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A COMPARISON OF VARIABLE AND
NON-VARIABLE STARS IN THE CEPHEID STRIP

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ABSTRACT

Spectra of three Cepheids (RX Aur, X Cyg and T Mon) and 11 non-variable supergiants are analyzed. Additional evidence is presented for the presence of non-variables within the Cepheid strip. Temperatures determined from H α line profiles and broad band photometry were used together with model atmospheres and line strengths from the spectra to determine the gravities, turbulent velocities and element abundances. The gravities of the Cepheids appear to be lower than for the non-variables at the same luminosity while the turbulent velocities are higher. No differences of significance are found in the element abundances. The gravity differences can be explained either in terms of mass loss in the Cepheids or very low helium abundance in the non-variables. The former interpretation fails to explain why some Cepheid strip stars do not vary while the latter raises problems in connection with the origin of the elements. If we assume that pulsation causes some mass loss a more consistent picture emerges in which only moderate helium deficiencies are required in the non-variables.

I. INTRODUCTION

It has been shown that there are non-variable supergiants which occupy the same region of the color-magnitude diagram as the classical Cepheids (Fernie and Hube 1971; Schmidt 1972a). This result is of interest in two connections. Firstly, the location of the hot edge of the instability strip is an important parameter from pulsation theory which can be compared with observational data (e.g., Iben and Tuggle 1972a,b). However, when there is not a clear separation between variable and non-variable stars, the location is ambiguous. Secondly, the fact that stars at the same luminosity and temperature may or may not vary provides evidence for differences between individual stars which are not evident from a superficial comparison. In this paper a detailed spectroscopic comparison is made between several Cepheids and a number of non-variables located within the Cepheid strip. The purpose of this study has been to search for empirical differences between the two sets of stars. Establishment of the existence of such differences might provide valuable insight concerning the reasons for the presence of stable stars within the instability strip.

To illustrate the type of effect we have looked for, we note that Cox, King and Tabor (1973) have pointed out that the helium abundance may be an important factor in causing the overlap of variable and non-variable stars. Since the edge of the instability strip depends on helium abundance, stars with little helium are stable within the instability strip for stars of higher helium abundance. Cox et al. show that if the mass fraction of helium is between 0.28 and 0.38 stars in the observed Cepheid strip will pulsate, while if the abundance is less than about 0.20 no star in the strip will pulsate. Although the helium abundance can not be measured directly, we can look to the gravity for an indication of anomalies. Two effects related

to the helium abundance may operate. The most important arises from the fact that the luminosity of a star evolving through the Cepheid region of the H-R diagram depends on the initial composition. Therefore, if we look at stars of the same luminosity and approximately the same effective temperature, the mass and, therefore, the gravity depend on the initial helium abundance. In particular, at a fixed luminosity a decrease in the initial helium abundance would require an increase in the mass and hence an increase in the surface gravity. The size of this effect can be quantitatively estimated from published theoretical calculations (Iben and Tuggle 1972a). The other effect depends on the atmospheric helium abundance. Since helium does not contribute appreciably to the opacity at the relevant temperatures, a low helium abundance will increase the mass absorption coefficient. The gas pressure of the line forming region of the atmosphere will then decrease and a lower spectroscopic gravity will be determined. Model atmospheres with low abundance of helium have been calculated in order to investigate this effect. In addition to the gravity, the abundances of the metals are investigated since any anomaly in helium abundance may well be accompanied by anomalies in other abundances as well.

It is also possible that pulsation could be inhibited by convection in the ionization zones (as was suggested by Parsons 1971). Since the hydrogen ionization zone is not very deep (at/optical depths of about 4.5 to 12.5 ^{Rosseland mean} pressure or about 0.25 to 1.6/scale heights below the photosphere) if there is an excessive amount of convection, it might show up at the surface as increased microturbulence. Thus, a comparison of turbulent velocities in Cepheids and non-variables might be useful in the consideration of this possibility.

Other factors which have been suggested as possibly being of importance are rotation (Preston 1965) and the direction of evolution (Schmidt 1972a).

Neither of these possibilities are investigated here. However, Kraft (1967) has argued that the supergiants in the region of the Cepheid strip are slow rotators. Therefore, it is unlikely that rotation is inhibiting pulsation in many stars. In a recent theoretical investigation of pulsational models, King, Cox, Eilers and Davey (1973) were unable to find evidence for hard oscillations in Cepheid models and therefore concluded that hysteresis effects are unlikely to be responsible for the presence of non-variables in the instability strip.

II. THE OBSERVATIONAL DATA

Spectra of 11 stars with spectral types between G0 and G5 and with luminosity classes Ib or Ia were analyzed. Additionally, spectra were used of three Cepheids which were each observed at several phases. These stars are listed in Table 1. The spectra were obtained with the coude spectrograph of the Kitt Peak National Observatory 2.1 meter telescope. The dispersion used was $17.8\text{\AA}/\text{mm}$. Some of the spectra were on Kodak 098-02 emulsion while the remainder were on Kodak IIa-F plates. Equivalent widths of about 110 lines (including 55 of FeI and 10 of FeII) were measured from these spectra following usual procedures. Most of these lines are in the region between the sodium D lines and H α . These equivalent widths together with the H α profiles from some of the spectra constitute the new data used in the present analysis. In the interests of space this data is not tabulated here but will be published separately.

III. TEMPERATURE DETERMINATIONS AND THE LOCATION OF NON-VARIABLES IN THE INSTABILITY STRIP

It has been shown that the determination of excitation temperatures from the metal lines can lead to systematic errors in the determination of

electron pressures or gravities (Schmidt 1971). To avoid such a problem, the effective temperatures of the stars studied here were determined independently of the metal lines. This was done using the $H\alpha$ profiles obtained from the spectra, the continuum scans of some of the stars which have been published previously (Schmidt 1972b) and published photometry on the UBVRI (Iriarte et al. 1965; Blanco et al. 1968; Wisniewski and Johnson 1968) and the 6-color (Kron 1958; Kron and Svolopoulos 1959) systems. The color excesses were obtained from the same sources as previously (Schmidt 1972c) and are listed in Table 1. The spectral types were not used in deriving these color excesses and provide a check on the values obtained for the non-variables. It was found that the color excesses indicated by the spectral types (using Johnson's 1966 spectral-type-color relation) differ by $0.^m.01 \pm 0.^m.08$ from those in Table 1. This seems to indicate that on the whole the adopted color-excesses are quite accurate.

The temperature derived previously for F and G supergiants and Cepheids (Schmidt 1972b,c) were used to obtain color-temperature relationships for the (B-V), (V-R), (R-I), (V-B)_{6-color} and (R-I)_{6-color} indices. The $H\alpha$ profiles were used in the same fashion as previously (Schmidt 1972b) to obtain estimates of effective temperature. The usable temperatures determined from each fitting are listed in Table 1 together with the adopted means. (B-V), (V-R) and (V-B)_{6-color} temperatures were omitted from the means in the case of the long period Cepheids because, as expected (Schmidt 1973), these temperatures disagree badly with those obtained from other colors. The temperatures from the various colors and from $H\alpha$ agreed well with systematic differences of no more than one percent. It should be noted that a different (B-V) — temperature relation was used for the Cepheids and the non-variables (Schmidt 1972c, 1973). However, in the range of temperatures of interest here, the two relations cross and there is little difference between them. Therefore, no error should arise from this source. We also note that the present

temperatures are based on the previously determined scale. Since this scale has been disputed (Böhm-Vitense 1972; van Paradijs 1973), its effect on the results will be considered in § IV.

The temperatures from Table 1 are plotted in Figure 1 against the spectral type. It can be seen that, as should be the case, they are well correlated. At any given spectral type the spread in temperature is only slightly greater than what would arise solely from the discreteness of spectral types.

Since doubt has recently been expressed concerning the strength of the evidence for the existence of non-variables in the instability strip (Cox et al. 1973), it is appropriate to review and extend the evidence at this point. For this purpose the temperature-^{absolute} magnitude diagram of the program stars is shown in Figure 2. The absolute magnitudes of the non-variables were obtained from K-line widths (Wilson and Bappu 1957), H α core widths (Kraft, Preston and Wolff 1964; Schmidt 1972b) and OI λ 7774 line strengths (Osmer 1972a). The period-luminosity-color relation of Sandage and Tammann (1969) was used for the Cepheids. The effective temperatures are from Table 1. The edges of the instability strip shown come from Schmidt (1972a) and are included for reference. It can be seen that the Cepheids and the non-variables chosen for this project actually occupy the same region of the diagram. Hence, a comparison should be meaningful.

In particular, Cox et al. have mentioned the uncertainty in the temperature scale in connection with the problem of the existence of non-variables within the Cepheid strip. We remark that while such uncertainty may lead to the stars being located incorrectly in the temperature-luminosity plane, Cepheids and non-variables are affected together. Hence, the conclusion that they occupy the same region is not dependent on the temperature scale. It is also true that the question of the ^{location of the} edges of the instability strip is not

relevant. Figure 1 shows that the non-variables surround the Cepheids in the temperature-luminosity diagram. Therefore, no matter how the edges are drawn, they must include some non-variables.

While discussing the temperatures of these stars, another question which should be raised is whether pulsation may affect the atmosphere of the Cepheids in some way which makes the static, radiative model atmospheres on which the temperatures are based inappropriate. In particular, if significant mechanical energy were dissipated in the atmosphere, it could change the temperature structure and cause the colors and the H α profiles to give wrong temperatures. However, pulsation calculations indicate that the mechanical energy of pulsation is only 5 to 10 percent of the luminosity (see Christy 1967). Of this, only a few percent is dissipated in the atmosphere with the bulk of it going into layers below the He ionization zone. Thus, we must expect the mechanical energy dissipated in the atmosphere to be less than 1/2 percent of the luminosity. For comparison, we note that line blanketing redistributes approximately 15 percent of the radiant energy in the atmospheres of these stars. Hence, pulsation should have little effect on the temperature structure of the atmospheres and there should be no systematic difference between the measured temperature scales of variables and non-variables (cf., Keller and Mutschlecner 1970).

Finally, regarding the absolute magnitudes, it can be seen from Figure 2 that some of the non-variables near the edge of the strip may have been placed in the strip due to errors in the determination of the absolute magnitudes of less than a magnitude. However, those near the center of the strip will need to be changed in M_V by more than one and a half magnitudes to bring them out of the strip. While the absolute magnitudes of some individual stars could conceivably be in error by that much, it is unlikely that of the present sample half a dozen should be. We also note that even though

the absolute magnitudes of the two groups of stars were found in different ways, the zero points of all the types of absolute magnitude determinations are ultimately based on the same Hyades distance modulus. It is therefore hard to see how a large systematic difference could arise from the use of different luminosity indicators. We conclude that doubts about the reality of non-variables in the Cepheid strip are poorly founded.

IV. MODEL CALCULATIONS, ANALYSIS AND RESULTS

Since we are interested in the gravities of the stars, the analysis of the line strengths using the traditional curve of growth method (with single layer curves of growth) is unsuitable. Furthermore, as previously noted, it has been shown (Schmidt 1971) that the use of excitation temperatures derived from metal lines can lead to systematic errors in derived gravities. On the other hand, the calculation of a model atmosphere for each star followed by line strength calculations with a variety of abundances and micro-turbulent velocities would require a prohibitive amount of computation. For this reason, an alternative scheme was adopted which incorporates the simplicity of the curve of growth but is based on more realistic model atmospheres. This method is similar to that used by van Paradijs (1973) except for the way of determining the temperatures.

A grid of model atmospheres was calculated with gravities in the range from 10 to 1000 cm sec^{-2} and effective temperatures from 5420°K to 6460°K.*

*The model temperatures have been corrected for backwarming since line blanketing was not included. For consistency with the effective temperature measurements (Schmidt 1972a,b) the correction was the following: $5040/T_{\text{eff}} = (5040/T_{\text{model}}) + 0.03$. All effective temperatures given in this paper contain this correction.

A helium abundance of 0.39 by weight was adopted. In addition, at one gravity ($\log g = 2$) models were calculated with low helium abundance (0.175) in order to test the effect of altering the helium content on the derived parameters. For each model, curves of growth were calculated for neutral and ionized iron with a turbulent velocity of 4 km/sec and excitation potentials of 1, 3 and 5 ev. The damping constants were chosen so as to give a curve with a damping parameter of $\log a = -3$. The results do not depend strongly on the damping parameter because the lines used are too weak. For all these calculations, the Smithsonian model atmosphere programs ATLAS5 and WIDTH5 were used.

In the analysis of the theoretical line strengths, a procedure was followed which is similar to that used previously (Schmidt 1971). For each model, the theoretical excitation temperature, $\theta_{\text{exc}} = 5040/T_{\text{exc}}$, was determined from the relative shifts of the curves for different excitation potentials. These values were then fitted to linear functions of effective temperature as follows:

$$\begin{aligned} \theta_{\text{exc}} &= 1.02 \theta_{\text{eff}} + 0.087 & \log g &= 1 \\ \theta_{\text{exc}} &= 1.06 \theta_{\text{eff}} + 0.057 & \log g &= 2 \\ \theta_{\text{exc}} &= 1.20 \theta_{\text{eff}} - 0.048 & \log g &= 3 \end{aligned} \quad (1)$$

These values of excitation temperature were found to be independent of other atmospheric parameters such as microturbulence, and the same for FeI and FeII. The theoretical curves of growth were then formed as plots of $\log W/\lambda$ against $X = \log \left(\frac{N_{\text{Fe}}}{N_{\text{H}}} g f \lambda \right) - \chi \theta_{\text{exc}}$. Since the shape of the curve did not differ significantly between the various models or between FeI and FeII, a standard curve was chosen to represent all of them. This was done for convenience in the use of the theoretical results. The standard curve is tabulated in Table 2 (normalized to $X = 0$ at $\log W/\lambda = -5$). The other theoretical curves were then fitted to the standard curve and the relative horizontal shifts for FeI and FeII, ΔX_{I} and ΔX_{II} , were found. It was also necessary to introduce

vertical shifts, ΔY , since the level of the flat portion of the curve of growth depended slightly on the gravity and temperature. The ΔX 's take into account the variations of ionization and continuous opacity with temperature and gravity. Their use is analogous to the use of electron pressure and ionization temperature with the Saha equation and the use of η_0 , the ratio of line to continuous opacity, in the classical curve of growth method. The ΔY takes into account the second order effects which arise from stratified models. It was found that the horizontal shifts could be represented by the following functions:

$$\Delta X_I = -85.296 + 20.964 \log T_{\text{eff}} - 0.141 \log g \quad (2)$$

$$\Delta X_{II} = 608.288 - 337.511 \log T_{\text{eff}} + 46.101 (\log T_{\text{eff}})^2 \quad \log g = 1$$

$$\Delta X_{II} = 621.812 - 342.850 \log T_{\text{eff}} + 46.588 (\log T_{\text{eff}})^2 \quad \log g = 2 \quad (3)$$

$$\Delta X_{II} = 635.337 - 348.188 \log T_{\text{eff}} + 47.074 (\log T_{\text{eff}})^2 \quad \log g = 3$$

The vertical shifts were fitted by the following:

$$\Delta Y = 6.939 - 1.823 \log T_{\text{eff}} - 0.025 \log g \quad (4)$$

No significant difference was found in ΔY between FeI and FeII. We note the fact that the level of the flat portions of the theoretical curves of growth are the same for FeI and FeII indicates that differences between microturbulent velocities of FeI and FeII which have been reported (e.g., Osmer 1972b) are probably the result of actual variations in the turbulence with atmospheric height rather than differences between the FeI and FeII curves of growth.

As anticipated, when the curves of growth calculated for the models with low helium abundance were compared with those for normal abundance, they gave the appearance of being normal models with slightly lower gravities. The differences were about 0.1 to 0.2 in $\log g$. Furthermore, since the effect of a change in the atmospheric helium abundance is similar to a

small change in the apparent gravity the temperatures obtained from the H α profiles should be relatively insensitive to the assumed helium abundance. Previous calculations (Schmidt 1972b) indicate that a change in $\log g$ of 1 produces a change in the temperature determined from H α of only about 60°K. Therefore reasonable variations in the helium abundance (which are equivalent to variations in $\log g$ much less than 1) do not produce any important uncertainty in the temperature determination.

Using the effective temperatures obtained in § III, the excitation temperatures were determined from equation (1) with $\log g = 2$. Plots of $\log W/\lambda$ vs. $\log (g\lambda) - \chi_{\text{exc}}^{\theta}$ were then constructed for the measured FeI and FeII line strengths. The effect of assuming the gravity at this point is small since neither the gravity nor the iron abundance is sensitive to the excitation temperature. Changing θ_{exc} by 0.03 would only alter $\log g$ by 0.09 and $\log N_{\text{Fe}}/N_{\text{H}}$ by 0.13. The variation of the excitation temperature between the various gravities is no larger than this. It should be noted that both of these derived parameters are, however, extremely sensitive to the choice of the effective temperature. This sensitivity arises from the temperature terms in the ΔX 's defined in equations (2) and (3).

Following a previous investigation (Schmidt 1971), the gf-values were obtained from the following sources: for FeI the Corliss and Warner (1964) oscillator strengths were corrected to the scale of Bridges and Wiese (1970); for FeII the astrophysical values of Groth (1961) or Warner (1966) (decreased by 0.88 in the log) were used. The observational curves were then fitted to the standard curve from Table 2. It was assumed that the vertical shift, $\delta Y = \log (W/\lambda)_{\text{obs}} - \log (W/\lambda)_{\text{std}}$, was the same for FeI and FeII. This assumption is not in fact correct in view of the differences between microturbulent velocities of FeI and FeII referred to above. Unfortunately there are too few FeII lines in our data to determine its microturbulence. However, from Osmer's data (1972b) we find that

differences in FeI and FeII microturbulence are not large for the luminosity class Ib stars considered here. Furthermore, if the effect of the stratification of the turbulent velocities is similar in the Cepheids and the non-variables, the differential comparison of the two groups should still be valid.

The horizontal shifts for FeI, $\delta X_I = (\log(gf\lambda) - \chi_{exc}^0)_{obs, FeI} - X_{std}$, and FeII, $\delta X_{II} = (\log(gf\lambda) - \chi_{exc}^0)_{obs, FeII} - X_{std}$, were then determined. The following relations were employed to determine the atmospheric parameters from the curve of growth shifts:

$$\log (N_{Fe}/N_H) = -\delta X + \delta Y + \Delta X - \Delta Y \quad (5)$$

$$\log v = \delta Y - \Delta Y + 0.602 \quad (6)$$

where ΔX and ΔY are defined by equations (2), (3), and (4). Since there are two equations of the form of (5) (one for FeI and one for FeII) and since the only unknowns in equation (5) are the iron abundance and the gravity, we can now determine these two parameters from an iterative procedure. The resulting value of the gravity was then inserted into equation (6) to determine the turbulent velocity. The values of the microturbulence, the gravity and the iron abundance determined from this analysis are listed in Table 3. In some cases, the FeII curves of growth were poorly defined. For one of the stars in Table 3 the gravity is omitted because the shift of FeII could not be determined. In this case, the Fe abundance from the FeI lines is given. In a number of other cases, the shift of the FeII curves was determined but the resulting gravities were uncertain. These values are indicated by colons. It should be noted that the scale errors which are possibly present in the gf-values will not affect the results of this analysis so long as we compare various stars with each other. Since the relations above are linear with respect to abundance and gravity, and since the gf-values enter them linearly

through the δX 's, scale changes in the gf -values will only produce scale changes in the gravity and iron abundance. Hence, relative values of these parameters will not be affected. Further discussion of this point will be presented later in this section.

Four of the stars in Table 3 were also analyzed by Osmer (1972b). A comparison of his turbulent velocities with the present values shows that in three cases they agree to better than 1 km/sec. This seems to be satisfactory in view of the errors involved.

The iron curve of growth was used to obtain abundances for the other elements. The oscillator strengths for these elements were taken from the sources listed by Tech (1971). If we assume that the turbulent velocities of all the ions are the same as that of iron and neglect differences in the level of line formation, then the horizontal shift required to place a line on the FeII curve of growth is just $\log N_i/N_{\text{FeII}}$. This is similar to the usual single layer assumption employed in coarse analyses. Since it has been previously shown that abundances derived from such studies are quite reliable (Schmidt 1971), this procedure should be justified. This is particularly true since differential abundances are all that are required. The ion abundances were corrected to give element abundances using the ionization temperatures and electron pressures derived previously for model atmospheres (Schmidt 1971). There were no significant differences in the abundances of elements among the non-variable stars.

Since most of the abundances are based on only a few lines and since we are interested in differences between the variables and the non-variables, the mean abundance for each element was calculated for all the non-variables and the abundances of the Cepheids were determined relative to these mean values. These differential abundances, $[N_i/N_{\text{Fe}}] = \log \frac{N_i}{N_{\text{Fe}}} (\text{Cepheid}) - \log \frac{N_i}{N_{\text{Fe}}} (\text{nonvariable})$, are listed in Table 4. The standard errors of the means were

determined from the scatter in the measurements between various phases and therefore include effects of temperature and gravity errors as well as errors of measurement. As previously noted, since these are differential abundances they should be unaffected by errors in the gf -values.

The gravities and iron abundances contained in Table 3 are plotted in Figures 3 and 4 as functions of $\log T_{\text{eff}}$ and M_V . Uncertain values are indicated by parentheses. The absolute scale of the gravities will be discussed below. It can be seen that there is no apparent trend in the abundance of iron or in the gravity with luminosity. On the other hand, there may be a slight tendency for smaller gravities and larger iron abundances at higher temperatures. Due to the paucity of points this is uncertain and may be of no significance.

As mentioned earlier, the temperatures used are on the scale derived by Schmidt (1972b) although this scale differs from more recent results given by Böhm-Vitense (1972) by between 300° and 800° for the temperature range of interest here. To determine what effect this difference in temperature scale would have, some of the stars were reanalyzed using the cooler scale. While the turbulent velocity is affected to only a small degree, the use of the Böhm-Vitense scale changes the gravities and abundances considerably. The logarithm of the gravity is lowered by 0.5 for stars with $\log T_{\text{eff}} = 3.80$ on the Schmidt scale and by 2.2 for stars with $\log T_{\text{eff}} = 3.75$. The logarithmic abundance of iron is lowered by 0.23 and 1.00 respectively at these two temperatures. Thus, using the Böhm-Vitense scale, the values of $\log g$ are lowered substantially and are perhaps more reasonable for stars of this type. Further, the possible slight trend in $\log g$ with $\log T_{\text{eff}}$ is removed. However, the relatively large change of the temperature scale introduces a strong dependence of the iron abundance upon temperature. For example, in the star T Mon, the iron abundance varies from -4.94 to -6.02 between the hottest

and coolest points which is obviously unreasonable. Therefore, the use of this temperature scale results in unacceptable trends in the iron abundance. Although this data favors the Schmidt temperature scale, the use of one scale or the other does not affect the relative values of the abundance and gravity. Hence, the conclusions of this paper are independent of the choice of temperature scale.

Finally, consideration of the data contained in Table 3 indicates that the absolute values of the gravities we have obtained are higher than those obtained in previous studies of similar stars (e.g., Parsons 1967, 1971; Osmer 1972b). Parsons (1967) has noted that substantially higher values of the surface gravities are obtained from analysis of the iron lines than from analysis of other metal lines and has suggested that this behavior may be due to systematic errors in the f -values which may still be present in the revised f -values used in the present study. Partial confirmation of this suggestion has been obtained from a determination of the gravity of the Sun using the same iron lines and oscillator strengths used in the study of the supergiants and the Cepheids. Data for this purpose were taken from Moore, Minnaert and Houtgast (1966). The solar gravity obtained from this analysis is approximately 0.2 dex higher than the gravity obtained from the solar mass and radius which suggests that the absolute scale of the gravities we have obtained ought to be lowered somewhat. In addition, the gravity scale could be lowered further by a slight (~ 100 - 200°K) decrease in the temperature scale bringing the present scale more into line with the ^{temperature} scale proposed by Parsons (1967, 1970, 1971). (The arguments against lowering the scale by the amount suggested by Böhm-Vitense have already been presented.) Therefore, the absolute scale of all of the gravities we have obtained could be lowered substantially by a combination of a revision in the oscillator strengths and a small change in the temperatures. Once again, however, we must emphasize

the point that it is the relative behavior of the Cepheids and the non-variables which is of interest in the present investigation and this relative behavior is not affected by manipulations producing scale changes in the derived parameters.

V. DISCUSSION

The data in Tables 3 and 4 and Figures 3 and 4 show both similarities and differences between the Cepheids and the non-variables. An examination of the gravities in Figure 3 indicates that the Cepheids have systematically lower values than the non-variables. If only the non-variables with absolute magnitudes between -4.4 and -5.4 are considered, the mean gravity is 2.7 ± 0.1 (s.e.). The numbers with colons in Table 3 were given half weight in determining this average. The Cepheids on the other hand have a mean gravity of 2.1 ± 0.2 (s.e.). The mean absolute magnitude of the Cepheids is -4.8 while the non-variables in this group have a mean of -4.9 . Thus, at almost the same luminosity there is a difference in the mean gravity of about 0.6 in the log in the sense that the Cepheids have lower gravities. There seems to be no significant difference between the abundances of the metals in the Cepheids and the non-variables. The iron abundances of the Cepheids average -4.63 ± 0.06 (s.e.) while the non-variables average -4.67 ± 0.10 (s.e.). In Table 4 it can be seen that the differential abundances of various elements do not differ significantly from zero in view of the standard errors quoted. The turbulent velocities in the Cepheids are seen to be somewhat higher than in the non-variables. This is possibly due to pulsational effects. These effects may include an apparent component in the microturbulent velocity due to a velocity gradient in the atmosphere and the excitation of turbulence from the pulsational motions. In any event, the suggestion that

convection or turbulence in the ionization zones inhibits pulsation in some stars finds no support in these results. While this does not rule out such an effect, it does make it seem unlikely.

The gravity difference between the two groups of stars is the most interesting part of the results of this analysis because it seems most likely to be related to the phenomenon of nonpulsation in the instability strip. However, this difference may be interpreted in several possible ways. One possibility which should be mentioned is that the pulsation itself affects the measured gravity. From the calculations of Keller and Mutschlechner (1970) it appears that for long period Cepheids the gravities indicated by atmospheric analyses should equal $GM/R^2 + \ddot{r}$. Analysis of typical radial velocity curves indicates that the acceleration changes $\log g$ in the Cepheids by no more than 0.1. Therefore, we conclude that the pulsations should have only a small effect on the results of this analysis. More directly, it may be that the lower gravities of the Cepheids are related to low masses. This would tend to support the low masses for Cepheids which have been found from a comparison of observations with pulsation theory (see, for example, Rodgers 1970). The mass deficiency indicated by the gravity differences is considerably larger than that indicated by previous work but in view of the various uncertainties in the analysis that may be acceptable. However, pulsation theory does not predict that the presence of pulsation should be so closely connected with stellar mass. If this interpretation of the data should prove correct, a theoretical explanation of the nonpulsation of some stars would still be necessary. One possible effect which has not been investigated in detail is the relation between mass and the rate at which the pulsational amplitude builds up. If the rate of amplitude increase were a strong function of mass, it is possible that the non-variables simply haven't had sufficient time to begin pulsating at large amplitude. However, the required e-folding

times would then have to be much larger than indicated by previous pulsation calculations (e.g., Baker and Kippenhahn 1962, 1965; Castor 1971; Iben 1971).

Another possibility is that the high gravities of the non-variables reflect low initial helium abundances (relative to the variables) in these stars. This would fit in with the suggestion of Cox et al. that the non-variables do not vary because they have low helium abundance. From the information given by Iben and Tuggle (1972a) we can estimate the size of this effect. For illustrative purposes if we assume that the helium abundance in the variables is 0.35, and that the non-variables had no helium at the time of formation and that no later mixing of helium formed in the core occurs, we would expect the surface gravity of the Cepheids to be lower by about 0.35 in the logarithm than that of the non-variables. Taking into account the effect of atmospheric helium abundance on the spectroscopic gravity lowers this to about 0.20. This is smaller than the observed difference, but again the uncertainties in the analysis may not make this unreasonable. Such an interpretation supports the suggestion of Cox et al. It should be noted that the metal abundances do not indicate any anomalies. Unfortunately there is no obvious reason why stars which are formed with low helium abundance should be endowed with normal population I composition as regards the metals.

A perhaps more attractive possibility is to combine the above suggestions and to suppose that the smaller gravity of the Cepheids is related to mass loss in these stars combined with their having a higher helium abundance than the non-variables. Invoking both effects would alleviate the problem that the gravity difference is larger than can easily be explained by either effect alone. Further, all of the facts can be explained at once. If we suppose that again the Cepheids are stars which have helium abundances of 0.35 and that the non-variables are stars with lower abundance, say about 0.15-0.20,

then the pulsation of some of the stars and not others is explained. If we further speculate that in addition to the lower mass of the more helium rich stars upon arrival in the instability strip (due to their higher initial helium content), there is mass loss due to pulsation, the gravities of the Cepheids are further reduced. This then agrees with the low masses of the Cepheids suggested by comparison of the pulsation theory with observations (however, see Iben and Tuggle (1972a,b) for another viewpoint) while at the same time allowing the explanation of the non-variables given by Cox et al. to stand. Further (although admittedly weak) support for the suggestion that the longer period Cepheids are in the process of losing mass is given by the masses calculated using temperatures obtained from the (R-I) colors (Schmidt 1973). In the lower part of Figure 3 of that paper it can be seen that among the longest period Cepheids there is a considerable spread in mass. While this is a tentative result for reasons given in that paper, it does nonetheless fit into the same general picture. This interpretation has the virtue of not requiring excessively small values of helium abundance in the non-variables.

It is obvious that various other possibilities exist in the interpretation of these observations. For example, we might consider mechanisms involving the enrichment of the envelopes of some stars with helium from the core. At present it is not possible to argue conclusively for any particular interpretation. However, it is clear that the phenomenon involved is not unusual among evolved supergiants. The stars in this project were chosen more or less at random from among bright G stars. Of the 11 stars, nearly half were found to be well inside the instability strip. Therefore, such stars are not at all rare. If the helium abundance turns out to be the cause of this phenomenon, then we would have to say that low helium abundances are common among evolved stars. This interpretation taken at face value raises

a number of obvious problems but if valid would be of great importance to theories concerning the chemical history of the galaxy. In this connection Shipman and Strom (1970) and Peterson and Shipman (1973) have presented evidence for variations in the helium content of stars located in different clusters and associations. On the other hand, if it is due solely or partly to mass loss, questions are raised in connection with the theories of stellar evolution and pulsation. Clearly, there is considerable theoretical and observational work remaining to be done before it is fully understood why in the Cepheid strip some stars/pulsate and others do not.

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TABLE 1

TEMPERATURES

A) Non-Variabiles

Star	H.D.	Spectral Type	M_V	E_{B-V}	θ_{eff}							$\log T_{\text{eff}}$	
					(B-V)	(V-R)	(R-I)	(V-B) ₆	(R-I) ₆	H α	Scans		Mean
	11544	G2Ib	-6. ^m 3	0. ^m 30	0.864						0.855:	0.861	3.766
	14662	F7Ib	-7.2	0.35	0.797			0.793	0.804	0.805	0.797	0.799	3.800
14 Per	16901	G0Ib	-4.5	0.21	0.833			0.830	0.833	0.856		0.838	3.779
μ Per	26630	G0Ib	-5.1	0.27	0.832	0.853	0.845			0.841		0.843	3.777
β Cam	31910	G0Ib	-5.4	0.21	0.838	0.788	0.826	0.838	0.810	0.819		0.820	3.789
ζ Mon	67594	G2Ib	-5.2	0.11	0.868	0.897	0.877					0.881	3.757
β Dra	159181	G2Ib	-6.4	0.16	0.862	0.853	0.865	0.858	0.860		.822	0.855	3.770
	187299	G5Iab-b	-4.2	0.66	0.882			0.890	0.897			0.890	3.753
β Aqr	204867	G0Ib	-4.9	0.13	0.837	0.832	0.832	0.838	0.815	0.847		0.836	3.780
α Aqr	209750	G2Ib	-5.1	0.12	0.866	0.862	0.877	0.868	0.857	0.858		0.864	3.766
ψ And	223047	G5Ib	-5.1	0.19	0.880			0.877	0.885	0.873 :		0.881	3.757

TABLE 1 (Continued)

TEMPERATURES

B) Cepheids

Star	Phase	M_V	E_{B-V}	θ_{eff}					Mean	$\log T_{\text{eff}}$
				(B-V)	(R-I)	(R-I) ϵ	H α	Scans		
RX Aur	0.34	-4.7	0.37	0.854			0.863		0.858	3.769
	0.47	-4.4		0.879			0.885		0.882	3.757
	0.64	-4.5		0.867			0.861		0.864	3.766
	0.72	-4.6		0.845					0.845	3.776
	0.93	-5.0		0.783			0.828	0.805	0.805	3.797
X Cyg	0.46	-4.5	0.40		0.871	0.857		0.875	0.866	3.763
	0.85	-4.5			0.829	0.827			0.828	3.784
T Mon	0.58	-5.0	0.40		0.877	0.858			0.868	3.764
	0.66	-4.9			0.877	0.858			0.868	3.764
	0.92	-5.8			0.800				0.800	3.799

TABLE 2
STANDARD CURVE OF GROWTH

$$V_{\text{micro}} = 4 \text{ km/sec}$$

log X	log W/ λ	log X	log W/ λ
-2.0	-6.833	1.0	-4.538
-1.5	-6.333	1.5	-4.430
-1.0	-5.838	2.0	-4.353
-0.5	-5.385	2.5	-4.238
0.0	-5.000	3.0	-4.098
0.5	-4.715		

TABLE 3

ATMOSPHERIC PARAMETERS

A. Non-Variables

B. Variables

Star H.D.	$\log \frac{N_{\text{Fe}}}{N_{\text{H}}}$	$\log g$	v km/sec	Star Name	Phase	$\log \frac{N_{\text{Fe}}}{N_{\text{H}}}$	$\log g$	v km/sec
11544	-4.68	2.20	5.0	RX Aur	0.34	-4.82	1.55	5.4
14662	-4.76	1.80	8.3		0.47	-5.11	2.20	6.4
16901	-4.43	2.55	5.9		0.64	-4.82	1.35	5.8
26630	-4.72	3.15:	5.6		0.72	-4.87	1.65	7.6
31910	-4.61	2.90	6.8		0.93	-4.71	2.10	6.5
67594	-5.06	2.35	4.9	X Cyg	0.46	-4.33	2.50	6.2
159181	-4.74	2.40	3.8		0.85	-4.51	2.25	7.6
187299	-4.82	3.15:	8.7	T Mon				
204867	-4.18	2.55	4.0		0.58	-4.51	2.40	9.1
209750	-4.58	3.10:	6.5		0.66	-4.43	2.95	8.5
					0.92	-4.15	2.05	9.6
223047	-4.83		6.9					

TABLE 4
 $[N_i/N_{Fe}]$ OF CEPHEIDS RELATIVE TO MEAN
 FOR NON-VARIABLES

Element	Number of lines	RX Aur		X Cyg		T Mon	
		$[N_i/N_{Fe}]$	S.E.	$[N_i/N_{Fe}]$	S.E.	$[N_i/N_{Fe}]$	S.E.
Al	2	+0.32	0.40	+0.04	0.30	+0.36	0.40
Ba	1	-0.71	0.27	+0.23	0.15	-0.03	0.23
Ca	4	+0.13	0.12	+0.11	0.10	+0.11	0.15
Ce	1	+0.10	0.22	+0.13	0.22	+0.05	0.17
Co	1	---	--	-0.05	0.31	-0.37	0.31
La	2	+0.44	0.39	+0.16	0.18	-0.65	0.25
Mn	3	+0.01	0.13	+0.18	0.07	+0.34	0.15
Ni	7	-0.11	0.12	-0.01	0.14	+0.20	0.15
Sc	3	0.00	0.31	+0.21	0.68	-0.14	0.26
Sm	2	+0.54	0.27	+0.24	0.27	+0.47	0.27
Ti	11	-0.03	0.12	+0.06	0.14	+0.10	0.11
V	5	+0.16	0.22	-0.14	0.36	+0.18	0.14

REFERENCES

- Baker, N., and Kippenhahn, R. 1962, Zs. f. Ap., 54, 114.
 _____ . 1965, Ap. J., 142, 868.
- Blanco, V. M., Demers, S., Douglass, G. G. and Fitzgerald, M. P. 1968, Pub. U.S. Naval Obs., 2nd Series, Vol. 21.
- Böhm-Vitense, E. 1972, Astr. and Ap., 17, 335.
- Bridges, J. M., and Wiese, W. L. 1970, Ap. J., 161, L71.
- Castor, J. I. 1971, Ap. J., 166, 109.
- Christy, R. F. 1967, in I.A.U. Symp. 28: 5th Symposium on Cosmical Gas Dynamics, ed. R. N. Thomas (New York: Academic Press), p. 105.
- Corliss, C. M., and Warner, B. 1964, Ap. J. Suppl., 8, 395.
- Cox, A. N., King, D. S. and Tabor, J. E. 1973, Ap. J., 184, 201.
- Fernie, J. D., and Hube, J. O. 1971, Ap. J., 168, 437.
- Groth, H.-G. 1961, Zs. f. Ap., 51, 231.
- Iben, I. 1971, Ap. J., 166, 131.
- Iben, I., and Tuggle, R. S. 1972a, Ap. J., 173, 135.
 _____ . 1972b, ibid., 178, 441.
- Iriarte, B., Johnson, H. L., Mitchell, R. I. and Wisniewski, W. K. 1965, Sky and Tel., 30, 21.
- Johnson, H. L. 1966, Ann. Rev. Astr. and Ap., 4, 193.
 and
 Keller, C. F.,/Mutschlecner, J. P. 1970, Ap. J., 161, 217.
- King, D. S., Cox, J. P., Eilers, D. D. and Davey, W. R. 1973, Ap. J., 182, 859.
- Kraft, R. P. 1967 in I.A.U. Symp. 28: 5th Symposium on Cosmical Gas Dynamics, ed. R. N. Thomas (New York: Academic Press) p. 207.
- Kraft, R. P., Preston, G. W. and Wolff, S. C. 1964, Ap. J., 140, 235.
- Kron, G. E. 1958, Pub. A.S.P., 70, 561.
- Kron, G. E. and Svolopoulos, S. N. 1959, Pub. A.S.P., 71, 126.
- Moore, C. E., Minnaert, M.G.J., and Houtgast, J. 1966, The Solar Spectrum 2935Å to 8770Å. (N.B.S. Monograph No. 61).

- Osmer, P. S. 1972a, Ap. J. Suppl., 24, 247.
 _____ . 1972b, ibid., 24, 255.
- Paradijs, J. van 1973, Astr. and Ap., 23, 369.
- Parsons, S. B. 1967, Ap. J., 150, 263.
 _____ . 1970, ibid., 159, 951.
 _____ . 1971, M.N.R.A.S., 152, 121.
- Peterson, D. M., and Shipman, H. L. 1973, Ap. J., 180, 635.
- Preston, G. W. 1965, 3rd Colloquium on Variable Stars, Kleine Veröff der Rameis Sternwarte Bamberg, Vol. 4, No. 40.
- Rodgers, A. W. 1970, M.N.R.A.S., 151, 133.
- Sandage, A. and Tammann, G. A. 1969, Ap. J., 157, 683.
- Schmidt, E. G. 1971, Ap. J., 170, 109.
 _____ . 1972a, ibid., 172, 679.
 _____ . 1972b, ibid., 174, 595.
 _____ . 1972c, ibid., 174, 605.
 _____ . 1973, M.N.R.A.S., 163, 67.
- Shipman, H. L., and Strom, S. E. 1970, Ap. J., 159, 183.
- Tech, J. L. 1971, A High Dispersion Spectral Analysis of the BaII Star HD 204075 (S Capricorni) (N.B.S. Monograph No. 119) p. 24.
- Warner, B. 1966, Comm. Univ. of London Obs., No. 70.
- Wilson, O. C., and Bappu, M. K. V. 1957, Ap. J., 125, 661.
- Wisniewski, W. Z. and Johnson, H. L. 1968, Comm. Lunar and Planetary Lab., 7, 57.

FIGURE CAPTIONS

Figure 1 - The mean temperatures from Table 1 plotted against spectral type for the non-variable stars.

Figure 2 - The location of the program stars in the temperature-magnitude diagram. The edges of the instability strip come from Schmidt (1972a). The symbols have the following meanings: ●

non-variable stars; ○ RX Aur; ■ X Cyg; ▢ T Mon. The large symbols for the Cepheids indicate the mean luminosity and temperature while the small symbols are these parameters at the phases of the spectra used here.

Figure 3 - The gravities of the stars plotted against the temperature and absolute magnitude. The symbols have the same meaning as in Figure 2.

Figure 4 - The abundance of iron plotted against the temperature and absolute magnitude. The symbols have the same meaning as in Figure 2.