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Key Points:

- Performance of the DSRC method is made more robust by incorporating ridge estimation to address shortcomings of the least squares approach
- The method is extended to account for spatial heterogeneity in the rainfall distributions
- Superiority and applicability of the improved method is demonstrated via synthetic and real-data studies

Supporting Information:

- Supporting Information S1
- Data Set S1
- Data Set S2
- Data Set S3
- Data Set S4
- Data Set S5
- Data Set S6
- Data Set S7
- Data Set S8

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Improved Dynamic System Response Curve Method for Real-Time Flood Forecast Updating

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Abstract The dynamic system response curve (DSRC) method has been shown to effectively use error feedback correction to obtain updated areal estimates of mean rainfall and thereby improve the accuracy of real-time flood forecasts. In this study, we address two main shortcomings of the existing method. First, ridge estimation is used to deal with ill-conditioning of the normal equation coefficient matrix when the method is applied to small basins, or when the length of updating rainfall series is short. Second, the effects of spatial heterogeneity of rainfall on rainfall error estimates are accounted for using a simple index. The improved performance of the method is demonstrated using both synthetic and real data studies. For smaller basins with relatively homogeneous spatial distributions of rainfall, the use of ridge regression provides more accurate and robust results. For larger-scale basins with significant spatial heterogeneity of rainfall, spatial rainfall error updating provides significant improvements. Overall, combining the two strategies results in the best performance for all cases, with the effects of ridge estimation and spatially distributed updating complementing each other.

1. Introduction

Real-time flood forecasting plays a significant important role in helping to reduce flood-related losses (Brisset et al., 2017; Hartnett & Nash, 2017; Lu et al., 2016; Wang et al., 2017). However, input and state estimation errors (Chen et al., 2011; Lee et al., 2011; Sun et al., 2018), parameter misspecification (Gupta et al., 2008; Pagano et al., 2011; Qin et al., 2018a, 2018b), and model structural inadequacy (Butts et al., 2004; Gupta et al., 2012) can lead to inaccuracies in flood forecasts. As the most important input to hydrological models, estimates of rainfall can directly affect the accuracy of flood forecasts (Jiang et al., 2016; Li & Liu, 2017). Therefore, real-time updating of rainfall estimates can be very powerful (Looper & Vieux, 2012).

The dynamic system response curve (DSRC) method was introduced by Si et al. (2015), based on the idea of using error feedback correction to obtain updated estimates of mean areal rainfall, and has been shown to improve the accuracy of real-time flood forecasts for several humid-region basins in China. Similarly, the hierarchical Bayesian inference method of Henn et al. (2015) is based on Bayesian Total Error Analysis (BATEA) methodology (Kavetski et al., 2006a; Kavetski et al., 2006b) coupled with a multimodel framework to infer basin mean precipitation from streamflow observations and has been applied to snow-dominated basins in the Sierra Nevada mountains of California. Results show that time series of inferred annual precipitation under different model and parameter assumptions are strongly correlated with one another, suggesting that this approach is capable of resolving year-to-year variability in basin mean precipitation. While the hierarchical Bayesian inference method has the similar goals to our DSRC method (Si et al., 2015), in that it is committed to addressing the uncertainty in the topographical representativeness of precipitation gages relative to the basin, the two methods focus on different time scales and are based on different theories. In particular, the ultimate goal of the DSRC method is to improve the accuracy of real-time flood forecasting during the flood season.

While the results of Si et al. (2015) indicated that rainfall estimate updating results in better improvements for larger basins than for small basins (Si et al., 2015), we have subsequently determined that this is due in part to problems associated with inversion of the normal equation coefficient matrix ($S^T S$). Closer examination of our results revealed that this condition tends to occur when the method is applied to smaller-sized basins and/or when the length of the time series of updated rainfall estimates is relatively short.

Accordingly, this paper proposes and tests two modifications to the DSRC method. In the first, we deal with ill conditioning of the normal equation coefficient matrix using ridge estimation (DSRC-R). In the second we improve the method to acknowledge the spatially distributed nature of the rainfall error, by accounting for heterogeneity in the spatial distribution of rainfall (DSRC-D). Of course, both modifications can be used concurrently when necessary and appropriate (DSRC-RD). Synthetic experiments are employed to evaluate the stability of the methods, following which we demonstrate the benefits of the improved methodology using real data.

2. Methodology

The methodology presented here builds upon the DSRC theory presented by Si et al. (2015). In brief, given a hydrological model $Q = f(P)$ that generates estimates of streamflow Q as a function of precipitation P , we can use a finite difference approach to approximate the system response to unit perturbations in rainfall. Based on this, an approximate estimate of the time series of estimation errors $\Delta P'$ of mean areal rainfall can be computed from the time series of streamflow errors ΔQ (differences between simulated and observed streamflows) via the method of least squares; this estimate is expressed as

$$\Delta P' = (S^T S)^{-1} S^T \Delta Q \quad (1)$$

Accordingly, the updated time series estimate P' of mean areal rainfall can be expressed as

$$P' = P_O + \Delta P' \quad (2)$$

where P_O is the original rainfall time series. In equation (1), S is the system response matrix defined as

$$S = \begin{bmatrix} \frac{\partial Q_1(P)}{\partial p_1} & \dots & \frac{\partial Q_1(P)}{\partial p_M} \\ \frac{\partial Q_2(P)}{\partial p_1} & \dots & \frac{\partial Q_2(P)}{\partial p_M} \\ \vdots & & \vdots \\ \frac{\partial Q_L(P)}{\partial p_1} & \dots & \frac{\partial Q_L(P)}{\partial p_M} \end{bmatrix} \quad (3)$$

where $p_1 \dots p_M$ are initial rainfall values, and the indices M and L represent the time series lengths of rainfall and streamflow, respectively. Once the updated time series estimate P' of mean areal rainfall is obtained, it can be used to improve the model simulations by rerunning the hydrological model using the updated rainfall estimate P' (instead of the original estimate P_O) to obtain the updated model-simulated estimate Q_U :

$$Q_U = Q(P') \quad (4)$$

Figure 1 presents a flow chart of DSRC updating method; for more details and the derivation of equation (1) please see Si et al. (2015).

2.1. Improving Stability of the DRSC Method

2.1.1. Causes for Instability in the DSRC Method

Let ΔP_O represent the true time series of rainfall error. Accordingly, the Euclidean distance D between the estimated rainfall error series $\Delta P'$ and the true values ΔP_O is expressed as

$$D^2 = (\Delta P' - \Delta P_O)^T (\Delta P' - \Delta P_O) \quad (5)$$

By taking the expected value of D we obtain:

Figure 1. Schematic diagram of the DSRC updating method. Q_O , Q_C , and Q_U represent the observed, calculated, and updated streamflow, respectively. E_Q is error series between simulated and observed streamflow. P_O , $\Delta P'$, and P' represent the original rainfall time series, the estimated rainfall error series, and the updated rainfall series, respectively. DSRC = dynamic system response curve.

$$E(D^2) = E[(\Delta P' - \Delta P_O)^T (\Delta P' - \Delta P_O)] \quad (6)$$

$$= \sigma^2 \text{tr}(S^T S)^{-1}$$

where $\sigma^2 = E[\Delta P_O - E(\Delta P_O)]^2$ is variance of the real rainfall error time series ΔP_O , and $\text{tr}(S^T S)^{-1}$ is the trace of matrix $(S^T S)^{-1}$.

From equation (6), we see that the expected Euclidean distance between the two time series is positively correlated with the trace of matrix $(S^T S)^{-1}$, meaning that the accuracy and stability of the rainfall error time series estimates obtained via the least square method depends on the properties of the trace of matrix $(S^T S)^{-1}$. Accordingly, under certain conditions, small perturbations to the right-hand side of equation (1) can cause instability of the solution, resulting in poor performance.

It has been pointed out by Tikhonov (Tikhonov et al., 2013) that instability of the least squares solution (equation (1)) occurs when the problem is ill posed, and when a problem is ill posed this can significantly affect the stability of the solution regardless of the numerical method employed (Delves & Mohamed, 1988; Hansen, 1998). Due to the positive correlation between the Euclidean distance of the two series and the trace of the matrix $(S^T S)^{-1}$, ill conditioning of $S^T S$ can result in poor accuracy of least squares estimation method.

2.1.2. Regularization via Ridge Estimation

There are several approaches to address ill-posed problems, such as the Galerkin method (Plato & Vainikko, 1990), dynamical systems method (Solodky, 2005), improved dynamical systems method (Luo & Li, 2010), and fast multiscale methods (Luo et al., 2011). A well-known way to address such problems is to use the form of regularization known as *ridge estimation* (Hoerl & Kennard, 1970), also known as ridge regression. In the machine learning literature, the method is known as weight decay. Due to multiple independent discoveries, the approach is also variously known as the Tikhonov-Miller method, the Phillips-Twomey method, the constrained linear inversion method, and the method of linear regularization (Groß, 2003). Based on the original mathematics of least squares estimation, the theoretical basis for ridge estimation involves adding a small positive value (called the ridge coefficient) to the diagonal of the coefficient matrix, so as to reduce the degree of correlation between the column vectors that make up the matrix. Applying this approach to the DSRC method, we obtain

$$\Delta P'_{\text{RE}} = (S^T S + \beta I)^{-1} S^T \Delta Q \quad (8)$$

$$W = (S^T S + \beta I)^{-1} \quad (9)$$

$$\Delta P'_{\text{RE}} = W S^T \Delta Q \quad (10)$$

where $\Delta P'_{RE}$ represents the estimated rainfall error series obtained via ridge estimation, the ridge coefficient β is a small positive number, and I is the identity matrix. It can be shown (see Appendix A) that:

$$\Delta P'_{RE} = Z \Delta P'_{LS} \quad (11)$$

where $\Delta P'_{LS}$ is the original least squares solution, and $Z = 1 - \beta(S^T S + \beta I)^{-1}$, which means that ridge estimation occurs by performing a linear transformation on the least squares estimates. Since Z is expressed in terms of the original normal equation coefficient matrix $S^T S$, it is apparent that it is a positive definite symmetric matrix, whose eigenvalues can be obtained via orthogonal mapping:

$$\lambda_i(Z) = \lambda_i / (\lambda_i + \beta), i = 1 \text{ to } M; \quad (12)$$

So, when $\beta > 0$, we get $\lambda_i(Z) < 1$.

Using equation (11), we can express the Euclidean distance of the ridge-estimated rainfall time series $\Delta P'_{RE}$ as

$$\Delta P'_{RE} T \Delta P'_{RE} = (Z \Delta P'_{LS})^T (Z \Delta P'_{LS}) \quad (13)$$

and substituting for Z we obtain:

$$\sum_{i=1}^M (\Delta P'_{RE})^2 = \sum_{i=1}^M \left(\frac{\lambda_i}{\lambda_i + \beta} \right)^2 (\Delta P'_{LS})^2 \quad (14)$$

Now, since $\frac{\lambda_i}{\lambda_i + \beta} < 1$, we obtain the following result:

$$\Delta P'_{RE} T \Delta P'_{RE} < (\Delta P'_{LS})^T (\Delta P'_{LS}) \quad (15)$$

Solving for the expected value of the Euclidean distance D between the ridge-estimated rainfall error series and the true values (similar to equations (5) and (6)), we obtain:

$$\begin{aligned} E(D_{RE}^2) &= E[(\Delta P'_{RE} - \Delta P_O)^T (\Delta P'_{RE} - \Delta P_O)] \\ &= \sigma^2 \text{tr}(S^T S + \beta I)^{-1} \end{aligned} \quad (16)$$

and comparing equations (6) and (16) we see that $\text{tr}(S^T S + \beta I)^{-1} < \text{tr}(S^T S)^{-1}$, which leads to the inequality

$$E(D_{RE}^2) < E(D_{LS}^2) \quad (17)$$

From this analysis we see that (i) rainfall error ridge estimation provides a compressed estimate based on the least squares estimate of rainfall error, and (ii) the mean-squared error of the ridge-estimated rainfall error is expected to be *smaller* than the value obtained using the original least squares approach. Theoretically, therefore, use of ridge estimation with DSRC can provide superior performance compared to the original DSRC method.

Note that the theoretical basis for the DSRC method is a quantifiable relationship between the errors in the model driver (independent) variables and the errors in the model output results. In our case (catchment systems), changes ΔQ in streamflow can be associated with changes ΔP in areal mean rainfall, expressed theoretically via the DSRC method as

$$\Delta Q = S \cdot \Delta P + E \quad (18)$$

where E represents the error (residual) associated with this relationship. Taking the average of both sides of this equation we obtain:

$$\overline{\Delta Q} = \overline{S} \cdot \Delta P + \overline{E} \quad (19)$$

and subtracting equation (19) from equation (18) we have

$$\Delta Q - \overline{\Delta Q} = (S - \overline{S}) \cdot \Delta P + E - \overline{E} \quad (20)$$

Figure 2. Schematic diagram of L curve.

Solving this equation via least squares estimation, we obtain the following estimate for the time series of rainfall errors:

$$\Delta P' = \frac{(S - \bar{S})^T (\Delta Q - \overline{\Delta Q})}{(S - \bar{S})^T (S - \bar{S})} \quad (21)$$

based on which we can show that

$$\Delta P' = r_{S, \Delta Q} \cdot \frac{\sqrt{(\Delta Q - \overline{\Delta Q})^T (\Delta Q - \overline{\Delta Q})}}{(S - \bar{S})^T (S - \bar{S})} = r_{S, \Delta Q} \cdot \frac{\sigma_{\Delta Q}}{\sigma_S} \quad (22)$$

where $\sigma_{\Delta Q}$ and σ_S are standard deviations of streamflow simulation error and rainfall error, respectively, and $r_{S, \Delta Q}$ is the correlation coefficient between S and ΔQ .

From equation (22), we see that the DSRC method uses the information contained in the correlation coefficient $r_{S, \Delta Q}$, which is proportional to the size of the error in the rainfall series. Accordingly, the larger the correlation coefficient, the more effective is the DSRC method at estimating the areal mean rainfall error.

2.1.3. Selection of the Ridge Coefficient

Of course, an important issue is the appropriate selection of the ridge coefficient β . While commonly used methods include Generalized Cross Validation (GCV; Golub et al., 1979) and the L Curve Criterion (Hansen & O'Leary, 1993), no theoretically correct method exists. Reports in the literature, based on numerical experiments, indicate that the L Curve Criterion method provides more stable performance than the GCV method (Hansen & O'Leary, 1993).

Accordingly, in this study, we use the L Curve Criterion method for selecting the ridge coefficient. We show in Appendix B, that this method is consistent with the theoretical basis for the DSRC method, provides unbiased least squares estimation, and that a suitable ridge coefficient β is one that does not significantly increase the deviation of P_{RE} from P_{LS} , while reducing the variance of the estimated rainfall error.

The L curve method proceeds as follows. Figure 2 shows a plot showing trade-offs between the residual size, expressed by the norm $\|S\Delta P'_{RE} - \Delta Q\|$ and the solution size, expressed by the norm $\|\Delta P'_{RE}\|$, for all valid values of ridge coefficient β . The underlying idea is to find values for β that achieve a suitable balance between minimizing these objectives (Hansen, 1992; Hansen & O'Leary, 1993).

The L curve is typically composed of two regions: a vertical (upper) part and a horizontal (right) part. From Figure 2 we can see that the solution is sensitive to perturbations of the right-hand side in equation (1) when the ridge coefficient value is small, which creates the upper part of the L curve. However, the solutions are dominated by estimation errors when the ridge coefficient value is relatively large, which creates the right part of L curve (Engl & Ramlau, 2015; Hansen, 1998). Therefore, the appropriate value for the ridge coefficient value is one that lies in the corner region (the maximum curvature region) of the curve. For further details, please see Hansen (Hansen, 1992; Hansen & O'Leary, 1993).

2.2. Rainfall Error Estimation Considering Spatial Distribution

The spatial distribution of rainfall estimation errors across a basin is generally not uniform. Accordingly, the use of a lumped strategy wherein we update the areal mean rainfall error of the entire basin, can be inappropriate (Moulin et al., 2009), and lead to not realizing the full potential of the DSRC method. To reflect the spatial distribution of rainfall (and hence rainfall estimation errors) across the basin in a relatively simple intuitive manner, we can proceed as follows:

A commonly used indicator of the heterogeneity of a statistical data series is the Gini coefficient, commonly used to measure the income gap of residents in a country or region (Anderson et al., 2017; Chen et al., 2018; Ward & Viner, 2017; Witlox, 2017). The indicator has also been used in

Figure 3. Boxplots showing the distributions of K_p values in the three study basins.

hydrometeorological research to measure the heterogeneity of observations (Konapala et al., 2017; Rajah et al., 2015). However, the Gini coefficient is only applicable when there is a large number of samples (Shi et al., 2013), whereas there is often only a relatively small number of rain gages in any given catchment (e.g., there are only six rain gages in WXG basin; see section 3.3.1).

In view of this, we propose here a simple index to reflect the spatial distribution of rainfall heterogeneity among stations in a basin. For real-time flood forecasting and management, the focus is on the *cumulative* rainfall during a flood event, as opposed to daily and hourly rainfall, based on which we define the index K_P of rainfall spatial distribution heterogeneity ($0 < K_P \leq 1$):

$$K_P = \frac{P_{\max} - P_{\min}}{P_{\max}} \quad (23)$$

$$P_{\max} = \max \left\{ \sum_{i=1}^T P_1(i), \dots, \sum_{i=1}^T P_k(i), \dots, \sum_{i=1}^T P_N(i) \right\}, i = 1 \text{ to } T; k = 1 \text{ to } N; \quad (24)$$

$$P_{\min} = \min \left\{ \sum_{i=1}^T P_1(i), \dots, \sum_{i=1}^T P_k(i), \dots, \sum_{i=1}^T P_N(i) \right\}, i = 1 \text{ to } T; k = 1 \text{ to } N; \quad (25)$$

where $P_{\max}$ is the maximum value of cumulative rainfall across all rain gages, $P_{\min}$ is the corresponding minimum value, N is the number of rain gages in the basin, and T is the duration of the flood event. Smaller values of K_P correspond to lower spatial distribution heterogeneity while larger values correspond to larger cumulative rainfall differences between rain gages (and hence the corresponding subregions).

For example, Figure 3 shows boxplots of K_P values obtained for three basins used later in section 3.3.1, these three basins are Qilijie (QLJ) basin, Jianyang (JY) basin, and Wuxigou (WXG) basin, respectively. Details of the calculations appear in Table S1 of the supporting information. We see that the index values (maximum, average, and minimum) increase with size of the basin (larger for QLJ than for JY, which has larger values than WXG). This result supports our intuition that spatial distribution of rainfall tends to be more heterogeneous in larger basins than in smaller basins.

Accordingly, it seems desirable to employ a spatially distributed strategy for updating rainfall estimates, especially when dealing with larger-scale basins having lower rain gage densities. However, a problem arises because there will typically be multiple rainfall error time series to be estimated, and we only have error information provided by only a single streamflow time series, resulting in a dimensional asymmetry with many more entities (rainfall values) to be estimated than there are corresponding informational entities (streamflow values).

To deal with this problem we will apply the simple regularizing assumption that the streamflow errors ΔQ can be allocated to each rainfall gage (subbasin) according to a weighting scheme, based on the fact that the original hydrological model (here XAJ, see section 3.1) computes simulated streamflow values $Q_i(t)$ separately for each (i th) subbasin before combining them to simulate the total streamflow $Q_C(t) = \sum_{i=1}^N Q_i(t)$ at the basin outlet (where N is the number of subbasins). Accordingly, we allocate the time series of weights $w_i(t) = \frac{Q_i(t)}{Q_C(t)}$ to each subbasin, so that the streamflow errors $\Delta Q(t)$ are allocated to each subbasin as $\Delta Q_i(t) = w_i(t) \cdot \Delta Q(t)$ at each time step.

Based on this assumption, the total flow error at the outlet can be partitioned among the contributing subbasins, and the corresponding mean areal rainfall error for each subbasin can be computed as

$$\begin{cases} \Delta P'_{RE,1} = (S_1^T S_1 + \beta_1 I)^{-1} S_1^T \Delta Q_1 \\ \vdots \\ \Delta P'_{RE,k} = (S_k^T S_k + \beta_k I)^{-1} S_k^T \Delta Q_k \\ \vdots \\ \Delta P'_{RE,N} = (S_N^T S_N + \beta_N I)^{-1} S_N^T \Delta Q_N \end{cases} \quad (26)$$

A flowchart illustrating the improved DSRC method, consisting of ridge-estimation (called DSRC-R) and spatially distributed rainfall updating (called DSRC-D) is shown in Figure 4, the subflowchart in the

Figure 4. Flowchart of DSRC-RD method. The rounded rectangles in the flowchart represent methods/models, and the gray rectangles and parallelograms represent important intermediate variables or final results. All letters and symbols correspond with the manuscript text. DSRC = dynamic system response curve.

dashed blue box indicates the process for calculating the S matrix, the subflowchart in the dashed red box indicates the strategy for streamflow error allocation, and the subflowchart in the dashed dark blue box indicates the DSRC-R method updating process.

3. Case Study Description

3.1. Model Description

The XAJ (Xinanjia) model adopted in this study is a conceptual hydrological model developed by Zhao (1992). It is used successfully and widely throughout China, especially in the southeastern regions. The most important input to the model is areal mean rainfall. Applications of the model have included flood forecasting, hydrologic station network design, and water availability estimation (Fang et al., 2017; Guo et al., 2016; Ju et al., 2009; Yang et al., 2012; Yao et al., 2012; Zhang, Liu, Cheng, et al., 2018; Zhang, Liu, Zhao, et al.,

Figure 5. Basin conceptualization used for the synthetic case studies.

2018). More details about the physical meaning and ranges of values for each parameter, and a parameter sensitivity analysis, appear in Zhao (1992), Song et al. (2013), and Bao et al. (2013). For the results reported here, all of the basins belong to humid regions located in southeastern China (section 3.3.1).

3.2. Synthetic Data Study

Si et al. (2015) provided a performance comparison of the original DSRC updating method with other error correction methods. Consequently, we focus here on comparing the improved versions (DSRC-R, DSRC-D, and DSRC-RD) of the DSRC method with the original version.

Performance was evaluated using several measures, including the Nash-Sutcliffe Efficiency (NSE) coefficient (Nash & Sutcliffe, 1970; Schaefli & Gupta, 2007; Yapo et al., 1996), and more comprehensively, the Kling-Gupta efficiency (KGE; Gupta et al., 2009) and its components: the correlation coefficient r , standard deviation ratio α , and mean ratio β , for which mathematical definitions are provided in Appendix C. Ideally, when there are no simulation errors, we would have $NSE = 1$ and $KGE = 1$, these being achievable only when $r = 1$, $\alpha = 1$, and $\beta = 1$ (Chen et al., 2015; Gupta et al., 2009).

In addition, to assess improvements to the NSE value resulting from the use of the DSRC method, the following normalized metric (percentage Improvement of NSE, INS) was also computed as proposed by Si et al. (2015):

$$INS = |NSE_{au} - NSE_{bu}| / NSE_{bu} \times 100\% \quad (27)$$

where NSE_{bu} indicates the NSE before updating, and NSE_{au} means the NSE after updating.

To verify that rainfall error information can be effectively (and with more stability) extracted from the streamflow series by the improved method, we designed a “typical” synthetic basin study. Different time series of rainfall errors were assigned to a given rainfall data set to simulate different scenarios. In evaluating performance, we assessed not just the ability to infer the rainfall error time series but also the accuracy of streamflow forecast obtained after rainfall updating.

Two different synthetic case studies were developed. In the first (Synthetic Case 1), the spatial heterogeneity K_p for the assigned rainfall and rainfall error series are assigned to be relatively small. In the second (Synthetic Case 2), the spatial heterogeneity K_p for the assigned rainfall and rainfall error series is assigned to be much larger than in Synthetic Case 1. We examined two scenarios for each case, a spatially lumped scenario and a spatially distributed scenario. In both cases, the inputs, outputs, parameters, initial states, and nature of the rainfall errors are all specified by us, and therefore known. We set the basin to have an area of $2,000 \text{ km}^2$ and five rainfall gages (and hence subbasins) as shown in Figure 5, and with the stream network consisting of three subreaches. We used the XAJ model to simulate the input-state-output response of this system, using the parameter values listed in Table S2 of the supporting information, and the Muskingum method to route flows through the stream network to the basin outlet.

The XAJ model was run with the “true” (uncorrupted) rainfall values P to obtain the “true” streamflow time series Q , and then with “corrupted” rainfall values $P + \Delta P$ to generate the “observed” streamflow time series Q_o by application of the following equation:

$$Q_o = Q(P + \Delta P) + \varepsilon \quad (28)$$

where ε is a zero-mean Gaussian error time-series whose magnitudes are limited to $\pm 5\%$ of the corresponding streamflow discharge $Q(P + \Delta P)$.

To quantify the difference between the “assigned” value of the rainfall error time series ΔP and its value $\Delta P'$ estimated by updating, we used the corresponding mean square error (MSE) and correlation coefficient r , defined as

$$MSE = \frac{1}{N} \sum_{i=1}^N (\Delta P_i - \Delta P'_i)^2 \quad (29)$$

$$r = \frac{E[(\Delta P - \overline{\Delta P})(\Delta P' - \overline{\Delta P'})]}{\sqrt{E[\Delta P - \overline{\Delta P}]^2} \sqrt{E[\Delta P' - \overline{\Delta P'}]^2}} \quad (30)$$

3.2.1. Synthetic Case 1

In general, since rainfall intensities vary with time, the size of the rainfall error will also vary in time. To ensure realism, we selected rainfall data corresponding to the 13 June 1998 flood event in the JY basin, and used data from 5 of the 16 available rain gages. For the spatially lumped scenario, we assigned the areal mean of the five rainfall time series to the entire basin and then corrupted it with a single rainfall error time series. For the spatially distributed scenario, each of the five subbasins was assigned its corresponding gage-based rainfall time series, and a different time series of rainfall errors was allocated to each gage (subbasin). The properties of the rainfall data and rainfall error series used for these studies are listed in Table S3 of the supporting information. In each case, the rainfall errors were randomly generated from zero-mean Gaussian distributions with the standard deviation selected such that the magnitude of each error value was limited to 20% of the corresponding rainfall value. In this synthetic case, spatial heterogeneities K_p for the assigned rainfall and rainfall error series are assigned to be quite small (0.09 and 0.22, respectively), which represents a relatively uniform spatial distribution of rainfall and rainfall error series.

3.2.2. Synthetic Case 2

To further evaluate the updating methods, we ran a second synthetic case in which the spatial heterogeneity of rainfall and rainfall error series was assigned to be much larger. This time we used rainfall data corresponding to the 16 July 1998 flood event in the JY basin. The method for allocating rainfall error is the same as that for Synthetic Case 1. For the spatially distributed scenario we used data from 5 of the 16 available rain gages, each of the five subbasins was assigned its corresponding gage-based rainfall time series, and a different time series of rainfall errors was allocated to each gage (subbasin). For the spatially lumped scenario, we assigned the areal mean of the five rainfall time series to the entire basin, and then corrupted it with a single rainfall error time series. The properties of the rainfall data and rainfall error series used for this case are listed in Table S4 in the supporting information. The spatial heterogeneities K_p for the assigned rainfall and rainfall error series are assigned to be much larger (0.797 and 0.835, respectively) than those in the Synthetic Case 1, thereby representing a much more heterogeneous spatial distribution of rainfall and rainfall error series.

3.3. Real Data Study

For the real data study, we selected three basins having different spatial scales (Figure 6). The hydrometeorological conditions are similar for these basins and therefore not the dominant factor influencing differences in spatial rainfall characteristics. This similarity in hydrometeorological characteristics helps to facilitate analysis and interpretation of the results.

3.3.1. Catchments

The QLJ basin, JY basin, and WXG basin represent a diversity of basin characteristics. Figure 6 shows where the basins are located, their topological geometry, and locations of rain gages, while their hydrometeorological characteristics are listed in Table 1.

The 14,787 km² QLJ basin (Figure 6a), located in Fujian Province in southeastern China has a dominantly subtropical monsoon climate, with rainfall and flooding events occurring mainly in the summer. The average annual temperature of the basin is 17 °C. Based on the continuous daily data record from 1988 to 1999, the average annual rainfall and pan evapotranspiration are 1,824 and 815 mm, respectively, while the average annual discharge is 671 m³/s, with a maximum discharge of 16,800 m³/s (19 June 1998). The land surface is mainly covered by shrubs, and human modifications (water conservancy projects and human activities) are minimal. There are 30 rainfall gages, three evapotranspiration gages, and one hydrologic streamflow gauging station located in the basin, such that the average subbasin area associated with each rain gage is ~493 km².

The 4,848 km² JY basin (Figure 6b) is located in the upstream reaches of the Min River in the Fujian province of southeastern China. Rainfall and flooding events occur mainly in summer. The average annual

Figure 6. The locations, topographic geometry, rain gauge distributions, and boundaries of the corresponding Thiessen Polygons for the three basins: (a) QLJ basin, (b) JY basin, and (c) WXG basin.

temperature of the basin is 18 °C and the average annual rainfall is about 1,700–2,400 mm (based on the continuous rainfall series from 1988 to 1999, the average annual rainfall is 1,993 mm), and the average annual pan evapotranspiration is 827 mm. The land surface has a high density of vegetation coverage and is not majorly affected by human activities such as water conservancy projects. The average annual discharge is 187 m³/s, with the maximum recorded discharge being 8,380 m³/s (22 June 1998). The basin has 16 rainfall gages, one evapotranspiration gage, and one hydrologic streamflow recording station (at the basin outlet), and the average subbasin area associated with each rain gage is ~285 km².

The 954-km² WXG basin (Figure 6c), located in the upstream reaches of the Songzi River in Hubei province in south central China, is characterized by a temperate continental monsoon climate type, with an average annual temperature of 14 °C. The terrain is mainly hilly with a high density of vegetation coverage. The average annual precipitation is 1,450 mm (based on the continuous rainfall series from 1998 to 2011), and the average annual pan evapotranspiration is 852 mm. The average annual discharge is 78 m³/s, with the maximum recorded discharge being 1,910 m³/s (6 July 1998). There are six rainfall gages, one evapotranspiration gage, and one hydrologic streamflow recording station in the basin, such that the average subbasin area of each rain gage is ~159 km².

3.3.2. Data

Two time scales of data, daily and hourly, were used in this study. The data include evapotranspiration, streamflow, and rainfall. Daily data were used to calibrate and evaluate the parameters of the evapotranspiration and runoff calculation modules of the XAJ hydrological model, while hourly rainfall and streamflow data were used to calibrate and evaluate the parameters of the flow concentration module.

To test the performance of the updating method for flood forecasting we used hourly (flood event) data. At the hourly time scale there are no evapotranspiration data due to equipment limitations, and so we used an

Table 1
Property Information and Data Series Available for the Three Basins

Basin name	Basin area (km ²)	Rain gauge number	Effective rain gauge area (km ² /Gauge)	Average annual rainfall (mm)	Average annual evapotranspiration (mm) and gauge number	Average annual temperature (°C)	Hydrologic station number	Average annual discharge (m ³ /s)	Maximum discharge (m ³ /s)
Basin property									
QLJ	14,787	30	493	1,824	815 and 3	17	1	671	16,800
JY	4,848	16	303	1,993	827 and 1	18	1	187	8,380
WXG	954	6	159	1,450	852 and 1	14	1	78	1,910
Basin name	Daily data (year)	Flood data (year)	Average flood event rainfall (mm)	Flood event number	Average flood runoff depth (mm)	Daily data for calibration and evaluation		Hourly data for calibration and evaluation	
						Calibration	Evaluation	Calibration	Evaluation
Data series									
QLJ	1988–1999	1988–2001	137.4	57	91.6	1988–1994	1995–1999	1988–1996 (37 flood events)	1997–2001 (20 flood events)
JY	1988–1999	1985–1999	165	48	119.8	1988–1994	1995–1999	1985–1993 (30 flood events)	1994–1999 (18 flood events)
WXG	1998–2011	1974–2011	134.4	50	97.5	1998–2006	2007–2011	1993–2005 (30 flood events)	2006–2011 (20 flood events)

hourly average computed from the daily evapotranspiration value for the day of the flood event. Since the daily data are a continuous time series while the hourly data exist only for the durations of the flood events, the lengths of the two kinds of data sets are different. Table 1 lists some information regarding these data series for the three basins.

3.3.3. Parameters for the XAJ Model

For each basin, the XAJ model parameters were calibrated and evaluated using both historical daily data (streamflow, rainfall, and pan evapotranspiration) and hourly data (streamflow and rainfall) during the flood events, as explained above. Both data sets (daily and hourly) were partitioned into calibration and evaluation sets (see Table 1). First, the daily data was used to calibrate and evaluate the parameters associated with evapotranspiration and runoff generation (K, WUM, WLM, and C). Then, the hourly flood event data were used to calibrate and evaluate the remaining parameters.

The Nash-Sutcliffe efficiency (NSE), Kling-Gupta efficiency (KGE) and its components, the relative error of peak flow (RPF), and the relative error of runoff depth (RRD) were used as objective functions during calibration and evaluation (Si et al., 2015). The RPF and RRD were used to calibrate and evaluate the parameters associated with evapotranspiration and runoff by using daily data. NSE and KGE were used to calibrate and evaluate the parameters associated with flow concentration (slope and river channel) by using hourly data during flood event. The calibrated parameter values are listed in Table S5 in the supporting information.

4. Results and Discussion

4.1. Synthetic Data Study

4.1.1. Synthetic Case 1

4.1.1.1. Results for the Lumped Scenario

Results for the spatially lumped scenario are shown in Table S6 of the supporting information for the original method (DSRC) and the improved ridge estimation method (DSRC-R). The correspondence between the “true” (assigned) and estimated rainfall error time series is shown using scatterplots in Figures 7a and 7b, the scatterplots for streamflow before and after updating are shown in Figures 7c and 7d.

From these results, we can see that whereas the total assigned rainfall error was 15.2 mm, the total estimated rainfall errors are 16.3 mm by the DSRC method (1.1-mm difference) and 14.8 mm by the DSRC-R method (0.4-mm difference). The corresponding MSE values are 0.026 (DSRC) and 0.009 (DSRC-R), while the corresponding correlation coefficients are 0.927 (DSRC) and 0.972 (DSRC-R).

Figure 7. Correspondence of rainfall error series and streamflow series for lumped scenario in synthetic case 1: (a) and (b) Comparison of the assigned with estimated rainfall error series for DSRC and DSRC-R methods, respectively; (c) and (d) comparison of the streamflow before and after updating for the DSRC and DSRC-R methods, respectively. DSRC = dynamic system response curve.

Figures 7c and 7d show that the streamflow after DSRC-R updating is closer to the 1:1 line than when using the DSRC method. The corresponding streamflow hydrographs are shown in Figure 8a, while more details are provided in Table S7 in the supporting information. Streamflow NSE values are 0.958, 0.994, and 0.999 for the original (nonupdated) XAJ model, DSRC-updated model, and the DSRC-R-updated model, respectively, with the INS statistic indicating a 3.7% improvement using DSRC and a 4.3% improvement using DSRC-R.

Clearly, the DSRC-R method provides better inversion and extraction of rainfall error information from the streamflow difference series and is more stable than the original DSRC method for this spatially lumped scenario in Synthetic Case 1.

4.1.1.2. Results for the Distributed Scenario

Results for the spatially distributed scenario are shown in Table S8 for the improved spatially distributed estimation method (DSRC-D) and the combined spatially distributed estimation method plus ridge estimation (DSRC-RD). The correspondence between the “true” (assigned) and estimated rainfall error time series is shown using scatterplots in Figures D1a and D1b (Appendix D); here we show all five sets of assigned and estimated rainfall errors together (i.e., for all five rain gages/subregions). To compare performance with regard to rainfall error extraction before and after using spatial distributed updating, we plot the results of the two different methods on the same scatterplot. The corresponding scatterplots for streamflow are shown in Figures D1c and D1d (Appendix D).

From Table 2, we see that the average MSE values obtained for rainfall error time series are 0.026 (DSRC), 0.009 (DSRC-R), 0.02 (DSRC-D), and 0.01 (DSRC-RD), while the corresponding integrated correlation

Figure 8. Comparison of the hydrographs obtained using different updating methods in the synthetic data study: (a) Lumped scenario in synthetic case 1. (b) Distributed scenario in synthetic case 1. (c) Lumped scenario in synthetic case 2. (d) Distributed scenario in synthetic case 2. Q_O and Q_C represent the “observed” and calculated streamflows, respectively.

coefficients are 0.927 (DSRC), 0.972 (DSRC-R), 0.945 (DSRC-D), and 0.975 (DSRC-RD). Comparing the two methods where spatial distribution of the rainfall errors is considered, Figure D1 shows that the results of DSRC-RD method fall slightly closer to 1:1 line than that of DSRC-D method, so while both methods are able to effectively infer the values of rainfall error associated with each subregion, the use of ridge estimation provides slightly better results.

Table 2
Performance Metrics for the Different Methods Tested in Synthetic Cases

Statistical variables	Indicators	DSRC	DSRC-R	DSRC-D	DSRC-RD
Synthetic case 1					
Rainfall error series	MSE	0.026	0.009	0.020	0.010
	r	0.927	0.972	0.945	0.975
Streamflow series	NSE	0.994	0.999	0.996	0.999
	INS	3.7	4.3	4.6	5.0
	KGE	0.958	0.978	0.968	0.982
	r	0.994	1.000	1.000	1.000
	α	0.992	0.990	1.021	1.012
	β	1.041	0.981	1.024	1.013
Synthetic Case 2					
Rainfall error series	MSE	0.086	0.063	0.025	0.020
	r	0.907	0.921	0.968	0.980
Streamflow series	NSE	0.975	0.980	0.998	0.999
	INS	2.8	3.4	5.3	5.4
	KGE	0.872	0.886	0.967	0.977
	r	0.964	0.971	0.998	1.000
	α	0.911	0.925	0.980	0.985
	β	0.915	0.920	0.975	0.983

The corresponding streamflow hydrographs shown in Figures 8b, indicates that the updated streamflow obtained using DSRC-RD is closer to the “observed” streamflow than that obtained using DSRC-D (more details are provided in Table S7 in the supporting information). Streamflow performance NSE values are 0.952 for the original (nonupdated) XAJ model, compared with 0.994 when using the original DSRC method and 0.999 when using DSRC-RD updating. Overall, the INS statistic indicates a 3.7% improvement using DSRC and a 5.0% improvement using DSRC-RD. Clearly, the DSRC-RD method provides the best results.

In summary, these results show that while all of the updating methods help to improve the streamflow forecasts, the best results are obtained using a combination of ridge estimation and accounting for the spatially distributed nature of the rainfall errors. However, consideration of the spatial heterogeneity of rainfall errors resulted in only relatively small benefits (compare DSCR-R and DSRC-RD). This is attributable to the fact that the values of spatial heterogeneity K_p for the assigned rainfall and rainfall error series are quite small (0.09 and 0.22, respectively), resulting in only minor differences.

4.1.2. Synthetic Case 2

4.1.2.1. Results for the Lumped Scenario

Results for the spatially lumped scenario are shown in Table S9 in the supporting information for the original method (DSRC) and the improved ridge estimation method (DSRC-R). The correspondence between the “true” (assigned) and estimated rainfall error time series is shown using scatterplots in Figures D2a and D2b (Appendix D).

From these results, we can see that whereas the total assigned rainfall error was 17.8 mm, the total rainfall error estimated by the DSRC method is 16.2 mm (1.6-mm difference) and by the DSRC-R method is 17.1 mm (0.7-mm difference). The corresponding MSE values are 0.086 (DSRC) and 0.063 (DSRC-R), while the corresponding correlation coefficients are 0.907 (DSRC) and 0.921 (DSRC-R).

Scatterplots comparing streamflow before and after updating are shown in Figures D2c and D2d (Appendix D), and the corresponding streamflow hydrographs are shown in Figure 8c; more details are provided in Table S10 in the supporting information. Streamflow performance NSE values are 0.948, 0.975, and 0.980 for the original (nonupdated) XAJ model, DSRC-updated model, and the DSRC-R-updated model, respectively, with the INS statistic indicating a 2.8% improvement using DSRC and a 3.4% improvement using DSRC-R.

Overall, the DSRC-R method provides better inversion and extraction of rainfall error information from the streamflow difference series for this spatially lumped scenario in synthetic case 2. However, the performance improvement achieved by DSRC-R is not as large as for the spatially lumped scenario in Synthetic Case 1.

4.1.2.2. Results for the Distributed Scenario

Results for the spatially distributed scenario in Synthetic Case 2 are shown in Table S11 for the improved spatially distributed estimation method (DSRC-D), and the combined spatially distributed estimation method plus ridge estimation (DSRC-RD). The correspondence between the “true” (assigned) and estimated rainfall error time series is shown using scatterplots in Figures D3a and D3b (Appendix D); here we show all five sets of assigned and estimated rainfall errors together and plot the results of two different methods on the same scatterplot. The corresponding scatterplots for streamflow are shown in Figures D3c and D3d (Appendix D).

From results of Synthetic Case 2 list in Table 2, we see that average MSE values obtained for rainfall error time series are 0.086 (DSRC), 0.063 (DSRC-R), 0.025 (DSRC-D), and 0.02 (DSRC-RD), while the corresponding integrated correlation coefficients are 0.907 (DSRC), 0.921 (DSRC-R), 0.968 (DSRC-D), and 0.980 (DSRC-RD). Comparing the two methods where spatial distribution of the rainfall errors is considered, Figures D3a and D3c show that the results of DSRC-D method fall closer to the 1:1 line than that of DSRC method. Similarly, Figures D3b and D3d show that the results of DSRC-RD method fall closer to the 1:1 line than that of DSRC-R method. While both methods are able to effectively infer the values of rainfall error associated with each subregion, the spatial distributed method provides better results.

The corresponding streamflow hydrographs are shown in Figure 8d, indicating that the updated streamflow obtained using DSRC-RD is closer to the observed streamflow than that obtained using DSRC-D; more details are provided in Table S10 in the supporting information. Streamflow performance NSE values are 0.948 for the original (nonupdated) XAJ model, compared with 0.975 when using the original DSRC method, and 0.999 when using DSRC-RD-updating. Overall, the INS statistic indicates a 2.8% improvement using DSRC and a 5.4% improvement using DSRC-RD. Again, the DSRC-RD method provides the best results for this synthetic case having larger spatial heterogeneity associated with the rainfall and rainfall error series.

4.1.3. Analysis of the Results

Note that the degree of spatial heterogeneity K_p associated with the rainfall and rainfall error series was quite different in these two synthetic cases (Figure 9).

As shown in Figure 9, the DSRC-RD method consistently provides the best results in both synthetic cases. In Synthetic Case 1 (blue bars), when spatial heterogeneity is small, the results of updating using ridge estimation (DSRC-R and DSRC-RD) are better than that of the methods not using ridge estimation DSRC and DSRC-D. However, for Synthetic Case 2 (red bars), when spatial heterogeneity is large, Figure 9 shows that the results of using distributed updating (DSRC-D and DSRC-RD) are better than that of the methods not using distributed updating (DSRC and DSRC-R).

Figure 9. Comparison of performance metrics for the two synthetic cases: (a) Mean square error (MSE) values of the updated rainfall error series. (b) Correspondence between assigned and estimated rainfall error series. (c) Nash-Sutcliffe efficiency (NSE) values of streamflow. (d) Inertial Navigation System (INS) values of streamflow. DSRC = dynamic system response curve.

4.2. Real Data Study

4.2.1. Results for the Largest Basin—QLJ

Detailed results obtained by application of the different versions of DSRC, compared with the results when DSRC updating is not performed, are provided in Tables S12 and S13 in the supporting information. Here, in Figures 10 and 11, we provide a summary of the results for all 57 flood events examined.

As can be seen, all versions of the updating method result in significant performance improvements, ranging from a minimum of 2% to a maximum of 30%. Figures 10b and 10d show that the spatial distributed updating methods (DSRC-D and DSRC-RD) consistently provide significantly better performance than the other updating methods, with median improvements of ~18%, while the original DSRC method provides a median improvement of ~14%. The KGE and its components indicate similar results as the NSE and INS: all versions of the updating method result in significant performance improvements, with DSRC-RD performing best (as shown by KGE gain in Figure S1a in the supporting information). Figures 11a and 11b show that KGE and r of the spatially distributed methods (DSRC-D and DSRC-RD) shift to the right, and the values are distributed in narrower ranges that are closer to 1. Meanwhile, the cumulative distributions of α and β (Figures 11c and 11d) after updating are more steep (less dispersed), and closer to 1, especially for DSRC-D and DSRC-RD. In some cases, the gain from using DSRC-RD is relatively small (e.g., events 2, 10, 41), while in several cases the gain is quite considerable (e.g., events 12, 15, 24).

Figure 12a shows the cumulative distribution function for INS gain relative to the original-DSRC for the QLJ basin. The INS gain associated with the spatially distributed updating methods is quite significant (also see Table S12). The nonupdated model provides an average $NSE = 0.788$ across all 57 flood events, and the original DSRC method provides an average $NSE = 0.889$ (improved by an average $INS = 12.9\%$). The use of ridge estimation (DSRC-R) provides an average $NSE = 0.911$ (average $INS = 15.7\%$), indicating that including ridge estimation provides significant benefits over the original method. However, the use of the spatially distributed methods (DSRC-D and DSRC-RD) provides an average $NSE = 0.930$ (improved by an average $INS = 18.3\%$), indicating that spatial distributed updating methods perform better than DSRC and DSRC-

Figure 10. The NSE and INS values, and their corresponding cumulative distribution functions for all 57 flood events in the QLJ basin: (a) NSE values; (b) Cumulative distribution of NSE; (c) INS values; and (d) Cumulative distribution of INS. DSRC = dynamic system response curve; NSE = Nash-Sutcliffe efficiency; INS = Inertial Navigation System.

Figure 11. Cumulative distribution functions for (a) Kling-Gupta efficiency (KGE) and its components (b) r , (c) α , and (d) β in QLJ basin. DSRC = dynamic system response curve; CDF = cumulative distribution function.

Figure 12. Cumulative distribution functions (CDF) for INS gain. DSRC = dynamic system response curve; INS = Inertial Navigation System.

R, and advantage of spatial distributed updating is more obvious in basins with large rainfall spatial heterogeneity.

As can be seen, for this basin, the difference between the DSRC-D and DSRC-RD methods, both of which provide significant improvements over the DSRC-R updating method, is minimal. This implies that the treatment of rainfall heterogeneity adds significant value to the updating of forecasts in this large basin and that taking rainfall error heterogeneity into account seems to obviate the need for ridge estimation in this case. Nonetheless, there is apparently no downside to including both strategies in the updating methodology.

Overall, it is clear that the accuracy of real-time flood forecasting can be significantly improved by considering the spatial heterogeneity of rainfall and rainfall errors in this large basin.

4.2.2. Results for the Midsized Basin—JY

Again, detailed results are provided in the supporting information (Table S14). Figure 13 provides a summary of the results for all 48 flood events examined.

As with the larger basin, all of the updating methods provide significant improvements in NSE over the non-updated model (average NSE improves from 0.812 to between 0.903 and 0.939). While DSRC-RD consistently provides the best overall result, the difference between DSRC-R and DSRC-D is, in this case,

Figure 13. The NSE and INS values and their corresponding cumulative distribution functions for all 48 flood events in the JY basin: (a) NSE values; (b) cumulative distribution functions (CDF) for NSE; (c) INS values; and (d) cumulative distribution functions for INS. DSRC = dynamic system response curve; NSE = Nash-Sutcliffe efficiency; INS = Inertial Navigation System.

Figure 14. The NSE and INS values, and their corresponding cumulative distribution functions for all 50 flood events in the WXG basin: (a) NSE values; (b) cumulative distribution functions for NSE; (c) INS values; and (d) cumulative distribution functions for INS. DSRC = dynamic system response curve; NSE = Nash-Sutcliffe efficiency; INS = Inertial Navigation System.

minimal (as shown in Figures 13b and 13d), both being better than the original DSRC. Overall, the combined benefits of accounting for spatial rainfall error heterogeneity while also using ridge estimation are clearly apparent (Figure 12b). The results of KGE and its components are consistent with those of NSE and INS; see Figure S2 and Table S15 in the supporting information. As shown in Figure S1b (KGE gain), DSRC-R and DSRC-D both result in significant performance improvements, and DSRC-RD also performs best.

4.2.3. Results for the Smallest Basin—WXG

Finally, we examine the smallest basin, which has only six rain gages. Detailed results are provided in the supporting information (Table S16). Figure 14 provides a summary of the results for all 50 flood events examined.

As shown in Figure 14, updating significantly improves the forecasting performance of the model. Again, KGE and its components (as shown in Figure S3 and Table S17 in the supporting information) show similar results. However, consistent with the relatively small size of the basin, the benefits of accounting for spatial rainfall error heterogeneity are relatively small while the benefits of implementing ridge estimation are quite significant (Figures 14b and 14d). While DSRC-D does improve slightly over the original DSRC (which does not account for spatial heterogeneity), the difference between DSRC-R and DSRC-RD is relatively insignificant (Figures 12c and S1c). However, there is no apparent downside to simultaneously trying to account for both spatial rainfall error heterogeneity while incorporating ridge estimation.

4.2.4. Summary of Results

Overall, the results from application of DSRC-based updating to these three basins of different sizes indicate that incorporation of ridge estimation provides (in all cases) a significant improvement over the original DSRC, by improving the stability of the solution procedure (see theoretical explanation in section 2.1.2). Clearly, as the size of the basin increases, the benefits of accounting for the spatial heterogeneity of rainfall and rainfall errors become significant. For the largest basin, we found no difference between applying DSRC-D and DSRC-RD suggesting that simply accounting for the spatial heterogeneity helped to

Figure 15. Statistical results characterizing performance of the different versions of the DSRC methodology in three different basins: (a) Distribution of average NSE value; (b) distribution of average INS value; (c) NSE values of DSRC-R method; and (d) box-plot of NSE values by DSRC-R method. DSRC = dynamic system response curve; NSE = Nash-Sutcliffe efficiency; INS = Inertial Navigation System.

sufficiently stabilize the results that the additional benefits of using ridge estimation were insignificant. This interpretation is supported by the fact that use of DSRC-R does significantly improve over the original DSRC, indicating that the original DSRC experienced some stability problems. Given that simultaneous use of ridge estimation while accounting for spatial rainfall error heterogeneity seems to consistently provide improvements over the original DSRC (at least for the three cases tested here), and the fact that there are no theoretical or practical reasons to not include both factors, we feel confident in recommending the use of the DSRC-RD approach in all cases.

4.3. Discussion

4.3.1. Discussion of the Results

We have proposed two improvements to the DSRC updating method, and demonstrated their value using synthetic experiments and real-data studies. Figures 15a and 15b show a comparison of performance achieved by different versions of the DSRC methodology for the different real-data case studies in terms of NSE and INS. These results show that for larger-scale basins with greater spatial heterogeneity of rainfall (e.g., the QLJ basin), the DSRC-D approach that accounts for the spatial distribution of rainfall heterogeneity results in superior performance, while for smaller basins (e.g., the WXG basin) with more spatially homogeneous rainfall fields, the use of ridge estimation (DSRC-R) provides significant benefits. Overall, the use of the combined DSRC-RD method is robust in its ability to provide improvements.

Figures 15c and 15d show the event-based NSE performance for each basin when the DSRC-R (ridge estimation) method is used. The event-based performance is most consistent for the smallest WXG basin (blue triangles, NSE range 0.925 to 0.96), somewhat less consistent for the mid-sized JY basin (red squares, NSE range 0.88 to 0.97), and most dispersed for the larger QLJ basin (gray circles, NSE range 0.79 to 0.96). Clearly the stability of ridge estimation results improves with the decrease of basin scale. In smaller basins, the distributed information is not enough for characterizing rainfall errors, which can result in a less well posed problem. However, ridge estimation helps to make the performance more robust.

Figure 16. Trend correlation between the K_p values and the improvement (INS) of DSRC-D method: (a) for QLJ basin; (b) for JY basin; (c) for WXG basin. INS = Inertial Navigation System.

Figure 16 shows the event-based percentage improvement in performance (INS) for each basin, plotted against the index of spatial heterogeneity K_p , when the DSRC-D (spatial correction) method is used. We see that the correlation between K_p and INS is strongest ($r = 0.825$) in the largest QLJ basin, decreases ($r = 0.821$) in the mid-sized JY basin, and is weakest ($r = 0.735$) in the smallest WXG basin. Further, the slope of the line increases with basin scale (from WXG to QLJ), consistent with the increase in spatial heterogeneity of rainfall.

4.3.2. Theoretical Discussion

The transformation from precipitation to runoff is typically nonlinear. To *reverse estimate* rainfall errors by using streamflow errors, we have assumed local (under current conditions) linearity between the precipitation and simulation errors. It is therefore fair to ask how the linear assumption impacts the results of this study.

First, hydrological forecasting studies the response relationships among inputs, state variables and outputs, as mediated by the model structure and parameters. Theoretically, there is a quantifiable response relationship between the errors in the model independent variables and the errors in the computed model results. The theoretical basis for our approach is the *local* (i.e., at current conditions) sensitivity of the system response to perturbations in the rainfall time series data. The hydrological model is treated as a general system, which can be expressed mathematically by a system response function. The system response function is determined by the inputs, outputs, state variables, and parameters. In this system response function, rainfall series P (input) is considered as an independent variable, and streamflow series Q (output) is considered as the dependent variable, and a first-order Taylor series expansion is used to linearize the nonlinear system response function. A finite difference approach is used to approximate the system response to unit perturbations in rainfall, and an approximate estimate of the rainfall errors is computed from the streamflow errors via minimum least squares optimization; these rainfall error estimates are then used to update the rainfall time series data and propagated through the model to update the streamflow forecast. Such an approximation helps to reduce the effect on the updated results caused by deviations from the local (at current time step) linearity assumptions.

Second, the main purpose of our research is to improve the accuracy of real-time flood forecasting. In view of the extensive application of XAJ model in real-time flood forecasting, the hydrological model we used both in our previous paper (Si et al., 2015) and this manuscript is the conceptual XAJ hydrological model (Xinjiang) developed by Zhao (1992), which uses areal mean rainfall as its most important input. It has been applied extensively for real-time flood forecasting, runoff simulation, and water resources planning all over the world. The model has four computing modules (runoff generation, runoff separation, slope flow concentration, and river routing). Runoff separation is computed by linear outflow coefficients of free water storage reservoir, KI is the outflow coefficient of interflow, KG is the outflow coefficient of groundwater, and $1 - (KI + KG)$ is the outflow coefficient of surface flow. Slope flow concentration is computed by linear reservoir, CS is the recession constant of surface flow, CI is the recession constant of interflow, and CG is the recession constant of groundwater. River routing is performed using the Muskingum method to successive subreaches. KE and XE are parameters of the Muskingum method, which can be initially determined by hydraulic formulas. All the abovementioned three computing modules are linear structures. So, only the runoff generation module is nonlinear; the characteristic feature being the concept of runoff formation upon depletion of storage, which means that runoff is not produced until the soil moisture content of the aeration

zone reaches field capacity, and thereafter runoff equals the rainfall excess without further loss. That means the runoff generation computing module is also linear when the entire basin-wide soil moisture content of the aeration zone reaches field capacity. Therefore, local (at current time step) nonlinearity of XAJ model tends to be low.

In summary, this linear assumption between precipitation and simulation errors in this study does not seem to have a significant impact on the updating results. We are committed to improve the accuracy during flood forecasting, and certainly the error is difficult to completely eliminate. However, the ability of the method to improve the accuracy of real-time flood forecasts is demonstrated by using both synthetic study and real data cases.

Of course, the influence of this linearity assumption on the applicability of the updating method may be different when used in combination with different hydrological models. For example, the application effect may be more affected when there are more nonlinear structures in the hydrological model. We did not specifically quantify the influence of this linearity assumption in this study, and it would be a good avenue to investigate through future work.

For spatial allocation of error magnitudes, we use the ratio of simulated streamflow of each subbasin and total streamflow at each time step. This is the first time that we have proposed to allocate the streamflow errors of each subbasin in this way. A characteristic feature of the XAJ model is to compute the simulated streamflow series separately for each subbasin before combining them together, and then to simulate the total streamflow at the basin outlet by Muskingum river routing method. That means the total streamflow is combined by the simulated streamflow of each subbasin. Similarly, the total streamflow error is also combined by the simulated streamflow error of each subbasin. Therefore, we have enough reason to believe that the streamflow error of each subbasin is more-or-less proportional to its streamflow value. To verify the feasibility of this hypothesis, we carried out distributed rainfall error scenarios in both Synthetic Case 1 and Synthetic Case 2, respectively. The results show that this streamflow error allocation method is effective. The spatially distributed updating method is able to effectively infer the values of rainfall error associated with each subregion.

Overall, the results of the real-data study show that the spatial distributed method provides better results than that of the original updating method. As the basin scale increases, the stability of the method (DSRC-D) can be significantly improved by accounting for the spatial heterogeneity of rainfall and rainfall error during practical application. Therefore, although this method is not supported by a large number of mature research theories, we believe that it is indeed an effective streamflow error allocation method in this study, and it needs to be further examined in future work.

5. Conclusions

Flood forecasting using hydrological models plays a very important role in the reduction of flood-related losses worldwide. Therefore, real-time updating methods to improve the accuracy of flood forecasts using error correction are an operational priority. In previous work (Si et al., 2015), we have shown that the DSRC input (rainfall) updating method provides a simple and effective error correction method that can result in significant improvements to forecasting accuracy. In this paper, we show that the accuracy and stability of the method can be further improved by use of ridge estimation and by accounting for spatial heterogeneity in the distribution of rainfall. Through synthetic and real-data testing we have shown the following:

1. The proposed improvements are effective, with all three versions (DSRC-R, DSRC-D, and DSRC-RD) providing better performance than the original DSRC method.
2. Ill-posedness of the original DSRC coefficient matrix can occur when the method is applied to smaller-scale basin or the length of the updating rainfall series is short. In such cases, ridge estimation (DSRC-R) results in more robust performance. Empirically, we note that the stability of the ridge estimation procedure improves with the decrease of basin scale.
3. As the basin scale increases, the stability of the method (DSRC-D) can be significantly improved by accounting for the spatial heterogeneity of rainfall and rainfall error during practical application.
4. Overall, the combined DSRC-RD method, which both accounts for spatial heterogeneity and implements ridge estimation provides the best results and is recommended for practical applications. However, if computational cost is an important consideration, then the use of the DSRC-R method is recommended for

smaller basins or where the length of updating rainfall series is short, while the use of the DSRC-D method is recommended for larger basins where the spatial distribution heterogeneity of rainfall is significant.

Future work will explore the benefits of this improved method in more general application, such as updating other sources of error, coupling with other hydrological models or environmental models, and for real-time updating of model parameters. As always, we encourage and invite collaboration on this and other issues related to hydrological model development, evaluation, and improvement.

Appendix A: Relationship Between Ridge Estimation and Least Squares Estimation

The relationship between ridge estimation and least squares estimation can be obtained by transforming equation (8).

$$\begin{aligned}\Delta P'_{RE} &= (S^T S + \beta I)^{-1} S^T \Delta Q \\ &= (S^T S + \beta I)^{-1} (S^T S) (S^T S)^{-1} S^T \Delta Q\end{aligned}\quad (A1)$$

According to equation (1), the estimated rainfall error series $\Delta P'_{LS}$ by using least squares estimation can be expressed as

$$\Delta P'_{LS} = (S^T S)^{-1} S^T \Delta Q \quad (A2)$$

then

$$\begin{aligned}\Delta P'_{RE} &= (S^T S + \beta I)^{-1} (S^T S) \Delta P'_{LS} \\ &= \left[(S^T S + \beta I) (S^T S)^{-1} \right]^{-1} \Delta P'_{LS} \\ &= \left[I + \beta (S^T S)^{-1} \right]^{-1} \Delta P'_{LS}\end{aligned}\quad (A3)$$

and then set the coefficient matrix Z expressed as follows:

$$\begin{aligned}Z &= \left[I + \beta (S^T S)^{-1} \right]^{-1} \\ &= (S^T S + \beta I)^{-1} (S^T S) \\ &= 1 - \beta (S^T S + \beta I)^{-1}\end{aligned}\quad (A4)$$

We can obtain a simplified relational expression between $\Delta P'_{RE}$ and $\Delta P'_{LS}$.

$$\Delta P'_{RE} = Z \Delta P'_{LS} \quad (A5)$$

Appendix B: Theoretical Consistency of L Curve and DSRC

The analysis and demonstration are as follows, take equation (11) into equation (16), we can get

$$\begin{aligned}E(D_{RE}^2) &= E \left[(Z \Delta P'_{LS} - \Delta P_O)^T (Z \Delta P'_{LS} - \Delta P_O) \right] \\ &= E \left[(Z \Delta P'_{LS} - Z \Delta P_O + Z \Delta P_O - \Delta P_O)^T (Z \Delta P'_{LS} - Z \Delta P_O + Z \Delta P_O - \Delta P_O) \right] \\ &= E \left\{ [Z (\Delta P'_{LS} - \Delta P_O) + Z \Delta P_O - \Delta P_O]^T [Z (\Delta P'_{LS} - \Delta P_O) + Z \Delta P_O - \Delta P_O] \right\} \\ &= E \left\{ (\Delta P'_{LS} - \Delta P_O)^T Z^T Z (\Delta P'_{LS} - \Delta P_O) + (\Delta P'_{LS} - \Delta P_O)^T Z^T (Z \Delta P_O - \Delta P_O) \right. \\ &\quad \left. + (Z \Delta P_O - \Delta P_O)^T [Z (\Delta P'_{LS} - \Delta P_O)] + (Z \Delta P_O - \Delta P_O)^T (Z \Delta P_O - \Delta P_O) \right\}\end{aligned}\quad (B1)$$

One of the characteristics of least squares estimation is that the mean value of the difference between the estimated series $\Delta P'_{LS}$ and the real rainfall error series ΔP_O is equal to zero, expressed as follows:

Figure D1. Results for Synthetic Case 1—Distributed scenario. (a) and (b) Comparison of the assigned and estimated rainfall error series: (a) DSRC and DSRC-D method; and (b) DSRC-R and DSRC-RD method. (c) and (d) Comparison of the streamflow before and after updating: (c) DSRC and DSRC-D method; and (d) DSRC-R and DSRC-RD method. DSRC = dynamic system response curve.

$$E(\Delta P'_{LS} - \Delta P_O) = 0 \quad (B2)$$

Combining equation (6) and equation (B2), the equation (B1) can be simplified as follows:

$$\begin{aligned} E(D_{RE}^2) &= \sigma^2 \text{tr} \left[(S^T S)^{-1} Z^T Z \right] + (Z \Delta P_O - \Delta P_O)^T (Z \Delta P_O - \Delta P_O) \\ &= \sigma^2 \text{tr} \left[(S^T S)^{-1} Z^T Z \right] + \Delta P_O^T (Z - I)^T (Z - I) \Delta P_O \end{aligned} \quad (B3)$$

Take equation (A4) into the above equation, we can get the following expression:

$$\begin{aligned} E(D_{RE}^2) &= \sigma^2 \text{trace} \left\{ (S^T S)^{-1} \left[I + \beta (S^T S)^{-1} \right]^{-1} \right\}^T \left[1 - \beta (S^T S + \beta I)^{-1} \right] \} + \beta^2 \Delta P_O^T (S^T S + \beta I)^{-2} \Delta P_O \\ &= \sigma^2 \text{trace} \left\{ \left[(S^T S + \beta I)^{-1} \right]^T \left[1 - \beta (S^T S + \beta I)^{-1} \right] \right\} + \beta^2 \Delta P_O^T (S^T S + \beta I)^{-2} \Delta P_O \\ &= \sigma^2 \text{tr} (S^T S + \beta I)^{-1} - \sigma^2 \beta \text{tr} (S^T S + \beta I)^{-2} + \beta^2 \Delta P_O^T (S^T S + \beta I)^{-2} \Delta P_O \\ &= \sigma^2 \sum_{i=1}^M 1/(\lambda_i + \beta) - \sigma^2 \beta \sum_{i=1}^M 1/(\lambda_i + \beta)^2 + \beta^2 \Delta P_O^T (S^T S + \beta I)^{-2} \Delta P_O \\ &= \sigma^2 \sum_{i=1}^M \lambda_i / (\lambda_i + \beta)^2 + \beta^2 \Delta P_O^T (S^T S + \beta I)^{-2} \Delta P_O = f_1(\beta) + f_2(\beta) \end{aligned} \quad (B4)$$

where $f_1(\beta)$ and $f_2(\beta)$ are different functions, with β as independent variable.

Figure D2. Results for Synthetic Case 2—Lumped scenario. (a) and (b) Comparison of the assigned and estimated rainfall error series: (a) DSRC method; and (b) DSRC-R method. (c) and (d) Comparison of the streamflow before and after updating: (c) DSRC method; and (d) DSRC-R method. DSRC = dynamic system response curve.

$$f_1(\beta) = \sigma^2 \sum_{i=1}^M \lambda_i / (\lambda_i + \beta)^2 \quad (\text{B5})$$

$$f_2(\beta) = \beta^2 \Delta P_O^T (S^T S + \beta I)^{-2} \Delta P_O \quad (\text{B6})$$

From equation (B4) we can clearly see that when the value of ridge coefficient β is zero, the ridge estimation is an unbiased least squares estimation.

$$E(D_{RE}^2) = E(D_{LS}^2) \quad (\text{B7})$$

Therefore, through equation (B4), we can see that the MSE of $\Delta P'_{RE}$ and ΔP_O is composed of two parts: One is the variance of ridge estimation expressed as $f_1(\beta)$; the other is the deviation of estimated value when replacing least squares estimation with ridge estimation value of ridge estimation expressed as $f_2(\beta)$. In order to select a suitable ridge coefficient, it is necessary to further study the change rate of $f_1(\beta)$ and $f_2(\beta)$ with ridge coefficient β . Taking β as independent variable, the following two equations can be obtained by deriving the equation and taking the limit, respectively.

$$\lim_{\beta \rightarrow 0^+} \left[\frac{df_1(\beta)}{d\beta} \right] = -2\sigma^2 \sum_{i=1}^M \left(\frac{1}{\lambda_i^2} \right) \quad (\text{B8})$$

$$\lim_{\beta \rightarrow 0^+} \left[\frac{df_2(\beta)}{d\beta} \right] = 0 \quad (\text{B9})$$

Figure D3. Results for Synthetic Case 2—Distributed scenario. (a) and (b) Comparison of the assigned and estimated rainfall error series: (a) DSRC and DSRC-D method; and (b) DSRC-R and DSRC-RD method. (c) and (d) Comparison of the streamflow before and after updating: (c) DSRC and DSRC-D method; and (d) DSRC-R and DSRC-RD method. DSRC = dynamic system response curve.

According equations (B8) and (B9), the $f_1(\beta)$ is a decreasing function of β and $f_2(\beta)$ is an increasing function of β . The criterion for choosing a suitable ridge coefficient in this study is to find a ridge coefficient which cannot increase the deviation significantly but also greatly reduce the variance of ridge estimation. This is consistent with the mechanism of finding the appropriate ridge coefficient by L curve method.

Appendix C.: Mathematical Definitions of NSE and KGE

The mathematical definitions of NSE expressed as follow:

$$NSE = 1 - \frac{\sum_{j=1}^L [Q_C(j) - Q_O(j)]^2}{\sum_{j=1}^L [Q_O(j) - \overline{Q_O}]^2} \quad (C1)$$

where Q_C is the computed discharge; Q_O is the observed discharge; $\overline{Q_O}$ is the mean value of observed discharge; L is the number of hours in a flood event (the time step in this study is hour).

The KGE is calculated as

$$KGE = 1 - ED \quad (C2)$$

$$ED = \sqrt{(r-1)^2 + (\alpha-1)^2 + (\beta-1)^2} \quad (C3)$$

$$\alpha = \sigma_S / \sigma_O \quad (C4)$$

$$\beta = \mu_S / \mu_O \quad (C5)$$

where ED is the Euclidian distance from the ideal point, r is the correlation coefficient between the simulations and the observations, μ_O and σ_O are the mean and standard deviation of the observations, μ_S and σ_S are the mean and standard deviation of the simulations, α is a measure of the relative variability in the simulated and observed values, and β is the ratio of the mean values of simulations and observations.

Appendix D: Figures of Synthetic Data Study

Three figures of synthetic data study are shown as follows. Figure D1 shows results for the distributed scenario of Synthetic Case 1 (section 4.1.1.2), Figure D2 shows results for the lumped scenario of Synthetic Case 2 (section 4.1.2.1), and Figure D3 shows results for the distributed scenario of Synthetic Case 2 (section 4.1.2.2).

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