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INFRARED SPECKLE OBSERVATIONS OF BINARY ROSS 614 AB:
COMBINED SHIFT-AND-ADD AND ZERO-AND-ADD ANALYSIS

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One-dimensional infrared speckle scans of Ross 614 AB were recorded at a wavelength of 2.2 μ m. For each scan an estimate of the instantaneous quality of the seeing was calculated and the scan was binned accordingly. The three bins corresponding to the three best seeing conditions were further processed by applying the shift-and-add algorithm to the set of images contained within each bin, thereby generating three shift-and-add images with differing shift-and-add point-spread-functions. After windowing the shift-and-add images (using edge-extension) to reduce the effect of contamination, we have obtained parameters corresponding to the separation and brightness ratio of a two component model of the double star Ross 614 AB by deconvolving the three shift-and-add images with the aid of the zero-and-add technique. Least squares analysis on the positions of the clusters of zeros found from zero-and-add yields a separation of 1.04 arcseconds and a brightness ratio of 4.3 for the binary system at this wavelength. An extension of the processing, which takes explicit account of the nonlinear motion of the scanning mechanism gives improved estimates of 1.04 arcseconds and 3.9 for the separation and brightness ratio, respectively.

We introduce a new method of processing speckle scans capable of compensating, not only for variable seeing, but also for variable distortions in the scanning mechanism.

Several speckle imaging methods have been proposed for overcoming the atmospherically induced degradation in imaging with large astronomical telescopes (cf. Bates 1982). All of these methods require some form of calibration to estimate the average effect of the atmosphere on the imaging process. Such calibration is commonly achieved by viewing an unresolved reference star under seeing conditions similar to those conditions applying when the resolved star is viewed. However, this can only take account of the long-time average or gross effects of the atmosphere, since the statistics describing the atmosphere are never exactly stationary.

Several researchers have noted that accurate assessment of the instantaneous state of the atmosphere at the instant at which a specklegram, or speckle image (Bates 1982), is recorded can increase the quality of the imaging (cf. Aime *et al.* 1978; von der Lhe 1984). An estimate of the instantaneous seeing is generated for each speckle image and the image is "binned" according to this value. The images contained in each bin are processed together to generate several estimates of the object of interest. Such preliminary processing has also been shown to be useful at infrared wavelengths (Mariotti *et al.* 1983; Christou, McCarthy and Cobb 1987b).

Speckle processing techniques have been successfully extended to infrared wavelengths (Sibille, Chelli and Lena 1979; McCarthy *et al.* 1982) by using a narrow slit which is scanned across the focal plane. All radiation passing through the slit is captured by a single detector. This produces a one-dimensional image which is essentially an integral projection (Bates and McDonnell 1986 §9) of the two-dimensional speckle image of the star.

Projections are conveniently defined in terms of two pairs of Cartesian coordinates (see Fig. 1). Suppose $a(\xi, \eta)$ is an image whose projections we wish to compute (note that we are taking the ξ, η coordinates to be fixed in the image). The angle ϕ between the ξ, η and x, y coordinates is arbitrary.

The projections in the y -direction (often called the projection at angle ϕ) are formed by calculating the line integral of $a(\xi, \eta)$ in the y -direction for all x between $-x_1(\phi)$ and $x_2(\phi)$ (refer to Fig. 1). On denoting this projection by $a(x)$, we see that

$$a(x) = \int_{-y_1(x, \phi)}^{y_2(x, \phi)} a(\xi, \eta) d\eta \quad (1)$$

Now consider an object, whose image resolved to the diffraction limit of the viewing instrument is $f(\xi, \eta)$, viewed under typical seeing conditions. During the m^{th} exposure (short enough for the seeing cells to remain effectively stationary throughout its duration) the seeing conditions are characterised by the point spread function (psf) $h_m(\xi, \eta)$. The m^{th} speckle image (formed during the m^{th} exposure), denoted by $s_m(\xi, \eta)$, is ideally the convolution of $f(\xi, \eta)$ and $h_m(\xi, \eta)$. The theory of image reconstruction from projections (cf. Bates and McDonnell 1986 chapter 5) ensures that the projection $s_m(x)$ of $s_m(\xi, \eta)$ is the convolution of the projections $f(x)$ and $h_m(x)$ of $f(\xi, \eta)$ and $h_m(\xi, \eta)$ respectively. So, allowing for the inevitable imperfections, we can write

$$s_m(x) = f(x) \odot h_m(x) + c_m(x) \quad (2)$$

where $\odot$ is the convolution operator and $c_m(x)$ is the "contamination", which accounts for detector noise and all departures from the simple isoplanatic model (cf. Bates 1982 §4). As regards the speckle images of Ross 614 AB, discussed herein, a significant source of contamination is the nonlinear scanning motion of the secondary mirror of the telescope (McCarthy, Cobb and Probst 1987). Even though the detailed form of this motion is unknown, because it is quasi-random, the technique we introduce in Section VIII is capable of compensating for it explicitly. It is worth emphasising that, provided the length $2L$ of the slit is large enough that $L > y_j(x, \phi)$, for $j = 1, 2$ and for all x and ϕ , and if each speckle image is scanned more quickly than the seeing cells can sensibly alter, the one-dimensional speckle image $s_m(x)$, which is typical of the speckle images currently being recorded by infrared astronomers, is a genuine speckle image, in that it is a (contaminated) convolution of a psf (characterising the seeing conditions) and a diffraction limited image of the object.

In this paper we combine five separate processing techniques to extract the true image from infrared speckle scans of the astrometric binary Ross 614 AB, an object of considerable interest to astrophysicists (Probst 1977).

The first processing step involves estimating the instantaneous seeing quality at the time when each of the speckle images was recorded. Each image is binned according to this estimated seeing quality (Section II). Next, shift-and-add (SAA) is applied to the set of images contained in each of the bins corresponding to the three best seeing conditions. This generates three SAA images of the same object. As is explained in Section III, each SAA image is the convolution of the true image with an unknown "composite psf". Then, edge-extension (Bates and McDonnell 1986 §15) is invoked to reduce the effect of contamination. Next, the zero-and-add (ZAA) technique (Davey, Sinton and Bates 1986), is adapted to extract the true image from the three edge-extended SAA images (Section IV). Finally, in Section V and Section VIII we introduce two complementary approaches to retrieving the brightness ratio and separation (of the two components) of the double star. Section V invokes a statistical technique due to Jeffreys (1980), while the technique developed in Section VIII takes explicit account of the above-mentioned nonlinear motion of the scanning mirror. This processing constitutes a species of "blind" (or automatic) deconvolution (cf. Lane and Bates 1987a,b) because, in the absence of *a priori* information concerning the detailed forms of the three composite psfs, it is able to remove the blurring due to them. In Section IX we assess the significance of our method and results.

II DATA RECORDING AND BINNING

McCarthy and Cobb (1986) show that two of the image sharpness parameters defined by Muller and Buffington (1974), namely S1 and S3, are related to the size of a speckle image, and therefore also to the quality of the seeing existing at the instant when the speckle image is recorded. These parameters

are defined as

$$S1 = \frac{\left(\int_{-\infty}^{\infty} s_m(x) dx \right)^2}{\int_{-\infty}^{\infty} s_m^2(x) dx} \quad (3)$$

and

$$S3 = \frac{\left(\int_{-\infty}^{\infty} s_m(x) dx \right)^2}{s_m(x_m)} \quad (4)$$

where x_m is the point of greatest brightness in the m^{th} speckle image $s_m(x)$; i.e.,

$$s_m(x_m) = \max_x s_m(x) \quad (5)$$

The average quality of the seeing can be usefully characterised by a single parameter, the Fried seeing parameter, r_0 (Roddier 1981 §4.3). When calculating the quality of the seeing for a single speckle image it is useful to introduce a second parameter r_e , which is an instantaneous estimate of r_0 . Statistical fluctuations in the seeing imply that r_e in general differs from r_0 .

Christou, McCarthy and Cobb (1987b) compute r_e by applying a least squares fit to the low-frequency component of the normal statistical model (Roddier 1981) of the turbulent atmosphere. They also note that it is much easier to compute the parameter S1 for each speckle image. Since there always seems to be close correlation between S1 and r_e , they suggest that it is appropriate to take the inverse of the former as an estimate of the instantaneous estimate of the seeing.

Christou, McCarthy and Cobb (1987b) demonstrated the usefulness of the parameter S1 by recording scans of several infrared sources and binning them. They recorded scans of the astrometric binary Ross 614 AB, and binned them according to the value of S1 computed for each of the images. They show that, when the SAA speckle processing algorithm was separately applied to each of the bins, the image produced from the bin corresponding to the best seeing, i.e. r_0 being greatest, was of superior quality to the SAA image generated by applying the algorithm to all of the recorded speckle images. Nine blocks, of 512 scans each, were recorded on the night of October 29, 1985, on the Steward Observatory 90 inch telescope using the standard photometric K band ($\lambda_0 = 2.2\mu\text{m}$, $\Delta\lambda = 0.6\mu\text{m}$). They were recorded over a period of approx 50 min. with a North-South scan direction and an image

scale of 76 milliarc-seconds per pixel.

III SHIFT-AND-ADD

The simplest speckle processing technique suggested to date is basic shift-and-add (SAA) (Bates and Cady 1980). The SAA image $f_{sa}(x)$ is given by

$$f_{sa}(x) = (s_m(x + x_m))_M \quad (6)$$

where $s_m(x)$ and x_m are defined in eqns. (2) and (5), and $(\dots)_M$ denotes averaging over all M values of m . The SAA image defined by eqn. (6) may be rewritten as

$$f_{sa}(x) = f(x) \odot \tilde{h}(x) + \tilde{c}(x) \quad (7)$$

$$\tilde{h}(x) = (h_m(x + x_m))_M \quad (8)$$

where $\tilde{h}(x)$ is a composite psf (it is composite because it represents the combined effect of M individual speckle psfs), here called the SAA psf and $\tilde{c}(x)$ incorporates all departures from the convolution.

When the data are assigned to K bins according to image quality, the SAA images so produced can be written as

$$f_{sa,k}(x) = f(x) \odot \tilde{h}_k(x) + \tilde{c}_k(x); \quad k = 1, \dots, K \quad (9)$$

where the subscript k indicates the k^{th} bin of data. $k = 1$ and $k = K$ correspond to the best and worst seeing conditions respectively. Christou, McCarthy and Cobb (1987b) used $K = 18$ for binning the Ross 614 AB data. The numbers of speckle images assigned to bins $k = 1, 2, 3$ are listed in Table 1.

We carry out SAA separately for each of the aforesaid bins, thereby generating several images comprising the convolution of the same $f(x)$ with an $\tilde{h}_k(x)$ which differs for each k . The SAA images corresponding to the three best seeing conditions are shown in Fig. 2. The image corresponding to the best seeing ($k = 1$) $f_{sa,1}(x)$ reveals the two components of the double star clearly resolved,

even with the small number of images assigned to that bin. The two components are also apparent in $f_{sa,2}(z)$ and $f_{sa,3}(z)$. The quality of $f_{sa,k}(z)$, for $k > 3$, deteriorates rapidly with increasing k (while the smaller component can be recognized with hindsight in the best of these SAA images, its apparent amplitude is significantly different for each $k > 3$), which is why we decided to process only the data in the three "best" bins.

The conventional method of obtaining an estimate of $f(z)$ from $f_{sa}(z)$ is to Wiener filter (Bates and McDonnell 1986 §16) $f_{sa}(z)$ with $\tilde{h}_{est}(z)$, an estimate of $\tilde{h}(z)$. The Wiener filter operation can be described as

$$f_{wien}(z) = \mathcal{F}^{-1} \left\{ \frac{F_{sa}(u)\tilde{H}_{est}(u)}{[|\tilde{H}_{est}(u)|^2 + \Phi(u)]} \right\} \quad (10)$$

where $\mathcal{F}^{-1}$ indicates inverse Fourier transformation, uppercase functions signify the Fourier transform of their lowercase counterparts, and $*$ denotes complex conjugation. $\Phi(u)$ is a measure of the contamination present in $F_{sa}(u)$ and is in general dependent upon the spatial frequency u . However, as it is difficult to predict the form of $\Phi(u)$ we take it to be a constant, called the filter constant Φ .

$\tilde{h}_{est}(z)$ is most straightforwardly generated by performing SAA on speckle images of an unresolved object, viewed under statistically similar seeing conditions. Blocks of 512 speckle images of the unresolved star SAO 133312, which is near Ross 614 AB, were recorded alternately with those of Ross 614 AB. The speckle images of SAO 133312 were binned according to their values of S1, and SAA was performed on the contents of each bin. The SAA images corresponding to the three best seeing conditions are shown in Fig. 3. Each of these images was used to Wiener filter its corresponding $f_{sa}(z)$ to obtain an estimate of the object. The separations $\Delta\theta$ and brightness ratios r found for the two components of Ross 614 AB using several filter constants are listed in Table 2.

These results exhibit appreciable dependence upon both k and Φ . The dependence upon k arises from the inexact matching of the seeing quality of the bins of speckle images of the resolved star with the corresponding bins for the unresolved reference star. The dependence upon Φ is a result of eqn. 10 not being a pure inverse filter. The Φ term adds a bias to the deconvolution and as Φ

becomes larger so does the bias, and hence the error in the result. Since it is virtually impossible to quantitatively estimate contamination levels with confidence, we cannot be certain of the true value for Φ . Because of the dependence of the values of separation and brightness ratio upon k and Φ we are unable to deduce their "best" values from the Wiener filter operation. There is the further point that $\tilde{h}(z)$ is object dependent, in that its form is determined both by the seeing conditions and the form of $f(z)$. This implies that, even if we could be certain of the best value for Φ and could match the seeing of the bins exactly, it might not be of much help, because we can expect there to be significant differences between $\tilde{h}(z)$ and $\tilde{h}_{est}(z)$. The processing described in Sections IV, V and VIII below overcomes all these difficulties.

Bates and Davey (1987) demonstrate that Fienup's (1982) iterative phase retrieval algorithms can be modified to blindly deconvolve true images from two-dimensional SAA images. Because of the inherent non-uniqueness of solutions to one-dimensional phase problems (Bates and McDonnell 1986 §22) we have had to develop alternative blind deconvolution techniques to process one-dimensional infrared speckle data. As explained in the Sections IV and VIII below, we invoke our ZAA technique to blindly deconvolve $f(z)$ from several SAA images simultaneously.

It is important to emphasise that SAA does not amplify the effects of contamination. This is because SAA is a species of first-order interferometry (Bates and Gough 1975). All other reported speckle imaging techniques (cf. Bates 1982) are of second or higher order, implying that they have increased sensitivity to additive contamination. Recent comparative studies by Freeman *et al.* (1987) suggest that SAA and triple-correlation (Weigelt and Wirtitzer 1983) may complement each other usefully for phase retrieval in the presence of additive contamination. However, it is as yet unclear how they compare under photon-limited conditions.

Zero-and-add (ZAA) is a deconvolution technique which was first introduced by Bates, Sinton and Minard (1985) and extended by Davey, Sinton and Bates (1986). It was originally proposed that ZAA be applied to astronomical situations in which one-dimensional speckle images are recorded. In principle the technique allows one to deconvolve $f(z)$ directly from the speckle images. We have now realised, however, that any ensemble of images, representable as some object convolved with different psfs, is deconvolvable in this way. The underlying theory of ZAA is outlined below because it must be grasped before one can understand our extension of it to the simultaneous blind deconvolution of several different SAA images.

Suppose $g_c(z)$ is any continuous one-dimensional image. The visibility $G_c(u)$, which is the spatial Fourier transform of $g_c(z)$, is defined by

$$g_c(z) \longleftrightarrow G_c(u) = \int_{-\infty}^{\infty} g_c(z) e^{-i2\pi uz} dz \quad (11)$$

where both z and u are one-dimensional real variables, and $\longleftrightarrow$ signifies Fourier transformation. Bates (1982 §7.2) shows that it is useful to generalise u to the complex variable

$$w = u + iv \quad (12)$$

which gives

$$G_c(w) = \int_{-\infty}^{\infty} g_c(z) e^{-i2\pi wz} dz \quad (13)$$

When considering sampled data, the image consists of discrete pixels. We write the pixelated image as

$$g(z) = \sum_{n=0}^N g_n \delta(z - n\Delta z) \quad (14)$$

where g_n are the N samples and Δz is their spacing. The discrete Fourier transform of eqn. 14 is then

$$G(w) = \sum_{n=0}^N g_n e^{-i2\pi n w \Delta z} \quad (15)$$

The visibility $G(w)$ of an image $g(z)$ whose extent is finite is an entire function of exponential type, and is therefore characterised completely by the particular values of w for which $G(w)$ is zero (Bates 1982 §7.2). The extent of an image is here defined as the length of the smallest continuous segment of the z -axis containing all the non-zero pixels of the object.

ZAA is based on the principle of the zero-map (Bates, Sinton and Minard 1985). The zero-map $Z\{G\}$ of $G(w)$ identifies all the points in the w -plane at which $G(w) = 0$. All the images of interest in optical and infrared astronomy are real, so that the zeros are distributed in pairs reflected in the v -axis (Bates and McDonnell 1986 §13). Consequently, only the right half of the complex w -plane is displayed in the zero-maps presented herein (see Figs. 4b, 6 and 7b).

When the image is pixelated it is more convenient to calculate the complex zeros of the z -transform where

$$z = e^{-i2\pi w \Delta z} \quad (16)$$

An image containing $N + 1$ pixels becomes a polynomial of degree N in the z -domain, with the coefficients of the polynomial given by the amplitudes of the image pixels. The zeros of this polynomial specify the locations on the complex z -plane where the z -transform of the function is zero. We locate these z -plane zero positions by invoking a polynomial zero finding routine due to Jenkins and Traub (1972) (Fright 1984; Davey, Sinton and Bates 1986). Once these zeros are computed they are mapped onto the complex w -plane using an inversion of eqn. 16

$$w = \frac{\ln(z)}{-i2\pi \Delta z} = \frac{-\text{phase}(z)}{2\pi \Delta z} + i \frac{\ln|z|}{2\pi \Delta z} \quad (17)$$

If $g(z)$ can be written as the convolution of two functions $f(z)$ and $h(z)$ i.e.

$$g(z) = f(z) \odot h(z) \quad (18)$$

then the Fourier transform of $g(z)$ can be written by invoking the convolution theorem (Bates and McDonnell 1986 §7) as the product

$$G(w) = F(w)H(w) \quad (19)$$

which is zero whenever either (or both) of $F(w)$ or $H(w)$ is zero. It follows that $Z\{G\}$ is the union of $Z\{F\}$ and $Z\{H\}$.

A double star, whose components are separated by Δl pixels and have an intensity ratio r , can be represented by

$$d(x) = r\delta(x) + \delta(x - \Delta l) \quad (20)$$

where $\delta(x)$ is the Dirac delta function. We assume that the components are individually unresolvable by the telescope (as is nearly always the case when observing at infrared wavelengths). To the scale used for recording our data one pixel corresponds to 76 milliarc-seconds. The zeros of $D(w)$, which is the Fourier transform of $d(x)$, are found by substituting eqn. 20 into eqn. 15 and equating $G(w)$ to zero. These zeros are located at the points

$$w = \frac{2n-1}{2\Delta l} + i \frac{\ln(r)}{2\pi\Delta l}; \quad n = -\infty, \dots, \infty \quad (21)$$

A representation of the projection of a two component star, for which $\Delta l = 14$ and $r = 5.0$, is shown in Fig. 4a. Its corresponding zero-map is depicted in Fig. 4b.

Suppose we are given L differently blurred versions $b_l(x)$ of $d(x)$, each formed by convolving $d(x)$ with a psf $\tilde{h}_l(x)$. The zeros of $B_l(w)$ are the combined zeros of $D(w)$ and $\tilde{H}_l(w)$. When all L of the $Z\{B_l\}$ are superimposed, the zeros of $D(w)$ reinforce each other while the zeros of the $\tilde{H}_l(w)$ are quasi-randomly scattered over the complex w plane. The zeros of $D(w)$ can thus be identified by inspection.

Although each speckle image of Ross 614 AB was recorded with 128 pixels, the useful data contained in each image exists only near the centre of the image. The same is true for the SAA images, while their extremes are dominated by contamination. We invoke edge-extension (Bates and McDonnell 1986 §15) to reduce the harmful effect of the contamination on the ZAA process. Edge-extension is applied by first truncating each SAA image where contamination has obviously affected the data. Then tails of a gaussian distribution are butted up to each side of the truncated data, maintaining continuity of the image and its first derivative with respect to x . Edge extension has the

added advantage of reducing the extent of the image. This is significant because our implementation of the polynomial zero finding routine (Jenkins and Traub 1972) is only stable for polynomials of order less than about 80. The edge-extended SAA image of Ross 614 AB corresponding to $k = 2$ is depicted in Fig. 5b. Comparison with the original image shown in Fig. 5a, which is taken from Fig. 2, illustrates the contamination suppression and extent reduction achieved by the edge-extension process.

The zero-maps of the three SAA images, after edge-extension, are shown superimposed in Fig. 6. The zero positions for the bin $k = 1$ are identified by the smallest set of open circles, with the two increasingly large sets of circles corresponding to the bins $k = 2$ and $k = 3$ respectively. Also plotted on Fig. 6 are the zero-maps corresponding to the three edge-extended SAA images of the reference star SAO 133312. These zero positions are identified by open triangles, with the increasing size of the marker again corresponding to increasing bin number (i.e. decreasing seeing quality).

It is apparent from inspection of the open circles in Fig. 6 that there are four clusters of three zeros, one zero from each of the SAA images of Ross 614 AB. Such clusters can be taken to correspond to the zeros of $F(w)$ since the zeros of $\tilde{H}_k(w)$ vary with k and are therefore spread over the w -plane. It is encouraging that these zero clusters appear to correspond to the zeros of a double star (cf. eqn. 21 and Fig. 4) since they nearly lie on a line parallel to the real axis of the w -plane. This interpretation is reinforced by none of the zeros corresponding to SAO 133312 showing any tendency to form similar clusters. Since SAO 133312 is a single star it should only give rise to zeros spread widely over the w -plane, like the zeros of $\tilde{H}_k(w)$. It is worth remarking that departures of the zeros in the clusters from a single line parallel to the u -axis are significant (as explained in Section VIII). Note that for large values of u the zeros which would be expected for a double star do not appear as clusters in Fig. 6. However these zeros correspond to high spatial frequencies, which tend to be far more severely affected by contamination than low spatial frequencies. Also, the diffraction limit of a 2.28m (90 inch) telescope operating at 2.2 μ m is approximately 200 milliarc-seconds. This limit is

almost three times the 76 milliarc-second sampling rate used for recording the data. Therefore, the information contained at high spatial frequencies is beyond the diffraction limit of the telescope. For both reasons the zeros at low spatial frequencies are expected to be more faithful than those at high frequencies. The diffraction limit is shown in Fig. 6 as u_c .

Sinton, Davey and Bates (1986) indicate how to apply ZAA to two dimensions by forming a series of one-dimensional projections (as defined by eqn. (1)) of each of the two-dimensional speckle images for several ϕ . In the context of this paper SAA and subsequently ZAA would be performed on separately binned one-dimensional projections so as to generate several zero-maps, each corresponding to a different ϕ . It is also worth mentioning here that Bates and Lane (1987) generalise ZAA to two dimensions by application of the zero-sheet concept, introduced by Lane, Fright and Bates (1987).

V STATISTICAL ANALYSIS

The statistical nature of the data, arising from both the atmospheric distortion and the presence of contamination, is dealt with in this section by invoking a least-squares analysis method of Jefferys (1980). We postulate that the effect of the contamination on the recorded images is to move the zero positions in a quasi-random manner. Thus a least-squares analysis on the zeros found in each cluster should provide a useful estimate of the true zero position and a measure of the error.

In the terminology of Jefferys (1980) the relations connecting ΔI and r with the real and imaginary parts of the complex zeros are called "equations of conditions", which he writes in vectorial form as $f(\mathbf{x}, \mathbf{a}) = 0$. The $\mathbf{x}$ here represent the real and imaginary parts of the zeros, and the components of $\mathbf{a}$ are the two parameters $a_1 \equiv \Delta I$ and $a_2 \equiv \ln r$.

If we have M SAA images, each giving N complex zeros (from Fig. 6 we see $M = 3$, $N = 4$), we can arrange the equations of conditions in component form as

$$0 = f_{\alpha} = \begin{cases} R_n^m - \frac{(2n-1)}{2\Delta I} & \alpha = 1, \dots, NM \\ I_n^m + \frac{\ln(r)}{2\Delta I} & \alpha = NM + 1, \dots, 2NM \end{cases} \quad (22)$$

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The R_n^m and I_n^m are the NM real and imaginary parts of the zeros, with $n = 1, \dots, N$; and $m = 1, \dots, M$. $n = n(\alpha)$ is the number of the zero. Both $n(\alpha)$ and $m(\alpha)$ are functions of α , the particular choice being a matter of convenience, as there is no unique way to enumerate the members of an $N \times M$ array.

In our case there are no constraints on the parameters $\mathbf{a}$, and so Jefferys' Lagrange multipliers λ disappear from the problem. We write the estimated values of R_n^m and I_n^m as $\hat{z}_{\alpha} = \hat{z}_{\alpha} + \hat{v}_{\alpha}$, where the $\hat{z}_{\alpha}$ are the measured values and the $\hat{v}_{\alpha}$ are the residuals.

The matrix σ is the covariance matrix of the $\hat{v}_{\alpha}$. For arbitrary σ the normal equations (Jefferys' eqns. (10)) may be solved exactly. Since $\partial f_{\alpha} / \partial \hat{z}_{\beta} = \delta_{\alpha\beta}$, his eqn. (10a) becomes

$$\mu_{\alpha} = - \sum_{\beta=1}^{2NM} \sigma_{\alpha\beta}^{-1} \hat{v}_{\beta} \quad (23)$$

for all α . His eqn. (10b) is

$$\sum_{\alpha=1}^{2NM} \frac{\partial f_{\alpha}}{\partial a_1} \mu_{\alpha} = \frac{1}{2a_1} \sum_{\alpha=1}^{NM} P_{\alpha} \mu_{\alpha} - \frac{a_2}{2\pi a_1^2} \sum_{\alpha=NM+1}^{2NM} \mu_{\alpha} = 0 \quad (24)$$

$$\sum_{\alpha} \frac{\partial f_{\alpha}}{\partial a_2} \mu_{\alpha} = \frac{1}{2\pi a_1} \sum_{\alpha=NM+1}^{2NM} \mu_{\alpha} = 0 \quad (25)$$

where we define $P_{\alpha} \equiv 2n(\alpha) - 1$. The individual sums in eqns. 24 and 25 each vanish. Using eqn. 24 in the equations of condition $f_{\alpha} = 0$ one may therefore derive the two equations

$$\frac{1}{2a_1} \sum_{\alpha,\beta=1}^{NM} P_{\alpha} P_{\beta} \sigma_{\alpha\beta}^{-1} - \frac{a_2}{2\pi a_1} \sum_{\alpha=1}^{NM} \sum_{\beta=NM+1}^{2NM} P_{\alpha} \sigma_{\alpha\beta}^{-1} = \sum_{\alpha=1}^{NM} P_{\alpha} \sigma_{\alpha\beta}^{-1} \hat{z}_{\beta} \quad (26)$$

$$\frac{1}{2a_1} \sum_{\alpha=NM+1}^{2NM} \sum_{\beta=1}^{NM} P_{\beta} \sigma_{\alpha\beta}^{-1} - \frac{a_2}{2\pi a_1} \sum_{\alpha=NM+1}^{2NM} \sum_{\beta=NM+1}^{2NM} \sigma_{\alpha\beta}^{-1} = \sum_{\alpha=NM+1}^{2NM} \sigma_{\alpha\beta}^{-1} \hat{z}_{\beta} \quad (27)$$

Given a set of observations $\hat{z}_{\alpha}$, a covariance matrix reciprocal σ^{-1} , and a convenient way of arranging $n(\alpha)$, these two equations may be solved for $a_1 \equiv \Delta I$ and $a_2 \equiv \ln r$.

In our case there seems to be no reason to consider a non-diagonal covariance matrix. Thus we suppose that the statistical deviations from the model fits of the complex zero locations are uncorrelated, and we write $\sigma_{\alpha\beta}^{-1} = \delta_{\alpha\beta} / \sigma_{\alpha}$, so that eqns. 26 and 27 become

$$a_1 \equiv \Delta I = \left(\sum_{\alpha=1}^{NM} \frac{P_{\alpha}^2}{\sigma_{\alpha}} \right) / \left(2 \sum_{\alpha=1}^{NM} P_{\alpha} \frac{\hat{z}_{\alpha}}{\sigma_{\alpha}} \right) \quad (28)$$

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VI TAKING EXPLICIT ACCOUNT OF NONLINEAR SCANNING MOTION

Because of the nonlinear nature of the motion of the secondary mirror (McCarthy, Cobb and Probst, 1987), the scale of the x-axis of each individual speckle scan is slightly distorted in some unknown manner. This means that the measured separation of the components varies from scan to scan. The mean of the measured separations is of course expected to equal the true separation.

It follows that the right hand side (RHS) of eqn. 7 is not a faithful representation of a SAA image. We should replace $f(x)$ by a distorted version $\tilde{f}(x)$ of the true image accounting for the aforementioned variable scaling distortion. In the case of a binary, for which $f(x)$ comprises two separated pixels (as in Fig. 4a), the pixel representing the fainter component of the binary should be spread out, in a plot of $\tilde{f}(x)$, into a number of contiguous pixels, as indicated by the simulated projections shown in Fig. 7a.

There are two noteworthy points concerning Fig. 7b. Firstly, the position of the zero closest to the origin is almost identical for each of the four zero-maps shown in Fig. 7b. Secondly, in the presence of distortion the complex zeros still appear to lie on a straight line, although the line is no longer parallel to the real axis.

The zeros in the zero-maps (shown in Fig. 6) computed from the data, corresponding to the elements of Ross 614 AB, are identified by arrows, as explained in Section IV above. Of these zeros, the position of the one closest to the origin is virtually identical for each of the three zero-maps. The other three zero positions do seem to lie on a straight line, as is roughly the case in Fig. 7b.

It is therefore plausible that we may compensate for the nonlinearity of the scanning by fitting the four groups of zeros with a straight line and then rotating the line until it is parallel to the real axis. In fitting the line, we weighted the points with the values in Table 2. The line was then rotated about a point defined by the intersection of the line itself with the average real part of the first group of zeros.

The nonlinear least-squares formalism of Section V was then used to find the brightness ratio

$$\frac{a_2}{a_1} \equiv \frac{\ln(r)}{\Delta l} = \left(-2\pi \sum_{\alpha=NM+1}^{2NM} \frac{\dot{z}_\alpha}{\sigma_\alpha} \right) / \left(\sum_{\alpha=NM+1}^{2NM} \sigma_\alpha^{-1} \right) \quad (29)$$

which are the equations that we use for our data reduction. We note that with σ diagonal, the binary separation Δl depends only on the real parts of the zeros, whilst the ratio $\ln(r)/\Delta l$ depends only on the imaginary parts.

Jefferys' (1980) analysis does not suggest any means of estimating the uncertainties in Δl and $\ln(r)$. We note, however, that we can calculate the differentials of eqns. 28 and 29. If δa is a small fluctuation in a engendered by fluctuations $\delta \dot{z}_\alpha$, we have from eqn. 28 that

$$\delta a_1 = -2(\Delta l)^2 \left(\sum_{\alpha=1}^{NM} P_\alpha^2 \frac{\delta \dot{z}_\alpha}{\sigma_\alpha} \right) / \left(\sum_{\alpha=1}^{NM} \frac{P_\alpha^2}{\sigma_\alpha} \right) \quad (30)$$

Since $\delta \dot{z}_\alpha/\sigma_\alpha$ has unit dispersion and since we assume that the $\delta \dot{z}_\alpha$ are uncorrelated, we find

$$\sigma(\Delta l) = 2(\Delta l)^2 \left(\sqrt{\sum_{\alpha=1}^{NM} P_\alpha^2} \right) / \left(\sum_{\alpha=1}^{NM} \frac{P_\alpha^2}{\sigma_\alpha} \right) \quad (31)$$

and likewise, from eqn. 29),

$$\delta a_2 = \delta a_1 \left(\frac{a_2}{a_1} \right) - \frac{2\pi a_1}{2NM} \sum_{\alpha=NM+1}^{2NM} \frac{\delta \dot{z}_\alpha}{\sigma_\alpha^{-1}} \quad (32)$$

and therefore

$$\sigma(\ln(r)) = \frac{\sigma(r)}{r} = \sqrt{\left(\frac{a_2}{a_1} \right)^2 \sigma^2(\Delta l) + \frac{4\pi^2 a_1^2 NM}{\left(\sum_{\alpha=NM+1}^{2NM} \sigma_\alpha^{-1} \right)^2}} \quad (33)$$

The results of applying the above analysis procedure to the zeros constituting the four clusters observed in Fig. 6 are presented in Table 3, which gives the brightness ratio and separation and its standard deviation in angular measure (i.e. $\Delta\theta$) rather than pixels (i.e. Δl). Although the analysis gives a standard deviation of the separation as 0.01 arcseconds, we feel it is unrealistic to specify an accuracy less than the 0.076 arcsecond image scale used in the sampling. This is confirmed by studying the fluctuations in the positions of the zeroes in simulated data, as in Figure 4. Thus we give a separation of 1.04 ± 0.07 arcseconds and a brightness ratio of 4.3 ± 0.4 .

and separation of Ross 614 AB. The result is 3.9 ± 0.4 and $1.04 \pm .01$ arcsec, respectively. The above value for the brightness ratio differs somewhat from that shown in Table 2, which was obtained from the unrotated data.

VII DISCUSSION

We have presented a new processing scheme for one-dimensional speckle images from which we deduce that the separation and brightness ratio in the infrared of the two components of Ross 614 AB are 1.04 arcsec and 3.9, respectively. These values are to be compared with the values 1.04 arcsec and 4.37, computed by applying triple correlation processing to the entire data set (Christou *et al.* 1987a).

Our processing scheme has the following advantages:

1. It avoids the need for observing a reference star under "statistically similar" seeing conditions.

This advantage is significant because it is difficult in practice to make accurate quantitative comparisons of different seeing conditions.

2. It overcomes the problem inherent in simple SAA that the composite psf is not determined solely by the seeing conditions, i.e. $\tilde{h}(z)$ depends both upon these conditions and the detailed form of $f(z)$.

3. It allows the quality of individual speckle images to be assessed so that the images can be graded and stored in separate bins. The grading does not have to be done with great precision because the sum of any number of differently blurred versions of a particular image (each blurred version being shifted by an arbitrary amount) can be expressed as the convolution of this image with another psf (i.e. a psf different from all of those causing the blurring in the aforesaid blurred version). The only crucial requirement on the grading is that the psfs implicit in the SAA images obtained from the different bins should themselves be significantly different.

This requirement is met by ensuring the statistical characters of the speckle images stored in different bins are as different as practicable.

4. It preserves the computational simplicity of basic SAA for the majority of the processing.
5. It allows straightforward compensation for nonlinear scanning in the absence of detailed knowledge of the scanning motion.

The technique we present in this paper has important practical implications for speckle processing because of its simplicity. Basic SAA is a simple technique and it can easily be performed on a microprocessor. The parameter S1 which provides an estimate of the seeing quality for binning the data is also computationally undemanding. By comparison with other extensions to SAA (Minard, Robinson and Bates 1985) and other speckle processing methods (Bates 1982; Freeman *et al.* 1987) which require sophisticated processing of each speckle image, our technique limits intensive computation (e.g. blind deconvolution by ZAA) to only a few SAA images, each of which is formed from the speckle images collected in a single bin.

Our processing scheme is most useful when we possess a valid *a priori* model of the object being viewed (e.g. a double star, as in the example with which this paper is illustrated). The model makes it easier to recognise those zeros (in the complex w -plane) that characterise the object.

Because of the apparent robustness of our scheme, observational astronomers might wish to consider applying it, in the manner of Sinton, Davey and Bates (1986), to two-dimensional speckle images. After binning and subjecting the contents of each bin to SAA, it may prove sufficient to process a single projection of each SAA image.

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1. Illustration of the projection defined by eqn. 1. The two-dimensional image $a(\xi, \eta)$ is projected, or integrated along the y axis from $-y_1(x, \phi)$ to $y_2(x, \phi)$ for each value of x between $-x_1(\phi)$ and $x_2(\phi)$, to produce the one dimensional image $a(x)$, which is called the projection at angle ϕ . The closed curve C is the perimeter of the image (i.e. there is negligible image intensity outside C). S represents the slit, of length $2L$, which scans the speckle images.
2. The SAA images generated by binning individual 1-D infrared speckle images of Ross 614 AB according to the instantaneous seeing quality, performing SAA on the images in each bin and scaling the result by the number of images in each bin. The three images corresponding to the three best conditions of seeing (i.e. $k = 1, 2, 3$) are depicted here (as published in Christou, McCarthy and Cobb 1987b). For increasing k (signifying decreasing seeing quality) the peak of the resulting SAA image decreases and the secondary peak becomes less pronounced. Curve amplitudes are normalized to have the same total power; the extent of the figure in the x -direction is 4.9 arcseconds (64 pixels) (same amplitude normalisation and x -extent conventions are adopted for subsequent images).
3. The result of the same processing as that invoked to produce Fig. 2 but for speckle images of SAO 133312. Here, as noted in Fig. 2, the image peak decreases with increasing k . Note final sentence of caption to Fig. 2.
4. Representation of a computer generated double star for which $\Delta l = 14$ and $r = 5.0$: (a) amplitude plot of the projection of the double star (note final sentence of caption to Fig. 2); (b) zero-map of the star's visibility, with the positions of the zeros indicated by circles. Note that in (b), as in all subsequent zero-maps presented in this paper, only the right half of the complex w -plane is plotted since the zero positions for a real image are symmetric about the imaginary axis. The u and v axes extend from 0.0 to 6.58 cycles/arcsec. (The image scale

- is 76 mas/pixel.) This also applies to Figs. 6 and 7b (note that a binary separation of 0.988 arcsecond (13 pixels) corresponds to a zero spacing in the u direction of 0.077 pixels⁻¹).
5. Edge extension of $f_{aa,2}(x)$ (i.e. the SAA image of Ross 614 AB produced from the second bin of data) which is shown in Fig. 2: (a) $f_{aa,2}(x)$; (b) truncated and edge-extended version of $f_{aa,2}(x)$. The edge extension process involved truncating the SAA image and fitting sections of Gaussian functions to the truncated regions, ensuring that the image and its first derivative at the junction are continuous. Note the reduction in the extent of the image and also the elimination of the contaminated parts of the SAA image. The scale is the same as in Fig. 2.
 6. Zero-maps of the three edge extended versions of the SAA images of Ross 614 AB shown in Fig. 2 (drawn as open circles) and the three of SAO 133312 shown in Fig. 3 (drawn as open triangles). The zero positions represented by markers (i.e. circles or triangles) of the same size and shape are due to the same SAA image, and increasing marker size corresponds to decreasing seeing quality (i.e. increasing k). Four clusters, each identified by an arrow and each containing one zero from all three of the SAA images of Ross 614 AB, are located in a line below the real axis. It is these four clusters which are used in the least squares reconstruction. The telescope diffraction limit is indicated on the u -axis ($u_c = D/\lambda \approx 5.03$ cycles/arcsec).
 7. Effect upon the zero-map of the spreading of the secondary peak of double star: (a) Three simulations of a double star with varying amounts of secondary peak spreading. Note that each $f(x)$ consists of the larger peak and one of the secondary peaks. Also, the sum of the pixels comprising the secondary peak is identical in each of the three images. (b) Zero-maps of the three images shown in (a). The zero positions are depicted by open circles, with the increasing size of the circle representing increasing spreading of the secondary peak. Note that the location of the zero closest to the origin is almost the same for each image. Note final sentence of captions to Figs. 2 and 4.

k	No. of images
1	9
2	25
3	102

Table 1: The number of speckle images of Ross 614 AB assigned to the bins corresponding to the three best conditions of seeing (i.e. $k = 1, 2, 3$).

γ	$k = 1$		$k = 2$		$k = 3$	
	$\Delta\theta$ (arc sec)	r	$\Delta\theta$ (arc sec)	r	$\Delta\theta$ (arc sec)	r
0.0003	1.04	6.31	1.02	5.24	1.03	4.97
0.001	1.04	6.16	1.02	5.11	1.03	4.70
0.003	1.03	6.04	1.02	5.10	1.03	4.39
0.01	1.03	5.86	1.02	4.95	1.06	4.37

Table 2: Values of the separation $\Delta\theta$ and brightness ratio r calculated by Wiener filtering the binned SAA images of Ross 614 AB with those produced by applying similar processing to speckle images of the unresolved source SAO 133312. The values shown here were obtained by least squares fits of quadratic functions to five pixels at the peaks corresponding to the components of the star. Note the large variation in both the separation and brightness ratio as a result of varying both k and γ . The filter constant is defined to be $\Phi = \gamma \max_u |\tilde{H}_{st}(u)|^2$.

σ				$\Delta\theta$ (arc sec)	r	$\sigma_{\Delta\theta}$ (arc sec)	σ_r
$n = 1$	$n = 2$	$n = 3$	$n = 4$				
0.002	0.004	0.006	0.008	1.04	4.3	0.01	0.4

Table 3: Results of applying the least squares analysis of Jefferys' to the zeros comprising the four clusters observed in Fig. 6.

Fig. 1

Fig. 2

$f(x)$

FIG. 3

$f(x)$

x

(a)

$iv \uparrow$

u

(b)

Fig 4

$f(x)$

(a)

$f(x)$

(b)

Fig. 5

Fig 6

FIG. 7