Thermo-mechanical strain rate dependent behavior of SMAs as vibration dampers and comparison to conventional dampers

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Abstract

A study on shape memory alloy (SMA) materials as vibration dampers is reported. An important component is the strain rate and temperature dependent constitutive behavior of SMA, which can significantly change its energy dissipation capacity under cyclic loading. The constitutive model used accounts for the thermo-mechanical, strain rate dependent behavior and phase transformation. With increasing structural flexibility the hysteretic loops size of SMA dampers increases due to increasing strain rates, thus further decreasing the response of the structure to cyclic excitation. The structure examined is a beam and its behavior with SMA dampers is compared to the same beam with conventional dampers. Parametric studies reveal the superior performance of the SMA over the conventional dampers even at the resonance frequency of the beam-damper system. An important behavior of the SMA dampers is discovered, in that they absorb energy from the fundamental and from higher vibration modes. In contrast the conventional dampers transfers energy to higher modes. For the same beam control, the stiffness requirement for the SMA dampers is significantly less than that of the conventional dampers. Response quantities of interest show improved performance of the SMA over the conventional dampers under varying excitation intensity, frequency, temperature, and strain rate.

Key words: Shape Memory Alloy damper; Thermo-mechanical model; Strain rate effects; Temperature effects; Non-linear dynamic analysis; Vibration control.

Introduction and review

Vibration control of structures using passive energy absorbers or passive dampers has received significant attention from the research as well as the design point of view. Dampers dissipate substantial amounts of input energy in order to reduce excessive structural vibration, and thus reduce the catastrophic failure susceptibility of structures subjected to extreme loads. Various types of dampers (such as viscoelastic, Maxwell, nonlinear viscous, friction, fluid, tuned-mass, metallic yield, and MR dampers) have been employed (Soong and Dargush, 1997; Crandall and Mark, 1973). Such dampers typically reduce some response quantities of the structure, while compromising others, and thus do not provide substantial reduction in vibration response. In this regard, the SMAs' super-elastic behavior and energy dissipation capability through wide hysteresis loops makes them attractive alternative damper materials. There are many variants of SMA, common ones being Nickel-Titanium, Nickel-

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Aluminum, Copper-Zinc-Aluminum, and Copper-Aluminum-Beryllium (Ozbulut et al, 2011). The Nickel-Titanium alloy, also known as Nitinol is a popular one. Nitinol properties such as super elasticity and shape memory through the reversible martensitic transformation are well documented in the literature, e.g. Buban and Frantziskonis, 2013.

Thomson et al, 1995, experimentally and theoretically demonstrated the potential of using SMAs as passive dampers, by utilizing the energy dissipation capacity of SMAs through their large hysteresis loop. SMA wires connected to a beam to reduce its vibration was the structure studied. Gandhi and Chapuis, 2002, studied the efficiency of SMA wires in controlling the flexural vibration of a beam by employing an amplitude-dependent complex modulus to model the wires. Results show significant control efficiency. After that work a number of studies have addressed the viability of applying SMAs as dampers in civil engineering structures subjected to seismic or wind loading. Han et al, 2003, report an experimental and finite element study of a two-story building with SMA cables as dampers to show the vibration suppression capability of the SMA. Zuo et al, 2006, performed an experimental study and observed that SMA dampers show practically no stiffness change and small energy dissipation at small displacements. However, at large displacements, stiffness changes are significant and energy dissipation capacity is very high. Sharabash and Andrawes, 2009, used SMA dampers in an analytical study to control the seismic vibration of a cable-stayed bridge, and observed that the vibration control efficiency of the damper depends strongly on the SMA hysteresis loop. Mishra et al, 2013, proposed an SMA-TMD (tuned mass damper) and performed a stochastic structural optimization study for seismic vibration control. Results show remarkable improvement over a conventional TMD.

The material models used in these studies do not address the coupled thermo-mechanical nature of the SMA hysteresis loops, the strain rate dependent SMA constitutive response, and the complexity of martensitic phase transformations. Dieng et al, 2013, in their damper work adopted a multi-axial SMA material model that captures the thermo-mechanical aspects of phase transformation in hysteresis loops. However, strain rate effects are not addressed in that study. Other relevant studies (Williams et al, 2002; Rustighi et al, 2005 and Saleeb et al, 2011) account for the SMA temperature induced phase transformation in designing non-linear, adaptive and tuned vibration absorbers. Temperature induced phase transformation changes the SMA mechanical properties and this changes the natural frequency of the structure-damper system, ultimately reducing the response of the structure. However, these damper designs require continuous energy supply, which is not cost effective. Moreover, if the input excitation contains a wide range of frequencies these dampers do not provide sufficient control efficiency.

An appropriate SMA material behavior model is important for damper design. Early attempts in constitutive modeling of SMAs were based on fitting uniaxial experimental data and used empirical relations for the evolution of martensite fraction (Graesser and Cozzarelli, 1991, 1994; Rengarajan et al, 1998). Such material models do not incorporate key aspects, i.e. phase transformation under stress or temperature change and strain rate effects. Other uniaxial models (Shariat et al, 2013) consider the effects of phase transformation under cyclic/dynamic load on the energy dissipation properties of SMAs. These models offer simplicity and implementation with fast computational algorithms, yet, do not consider the effect of strain rate on phase transformation and on the stress-strain hysteresis loops. Recent works (Prahlad and Chopra 2003; Iadicola and Shaw 2004; Vitiello et al, 2005; Auricchio et al, 2008; Morin et al, 2011; Mirzaeifar et al, 2011, 2012) incorporate the effects of strain rate on the stress-strain SMA behavior by considering the thermo-mechanical aspects of phase transformation and evolution of martensitic phase fraction and thus develop experimentally verified uniaxial strain rate and temperature dependent phenomenological models.

A few SMA multi-dimensional constitutive models for complex thermo-mechanical loading have been proposed (Helm and Haupt, 2003; Moumni et al, 2008; Grabe and Bruhns, 2008; Morin et al, 2011; Saleeb et al, 2011 and Andani and Elahinia, 2014). Most of these models incorporate the thermo-mechanical phenomena related to phase transformation as well as the effects of strain rate. The martensitic volume fraction is considered as an internal variable and properly identified functions are used to describe the thermomechanical aspects of phase transformation. To characterize the strain and temperature path dependence of phase transformation, strain and temperature controlled multi-axial experiments that cover the entire temperature transformation regime are required. The thermomechanics based SMA material model of Helm and Haupt, 2003 has been verified extensively with multi-dimensional experiments and is employed in the present study. This model incorporates stress and temperature induced phase transformation of the SMA, and strain rate effects.

This paper emphasizes the strain rate and thermo-mechanical transformation properties of SMAs, an important part for understanding their behavior as dampers. Nonlinear time-history analyses are performed to determine the control efficiency SMA dampers over the conventional linear viscous dampers. An extensive parametric study is performed considering a wide range of dampers and beam parameters, as well as various loading scenarios and temperature.

A natural complement to the present study would be the design for the SMA fatigue life. Since superelastic SMAs are used heavily in the medical industry for nonvascular and vascular stents, which encounter a large number of cycles within a human body, methodologies for predicting the fatigue failure life have been researched extensively and guidelines for design engineers are available in the literature. The reader is referred to citations [3-23] in Buban and Frantziskonis, 2013 for a review on this issue. However, SMA fatigue life and design is out of the scope of the present paper.

Motivation and extended applications

Cyclic load of even constant frequency and amplitude involves a range of strain rates, from zero to an alternating positive/negative maximum value that depends on the amplitude and the frequency of the load. Furthermore, for a structure and damper system, increase in the structure's flexibility implies increase in the rate of change of displacement and thus increase in the strain rate in the damper. For SMA spring dampers, as will be shown, the stress in the SMA spring increases with increasing strain rate. This causes increase in the developed force and thus increases in the size of the hysteretic loop in the SMA that ultimately increases the dissipation of input excitation energy and decreases the response of the structure. Thus, the inclusion of proper strain rate effects in SMA models is crucial when studying their performance as dampers; the same holds for the design of SMA dampers. Further, temperature effects are important. A verified SMA model, that of Helm and Haupt, 2003, is adopted. This model accounts for the thermo-mechanical hysteretic behavior, strain rate and temperature dependence, phase transformation, and shape memory and super elasticity.

In addition to the applications of SMA dampers described in the "Introduction and review" section, SMA dampers as the ones examined herein could be used to control the vibration of buildings due to wind or seismic loading. Also, they could be employed between two connected buildings to reduce the pounding effect due to seismic excitation. Furthermore, as will be shown, the SMA dampers dissipate significant amount of input excitation energy through their hysteresis loop, and such dissipation increases with increasing strain rate. Thus SMA dampers could be used as shock absorbers or to protect structures from blast or plus type loading.

Thermo-mechanical material model of SMA

Super-elasticity and shape memory are two very well-established SMA properties. SMA remains in the austenite phase above the austenite finish temperature and due to loading it transform to martensite through forward transformation. Upon unloading the martensite SMA recovers its deformation by gradual reverse transformation from the martensite to austenite. During loading-unloading cycles, the SMA dissipates significant energy through its flag-shaped hysteresis loop, which is very important for damper design. The size and shape of the hysteresis loop depends strongly on the temperature and strain rate. Moreover, SMAs are capable of recovering significant percentage of strain upon heating beyond a certain temperature (austenite finish temperature).

The Helm and Haupt, 2003 model has been widely employed for studying the SMA behavior under cyclic loading. It is based on the free energy formulation and utilizes evolution equations for internal variables such as the inelastic strain and martensite fraction, which play a crucial role in energy dissipation during cyclic loading. In the present study the multi-axial constitutive material model is reduced into a uniaxial stress case, expressed as follows

$$\sigma = \sigma(\varepsilon, \varepsilon_{in}, \theta) \quad (1.a)$$

$$\sigma_{in} = \sigma_{in}(\varepsilon_{in}, \dot{\varepsilon}_{in}, Z, \dot{Z}, \theta) \quad (1.b)$$

$$\dot{\varepsilon}_{in} = \dot{\varepsilon}_{in}(\sigma, \sigma_{in}, Z, \theta) \quad (1.c)$$

$$Z = Z(\varepsilon_{in}, \dot{\varepsilon}_{in}, \theta) \quad (1.d)$$

$$\dot{\theta} = \theta(\sigma, \varepsilon, \dot{\sigma}_{in}, \dot{\varepsilon}_{in}, Z) \quad (1.e)$$

Equations 1a is such that the developed stress σ is a nonlinear function of the applied strain ε , developed internal strain ε_{in} and ambient temperature θ . The internal stress σ_{in} is expressed as a function of ε_{in} , internal strain rate $\dot{\varepsilon}_{in}$, martensite fraction Z and its rate $\dot{Z}$ and θ , Eqn. 1b. Also the time rate of the developed internal strain $\dot{\varepsilon}_{in}$ is a nonlinear function of σ, σ_{in}, Z and θ , Eqn. 1c. Equation 1d provides the evolution of martensite fraction Z , as a nonlinear function of $\varepsilon_{in}, \dot{\varepsilon}_{in}$ and θ . Finally, the rate of temperature change $\dot{\theta}$ is expressed through Eqn. 1e, as function of $\sigma, \varepsilon, \dot{\sigma}_{in}, \dot{\varepsilon}_{in}$ and Z . Equations 1a-e forms a strongly non-linear system of equations that is solved iteratively for the developed stress given the applied strain, as shown in appendix A.

In the present study, the martensite start M_s and finish M_f temperatures are 285K and 265K, respectively, whereas austenite start A_s and finish A_f temperature are 295K and 315K, respectively. Figure 1, obtained by using the model described above, shows the effects of temperature and strain rate on the hysteresis loops and dissipated energy under uniaxial cyclic load conditions. In Fig. 1 the stress and energy per unit volume are normalized with respect to the SMA transformation strength. Figure 1a shows that with decreasing temperature, a lower level of stress is required to trigger phase transformation and low enough temperature results in considerable residual deformation. At higher temperatures there is no residual strain. Figure 1c shows that with increasing temperature the hysteresis loop size increases, thus increasing the energy dissipation capacity of the SMA damper. After the austenite finish temperature the hysteresis loop size decreases slightly, thus preventing further increase in energy dissipation capacity. Thus, the benefit of the super-elasticity and energy dissipation capacity of SMA can only be attained within a certain temperature range

(270K to 330K for this SMA) and therefore temperature plays a crucial role in designing a SMA damper system.

Figure-1: Load-deformation behavior of super-elastic SMA. (a) under different temperatures and (b) under different strain rates. Dissipated energy by SMA damper for one load cycle with respect to (c) temperature and (d) strain rate.

Figure 1b shows the hysteresis loops for different strain rates and Fig. 1d shows that with increasing strain rate the hysteresis loop size increases, thus the energy dissipation capacity of the damper increases. Increasing strain rate increases the forward transformation stress, whereas the backward transformation stress remains practically constant. The transformation strains, both forward and backward, remain practically constant with increasing strain rate. Overall, the energy dissipation capacity of the SMA remains almost constant at low to moderate strain rates and shows significant increase at high strain rates.

Dynamic response of beam with dampers

A two-dimensional model of a beam with dampers (either conventional or SMA) is considered. Figure 2 shows a schematic of a beam with conventional and SMA dampers, respectively. Here k_d, c_d denote the spring constant and the damping constant of the conventional damper, respectively. The beam is modeled with conventional beam (finite) elements, assuming an Euler-Bernoulli beam with n degrees-of-freedom (n -dof) and its constitutive behavior is considered linear. As dampers substantially reduce the beam's response, its behavior can reasonably be considered linear. The conventional damper is a spring-dashpot connected in parallel system, where both the spring and damper are linear. However, the behavior of the SMA damper is substantially nonlinear as the energy is dissipated through reversible phase transformation, triggered by cyclic loading-unloading. The beam is excited by vertical sinusoidal excitation at mid-span. The equation of motion for the beam with dampers (either conventional or SMA damper) reads

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} + F_d\{r\} = P\{r\} \quad (2)$$

where $[M]$, $[C]$ and $[K]$ denote the mass, damping and stiffness matrices for the beam of n degrees of freedom respectively and an over-dot denotes time rate. P denotes the excitation force and F_d the damper force. Vector $\{r\}$ denotes the influence vector where all its elements are zero except at the node where loading is applied or the damper is connected its value is unity. The damping matrix $[C]$ is considered as mass and stiffness proportional Rayleigh damping, thus expressed as

$$[C] = \alpha[M] + \beta[K] \quad (3)$$

where $\alpha = 2\xi_b/(\omega_1 + \omega_2)$ and $\beta = 2\xi_b\omega_1\omega_2/(\omega_1 + \omega_2)$. Here, ξ_b denotes the damping ratio of the beam material considered the same for all vibration modes, $\omega_1 = 2\pi/T_1$ and $\omega_2 = 2\pi/T_2$ denote the first and second mode frequencies, respectively, and T_1 and T_2 the first and second mode time periods, respectively.

Figure-2: Model of beam supplemented with (a) conventional linear dampers and (b) SMA dampers.

For the conventional linear damper, the force displacement relation of the damper reads

$$F_d = c_d \dot{u}_d + k_d u_d \quad (4)$$

where $\dot{u}_d$ and u_d denote the velocity and displacement of the node where the damper is connected to the beam. The damping coefficient c_d and stiffness k_d of the linear damper can be expressed in terms of the damping ratio ξ_d and stiffness ratio S_r , first mode time period of the beam T_1 and modal mass corresponding to first mode $m_1 = \{\phi_1\}^T [M] \{\phi_1\}$, i.e.

$$c_d = \frac{4\pi m_1 \xi_d}{T_1} \quad (5.a)$$

$$k_d = \frac{4\pi^2 m_1 S_r}{T_1^2} \quad (5.b)$$

Here, $\{\phi_1\}$ denotes the first mode of the beam, ρ the beam material density and L, b, d the length, depth and width of beam, respectively. The effect of lumped mass $m_l = \rho b d L / 2$ that is present at the center of the beam is already incorporated in the mass matrix.

For the SMA damper the force displacement relation is strongly non-linear, expressed as

$$F_d = F_{SMA}(u_d, u_{in}) + k_d u_d \quad (6)$$

where u_d denotes the displacement of the node where the SMA damper is connected, and u_{in} the internal deformation that develops from the phase transformation. The internal deformation of the SMA spring depends on its displacement and developed force. Thus, $F_{SMA}(u_d, u_{in})$ becomes strongly non-linear requiring an iterative technique at each time step. Considering the temperature of the SMA spring to be the same as the ambient temperature, the hysteresis force of SMA spring can be expressed as (derivation of Eq. (7) is provided in appendix B).

$$F_{SMA}(u_d, u_{in}) = \left(\frac{m_l g F_{0SMA}}{q_{SMA}} \right) \left[(u_d - u_{in}) - \left(\frac{9 \alpha k L_{SMA}}{4 m + k} \right) (q - q_0) \right] \quad (7)$$

where $F_{0SMA} = F_y / m_l g$ denotes the normalized transformation strength, q_{SMA} is the transformation displacement, L_{SMA} the length of the spring expressed as $(4\mu + \kappa)q_{SMA} / 3\sigma_y$, and F_y the transformation strength. Also, μ and κ denote the shear and compression modulus of the spring, respectively, θ the temperature of the SMA spring, θ_0 the ambient temperature, and α the linear coefficient of thermal expansion. It is noted that the transformation strength F_y and transformation displacement q_{SMA} correspond to the yield stress σ_y and yield strain ε_y in the Helm-Haupt, 2003, paper.

When the developed stress in the SMA spring crosses its yield stress, internal strain starts to develop due to phase transformation, i.e. that point on the stress-strain curve of the SMA is the starting point of phase transformation. Moreover to maximize the energy dissipation capacity of the SMA spring it is important that it transforms at a low strain level. The corresponding transformation displacement q_{SMA} of SMA spring is very low (0.00035 m). Also, according to the adopted material model the ratio of post transformation stiffness over initial stiffness is very small (almost 0.0001). Thus, one can neglect the post transformation stiffness of the SMA spring.

Numerical simulation

Response evaluation and parametric study for each damper is performed through numerical simulation of the beam with the dampers, Fig. 2. The entire beam is discretized into 100 two-node elements, and each node has two degrees-of-freedom, vertical displacement and rotation. Length L and cross section (depth d and width b) of the beam are adjusted so that the desired time period of the beam T_b is obtained. Another parameter that controls the dynamic response of the beam is its material damping ratio ξ_b . All the parameters and their default values are shown in Table-1. For the parametric study, a range of realistic parameter variations has been considered. Stiffness ratios for both dampers are adopted in such a way that the effect of viscous damping or SMA hysteresis damping can be studied. However, one can use very high value of stiffness ratio, but that will increase the cost and require large space, which will limit the application. Also in the later part of this study it is observed that

even with high stiffness ratio, the conventional damper cannot provide the same level of control efficiency that can be obtained with small stiffness ratio in the SMA damper. These statements are detailed in the parametric study part of the paper. Relevant studies on viscoelastic dampers (Min et al, 2004; Saidi et al, 2011; Moliner et al, 2012) considered damping ratios within a similar range (within 7% to 18%). In the case of the SMA damper, the specified value of normalized transformation strength yields minimum response, and thus provides maximum efficiency. Also, for other parameters of the SMA damper, reference is given to a recent paper on SMA-TMD by Mishra et al, 2013. Important parameters for the conventional damper are the stiffness ratio and the damping ratio, whereas the stiffness ratio, normalized transformation strength, strain rate and ambient temperature are the most important parameters for the SMA damper.

Table-1: Parameters adopted for the beam, dampers and force excitation

Properties of the beam	Properties of the dampers		Excitation parameters
	Conventional damper	SMA damper	
Time Period = 1 sec Damping ratio = 2 %	Stiffness ratio = 0.15 Damping ratio = 15 %	Stiffness ratio = 0.15 Normalized transformation strength = 0.15 yield displacement = 0.00035 m Strain rate = 0.0175 /s Temperature = 300K	Excitation acceleration intensity = 0.5g Excitation acceleration normalized frequency = 1.0

Sinusoidal acceleration with no phase lag is applied on the lumped mass at the mid-point of the beam. Input excitation is characterized by its peak acceleration intensity (in terms of g) and normalizing excitation frequency, which is the ratio of excitation frequency to the first mode frequency of the beam with damper system. The excitation amplitude is modulated with a time dependent modulation function, which initially increases exponentially and after attending a peak value of unity decreases exponentially. The time dependent sinusoidal loading is expressed as

$$a(t) = a_{\max} A(t) \sin \left[2\pi\lambda \left(\frac{t}{T} \right) \right] \quad (8)$$

where $a_{\max}$ denotes the peak amplitude of the acceleration, $A(t)$ the time dependent modulating function, λ the normalizing excitation frequency and T the time period of the beam or beam with damper system. The modulation function can be expressed as

$$A(t) = \frac{\exp(-\alpha_1 t) - \exp(-\alpha_2 t)}{\max|A(t)|} \quad (9)$$

where, α_1 and α_2 are constants and equal to 0.35 and 0.65, respectively. The denominator in (9) is such that the modulation function will finally reach a maximum value of unity. Using (8) and (9) the applied force at the midpoint of the beam is obtained as

$$P(t) = m_l a(t) = a_{\max} \left(\frac{\rho b d L}{2} \right) \left[\frac{\exp(-\alpha_1 t) - \exp(-\alpha_2 t)}{\max|A(t)|} \right] \sin \left[2\pi\lambda \left(\frac{t}{T} \right) \right] \quad (10)$$

The modulating function reflects a general scenario of loading, where load gradually increases and then gradually decreases, and is more realistic than a constant amplitude sinusoidal load. Also, both dampers are passive in nature, thus the sudden application of high amplitude loading will not give enough time for the dampers to work at their maximum efficiency, and thus consideration of such modulating function is justifiable.

To obtain the response of the beam under the assumed dynamic loading the step-by-step Newmark-beta (average acceleration technique) numerical integration method is used with the time step Δt of 0.005 seconds. Since the beam material is linear-elastic, the response of the beam with the conventional linear dampers can be obtained directly, without any iteration. The force-deformation hysteresis loops in the SMA imply strongly non-linear response, thus the response of the beam with SMA dampers is obtained by performing iterations until convergence.

Response assessment

In this study the excitation frequency is considered to be the same as the fundamental frequency of the beam with damper system. Figure 3 shows the response quantities of interest, i.e. the time history of vertical acceleration and displacement, normalized bending moment and normalized shear force at the point where maximum values of these variables are observed. The dynamic bending moment and shear force are normalized with respect to the static bending moment and shear force values. Also, the force deformation hysteresis curves for both dampers are shown in Fig. 3e. Figure 3a shows that vertical acceleration of the beam with the SMA damper is substantially reduced, about 55% less than that of the beam with conventional damper. The maximum vertical displacement of the beam with SMA damper is 60% lower than that in the beam with conventional damper. Similar results hold for the normalized bending moment and shear force, Figs. 3b,c. Here, the bending moment reduces almost 66%, whereas the shear force reduces almost 76%. This rather remarkable reduction in the response can be explained with the help of Figs. 3e,f that show the associated force-deformation characteristics of the conventional viscous and the SMA damper and the FFT amplitude of acceleration at midpoint of the beam, respectively. The hysteresis loop of the SMA damper is much larger than that of the conventional damper, hence, qualitatively; the SMA damper has better energy dissipation capability than the conventional damper. Figure 3e shows that peaks of the FFT amplitude are observed at different frequencies, and this is because the beam-damper system is excited at its fundamental mode frequency. The fundamental frequencies are: (i) for the beam without any damper 1.00 Hz; (ii) the beam with conventional damper 1.08 Hz; and (iii) for beam with SMA damper 1.03Hz. Although the linear stiffness for both dampers is the same, the enhanced damping provided by the SMA hysteresis loop causes less of a shift in the fundamental mode frequency than the fundamental mode frequency of the beam with conventional damper. Moreover, the FFT amplitude of the acceleration of the beam with SMA damper is much lower than that of the beam with the conventional damper. Since the SMA damper dissipates a significant part of the input excitation energy, only a very small part of it transfers to the beam. The conventional damper is not able to dissipate such high excitation energy.

In conclusion, even at the resonating frequency of the beam-damper system, the SMA damper shows much higher level of efficiency than the conventional damper. To analyze further, a wide range of system parameters as well as different scenarios of loading are considered.

Figure-3: Time-history of (a) vertical acceleration, (b) vertical displacement, (c) normalized bending moment and (d) normalized shear force of beam with conventional linear damper and SMA damper; (e) force-deformation hysteresis loop of both the dampers and (f) FFT of beam and beam-damper system acceleration.

Parametric study

A comprehensive parametric study is performed by considering various scenarios for the damper parameters, beam properties and input excitations. This also identifies the governing design variables for the SMA damper. Responses are normalized by the respective response of the beam without any dampers.

The stiffness ratio is one of the most important parameters in the design of a damper. Three response variables of interest for various stiffness ratios for the damper are shown in Fig. 4. In particular, Fig. 4a shows that for the same stiffness ratio for the damper, the maximum vertical displacement of the beam can be reduced substantially by replacing the conventional dampers with SMA dampers. As a function of stiffness ratio, reduction in beam displacement in the range of 45%-65% can be achieved. As shown in Fig. 4b and 4c, similarly to the displacement ratio, both moment and shear force ratio significantly decrease with the SMA damper. As a function of stiffness ratio, improvement in maximum moment ratio by using SMA over conventional damper is in the range of 50% to 70%, and for shear force ratio in the range of 55% to 83%. Although the loading frequency is the same as the resonant frequency of the beam-damper system, substantial amount of input excitation energy is dissipated by the SMA hysteretic loop, not present in the conventional damper. In addition, the reduction in the moment or shear force implies reduction in material used for the beam.

Figure-4: (a) maximum vertical displacement ratio, (b) maximum moment ratio and (c) maximum shear force ratio with respect to damper stiffness ratio.

Both cases show monotonic decrease in response quantities with increasing stiffness ratio. However, to achieve a similar level of control efficiency, the requirement on stiffness ratio is reduced substantially in the SMA damper as compared to the conventional linear damper. Almost identical level of efficiency could be achieved with a mere 0.05 stiffness ratio in the SMA damper, whereas the conventional damper would require more than 0.25 stiffness ratio.

Figure-5: (a) maximum vertical displacement ratio, (b) maximum moment ratio, and (c) maximum shear force ratio as a function of damping ratio for the conventional damper. (d) maximum vertical displacement ratio, (e) maximum moment ratio, and (f) maximum shear force ratio as a function of the normalized transformation strength ratio for the SMA damper for various strain rates.

Another important design parameter is the damping ratio for the conventional damper or the normalized transformation strength for the SMA damper. Figures 5a-c show response parameters as a function of the damping ratio for the beam with conventional damper, whereas Figs. 5d-f show the same parameters as a function of the SMA damper transformation strength. From Fig. 5a, it follows that the maximum vertical displacement ratio of the beam with the conventional damper decreases with increasing damping ratio. A

similar trend is observed in Figs. 5b and 5c for the bending moment and shear force ratio. There does not exist an optimum value of damping ratio for the conventional damper that will maximize the control efficiency. Thus, by providing high level of damping, substantial reduction in the response could be obtained. However, a large size damper hinders its applicability and increases its cost. Whereas, for the SMA damper there exist an optimal transformation strength of the SMA, that minimizes the response of the structure, and thus maximizes the control efficiency. It is clear that the SMA damper provides much higher control efficiency than the conventional damper. At low values of transformation strength, due to the applied load the SMA material experiences stress induced phase transformation and thus dissipates a considerable part of the input energy. This trend persists with increasing transformation strength (for a given level of excitation) until the transformation strength reaches a critical value where input excitation becomes insufficient to move the SMA material to the stress induced phase transformation zone. Thus at the high level of transformation strength the SMA damper does not depict any hysteretic energy dissipation through the phase transformation, and thus behaves as a linear spring. Another important aspect is the effect strain rate on the optimal transformation strength of SMA and the optimal as well as overall responses of the beam with the SMA damper. With increasing strain rate the optimal transformation strength value shifts to the left, i.e. the required optimal transformation strength value decreases. Strain rate increase reduces both the optimal and overall response of the beam; however these reductions become practically negligible beyond the optimal transformation strength of the SMA damper. This is because with increasing strain rate the hysteresis loop size increases and thus for the same value of transformation strength, energy dissipation is higher at high strain rate, and this causes reduction in the response of beam and shift in the optimal transformation strength values. However, beyond the optimal transformation strength, the input excitation becomes insufficient for stress induced phase transformation. In this case the SMA spring behaves almost as a linear damper and this implies almost no change in beam response with increasing strain rate.

Figure-6: Frequency response functions of input and output acceleration of beam with (a) convention damper and (b) SMA damper.

The frequency response function (FRF) of input and output acceleration of the beam and beam with conventional or SMA damper is plotted in Fig. 6. Figure 6a shows that in the case of conventional damper, near the fundamental frequency mode (of the beam-damper system) the amplitude of FRF decreases monotonically with increasing damping ratio, however the FRF amplitude increases near the higher modes frequency of the beam. Thus, the conventional damper in the beam transfers the input excitation energy from the fundamental modes to higher modes, which ultimately excites the higher modes and thus is not able to

substantially reduce the beam response. However, the SMA damper shows different trend in FRF amplitude. Near the fundamental frequency of the beam-damper system the FRF amplitude initially decreases with the increasing transformation strength and after a critical value ($F_{OSMA} = 0.15$) it starts to increase (Fig. 6b). A similar pattern holds near the higher mode frequencies, however in the beam with SMA damper the FRF amplitude near the higher mode frequencies decreases. Thus significant part of the input excitation energy has been dissipated by the SMA phase transformation, not transfer to higher modes, which ultimately enhances the performance of the beam. This is a striking attribute of the SMA material, and emphasizes the fact that using very high level of damping ratio for the conventional damper does not yield similar level of control efficiency, as obtained by a smaller SMA transformation strength.

Figure-7: Displacement ratio, moment ratio, and shear force ratio as a function of temperature (a)-(c), and strain rate (d)-(f) for the SMA damper.

Now, the effect of temperature and strain rate on the control efficiency of SMA damper is addressed. At constant temperature, phase transformation in SMA is governed by the applied stress. At the natural frequency of the beam-damper system, Figs. 7a-c shows the change in the beam response as a function of temperature. At low temperature (below the austenite start temperature) the martensite phase is stable. Thus, at low temperature due to the application of loading, self-accommodating martensite transforms into orientated martensite and a relatively small amount of energy is dissipated resulting into a small hysteresis loop as well as a large residual deformation. With increasing temperature (yet within the austenite start and finish temperature range) the austenite phase becomes more stable. At this range of temperature, application of sufficient loading imposes full transformation from austenite to orientated martensite phase, which results into a large size flag-shaped hysteresis loop, thus, a high level of input excitation energy is dissipated. However, after a critical value of temperature (higher than the austenite finish temperature) the austenite phase becomes very stable and thus a high level of energy is required to transform austenite to martensite. Due to the absence of such high input energy, full phase transformation does not occur at high temperature. Thus the

efficiency of the SMA damper degrades. To demonstrate the coupled effects of temperature and strain rate on the control efficiency of the SMA damper, simulations are performed at different strain rates, under varying temperature. As mentioned earlier, with increasing strain rate the hysteresis loops size of SMA increases, which then increases the energy dissipation capacity of the SMA damper. Also, with increasing temperature the SMA hysteresis loop size increases and thus the energy dissipation capacity of SMA damper increases. Thus, the coupling effect of temperature and strain rate increase enhances the energy dissipation capacity of the SMA damper, which further causes improvement in control efficiency at moderate to high strain rates. However, such an increase is small at low strain rates. Thus the beneficial effect of strain rate occurs only at moderate to high levels of strain rate. Figures 7d-f show the effect of strain rate on the control efficiency of the SMA damper at constant values of transformation strength and temperature. It is observed that various beam response parameters decrease monotonically with increasing strain rate. This can be explained with the help of Fig. 1, where it is shown that increasing strain rate increases the size of hysteresis loop. At constant temperature, as the strain rate from imposed loading increases, the hysteresis loop size of the SMA increases and this increases the energy dissipation capacity of the SMA damper.

Performance for different beam flexibilities is shown in Fig. 8, where it is seen that the SMA damper largely reduces the beam response, much more than the conventional damper. With increasing beam flexibility, the response of the beam with conventional damper increases, yet for the beam with SMA damper the response decreases. This improvement in the control efficiency of the SMA damper is due to the increase in applied strain rate from the increased beam flexibility. With increasing beam flexibility the deformation in the beam increases (at a constant excitation intensity and frequency), which results in increased strain rate in the damper. Since the hysteresis loop or energy dissipation capacity of the conventional damper is independent of strain rate, the beam response parameters increase with increasing beam flexibility. In the case of SMA damper, the hysteresis loop size depends strongly on the applied strain rate (Fig. 1b). Also, energy dissipation capacity of SMA damper increases with increasing strain rate (Fig. 1d). Thus, increasing strain rate leads to more input energy dissipation and thus less amount of force is transmitted to the beam. Improvement of the vertical displacement of the beam is in the 50%-70% range. Improvement in bending moment is in the 57%-77% range, and shear force in the 59%-86% range.

Figure-8: (a) maximum vertical displacement ratio (b) maximum moment ratio and (c) maximum shear force ratio as a function of the natural time period of the beam.

The beam response as a function of damping ratio is shown in Fig. 9. As the structural damping increases, the control efficiency of both the conventional and SMA dampers decreases. However, the relative improvement in displacement reduction of beam decreases from 66% to 48% with increasing structural damping ratio. Similarly, the relative

improvement in moment ratio decreases from 72% to 59% and shear force ratio decreases from 80% to 68%. The hysteretic damping provided by the SMA depends strongly on the level of applied strain or displacement. With increasing beam damping ratio the displacement of the beam decreases which reduces the size of the hysteresis loop of the SMA damper.

Figure-9: (a) maximum vertical displacement ratio (b) maximum moment ratio and (c) maximum shear force ratio as a function of the damping ratio of the beam.

Performance of the dampers is also examined under different input acceleration conditions, i.e. peak intensity of acceleration and dominant frequency. Excitation intensity is characterized by the peak acceleration applied to the beam. Values of various response parameters with respect to load intensity are shown in Fig. 10. For the beam with the conventional dampers, all response parameters remain almost identical irrespectively of the peak intensity of the excitation. This is a direct consequence of the linearity in the conventional dampers and beam. However, the beam with SMA dampers shows slight variation in the response parameter with changing peak excitation intensity, because of its nonlinear nature.

Figure-10: (a) maximum vertical displacement ratio (b) maximum moment ratio and (c) maximum shear force ratio as a function of the input excitation peak acceleration.

The performance of the dampers is also examined for beam excitations of various frequencies, with input excitation according to Eqn. (10). In all previous results the excitation frequency is the same as the fundamental frequency of the beam-damper system. Figure 11 shows the beam response in terms of the maximum displacement ratio, maximum moment and shear force ratio as a function of the normalized load frequency. The SMA damper substantially reduces the beam response throughout the examined frequency range, while this is not the case for the conventional damper. Notably, for both dampers the response reduction capability becomes constant after a certain excitation frequency. Both dampers show reduction in efficiency when the normalized load frequency is unity, i.e. when the load frequency matches that of the natural frequency of the beam-damper system. Relative

improvement in the displacement reduction remains almost constant, approximately 60%. Similar level of improvement in the moment and shear force ratios is observed.

Figure-11: (a) maximum vertical displacement ratio (b) maximum moment ratio and (c) maximum shear force ratio as a function of normalized excitation frequency.

Conclusions

It is concluded that in examining SMAs as dampers it is important to account for the temperature and strain rate dependent SMA behavior. With increasing temperature the hysteresis loop size increases, thus the energy dissipation capacity of the SMA damper also increases, which causes reduction in beam response. However, since increase in temperature increases the transformation stress of the SMA, at very high temperatures the SMA damper does not provide effective hysteretic energy dissipation. Also, the super elasticity and thus the beneficial damping effects is only available within a certain temperature range, and for the particular material examined here in the 270 to 330 K range.

With increasing strain rate the hysteresis loop size increases, which in turn increases the energy dissipation capacity of the SMA damper. Increase in damping efficiency is noticeable at moderate to high strain rates i.e. more than 0.04/s strain rate for the particular material examined. Under cyclical load, with increasing structure flexibility, the strain rate imposed on the damper increases, which increases the hysteretic loop size, thus increasing the SMA damping efficiency. Whereas for the conventional dampers the force is independent of strain rate and thus with increasing beam flexibility its damping efficiency decreases.

It has been shown that the stiffness ratio and the normalized transformation strength are important parameters for damping efficiency. Optimum SMA damper parameters result in as much as 60% more response reduction than that in the same beam with conventional damper. One noticeable observation in this study is that using similar stiffness ratio for the dampers, high response reduction can be achieved by the SMA as compared to that by the conventional damper. This is significant because it eliminates the requirement of a large conventional damper, which limits its applicability. This study also reinforces the fact that phase transformation is an important attribute of the SMA in obtaining an optimal performance for the beam with SMA damper.

Even at the resonating frequency of the beam damper system, the SMA damper shows much higher control efficiency than the conventional damper. Thus, the SMA damper acts as a “perfect” absorber, i.e. it can absorb energy starting from fundamental mode, all the way to higher modes of a structure. However, as is known, the conventional damper transfers the energy from the fundamental to higher modes. Finally, the paper shows that the overall performance advantages of the SMA dampers over conventional dampers are retained with changes in the various beam parameters, as well as different excitation scenarios. Also, to

obtain optimal control efficiency for SMA damper, strain rate and temperature effects are important for design.

Appendix-A: Time marching iterative algorithm for SMA stress strain loop

In addition to equations (1) a yield surface is defined that serves as a criterion whether the stress strain relation is in the elastic or inelastic regime. That yield surface is expressed as

$$f = f(\sigma, \sigma_{in}) \quad (\text{A.1})$$

A step-by-step time marching method has been employed to evaluate the developed stress S as a function of strain at the current time step, using the implicit first order Runge–Kutta method. This iteration process continues until the error in the stress σ , internal strain ε_{in} , internal stress σ_{in} , martensite fraction Z , and temperature θ at the current time step become less than or equal to a tolerance limit d'_{tol} , which is taken to be 0.00001. The iteration steps are as follows:

Step-1: Compute the developed stress and yield function for time step $n+1$ and first iteration, $j=1$ using Eqns. 1a and A.1.

$$S_{(n+1)}^{(j)} = S\left(\varepsilon_{(n+1)}, \varepsilon_{in,(n+1)}^{(j)}, q_{(n+1)}^{(j)}\right) \quad (\text{A.2})$$

$$f_{(n+1)}^{(j)} = f\left(\sigma_{(n+1)}^{(j)}, \sigma_{in,(n+1)}^{(j)}\right) \quad (\text{A.3})$$

Step-2: If, $f_{(n+1)}^{(j)} \leq 0$ then the stress strain is in elastic regime, thus move to next time step, i.e. go to step-1. Else stress-strain is in inelastic regime, thus go to step-3 for iterative solution and use first order implicit Runge–Kutta method.

Step-3: Compute the internal stress due to applied strain and temperature, internal strain, stress induced martensite fraction, temperature induced martensite fraction and temperature at time step $n+1$ and iteration j .

$$\sigma_{in,(n+1)}^{(j)} = \sigma_{in}\left(\varepsilon_{in,(n+1)}^{(j)}, \dot{\varepsilon}_{in,(n+1)}^{(j)}, Z_{(n+1)}^{(j)}, \dot{Z}_{(n+1)}^{(j)}, \theta_{(n+1)}^{(j)}\right) \quad (\text{A.4})$$

$$\dot{\varepsilon}_{in,(n+1)}^{(j)} = \dot{\varepsilon}_{in}\left(\sigma_{(n+1)}^{(j)}, \sigma_{in,(n+1)}^{(j)}, Z_{(n+1)}^{(j)}, \dot{Z}_{(n+1)}^{(j)}, \theta_{(n+1)}^{(j)}\right) \quad (\text{A.5})$$

$$Z_{(n+1)}^{(j)} = Z\left(\varepsilon_{in,(n+1)}^{(j)}, \dot{\varepsilon}_{in,(n+1)}^{(j)}, \theta_{(n+1)}^{(j)}\right) \quad (\text{A.6})$$

$$\dot{\theta}_{(n+1)}^{(j)} = \theta\left(\sigma_{(n+1)}^{(j)}, \varepsilon_{(n+1)}^{(j)}, \dot{\sigma}_{in,(n+1)}^{(j)}, \dot{\varepsilon}_{in,(n+1)}^{(j)}, Z_{(n+1)}^{(j)}, \theta_{(n+1)}^{(j)}\right) \quad (\text{A.7})$$

Step-4: Evaluate error and whether iterations have converged, i.e. check whether $abs(X_{(n+1)}^{(j+1)} - X_{(n+1)}^{(j)}) \leq X_{(n+1)}^{(j)} \delta_{tol}$ is satisfied, where X denotes any of the relevant variables, i.e. $X = \sigma, \sigma_{in}, \varepsilon_{in}, Z, \theta$. If convergence has occurred increase the time step by one unit. If not, go back to step-1 for next iteration.

Appendix-B: Formulation for different design parameters of SMA damper

Here the basic equations describing the SMA constitutive model used in this study are presented. However, for detailed understanding of the SMA model the reader is referred to

the paper by Helm and Haupt, 2003. The one-dimensional stress-strain relation of this SMA material model can be expressed as

$$\sigma = \left[\frac{(4\mu + \kappa)}{3} \right] [\varepsilon - \varepsilon_{in}] - 3\alpha\kappa(\theta - \theta_0) \quad (\text{B.1})$$

The yield strength of SMA damper is expressed as

$$F_y = \sigma_y A = m_1 g F_{0SMA} \quad (\text{B.2})$$

where, F_y is the yield strength of SMA spring, σ_y is the yield stress of SMA, A is the cross-section area of the spring, m_1 the first mode modal mass of the spring, g is the acceleration of gravity, and F_{0SMA} is the normalized transformation strength of the SMA spring. The cross-sectional area and length of SMA spring are expressed as

$$A = (m_1 g F_{0SMA} / \sigma_y) \quad (\text{B.3})$$

$$L_{SMA} = (4\mu + \kappa) q_{SMA} / 3\sigma_y \quad (\text{B.4})$$

Here, q_{SMA} is the yield displacement of SMA. Now considering Eqns. (B.1) and (B.3) the developed force in the SMA spring is obtained as

$$F_{SMA} = \sigma A = \left(\frac{m_1 g F_{0SMA}}{\sigma_y} \right) \left[\left\{ \frac{(4\mu + \kappa)}{3} \right\} (\varepsilon - \varepsilon_{in}) - 3\alpha\kappa(\theta - \theta_0) \right] \quad (\text{B.5})$$

Before the onset of yielding the internal strain is zero and thus the linear elasticity relation holds (considering the initial temperature θ of the SMA spring is the same as the ambient temperature θ_0)

$$\sigma_y = \left\{ \frac{(4\mu + \kappa)}{3} \right\} \varepsilon_y \quad (\text{B.6})$$

Finally, using Eqns. (B.1), (B.5) and (B.6) the following expression for the developed force in the SMA spring is obtained

$$F_{SMA} = \left(\frac{m_1 g F_{0SMA}}{e_y} \right) \left[(e - e_{in}) - \left\{ \frac{9ak}{(4m + k)} \right\} (q - q_0) \right] \quad (\text{B.7})$$

and, in terms of displacement, Eq. (7) results.

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