0635 (R)	2.70
0.4064	2.61	0.3302	0.1270 (R)	2.71
0.4826	2.49	0.3301	0.1903 (R)	---
0.5587	2.36			
0.6350	2.22	0.6351	0.1270 (L)	2.24
0.7113	2.05	0.6350	0.0634 (L)	2.23
0.7874	1.83	0.6349	0.0635 (R)	2.23
0.8636	1.55	0.6350	0.1270 (R)	2.25
0.9398	1.15			
1.0161	0.67	0.9398	0.0635 (L)	1.18
1.0922	0.00	0.9397	0.0635 (R)	1.17

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 8.913 V
 VOLTAGE, E_o , = 8.588 V
 SHEAR STRESS, τ_w^o = 0.96 Pa

RUN T-19 FLOW RATE = 1.00 m³/s HALF ANGLE = 8.0 DEG

TEMPERATURE = 22.8°C ATMOSPHERIC PRESSURE = 599.6 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	1.72			
0.1017	1.69	0.3301	0.1904 (L)	1.57
0.1778	1.64	0.3302	0.1271 (L)	1.56
0.2541	1.60	0.3301	0.0635 (L)	1.56
0.3302	1.55	0.3303	0.0635 (R)	1.55
0.4064	1.50	0.3302	0.1270 (R)	1.56
0.4826	1.43	0.3301	0.1903 (R)	---
0.5587	1.36			
0.6350	1.28	0.6351	0.1270 (L)	1.29
0.7113	1.18	0.6350	0.0634 (L)	1.28
0.7874	1.05	0.6349	0.0635 (R)	1.29
0.8636	0.89	0.6350	0.1270 (R)	1.29
0.9398	0.67			
1.0161	0.38	0.9398	0.0635 (L)	0.68
1.0922	0.00	0.9397	0.0635 (R)	0.67

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 8.824 V
VOLTAGE, E_o , = 8.588 V
SHEAR STRESS, τ_w^o = 0.53 Pa

RUN T-20 FLOW RATE = 0.665 m³/s HALF ANGLE = 8.0 DEG

TEMPERATURE = 22.8°C ATMOSPHERIC PRESSURE = 599.6 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP
LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	0.77			
0.1017	0.75	0.3301	0.1904 (L)	0.69
0.1778	0.73	0.3302	0.1271 (L)	0.69
0.2541	0.71	0.3301	0.0635 (L)	0.69
0.3302	0.69	0.3303	0.0635 (R)	0.69
0.4064	0.66	0.3302	0.1270 (R)	0.69
0.4826	0.63	0.3301	0.1903 (R)	---
0.5587	0.60			
0.6350	0.56	0.6351	0.1270 (L)	0.57
0.7113	0.52	0.6350	0.0634 (L)	0.56
0.7874	0.46	0.6349	0.0635 (R)	0.57
0.8636	0.39	0.6350	0.1270 (R)	0.57
0.9398	0.30			
1.0161	0.17	0.9398	0.0635 (L)	0.30
1.0922	0.00	0.9397	0.0635 (R)	0.30

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E_s = 8.824 V
 VOLTAGE, E_o = 8.588 V
 SHEAR STRESS, τ_w = 0.20 Pa

RUN T-21

FLOW RATE = $0.283 \text{ m}^3/\text{s}$

HALF ANGLE = 8.0 DEG

TEMPERATURE = 22.8°C

ATMOSPHERIC PRESSURE = 599.6 mm of Hg

PRESSURE DIFFERENCES

(ALL PRESSURE DIFFERENCES MEASURED WITH RESPECT TO THE VERTICAL TAP LOCATED 1.0922 m FROM INLET)

<u>VERTICAL TAPS</u>		<u>ADDITIONAL HORIZONTAL TAPS</u>		
Distance From Inlet (m)	Press. Diff. (mm of Hg)	Distance From Horizontal Tan- gent at Inlet (m)	Distance Left or Right (L,R) From Vertical Axis (m)	Press. Diff. (mm of Hg)
0.0254	0.157			
0.1017	0.153	0.3301	0.1904 (L)	0.138
0.1778	0.148	0.3302	0.1271 (L)	0.137
0.2541	0.143	0.3301	0.0635 (L)	0.137
0.3302	0.137	0.3303	0.0635 (R)	0.137
0.4064	0.132	0.3302	0.1270 (R)	0.138
0.4826	0.125	0.3301	0.1903 (R)	---
0.5587	0.117			
0.6350	0.110	0.6351	0.1270 (L)	0.111
0.7113	0.101	0.6350	0.0634 (L)	0.111
0.7874	0.090	0.6349	0.0635 (R)	0.111
0.8636	0.075	0.6350	0.1270 (R)	0.110
0.9398	0.056			
1.0161	0.032	0.9398	0.0635 (L)	0.057
1.0922	0.000	0.9397	0.0635 (R)	0.057

TIME MEAN WALL SHEAR STRESS PROBE QUANTITIES

VOLTAGE, E , = 8.720 VVOLTAGE, E_o , = 8.588 VSHEAR STRESS, τ_w = 0.034 Pa

NOMENCLATURE

$\bar{A}^1$	constant in shear stress-heating rate relationship, equation (C-9), or flow area, or coefficient in finite difference equation (B-2)
A^+	constant in van Driest mixing length equation (6-2)
$\bar{a}$	amplitude of disk half-spacing waviness
$\bar{B}$	coefficient in finite difference equation (B-2)
$\bar{b}$	width of rectangular flow duct in spanwise direction
C	integration constant in "logarithmic law"
$\bar{C}$	coefficient in finite difference equation (B-2)
C_D	venturi discharge coefficient
$\bar{c}$	specific heat at constant pressure
$\bar{D}$	coefficient in finite difference equation (B-2)
$\bar{D}_h$	equivalent hydraulic diameter, four times flow area/peripheral length
$\bar{E}$	shear stress probe voltage
f	friction factor, $2\bar{\tau}_w/\bar{\rho}\bar{V}^2$
$\bar{f}$	cut-off frequency of shear stress anemometer
$\bar{f}_b$	bursting frequency
H	shape factor, δ_1/δ_2
$\bar{h}$	length of shear stress film in flow direction
h^+	non-dimensional length of film, $\bar{h} \bar{U}_\tau/\bar{v}$

1. Overscored variables are dimensional; those without overbars are dimensionless.

$I(x)$	mathematical function, $[\sin x/x^2 - \cos x/x - \text{Si}(x)]/2$
K_1	velocity gradient parameter, $(\bar{v}/\bar{U}_\infty^2) d\bar{U}_\infty/d\bar{r}$ $[(\bar{v}/\bar{V}^2) d\bar{V}/d\bar{r}$ for internal duct flows]
K_2	pressure gradient parameter, $(\bar{v}/\bar{\rho}\bar{U}_T^3) d\bar{P}/d\bar{r}$
K_3	shear stress gradient parameter, $(\bar{v}/\bar{\rho}\bar{U}_T^3) (\partial\bar{\tau}/\partial\bar{z})_w$
k	integer controlling radial location where numerical results are printed
$\bar{\ell}$	mixing length
ℓ^+	dimensionless mixing length, $\bar{\ell} \bar{U}_T/\bar{v}$
M	mass transfer rate in finite difference equation (B-3)
n	total number of nodes
n'	number of nodes in each boundary layer
$\bar{P}$	pressure, or power required to maintain shear stress film at temperature T_o
P	dimensionless pressure, $4\pi\bar{\tau}^3(\bar{P}_i - \bar{P})/\bar{\mu}\bar{Q}$
Pr	Prandtl number, $\bar{\mu}\bar{c}/\bar{\lambda}_f$
$\bar{Q}$	volumetric flow rate
$\bar{q}''$	heat flux required to maintain shear stress film at temperature T_o
R	local Reynolds number, $\bar{V} \bar{D}_h/\bar{v}$ ($4\bar{V}\bar{t}/\bar{v}$ for circular disks)
R_o	overall Reynolds number, $\bar{Q}/4\pi\bar{v}\bar{t}$
R_2	Reynolds number based on momentum boundary layer thickness, $\bar{U}_\infty \delta_2/\bar{v}$
$\bar{r}$	radial coordinate
r	dimensionless radius, $\bar{r}/(\bar{t}/R_o)$
$\bar{s}$	spanwise width of shear stress film
$\text{Si}(x)$	integral sine function, $\int_0^x \frac{\sin u}{u} du$

$\bar{T}$	temperature
$\bar{T}_0$	temperature of shear stress film
$\bar{t}$	half disk spacing
$\bar{U}_\infty$	free stream velocity
$\bar{U}_\tau$	shear velocity, $\sqrt{\bar{\tau}_w/\bar{\rho}}$
$\bar{u}$	radial velocity
$\bar{u}'$	fluctuating component of $\bar{u}$
u	dimensionless radial velocity, $\bar{u}/\bar{V}$
u^+	dimensionless radial velocity, $\bar{u}/\bar{U}_\tau$
$\bar{V}$	mean velocity, $\bar{Q}/4\pi\bar{r}\bar{t}$
$\bar{w}$	transverse velocity
w	dimensionless transverse velocity $\bar{w}\sqrt{R_0}/\bar{V}$
$\bar{x}$	downstream coordinate
x^+	downstream distance parameter, $\bar{x}/(\bar{t}R)$
$\bar{y}$	distance from wall, $\bar{t} - \bar{z}$
y	dimensionless wall distance, $\bar{y}\sqrt{\bar{U}_\infty/\bar{r}\bar{v}}$
y^+	dimensionless wall distance, $\bar{y}\bar{U}_\tau/\bar{v}$
$\bar{z}$	transverse coordinate
z	dimensionless transverse coordinate, $\bar{z}/\bar{t}$

Greek Symbols

α	proportionality constant, $\bar{\lambda}/\bar{\delta}$
$\bar{\alpha}$	thermal diffusivity, $\bar{\lambda}_f/(\bar{\rho}\bar{c})$
Δ	difference
$\bar{\delta}$	boundary layer thickness, location where velocity is $0.995 \bar{U}_\infty$
δ	dimensionless boundary layer thickness, $\bar{\delta}/\bar{t}$

$\bar{\delta}_1$	displacement boundary layer thickness, $\int_0^t (1 - \bar{u}/\bar{U}_\infty) d\bar{y}$
δ_1	dimensionless displacement thickness, $\bar{\delta}_1/\bar{t}$
$\bar{\delta}_2$	momentum boundary layer thickness, $\int_0^{\bar{t}} \bar{u}/\bar{U}_\infty (1 - \bar{u}/\bar{U}_\infty) d\bar{y}$
δ_2	dimensionless momentum thickness, $\bar{\delta}_2/\bar{t}$
$\bar{\delta}_t$	thermal boundary layer thickness, defined as location where $\bar{T}$ is within 0.995 of its undisturbed value
δ_t^+	dimensionless thermal boundary layer thickness $\bar{\delta}_t \bar{U}_t / \bar{\nu}$
θ	half angle of flow convergence, or azimuthal included angle of computation cell in Figure 32
κ	constant in mixing length formula
$\bar{\lambda}$	wavelength of surface waviness
$\bar{\lambda}_f$	thermal conductivity
$\bar{\mu}$	viscosity
$\bar{\nu}$	kinematic viscosity, $\bar{\mu}/\bar{\rho}$
$\bar{\rho}$	density
σ	weighting factor in finite difference formulation
$\bar{\tau}$	shear stress
$\bar{\tau}_a$	anemometer response time used in estimating $\bar{f}$
ϕ	laminarization indicator, defined in equation (8-3)
$\bar{\psi}$	stream function
ψ	dimensionless stream function, $-\bar{\psi} \sqrt{4\pi\bar{t}/\bar{\nu}\bar{Q}}$

Superscripts

- * indicates that value of parameter is estimated
- ' indicates that half spacing includes waviness

Subscripts

avg	average
c, ϕ	centerplane or midplane
eddy	turbulent eddy
eff	effective
entry	position corresponding to entry location or position downstream of entry effects
d	indicates a dimensionless variable in Appendix D
i	inlet, or subscript denoting node variation in radial direction
j	subscript denoting node variation in $\bar{z}$ -direction
l	laminar
meas	measured
o	non-flow value
p	based upon pressure differences
pred	predicted
probe	position corresponding to shear stress probe location
t	turbulent
w	wall
τ	based upon shear stresses

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