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INCOHERENT OPTICAL PROCESSOR FOR X-RAY TRANSAXIAL
TOMOGRAPHY

The University of Arizona

PH.D.

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INCOHERENT OPTICAL PROCESSOR

FOR

X-RAY TRANSAXIAL TOMOGRAPHY

by

John Edward Greivenkamp, Jr.

A Dissertation Submitted to the Faculty of the

COMMITTEE ON OPTICAL SCIENCES (GRADUATE)

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

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As members of the Final Examination Committee, we certify that we have read
the dissertation prepared by John Edward Greivenkamp, Jr.
entitled Incoherent Optical Processor for X-Ray Transaxial Tomography

and recommend that it be accepted as fulfilling the dissertation requirement
for the Degree of Doctor of Philosophy.

W. J. Wardell

4-25-80

Date

H. W. Banett

7-25-80

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M. P. Capp

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TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS.	vii
LIST OF SYMBOLS.	xi
ABSTRACT	xii
1. INTRODUCTION	1
2. THE RECONSTRUCTION PROCESS	3
The Data Set	3
Existence of a Reconstruction.	6
Back Projection and Summation.	6
Filtered Back Projection	11
Convolution Algorithm.	12
Dose Considerations.	19
Digital Implementations.	19
Analog Implementations	23
3. LOOP PROCESSOR--DATA RECORDING	25
Recording Geometry	25
Film-Screen Detectors.	29
Dynamic Range Expansion.	39
Required Signal-to-Noise Ratio	50
Recording in Fan Beams	50
Recording Apparatus.	55
4. LOOP PROCESSOR--BASIC DESIGN	59
Unfiltered Reconstruction.	59
Film Loop.	61
The Reason for Filtering	64
Producing Masks.	66
Complete System for Unfiltered Reconstructions	66
5. LOOP PROCESSOR--FILTERING.	69
Two-Channel Systems	69
Gray-Scale Masks	71
OTF Synthesis.	73
Two-Channel System with OTF Synthesis.	79
Time-Modulated System.	85

TABLE OF CONTENTS--(Continued)

	Page
6. LOOP PROCESSOR--PRACTICAL CONSIDERATIONS.	92
Film Transport.	92
Loop Production	95
Optical System.	95
Electronics	101
Complete System	104
7. LOOP PROCESSOR--RESULTS	106
Phantom Studies	106
Animal Studies	115
Comparison to Other Reconstruction Methods.	120
Historical Development.	123
Artifacts	128
Alignment	130
Weighting of Projections.	136
8. DISCUSSIONS AND CONCLUSIONS	138
Future Work	138
Resolution-Density Discrimination Trade-Off	149
System Utilization.	149
Conclusions	150
APPENDIX A: CENTRAL-SLICE THEOREM.	151
APPENDIX B: FAN-BEAM CORRECTION.	153
APPENDIX C: LENS DISTORTION.	159
APPENDIX D: DOSE AND EXPECTED PERFORMANCE.	162
REFERENCES.	165

LIST OF ILLUSTRATIONS

Figure	Page
2.1. The projection $f_{\phi}(x')$ of an object $\mu(x,y)$	4
2.2. Summation image for a simple object.	8
2.3. The point spread function for a summation image.	10
2.4. The frequency-domain representation of the convolution filter	16
2.5. The filter required for the convolution algorithm.	16
2.6. The point spread function in a filtered image.	18
2.7. Data collection in a first-generation CT scanner	20
2.8. Data collection in recent CT scanners.	22
3.1. A device for recording the one-dimensional x-ray projections of an object on film	26
3.2. A three-point object and its resulting sinogram.	27
3.3. The projection of a helical absorber as it is rotated about its axis	30
3.4. An x-ray film cassette	31
3.5. A set-up for measuring the characteristics of a film-screen detector.	33
3.6. Transmission vs. absorber thickness for a film-screen detector	35
3.7. The characteristic curve predicted by Eq. 3.9.	38
3.8. A water bath	42
3.9. The steps in producing an x-ray dodger from the equivalent water bath.	43
3.10. The dodger corresponding to an annular ring.	45
3.11. The effect of a two-level halftone screen on the response of a single-emulsion x-ray film.	47

LIST OF ILLUSTRATIONS--(Continued)

Figure		Page
3.12.	An x-ray cassette incorporating two halftone screens. . . .	48
3.13.	An experimental characteristic curve produced with halftone screens.	49
3.14.	The geometry for fan-beam recording	52
3.15.	The distorted sine curve produced by fan-beam recording . .	53
3.16.	The tilted-cassette correction for recording in fan beams .	54
3.17.	Error vs. tilt angle.	56
4.1.	An incoherent optical system for producing unfiltered reconstructions	60
4.2.	The slit required to reconstruct a point with coordinates (r, θ)	62
4.3.	The processing mask	63
4.4.	The reason for filtering.	65
4.5.	The method used to produce the processing loops	67
4.6.	The complete system for unfiltered reconstructions.	68
5.1.	A two-channel optical system.	72
5.2.	The optical transfer function produced by OTF synthesis . .	76
5.3.	The optical point spread function produced by OTF synthesis	78
5.4.	Logarithmic phase plate	80
5.5.	The convolution filter produced by OTF synthesis.	84
5.6.	The transfer function corresponding to the filter in Fig. 5.5	84
5.7.	The measured transfer function synthesized by the Ronchi pupil	86

LIST OF ILLUSTRATIONS--(Continued)

Figure		Page
5.8.	A sinusoidal cutout.	87
5.9.	The concentric Ronchi ruling disc.	89
5.10.	The electronic processing performed by the switching amplifier.	90
6.1.	The film transport	93
6.2.	A film gate.	94
6.3.	Scotch-yoke mechanism.	96
6.4.	The effect of lens distortion on a processing mask	98
6.5.	The effect of lens distortion on a sine curve in a sinogram	98
6.6.	The optical system with a field lens	100
6.7.	The circuit diagram of the switching amplifier	102
6.8.	Block diagram of the display electronics	103
6.9.	The complete single-channel optical system used to reconstruct an object from its corresponding sinogram. . .	105
7.1.	The commercial EMI phantom	108
7.2.	A sinogram of the EMI phantom.	109
7.3.	The optical reconstruction of the EMI phantom.	110
7.4.	A phantom of high spatial regularity	111
7.5.	The optical reconstruction of the phantom in Fig. 7.4. . .	112
7.6.	A low-contrast phantom	113
7.7.	The optical reconstruction of the phantom in Fig. 7.6. . .	114
7.8.	A sinogram of a slice through the thorax of a dog.	116

LIST OF ILLUSTRATIONS--(Continued)

Figure	Page
7.9. The optical reconstruction of the sinogram in Fig. 7.8. . .	117
7.10. The optical reconstruction of a slice through the abdomen and front paws of a dog	118
7.11. The optical reconstruction of a slice through the lower abdomen and folded hind legs of a dog	119
7.12. A reconstruction of the EMI phantom produced by digitiz- ing the sinogram shown in Fig. 7.2.	121
7.13. A reconstruction of the EMI phantom produced by a commercial CT scanner	122
7.14. Two-channel optical reconstructions with gray-scale masks .	124
7.15. Two-channel optical reconstructions with OTF synthesis. .	126
7.16. Time-modulated optical reconstructions without transition blanking.	127
7.17. Optical reconstruction of a uniform data set.	129
7.18. The effect of sinogram shift on the reconstruction. . . .	132
7.19. The effect of the amount of sinogram viewed on the reconstruction	133
7.20. The effect of optical magnification on the reconstruction .	135
8.1. The polarization-coded split pupil.	140
8.2. A polarization-coded Ronchi ruling.	141
8.3. An etched-calcite phase Ronchi ruling	142
8.4. The output state of the PEM as a function of time	144
8.5. An optical system for use with the PEM.	146
8.6. The split pupil-split detector optical system	148

LIST OF SYMBOLS

General

Fourier transform pairs are represented by lower and upper case versions of a single character.

$\hat{}$ indicates an estimate of a function.

$\sim$ indicates a filtered function.

Functions are written with various arguments to represent the same distribution. For example, $\mu(x,y)$ and $\mu(r,\theta)$ are the same spatial distribution of the absorption coefficient.

$$\mathcal{F}_2 \left\{ g(x,y) \right\} = G(\xi,\eta) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \, g(x,y) e^{-i2\pi(\xi x + \eta y)}$$

Two-Dimension Fourier Transform

$$\mathcal{F}_2^{-1} \left\{ G(\xi,\eta) \right\} = g(x,y) = \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta \, G(\xi,\eta) e^{i2\pi(\xi x + \eta y)}$$

Two-Dimensional Inverse Fourier Transform

ABSTRACT

A single-channel incoherent optical processor that reconstructs an object from its one-dimensional x-ray projections (transaxial tomography) has been developed. The projection data are recorded on x-ray film with an x-ray film-screen detector in the sinogram format. Every point in the object produces a sine curve on the film. The amplitude and phase of the curve are related to the radial and azimuthal coordinates of the point. The detector automatically performs the logarithmic conversion from x-ray flux to the effective thickness of the object. After development, the film is used as the input transparency for the optical processor. The convolution algorithm, the same procedure used by commercial computed tomography (CT) units, is optically implemented to reconstruct the object from the sinogram. The totally analog processor employs time-modulated optical transfer function synthesis to achieve the necessary bipolar filtering operations. This technique involves a time-varying pupil plane mask in conjunction with synchronous demodulation of a single detector output to obtain the desired response. A Ronchi ruling (an amplitude square-wave grating) generates an optical point spread function which appropriately blurs the projection data, so that when this image is subtracted from an unblurred image of the projection data, the result is properly filtered projection data. The object slice is then reconstructed in a serial fashion by a moving processing mask in the image plane. The mask contains a sinusoidal

slit of varying amplitude to choose a particular sine curve in the filtered image of the projection data. The total operation of the system is discussed, and results are presented that are similar in quality to reconstructions produced by CT scanners. The advantage of this optical system is that it should offer equivalent performance at a reduced cost.

CHAPTER 1

INTRODUCTION

Since its introduction in 1971, x-ray transaxial tomography or computed tomography (CT) has been heralded as a major advance in the field of diagnostic radiology. The advantage of CT over conventional radiography is that a two-dimensional map of the x-ray linear attenuation coefficient of a single slice through the body is produced. A radiograph, on the other hand, is a two-dimensional projection of a three-dimensional object. This shadowing in a radiograph removes depth discriminability and reduces the detail that can be discerned within the object. CT images are not degraded by adjacent planes, so that depth information and high-resolution, low-contrast details are recovered (Swindell and Barrett 1979).

In spite of acceptance by the medical community, CT scanners are not without drawbacks. They are complicated instruments and are expensive to purchase and to maintain. The expense of the machine has limited its availability basically to large hospitals. Some of the features responsible for the cost of a system are an array of a large number (200-600) of discrete x-ray detectors to measure the necessary projection data, a digital computer to process the data, and a gantry to rotate the x-ray source and detector array around the patient.

Since a gantry and some sort of computing hardware will be necessary for any transaxial tomography system, the only item that can be changed to generate a large cost reduction is the detector array. With this in mind, a completely new system, which implements all of the operations inherent to transaxial tomography, has been developed. The detector array is replaced by an x-ray film-screen detector. After development, the x-ray film is used as the input to an incoherent optical processor. The optical processor makes use of time-modulated optical transfer function synthesis and performs all of the operations usually reserved for a digital computer.

In addition to cost, an additional side benefit of the use of film is a possible increase of the spatial resolution in the images. The discrete detectors of CT scanners limit this resolution. Film-screen detectors record the data at an inherently higher spatial resolution, so that the processing conditions can be determined by the attending physician for a particular application.

In subsequent chapters, the advantages, as well as the limitations of this processor, will be discussed in detail as the operation of this totally analog system--the "loop processor"--is described.

CHAPTER 2

THE RECONSTRUCTION PROCESS

The Data Set

The purpose of a transaxial tomography system is to produce a map of the linear x-ray attenuation coefficient $\mu(x,y)$ in a plane through a body. The development used in this chapter follows Barrett and Swindell (1977), Gordon (1977), and Brooks and DiChiro (1976a). The data used to reconstruct this slice are the one-dimensional projections of the slice. The various projections are catalogued by the projection angle ϕ and are defined by the line integrals

$$f_{\phi}(x') = \int_{-\infty}^{\infty} \mu(x',y') dy', \quad (2.1)$$

where the x' - y' axes are rotated counterclockwise from the x - y axes by the angle ϕ . This geometry is illustrated in Fig. 2.1. For processing, a complete set of projection data must be collected, that is, ϕ must vary from 0 to 180°.

When a monoenergetic x-ray beam goes through matter, it is exponentially attenuated. If a parallel x-ray beam (point source at infinity) is passed through the desired slice in the y' direction, the exiting flux $I_{\phi}(x')$ can be written as

Fig. 2.1. The projection $f_\phi(x')$ of an object $\mu(x, y)$.

$$I_{\phi}(x') = I_0 e^{-\int_{-\infty}^{\infty} \mu(x', y') dy'} , \quad (2.2)$$

where I_0 is the incident x-ray flux. This beam strikes some sort of detector array, where a logarithmic conversion is necessary to recover the projection data from the x-ray flux:

$$f_{\phi}(x') = -\ln(I_{\phi}(x')/I_0). \quad (2.3)$$

In practice, the x-ray source will produce a beam with a broad energy spectrum (a bremsstrahlung spectrum). The attenuation coefficient μ is a function of energy so that the beam spectrum changes as the beam passes through the object. Since the low energy x rays are rapidly attenuated, this effect is known as beam hardening. What is actually measured as a projection is

$$f_{\phi}(x') = -\ln \left[\frac{\int_0^{\infty} dE I_E(E) e^{-\int_{-\infty}^{\infty} \mu(x', y', E) dy'}}{\int_0^{\infty} dE I_E(E)} \right], \quad (2.4)$$

where $I_E(E)$ is the incident source spectrum. The artifacts in the reconstruction produced by this polychromaticity and possible corrections are discussed in the literature (see for example Brooks and DiChiro 1976b, Macovski et al. 1976, or McCullough 1975). For the remainder of this discussion, a monoenergetic beam is assumed.

Existence of a Reconstruction

Now that a complete set of projection data has been collected, a question remains as to whether this data is sufficient to reconstruct the object slice. The proof makes use of the central-slice theorem, which is derived in Appendix A:

$$F_{\phi}(\xi') = M(\xi', \eta') \Big|_{\eta'=0}. \quad (2.5)$$

This theorem states that the one-dimensional Fourier transform of a projection $F_{\phi}(\xi')$ is equal to a slice through the two-dimensional transform of the object $M(\xi', \eta')$. The line $\eta'=0$ is rotated by the angle ϕ from the ξ -axis. By making use of projections taken from 0 to 180°, the complete transform of the object $M(\xi, \eta)$ can be evaluated. A two-dimensional inverse Fourier transform yields the object slice.

Because of the two-dimensional transform and the interpolation necessary to evaluate $M(\xi, \eta)$ on a rectangular grid, this reconstruction method is not used in commercial CT scanners.

Back Projection and Summation

A crude estimate of the object can be obtained with the simple operations of back projection and summation. Back projection is, in a sense, the inverse of the projection operation. A two-dimensional function is generated by smearing a projection uniformly in the y' direction. The back projection may be written as

$$b_{\phi}(x', y') = f_{\phi}(x'), \quad (2.6)$$

or

$$b_{\phi}(r, \theta) = f_{\phi}(r \cos(\theta - \phi)). \quad (2.7)$$

A back projection can be performed for all values of the projection angle ϕ . Back projection is not the inverse of measuring a projection because this procedure does not reproduce the object slice. Summation is the process of combining all of the back projections to obtain an estimate of the object. If a discrete number of projections is taken, the summation image would be the algebraic sum of the back projections. For a continuous data set, the summation image is

$$\hat{\mu}_u(r, \theta) = \frac{1}{\pi} \int_0^{\pi} b_{\phi}(r, \theta) d\phi, \quad (2.8)$$

or

$$\hat{\mu}_u(r, \theta) = \frac{1}{\pi} \int_0^{\pi} f_{\phi}(r \cos(\theta - \phi)) d\phi, \quad (2.9)$$

where $\hat{\mu}(r, \theta)$ is the object estimate, and the subscript u indicates that this is an unfiltered image. The integration need only extend to π since additional projections would be redundant. The process of forming a summation image is illustrated in Fig. 2.2 for the case of four projections.

Since all of the operations being performed are linear and shift-invariant, a point spread function can be found for the reconstruction to examine the quality of the image. This evaluation can be performed by considering a point absorber located at the origin. All of the projections are then equal, and $f_{\phi}(x') = \delta(x')$, where $\delta(x')$ is

Fig. 2.2. Summation image for a simple object.

The object consists of two small discs located at the intersections of the back projections. The four projections used for this image are also shown.

the one-dimensional Dirac delta function. The unfiltered point spread function $p_u(r, \theta)$ can be found from Eq. 2.9:

$$p_u(r, \theta) = \frac{1}{\pi} \int_0^\pi \delta(r \cos(\theta - \phi)) d\phi. \quad (2.10)$$

This integral can be evaluated by use of the identity

$$\delta(y(\phi)) = \left| \frac{\partial y(\phi)}{\partial \phi} \right|^{-1}_{\phi=\phi_0} \delta(\phi - \phi_0), \quad (2.11)$$

where $y(\phi_0) = 0$. Since there is only one root in the interval $0 < \phi \leq \pi$, Eq. 2.10 becomes

$$p_u(r) = (\pi r)^{-1}. \quad (2.12)$$

This result can be visualized by considering the summation image formed by a limited number of projections of the point absorber (see Fig. 2.3). A spoke pattern is produced, and the density of spokes represents the point spread function. The $\frac{1}{r}$ behavior follows in the limit as the number of projections goes to infinity.

Since a point spread function has been determined, the unfiltered image can be written in terms of the original object:

$$\hat{\mu}_u(r, \theta) = \mu(r, \theta) ** (1/\pi r), \quad (2.13)$$

where (**) represents a two-dimensional convolution. While the back projection-summation procedure produces a recognizable image, the slow

Fig. 2.3. The point spread function for a summation image.

falloff of the point spread function blurs the image and reduces its contrast. For these reasons, this procedure is not an acceptable method but does suggest methods of filtering to obtain better images.

Filtered Back Projection

The most obvious filtering method is to restore the blurred image by frequency-domain filtering. The Fourier transform of Eq. 2.13 yields

$$\hat{M}_u(\vec{\rho}) = M(\vec{\rho}) \frac{1}{\pi\rho} , \quad (2.14)$$

where $\rho = (\xi^2 + \eta^2)^{\frac{1}{2}}$ and $\mathcal{F}_2\left\{\frac{1}{r}\right\} = \frac{1}{\rho}$. The form of the two-dimensional filter $H_2(\rho)$ can now be determined, and

$$H_2(\rho) = \pi \rho A_2(\rho), \quad (2.15)$$

where $A_2(\rho)$ is a slowly varying apodizing function which is applied to a basic "rho-filter". The apodizing function is necessary since, in practice, there will be a limit to the extent of the frequency domain that is useful. This limit is due to the presence of noise in the data. The spectrum of the filtered image is given by

$$\hat{M}(\vec{\rho}) = \hat{M}_u(\vec{\rho}) H_2(\rho) , \quad (2.16)$$

or

$$\hat{M}(\vec{\rho}) = \left(M(\vec{\rho}) \frac{1}{\pi \rho} \right) (\pi \rho A_2(\rho)) = M(\vec{\rho}) A_2(\rho). \quad (2.17)$$

Transforming back to the space domain produces

$$\hat{\mu}(r, \theta) = \mu(r, \theta) ** a_2(r). \quad (2.18)$$

The point spread function of the reconstructed image can be identified as $a_2(r)$:

$$p(r) = a_2(r) = \mathcal{F}_2^{-1} \{A_2(\rho)\}. \quad (2.19)$$

Because the choice of $a_2(r)$ is open, the reconstructed image can be made arbitrarily close to the original object in the absence of noise.

Convolution Algorithm

A simplification of the two-dimensional filtering described above can be obtained by filtering each projection before the back-projection operation. These are one-dimensional operations, and the filtered projection is written as

$$\tilde{f}_{\phi}(x') = f_{\phi}(x') * h(x') \quad (2.20)$$

where $(*)$ indicates a one-dimensional convolution, and $h(x)$ is a convolution filter. The operations of back projection and summation are now performed on these new "projections" to obtain a filtered estimate of the object:

$$\hat{\mu}(r, \theta) = \frac{1}{\pi} \int_0^{\pi} f_{\phi}(r \cos(\theta - \phi)) d\phi \quad (2.21)$$

or

$$\hat{\mu}(r, \theta) = \frac{1}{\pi} \int_0^{\pi} d\phi \int_{-\infty}^{\infty} dx'_0 f_{\phi}(x'_0) h(r \cos(\theta - \phi) - x'_0) . \quad (2.22)$$

This is the so-called convolution algorithm or filtered-back-projection algorithm for computed tomography. The point spread function for this method can be found by considering a point absorber at the origin. Once again, $f_{\phi}(x') = \delta(x')$, and

$$p(r, \theta) = \frac{1}{\pi} \int_0^{\pi} d\phi \int_{-\infty}^{\infty} dx'_0 \delta(x'_0) h(r \cos(\theta - \phi) - x'_0) , \quad (2.23)$$

or

$$p(r, \theta) = \frac{1}{\pi} \int_0^{\pi} h(r \cos(\theta - \phi)) d\phi . \quad (2.24)$$

The point spread function for this method is nothing more than the summation image of the filter function (see Eq. 2.9).

Since all of the operations that are being performed are linear, it would be reasonable to assume that the same freedom that exists in tailoring the point spread function with frequency-domain filtering (Eq. 2.19) should exist for the convolution method. Only the order of the operations differs for the two methods. There should also be some connection between the two methods which will indicate the form of $h(x)$. Eq. 2.17 represents the desired image obtained with frequency-domain filtering:

$$\hat{M}(\vec{\rho}) = \left(M(\vec{\rho}) \frac{1}{\pi\rho} \right) (\rho \pi A_2(\rho)) , \quad (2.25)$$

where $M(\vec{\rho})$ is the object spectrum, $\frac{1}{\pi\rho}$ is associated with the back projection-summation operation, and $\pi\rho A_2(\rho)$ is the filter. Consider a slice through the frequency plane along the line $\eta' = 0$:

$$\hat{M}(\xi', 0) = \left(M(\xi', 0) \frac{1}{\pi|\xi'|} \right) (\pi|\xi'| A_2(\xi', 0)) . \quad (2.26)$$

The absolute value signs are necessary because ρ can take on positive values only. By the central-slice theorem (Appendix A),

$$M(\xi', 0) = F_{\phi}(\xi') , \quad (2.27)$$

and Eq. 2.26 can be rewritten as

$$\hat{M}(\xi', 0) = \left(F_{\phi}(\xi') \frac{1}{\pi|\xi'|} \right) (\pi|\xi'| A_2(\xi', 0)) . \quad (2.28)$$

Reordering the terms leads to

$$\hat{M}(\xi', 0) = \left(F_{\phi}(\xi') \pi|\xi'| A_2(\xi', 0) \right) \frac{1}{\pi|\xi'|} . \quad (2.29)$$

The term in the parentheses is a filtered projection, and $\frac{1}{\pi|\xi'|}$ can be associated with the blur that occurs in the back projection-summation procedure. The frequency-domain representation of the convolution filter is then

$$H(\xi) = \pi |\xi| A(\xi), \quad (2.30)$$

where $A(\xi)$ is a slice through $A_2(\rho)$. The filtered projections can be reassembled by means of a summation image to obtain the same result as with frequency-domain filtering.

The filter $H(\xi)$ is shown in Fig. 2.4 and consists of a linear frequency boost out to some frequency ξ_0 , where the apodization begins to dominate. The form of the convolution filter $h(x)$ will vary with the choice of apodization, but its general characteristics can be determined. It is a zero-mean filter with a large positive central core of width 2ϵ and negative side lobes that fall off as $-1/x^2$ or oscillate within a $1/x^2$ envelope depending on the apodization. The convolution filter is shown in Fig. 2.5. The parameters ξ_0 and ϵ are related and are indicative of the spatial resolution that will be present in the reconstructed image. In the absence of noise, the image can be made arbitrarily close to the object.

The point spread function in the reconstruction for the filter shown in Fig. 2.5 can be evaluated by the use of Eq. 2.24. The filter function can be written as

$$h(x) = \begin{cases} 1, & |x| \leq \epsilon \\ -\epsilon^2/x^2, & |x| < \epsilon \end{cases} \quad (2.31)$$

Since $h(x)$ is even, the point spread function will be radially symmetric and will be evaluated for $\theta = 0$. By use of the change of variable $x_0 = r \cos \phi$, Eq. 2.24 becomes

Fig. 2.4. The frequency-domain representation of the convolution filter.

Fig. 2.5. The filter required for the convolution algorithm.

$$p(r) = \frac{2}{\pi} \int_0^r \frac{h(x_o) dx_o}{\sqrt{r^2 - x_o^2}} \quad (2.32)$$

Straightforward evaluation of this integral produces

$$p(r) = \begin{cases} 1, & r \leq \epsilon \\ \frac{2}{\pi} \sin^{-1} \left(\frac{\epsilon}{r} \right) - \frac{2\epsilon(r^2 - \epsilon^2)^{\frac{1}{2}}}{\pi r^2} & r > \epsilon, \end{cases} \quad (2.33)$$

For $r \gg \epsilon$, the two terms in $p(r)$ cancel and result in a compact point spread function. The deletion of the negative lobes of this filter yields a point spread function of the form

$$p_+(r) = \begin{cases} 1, & r \leq \epsilon \\ \frac{2}{\pi} \sin^{-1} \left(\frac{\epsilon}{r} \right), & r > \epsilon. \end{cases} \quad (2.34)$$

This operation is basically an unfiltered back projection, and the point spread function tends to $1/r$ for large r . These two point spread functions are shown in Fig. 2.6.

All of the filtering operations required by the convolution algorithms are one dimensional. This property makes this an easy algorithm to implement and is the choice for most commercial CT units.

Fig. 2.6. The point spread function in a filtered image.

The filtered point spread function is compared to the point spread function of an unfiltered image.

Dose Considerations

As alluded to in the preceding sections, noise in the data set can significantly influence the quality of the reconstruction. There are many possible sources for this noise, but in a well-designed radiographic system, the limiting noise should be due to the discrete nature of the x-ray photons. This "quantum noise" was discussed by Barrett, Gordon and Hershel (1976). They found that the dose delivered to the center of the object is

$$D_{\text{center}} \propto \frac{(\text{SNR})^2}{\eta \epsilon^3 t}, \quad (2.35)$$

where SNR is the signal-to-noise ratio in the image, η is the quantum efficiency of the detector, and t is thickness of the x-ray beam. There is therefore a strong trade-off between patient dose, spatial resolution, and density discrimination (related to the SNR in the image). Since the use of x rays is harmful to the patient, the dose must be limited, and this practical consideration restricts the quality of the reconstruction.

Digital Implementations

The first generation of CT scanners was introduced in 1971 by E.M.I., Ltd. (Hounsfield 1973). This device uses a pencil beam of x-rays and a single detector to scan the slice (see Fig. 2.7). The detector measures individual line integrals and collects a projection as the beam translates through the object. The apparatus is rotated

Fig. 2.7. Data collection in a first-generation CT scanner.

The pencil beam measures a single projection as it is translated through the object. The apparatus is rotated to collect other projections.

about the patient to a new projection angle and the process is repeated. This system requires about 4-5 minutes to collect all of the data necessary for a scan.

The trend for current CT scanners has been to reduce the time required for data collection to less than 5 seconds. This increase in speed is accomplished by the use of a more intense x-ray source and a one-dimensional array of up to 600 discrete detectors (see Fig. 2.8). The fan of the beam is sufficient to cover the entire object, and the only motion required is rotation. All of the necessary line integrals can be measured with a rotation of 180° plus the fan angle, but the data must be reordered to obtain parallel projection data from the fan-beam geometry. For the well-being of the patient, the source and detectors are moved about a stationary patient.

The detectors used in the digital system consist of either scintillation crystals with photomultipliers or photodiodes, or xenon gas ionization chambers. These detectors respond directly to x-ray flux, and the logarithmic conversion (Eq. 2.3) is accomplished digitally or with a logarithmic amplifier on the output of the detector. The width of the discrete detectors ultimately limits the spatial resolution of these systems to about 0.5-2 mm, and differences in attenuation coefficient of 0.5% or more can be measured. Because of the dose consideration, these two results cannot be observed simultaneously. The problem of the discrete detector size has been partially overcome by masking off part of each detector to produce narrower detectors. The detector array is then translated to fill in the gaps

Fig. 2.8. Data collection in recent CT scanners.

Only rotation is needed to collect all the projection data.

in the data set. This technique does not efficiently utilize patient dose as a portion of the available x-rays are blocked by the detector masks, although the blockers can be placed in front of the patient to reduce the problem. The data processing is performed on special-purpose minicomputers that implemented the convolution algorithm (Eq. 2.22). The algorithm used is modified to take into account the fan-beam geometry of the unit. A recent review of the state of the art is given by Kak (1979).

Analog Implementations

Enticed by its possible advantages (as well as the interesting nature of the problem), many researchers have examined the use of optical data processing for transaxial tomography. Included in the proposals are systems that produce either filtered or unfiltered images, systems that record the projection data on x-ray film or with an image intensifier-video network, systems in which the image is produced in a serial or a parallel fashion, systems which perform the optical filtering in either coherent or incoherent light, and systems which use electro-optics or mechanical motions to implement the back projection operation. A recent review of this work is presented by Gmitro et al. (1980).

The major advantage of an analog system over its digital counterpart is a possible reduction in cost. This savings could be achieved by replacing the detector array and its associated electronics (which can constitute as much as 50% of the production cost of a basic CT scanner (D'Haenens 1980)) with an x-ray film-screen detector. In

addition to this benefit, a film system will allow the raw data to be stored for future reference and/or reprocessing with the standard radiographs of the patient. The patient's record is consolidated, and there is no need for storage facilities for magnetic tapes or discs. One additional consideration is that an analog system can overcome the limitations imposed by the discrete detectors of a CT scanner. Since the projection data can be recorded at high spatial resolution with film or an image intensifier, the image can be processed to that resolution (with the corresponding decrease in contrast required by the x-ray dose).

The remainder of this discussion will center on one of these analog systems--the "loop processor". This device records the data directly on x-ray film, which is used as the input transparency for an incoherent optical processor. The convolution algorithm is implemented in a serial fashion to reconstruct the object slice.

CHAPTER 3

LOOP PROCESSOR--DATA RECORDING

Recording Geometry

The projection data necessary for the loop processor are recorded with an x-ray film-fluorescent screen detector. Each projection is recorded as a narrow strip on the film, and the strips are stacked adjacent to each other. A device for performing this operation is shown in Fig. 3.1 (Barrett and Swindell 1977). An x-ray beam, which has been collimated by a lead slit, passes through the object and impinges on the film. The beam is confined to the plane of interest. For now, let the source be far away so that a parallel x-ray beam is produced. One projection appears on the film. The object is then rotated through a small angle, and the film is translated vertically to record a second projection. This process continues until a complete set (180°) of data is collected. In practice, the motions used are continuous, and in a clinical setting, the apparatus must rotate around a stationary patient.

The developed film, which is called a sinogram for reasons which will soon be clear, has encoded on it the object in terms of its projections. A three-point object and the resulting sinogram are shown in Fig. 3.2. Each point produces a sine curve on the film, and

Fig. 3.1. A device for recording the one-dimensional x-ray projections of an object on film.

TOP VIEW OF OBJECT

DEVELOPED SINOGRAM

Fig. 3.2. A three-point object and its resulting sinogram.

all of the curves have the same period, which is twice the sinogram length. The amplitude of a curve is proportional to the radius r at which the point is located, and the phase of the sine curve is related to the azimuth angle θ of the point. In Fig. 3.2, points A and B, which are at the same radius, produce curves of equal amplitude but different phase. The curves produced by points B and C, which are at the same azimuth angle, differ only in their amplitude. There is thus a one-to-one correspondence between object points and sine curves on the sinogram.

In the preceding discussion, the object slice has been assumed to have zero thickness, which is obviously not the case. To display a two-dimensional image, an averaging process must therefore be performed along some family of lines through the slice. Since a digital CT scan samples the projection data in angle and the detectors integrate in the vertical direction, the averaging is performed perpendicular to the slice. If the object slice has no variations in the vertical direction, it will be reconstructed by a CT scanner without blurring. The continuous data-taking procedure of the loop processor will handle this special object differently. Any point absorber (a point in all three directions) will produce a true sine curve on the sinogram, independent of its vertical location. A vertical line will therefore produce a sine curve on the film that has been smeared in the ϕ direction, where the length of the smear is the x-ray beam height. This object will be reconstructed by the loop processor as an arc, and

its angular size $\Delta\theta$ is related to the ratio of the beam height t to the sinogram length:

$$\Delta\theta = \pi t/L. \quad (3.1)$$

On the other hand, a line through the object slice which reconstructs as a point, i.e., produces an undistorted sine curve on the sinogram, is the line of integration for the averaging process of the loop processor. A segment of a helix is such a line (see Fig. 3.3). The projections of this helix are all portions of a sine curve. Regardless of the beam thickness, this object will reconstruct as a point, and the object slice is averaged along this family of helices. This helical line would be reconstructed as an arc by a CT scanner. Since the human body has no predominant preferential directions, it does not matter if the averaging is performed along a vertical line or a helical line. The slice thickness is generally less than 5 mm.

Film-Screen Detectors

A question yet to be answered is the suitability of an x-ray film-screen combination as a detector for transaxial tomography. This detector system consists of a high-speed, double-emulsion film sandwiched between two fluorescent intensifying screens in a light-tight cassette (see Fig. 3.4). An incident x-ray photon is absorbed by the screen, and the resulting fluorescence generates a large number of optical photons which expose the film. A single x-ray interaction

Fig. 3.3. The projection of a helical absorber as it is rotated about its axis.

All of its projections are portions of a sine curve, and this object will be reconstructed as a point.

Fig. 3.4. An x-ray film cassette.

The film is sandwiched between two fluorescent intensifying screens in a light-tight cassette.

can render many silver halide grains in the film latent. The primary benefit from the use of fluorescent screens over bare film is the reduced patient dose derived from the increase in the quantum efficiency of the detector (see Eq. 2.35). The stopping power of a pair of screens can be controlled by varying the thickness and composition of the screens. The detective quantum efficiency (DQE) of bare film is only a few percent, the use of standard calcium turnstate screens increases the DQE to about 15%, and with the new rare earth screens, a DQE of up to about 60% can be obtained (Wagner 1977). The DQE of the rare earth screens is comparable to that of the detectors used in CT scanners, so that the use of a film-screen detector does not require extra patient dose. The DQE's quoted were measured at 80 KVP.

DQE is a measure of the effective quantum efficiency and includes noise sources in the detector which reduce the quality of the output. One definition of DQE is that it is the ratio of noise equivalent quanta to exposure quanta. The noise equivalent quanta is the number of input quanta needed with a perfect detector to produce the same output noise as the detector being measured. Two of the sources of this additional noise are the diffusion and absorption of the optical photons within the screen and film granularity.

Film is notorious for its nonlinear response to a stimulus, and the characteristic curve for a film-screen combination must be determined before film can be effectively used as a detector. A set-up for measuring the film response is shown in Fig. 3.5 and consists of an x-ray beam traversing various thicknesses of Plexiglas,

Fig. 3.5. A set-up for measuring the characteristics of a film-screen detector.

The absorber thickness varies for different paths through the Plexiglas triangle. The lead slit reduces the amount of scattered radiation reaching the film.

which are provided by the Plexiglas triangle. The attenuation coefficient of Plexiglas is very nearly that of water ($\mu_{\text{H}_2\text{O}}=0.2/\text{cm}$). A path-length correction is made for the divergence of the beam. The developed film is measured with a densitometer to produce a transmission-versus-thickness (T-x) curve. This procedure allows a system measurement to be made. The effects of the film, the intensifying screens, the spectrum of the source, and the energy dependence of the attenuation coefficient are all taken into account with this technique. An experimental T-x curve is shown in Fig. 3.6, and over some limited dynamic range, the developed film has a transmission that is linear with the thickness of material in the beam:

$$T = T_0 + G\mu x , \quad (3.2)$$

where T_0 is a constant, and G is related to the slope. The factor G is relatively independent of the film and screen types and is equal to about 0.5. The dynamic range of this detector is roughly 8 cm. When exposed in the linear portion of its characteristic curve, the film-screen detector automatically performs the logarithmic conversion necessary to recover the projection data (effective thickness) from the x-ray flux (Eq. 2.3).

Methods of increasing the dynamic range of the detector are discussed in the next section. The constant T_0 in Eq. 3.2 can be ignored because the convolution algorithm (Eq. 2.22) used to process

Fig. 3.6. Transmission vs. absorber thickness for a film-screen detector.

This curve is measured for DuPont Quanta II intensifying screens with DuPont Cronex IV film. The dynamic range of this combination is roughly 8 cm. The measurement was made with a 120 KVP x-ray source.

the data makes use of a zero-mean filter, and this filter has no response to constant signals.

A possible explanation of this linear response can be had by making the assumption that each incident x-ray quanta exposes, on the average, a certain number of photographic grains. The total number of grains per unit area N/A is thus proportional to the x-ray exposure E . Nutting's equation (Dainty and Shaw 1974) relates the optical density D to the number of grains per unit area:

$$D = .434 \frac{N\bar{a}}{A}, \quad (3.3)$$

where $\bar{a}$ is the average area of a grain, and by definition,

$$D = -\log T. \quad (3.4)$$

In terms of the exposure,

$$D = .434 \bar{a} \tau E, \quad (3.5)$$

or

$$T = 10^{-.434\bar{a} \tau E}, \quad (3.6)$$

where τ is a constant. The exposure can be found in terms of the object thickness by Eq. 2.2 (the exposure time must also be included at this step to make the change from flux to exposure):

$$E = E_0 e^{-\mu x} . \quad (3.7)$$

where E_0 is the x-ray exposure when there is no material in the beam.

Combining the last two equation produces

$$T = 10^{-.434 \bar{a} \tau E_0} e^{-\mu x} \quad (3.8)$$

or

$$T = 10^{-\tau' e^{-\mu x}} \quad (3.9)$$

where $\tau' = .434 \bar{a} \tau E_0$. This expression is plotted for several values of τ' in Fig. 3.7 and strongly resembles the experimental curve (Fig.3.6). The approximation this equation is based upon will break down when the clusters of grains begin to interact. This condition occurs for small values of absorber thickness (high density films), and indeed this curve departs from the experimental result in this region. For a true comparison, Eq. 3.9 must be multiplied by the transmission losses of the film base and fog level.

Another concern in choosing a detector for a system is its uniformity of response. The x-ray film itself has been found to be extremely uniform, but variations have been measured in response of the intensifying screens. The size of these variations changes from screen to screen. Screens have been obtained where the variation in transmission is less than $\pm 3\%$. Most of this variation is a slow shading of the film from side to side.

Fig. 3.7. The characteristic curve predicted by Eq. 3.9.

The expression is plotted for several values of the constant τ' , and is compared to an actual film curve. The curves are normalized to a maximum of 1 and assume $\mu = .22/\text{cm}$.

A possible advantage of a film-screen system is that its sensitivity and uniformity will not change from run to run. This is sometimes a problem with the detector arrays in CT units, as on occasion a calibration must be performed. Even if the absolute sensitivity of the film changes, high-spatial-frequency variations will not result.

Because the patient dose must be utilized efficiently, the intensifying screens with the highest available quantum efficiency should be used. The dose at which a reconstruction is to be made therefore determines the speed of the x-ray film. The film must be chosen to place the linear portion of the characteristic curve at the desired x-ray exposure level. For a low-dose picture, a fast film is necessary, and for a high-dose picture, a slow film is required. The film speed may be varied by changing film type or the development procedure, or by a spectral mismatch between the film and the screens. Table 3.1 lists various film-screen combinations and their relative speeds. The pairs are listed in approximate order of increasing quantum efficiency of the screens, and the dynamic range of each combination is also given.

Dynamic Range Expansion

The small dynamic range of the film-screen detectors examined in the preceding section is a severe limitation for a practical transaxial tomography system. An average human head or body is much thicker than 10 cm. Two approaches have been proposed to reduce this restriction. The first involves reducing the apparent dynamic range of the object, and the second aims to improve the detector itself.

Table 3.1. Characteristics of Various Film-Screen Detectors.

Measurements made at 120 KVP		Relative* Speed	Dynamic Range (in cm of Plexiglas)
Screen	Film		
DuPont Par	3M R	0.28	8
DuPont Lightning Plus	3M R	0.80	8
3M SX-68	3M R	0.93	8
DuPont Quanta II	Kodak Ortho G	0.80	9
	3M R	1.00	8
Kodak Lanex Regular	Kodak RP/V	0.025	8
	Kodak TL	0.058	8
	DuPont Cronex IV	0.60	8
	3M XUD	0.80	8
	Kodak Ortho G	1.18	10
	3M XD	2.03	8
	3M XM	2.86	8
DuPont Quanta III	Kodak RP/V	0.076	9
	Kodak TL	0.17	8
	3M XUD	0.73	8
	Kodak Ortho G	0.95	10
	3M XM	1.02	8
	DuPont Cronex IV	1.38	9
	3M R	1.46	9
	3M XD	1.88	8

* DuPont Quanta II Screens with 3M R film is defined to be 1.00.

The dynamic range required of a detector is the difference in the line integrals of the attenuation coefficient along the lines through an object and that of the line passing outside the object. This difference can be reduced by placing the body in a water bath (see Fig. 3.8). All of the rays passing through the object now receive approximately the same amount of attenuation. The object rotates within the water bath. The dynamic range required by the detector now is determined only by differences in attenuation coefficient within the object, not thickness variations. If the object is water equivalent, there will be no variations in x-ray irradiance at the output. The early CT scanners used this technique (actually water bags stacked around the patient) to reduce the dynamic range. Also, since thickness variations can produce beam-hardening artifacts (Eq. 2.4), the use of a water bath reduces this type of artifact in the reconstruction.

For practical reasons, it is desirable to avoid the use of a water bath. An alternative is the use of an x-ray "dodger" to modify the beam intensity before it reaches the object. It is as if the water bath, with a hole corresponding to the object, is placed in front of the object. The hole can be collapsed, and the dodger can be made out of a material with a larger attenuation coefficient than water to reduce its thickness. The steps used in going from a water bath to a dodger are illustrated in Fig. 3.9. For a uniform circular object, the dodger can completely compensate for the object, and no flux variations will be seen by the detector.

There is another class of dodgers which reconstructs as an annulus outside the object. The form of the dodger is the projection

Fig. 3.8. A water bath.

The object rotates within the rectangular water bath, and all rays passing through the object receive approximately the same attenuation.

(A)

(B)

(C)

Fig. 3.9. The steps in producing an x-ray dodger from the equivalent water bath.

The x-rays are incident from the top.

- (A) The water bath with a hole for the object.
- (B) The hole has been collapsed to the edge of the bath.
- (C) The resulting dodger produced by replacing the water with a material of higher absorption coefficient.

of the annular ring. When a non-circular object is used with this dodger, the thickness variations seen by the detector are reduced over the extent of the object. The dynamic range required of the detector is therefore reduced. This condition is shown in Fig. 3.10. The dodger in Fig. 3.9 can also be used with non-circular object. When the data is recorded with a fan beam (the x-ray source is not at infinity), the dodger can be distorted to obtain the same result. Some CT units are equipped with a selection of dodgers to compensate for various size objects.

A more direct approach to the problem of limited dynamic range is to improve the detector. Glaser and Barrett (1979) propose the use of halftone screens to modify the response of the detectors. This technique trades some of the extra spatial resolution of film for increased dynamic range. For now, consider a single intensifying screen and a single-emulsion film with a halftone screen between. The halftone is one dimensional, with a period of d in the x -direction, and has a transmission profile

$$T_s = \begin{cases} T_a, & 0 \leq x < a \\ 1, & a \leq x < d \end{cases}, \quad (3.10)$$

where T_a and a are constants. The distance d should be well below the spatial resolution to be measured on the film, but this distance must be resolved by the film. This halftone acts as a neutral density filter for modulating the exposure received by the film from the intensifying screen. If $T(E)$ is the response of the film to light photons from the intensifying screen (the light output of the screens is very linear with

Fig. 3.10. The dodger corresponding to an annular ring.

- (a) The annular ring containing a non-circular object.
- (b) A projection of (a). The projection of the ring corresponds to the shape of the dodger.

x-ray flux, so that $T(E)$ also describes the T - x curve for the detector), the area-weighted transmission of the film-half-tone system is

$$\bar{T}(E) = (a/d)T(T_a E) + ((d-a)/d)T(E). \quad (3.11)$$

This result is shown in Fig. 3.11, where curves A and B are the two terms in the equations, and curve C is the net result. The scale on the horizontal axis is not exposure but thickness, and takes into account the exponential attenuation of the x-ray beam. The dynamic range of the detector has been essentially doubled by this procedure. The kink in curve C, due to the crossover from curve A to curve B, can be reduced or eliminated by the use of a three-or-more-level half-tone screen or possibly by careful choice of the parameters a and T_a .

In practice, the half-tone technique is complicated by existence of two intensifying screens and two film emulsions. Each fluorescent screen exposes both emulsions as the film base is not opaque. Half-tone screens are placed on both sides of the film (see Fig. 3.12). The spatial frequency of the half-tone is also chosen so that the half-tone is not resolved by the far emulsion. This condition is possible because of the spread of the light in the film base. It is, however, critical that the near emulsion maintain good contact with the half-tone. Results obtained with this technique are presented in Fig. 3.13, and show that the dynamic range of the film-screen detector has been expanded by about a factor of two.

The use of the half-tone technique with a properly designed x-ray dodger should provide a film-screen detector with sufficient dynamic

Fig. 3.11. The effect of a two-level halftone screen on the response of a single-emulsion x-ray film.

Curve A is exposed through the dense portion of the halftone, Curve B through the clear part, and Curve C is the area weighted response.

Fig. 3.12. An x-ray cassette incorporating two halftone screens.

Fig. 3.13. An experimental characteristic curve produced with halftone screens.

The dynamic range of the detector has essentially doubled by the use of halftone screens. The screens are DuPont Detail, and the film is DuPont Cronex IV. The curves are normalized to a maximum of 1.0.

range for human studies. Under these conditions, a water bath would not be necessary.

Required Signal-to-Noise Ratio

The preceding section examined the range of signals to which the detector must respond. In this section, the noise levels required for the system to see small, low-contrast details within the object are examined. A typical small object would be 2 mm in diameter and would differ by 1% in attenuation coefficient from the surrounding material. The change in transmission this detail would record on the x-ray film can be determined by Eq. 3.2 in differential form:

$$\Delta T = G \Delta \mu x. \quad (3.12)$$

If the surrounding material is water, as is often the case, $\mu \approx 0.2/\text{cm}$ and $\Delta \mu = 0.002/\text{cm}$ for a 1% difference. Since $G \approx 0.5$,

$$\Delta T = 0.5 (0.002/\text{cm})(0.2 \text{ cm}) = 2 \times 10^{-4}. \quad (3.13)$$

This quantity is the change in transmission of the film produced by the ray going through the center of the 2mm object and is the accuracy to which the transmission of the sinogram must be measured. It indicates that the optical processor yet to be described must have a signal-to-noise ratio yet to be defined of at least 5000:1.

Recording in Fan Beams

The projection data must be recorded with a fan beam as indicated in Fig. 3.1. What is actually recorded on the film by each point in the object is not a sine curve but is a distorted sine curve.

The geometry of this situation is shown in Fig. 3.14, and the shape of the curve actually produced is

$$x = \frac{d_1 r \sin (\pi y/L)}{d_2 + r \cos (\pi y/L)} \quad , \quad (3.13)$$

where y is the coordinate along the length of the sinogram (increasing down the length of the sinogram), r is the radius of the point in question, d_1 and d_2 are defined in Fig. 3.14, and L is the sinogram length. This expression is derived in Appendix B and is plotted in comparison with a sine curve in Fig. 3.15. The maximum excursion of the recorded curve occurs at

$$y_m = \frac{L}{\pi} \cos^{-1} (-r/d_2) \quad (3.14)$$

instead of at $L/2$ or $3L/2$.

Barrett and Swindell (1977) propose correcting this error by tilting the x-ray cassette during the recording process (see Fig. 3.16). When there is no tilt, a point on one side of the object is recorded late on the sinogram, and a point on the other side is recorded early. Because of this effect, skewing or reordering the data by tilting the cassette makes intuitive sense. The expression for the shape of the resulting curve for each point is parametrically described by

$$x = \frac{d_1 r \sin \theta \cos \alpha}{d_2 + r \cos \theta} \quad , \quad (3.15)$$

and

Fig. 3.14. The geometry for fan-beam recording.

Fig. 3.15. The distorted sine curve produced by fan-beam recording.

The distorted curve is plotted for the conditions $d_1 = 1 \text{ m}$, $d_2 = 0.75 \text{ m}$, $r = 12.5 \text{ cm}$ and $L = 0.38 \text{ m}$. A true sine curve is shown for comparison.

Fig. 3.16. The tilted-cassette correction for recording in fan beams.

$$y = L\theta/\pi - \frac{d_1 r \sin \theta \sin \alpha}{d_2 + r \cos \theta} \quad (3.16)$$

where α is the tilt angle, and θ is the rotation angle of the point in question. This expression is also derived in Appendix B. The RMS difference between this curve and a best fit sine curve is plotted versus the tilt angle for the case $d_1 = 1$ m, $d_2 = 0.75$ m, $r = 12.5$ cm and $L = 0.38$ m in Fig. 3.17. The minimum error is 0.43 mm and occurs at a value

$$\alpha_{\min} = \frac{L}{\pi d_1}, \quad (3.17)$$

as predicted by Barrett and Swindell (1977). The RMS error without the tilt correction for the same parameters as above is 10.1 mm. The corrected curve differs by less than 25% of the desired resolution (2 mm) for even a rather short 1 m source-to-film distance. In order to collect a complete set of projection data, the object must be rotated through an angle of 180° plus the fan angle.

Recording Apparatus

A device has been constructed by Scott Gordon and Arthur Gmitro to produce sinograms, and is quite similar to the system diagrammed in Fig. 3.1. Provisions have been made for tilting the film cassette to implement the fan-beam correction. An additional slit has been added to the system. The first slit, or pre-slit, is located in front of the object and collimates the beam to the plane of interest. The second slit is located immediately in front of the x-ray cassette and defines the beam height used for the data collection. This adjustable slit also

Fig. 3.17. Error vs. tilt angle.

The RMS difference between the recorded sine curve and a true sine curve is plotted as a function of tilt angle for the case $d_1 = 1$ m, $d_2 = 0.75$ m, $r = 12.5$ cm and $L = 0.38$ m. The minimum occurs at the value $\alpha = L/\pi s_1$.

prevents out-of-plane scattered radiation from exposing the film. The object sits on a rotary table, and the film cassette is translated vertically by means of a lead screw. Both of these motions are driven by stepper motors, and their relative speeds are controlled by a master oscillator connected through two frequency dividing circuits. Perfect synchronization is maintained between the rotation and the translation. The source used for these experiments is a standard rotating-anode x-ray tube operating in the fluoroscopic mode. A standard 35 x 43 cm (14 x 17") film cassette is normally used, but the apparatus can accommodate up to a 14 x 36" cassette.

Because of the limited dynamic range of the film-screen detector, a water bath can be placed around the object. If temporal variations in the flux from the x-ray tube, which would produce changes in the transmission of the sinogram along its length, become a problem, the system is equipped with a feedback loop to reduce this deleterious effect. An ionization chamber is used to measure the beam strength, and its output signal runs a voltage controlled oscillator which produces the master frequency. The data-taking procedure slows down or speeds up with the x-ray flux to maintain a constant exposure to the film. It has not been found necessary to use this option for data recording. Small side-to-side variations in the uniformity of the intensifying screens can be corrected to first order by narrowing the second slit at one end only. This results in a linearly increasing x-ray exposure across the

film, which can compensate for the non-uniformity. Fiducial marks, used to determine the center of the projection data, are recorded on each edge of the film by lead blockers placed next to the object. These blockers are located equidistant from the center of rotation of the object.

CHAPTER 4

LOOP PROCESSOR--BASIC DESIGN

Unfiltered Reconstruction

The recorded sinogram is a linear superposition of sine curves--each point in the object corresponds to a unique sine curve on the film. This property suggests a simple system to produce an unfiltered reconstruction or summation image (see Eq. 2.9):

$$\hat{\mu}_u(r, \theta) = \frac{1}{\pi} \int_0^{\pi} f_{\phi}(r \cos(\theta - \phi)) d\phi . \quad (4.1)$$

To reconstruct the object at a particular point (r, θ) , all that need be done is to sum the projection data located along a sine curve. Not surprisingly, the line of integration is exactly the sine curve traced out by the point in question during the data recording.

An incoherent optical system to perform the above operation is shown in Fig. 4.1. A sinogram, transilluminated with white light, is imaged onto an image-plane mask, and the transmitted light is measured by a photodetector. The signal from the detector is proportional to the value of the reconstruction at the point under consideration. The form of the mask must be chosen so that it performs the operations required by Eq. 4.1. The mask, which defines the line of integration is nothing more than a sinusoidal slit. The shape of the slit is dependent on the

Fig. 4.1. An incoherent optical system for producing unfiltered reconstructions.

values of r and θ being reconstructed, and the displacement of the slit from a straight line is given by

$$d(y) = mr \sin (\pi y/mL - \theta), \quad (4.2)$$

where L is the sinogram length, m is the optical magnification, and y is measured along the length of the sinogram and increases down its length (see Fig. 4.2). The width of the slit defines the resolution in the reconstructed image.

Film Loop

In order to reconstruct the entire image by this method, a large ensemble of image-plane masks ($\approx 10^5$ for an image width of 250 pixels) for all possible combinations of r and θ is required. Fortunately, all of these masks can be combined onto a single piece of film that contains many cycles of sinusoidal displacement, as shown in exaggerated form in Fig. 4.3. The displacement of the slit on this mask is

$$d(y) \propto y \sin (\pi y/mL) . \quad (4.3)$$

The period of each cycle is constant and equal to the period of the sine curves in the image of the sinogram. In practice, there will be 100 or more cycles along the mask. The amplitude of the displacement is slowly increased from zero along the length of the mask and can be considered constant over any single period. This mask is now pulled past the image of the sinogram, and every position of the mask corresponds to one in the ensemble of possible image plane masks. Since θ varies rapidly along

Fig. 4.2. The slit required to reconstruct a point with coordinates (r, θ) .

Fig. 4.3. The processing mask.

The entire ensemble of image plane masks is contained on this long processing loop.

the mask's length and r varies slowly, the object will be reconstructed along a spiral raster. The number of turns in the raster will be equal to the number of periods of the sine curve on the mask. This film is formed into a continuous loop for convenience, and hence the name given to this processor--the loop processor.

The Reason for Filtering

Even though the processor described so far produces an unfiltered image that is degraded by an unacceptable $1/r$ point spread function (for a slit of finite width, the actual point spread function is given in Eq. 2.34, where the slit width is 2ϵ), much insight can be gained about the requirement of filtering by studying its operation. This device could best be described as a sine-curve correlator. As the loop is pulled through the image plane, it systematically searches for sine curves on the sinogram. When an exact match is found--the true location of the object point--the sinusoidal slit will be filled by the sine curve in the data set, and a maximum signal will be produced by the detector. However, there are other sine curves on the sinogram, and even at this instant, they will contribute to the signal as some portion of these curves will lie within the slit. This situation is shown in Fig. 4.4. The reconstructed value at one point is therefore influenced by the other points in the object, and this influence results in the $1/r$ point spread function. Filtering, which is described in Chapter 5, removes this degradation by subtracting from the signal an amount equal to the extra contributions.

Fig. 4.4. The reason for filtering.

The contributions of unwanted sine curves within the slit produce the $1/r$ point spread function.

Producing Masks

The method used to produce the masks is shown in Fig. 4.5. The process very closely mimics the data-recording procedure. An illuminated horizontal slit with a movable obscuration is imaged onto unexposed film. Since a high-contrast mask is desired, 35 mm Kodalith film is used. The obscuration is moved in the desired pattern while the film is translated vertically. The two motions are coupled by means of a master oscillator and stepper motors. The masks are essentially exposed one horizontal line at a time.

Complete System for Unfiltered Reconstructions

A diagram of the complete system for unfiltered reconstructions is given in Fig. 4.6. The film loop is pulled through the image plane of the optical system at constant speed by a stepper motor. The pulses driving the motor are counted to produce signals proportional to r and θ . These signals are fed into a coordinate conversion module to generate x and y signals that deflect the electron beam of a display oscilloscope to write out a spiral raster. Since each turn of the raster will be written in the same amount of time--the time required for one period of the loop to pass through the image plane--the electron beam current must be increased with radius to produce a uniform raster. This write-speed correction is accomplished by multiplying the detector output by a signal proportional to r . The net signal drives the z -input (brightness) of the display. The picture is integrated on Polaroid film.

Fig. 4.5. The method used to produce the processing loops.

Fig. 4.6. The complete system for unfiltered reconstructions.

CHAPTER 5

LOOP PROCESSOR--FILTERING

Two-Channel Systems

Up to this point, the discussion of the loop processor has centered on producing unfiltered reconstructions. It is the purpose of this chapter to describe how the system can be modified to perform the necessary filtering operations, as required by the convolution algorithm (Eq. 2.22):

$$\hat{\mu}(r, \theta) = \frac{1}{\pi} \int_0^{\pi} d\phi \int_{-\infty}^{\infty} dx'_0 f_{\phi}(x'_0) h(r \cos(\theta - \phi) - x'_0) \quad (5.1)$$

This expression was derived for reconstructing the object in a parallel fashion (back projecting filtered projections), but it can also be interpreted as a serial reconstruction. The algorithm states that the value of the reconstruction at a particular point (r, θ) in the object is found by multiplying each projection by a shifted bipolar filter function and integrating to obtain one point in the convolution of $h(x)$ and $f_{\phi}(x)$. The results for each projection are summed to obtain the final result. The shifts that are required place the center of the filter along the line occupied by the slit in the preceding chapter. In reference to Fig. 4.4, the negative wings of the filter subtract a portion of the unwanted sine curves from the signal produced by the

positive central slit. This operation cancels the influence of all but the desired sine curve on the signal, and the point spread function in the reconstruction is improved.

These operations could be performed by the loop processor by replacing the sinusoidal slit by a bipolar transparency. However, with incoherent light, only positive quantities (intensity and transmission) can be realized. Biasing the filter so that it is now all above zero is a possibility, although biases should be avoided or minimized because of the noise associated with them. The bias term applied to the filter produces a constant signal at the detector in addition to the desired signal. The fluctuations due to the statistics of the bias term often exceed or are equal to the signal to be measured. Biasing also places additional demands on the dynamic range of the detector. A more attractive alternative is some sort of two-channel system. The bipolar filter function may be written as the difference between two non-negative functions:

$$h(x) = h_+(x) - h_-(x) . \quad (5.2)$$

The positive portion of the filter is represented by $h_+(x)$ and is usually close to a delta function, while $h_-(x)$ corresponds to the negative sidelobes of the filter. Since only the difference between $h_+(x)$ and $h_-(x)$ is important, there may be a common-mode component to the two filters.

Gray-Scale Masks

The most straightforward implementation of this filter function involves a system with two imaging lenses and two processing loops (Fig. 5.1) (Barrett and Swindell 1977). The mask in the positive channel is a sinusoidal slit as described in Chapter 4. The negative channel loop has a transmission profile given by $h_-(x)$ --a sinusoidal opaque strip with transmissive areas to both sides of the strip that fall off as $1/x^2$. If the two loops are optically superimposed, the slit on the positive mask will cover the opaque strip of the negative mask. Two photodetectors are used to collect the light transmitted through these masks as they are pulled together past the sinogram images. A differential amplifier subtracts the two detector signals, and the output is proportional to the filtered reconstruction of the object.

While this two-channel approach is theoretically sound, it is plagued by several technical problems. As indicated in Chapter 3, small low-contrast objects produce a sine curve on the sinogram that differs by 0.02% of the transmission of the surrounding film. When this curve lines up with the film loop (i.e., the correct values of r and θ to reconstruct the point), the positive channel signal will change by 0.02% of its nominal value. The two optical channels must therefore have a signal-to-noise ratio of 5000:1 and be balanced to this same precision. This fact requires that the two loops track with near-perfect registration over their entire lengths. A second complication is the form required by the negative-channel mask. Since its transmission must decrease as $1/x^2$, the mask must have a controllable contrast and a

Fig. 5.1. A two-channel optical system.

The outputs of the two optical channels are subtracted to implement bipolar filtering operations.

dynamic range of 10^4 in transmission, or a maximum density of greater than 4, to filter the entire projection. This unusual response has been obtained with Kodalith film developed in dilute D-76, but the uniformity of these masks was not good. A third problem with this two-channel system is that the two imaging lenses are "off-axis", and each sees a different intensity distribution on the light box. These differences arise from $\cos^4 \theta$ obliquity factors and vignetting. Image plane filters to correct for these obliquity factors were produced. The image quality was improved, but significant tracking errors remained. For these reasons, this approach does not seem to be feasible.

OTF Synthesis

The second of these problems, the form of the negative-channel mask, can be eliminated by the use of optical transfer function (OTF) synthesis. This technique has recently been discussed by Gorlitz and Lanzl (1977), Lohmann (1977), Rhodes (1977), Lohmann and Rhodes (1978) and Stoner (1978). The operations that can be performed by pupil-plane filtering in a single incoherent optical channel are very limited. In a coherent system this is not the case. The coherent transfer function is a scaled version of the pupil mask $p(x,y)$, and the point spread function is the Fourier transform of the transfer function. There are no restrictions on this point spread function--it can be positive or negative, real or complex. Since an incoherent optical system is linear in intensity, not amplitude, its point spread function is the modulus squared of the coherent point spread function (Gaskill 1978):

$$h'(x,y) \propto \left| \mathcal{F}_2\{p(x,y)\}_{\xi=x/\lambda f, \eta=y/\lambda f} \right|^2, \quad (5.3)$$

where λ is the wavelength, and f is the focal distance of the lens. The incoherent point spread function is constrained to be positive and real. The corresponding transfer function is

$$H'(\xi,\eta) \propto [p(x,y) \star \star p^*(x,y)]_{x=\xi\lambda f, y=\eta\lambda f}, \quad (5.4)$$

where $(\star\star)$ represents a two-dimensional correlation. The transfer function is Hermitian and will always have a maximum at zero spatial frequency. By placing the appropriate pupil plane masks in the imaging lenses of a two-channel optical system, where the outputs of the two channels are subsequently subtracted, the above restrictions can be removed and a general transfer function or point spread function can probably be obtained (the universality of this statement is a subject of current research).

The advantage gained by the use of a two-channel incoherent optical system for frequency-domain filtering over an apparently simpler coherent system is primarily a reduction of processing noise. An incoherent system can have a signal-to-noise ratio perhaps 1000 times better than that of its coherent counterpart (Chavel and Lowenthal 1978). This property is due to the redundant or multichannel nature of the incoherent system which makes it insensitive to dirt and blemishes

on the optics. The system also does not measure phase variations on the object and does not require the high accuracy in filter positioning needed in coherent systems. There is also no coherent speckle. These advantages, when coupled with the flexibilities generated by OTF synthesis, make incoherent optical filtering a very attractive alternative to coherent filtering.

A point spread function useful for transaxial tomography can be formed by placing a Ronchi ruling (an amplitude square-wave grating) in one pupil and leaving the other unobstructed (Barrett et al. 1979b). The channel with the Ronchi ruling is to be considered the negative channel. An intuitive understanding of this situation can be had by recalling that the intensity of the diffraction orders from a 50% duty cycle Ronchi ruling decrease as $1/n^2$, where n is the order number (Schulman 1970). The point spread function of the negative channel, which is a scaled version of the diffraction pattern, falls within the required $1/x^2$ envelope. The positive channel (no pupil mask) maintains its delta-function like behavior, so that the composite point spread function has the desired functional form.

The Ronchi pupil can also be examined in the frequency domain by the use of Eq. 5.4. When a 50% Ronchi ruling of spatial frequency $1/d$ (Fig. 5.2a) is placed in the pupil plane, the resulting transfer function $H'(\xi, \eta)$, calculated by an autocorrelation, is a series of triangular passbands (Fig. 5.2b). Note there are many Ronchi lines across the lens pupil, so that the transfer function due to the lens aperture acts as an envelope for the Ronchi transfer function. The positive channel

Fig. 5.2. The optical transfer function produced by OTF synthesis.

- (A) The Ronchi ruling placed in the negative channel,
- (B) the transfer function produced by the Ronchi,
- (C) the positive-channel transfer function, and
- (D) the net transfer function for the synthesized optical system.

contains no filter, so its transfer function $H'_+(\xi, \eta)$ is simply that of the lens aperture (Fig. 5.2c). In order to achieve a net transfer function with zero response at zero spatial frequency, or equivalently a zero-mean point spread function, the detector output of the positive channel is reduced by 50% (this step could also be accomplished with a neutral density filter in front of the imaging lens). The outputs of the two detectors are now subtracted, and the synthesized transfer function $H'(\xi, \eta)$ consists of triangular passbands (Fig. 5.2d). However, at low spatial frequencies, $H'(\xi, \eta)$ increases linearly with $|\xi|$ as required (see Eq. 2.30). By properly choosing the spatial frequency of the Ronchi ruling, the range of frequencies that are properly filtered can be controlled. As can be seen, the transfer function of the lens has little effect and will be ignored. The higher-order passbands do not affect the images since they will be apodized by a slit in the image plane of the optical system. Since the Ronchi ruling has no vertical structure, the filter performs no operations in the y-direction. Therefore, the corresponding point spread function is one-dimensional, and is given by (see Fig. 5.3):

$$h'(x) = \delta(x) - \sum_{n \text{ odd}} \left(\frac{2}{\pi n} \right)^2 \delta(x - n\lambda f/d), \quad (5.5)$$

The even orders are not present, and the odd orders decrease in intensity as $1/n^2$. If the transfer function of the lens aperture had been included, the δ -functions in the above equation would be replaced by the Airy disc of the lens.

Fig. 5.3. The optical point spread function produced by OTF synthesis.

It is interesting to note that the point spread function obtained by OTF synthesis is of exactly the same form as a discrete convolution filter often used in commercial CT scanners. This is the Ramachandran-Lakshminarayanan or "Ram Lak" filter that yields an exact reconstruction for noise-free band-limited data and is an excellent approximation for real data (Ramachandran and Lakshminarayanan 1971).

The Ronchi ruling is by no means a unique pupil mask for trans-axial tomography. A phase Ronchi ruling (where the opaque bars are replaced by areas of π phase retardation) is also useful and has the advantage of increased optical throughput. The point spread function can also be tailored to a desired form by creatively controlling the aberrations (phase variations) of the imaging lens. This procedure is discussed by Barrett et al. (1979a), and they find that a logarithmic phase plate (see Fig. 5.4) in the pupil will produce a point spread function with $1/x^2$ wings.

Two Channel System with OTF Synthesis

When a sinogram is imaged by this two-channel incoherent optical system with a Ronchi ruling in one of the pupil planes, the composite image plane contains a synthesized version of the convolution of the $f_{\phi}(x)$'s and $h'(x)$. The convolution algorithm (Eq. 2.22) requires that only one point from each convolution is summed for a particular r and θ . This selection is accomplished by using identical sinusoidal slits placed in the image planes of the two optical channels.

Fig. 5.4. Logarithmic phase plate.

The Ronchi ruling could be replaced by a logarithmic phase plate to obtain the same point spread function. The height of the profile is greatly exaggerated for clarity.

The two-channel system with OTF synthesis reconstructs the object by pulling two identical sinusoidal slits through the image plane. The transmitted light is collected by two photodetectors, and a difference signal is generated that is proportional to the reconstructed value of the attenuation coefficient. This signal drives the display electronics shown in Fig. 4.6. As in the two-channel system with gray-scale masks, the two processing loops must be identical, and even though the two masks are now functionally the same, this is a very difficult task to perform. The system is also a two-channel system and possesses all of the problems associated with using the imaging lenses off-axis. These problems place a severe limitation on the performance of this system, but this system merits further discussion as it is a logical step in arriving at a workable processor.

The effect of the slits can be found by examining the amount of light reaching the detector as a function of r and θ (actually the difference signal in the composite optical system):

$$\phi(r, \theta) \propto \int_0^\pi d\phi \int_{-\infty}^{\infty} dx_0 s(r \cos(\theta - \phi) - x_0) \int_{-\infty}^{\infty} dx'_0 f_\phi(x'_0/m) h'(x_0 - x'_0) . \quad (5.6)$$

In this equation, $s(x)$ is the cross-sectional profile of the slit, m is the optical magnification, and the final integral (dx'_0) represents the image of the projections $(f_\phi(x/m) * h'(x))$. By reversing the order of integration and performing a simple change of variables ($x''_0 = r \cos(\theta - \phi) - x'_0$), this equation becomes

$$\Phi(r, \theta) \propto \int_0^\pi d\phi \int_{-\infty}^\infty dx'_0 f_\phi(x'_0/m) \quad (5.7)$$

$$\int_{-\infty}^\infty dx''_0 s(x''_0) h'(r \cos(\theta - \phi) - x'_0 - x''_0) ,$$

or

$$\Phi(r, \theta) \propto \int_0^\pi d\phi \int_{-\infty}^\infty dx'_0 f_\phi(x'_0/m) h(r \cos(\theta - \phi) - x'_0) , \quad (5.8)$$

where

$$h(x) = s(x) * h'(x). \quad (5.9)$$

Eq. 5.8 should be recognized as the convolution algorithm (Eq. 2.22) with the projection data scaled by the optical magnification. The convolution filter $h(x)$ can be identified as the optical point spread function convolved with the slit function. In this case, $s(x)$ is a

rectangle function, and the net filter is shown in Fig. 5.5. The width of the slit must be chosen in conjunction with the spatial frequency of the Ronchi ruling, as both influence the resolution in the processed image. In Fig. 5.5, the width w equals $\lambda f/d$, although in practice there is a range of acceptable Ronchi frequencies. The resolution in the reconstruction is w/m and is relatively independent of the Ronchi frequency. The transfer function $H(\xi)$ corresponding to this convolution filter is shown in Fig. 5.6. The higher-order triangular pass bands in the optical transfer function have been apodized by the presence of the slit, which effectively multiplies $H'(\xi)$ by a broad sinc function $\left(\frac{\sin \pi \xi}{\pi \xi}\right)$.

A few words should be said about using the OTF synthesis technique with a broadband light source. For both the filter function and the transfer function, wavelength is merely a scale factor. To find the true response, an average over the source spectrum and detector responsivity must be performed. In this particular case, the averaging process will change some of the details of the two functions, but it will not alter their important features. For a single wavelength, the transfer function at low spatial frequencies is given by

$$H(\xi) \propto \frac{\lambda f}{d} |\xi| . \quad (5.10)$$

If $W(\lambda)$ is the spectral weighting of the source-detector combination, the net transfer function is

Fig. 5.5. The convolution filter produced by OTF synthesis.

Fig. 5.6. The transfer function corresponding to the filter in Fig. 5.5.

$$\bar{H}(\xi) \propto \frac{\int_0^\infty W(\lambda) \frac{\lambda f}{d} d\lambda}{\int_0^\infty W(\lambda) d\lambda} = \frac{\bar{\lambda} f}{d} |\xi| \quad , \quad (5.11)$$

where $\bar{\lambda}$ is the effective wavelength. The response is still linear with $|\xi|$ but at an average slope. Likewise, a $1/x^2$ function that is modified by a scale factor will still decrease as $1/x^2$, so that the filter function will continue to have a large positive core with negative side-lobes that fall off within the desired envelope. It is sufficient for these discussions to use an average wavelength.

The synthesized transfer function obtained with the Ronchi pupil (Fig. 5.7) was measured by recording the response of the system to a single sinusoidal component of a projection. Straight slits were used in the image plane, and various sinusoidal cutouts (Fig. 5.8) were placed on the light box. Because of the slits in the image planes, the system integrates in the vertical direction, and the cutout is equivalent to a mask having a transmission that varies sinusoidally in the horizontal direction. A single cutout is used to generate one point on the transfer function curve.

Time-Modulated System

A third configuration of the loop processor uses a single film loop and a single imaging lens to overcome the difficulties of the previous system. Instead of having two optical channels side-by-side in space, the two optical systems are produced by temporally switching between the two desired states. This processor, which makes use of

Fig. 5.7. The measured transfer function synthesized with the Ronchi pupil.

The spatial frequencies are measured in the sinogram plane. For these measurements, $w = 0.14$ mm, $d = 0.25$ mm, and $m = 1/14$.

Fig. 5.8. A sinusoidal cutout.

A series of these cutouts were used to measure the transfer function in Fig. 5.7. When integrated in the vertical direction, this mask is equivalent to a transparency with a sinusoidal variation in transmission. The actual cutouts cover the entire light box.

time-modulated OTF synthesis, operates in a manner very similar to its two-channel predecessor. By rapidly flipping a Ronchi ruling in and out of the pupil plane of the imaging lens, the state of the optical system changes from the positive channel to the negative channel. The detector signal is synchronously demodulated to produce a signal proportional to the absorption coefficient. The advantage of a single-channel system is that imbalances between the positive and negative channels of a two-channel system are not present.

During the reconstruction, the pupil mask must be switched through a full cycle for each pixel in the image. To minimize the reconstruction time, the switching rate must be as fast as possible. One way to switch the pupil is to spin the transparency shown in Fig. 5.9 in front of the lens. The disc consists of sections of concentric Ronchi ruling and clear sections. The concentric Ronchi approximates a linear Ronchi in the pupil of the lens and performs the correct filtering operation. A reference signal is picked-off the edge of the rotating disc by a second detector. It has been found that deleterious signals are present when the transition between Ronchi and clear regions passes in front of the lens, and the reference signal is used to "blank" the electronics during this period.

Because the positive channel signal must be attenuated by 50% in order to achieve a zero-mean convolution filter, a simple lock-in amplifier is not sufficient to demodulate the detector signal. The required signal processing is detailed in Fig. 5.10. The reference and detector signals are shown in (a) and (b) respectively. These signals

Fig. 5.9. The concentric Ronchi ruling disc.

The curved lines in the lens pupil approximate a linear Ronchi ruling. The disc is spun in front of the imaging lens to implement time-modulated OTF synthesis.

Fig. 5.10. The electronic processing performed by the switching amplifier.

are then sent to a switching amplifier that reduces the positive portions of the detector signal by a factor of two and inverts the negative portions (c). The crossover transients are now removed by a blanking circuit (d). This output signal is low-pass filtered to measure its DC level. This step is equivalent to subtracting the positive and negative signals in the two-channel systems. The resulting signal is a measure of the reconstructed x-ray attenuation coefficient and is used to drive the display electronics in Fig. 4.6.

CHAPTER 6

LOOP PROCESSOR--PRACTICAL CONSIDERATIONS

Film Transport

The processing loop for the current version of the loop processor is roughly 8 m in length and contains 135 periods of sinusoidally displaced slit. The purpose of the film transport is to hold this length of film, move the film through the image plane, and position it accurately in the image plane. The film loop is accommodated by a series of turns around film rollers (see Fig. 6.1). The film is driven by sprocketed film rollers located on each side of the image plane, and the motion is generated by means of a geared-down stepper motor (4:1 gearing producing 1600 steps per revolution). The lateral positioning of the film is also performed by the drive sprockets (Positive Control Sprockets; LaVezzi Machine Works, Elmhurst, IL). One row of teeth on the sprockets is designed to match the width of the sprocket holes in the film, and this condition resists the side movement of the film. The position of the processing loop along the optic axis is provided by a specially designed film gate (diagrammed in Fig. 6.2). The film is held only along its edges with spring-loaded pressure plates so that light can be transmitted through the film. This arrangement has proved to be quite satisfactory. A large access hole is provided in the film transport for the placement of lenses and detectors behind the image plane.

Fig. 6.1. The film transport.

The tensioning of the film loop is provided by a spring.

Fig. 6.2. A film gate.

The film is supported only along its edges, so that light can be transmitted through the film. The film is held in place by spring loaded pressure plates.

Loop Production

The device used to produce the sinusoidal motion for mask production (Fig. 4.5) is a modified Scotch-yoke mechanism (Martin 1969). It consists of an eccentric pin on a rotating disc, where the pin is placed in a slot (Fig. 6.3). The slot is constrained to move only in the horizontal direction, and the rotation of the pin produces sinusoidal motion. The amplitude of the motion is determined by the radius of the pin and is varied by mounting the pin on a lead screw. For smoother operation, the slot has been replaced with linear ball bearings on a ground rod. Both motions, the rotation of the pin and the lead screw rotation, are produced by stepper motors and can be controlled in synchronism with the film transport, which holds the unexposed film.

Optical System

There are four major components to the optical system--the light box, the imaging lens, the detector, and the field lens. The purpose of the light box is to uniformly illuminate the sinogram. The light is provided by four quartz-halogen projection bulbs (type FCS - 150 W each), which are powered by a voltage-regulated DC power supply. The sinogram is mounted in front of a white Plexiglas diffuser. To improve the light efficiency of the system and to provide better uniformity of illumination across the sinogram, the light sources are placed in a mirror tunnel comprised of four front surface planar mirrors placed at right angles to one another. From the viewpoint of the sinogram, a very large array of sources (the virtual images of the projection bulbs) is produced, and a uniform brightness results.

Fig. 6.3. Scotch-yoke mechanism.

The optical magnification m required for the system must place the image of the 35 cm dimension of the sinogram onto the 24 mm that is usable on the 35 mm processing loop. The magnification is then $1/14$, and a 50 mm imaging lens was chosen. The presence of an image-plane slit reduces the requirements placed on the imaging lens. Blurring aberrations (spherical, coma, or astigmatism) are not a serious problem, and a high-quality photographic lens is satisfactory. Since image distortion could cause a mismatch between the sinogram and the loop, an extensive analysis was performed to determine how much distortion could be tolerated. Distortion can be modeled in the image plane by

$$r'_o = m r_o + \beta r_o^2, \quad (6.1)$$

where r_o is the distance of the object point from the optic axis, r'_o is the radius of the corresponding image point, and β is a distortion coefficient. For a perfect lens, β equals zero. When β is negative, barrel distortion occurs, and a positive β produces pincushion distortion. The concern is that although the loop and sinogram are imaged through the same lens, the mask will be imaged one line at a time when it is produced and will suffer only a one-dimensional distortion (Fig. 6.4), while the sinogram will be imaged all at once and will undergo radial distortion (Fig. 6.5). There is no guarantee that after imaging the mask and sine curves on the sinogram will have the same shape.

Fig. 6.4. The effect of lens distortion on a processing mask.

The desired curve is distorted in one dimension only.

Fig. 6.5. The effect of lens distortion on a sine curve in a sinogram.

The curve is imaged with radial (two-dimensional) distortion.

A series of calculations for a lens with a distortion coefficient $\beta = -5 \times 10^{-6}/\text{mm}$ was performed. The method of determining β is given in Appendix C. Various distorted sine curves from a sinogram were compared to distorted masks. The RMS error between the sine curve and a best-fit mask (the amplitude of the mask can be varied for different amplitude and phase sine curves) was found to be less than 0.5 mm referenced back to the sinogram plane. This error should not affect the operation of the processor at 2 mm resolution. A 50 mm focal length, F/2, Auto-Nikkor was chosen for the objective lens. This lens has a distortion coefficient of $-2 \times 10^{-6}/\text{mm}$ and will produce even less error than the above case. The lens is mounted on the front of the film transport, and extensive light shielding is provided to reduce stray light.

The optical detector for the loop processor is a large-area silicon photodiode (PIN-25; 6 cm² active area; United Detector Technology, Inc., Santa Monica, CA). It is used in an unbiased mode and is followed by a preamplifier which converts the output current to a voltage. The average wavelength for this detector-source combination is roughly 0.9 μm .

The optical system is completed by the use of a field lens which images the pupil of the imaging lens onto the detector. Its use is shown in Fig. 6.6, and the field lens acts primarily to conserve light. All of the light that gets through the loop is imaged onto the detector. The field lens also removes the effects of sensitivity variations across the detector. As shown, two plano-convex lenses are used at near infinite conjugates to accomplish the 1:1 imaging of the pupil onto

Fig. 6.6. The optical system with a field lens.

the detector. The diameter of the lenses is determined by the sinogram image size, and the field lens assembly has an effective f-number of 0.6.

Electronics

There are two circuits used in the loop processor which require further discussion. The first is the switching amplifier that performs the operations shown in Fig. 5.10. This device is used to demodulate the detector signal in the time-modulated system and is diagrammed in Fig. 6.7. This circuit is built around two analog switches (AH0014CD; National Semiconductor) which perform the blanking operation and switch the input signal from the detector to a path of gain equal 0.5 (positive channel) or a path of gain equal -1 (negative channel).

The second circuit performs the write-speed correction for the video display (Fig. 4.6). A block diagram of this circuit appears in Fig. 6.8. The multiplication of the detector signal (low-pass filtered output from the switching amplifier) and the radius signal is performed in two steps. The radius signal is sent to a voltage controlled oscillator which puts out a pulse train whose frequency is proportional to radius. The width of each pulse is constant so that this signal will write a uniform raster on the display. The detector signal is included in the video signal by modulating the amplitude of the individual pulses. The intent of the circuit is to produce a composite signal that is linear in both radius and reconstructed attenuation coefficient, and visual inspection of the images indicates that the circuit performs adequately.

Fig. 6.7. The circuit diagram of the switching amplifier.

Fig. 6.8. Block diagram of the display electronics.

The *r* signal drives the VCO, and the one-shot guarantees that all the pulses have the same width. The pulse height of the video signal is modulated by switching between the desired signal and ground in synchronism with the pulses.

Complete System

The complete optical system used to reconstruct an object from its x-ray projections is diagrammed in Fig. 6.9. The sinogram is recorded on standard 35 x 43 cm x-ray film and is mounted on the light box. In the current system, the processing loop contains 135 frames, which produces 270 raster lines across the image. The reconstructed field is 25 cm. The spatial frequency of the Ronchi ruling $1/d$ is 4 lp/mm, and the concentric Ronchi is bonded to a clear Plexiglas disc (200 mm diameter). The disc can be spun at 9000 RPM (150 Hz). This rate limits the reconstruction time to about 7 minutes. The images are processed at 2 mm spatial resolution ($\epsilon = 1\text{mm}$), although this resolution could be varied by changing the processing loop and the Ronchi ruling. A 2 mm resolution was chosen because it represents the resolution normally used in clinical examinations. A review of other important system parameters is as follows: the optical magnification m is $1/14$, the focal length of the imaging lens f is 50 mm, the sinogram length L is 38 cm, and the effective wavelength of the system λ is $0.9\text{ }\mu\text{m}$.

Fig. 6.9. The complete single-channel optical system used to reconstruct an object from its corresponding sinogram.

CHAPTER 7

LOOP PROCESSOR--RESULTS

The projection data for the results presented in this chapter are recorded on standard 35 x 43 cm x-ray film. DuPont Cronex IV film was used in conjunction with DuPont Quanta-II intensifying screens. This is a rather fast combination (see Table 3.1), and as such required a center dose of less than 0.3 Rad for a complete sinogram. This is a factor of two to four less than that delivered for an average CT scan, and a factor of more than ten less than a high-dose scan. The method used to determine the dose is described in Appendix D. At this dose level, changes in density of at least 1.4% can be expected to be measured when the image is processed at 2 mm resolution. This calculation is also included in Appendix D. The beam height for these data is 2-4 mm, and the data were recorded without the use of halftone screens or dodgers. A water bath was used to reduce the variations in the transmitted x-ray beam to within the dynamic range of film-screen detector. A 120 KVP x-ray source was used with about a 2 m source-film distance. The tilted-cassette fan-beam correction was utilized.

Phantom Studies

A series of studies using objects of known dimensions and composition (phantoms) has been done to evaluate the system performance. The first of these studies is with the commercial EMI phantom (see

Fig. 7.1). It consists of several plastic cylinders simulating various types of body tissue and a crow's foot resolution target. The bars in this target are 1 cm wide at the edge, and by measuring how far into the target the bars are visually separated, the spatial resolution of the system can be determined. The sinogram of this object is reproduced in Fig. 7.2, and the sinusoidal nature of the data set is readily apparent. The time-modulated optical reconstruction of this phantom is shown in Fig. 7.3. The density differences between the materials are easily seen, and the resolution target indicates that the spatial resolution in the image is 2 mm. This resolution is as expected for the processing loop.

A phantom with high spatial regularity is shown in Fig. 7.4. It consists of a hexagonal array of glass sample tubes. All of the tubes are filled with water except one row which is alcohol filled. The system spatial resolution is once again confirmed in the optical reconstruction (Fig. 7.5), as the spaces between the bottles are clearly visible. The difference in attenuation coefficient between alcohol and water is also apparent.

An object with low-contrast details is diagrammed in Fig. 7.6. This phantom was borrowed from Dr. E. C. McCullough of the Mayo Clinic, Rochester, MN. Lexan pins of various diameters are contained in a Plexiglas cylinder. The smallest pin is 6 mm in diameter, and the difference in the x-ray attenuation coefficient is 1.5%. In the optical reconstruction (Fig. 7.7), all of the pins are visible, and this sinogram is a low-dose data set. The result is consistent with the expected

Fig. 7.1. The commercial EMI phantom.

Fig. 7.2. A sinogram of the EMI phantom.

Fig. 7.3. The optical reconstruction of the EMI phantom.

Fig. 7.4. A phantom of high spatial regularity.

Fig. 7.5. The optical reconstruction of the phantom in Fig. 7.4.

Fig. 7.6. A low-contrast phantom.

Fig. 7.7. The optical reconstruction of the phantom in Fig. 7.6.

performance at this dose level (Appendix D). In high-dose reconstructions of a lower contrast phantom, the system has detected changes in attenuation coefficient of less than 0.5%.

Animal Studies

To test the system with biological samples, three sections through a dog have been imaged. The dog was medium sized (20 kg) and was anesthetized during the data-taking procedure. The first slice is through the thoracic cavity, and the sinogram for this section is in Fig. 7.8. The dark areas correspond to lung and the white areas are bones. The optical reconstruction (Fig. 7.9) shows the rib cage and spine surrounded by layers of muscle, fat, and skin. Inside the rib cage are the heart (gray) and lungs. A second reconstruction (Fig. 7.10) images a slice through the abdomen and front paws of the dog. The spine, ribs, and front paws are shown as well as a great deal of soft tissue detail. The dog's kidneys, liver, and intestines are clearly visible. Details can also be seen within the spine. The final animal study (Fig. 7.11) is taken through the lower abdomen and folded hind legs of the dog. Of primary interest in this picture are the bones--the spine and the leg bones (femurs, tibiae, and fibulae). The spatial resolution of the system is exemplified by the appearance of the bone marrow in the leg bones. The soft tissues in this reconstruction are the leg muscles and the bladder (center).

Fig. 7.8. A sinogram of a slice through the thorax of a dog.

Fig. 7.9. The optical reconstruction of the sinogram in Fig. 7.8.

Fig. 7.10. The optical reconstruction of a slice through the abdomen and front paws of a dog.

Fig. 4.11. The optical reconstruction of a slice through the lower abdomen and folded hind legs of a dog.

A problem common to this system and CT scanners is motion blurring. If the patient moves during the collection of the projection data, artifacts are introduced into the reconstruction. This blurring appears as streaks around the dense objects (i.e., bones) that moved. An example of this artifact is in Fig. 7.11, and is especially obvious around the dog's femurs.

Comparison to Other Reconstruction Methods

In order to verify that all of the data on the sinograms are being processed, sinograms have been digitized on a scanning microdensitometer and reconstructed by implementing the convolution algorithm in a digital fashion. An image of the EMI phantom (Fig. 7.1 and 7.3) reconstructed in this manner appears in Fig. 7.12. There is very little difference between the optical and digital reconstructions, and it appears that all of the information on the sinogram is being utilized. There are some changes in the artifacts around the bone cylinder, and these artifacts are discussed in a later section.

Another test of the loop processor is to compare its images with those produced by a commercial CT scanner. A comparison image can be found in Fig. 7.13, and is an image of the EMI phantom by the EMI Model CT 5005 scanner. This image was produced at a slightly higher dose than the optical image and therefore has better noise properties. It is also displayed at a higher contrast. Nevertheless, the CT and optical images are quite comparable, and indicate the ability of the analog

Fig. 7.12. A reconstruction of the EMI phantom produced by digitizing the sinogram shown in Fig. 7.2.

Fig. 7.13. A reconstruction of the EMI phantom produced by a commercial CT scanner.

system to match the performance of the currently available digital machines.

Historical Development

The optical reconstructions that have been presented are the result of three years of development that was iterative in nature. The process closely followed the order of development presented in Chapter 5--a two channel system with gray-scale masks, a two-channel system with OTF synthesis, and a single-channel system with time-modulated OTF synthesis. It is the purpose of this section to trace the improvements in image quality through the process. The best results obtained at each stage of development will be presented for four different objects when available. The objects are a uniform data set (clear sinogram), the EMI phantom (Fig. 7.1), the hexagonal array of glass sample tubes (Fig. 7.4), and the slice through the abdomen and front paws of a dog (Fig. 7.10).

The reconstructions obtained with the two-channel system that utilized gray-scale image plane masks are shown in Fig. 7.14. The images are just barely visible, and these results could best be described as lousy. The processing loop contained 45 frames, and the spatial resolution in the images is 4 mm. The largest single problem in this case is tracking between the two loops. The loops contain transmission variations along their lengths, and these mismatches produce the ring structures.

The first major improvement in image quality occurred with the switch to OTF synthesis. Two-channel reconstructions at 2 mm spatial

(a)

(b)

(c)

Fig. 7.14. Two-channel optical reconstructions with gray-scale masks.

(a) Uniform data set, (b) Hexagonal array of glass sample tubes, and (c) Dog abdomen.

resolution are presented in Fig. 7.15. The two processing loops contain 60 frames and are now supposed to be identical. Recognizable structures are visible in the images. The ring structures have been greatly reduced since the masks showed greater similarity than before. The side-to-side variations in the pictures are due to the obliquity factors present with off-axis imaging optics.

The introduction of a time-modulated single-channel system removed both the obliquity factors and the tracking errors (Fig. 7.16). These reconstructions were also made at 2 mm resolution with 60 frames of processing loop. At this point, the switching amplifier (Figs. 5.10 and 6.7) did not possess the provision for blanking the transition "ears" on the switched signal. These crossover transients, caused by the transition between Ronchi ruling and clear passing through the pupil, were found to be responsible for the background variations in the images. Details of the objects are now clearly discernible in these reconstructions. The spiral pattern in the reconstructions is the chopping frequency leaking through the low pass filter, and the one radial line in the pictures is an artifact produced by the r, θ to x, y coordinate conversion module.

The only fundamental change in the system from these last results to the current results is the introduction of transition blanking in the switching amplifier. Other changes include using a longer processing loop to decrease the raster spacing, a higher chopping frequency to remove the spiral pattern, a better alignment of the optical system

(a)

(b)

(c)

(d)

Fig. 7.15. Two-channel optical reconstructions with OTF synthesis.

(a) Uniform data set, (b) EMI phantom, (c) Hexagonal array of glass sample tubes, and (d) Dog abdomen.

(a)

(b)

(c)

(d)

Fig. 7.16. Time-modulated optical reconstructions without transition blanking.

(a) Uniform data set, (b) EMI phantom, (c) Hexagonal array of glass sample tubes, and (e) Dog abdomen.

with the sinogram, and a sizable reduction in noise (vibrations and noise in the electronics). The noise reduction was non-trivial and required a great deal of effort. Images from the current system are found at the beginning of this chapter. An image of a uniform data set processed with transition blanking is presented in Fig. 7.17.

The success of the loop processor is clearly due to two innovations--the use of OTF synthesis, and the use of a time-modulated incoherent optical system. It is unlikely that the processor would have produced images anywhere near those of digital CT scanners without both of these techniques. The requirement that the system measure difference signals that are 0.02% of the individual channel signals (or the peak value of the time-modulated signal) is an extremely difficult condition to achieve. Virtually all sources of imbalance must be removed for an operational system.

Artifacts

Two basic types of processing artifacts appear in the optical reconstructions. The first is caused by the limited dynamic range of the film-screen detector, and an example is the spoke pattern seen at the edge of the reconstruction in Fig. 7.5. Along the dark lines, the x rays must pass through a large amount of glass (large effective thickness), and the corresponding points in the projection data are not properly recorded. The x-ray flux is outside the linear response of the detector. This same effect will produce "cupping" in large high-density objects. Except for a saturation of the bright areas in the display, the bone cylinder in the EMI phantom reconstructs with a dark hollow in the center.

Fig. 7.17. Optical reconstruction of a uniform data set.

The second artifact is exemplified by the shadow next to the bone cylinder in the EMI reconstruction (Fig. 7.3). While the cause of this artifact is not known, it is suspected to be due to the curvature of the Ronchi ruling lines. For large or high-density objects, the errors in filtering caused by the slight departure of the concentric Ronchi from a linear Ronchi in the lens pupil may become apparent and result in this shadow. A second possible source of this artifact is nonlinearities in the detector and the electronics. As the optical system is chopped, the intensity reaching the detector varies by 50% (the transmission of the Ronchi ruling) and nonlinearities would become apparent around bright structures in the image. This shadow artifact was also found to rotate as the phase of the sinogram (the projection angle at the top or bottom of the sinogram) was changed. The shadow lies along a line parallel to the projection direction of the data at the top of the light box.

Alignment

The physical alignment (or misalignment) of the sinogram and the optical system can introduce new artifacts into the reconstructions. The tolerances that are quoted in this section have all been experimentally determined, and most are interrelated. The alignment artifacts will appear around large or high-contrast objects for small errors, and there is some interaction with the system artifacts described in the previous sections.

The first requirement is that the center of the sinogram matches the centerline of the loop to about one-quarter of a resolution element

(accurate to 0.5 mm in the sinogram plane for 2 mm spatial resolution). An error produces an artifact around bright objects. It consists of a dark shadow to one side of the object with an opposing bright streak. This artifact is demonstrated in Fig. 7.18. Three reconstructions are shown where the sinogram has been shifted by 0.75 mm for each image (resolution equals 2 mm). The artifact is especially obvious in Fig. 7.18c. The direction of the streak is parallel to the projection direction ϕ for projections at the top or bottom of the light box ($\phi = 0^\circ$ or $\phi = 180^\circ$). For these particular reconstructions, this direction is perpendicular to the projection direction at the ends of the data set for Fig. 7.3 (different sinograms were used for Figs. 7.3 and 7.18, and they differ in phase by 90°).

Processing the wrong number of projections produces an artifact that runs in the same direction as the shift artifact. It has been found that the viewed sinogram must represent $180^\circ \pm 2^\circ$ worth of projection data (± 4 mm for a 38 cm sinogram). For a bright object and insufficient projection data, a dark streak appears across the entire image. For excess projections, this streak is bright. The artifact is illustrated in Fig. 7.19, where an additional 1 cm of the sinogram has been masked off for each image. Fig. 7.19a shows a bright streak, and Fig. 7.19c shows a dark streak. Fig. 7.19b is close to the correct sinogram length. Since this artifact, the shift artifact, and the system artifact (the shadow described in the previous section), all occur along the same direction, it can be very time-consuming to properly align the system. The three artifacts interact, and it is difficult to separate the three effects.

(a)

(b)

(c)

Fig. 7.18. The effect of sinogram shift on the reconstruction.

(a) No shift, (b) 0.75 mm shift, and (c) 1.50 mm shift.

(a)

(b)

(c)

Fig. 7.19. The effect of the amount of sinogram viewed on the reconstruction.

(a) 1 cm additional sinogram, (b) close to the correct amount of projection data, and (c) 1 cm too little sinogram.

The optical magnification must also be controlled. This adjustment controls how well the period of the processing loop matches the period of the sine curves on the sinogram. The magnification should not vary by more than 1% (at 2 mm spatial resolution), or a geometric distortion will appear in the reconstruction. For the current system, this requirement implies that the distance between the sinogram and the image plane must be accurate to about 2 mm. The geometric distortion occurs along the same line as the other artifacts--the line corresponding to the projection data at the top or bottom of the light box. This distortion is shown in Fig. 7.20, and the line of distortion extends from the bone cylinder to the center of the reconstruction and through the crow's foot resolution target. Figure 7.20b was produced at the approximately correct magnification, and the resolution target has six fingers (as does the phantom). When the distance between the sinogram and the image plane is reduced by 3 cm (a far larger shift than that which is detectable in the image), one of the fingers disappears from the reconstruction (Fig. 7.20a), and leaves only five. This lost finger can be recovered, however, by increasing the distance between the sinogram and the image plane. Figure 7.20c was produced 3 cm farther away than the correct position (Fig. 7.20b), and seven fingers are reconstructed. Severe artifacts also appear around the bone cylinder.

While strictly speaking not an alignment problem, errors in the positive-negative channel balance can degrade the reconstructions. A zero differential signal is always desired, but signals up to about 0.5% of the individual channel signals can be tolerated. Above this level,

(a)

(b)

(c)

Fig. 7.20. The effect of optical magnification on the reconstruction.

(a) Imaging distance 3 cm too short, (b) Imaging distance correct, and (c) Imaging distance 3 cm too long.

blurring or edge enhancement occurs (depending on the sign of the difference signal), and the transmission variations of the processing loop are not removed from the video signal since the processing filter $h(x)$ is no longer zero-mean. Ring artifacts, like those of the two-channel systems, enter into the reconstruction. The level at which these degradations occur depends strongly on the contrast at which the image is produced.

Weighting of Projections

As mentioned earlier in Chapter 5, vignetting in the imaging lens and $\cos^4 \theta$ obliquity factors can influence the amount of light received in the image plane from a particular point on the light box. It has been found that the illumination, as viewed by an on-axis imaging lens, drops by as much as 30% from the center of the light box to the edge of the field. This produces a multiplicative weighting that is applied to the projection data. Projections at the center of the light box are more heavily counted than those at the top or the bottom of the light box. Individual projections are also weighted across the light box. An "anti-vignetting" filter was produced that when mounted on the light box, produced a uniform response in the image plane. This will equally weight the projections. No improvement in image quality was found by using this filter, and because of the light losses incurred by its use, reconstructions are now made with the weighted data. This effect is not totally understood. A possible explanation is that even though the convolution filter has a large spatial extent, most of

its response is localized, and accordingly, the filter would not respond to slowly varying distributions. The filter is also zero-mean and has no response at zero spatial frequency.

CHAPTER 8

DISCUSSIONS AND CONCLUSIONS

Future Work

The loop processor as described to this point, produces tomographic reconstructions comparable in quality to those of commercial CT scanners. Two practical problems must be overcome before the optical system can be a commercially viable product. First, the water bath, used to produce the reconstructions shown, must be eliminated by expanding the dynamic range of the film-screen detector. More work is required to implement the halftone and dodger techniques mentioned in Chapter 3. The second problem is the reconstruction time. The current system requires about 7 minutes to process an image, and this time must be reduced to 1-2 minutes for a useful system. This time requirement will be discussed in greater detail in a later section. The reconstruction time is limited by the rate at which the pupil is switched in the time-modulated system. The rotating Ronchi disc produces a mechanical chopping rate of 300 Hz. This frequency could conceivably be increased by the necessary factor of 2 to 4 by the use of a better mechanical design to spin the disc faster and more segments on the disc. A better solution would be to use some sort of electro-optic pupil switching device (in particular, polarization switching)

or a spatially multiplexed pupil. Possible techniques are discussed in the next few paragraphs.

Before polarization switching can be implemented, some way must be found to polarization code the two pupil masks onto a single filter. Three schemes have been devised. The first (Fig. 8.1) involves splitting the pupil into two halves and covering each half with an orthogonal linear polarizer. One of the polarizers is then covered with a linear Ronchi ruling. For one polarization, half the pupil is clear (positive-channel mask) and the other half is opaque. The orthogonal polarization will allow the Ronchi ruling (negative-channel mask) to be used and will blacken the clear portion. A second similar approach is to produce a polarization-coded Ronchi ruling (Fig. 8.2), where the bars of a Ronchi are made out of linear polarizers. The bars will be present for one polarization (negative channel) and clear for the other (positive channel). This Ronchi ruling can be produced on vectograph film (Richard 1973). Vectograph consists of an oriented polyvinyl alcohol (PVA) film applied to a base. The PVA layer can be selectively dyed by a photographic process to form an image as a polarization pattern. The third method to produce a polarization coded Ronchi ruling is shown in Fig. 8.3. It consists of a crystal of calcite or other birefringent material etched to produce a phase Ronchi ruling. The two indices of refraction of the crystal are n_o and n_e . Let the etched volume be filled with an index matching fluid of index n_o (or alternatively n_e). For one polarization, there is no phase shift, and the pupil is clear.

Fig. 8.1. The polarization-coded split-pupil.

Fig. 8.2. A polarization-coded Ronchi ruling.

The bars of the Ronchi are linear polarizers.

Fig. 8.3. An etched-calcite phase Ronchi ruling.

For the other polarization, the phase shift is

$$\delta = 2\pi(n_o - n_e) d' / \lambda \quad , \quad (8.1)$$

where d' is depth of the etch. If d' is properly chosen, a π -phase shift for alternate bars occurs with the orthogonal polarization, and the desired $1/x^2$ point spread function is produced. This technique could also be implemented without an index matching fluid by choosing the etch depth such that the phase change is an even number of half waves for one polarization and an odd number for the other.

A method to rapidly switch between the two polarization states required for these polarization-coded pupils can be provided by the use of a photo-elastic modulator (PEM). The PEM is an oscillating wave-plate whose phase retardation can be expressed as

$$\delta = \delta_o \sin (2\pi \nu t) \quad (8.2)$$

where δ_o is an adjustable amplitude and the oscillation frequency ν is fixed. For a particular device produced by Hinds International, Inc. (Portland, OR), the oscillation frequency is 42 KHz. If linearly polarized light is incident at some angle to the optic axis of the crystal, the output polarization state will be time varying. If $\delta_o = \pi$ and the angle equals 45° , the output will smoothly change from the input polarization through circular polarization to the orthogonal linear polarization and back (Fig. 8.4). The switching will occur at twice the oscillation frequency.

Fig. 8.4. The output state of the PEM as a function of time.

The input is linearly polarized in the vertical direction, and the optic axis of the crystal is at 45° ($\delta_0 = \pi$).

This device can therefore be used to switch between the two linearly polarized states, but generally both states will be present at any one time (circularly or elliptically polarized outputs). Instead of abrupt switches (as with the rotating Ronchi disc), gradual shifts between the two output states (positive and negative channels) are obtained. This discussion is further complicated by the use of white light--only an average wavelength can be defined. Since the net output is a linear combination of the two desired states, the synthesized transfer function and point spread function will have the desired form. The only requirement for this result is that the electronic processing applied to the detector output balances the positive and negative portions of the signal for zero response at zero spatial frequency, i.e., overcomes any transmission differences between the two masks.

A possible optical system for use with the PEM is shown in Fig. 8.5. The PEM has been placed near the detector plane which contains an image of the pupil formed by the field lens. The polarization-coded Ronchi ruling is placed in the stop of the imaging lens. It may be easier to understand this system by considering light going through the system backwards. With the PEM, it is necessary to use a photomultiplier tube to measure the rapidly changing signal, as a large-area photodiode used in the unbiased mode is too slow due to its internal capacitance. In the biased mode, the diode's speed is increased, but a dark current is produced that serves as an unacceptable noise source.

Fig. 8.5. An optical system for use with the PEM.

An alternative method of reducing the reconstruction time is diagrammed in Fig. 8.6, and is actually a step back to the two-channel systems (Wu 1980). It contains a spatially multiplexed pupil and uses only one imaging lens and one processing loop. Its two-channel nature arises from the imaging lens and detector being split into halves, and the two halves are used independently of each other. One half of the pupil contains the positive-channel mask (clear) which is imaged onto half of the detector by the field lens. The other half of the pupil contains the negative-channel mask (Ronchi ruling) and is imaged onto the other half of the detector. The two detector outputs are the positive and negative signals, and a difference signal is the desired output. The detector could be a monolithic quadrant detector or adjacent discrete detectors. The advantage of this split pupil-split detector system is that there is no modulator. The system would be very simple and would not require polarizers, choppers, or complicated processing electronics. It would require good imaging properties for the field lens.

Either of these systems would allow the loop processor to reconstruct images in an acceptable time. Another advantage of these implementations is that linear Ronchi rulings are used, and the artifacts possibly caused by the curved lines in the concentric Ronchi ruling may disappear.

Fig. 8.6. The split pupil-split detector optical system.

The net output is the difference in the two detector signals.

Resolution-Density Discrimination Trade-Off

As mentioned in Chapter 2, there is a possible performance benefit to be gained by using a film-screen detector. In a CT scanner, the spatial resolution in the image is defined by the size or spacing of the discrete x-ray detectors. Film is basically a continuous detector array, so that the data is recorded at high spatial resolution. The resolution limit in the optical reconstruction is introduced in the processor, and within the constraints of the patient dose, the trade-off between density discrimination and spatial resolution (Eq. 2.35) can be utilized for a particular application. This was indeed the case in a study of renal failure in rats (Kujoory, Hillman, and Barrett 1980), where sinograms were digitized and processed to yield a resolution of 150 μm . This trade-off can be accomplished with the loop processor by changing the width of the slit on the loop and the period of the Ronchi ruling.

System Utilization

The loop processor has been designed to be a cost-effective alternative to digital CT scanners. The tomographic images produced by the two systems are quite comparable, but the ease of operation is not. A CT scanner is a push-button operation--push a button and wait for a picture. In some systems, this total operation consumes less than 30 seconds. With the loop processor, film must be loaded into the x-ray cassette before the data is taken, and the film must be developed and mounted in the optical processor prior to processing. The total time

from the start of a scan until an image is obtained will be three to five minutes. This calculation assumes that the reconstruction time is reduced as described at the start of this chapter.

Because of this time factor, the loop processor would only rarely be in direct competition with the expensive CT scanners. Large hospitals will continue to demand the high patient turnover and ease of operation offered by the digital machines. There is, however, another class of institutions with a definite need for tomographic imaging but unable to justify the extremely large capital investment required for a CT unit. These small hospitals and clinics may be very willing to trade convenience and time savings for a large monetary savings. It is for these institutions that the loop processor is intended, and it appears that this analog system would fill their needs very well.

Conclusions

An incoherent optical system that reconstructs an object from its x-ray projections has been developed. The projection data are recorded directly on x-ray film in the sinogram format, and the developed film is used as the input transparency for the single-channel optical processor. The convolution algorithm for transaxial tomography is used to reconstruct the object slice, and the bipolar filtering operations are implemented by the use of time-modulated OTF synthesis. The performance of the loop processor has been shown to match that of digital CT scanners.

APPENDIX A

CENTRAL-SLICE THEOREM

The central-slice theorem relates a one-dimensional projection of an object to the spatial frequency distribution of the object.

Consider a projection

$$f_{\phi}(x') = \int_{-\infty}^{\infty} \mu(x', y') dy', \quad (\text{A.1})$$

where the y' direction is arbitrary. A one-dimensional Fourier transform yields:

$$F_{\phi}(\xi') = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \mu(x', y') e^{-i2\pi\xi'x'}, \quad (\text{A.2})$$

where ξ' corresponds to the x' direction. A simple substitution produces

$$F_{\phi}(\xi') = \left\{ \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \mu(x', y') e^{-i2\pi\xi'x'} e^{-i2\pi\eta'y'} \right\}_{\eta'=0}, \quad (\text{A.3})$$

which is recognized as a two-dimensional Fourier transform:

$$F_{\phi}(\xi') = M(\xi', \eta') \Big|_{\eta'=0}, \quad (\text{A.4})$$

where $\mu(x,y)$ and $M(\xi,\eta)$ form a transform pair. The result, the central-slice theorem, states that the one-dimensional Fourier transform of a projection taken at the angle ϕ , is equal to a slice through the two-dimensional Fourier transform of the object. The slice is taken along the line $\eta'=0$, and the angle ϕ has been included in the rotated ξ' - η' coordinate system.

APPENDIX B

FAN-BEAM CORRECTION

It is the purpose of this appendix to derive the shape of the curve on a sinogram produced by a point for the geometry shown in Fig. 3.16. This figure shows the tilted-cassette correction for fan-beam recording of projection data. The tilt angle is α , and a detailed geometry is shown in Fig. B.1. The x' - y' axes are tilted by α with respect to the x'' - y'' axes (see Fig. B.2), so that the x-ray beam is confined to the $y'=0$ plane. The x'' - y'' axes define the film cassette plane. The transformation between the two coordinate systems is

$$x' = x'' \cos \alpha + y'' \sin \alpha , \quad (B.1)$$

$$y' = -x'' \sin \alpha + y'' \cos \alpha , \quad (B.2)$$

and

$$z' = z'' - d_1 . \quad (B.3)$$

The coordinates of the point located at (r, θ) in the x' - y' - z' coordinate system are given by

$$(x', y', z') = (r \sin \theta, 0, -d_2 - r \cos \theta) . \quad (B.4)$$

The equations of the line from the source through this point are therefore:

$$x'(d_2 + r \cos \theta) + z'(r \sin \theta) = 0 \quad (B.5)$$

Fig. B.1. The geometry of the tilted-cassette correction for fan-beam recording.

Fig. B.2. The relationship between the x' - y' axes and the x'' - y'' axes.

and

$$y' = 0. \quad (\text{B.6})$$

Converting these equations to the $x''-y''-z''$ system yields:

$$(x'' \cos \alpha + y'' \sin \alpha)(d_2 + r \cos \theta) + (z'' - d_1)(r \sin \theta) = 0, \quad (\text{B.7})$$

and

$$y'' \cos \alpha = x'' \sin \alpha .$$

The intersection of this line and the cassette ($z''=0$) defines where the point will be recorded for a particular r and θ . With some algebra, the result is obtained:

$$x'' = \frac{d_1 r \sin \theta \cos \alpha}{d_2 + r \cos \theta} , \quad (\text{B.8})$$

and

$$y'' = \frac{d_1 r \sin \theta \sin \alpha}{d_2 + r \cos \theta} . \quad (\text{B.9})$$

This position must now be related to an absolute position on the sinogram as it translates through the $x''-y''$ plane. The coordinates (x,y) on the sinogram are shown in Fig. B.3. Since for every π change in θ , the sinogram will translate by L , the sinogram length, the final coordinate transform is

$$x = x'' \quad (\text{B.10})$$

and

Fig. B.3. The coordinates on the sinogram.

A sinogram of length $2L$ is shown for clarity.

$$y = L\theta/\pi - y'' . \quad (\text{B.11})$$

The shape of the recorded curve is then parametrically described by the following two equations:

$$x = \frac{d_1 r \cos \theta \cos \alpha}{d_2 + r \cos \theta} , \quad (\text{B.12})$$

and

$$y = L\theta/\pi - \frac{d_1 r \sin \theta \sin \alpha}{d_2 + r \cos \theta} , \quad (\text{B.12})$$

where θ is the parameter.

A special case of this discussion occurs for $\alpha = 0$ (no tilt).

The two above equations can then be solved for the shape of the distorted sine curve:

$$x = \frac{d_1 r \sin (\pi y/L)}{d_2 + r \cos (\pi y/L)} . \quad (\text{B.14})$$

The maximum excursion of this recorded curve will occur when the radius vector is perpendicular to the x-ray beam, which occurs for $\theta = \cos^{-1} (-r/d_2)$. The sinogram position is therefore

$$y_m = \frac{L}{\pi} \cos^{-1} (-r/d_2) , \quad (\text{B.15})$$

instead of $L/2$ or $3L/2$.

While this derivation has assumed that $\theta = 0$ at the top of the sinogram, it is completely general, and can handle other phase sine curves by adding the appropriate offset to the value of y .

APPENDIX C

LENS DISTORTION

To measure the distortion of an imaging lens, the actual image can be compared to a theoretical perfect image (i.e., paraxial). This procedure was done by photographing an object the same size as a sinogram at the desired magnification m with the lens being measured. The resulting image is examined, and the geometry is shown in Fig. C.1 for barrel distortion. In this figure, $2 r_2 = L$, and the bow Δr of the edge of the image is measured. Distortion can be modeled by

$$r'_o = m r_o + \beta r_o^2, \quad (C.1)$$

where β is the distortion coefficient, unprimed variables are measured in object space, and primed variables are in the image. Using this relationship for two lengths in the image produces:

$$r' = m r_1 + \beta r_1^2 \quad (C.2)$$

and

$$r'_2 = m r_2 + \beta r_2^2. \quad (C.3)$$

The angle θ can also be determined:

$$\cos \theta = \frac{r'_2 - \Delta r}{r'_1} = \frac{r_2}{r_1}. \quad (C.4)$$

Fig. C.1. A distorted image compared to a paraxial image.

These three equations can be solved for the distortion coefficient in terms of Δr :

$$\beta = \frac{\Delta r}{r_2^2 - r_1 r_2} \quad (C.5)$$

For a bow of 1/14 mm and a 35 x 43 cm sinogram, the result is

$$\beta = 5 \times 10^{-6} / \text{mm}.$$

APPENDIX D

DOSE AND EXPECTED PERFORMANCE

The dose delivered to the center of the object during the recording of a sinogram will be determined by using an exposure chart for Bremsstrahlung radiation modified for the geometry and operating conditions of the experiment. This chart and various other parameters needed for this calculation can be found in the Radiological Health Handbook (1970). The specific geometry to be considered is that used for the animal studies (Figs. 7.8-7.11). A 120 KVP x-ray source, operating with a 20 mA beam current and filtered with 0.2 mm of copper, was located 2.4 m from the film and 2.0 m from the center of the object. The object was located at the center of a 28 cm water bath. The beam height was 2 mm, and the total exposure time was 330 sec. Very similar geometries were used for the phantom studies.

The exposure charts relate the dose delivered for various KVP's at different source to object distances and different filtrations. The exposure is expressed as mRad/mAs , where mAs is the product of beam current and exposure time. Since all of the charts express the filtration in mm of aluminum, the amount of aluminum corresponding to 0.2 mm of copper must be determined. At an average energy for the source (approximately 80 KeV), the attenuation coefficient is 0.55/cm for aluminum and is 6.92/cm for copper. Equal attenuation for copper and aluminum will occur when

$$x_{Al} = \frac{\mu_{cu}}{\mu_{Al}} x_{cu} , \quad (D.1)$$

where the x 's are the thicknesses of the filters. In this case, $x_{Al} = 2.5$ mm. The exposure, as determined from the chart, is then 3.7 mRad/mAs. for a 2.0 m source-object distance and a 120 KVP source.

Two corrections must be applied to this exposure before the dose can be evaluated. At diagnostic energies, the predominant attenuation mechanism is Compton scattering. About 20% of the incident energy is absorbed at the scattering site, and the remaining energy is carried off by the scattered photon. This scattered energy is generally not deposited in the slice of interest. The exposure chart assumes a large-area irradiation where most of the scattered radiation will be absorbed in the body. The value from the exposure chart must therefore be reduced by a factor of about 5 to account for this Compton effect. The second correction accounts for the attenuation of the x-ray beam in reaching the center of the object. The beam will be exponentially attenuated by half the water-bath thickness ($\mu_{H_2O} = .2/\text{cm}$).

With the above information, the dose delivered to the center of the object can now be found:

$$D_{\text{center}} = (3.7 \text{ mRad/mAs}) (20 \text{ mA}) (330 \text{ sec}) e^{-(.2/\text{cm})(14 \text{ cm})} / 5, \quad (D.2)$$

or

$$D_{\text{center}} = 0.30 \text{ Rad.} \quad (D.3)$$

This calculation requires the conversion factor of 1 Rad = 0.01 J/Kg. This dose was verified with exposure measurements using an ionization chamber and is consistent with results from a thermal luminescent dosimeter.

The system performance that can be expected at this dose level can be determined by using Eq. 2.35 (Barrett, Gordon and Hershel 1976). With the constants of proportionality, the relationship is

$$D_{\text{center}} = \frac{2.25 E_x (\text{SNR})^2 f_c \mu_{\text{av}} d/2}{\rho \mu_{\text{av}} \eta \epsilon^3 t}, \quad (\text{D.4})$$

where E_x is the average photon energy (~ 80 KeV), ρ is the density (1 g/cm^3), μ_{av} is the average attenuation coefficient ($.2/\text{cm}$), η is the question efficiency of the detector ($\sim 50\%$), ϵ is the resolution to which the image is processed (2 mm), t is the x-ray beam height (2 mm), d is the object diameter (28 cm), and f_c is a Compton "fudge factor" (0.2) which accounts for the fact the scattering deposits only about 20% of the energy removed from the beam in the slice of interest. With a center dose of 0.3 R, the SNR in the image can be evaluated:

$$\text{SNR} = 71. \quad (\text{D.5})$$

The minimum density difference $\Delta\mu/\mu$ corresponding to this SNR depends on the number of resolution cells that contain the change. For a large object (four or more pixels), $\Delta\mu/\mu = 1/\text{SNR} = 1.4\%$. For objects covering only one pixel, the minimum detectable change will be several times this level.

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