Behavior in Sealed-Bid Second-Price Auctions with Correlated Private Values and Discrete Bidding

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Abstract

Kremer's 1997 article suggest that we use sealed-bid second-price auctions to determine the private value of patents. This requires that bidders bid their private value in second-price auctions, which is what the theory has determined is the dominant strategy. However, in laboratory experiments, bidders tend to overbid or underbid and rarely follow the dominant strategy. The experiment discussed in this article was designed to attempt to correct this behavior. Though bidders in this experiment performed better than bidders in previous experiments, over- and under-bidding was not fully corrected. This may be attributed to the fact that negative feedback (losses upon overbidding) and positive feedback (profits from not bidding value) occurred with equal frequencies, convincing the subject that there is a trade-off between profits and losses when overbidding and not that overbidding is unwise.

1 Motivation and Background

Patents are designed to provide incentives for investing in research and development as well create new ideas, which in turn can lead to economic growth. However, the current system gives inventors monopolies over the idea and goods produced using the idea (creating a deadweight loss), while also creating under-investment in research, as well as wasteful funding towards creating substitutions of the idea. At a monopoly's prices, some people who value the good above the marginal cost of production will not consume the good. The quantities consumed at the monopoly price create a deadweight loss, preventing consumers and producers from capturing all of the potential surplus in the market.

Because patented ideas are a public good and the marginal cost of providing it to more people is significantly less than the marginal social benefits it brings (once an idea is invented, reproducing it is much less costly), inventors are given monopoly pricing and exclusiveness of the idea to encourage inventions and research. However even given monopoly pricing, researchers still do not have enough incentive to undertake original research. They do not take into consideration the social benefits of their invention; nor are they rewarded for them. Since inventors only consider the private values, they under-invest in research that is valued more socially. These social benefits may be from positive externalities they create for other researchers from knowledge spill-overs, or from the prevention and elimination of a disease (to take a pharmaceutical example). In previous studies by Nadiri [1993], Mansfield et al [1977], and Trajtenberg [1990], the social rates of return to research and development range from 25% to 270% of the private returns. Since these are calculated from the quantities consumed under monopoly pricing, the social rate of return would be even greater if priced at marginal cost since the deadweight loss would be avoided. The monopoly pricing gives others incentive to create substitutes. In the process of doing so, research funds are wasted on reverse engineering and inventing around patents. Additionally, there is not enough incentive to fund research to create complements to new inventions. Complements could increase the speed at which progress is made, but currently companies must sink costs into creating complementary inventions before even knowing the conclusion of the license agreement with the original patent owners.

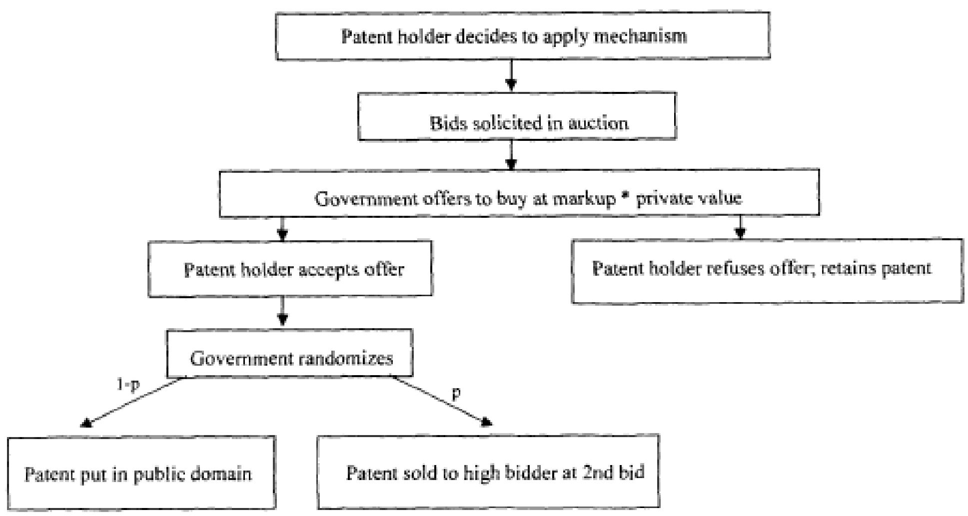

Figure 1: Kremer, 1997

In his 1997 article, Kremer suggests a different way of approaching the patent system. He proposed that the inventor can choose to sell his patent in a sealed-bid second-price auction, which will determine the private value of the patent. The government will buy the patent (if the patent owner accepts the offer) at the private value determined from the auction, plus a mark-up to account for the social value of the invention, and release the patent to the public domain. Since auction participants need incentive to bid their true values, the patent would have to be awarded to the winner of the auction with some small probability. This process is mapped out in Figure I, from Kremer's article. The reason for using a sealed-bid second-price auction for the patent is because the dominant strategy is to bid exactly your private value (Vickrey 1961). In the case of the patent, the bidder's private value is their prediction or estimate of the value of the patent, which will reveal the private value of the patent.

It is useful to examine whether this theory holds true in experiments in order to determine if a second-price auction will reveal the true private value of the patent. Laboratory experiments in second-price auctions have generally revealed that bidders tend to bid over their value or bid under their value, while rarely bidding their value. Strides are made in understanding behavior in second-price auctions (SPAs) as experiments are done with modifications to the rules of the auction. Kagel, Harstad, and Levin and Harstad (2000) ran experiments with "affiliated" private values. They drew a parameter C from a uniform distribution (the distribution was known to the bidders) and each bidder's private value was subsequently drawn, independently, from the uniform distribution $[C-R, C+R]$ for different $R's$ in different periods of the experiment. Both of these experiments showed overbidding in these auctions. In an experiment conducted by Kagel and Levin (1993), subjects had independent private values drawn from a uniform distribution which was known to the bidders. After each period, the bids and values were reported to each of the five or ten subjects. In the auctions with 5 participants, only an average of 27 percent of bids were value bids, while 67.2 percent were overbids. In the auctions with 10 participants, an average of 32.5 percent of bids were value bids and 57.9 percent were overbids. [1] In addition, they found that overbidding was independent of the private values. From this, they concluded that overbidding cannot be attributed to rivalistic behavior. They also found there was no evidence of subjects adapting to the dominant strategy even after experiencing losses.

Shogren, Parkhurst and McIntosh (1996) tried a "tournament" type distribution of winnings, where bidders win points during auctions, and are rewarded by first, second, third place. They claim that overbidding has a much greater adverse effect in this type of reward system, However, Kagel and Levin (2011) point out that the tournament structure actually gives motivation for bidders to overbid in order to reduce the winner's profits.

Garratt, Walker and Wooders (2004) used subjects who were frequent eBay auction participants to determine whether field experience improves the chances that bidders will bid their value. Once again, the majority of the participants did not bid their value, with 41 percent of participants

[1] Some of these auctions were split into two separate markets, with fewer number of bidders so this information does not necessarily reflect the relationship between number of bidders and bidding behavior.

underbidding and 37 percent overbidding. After analyzing the data in terms of buyers versus sometimes sellers, they found that the sellers tended to underbid more frequently than the buyers did. One explanation for this is that people who have sold on eBay are accustomed to the habit of buying only if the price is below their value, so as to make a profit from resale.

Georganeous, Levin, and McGee (2010) added the characteristic of additional penalties for deviating from the dominant strategy in their experiment. Losses would be multiplied by a factor β, which was either 1, 0.1, or 20, and subjects were informed of when this β changed. While the subjects still failed to discover the dominant strategy, it was discovered that they do respond to changes in β. For $\beta = 20$, subjects bid closer to their value while for $\beta = 0.1$, subjects bid further from their value. This seems to suggest that bidders believe (incorrectly) that they are confronted with a tradeoff between higher bidding (and consequently winning) and the costs of higher bidding, with changes in β altering potential costs of winning.

Each of these experiments led the experimenters to conclude that, because of the sealed nature of the bidding and the second-price rule, the probability of receiving negative feedback when overbidding is extremely low as it is possible to overbid with no cost if the second price that's paid is still below their value. In fact, when this happens and bidders make a profit from overbidding, they are actually receiving positive feedback, which may cancel out any negative feedback they have received. Most of the those previously referenced believe that when subjects begin the experiment believing that higher bidding will lead to winning with no adverse effect on profits, it is hard to correct this belief. This would require better feedback from the auction or that the bidder ask himself what is to be gained from overbidding in comparison to bidding the private value? It seems it is unnatural for subjects to ask themselves this question, with more reliance on their experience in the auction.

2 Experiment Set-up and Procedure

Set-up

If these are the reasons SPA experiments do not conform to the theory, then it is natural to try and design an experiment to counter them and diminish the possibilities of bidding error or lead subjects to ask themselves the question comparing differences of overbidding and bidding value. We ran two 2-player second-price sealed-bid experiments: One with correlated resale values and one without[2]. Each experiment consisted of a series of 15 rounds and six subjects. Subjects were told that each round they would be bidding for either a blue, red, or white poker chip. The color of the chip being auctioned alternated by round, totaling 15 rounds, 5 of each color. Unlike previous experiments that drew the private values from an essentially continuous uniform distribution, the subjects were assigned pre-determined discrete values of \$1, \$2, and \$3. They were told how much a blue chip was worth to them, how much a red chip was worth to them, and how much a white chip was worth to them (one worth \$1, one worth \$2, and one worth \$3). They were also informed that the other subjects may have different values assigned to a certain color, but that it would also be \$1, \$2, or \$3. Each round, the six subjects were paired up so that there were three auctions per round. In both experiments, each unique pairing occurred exactly three times in the 15 rounds (even with the fact that in the correlated values experiment, the pairing is slightly more complicated). Subjects were told they would unlikely be paired with the same person in two sequential auctions (to discourage collusion or other repeated game effects).

Limiting each auction to two players increases the likelihood of a bidder winning an auction so that they may receive feedback more frequently. Unlike in previous affiliated value experiments, where the private values were drawn from the uniform distribution $[C - R, C + R]$, the resale values of the bidders are correlated by pairing together subjects with the same values more frequently. (Figure 2) This means that, out of five rounds for one color chip, in three of those rounds a subject is paired with a second subject whose

[2] The experiment "without" correlation actually has a slight negative correlation of $r = -0.2$ due to the nature of the pairings. Pairings with the same value only occur once per value, but pairings with different values are duplicated.

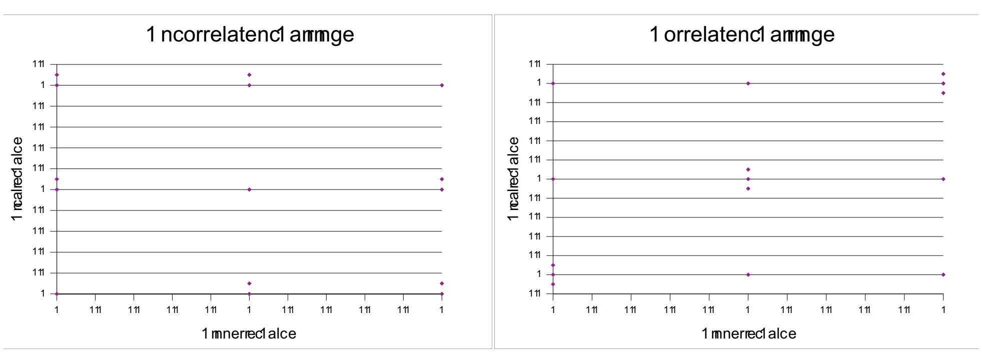

Figure 2: Pairings

value for the chip is the same as his. The other two rounds he is paired with subjects with the other two values. For example, if a bidder's resale value for a blue chip was $3, then within the 5 rounds with a blue chip, he will be paired with someone whose value for a blue chip is also $3 *three* times, someone whose value for a blue chip is $2 *once*, and someone whose value for a blue chip is $1 *once*. This was done so that they are more likely to receive *negative* feedback due to overbidding. Bidders were only allowed to submit discrete bids equal to $0, $1, $2, $3, $4. The subjects were undergraduate students at the University of Arizona, recruited through the Economic Science Lab.

PROCEDURE

Each bidder starts with 20 fake one-dollar bills. Bids are submitted physically, on sheets of paper called "bid sheets". This was to create a sense of real value for the chip they are bidding on and for the dollars they are using to bid with. The bid sheets list their resale value for each round, the possible bids values (subjects were told to circle their bid), as well as information regarding the results of the previous round. This includes their resale value, their bid, their rival's bid, who won the auction, and profit (which was shown explicitly as resale value minus rival's bid if they won). Each subject was also given a "record sheet" so they could keep track of their earnings and observe their profits relative to their bidding. With the instructions, they were given sample bid sheets and corresponding examples on the record sheets so they could a) visually comprehend the second-price rule and b) study how to record information from the bid sheet. Ties were determined by a coin flip, giving the win to the subject with a higher "subject number" (for identification) if heads and lower "subject number" when tails.

3 Experiment Results

3.1 Uncorrelated vs. Correlated

Bidders in the uncorrelated experiment were more inclined to bid their value while bidders in the correlated experiment continued to overbid, with a little underbidding as well. As seen in Table 1, 71% of the bids in the uncorrelated experiment were equal to the bidder's value, compared to the 44% of bids in the correlated experiment. Most of the overbids were over by $1, with a only a total of four over by $2 (one in the uncorrelated data, three in the correlated).

	Underbid	Bid Value	Overbid
Uncorrelated	12 (13.33%)	64 (71.11%)	14 (15.56%)
Correlated	11 (12.22%)	40 (44.44%)	39 (43.33%)

Table 1: Bidding Frequencies

To test the differences between the uncorrelated and correlated experiments, I will use the hypothesis that the probability of bidding the exact value was the same for the uncorrelated and correlated experiments. Data from each of the two experiments will be represented as random draws from binomial process: "success" for bidding value, "failure" for not bidding value (overbid or underbid). The two binomial parameters are unknown, but can be estimated given the data.

Pearson's chi-square goodness-of-fit test can be used to test the equality of the two distributions (of binomial processes). Let n_u denote the number of bids in the uncorrelated experiment and n_c denote the number of bids in the correlated experiment. Since both experiments had 15 rounds with 6 bidders, $n_u = n_c = 90$. Let N_u and N_c denote the number of value bids (or successes) in the uncorrelated and correlated experiments, respectively, 64 and 40. Let the null hypothesis be $H_0 : p_u = p_c$, where p_u is the probability of a value bid (or success) in the uncorrelated experiment and p_c is the probability of a value bid (or success) in the correlated (therefore 1-p_u and 1-p_c are the probabilities of failure. The null hypothesis states that the probability of bidding value is the same in both experiments, or equivalently, that there is a number p such that $p_u = p$ and $p_c = p$. At this time, since p is unknown, we estimate it to be the average number of successes, $\frac{N_u+N_c}{n_u+n_c}$, or $\frac{64+40}{90+90} = \frac{104}{180} \approx .5778$. The Pearson statistic is given by

$$Q = \sum_{j\in\{u,c\}} \left[\frac{(N_j - n_j p)^2}{n_j p} + \frac{((n_j - N_j) - n_j(1-p))^2}{n_j(1-p)} \right]$$

Using the values from the experiment, the test statistic Q is found to be 13.12 with p-value 0.00029263. The p-value represents the probability of obtaining at least this many successes from a chi-square distribution. Given the p-value, we reject the null hypothesis that $p_u = p_c$ at a less than a 1% significance level and conclude that the probability of value-bidding is different between the uncorrelated and correlated experiments, and therefore the binomial distributions are different. Running a Pearson's chi-square goodness-of-fit test to test the equality of our experiments against Kagel and Levin's (1993) gives us the following table that lists test statistic Q's and p-values between the three experiments.

Experiments	Pearson's Test Statistic Q	p-values
Uncorrelated vs. Correlated	13.12	0.00029263
Uncorrelated vs. K&L	65.335	0
Correlated vs. K&L	10.957	0.00093251

Table 2: Pearson's Test Statistics and p-values

The differences in these binomial distributions are statistically significant at the $< 1\%$ level Since the probability of success is higher in both the uncorrelated and correlated compared to the Kagel and Levin experiment, some aspect of the experiment must contribute to the improvement, whether it be the discrete bids, valuations, or the physical exchanges of poker chips.

3.2 Negative and Positive Feedback

Negative feedback (when the bidder received a loss due to overbidding) occurred 4 times out of the 14 overbids in the uncorrelated experiment and 5 out of the 39 times in the correlated experiment. (Table 3) Of these, only two people (one in each experiment) experienced multiple losses, with a maximum of 2 losses for each bidders. The two losses experienced by the bidder in the uncorrelated experiment occurred in two sequential rounds, after which he seemed to "learn" and only bid his value. The two losses experienced by the bidder in the correlated experiment occurred with 10 rounds in between, during which he overbid in half of those rounds. Two of the other five bidders who experienced losses seemed to "learn" from the negative feedback and only bid their value after, but the other three continued to overbid.

Positive feedback (when the bidder receives a profit despite over- or underbidding) occurred 7 times in the uncorrelated experiment and 15 in the correlated. For two bidders, the only time they made profit was from an overbid (they received losses from overbidding as well). One of these bidders bid value 6 times within the 15 rounds and overbid the other 9. He made zero profit for every value-bid, but made $7 from overbidding. The other bidder started the experiment bidding value, but after two rounds without making profit, changed their strategy to overbidding and made $2 per round for two rounds, and $3 for another.

Using Fisher's exact test on this contingency table[3] will analyze the statistical significance of the association between positive, negative, and no feed-

[3] I use a Fisher's test because the values in this contingency table are relatively small, and Fisher's test will give a more exact p-value than a chi-square test.

	Negative	Positive	None	Total Overbids
Uncorrelated	4 (28.57%)	7 (50.00%)	3 (21.43%)	14
Correlated	5 (12.82%)	15 (38.46%)	19 (48.72%)	39

Table 3: Frequencies of Overbids by Type of Feedback Received

back for correlated and uncorrelated values. Doing so yields a p-value of 0.2013, showing we cannot reject that these feedbacks do not occur with equal frequencies, i.e. negative feedback occurs as often as positive feedback and vice versa. Ideally, we want negative feedback for overbidding to occur much more frequently than positive feedback so as to make bidders reconsider overbidding.

3.3 Learning and Value's Affect on Bidding

Running the following regression equation shows that there was some learning (Table 5), since the "odds", or the ratio of probability of non-optimal bidding to probability of bidding value decreases as round number increases. Each round, the odds decrease by a factor of $e^{0.09021} \approx 1.0944$. This implies that subjects did start to bid value (to a certain extent) as the experiment progressed.

Regression Equation:

$$logit(p) = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \beta_3 * X_3$$

where:
$logit(p) = log(odds)$ of the binomial dependent, 0 if bid value, 1 if did not bid value
X_1 = bidder's private value
X_2 = round number (1-15)
X_3 = dummy variable with
1 if from correlated experiment
0 if from uncorrelated experiment

The regression also shows that bidders deviated from value-bidding the higher their value: increasing the value by \$1 increased the odds by a factor of $e^{0.46545} \approx 1.5927$. For the correlated data, running a linear regression of

$$B(y) = \beta_0 + \beta_1 * Y$$

Min	1Q	Median	3Q	Max
-1.7312	-0.9754	-0.6450	1.0719	2.0488

Table 4: Deviance Residuals

	Estimate	Std. Error	z value	Pr(> $\|z\|$)
(Intercept)	-1.17044	0.54977	-2.129	0.033255 *
value	0.46545	0.20087	2.317	0.020496 *
round	-0.09021	0.03812	-2.366	0.017964 *
corDummy	1.20019	0.32797	3.659	0.000253 ***

Table 5: Regression Results

where B(y) = bid and Y = resale value shows that there is a positive relation between value and bid differences. The higher values lead to overbids and lower values lead to underbids (Table 6) since the slope of the regression line, 1.2000 is steeper than 1 (bidding value). Doing the same for the uncorrelated data, we find the slope is barely less steep as the correlated, 1.1667. (Table 7)

	Estimate	Std. Error	z value	Pr(> $\|z\|$)
(Intercept)	-0.05556	0.20146	-0.276	0.783
Value	1.20000	0.09326	12.867	<2e-16 ***

Table 6: Bid vs. Value Regression Results: Correlated

A majority, 52% (12/23) of the underbids occurred when the bidder's value was \$1. These are the only values during which bidders bid \$0. This may be because bidders realized they have no chance of making any profit when their value is \$1 (except if their opponent bids \$0, or they both bid \$0 and they win the tie). Realizing this, bidders may be altruistic in letting their opponent profit an addition dollar from their bidding \$0.

Table 8 shows the expected bidder surplus had everybody bid their value every round versus the average bidder surplus from the actual bids. As expected from non-optimal bidding, the average bidder surplus is lower than

	Estimate	Std. Error	z value	Pr(> $\|z\|$)
(Intercept)	-0.31111	0.14652	-2.123	0.0365 *
Value	1.16667	0.06782	17.201	<2e-16 ***

Table 7: Bid vs. Value Regression Results: Uncorrelated

the bidder surplus given value bidding.

	Value Bidding	Actual Bids
Uncorrelated	8	7.17
Correlated	4	3.83

Table 8: Differences in Bidder Surplus due to Non-Optimal Bidding.

4 Conclusion

Even with discrete valuations and bids, two-person auctions, and specially designed physical aspects of the auction, the experiment did not result in subjects bidding their values, as the theory suggest should be the case. Furthermore, contrary to what was hypothesized, the uncorrelated pairings performed overall better than the correlated pairings and the probability of value-bidding is higher in the uncorrelated experiment. This may be attributed to the higher number of ties in the correlated pairings, which was expected. What we didn't account for was the possible frustration of tying so frequently and, even upon winning a tie, making zero profit (when bidding value). This may have increased their utility of winning and led bidders to overbid, in an attempt to win a round, or underbid, to make a profit when winning a tie.

Bidding behavior also changed depending on their value: bidders tended to overbid when private values were $3, and underbid when private values were $1. This may have been because bidders wanted to win more when their values were the highest (and consequently, when they had the chance to profit the most) so they overbid to guarantee a win, while when their values were the lowest value ($1), they felt there was no profit to be made, so they bid $0 to allow their rivals to profit.

It is clear that negative feedback (from overbidding) still does not outweigh the positive feedback in the experiment. Additionally, feedback still does not occur very frequently as many times bidders experience a profit/loss of \$0 (either from winning and making no profit or loss because their resale value was equal to their rival's bid or from losing the auction despite an overbid).

These experiments *do* show an improvement from previous experiments, namely Kagel and Levin's 1993 experiment, so some characteristic or combination of characteristics of the auction may lead bidders to question the validity of not bidding their resale value. It may be useful to run experiments changing specific characteristics to see how it will change the results.

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