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Yufeng Yan, Jose Sasian, "Photographic zoom fisheye lens design for DSLR cameras," *Opt. Eng.* **56**(9), 095103 (2017), doi: 10.1117/1.OE.56.9.095103.

Photographic zoom fisheye lens design for DSLR cameras

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Abstract. Photographic fisheye lenses with fixed focal length for cameras with different sensor formats have been well developed for decades. However, photographic fisheye lenses with variable focal length are rare on the market due in part to the greater design difficulty. This paper presents a large aperture zoom fisheye lens for DSLR cameras that produces both circular and diagonal fisheye imaging for 35-mm sensors and diagonal fish-eye imaging for APS-C sensors. The history and optical characteristics of fisheye lenses are briefly reviewed. Then, a 9.2- to 16.1-mm $F/2.8$ to $F/3.5$ zoom fisheye lens design is presented, including the design approach and aberration control. Image quality and tolerance performance analysis for this lens are also presented. © 2017 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.OE.56.9.095103]

Keywords: fisheye lenses; zoom lenses; two group zooms; lens design; aberration compensation.

Paper 170893 received Jun. 12, 2017; accepted for publication Sep. 6, 2017; published online Sep. 27, 2017.

1 Introduction

Among the various types of imaging lenses that exist, fisheye lenses are outstanding for their ultrawide-angle field of view and for their unusual image mapping. Fisheye lenses can cover more than a hemispherical (180 deg) field of view and were initially used for meteorology purposes to record the entire visible sky. Today, fisheye lenses are utilized for many applications such as creative photography, surveillance, photogrammetry, art, and advertising. Fisheye lenses suffer from a large amount of barrel distortion, which is used in creative photography. Some photographic fisheye lenses with extreme fields of view have been commercially manufactured, such as the Nikon 6-mm fisheye lens with a hyper 220-deg full field of view.¹ However, many photographers have come to realize that a full field of view of 180 deg is most desirable for photography and this field has become a standard. An important specification of a photographic fisheye lens is how the object hemisphere is imaged on the active area of the camera sensor. There are currently two types of fisheye lenses on the market, these are diagonal fisheye lenses that cover the full field of view along the diagonal of the sensor so that the sensor is fully illuminated with the image, and the circular fisheye lenses that project an entire circular image within the camera sensor. This imaging is shown in Fig. 1.

To provide different image sizes, these fisheye lenses require different focal lengths for a given sensor size. In addition, fisheye lenses for cameras having different image formats, such as 35-mm format and APS-C format [a smaller sensor format that is used on many compact digital single-lens reflex camera (DSLR) cameras], also call for a different focal length. Thus, a fisheye lens with a variable focal length would be convenient and desirable for photographers as to provide full sensor imaging or circular imaging. However, the extreme field of view of a fisheye lens, its optical asymmetry, and the required long back focal distance (BFD) make correcting off-axis aberrations, such as field

curvature, oblique spherical aberration, and lateral color challenging. These challenges become greater when the lens must be optimized for different focal lengths, this is for a zoom lens. Over the last decade, a few zoom fisheye lens designs have been patented. For example, the lens in U.S. Patent #6,987,623² assigned to Nikon, the lens in U.S. Patent #7,317,581³ assigned to Pentax, and the lens in U.S. Patent #8,456,751⁴ assigned to Canon. Among these patents, the design by Canon has been mass-produced. Apparently, this lens is the only photographic zoom fisheye lens on the market that maintains its field of view at all zoom positions. Furthermore, this lens provides a focal length that varies in the range of 8 to 15 mm at a relative aperture of $F/4$ across the zoom range, which is relatively slow compared to some fixed focal length fisheye lenses.

Lens design details for zoom fisheye lenses are scarce in the literature. However, this paper presents design details of a zoom fisheye for diagonal and circular imaging with an aperture of $F/2.8$ at the shortest focal position of 9.2 mm and $F/3.5$ at the longest focal position of 16.1 mm. The highlights of the design are its simplicity, aberration control, and image quality. This paper begins with an overview of fisheye lenses, discusses relevant optical characteristics, presents the design philosophy, provides lens data and analysis, and then concludes.

2 Fisheye Lenses

The concept of a fisheye lens was inspired by considering the eyes of fish under water. A fisheye model was first introduced by Wood⁵ [Fig. 2(a)]. He placed a photographic plate in a water-tight box filled with water with a pinhole on the top. Then, he added a cover glass on the top of the pinhole to seal the water box so it could be pointed horizontally. Thus, the optical system was simply a water-filled pinhole camera with no lens involved. Some sample photos taken by this camera were published in Wood's paper. Wood also pointed out that this kind of wide-angle camera could be used for sky recording.

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Fig. 1 Images taken by (a) a diagonal fisheye lens and (b) a circular fisheye lens.

Fig. 2 Early development of fisheye lenses: (a) pinhole camera with water tank by Wood, (b) pinhole camera with hemispherical lens by Bond, (c) Hill sky lens, and (d) Allgemeine Elektrizitäts-Gesellschaft (AEG) fisheye lens.

The fisheye pinhole camera was then improved to a more practical design by Bond⁶ [Fig. 2(b)]. A single hemispherical lens was used to replace the water in Wood's design, so the new design contained no water. In this design, all light would go through the small aperture located at the center top of the lens and form the image onto an almost hemispherical image surface. Bond's design should be still considered as a fisheye pinhole camera, rather than as a fisheye lens. Due to the lack of aberration control in this system, the aperture needed to be very small, which made the entire system so slow that it could be operated only at about $F/50$. Also, field curvature of the image was so large that the entire image could never been fully focused on a single photographic plate.

Shortly after Bond's fisheye pinhole camera was introduced, Robin⁷ published a paper on his famous "Hill sky lens" [Fig. 2(c)]. Instead of a pinhole, he used a negative meniscus lens in the front to guide light into the stop aperture, then he used two additional lenses behind the stop for imaging and aberration control. This lens was later mass-produced as a sky recording device by Beck of London.⁸ With the control of coma, astigmatism, and field curvature, the lens could operate at $F/22$. Lack of control of spherical aberration became the limiting factor on aperture size. Color correction was non-existent as well, so the lens could produce monochromatic images only with a color filter. Nevertheless, Hill's fisheye lens was truly a milestone in the history of

fisheye lens design. It was the first actual fisheye lens in the world. The optical structure of his design became common to later fisheye lens designs. The negative meniscus shape of the first element has become common on all modern fisheye lenses.

Another remarkable design that rarely is mentioned is the AEG fisheye lens⁹ [Fig. 2(d)]. It was patented by the AEG company in Berlin in 1932. The lens is a more elaborate design than the Hill's fisheye lens. Along with the control of all monochromatic aberrations, the achromatic doublet was introduced to correct axial color. With better aberration control, this lens could produce polychromatic images at a maximum speed of $F/6.3$. Germany later shared this patent with its ally Japan during World War II. The lens was then modified by Nikon and become the foundation of modern photographic fisheye lenses.

3 Optical Characteristics of a Fisheye Lens

Technically, a fisheye lens is a reverse telephoto lens in that it can be divided into a negative optical power front lens and a positive power rear lens. This configuration makes it possible to achieve large fields of view, a large BFD, at the expense of lens length and an image mapping that departs from the standard $Y = f \tan(\theta)$, where f is the focal length and θ is the semifield of view angle.

Assuming a Lambertian scene, telecentricity in image space $\bar{u}' = 0$ and uniform relative illumination, the optical flux $\Phi = \pi \cdot y^2 \sin^2(\theta)$ entering a fisheye lens must be equal to the optical flux $\Phi' = \pi Y^2 NA^2$ forming the image. Here, y is the radius of the entrance pupil, Y is the image height, and NA is the numerical aperture. Then, it follows that when $NA \cong y/f$, the image height Y is related to the semifield of view angle by $Y = f \sin(\theta)$. This mapping is known as orthographic and depending on the amount of pupil coma/image distortion, fisheye lenses depart from adhering to this mapping.

Other possible mappings are the equidistant projection $Y = f\theta$, the stereographic projection $Y = 2f \tan(\theta/2)$, and the equisolid angle projection $Y = 2f \sin(\theta/2)$. For

photographic fisheye lenses, image quality usually controls the design, and the resulting mapping can be described by

$$Y = a \cdot f \sin(\theta/b), \tag{1}$$

where a and b are the coefficients that define the mapping.

For a lens to cover the diagonal of a 35-mm camera and a field of view of 180 deg, it is noted that for orthographic mapping the focal length must be 21.633 mm. However, in practice and due to the actual mapping achieved, the focal length of a diagonal fisheye lens varies around 16 mm and for a circular fisheye lens around 9 mm. Thus, fisheye lenses have comparatively small focal lengths resulting in a large depth of field, which is another highlight of fisheye lenses.

The aperture stop in a fisheye lens is located near, or at the rear positive lens group. The front lens group, being negative and usually formed by one or more meniscus lenses,

Fig. 3 (a) Sensor sizes and their comparison with the image circle at (b) wide-angle zoom position, (c) intermediate zoom position, and (d) telephoto zoom position.

Table 1 Design specification of the zoom fisheye lens at different zoom positions.

Zoom position	Wide angle	Intermediate	Telephoto
Focal length (mm)	9.2	10.8	16.1
$F/\#$	$F/2.8$	$F/3$	$F/3.5$
FOV (deg)	180	180	180
Half image height (mm)	12	14.1	21.6
Design spectrum	Visible light (F, d, C)		
Mapping	Equisolid angle to within 10%		
BFD	38 mm		
Total track	140 mm		
Maximum lens clear aperture	68.6 mm		
Object location	At infinity

Fig. 4 Lens construction at extreme zoom positions.

contributes a substantial amount of negative pupil spherical aberration $\overline{W}_{040}$. The consequence is that the image of the aperture stop, this is the entrance pupil, as seen at oblique angles, appears to tilt and move forward, off the optical axis. Effectively, in the meridional plane, the entrance pupil follows the external caustic sheet for the entrance pupil's spherical aberration. This phenomenon is known as pupil walking and notably allows fisheye lenses to cover hyper-hemispherical fields of view, which are in excess of 180 deg.

Currently, there are very few zoom fisheye lenses on the market. The Canon 8- to 15-mm zoom fisheye lens⁴ has become famous. It is a thirteen elements design with a

constant maximum aperture of $F/4$ throughout the zooming range. The control of aberration and image contrast for such a large field of view at multiple focal lengths requires the use of several design techniques.

4 Design of a Zoom Fisheye Lens

4.1 Specifications

This section presents the design specification for a zoom fisheye lens that features simplicity while providing excellent image quality. The focus of the discussion is in the focal length and image mapping. The specifications are given in Table 1.

Table 2 Lens data.

Surface number	Radius of curvature		Thickness	Material		
1	92.031		2.237	N-LAK34		
2	17.647		22.819			
3	−43.924		1.999	N-PSK53A		
4	25.754		6.788	SF6		
5	245.219		4.811			
6	−23.732		8.001	N-LASF45		
7	−34.205		T7 (variable)			
8	21.396		2.000	N-SSK8		
9	40.422		5.122			
10 (stop)	Infinity		0.964			
11	175.333		0.976	N-LAK10		
12	43.311		0.000			
13	29.529		7.749	N-BK10		
14	−10.137		0.999	N-LASF44		
15	−41.349		2.504	SF2		
16	−13.753		0			
17	−147.830		3.000	N-PK52A		
18	−16.121		2.000	SF6		
19	−36.900		BFD			
Multiconfiguration data	Wide angle		Intermediate	Telephoto		
Focal length (mm)	9.2		10.8	16.1		
T7	24.985		17.300	2.009		
BFD	38.000		40.791	50.463		
Surface	Conic	8th order	4th order	10th order	6th order	12th order
2	−0.183	$−1.188 \times 10^{-10}$	$−2.772 \times 10^{-6}$	3.339×10^{-13}	1.760×10^{-8}	$−4.160 \times 10^{-16}$
11	0.000	2.457×10^{-10}	$−2.944 \times 10^{-5}$	0.000	$−6.658 \times 10^{-8}$	0.000

The focal length of an optical system is an important design specification; however, defining the focal length(s) for a photographic zoom fisheye lens is not straightforward. As mentioned before, a variation of equisolid angle mapping is often used to characterize photographic fisheye lenses. This mapping varies depending on fisheye lens design, and therefore, to provide the same image size and field of view (FOV), slightly different focal lengths result. For example, the Canon,¹⁰ Sigma,¹¹ Nikon,¹² and Sony¹³ lenses are diagonal fisheye lenses for 35-mm format cameras. Despite the fact that they all image a 180-deg FOV, the Canon and the Sigma lenses have a 15-mm focal length, whereas the Nikon and the Sony lenses have a 16-mm focal length. For the design that is presented in this paper, a certain amount of freedom on the mapping and focal length is allowed in order to achieve better optical performance. However, the mapping is limited so the departure from equisolid mapping is less than 10%.

This design can also be used as a diagonal fisheye lens for APS-C format cameras at its intermediate zoom position where the image height is equal to the diagonal of an APS-C format sensor. Figure 3(a) shows the size comparison between a 35-mm sensor and an APS-C sensor. Figure 3(b) shows the image circle (red) at wide-angle zoom position. Figure 3(c) shows the image circle at intermediate zoom position for a diagonal APS-C fisheye image. Figure 3(d) shows the image circle at telephoto zoom position. For zoom lens terminology usage, the wide-angle zoom position and telephoto zoom position refer to the positions of the lens groups at the shortest and longest focal lengths. The term “telephoto” here does not imply that the lens has a telephoto construction since the fisheye lens is a reverse telephoto lens throughout the entire zoom range. The diagonal of an APS-C sensor is not standardized among different camera companies; the size of the Nikon DX sensor (23.5 mm × 15.6 mm) is used as the reference for this design.

4.2 Lens Construction

Figure 4 shows the construction of the zoom lens at its extreme zoom positions. The front group consists of one doublet and two meniscus lenses; the rear group consists of a singlet lens, a doublet lens, and a triplet lens. The rear lens group, includes the aperture stop, acts as the variator and produces the majority of the lens movement to change the power of the lens, whereas the front group acts as the compensator that moves slightly to keep the image plane stationary. The lens prescription data are provided in Table 2. Linear units are in millimeters, and glass materials are chosen from the Schott glass catalogue.

The physical motion of the lens groups is shown in the cam curve shown in Fig. 5. The rear group moves linearly, whereas the front group moves nonlinearly. The zoom is achieved by varying both BFD and space between front and rear groups.

4.3 Aberration Control

The design of the final lens is shown in Fig. 4. It consists, for maximum simplicity, of two zooming groups. The focal length of the groups is −16 and +30 mm, respectively. Changing the group separation changes the focal length, and moving both groups allows maintaining the focus.

Since there are only two groups, it is required to independently correct each group, or nearly correct, for its fourth-order aberrations, except distortion and Petzval field curvature, and to also correct the second group for invariance of aberrations as its virtual object changes position. If upon zooming the aberrations of the rear group remain invariant, then the entire lens remains corrected upon zooming provided that it has been corrected for one zoom position. The equations for the aberration coefficients upon object shift¹⁴ are given in Table 3.

For invariance of astigmatism $W_{222}^* - W_{222} = 0$, there must be negligible pupil spherical aberration $\overline{W}_{040}$, no image distortion W_{311} , and the chief ray must nearly pass by the nodal points as to have $\Delta(\overline{u}^2) = 0$, or alternatively

Fig. 5 Cam curve of the zoom lens.

Table 3 Aberration coefficients upon object shift according to the object shift parameter S .

$$W_{040}^* = W_{040} + \left[W_{131} + \frac{1}{8} \mathcal{K} \Delta(u^2) \right] S + \left[\frac{3}{2} W_{222} + \frac{3}{8} \mathcal{K} \Delta(u\overline{u}) + W_{220P} \right] S^2 + \left[W_{311} + \frac{3}{8} \mathcal{K} \Delta(\overline{u}^2) \right] S^3 + \overline{W}_{040} S^4 \quad (2)$$

$$W_{131}^* = W_{131} + \left[3W_{222} + \frac{1}{2} \mathcal{K} \Delta(u\overline{u}) + 2W_{220P} \right] S + [3W_{311} + \mathcal{K} \Delta(\overline{u}^2)] S^2 + 4\overline{W}_{040} S^3 \quad (3)$$

$$W_{220P}^* = W_{220P} \quad (4)$$

$$W_{222}^* = W_{222} + [2W_{311} + \mathcal{K} \Delta(\overline{u}^2)/2] S + 4\overline{W}_{040} S^2 \quad (5)$$

$$W_{311}^* = W_{311} + 4\overline{W}_{040} S \quad (6)$$

$4W_{311} + \mathcal{K}\Delta(\bar{u}^2) = 0$. In Table 3, $\mathcal{K}$ is the Lagrange invariant and $\bar{u}$ is the chief ray slope.

Table 4 provides the fourth-order aberration coefficients of the first group, the second group, and the complete zoom lens for the wide field and telephoto positions. The field of view used is 30 deg as aberration coefficients depend on $\bar{u}$, which is undefined for $\theta = 90$ deg, and the $f/\#$ used is $f/3.5$ for the wide-field position and $f/6.1$ for the telephoto position. These $f/\#$ s make the passage of light in the front group about the same for both positions and so the front group aberrations are about the same. Furthermore, for both positions, the rear group aberrations coma, field curvature, and distortion are similar indicating that Eqs. (3), (4), and (6) in Table 3 are satisfied to some extent for invariance.

Although spherical aberration relatively changes in the rear group, the total of this aberration is not significant for either position. Note that except for distortion the total fourth-order aberrations, for the 30-deg field of view and the corresponding F -numbers, are on the order of one wave or less, which is considered a small amount. Equation (5) in Table 3 for invariance of astigmatism is satisfied to some extent. However, a second compensation mechanism takes place as there is stop shifting for the front group, which in the presence of coma changes the astigmatism from the front group to compensate the small change of astigmatism upon zooming from the rear group.

The above analysis about the invariance of fourth-order aberrations for the rear group explains how the bulk of

Table 4 Zoom lens aberration coefficients in waves for a semifield of 30 deg.

Group	Wide field			Telephoto		
	Front	Rear	Total	Front	Rear	Total
W_{040}	-0.017438	0.68148	0.664042	-0.0174	0.372954	0.355554
W_{131}	-0.978209	1.327662	0.349453	-1.020593	1.174717	0.154124
W_{222}	1.333673	0.422844	1.756517	0.069917	1.548848	1.618765
W_{220P}	-8.234249	8.418402	0.184153	-8.225243	8.409195	0.183952
W_{311}	143.42975	3.653452	147.083202	134.292114	7.697213	141.989327

Fig. 6 Image space CRA vs. HFOV at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position.

the aberrations are controlled in such a simple two-group zoom lens. For this analysis, we maintained the size of the entrance pupil the same for both zoom positions. In practice, the entrance pupil size varies and aberration balance between the front and rear groups is still necessary for sharp imaging. This balancing is achieved by real ray tracing and optimization with computer software.

Due to the lack of symmetry about the aperture stop, lateral color aberration does not tend to cancel and requires special attention for its correction. Kumler and Bauer¹⁵ have shown that many fisheye lenses designed for 35-mm sensors have significant lateral color near the field edge, usually larger than $30\text{ }\mu\text{m}$. Since the distance between the zoom groups changes in a zoom fisheye lens, it is essential to achromatize each group independently. One method to control lateral color is by the use of achromatic doublets in each group. In this design, ED glass from the Schott glass catalogue (N-PK52A) was used to help control lateral color aberration.

Field curvature is sometimes controlled by balancing Petzval curvature and astigmatism. For example, in a Cooke triplet, the Petzval radius can be about 2.4 times the focal length. The residual field curvature is balanced against some residual astigmatism. Given the very large field of view of fisheye lenses, and the possible change of astigmatism upon zooming, then it is necessary to well correct Petzval field curvature with no substantial residual to be balanced with astigmatism. Higher order field curvature also

needs to be controlled, and one technique that is implemented for this correction is the use of a conic surface in the front negative meniscus lens. The use of field flattener's lenses is not allowed as for SLR cameras there needs to be space for a folding mirror. Field curvature aberration is also mitigated by the use of high index glass for the crown lenses and low index glass for the negative lenses.

The use of special glasses with very low dispersion is becoming popular for modern camera lens designs. Achromatic doublets made with such glasses can control lateral color effectively. However, these special glasses are usually much softer than normal crown glass and issues can arise due to manufacturability and durability; these glasses comparatively cost much more. The size of the lens elements with special glasses needs to be constrained to reduce cost. This can be done by putting such elements in the rear group close to the aperture stop. Another glass choice issue for fisheye lenses is the glass selection for the large negative meniscus lens in the front. Previous references suggest that regular crown glasses, such as BK7, should be used for the front element because of their low cost and low chromatic dispersion.^{16,17} However, the regular crown glasses may not be the best glass choice today for modern fisheye lens design, when size and weight are considered. Using glasses with increasing index of refraction, such as flint glasses, for the front meniscus lens can result in a decreased diameter at the expense of making lateral color aberration harder to correct.

Fig. 7 Optical path difference for 0 deg, 30 deg, 60 deg, and 90 deg half field at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position. Scale is ± 2 waves.

In this design, the aperture stop is located at the rear lens group. While an aspheric surface at, or near the stop effectively controls spherical aberration, it is also used to control higher order coma W_{151} and oblique spherical aberration W_{240} ; these aberrations also depend on the asphericity and can effectively be influenced. The Petzval radius of this design is 1352 mm or more than 100 times the focal length, and as mentioned before, a conic surface is used in the front meniscus lens to control higher order Petzval field curvature.

4.4 Optimization

Zemax OpticStudio was used to optimize the lens. The error function first was defined with root mean square (RMS) optical path difference (OPD), and then with modulation transfer function (MTF) operands. Given the lens large field of view, 12 field positions were used during the lens optimization to

properly sample the field. During the MTF optimization, the MTF versus field plot was used as a reference to adjust the weight of each field. Multiple focal length configurations were also used during the optimization to obtain even performance across the zoom range. Constraints on distortion aberration and to avoid physical lens interference were used. In addition, the chief ray angle (CRA) in image space was constrained to meet the maximum CRA requirement of the digital sensor, which is usually no more than 30 deg. This maximum image space CRA constraint must be followed throughout the entire zoom range since the relationship between FOV and CRA varies at different zoom positions. The plots of CRA versus HFOV in image space at different zoom positions are provided in Fig. 6. Furthermore, a constraint on image space CRA beneath the 30-deg sensor limitation also benefits the relative

Fig. 8 Astigmatic field curves at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position.

Fig. 9 Longitudinal aberration at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position.

illumination at the sensor corners. Toward the end of the design stage, glass substitution was used with hammer optimization in Zemax OpticStudio to improve glass selection of the achromatic doublets.

4.5 Performance Evaluation

The zoom lens was evaluated using wavefront OPD plots as shown in Fig. 7. Plots of astigmatism, longitudinal aberration, distortion from equisolid angle projection, and lateral color were also used and are shown in Figs. 8–11. Astigmatism and distortion are analyzed at 588-nm wavelength (d-light). All evaluations in this section are analyzed at the maximum aperture setting for each zoom position.

The peak to valley wavefront OPD deviations are mostly under two waves for all zoom positions. The lens shows better aberration control at the wide angle and intermediate zoom positions, where the focal lengths are smaller. At the telephoto zoom position, the image size is larger and the aberration control becomes more difficult. Some vignetting is introduced to remove largely aberrated rays. The RMS wavefront error across the field is controlled under 0.5 waves at the wide angle and intermediate zoom positions and is under 1 wave at the telephoto position.

Breaking down to individual aberrations, the lens has good field curvature and astigmatism performance at the wide angle and intermediate zoom positions. At the telephoto zoom position, astigmatism becomes more significant. However, astigmatism and field curvature are balanced at the

Fig. 10 Departure from equisolid angle mapping at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position.

Fig. 11 Lateral color at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position.

edge of the field. So, the overall field curvature and astigmatism performance does not degrade significantly. The longitudinal aberration plot evaluates both spherical aberration and axial color. The lens shows some spherochromatism, which is balanced with axial color aberration.

The distortion plot shows how much the lens projection is deviated from equisolid angle mapping. This is maintained under 10% at the edge of the field. The lateral color performance of this lens is excellent. The lateral color plot is

curved toward zero at the edge of the field, making the maximum lateral color smaller than $8\ \mu\text{m}$ at the telephoto zoom position. At both the wide-angle zoom position and intermediate zoom position, the maximum lateral color is controlled under $5\ \mu\text{m}$, which is the size of a single pixel in many modern 35-mm DSLR cameras.

Figure 12 shows MTF plots for the design at all three zoom positions. Twelve equal-area fields were used to analyze each zoom position. For photographic lenses, a different

Fig. 12 MTF versus spatial frequency at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position.

Fig. 13 MTF versus field at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position.

MTF plot is often used. This evaluates the contrast versus the field of view in image space at different spatial frequencies. This MTF plot directly shows how contrast is varied from the center of the image toward the edge, and these data are often provided by photographic lens manufactures with their products. Spatial frequencies 10 and 30 lp/mm are typically used for this evaluation to cover the frequency range for normal photographic use. The MTF versus field plot is provided in Fig. 13. Note the field of view in image space is represented by the half image height.

At the telephoto zoom position, the contrast at high spatial frequency varies significantly over the field. The contrast is more uniform across the field at the wide-angle zoom position and at the intermediate zoom position. The MTF versus field plot gives a better understanding on how contrast varies across the image. This lens has a consistent contrast across the field at the wide angle and intermediate zoom positions.

At the telephoto zoom position, the contrast performance tends to degrade. Practically, at this zoom position, only the image diagonal achieves full 180-deg field of view, and most of the image portion at the large field is cut off by the rectangular shape of the sensor. Thus, some contrast performance at the edge of the image circle is sacrificed to provide best contrast at the field center of the telephoto zoom position.

The relative illumination measures the illumination intensity level normalized to the maximum intensity across the field. It is highly dependent on image space CRA, effective size of entrance/exit pupil, or equivalently on distortion aberration. The entrance pupil shape of this design and its angle dependence are presented in Fig. 14. The total relative illumination is calculated by real ray tracing in Zemax OpticStudio and is based on the method that is described by Rimmer.¹⁸ Such relative illumination plot is provided

Fig. 14 Entrance pupil shape at its maximum at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position. Plot scale is 7 mm.

Fig. 15 Relative illumination at (a) wide-angle zoom position, (b) intermediate zoom position, and (c) telephoto zoom position.

Fig. 16 Effect of 1-arc min surface/element tilt, and 10- μm surface/element decenter, at all three critical zoom positions, based upon the root-sum-square method.

in Fig. 15. Vignetting of the system is set to maintain at least 50% of relative illumination at the field edge.

4.6 Tolerance Analysis

During the manufacturing and assembling stage, tilt and decenter of surfaces and lens elements usually have the greatest effect on final lens performance. For the tolerance analysis, the effect of an element and surface decenter of 10 μm , and a tilt of 1 arc min were evaluated. The lens is axially symmetric, so only decenters along Y -axis and tilts about the X -axis were evaluated for simplicity. Twelve fields were used to sample the field of view. The estimated RMS wavefront changes based upon the root-sum-square method at each zoom position were calculated using Zemax OpticStudio and are summarized in Fig. 16, which shows the nominal RMS wavefront error in dark color and its estimated change in light color.

The sensitivity results show that surface and element decenter impact most of the RMS wavefront lens performance. However, as two surfaces make a lens and are coupled, the element decenter has less impact than the surface decenter. The impact from tilt and decenter is uniform across the zoom range. This sensitivity analysis provides a first estimate about how tilts and decenters would affect the RMS wavefront performance, and provides a first useful estimate about the order of the tolerances needed during manufacturing and assembly.

5 Conclusion

This paper discusses fisheye lenses and design details of a zoom lens for circular and diagonal fisheye imaging. The lens features a large aperture compared to current designs

on the market while maintaining constructional simplicity as it uses only 11 lens elements in two groups. The aberration control of this design is explained using fourth-order aberration theory, and computer optimization details are also provided. The optical performance of the lens is illustrated with aberration and MTF plots. The analysis indicates excellent imaging performance considering the F -numbers at which the lens works. Other critical design issues, such as image space CRA, relative illumination and manufacture sensitivities are also addressed in this paper. Overall, a zoom fisheye lens design with good optical performance is provided and explained in detail.

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