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Optimal geometry for off-axis, freeform illumination design

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Abstract. A thorough investigation into the efficiency and performance of off-axis, freeform illumination systems has been conducted, and a method for arriving at the optimal tilt is established for both reflective and refractive designs. The optimal tilt for refractive designs is determined by minimizing the total light lost due to Fresnel reflections across the lens, while the optimal tilt for reflective designs ensures the source is not obstructing the reflected ray bundle. For uncoated lenses, utilizing the proposed tilt can improve efficiency by more than 150%. © 2018 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.OE.57.6.065102]

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1 Introduction

Over the past couple of months, we have investigated the design of off-axis, freeform illumination systems. Although our efforts focused mainly on the ray mapping method,¹ when we constructed an optical surface to enforce our calculated mapping we found it unclear how the source should be aimed with respect to the target. While it is natural to assume the source should be pointed directly at the target in symmetric systems, when the source is translated off-axis, the solution is not as straightforward.

This problem is demonstrated in Fig. 1, where we show the ray diagram for three possible geometries that in general are mutually exclusive. When designing configuration **a**, the source is tilted such that the outermost ray on the right is directed at the right edge of the target, i.e., no optical power is required to redirect the ray. Configuration **b** tilts the source so that the central ray is aimed at the center of the target, and configuration **c** means tilting so that the outermost ray on the left is directed at the left side of the target. These three geometries are the most intuitive choices when approaching the problem, but in general, there are many more equally viable options and it is not obvious which one is best. In this paper, we solve that problem by proposing direct analytical methods to determine the optimal starting geometry for both refractive and reflective off-axis systems.

2 Refractive Surfaces

We start by designing refractive systems because the problem is slightly easier in this case. Since the main source of loss in refractive lenses is by reflection at large incident angles, efficiency and tolerances can be simultaneously improved by minimizing a variation of the ray deviation at each point on the optical surface. We do this by looking at the Fresnel reflection coefficients²

$$\begin{aligned} R_s &= \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right|^2 \\ R_p &= \left| \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right|^2, \end{aligned} \quad (1)$$

where n_1 and n_2 are the indices of refraction of the first and second media, respectively, θ_i is the angle the incident ray makes with respect to the surface normal, and θ_t is the outgoing angle that the refracted ray makes with the surface normal. Typically, in illumination systems, we work with randomly polarized light, so for this investigation the reflection coefficient used will be the average of that for the s and p light, $R = \frac{1}{2}(R_s + R_p)$.

It is convenient to rewrite the angles using dot products between vectors $\cos \theta_i = \mathbf{I} \cdot \mathbf{N}$ and $\cos \theta_t = \mathbf{O} \cdot \mathbf{N}$. Inserting these quantities into Eq. (1), we obtain the vector form of the Fresnel reflection coefficients

$$R_s = \left| \frac{\mathbf{N} \cdot (n_1 \mathbf{I} - n_2 \mathbf{O})}{\mathbf{N} \cdot (n_1 \mathbf{I} + n_2 \mathbf{O})} \right|^2 \quad R_p = \left| \frac{\mathbf{N} \cdot (n_1 \mathbf{O} - n_2 \mathbf{I})}{\mathbf{N} \cdot (n_1 \mathbf{O} + n_2 \mathbf{I})} \right|^2, \quad (2)$$

where $\mathbf{N}$ denotes the surface normal, $\mathbf{O}$ is the vector between the point on the surface, $r\mathbf{I}$, and the point on the target, $\mathbf{T}$, and $\mathbf{I}$ is the unit vector denoting the direction of the ray leaving the source. This geometry is laid out in Fig. 2, where x_0 denotes the lateral distance between the source and center of the target.

To quantitatively measure the efficiency of our design, we introduce a term comprised of the reflection coefficient at each point multiplied by the corresponding intensity of the ray (I_e) incident on that point. Effectively, this is the total amount of energy lost due to reflections on the lens. The optimum design can then be selected as the one that minimizes that loss, as shown below

$$\min \sum_{i,j} I_{e_{ij}} R_{i,j}, \quad (3)$$

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Fig. 1 Three of the infinite potential design configurations for off-axis illumination systems.

Fig. 2 Diagram showing geometry and vectors used in the calculations of Sec. 2.

where i, j refer to the indices of discrete points along the parametric lens surface. Assuming a Lambertian source, we can write the ray intensity in terms of vector quantities

$$I_e = \mathbf{N}_{\text{source}} \cdot \mathbf{I}, \quad (4)$$

and we can calculate the output vector as the distance between the surface and the target

$$\mathbf{O} = \frac{\mathbf{T} - r\mathbf{I}}{|\mathbf{T} - r\mathbf{I}|}, \quad (5)$$

where r gives the position of the optical surface along the input vector. In this analysis, we assumed a small, spherical surface of constant radius equal to half the size of the lens.

Both $\mathbf{N}_{\text{source}}$ and $\mathbf{I}$ change as the source is tilted by an angle, θ , such that

$$\mathbf{I} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} I_{0,x} \\ I_{0,y} \\ I_{0,z} \end{bmatrix}$$

$$\mathbf{N}_{\text{source}} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (6)$$

and the surface normals, $\mathbf{N}$, can be calculated using the output and input vectors

$$\mathbf{N} = \frac{n_2\mathbf{O} - n_1\mathbf{I}}{|n_2\mathbf{O} - n_1\mathbf{I}|}. \quad (7)$$

Putting this all together, we can arrive at a model for the total intensity lost due to Fresnel reflections as a function of the tilt angle, where the only requirements are the input

vectors and target points, $\mathbf{I}$ and $\mathbf{T}$, which are often calculated prior to surface construction. In this example, we used the double-pole mapping method described in Ref. 3 to calculate the target points aiming an entire hemisphere of a Lambertian point source onto a uniform square target 2-m $\times$ 2-m wide, 1 m away, and laterally offset by 1 m. The lens material in this example and throughout the rest of the paper is poly(methyl methacrylate)

$$\text{Loss} = \sum_{i,j} \left(\frac{1}{2} \left\{ \left| \frac{[\mathbf{T} - ((n_1 + r)\mathbf{I})] \cdot [(n_1 + n_2r)\mathbf{I} - n_2\mathbf{T}]}{[\mathbf{T} - ((n_1 + r)\mathbf{I})] \cdot [(n_1 - n_2r)\mathbf{I} + n_2\mathbf{T}]} \right|^2 + \left| \frac{[\mathbf{T} - ((n_1 + r)\mathbf{I})] \cdot [n_1\mathbf{T} - (n_1r + n_2)\mathbf{I}]}{[\mathbf{T} - ((n_1 + r)\mathbf{I})] \cdot [n_1\mathbf{T} - (n_1r - n_2)\mathbf{I}]} \right|^2 \right\} \times \mathbf{N}_{\text{source}} \cdot \mathbf{I} \right). \quad (8)$$

To examine the accuracy of this model, we tested our predictions from the above equation against calculated throughput values of lenses made with the prescribed tilt. The loss of the lens was calculated according to

$$\text{Loss} = \frac{\sum_{i,j} I_{e,i,j} R_{i,j}}{\sum_{i,j} I_{e,i,j}}. \quad (9)$$

As you can see in Fig. 3, the results are extremely close. From Fig. 3, it is clear that the efficiency of optical designs can be extremely sensitive to the tilt angle of the source. To aid in the understanding of why tilting has

Fig. 3 Comparison of the theoretical model and calculated values on the lenses for total loss due to Fresnel reflections as a function of tilt angle.

Fig. 4 Lenses made for the same geometry as in Fig. 3 at the indicated tilt values with the light lost due to reflections plotted in color along the surface where the loss takes place. The loss was normalized across all four plots for easy comparison.

such a large effect on efficiency, we show some sample lens designs at various tilt angles in Fig. 4. In this figure, we can see that tilting the lens reduces the bend angle required for the rays on the left side, which minimizes loss. Eventually, we reach a midpoint (somewhere between 30 deg and 40 deg for this geometry according to Fig. 3), where this effect switches sides, causing too much bending on the right side of the lens due to too much tilting. The optimal tilt is the region between these two values, where we are neither tilted too far nor too little.

To investigate how this sensitivity changes with system geometry, a parametric analysis of the optimal surface tilt as a function of the offset and the target size was performed. For all of the analyses done in this section, the source-to-target distance was held at 1 m, the target was a uniform square, and the grid size was 101×101 . A specific example of one geometry investigated in the analysis is demonstrated in Fig. 5.

The results are detailed below. In Fig. 6(a), we see the calculated optimal tilt angle for each target size and lateral

Fig. 5 Diagram explaining geometry used for the parametric analysis. In this example, a lens with an offset of 750 mm is tilted 20 deg toward a 2-m $\times$ 2-m target. The inner surface is a sphere concentric to the source so it contributes zero optical power.

Fig. 6 (a) The optimal tilt angle and (b) the throughput increase that can be obtained by implementing the recommended tilt as a function of the lateral offset between the source and target, x_0 , and the dimensions of the square target.

offset. For the example geometry shown in Fig. 5, we can see that with a target width of 2000 mm and a lateral offset of 750 mm the optimal tilt would be around 20 deg. To determine this optimal tilt, we calculated Eq. (8) as a function of tilt angle and determined the minimum by solving for the zero point of the numerical derivative. In Fig. 6(b), we have plotted the potential throughput increase that can be achieved by implementing this calculated optimal tilt instead of not tilting the lens at all. In the example of Fig. 5, the throughput increase was around 35%.

However, as mentioned in Sec. 1, the intuitive solution is to tilt the source such that the center of the lens is aimed at the center of the target. Although this improves the system performance dramatically compared to no tilt, this is still not optimal as we see in Fig. 7 where the optimal tilt angle is compared to aiming the source at the center of the target, where results differ by as much as 25 deg.

In general, we found that the optimal tilt aimed the lens slightly above the center of the target (i.e., it was tilted less than expected) for large targets relative to the offset and slightly above the center (or tilted more than expected) for small targets relative to the offset. Intuitively, we can explain this by recognizing that if the target size is larger than the offset distance, there will still be some target behind the source we need to illuminate, which increases the cost of tilting the source and results in a smaller tilt angle. Conversely, if the target is small relative to the offset distance, the rays pointed directly away from the target have to bend a lot more to hit it than those pointed toward the

target, incentivizing a larger tilting of the source, and resulting in a greater tilt angle.

3 Reflective Surfaces

Metals in reflective systems also exhibit Fresnel reflection characteristics, so we can model the efficiency in a similar manner as above. However, as we will see later, these systems are much more limited by geometry than anything else, so the analysis on efficiency and manufacturing is intentionally more qualitative than quantitative. Assuming our light source is at the d line and our mirror is made of aluminum, the dielectric constant is approximately $\epsilon = -44.958 + 14.909i$.⁴ The new equation for R is then calculated by taking the complex magnitude as below

$$R_s = \left| \frac{\mathbf{N} \cdot (n_1 \mathbf{I} - \sqrt{\epsilon} \mathbf{A})}{\mathbf{N} \cdot (n_1 \mathbf{I} + \sqrt{\epsilon} \mathbf{A})} \right|^2 \quad R_p = \left| \frac{\mathbf{N} \cdot (n_1 \mathbf{A} - \sqrt{\epsilon} \mathbf{I})}{\mathbf{N} \cdot (n_1 \mathbf{A} + \sqrt{\epsilon} \mathbf{I})} \right|^2, \quad (10)$$

where $\mathbf{A}$ is the refracted ray entering the metal, calculated according to a modified version of Snell's law in vector form below⁵

$$\mathbf{A} = \frac{1}{n} [\mathbf{N} \times (-\mathbf{N} \times \mathbf{I})] - \mathbf{N} \sqrt{1 - \frac{1}{n^2} (\mathbf{N} \times \mathbf{I}) \cdot (\mathbf{N} \times \mathbf{I})} \quad (11)$$

and $n = \text{Real}[\sqrt{\epsilon}]$. Figure 8 shows the layout of the vectors used in the reflective system.

Fig. 7 (a) Angular difference between aiming the lens at the center of the target and the optimal tilt. Positive values indicate that the optimal tilt is less than the central tilt and negative values indicate the opposite. (b) The throughput increase that can be obtained by implementing the recommended tilt instead of aiming the lens at the center of the target.

Fig. 8 Diagram showing geometry and vectors used in the calculations of Sec. 3.

Using the reflection values calculated for aluminum, we establish a merit function in the same manner as in Sec. 2, with the exception that in reflective systems loss is defined as the amount of light not reflected by the lens

$$\min \sum_{i,j} I_{e_{i,j}} (1 - R_{i,j}). \quad (12)$$

Testing this model against calculated values on constructed surfaces we find again in Fig. 9 that the results are very close. Although the change in efficiency is close to zero, surprisingly, it turns out that the optimal tilt for system efficiency is pointing away from the target.

Since the loss of efficiency from tilting the source is negligible, it seems that sensitivity might serve as more practical criteria for determining the optimal tilt. The measure we selected to represent sensitivity was the incident angle light rays make with the optical surface, since higher angles exaggerate deviations caused by imperfections in the manufactured surface. As mentioned in Sec. 2, we can calculate this angle using the dot product between the incident ray and the surface normal

$$\cos \theta_i = \mathbf{I} \cdot \mathbf{N}. \quad (13)$$

Fig. 9 Comparison of the theoretical model and calculated values on the constructed surface for total loss due to Fresnel reflections as a function of tilt angle.

Fig. 10 Calculation of the average and peak incidence angle required as the source is tilted toward and away from the target.

After calculating both the peak and average incidence angles as a function of the tilt angle in Fig. 10, we can see that again the optimal tilt is actually directed away from the target by almost 20 deg.

Despite the fact that mathematically the optimal design is tilting away from the target, in practice, this is usually not possible. Often, a forward tilt is necessary to avoid obstruction from the source, so the best we can do is find the minimum tilt such that the source is outside of the ray path by some distance d . Looking back at Fig. 8, we can see that if the source is outside of the path of the ray aimed farthest away from the target, the rest of the light will pass unobstructed as well. If we calculate the line from the source that is perpendicular to this vector, its magnitude and direction tell us the distance our source is from the ray. We can find this by first calculating the vector, $\mathbf{v}$, perpendicular to $\mathbf{O}$ and solving the system of parametric equations from the starting points $\mathbf{P}_1 = \mathbf{O}$ and $\mathbf{P}_2 = r\mathbf{I}$ for where the two intersect

$$\mathbf{P}_1 - \mathbf{P}_2 = t_2 \mathbf{O} - t_1 \mathbf{v}. \quad (14)$$

A diagram of this is shown in Fig. 11.

Fig. 11 Geometry used to solve for the vector, $\mathbf{v}$, where the ray bundle is demonstrated by the two outermost rays. If $\mathbf{v}_x$ is negative, then $\mathbf{v}$ points to the left, indicating the source is inside of the ray bundle leaving the lens. If $\mathbf{v}_x$ is positive, the source is outside of the ray bundle and the system is not obscured.

Fig. 12 Distance between the outermost ray and the source. Negative values indicate distance inside of the ray bundle and positive values indicate distance outside of the ray bundle.

By either solving the above system for the angle at which $t_1 \mathbf{v}_x \geq d$ as we do in the Appendix or plotting the x and z components of $t_1 \mathbf{v}$ as a function of the tilt angle as in Fig. 12, we can determine the minimum tilt that provides the required clearance. In the example of Fig. 12, if $d = 4$, the recommended tilt would be around 28 deg.

4 Conclusion

In this paper, we develop a complete method for determining the optimal tilt for both refractive and reflective, off-axis illumination systems. For refractive lenses, the optimal tilt is determined by efficiency and can be found using numerical methods to determine the minimum of Eq. (8) as a function of the tilt angle, θ , which is hidden in the input vector, $\mathbf{I}$, according to Eq. (6). The optimal tilt for reflective lenses is theoretically aimed away from the target, but because this orientation will usually be blocked by the source we use Eqs. (14), (20), or (24) to calculate the minimum tilt that misses the source by a distance, d . By utilizing the proposed methods, designers can enjoy efficiency improvements of up to 150% in some cases, which is a substantial increase for such a small modification. Although optical coatings can also be used to greatly reduce losses in illumination systems such as these, their utility still depends on the initial performance of the uncoated system and best performance will be achieved by implementing the optimal tilt alongside a coated optic.

It should be noted that all of the analyses, here, were performed for a system that is only off-axis in one direction. If, for example, a design needs to be made where a lens is shifted in both x and y , then an appropriate change of reference will need to be made such that the shift axis used in this analysis is aligned with the direction of displacement from the center of the target

$$\text{Axis} = \vec{\Delta x} + \vec{\Delta y}. \quad (15)$$

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Appendix A: Solution to the Reflective Tilt

To arrive at an explicit solution to Eq. (14), we can start by solving its z component for t_2

$$t_2 = \frac{\mathbf{P}_{1,z} - \mathbf{P}_{2,z} + t_1 \mathbf{v}_z}{\mathbf{O}_z}. \quad (16)$$

Plugging this result into the x component of Eq. (14) and solving for t_1 yields

$$t_1 = \frac{\mathbf{P}_{1,x} - \mathbf{P}_{2,x} - (\mathbf{P}_{1,z} - \mathbf{P}_{2,z}) \frac{\mathbf{O}_x}{\mathbf{O}_z}}{\mathbf{v}_z \frac{\mathbf{O}_x}{\mathbf{O}_z} - \mathbf{v}_x}. \quad (17)$$

Substituting our known values into the above and simplifying, we end up with

$$t_1 = |\mathbf{T} - r\mathbf{I}|r(\mathbf{I}_x \mathbf{T}_z - \mathbf{I}_z \mathbf{T}_x). \quad (18)$$

The intersection point can be found by traveling a distance t_1 along the vector $\mathbf{v}$ from $\mathbf{P}_1$

$$\mathbf{V} = \mathbf{P}_1 + t_1 \mathbf{v}. \quad (19)$$

To determine the optimum tilt, we have to solve the above equation for the angle, θ , where the intersection point $\mathbf{V}_x$ is at least a distance d away, where t_1 is found using Eq. (18). Plugging in t_1 and simplifying, we arrive at the following:

$$\mathbf{V}_x = d = r(\mathbf{I}_x \mathbf{T}_z^2 - \mathbf{I}_z \mathbf{T}_x \mathbf{T}_z - r\mathbf{I}_x \mathbf{I}_z \mathbf{T}_z + r\mathbf{T}_x \mathbf{I}_z^2). \quad (20)$$

In practice, we solve the rest of this problem by calculating $\mathbf{V}_x$ as a function of tilt angle and numerically solving for the point when $\mathbf{V}_x = d$. But we can continue by replacing $\mathbf{I}$ with the rotated vector from Eq. (6) and simplifying using the assumption that $\mathbf{T}_z \gg r\mathbf{I}_z$ (the distance from the lens to the target is larger than the distance from the source to the lens) to arrive at an equation in terms of θ

$$\frac{d}{r\mathbf{T}_z} = \cos \theta (\mathbf{I}_{0,x} \mathbf{T}_z - \mathbf{I}_{0,z} \mathbf{T}_x) - \sin \theta (\mathbf{I}_{0,z} \mathbf{T}_z - \mathbf{I}_{0,x} \mathbf{T}_x), \quad (21)$$

which we can rewrite to be

$$A \cos \theta + B \sin \theta = C, \quad (22)$$

where

$$A = (\mathbf{I}_{0,x} \mathbf{T}_z - \mathbf{I}_{0,z} \mathbf{T}_x) \quad B = (\mathbf{I}_{0,x} \mathbf{T}_x - \mathbf{I}_{0,z} \mathbf{T}_z) \quad C = \frac{d}{r\mathbf{T}_z}. \quad (23)$$

Solving Eq. (22) for θ yields the optimal tilt angle

$$\theta = \cos^{-1} \left(\frac{C}{\sqrt{A^2 + B^2}} \right) + \tan^{-1} \left(\frac{B}{A} \right). \quad (24)$$

Fig. 13 Comparison of the optimal tilt calculated from the actual surface points in black and using the model from Eq. (8) in red. Ratio was calculated as $R = \frac{D_{\text{target}}}{H_{\text{lens}}}$.

Fig. 14 Predicted value of the optimal tilt and mean surface error as a function of grid size. The true optimal tilt determined by fitting the point clouds to NURBS, ray tracing with Fresnel reflection, and calculating the loss is plotted in blue.

Appendix B: On Integrability and Validity

In this analysis, we have assumed the target points $\mathbf{T}$ can be calculated independent of the optical surface. In general, this is not exactly true; usually, there is some curl in the mapping relationship leading to errors in the final locations of the vectors. Herein, is an investigation into the impact this assumption has on the validity of our method. For all the analyses in this section, we used the double-pole mapping method to direct an entire hemisphere of light onto a uniform square target. The source-to-target distance was 1 m, the

offset was 1 m, and the target was 1-m $\times$ 1-m, making the center of the target 45 deg from the source.

In our derivation of Eqs. (1)–(8), we intentionally left out the shape of the surface, electing to choose some representative shape of a small sphere instead. The purpose of this was twofold: first, it allowed us to approximate the final system behavior without having to actually construct a surface, and second, it allowed us to ignore any effect that the target points might have on surface shape. As shown in Fig. 3, approximating the surface has a negligible effect on the capabilities of the method in determining the optimal tilt for a reasonably far target distance. To investigate the range of validity for this approximation, we plot the calculated optimal tilt using the true surface points alongside the predicted optimal tilt from the model as a function of the ratio of the lens size to the target distance in Fig. 13. This was accomplished by scaling down the target size, offset distance, and distance to the target equally while keeping the lens size constant. The model is almost exact even for ratios as large as 0.6, which for our investigation meant a 20-mm lens, 33 mm from the target at the base and only 13 mm at the apex.

To investigate the effect of deviations between the true target points and the calculated ones caused by a violation of the integrability condition, we calculated the optimum tilt and mean surface error in Fig. 14. The surface error at each point, E , was calculated according to $E = (\nabla \times \mathbf{N}) \cdot \mathbf{N}$, and the mean was determined by summing over and then dividing by the total number of points.

Although this method is limited by the integrability of the mapping method selected, with a reasonable grid size (or better mapping relationship), the optimal tilt can be calculated to within a fraction of a percent despite the fact that there will be error in the target point locations. As the mean surface error decreases, we see the predicted optimal tilt appears to approach the value determined from ray tracing. As better methods for determining an initial mapping relationship are developed, this error will continue to shrink, making the study proposed in this paper even more accurate.

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