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OBSERVED DOPPLER SPECTRA OF HAIL

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ABSTRACT

Observations of Doppler spectra from particles 610 m above the ground were obtained by means of a vertically pointing X-band radar during a period when large hail was falling at the ground. It was found that the variance of the Doppler spectrum was a fairly good indicator of maximum hailstone sizes. Calculations of ice-particle size spectra were made on the basis of assumptions of particle shape, composition and fall speed. The many assumptions needed to bring calculated ice-sphere spectra into approximate conformity with observations indicates the complexity of this procedure and the need for at least one more independent observation of the properties of the hailstones. Measurements of depolarization might yield an independent estimate of particle shape. It is concluded that the pronounced minima in the back-scattering curves of large ice spheres should serve as an independent indicator of the speed of vertical air motions.

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1. Introduction

Various authors have investigated the Doppler spectra of hail (Donaldson and Wexler, 1969; Boston and Rogers, 1969). For the most part they have taken an empirically derived hailstone size distribution, assumed that the hailstones were dry or wet spheres, employed an expression relating sphere diameter to terminal speed and calculated the resulting Doppler spectra as a function of maximum ice-sphere diameter. Having the spectra, calculations have been made of mean Doppler velocities and variances of Doppler spectra. Battan and Theiss (1968) reversed the procedure. They started with an observed Doppler spectrum, and by means of a series of assumptions, similar to those already mentioned, they calculated an ice-sphere diameter spectrum.

None of these studies had the benefits of simultaneous observations of Doppler spectra and of the characteristics of the hail. This article deals with the analysis of Doppler radar observations made during the fall of hail. The data on hailstone sizes consists of information on the maximum stone sizes as a function of time. Unfortunately, data on size distribution or hailstone shape were not obtained. Nevertheless, the acquisition of Doppler spectra as a function of time just above the radar site during the fall of large hail reveals some information worth reporting.

2. Observations

On 10 August 1966, the University of Arizona, X-band pulsed radar was located on the summit of Mount Lemmon, just northeast of Tucson, at an elevation of about 2,800 m. The antenna was pointing towards the zenith.

Thunderstorms developed and moved overhead in early afternoon. There was light intermittent rain starting at about 1330 MST. The first hailstones observed at 1451 MST were small, having maximum diameters of about 6 mm. Intermittent showers of hail of varying sizes and intensities continued until at least 1705 MST. At times, hailstone diameters were as large as 25 mm. Accumulated hail reached a depth of more than 5 cm on the ground. The analysis described in this report deals with Doppler radar measurements made between 1502 and 1511 MST. The specific weather observations reported during this period at the radar site are given in Table 1.

As noted in earlier reports, the University of Arizona Doppler radar yields spectra at altitude intervals of about 152 m. Because of the recovery time of the receiver, observations closer than about 500 m are not reliable. For this reason, the radar data from the next two altitude intervals, i.e., 610 and 760 m, were examined. Since the radar beam width was 1.3° and the pulse length was $0.25 \mu\text{s}$, at the lower altitude, the radar was observing the hailstones in a volume about 14 m in diameter and 38 m high (about $6 \times 10^3 \text{ m}^3$).

Table 1. Weather observations during period of radar analysis.

Time - MST	Observation
1459	Close thunder; rain steady.
1500	Echo up to 35,000 ft.
1503	Trace of hail--about 1/4" but in small numbers.
1505	Intensity of hail picking up; updrafts; clouds are low and visibility poor.
1506	Hail heavy now with sizes up to 3/8".
1506.5	Hail size now includes a few 1/2", ground covered.
1508	Hail to 3/4"; <u>very</u> close lightning.
1509	Hail to 1" in size; very heavy and intense; sounds like a popcorn popper here in van.
1513	Hail not so hard; sizes about 1/2".

Figures 1 a and b give the following information yielded by the radar: total Z_e ; mean Doppler velocity, $\bar{V}_r$; variance of the Doppler spectrum, σ_v^2 ; updraft velocity, W_a , calculated from Rogers (1964) equation, relating updraft speed to Z_e ; the quantity $\bar{W}_T = (W_a - \bar{V}_r)$ a measure of the mean Doppler velocity in still air.

Several general comments can be made. First of all, the spectra at the two altitudes at the same time were similar in many respects. At both altitudes, Z_e was about $3 \times 10^2 \text{ mm}^6/\text{m}^3$ at the start of the observations and was about three orders of magnitude greater at 1511 MST. Variances ranged from about 3 to over $12 \text{ m}^2/\text{sec}^2$. The estimated mean "still-air" Doppler velocities ($W_a - \bar{V}_r$) ranged from as low as 4 m/sec at the start to a maximum of about 9 m/sec near the end of the observations.

Unfortunately the radar set is designed in such a fashion that the Doppler spectrum is cut off at -19.5 m/sec (downward velocities). As a result, part of the spectrum was cut off after about 1508 MST. The total amount of power lost does not seem to be appreciable however, except in the case of the last two sets of observations. If the fall velocity of the hailstones is taken to be the quantity ($W_a - \bar{V}_r$), the time required for the hail distribution to descend from 760 m to the ground would range from about 1.4 to 3.2 min corresponding to velocities of 9 and 4 m/sec, respectively. The time for the hailstones to fall 610 m at these speeds would be 1.2 and 2.5 min, respectively. In the analysis

to follow, it would be appropriate to assume such time lags in comparing the Doppler spectra with the observed hail data. The observations in Table 1 indicate an uninterrupted fall of hail from 1503 MST to the end of the period. It seems safe to assume that the radar was detecting hailstones somewhat similar to those reaching the radar from 1 to 3 minutes later.

3. Analysis

The observed quantities given in Table 1 and Figs. 1 a and b can be compared with other observations and with theoretical predictions. Table 2 presents a listing of various data, at the altitude of 610 m, for each time shown in Fig. 1. Since the data at the two levels are similar in many respects, it seems reasonable to assume that either one is a representative measure of the true state of the precipitation just above the ground.

Various authors have published data on the radar reflectivity of hailstorms. Most recently Dennis et al. (1970) reported the distribution of Z_e in hailstorms in South Dakota. They reported a maximum 3-cm Z_e in the "median hailstorm" to be about $10^5 \text{ mm}^6 \text{ m}^{-3}$ while cross sections through their echoes show values ranging from 10^2 to $10^6 \text{ mm}^6 \text{ m}^{-3}$ in reasonable agreement with the data in Table 2. Their radar observations were obtained by scanning through thunderstorms located away from the radar. Observations of hail at the ground could not be readily associated with specific radar data.

Table 2. Doppler radar observations and some inferred hailstone properties.

Time MST	Max. reported hail diameter cm	z_e mm ⁶ m ⁻³	$\bar{W}_T$ ⁽¹⁾ m sec ⁻¹	σ_V^2 m ² sec ⁻²	D _{max} inferred ⁽²⁾ from $\bar{W}_T$ -cm		D _{max} inferred ⁽²⁾ from σ_V^2 -cm	
					Dry	Wet	Dry	Wet
150239		4.1x10 ²	5.9	5.2	0.2	0.2	1.6	2.1
0328	0.6	6.1x10 ²	6.0	3.5	0.2	0.2	1.4	1.5
0418		1.6x10 ³	6.4	3.4	0.2	0.2	1.4	1.5
0507		1.9x10 ⁴	7.7	4.4	0.3	0.3	1.5	1.8
0556	1.0	1.4x10 ⁴	7.5	5.0	0.3	0.3	1.6	2.0
0645	1.3	1.2x10 ⁴	7.4	4.5	0.3	0.3	1.5	1.8
0735		1.7x10 ⁴	7.7	6.4	0.3	0.3	1.8	2.4
0824	1.9	1.9x10 ⁴	7.7	6.3	0.3	0.3	1.8	2.3
0913	2.5	1.5x10 ⁴	7.6	8.3	0.3	0.3	2.0	∞
1002		5.8x10 ⁴	8.4	6.5	0.35	0.35	1.8	2.5
1030		2.0x10 ⁵	9.1	12.2	0.4	0.4	2.4	∞
151100		2.0x10 ⁵	9.4	10.0	0.4	0.4	2.4	∞

(1) The quantity $\bar{W}_T = W_a - V_r$ where W_a was determined from Rogers (1964) equation.

(2) The values of D_{max} were inferred from $\bar{W}_T$ and σ_V^2 by means of graphs published by Boston and Rogers (1969).

Several authors, notably Atlas and Ludlam (1961) and Douglas (1964) observed hailstones at the ground, calculated size distributions and values of Z_e . Douglas (1964) proposed a mean hailstone size distribution.

$$N = N_0 e^{-3.09D}, \quad (1)$$

where N is the number of hailstones per cubic meter in a size range from D to $D + \Delta D$. If ΔD is taken to be 0.3 cm, N_0 has the value $31 \text{ cm}^{-1} \text{ m}^{-3}$. Douglas used the following fall-speed law:

$$W_T = 16.2 D^{1/2}, \quad (2)$$

where D is in centimeters, W_T is in m sec^{-1} . This expression can be derived from the well known equation for the fall speed of rigid spheres

$$W_T^2 = \frac{4}{3} \frac{g D \rho_s}{C_D \rho_a} = K D, \quad (3)$$

where ρ_s and ρ_a are the densities of the sphere and air respectively and C_D is the drag coefficient. Equation (2) can be obtained by taking $\rho_s = 0.9 \text{ g/cm}^3$, $C_D = 0.6$ and $\rho_a = 0.75 \times 10^{-3} \text{ g/cm}^3$. Obviously the quantity K can vary widely depending on the densities and the drag coefficient as shown recently by Lhermitte (1971).

Douglas (1964) calculated the 3.3-cm Z_e for each of 67 hail samples assuming them to be either dry or "wet." In the former, in reasonable agreement with Table 2, Z_e ranged from about 2×10^2 to $6.5 \times 10^6 \text{ mm}^6 \text{ m}^{-3}$ with a distinct frequency maximum at about $4.3 \times 10^4 \text{ mm}^6 \text{ m}^{-3}$. In the case of wet hailstones, the frequency distribution of Z_e was shifted toward slightly higher Z_e .

Recently Battan (1971) made some new calculations of 3.21-cm Z_e employing equation (1) and various values of $D_{\max}$ and water coatings on ice spheres. It was found that as long as $D_{\max}$ is at least 1 cm, further increases of $D_{\max}$ do not greatly affect Z_e . Over a range of $D_{\max}$ from 1 to 4 cm, Z_e increases from about 2×10^5 to $7 \times 10^5 \text{ mm}^6 \text{ m}^{-3}$.

As shown in Table 2, at about 1506 MST hailstones about 1 cm in diameter were observed at the ground. One to two minutes earlier, at an altitude 610 m above the radar, the observed Z_e values were 10 to 100 times smaller than $10^5 \text{ mm}^6 \text{ m}^{-3}$. This result suggests that the Douglas distribution (equation 1) does not apply or that the assumptions of spherical wet hailstones is seriously in error.

Boston and Rogers (1969) employed equations (1) and (2) to calculate curves of the mean Doppler velocities and Doppler variances as a function of $D_{\max}$. Their curves were used to obtain the quantities shown in the last four columns of Table 2. In their analyses, Boston and Rogers assumed the hailstones were falling in still air. If the effects of small scale turbulence are neglected, the effects of vertical air motions do not affect

the variance of the Doppler spectrum. On the other hand the mean Doppler velocity is a measure of the reflectivity-weighted terminal speed of the hailstones plus the updraft speed. As already noted, the latter quantity was estimated in the first instance from Rogers (1964) equation relating W_a to Z . Rogers derived it for the case of an exponential distribution of raindrops, hence it would be surprising if it were a good measure of W_a in a hailstorm.

This conclusion appears to be supported by the results in Table 2. The value of $D_{\max}$ inferred from $\bar{W}_T$ are much too small throughout the period of observation. The calculation of Boston and Rogers (1969) indicate that values of $D_{\max}$ between 1 and 2 cm correspond to values of $\bar{W}_T$ of about 16 m/sec and 15 m/sec in the cases of dry and wet hail, respectively. Since the quantity $\bar{V}_r$ can be taken as being reliable, it seems reasonable to speculate that the calculated values of W_a are too negative. On the other hand it must be borne in mind that the discrepancies may be attributed to the assumption of sphericity and the assumption of equations (1) and (2).

The estimate of $D_{\max}$ from observations of the Doppler variances are reasonably good, particularly if it assumed the hailstones were scattering as if they were dry ice spheres. In the early part of the record, the inferred $D_{\max}$ values were too large by a factor of about 2. During the last half of the observations the observed maximum hailstone diameters are in good agreement with the inferred diameters. It might have been expected that the hailstones would act as if they were wet, but it is seen that the observed variances of $8 \text{ m}^2 \text{ sec}^{-1}$ or greater, exceed the largest calculated by Boston and Rogers (1969).

The Doppler spectra were employed to calculate the size spectra of ice spheres. The data at 150239, 150328 and 150418 MST were combined to yield one average spectrum shown in Fig. 2. As shown in Fig. 1 the values of $\bar{V}_r$, W_a and $(W_a - \bar{V}_r)$ did not vary much during the period. The updraft velocity was taken to be -1.8 m/sec, the mean value of W_a . Equation (2) was employed to calculate the value of D corresponding to any value of W_T ($= V_r - W_a$). Knowing the backscattering cross section of ice spheres as a function of D (Battan, et al., 1970), calculations were made of the size distributions of dry ice spheres. From the curves in Fig. 3 it is evident that for diameters between about 0.3 and 1.3 cm, concentrations of wet spheres would be within a factor of 3 of the concentrations of dry spheres. In this analysis a wet sphere is one composed of a solid ice sphere coated with a layer of water 0.01 cm thick.

Calculated size distribution of dry ice spheres for the average Doppler spectrum shown in Fig. 2 is shown in Fig. 4. It appears that the curve might be accounted for if it were assumed that the radar were detecting two distributions of particles, a distribution of small ice spheres or raindrops ranging in diameter up to about 6 mm and a distribution of larger ice particles ranging in diameter from about 7 to 10 mm. Note that if the particles having diameters smaller than 7 mm were assumed to be water drops their concentrations would be about 5 times smaller than the values shown in Fig. 4.

A maximum ice sphere diameter of about 1 cm was observed at 150536 MST, which was about 1.6 minutes after the last of the 3 Doppler spectra included in the average depicted in Fig. 2. The curve in Fig. 4 shows a very small concentration of ice spheres having a diameter of 1 cm. The size distribution curve differs markedly from the Douglas curve which is also shown in Fig. 4.

A second series of calculations of ice-sphere size distributions were made from an average of the Doppler spectra at 150913 and 151002 (Fig. 5). When the vertical air motion was taken to be that calculated from the Rogers equation, -4.6 m sec^{-1} , the resulting size distribution for dry ice spheres is shown in Fig. 6. Maximum sphere diameters were smaller than 0.9 cm. This result is clearly in disagreement with visual observations of hailstones some 2.5 cm in diameter. It can also be seen in Fig. 6 that the calculated distribution curve differs markedly from the Douglas curve.

The results of the calculations and analyses already reviewed can be summarized briefly. The variance of the Doppler spectrum was reasonably well correlated with the maximum hailstone size. On the other hand, mean Doppler velocity was not well correlated. Also, calculated ice-sphere size distribution did not adequately account for the large hailstones observed and the calculated distribution curves had slopes very much larger than one might have expected. These results suggested that the estimated

updraft velocity given by the Rogers equation, as might be expected in the case of hail, was in error. If instead of downdrafts of the indicated values (e.g., -4.6 m sec^{-1}) there were weak updrafts, the calculated ice sphere size spectra would be in better agreement with observations. The results already discussed also brought into question the general validity of the fall-speed law given as equation (2).

In order to test the effects of a different fall speed law, calculations were made employing a relation graphed by Lhermitte (1971) for the case of ice particles having a density of 0.9 g cm^{-3} falling in air having a density of $0.8 \times 10^{-3} \text{ g cm}^{-3}$ and having a nonsphericity index $\alpha = 3$. This quantity depends on the ratio of hailstone volume to cross-sectional area. Unfortunately, appropriate backscattering cross sections for such nonspherical particles are not known, and therefore, it was assumed that the particles were scattering as if they were spheres having the same mass as the actual hailstones. For this reason, the calculated size-distribution curves shown in Fig. 7 have equivalent diameter as the abscissa. The curves for wet and dry ice spheres are shown as solid lines. Even though it was assumed that the particles were in a 4.6 m sec^{-1} downdraft as indicated by the Rogers equation, the new calculation appear more reasonable than the earlier ones shown in Fig. 6. The slopes of the curves are not exceedingly large and the curves indicate the presence of hailstones as large as 2.5 cm in diameter.

Both curves show an anomalous peak--the "dry-hail" curve at 1.5 cm and the "wet-hail" curve at 1.9 cm. These peaks are a result of the dips in the backscattering cross sections shown in Fig. 3. It would be expected that the hailstone size distribution curves would not show pronounced maxima and minima when the atmospheric volume which is sampled is as large as about 10^4 m^3 .

The pronounced maximum in the "wet-hail" curve can be greatly reduced by shifting the entire Doppler spectrum 2 m sec^{-1} . This is equivalent to assuming that there was a downdraft of -2.6 m sec^{-1} rather than the -4.6 m sec^{-1} given by Roger's equation. The size distribution obtained when the weaker downdraft is assumed is shown as the dashed line in Fig. 7. It appears to be in reasonable agreement with the observation of hail size.

Conclusions

There is little question that Doppler radar can yield semi-quantitative information about the presence of hail. This analysis supports conclusions of Donaldson and Wexler (1969) and Boston and Rogers (1969) that the variance of the spectrum of vertical Doppler velocity can be used to yield estimates of maximum hailstone size. On the other hand, it has been shown that at least on some occasions, measurements of mean Doppler velocity are of little value in inferring the presence and sizes of hail. This appears to be largely a result of difficulties in taking into account vertical motion of the air and the terminal velocities of the hailstones.

The problem of obtaining the size distribution of hailstones is more complicated than had been suspected earlier. The difficulties arising because of uncertainties in the refractive indices of the hailstones have been recognized for some time. Now it appears that the non-sphericity of the hailstones is more important than had been realized. It crucially affects the fall speeds and to an unknown extent the scattering cross sections. It appears that if there were independent measurements, possibly by means of depolarization observations, it should be possible to improve the estimates of equivalent hailstone size.

The recognition that the minima in the curves of backscattering cross section should lead to a minimum in the Doppler spectra produced by ice spheres offers another independent means of inferring the updraft speed in a hailstorm and deserves further investigation.

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LEGENDS FOR FIGURES

- Fig. 1 a and b Doppler spectra at altitudes about 610 and 760 m above the vertically pointing radar which was located on a mountain peak at an elevation of 2,900 m.
- Fig. 2 Average Doppler spectrum obtained by combining the spectra at 150239, 150328 and 150418 MST at altitude 610 m above the radar.
- Fig. 3 Normalized backscattering cross sections of dry and wet ice spheres. The latter are taken to be ice spheres coated with a layer of water 0.01 cm thick.
- Fig. 4 Calculated size spectrum of dry ice sphere corresponding to Doppler spectrum in Figure 2 when the vertical air velocity is taken to be -1.8 m sec^{-1} .
- Fig. 5 Average Doppler spectrum obtained by combining the spectra at 150913 and 151002 MST at altitude 610 m above the radar.
- Fig. 6 Calculated size spectrum of ice spheres corresponding to the Doppler spectrum in Figure 5 when the vertical air velocity was taken to be -4.6 m sec^{-1} .

Fig. 7

Calculated size spectrum of ice spheres corresponding to the Doppler spectrum in Figure 5 but with a fall-speed law given by Lhermitte (1971). The solid curves are for a downdraft of 4.6 m sec^{-1} ; the dashed curve for a downdraft of 2.6 m sec^{-1} .

Figure 1a and 1b

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7