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Aspheric/freeform optical surface description for controlling illumination from point-like light sources

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Abstract. We present an optical surface in closed form that can be used to design lenses for controlling relative illumination on a target surface. The optical surface is constructed by rotation of the pedal curve to the ellipse about its minor axis. Three renditions of the surface are provided, namely as an expansion of a base surface, and as combinations of several base surfaces. Examples of the performance of the surfaces are presented for the case of a point light source. © 2016 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.OE.55.11.115104](https://doi.org/10.1117/1.OE.55.11.115104)]

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1 Introduction

Optical design depends on optical surface description, thus it is important to count on surfaces that can provide solutions to imaging and nonimaging problems. For imaging problems, the well-known axially symmetric conic and polynomial surface of Eq. (1) provides solutions for a vast number of problems

$$z(r) = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}} + A_2r^2 + A_4r^4 + A_6r^6 + A_8r^8 \dots \quad (1)$$

The case of nonimaging optics also requires surfaces that typically cannot be represented by conic/polynomial type expansions. Therefore, several surface descriptions have been proposed to solve illumination optics problems including splines, implicitly defined surfaces, surfaces based on Bernstein polynomials, and freeform surfaces.^{1–8} On the other hand, most illumination problems are solved numerically and the resulting data points are interpolated for ray tracing purposes. Numerical methods provide a solution surface as a set of data points. However, in some applications it is desirable to describe the solution surface in closed form so as to be specific about the nature of the surface. Thus, there is a need for surface descriptions that effectively solve illumination problems and that can ideally be expressed in closed analytical form.

Some advantages of using closed-form surface descriptions are that the actual surface can be precisely specified, that some tolerancing analyses can be produced, and that parametric studies can be conducted.

U.S. Patent 4,734,836 describes a lens for uniform illumination on a planar target using an approximate point source. The first surface of this axially symmetric lens is spherical in shape and is concentric with the point-like source. The second surface is concave near the center and turns smoothly into convex toward the lens edge. Thus, the lens spreads out the light to avoid a bell-shaped illumination

profile. The design of the concave–convex surface can be extreme as rays may come along the optical axis with up to 75 deg or more degrees of inclination with respect to this optical axis.

It is noted that the pedal curve to the ellipse resembles the concave–convex profile that is required in such a lens. Thus, in this paper, using such a pedal curve we construct and demonstrate surfaces in closed form that substantially describe the desired surface for uniform illumination on a target surface. We provide examples about the performance of three surfaces that we construct using the concept of base surface and discuss some of their properties. Previously, we have reported surfaces that are useful for solving other optical design problems.^{9–12}

2 Aspheric and Freeform Surface Construction

The pedal curve to the ellipse is shown in Fig. 1 and is given analytically as

$$a^2x^2 + b^2y^2 = (x^2 + y^2)^2, \quad (2)$$

where a is the major axis of the ellipse and b is the minor axis. The sag $S(r)$ of the surface can be obtained by rotation of the pedal curve about the z -axis and is written as

$$S(r) = b - \sqrt{\frac{b^2 - 2r^2 + \sqrt{b^4 + 4(a^2 - b^2)r^2}}{2}}, \quad (3)$$

where $r = \sqrt{x^2 + y^2}$ is the radial distance from the optical axis or z -axis. We call $S(r)$ a base surface.

Now we construct an aspherical surface $z_1(r)$ by constructing a polynomial on the base surface $S(r)$ as

$$z_1(r) = A_1S(r) + A_2S^2(r) + A_3S^3(r) + A_4S^4(r) + A_5S^5(r) + A_6S^6(r) + A_7S^7(r) + A_8S^8(r) + \dots, \quad (4)$$

where A 's represent the deformation coefficients.

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Fig. 1 (a) Pedal curve of the ellipse and (b) coordinate axes on half the pedal curve. Note the change of curvature from the curve center to the edge.

We also construct a freeform $z_2(r)$ surface by a superposition of base surfaces as

$$z_2(r) = A_1 S_1(r) + A_2 S_2(r) + A_3 S_3(r) + \dots, \quad (5)$$

where each of the $S(r)$ terms has its own independent a , b , and A coefficients. These two surfaces $z_1(r)$ and $z_2(r)$ were programmed as user-defined surfaces in Zemax OpticStudio optical design software.¹³

To design a surface, we assume the light source to be point like and Lambertian, so that the intensity decreases as the cosine of the angle of ray emission θ with respect to the z -axis of Fig. 1. The source is located on the optical axis, and the target surface is flat and perpendicular to the optical axis.

The optical flux $\Phi(\theta)$ from a Lambertian point source L_0 as a function of angle θ is given as

$$\Phi(\theta) = 2\pi L_0 \int_0^\theta \cos(\theta) \sin(\theta) d\theta = \pi L_0 \sin^2(\theta). \quad (6)$$

For uniform illumination on a flat surface, conservation of optical flux requires that for a given ray emitted at angle θ with respect to the optical axis, the ray intersection Y at the target surface should satisfy

$$Y = \frac{Y_{\max} \sin(\theta)}{\sin(\theta_{\max})}, \quad (7)$$

where $\theta_{\max}$ is the maximum angle of emission and $Y_{\max}$ is the maximum ray intersection at the target surface. This comes about because the fractional optical flux from a Lambertian source and from a flat surface that is uniformly illuminated are given by $\sin^2(\theta)/\sin^2(\theta_{\max})$ and $Y^2/Y_{\max}^2$, respectively.

For the actual surface design, 20 rays from the source were traced equally spaced in angle of emission from $\theta = 0$ to $\theta = \theta_{\max}$. An error function was created as the sum of the squares of the differences of ray intercepts and the theoretical ray intercept Y . Then using the damped least square and the orthogonal descent optimization methods, the error function was minimized. It was noted that when ray total internal reflection took place, the ray tracing stopped for that ray and the optimization process stagnated. This indicated that no physical solution was possible due to the index of refraction value or to a target surface in close proximity to the lens. The surface coefficients were used by the optimizer as degrees of freedom to reduce the error function. The coefficients were released as variables in sets of two and as the optimizer proceeded, more coefficients were released.

3 Aspheric and Freeform Lens Examples

To illustrate the performance of the surfaces, we designed two lenses made out of polycarbonate plastic ($n = 1.58546992$ at $\lambda = 587.5618$ nm). The first lens uses the aspheric surface profile $z_1(r)$ with a maximum ray angle of $\theta_{\max} = 75$ deg. The second lens uses the freeform surface profile $z_2(r)$ also with a maximum ray angle of $\theta_{\max} = 75$ deg.

For both lenses, the marginal ray (at $\theta_{\max}$) angle of incidence on the surface was constrained to 0 deg with respect to the optical axis. In addition, the distance from the

Fig. 2 Aspheric lens with profile $z_1(r)$ and ray trace to the target surface.

Fig. 3 Freeform lens with surface profile $z_2(r)$ and ray trace to the target surface.

Table 1 Coefficients defining the surface profile $z_1(r)$.

$z_1(r)$	a (mm)	b (mm)	A_1	A_2
	10.209629	4.1375154	-0.76745888	-0.0076601472
	A_3	A_4	A_5	A_6
	0.0032823004	$3.9058491 \times 10^{-005}$	-0.00036512352	$8.911191 \times 10^{-005}$

Table 2 Coefficients defining the surface profile $z_2(r)$.

$z_2(r)$	a (mm)	b (mm)	A
$S_1(r)$	27.089235	5.7955684	-0.34716392
$S_2(r)$	10.330664	11.189056	-0.55695918
$S_3(r)$	117.56108	71.18933	-0.5932761

Lambertian point source to the aspheric/freeform surface is 5 mm and the distance from the source to the target surface is 20 mm. The lenses and ray trace are shown in Figs. 2 and 3, respectively, and the surface descriptions are given in Tables 1 and 2, respectively.

4 Aspheric and Freeform Lens Performance

To evaluate the performance of the aspheric and freeform surfaces, plots of the relative illumination and transverse ray error on the target surface were produced as shown in Figs. 4 and 5, respectively.

It is clear from examination of the relative illumination and transverse ray error plots that the freeform surface profile of the lens in Fig. 3 is able to best model the ideal surface. In contrast, the relative illumination plot for the aspheric surface varies from about 0.5 to 1 and this is not a good surface match to the ideal surface.

Noteworthy is that the freeform lens plots do not exhibit significant oscillation at the edge of the target. This result can be explained by the absence of higher order terms in the surface description. This lack of oscillation in fact is a useful outcome as many aspheric and freeform surfaces constructed by superposition of higher order polynomials are subject to produce oscillation on the ray behavior. Lens systems that contain lens elements that use higher order aspheric terms are susceptible to creating imaging/nonimaging artifacts when they are slightly misaligned. These artifacts are because the higher order terms may create bumps and dips that under perfect registration cancel but that under a slight misalignment add, thus creating the artifacts.

5 Extended Freeform Solution Lens Example

A third approach to find a solution was using the surface $z_3(r)$ described as

Fig. 4 (a) Relative illumination and (b) transverse ray error in mm of the aspheric lens at the target surface for lens of Fig. 2. Transverse ray error plot scale is ± 3 mm.

Fig. 5 (a) Relative illumination and (b) transverse ray error in mm of the freeform lens at the target surface for lens of Fig. 3. Transverse ray error plot scale is ± 0.3 mm.

Table 3 Coefficients defining the surface profile $z_3(r)$.

$z_3(r)$	a (mm)	b (mm)	$A1$	$A2$	$A3$
$S_1(r)$	10.34386	3.5546671	−0.61768575	0.0036149339	−0.0008041664
	a (mm)	b (mm)	$B1$	$B2$	$B3$
$S_2(r)$	10.299418	10.299418	−0.020104562	−0.01061093	0.00023346129

Fig. 6 (a) Relative illumination and (b) transverse ray error in mm at the target surface for the lens in Table 3. Transverse ray error plot scale is ± 0.3 mm.

$$z_3(r) = A_1 S_1(r) + A_2 S_1^2(r) + A_3 S_1^3(r) + B_1 S_2(r) + B_2 S_2^2(r) + B_3 S_2^3(r). \quad (8)$$

An improved solution was found with the coefficients in Table 3. As with the freeform surface, the maximum ray angle was $\theta_{\max} = 75$ deg, the material was polycarbonate, the lens axial thickness 5 mm, and the axial distance from the source to the target surface 200 mm.

The surface $z_3(r)$ has a polynomial surface $S_2(r)$ using as a base surface a sphere given that $a = b$. The performance illustrated in Fig. 6 and it is the best performer of the surfaces as the root-mean-square of the transverse ray error was the lowest. However, the polynomial nature of the surface led to a very smooth oscillation, albeit smooth, near the edge of the relative illumination plot.

It is worth mentioning that two factors that contribute to complicate or prevent finding solutions are failure of the ray-tracing algorithm to find the ray intersection point on the surface as it becomes steep, and ray total internal reflection. Further, in this type of solution, the angle of incidence can be large near the inflection point of the surface and Fresnel light reflection losses can be significant.

6 Conclusion

We have introduced the concept of a base surface from which an aspheric polynomial surface can be constructed by power expansion of the base term, and from which a freeform surface can also be constructed by superposition of several base surfaces having different parameter values. The surfaces $z_1(r)$ and $z_2(r)$ that we present in this paper are useful for

providing specific illumination distributions. Notably, the freeform surface $z_2(r)$ exhibits little oscillation in the relative illumination or transverse ray error. A surface $z_3(r)$ further illustrates the concept of base surface and also provides useful solutions.

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