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This article is a companion to Barchyn (2018) <https://doi.org/10.1002/2018JF004629>, Pelletier et al. (2018) <https://doi.org/10.1002/2017JF004461>.

Key Points:

- Yardangs with an aspect ratio (AR) of 4 are not minimum-drag forms
- Yardang ARs are controlled by bedrock structure and the aerodynamics of turbulent jets
- Numerical modeling illustrates the importance of water-driven processes and bedrock structure

Supporting Information:

- Supporting Information S1
- Movie S1
- Movie S2

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Controls on Yardang Development and Morphology: 2. Numerical Modeling

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Abstract Here I present a set of mathematical modeling results, constrained by the results of the companion paper, aimed at improving our understanding of yardang development and controls on yardang morphology. The classic model for yardang development posits that yardangs evolve to an aspect ratio of ≈ 4 in order to minimize aerodynamic drag. Computational fluid dynamics model results presented here, however, demonstrate that yardangs with an aspect ratio of 4 do not minimize drag. As an alternative, I propose that yardang aspect ratios are primarily controlled by the lateral downwind expansion of wind and wind-blown sediments focused into the troughs among yardangs, which can be quantified using previous studies of wall-bounded turbulent jets. This approach predicts yardangs with aspect ratios in the range of 5 to 10, that is, similar to those of natural yardangs. In addition to aerodynamics, yardang aspect ratios are influenced by the strikes and dips of strata, as demonstrated in the companion paper. To better understand the aerodynamic and bedrock structural controls on yardang morphology, I developed a landscape evolution model that combines the physics of boundary layer flow and abrasion by eolian sediment transport with a model for the erosion of the tops and lee sides of yardangs by water-driven erosional processes. Yardang formation in the model is enhanced in substrates with greater heterogeneity (i.e., alternating strong and weak strata). Yardang morphology is controlled by the strikes and dips of strata as well as the topographic diffusivity associated with water-driven erosional processes.

1. Introduction

1.1. Problem Statement

Yardangs are streamlined ridges, carved into bedrock or cohesive sediments, that form parallel to the prevailing wind direction. The leading conceptual model for yardang development posits that wind and wind-blown sediments are focused into preexisting substrate weaknesses, leading to further deepening and the accommodation of wind and wind-blown sediments in the incipient troughs among yardangs (Laity, 2011; Laity & Bridges, 2009). This conceptual model is well supported by previous studies, but basic questions remain regarding how yardangs develop and what variables control their morphology.

The classic theoretical model for yardang form posits that yardangs evolve to an aspect ratio of ≈ 4 because this aspect ratio minimizes aerodynamic drag (Figure 1) (Ward & Greeley, 1984). Aerodynamic drag is the sum of pressure and skin friction drag. Pressure drag scales with the difference in pressure between windward and leeward sides of an obstacle or bedform; it tends to be higher for less well streamlined bedforms that generate larger zones of flow separation in their wakes, and smaller for more tapered bedforms that generate smaller or less well developed zones of flow separation. To minimize flow separation and hence pressure drag, a bedform can evolve to a more elongated or tapered shape. However, the reduction in pressure drag resulting from a more tapered shape comes at the cost of an increase in skin friction drag because new surface area is created where the wind velocity must come to zero (i.e., the no-slip boundary condition). In invoking the minimum-drag model for yardangs, past research has relied on calculations performed for streamlined free bodies (i.e., bodies completely surrounded by a free stream). Streamlined bedforms are not free bodies, however, but instead reside within a rough turbulent boundary layer and are thus within the class of objects referred to as streamlined half-bodies. Additional testing of the minimum-drag model is thus necessary.

An aspect ratio of 4 is, in any case, broadly consistent with the aspect ratio of many streamlined bedforms on Earth and other planets based on the data sets of Komar (1983, 1984) (Figure 2). As such, a new understanding of how yardangs commonly evolve to an aspect ratio ~ 4 (i.e., in the range of approximately 2 to 10) is needed if yardangs do not minimize drag. Here I propose that yardang aspect ratios are controlled in part

Figure 1. Conceptual model for the aerodynamic control on yardang aspect ratios (after Ward & Greeley, 1984). At small aspect ratios, pressure drag dominates over skin friction drag. As yardangs become more tapered or streamlined, pressure drag decreases but skin friction drag increases. The sum of skin friction and pressure drag has a minimum for streamlined free bodies at an aspect ratio of ≈ 4 (e.g., Young, 1939).

by the lateral downwind expansion of wind and wind-blown sediment focused into the troughs among yardangs. The turbulent dispersion downwind from troughs is analogous to the classic fluid mechanics problem of a 3-D wall-bounded turbulent jet. Experiments and numerical models of 3-D wall-bounded turbulent jets have established that such jets disperse laterally with an aspect ratio in the range of ≈ 5 to 10, approximately independent of Reynolds number or other factors. The strikes and dips of bedrock strata can also exert control on yardang aspect ratios and may explain deviations from typical values (Hu et al., 2017; Pelletier et al., 2018).

Yardang aspect ratios have received the most attention in the literature, yet what controls the widths/spacings of yardangs is also a fundamental unanswered question in eolian geomorphology. Yardangs, if grouped with microyardangs and megayardangs, can develop over a wide range of spatial scales. However, within a given study site and level within the hierarchy of microyardangs to megayardangs, yardang sizes (widths, spacings, and heights) are often quite consistent. Several candidate length scales in the physics of yardang development may control yardang size. For example, studies have emphasized that water-driven erosional processes are important and perhaps dominant processes controlling erosion rates on the tops and lee sides of yardangs (e.g., Brookes, 2001, 2003; Dong et al., 2012). Water-driven erosional processes on hillslopes, including freeze-thaw-driven creep, rain splash, and interrill and rill erosion, can be approximately represented using a diffusion equation for topography (Culling, 1960). The literature supports the utility of the diffusion equation for quantifying hillslope evolution by freeze-thaw-driven creep and rain splash, provided that the slope is transport-limited (i.e., soil is present) and slope gradients are modest (Dunne et al., 2010; Furbish et al., 2009; McKean et al., 1993). Pelletier et al. (2018) documented how empirical models for interrill and rill erosion may also be approximated by the diffusion equation. The topographic diffusivity is the fundamental parameter of the diffusion equation, representing the efficiency of water-driven sediment flux for a given slope. The diffusion length scale, that is, the spatial scale that can be diffused over a given time scale, is thus a potential length scale for controlling yardang morphology.

Figure 2. Images of streamlined bedforms on Earth and Mars. (a) Shaded-relief image of a ground-based lidar survey of a pair of yardangs in Ocotillo Well State Vehicular Recreation Area (OWSVRA). (b) Thermal Emission Imaging System (THEMIS) image #PIA03825 of streamlined islands in Ares Valles, Mars, likely formed by catastrophic flooding. (c) Streamlined loess hills (after Baker and Milton, 1974) formed by catastrophic flooding in the Channelled Scablands region, Washington state. (d) Mars Observing Camera image #PIA03661 of yardangs in Aeolis, Mars.

Laity (2011) proposed that yardangs develop “when air enters a passageway established by a channel or fracture,” suggesting that the characteristic spacing of joints or first-order fluvial channels are also a candidate length scales for the control of yardang morphology. However, many geomorphic studies of the past few decades have demonstrated that when an instability mechanism is at work, a macroscopic preexisting weakness such as a channel or fracture may not be necessary to initiate the feedback. Studies have shown that any heterogeneity, no matter how small, can sometimes trigger the initiation of a positive feedback or feedbacks that lead to the development of quasiperiodic topography from an initial condition with no periodicity. Examples include coastal spits (Ashton & Murray, 2006), dryland vegetation bands (e.g., Pelletier et al., 2012), and mima mounds (Gabet et al., 2014). In all of these examples, the length scale of the quasiperiodic topography is set by characteristic spatial scales in the physics of the feedback process(es). In the absence of a more quantitative understanding of how yardangs are initiated and how the physics of the processes affect the morphology of the resulting yardangs, we are left with the default hypothesis that the troughs among yardangs are initiated by periodic channels or fractures. This default hypothesis is often hard to test because we generally cannot constrain the initial conditions of yardang development. Numerical modeling can, however, be used to investigate the range of possible initial conditions that can give rise to semiperiodic yardangs. This paper proposes such a model. The model complements that of Barchyn and Hugenholz (2015), who emphasized the feedbacks that may occur among rates of eolian abrasion and the associated release of loose sediment, the coarser fraction of which may protect the troughs from erosion and hence slow yardang development.

1.2. Summary of Results From the Companion Paper and Goals of This Paper

The models presented in this paper are motivated by observations and data from the yardangs fields of Ocotillo Wells State Vehicular Recreation Area (OWSVRA; Pelletier et al., 2018). Field observations at OWSVRA indicate that yardangs are more likely to form in the more heterogeneous Borrego Formation (composed of alternating sandstones and mudstones) than in other stratigraphic units in the study area. The modeling of this paper seeks to determine how substrate heterogeneity may enhance yardang development. Nearly all of the tops and lee sides of yardangs at OWSVRA are soil-covered. Rill-wash deposits at the base of many rills indicates that the evolution of yardang slopes is predominantly transported-limited (i.e., sediment tends to be deposited in zones of concavity). This observation forms a basis for adopting a diffusion model for the evolution of yardang slopes by water-driven processes. Estimates of yardang erosion rates over short and long time scales, combined with measurements of mean topographic curvature, constrain the topographic diffusivity to be ~ 1 m/kyr, assuming that the topographic curvature measured is representative of the curvature over geologic time scales. Aerodynamic measurements constrain the threshold shear velocity, u_{*t} , and aerodynamic roughness length, z_0 , to be 0.2 m/s and 1 mm, respectively. These data provide useful constraints on model input parameters. Sediment flux data indicate that sediment is predominantly transported close to the saltation-suspension transition. Sediment fluxes are largest in the narrowest portions of troughs among yardangs and are lowest in the wake of the yardang ridge.

Yardang aspect ratios in OWSVRA generally deviate from the proposed ideal aspect ratio of 4, with the largest values occurring in strata that strike nearly perpendicular to the wind and the smallest values occurring in areas with steep dips where the strata strike nearly perpendicular to the prevailing wind direction. Yardang spacing is proportional to width. The modeling of this paper aims to reproduce these patterns of structural control and correlation among yardang morphologic variables. Yardangs are well represented as asymmetric Gaussians. The asymmetric Gaussian function honors the concave zones at the bases of yardangs documented in Figures 5b, 7d, and 12c of Pelletier et al. (2018). In section 3.2.1 I propose a steady state analytic model that links the spatial pattern of sediment flux around yardangs to the asymmetric Gaussian function. Because the models of this paper are motivated and constrained by observations and data from OWSVRA, they are not applicable to all yardang fields. In section 4, I discuss aspects of the numerical model that would likely have to be modified for application to other yardang fields.

In addition to these site-specific goals, the modeling of this paper also seeks to understand yardang development more generally by quantifying the drag of idealized yardangs of varying aspect ratio to test the minimum-drag model of Ward and Greeley (1984). I also propose an alternative conceptual model for the typical range of aspect ratios based on fluid-mechanical studies of the downwind lateral dispersion of wall-bounded turbulent jets.

2. Methods

2.1. Computational Fluid Dynamics Modeling and Drag Computation

The goals of the computational fluid dynamics (CFD) modeling of this paper are to quantify the dependence of the drag coefficient on yardang aspect ratio and to solve for the 3-D mean wind velocities over evolving topography in the landscape evolution model. I used the 2013 version of the PHOENICS CFD model (Ludwig, 2011) for these purposes. PHOENICS computes the time-averaged flow velocities associated with neutrally stratified, fully developed rough turbulent boundary layer flow over prescribed topography. PHOENICS solves the flow equations using the iterative SIMPLEST algorithm of Spalding (1980), which is a variant of the SIMPLE algorithm of Patankar and Spalding (1972). For all of the calculations of this paper I used the renormalization group variant of the k - ϵ closure scheme first proposed by Yakhot and Orszag (1986) and later updated by Yakhot et al. (1992), which is optimized for highly sheared/separated flows.

Inputs to the PHOENICS model runs described in this section include terrain models representing idealized yardangs with a wide range of aspect ratios, an aerodynamic roughness length (z_0), and a prescribed horizontal velocity at a reference height far from the ground ($u_r = 10$ m/s at $z_r = 100$ m was used for all of the model runs of this section). The ground surface is prescribed to be a fully rough boundary, that is, one that results in a law-of-the-wall velocity profile, that is,

$$u(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right) \quad (1)$$

where $u(z)$ is the wind velocity at a height z above the bed, u_* is the shear velocity constrained using the reference velocity u_r , κ is the von Kármán constant (0.41), and z_0 is the aerodynamic roughness length (assumed to be 1 mm for all of the model runs associated with this section, i.e., at the upper end of values measured in OWSVRA; Pelletier et al., 2018). At the upwind boundary of the model domain an inlet law-of-the-wall velocity profile is prescribed with u_* and z_0 values consistent with the reference velocity, reference height, and the z_0 value prescribed for the ground-surface boundary condition within the model domain. At the downwind boundary (i.e., the outlet), a fixed-pressure boundary condition is used.

The computational grid consists of a 3-D terrain-following coordinate system with 256 and 128 grid points in the along-wind and cross-wind directions, respectively. Fifty logarithmically spaced grid points were used in the vertical direction, ranging from 0.003 to 100 m above the bed. The rule in atmospheric boundary layer simulations is that the top boundary should be ~ 30 times larger than the relief of the topography in order to capture the effects of changes within the roughness sublayer (such as flow separation) on the large-scale flow. Yardangs 3 m in height were input to the model; hence, a top boundary of 100 m is 30 times larger than the topographic relief.

For the purposes of computing the dependence of the yardang drag coefficient on the aspect ratio, yardangs were modeled as asymmetric Gaussians, that is,

$$\begin{aligned} z &= z_{\max} \exp\left(-\left(\frac{x}{x_w}\right)^2 - \left(\frac{y}{x_w}\right)^2\right) \quad \text{if } x < 0 \\ &= z_{\max} \exp\left(-\left(\frac{x}{x_l}\right)^2 - \left(\frac{y}{x_w}\right)^2\right) \quad \text{if } x > 0 \end{aligned} \quad (2)$$

where z is elevation above the base of the yardang, $z_{\max}$ is the height of the yardang crest (assumed to be 3 m for all of the cases associated with this section), x is the distance downwind from the yardang crest along the yardang axis, y is the distance from the yardang axis along a direction perpendicular to the axis, and x_l and x_w are characteristic dimensions of the yardang in the directions parallel and perpendicular to the wind, respectively. The companion paper (Pelletier et al., 2018) demonstrated the adequacy of equation (2) for representing the first-order morphology of yardangs in OWSVRA. The yardang width scales with $2x_w$, while the length scales with $x_w + x_l$. Therefore, the aspect ratio is given by $(x_l + x_w)/2x_w$.

The drag force on a surface interacting with a rough turbulent boundary layer flow is obtained by integrating the product of the pressure and the cosine of the angle of the surface with respect to the horizontal (i.e., the pressure drag) over the surface, plus the integral of the product of the shear stress and the sin of the angle with respect to the flow (i.e., the skin friction drag):

$$F_D = \int_A P \cos \theta \, dA + \int_A \tau_b \sin \theta \, dA \quad (3)$$

where P is the pressure in excess of hydrostatic, θ is the angle between the ground surface and the horizontal, dA is the surface element, τ_b is the bed shear stress, and the integral is computed over the surface A . The drag coefficient is defined as

$$c_D = \frac{F_D - F_{D,fs}}{\frac{1}{2} \rho A u_r^2} \quad (4)$$

where $F_{D,fs}$ is the drag associated with a horizontal surface (which has only a skin drag component), ρ is the fluid density, and u_r is the velocity at a reference distance far from the surface.

The subtraction, in equation (4), of the drag associated with a flat horizontal surface differs from most drag coefficient definitions. I included this subtraction in the calculations because when computing the drag associated with the development of a bedform, it is necessary to subtract the drag that would be present if there was no bedform, that is, the drag for a horizontal surface. If this subtraction is not included, the resulting drag coefficient would be a function not only of the shape of the yardang or other bedform type but also of the spatial extent of the surrounding flat horizontal surface. By subtracting the drag associated with a horizontal surface, the increase in drag associated with yardang development is isolated. Note that drag coefficients reported in the literature can differ in their absolute values since different conventions are used for normalizing the drag force in equation (4). However, as long as the definition is kept consistent within a given study, the relative differences in the drag coefficient obtained are independent of any such differences in convention.

2.2. Landscape Evolution Modeling

2.2.1. Boundary Layer Flow Model Component

In addition to providing the computational tool for determining how drag varies with yardang aspect ratio, PHOENICS also provides the boundary layer flow component of the landscape evolution model. During each iteration of the landscape evolution model, PHOENICS solves for the mean wind velocities given the topography resulting from the previous model iteration and an aerodynamic roughness length as input. As a commercial code, I cannot include PHOENICS in the model source code provided in the supporting information. As such, the source code accepts, as input, the mean wind horizontal velocities for each iteration of the model (calculated by PHOENICS or, if preferred, an alternative boundary layer flow model can be used) and uses those velocities to advect and diffuse the sediment horizontally, perform abrasion of the substrate, and update the topography for input into PHOENICS during the next iteration of the model. In all of the landscape evolution model results presented in this paper, the flow equations were solved using a 3-D terrain-following coordinate system with 256 and 128 grid points in the along-wind and cross-wind directions, respectively. Thirty logarithmically spaced grid points were used in the vertical direction, ranging from 0.01 to 30 m above the bed.

2.2.2. Sediment Transport and Abrasion Model Component

In this section I describe the model component that computes the eolian sediment flux and the resulting abrasion rate. An appropriate starting point is the calculation of the vertical profile of eolian sediment concentration, $c(z)$. The concentration profile over a horizontal surface predicted by the advection-diffusion-settling model of eolian sediment transport is (Anderson & Hallet, 1986)

$$c(z) = c(z_0) \left(\frac{z_0}{z} \right)^{R_\#} \quad (5)$$

where $R_\#$ is the Rouse number, defined as w_s/ku_* where w_s is the particle settling velocity. The companion paper (Pelletier et al., 2018) demonstrated the efficacy of equation (5) for characterizing measured sediment flux data in the vicinity of yardangs at OWSVRA.

Given values for the vertical profiles of the mean wind velocities in the along-wind and cross-wind directions, u and v , predicted by PHOENICS, the model divides the vertical concentration profile at each (x,y) grid point into the same 30 logarithmically spaced height intervals (assumed to be the same intervals used to compute the wind velocity field). The maximum height at which sediment is transported can be changed to be larger or smaller than 30 m by modifying the value of “maxheight” in the code, but should be large enough that abrasion at that height is negligible. Within each height interval, the model computes the fraction of the

total sediment concentration in that height interval using equation (5). The fraction of sediment concentration within each height interval is then transported to neighboring grid points according to the steady state advection-diffusion equation:

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D_{\text{turb}} \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) \quad (6)$$

where D_{turb} is the horizontal diffusivity of the sediment in transport. In this way, the sediment concentration increases in troughs where winds converge and decreases above topographic highs where winds diverge.

Using the experimental data of Routbort et al. (1980) and Scattergood and Routbort (1983), Anderson (1986) demonstrated (his eqn. (6)) that the rate of eolian abrasion is proportional to the kinetic energy flux, which scales with the product of sediment concentration and the cube of the particle velocity in the downwind direction:

$$E = Kcu^3 \quad (7)$$

where K is an erodibility coefficient that combines the effects of the substrate's susceptibility to abrasion, the impact angle, and the density and diameter of the particles abrading the surface.

Abrasion is the result of particles with a wide range of trajectories impacting the bed, including a relatively large flux of particles transported close to the bed traveling at low speeds and a smaller flux of particles traveling higher in the air column where they attain higher speeds before coming back down to impact the bed. In the numerical model, I include the net abrasion from this wide range of particle trajectories by vertically integrating the kinetic energy profile:

$$E_a = \int_{z_0}^{\infty} E(z) dz \quad (8)$$

This approach honors the spatially distributed nature of eolian sediment transport, that is, the fact that wind speeds and particle trajectories at a range of heights influence abrasion of the bed. Integration of the profile may seem entirely inappropriate for the steep windward face of yardangs where the upper portion of the yardang interacts only with the upper portion of the sediment flux profile. However, the advection of sand by the wind (equation (6)) has the effect of deflecting sand transported close to the bed around yardangs so that such sand does not factor into the abrasion of the upper portion of the yardang when the concentration profile is integrated vertically in equation (8). More complex 3-D models that explicitly model the interaction of particles with the 3-D topography are possible, but equations (6)–(8) provide a useful 2-D approach to quantifying the spatial variations in abrasion rate in complex topography.

The value of E_a at each time step of the model is multiplied by the model time step, and the result is subtracted from the topography at the current time step to compute a new topography for use in the next time step of the model. Note that erosion and deposition by diffusive hillslope processes are also performed, as described in section 2.2.3.

The troughs between adjacent yardangs can episodically store and release sediment, preventing abrasion of the bed for time scales on the order of years (e.g., Pelletier et al., 2018). In the model this effect is represented by backfilling closed depressions in the topography using an algorithm originally designed for the hydrologic correction of digital elevation models (Pelletier, 2008). At the next time step of the model all areas of the model domain are assumed to be susceptible to erosion. This approach honors a likely negative feedback in the system: trough scouring creates a zone of flow expansion that can promote episodic sediment deposition, thus limiting further deepening of the trough until other low-relief portions incise to the same level.

I assume for the purposes of this paper that the particle velocities u and v in equations (6) and (7) can be approximated by the wind velocity. This assumption is strictly valid only for suspended sediment. The breakdown of this assumption is one reason why Anderson (1986) developed a separate set of equations for abrasion by saltating particles and applied equation (6) only to suspended particles. However, in the companion paper (Pelletier et al., 2018), we showed that the concentration and flux profiles obtained in OWSVRA are well represented by an advection-diffusion-settling model at the saltation-suspension transition. Moreover, in section 3.3 of this paper, I demonstrate that the advection-diffusion-settling model predicts abrasion profiles consistent with those measured by Sharp (1980) in the Coachella Valley. Also, as noted in the companion

Table 1
Variables Used in This Paper

Variable	Description	Reference value
x	along-wind distance	
y	cross-wind distance	
z	vertical distance above the ground	
u	horizontal velocity in along-wind direction	
v	horizontal velocity in cross-wind direction	
κ	von Kármán constant	0.41
u_*	shear velocity	0.8 m/s
u_{*t}	threshold shear velocity	0.2 m/s
z_0	aerodynamic roughness length	0.001 m
$z_{\max}$	height of yardang crest in drag calculations	3
x_w	scale factor in asymmetric Gaussian model	
x_l	scale factor in asymmetric Gaussian model	
F_D	drag force on yardang	
$F_{D,fs}$	drag force on flat surface	
P	air pressure in excess of hydrostatic	
θ	angle between the surface and the horizontal	
τ_b	bed shear stress	
A	surface area of yardang	
ρ	air density	1.225 kg/m ³
u_r	reference velocity	
c	sediment concentration	
$R_{\#}$	Rouse number	2.5
w_s	settling velocity	
D_{turb}	lateral turbulent diffusivity	
K	bedrock erodibility coefficient	0.7 s ³ · m ⁻² · kyr ⁻¹
E_a	eolian abrasion rate	
K_h	erodibility coefficient of more resistant ("hard") strata	
η	random field (white noise distributed between 0 and 1)	
a	coefficient defining erodibility contrast between hard and soft strata	
D_{topo}	topographic diffusivity	1 m ² /kyr
t	elapsed time	
$y_{1/2}$	distance in the cross-wind direction where the velocity of a wall-bounded turbulent jet falls to half of its maximum value	
s	lateral turbulent dispersion coefficient	
Pe	Péclet number	
T_a	turbulent advection time scale	
T_d	turbulent diffusion time scale	
$E_{a,\max}$	maximum eolian abrasion rate	
d	particle diameter	
α	coefficient relating z_0 to excess u_*	0.063
g	acceleration due to gravity	9.81 m/s

paper (Pelletier et al., 2018), differences among wind and particle velocities at a given height above the surface are likely to be small at the range of heights above the ground where abrasion is maximized. For example, Rasmussen and Sørensen (2008), measured the ratio of particle to wind speed to be approximately 0.5 close to the surface (5 mm) in a wind tunnel study characterized by relatively coarse texture (particle diameters of 242 and 320 μm), with values increasing to nearly 1 (i.e., 0.8–0.9) at heights of only 8 cm above the bed. The results of Rasmussen and Sørensen (2008) suggest that for the purposes of modeling abrasion (which is generally dominated by particles traveling at least a few centimeters above the ground; e.g., Sharp, 1964, 1980), it is reasonable in eolian systems dominated by fine sediment (e.g., 80–120 μm ; Pelletier et al., 2018) to assume that wind and wind-blown sediments have the same velocity.

Spatial variations in erodibility in the model (i.e., "hard" versus "soft" strata) are prescribed in the model via the geometry (strike, dip, spacing, and thickness) of the strata. This framework limits the model to lithologies with bedded or banded rocks (i.e., sedimentary and some types of metamorphic rocks with banding). In addition, I assume that the hard strata have white-noise random variations in erodibility defined via

Figure 3. Plot of the drag coefficient, C_d , as a function of yardang aspect ratio. Contrary to the classic model (Figure 1), drag does not increase with increasing aspect ratio above an aspect ratio of 4. Instead, drag is essentially independent of aspect ratio from values larger than 5.

long-term erosion rate of 3–4 mm/year at OWSVRA with the average curvature on the tops and lee sides of yardangs (equal to $-0.27/\text{m}$; the ratio of these quantities is $\approx 1 \text{ m}^2/\text{kyr}$). Equation (10) is solved in the model using the Alternative-Direction-Implicit version of the Crank-Nicholson method (Crank & Nicolson, 1947; Peaceman & Rachford, 1955).

2.2.4. Summary of Model Parameters

In addition to the initial topography and the stratal architecture, each model run is characterized by six parameters: u_* (m/s), z_0 (m), a Péclet number that defines the ratio of advective to diffusive time scales in equation (6) (explained in section 3.2), K ($\text{s}^3 \cdot \text{m}^{-2} \cdot \text{kyr}^{-1}$), D_{topo} (m^2/kyr), and a (unitless) (Table 1). The values of u_* and z_0 can be constrained for specific study sites with measurements of the wind velocity profile (e.g., the companion paper presents a range of values for u_* and z_0 from OWSVRA). I did not attempt to relate elapsed time in the model to the frequency of wind storms or any other aspect of the temporal history of wind forcing. Instead, the value of K was chosen so that the E_a value for the horizontal surface at the upwind end of the model domain equals an erosion rate consistent with independent constraints from the study site (3.5 m/kyr is used here to be consistent with the 3–4 mm/yr measured at OWSVRA; Pelletier et al., 2018). The

value of K was defined using a convention that the vertical integral of the sediment concentration at the upwind boundary of the model domain equals 1. According to equation (7), therefore, K converts the cube of a velocity to an erosion rate (also a velocity) and thus has units of T^2/L^2 . All of the model results presented in this paper have 256 pixels in the along-wind direction and 128 pixels in the cross-wind direction. In the model results of section 3.2.1, each grid point represents 0.33333 m (thus, the domain is approximately 85-m long by 43-m wide), while in the results of section 3.2.1 each grid point represents 0.5 m (domain is 128-m long by 64-m wide).

3. Results

3.1. Are Yardangs Minimum-Drag Forms?

The results of the PHOENICS model runs indicate that the drag coefficient of yardangs, C_D , decreases with increasing aspect ratio until an aspect ratio of ≈ 5 (Figure 3). For larger aspect ratios, the value of C_D stays approximately constant as aspect ratio increases up to 20 (Figure 3). These results indicate that yardangs are not minimum

Figure 4. Conceptual model of the differences between streamlined free bodies (upper image) and streamlined half-bodies (lower image).

Figure 5. (a) Conceptual diagram of a 3-D wall-bounded turbulent jet with orifice dimensions w_0 and h_0 . (b) Yardang aspect ratio with respect to the opening angle of a wall-bounded turbulent jet.

drag forms. Instead, drag is nearly independent of aspect ratio over a large range that includes aspect ratios that are typical for yardangs (≈ 5 – 10) and aspect ratios that are not typical (i.e., ≈ 10 – 20). Drag coefficients depend weakly on shear velocity and z_0 (e.g., Gollos, 1953; Hogg et al., 1997). As such, the results in Figure 3 are specific to the parameters I used ($u_* = 0.26$ m/s and $z_0 = 1$ mm). However, I verified that the relative dependence of drag coefficients on aspect ratio is similar to that plotted in Figure 3 for a range of shear velocities (by varying the velocity 100 m above the ground from 3 to 30 m/s) and a range of z_0 values ranging from 0.1 to 10 mm.

The results in Figure 3 can be understood conceptually by considering the differences between a streamlined free body and a streamlined half-body (illustrated conceptually in Figure 4). In the case of a streamlined free body (i.e., a body completely surrounded by flow moving at a uniform far-field velocity), drag is minimized at the relatively small aspect ratio of ≈ 4 (e.g., Young, 1939) because there is a relatively large skin drag penalty associated with tapering of the obstacle. Flow separation is triggered by the presence of a relatively abrupt change in surface slope and/or curvature on the lee side of an obstacle (e.g., Lamballais et al., 2010; Simpson, 1989). Tapering of the obstacle reduces the slope and curvature on the lee side and hence reduces flow separation. A reduction in flow separation, in turn, reduces the pressure difference between the windward and lee sides of the obstacle and hence the pressure drag. However, tapering creates substantial new surface area where the flow must come to zero to honor the no-slip boundary condition at the surface (Figure 4). For a yardang or other streamlined half-body, in contrast, a surface exists on the lee side of the obstacle whether or not the half-body is blunt or tapered.

As a result, the skin drag penalty associated with tapering is not as large in the case of a streamlined half-body as in the case of a tapered free body.

A more general problem with the minimum-drag model for yardangs and other streamlined bedforms is that there is no a priori reason why drag should be minimized during bedform development. The drive for optimal efficiency intrinsic to natural selection provides a clear physical reason why drag should be minimized for many streamlined organisms (e.g., fish and birds). However, there is no analogous selection process or optimization principle for bedform development. That is not to say that drag minimization does not occur in bedform development, just that there is no basis for the hypothesis that drag minimization is required for bedform development.

3.2. Alternative Conceptual Model for the Aerodynamic Control of Yardang Aspect Ratios

The fact that yardangs are not minimum-drag forms begs the question of what controls yardang aspect ratios. The companion paper (Pelletier et al., 2018) demonstrated that yardang aspect ratios can be controlled by stratal architecture (i.e., the strikes and dips of resistant strata). However, there must be some aerodynamic control on yardang aspect ratios since yardangs with aspect ratios of ≈ 4 can and do form in laboratory and field conditions that lack structural control (e.g., Ward & Greeley, 1984).

A key process in yardang development is the focusing of wind and wind-blown sediments into the incipient troughs among yardangs. I posit that this focusing can be considered to be analogous to the case of a 3-D wall-bounded turbulent jet (Figure 5a). In this section I take advantage of decades of research on the mechanics of wall-bounded turbulent jets to develop a conceptual model for yardang aspect ratios based on the lateral expansion of turbulent flows focused into the troughs among yardangs.

A three-dimensional (3-D) turbulent jet refers to the expanding cone of fast-moving fluid exiting an orifice or zone of confinement (usually taken to be circular or rectangular in shape in experiments and CFD model calculations). As the fluid exits the orifice, a velocity gradient is established between the fast-moving fluid

exiting the orifice and the ambient fluid that the jet interacts with (which some experiments make stagnant while others prescribe a downstream velocity). This velocity gradient drives turbulent mixing of momentum, causing the jet to expand with increasing distance downwind from the orifice. A wall-bounded turbulent jet is simply a jet with one side of the orifice adjacent to a wall.

In both yardangs and wall-bounded turbulent jets, flow accelerates as it becomes spatially confined in the downstream direction. In the wall-bounded turbulent jet case, that restriction is a rectangular or circular orifice, while in the case of yardangs it is the semicircular trough between adjacent yardangs. This is a significant difference that renders the wall-bounded turbulent jet only an approximate model system for flows between yardangs. In both cases, the resulting confined flow or jet interacts with a no-slip boundary as it expands downstream from the zone of flow constriction. Some experiments of wall-bounded turbulent jets include the full range of velocities of the downstream fluid into which the jet flows, from zero (a stagnant fluid) to the velocity of the jet itself (e.g., Figure 8 of Launder & Rodi, 1983). Experiments in which the jet interacts with a moving fluid are the most similar to flow in yardang troughs.

The research on 3-D wall-bounded turbulent jets has demonstrated that the mean velocity spreads laterally (i.e., in the cross-stream direction) as a Gaussian. The width of the jet in the cross-wind direction is defined as twice the cross-wind distance between the axis of the jet and the point where the velocity falls to half of its maximum value, $y_{1/2}$ (Figure 5). Experiments and CFD model calculations demonstrate that the value of $y_{1/2}$ increases linearly with distance, x , downwind from the orifice with a constant value, s , equal to approximately 0.1 for a range of Reynolds numbers (Launder & Rodi, 1983):

$$\frac{dy_{1/2}}{dx} = s \approx 0.1 \quad (11)$$

Equation (11) implies that a 3-D wall-bounded turbulent jet has a triangular shape in map view with an opening angle equal to $2\text{-atan}(s)$ (Figure 5b). Assuming that the tapering of yardangs is approximately related to the expansion of a 3-D wall-bounded turbulent jet in the trough between adjacent yardangs, yardangs will have a shape in map view that is a mirror image of the shape of the trough (Figure 5b); that is, the opening angle of the tapered end of the yardang will also be $2\text{-atan}(s)$. This implies an aspect ratio l/w of approximately $1/2s$, or approximately 5 assuming $s \approx 0.1$.

The value $s \approx 0.1$ is the maximum value in the range reported by Launder and Rodi (1983) (approximately 0.05 to 0.1) in their Figure 8. Variations in the value of s are associated with differences in the downstream velocity of the fluid that the jet interacts with. In addition, s may have some dependence on shear velocity and/or roughness length (such a dependence has not been documented, to my knowledge, but the limited range of conditions under which studies of wall-bounded turbulent jets have been conducted cannot rule out such dependence). Values of s between 0.05 and 0.1 imply yardang aspect ratios in the range of 5 to 10, that is, similar to those observed in nature in the absence of structural control.

The constant s in equation (11) is the inverse of a Péclet number, defined as the ratio of the advection time scale T_A , to the diffusion time scale, T_D . For a length scale equal to the model pixel size, Δx , the Péclet number is defined by

$$\text{Pe} = \frac{T_A}{T_D} = \frac{u\Delta x}{D_{\text{turb}}} = \frac{1}{2s} \quad (12)$$

Values of s between 0.05 and 0.1 thus imply Péclet numbers of between 5 and 10. In the landscape evolution model of this paper, a reference value of $\text{Pe} = 5$ will be used, consistent with $s \approx 0.1$. The sensitivity of the model results to Pe is also investigated.

3.2.1. Theoretical Motivation for the Asymmetric Gaussian Shape of Yardangs

Before I document the results of the landscape evolution model, let us first explore how an asymmetric Gaussian model for the shape of individual yardangs can be motivated theoretically. The asymmetric Gaussian has been shown to accurately represent the morphology of yardangs in OWSVRA (Pelletier et al., 2018). In section 3.2.1, I noted that the lateral expansion of a 3-D wall-bounded turbulent jet also follows a Gaussian. In this section I consider how an asymmetric Gaussian is consistent with a spatial pattern of eolian abrasion that is qualitatively consistent with measured spatial variations in sediment fluxes around yardangs documented in the companion paper (Pelletier et al., 2018).

Figure 6. Model for the asymmetric Gaussian shape of yardangs. (a) Color map of hypothesized eolian abrasion rate, E_a (equation (15)). (b) Color map of yardang topography associated with the asymmetric Gaussian model (equation (2)). (c) Plots of elevation and eolian erosion rate for the cross-sectional profiles shown in (a) and (b).

The erosion rate is the sum of water-driven hillslope sediment transport processes governed by equation (10) and eolian abrasion governed by equation (8):

$$\frac{\partial z}{\partial t} = D_{\text{topo}} \nabla^2 z - E_a. \quad (13)$$

The yardang-forming instability tends to increase relief until an approximate steady state condition is achieved, for which equation (13) can be written as

$$E_a = D_{\text{topo}} \nabla^2 z \quad (14)$$

Figure 6a provides a color map of E_a with $z(x,y)$ assumed to follow an asymmetric Gaussian (equation (2), shown as a color map in Figure 6b). According to equation (14), profiles of E_a in the cross-wind direction should scale with the second derivative of a Gaussian if the topographic profile is itself a Gaussian, that is,

$$E_a \propto \left(y^2 - \frac{x_w^2}{2} \right) \exp \left(- \left(\frac{y}{x_w} \right)^2 \right) + \frac{x_w^2}{2} \quad (15)$$

This function is similar to a Gaussian (inverted so that the $[x,y] = [0,0]$ has low rather than high values) but modified so that it has a band of high erosion rates flanking the obstacle (Figure 6). When wind and wind-blown sediments are deflected around yardangs, the highest sediment fluxes are found in a band that surrounds each yardang as a result of the deflection of wind-blown sand around the yardang (Pelletier et al., 2018). Equation (15) is qualitatively consistent with this pattern. As such, this section demonstrates that a spatial pattern of eolian abrasion around yardangs similar to the pattern of sediment flux documented in the companion paper (Pelletier et al., 2018) is consistent with an asymmetric Gaussian model for yardang morphology. More broadly, the model of this section demonstrates that a steady state solution exists in which incision of the troughs by eolian abrasion is balanced by diffusive processes that limit yardang relief.

Figure 7. Plots of relative kinetic energy flux as a function of distance above the ground for two values of the shear velocity, u_* , and a range of values of the particle diameter, d .

3.3. The Eolian Abrasion Profile

Figure 7 plots the relative abrasion rate as a function of the distance above the ground for a range of particle diameters, d , and shear velocities using the equations of section 2.2.2. The aerodynamic roughness length for these calculations was estimated from the shear velocity using the empirical relationship of Sherman (1992) and Sherman and Farrell (2008):

$$z_0 = (\alpha/g)(u_* - u_{*t})^2 \quad (16)$$

where g is the acceleration due to gravity. I adopted the values $\alpha = 0.063$ and $u_{*t} = 0.2 \text{ m/s}$ documented in the companion paper (Pelletier et al., 2018). I used the formula of Farrell and Sherman (2015) to estimate the settling velocity w_s of each particle size.

The results plotted in Figure 7a show a maximum in the abrasion profile at a finite distance above the ground equal to a few centimeters for most of the parameter sets considered, consistent with the results of long-term abrasion studies (e.g., Sharp, 1964, 1980). Above the maximum, abrasion rate falls off rapidly with height such that the abrasion rate 1 m above the surface is generally $\sim 10\%$ or less than the maximum value. Specifically, for the intermediate-particle-size case of $100 \mu\text{m}$, the abrasion rate 1 m above the ground is more than 10 times smaller than the maximum abrasion rate for $u_* = 0.8 \text{ m/s}$, decreasing to ~ 100 times smaller than the maximum for $u_* = 0.6 \text{ m/s}$.

Figure 7b compares the model prediction to the data reported by Sharp (1980) for the abrasion of lucite rods placed vertically in the ground in the Coachella Valley from 1952 to 1967. Sharp's (1980) data were acquired along the direction in which the maximum abrasion was measured (NE-SW) (his Table 1). The best fit was obtained using a Rouse number of 1.31 and a z_0 value of 9 mm. The most common particle sizes in transport in the Coachella Valley range from 125 to $250 \mu\text{m}$ (Table 3 of Sharp, 1964). If the characteristic particle size is assumed to be $125 \mu\text{m}$, then the best fit Rouse number of 1.31 implies $u_* = 1.3 \text{ m/s}$. This value is on the high end of shear velocities that have been measured in nature, but not unreasonable

given that Sharp chose his field area because it was known to be an area subject to "frequent strong winds directly out of San Geronio Pass" (Sharp, 1964). Assuming $u_* = 1.3 \text{ m/s}$, equation (16) predicts $z_0 = 8 \text{ mm}$, that is, consistent with the z_0 value of 9 mm determined from a least squares fit to the Sharp (1980) data. Note that a range of particle sizes are always in transport and median particle sizes generally decrease with increasing height above the bed (e.g., Figure 5 of Sharp, 1964; Pelletier et al., 2018). As such, assuming any single particle size is an approximation. Nevertheless, Figure 7b demonstrates that the equations of section 2.2.2 predict abrasion profiles consistent with those measured in the field.

Table 2
Parameters and Initial Conditions of the Two Sets of Model Runs

	Model set #1 (section 3.4.1)	Model set #2 (section 3.4.2)
Initial topography	small (0.1 m) periodic hills	flat, horizontal
Substrate characteristics	uniform erodibility characterized by K	nonuniform (defined by strike, dip, spacing, and thickness of resistant strata plus erodibility contrast between hard and soft strata)
u_*	0.8 m/s	0.8 m/s
z_0	0.001 m	0.001 m
Pe	5 (also varied)	5
K	$0.7 \text{ s}^3 \cdot \text{m}^{-2} \cdot \text{kyr}^{-1}$	$0.7 \text{ s}^3 \cdot \text{m}^{-2} \cdot \text{kyr}^{-1}$
D_{topo}	$1 \text{ m}^2/\text{kyr}$ (also varied)	$1 \text{ m}^2/\text{kyr}$ (also varied)
a	1	1 (also varied)

Figure 8. Color maps of sediment concentration, c , for three different values of the Péclet number: (b) $Pe = 5$ (which agrees most closely with the fluid mechanics literature), (c) $Pe = 20$, and (d) $Pe = 1.25$. (a) Color map of the initial topography.

3.4. Landscape Evolution Model Results

In this section I describe the results of two sets of landscape evolution model runs. In the first set of runs (section 3.4.1 and Table 2) the substrate has uniform erodibility. The yardang-forming instability in these runs is seeded with small (0.1 m tall) periodic hills. Such an initial condition is unrealistic because periodic yardangs are most likely generated from an aperiodic or otherwise structureless initial condition. However, this is a useful case to consider in order to gain intuition about how the model behaves in a relatively simple scenario and to demonstrate how the aspect ratio of yardangs is set early on by the interaction of wind and wind-blown sediments with topography. In the second set of model runs (section 3.4.2 and Table 2) I consider the case of an initially horizontal topography. In these runs the yardang-forming instability is triggered by spatial variations in the substrate erodibility. These variations are defined by the strike, dip, spacing, and

Figure 9. Time evolution of the model with initially small periodic hills. (a and c) Color map of model topography after 1 and 2 kyr. (b and d) Color map of eolian abrasion rate after 1 and 2 kyr.

Figure 10. Dependence on the relief on time for the version of the model with initially small periodic hills, for a range of values of the (a) topographic diffusivity D_{topo} and (b) the initial relief, R_{init} . Yardangs are more likely to develop for smaller values of D_{topo} and/or larger values of R_{init} .

thickness of strata. In addition, I assume random spatial variability in the erodibility coefficient K_h that, over time, generates microtopographic variations that, under certain conditions, can initiate the yardang-forming instability. This second set of model runs speaks directly to how periodic topography can self-organize from initial conditions that are aperiodic in the cross-wind direction.

3.4.1. Model Results With Initially Small Periodic Hills

Figures 8b–8d present color maps of the eolian abrasion rate during the first time step of the model given an initial topography with small (0.1 m) periodic hills (Figure 8a). Figures 8b–8d differ only in the value of Pe : Figure 8b is run with $Pe = 5$. Figures 8c and 8d are run with values of Pe four times higher and four times lower than the reference value: $Pe = 20$ and $Pe = 1.25$, respectively. The presence of small periodic hills causes wind and wind-blown sediments to be deflected around the hills. The aspect ratio of the zone of low eolian abrasion rate in the wake of each hill depends on the value of Pe . For $Pe = 5$, the zone of low eolian abrasion rate has an aspect ratio of ≈ 10 . For $Pe = 20$ and $Pe = 1.25$, the zone of low eolian abrasion rate has aspect ratios of ≈ 25 and ≈ 3 , respectively. Note that no erosion has yet taken place in the model; that is, the wind flow and the resulting abrasion rate have been computed based on the initial topography but the initial topography has not yet been modified for any of the results illustrated in Figure 8. The fact that the model exhibits a yardang-shaped zone of low eolian abrasion rate in the wake of every obstacle demonstrates that the characteristic streamlined form of yardangs is present in the model from the beginning.

The time evolution of the model with initially small periodic hills is illustrated in Figure 9 (an animation is provided as dynamic content). The model parameters are $u_* = 0.8$ m/s, $z_0 = 1$ mm, $Pe = 5$, $K = 0.7$ s³ · m^{−2} · kyr^{−1}, and $D_{\text{topo}} = 1$ m²/kyr (Table 2). The values of u_* and z_0 were chosen to be within the range of values measured during wind storms at OWSVRA. The value of K was chosen so that the resulting eolian abrasion rate, E_a , equals 3.5 m/kyr, that is, in the middle of the 3–4 m/kyr range of values inferred from geologic constraints and repeat ground-based lidar surveys at OWSVRA (Pelletier et al., 2018). The value of D_{topo} corresponds to the value inferred from OWSVRA (Pelletier et al., 2018).

Figures 9a and 9b and 9c and 9d illustrate the topography and eolian abrasion rate, respectively, following 1 and 2 kyr of model evolution. Over time, the zone of low erosion in the wake of the initial topographic obstacles generates a streamlined form similar to that of natural yardangs.

Figure 10 plots the relief as a function of time for a range of values of the initial relief and D_{topo} . For cases with a sufficiently low D_{topo} and/or high initial relief, relief increases nonlinearly in time as incipient troughs are lowered and wind and wind-blown sediment are focused into troughs and deflected away from topographic highs in a positive feedback. As the landscape relief increases, the rate of water-driven sediment transport from hillslopes increases, reducing the rate of increase in relief over time. An approximate steady state condition is developed in which incision of the troughs is balanced by diffusive erosion of the upper portions of yardangs. Conversely, for cases with a sufficiently large D_{topo} and/or small initial relief, yardangs can fail to develop; that is, the initial relief can decrease over time or increase so slowly that they do not develop into anything larger than microtopographic features. Note that there is a limit to the increase in relief generation with decreasing D_{topo} , that is, as D_{topo} approaches zero the rate of relief generation reaches a maximum.

3.4.2. Model Results With Initially Flat Topography and Substrate Heterogeneity

In this section I consider model runs with an initially horizontal topography (i.e., $z[x,y] = 0$). The yardang formation instability is seeded by resistant strata that have random variations in erodibility (equation (9)).

Figure 11. Time evolution of the model with initially flat, horizontal topography and resistant layers with random variations in erodibility. (a and c) Color map of model topography after 1.5 and 2 kyr. The typical yardang has an aspect ratio of ≈ 5 . (b and d) Color map of eolian abrasion rate after 1.5 and 2 kyr. As the relief increases, the erosion becomes more localized in troughs in a positive feedback.

As initial conditions considered in this section have no characteristic spatial scale in the cross-wind direction, the emergence of yardangs with a characteristic width/spacing reflects the preferential development of yardangs at a particular range of spatial scales set by the physics of the model rather than any characteristic spacing of preexisting channels or substrate weaknesses.

Relief is generated in this set of model runs by prescribing a sequence of parallel strata that are relatively resistant to erosion (Figure 11). Portions of the substrate with smaller values of K_h tend to become

Figure 12. Time evolution of the model illustrating bedrock structural control on yardang morphology. (a and c) Color map of model topography after 2 kyr with a dip of 60° and a strike (a) perpendicular to the wind and (c) offset 10° from the prevailing wind direction. For yardangs that strike nearly perpendicular to the wind (as in (a)), a steeper dip creates resistant strata that are more closely spaced, thereby limiting the lengths of yardangs and hence their aspect ratios. For yardangs that strike nearly parallel to the wind, wind and wind-blown sediment are diverted around the resistant bed where they incise readily into the weaker beds flanking the resistant bed. The result is yardangs with a large aspect ratio that parallels the bedrock structure for a long distance.

Figure 13. Dependence on the relief on time for the version of the model with flat, horizontal initial topography and resistant strata with random variations in erodibility, for a range of values of the (a) topographic diffusivity D_{topo} and (b) the parameter a that defines the erodibility contrast between hard and soft strata. Yardangs are more likely to develop for smaller values of D_{topo} and/or large values of a .

$D_{\text{topo}} = 2 \text{ m}^2/\text{kyr}$. Figure 14 demonstrates that larger values of D_{topo} result in longer and wider yardangs. This result can be understood by considering the time scale of yardang development and the amount of topographic smoothing by water-driven sediment transport processes that can occur over that time scale. Yardang development is an instability that operates over a time scale of ≈ 2 kyr for this set of model parameters. Based on the model results of Figure 14, topographic variations that are diffused over a time scale of 2 kyr do not grow into yardangs, while larger variations tend to develop into yardangs. The length scale of topographic smoothing, which is associated the half-width of a diffusive profile, is given by

$$\lambda \approx 2\sqrt{D_{\text{topo}}t} \quad (17)$$

For $D_{\text{topo}} = 2 \text{ m}^2/\text{kyr}$, $\lambda \approx 4$ m. Conversely, for $D_{\text{topo}} = 0.5 \text{ m}^2/\text{kyr}$, $\lambda \approx 2$ m. Note that the largest yardangs in Figure 14a are comparable to the largest yardangs in Figure 14b. The main difference between the two figures is the absence of small yardangs, that is, those less than ≈ 8 m in width, in Figure 14b that are present in Figure 14a. Such yardangs are absent because diffusion smoothed these features before the yardang-forming instability had time to develop them into yardangs.

topographic highs over time; hence, they experience less eolian abrasion than topographic lows where abrasion is localized. This generates a positive feedback in which wind and wind-blown sediments are further deflected around topographic highs and into lows, thus increasing erosion rates in the troughs. If the troughs become overdeepened, the overdeepenings are episodically backfilled with sediments, thus allowing other portions of the landscape to keep pace with the troughs. The topography in the model self-organizes into yardangs with aspect ratios similar to those observed in nature (an animation of this case is provided as dynamic content).

Yardang geometry in the model is strongly controlled by stratal architecture. Figure 12 illustrates the model topography and erosion rate after 2 kyr of evolution. These cases are governed by the same parameters as the model run illustrated in Figure 11 except that the dip angle has increased from 20° to 60° . Figures 12a and 12b differ only in the strike direction; the strike is perpendicular to the wind in Figure 12a and is offset 10° from the wind in Figure 12b. The result illustrated in Figure 12a demonstrates that for strata striking nearly perpendicular to the wind, steeper dips generate shorter yardangs, consistent with observations at OWSVRA (Pelletier et al., 2018). Yardang width stays approximately constant, thereby decreasing the aspect ratio. Figures 12c and 12d demonstrate that the yardang aspect ratios tend to increase when the strata strike more nearly parallel to the prevailing wind direction, a result that is also consistent with data from OWSVRA (Pelletier et al., 2018).

In Figures 13a and 13b I illustrate how topographic diffusivity and the erodibility contrast between hard and soft strata (quantified by a) control relief generation in the model. Figures 13a and 13b plot the relief as a function of time for a range of values of D_{topo} and a . For relatively low values of a and/or high values of D_{topo} , yardangs do not form (the microtopography remains small). The fact that greater heterogeneity (i.e., a large value of a) promotes yardang development is consistent with the observation that yardang development is enhanced in the more heterogeneous Borrego Formation of the OWSVRA (Pelletier et al., 2018).

Figure 14 illustrates the role of D_{topo} in controlling yardang size. This figure illustrates color maps of the model topography using the same parameters as Figure 11 except for (a) $D_{\text{topo}} = 0.5 \text{ m}^2/\text{kyr}$ and (b)

Figure 14. Model results designed to illustrate the role of D_{topo} in controlling the lengths and widths of yardangs in the version of the model with flat, horizontal initial topography and resistant strata with random variations in erodibility. Color maps of the model topography for the same parameters as the case illustrated in Figure 11 except (a) $D_{\text{topo}} = 0.5 \text{ m}^2/\text{kyr}$ and (b) $D_{\text{topo}} = 2 \text{ m}^2/\text{kyr}$. Larger values of D_{topo} filter out smaller yardangs, resulting in longer and wider yardangs for otherwise similar conditions.

4. Discussion

4.1. Implications for the Minimum-Drag Model of Streamlined Bedforms

The analysis of this paper demonstrates that yardangs are not minimum-drag forms. A key assumption of the minimum-drag model is the similarity between the aerodynamics of streamlined free bodies and half-bodies. That streamlined free bodies minimize drag at an aspect ratio of ≈ 4 is a well-established fact that has long been used in torpedo design among other applications. Figure 4, however, demonstrates that important differences exist between the aerodynamics of streamlined free bodies and half-bodies. In streamlined free bodies, a relatively large skin drag penalty is incurred as the aspect ratio is increased because new surface area is created at which the fluid moving past the obstacle must come to zero speed. These new areas of stagnant flow induce drag in nearby portions of the flow which, in turn, contribute to the total drag that the obstacle induces on the flow. In contrast, streamlined half-bodies reside with a boundary layer in which the flow has to come to zero speed regardless of how tapered the bedform is. As a result, the penalty incurred by increasing the aspect ratio of streamlined half-bodies is much lower than in the case for free bodies, a result borne out by the CFD model results illustrated in Figure 3.

More data are needed to determine the most common aspect ratios of yardangs and the variables that exert control on these values. However, the analysis presented in the companion paper (Pelletier et al., 2018) demonstrates that 4 is not a common aspect ratio in OWSVRA. It was shown that this is in part because bedrock structure plays an important role in controlling yardang morphology in this yardang field. That said, 4 is the correct order of magnitude for yardang aspect ratio; that is, many yardangs are in the range of 2 to 10.

In this paper I proposed that aspect ratios in the range of 5–10 can result from the lateral dispersion of turbulent momentum downwind from the zone of confined flow, a dispersion that can be quantified by a Péclet number. This idea is consistent with studies of 3-D wall-bounded turbulent jets, which have shown that confined flows expand linearly with distance downwind with aspect ratios in the range of 5 to 10, depending primarily on the velocities of the downstream flows that the jets interact with. Yardangs with smaller (e.g., 2) and larger (e.g., >10) aspect ratios appear to be associated with yardangs that dip steeply and strike nearly perpendicular to the prevailing wind direction (resulting in small aspect ratios) and those that strike subparallel to the prevailing wind direction (large aspect ratios), consistent with data from OWSVRA (Pelletier et al., 2018).

4.2. Wind and Water Both Play Crucial Roles in Yardang Development

The analyses of this paper and the companion paper demonstrate the important role played by water-driven processes, including freeze-thaw-driven creep, rain splash, and interrill and rill erosion. This conclusion is not novel (e.g., Brookes, 2001, 2003; Dong et al., 2012). However, the numerical model demonstrates quantitatively how the efficiency of water-driven erosional processes controls yardang morphology, that is, the value of the topographic diffusivity, D_{topo} , features prominently in many of the key equations of this paper. A larger value of the topographic diffusivity increases the width and spacing of yardangs (Figure 14), all else being equal.

4.3. Implications for the Relative Importance of Abrasion by Saltating and Suspended Particles

There is an active debate within the geomorphic community regarding the relative importance of saltating versus suspended particles in driving eolian abrasion. This debate is a subset of the larger question of how abrasion rate varies with distance above the ground. Clearly, there is no universal answer to this question because the particle size distribution of the bed and the wind climatology vary greatly. Nevertheless, the results of this and the companion paper suggest an important role for abrasion by particles near the transition from saltation to suspension. Data from OWSVRA indicate that eolian sediments are transported close to the transition from saltation to suspension (i.e., the concentration profile is a power law with negative exponent [the Rouse number] of 2.5). The numerical model of this paper indicates that transport at the transition between saltation and suspension is consistent with observed vertical abrasion profiles (Figure 7).

4.4. Limitation of the Numerical Model

The numerical model of this paper includes a number of simplifications that should be improved upon in future work. The complex interaction between the bed and particles in transport is approximated by a set of equations (equations (6)–(8)) in which particles are assumed to be advected by the wind (thus steering sediment around topographic highs and into topographic lows) and the contributions from particles transported at different height above the bed is assumed to be captured by integrating the product of sediment concentration and kinetic energy. This approach is simplified and neglects, among other factors, differences in incidence angle. Abrasion rate scales with the sine of incidence angle (Routbort et al., 1980; Scattergood & Routbort, 1983), which for a horizontal surface subject to saltating particles is in the range of 8–15° (Mitha et al., 1986). The steep portion of the windward face of yardangs is likely subject to impacts with a higher mean incidence angle, thus increasing the erosion rate there. I did not include this effect in the model because specifying incidence angles is difficult for particles near the saltation-suspension transition and because neglecting the effect of a higher mean incidence angle on the windward face is likely partially offset by another simplification of the model, that is, that the yardang is assumed to be uniformly soil-covered. Abrasion strips soil from the majority of the windward face of yardangs at OWSVRA, exposing more resistant bedrock and thereby slowing the erosion that would otherwise be promoted by a higher mean incidence angle. Future work should explicitly include soil thickness (allowing bedrock to be exposed on the windward face) and spatial variations in incidence angle.

The model of this paper includes random variations in substrate erodibility using the erodibility coefficient of soft strata, K , as an upper bound and assuming that each grid point where hard strata are exposed has an erodibility coefficient K_r equal to the product of K and a random number equally distributed between $(1 - a)$ and 1 (equation (9)). This approach is just one of many that could be used to implement erodibility variations in the model to seed the yardang-forming instability. Equation (9) has the advantage that it requires just one additional parameter (i.e., a) and hence is a parsimonious approach to introducing

random variability into the model. However, this approach limits the difference between the mean values of the erodibility coefficient for the hard and soft strata to be a factor of 2 and provides no flexibility for prescribing or varying the distributions of the erodibility coefficients. Future applications of the model should consider a more comprehensive or general approach to quantifying spatial variations in erodibility, ideally one that is guided by data.

The diffusion model of this paper assumes a uniform diffusivity, which is a crude approximation for interrill and rill erosion especially. Sediment transport by interrill and rill flow depends on topographic position, with larger sediment fluxes occurring on the lower portions of slopes and/or on longer slopes, all else being equal (McCool et al., 1989). Future studies of yardang evolution should explicitly model rill formation and evolution (e.g., McGuire et al., 2013) or allow D_{topo} to vary with topographic position and/or soil thickness.

4.5. Future Applications of the Numerical Model

The numerical model of this paper was motivated by observations and measurements at OWSVRA. As such, its application to other yardang fields will require modification. On Mars, for example, modern hillslope transport is dominated by gravity-driven rather than water-driven processes. In such cases, a model of mass wasting that explicitly includes weathering and abrupt mass movements would be more appropriate than the diffusion equation. However, a model with sediment transport that depends linearly on slope and one that is based on a threshold angle for stability both result in a limit on yardang height that is a function of noneolian processes. As such, many general/conceptual results from the models of this paper, such as the fact that yardang height can be limited by noneolian processes, likely apply to other yardang fields.

Yardangs form over a wide range of spatial scales (e.g., Goudie, 2007). The model of this paper should prove useful as a starting point for investigating the feedbacks among wind erosion and topography at larger spatial scales. Basic questions remain about how the largest scale of yardangs, that is, megayardangs with heights of ~10–100 m and lengths of ~1 km (Goudie, 2007), form. Megayardangs may be associated with extreme shear velocities that are attainable at only a few locations on Earth. In the numerical model of this paper, more extreme windstorms are associated with larger values of u_* and z_0 , which have the effect of limiting topographic steering of wind and wind-blown sand to larger spatial scales. Megayardangs may in some cases be a result of preexisting low-drainage-density fluvial systems in the troughs among yardangs. More research, likely involving field work at new study sites, is needed to test these hypotheses.

Feedbacks may also exist between wind erosion and tectonic exhumation. In the Qaidam Basin of China, for example, anticlines have developed with steep forelimbs oriented perpendicular to the wind (Kapp et al., 2011, their Figure 5). Such patterns suggest that the growth of fault propagation folds may be enhanced where wind erosion reduces the topographic overburden that would otherwise limit fault growth. Future studies should test the feasibility of this and other process-based hypotheses for the feedbacks between wind erosion and tectonic exhumation.

5. Conclusions

In this paper I presented models, motivated by the observations and measurements at OWSVRA presented in the companion paper (Pelletier et al., 2018), aimed at a better understanding of yardang development and controls on yardang morphology. Using the results of computational fluid dynamics (CFD) modeling, I demonstrated that yardangs with an aspect ratio of ≈ 4 are not minimum-drag forms. As an alternative, I proposed that the downwind expansion of wind and wind-blown sediment focused into the troughs between adjacent yardangs can be modeled as a 3-D wall-bounded turbulent jet. This model yields yardangs with aspect ratios in the range of 5–10. To better understand the aerodynamic and structural controls on yardang morphology, I developed a landscape evolution model that combines the physics of boundary layer flow and abrasion by eolian sediment transport with a model for the erosion of the tops and lee sides of yardangs by water-driven erosional processes. This model suggests that yardang formation is enhanced in substrates with greater heterogeneity and lower rates of erosion by water-driven processes. The results of this paper place the geomorphology of yardangs on a more quantitative foundation and provide a basis for future model-based investigations, including how megayardangs form, the interaction of wind and water in the formation of wind-sculpted landscapes, and the interaction between eolian erosion and tectonic exhumation.

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