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THE FLATTENING OF THE GALACTIC SPHEROID

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Summary. In the Solar neighborhood, the velocity dispersion of metal-poor halo stars is much smaller perpendicular to the Galactic plane than in other directions. This dispersion component appears to remain small in distant samples near the Galactic poles. A simple argument based on the tensor virial theorem shows that this situation cannot be globally valid for any population with the modest flattening usually assigned to metal-poor Population II. Either the observed kinematic samples are drawn from a highly flattened population, or, more probably, the kinematics of the regions sampled are atypical of the population as a whole.

1 INTRODUCTION

In his review of the properties of the Galactic spheroid Freeman (1987) notes an apparent contradiction between kinematic and star count data for metal-poor members of Population II. All the count data are consistent with a near-spherical distribution; a mean flattening $b/a = 0.8$ is suggested by most analyses. In the Solar neighborhood the velocity ellipsoid for this population is highly anisotropic with dispersions, σ_R , σ_ϕ and σ_z , of about 140, 100 and 75 km/s. Data in several fields for more distant objects are consistent with these dispersions being universal in cylindrical coordinates (i.e. the large dispersion everywhere pointing at the rotation axis) but not with them being universal in spherical coordinates (i.e. the large dispersion everywhere pointing at the Galactic centre). In particular, σ_z appears to remain small out to 20 kpc in the direction of the Galactic poles. Early models of this behaviour suggested that the observed population must be highly flattened (White 1985; Levison & Richstone 1986; Binney & May 1986). Sommer-Larsen (1987) subsequently found nearly round models which were *consistent* with the data in the form presented by Levison & Richstone (1986). However, his predicted σ_z was considerably larger, and his velocity anisotropies considerably smaller than those advocated by Ratnatunga & Freeman (1985, 1989). The purpose of the present note is to give a simple argument which links the global dispersion behaviour of any halo population to its overall flattening. This shows explicitly that the simple cylindrical model must be strongly violated by any near-spherical population.

2 APPLICATION OF THE TENSOR VIRIAL THEOREM

Consider a population of stars in equilibrium within the spherical, “isothermal” potential,

$$\Phi(r) = -V_c^2 \ln(r/r_o). \quad (1)$$

A flat rotation curve is a reasonable approximation to that of our Galaxy. The assumed spherical symmetry will result in an *underestimate* of the flattening that is associated with any given velocity anisotropy in a realistically oblate potential. This potential leads to the following, simple form for the tensor virial equations:

$$\sum_i \dot{\mathbf{x}}_i \dot{\mathbf{x}}_i = V_c^2 \sum_i \mathbf{x}_i \mathbf{x}_i / r^2 \quad (2)$$

The global dispersion tensor is thus just proportional to a modified inertia tensor calculated using unit vectors in the direction of each star. For an axisymmetric population this tensor will have two independent principal values, $\langle v_z^2 \rangle$ and $\langle v_e^2 \rangle$, along and perpendicular to the symmetry axis. From (2) these are given by

$$\langle v_z^2 \rangle / V_c^2 = \langle \cos^2 \theta \rangle, \quad \langle v_e^2 \rangle / V_c^2 = \langle \sin^2 \theta \rangle / 2, \quad (3)$$

where θ is the usual polar angle of a spherical coordinate system. For a system stratified on similar concentric spheroids of eccentricity, $e = \sqrt{1 - b^2/a^2}$, the average is easily carried out to yield

$$\langle v_z^2 \rangle / V_c^2 = 1 - 2\langle v_e^2 \rangle / V_c^2 = e^{-3}(1 - e^2) \left[\frac{1}{2} \ln \frac{1+e}{1-e} - e \right]. \quad (4)$$

These two ratios, as well as the ratio, $\langle v_z^2 \rangle^{1/2} / \langle v_e^2 \rangle^{1/2}$, are shown in Fig. 1 as a function of flattening. An expression for the latter quantity can also be obtained for the case where the underlying *mass* distribution is assumed spheroidal. The formulae of Roberts (1962) and Chandrasekhar & Lebovitz (1962) give

$$\langle v_z^2 \rangle / \langle v_e^2 \rangle = 2 \left[e\sqrt{1 - e^2} - (1 - e^2) \sin^{-1} e \right] / \left[\sin^{-1} e - e\sqrt{1 - e^2} \right]. \quad (5)$$

This function is also shown in Fig. 1. A careful study of Roberts’s analysis shows that (5) is valid independent of the density profiles of the stars and the mass which can be different. The only requirement is that both density distributions be stratified on concentric spheroids with the same axis ratio. (A further consequence of this is that the famous Binney diagram relating rotation and flattening of elliptical galaxies (e.g. Binney 1982) does not require M/L to be constant throughout an elliptical; it is sufficient for it to be constant on surfaces of equal luminosity density.)

Fig. 1 shows that a halo population with the flattening, $b/a \sim 0.8$, inferred from count data is expected to have $\langle v_z^2 \rangle^{1/2} / \langle v_e^2 \rangle^{1/2} \approx 0.85 - 0.9$. Each global

dispersion should be about 130 km/s if the Galactic potential is approximately isothermal with circular velocity 220 km/s. This global isotropy is difficult (but not, perhaps, impossible) to reconcile with the large dispersion anisotropies seen locally and in distant Population II samples. It is clearly incompatible with the simple cylindrical and anisotropic model which Freeman (1987) and Ratnatunga & Freeman (1989) show to fit these data well. The *local* dispersion ellipsoid, $(\sigma_R, \sigma_\phi, \sigma_z) \simeq (140, 100, 75)$ km/s, cannot apply throughout the Galaxy in any coordinate system because of the constraint, $\langle v_x^2 \rangle + 2\langle v_e^2 \rangle = V_c^2$. If the dispersion ellipsoid is assumed to align with a cylindrical coordinate system and to have axial ratios independent of position, then $\langle v_x^2 \rangle^{1/2} / \langle v_e^2 \rangle^{1/2} = 0.62$, giving $b/a = 0.47$ if the potential is assumed spherical, or $b/a = 0.33$ if the mass and the observed distribution have the same flattening. Counts of metal-poor Population II objects are not consistent with such extreme mean flattenings.

Fig. 1. Ratios of global velocity dispersions derived by applying the tensor virial theorem to a spheroidal halo population are plotted as a function of axial ratio, b/a . The solid curves apply to a population in equilibrium in a spherical singular isothermal potential of circular velocity V_c . The dashed curve applies to a population orbiting within a spheroidal mass distribution with flattening identical to that of the observed population. Curve 1 shows $\langle v_x^2 \rangle^{1/2} / V_c$, curve 2 shows $\langle v_e^2 \rangle^{1/2} / V_c$, and the two curves 3 show $\langle v_x^2 \rangle^{1/2} / \langle v_e^2 \rangle^{1/2}$.

An even more extreme apparent contradiction arises from the following argument. Ratnatunga & Freeman (1989) argue that σ_z for metal-poor halo stars averaged over a cylindrical shell at the Solar radius is about 75 km/s. Hence for this shell $\langle v_x^2 \rangle^{1/2} / V_c \simeq 0.34$. If this value applied to shells of all radii then Fig. 1 shows the mean flattening of the population would be 0.35 for a spherical

potential well. An even more extreme flattening would be implied for a realistically oblate potential. Once again this result is quite incompatible with the count data. For a nearly round population the globally averaged z -dispersion must be about 130 km/s.

3 CONCLUSIONS

There is a clear contradiction between count data which imply that the halo of our Galaxy is nearly round, and the hypothesis that σ_z is small over most of the halo. Several resolutions are possible. The populations sampled by the two types of observation may be distinct and the kinematically sampled population may indeed be flat. The rather small samples of stars near the Galactic pole may be statistically unrepresentative, the true z -dispersion being higher. The kinematic properties of the cylindrical shell containing the sun may be very different from those of shells at other radii. The first two alternatives do not seem very plausible. A version of the first might be that the halo population is poorly modeled by a single spheroidal component; the positional and kinematic determinations of shape might then assign different weights to components of different effective flattening. The third “resolution” seems more promising but will require detailed modelling. Only the models of Sommer-Larsen (1987) have come close to reconciling a near-spherical distribution with the kinematic data. However, even they do not show the small polar dispersion and the large low latitude dispersions found by Ratnatunga & Freeman (1989). In addition, they have the unexpected, and perhaps implausible property that stars at large radii are on predominantly circular orbits. The final resolution of these contradictions will probably contain elements of all three possibilities. It may well imply an orbital structure for the metal-poor halo quite different from that of the classical collapse model.

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References

- Binney, J., 1982. in *Morphology and Dynamics of Galaxies*, (ed. Martinet, L. and Mayor, M.), Geneva Observatory.
- Binney, J. & May, A., 1986. *Mon. Not. R. astr. Soc.*, **218**, 743.
- Chandrasekhar, S. & Lebovitz, N.R., 1962. *Astrophys. J.*, **136**, 1037.
- Freeman, K.C., 1987. *Ann. Rev. Astr. Astrophys.*, **25**, 603.
- Levison, H. & Richstone, D.O., 1986. *Astrophys. J.*, **308**, 627.
- Ratnatunga, K. & Freeman, K.C., 1985. *Astrophys. J.*, **291**, 260.
- Ratnatunga, K. & Freeman, K.C., 1989. *Astrophys. J.*, in press.
- Roberts, P.H., 1962. *Astrophys. J.*, **136**, 1108.
- Sommer-Larsen, J., 1987. *Mon. Not. R. astr. Soc.*, **227**, 21P.
- White, S.D.M., 1985. *Astrophys. J.*, **294**, L99.