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A new wave front shape-based approach for acoustic source localization in an anisotropic plate without knowing its material properties

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ABSTRACT

Estimating the location of an acoustic source in a structure is an important step towards passive structural health monitoring. Techniques for localizing an acoustic source in isotropic structures are well developed in the literature. Development of similar techniques for anisotropic structures, however, has gained attention only in the recent years and has a scope of further improvement. Most of the existing techniques for anisotropic structures either assume a straight line wave propagation path between the source and an ultrasonic sensor or require the material properties to be known. This study considers different shapes of the wave front generated during an acoustic event and develops a methodology to localize the acoustic source in an anisotropic plate from those wave front shapes. An elliptical wave front shape-based technique was developed first, followed by the development of a parametric curve-based technique for non-elliptical wave front shapes. The source coordinates are obtained by minimizing an objective function. The proposed methodology does not assume a straight line wave propagation path and can predict the source location without any knowledge of the elastic properties of the material. A numerical study presented here illustrates how the proposed methodology can accurately estimate the source coordinates.

KEY WORDS: acoustic source localization; anisotropic structures; elastic wave propagation; wave front shape; elliptical wave front; non-elliptical wave front

1. Introduction

Structural health monitoring (SHM) has now become an integral part of infrastructure engineering in order to detect and mitigate the damage occurring in a structure at any point of its lifetime. Various events such as impact of a foreign object, formation of cracks, failure of a structural element, etc. can act as an acoustic source in a structure that generates acoustic signals which propagate through the system. An important step towards the passive health monitoring of structures is the prediction of the acoustic source location by capturing and analyzing the wave-signals generated by the source and recorded by a group of ultrasonic sensors installed at specific locations of the structure. Several studies in the past have proposed methods for acoustic source localization in isotropic structures [1–13]. A large portion of modern infrastructure elements, however, exhibit anisotropy, and development of techniques for acoustic source localization in such anisotropic structures has received attention only in the recent past. Nakatani et al. [14,15] considered the beamforming technique which is commonly applicable to isotropic plates and extended the concept to be applicable to anisotropic plates. They modified the expression of beamforming array by incorporating direction dependent wave speed and proposed a source localization technique using delay-and-sum algorithm. This method requires the knowledge of the direction-dependent velocity profile in an anisotropic plate. Kundu et al. [16,17] proposed an optimization-based technique that uses minimization of an objective function containing time difference of arrival of waves between sensors and direction-dependent velocity information. For anisotropic materials with unknown material properties (and therefore unknown direction-dependent velocity profiles), Kundu [18] and Kundu et al. [19] proposed a

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technique where the source location can be estimated by mounting sensors in several right-angled clusters on some pre-selected locations. Time difference of arrival of waves between the sensors in a cluster needs to be evaluated as accurately as possible. It was done by cross-correlating signals recorded by the sensors. This method does not require any solution of nonlinear equations. Yin et al. [20] extended this concept to propose a localization technique using “Z”-shaped sensor clusters, thus requiring fewer sensors for similar accuracy. Ciampa et al. [21,22] proposed a technique for the similar problem where one needs to solve nonlinear equations. Kundu et al. [23] proposed a hybrid technique that estimates an initial source location by the method proposed in [19] and then with thus obtained source location as the initial guess, an objective function was minimized to arrive at a more accurate estimate of the source coordinates. Simone et al. [24] proposed a linearized algorithm to predict the source location in a composite plate using only four sensors. Sabzevari and Moavenian [25] also proposed a four sensor-based method for anisotropic plates using attenuation analysis. Use of two arrays of sensors in mutually perpendicular directions has been proposed by Xiao et al. [26] to localize the acoustic source for a plate-type structure with unknown material properties by the beamforming technique.

Most of these studies for anisotropic structures, however, assume that the wave propagates in a straight line from the source to a sensor. This may not be a realistic assumption for an anisotropic material [27]. Baxter et al. [28] proposed a source localization technique in a complex structure with unknown properties where due to complexity in the structure the wave may not propagate along a straight line from the source to the sensor. This technique requires the structure to be initially trained by a set of artificial training data and then the source location is predicted from the comparison between the recorded time of arrival difference between a pair of sensors and the time of arrival difference between that pair of sensors from the training data. Hensman et al. [29] improved this technique by introducing probabilistic interpretation and thus requiring less training data. Grabowski et al. [30] presented a source localization technique in thin plates using time-distance domain transformation. Their method requires the material properties to be known. Gollob et al. [31] proposed a source localization approach in a heterogeneous medium based on a modified Dijkstra algorithm. Although their method does not assume a straight line wave propagation path between the source and the sensor, one needs to know the direction-dependent velocity profile. Park et al. [32] introduced a technique for localizing the acoustic source based on the shape of the wave front in an anisotropic plate without knowing its material properties. Their approach does not assume that the wave travels along a straight line. They assumed two different geometrical shapes, namely, rhombus and ellipse, as the wave front shapes, and proposed a source localization technique by performing geometrical analyses on those shapes and incorporating the sensor cluster data.

In the present study, the elliptical wave front-based formulation proposed in [32] is modified to accommodate different other shapes of wave fronts. A more general shape of the wave front in an anisotropic plate is developed by considering a parametric curve as the wave front. Various wave front shapes can be modeled by changing the parameters of the curve. The technique solves for a set of unknown variables including the coordinates of the acoustic source by minimizing an objective function. It is demonstrated through a numerical example that both ellipse-based and the parametric curve-based approaches can predict the source location with sufficient accuracy. When the wave front does not look like an ellipse or rhombus then the technique proposed in reference [32] cannot be applied. In those situations the only way one can predict the acoustic source accurately is by using parametric curve based non-elliptic wave fronts proposed in this paper. Even when the numerically generated wave front looks approximately like an ellipse the assumption of an elliptical wave front does not predict the acoustic source as accurately as the proposed new parametric curve based technique does.

In the following sections, the sensor cluster configuration to be used in this study is described first. Then the elliptical wave front shape-based method proposed in [32] for localizing the acoustic source in an anisotropic plate is modified by properly relating the wave front geometry with the recorded signal data. Following this, a parametric curve is proposed as the assumed shape of the wave front and a source localization technique is proposed for this wave front shape. The solution algorithm to solve for the unknown parameters for both these approaches is discussed. Finally, A numerical study considering the same anisotropic plate as in [32] shows the effectiveness of the proposed methodology.

2. Tangent estimation for incident wave-angle at sensor clusters

In order to estimate the tangent of the angle at which the wave generated by an acoustic event arrives at a certain location in an anisotropic plate, Kundu [18] and Kundu et al. [19] proposed a sensor cluster-based technique that makes use of a cluster consisting of three receiving sensors mounted on the plate in a right-angled L-shape configuration at the specific location of interest. Park et al. [32] considered two different orientations of such a sensor cluster as shown in Figs. 1(a) and 1(b), denoted in the present study as Type-a and Type-b sensor clusters, respectively. It may be noted that the key difference between these two orientations lies in the relative position of the three sensors A, O and B within a sensor cluster. As can be seen from Fig. 1, Type-a sensor cluster is designed to receive the incident wave-angle in the form of an acute angle, while Type-b sensor cluster is designed to receive the incident wave-angle in the form of an obtuse angle. It is shown later in this section that this basic difference between the two types of sensor clusters leads to tangents of the incident wave-angles given by the opposite signs of the same measured quantity. Both these types of sensor clusters are mounted on a plate such that the lines AO and OB are parallel to the x and y -axes of the assumed coordinate system, respectively. The distance (d) between sensors A and O is same as that between sensors O and B, and should be small enough to ensure insignificant variation of the signal characteristics among the three sensors in a cluster. Further, d should be much smaller than the distance between the acoustic source and the sensor cluster [32]. Thus, one may reasonably assume that the wave front encountered by a sensor cluster is planar in nature and therefore, a single wave speed (v) can be considered for the entire cluster even though the plate material is anisotropic. It may be noted here that because of the small value of d , the coordinates of any one of the three sensors can be regarded as the coordinates of the cluster. In this study, the coordinates of sensor O are considered as the sensor cluster coordinates.

Fig. 1. (a) Type-a and (b) Type-b sensor clusters

The angle θ is the incident angle of the planar wave front striking the sensor cluster. The wave front first arrives at sensor O, then it strikes either sensor A or B. If t_O , t_A and t_B denote the times of arrival (TOAs) of the signal from the acoustic source to the sensors O, A and B, respectively, for Type-a configuration (see Fig. 1(a)), $\tan \theta$ can be expressed as

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{v(t_B - t_O)/d}{v(t_A - t_O)/d} = \frac{t_B - t_O}{t_A - t_O} \quad (1)$$

Similarly, for Type-b configuration (see Fig. 1(b)),

$$\tan \theta = -\tan \alpha = -\frac{\sin \alpha}{\cos \alpha} = -\frac{v(t_B - t_O)/d}{v(t_A - t_O)/d} = -\frac{t_B - t_O}{t_A - t_O} \quad (2)$$

In Eqs. (1) and (2) v is the velocity of the wave. In order to obtain a sufficiently accurate estimation of $\tan \theta$, TOAs (i.e., t_O , t_A and t_B) need to be measured as accurately as possible. This can be achieved by using Akaike Information Criterion (AIC) [12,27] given by

$$AIC(k) = k \ln[\text{var}\{U(1:k)\}] + (K - k - 1) \ln[\text{var}\{U(k+1:K)\}] \quad (3)$$

where k is the sample number corresponding to time t in the signal recorded by a sensor, K is the total number of samples recorded, U is the recorded signal amplitude and $\text{var}\{U(r:s)\}$ denotes the variance of all signal amplitude values between (and including) r^{th} and s^{th} samples. If AIC is plotted against k , two distinct valleys are observed. The first valley represents the arrival of the first wave group, while the second valley represents the arrival of the second wave group [27]. The values of t corresponding to the first and second valleys are the required TOAs at the sensor for the first and second wave groups, respectively. In this study, only the second wave groups that correspond to the first antisymmetric mode [27] are considered to develop the proposed approach for localizing the acoustic source in an anisotropic plate. Hence, only the TOAs corresponding to the second wave groups will be used.

3. Proposed techniques for acoustic source localization

The waves generated in anisotropic plates due to an acoustic event do not usually propagate along straight lines from the source to a sensor mounted on the plate [27, 32]. This leads to the generation of non-circular wave fronts. Park et al. [32] attempted to localize the source by analyzing the wave front shapes. Based on a numerical study on an anisotropic plate subject to an acoustic event, it was concluded that the first wave group leads to a wave front shape that can be idealized as a rhombus, whereas the second wave group results in a wave front shape that can be idealized as an ellipse. Even though the rhombus wave front was found to perform well in predicting the source location, the localization technique for an elliptical wave front, however, was shown to have a scope of further improvement. In this section, the elliptical wave front-based method presented in [32] is modified by properly accounting for the relationship between the tangent of the incident wave-angle (measured using Eqs. (1) and (2)) and the derivative of the shape of the wave front. A more general three parameter-based curve is also proposed to have the capability of modeling other wave front shapes. A source localization technique is proposed for both elliptical and non-elliptical wave front shapes.

3.1. Modified formulation for elliptical wave front

The equation of an elliptical wave front as shown in Fig. 2 is given by

$$\frac{(x-C_x)^2}{a^2} + \frac{(y-C_y)^2}{b^2} = 1 \quad (4)$$

where (x, y) are the coordinates of a point on the ellipse, (C_x, C_y) are the coordinates of the acoustic source, and a and b are the lengths of the semi-major and semi-minor axes, respectively. Taking derivative of Eq. (4) with respect to x leads to

$$\frac{dy}{dx} = -\frac{b^2}{a^2} \frac{x-C_x}{y-C_y} \quad (5)$$

It may be noted that an acoustic event generates wave fronts that may be considered as concentric in nature [32]. Therefore, the ratio b^2/a^2 can be replaced by a single (positive) unknown λ . Thus, Eq. (5) may be re-written as

$$\lambda(x-C_x) + m(y-C_y) = 0 \quad (6)$$

where $m = \frac{dy}{dx}$. Since the wave fronts pass through the sensor clusters, for the i^{th} sensor cluster, Eq. (6) gives

$$\lambda(x_i - C_x) + m_i(y_i - C_y) = 0 \quad (7)$$

with (x_i, y_i) denoting the coordinates of the i^{th} sensor cluster and m_i is m measured at that sensor cluster. It may be noted that m_i physically represents the slope of the tangent line to the wave front at (x_i, y_i) , i.e.,

$$m_i = \tan \psi_i \quad (8)$$

where ψ_i is the angle ψ (see Fig. 1) for the i^{th} sensor cluster. It follows from Fig. 1 that ψ_i is equal to $\theta_i + \frac{\pi}{2}$ and $\theta_i - \frac{\pi}{2}$ for Type-a and Type-b sensor clusters, respectively. Therefore, Eq. (8) can be re-written as

$$m_i = -\frac{1}{\tan \theta_i} \quad (9)$$

where $\tan \theta_i$ can be obtained using Eq. (1) or (2) for the i^{th} sensor cluster. Thus, Eq. (7) consists of three unknowns: λ , C_x and C_y . Considering n number of sensor clusters, Eq. (7) leads to n simultaneous nonlinear equations. These equations can be solved using Levenberg–Marquardt algorithm which minimizes the following objective function

$$\Phi = \sum_{i=1}^n [\lambda(x_i - C_x) + m_i(y_i - C_y)]^2 \quad (10)$$

A minimum value of 3 is needed for n . The Levenberg–Marquardt algorithm to solve for the three unknowns by minimizing Φ is described next.

Fig. 2. Schematic diagram of an elliptical wave front

Let $\boldsymbol{\beta} = [\lambda \ C_x \ C_y]^T$, $f_i(\boldsymbol{\beta}) = \lambda(x_i - C_x) + m_i(y_i - C_y)$ and $\mathbf{f}(\boldsymbol{\beta}) = [f_1 \ f_2 \ \dots \ f_n]^T$. To start the Levenberg–Marquardt algorithm, an initial guess of $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ needs to be made. Then the following steps can be performed to solve for $\boldsymbol{\beta}$ iteratively:

Step 1: Compute the Jacobian $\mathbf{J}$ as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial \lambda} & \frac{\partial f_1}{\partial C_x} & \frac{\partial f_1}{\partial C_y} \\ \frac{\partial f_2}{\partial \lambda} & \frac{\partial f_2}{\partial C_x} & \frac{\partial f_2}{\partial C_y} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial \lambda} & \frac{\partial f_n}{\partial C_x} & \frac{\partial f_n}{\partial C_y} \end{bmatrix} = \begin{bmatrix} x_1 - C_x & -\lambda & -m_1 \\ x_2 - C_x & -\lambda & -m_2 \\ \dots & \dots & \dots \\ x_n - C_x & -\lambda & -m_n \end{bmatrix} \quad (11)$$

Step 2: Evaluate $\mathbf{f}(\boldsymbol{\beta})$.

Step 3: Find the increment of $\boldsymbol{\beta}$ as

$$\Delta \boldsymbol{\beta} = -[\mathbf{J}^T \mathbf{J}]^{-1} \mathbf{J}^T \mathbf{f} \quad (12)$$

Step 4: Calculate a new $\boldsymbol{\beta}$ by adding $\Delta \boldsymbol{\beta}$ to the current $\boldsymbol{\beta}$.

Step 5: Calculate the Euclidean norm of $\Delta \beta$. If the result is more than a pre-defined tolerance level, then go back to step 1. Otherwise, exit the algorithm and report the latest β as the final solution.

Thus, λ, C_x and C_y can be obtained systematically and the predicted location of the acoustic source is given by (C_x, C_y) .

3.2. Wave front represented by a parametric curve

The numerical study conducted by Park et al. [32] indicates that the shape of the wave fronts generated by the second wave group in general deviates from an exact elliptical shape. Hence, it is proposed here to consider a curve that is more general in shape than an ellipse. To model the wave front more accurately and thus increase the accuracy of the source prediction the new parametric curve is considered. By including some additional parameters, Eq. (4) can be modified to obtain the following expression for the parametric curve:

$$\gamma \frac{|x - C_x|^{p+1}}{p+1} + \frac{|y - C_y|^{q+1}}{q+1} = c \quad (13)$$

where γ , p and q are three unknown positive numbers and c is an arbitrary constant. Taking derivative of Eq. (13) with respect to x leads to

$$m = (-1)^{quad} \gamma \frac{|x - C_x|^p}{|y - C_y|^q} \quad (14)$$

where *quad* denotes the quadrant number of the point (x, y) with respect to the straight lines $x = C_x$ and $y = C_y$. Thus,

$$\begin{aligned} quad &= 1 \quad \text{if } x > C_x \text{ and } y \geq C_y \\ &= 2 \quad \text{if } x \leq C_x \text{ and } y > C_y \\ &= 3 \quad \text{if } x < C_x \text{ and } y \leq C_y \\ &= 4 \quad \text{if } x \geq C_x \text{ and } y < C_y \end{aligned} \quad (15)$$

It may be noted from Eq. (13) that for the wave front shapes numerically generated by Park et al. [32], the parametric curve is symmetric with respect to the straight lines $x = C_x$ and $y = C_y$. Further, from Eqs. (14) and (15), it follows that the derivatives of the curve at four points on these two lines (two points along $x = C_x$ and two along $y = C_y$) are same as those for an ellipse at the corresponding points. Eq. (13) indicates that the parametric curve becomes an ellipse when both p and q are 1.0. Hence, ellipse is a special case of the family of curves represented by Eq. (13). By assuming (C_x, C_y) as $(0,0,0)$ along with different combinations of γ as 1.0 and 1.5, p and q as 0.6, 1.1 and 1.6, and c as 10, 15 and 20, a wide range of curves can be generated from Eq. (13) as shown in Fig. 3. Each of the 18 plots of Fig. 3 considers a constant set of p , q and γ , and shows three concentric curves corresponding to three different values of c but for the same values of the other parameters p , q and γ . In order to demonstrate the generality of Eq. (13) as the representation of a wave front due to an acoustic event, the numerically generated

wave front in the anisotropic plate in [32] is considered, and by trial and error the parameters of Eq. (13) are adjusted so that it matches closely with the wave fronts. This comparison is shown in Fig. 4. This figure shows the best-fitted ellipse and the best-fitted parametric curve. The parameters chosen for the parametric curve in Eq. (13) to generate Fig. 4 are: $(C_x, C_y) = (250, 250)$, $p = 0.6$, $q = 0.6$, $\gamma = 0.46$ and $c = 550$, whereas for the elliptical curve the chosen parameters are: $(C_x, C_y) = (250, 250)$, $p = 1.0$, $q = 1.0$, $\gamma = 0.4$ and $c = 2200$. Although two curves in Fig. 4 look almost identical, a close observation of the two curves reveals some differences. Compared to the ellipse the parametric curve is extended more along the major and minor axes. Fig. 4 shows that the wave front is also expanded more along these two axes. It is shown in this paper that even this small difference can produce noticeable improvement in the source localization.

(g)

(h)

(i)

(j)

(k)

(l)

(m)

(n)

Fig. 3. Generated parametric curves given by Eq. (13) considering (a) $p = 0.6$, $q = 0.6$ and $\gamma = 1.0$, (b) $p = 0.6$, $q = 1.1$ and $\gamma = 1.0$, (c) $p = 0.6$, $q = 1.6$ and $\gamma = 1.0$, (d) $p = 1.1$, $q = 0.6$ and $\gamma = 1.0$, (e) $p = 1.1$, $q = 1.1$ and $\gamma = 1.0$, (f) $p = 1.1$, $q = 1.6$ and $\gamma = 1.0$, (g) $p = 1.6$, $q = 0.6$ and $\gamma = 1.0$, (h) $p = 1.6$, $q = 1.1$ and $\gamma = 1.0$, (i) $p = 1.6$, $q = 1.6$ and $\gamma = 1.0$, (j) $p = 0.6$, $q = 0.6$ and $\gamma = 1.5$, (k) $p = 0.6$, $q = 1.1$ and $\gamma = 1.5$, (l) $p = 0.6$, $q = 1.6$ and $\gamma = 1.5$, (m) $p = 1.1$, $q = 0.6$ and $\gamma = 1.5$, (n) $p = 1.1$, $q = 1.1$ and $\gamma = 1.5$, (o) $p = 1.1$, $q = 1.6$ and $\gamma = 1.5$, (p) $p = 1.6$, $q = 0.6$ and $\gamma = 1.5$, (q) $p = 1.6$, $q = 1.1$ and $\gamma = 1.5$ and (r) $p = 1.6$, $q = 1.6$ and $\gamma = 1.5$

Fig. 4. (a) A comparison between a numerically generated wave front (adapted from [32]), and wave fronts modeled by the parametric curve (thin continuous line) and ellipse (thick dashed line), (b) zoomed-in view of Fig. 4(a). It may be observed that compared to the ellipse the parametric curve is extended more along the major and minor axes.

As in the case of an elliptical wave front (see subsection 3.1), n sensor clusters can be considered to solve for the unknowns. For the i^{th} sensor cluster, Eq. (14) may be re-written as

$$(-1)^{quad_i} \gamma |x_i - C_x|^p - m_i |y_i - C_y|^q = 0 \quad (16)$$

where $quad_i$ denotes the $quad$ value (1, 2, 3 or 4, as defined in Eq. (15)) for the i^{th} sensor cluster. Eq. (16) consists of five unknowns: p , q , γ , C_x and C_y . Thus, n simultaneous nonlinear equations are obtained from Eq. (16). The following objective function S can be minimized to solve for the unknowns.

$$S = \sum_{i=1}^n [(-1)^{quad_i} \gamma |x_i - C_x|^p - m_i |y_i - C_y|^q]^2 \quad (17)$$

It has been observed that if both exponents p and q are considered to be unknown variables, the minimization problem often fails to converge. To avoid this, p is considered to be an unknown parameter while q is set at a constant value. Different constant values for q can be considered to arrive at the best fit in order to obtain an accurate acoustic source prediction. Nelder-Mead simplex algorithm [33] can be used to minimize S . In this study, the version of the algorithm given in [34] is followed and is described next.

Let $\mathbf{x} = [p \ \gamma \ C_x \ C_y]^T$. Five initial vertices for $\mathbf{x}$ (i.e., five different initial guesses for $\mathbf{x}$) are considered and S is calculated using Eq. (17) for all vertices. Then the following steps are performed to iteratively solve for the unknowns.

Step 1: Order the five vertices so that $S(\mathbf{x}_1) \leq S(\mathbf{x}_2) \leq S(\mathbf{x}_3) \leq S(\mathbf{x}_4) \leq S(\mathbf{x}_5)$.

Step 2: Compute the reflection point $\mathbf{x}_r$ by

$$\mathbf{x}_r = \bar{\mathbf{x}} + \rho(\bar{\mathbf{x}} - \mathbf{x}_5) \quad (18)$$

where $\bar{\mathbf{x}} = \sum_{i=1}^4 \mathbf{x}_i / 4$ is the centroid of the four best points (i.e., all vertices except for $\mathbf{x}_5$), and ρ is the coefficient of reflection chosen as 1. Compute $S(\mathbf{x}_r)$. If $S(\mathbf{x}_1) \leq S(\mathbf{x}_r) < S(\mathbf{x}_4)$, replace $\mathbf{x}_5$ with $\mathbf{x}_r$ and go to step 1. Otherwise, go to step 3.

Step 3: If $S(\mathbf{x}_r) < S(\mathbf{x}_1)$, compute the expansion point $\mathbf{x}_e$ as following, otherwise go to step 4.

$$\mathbf{x}_e = \bar{\mathbf{x}} + \chi(\mathbf{x}_r - \bar{\mathbf{x}}) \quad (19)$$

where χ is the coefficient of expansion chosen as 2. Evaluate $S(\mathbf{x}_e)$. If $S(\mathbf{x}_e) < S(\mathbf{x}_r)$, replace $\mathbf{x}_5$ by $\mathbf{x}_e$ and go to step 1. Otherwise, (i.e., if $S(\mathbf{x}_e) \geq S(\mathbf{x}_r)$), replace $\mathbf{x}_5$ with $\mathbf{x}_r$ and go to step 1.

Step 4: If $S(\mathbf{x}_r) \geq S(\mathbf{x}_4)$, go to step 4a.

Step 4a: If $S(\mathbf{x}_4) \leq S(\mathbf{x}_r) < S(\mathbf{x}_5)$, perform an outside contraction by calculating $\mathbf{x}_c$ as shown in Eq. (20). Otherwise, if $S(\mathbf{x}_r) \geq S(\mathbf{x}_5)$, go to step 4b.

$$\mathbf{x}_c = \bar{\mathbf{x}} + \eta(\mathbf{x}_r - \bar{\mathbf{x}}) \quad (20)$$

where η is the coefficient of contraction chosen as $\frac{1}{2}$. Evaluate $S(\mathbf{x}_c)$. If $S(\mathbf{x}_c) \leq S(\mathbf{x}_r)$, replace $\mathbf{x}_5$ with $\mathbf{x}_c$ and go to step 1. Otherwise, go to step 5.

Step 4b: Perform an inside contraction by calculating $\mathbf{x}_{cc}$ as

$$\mathbf{x}_{cc} = \bar{\mathbf{x}} - \eta(\bar{\mathbf{x}} - \mathbf{x}_5) \quad (21)$$

Evaluate $S(\mathbf{x}_{cc})$. If $S(\mathbf{x}_{cc}) < S(\mathbf{x}_5)$, replace $\mathbf{x}_5$ with $\mathbf{x}_{cc}$ and go to step 1. Otherwise, go to step 5.

Step 5: Perform a shrink by replacing all points except $\mathbf{x}_1$ with $\mathbf{x}_i = \mathbf{x}_1 + \sigma(\mathbf{x}_i - \mathbf{x}_1)$, $i = 2, 3, 4$ and 5. σ is the coefficient of shrinkage chosen as $\frac{1}{2}$.

The above-mentioned steps can be performed until the standard deviation of the computed values of S at the current vertices becomes less than a pre-defined tolerance level. At this stage, the algorithm is terminated and the latest $\mathbf{x}_1$ is reported as the final solution of the unknown vector $\mathbf{x}$. The predicted location of the acoustic source is then given by (C_x, C_y) .

4. Numerical study

To illustrate the effectiveness of the proposed approach for acoustic source localization, the same numerical example shown in [32] is considered in this section. The material properties of the anisotropic plate along with its geometry and the details on numerical modelling, actual location of the acoustic source and the coordinates of the sensor clusters can be found in [32]. The schematic diagram of the plate and the location of the source and the six sensor clusters are reproduced from [32] and is shown in Fig. 5. The bottom left corner of the plate is considered as the origin (0.0, 0.0) of the coordinate system. It may be observed that the sensor clusters 1, 2 and

3 are of Type-a, whereas 4, 5 and 6 are of Type-b (see Section 2). The TOAs of the second group of waves are estimated at all sensor clusters using *AIC* as discussed in Section 2, and then from Eqs. (1) and (2) $\tan \theta$ is estimated at these cluster locations. m_i is then computed from Eq. (9) for $i = 1, 2, \dots$ and 6. These computed values are $-1.50, -0.47, -4.00, 2.88, 0.60$ and 1.50 for sensor clusters 1, 2, 3, 4, 5 and 6, respectively. The acoustic source is estimated following the proposed techniques based on modified elliptical wave front (see subsection 3.1) and wave front represented by parametric curves (see subsection 3.2). For the parametric curve representation of the wave front, it has been observed after checking with several constant positive values for the parameter q that a consideration of $q = 1.0$ leads to a sufficiently close matching of the wave front with the parametric curve resulting an accurate estimation of the acoustic source location without causing any problem in the convergence of the solution algorithm. Hence, in this study, it is proposed to consider q as 1.0 for the parametric curve formulation and the same has been used in the numerical example.

Fig. 5. Schematic diagram of the anisotropic plate used in the numerical study along with the coordinates of the acoustic source and six sensor clusters (reproduced from [32])

If only three sensor clusters (i.e., $n = 3$) are used to predict the acoustic source, a total of 20 combinations of three sensor clusters are possible from the six sensor clusters shown in Fig. 5. The acoustic source location is predicted for all these combinations using both wave front models – elliptic and non-elliptic. Table 1 shows the coordinates of the predicted source location along with the error in the predictions with respect to the true source location (i.e., at (250 mm, 250 mm)). The values of λ for all sensor cluster combinations are also shown. It may be observed from Table 1 that for some sensor cluster combinations the ellipse-based approach leads to a large (even unrealistic in some cases) error in the source prediction. It is observed that for the cluster combination numbers 8, 13 and 14 (see Table 1), λ is negative, whereas it is expected to be positive always (see subsection 3.1). For the cluster combination number 3 (in Table 1), λ is 14.22, which is much greater than its value for the rest of the sensor cluster combinations. This wide scattering of λ value can be reduced by considering more sensor clusters (i.e., for $n > 3$) and is discussed in the next paragraph.

When the parametric curve-based approach is followed for localizing the acoustic source, Table 1 shows that the predictions improve significantly over the ellipse-based approach for most cases. The errors are less than those obtained from the ellipse-based approach for most sensor cluster combinations. Besides, the parametric curve-based approach performs satisfactorily even for the combination numbers 3, 8, 13 and 14 which led to large or unrealistic prediction errors for the ellipse-based prediction. The mean and standard deviation of the errors (excluding combination nos. 3, 8, 13 and 14) are also shown in Table 1. Disregarding

combinations 3, 8, 13 and 14, the predicted results are shown graphically in Fig. 6. The average of the predicted source coordinates from the two models (again excluding combinations 3, 8, 13 and 14) are shown in Fig. 7. It can be observed that the average location of the source predicted by the parametric curve-based approach is closer to the actual source location.

Table 1
Predicted source localization results with $n = 3$ using the techniques proposed in Section 3

Combination No.	Sensor cluster combinations	Modified ellipse-based approach				Parametric curve-based approach				
		λ	C_x (mm)	C_y (mm)	Error (mm)	p	γ	C_x (mm)	C_y (mm)	Error (mm)
1	1, 2 and 3	1.59	234.31	245.46	16.33	1.14	0.71	244.27	243.60	8.59
2	1, 2 and 6	2.90	250.00	436.98	186.98	1.15	0.74	250.00	272.73	22.73
3	1, 2 and 5	14.22	265.14	2090.26	1840.32	1.28	0.37	261.03	279.73	31.71
4	1, 2 and 4	2.00	241.43	305.37	56.03	1.19	0.62	253.74	278.81	29.05
5	1, 3 and 6	1.49	250.00	248.35	1.65	1.12	0.76	250.00	245.07	4.93
6	1, 3 and 5	1.34	276.79	252.50	26.90	1.15	0.62	268.84	247.65	18.99
7	1, 3 and 4	1.26	293.54	254.70	43.79	1.19	0.45	290.51	249.83	40.51
8	1, 6 and 5	-1.24	250.00	-114.71	364.71	1.57	0.09	250.00	296.52	46.52
9	1, 6 and 4	1.82	250.00	293.31	43.31	1.17	0.72	250.00	284.22	34.22
10	1, 5 and 4	1.44	275.88	267.07	31.00	1.20	0.52	265.12	267.07	22.80
11	2, 3 and 6	1.35	238.57	239.94	15.22	1.06	1.00	241.86	240.52	12.49
12	2, 3 and 5	0.87	253.74	229.17	21.16	1.10	0.59	255.76	231.94	18.95
13	2, 3 and 4	-0.30	84.11	202.66	172.51	1.78	0.05	261.02	277.24	29.39
14	2, 6 and 5	-3.03	269.38	-315.39	565.72	1.45	0.17	262.53	275.76	28.65
15	2, 6 and 4	1.79	243.83	295.90	46.31	1.16	0.77	251.08	284.37	34.38
16	2, 5 and 4	1.14	256.62	267.07	18.31	1.10	0.75	258.20	267.07	18.94
17	3, 6 and 5	2.64	297.99	317.44	82.77	1.25	0.55	279.20	278.44	40.76
18	3, 6 and 4	2.01	278.66	279.79	41.34	1.23	0.58	274.48	270.76	32.10
19	3, 5 and 4	1.63	284.49	267.07	38.48	1.23	0.53	276.05	267.07	31.15
20	6, 5 and 4	2.19	301.15	267.07	53.93	1.25	0.53	280.28	267.07	34.76
Mean error (mm)			45.22			25.33				
Standard deviation of errors (mm)			42.61			10.98

Fig. 6. Coordinates of the actual acoustic source (denoted by ‘actual’) and those predicted by the modified ellipse-based approach (denoted by ‘ellipse’) and parametric curve-based approach (denoted by ‘parametric’) for $n = 3$

Fig. 7. Coordinates of the actual acoustic source (denoted by ‘actual’), average (excluding combination numbers 3, 8, 13 and 14 in Table 1) of the predicted coordinates by the modified ellipse-based approach (denoted by ‘ellipse’) and average (excluding combination numbers 3, 8, 13 and 14 in Table 1) of the predicted coordinates by the parametric curve-based approach (denoted by ‘parametric’) with $n = 3$

Similarly, four sensor clusters can be considered (i.e., $n=4$) for predicting the acoustic source leading to 15 possible sensor cluster combinations. The prediction results for the two wave front models are shown in Table 2. This table indicates that the ellipse-model performs

much better for $n=4$ compared to $n=3$. Further, for most sensor cluster combinations, the parametric curve-based approach gives better predictions than the ellipse-based approach. Combination number 4 seems to be the only case with a large localization error. However, even in this case, the error for the parametric curve model is comparatively less than the elliptical wave front model. Table 2 also lists the mean and standard deviation of the errors (excluding combination no. 4). Fig. 8 shows the source prediction results graphically for $n=4$. The average source coordinates are calculated after disregarding the combination number 4, and Fig. 9 shows the source localization results with these average coordinates. This figure shows that for $n=4$ also, the parametric curve-based approach performs better than the ellipse-based approach.

When five sensor clusters are considered for the acoustic source prediction (i.e., $n=5$), 6 different sensor cluster combinations are possible. For all these combinations the acoustic source is localized using the two models. The predicted source coordinates and the corresponding errors along with the mean and standard deviation of the errors are listed in Table 3, and the same results are shown graphically in Fig. 10. These results indicate that both models give accurate estimates of the source coordinates for $n=5$. Again, the parametric curve-based method gives better predictions for most sensor cluster combinations. The average source coordinates are evaluated and are shown in Fig. 11.

Finally, when all six sensor clusters are considered simultaneously (i.e., $n=6$), only one combination of sensor clusters is possible that includes all sensor clusters 1 to 6. In this case, the ellipse-based technique predicts the source coordinates as (265.80 mm, 262.24 mm), giving an error of 19.98 mm, whereas the coordinates predicted by the parametric curve-based approach is (260.39 mm, 251.70 mm), giving an error of 10.53 mm. For the ellipse-based technique λ is obtained as 1.53, and for the parametric curve-based approach p and γ are obtained as 1.43 and 0.15, respectively. Therefore, both models give satisfactory results, although the parametric curve-based method localizes the source more accurately than the ellipse-based method. These results are shown graphically in Fig. 12. It may be noted from Tables 1–3 and this paragraph that the values obtained for the curve parameters from the minimization process may not match well with those used to generate Fig. 4. This may be because Fig. 4 demonstrates the generality of the proposed curve as a wave front shape and hence trial and error is used to obtain the figure, whereas a systematic optimization algorithm is employed to solve for those parameters during the source localization. Besides, the parameter q is assumed constant in the optimization process. Another observation is that although the parametric curve-based approach gives more accurate results than the modified ellipse-based approach in most cases, for a few cases the prediction errors using the parametric curve-based approach are quite large (e.g., combination no. 4 in Table 2) or are more than the corresponding errors using the modified ellipse-based approach (e.g., combination nos. 1, 3, 7, 9, 11 in Table 2 and combination no. 1 in Table 3). This is because for some sensor cluster combinations the shape of the wave fronts passing through the sensor cluster positions may appear closer to an ellipse than to the parametric curve. However, this should be overlooked in favor of the observation that in most cases the parametric curve-based approach outperforms the ellipse-based approach.

To graphically show how the mean and standard deviation of the errors vary with the number of sensor clusters, the mean and standard deviation of the errors listed in Tables 1–3 are considered and plotted against the number of sensor clusters in Figs. 13 and 14, respectively. It may be observed that as the number of sensor clusters increases, the mean error in source localization decreases for both proposed approaches. Also, the parametric curve-based approach gives lower mean error than the ellipse-based model. The standard deviation of errors with the parametric curve-based approach is less than that for the ellipse-based model when three and five sensor clusters are considered, and is marginally more than the ellipse-based model for four sensor clusters.

Table 2
Predicted source localization results with $n = 4$ using the techniques proposed in Section 3

Combination No.	Sensor cluster combinations	Modified ellipse-based approach				Parametric curve-based approach				
		λ	C_x (mm)	C_y (mm)	Error (mm)	p	γ	C_x (mm)	C_y (mm)	Error (mm)
1	1, 2, 3 and 6	1.46	245.93	246.20	5.57	1.22	0.45	250.00	242.74	7.26
2	1, 2, 3 and 5	1.31	265.66	247.60	15.84	1.35	0.21	259.94	241.48	13.09
3	1, 2, 3 and 4	1.29	279.53	255.23	29.99	1.74	0.02	289.84	238.90	41.36
4	1, 2, 6 and 5	-0.22	275.00	27.60	223.80	2.08	0.002	250.00	147.35	102.65
5	1, 2, 6 and 4	1.84	247.46	295.45	45.52	1.13	0.88	250.00	286.08	36.08
6	1, 2, 5 and 4	1.49	265.43	273.82	28.39	1.30	0.31	260.72	267.07	20.16
7	1, 3, 6 and 5	1.46	260.83	251.15	10.89	1.67	0.04	250.00	233.55	16.45
8	1, 3, 6 and 4	1.61	267.22	264.93	22.79	2.14	0.003	255.36	237.23	13.85
9	1, 3, 5 and 4	1.35	284.11	257.48	34.92	0.61	10.87	308.17	267.07	60.62
10	1, 6, 5 and 4	1.69	261.01	281.29	33.17	1.61	0.07	250.00	267.07	17.07
11	2, 3, 6 and 5	1.33	250.67	243.11	6.92	1.50	0.12	261.76	248.73	11.83
12	2, 3, 6 and 4	1.71	265.50	268.53	24.16	1.74	0.04	269.92	256.21	20.86
13	2, 3, 5 and 4	1.12	277.73	249.05	27.74	1.76	0.04	264.41	267.07	22.34
14	2, 6, 5 and 4	1.66	254.41	284.47	34.75	1.46	0.15	262.31	267.07	21.05
15	3, 6, 5 and 4	1.97	281.71	278.56	42.68	1.34	0.32	273.08	267.07	28.71
Mean error (mm)			25.95			23.62				
Standard deviation of errors (mm)			12.46			14.14

Fig. 8. Coordinates of the actual acoustic source (denoted by 'actual') and those predicted by the modified ellipse-based approach (denoted by 'ellipse') and parametric curve-based approach (denoted by 'parametric') with $n = 4$

Fig. 9. Coordinates of the actual acoustic source (denoted by ‘actual’), average (excluding combination number 4 in Table 2) of the predicted coordinates by the modified ellipse-based approach (denoted by ‘ellipse’) and average (excluding combination number 4 in Table 2) of the predicted coordinates by the parametric curve-based approach (denoted by ‘parametric’) with $n = 4$

Table 3

Predicted source localization results with $n = 5$ using the techniques proposed in Section 3

Combination No.	Sensor cluster combinations	Modified ellipse-based approach				Parametric curve-based approach				
		λ	C_x (mm)	C_y (mm)	Error (mm)	p	γ	C_x (mm)	C_y (mm)	Error (mm)
1	1, 2, 3, 6 and 5	1.42	255.33	247.67	5.81	1.46	0.12	252.76	238.44	11.89
2	1, 2, 3, 6 and 4	1.58	261.98	263.68	18.19	1.45	0.14	260.44	251.17	10.50
3	1, 2, 3, 5 and 4	1.32	277.13	256.11	27.81	1.32	0.25	271.41	250.30	21.41
4	1, 2, 6, 5 and 4	1.69	256.21	284.36	34.92	1.39	0.21	253.71	271.12	21.44
5	1, 3, 6, 5 and 4	1.57	270.72	263.89	24.95	1.51	0.10	259.02	249.74	9.02
6	2, 3, 6, 5 and 4	1.65	268.98	266.37	25.06	1.56	0.10	268.55	259.40	20.79
Mean error (mm)				22.79		15.84				
Standard deviation of errors (mm)				9.91		5.96

Fig. 10. Coordinates of the actual acoustic source (denoted by 'actual') and those predicted by the modified ellipse-based approach (denoted by 'ellipse') and parametric curve-based approach (denoted by 'parametric') with $n = 5$

Fig. 11. Coordinates of the actual acoustic source (denoted by 'actual'), average of the predicted coordinates by the modified ellipse-based approach (denoted by 'ellipse') and average of the predicted coordinates by the parametric curve-based approach (denoted by 'parametric') with $n = 5$

Fig. 12. Coordinates of the actual acoustic source (denoted by 'actual') and those predicted by the modified ellipse-based approach (denoted by 'ellipse') and parametric curve-based approach (denoted by 'parametric') with $n = 6$

Fig. 13. Mean error for the predicted source coordinates using modified ellipse-based approach (denoted as 'ellipse') and parametric curve-based approach (denoted as 'parametric') for $n = 3$ (see Table 1), $n = 4$ (see Table 2) and $n = 5$ (see Table 3)

Fig. 14. Standard deviation of the errors for predicted source coordinates using modified ellipse-based approach (denoted as ‘ellipse’) and parametric curve-based approach (denoted as ‘parametric’) for $n = 3$ (see Table 1), $n = 4$ (see Table 2) and $n = 5$ (see Table 3)

It is desirable to know how sensitive the source localization results are to the initial guesses for β for the modified ellipse-based approach (see subsection 3.1) and for $\mathbf{x}$ in the parametric curve-based approach (see subsection 3.2). To generate the results shown thus far in this section, the initial guess for β has been chosen as $[1.0 \ 240.1 \ 230.3]^T$, while the five different initial vertices for $\mathbf{x}$ have been set at $[1.1 \ 0.5 \ 240.1 \ 290.1]^T$, $[1.5 \ 1.2 \ 200.1 \ 230.1]^T$, $[1.2 \ 0.5 \ 270.1 \ 270.1]^T$, $[1.2 \ 0.7 \ 220.1 \ 210.1]^T$ and $[1.8 \ 0.2 \ 260.1 \ 280.1]^T$ with the last two elements (i.e., the source coordinates) for both β and $\mathbf{x}$ having units in ‘mm’. In order to investigate how different initial guesses may affect the source localization results, five more initial guesses for β (different from the one mentioned above) and five more initial guesses for the five vertices of $\mathbf{x}$ (different from the ones mentioned above) are considered and the source localization process applying both proposed techniques is repeated for these new initial guesses. Table 4 lists the new initial guesses. The obtained source localization results are shown in Tables 5–8 for n equal to 3, 4, 5 and 6, respectively. For the sake of brevity, only the errors in the prediction of the source coordinates are listed in these tables.

The column ‘Combination No.’ in Tables 5–7 refers to the same sensor cluster combinations shown in Tables 1–3. The mean and standard deviation (disregarding the same sensor cluster combinations as mentioned in the preceding paragraphs of this section) of the errors are also shown in Tables 5–7. Tables 5–8 indicate that for $n = 3$ and modified ellipse-based approach all five new initial guesses lead to the same source localization results, and these results are identical to those obtained earlier in this section (see Tables 1–3). Therefore, the source localization results are not much sensitive to the initial guesses when the modified ellipse-based approach is followed. However, to ensure convergence of the Levenberg–Marquardt algorithm, it is recommended to keep the initial guess for λ between 0.5 and 3.0, and the initial source coordinates between 200 mm and 270 mm. Also, considering accuracy of the predictions, a minimum value of 4 is recommended for n when the modified ellipse-based

approach is used. When the source coordinates are predicted with five new guesses using the parametric curve-based approach, Table 5 indicates that with $n=3$ the results are different for different initial guesses, and they do not match with those shown in Table 1. This discrepancy is attributed to the fact that in this case only three simultaneous equations are available to solve for four unknowns and thus the solutions are not expected to be unique. When four sensor clusters are used, Table 6 indicates that the parametric curve-based approach with the new initial guesses gives same source localization results for all sensor cluster combinations and they match with the results listed in Table 2, except for combination no. 4 and 15 with IG1. For $n=5$ and $n=6$, from Tables 7 and 8 it is observed that same results are obtained with all new initial guesses. These results are identical to those shown in Table 3. Hence, for parametric curve-based approach the source localization results are not sensitive to the initial guesses if a minimum of four sensor clusters are used. For this reason and also to achieve sufficient accuracy in the predictions, a minimum value of 4 is recommended for n when the parametric curve-based approach is used. In order to ensure convergence of the Nelder-Mead algorithm, it is recommended to keep the initial values of p between 1.0 and 2.0, γ between 0.5 and 1.5, and the source coordinates between 200 mm and 270 mm.

Table 4
Six initial guesses for β and five vertices of $\mathbf{x}$

	β	$\mathbf{x}$
Initial guess no. 1 (IG1)	$[0.5 \ 200.5 \ 270.3]^T$	$[1.6 \ 2.5 \ 270.2 \ 220.1]^T$ $[0.5 \ 1.6 \ 215.6 \ 200.1]^T$ $[1.2 \ 1.9 \ 210.1 \ 280.1]^T$ $[2.2 \ 1.5 \ 280.9 \ 217.5]^T$ $[2.8 \ 1.2 \ 235.1 \ 230.1]^T$
Initial guess no. 2 (IG2)	$[2.0 \ 210.7 \ 225.0]^T$	$[0.6 \ 0.5 \ 210.3 \ 225.9]^T$ $[0.5 \ 1.1 \ 255.2 \ 230.1]^T$ $[1.9 \ 1.2 \ 290.1 \ 240.7]^T$ $[1.3 \ 2.5 \ 220.9 \ 211.1]^T$ $[1.1 \ 0.2 \ 200.1 \ 260.1]^T$
Initial guess no. 3 (IG3)	$[1.5 \ 260.7 \ 180.0]^T$	$[1.3 \ 0.3 \ 210.3 \ 219.3]^T$ $[1.9 \ 2.1 \ 215.2 \ 270.2]^T$ $[1.5 \ 0.2 \ 190.1 \ 245.2]^T$ $[1.8 \ 2.1 \ 230.2 \ 231.1]^T$ $[1.2 \ 1.2 \ 240.1 \ 255.1]^T$
Initial guess no. 4 (IG4)	$[0.8 \ 190.5 \ 240.0]^T$	$[1.1 \ 1.3 \ 240.3 \ 219.3]^T$ $[1.3 \ 1.1 \ 235.2 \ 230.2]^T$ $[1.6 \ 1.1 \ 205.1 \ 245.2]^T$ $[1.9 \ 1.4 \ 220.2 \ 231.1]^T$ $[1.0 \ 1.8 \ 280.1 \ 255.1]^T$
Initial guess no. 5 (IG5)	$[2.8 \ 290.7 \ 215.1]^T$	$[1.2 \ 0.5 \ 210.3 \ 269.3]^T$ $[1.3 \ 0.6 \ 225.2 \ 270.2]^T$ $[1.4 \ 1.9 \ 235.1 \ 285.2]^T$ $[1.5 \ 1.5 \ 240.2 \ 291.1]^T$ $[1.6 \ 1.7 \ 250.1 \ 205.1]^T$

Table 5

Errors (in mm) in predicted source localization results for $n = 3$ using the techniques proposed in Section 3 with the initial guesses listed in Table 4

Combination No.	Modified ellipse-based approach					Parametric curve-based approach				
	IG1	IG2	IG3	IG4	IG5	IG1	IG2	IG3	IG4	IG5
1	16.33	16.33	16.33	16.33	16.33	18.95	15.82	10.31	13.52	10.22
2	186.98	186.98	186.98	186.98	186.98	277.04	2.74	125.66	82.74	94.29
3	1840.32	1840.32	1840.32	1840.32	1840.32	189.94	72.12	427.25	195.75	41.03
4	56.03	56.03	56.03	56.03	56.03	49.76	26.54	44.52	46.92	18.74
5	1.65	1.65	1.65	1.65	1.65	3.32	6.12	2.92	2.03	4.59
6	26.90	26.90	26.90	26.90	26.90	52.30	40.60	32.13	30.30	12.79
7	43.79	43.79	43.79	43.79	43.79	45.45	39.33	47.36	46.32	40.56
8	364.71	364.71	364.71	364.71	364.71	216.12	25.50	335.87	216.81	286.41
9	43.31	43.31	43.31	43.31	43.31	2.53	22.45	40.95	42.36	20.98
10	31.00	31.00	31.00	31.00	31.00	17.71	26.47	33.57	33.52	17.11
11	15.22	15.22	15.22	15.22	15.22	19.29	15.66	12.13	14.42	6.89
12	21.16	21.16	21.16	21.16	21.16	97.56	11.90	21.84	23.71	14.10
13	172.51	172.51	172.51	172.51	172.51	56.01	131.93	97.41	9137.34	100.73
14	565.72	565.72	565.72	565.72	565.72	188.57	11.98	237.96	64.54	20.93
15	46.31	46.31	46.31	46.31	46.31	67.05	19.99	41.98	43.20	20.43
16	18.31	18.31	18.31	18.31	18.31	17.08	19.37	18.17	18.06	18.95
17	82.77	82.77	82.77	82.77	82.77	89.03	20.33	545.38	79.76	12.39
18	41.34	41.34	41.34	41.34	41.34	18.67	22.24	42.27	41.15	25.52
19	38.48	38.48	38.48	38.48	38.48	50.86	17.63	78.67	39.99	28.29
20	53.93	53.93	53.93	53.93	53.93	101.99	51.29	58.49	54.34	20.56
Mean (mm)	45.22	45.22	45.22	45.22	45.22	58.04	22.41	72.27	38.27	22.90
St. dev. (mm)	42.61	42.61	42.61	42.61	42.61	66.72	12.59	129.60	22.14	20.93

Table 6

Errors (in mm) in predicted source localization results for $n = 4$ using the techniques proposed in Section 3 with the initial guesses listed in Table 4

Combination No.	Modified ellipse-based approach					Parametric curve-based approach				
	IG1	IG2	IG3	IG4	IG5	IG1	IG2	IG3	IG4	IG5
1	5.57	5.57	5.57	5.57	5.57	7.26	7.26	7.26	7.26	7.26
2	15.84	15.84	15.84	15.84	15.84	13.09	13.09	13.09	13.09	13.09
3	29.99	29.99	29.99	29.99	29.99	41.36	41.36	41.36	41.36	41.36
4	223.80	223.80	223.80	223.80	223.80	195.04	102.65	102.65	102.65	102.65
5	45.52	45.52	45.52	45.52	45.52	36.08	36.08	36.08	36.08	36.08
6	28.39	28.39	28.39	28.39	28.39	20.16	20.16	20.16	20.16	20.16
7	10.89	10.89	10.89	10.89	10.89	16.45	16.45	16.45	16.45	16.45
8	22.79	22.79	22.79	22.79	22.79	13.85	13.85	13.85	13.85	13.85
9	34.92	34.92	34.92	34.92	34.92	60.62	60.62	60.62	60.62	60.62
10	33.17	33.17	33.17	33.17	33.17	17.07	17.07	17.07	17.07	17.07
11	6.92	6.92	6.92	6.92	6.92	11.83	11.83	11.83	11.83	11.83
12	24.16	24.16	24.16	24.16	24.16	20.86	20.86	20.86	20.86	20.86
13	27.74	27.74	27.74	27.74	27.74	22.34	22.34	22.34	22.34	22.34
14	34.75	34.75	34.75	34.75	34.75	21.05	21.05	21.05	21.05	21.05
15	42.68	42.68	42.68	42.68	42.68	37.31	28.71	28.71	28.71	28.71
Mean (mm)	25.95	25.95	25.95	25.95	25.95	24.24	23.62	23.62	23.62	23.62

St. dev. (mm)	12.46	12.46	12.46	12.46	12.46	14.56	14.14	14.14	14.14	14.14
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Table 7

Errors (in mm) in predicted source localization results for $n = 5$ using the techniques proposed in Section 3 with the initial guesses listed in Table 4

Combination No.	Modified ellipse-based approach					Parametric curve-based approach				
	IG1	IG2	IG3	IG4	IG5	IG1	IG2	IG3	IG4	IG5
1	5.81	5.81	5.81	5.81	5.81	11.89	11.89	11.89	11.89	11.89
2	18.19	18.19	18.19	18.19	18.19	10.50	10.50	10.50	10.50	10.50
3	27.81	27.81	27.81	27.81	27.81	21.41	21.41	21.41	21.41	21.41
4	34.92	34.92	34.92	34.92	34.92	21.44	21.44	21.44	21.44	21.44
5	24.95	24.95	24.95	24.95	24.95	9.02	9.02	9.02	9.02	9.02
6	25.06	25.06	25.06	25.06	25.06	20.79	20.79	20.79	20.79	20.79
Mean (mm)	22.79	22.79	22.79	22.79	22.79	15.84	15.84	15.84	15.84	15.84
St. dev. (mm)	9.91	9.91	9.91	9.91	9.91	5.96	5.96	5.96	5.96	5.96

Table 8

Errors (in mm) in predicted source localization results for $n = 6$ using the techniques proposed in Section 3 with the initial guesses listed in Table 4

Modified ellipse-based approach					Parametric curve-based approach				
IG1	IG2	IG3	IG4	IG5	IG1	IG2	IG3	IG4	IG5
19.98	19.98	19.98	19.98	19.98	10.53	10.53	10.53	10.53	10.53

5. Conclusions

In this study, an attempt has been made to predict the acoustic source location in an anisotropic plate without knowing its material properties by assuming a geometric curve as the shape of the wave front generated from an acoustic event. The proposed technique does not assume that waves propagate along a straight line from the source to a sensor. The methodology uses the sensor cluster configuration proposed by Kundu [18] and Kundu et al. [19] for estimating the tangent to the incident wave path which is the direction perpendicular to the wave front at a cluster location. The time of arrival of the acoustic wave at a sensor has been computed using Akaike Information Criterion. Two wave front shapes have been considered: an elliptical shape and a non-elliptical shape represented by a parametric curve. For developing the elliptical shape-based technique, the technique proposed by Park et al. [32] has been improved for a better adherence to the relationship between the tangent of the wave-path estimated by the installed sensor clusters and the derivative of the geometrical shape. The unknown variables including the coordinates of the acoustic source have been obtained by minimizing an objective function. The solution algorithm that is adopted for performing this minimization is briefly discussed. For the parametric curve-based approach, an assumed geometrical shape with three unknown parameters has been assumed to be the wave front shape. Such a parametric curve is more general in nature than an elliptical shape and hence has the capability of representing a wave front more accurately than an ellipse. By considering different sets of parameter values, it has been demonstrated that a wide variety of wave front shapes can be modeled using such parametric curves. A relationship between the derivative of a parametric curve and the observed tangent of the incident angle at a sensor cluster has been derived. The unknown variables including the source coordinates have been obtained by minimizing an objective function. While performing this minimization, one of the three unknown parameters in the expression of the parametric curve has been held constant at a chosen (positive) value in order to ensure

convergence of the minimization process. The simplex algorithm adopted for this minimization has been discussed briefly.

The effectiveness of both proposed techniques to predict the source location has been illustrated with the help of a numerical example as considered by Park et al. [32]. Four different numbers of sensor clusters, namely 3, 4, 5 and 6 have been considered simultaneously from a set of six clusters, leading to 20, 15, 6 and 1 combinations of sensor clusters, respectively. For all these combinations, the acoustic source has been estimated using both wave front models – elliptic and non-elliptic. It has been observed that while both models can predict the source with reasonable accuracy in most cases, the overall performance of the parametric curve-based non-elliptic wave front performed better than the ellipse-based approach in terms of the accuracy of the source localization. It may be noted here that to develop the proposed approach the axes of anisotropy in a plate have been assumed to be known *a priori*. In this investigation these axes have been assumed to be parallel to the coordinate axes x and y . For an anisotropic plate with unknown orientation of anisotropy one needs to evaluate another unknown parameter – the inclination angle of the curve with respect to the coordinate axes. Due to the increase of the number of unknown parameters the convergence of the acoustic source location to the right values is expected to be more challenging. It is currently under investigation.

Since in this study one of the parameters in the parametric curve-based approach has been chosen to be constant at a suitable value, there is further scope of research for evaluating this parameter in a more judicious manner. Also, a more general expression of the parametric curve may be developed to model the wave front shape more closely and thus to increase the accuracy of the source prediction technique further. One may also consider improving the efficiency of the presented methodology to reduce the total number of sensors required. Since this study has illustrated the effectiveness of the proposed approach through a numerical example, there is scope of further research focusing on the experimental validation of this technique. Also, the capability of the presented approach to localize the acoustic source in structures with more complex anisotropies than what has been considered in this paper is worth investigating.

References

- [1] A. Tobias, Acoustic emission source location in two dimensions by an array of three sensors, *Nondestructive Testing* 9 (1976) 9–12.
- [2] T. Kundu, Acoustic source localization, *Ultrasonics* 54 (2014) 25–38.
- [3] G.C. McLaskey, S.D. Glaser, C.U. Grosse, Beamforming array techniques for acoustic emission monitoring of large concrete structures, *Journal of Sound and Vibration* 329(12) (2010) 2384–2394.
- [4] T. He, Q. Pan, Y. Liu, X. Liu, D. Hu, Near-field beamforming analysis for acoustic emission source localization, *Ultrasonics* 52(5) (2012) 587–592.
- [5] H.M. Matt, F. Lanza di Scalea, Macro-fiber composite piezoelectric rosettes for acoustic source location in complex structures, *Smart Materials and Structures* 16 (2007) 1489–1499.
- [6] S. Salamone, I. Bartoli, P. di Leo, F. Lanza di Scalea, A. Ajovalasit, L. D’Acquisto, J. Rhymer, H. Kim, High-velocity impact location on aircraft panels using macrofiber composite piezoelectric rosettes, *Journal of Intelligent Material Systems and Structures* 21 (2010) 887–896.
- [7] S. Salamone, I. Bartoli, J. Rhymer, F.L. di Scalea, H. Kim, Validation of the Piezoelectric Rosette Technique for Locating Impacts in Complex Aerospace Panels, in: T. Kundu (Ed.), *Health Monitoring of Structural and Biological Systems*. Pub. Bellingham, Wash, 18th Annual International Symposium on Smart Structures/NDE, vol. 7984, March 7–10, 2011, San Diego, California, 2011, pp. 79841E1-11.
- [8] D.C. Betz, G. Thursby, B. Culshaw, W. Staszewski (2007), Structural damage location with fiber Bragg grating rosettes and lamb waves, *International Journal of Structural Health Monitoring* 6(4) (2007) 299–308.
- [9] J. Jiao, C. He, B. Wu, R. Fei, X. Wang, Application of wavelet transform on modal acoustic emission source location in thin plates with one sensor, *International Journal of Pressure Vessels and Piping* 81(5) (2004) 427–431.

- [10] M. Surgeon, M. Wevers, One sensor linear location of acoustic emission events using plate wave theories, *Materials Science and Engineering A265* (1999) 254–261.
- [11] N. Toyama, J.H. Koo, R. Oishi, M. Enoki, T. Kishi, Two-dimensional AE source location with two sensors in thin CFRP plates, *Journal of Materials Science Letters* 20(19) (2001) 1823–1825.
- [12] A. Ebrahimkhanlou, S. Salamone, Acoustic emission source localization in thin metallic plates: A single-sensor approach based on multimodal edge reflections, *Ultrasonics* 78 (2017) 134–145.
- [13] T. He, Y. Xie, Y. Shan, X. Liu, Localizing Two Acoustic Emission Sources Simultaneously Using Beamforming and Singular Value Decomposition, *Ultrasonics* (2017), doi: <https://doi.org/10.1016/j.ultras.2017.10.019>.
- [14] H. Nakatani, T. Hajzargarbashi, K. Ito, T. Kundu, N. Takeda, Impact localization on a cylindrical plate by near-field beamforming analysis, in: M. Tomizuka (Ed.), *Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems*, SPIE's 2012 Annual International Symposium on Smart Structures and Nondestructive Evaluation, vol. 8345, San Diego, California, March 12–15, 2012, 2012.
- [15] H. Nakatani, T. Hajzargarbashi, K. Ito, T. Kundu, N. Takeda, Locating point of impact on an anisotropic cylindrical surface using acoustic beamforming technique, in: 4th Asia-Pacific Workshop on Structural Health Monitoring, Melbourne, Australia, December 5–7, 2012.
- [16] T. Kundu, S. Das, K.V. Jata, Point of impact prediction in isotropic and anisotropic plates from the acoustic emission data, *Journal of the Acoustical Society of America* 122(4) (2007) 2057–2066.
- [17] T. Kundu, S. Das, S.A. Martin, K.V. Jata, Locating point of impact in anisotropic fiber reinforced composite plates, *Ultrasonics* 48(3) (2008) 193–201.
- [18] T. Kundu, A new technique for acoustic source localization in an anisotropic plate without knowing its material properties, in: 6th European Workshop on Structural Health Monitoring, Dresden, Germany, July 3–6, 2012.
- [19] T. Kundu, H. Nakatani, N. Takeda, Acoustic source localization in anisotropic plates, *Ultrasonics* 52(6) (2012) 740–746.
- [20] S. Yin, Z. Cui, T. Kundu, Acoustic source localization in anisotropic plates with “Z” shaped sensor clusters, *Ultrasonics* 84 (2018) 34–37.
- [21] F. Ciampa, M. Meo, A new algorithm for acoustic emission localization and flexural group velocity determination in anisotropic structures, *Composites Part A* 41 (2010) 1777–1786.
- [22] F. Ciampa, M. Meo, E. Barbieri, Impact localization in composite structures of arbitrary cross section, *Structural Health Monitoring: An International Journal* 11 (2012) 643–655.
- [23] T. Kundu, X. Yang, H. Nakatani, N. Takeda, A two-step hybrid technique for accurately localizing acoustic source in anisotropic structures without knowing their material properties, *Ultrasonics* 56 (2015) 271–278.
- [24] M.E.D. Simone, F. Ciampa, S. Boccardi, M. Meo, Impact source localisation in aerospace composite structures, *Smart Mater. Struct.* 26 (2017) 125026.
- [25] S.A.H. Sabzevari, M. Moavenian, Sound localization in an anisotropic plate using electret microphones, *Ultrasonics* 73 (2017) 114–124.
- [26] D. Xiao, T. He, Q. Pan, X. Liu, J. Wang, Y. Shan, A novel acoustic emission beamforming method with two uniform linear arrays on plate-like structures, *Ultrasonics* 54 (2014) 737–745.
- [27] W.H. Park, Acoustic source localization in an anisotropic plate without knowing its material properties, PhD dissertation, Department of Aerospace and Mechanical Engineering, The University of Arizona (2016).
- [28] M.G. Baxter, R. Pullin, K.M. Holford, S.L. Evans, Delta T source location for acoustic emission, *Mech. Syst. Signal Process.* 21 (2007) 1512–1520.
- [29] J. Hensman, R. Mills, S.G. Pierce, K. Worden, M. Eaton, Locating acoustic emission sources in complex structures using gaussian processes, *Mech. Syst. Signal Process.* 24 (2010) 211–223.
- [30] K. Grabowski, M. Gawronski, I. Baran, W. Szychalski, W.J. Staszewski, T. Uhl, T. Kundu, P. Packo, Time–distance domain transformation for Acoustic Emission source localization in thin metallic plates, *Ultrasonics* 68 (2016) 142–149.
- [31] S. Gollob, G.K. Kocur, T. Schumacher, L. Mhamdi, T. Vogel, A novel multi-segment path analysis based on a heterogeneous velocity model for the localization of acoustic emission sources in complex propagation media, *Ultrasonics* 74 (2017) 48–61.
- [32] W.H. Park, P. Packo, T. Kundu, Acoustic source localization in an anisotropic plate without knowing its material properties – A new approach, *Ultrasonics* 79 (2017) 9–17.

- [33] J.A. Nelder, R. Mead, A simplex method for function minimization, *Computer Journal* 7 (1965) 308–315.
- [34] J.C. Lagarias, J.A. Reeds, M.H. Wright, P.E. Wright, Convergence properties of the Nelder–Mead simplex method in low dimensions, *SIAM J. Optim.* 9(1) (1998) 112–147.

Research Highlights

- Acoustic source localization technique in highly anisotropic plate is presented
- This technique avoids the straight line wave propagation assumption from source to sensor
- The wave front of a general shape typically observed in highly anisotropic plates is analyzed.