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THE IONIZATION AND THERMAL EQUILIBRIUM OF A GAS
EXCITED BY ULTRAVIOLET SYNCHROTRON RADIATION

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THE IONIZATION AND THERMAL EQUILIBRIUM OF A GAS EXCITED
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ABSTRACT

The ionization and thermal balances are considered for a gas that is ionized by a dilute radiation field, taking into account the diffuse ionizing radiation produced by the gas. A number of models are constructed in which the electron temperature and the ionization of the elements H, He, C, N, O, Ne, and Mg are determined for optically thin and optically thick gases ionized by ultraviolet synchrotron radiation under different conditions. Conclusions are then drawn about the general characteristics of ionization by synchrotron radiation. It is shown that, in an optically thin gas, because of the insensitive frequency-dependence of synchrotron radiation each element occupies a number of different stages of ionization at any one point in the gas. It is also shown that in an optically thick gas the heavy elements remain ionized to much greater distances from the source than hydrogen and helium, and that the gas becomes thermally unstable when H and He have become almost completely neutral.

In addition, observations of the emission-line intensities of the Crab Nebula are compared with a model of this object. Considerable disagreement exists between the observed and predicted intensities, and possible reasons for the discrepancy are discussed.

I. INTRODUCTION

Within the past decade the optical continuum emission of several astronomical objects has been attributed, either in part or in whole, to synchrotron radiation. These objects consist of the Crab Nebula (Oort and Walraven 1956; Woltjer 1958), a supernova remnant; M82 (Lynds and Sandage 1963), an exploding galaxy; and the anomalous jet in M87 (Baade 1956; Bless 1962), an elliptical galaxy. Since this type of environment may be common in galaxies and, perhaps, important in the emission processes in quasi-stellar sources (Matthews and Sandage 1963; Greenstein and Schmidt 1964; and Oke 1965) and Seyfert galaxies (Osterbrock and Parker 1965), an understanding of the conditions of an ionized gas under the influence of such a radiation field is of considerable interest. To this end, we will consider the ionization and thermal equilibrium of a gas which is ionized by ultraviolet synchrotron radiation, utilizing the results of previous work that has been done on the physical conditions of gas that is ionized by radiation and applying them to synchrotron objects.

Zanstra (1951) first showed that the ionization of a gas could be obtained without having to determine the diffuse radiation field if the gas were optically thick to the ionizing radiation. In such a case the diffuse radiation, which is caused by recombinations of ions and electrons to the ground state, immediately causes re-ionization of the neutral gas. Hence, the ionization structure of the gas can be determined by neglecting the diffuse radiation if recaptures to only the excited states of the atom are considered in the equilibrium process. This is known as the "on the spot" approximation. Although it may be quite good when applied to hydrogen and helium, this

approximation cannot be used to determine the ionization of any of the heavy elements because their abundances are so much less than that of hydrogen that they do not contribute to the continuous opacity of the gas. For this reason few detailed calculations of the ionization of these elements have been made. Recently, however, in several articles concerning the structure of planetary nebulae, Hummer and Seaton (1963, 1964) have considered this problem. They calculate the diffuse radiation produced by a gas and apply their results to a model nebula composed of H and He. In this paper, we will extend their work by considering the diffuse field as it affects both the ionization and thermal balances of a gas, taking into account the elements H, He, C, N, O, Ne, and Mg.

II. THEORY

a) The Ionization Equilibrium

The ionization equation relating the abundances of the different ions of an element is given by the relation (Aller 1956)

$$\frac{\chi(i+1)}{\chi(i)} = \frac{\int_{\nu_i}^{\infty} \frac{4\pi J_{\nu} a_{i,\nu}(i) d\nu}{h\nu}}{N_e \sum_{n=p}^{\infty} \alpha_n(T_e)}, \quad (1)$$

where $\chi(i)$ is the relative abundance of the element in the i th stage of ionization, J_{ν} is the mean intensity of ionizing radiation, $a_{i,\nu}(i)$ is the ground state photoionization cross section of the ion i , ν_i is the ionization threshold frequency, $\alpha_n(T_e)$ is the recombination coefficient of electrons to the n th level of the i th ion, p is the quantum number

of the lowest orbital shell to which an electron can recombine, T_e is the electron temperature, and N_e is the electron density of the gas, which is taken to be

$$N_e = \frac{N}{1+Y} [\chi(HII) + Y \chi(HeII) + 2Y \chi(HeIII)], \quad (2)$$

where the density of the gas, N , is the sum of the densities of hydrogen and helium nuclei, and Y is the helium to hydrogen abundance ratio by number of atoms. The recombination coefficients computed by Seaton (1959) in the hydrogenic approximation have been used for all elements in the calculations.

b) The Thermal Equilibrium

The electron temperature of a nebula is determined by a balance between the energy gained by the electrons per unit time and the amount lost. The rate at which energy is given to the electrons by the radiation field is (Spitzer 1948)

$$G = \sum_{j=1}^3 N(j) \int_{\nu_j}^{\infty} \frac{4\pi J_{\nu} a_{j,\nu}(j) d\nu}{h\nu} h(\nu - \nu_j), \quad (3)$$

where the summation index represents the ions HI, HeI, and HeII.

The interactions through which the free electrons lose energy to the diffuse radiation field have been thoroughly investigated. We will merely quote the appropriate expressions giving the rate of energy loss per unit volume for each of the important mechanisms.

Collisional Excitation of the Heavy Elements (Hebb and Menzel 1940):

$$L_c = \sum_{i, \beta} 8.63 \times 10^{-6} N_e N_i \frac{\Omega_{i\beta} \chi_{i\beta}}{\omega T_e^{1/2}} e^{-\chi_{i\beta}/kT_e}, \quad (4)$$

where the summation is performed over all low-lying levels of the ions of the heavy elements. A list of data for the collisional transitions occurring in the heavy elements that were used in this work is given in Table 1.

Free-Bound and Free-Free Radiation (Seaton 1959, 1960):

The energy lost by electrons upon recombination and in free-free encounters with ions may be expressed as

$$L_f = \sum_{j=1}^3 N_e N_j k T_e \left[\beta_{ff}(T_e) + \sum_{n=1}^{\infty} \beta_n(T_e) \right], \quad (5)$$

where the coefficients $\beta_{ff}(T_e)$ and $\beta_n(T_e)$ have been determined by Seaton for hydrogenic ions, and electron recaptures with the ions HII, HeII, and HeIII are considered.

Collisional Excitation of Hydrogen (Hummer 1963):

Because of its larger cross section and smaller threshold, we consider only the 1s - 2p transition. The energy loss can be written as

$$L_h = N_e N(\text{HI}) \chi_{2,1} g(1s-2p), \quad (6)$$

where $\chi_{21} = 10.19 \text{ eV}$, and the collision coefficient $q(1s-2p)$ for the reaction has been evaluated by Hummer.

The equilibrium electron temperature at a point in the gas is that value of T_e for which the equation of thermal equilibrium,

$$G = L \equiv L_c + L_f + L_h, \quad (7)$$

is satisfied.

c) The Radiation Field

We take hydrogen and helium to be the main contributors to the opacity of the gas, hence, the volume absorption coefficient is

$$K_\nu = N(\text{HI}) a_{i,\nu}(\text{HI}) + N(\text{HeI}) a_{i,\nu}(\text{HeI}) + N(\text{HeII}) a_{i,\nu}(\text{HeII}) \quad (8)$$

The diffuse ionizing radiation emitted by the gas is created almost entirely by the recaptures of electrons by hydrogen and helium. This recombination spectrum consists of the continua produced by recombinations to the ground states of HI, HeI, and HeII, the Balmer continuum of HeII, and the HeI and HeII $1s - 2p$ resonance lines. Since the continuous emissivity of the gas is simply the energy radiated by electrons when they recombine, the continuous emission coefficient of radiation caused by electron recaptures to the m th level of one of the above ions i is

$$j_\nu^c = \frac{N_e N_i \alpha_m^\nu(T_e)}{4\pi} h\nu, \quad (9)$$

where it can be shown from the definition of $\alpha_n(T_e)$ that (Hummer and Seaton 1963)

$$\alpha_n^{\nu}(T_e) = 4\pi \frac{\omega_i}{\omega_{i+1}} e^{X_n/kT_e} \left(\frac{h^2}{2\pi m_e kT_e} \right)^{3/2} \left(\frac{\nu}{c} \right)^2 e^{-\frac{h\nu}{kT_e}} a_{n,\nu}(i), \quad (10)$$

where X_n is the ionization potential of the n th level. In the same manner, the amount of radiation emitted in each of the HeI and HeII 1s - 2p resonance lines is given by

$$j_{\nu}^{\ell} = \frac{N_e N_i \sum_{n=2}^{\infty} \alpha_n(T_e)}{4\pi} \eta h \nu_{\alpha}, \quad (11)$$

where ν_{α} is the frequency of the line. The factor η represents the fraction of all those recombinations to states $2 \leq n < \infty$ that result in the production of a resonance-line photon. Consider the situation where the gas is optically thick in the ground state continuum of helium. Because the HeII ion is hydrogenic, all recaptures will result in the eventual population of the $n=2$ level (Zanstra 1927). Of these, approximately 1/3 of the ions will populate the 2^2S state (Burgess 1958) and subsequently give rise to two-photon emission. The remainder of the ions belong to the 2^2P state, and cause Lyman- α radiation. Thus, $\eta = 0.66$ for HeII. In the case of HeI, about 3/4 of the recombinations to excited states go to triplet levels, and these eventually cascade to the 2^3S state. The 2^3S state is depopulated by several processes, depending upon the density of the gas. At higher densities

the most important process is the $2^3S - 2^1S$ collisional transition, while at lower densities two-photon emission depopulates 2^3S . Thus, in either case no recaptures to triplet levels ever result in resonance-line radiation. Of the remaining recaptures that occur to singlet levels, about 1/3 cascade to state 2^1S and 2/3 to state 2^1P , producing the resonance line. Consequently, the efficiency of resonance-line production in HeI is $\eta = 0.17$.

We will consider synchrotron radiation as being incident upon the gas. The theory of this form of radiation has been worked out by Oort and Walraven (1956) and Westfold (1959), among others. Results show that the emergent intensity of radiation from a group of relativistic electrons having a power-law energy distribution with a high energy cut-off at the frequency ν_2 is expressed by the relation

$$I_\nu = D \left(\frac{\nu}{\nu_0} \right)^{-\gamma} \frac{G(\nu/\nu_2)}{G(0)}, \quad (12)$$

where D and γ are constants depending upon conditions in the source, and the function $G(\nu/\nu_2)$ has been tabulated for different frequencies by Westfold (1959) and Bless (1966). In order to give D the units of intensity, equation (12) has been written in terms of an arbitrary frequency, ν_0 , which has been chosen to be 8.82×10^{14} c/s (3400Å). For all frequencies $\nu \ll \nu_2$, the intensity has a power-law dependence upon the frequency, whereas for $\nu \gg \nu_2$, the intensity decreases exponentially with increasing frequency.

d) Cross Sections

The absorption cross section of HI (and HeII) has been taken from

Menzel and Pekeris (1935), taking into account the variation of the Gaunt factor. The cross section used for HeI is the one derived by Huang (1948), as experimentally verified by Baker, Bedo, and Tombouliau (1961). The cross sections for all ions with outer shell $2p^q$ ground state configurations have been taken from Seaton (1958), who fits the absorption coefficients to the formula

$$a_{\nu}(i) = 10^{-18} A \times B \left[\alpha \left(\nu/\nu_i \right)^{-s} + (1-\alpha) \left(\nu/\nu_i \right)^{-s-1} \right] \text{ cm}^2, \quad (13)$$

where A , B , and α are constants, depending upon the ion, that Seaton evaluates, and ν_i is the absorption limit of the ion. The ions included in this category are CI-II, NI-III, OI-IV, NeI-V, and Mg III-V.

Unfortunately, little information exists about absorption cross sections for the higher stages of ionization of the above elements. However, since these ions have $2s^2$ and $2s$ configurations, which have relatively high ionization potentials, they do not sensitively affect the ionization equilibrium of the gas. For our purposes it is sufficient to fit such cross sections with the formula $a_{\nu}(i) = B (\nu/\nu_i)^{-2} \text{ cm}^2$, where B is the value of the cross section at the absorption head. Now, for some of the ions in question, the constant B is undetermined. Hence, on the basis of data given by Ditchburn and Öpik (1962), we have assumed for all of the ions in an isoelectronic sequence that the absorption coefficient at the absorption limit varies inversely as the effective nuclear charge, $B = K/(Z + 1)$, where K is a constant characteristic of all the ions in a given isoelectronic sequence, and can be determined from the least ionized members of the sequence.

Thus, for all ions with an outer $2s^2$ configuration (CIII, NIV, and OV), the constant β is determined from BeI whose threshold cross section has been measured to be $K = 8.2 \times 10^{-18} \text{ cm}^2$ (Ditchburn and Öpik 1962). Similarly, all $2s$ ions (CIV and NV) have been assumed to have $K = 2.5 \times 10^{-18} \text{ cm}^2$, as determined from LiI.

The photoionization cross sections of the lower stages of ionization of magnesium fall into none of the previous categories, and had to be considered separately. The experimental results of Ditchburn and Marr (1953) were used for MgI in the region $1620\text{\AA} > \lambda > 1450\text{\AA}$. Below this region the work of Ditchburn and Marr was extrapolated using the form of the cross section derived by Burgess and Seaton (1960). Calculations for the absorption of MgII exist by Biermann and Lübeck (1949), who evaluated the cross section at eight wavelengths. A smooth curve was fit to their values to obtain results over the entire absorbing region.

III. CONSTRUCTION OF THE MODELS

The relative importance of the radiative processes that determine the conditions in a gas depend upon the optical depth of the gas. In one limit, the gas is optically thin to radiation in the hydrogen Lyman lines (Menzel and Baker Case A). Since hydrogen is the primary contributor to the continuous opacity of the gas, the probability that any ion absorbs an ionizing photon is quite small. Thus, the diffuse ionizing radiation caused by recombinations of electrons to the ground states of ions that have been photoionized is negligible in comparison with the incident radiation causing the ionization, and may be neglected. There is no radiative transfer problem

to be solved in this case, since the presence of the gas does not perturb the radiation field. In this situation, it is convenient to assume the synchrotron source completely surrounds the gas, and emits isotropically.

At the other extreme is a gas which is optically thick to Lyman continuum radiation (Case B). Such a gas completely absorbs the incident ionizing photons, and necessitates solving the transfer equation throughout the gas, including the contribution of the diffuse ionizing radiation field, in order to determine the mean intensity of radiation everywhere in the gas. Let us consider the gas to be optically thick to radiation over a distance scale that is less than the dimensions of the gas. The fact that the dimensions of synchrotron emitters are comparable to the sizes of the gases they excite means that most of the incident radiation at a region within a gas has entered the gas through the surface nearest that point. On this scale, the ionization of the gas may be determined by assuming the gas and the source to have a plane-parallel configuration.

a) Optically Thin Models

Because the incident radiation field is isotropic and surrounds the gas, the mean intensity of radiation throughout the optically thin gas is $J_\nu = I_\nu^\circ$, where I_ν° is given by equation (12). The ionization equation becomes

$$\frac{\chi(i+1)}{\chi(i)} = \frac{4\pi(1+\gamma)\nu_0^\gamma \int_{\nu_i}^{\infty} \nu^{-\gamma-1} a_{i,\nu}(i) G(\nu/\nu_2) d\nu}{G(0)h[\chi(HII) + \gamma\chi(HeII) + 2\gamma\chi(HeIII)] \sum_{n=p}^{\infty} \alpha_n(T_e)} \times \frac{D}{N}. \quad (14)$$

The simultaneous solution of this equation for all ions in the gas along

with the thermal equation determines the statistical equilibria of the elements. Rather than attempt a direct simultaneous solution of the ionization and temperature equations, an iterative technique has been developed. The sole dependence of equation (14) upon the electron temperature is in the recombination coefficient, which is insensitive to T_e . Thus a good preliminary knowledge of the ionization is obtained by initially assuming a certain temperature - 10^4 °K, say - and solving equation (14) for the $\chi(i)$. The abundances of hydrogen and helium ions are obtained first because they determine the electron density. Some characteristic values of $\chi(\text{HII})$, $\chi(\text{HeII})$, and $\chi(\text{HeIII})$ are initially chosen and substituted into the right side of equation (14), which is then solved to give a preliminary knowledge of the abundance of the H and He ions. This information enables the ionization of the remaining elements in the gas to be calculated straightforwardly.

The electron temperature of the gas is determined from equation (7) once the ionization equilibrium has been calculated. The complexity of the dependence of the heating and cooling rates on the temperature does not allow a direct solution to be made. Instead, an electronic computer calculates the rates for various values of T_e , searching for that particular value which will result in equality of the heating and cooling rates. After the electron temperature has been determined, the iterative process has been completed, and need only be repeated. The preliminary values of the $\chi(i)$ and T_e are substituted back into the ionization equation, from which new results for the ionization of the elements are obtained. These, in turn, are used to recalculate the temperature. The entire procedure is repeated until identical results are obtained after one whole cycle.

Several models have been constructed for various values of the free parameters, and appear in Figures 1-4. Each model is characterized by a certain spectral distribution of the ionizing synchrotron radiation, and the parameters used are the following: (1) $\gamma = 0.50$, $\nu_2 = \infty$; (2) $\gamma = 1.00$, $\nu_2 = \infty$; (3) $\gamma = 0.33$, $\nu_2 = 6.00 \times 10^{14}$ c/s. The relative abundances of all the elements except helium are taken to be those given by Aller (1961) for interstellar space, where the logarithmic abundances, by number, for H, C, N, O, Ne, and Mg are 12.00, 8.60, 8.05, 8.95, 8.70, and 7.40 respectively. The helium to hydrogen ratio $\text{He}/\text{H} = 0.15$ is used, which differs slightly from Aller.

Aside from the relative abundances of the elements the only other parameters that must be specified for each model are the intensity of the radiation field, which is given by the constant $\mathcal{D}$, and the density of the gas N . Because the variables $\mathcal{D}$ and N appear in the ionization equation in the ratio $\mathcal{D}/N$, we can account for them both in the form of this one parameter, which is a measure of ionization conditions in the gas. The variable $\frac{\mathcal{D}}{N}$ is proportional to the photoionization rate, while the density determines the relative importance of the recombination rate. Hence, when $\mathcal{D}/N$ is large the gas is highly ionized; when it is small the reverse is true, and there is little ionization. In Figures 1-3 are plotted for each of the models the relative abundances of the ions of each element as a function of $\mathcal{D}/N$, which is varied such that conditions range from those of low to those of high ionization. The electron temperatures of each of the models are given in Figure 4.

b) Optically Thick Models

It has previously been shown that the ionization and temperature of the optically thick gas may be calculated by assuming the gas to be plane-parallel. The gas may be depicted as consisting of a large number of parallel slabs in which the conditions within any given increment are essentially uniform. Suppose now that the gas is ionized, and is emitting ionizing radiation upon recombination. The intensity of the diffuse radiation I_ν^d at a point in the gas from the emission at an angle $\cos^{-1}\mu$ to the normal, from any one slab is

$$I_\nu^d = S_\nu(t_\nu) e^{-t_\nu/\mu} [1 - e^{-T_\nu/\mu}], \quad (15)$$

where t_ν is the optical depth measured from the point to the slab, T_ν is the optical thickness of the slab, and $S_\nu(t_\nu)$ is the source function of the emitting gas, which is constant within the interval. The mean intensity of radiation from the slab at the point is easily shown to be

$$J_\nu^d = \frac{S_\nu(t_\nu)}{2} [E_2(t_\nu) - E_2(t_\nu + T_\nu)], \quad (16)$$

where E_2 is the second exponential integral function.

The mean intensity of incident radiation from the synchrotron source can be obtained in exactly the same manner used above by considering the plane-parallel source to be emitting radiation isotropically, I_ν^o . At a point in the gas where the optical depth, as measured from the surface, is τ_ν , the mean intensity is then

$$J_\nu^0 = \frac{I_\nu^0}{2} E_2(\tau_\nu). \quad (17)$$

The mean intensity at a point from all sources of radiation may now be found from equations (16) and (17), where equation (16) is summed over all slabs in the gas. This enables the ionization equation for the optically thick gas to be written

$$\frac{\chi(i+1)}{\chi(i)} = \frac{2\pi(1+\gamma) \int_{\nu_i}^{\infty} \left\{ I_\nu^0 E_2(\tau_\nu) + \sum S_\nu(t_\nu) [E_2(t_\nu) - E_2(t_\nu + \tau_\nu)] \right\} a_{i,\nu}(i) \frac{d\nu}{\nu}}{N h [\chi(\text{HII}) + \gamma \chi(\text{HeII}) + 2\gamma \chi(\text{HeIII})] \sum_{n=p}^{\infty} \alpha_n(T_e)} \quad (18)$$

The solution to this equation must be obtained by iteration because the degree of ionization is dependent upon the source function S_ν , i.e., the diffuse radiation, of the material, which can not be determined unless the ionization equilibrium is already known. This problem may be circumvented by initially calculating the ionization of H and He using the "on the spot" approximation, which equates the number of ionizations due to the incident radiation to the number of electron recaptures to the excited states of an ion, or

$$\frac{\chi(i+1)}{\chi(i)} = \frac{2\pi(1+\gamma) \int_{\nu_i}^{\infty} I_\nu^0 E_2(\tau_\nu) a_{i,\nu}(i) \frac{d\nu}{\nu}}{N h [\chi(\text{HII}) + \gamma \chi(\text{HeII}) + 2\gamma \chi(\text{HeIII})] \sum_{n=2}^{\infty} \alpha_n} \quad (19)$$

This equation is similar to the ionization equation of an optically thin gas and is solved in much the same manner except that it is repeated for each interval in the gas. Starting at the boundary of the gas, where $\tau_\nu = 0$, initial values for $\chi(\text{HII})$, $\chi(\text{HeII})$, $\chi(\text{HeIII})$, and the electron temperature are assumed and substituted into the right side of equation (19), which is solved directly for the $\chi(i)$ of H and He. The values determined

at the surface of the gas are assumed to apply to the first slab, from which the optical depth of the interval is calculated. Once the optical thickness of the first slab is known, the radiation field can be determined at the surface of the next interval in the gas. The values of the ionization obtained for the first slab are used in the right side of the ionization equation, which is then solved for the ionization at the next point in the gas. This procedure is repeated from interval to interval until the gas becomes neutral.

The calculations of the ionization of H and He using the "on the spot" approximation enable the diffuse emission from each increment in the gas to be determined. The mean intensity of radiation can then be obtained at each point in the gas from equations (16) and (17). This information is used in equation (18) to yield the ionization equilibrium of all the elements in the gas, including a redetermination of the ionization of H and He.

Up to this point the nebula has been assumed to be isothermal. No temperature determination is possible until the ionization concentration of the heavy elements is known. Once this is obtained, the equation of thermal balance is solved numerically for the electron temperature of each increment. When this procedure is finished one iterative cycle has been completed. Approximate values for the temperature and ion abundances are known. They are then substituted back into the ionization and temperature equations for their redetermination at all points in the gas. This iteration process continues until a convergence is obtained.

Originally it was planned to carry the calculations as far into the gas as there was any ionization present. However, in the outer regions

of the gas where hydrogen and helium are largely neutral, the ionization conditions in the gas led to a physical instability of the temperature. At this point the nebula could no longer achieve a thermal equilibrium, so the calculations were terminated. The development of the instability will be dealt with more fully in the next section.

The results of two optically thick models are shown in Figures 5-8, which give the abundances of all the ions and the electron temperature for each model as a function of distance into the gas. The parameters of the first optically thick model (Model 4) are $\gamma = 1.10$, $\nu_2 = \infty$, $N = 10^3 \text{ cm}^{-3}$, $\mathcal{D} = 6.32 \times 10^{-17} \text{ erg cm}^{-2} \text{ sec}^{-1} (\text{c/s})^{-1} \text{ ster}^{-1}$, $\text{He/H} = 0.45$, and were chosen to correspond to conditions in the Crab Nebula. The parameters of the remaining model (Model 5) were arbitrarily picked to be: $\gamma = 2.00$, $\nu_2 = \infty$, $N = 10^3 \text{ cm}^{-3}$, $\mathcal{D} = 4.91 \times 10^{-16} \text{ erg cm}^{-2} \text{ sec}^{-1} (\text{c/s})^{-1} \text{ ster}^{-1}$ and $\text{He/H} = 0.15$.

IV. RESULTS

Except when the critical frequency of radiation ν_2 falls near the visible region of the spectrum, the spectral distribution of ultraviolet synchrotron radiation differs considerably from that of black body radiation above the Lyman limit of hydrogen. Hence, the ionization structure of the gases we are considering is quite unlike the classically studied ionization from a black body.

a) Optically Thin Models

Perhaps the most striking features of the optically thin models (where $\nu_2 = \infty$) are the simultaneous existence of many different stages

of ionization for a given element, and the presence of neutral elements in an otherwise highly ionized gas. Both of these effects may be directly attributed to the relatively insensitive frequency-dependence of synchrotron emission. The number of photons that can ionize neutral atoms is not significantly greater than the number that can ionize elements in the third and fourth stages of ionization. Consequently, the sizes of the photoionization cross sections and the recombination coefficients play an important role in the statistical equilibrium. The net result in the cases of HI, HeI, and MgI, which all have very small absorption cross sections compared with the other elements at high frequencies, is that these elements become neutral before any of the other elements in optically thin gases at low intensities of the radiation field. This occurs to MgI in spite of the fact that its recapture cross section is the lowest of all of the ions, and, since it also has the lowest ionization potential of any ion (7.64 eV), the intensity of radiation which ionizes MgI is therefore larger than that for any other ion.

Model 3, which, with a cut-off frequency of 6.0×10^{14} c/s, has the same far-ultraviolet spectral distribution as a black body of temperature 28800°K, possesses completely different characteristics than the first two models. The exponential decrease in intensity at high frequencies for this model produces essentially one stage of ionization for each of the elements over a wide range in absolute intensities of the radiation field. In addition, the absence of very high energy photons in the radiation field leads to relatively low electron temperatures for this model, in contrast to the temperatures encountered in Models 1 and 2.

b) Optically Thick Models

Models 4 and 5 show the statistical equilibrium of an optically thick plane-parallel gas with radiation incident upon one side. The results illustrate the stratification in ionization typical of such a situation, with the highest degree of ionization occurring near the synchrotron source and diminishing with distance from it. In addition, however, several interesting features appear which deserve comment:

(1) The sizes of the transition zones in which the ions H^+ and He^+ become neutral are relatively large. One of the established results of the investigations of the ionization equilibrium of circumstellar gas is that the transition between the ionized and neutral states of H and He occurs in a very short distance compared to the sizes of the ionized regions (Strömgren 1939; Hummer and Seaton 1964). This is due to the exponential frequency-dependence of the stellar continuum in the Wien approximation. For all but the highest stellar temperatures, the preponderance of the radiation capable of causing photoionization of the gas is emitted near the ionization limits, where the opacity of the ions is largest and the probability of ionization the highest. When a distance into the gas is reached where the number of photons near the ionization limit is sharply reduced, the ionization cannot be sustained, and the ions suddenly become neutral. Such is not the case for synchrotron radiation. The power-law spectrum contains sufficient far-ultraviolet energy with respect to frequencies near the ionization limit so that a significant number of the photoionizations of an ion at any point in the gas originate from the higher energy photons. As a consequence, when the incident radiation near the ionization limit has been absorbed, a considerable

amount of high-frequency radiation remains to sustain an appreciable fraction of the ionization.

(2) The size of the He^+ region is larger than the H^+ region of the gas. The relatively insensitive frequency-dependence of the power-law spectrum of synchrotron radiation results in a comparable number of photons being emitted between the H I and He I ionization limits and above the He I ionization limit. The latter radiation is capable of ionizing hydrogen, however, at these frequencies helium is a more effective absorber than hydrogen, in spite of its lesser abundance, because of its greater absorption cross section. As a result, the more abundant hydrogen depletes its supply of ionizing photons before helium and becomes predominantly neutral.

(3) The heavy elements remain ionized to much greater distances than H or He . Even when the latter are mostly neutral, the elements C , N , O , Ne , and Mg are all predominantly singly ionized. Several factors contribute to this effect. First, because the absorption coefficients of the neutral heavy elements vary approximately as ν^{-1} , rather than ν^{-3} , they are much larger than the cross sections of H and He at high frequencies, and cause effective absorption of the synchrotron radiation in this spectral region. Furthermore, the entire first electron shell and parts of the second shell are filled for these ions, giving rise to smaller recombination coefficients. At distances beyond the H^+ and He^+ regions the gas is deprived of most of the free electrons with which the ions of the heavier elements can recombine. Hence, the statistical equilibrium tends to favor ionization under these conditions since radiation from around $100\text{\AA}$ can easily support ionization of the heavy elements as long as the electron density is very low. In the cases

of carbon and magnesium, which have ionization potentials less than 13.6 eV, the radiation near the ionization limits of these ions is not absorbed by H and He, therefore the ionizing radiation for these elements is strong everywhere in the gas, and these two elements do not become neutral.

(4) When the H and He are almost completely neutral, the gas becomes thermally unstable. In order to understand this phenomenon the functional dependence of the heating and cooling rates upon the temperature must be examined. For the sake of simplicity, we will consider the heating of the gas from hydrogen alone. The results may be generalized to include the ions of helium since the physical processes involved are identical. According to expression (3) the kinetic energy expended on the electrons by the radiation field at a point in the gas is

$$G = N(HI) \int_{\nu_1}^{\infty} \frac{4\pi J_{\nu} a_{1,\nu}(HI) d\nu}{h\nu} h(\nu - \nu_1). \quad (20)$$

The mean intensity of radiation J_{ν} is determined primarily by the conditions in the gas between the source and the point in question, therefore the only quantity in this equation which has any local temperature-dependence is the density of neutral hydrogen. It is clear from the ionization equation of hydrogen that the fraction of hydrogen which is neutral at a point is almost entirely independent of T_e in the limit of very low ionization. In this limit, we take the heating rate G to be independent of the electron temperature.

The temperature-dependence of the cooling rate is a complicated one, as seen from equations (4) through (6). We have chosen from Model 5 a typical point in the gas and plotted the heating and cooling rates in

Figure 9. The cooling rate has been calculated using the abundances obtained from the ionization equilibrium for that point, which is at a distance from the inner surface of the gas of 0.37 pc. At this distance, the H and He are largely neutral, so the heating curve has been taken to be independent of the temperature. The equilibrium value of the electron temperature is seen to be about 7000°K. At points in the gas nearer the source the temperature is considerably higher, however, at points further out in the gas the heating rate decreases faster than the cooling rate, resulting in lower temperatures (see Figure 8). It is the migration of the solution of the equation of thermal balance to the left along the cooling curve that eventually results in the gas becoming thermally unstable. Weymann (1960) has investigated the stability of a compressible gas which is in thermal equilibrium at some temperature T , and whose heating rate is fixed. Assuming the gas to have an arbitrary cooling rate, he considers the effect of short wavelength pressure and thermal disturbances upon the gas. He finds the gas is unstable to these perturbations for temperatures where $\alpha \equiv d \log L / d \log T < 1$, where L is the rate of cooling, defined in equation (7), and plotted in Figure 9. Such a solution to the thermal equilibrium is unstable because the density and temperature of the gas change in such a way from the effect of the perturbation that the pressure equilibrium of the gas cannot be maintained unless the perturbation grows. Thus, the gas will either contract and become cooler, or expand and get hotter until an equilibrium situation is again reached.

If this stability criterion is now applied to the solutions of the

thermal equilibrium at all points in the nebula, it is seen that the gas is stable out to a distance where the heating curve intersects the cooling curve where $\alpha < 1$ (that portion of the curve lying between the two dot marks in Figure 9). What happens to the gas beyond this point cannot be determined from the theory used in this work inasmuch as the pressure equilibrium of the gas, which we have neglected, would have to be taken into account in order to describe the state of the gas under these conditions. Consequently, the calculations extend only out to the distance where the instability occurs.

c) The Crab Nebula

One of the purposes of this investigation is to compare predicted emission-line intensities obtained from the models with observations of synchrotron sources. The Crab Nebula (NGC 1952) is the brightest such source, and the only one for which any detailed photometry has been done. Woltjer (1958) has studied the spectra obtained by Mayall of a number of the filaments and has determined the relative intensities of some of the more important lines. Table 2, which is reproduced from his paper, gives the mean emission-line intensities relative to $[\text{OII}] \lambda 3727$ (=100).

The parameters of the synchrotron emission from the amorphous mass have been fairly well established. The radiation in the X-ray region appears to be an extension of the visual synchrotron emission, and it has been suggested by Woltjer (1964) that the far-ultraviolet spectral distribution can be represented by the spectral index $\gamma = 1.10$. Measurements of the continuum flux by O'Dell (1962) show that $\log F_{\nu} = -25.50 \text{ watt m}^{-2} (\text{c/s})^{-1}$ at $3210\text{\AA}$, where a correction has been made for an interstellar absorption of $A_{\nu} = 1.59$. This value of the extinction has been chosen because it gives the best fit to the rocket observations when the visual spectrum is extrapolated down into the far-ultraviolet region. If the assumption is made that the synchrotron

emission is isotropic and if the average angular radius of the amorphous mass is taken to be $1/4$, it is straightforward to compute that the emergent intensity of synchrotron radiation from NGC 1952 at $\nu_0 = 8.82 \times 10^{14}$ c/s is $\mathcal{D} = 6.32 \times 10^{-17}$ erg cm⁻² sec⁻¹ (c/s)⁻¹ ster⁻¹. We shall consider this to be a representative value of the radiation field that ionizes the filaments. The average density in the filaments has been determined by Osterbrock (1957) to be $N_e = 1000$ cm⁻³, where the typical diameter of the brighter filaments is 0.01 pc; however, there is some variation in both of these quantities. These data were used to construct Model 5. In addition, the helium rich abundance suggested by Woltjer (1958), where He/H = 0.45 by number, was used, and the gas was assumed to have the normal cosmic heavy element abundance determined by Aller (1961).

In comparing observations of the Crab Nebula with calculations based on the model we must account for the fact that the filaments do not absorb all of the ionizing radiation that is incident upon them. That is, they are structures that are density-bounded and not ionization-bounded. In our calculations, we have assumed the gas to be arbitrarily thick, and then have determined the ionization out to a point where the ionizing radiation is completely absorbed. Thus, in applying the ionization conditions and the line intensities we predict to the Crab Nebula, we will be concerned with the results of the models only out to a distance which is equal to the mean diameter of the filaments, about 0.01 pc.

The difficulty in comparing any theoretically computed line intensities with the observed ones is that the line strengths ultimately depend upon the

relative abundances of the elements, and these are not independently known. To bypass this uncertainty, the only meaningful comparisons that can be made are those between lines where the abundances of the elements cancel out, i.e., lines of the same element in different stages of ionization. Woltjer's observations include two such sets of lines: λ 3727 of OII and λ 5007 of OIII, and λ 4471 of HeI and λ 4686 of HeII. It is the intensity ratios of these two sets of lines that we will attempt to verify.

The model is optically thin in all of the observed emission lines, therefore the intensity of each line is obtained by integrating the emission coefficient along the line of sight through the gas. The volume emission coefficients for the lines of the HI Balmer and HeII Paschen series have been given by Pengelly (1964) in terms of the ion and electron densities for different values of the electron temperature. The same information is given by Seaton (1960) for the HeI recombination lines. We have fitted the expressions for the emissivity of the lines to interpolation formulae which are valid for all temperatures and have then used the results of the ionization and temperature of the model to calculate the line intensities. The same procedure has been used to determine the intensities of the forbidden lines from their emission coefficients (Aller 1956). The results of these calculations for Model 5 are shown in Table 3 for a number of lines, each of which is given for several widths, R , of the gas.

The assumed He/H abundance ratio of the model may be checked by comparing the predicted intensity ratios of the hydrogen and helium lines to those observed by Woltjer. Our model gives $H\beta$ /HeI λ 4471 = 5.0, which agrees well with Woltjer's value of 5.4. The intensity ratio of $H\beta$ /HeII λ 4686,

which is 2.8 in the model, is observed to be 1.4. The disagreement of a factor of 2 is not surprising because the observed intensity of λ 4686 is made uncertain by a blend with the $[\text{FeIII}]$ λ 4658 line. The possibility that the H and He line ratios are not sensitive to their relative abundances was checked by constructing a model identical to Model 5, with the exception that the He/H abundance was taken to be 0.15. The resulting intensity ratios of both of the above lines were decreased by a factor of approximately 3, which corresponds to the difference in abundance ratios between the two models, even though the ionization conditions were somewhat different. This leads us to conclude that the $\text{H}\beta$ / λ 4471 ratio is not sensitive to the ionization conditions in the gas, but accurately reflects the relative abundances of these two elements. On the basis of Model 5, then, it appears that the helium to hydrogen abundance in the filaments is well represented by the value $\text{He}/\text{H} = 0.45$.

In order to compare the ionization of the model with actual conditions within the Crab Nebula, we now consider the intensity ratios of the two oxygen lines and the two helium lines. The predicted ratio of $\text{HeII } \lambda$ 4686/ $\text{HeI } \lambda$ 4471 = 1.8 is somewhat below the observed value of 3.9, however this difference may again be due to the uncertain intensity of λ 4686. On the other hand, the oxygen line intensity ratio reveals a discrepancy that is difficult to explain. Woltjer observes the ratio $(\lambda$ 5007 + λ 4959)/ λ 3727 = 1.5, from which it can be deduced that the intensities of λ 5007 and λ 3727 should be approximately equal, since 3/4 of the transitions from the $^1\text{D}_2$ state of OIII go to $^3\text{P}_2$. The calculations for Model 5 predict a ratio of λ 5007/ λ 3727 = 5.3. Thus, the observations

call for similar concentrations of OII and OIII, whereas the model shows the OIII to be approximately six times more abundant than the OII. There is no doubt that the attempt to represent the chaotic structure of the filaments by a plane-parallel geometry is at best a crude approximation, and in that sense disagreement is to be expected. Nevertheless, it seems unlikely that this would affect the general nature of the ionization of the gas in such a pronounced fashion. The discrepancy between the observed and predicted oxygen line intensities is probably also partly due to the choice of incorrect values for the input parameters necessary for the calculations. For instance, the density of the filaments may be higher than we have assumed, or perhaps the synchrotron continuum is less intense than measurements indicate. Either of these effects would tend to lower the degree of ionization, although to what extent could not be determined without the construction of more models. Alternatively, it is conceivable that the mean size of the filaments is larger than we have taken it to be. This would decrease the $[OIII] / [OII]$ line ratio because at larger optical depths the oxygen would predominantly occupy the second stage of ionization. On the other hand, the relative intensities listed in Table 3 for different sizes of the filaments show that this effect is small.

Another plausible explanation for the discrepancy in ionization conditions in the Crab Nebula and the model involves our assumptions concerning the far-ultraviolet ionizing radiation from the synchrotron source. It has been assumed that the synchrotron continuum at ionizing frequencies

is represented by an extrapolation of the visual continuum down into the X-ray region so as to obtain a fit with rocket measurements. Quite possibly, this gives a poor representation of the spectral distribution in this region. It may be that the synchrotron radiation drops off sharply at some point before the OII ionization threshold is reached, giving rise to less ionization of OII than we predict. At the same time, the radiation producing the X-ray radiation may come from an entirely different source. Greenstein (1964) has postulated the existence of multiple sources of synchrotron radiation for such objects as M82 and certain quasi-stellar sources which show NeV in emission, yet which on the basis of their optical continua do not appear to possess sufficient ultraviolet energy to produce such high ionization. Perhaps the ultraviolet synchrotron radiation from the Crab Nebula is complicated in a similar manner.

In conclusion, it appears that a more thorough knowledge of the ionizing radiation field and a more sophisticated geometrical representation of the Crab Nebula are necessary before ionization conditions can be satisfactorily predicted for this object. It should also be noted that it would be very desirable to have more complete information on the intensities of lines originating from different ions of the same element with which to compare calculations. The emission lines of neon afford such an opportunity since Woltjer has already measured the intensity of $[\text{NeIII}] \lambda 3869$. Future observations of either $[\text{NeV}] \lambda 3426$ or the $[\text{NeII}]$ line at 12.79μ could be used with $\lambda 3869$ to serve this purpose.

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TABLE 1

COLLISION PARAMETERS FOR LOW-LYING LEVELS OF THE HEAVY ELEMENTS

Ion	Transition	χ (eV)	Ω/ω	Ref*	Ion	Transition	χ (eV)	Ω/ω	Ref*
CII	$2p^2 P_{1/2} - 2p^2 P_{3/2}$	0.0079	0.64	1	OIII	$2s^2 2p^3 P - 2s2p^3 S$	7.48	0.056	2
CII	$2s^2 2p^2 P - 2s2p^2 P$	5.33	0.20	2	OIV	$2p^2 P_{1/2} - 2p^2 P_{3/2}$	0.0479	0.45	4
CII	$2s^2 2p^2 P - 2s2p^2 D$	9.29	1.64	2	OIV	$2s^2 2p^2 P - 2s2p^2 P$	8.84	0.20	2
CIII	$2s^2 1S - 2s2p^3 P$	6.49	0.30	2	NeII	$2p^5 P_{3/2} - 2p^5 P_{1/2}$	0.0969	0.075	4
CIV	$2s^2 S - 2p^2 P$	8.00	1.5	2	NeIII	$2p^4 P_2 - 2p^4 P_1$	0.0806	0.076	4
NII	$2p^2 P_0 - 2p^2 P_1$	0.0061	0.375	3	NeIII	$2p^4 P_2 - 2p^4 P_0$	0.1149	0.02	4
NII	$2p^2 P_0 - 2p^2 P_2$	0.0163	0.30	4	NeIII	$2p^4 P - 2p^4 D$	3.20	0.0844	1
NII	$2p^2 P - 2p^2 D$	1.90	0.241	1	NeIII	$2p^4 P - 2p^4 S$	6.91	0.0086	1
NII	$2p^2 P - 2p^2 S$	4.05	0.0344	1	NeIV	$2p^3 P - 2p^3 D$	5.08	0.17	1
NII	$2s^2 2p^3 P - 2s2p^3 S$	5.85	0.056	2	NeIV	$2p^3 P - 2p^3 P$	7.71	0.056	1
NIII	$2p^2 P_{1/2} - 2p^2 P_{3/2}$	0.0216	0.40	4	NeV	$2p^2 P_0 - 2p^2 P_1$	0.0513	0.18	4
NIII	$2s^2 2p^2 P - 2s2p^2 P$	7.10	0.20	2	NeV	$2p^2 P_0 - 2p^2 P_2$	0.1378	0.22	4
NIV	$2s^2 1S - 2s2p^3 P$	8.33	0.30	2	NeV	$2p^2 P - 2p^2 D$	3.76	0.0933	1
NV	$2s^2 S - 2p^2 P$	9.98	0.97	2	NeV	$2p^2 P - 2p^2 S$	7.92	0.0177	1
OII	$2p^3 P - 2p^3 D$	3.32	0.32	1	MgII	$3s^2 S - 3p^2 P$	4.43	10.5	6

TABLE 1 (continued)

Ion	Transition	χ (eV)	Ω/ω	Ref*	Ion	Transition	χ (eV)	Ω/ω	Ref*
OII	$2p^3 4s - 2p^3 2p$	5.02	0.145	1	MgII	$3s^2 s - 4s^2 s$	8.65	0.10	2
OIII	$2p^2 3p_0 - 2p^2 3p_1$	0.0141	0.31	5	MgII	$3s^2 s - 3d^2 d$	8.86	1.7	2
OIII	$2p^2 3p_0 - 2p^2 3p_2$	0.0380	0.25	5	MgV	$2p^4 3p - 2p^4 1d$	4.51	0.060	1
OIII	$2p^2 3p - 2p^2 1d$	2.51	0.177	1	MgV	$2p^4 3p - 2p^4 1s$	9.63	0.0124	1
OIII	$2p^2 3p - 2p^2 1s$	5.35	0.024	1					

*1 = Seaton (1958)

2 = Osterbrock (1963)

3 = Burbidge, Gould, and Pottasch (1963)

4 = Osterbrock, private communication. Since these values were taken from preliminary calculations, they may differ slightly from the published results (Osterbrock 1965).

5 = Seaton (1955)

6 = van Regemortel (1960)

TABLE 2

OBSERVED INTENSITIES OF THE EMISSION LINES OF THE CRAB NEBULA

O II	Ne III	He I	He II	H I	O III	H I	N II
3727	3869	4471	4686	4861	(4959 5007)	6563	(6548 6584)
100	15.5	1.9	7.5?	10.3	151	35	137

TABLE 3

PREDICTED EMISSION-LINE INTENSITIES OF THE CRAB NEBULA

Line Intensities Relative to $[O II] \lambda_{3727} (=100)$									
R(pc)	Ne V 3426	O II 3727	Ne III 3869	He I 4471	He II 4686	H I 4861	O III 5007	He I 5876	Ne II 12.79 μ
0.005	7.69	100	120	6.99	22.0	38.5	859	20.8	19.1
0.010	3.10	100	73.7	5.37	9.62	27.1	529	16.2	20.7
0.020	1.77	100	47.0	3.47	5.56	16.3	330	10.4	20.0

IONIZATION EQUILIBRIUM FOR MODEL 1: $\gamma = 0.50$, $\nu_2 = \infty$

Fig. 1. The degree of ionization of each of the elements of high cosmic abundance is given as a function of the intensity of the ionizing radiation field and gas density for an optically thin gas. The spectral index, $\gamma = 0.50$, is constant for all frequencies.

IONIZATION EQUILIBRIUM FOR MODEL 2: $\gamma=1.00, \nu_2 = \infty$

Fig. 2. The relative abundances of the ions of each element for an optically thin gas for various values of the density and intensity of the radiation field. The spectral index, $\gamma = 1.00$, is constant for all frequencies.

IONIZATION EQUILIBRIUM FOR MODEL 3: $\gamma=0.33, \nu_2=6.00 \times 10^{14}$ c/s

Fig. 3. The degree of ionization of the elements is plotted as a function of the radiation intensity and gas density for an optically thin gas. In this model the synchrotron radiation has a high-frequency cut-off at 6×10^{14} c/s; hence, for frequencies above the Lyman limit of hydrogen, the spectral distribution of radiation corresponds to that of black body radiation of temperature 28800°K. The ionization differs markedly from that of the previous models due to the exponential decrease in radiation with increasing frequency. For any one element at a given intensity there is only one predominant stage of ionization, which exists over a wide variation in the intensity.

Fig. 4. The electron temperatures of each of the three optically thin models is given as a function of the intensity of the radiation field and density of the gas. The first two models show a correlation between the temperature and the intensity of radiation. The third model has relatively low temperatures for all intensities because of the absence of very high energy photons.

IONIZATION EQUILIBRIUM FOR MODEL 4: $\gamma=2.00, \nu_2=\infty$

Fig. 5. The ionization equilibrium for a plane-parallel optically thick gas as a function of distance into the gas. The data for this model are given in the text. The heavy elements remain ionized out to a distance considerably beyond the point where hydrogen and helium become neutral. The high abundance of CII and MgII for distances greater than 0.01 pc is due to the fact that their ionization potentials are less than 13.6 eV. Presumably, they remain ionized even after the temperature instability causes the temperature to drop, and the rest of the elements have become neutral.

Fig. 6. The electron temperature given as a function of distance into the gas for Model 4. The local maximum in temperature occurs in the gas when the hydrogen becomes neutral. The increase in T_e at this point is due to the sudden decrease in the density of free electrons, whose inelastic collisions with the heavy elements provide the main source of cooling of the gas.

IONIZATION EQUILIBRIUM FOR MODEL 5: $\gamma = 1.10, \nu_2 = \infty$

Fig. 7. The relative ion abundances for Model 5, which represents conditions in the filaments of the Crab Nebula. As in the previous optically thick model, the hydrogen and helium become neutral after a very short distance compared to the other elements.

Fig. 8. The electron temperature is plotted for each point in the gas for Model 5.

Fig. 9. The heating and cooling rates for a typical point in an optically thick gas where the H and He are primarily neutral, plotted as a function of the electron temperature. The intersection of the curves determines the equilibrium value of T_e . The two dots on the cooling curve mark the points where $d \log L / d \log T_e = 1$. Any solution occurring between these two points is unstable.