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STIMULATED LINEAR ACCELERATION RADIATION:
A PULSAR RADIO EMISSION MECHANISM

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ABSTRACT

The Einstein coefficient method is used to calculate the absorption coefficient κ for a steady flux of electrons undergoing acceleration in a uniform electric field. Under the approximation that the motion is ultrarelativistic, the emission frequency is low, and the emission direction is nearly parallel to the direction of motion, we find that κ is negative for certain emission directions. The formalism is applied to pulsars, and it is shown that strong maser action is expected. If the emission takes place near the velocity-of-light cylinder, the computed collisional Bremsstrahlung may be strong enough to produce the recently observed high-energy γ -rays.

I. INTRODUCTION

In this paper, we examine the possibility that the radio emission from pulsars is stimulated linear acceleration radiation. It is well-known that magnetized rotating neutron stars are theoretically capable of accelerating charged particles to very high energies, and we here take the point of view that the electric fields involved in the acceleration process are quasi-static (Gold 1969, Goldreich and Julian 1969) and are not of the low-frequency rotating dipole radiation type considered by some authors (Pacini 1967, 1968; Gunn and Ostriker 1971).

Ordinarily, very little radiation is emitted by a charged particle performing motion in one dimension, but in our treatment we consider the induced emission and absorption appropriate to a flux of charged particles. The absorption coefficient κ is found by the Einstein A- and B-coefficient method, and we show that for ultrarelativistic motion and low frequencies, the absorption coefficient is negative for certain ranges of the angle of emission with respect to the field direction.

For pulsars, we are particularly interested in the case where the acceleration is along a magnetic field line. Here, the particle motion transverse to the magnetic field is strongly damped by synchrotron radiation and quantization effects, and κ is different from the pure electric field case. Again, however, κ is negative for certain ranges of the emission angle, and for parameters appropriate to pulsars, the total optical depth τ of the acceleration region satisfies $\tau \ll -1$. Thus, strong maser amplification can occur.

This conclusion is not critically dependent on one's choice of pulsar magnetosphere model, and seems to hold for models where the pulsed radiation is generated either near the surface of the neutron star or near the velocity-of-light cylinder.

In Sec. II, we develop the basic formulation of the Einstein coefficient method as applied to particles undergoing acceleration in a uniform field. This formalism is developed further in Sec. III under the approximation that the emission frequency is low, the angle of emission is small, and the relativistic factor $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ satisfies $1 \ll \gamma_{\text{final}} \gg \gamma_{\text{initial}}$.

Section IV applies the approximate theory to radio emission from pulsars, where we consider the two cases of emission near the surface of the rotating neutron star and near the velocity-of-light cylinder. In the latter case, for the Crab Nebula pulsar NP 0532, the total quantity of charge involved can be evaluated from observations of the nebular synchrotron radiation from the Crab Nebula and is shown to be much too large to allow much charge separation. Thus one may have counter-streaming charge populations of opposite sign in the same acceleration region. The resultant collisional Bremsstrahlung is estimated and shown to be perhaps large enough to account for the recent observations of high-energy pulsed γ -rays from NP 0532.

II. THE ABSORPTION COEFFICIENT

We now consider in detail the calculation of the absorption coefficient κ of the region in which the quasi-static electric acceleration operates. We assume that the field is uniform and straight, such as between two infinite, uniformly charged capacitor plates separated by a distance L . This motion along a straight field line applies also to the case where the electron is accelerated along a straight magnetic field line, with $\vec{E} \cdot \vec{B} \neq 0$, $E \ll B$, where E is the electric field strength in esu, and B is in gauss. This case, which is the one of astrophysical interest, is discussed further in Sec. III.

We use the Einstein coefficient method to calculate the absorption coefficient (Einstein 1917, Twiss 1958, Bekefi et al 1961, Bekefi 1966). We compute the A-coefficient under the assumption that the only "spontaneous" emission is the usual linear acceleration radiation: A uniform flux of electrons moves through the negatively charged capacitor plate and is accelerated to ultrarelativistic energies by the electric field, which acceleration results in the emission of a certain amount of radiation according to the usual classical electrodynamic expressions (Jackson 1962). Except for this radiation the total energy (kinetic and potential) of each electron is conserved, and the electrons may be thought of as making radiative transitions while in the acceleration region.

Since the initial kinetic energy of each electron is non-negative, they can make transitions to states which have classical turning points inside the acceleration region, and thus the flux of electrons into the acceleration region constitutes a "population inversion," which is one of the necessary conditions for maser action.

The use of the Einstein coefficient method implies that the relative change in the radiation intensity across the acceleration region is small. We show in Sec. IV, however, that for the parameters appropriate to pulsar radio emission, there are directions of propagation such that $\kappa < 0$ and $\tau = \kappa L \ll -1$. Thus the Einstein coefficient formalism is in principle not capable of solving this problem completely.

We define the Einstein coefficients in the following way: Let us label the electron states according to their initial momenta $\underline{p}_i$, and let $\phi(\underline{p}_i)$ be the electron column density per unit volume in initial-momentum space, so that $\phi(\underline{p}_i)d^3\underline{p}_i$ has units electrons/cm². Then if $I(\underline{k})$ is the specific intensity at photon wave-vector $\underline{k}$ in erg cm⁻² sec⁻¹ Hz⁻¹ ster⁻¹, the induced change in intensity ΔI over the region is

$$\Delta I(\underline{k}) = h\nu I(\underline{k}) \int [B(\underline{p}_i \rightarrow \underline{p}_i - \hbar \underline{k}) \phi(\underline{p}_i) - B(\underline{p}_i - \hbar \underline{k} \rightarrow \underline{p}_i) \phi(\underline{p}_i - \hbar \underline{k})] d^3 \underline{p}_i \quad (1),$$

where $B(\underline{p}_1 \rightarrow \underline{p}_2)$ is the B-coefficient for an electron to make an induced radiative transition from the state characterized by initial momentum $\underline{p}_1$ to the state characterized by initial momentum $\underline{p}_2$, and where

$\nu = ck/2\pi$. The Einstein relations are (Einstein 1917)

$$B(\underline{p}_1 \rightarrow \underline{p}_2) = B(\underline{p}_2 \rightarrow \underline{p}_1) = A(\underline{p}_1 \rightarrow \underline{p}_2) c^2 / h\nu^3 \quad (2),$$

The A-coefficient is the spontaneous transition rate per second per electron per Hz per ster. The classical single-particle radiation formula (Jackson 1962), however, gives us the total radiation $w(\underline{p}_i, \underline{k})$ in ergs per electron per Hz per ster produced across the entire path. Therefore, for our steady flux of electrons,

$$A(\underline{p}_i \rightarrow \underline{p}_i - \hbar \underline{k}) = w(\underline{p}_i, \underline{k}) / [T(\underline{p}_i) h \nu] \equiv A(\underline{p}_i, \underline{k}) \quad (3),$$

where $T(\underline{p}_i)$ is the total travel time across the acceleration region.

We assume that the total amount of energy radiated by the electrons from this spontaneous process is much less than the total kinetic energy gained in the acceleration region. Thus $w(\underline{p}, \underline{k})$ and $T(\underline{p})$ are computed simply by solving the Lorentz force equation without the radiation reaction terms.

The optical depth is then found from Eqns. (1) and (2) as

$$\tau(\underline{k}) = \frac{\Delta I}{I} = \frac{c^2}{\nu^2} \int A(\underline{p} + \hbar \underline{k} \rightarrow \underline{p}) [\phi(\underline{p}) - \phi(\underline{p} + \hbar \underline{k})] d^3 \underline{p} \quad (4).$$

To simplify the problem further, we now suppose that all the electrons have the same initial momentum $\underline{p}_0$, and we write $\phi(\underline{p}) = nL\delta(\underline{p} - \underline{p}_0)$, where n is the average electron density in the acceleration region. Eqns. (3) and (4) then imply

$$\kappa = \frac{nc^2}{\nu^2} [A(\underline{p}_0 + \hbar \underline{k}, \underline{k}) - A(\underline{p}_0, \underline{k})] \approx \frac{nc^2}{\nu^2} \hbar \underline{k} \cdot \nabla_{\underline{p}_0} A(\underline{p}_0, \underline{k}) \quad (5)$$

Note that we have neglected the effects of a possibly anisotropic index of refraction in the medium (Twiss 1958). In Sec. III we compute the anisotropic plasma frequency in the medium and point out that it may provide a natural low-frequency cut-off. For pulsars, maser saturation is shown in Sec. IV to occur from beyond this cut-off up to two or three decades higher in frequency, where dispersion effects of this sort may certainly be neglected. In the present work we ignore the effects of magnetic fields on the index of refraction, which hopefully will be treated in a later paper. We comment further on this point in Sec. V.

It is now necessary to compute the function $w(\underline{p}_i, \underline{k})$, and to do this we must solve the Lorentz force equation for an electron with arbitrary initial momentum $\underline{p}_i$. This is an elementary exercise in relativistic mechanics, and we let e be the electronic charge in Gaussian esu, m the electron rest mass, and E the electric field strength (assumed to be in the x -direction). We define $a \equiv mc^2/eE$, $\beta \equiv v/c$, $\pi \equiv \underline{p}_i/mc$, $K^2 \equiv 1 + \pi_y^2 + \pi_z^2$, $\zeta \equiv ct/a$, where t is the coordinate time. One can show that for a uniform electric field in the x -direction, the Lorentz force equation can be solved to find the velocity components as a function of time as $\beta_x = (\zeta + \pi_x)/\gamma$, $\beta_y = \pi_y/\gamma$, and $\beta_z = \pi_z/\gamma$, where $\gamma^2 = (\zeta + \pi_x)^2 + K^2$. One may obtain the position as a function of ζ as $x = a\gamma$, $y = \pi_y a \ln [(\zeta + \sqrt{\zeta^2 + K^2})/K]$, $z = \pi_z y / \pi_y$.

It is then straight-forward to find the total crossing time $T(\pi)$ as

$$T(\pi) = \frac{1}{c} [L^2 + 2aL(\pi_x^2 + \pi_y^2)^{\frac{1}{2}} + a^2 \pi_x^2]^{\frac{1}{2}} - \frac{a}{c} \pi_x \quad (6).$$

The function $w(\underline{p}_i, \underline{k})$ may be written from classical electro-dynamics as (Jackson 1962)

$$w(\underline{p}_i, \underline{k}) = \frac{e^2}{4\pi c} |\underline{Q}|^2 \quad (7),$$

where

$$\underline{Q}(\underline{p}_i, \underline{k}) \equiv \int_0^Z \frac{\hat{n} \chi [(\hat{n} - \beta) \chi \beta'] e^{i\tilde{\omega}(\zeta - \hat{n} \cdot \lambda)}}{(1 - \beta \cdot \hat{n})^2} d\zeta$$

with the definitions $Z \equiv cT/a$, $\lambda \equiv [r - r(0)]/a$, $\tilde{\omega} \equiv a\omega/c$, $\hat{n} \equiv \underline{k}/k$, and the prime indicating $d/d\zeta$. It is convenient to do an integration by parts and write

$$Q = \left[e^{i\tilde{\omega}(\zeta - \hat{n} \cdot \lambda)} \frac{\hat{n} \cdot \chi(\hat{n} \chi \beta)}{1 - \beta \cdot \hat{n}} \right]_0^Z - i\tilde{\omega} \int_0^Z \hat{n} \cdot \chi(\hat{n} \chi \beta) e^{i\tilde{\omega}(\zeta - \hat{n} \cdot \lambda)} d\zeta \quad (8),$$

where we must retain the boundary terms because of the assumed smallness of $\tilde{\omega}$. The above formalism has lumped together the different polarization states of the radiation, but for $\gamma_{\text{final}} \gg \gamma_{\text{initial}}$, the radiation is almost completely linearly polarized in the x-direction.

We are now equipped to find the absorption coefficient by substituting the above expressions for $r(\zeta)$, $\beta(\zeta)$, and $T(\pi)$ into Eqns. (7) and (8) and then into Eqn. (5) via Eqn. (3). Unfortunately, it would be impossible to carry out the indicated integrations, and thus we proceed to an approximate treatment.

III. APPROXIMATE TREATMENT

In this section we evaluate K with the help of several approximations; namely, for small $\tilde{\omega}$, large $\gamma_{\text{final}} \approx Z$, small angles of emission, and $\gamma_{\text{final}} \gg \gamma_{\text{initial}}$.

First, let us evaluate the argument of the exponential in the boundary term of Eqn. (8). For $Z \equiv cT/a \approx eEL/mc^2 \approx \gamma_{\text{final}}$, $\hat{n}_x = \cos\theta \approx 1 - \frac{1}{2}\theta^2$, and $\pi = 0$, we have $\tilde{\omega} [Z - \hat{n} \cdot \lambda(Z)] = \tilde{\omega}(Z - \hat{n}_x \gamma_{\text{final}}) \approx \frac{\tilde{\omega}}{2Z} (\theta^2 Z^2 - 1)$, where the approximation $\gamma_{\text{final}} = (1 + Z^2)^{\frac{1}{2}} \approx Z(1 + \frac{1}{2}Z^{-2})$ has been employed. Then the exponential may be approximated by unity, provided that

$$\frac{\tilde{\omega}}{2Z} |\theta^2 Z^2 - 1| \ll 1 \quad (9).$$

The denominator in Eqn. (8) may be approximated by

$$1 - \beta \cdot \hat{n} \approx 1 - Z(Z^2 + 1)^{-\frac{1}{2}}(1 - \frac{1}{2}\theta^2) \approx \frac{1}{2}(\theta^2 + Z^{-2}) \quad (10),$$

and the derivatives of $T(\pi)$ may be evaluated by means of Eqn. (6) as

$$\left[\frac{\partial T}{\partial \pi_x} \right]_0 = -1, \quad \left[\frac{\partial T}{\partial \pi_y} \right]_0 = \left[\frac{\partial T}{\partial \pi_z} \right]_0 = 0 \quad (11),$$

where the subscript 0 means evaluation at $\pi = 0$.

Finally, one may differentiate Q , set $\pi = 0$, and then use Eqns. (9), (10), and (11), together with the assumption $\tilde{\omega} \lesssim 1$. After rather lengthy algebraic manipulations, one then finds that, to highest order in Z and lowest order in θ , the integral term in Eqn. (8) may be neglected altogether and one may write, setting $n_y \approx \theta$ so that the propagation vector lies in the x-y plane

($\hat{n}_z = 0$), with $F \equiv (1 + \theta^2 Z^2)^{-1}$,

$$Q_{xo} \approx -2\theta^2 Z^2 F, \quad Q_{yo} \approx 2\theta Z^2 F, \quad Q_z = 0 \quad (12).$$

Further, we have

$$\left[\frac{\partial Q_x}{\partial \pi_x} \right]_o \equiv Q_{xxo} \approx \theta^2, \quad \left[\frac{\partial Q_y}{\partial \pi_x} \right]_o \equiv Q_{yxo} \approx -\theta \quad (13),$$

where we have used the relation $[\partial(Z + \pi_x)/\partial \pi_x]_o = 0$.

In addition, one can show that

$$Q_{xyo} \approx 2Z\theta(1 - \theta^2 Z^2)F^2 - \theta, \quad Q_{yyo} \approx 2Z(\theta^2 Z^2 - 1)F^2 + 1 \quad (14).$$

The quantities

$$\hat{n} \cdot \nabla_{\pi} \left[\frac{|Q|^2}{Z} \right]_o \approx \frac{\partial}{\partial \pi_x} \left[\frac{|Q|^2}{Z} \right]_o + \theta \frac{\partial}{\partial \pi_y} \left[\frac{|Q|^2}{Z} \right]_o \equiv f_x + f_y$$

may now be evaluated, so that one obtains from Eqns. (11)-(14)

$$f_x \approx 4\theta^2 Z^2 (1 - \theta^2 Z) F^2$$

$$f_y \approx 8\theta^2 Z^2 (\theta^2 Z^2 - 1) F^3 + 4\theta^2 Z F.$$

Then since $L/a = eEL/mc^2 \approx Z$, Eqn. (5) implies $\tau = \frac{1}{4} R Z (f_x + f_y) \equiv \tau_x + \tau_y$, where $R \equiv ne^2/(\pi^2 m v^2)$, and

$$\tau_x = R\theta^2 Z^3 (1 - \theta^2 Z) F^2 \quad (15),$$

$$\tau_y = 2R\theta^2 Z^3 (\theta^2 Z^2 - 1) F^3 + R\theta^2 Z^2 F \quad (16).$$

We have kept the x- and y- contributions to τ separate because of the suppression of the electron motion transverse to the magnetic field line by synchrotron radiation or quantization effects. In this case we may throw τ_y away, and note that $\tau \approx \tau_x < 0$ for $\theta > Z^{-\frac{1}{2}}$. Then parametrizing $\theta = \alpha Z^{-\frac{1}{2}}$, one can write for $Z \gg 1$, and $\alpha \gtrsim 1$.

$$\tau \approx \tau_x \approx R(1 - \alpha^2)/\alpha^2 \approx 3 \times 10^7 n (1 - \alpha^2)/(\alpha v)^2 \quad (17),$$

where n is in cm^{-3} and ν in Hz.

If, on the other hand, the magnetic field does not suppress the transverse motion, one can easily show that the dominant terms are

$$\tau = \tau_x + \tau_y \approx R\theta^2 Z^3 (3\theta^2 Z^2 - 1) F^3 \quad (18).$$

Note that the regime of negative τ has now shifted to $\theta < (3^{1/2} Z)^{-1}$.

We now discuss the relationship of our parameter R to the plasma frequency. Indeed, the quantity $\pi R \nu^2 = ne^2/\pi m \equiv \tilde{\nu}_p^2$ is the square of the plasma frequency for a cold plasma with electron density n . In our case, however, the electrons are streaming with ultrarelativistic energies, and hence one must evaluate the plasma frequency in the rest frame of the electrons. One can show that in a plasma where there are excess charges of one sign, the plasma frequency ν_{po} in the rest frame of the electrons is still $\nu_{po}^2 = n_o e^2/\pi m$, where n_o is the total electron rest density. The transformed plasma frequency ν_p in our laboratory frame is then given by the Doppler formula $\nu_p = \nu_{po}/[\gamma(1-\beta\cos\theta)] \approx 2Z\nu_{po} F$. But $n_o = n/\gamma \approx n/Z$, and hence

$$\nu_p^2 \approx 4Z \tilde{\nu}_p^2 F^2 \quad (19).$$

We may therefore write

$$R \approx \nu_p^2 / (4\pi Z \nu_F^2) \quad (20),$$

where ν_p is the real, anisotropic plasma frequency in the laboratory frame.

The question to be answered is whether or not ν_p provides a low-frequency cut-off for the radiation. If it does, then we must have

$$\nu > \nu_p, \text{ and for the pulsar case where } \tau \approx \tau_x, \theta = \alpha Z^{-1/2} \text{ gives via Eqn. (17)}$$

for $\alpha \gtrsim 1$

$$\tau \approx \nu_p^2 Z \alpha^2 (1 - \alpha^2) / (4\pi \nu^2) \quad (21).$$

Then for $Z \approx \gamma_{\text{final}} \gg 1$, we can still have strong amplification for $\nu > \nu_p$. Indeed, one is tempted to write $\Delta I/I = \exp(-\tau) \approx \exp(Z \nu_p^2 / \nu^2)$! However, for large Z this is certainly meaningless and shows only that the "maser" must be saturated.

But if the transverse particle motion is not suppressed, the domain of negative τ is for $\theta Z < 3^{-\frac{1}{2}}$, and Eqns. (18) and (20) imply that $\tau \approx -C \nu_p^2 / (4\pi \nu^2)$, where C is less than one. In this case the amplification will be inappreciable for $\nu > \nu_p$, and maser action cannot occur.

However, one can ask whether or not ν_p actually provides such a low-frequency cut-off. The quantity assumed small in Eqn. (9) is, for $\theta \lesssim Z^{-\frac{1}{2}}$, larger than, or of the order of, the number of wave-lengths of the wave actually seen by the plasma, and in assuming that this quantity is small, we have limited our treatment to frequencies such that the Doppler-shifted wave length seen by the plasma is much greater than the Lorentz-contracted length of the acceleration region. Since the accelerated plasma only sees a small fraction of the phase of the wave, it is not clear whether or not the concept of a "plasma frequency" has any real meaning. Any curvature in the field line outside the acceleration region would quickly make $\theta \gg Z^{-1}$, resulting in a sharp drop in ν_p along the direction of propagation.

On the other hand, in the non-suppressed case, any appreciable curvature of the electric field inside the accelerating region would restrict the amount of amplification, since radiation produced in the rear of the layer would

intersect the field lines at successively greater angles until $\theta Z < 3^{-\frac{1}{2}}$ is violated.

But for the suppressed case (Eqn. (21)), which is the one of astrophysical interest, amplification occurs for $\theta > Z^{-\frac{1}{2}}$, and thus field-line curvature away from the propagation direction does not hamper the amplification.

IV. APPLICATION TO PULSARS.

It is quite difficult to apply our results directly to pulsars, since there is no general agreement on the basic structure of their magnetospheres. We here consider two cases: Case I, with quasi-static acceleration taking place near the neutron star surface, where the electrons and protons are completely charge-separated, with densities given by the electrodynamic models of Goldreich and Julian (1969) and Sturrock (1971); and Case II, with the quasi-static acceleration taking place near the velocity-of-light cylinder. In the latter case, for NP 0532, one can estimate the charge densities and energies from the observed synchrotron spectrum from the nebula itself. Thus γ may be estimated for both cases.

Case I: We assume, as does Sturrock (1971), that the radio emission is generated near the surface of the rotating neutron star. In Sturrock's model, the charge density near a polar cap is given by his Eqn.(3.2), which we write as $n = B/(ecP)$, where P is the pulsar rotational period. This is the same relation as the one derived by Goldreich and Julian (1969), but in their theory it is meant to apply to the equatorial, strictly co-rotating part of the magnetosphere. Evaluating this expression for NP 0532, for which $P = 0.033$ sec, and using $B=10^{12}$ gauss, we obtain $n = 2 \times 10^{12} \text{ cm}^{-3}$. Thus $\tilde{\gamma}_p^2 = 2 \times 10^{20} \text{ Hz}^2$.

Sturrock's model also furnishes us with a value for the maximum energy attained by electrons in this region. From his Eqn. (3.14) we find, using $R = 10^6 \text{ cm}$, that $E_{\text{max}} \approx 10^{14} \text{ eV}$, corresponding to $Z \approx 2 \times 10^8$, which is quite high. The plasma frequency may be found for $\theta = \alpha Z^{-\frac{1}{2}}$ ($\alpha \gtrsim 1$) from Eqn. (19) as

$$\gamma_p^2 \approx 4 \tilde{\gamma}_p^2 / \alpha^4 Z \quad (22),$$

whereupon $\nu_p \approx 2 \times 10^6 / \alpha^2$, which is below present observational limits.

We note that pulsed radiation from NP 0532 has been observed down to less than 10^8 Hz, and that it is very likely that the low-frequency compact radio source in the Crab Nebula is simply the pulsed radiation smeared out by propagation effects in the interstellar medium (Drake 1971). This compact source has been observed down to 2×10^7 Hz (data by Bridle, quoted by Drake 1971).

The general dependence of ν_p on pulsar period according to Sturrock's model may easily be found. For $P \lesssim 1$ sec, the electron energies are radiation-reaction limited, and one may show that, for $\theta \gtrsim Z^{-1/2}$, $\nu_p \propto P^{-7/16}$. For $P \gtrsim 1$ sec, the electrons are not radiation limited and his Eqn.(3.15) applies. Then $\nu_p \propto P^{1/2}$, and for $P = 3$ sec, we have $Z \approx 3 \times 10^6$ and $\nu_p \approx 10^6 / \alpha^2$. Thus the ν_p for presently observed pulsars are all below the ionospheric cut-off.

The "amplification factors" $\exp(-\tau)$ are thus typically very large. For example, at $\nu = 10^8$ Hz and $\alpha = 2$, we have for NP 0532 $\tau \approx -5 \times 10^3$, and for $\nu = 10^{10}$ Hz, $\tau \approx -1/2$. Thus the amplification cuts off at high frequencies. We note that the position of the high-frequency cut-off is proportional to $n^{1/2}$, which in Sturrock's theory is proportional to $P^{-1/2}$. It would be interesting to see whether or not short-period pulsars tend to have these cut-offs at higher frequencies than long-period ones. Unfortunately, extensive and reliable data on pulsar spectra is not yet available (see, for example, Hewish 1970, Smith 1972).

We must check to see whether our low-frequency condition Eqn. (9) is satisfied for Case I. For $\theta = Z^{-1/2}$, we must have

$$\tilde{\omega}(\theta^2 Z^2 - 1)/2Z \approx \frac{1}{2} \tilde{\omega} = a\omega/2c \approx \pi L/Z\lambda \ll 1 \quad (23),$$

since $a = mc^2 L / (eEL) \approx L/Z$. Then for $L = 10^6$ cm, $Z = 10^6$, and $\lambda = c/10^8$ Hz = 300 cm, $\pi L/Z\lambda \approx 10^{-2}$. Thus the condition is satisfied.

Case II: We now investigate the possibility that the radio emission is produced near the velocity-of-light cylinder. The parameters pertaining to this case are more uncertain than those of Case I since there are no quantitative models at all for this part of the magnetosphere. For NP 0532, however, the parameters n and Z can be estimated by considering the synchrotron spectrum of the Crab Nebula, since the nebular optical and X-ray emission must be from particles that are continually being injected by the pulsar. The optical and X-ray spectral index is about $\alpha = -1$ (Baldwin 1971), and the corresponding electron energy distribution index is $\epsilon = 2\alpha = -2$ (Kardashev 1962). The usual relation $\epsilon = 2\alpha - 1$ does not apply here because of the high synchrotron energy loss rate. Now there must be a low energy cut-off to the injected electron energy distribution, since otherwise the radio spectrum would have a steeper slope than observed. This puts most of the injected electrons, and the average injected electron energy, near an energy such that the corresponding critical frequency ν_c is between 10^{13} and 10^{15} Hz, since for an energy spectrum $N(E) = KE^{-2}(E_1 < E < E_2, E_2 \gg E_1)$, $\langle E \rangle = E_1 \ln(E_2/E_1)$. Further, it is known that the magnetic field strength in the nebula is about 3×10^{-4} G (Shklovsky 1968). It therefore follows from the well-known relation $\nu_c \approx eB^2/mc$ (Jackson 1962) that $Z = 10^{6 \pm 1}$ for the region where these electrons are accelerated.

Now, the total energy loss from the pulsar in high-energy particles is about 2×10^{38} erg/sec (Finzi and Wolf 1969), and thus the total particle flux is, for electrons, $F = 2 \times 10^{38 \pm 1} \text{ sec}^{-1}$. For NP 0532, the velocity-of-light region is at $R \approx 10^8$ cm, and this implies a particle density of $n \approx F/R^2 c = 5 \times 10^{11 \pm 1} \text{ cm}^{-3}$, where we have left out a factor of 4π to allow for beaming effects. Hence, $\tilde{\nu}_p^2 = 5 \times 10^{19 \pm 1}$, and at $\theta = \alpha Z^{-1/2}$, Eqn.(22) gives $\nu_p = 1.5 \times 10^{7 \pm 1} / \alpha^2$, and at 10^8 Hz and $\alpha = 2$, one finds $\tau = -200$ for $Z = 10^7$ and,

$\tau = -2 \times 10^4$ for $Z = 10^5$. Thus the conditions are still appropriate for maser action, but perhaps with a lower high-frequency cut-off than for Case I. NP0532 has been detected at 1664 MHz (Manchester 1971), but not at higher frequencies.

We must again check to see whether we have violated our low frequency condition $\frac{1}{2}\tilde{\omega} = \frac{\pi L}{Z\lambda} \ll 1$. For $L=10^8$ and $Z=10^6$ we see that $\frac{1}{2}\tilde{\omega} = 1$ for 10^8 Hz. Thus our approximation breaks down for frequencies this high.

With $F \approx 2 \times 10^{38} \text{ sec}^{-1}$, the gross current flowing from the pulsar would be 10^{29} esu/sec. Such an enormous flow of charge could not possibly be totally charge-separated, since otherwise the corresponding electric field would be $E \approx eF/Rc \approx 3 \times 10^{10}$ esu over a scale of 10^8 cm. This would accelerate particles to energies $\gamma_{mc}^2 \approx eER \approx e^2 F/c \approx 2 \times 10^9$ erg, which is much too large. Therefore, in the acceleration region, charges of opposite sign would be streaming in opposite directions with respect to each other, or at least would be merging together. This would subject the plasma to relativistic two-stream instabilities, which might be the cause of the observed intensity fluctuations in many pulsars, or of the "marching subpulse" phenomenon.

Another interesting consequence of two counter-streaming charge systems is the production of γ -rays by collisional Bremsstrahlung. Following arguments given by Jackson (1962), one can show that the total energy cross-section Σ for the production of photons from collision between heavy particles of energy $\gamma' m' c^2$ and light particles of energy $\gamma m c^2$ is, for $\gamma \gg \gamma' \gg 1$, $\Sigma \approx 5 \alpha r_0^2 \gamma m c^2 [\ln(\gamma' \gamma) + 1]$, where $r_0 = e^2/mc^2$ and here α is the fine-structure constant. For $\gamma' = 10^3$ and $\gamma = 10^{6 \pm 1}$, we have $\Sigma = 5 \times 10^{-26 \pm 1}$ erg cm². If the total flux $F = 2 \times 10^{38 \pm 1} \text{ sec}^{-1}$ engages in this process, the total luminosity is $L \approx \Sigma n R F \approx \Sigma F^2 / Rc = 7 \times 10^{32 \pm 1} \text{ erg/sec}$, where the positive sign is for $Z=10^5$ and the negative for $Z=10^7$. Grindlay (1972) has observed a pulsed high-energy γ -ray flux of $10^{-10} \text{ photons cm}^{-2} \text{ sec}^{-1}$ for

$h\nu > 5 \times 10^{11} \text{ eV} \approx 0.8 \text{ erg}$. If the pulsar is at a distance of 2 kpc and if we allow a factor of 10 for beaming, one obtains a total observed high-energy luminosity of $3 \times 10^{33} \text{ erg/sec}$. But for $Z = 10^5$ there are no photons in this energy range, and for $Z = 10^6$ the theoretical luminosity is $7 \times 10^{32} \text{ erg/sec}$. Thus our Bremsstrahlung mechanism appears to be possibly adequate for producing the observed γ -rays.

Note from Grindlay's Fig. 5 that the high-energy data are considerably higher than one would expect from a direct extrapolation of the 1 MeV data of Kurfess (1971). Thus the high-energy photons might be associated with a process quite distinct from the X-ray and 1 MeV photons.

V. DISCUSSION

We have seen that our stimulated linear acceleration process is capable of producing copious amounts of radio emission for parameters appropriate to pulsars. Since the negative absorption coefficient for $\theta > Z^{-\frac{1}{2}}$ sets up a maser condition which does not depend on the field lines being straight, one is forced to conclude that if the particles are being accelerated quasi-statically, maser action of this sort must be taking place.

Since we know from observations of the Crab Nebula that particles are indeed being accelerated by fields generated by the pulsar, the only way in which this argument might fail would be if the acceleration were not quasi-static, as in the non-linear inverse Compton acceleration of Gunn and Ostriker (1971), or if the particle acceleration were taking place so far away from the pulsar that the particle density would be too low to allow maser action at the observed frequencies. We note that "Case I" acceleration is predicted by electrodynamic pulsar magnetosphere models (Goldreich and Julian 1969, Sturrock 1971, Cohen and Rosenblum 1972).

There are, however, some indirect reasons for believing that the pulsed radiation comes from near the velocity-of-light cylinder. First, it seems possible to produce the very sharp peak in the main pulse of the optical emission of the Crab pulsar, which is unresolved to about $30 \mu \text{ sec}$ (Horowitz et al 1972), and the observed displacement of the polarization minima (Cocke et al 1970, Kristian et al 1970), by compression and beaming from a relativistically moving source (Smith 1970, Ferguson 1971). Secondly, the time-dependent linear polarization of the pulsar is fit quite well by a relativistic rotating vector model (Ferguson 1973, Cocke et al 1973), with velocities of the emission regions near $\beta = \frac{1}{2}$. This implies $R \approx 8 \times 10^7 \text{ cm}$.

Thirdly, Zheleznyakov and Shaposhnikov (1972) have fit the optical, X-ray, and γ -ray spectrum of the Crab pulsar with synchrotron radiation from a source orbiting with a speed $\beta = 0.725$. It is well-known (Drake 1971) that the optical and radio pulses of the Crab pulsar must originate within 6×10^6 cm of each other, so that the location of the radio pulse source is pretty well pinned down by knowing the location of the optical source.

Thus there is some reason to feel that, at least for NP 0532, the radiation is coming from near the velocity-of-light cylinder. Unfortunately, we have seen that our approximation based on $\tilde{\omega} \ll 2$ breaks down in this case, and hence it will be necessary to extend the calculations.

Many pulsars, such as CP 0834 and CP 1133, show double-humped pulses, with the component appearing first in time being usually the stronger (Lyne et al 1971, Fig. 11). If the field lines are curved so that they "trail" the rotation, then the radiation originating in the leading half of the region $\theta > Z^{-\frac{1}{2}}$ remains always in this region as it proceeds outward, whereas radiation from the trailing half has to cross into the region $\theta < Z^{-\frac{1}{2}}$ on its way out. Thus the leading pulse component would be expected to be more intense.

It is a common belief that pulsar radiation is essentially a broad-band phenomenon (e.g., Smith 1970). However, there is considerable evidence (Hewish 1970, Drake 1972) that individual pulses show a great deal of fine structure in their spectra. In any case it is necessary to carry out a proper maser-theory calculation for our stimulated process in order to see whether the theoretical emission spectrum is broad-band, or whether the emission is very sharply peaked in frequency space.

As we have mentioned, the polarization produced by our mechanism is at first sight strictly linear. However, the strong magnetic fields involved might suppress the extraordinary propagation mode, with the result that the

emerging radiation would be predominantly circular, but quantization effects transverse to the field might deactivate this mechanism.

It is possible that the large negative optical depths encountered here would render the medium so unstable that the steady electron flux would be broken up, with resulting strong coherent radiation from bunches or sheets. This would probably depend on the relative strength of the maser radiation field produced, since the bunching would be limited by Coulomb forces. Many pulsars show very stable intensity and polarization patterns, whereas others are much more irregular (Lyne et al 1971). Perhaps the irregular ones are afflicted by instabilities of this sort.

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