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EVIDENCE OF TIDAL EFFECTS IN SOME
PULSATING STARS. I. CC ANDROMEDAE
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ABSTRACT

Analyses of the light variation of the δ Scuti star CC Andromedae and of the radial velocity variation of the β Canis Majoris star σ Scorpii, a single-line spectroscopic binary, indicate that the long period modulations exhibited are caused by tidal deformations induced in the hydrogen and/or helium ionization zones of each primary by a faint companion, resulting in surface zonal variations of the amplitude and phase of each primary's normal radial pulsations.

The variations in the tide raising potential calculated at the center of the apparent disk of σ Scorpii correlate very strongly with the observed variations in the phase zero-point of the fundamental pulsation.

It is suggested that all the β Canis Majoris and δ Scuti stars exhibiting long period modulation, and probably also the RR Lyrae stars showing a Blazhko effect, do so because of tidal perturbations induced by faint companions.

I. INTRODUCTION

Among the shorter period, regularly pulsating stars, the occurrence of long period beats ($P_L \geq 30 P_0$, say, where P_L signifies the long period and P_0 the fundamental) is fairly common. In particular, many of the β Canis Majoris stars (Ledoux and Walraven 1958), the δ Scuti stars (Eggen 1956a, 1956b), and the RR Lyrae stars (Detre 1956) exhibit repetitive variations in the phase and amplitude and/or shape of their light, color, and velocity curves, and a variety of explanations of these modulation phenomena have been advanced. Thus Struve (1950, 1951) suggested that the beats observed in the β Canis Majoris stars could be the result of two nearly equal periods, one due to a very close satellite and the other to atmospheric oscillations in the primary induced by the satellite, while Ledoux (1951) investigated the possibility that the beats in these stars resulted from the excitation by rotation of two nonradial modes of nearly equal periods. For these same objects van Hoof (1961a, 1961b) presented arguments favoring the interpretation of the periodicities as due to simultaneous excitation of a large number of radial modes by resonance interactions, while Chandrasekhar and Lebovitz (1962) have suggested the multiple frequencies and beats observed may be due to rotational coupling of a purely radial and a purely nonradial oscillation mode. Fitch (1960a, 1960b) attempted to account for the beat phenomena in some δ Scuti stars as due to resonance coupling between fundamental radial mode and an overtone, while two recent explanations of the Blazhko effect in the RR Lyrae stars are those of Balazs-Detre (1960), who considered the modulation as due to an oblique rotator, and of Christy (1966a), who suggested that the 41-day modulation exhibited by RR Lyrae may be due to the essentially radial pulsations of a nonspherical star which

rotates and presents different aspects of its nonspherical appearance during the course of the rotation period.

Objections may be raised to most of the hypotheses just mentioned, and it is the purpose of the present paper, expected to be one of a series, to suggest still another explanation for the modulation phenomena observed. Specifically, results of analyses of the δ Scuti star CC Andromedae and the β Canis Majoris star σ Scorpii argue strongly in favor of a tidal explanation of the modulations.

II. PERIODOGRAM ANALYSIS

Periodogram calculation provides a powerful tool for the analysis of the variability of short period variable stars, and Wehlau and Leung (1964) have given a full discussion of the techniques involved and the problems encountered. However, the simplest procedures for periodogram computation are quite wasteful of computer time, so we wish to present briefly a shortcut which has proven useful and may be of interest to others.

Assume that at N discrete times t_i measures $m(t_i)$ are made to sample a continuous function $f(t)$, so that $m(t_i) = f(t_i)W(t_i)$, where the observing window $W(t) = 1$ when $t = t_i$ and is otherwise zero. To ascertain the presence of a periodic component of discrete frequency n_j we can calculate the periodogram $P(m,n) = P(fW,n)$ over some range of frequencies $n_1 \leq n_j \leq n_2$, as

$$P(m,n) = \frac{2}{N} \left\{ \left[\sum_{i=1}^N m(t_i) \sin 2\pi n t_i \right]^2 + \left[\sum_{i=1}^N m(t_i) \cos 2\pi n t_i \right]^2 \right\}^{1/2} \quad (1)$$

Were the sampling performed in a nearly continuous fashion over an indefinitely long time interval, we would have, in effect, $W \equiv 1$, and $P(m,n) = P(f,n)$ as the desired periodogram whose peak amplitudes $P(f,n_j)$ would be

the amplitudes of the frequencies n_j present in $f(t)$, assuming no observational error. However, these conditions on $W(t)$ cannot be met for stellar observations made at a single, ground-based observatory and there will be present in $P(m,n)$ subsidiary peaks flanking $P(f,n)$, due to $P(W,n)$. To illustrate this point we show in Figure 1 a portion of the periodogram $P(\ell W,n)$ for the principal component in the light variation $\ell(t)$ of CC Andromedae (Fitch 1960b), and directly below it the function $1/2P(W,n)$. From Figure 1 it is immediately apparent that the detailed structure of the periodogram in the neighborhood of any real frequency is determined, apart from observational noise, entirely by the observer's choice of times of observation, and that to avoid the possibility of missing a real peak the computation $P(fW,n)$ must be carried out in frequency steps no larger than, say, $1/5$ of the spacing between $P(W,0)$ and its first side lobe. In the present example, since observations were made in two consecutive observing seasons, the peak spacing will be at an interval of 1 cycle/year = 0.0027 cycles/day (c/d), and therefore to scan 2000 data points for frequencies in the range from 0 to 20 c/d requires about 40,000 frequency steps and the calculation of 80 million sines and cosines. Moreover, to find a second component the entire calculation must be repeated after pre-whitening the data for each term already located. The computational effort may be substantially reduced if the main peaks can be artificially broadened and the side lobes suppressed, but since each side lobe pattern is a direct consequence of the observational time sequence, which varies from star to star, it appears that no general apodizing function exists.

The following procedure has been successfully applied as a substitute for a general apodizing function. Subdivide the entire data into k groups of N_j observations each, such that in each group $t_{N_j} - t_1 \leq \Delta t$, where Δt is

an appropriately chosen time interval, and such that all observations are used once only. Writing

$$p_j(m,n) = \left\{ \left[\sum_{i=1}^{N_j} m(t_i) \sin 2\pi n t_i \right]^2 + \left[\sum_{i=1}^{N_j} m(t_i) \cos 2\pi n t_i \right]^2 \right\}^{1/2} \quad (2)$$

as the j^{th} partial periodogram corresponding to observations made in the j^{th} time interval, we now put

$$P(m,n) = \frac{2}{N} \sum_{j=1}^k p_j(m,n) \quad (3)$$

where $N = \sum_{j=1}^k N_j$. The breadth of the main peak in the periodogram will now be inversely proportional to Δt and therefore the frequency step size Δn to be employed in the scan may also be taken as inversely proportional to Δt . Commencing the computation with $\Delta t = 0.5$ day and $\Delta n = 0.5$ c/d to locate the center of the first broad peak, one proceeds to calculate successively sharper peaked periodograms by increasing Δt and decreasing both Δn and the total frequency span, until the sharpest possible peak is obtained with $\Delta t \geq$ the total time span of the observations. The computation may be carried out fairly efficiently in a completely automatic fashion, requiring only a preassignment of the time intervals and frequency steps to be used at each stage of the calculation, from an inspection of the observational time sequence. The entire procedure is illustrated by the summary in Table 1 of the calculation for the main frequency component in the 2084 measures of the light variation of CC Andromedae, where only 150 frequencies were needed to isolate $n_0 = 8.0060$ c/d in about 7 minutes operating time on an IBM 1401-7072 system.

III. THE LIGHT VARIATION OF CC ANDROMEDAE

When these techniques were applied to the published observations of CC Andromedae (Fitch 1960b), the frequencies of the four strongest terms listed in Table 2 were found before the automatic search routine was rendered ineffective by the observational noise level. In the data for CC Andromedae in Table 2, the first two terms found here were known previously (Fitch 1960b), the third and fourth terms were found in the present investigation, and the existence of the fifth and sixth terms was not found by periodogram analysis but rather was verified by least-squares fitting after the respective primary frequencies had been identified. Table 2 shows that the frequency of the second strongest term differs from that of the fundamental by nearly twice as much as does the frequency of the third term, recalling the observation of Opolski and Ciurla (1962) that the results of periodogram analyses performed by van Hoof (1961a, 1961b) on observations of the β Canis Majoris star γ Eridani may best be interpreted as evidence that the fundamental (0.1735 day period) is modulated by a single long period (15.9 days) rather than by beats with many nearly equal periods, and to illustrate this point we reproduce in Table 2 van Hoof's (1961a, 1961b) data for the five strongest frequencies in γ Eridani, together with our description of these frequencies.

Experience with the analytic representation of the nonlinear intermodal coupling present in the light variation of VX Hydrae (Fitch 1966) prompted an attempt to reproduce the observed light variation of CC Andromedae by least-squares fitting of 29 combination terms involving sums and differences of the basic frequencies $n_0 = 8.0059$, $n_L = 0.09555$, and $n_1 = 13.3463$ c/d, with the result that only 8 of these terms had a significant amplitude

and that the light variation predicted by them gave only a slightly better approximation to the observations than did the original representation (Fitch 1960b).

Since the observations on any one night never extend over more than 0.31 day = 0.03 of the long period (10.466 days), it was decided to fit by least-squares the yellow light variation observed on individual nights to the 3 frequencies $n_0 = 8.0059$, $2n_0 = 16.0118$, and $n_1 = 13.34625$ c/d, where the origin of time T is taken at Hel. J.D. 2435000.0 and the equation of condition used is

$$\Delta m(t) = +0.2626 - 2.5 \log \left\{ 1 + \sum_{j=1}^3 \left[a_j \sin 2\pi n_j (t-T) + b_j \cos 2\pi n_j (t-T) \right] \right\} \quad (4)$$

The results of fitting to observations on 27 nights are summarized by the amplitudes $A_j = (a_j^2 + b_j^2)^{1/2}$ and phase zero-points $\alpha_j = (2\pi)^{-1} \tan^{-1}(b_j/a_j)$ listed in Table 3 together with the mean time of observation $\langle t \rangle$ on each night and the corresponding phases ϕ_L of the long period given by $\phi_L = 0.09555 (\langle t \rangle - T)$. The dependence of the A 's and α 's on ϕ_L is displayed in Figure 2, where the solid circles and crosses represent observations during 1956 and 1957, respectively, and the full-drawn curves at the top of the figure were calculated from coefficients obtained by least-squares fitting of the A 's and α 's to harmonic expansions in the long period, with terms through frequencies $6n_L$. Thus

$$\alpha_0 = +0.9229 + \sum_{j=1}^6 \left[c_j \sin 2\pi j n_L (\langle t \rangle - T) + d_j \cos 2\pi j n_L (\langle t \rangle - T) \right] \quad (5)$$

with a similar expression for A_0 , and the coefficients c_j and d_j , used later for correction of the phase variations, are collected in Table 4. The small waves in the full-drawn curves of Figure 2 are almost certainly due to

observational errors and incomplete data (since there are no measures in the range $0.86 < \phi_L < 1.00$), but the two minima in α_0 are well defined and distinctly different. The members of the pairs of maxima of α_0 and of the minima and maxima of A_0 , while less precisely determined, nonetheless differ from each other. In Figure 2 the amplitude of the harmonic is seen to be nearly zero at the minima of A_0 and to go through maxima with A_0 , so that it represents a nonlinear term in the fundamental, whose primary effect is to produce a slight skewness in the large amplitude light variation similar to that characteristic of the RR Lyrae stars, while at small amplitude the light variation is very nearly sinusoidal, as would be predicted by linear theory. The variations in phase zero-point of the harmonic are very poorly defined near the (approximately zero) amplitude minima but are fairly well delineated near amplitude maxima, and within the limits of error may be taken as just twice that of the fundamental, and since the frequency of the harmonic is also just twice that of the fundamental, the phase variations observed in fundamental and harmonic are actually real variations in time of occurrence of a single, nonlinear pulsation mode.

The term $n_1 = 13.34625$ c/d, designated as overtone in Figure 2, is of very small amplitude and does not appear to share in the modulation by the long period P_L , so its reality might be questioned were it not that a similar period ratio $P_1/P_0 = 0.635$ is observed in the δ Scuti star DQ Cephei (Fitch and Wehlau 1965) and the identical ratio $P_1/P_0 = 0.600$ is observed in δ Scuti itself (Fitch 1965). If P_0 and P_1 are in fact fundamental and first overtone in this star, then their ratio implies a stellar model outside the range thus far found by Christy (1966a, 1966b) to be capable of radial pulsation. It may then be that n_0 and n_1 are both overtones, in the manner suggested by Christy (1966b), or that n_0 is the fundamental radial

pulsation mode and n_1 a nonradial mode, but further consideration of this question is postponed to a later investigation.

With the phase zero-point variation of the fundamental expressed (from eq. [5]) as $\Delta\phi_0 = \alpha_0 - 0.9229$ and writing $\phi_0 = n_0(t-T) + \Delta\phi_0$ and $\phi_1 = n_1(t-T)$ as the phases of fundamental and overtone, respectively, the observed light variation of CC Andromedae was fitted by least-squares to 19 frequencies including n_1 and a doubly-harmonic expansion in the long period and the phase-corrected fundamental as

$$\Delta m(t) = -2.5 \log \left\{ +0.7848 + a_1 \sin 2\pi\phi_1 + b_1 \cos 2\pi\phi_1 + \sum_{j=0}^2 \sum_{k=0}^4 \left[a_{jk} \sin 2\pi(j\phi_0 \pm k\phi_L) + b_{jk} \cos 2\pi(j\phi_0 \pm k\phi_L) \right] \right\} \quad (6)$$

where the coefficients a, b found in the fitting are listed in Table 5.

This analytic representation, illustrated by the full-drawn curves in Figure 3, reproduces the observations much more accurately than did the initial (Fitch 1960b) representation, though there are still systematic discrepancies at some minima and maxima, perhaps due to irregular fluctuations in the star.

Having established that the fundamental pulsation in CC Andromedae is modulated by a long (10.466 day) period, with both amplitude and phase zero-point exhibiting pairs of distinctly different maxima and minima during the long period, and observing that the phase variation in the fundamental is accurately repeated over an interval of some 38 long periods, we consider that a tidal modulation of the fundamental by a faint companion in a 10.466 day orbit provides the most likely explanation of the modulation phenomena observed. Christy (1966a) has shown that the amplitude, phase, and shape of the light and velocity variations of stars in the Cepheid instability strip is determined in a highly nonlinear fashion by

the physical conditions in and above the H and He ionization zones in these stars, so that small variations in the structure of the outer layers of these stars could easily lead to significant changes in pulsation properties. Should a faint companion produce a sufficiently nonspherical symmetry to the outer layers by tidal action, then the amplitude, shape, and phase of the fundamental radial pulsation mode will be caused to vary in a zonal manner over the stellar surface, and as the companion describes its orbit about the primary it will cause the aspect of the tidal zones to vary, so that the primary pulsation will ordinarily appear to undergo two modulation cycles during one orbital period. If the orbit is circular, successive cycles will be identical but if the orbital eccentricity is appreciable the strength as well as the aspect of the tides will vary and successive cycles will appear distinctly different, as in the case of CC Andromedae (and, probably, ν Eridani). The mechanism we have here described differs from Christy's (1966a) rotational aspect explanation of the modulation only in that the companion here assumed is the cause of both the nonspherical figure of the pulsating star and its changing aspect also.

IV. THE RADIAL VELOCITY VARIATION OF SIGMA SCORPII

It seemed desirable to perform on van Hoof's (1960a) observations of ν Eridani the same type of analysis as described in § III, but these observations will not be available in published form until 1967 (van Hoof 1966), and since the published radial velocity measures of ν Eridani (Struve, McNamara, Kung, Kraft, and Williams 1952; Struve and Abhyankar 1955) are not suitably distributed in phase of the long period for our type of discussion, another test case was sought and the β Canis Majoris (β Cephei) star σ Scorpii was chosen. The binary nature of this star was discovered by

Henroteau (1918), but the velocities here analysed are those of Levee (1952) and of Struve, McNamara, and Zeberg (1955). From Struve, et al. (1955) we take the fundamental pulsation period $P_0 = 0.246844$ day ($n_0 = 4.05114$ c/d) and fit the velocities on individual nights by least-squares to an equation of the form

$$v(t) = \langle v(t) \rangle + \sum_{j=1}^2 \left[a_j \sin 2\pi j \phi_0 + b_j \cos 2\pi j \phi_0 \right] \quad (7)$$

where $\phi_0 = n_0(t - \text{JD}2433000.0)$, and in Table 6 are compiled the resulting values of $\langle v(t) \rangle$ and also in the manner described in § III, the values of the amplitudes A , phase zero-points α , mean times of observation $\langle t \rangle$, and mean long period phases $\phi_L = 0.03018(\langle t \rangle - \text{JD}2433468.5)$, and the weights assigned to these values. Nights on which the observations extended over less than $3/4$ of a fundamental period were assigned $1/2$ weight, except that on two nights a single maximum or single minimum was observed, and the meaningless solutions obtained by the least-squares fitting were given weight zero. According to Struve, et al. (1955), the orbital period of the single-line spectroscopic binary is either 33.13 or 34.23 days, and they adopt the latter value to agree with Henroteau's (1918) original estimate, but we have chosen the smaller value since it gives a much better representation of the orbital motion through the values of $\langle v(t) \rangle$ that we have derived from Levee's (1952) measures. Through the courtesy of the staff of the Stellar Division of the Kitt Peak National Observatory the orbital elements were derived from the mean velocities $\langle v(t) \rangle$ and mean epochs $\langle t \rangle$ by a calculation performed with the Kitt Peak spectroscopic binary orbit program, and these elements with their probable errors are displayed in Table 7. In the derivation of these elements only the measures of Levee were employed because they constitute a homogeneous group fairly well distributed over the

phases of the orbital period, while most of the observations reported by Struve, et al. (1955) fall in the orbital phase interval $0.44 \leq \phi_L \leq 0.72$. The calculated orbital velocity variation, which gives an extremely accurate representation of Levee's measures, is illustrated by the full-drawn curve in the top section of Figure 4.

In order to obtain an unbiased representation of the observed variations in the pulsation amplitudes and phase zero-points, all the measures in Table 6 for the fundamental pulsation, with their weights, were used to evaluate the Fourier coefficients in harmonic expansions in the orbital period (cf. eq. (5) for CC Andromedae), and the analytic representations obtained are illustrated by the full-drawn curves in the lower sections of Figure 4. Note that the calculated variations in the phase interval $0.15 < \phi_L < 0.29$ are uncertain because of the lack of observations there, and that the very small waves particularly evident in the calculated amplitude variation are probably not real but rather the result of observational noise. From Figure 4 it may be seen that around the time of periastron passage the phase zero-point of the fundamental varies through a range of about $0.1 P_o = 36$ minutes, which far exceeds the observational scatter. Since corrections for light time in the projected orbit of the primary about the center of mass of the system never exceed $0.003 P_o$, we must regard the observed phase variations as real.

If the observed variations in amplitude and phase zero-point of the fundamental pulsation are indeed caused by tidal perturbations in the surface layers of the primary, we should expect correlations between them and the variations of the average tide raising potential over the visible surface of the primary. According to the development presented by Danby (1962), if the origin of a spherical polar coordinate system be at the center of

mass of a primary mass M_1 having radius R_1 and if a secondary point mass M_2 be situated at the point $(r, 0, \phi)$, where $r > R_1$, then the tide raising potential U_T at the point (R_1, θ, ϕ) will be given by

$$U_T = - \frac{GM_2}{r} \sum_{j=2}^{\infty} \left(\frac{R_1}{r} \right)^j P_j (\cos \theta) \quad (8)$$

where the P_j are Legendre polynomials. Further, Kopal (1959) has shown that to an accuracy consistent with the point-mass approximation for M_2 , only terms through $j=4$ need be included in the summation. If we express U_T in units of the unperturbed potential $U_o = -GM_1/R_1$, then equation (8) becomes

$$\frac{U_T}{U_o} = \frac{M_2}{M_1} \sum_{j=2}^4 \left(\frac{R_1}{r} \right)^{j+1} P_j (\cos \theta) \quad (9)$$

The observed pulsation characteristics result from measures of the integrated distribution of velocities over the disk of the primary, weighted by the distribution of surface brightness, and a proper comparison of U_T with these characteristics would therefore require an integration of U_T over this disk, calculated with some appropriate but presently unknown weighting function. To obviate this difficulty we assume that the value of U_T at the center of the apparent disk is an adequate approximation to the correct average value over the disk, and with this restriction θ becomes the angle at the center of the primary between the direction to the earth and the radius vector to the center of the secondary. Then it is easily seen that $\cos \theta = \sin(v_1 + \omega) \sin i$, where v_1 is the true anomaly of the primary, ω is the longitude of periastron, and i is the orbital inclination.

Prompted in part by the series of analyses in which van Hoof (1961a, 1961b, 1962a, 1962b, 1962c, 1962d, 1962e, 1964) found a large number of

periodicities in various β Canis Majoris stars and identified them as arising from resonance interactions involving the simultaneous excitation of numerous radial pulsation modes in the different stars, Stothers (1965) considered the theoretical relationships between the periods of the first several radial pulsation modes and such parameters as mass and radius. While we believe that the identifications made by van Hoof are erroneous, the mass-radius-period relations discussed by Stothers are assumed to be correct, and from his Tables 3 and 4 we adopt $M_1 = 20M_{\odot}$ and $R_1 = 7.8R_{\odot}$ as consistent with $P_0 = 5.9$ hours in σ Scorpii.

If we set $M_2 = \beta M_1$, then the mass function $f(M)$ is given by

$$f(M) = \frac{M_1 \beta^3 \sin^3 i}{(1 + \beta)^2} \quad (10)$$

and with our adopted value of M_1 and the observed value of $f(M)$ (Table 7) we find $M_2 \approx 4M_{\odot}$ for $i = 90^\circ$. Thus the secondary probably lies on the early main sequence, and from the data given in Strömberg's (1965) Table 4, we find that a zero age star on the upper main sequence obeys, approximately,

$$\log(R/R_{\odot}) = +0.06 + 0.54 \log(M/M_{\odot}). \quad (11)$$

If $M_2 \approx 4M_{\odot}$, then $R_2 \approx 2.4R_{\odot}$, and if $i = 90^\circ$ a primary eclipse of depth $\Delta m = 0.11$ mag. should be observed. No eclipse of σ Scorpii has been reported so, putting $R_2 = 3R_1$, we find the condition that no eclipse occurs may be expressed as

$$\tan i \leq \frac{K_1 P (1 + \beta) (1 - e^2)^{3/2}}{2 \pi R_1 \beta (1 + \delta) (1 + e \sin \omega)}. \quad (12)$$

With the known orbital elements and the adopted values of M_1 and R_1 , equations (10), (11), and (12) may be solved by successive approximations to

yield $i \leq 86^\circ.3$. Choosing the equality sign here we obtain $M_2 = 4.00M_\odot$ and $R_2 = 2.43R_\odot$, and can then use equation (9) to calculate U_T/U_o , with the results illustrated by the dashed curves in Figure 5, where the full-drawn curves representing the observed variations in amplitude and phase zero-point have been repeated from Figure 4.

In Figure 5 the scale of ordinates for U_T/U_o depends sensitively upon the mass ratio β and is, probably, quite uncertain, but the variations in U_T/U_o are determined almost entirely by the orbital elements of Table 7 and are therefore accurately known. The striking similarity between the variations in phase zero-point and variations in U_T/U_o provides fairly convincing support to our arguments favoring a tidal explanation of the long period modulation. A positive correlation between variations in fundamental amplitude and tidal potential is probably present but is much less certain due to the scatter in the amplitude determinations.

V. DISCUSSION

a) The β Canis Majoris Stars

According to Slettebak and Howard (1955) the mean rotational velocity $\langle v \sin i \rangle$ for luminosity class III and IV stars in the spectral range B1-B3 is about 100 km/sec, while McNamara and Hansen (1961) find that the β Canis Majoris stars, which occupy nearly this same region of the H-R diagram, in no case show $v \sin i > 40$ km/sec. Moreover, McNamara and Hansen observe that among the β Canis Majoris stars there appears to be a correlation between $v \sin i$ and the existence of beat periods, such that the pulsators with largest $v \sin i$ generally exhibit a beat period or variable velocity range while those with zero $v \sin i$ are ordinarily of constant velocity amplitude, and they conclude that high rotational

velocities inhibit pulsation in these stars so that the β Canis Majoris group consists of all stars of sufficiently low rotational velocities in their portion of the H-R diagram.

Now the theory of rotational splitting of radial and nonradial modes, suggested by Chandrasekhar and Lebovitz (1962) as a possible explanation of the beats observed, requires fairly high rotational velocities, and Mrs. Böhmer-Vitense (1963) concluded that for the theory to apply the stars exhibiting beats must be observed pole-on, while the statistical analyses of McNamara and Hansen are more easily interpreted as implying that these stars are seen equator-on. More recently Clement (1965, 1966) has examined the Chandrasekhar-Lebovitz mechanism and concluded that it remains a possible explanation of the long period beats but appears inconsistent with several observational characteristics of these stars.

Abt and Hunter (1962) have found very strong evidence indicating that among the B stars, members of binaries have much lower rotational velocities than do single stars, and Abt (1965), finding the same tendencies among the A stars, suggests that in binary systems of orbital periods under 100 days, tidal interactions will act to slow down rotation. Thus the low rotational velocities found by McNamara and Hansen (1961) for the β Canis Majoris stars give indirect evidence that they are all probably members of binary systems, and if we assume their rotational axes parallel their orbital angular momentum vectors we would interpret a rotational $v \sin i = 0$ as evidence for orbital inclination $i \approx 0$ and little or no observational indications of long period modulation, because the aspect of the tidal zones would then remain constant and only the variations due to changing tidal strength through orbital eccentricity could be seen (i.e., we suppose all β Canis Majoris stars would exhibit modulations were their orbital inclinations or

eccentricities large enough). This explanation of the observed correlation between $v \sin i$ and the presence of beats gives further support to our argument, as does also the fact that two members of the group, σ Scorpii and 16 Lacertae (Struve, McNamara, Kraft, Kung, and Williams 1952), are known single-line spectroscopic binaries. Finally, Huang and Struve (1955) find that the broadening or splitting of spectral lines observed at some phases of the long period in these stars is accompanied by a constancy of the total equivalent widths, and they conclude that the separate line components sometimes observed must originate over different regions of the stellar surface, consistent with our zonal interpretation of the modulation phenomena.

In all of the preceding discussion we have implicitly assumed that the cause of the pulsational instabilities in these stars resides in the structure of each's second He ionization zone, but according to Christy (1966a) there is as yet no theoretical indication of radial pulsational instability for $T_e \gtrsim 7500^\circ$. Therefore we note only that some type of pulsational instability is observed and that, excepting the periods, the observational characteristics of these pulsations are probably strongly influenced by conditions in the surface layers.

b) The δ Scuti Stars

McNamara (1961) noted that $v \sin i$ for the δ Scuti stars is appreciably smaller than that found by Slettebak (1955) to be characteristic of stars of similar spectral type and luminosity, so that Abt's (1965) correlation between the binary state and low $v \sin i$ may again be adduced as indirect evidence of the binary nature of these stars. Further, it appears very difficult to account for the modulation characteristics observed in CC Andromedae (δ III) as arising from other than tidal effects, and,

finally, the δ Scuti star δ Delphini, which has been shown by the analysis of Wehlau and Leung (1964) to exhibit long period modulation, was recently found by Preston (1966) to be a double-line spectroscopic binary.

c) The RR Lyrae Stars

None of the preceding discussion has born directly upon the RR Lyrae stars, except insofar as the δ Scuti stars constitute a proper subgroup of the former, but present views on the evolutionary history of stars would indicate that at some time after leaving the main sequence all stars in the appropriate mass range will pass at least once through an RR Lyrae stage, excepting probably only such very close binaries as the W Ursae Majoris stars. Since a large fraction of all stars occur in binary systems, we expect that many RR Lyrae stars will have (probably faint) companions. Should any of these hypothetical companions be close enough to their respective primaries ($P_{\text{orbit}} < 200$ days, say) to cause significant tidal perturbations, modulations could be produced similar to those observed (Detre 1956). According to Detre (1962), the magnetic field variations found in RR Lyrae by Babcock (1958) correlate with the phases of the 41-day period, and as such they seem to us suggestive of Babcock's (1958) discovery that in Abt's (1953) star HD98088, a gF0p spectroscopic binary of period 5.905 days, the effective magnetic field varies in synchronism with the orbital motion.

Thus it seems likely, though not certain, that the Blazhko effect observed in many RR Lyrae stars results from tidal modulations of the normal radial pulsation mode.

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TABLE 1

AUTOMATIC PERIOD DETERMINATION FOR CC ANDROMEDAE

Approximation No.	No. of Frequencies	Δt (days)	Δn (c/d)	Initial n (c/d)	Final n (c/d)	Peak n (c/d)	Peak Amplitude
1	40	0.5	0.5	0.0	19.5	7.97	0.0521
2	30	2.5	0.03	7.52	8.39	7.985	0.0518
3	20	10.5	0.01	7.885	8.075	8.0007	0.0494
4	20	40.5	0.0025	7.976	8.023	8.0050	0.0486
5	20	160.5	0.0005	8.000	8.010	8.00572	0.0485
6	20	540.5	0.0001	8.0047	8.0066	8.00598	0.0485

TABLE 2

FREQUENCY ANALYSIS OF CC ANDROMEDAE AND ν ERIDANI

Star	Frequency Description	Frequency n_i (c/d)	$n_i - n_o$ (c/d)	Amplitude	
				(mag)	(km/sec)
CC And	n_o	8.0059	0.0000	0.065	
	$n_o - 2n_L$	7.8148	-0.1911	0.030	
	$n_o + n_L$	8.1010	+0.0951	0.010	
	n_1	13.3462	$P_1/P_o = 0.600$	0.008	
	$2n_o$	16.0118		0.007	
	$2(n_o - n_L)$	15.8207		0.006	
ν Eri (after van Hoof)	n_o	5.7643	0.0000	0.106	29
	$n_o - 2n_L$	5.6375	-0.1268	0.060	17
	$n_o - n_L$	5.7019	-0.0624	0.040	13
	$n_o + n_L$	5.8275	+0.0632	0.030	9
	n_1	8.5698	$P_1/P_o = 0.672$	0.027	6

TABLE 3
AMPLITUDES AND PHASE ZERO-POINTS OF
THE LIGHT VARIATION OF CC ANDROMEDAE

$\langle t \rangle$ (JD2430000+)	n_L	n_o		$2n_o$		n_1	
	ϕ_L	A_o	α_o	A_{20}	α_{20}	A_1	α_1
5724.82	69.26	0.0423	0.972	0.0052	0.326	0.0098	0.514
5725.85	69.35	.0507	1.022	.0038	.949	.0094	.622
5728.80	69.64	.0816	0.822	.0149	.500	.0070	.536
5729.80	69.73	.0513	0.817	.0098	.483	.0148	.533
5735.80	70.31	.0443	0.984	.0029	.635	.0111	.639
5751.74	71.83	.0541	0.955	.0009	.731	.0085	.426
5753.73	72.02	.1117	0.927	.0223	.709	.0080	.423
5759.74	72.59	.0774	0.862	.0159	.561	.0068	.737
5768.73	73.45	.0716	0.945	.0140	.639	.0066	.199
5771.69	73.73	.0467	0.858	.0060	.479	.0048	.632
5785.72	75.08	.0894	0.888	.0185	.659	.0111	.469
5799.73	76.41	.0774	1.014	.0061	.135	.0144	.513
5802.66	76.69	.0623	0.813	.0051	.510	.0149	.540
5807.68	77.17	.0476	0.857	.0089	.455	.0034	.644
5810.69	77.46	.0866	0.962	.0124	.761	.0047	.596
5821.66	78.51	.0951	0.914	.0139	.646	.0122	.504
5823.66	78.70	.0662	0.789	.0106	.435	.0053	.567
5828.64	79.18	.0622	0.876	.0127	.638	.0096	.599
5831.63	79.46	.0982	0.945	.0241	.756	.0023	.224
6096.85	104.80	.0304	0.910	.0036	.809	.0115	.474
6098.86	105.00	.0922	0.924	.0131	.699	.0047	.456
6100.75	105.18	.0652	0.888	.0302	.621	.0192	.354
6101.75	105.27	.0422	1.000	.0060	.837	.0004	.292
6102.85	105.38	.0676	0.995	.0064	.796	.0068	.575
6107.85	105.86	.0475	0.974	.0047	.366	.0020	.885
6110.83	106.14	.0870	0.872	.0157	.659	.0102	.614
6124.74	107.47	0.0846	0.983	0.0084	0.658	0.0118	0.049

TABLE 4
FOURIER COEFFICIENTS OF THE PHASE VARIATION

j	$\frac{c}{a_j}$	$\frac{d}{b_j}$
1	+0.0390	+0.0058
2	-0.0770	+0.0166
3	-0.0179	+0.0006
4	+0.0059	-0.0046
5	+0.0043	-0.0031
6	+0.0022	-0.0105

TABLE 5

EXPANSION COEFFICIENTS FOR THE LIGHT VARIATION
OF CC ANDROMEDAE

j,k	Frequency	a_{jk}	b_{jk}	j,k	Frequency	a_{jk}	b_{jk}
1,+4	8.38810	-0.0003	+0.0007	2,+2	16.20290	-0.0024	-0.0017
1,+3	8.29255	+0.0006	-0.0020	2,+1	16.10735	+0.0008	-0.0006
1,+2	8.19700	+0.0064	-0.0080	2, 0	16.01180	-0.0015	-0.0064
1,+1	8.10145	+0.0001	+0.0002	2,-1	15.91625	+0.0008	0.0000
1, 0	8.00590	+0.0477	-0.0256	2,-2	15.82070	+0.0022	-0.0028
1,-1	7.91035	+0.0010	+0.0004	0,+1	0.09555	+0.0003	-0.0029
1,-2	7.81480	+0.0113	-0.0006	0,+2	0.19110	+0.0017	-0.0004
1,-3	7.71925	+0.0013	-0.0005	0,+3	0.28665	+0.0007	0.0000
1,-4	7.62370	-0.0005	-0.0005	0,+4	0.38220	+0.0003	-0.0014
n_1	13.34625	-0.0062	-0.0015				

TABLE 6
AMPLITUDES AND PHASE ZERO-POINTS OF THE
RADIAL VELOCITY VARIATION OF SIGMA SCORPII

$\langle t \rangle$ (JD2430000+)	ϕ_L (Periods)	$\langle V(t) \rangle$ (km/sec)	A_o (km/sec)	α_o (Periods)	A_{20} (km/sec)	α_{20} (Periods)	Wt.
3478.66	0.31	+5.7	18.0	0.193	10.5	0.606	1
3486.71	0.55	+133.4	223.6	.058	63.7	.827	0
3754.93	8.64	-22.5	43.5	.184	4.3	.163	2
3763.90	8.92	-9.6	46.0	.186	7.2	.106	2
3782.86	9.49	-5.2	42.3	.183	4.6	.003	2
3792.82	9.79	-25.1	47.7	.158	5.4	.885	2
3798.76	9.97	+12.2	42.5	.204	5.7	.113	2
3804.81	10.15	+34.9	57.2	.162	11.2	.017	2
3811.81	10.36	+4.9	41.5	.160	3.4	.022	2
3812.75	10.39	-3.1	34.2	.173	4.2	.409	1
3821.80	10.66	-20.4	56.4	.159	7.0	.950	2
3822.77	10.69	-18.0	61.0	.163	11.0	.833	1
3828.78	10.87	-19.9	50.3	.164	7.7	.031	2
3835.77	11.08	+44.5	54.6	.111	13.6	.803	2
3842.76	11.30	+19.0	38.1	.165	6.2	.029	2
3843.76	11.33	+11.0	35.7	.170	6.7	.003	2
3857.74	11.75	-21.1	51.2	.152	8.0	.873	1
4861.88	42.05	+36.5	51.4	.144	6.2	.110	2
4874.88	42.44	-9.8	37.7	.151	3.1	.948	2
4875.86	42.47	-10.6	40.9	.158	4.8	.962	2
4876.88	42.50	-11.1	42.5	.144	7.0	.947	2
4877.78	42.53	+299.7	426.2	.583	171.9	.943	0
4878.86	42.56	-20.3	53.7	.165	10.0	.994	2
4879.86	42.59	-16.0	46.2	.162	3.3	.983	2
4880.83	42.62	-22.2	53.0	.185	6.0	.184	2
4881.85	42.65	-18.9	43.7	.154	4.8	.035	2
4882.84	42.68	-27.5	45.0	.155	4.0	.934	2
4883.85	42.72	-23.6	53.3	.151	9.6	.942	2
4908.79	43.47	-4.8	44.9	.154	7.1	.910	2
4910.75	43.53	-18.9	50.3	.157	9.8	.014	2
4911.82	43.56	-16.7	47.6	.161	11.8	.084	2
4913.77	43.62	-14.1	41.5	0.167	2.8	0.992	2

TABLE 7
ORBITAL ELEMENTS OF SIGMA SCORPII

P = 33.13 days (assumed)	e = 0.40 ± 0.02
$\gamma = +2.8 \pm 1$ km/sec	$\omega = 301^\circ \pm 3^\circ$
$K_1 = 34.7 \pm 1$ km/sec	$a_1 \sin i = 1.45 \times 10^7$ km
$T_1 = \text{JD}2433799.8 \pm 0.4$	$f^1(M) = 0.11 M_\odot$

Fig. 1. - Comparison of a portion of the periodogram for the principal component in the light variation of CC Andromedae (above) with the corresponding portion of the periodogram of the observing windows (below).

Fig. 2. - Phase zero-points and light amplitudes of the fundamental n_0 (top), its first harmonic $2n_0$ (center), and the overtone n_1 (bottom), determined by fits to observations on individual nights. The solid circles and crosses represent measures made in 1956 and 1957, respectively, and the full-drawn curves result from least-squares fitting to Fourier expansions in the 10.466 days period.

Fig. 3. - Comparison of observed and computed light variations of CC Andromedae in 1957.

Fig. 4. - The orbital radial velocity variation of the primary component of σ Scorpii (top) and the corresponding variations in the phase zero-point (center) and amplitude (bottom) of the fundamental pulsation. The full-drawn curves (center and bottom) result from least-squares fitting to Fourier expansions in the 33.13 days orbital period.