FINANCIAL FRICTIONS AND CAPITAL STRUCTURE CHOICE:
A STRUCTURAL DYNAMIC ESTIMATION

by

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DEDICATION

To my parents, Nilia and Amilcar, my brothers, Gustavo and Nicolas, and my wife, Natalia, for giving me all the support and love I needed to reach far.
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This thesis studies different aspects of firm decisions by using a dynamic model. I estimate a dynamic model of the firm based on the trade-off theory of capital structure that endogenizes investment, leverage, and payout decisions. For the estimation of the model I use Efficient Method of Moments (EMM), which allows me to recover the structural parameters that best replicate the characteristics of the data. I start analyzing the question of whether target leverage varies over time. While this is a central issue in finance, there is no consensus in the literature on this point. I propose an explanation that reconciles some of the seemingly contradictory empirical evidence. The dynamic model generates a target leverage that changes over time and consistently reproduces the results of Lemmon, Roberts, and Zender (2008). These findings suggest that the time-varying target leverage assumption of the big bulk of the previous literature is not incompatible with the evidence presented by Lemmon, Roberts, and Zender (2008). Then I study how corporate income tax payments vary with the corporate income tax rate. The dynamic model produces a bell-shaped relationship between tax revenue and the tax rate that is consistent with the notion of the Laffer curve. The dynamic model generates the maximum tax revenue for a tax rate between 36% and 41%. Finally, I investigate the impact of financial constraints on investment decisions by firms. Model results show that investment-cash flow sensitivity is higher for less financially constrained firms. This result is consistent with Kaplan and Zingales (1997). The dynamic model also rationalizes why large and mature firms have a positive and significant investment-cash flow sensitivity.
Chapter 1

Financial Frictions and Capital Structure
Choice: A Structural Dynamic Estimation

1.1 Introduction

The trade-off theory of capital structure suggests that, by analyzing the costs and benefits of debt, optimizing firms should be able to select the target debt-equity ratios that maximize their value. This theory predicts that when firms deviate from their optimal leverage, they will return to their target level. The literature has two seemingly opposite positions regarding the evolution of target leverage over time. Hovakimian, Opler, and Titman (2001), Flannery and Rangan (2006), and Huang and Ritter (2009) suggest that target leverage is time-varying because firm characteristics and market conditions change over time. On the contrary, Lemmon, Roberts, and Zender (2008) argue that firms rebalance their leverage ratios toward a largely time-invariant target, and that time-dependent firm characteristics play a small role in the determination of target leverage. Furthermore, the speed of mean-reversion of leverage is still subject to great debate in the literature.\(^1\)

By estimating a dynamic structural firm model based on the trade-off theory of capital structure, I study the leverage decisions made by the value maximizing firm. I find that target leverage varies over time, as Hovakimian, Opler, and Titman (2001), Flannery and Rangan (2006), and Huang and Ritter (2009) suggest. However, I also find that leverage decisions oscillate around a constant level (that is, they are mean-

\(^1\)See Fama and French (2002), Flannery and Rangan (2006), Lemmon, Roberts, and Zender (2008), and Huang and Ritter (2009).
reverting), which generates the results of Lemmon, Roberts, and Zender (2008). In other words, leverage decisions generated by the dynamic model suggest that the apparently contradictory positions can reflect different views of the same decision making process.

I reproduce the experiments of Lemmon, Roberts, and Zender (2008) and I find that the dynamic model consistently replicates their results. However, I provide a different explanation for the observed convergence of leverage to more moderate levels over time. In the dynamic model, this pattern of leverage is largely due to the fact that leverage decisions are a function of a mean-reverting profitability shock. This mean-reverting property is inherited by leverage and, when averaged into the four portfolios described by Lemmon, Roberts, and Zender (2008), is the cause of the slow decay of leverage to more moderate ratios over time. That is, mean-reversion of leverage in the dynamic model is mechanical and not the result of firms actively trying to rebalance their leverage ratios back to a largely constant target.

Lemmon, Roberts, and Zender (2008) also find that while leverage tends to moderate, it is persistent, that is, firms with relatively high (low) leverage tend to keep relatively high (low) leverage for long periods of time. They find that unobserved firm characteristics are the most important determinants of leverage decisions and show up as important firm-specific fixed effects in leverage regressions. Thus, they suggest that those unobserved firm characteristics are the cause of the observed persistence. I use the structural approach to better understand this phenomenon. I reproduce their experiments, but when I control for all sources of heterogeneity among firms I find that the persistence of leverage completely disappears.

Finally, I find that, for the firms in the panel, book leverage mean-reverts to the long-run constant level at a speed of 33.50% per year - implying a half-life of 1.70 years. This estimate is close to the empirical evidence of Flannery and Rangan (2006) and DeAngelo, DeAngelo and Whited (2010), who suggest speeds of adjustment of 34.40% and 37.80% per year, respectively.
I build the dynamic firm model using the trade-off theory of capital structure. This theory starts with the irrelevance proposition of Modigliani and Miller (1958), which shows that the value of the firm is independent of its capital structure when the firm’s investment policy is fixed and there are no taxes and contracting costs. Since that result, researchers have relaxed some of the assumptions and studied whether capital market frictions create a link between firm value and financing decisions. Modigliani and Miller (1963) show that the interest tax shield created by debt increases firm value, and thus it may be beneficial for firms to issue debt. Furthermore, Jensen and Meckling (1976) and Jensen (1986) show that agency conflicts between managers and shareholders lead managers to misuse free cash flows in unprofitable investments and excessive personal benefits. By increasing debt, these agency costs can be reduced because managers are forced to pay out a larger proportion of the free cash flows as interest.

However, issuing debt may also imply some costs. Kraus and Litzenberger (1973) suggest that increasing debt implies higher expected bankruptcy costs, which reduces firm value. In addition, Jensen and Meckling (1976) and Myers (1977) show that the issuance of risky debt induces agency problems between shareholders and bondholders, such as underinvestment, asset substitution and excessive dividends. Thus, issuing debt might also reduce firm value.

According to the trade-off theory, firms should weigh the costs and benefits of debt in order to select the target leverage that maximizes their value. Furthermore, the theory also makes similar optimizing predictions about dividends, that is, firms should choose dividend levels that maximize their value, which occurs when the marginal benefit of dividends just offsets the marginal cost.

An alternative explanation of financing decisions is the pecking order theory developed by Myers (1984). The basic assumption of this theory is that asymmetric information problems between managers and investors overwhelm the trade-off forces. However, if the costs of issuing new securities do not overpower those forces, then the
trade-off theory survives and includes asymmetric information problems as one of the forces that define optimal leverage and dividends.

Historically, studies of capital structure decisions have used static specifications. Since empirical support for the static trade-off hypothesis is not strong (e.g., Titman and Wessels (1988), Shyam-Sunder and Myers (1999), Welch (2004)), this paper adds the dynamic dimension to the model.\(^2\) Lemmon, Roberts, and Zender (2008) suggest the use of a dynamic structural model to address leverage decisions. They regress leverage on traditional determinants of financing decisions, such as profitability and the market-to-book ratio, and find that most of the variation in leverage is explained by unobserved firm-specific fixed effects. They suggest that structural modeling is a natural way to address unobserved heterogeneity. In addition, they find that convergence over time of leverage ratios is partially explained by active management of leverage toward desired levels, and argue that a dynamic model can rationalize this behavior. Dynamic models are a more sensible representation of the firm’s decision making process, as opposed to static ones. First, dynamic models capture the fact that a firm’s choices depend on its history of states, which is the outcome of the idiosyncratic uncertainty faced by the firm in the past, and the decisions it made in consequence. Second, these models also treat firms as forward-looking agents, that is, firms consider the impact of current investment, leverage, and payout decisions on future states and decisions.

Therefore, I estimate a dynamic structural model that explicitly captures the effect of leverage on firm value introducing the benefits (i.e., interest tax shields) and

costs (i.e., costs of financial distress) of debt. I include costs of external financing to rationalize the direct evidence of the importance of these expenses, as reported by Altinkilic and Hansen (2000). I also introduce partial reversibility of capital to generate a positive asymmetry in the distribution of investment rates, as documented by Cooper and Haltiwanger (2006) at the firm level.

One of the biggest benefits of the structural approach is the ability to do sensitivity analysis. In the dynamic model, every period the firm chooses the level of leverage that maximizes its value. The determination of leverage in the model includes the costs of adjustment, namely, debt and equity issuance costs. Therefore, I do sensitivity analysis and study the evolution of leverage decisions over time when there are no adjustment costs. This experiment allows me to recover the target leverage and study its evolution over time. I find that target leverage varies over time and fluctuates around a long-run constant level. Furthermore, the speed of mean-reversion of book leverage is now 50.48%, which means a half-life of about 0.99 years. This higher speed implies that, with no issuance costs, the firm adjusts its leverage to exogenous shocks (e.g., profit shocks) in a faster way. The previous results also imply that firms in the dynamic model do not have a constant target in the long-run sense toward which they try to adjust their current leverage. Hennessy and Whited (2005) develop a dynamic model and also find that there is no target leverage in the long-run sense. Nevertheless, Graham and Harvey (2001), Leary and Roberts (2005), Hovakimian (2006), Kayhan and Titman (2007), and Lemmon, Roberts, and Zender (2008) suggest that mean-reversion of leverage is partly due to the active attempt of firms to maintain their leverage ratios relatively close to the target level. DeAngelo, DeAngelo and Whited (2010) estimate a dynamic model in which firms move temporarily away from target by issuing transitory debt to take advantage of investment opportunities. They find that firms have a target leverage in the long-run sense.

I also study the consistency of the dynamic model with respect to predictions of the trade-off theory of capital structure and the empirical evidence. The trade-off
theory predicts that more profitable firms have higher leverage. In agreement with
the theory, the dynamic model generates a positive relationship between leverage and
profitability. However, this prediction is in contrast with the evidence of Titman and
Fama and French (2002) and Flannery and Rangan (2006) that more profitable firms
have, in fact, less leverage. Hennessy and Whited (2005) also report a negative
relationship between leverage and profitability.

The model also generates a negative relationship between leverage and investment
opportunities, which is consistent with the prediction of the trade-off theory and the
and Lemmon, Roberts, and Zender (2008). Hennessy and Whited (2005) also find a
negative relationship between leverage and investment opportunities.

Also confirming the trade-off theory, the dynamic model generates a negative
relationship between leverage and both dividends and non-debt tax shields such as
depreciation. Results reported by Hovakimian, Opler, and Titman (2001), Fama and
French (2001, 2002), Flannery and Rangan (2006), and Lemmon, Roberts, and Zender
(2008) agree with these predictions.

Regarding dividend decisions, the dynamic model generates a positive relationship
between dividends and profitability, and a negative relationship between dividends
and investment opportunities. These results are consistent with the predictions of

I also analyze the impact of changes in the corporate income tax rate on the
optimal policies generated by the firm model. Specifically, I investigate the evolution
of corporate income tax payments and leverage decisions as the government changes
the tax rate. I find that as the tax rate increases, income tax revenues initially
increase, but after some point income tax payments start to decrease. That is, the
model generates a bell-shaped relationship between tax revenue and the tax rate
that is consistent with the notion of the Laffer curve. Furthermore, I study how
taxes impact leverage decisions. Corporate income taxes are a key part of the trade-off theory of capital structure. According to this theory, a higher income tax rate increases optimal leverage. Consistently with this prediction, optimal leverage in the dynamic model increases with the tax rate.

The last section of this dissertation investigates the effects of financial constraints on firm decisions. In particular, I study the impact of financial constraints on investment decisions in the context of the debate between Fazzari, Hubbard, and Petersen (1988) and Kaplan and Zingales (1997). The optimal policies generated by the dynamic model support the results of Kaplan and Zingales (1997) in the sense that investment-cash flow sensitivity is higher for less financially constrained firms.

From an econometric standpoint, I recover the structural parameters by using the Efficient Method of Moments (EMM) developed by Gallant and Tauchen (1996). EMM is a systematic approach for selecting moments when estimating a structural model using Generalized Method of Moments (GMM). The key idea of this approach is to match the moments implied by the structural model to the moments implied by the data. The estimation method is particularly useful when the likelihood function of the structural model does not have a closed form, but the dynamic system can be easily simulated. This is exactly my situation.

EMM estimation can be regarded as a two-stage procedure. In the first stage, I use a semi-nonparametric (SNP) density function to describe the statistical properties of the data, namely, investment and leverage decisions. The outcome of this stage is a family of density functions for the joint distribution of the data, and I choose the one that best fits the sample in the most parsimonious way.

I recover the structural parameters of the dynamic firm model in the second stage. These estimators are those that generate simulations of investment and leverage decisions as close as possible to the observed data. The estimates of the structural parameters are consistent with findings in previous studies, have the correct sign according to economic theory and are statistically significant. In particular, the estimates of
the parameters that capture the costs of financial distress imply that these costs play a very important role in the firm’s financing decisions and the determination of firm value.

The paper is also related to the following articles. Cooley and Quadrini (2001) study how financial-market frictions affect firm dynamics such as growth, job reallocation, and exit. They show that the combination of persistent profitability shocks and financial frictions can explain the dependence of firm dynamics on size and age. In their model, financial-market frictions arise from costly equity issuance and deadweight bankruptcy costs while the debt is a one-period contract signed with a financial intermediary. The model of risky debt in my paper builds on their debt model.

Hennessy and Whited (2005) develop a dynamic model of the firm and explain empirical findings apparently inconsistent with the trade-off theory. For example, they show that a rational model can generate results consistent with market timing evidence. The main difference with my article is that I aim to study the evolution of target leverage over time and its relationship with Lemmon, Roberts, and Zender (2008). Hennessy and Whited (2007) use a dynamic model to estimate the costs of external finance. They propose a dynamic model that endogenizes investment, leverage, cash distributions, and default. Their paper is different from mine in that their primary objective is to estimate the magnitude of bankruptcy costs and the equity flotation costs. Gamba and Triantis (2008) simulate a dynamic model of the firm under reasonable parameterizations. They study the value and optimal management of financial flexibility.

Shyam-Sunder and Myers (1999) and Chang and Dasgupta (2009) suggest that pecking order or random financing schemes, respectively, coupled with mean reversion in cash flows can generate leverage patterns similar to those reported by Lemmon, Roberts, and Zender (2008). This mean-reversion is called mechanical because it is driven by the mean-reverting profit shocks, and not by the active efforts of the firm to rebalance its leverage toward a largely constant level. In this paper, I have
a financing scheme based on the trade-off theory of capital structure, coupled with mean reversion in the profit shock. As I mentioned before, leverage inherits this mean-reverting property from the shock. That is, the mean-reversion of leverage is not induced by the active efforts of firms to rebalance their leverage to a constant target. In this sense, the mean-reversion in this article is mechanical too.

The paper is organized as follows. Section 2 describes the model. In Section 3, I describe the data used for estimation. Section 4 examines the EMM procedure and presents the estimation results. In Section 5, I discuss the identification strategy. Section 6 analyzes the predictions of the model about target leverage and presents the main results of the paper. In Section 7, I study the consistency of the optimal decisions generated by the model. Section 8 presents the results of sensitivity analysis of corporate tax revenue with respect to the income tax rate, while Section 9 shows the results of sensitivity analysis of investment decisions with respect to financial constraints. Section 10 concludes.

1.2 The Model

The methodology I use to develop the model is discrete time, infinite horizon, stochastic dynamic programming. The objective of the firm is to maximize its value, which is achieved by maximizing the expected discounted sum of cash flows to investors (debt-holders and shareholders). Before formally stating the maximization problem of the firm, I start by carefully describing each of its components.

Agents are risk-neutral and use the risk-free rate of interest, \( r_f \), as their discount rate. State variables with primes indicate values in the next period, while state variables with minus signs indicate values in the previous period. There are two endogenous state variables: capital \( k \) and debt \( d \). The capital of the firm, \( k \), is used for production and can vary over time because of investment decisions. In each period, capital depreciates at a constant rate \( \delta > 0 \).
The debt of the firm, \( d \), matures in one period and is rolled over every period. This structure of debt is similar to a perpetual bond with a floating rate. The total amount of debt can be increased or decreased over time according to the debt decisions. The debt contract includes a positive net worth covenant by which if, in any period, the value of the firm falls below the nominal value of the debt, the firm goes into bankruptcy and is liquidated. Therefore, the bond yield required by debt-holders will depend on the probability of bankruptcy. The impact of risky debt on the determination of the bond yield will be fully explained below.

There is one source of uncertainty that drives the optimal policies of the firm, the profitability shock, \( z \). This shock changes the marginal profitability of capital, making it more or less attractive to invest. Therefore, high realizations of \( z \) are followed by large investment decisions, and vice versa. I assume state variable \( z \) follows an AR(1) process

\[
\begin{align*}
    z' &= c + \rho z + \varepsilon' \\
    \varepsilon' &= \gamma + \varsigma \\
    \varsigma &= \eta + \nu \\
    \eta &= \mu + \varepsilon
\end{align*}
\]

where \( \varepsilon \) is assumed to be an \( iid \) truncated normal random variable with mean 0 and variance \( \sigma^2_\varepsilon \). Therefore, shock \( z \) takes values in the compact set \( \mathcal{Z} = [\underline{z}, \bar{z}] \).

The operating profits of the firm, \( \pi(k, z) \), depend on the capital in place, \( k \), and the profitability shock, \( z \), in the following way

\[
\pi(k, z) = zk^\alpha
\]

where \( \alpha \in (0, 1) \). Equation (1.2) shows that the operating profit function is of the Cobb-Douglas form with decreasing marginal profitability. Cooper and Ejarque (2003) argue that the function \( \pi(k, z) \) is a reduced form profit function that is obtained after the firm maximizes over the flexible factors, such as labor.⁴ Decreasing

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³It is common in the literature to assume \( z \) follows an AR(1) process in logs as in Gomes (2001), Moyen (2004, 2007), Hennessy and Whited (2005, 2007). My choice of the process also allows for negative shocks that might, eventually, make the firm go into bankruptcy.

⁴Cooper and Ejarque (2003) show that the reduced form profit function \( \pi(k, z) \) can be obtained
marginal profits are likely to arise when firms compete in an imperfect way and/or when firms have limited managerial or organizational resources. The former argument is suggested by Hennessy and Whited (2005) while the latter is suggested by Lucas (1978).

I assume issuing debt and equity is costly. This assumption incorporates the evidence provided by Altinkilic and Hansen (2000) on the importance of underwriter fees. Specifically, I assume the external finance cost function is

\[ c_i(x'^d, x'^e) = \phi_d x'^d + \phi_e x'^e \]  

(1.3)

where \( x'^d \) and \( x'^e \) are the two decision variables. The former refers to debt reductions (-) and increments (+) while the latter refers to equity payouts (-) and issuances (+). Parameters \( \lambda^d \) and \( \lambda^e \) reflect the per-unit costs of issuing debt and equity, respectively. The indicator function \( \phi_d \) equals 1 if \( x'^d > 0 \), and 0 otherwise. Similarly, the indicator function \( \phi_e \) equals 1 if \( x'^e > 0 \), and 0 otherwise. I assume that the costs of external finance can be deducted from earnings for tax purposes.

Cooper and Haltiwanger (2006) show that the investment rate distribution at the micro-level is highly skewed to the right, and that a dynamic model that includes capital irreversibility creates such a positive asymmetry. Accordingly, I introduce a cost function of reversing capital

\[ c_p(k', k) = \phi_p \lambda^p |i| \]  

(1.4)

where \( i = k' - k(1 - \delta) \) is current investment and the indicator function \( \phi_p \) equals 1 if \( i < 0 \), and 0 otherwise. Parameter \( \lambda^p \) captures the per-unit costs of reversing capital.

in the following context: let \( p = Sy^{-\eta} \) be the demand function where \( S \) is a demand shock and \( \eta \) is the inverse of the demand elasticity, and let \( y = \omega K^{\varphi}(1 - \varphi) \) be the production function. Then maximizing the profit function over the flexible factor, \( l \), leads to a reduced form profit function like equation (1.2) where \( z \) reflects changes in demand, factor prices and technology. In that context, parameter \( \alpha \) is given by \( \alpha = \varphi(\eta - 1)/(1 - \varphi)(1 - \eta) - 1 \), and it reflects both the elasticity of demand and the marginal productivity of capital in the production function.
I also assume that the costs of partial reversibility of capital can be deducted from earnings for tax purposes.

Corporate earnings are taxed at rate \( \tau_c \). Therefore, the firm’s net income is defined by

\[
NI = \left[ \pi (k, z) - \delta k - c_i \left( x^d, x^e \right) - c_p (k', k) - r_d (k, d, z^{-}) d \right] (1 - \tau_c) \tag{1.5}
\]

where \( r_d(k, d, z^{-}) \) is the bond yield required by debt-holders during the last period. It is noticeable that the bond yield \( r_d \) depends on the previous-period values of \( z \) (i.e., \( z^{-} \)). The reason for this will be shown below.

Equation (1.5) shows a key component of the dynamic model: the benefits of debt created by the interest tax shield. Every period, the dynamic model trades-off these benefits against the costs of debt (i.e., bankruptcy costs) and decides optimal investment, leverage, and payout to shareholders. Therefore, these two components capture the fundamental driving forces of the trade-off theory.

Finally, the internal cash flow of the firm is

\[
ICF = NI + \delta k
\]

\[
= \left[ \pi (k, z) - \delta k - c_i \left( x^d, x^e \right) - c_p (k', k) - r_d (k, d, z^{-}) d \right] (1 - \tau_c) + \delta k \tag{1.6}
\]

and its utility in each period, or cash flow to investors, is

\[
u (k, d, z, z^{-}, x^d, x^e) = ICF - i + r_d (k, d, z^{-}) d
\]

\[
= -x^d + r_d (k, d, z^{-}) d - x^e. \tag{1.7}
\]

This is the cash flow received (+) or provided (-) by firm investors (i.e., debt-holders and shareholders) in each period.

I now define the transition functions and state space of the two endogenous state variables. The state equation of capital \( k \) satisfies the accounting cash-flow equation and is defined as

\[
k' = k (1 - \delta) + ICF + x^d + x^e. \tag{1.8}
\]
The capital of the firm, $k$, takes values in the compact set $K = [0, \bar{k}]$ where $\bar{k}$ is the maximum level of capital. Following Gamba and Triantis (2008), I assume that under the highest profitability shock, $\bar{\pi}$, capital $\bar{k}$ satisfies

$$\pi(\bar{k}, \bar{\pi}) - (\delta + r_f) \bar{k} = 0. \tag{1.9}$$

Intuitively, at capital level $\bar{k}$, operating profits just cover depreciation and the opportunity cost of capital. Therefore, it is not economically profitable to accumulate capital to a level $k > \bar{k}$.

The transition function of debt $d$ is defined as

$$d' = d + x^d. \tag{1.10}$$

The variable $d$ takes values in the compact set $D = [0, \bar{k}]$.

The previous restrictions on the state space of $k$ and $d$ bind the decision variables $x^d$ and $x^e$ to a compact set. Specifically,\footnote{This formulation of debt allows for the possibility that $d = k$. However, this is without any loss of generality because that event is never optimal in the dynamic model at the estimated values of the parameters.}

$$x^d \in [-d, \bar{k} - d] \text{ and } x^e \in [- (k - d), \bar{k} - (k - d)]. \tag{1.11}$$

I can now describe the dynamic behavior of the firm over time. During period $t$, the firm operated with capital $k$ and had debt $d$. At the end of period $t$ (i.e., at moment $t$) the profitability shock $z$ is realized and the complete state vector, $(k; d, z, \bar{z})$, is available to determine the value of the firm. If that value is below the nominal value of the debt, the firm goes into bankruptcy and is liquidated, that is, the dynamic process stops. Otherwise, it continues in the business and selects $x^d$ and $x^e$. These decisions maximize firm value and determine capital $k'$ and debt $d'$ to be used during period $t + 1$. Figure 1.1 shows the model’s time line.

The firm model in the present study is based on the trade-off theory of capital structure, which suggests that the use of debt to finance investments has costs and
bene\textbf{f}its. In the model, the benefits of debt include the interest tax shield, while the costs of debt include the expected bankruptcy costs. Thus, every period, after the firm receives a profit shock, it reassesses the costs and benefits of debt in this new state and determines the optimal leverage for the next period. Then, it adjusts leverage (i.e., selects $x^d$ and $x^e$) from the current ratio to the optimal level. Figure 1.2 shows firm value (normalized by the value of the all-equity firm) for different levels of leverage after receiving a new profit shock. In that specific case, firm value would be maximized at a leverage ratio of around 45%. Therefore, the firm in the dynamic model would choose $x^d$ and $x^e$ so that leverage becomes 45% during the next period.

As stated above, the objective of the firm is to maximize its value, which is achieved by maximizing the expected discounted sum of cash flows to investors. Therefore, the maximization problem faced by the firm is

$$v_0 = \sup_{x^d_t, x^e_t} E \sum_{t=0}^{\infty} \frac{1}{(1 + r_f)^t} y_t \text{ subject to } d_t < k_t$$  \hspace{1cm} (1.12)

where $v_0$ is the current value of the firm, $E$ is the expectation given current information (i.e., initial capital stock, debt and profitability shock), and $y_t$ is defined as

$$y_t = u \left( k_t, d_t, z_t, z_{t-1}, x^d_t, x^e_t \right) \text{ if } v_t > d_t$$

$$y_t = L_t, \quad y_{t+1} = y_{t+2} = \ldots = 0 \text{ if } v_t \leq d_t$$  \hspace{1cm} (1.13)

where $L_t$ is the liquidation value of the firm at moment $t$ as described below by equation (1.14). The interpretation of the maximization problem is as follows: every period the firm must choose $x^d_t$ and $x^e_t$ in such a way that it maximizes the expected discounted sum of cash flows to investors (this refers to the first line of equation (1.13)). However, if in any period the value of the firm falls below the nominal value of the debt, it goes into bankruptcy, receives its liquidation value and zero thereafter (this refers to the second line of equation (1.13)).

The model assumes the debt is protected with a positive net worth covenant, and thus the bankruptcy-triggering event consists in the value of the firm falling
below the nominal value of the debt. In that case, the firm goes into bankruptcy and is liquidated. In this formulation of the model, bankruptcy would mean straight liquidation under Chapter 7 of the Federal Bankruptcy Reform Act (FBRA) of 1978. Accordingly, the (liquidation) value of the firm that goes into bankruptcy is

$$ v (k, d, z, z^-) = L = [k (1 - \delta) + ICF_b] (1 - \xi) $$

(1.14)

where $ICF_b = [\pi (k, z) - \delta k] (1 - \tau_c) + \delta k$ is the internally generated cash flow in the period previous to bankruptcy and $\xi$ reflects the direct costs of bankruptcy. Intuitively, the liquidation value of the firm is the realization into cash of total assets (depreciated capital plus internal cash flow) minus the direct costs of bankruptcy. Equation (14) shows another key component of the dynamic model: the costs of debt created by the bankruptcy costs. Every period, the firm trades-off the benefits of debt (i.e., the interest tax shield) against these costs of debt and decides optimal investment, leverage, and payout to shareholders. As I mentioned before, these two components represent the essential driving forces of the trade-off theory in the dynamic model.

In the event of bankruptcy, the recovery amount accruing to debt claimants is

$$ R (k, d, z) = \min \{L, d\} = \min \{[k (1 - \delta) + ICF_b] (1 - \xi), d\} . $$

(1.15)

Equation (1.15) means that the value accruing to debt-holders is the minimum between the liquidation value of the firm and the nominal value of the debt. Accordingly, fair pricing of the debt requires the fulfillment of the following equation

$$ d' = \frac{1}{1 + r_f} \int [(1 - \phi_b) d' (1 + r_d) + \phi_b R (k', d', z')] F (dz' | z) . $$

(1.16)

This equation means that debt-holders require a bond yield, $r_d$, which equates the nominal value of the debt (left-hand side) to the expected discounted payoff of debt in next period (right-hand side). If the firm avoids bankruptcy, the debt payoff is the nominal value plus the promised yield, $d' (1 + r_d)$. On the contrary, if the firm does go
into bankruptcy, the debt payoff is the recovery amount, \( R(k', d', z') \). Furthermore, the bond yield required by debt claimants can be solved for explicitly and is given by

\[
r_d(k', d', z) = \frac{1 + r_f - \frac{1}{1 + r_f} \int \phi_b R(k', d', z') F(dz' | z)}{\int (1 - \phi_b) F(dz' | z)} - 1.
\] (1.17)

The last step to complete the dynamic model of the firm is to describe its recursive formulation. Let \( v = v(k, d, z, z^-) \). Then, the value of the firm that does not go into bankruptcy is given by the following Bellman equation\(^6\)

\[
v = \sup_{x^d, x^e} \left\{ u(k, d, z, z^-, x^d, x^e) + \frac{1}{1 + r_f} \int [(1 - \phi_b) v' + \phi_b L'] F(dz' | z) \right\}
\] subject to \( d' < k' \)

where the indicator function \( \phi_b \) equals 1 if \( v(k', d', z', z) \leq d' \), and 0 otherwise. The value function is the maximized value of the firm, namely, the maximized expected discounted sum of cash flows to investors.

The implementation of the model is described in detail in Appendix I. Finally, the discount factor is \( \beta = 1 / (1 + r_f) \) and the complete vector of model parameters is \( (\beta, \delta, \rho_z, \sigma_z, \alpha, \lambda^d, \lambda^e, \lambda^p, \alpha_c, \xi) \).

1.3 Data

The data used in this study are from the yearly Standard & Poor’s Compustat industrial files. The sample includes all firms in the database and covers the period 1988 to 2009. I choose to start the sample in 1988 to avoid the structural break implied by the Tax Reform Act of 1986. To select the sample, I follow a procedure similar to that of Hennessy and Whited (2007). I delete firm-year observations with missing or negative data. Furthermore, I include in the sample only firms that have at least two consecutive years of data. Finally, I exclude regulated, financial or public service

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\(^6\)The recursive formulation of the firm problem is equivalent to the maximization problem given by equation (1.12).
firms, that is, I exclude all firms whose primary SIC code is between 4900 and 4999, between 6000 and 6999, or greater than 9000. After this procedure, the final sample includes 14,465 different firms and 111,944 firm-year observations. The final sample is an unbalanced panel with a minimum of 4,295 and a maximum of 6,457 observations per year.

Data variables are defined in the following way: capital $k$ is Book Assets - Total (variable “at” or data item 6) and debt $d$ is Liabilities - Total (variable “lt” or data item 181). For estimation purposes, I compute the following two decision variables: investment ratio, $i$, and book leverage, $l$. I choose the investment decision to be a ratio because that allows me to homogenize the investment decision of firms of different size. Therefore, these decision variables are computed as

\[
    i = \frac{[k' - k(1-\delta)]}{k(1-\delta)} = \frac{ICF + x^d + x'}{k(1-\delta)}
\]

\[
    l = \frac{d'}{k'} = \frac{d + x^d}{k(1-\delta) + ICF + x^d + x'}.
\] (1.19)

Table 1.1 shows summary statistics of the sample. The investment ratio decision has a mean value of 44.53% per year, while the book leverage decision has a mean value of 46.35%. Investment is generally positive, it has a right skew, and is leptokurtic. These findings are consistent with the evidence documented by Cooper and Haltiwanger (2006). By contrast, the distribution of book leverage is closer to the normal distribution. Leverage skewness is slightly positive at 0.06537. Since leverage is bounded between 0 and 1, its distribution is platykurtic.

The Jarque-Bera (1987) test takes the sample skewness and kurtosis and evaluates the null hypothesis that the data are from a normal distribution. The Jarque-Bera test statistic has an asymptotic chi-square distribution with two degrees of freedom and the 90% critical value for a $\chi^2(2)$ random variable is 5.99. The large statistics, 1,620,612,973,856.78 for investment and 2,996.77 for book leverage, flatly reject the normality assumption for both decisions.

Figures 1.3 and 1.4 graphically display the previous summary statistics of the mar-
ginal distributions of investment and leverage decisions, respectively, for the sample.

Table 1.2 shows summary statistics for a subset of the sample. I eliminate observations with investment greater than 200% to make clear the distribution of investment decisions around zero. After this filter, the subset of the sample has 109,049 firm-year observations. These summary statistics can be graphically observed in Figures 1.5 and 1.6, which show the marginal distributions of investment and leverage decisions, respectively, for the subset of the sample. It is apparent that the investment distribution is relatively smooth and skewed to the right, which is a feature that the model should be able to reproduce.

1.4 Estimation

This section presents the estimates of the structural parameters of the dynamic model. The estimation technique I use is the Efficient Method of Moments (EMM) developed by Gallant and Tauchen (1996). The underlying idea of the methodology is to match the moments generated by the simulation of the model to those implied by the observed data. An important benefit of EMM is that I do not have any discretion in the selection of the moments for the estimation of the structural model, that is, it is a systematic approach for choosing moments when using Generalized Method of Moments (GMM).

EMM estimation consists of two steps. In the first step, I characterize the statistical properties of the data and select the density function that best fits the sample. In the second step, I recover the structural parameters of the dynamic model.

1.4.1 Description of the Statistical Properties of the Data

I use the semi-nonparametric (SNP) density function, as proposed by Gallant and Tauchen (1989), to describe the statistical features of the data (i.e., investment and leverage decisions). The SNP density is a general-purpose model with which I generate
a family of density functions for the joint distribution of the data. I then select the
density function that best characterizes the data in the most parsimonious way.

Appendix 2 describes the construction of the SNP density for the present study.
I find that the SNP density function that best fits the sample of investment and
leverage decisions is a bivariate normal density function with a VAR(1) structure for
the mean and an ARCH(3) structure for the variance. The significant conditional
heteroskedasticity parameters support the sample evidence that investment rate has
a kurtosis of 8.08297, well above that of the normal distribution, 3.

1.4.2 Estimation of Structural Parameters

In the second step, I estimate the vector of parameters of the dynamic model, \( \rho = (\delta, \rho_{c}, c_{v}, \sigma_{v}, c, \alpha, \lambda^{d}, \lambda^{e}, \lambda^{p}, \xi) \). From the previous step, I have the SNP density that
best characterizes the data. The score function of this density is used to construct the
vector of moments that will be used in the GMM objective function. For a candidate
set of parameters, I simulate the model and compute the objective function. Then, by
using a nonlinear optimizer, I search for the parameter values that minimize the GMM
criterion. I use the simulated annealing minimization algorithm, which is robust to
local minima.

After simulating the decisions of the firm, the moment equations are computed
as\(^{7}\)

\[
m(\rho, \tilde{\theta}_{n}) = \frac{1}{N} \sum_{t=1}^{N} \frac{\partial}{\partial \theta} \log f \left( \tilde{y}_{t} | \tilde{x}_{t-1}, \tilde{\theta}_{n} \right)
\]

(1.20)

where \( \{\tilde{y}_{t}\}_{t=1}^{N} \) are the simulated data, \( N \) is the length of the simulation, and \( \tilde{\theta}_{n} \) is the
quasi-maximum likelihood estimate of the parameter vector, \( \theta \), of the SNP density
function selected in the first step.

\(^{7}\)The definition of function \( f(y_{t}|x_{t-1}, \theta) \) is in Appendix 2.
The EMM estimator is

\[ \hat{\rho}_n = \arg \min_{\rho} m \left( \rho, \tilde{\theta}_n \right)' \left( \tilde{I}_n \right)^{-1} m \left( \rho, \tilde{\theta}_n \right) \]  

(1.21)

with \( \tilde{I}_n = \frac{1}{n} \sum_{t=1}^{n} \left[ \frac{\partial}{\partial \theta} \log f \left( y_t | x_{t-1}, \tilde{\theta}_n \right) \right] \left[ \frac{\partial}{\partial \theta} \log f \left( y_t | x_{t-1}, \tilde{\theta}_n \right) \right]' \), where the weighting matrix \( \tilde{I}_n \) assumes the SNP density function closely approximates the true stochastic process of \( y_t \).

Under standard regularity conditions, the EMM estimator is consistent and asymptotically normal.\(^8\) Furthermore, if the SNP density exactly replicates the true data generating process, then the EMM estimator is fully efficient. If the SNP density closely approximates the true data generating process, then the efficiency of the EMM estimator will be close to that of maximum likelihood.

A concern with the model described by the Bellman equation (1.18) is that it refers to only one firm (i.e., a single agent) while the sample described in Section 3 comprises a panel of firms. Lemmon, Roberts, and Zender (2008) find important unobserved firm-specific heterogeneity related to leverage decisions in the Compustat sample. One possible solution to this issue is to either take the unobserved heterogeneity out of the sample, or add structural heterogeneity to the model. Given the current formulation of the model, I can easily follow the latter approach and add random effects to parameter \( \sigma^\epsilon \), which captures conditional standard deviation of the profitability shock. Under that specification, \( \sigma^\epsilon \) becomes

\[ \sigma^\epsilon = c_{\epsilon} + \nu, \ \nu \sim N \left( 0, \sigma_{\nu} \right) \]  

(1.22)

This modification allows me to simulate a panel of firms and reproduce the observed variation of investment and leverage decisions in a more accurate way. By adding random effects to parameter \( \sigma^\epsilon \), I create a panel of 11 different independent firms. For each firm, I set a simulation length of 100,000 periods after discarding

\(^8\)Gallant and Tauchen (1996) and Gallant and Long (1997) provide the proofs of the asymptotic properties of the EMM estimator.
the first 5,000 periods to avoid the influence of starting values. As before, I assume $\beta = 0.98$ and $\tau_c = 40\%$. Therefore, the vector of model parameters to estimate is $\rho = (\delta, \rho_z, c_z, \sigma_\nu, c, \alpha, \lambda^d, \lambda^e, \lambda^p, \xi)$. Table 1.3 exhibits EMM point estimates of model parameters, standard errors and t-statistics. All estimated parameters are highly statistically significant, as suggested by the large t-statistics.

The estimate of the depreciation parameter, $\delta$, at 0.05728 per year is a reasonable value considering that the mean investment ratio, shown in Table 1.1, is 0.06215 per year. The estimate of the persistence parameter of the profitability shock process, $\rho_z$, is 0.62957. This value implies that the profitability shock is positively serially correlated and that the current shock has an important influence on the shock in next period. In addition, the estimate is similar to that reported by Hennessy and Whited (2007). The estimated mean value for the conditional standard deviation of the profitability shock, $c_z$, is 1.13021 and is close to the value estimated by Cooper and Ejarque (2003). The estimate for the standard deviation of the conditional standard deviation of the profitability shock, $\sigma_\nu$, is 0.12562. I find parameter $c$ to be 0.90827, which implies that the unconditional mean of profitability shock is positive (given that $\rho_z$ is also positive).

The estimate of the parameter related to the concavity of the profit function, $\alpha$, is 0.61616 - consistent with the value found by Cooper and Haltiwanger (2006), and Hennessy and Whited (2007). Parameters $\lambda^d$ and $\lambda^e$ are estimated at 0.00523 and 0.10100, respectively. These values are close to those reported by Altinkilic and Hansen (2000), and Hennessy and Whited (2007). My estimate of the parameter that captures the per-unit costs of partial reversibility of capital, $\lambda^p$, is 0.05086, which is slightly above the value documented by Cooper and Haltiwanger (2006). Finally, the estimated value of the direct costs of bankruptcy parameter, $\xi$, is 0.15333, which is slightly above that reported by Hennessy and Whited (2007).
1.5 Identification

I discuss the sources of identification of $\rho = (\delta, \rho_z, c_z, \sigma_z, c, \alpha, \lambda^d, \lambda^e, \lambda^p, \xi)$, the vector of structural parameters of the model that will be estimated. These parameters are the capital depreciation rate ($\delta$), the persistence parameter of the profitability shock ($\rho_z$), the parameters that capture the scale of the profitability shock ($c_z, \sigma_z$), the constant coefficient of the AR(1) process of the profitability shock ($c$), the concavity of the profit function parameter ($\alpha$), the per-unit costs of issuing debt ($\lambda^d$) and equity ($\lambda^e$), the per-unit costs of partial reversibility of capital ($\lambda^p$), and the direct costs of bankruptcy parameter ($\xi$). To achieve identification of these ten parameters, I assume the other two model parameters have the following values: $\beta = 0.98$, $\tau_c = 40\%$. The value of $\beta$ implies a risk-free rate of interest, $r_f$, of 2.04% per year. I fix the value of the tax rate, $\tau_c$, because it was relatively constant during the period 1988-2009.

Variability of firm investment is informative about the concavity of the profit function parameter, $\alpha$. The lower the parameter, the lower the marginal profitability of the firm, which means that firm investment should respond less aggressively to profitability shocks. The mean of investment is informative about the capital depreciation rate, $\delta$. A larger depreciation rate should imply more investment on average. The magnitude of capital reductions (negative investments) helps me identify the per-unit costs of partial reversibility of capital ($\lambda^p$). Larger costs of reversing capital should induce smaller capital reductions.

The relative size of debt and equity issuances helps me identify the per-unit costs of issuing debt, $\lambda^d$, and equity, $\lambda^e$. Larger costs of issuing debt compared to equity should induce relatively larger issuances of equity, and vice versa. The maximum level of firm leverage is informative of the direct costs of bankruptcy, $\xi$. Higher maximum levels of leverage imply lower direct costs of bankruptcy. Finally, the parametric assumptions about profitability shock (i.e., state variable $z$ follows an AR(1) process) plus the joint variation in investment and leverage (e.g., the persistence in both
decisions) help me identify the remaining parameters of the process, \((\rho_z, c_z, \sigma_\nu, c)\).

1.6 Model predictions about capital structure choice

The literature has two seemingly opposite positions about the behavior of target leverage over time. On one hand, Hovakimian, Opler, and Titman (2001), Flannery and Rangan (2006), and Huang and Ritter (2009) suggest that firms adjust their leverage toward time-varying targets. They argue that these targets vary over time as firm characteristics and market conditions, such as profitability and growth opportunities, change. On the other hand, Lemmon, Roberts, and Zender (2008) suggest that firms rebalance their leverage toward a largely time-invariant target, and that the role of time-varying factors in the determination of target leverage is negligible.

I investigate these issues in the following subsections and find that the dynamic model provides support for the former position. First, target leverage is state-dependent, that is, it varies over time as firm characteristics and market conditions change, which is consistent with the first group. Second, leverage decisions mean-revert to a long-run constant level, which is the mean of the unconditional distribution of leverage decisions. This property of the model generates the results of Lemmon, Roberts, and Zender (2008). However, the difference with these authors is in the conceptual interpretation of the long-run constant level. In the context of the dynamic model, that level is not the time-invariant target leverage, but the mean of the asymptotic distribution around which target leverage oscillates. The firm does not have incentives to converge to that mean value, that is, the firm does not have a target leverage in the long-run sense. The observed mean-reversion of leverage is mainly due to the fact that, in the dynamic model, leverage decisions inherit the mean-reverting property from the profitability shock.

Overall, these findings suggest that the apparently opposite empirical evidence presented by both positions might be reflecting different aspects of the same decision
making process.

1.6.1 Evolution of leverage decisions over time

In this subsection, I describe the behavior of leverage over time for a single firm. I start analyzing the unconditional distribution of leverage decisions generated by the dynamic model. In the dynamic model, the firm decides its optimal level of leverage in each period. The firm makes this choice as a function of the current state of the system (i.e., previous-period profitability shock and current debt, assets, and profitability shock). This implies that optimal leverage changes from period to period as the firm receives different profitability shocks over time. Thus, by simulating a long sequence of profitability shocks, I can recover the asymptotic distribution of optimal leverage choices by the firm. I proceed to simulate that distribution and the unconditional mean of leverage decisions turns out to be 44.18%. These results come from the simulation of a single firm at the values of the parameters estimated in Section 4; in particular, the firm is simulated with \( \sigma_\epsilon \) at the mean of the distribution (i.e., at \( \sigma_\epsilon = c_\epsilon \)).\(^9\) Figures 1.7 and 1.8 show the unconditional distributions of optimal investment and leverage decisions generated by Next, I stuthe dynamic model for the single firm, respectively.

Equation (1) defines the profit shock, \( z \), as an AR(1) process and the estimate of the persistence parameter, \( \rho_z \), is 0.62957, which implies that the profitability shock follows a mean-reverting process with a speed of mean-reversion of about 37% per year. Figure 1.9 shows the evolution of profit shocks over time at the values of the parameters estimated in Section 4. The sequences of high shocks represent periods (years) of economic boom, while those sequences of low shocks represent periods (years) of economic depression. It is apparent that profit shocks follow a mean-

\(^9\)Simulating firm decisions with \( \sigma_\epsilon \) at different values of the support does not qualitatively change the previous results.
reverting process. Figure 1.10 shows the unconditional distribution of profit shocks, which is a normal distribution.

Next, I investigate whether leverage, as a function of the profit shock, inherits its mean-reverting property, that is, whether leverage oscillates around a long-run mean level. A standard autoregressive model of book leverage is

$$\frac{d_{t+1}}{k_{t+1}} - \frac{d_t}{k_t} = a_1 \left( b_{ML} - \frac{d_t}{k_t} \right) + \varepsilon_{t+1}$$  \hspace{1cm} (1.23)

where $d_{t+1}/k_{t+1}$ is book leverage, $b_{ML}$ is the long-run mean book leverage, and $a_1$ is the fraction of the gap between current book leverage and the long-run mean level that the firm closes every period. After rearranging terms, I estimate the following model

$$\frac{d_{t+1}}{k_{t+1}} = a_1 b_{ML} + (1 - a_1) \frac{d_t}{k_t} + \varepsilon_{t+1}.$$  \hspace{1cm} (1.24)

The estimates suggest that firm leverage is, indeed, mean-reverting to the long-run constant value. The coefficient $a_1$ is 0.3350 with a $t$-statistic of 898.38. This estimate implies a moderate speed of adjustment of 33.50% per year and a half-life of about 1.70 years, that is, it takes the firm approximately 1.70 years to close half the gap between its current book leverage and the long-run mean level. This estimated speed of mean-reversion is close to that found by Flannery and Rangan (2006), who suggest a speed of adjustment of 34.40% per year. DeAngelo, DeAngelo and Whited (2010) also report a similar speed of adjustment of 37.80% per year. For market leverage, $d_{t+1}/v_{t+1}$, coefficient $a_1$ is 0.2802 with a $t$-statistic of 806.62, which means a speed of mean-reversion of 28.02% per year and a half-life of about 2.11 years.\(^\text{10}\) The estimated long-run constant value, $b_{ML}$, is 0.4418, which implies that book leverage oscillates around 44.18%. This mean level is exactly equal to the mean of the unconditional distribution of leverage decisions described above. Furthermore,

\(^{10}\)The denominator of market leverage, $v_{t+1}$, is the value function calculated in the Bellman equation (1.18).
this constant value does not depend on the current state of the system (i.e., previous-period \( z \) and current \( d, k, \) and \( z \)) or on functions of that state (e.g., operating profits or market-to-book ratio), but on the structural assumptions about the firm, such as the functional form of the utility, the persistence of profitability shocks, the concavity of the profit function, etc. Figure 1.11 shows the evolution of book leverage decisions over time for the single firm. For completeness, Figure 1.12 shows the evolution of investment decisions over time for the single firm.

To summarize, the results of this subsection show that, in the dynamic model, leverage varies over time and oscillates around a constant value, which is the mean of the unconditional distribution of leverage decisions.

### 1.6.2 Sensitivity analysis: leverage decisions

Every period, the firm selects the optimal leverage, which attains its maximum value. This maximization process includes the costs of debt and equity issuance. The literature considers that target leverage is the optimal leverage ratio that the firm would choose in the absence of adjustment costs. Given that the costs of adjustment are always positive, target leverage is unobserved. Therefore, I do sensitivity analysis and study the evolution of leverage decisions over time for a single firm under zero costs of issuance. By doing this, I recover the target leverage and investigate its evolution over time. Additionally, I study the effect of changes in the costs of debt and equity issuance on the speed of mean-reversion of leverage and on the mean value of the unconditional distribution of leverage decisions.

I start solving the model for a single firm at the estimated values of the parameters, an income tax rate \((\tau_c)\) of 40\% and a discount factor \((\beta)\) of 0.98. This is the base case used to analyze the optimal leverage decisions described in Subsection 6.1, and includes per-unit debt \((\lambda^d)\) and equity \((\lambda^e)\) issuance costs of 0.00523 and 0.10100, respectively. I then generate a long simulation of optimal policies and calculate the
speed of mean-reversion and mean book leverage for this specification. I set the simulation length at 4,000,000 observations and eliminate the first 1,000 observations to remove the effects of an arbitrary starting point. Table 1.4 shows these values in column (6) under heading “100%”.

Then I change both debt and equity issuance costs in the same proportion, solve the model again, generate a new long simulation of optimal policies and proceed to calculate the speed of mean-reversion and mean book leverage for the new specification. For example, in column (7) I increase both adjustment costs by 20%. This increment reduces the speed of mean-reversion of book leverage by almost 2% from 33.50% to 31.64% per year, while the mean of book leverage slightly diminishes to 44.06%. These results are confirmed when I increase the adjustment costs even further in columns (8) through (11).

On the other hand, if I reduce debt and equity issuance costs, both the speed of mean-reversion and the mean of book leverage decisions increase. For example, reducing both adjustment costs by 20% in column (5) makes the speed of mean-reversion of leverage go up to 35.18% per year, while the mean of book leverage barely increases to 44.48%. I verify these results by reducing the adjustment costs even further in columns (4) through (1). In particular, Table 1.4 shows that, with zero issuance costs, firm leverage is mean-reverting to a long-run constant value at a considerably higher speed of adjustment of 50.48% per year, which implies a half-life of 0.99 years. This result implies that target leverage oscillates around a constant value. That is, target leverage is varying over time as firm characteristics and market conditions change.

The negative relationship between issuance costs and speed of mean-reversion of book leverage is due to the fact that, when adjustment costs are lower, the firm adjusts its leverage to profitability shocks in a more prominent way. For example, if the firm receives a positive profit shock, it will increase its leverage, but the increment in leverage will be more significant if issuance costs are lower. On the contrary,
the impact of these costs on the mean of the unconditional distribution of leverage decisions is negligible.

Figure 1.13 graphically shows these results. The speed of mean-reversion of book leverage decreases as the costs of issuing debt and equity increase. Furthermore, the line is convex for adjustment costs above 20% of the estimated ones. Below that threshold, the speed of mean-reversion tends to stabilize around 50% per year because the impact of the adjustment costs becomes insignificant in the context of the current discretization of the state space.

Figure 1.14 shows the mean of the unconditional distribution of book leverage decisions as a function of debt and equity issuance costs. This line is almost horizontal because, as noted above, the effect of adjustment costs on the mean of book leverage is negligible. Figure 1.15 shows the evolution of book leverage decisions over time for a single firm when there are no adjustment costs.

Overall, these results show that, in the dynamic model, target leverage is time-varying and fluctuates around a long-run time-invariant level. That is, firms in the dynamic model do not have a constant target in the long-run sense to which they try to adjust their current leverage.

1.6.3 Lemmon, Roberts, and Zender (2008) leverage portfolios

I study whether the dynamic model can reproduce the characteristics of the data reported by Lemmon, Roberts, and Zender (2008). To this end, I construct a panel of firms, for which I need to introduce heterogeneity to the model. The way in which the literature (e.g., DeAngelo, DeAngelo and Whited (2010)) has done this is by simulating the model for different values of a parameter. Then, the panel is composed of different types of firms. Each type is different in the conditional standard deviation of the profitability shock, \( \sigma_e \), which implies that each firm type has a different variance of returns. Figure 1.16 shows the book leverage decisions of
two different firm types. The upper line shows leverage decisions of a firm with low conditional standard deviation of the profitability shock, while the lower line shows leverage decisions of a firm with high variance of returns. The panel is composed of 10 types or groups of firms and there are 100 identical firms in each group. Each type is different in parameter $\sigma_e$, which ranges from 0.7 to 1.5. This source of heterogeneity introduces variation in the mean value of the asymptotic distribution of leverage decisions. The complete panel is then composed of 1,000 firms, and the remaining parameter values used to generate the panel are those estimated in Section 4. Each firm is simulated over 1,000 periods after discarding the first 1,000 observations to eliminate the influence of an arbitrary initial point.

Figure 1.17 shows the actual book leverage paths generated by a panel of firms created by the dynamic model. I follow the procedure used by Lemmon, Roberts, and Zender (2008) to construct the four portfolios (i.e., “very high”, “high”, “medium” and “low” leverage portfolios). It is evident in Figure 1.17 that the dynamic model, which creates time-varying target leverage, generates leverage decisions characterized by both a transitory and a permanent component, as described by Lemmon, Roberts, and Zender (2008) in their Figure 1.1. The former component refers to the fact that firms with relatively high (low) leverage are likely to return to more moderate levels of leverage, while the latter component implies that firms with relatively high (low) leverage tend to keep relatively high (low) leverage for long periods of time (more than 20 years). In the context of the dynamic model, the values to which these lines converge in the long-run are the average of the means of the asymptotic distributions of leverage decisions of all the firms included in the corresponding portfolio.

Overall, the dynamic model consistently replicates the features of leverage (i.e., convergence and persistence) described by Lemmon, Roberts, and Zender (2008). I study these characteristics in the following subsections.
1.6.4 Convergence of leverage to more moderate levels

A prominent feature of the data reported by Lemmon, Roberts, and Zender (2008) is the observed convergence of leverage to more moderate levels over time, which is consistently reproduced by the dynamic model. I study whether this firm behavior in the dynamic model is due to active rebalancing of leverage toward a long-run time-invariant target. This possibility is suggested by Graham and Harvey (2001), Leary and Roberts (2005), Hovakimian (2006), and Kayhan and Titman (2007). The observed decay of leverage over time in the dynamic model is due to the fact that leverage reverts to the mean of the asymptotic distribution of leverage decisions. In Subsection 6.1, I showed that leverage is mean-reverting because it is a function of the profitability shock, which is defined as mean-reverting. Therefore, leverage is mean-reverting because it inherits that property from the profitability shock. In other words, the mean-reversion of leverage is mechanical because it is driven by the mean-reverting profit shocks, and not by the active efforts of the firm to rebalance its leverage toward a largely constant level.

Following Lemmon, Roberts, and Zender (2008), I study the net security (i.e., debt and equity) issuance activity for the four unexpected leverage portfolios. Net debt issued is computed as the change in total debt scaled by beginning-of-period assets, while net equity issued is computed as the change in total equity scaled by beginning-of-period assets.

Figure 1.18 exhibits the net debt issuing activity of the firms in the four portfolios. It is clear that, initially, there is a negative relationship between firm leverage and the propensity to issue net debt. As we move from the Low portfolio to the Very High portfolio, the average issuance of net debt decreases monotonically. This different behavior among the portfolios tends to disappear after 5 years. These results are similar to those of Lemmon, Roberts, and Zender (2008).

The net equity issuing activity of the firms in the four portfolios is shown in
Figure 1.19. The pattern of net equity issuing activity is initially analogous to that of net debt issuing activity. That is, the propensity to reduce net equity is negatively related to firm leverage. As we move from the Very High portfolio to the Low portfolio, firms reduce progressively more equity on average. Figure 1.19 also shows that this different behavior remains on average in the long-run, which is due to the fact that net equity issuing activity also reflects the impact of dividend payments. As I explain in detail in Subsection 7.1 below, the dynamic model generates a negative relationship between leverage and dividends, which creates the long-run pattern of net equity issuing activity displayed in Figure 1.19.

Even though the previous results might be interpreted as firms actively trying to rebalance their leverage to more moderate levels, this is not the case in the dynamic model. As I showed in Subsection 6.2, firms in the dynamic model do not have a constant target in the long-run sense to which they try to adjust their current leverage. Thus, the described patterns of net security issuance activity are due to the mean-reverting property of leverage, which, in turn, is the consequence of leverage being a function of a mean-reverting profitability shock. In other words, the leverage of firms that are initially in the Low portfolio tends to mean-revert to more moderate levels and, to achieve that objective, firms issue more debt and reduce less equity on average.

Finally, I study the impact of introducing time-varying determinants to the autoregressive model described in equation (23) on the speed of mean-reversion of book leverage. As Lemmon, Roberts, and Zender (2008) suggest, the underlying idea of this exercise is that if the traditional determinants of book leverage play an important role in the determination of the long-run value around which book leverage oscillates, then their exclusion should considerably decrease the speed of mean-reversion of leverage. The reason for this is that firm leverage would be mean-reverting to a value that differs from the one specified by the econometrician.
Accordingly, I estimate the following autoregressive model of book leverage

\[
\frac{d_{i,t+1}}{k_{i,t+1}} = a_1 \left[ b_0 + b_1 \frac{d_{i,0}}{k_{i,0}} + b_2 \left( \frac{1}{T} \sum_{t=0}^{T-1} \frac{d_{i,t}}{k_{i,t}} \right) + b_3 \sigma_{\varepsilon_t} + b_4 \frac{\pi(k_{i,t}, z_{i,t})}{k_{i,t}} + b_5 \frac{v_{i,t}}{k_{i,t}} \right] - (1 - a_1) \frac{d_{i,t}}{k_{i,t}} + \varepsilon_{i,t+1}
\]  

where \(d_{i,t+1}/k_{i,t+1}\) is current book leverage, \(d_{i,0}/k_{i,0}\) is the initial value of book leverage, \((\sum_{t=0}^{T-1} d_{i,t}/k_{i,t})/T\) is mean book leverage, \(\sigma_{\varepsilon_t}\) is the conditional standard deviation of profitability shocks, \(\pi(k_{i,t}, z_{i,t})/k_{i,t}\) is profitability (operating profits), \(v_{i,t}/k_{i,t}\) is the market-to-book ratio (investments opportunities), and \(div_{i,t}/k_{i,t}\) is lagged dividends. Index \(i\) refers to firm \(i\), while index \(t\) refers to time period \(t\). Results are presented in Table 1.5, where the left panel shows the results for the Compustat data, while the right panel shows the results for the simulated panel. In the context of the dynamic model, the correct specification of the long-run value to which leverage mean-reverts should include the mean of the unconditional distribution of leverage decisions. Column (3) shows the results from the specification that includes mean leverage. The speed of mean-reversion is 54.35% per year and the coefficient for mean leverage is close to 1, as expected. In column (2), I exclude mean leverage from the previous model, leaving initial leverage as a proxy for mean leverage, and the speed of mean-reversion decreases considerably to around 31.46% per year as the specification of the constant value is less accurate. The adjusted R-squared also diminishes. Moreover, column (1) shows that if initial leverage is also excluded, then the speed of mean reversion falls even further to 27.77% per year.

Column (6) of Table 1.5 exhibits the estimation results for a specification analogous to that of column (3), but for the simulated panel. The speed of mean-reversion is 38.19% per year, which implies a half-life of about 1.44 years. This result is close to those reported by Flannery and Rangan (2006) and DeAngelo, DeAngelo and Whited (2010), who find speeds of adjustment of 34.40% and 37.80% per year, respectively.

Consistent with the previous claim, column (5) shows that excluding mean leverage (i.e., the most important determinant) substantially reduces the speed of mean
reversion to 20.39% per year. Moreover, column (4) shows that if initial leverage (i.e.,
the proxy for mean leverage) is also excluded, then the speed of mean reversion falls
even further to 11.46% per year. This last result is consistent with those of Fama and
French (2002), who use specifications similar to that in column (4) and find speeds
of mean reversion of book leverage between 10% and 18% per year.

Finally, column (7) presents the results of varying the parameters with which I
simulated the panel, instead of mean leverage. The results are roughly the same, that
is, the speed of mean-reversion of book leverage slightly decreases to 37.92% per year,
the traditional regressors keep their coefficient values and the model fit is the same.
This column exhibits the economic driver of the different mean levels of leverage
decisions, that is, the different conditional standard deviation of the profitability
shock of each type of firm.

The evidence reported in this subsection is consistent with that of Lemmon,
Roberts, and Zender (2008) in the sense that book leverage tends to revert to more
moderate levels over time. However, the leverage behavior created by the dynamic
model suggests that firms do not actively manage their leverage ratios toward long-run
target levels. On the contrary, the convergence of leverage generated by the dynamic
model is a consequence of the mean-reverting property of leverage as a function of
the profitability shock. That is, it is due to mechanical mean-reversion of leverage
and not to active leverage management.

1.6.5 Long-run stability of leverage ratios

In this subsection I study the persistence of leverage, that is, the observation that firms
with relatively high (low) leverage tend to maintain relatively high (low) leverage in
the long-run. I start constructing the “unexpected leverage” portfolios as described by
Lemmon, Roberts, and Zender (2008). First, I regress book leverage on profitability,
market-to-book ratio, and lagged dividends. Second, I use the regression residuals
(i.e., the unexpected leverage) to sort firms into four portfolios. Finally, I follow the average actual leverage of each portfolio during the next 20 years. The objective of this procedure is to remove the observable heterogeneity associated with traditional determinants of leverage. Figure 1.20 presents the unexpected long-run book leverage paths generated by the panel of firms and shows that the results are almost identical to those of Figure 1.17. This finding suggests that the regression residual explains most of the variation in leverage across firms in the panel and that the usual time-varying determinants have little power to explain such variation.

Finally, Figure 1.21 shows the unexpected book leverage portfolios controlling for the conditional standard deviation of the profitability shock, $\sigma_x$. That is, I reproduce the previous experiment but control for the only source of heterogeneity among types of firms. Convergence is complete (i.e., persistence disappears) because, in the regression, the exogenous variables remove all the variation in leverage and the residual is pure noise. Thus, given that the sorting of the portfolios is based on the residual, which is pure noise, the average leverage of the four portfolios completely converge to a single ratio. In unreported figures, the actual and unexpected market leverage portfolios exhibit the same pattern described above.

Next, I use the panel of firms created by the dynamic model to study the possible factors that underlie the persistence of leverage. Each type of firm in the panel has a different mean value around which leverage oscillates. That is, the different structural assumptions of each type of firm create a type-specific constant value toward which leverage mean-reverts. This feature of the dynamic model creates the observed long-run stability of leverage ratios.

First, following Lemmon, Roberts, and Zender (2008), I investigate the role of different exogenous variables in the determination of future leverage. Accordingly, I
estimate the following regression

\[
\frac{d_{i,t+1}}{k_{i,t+1}} = b_0 + b_1 \frac{d_{i,0}}{k_{i,0}} + b_2 \left( \frac{1}{T} \sum_{t=0}^{T-1} \frac{d_{i,t}}{k_{i,t}} \right) + b_3 \sigma_{\epsilon_i} + b_4 \frac{\pi(k_{i,t}, z_{i,t})}{k_{i,t}} + b_5 \frac{v_{i,t}}{k_{i,t}} + b_6 \frac{div_{i,t}}{k_{i,t}} + \varepsilon_{i,t+1}
\]

(1.26)

where the variables are those defined in equation (25).

Table 1.6 shows the results from estimating different specifications of equation (25). The left panel (i.e., columns (1) through (3)) refers to Compustat data while the right panel (i.e., columns (4) through (7)) refers to data simulated with the firm model. Column (1) shows the coefficients corresponding to the regression of leverage on its usual determinants (e.g., profitability, market-to-book ratio, lagged dividends). Even though they are all significant, their explanatory power is roughly zero. The finance literature usually includes other determinants available in Compustat in the regression (e.g., tangibility, industry mean leverage), but the adjusted R-squared barely increases to more than 5%. Lemmon, Roberts, and Zender (2008) add initial leverage as a regressor and the fit increases to more than 20%. They suggest that initial leverage is a proxy for unobserved firm characteristics. However, because leverage decisions are mean-reverting, I introduce the mean of leverage decisions to the specification and the adjusted R-squared increases to around 50%. Nevertheless, initial and mean leverage are tautological in the sense that they are not informative about the economic reasons that drive leverage decisions.

Then, I reproduce the experiment using the simulated panel. The usual determinants (column (4)) explain very little of the variation in leverage with an adjusted R-squared of 11%. Adding initial leverage increases the model fit to 47%, while adding mean leverage increases the adjusted R-squared even further to 70%. However, in the case of the simulated panel, I introduced firm heterogeneity by simulating the model for different values of the parameter that governs the conditional standard deviation of the profitability shock, \( \sigma_\epsilon \), which ranges from 0.7 to 1.5. Thus, in column (7) I substitute mean leverage by the values of the parameters with which I generated the
panel and the results are the same. The traditional regressors keep their significance and coefficient values, and the model fit is the same. But this specification highlights the economic driver of the different mean levels of leverage decisions, i.e., the different conditional standard deviation of the profitability shock of each type of firm. The (unreported) results for market leverage regressions are analogous.

Second, I perform a variance decomposition of book leverage to ascertain the explanatory power of the determinants in the different specifications of equation (25). I use the Type III sum of squares for each effect in the alternative models because it is insensitive to the ordering of the regressors and the fact that the panel is unbalanced. I also normalize the estimates to add up to one.

Table 1.7 shows the fractions of the model sum of squares that is attributable to each variable. As before, the left panel shows the results for the Compustat data, while the right panel shows the results for the simulated panel. Column (1) shows the variance decomposition for the regression of leverage on its usual determinants (e.g., profitability, market-to-book ratio, lagged dividends) excluding initial leverage. In this specification, each of the three variables account for an important part of the explained sum of squares. Column (2) presents the results of adding initial leverage, which accounts for 99.74% of the explained variation of book leverage, while the other determinants account for almost nothing. Column (3) adds mean leverage to the specification in column (2). Not surprisingly, initial leverage loses almost all its explanatory power, which goes to mean leverage. In this specification, initial leverage accounts for only 1.60% of the explained variation of leverage, while mean leverage accounts for 98.21% of the explained sum of squares. Furthermore, the explanatory power of profitability, market-to-book, and lagged dividends is almost zero.

Column (4) shows the variance decomposition for the regression of leverage on its usual determinants (e.g., profitability, market-to-book ratio, lagged dividends) for the simulated panel. In this specification, each of the three variables accounts for an important part of the explained sum of squares. Column (5) presents the results
of adding initial leverage, which accounts for 82.77% of the explained variation of book leverage, while profitability, market-to-book, and lagged dividends account for only 7.29%, 6.27%, and 3.67%, respectively. Column (6) adds mean leverage to the specification in column (5). As with Compustat data, initial leverage loses almost all its explanatory power, which is captured by mean leverage. That is, initial leverage accounts for 0% of the explained variation of leverage, while mean of leverage accounts for 89.65% of the explained sum of squares. In addition, the explanatory power of profitability, market-to-book, and lagged dividends diminishes to 5.68%, 3.39%, and 1.29%, respectively. Column (7) shows the results of the substitution of mean leverage by the values of the parameters with which I simulated the panel. As before, the results are the same, that is, the traditional regressors keep roughly an identical explained sum of squares and the model fit is the same. Again, this specification shows that the economic driver of the different mean levels of leverage decisions in the simulated panel is the different conditional standard deviation of the profitability shock of each type of firm. Unreported results for market leverage regressions show similar conclusions.

To conclude, the dynamic model allows me to better understand the persistence of leverage reported by Lemmon, Roberts, and Zender (2008). I reproduce their experiments but controlling for all sources of heterogeneity among firms, something that they cannot do with Compustat database, and I find that persistence of leverage completely disappears. Therefore, the results I find about leverage persistence go in line with Lemmon, Roberts, and Zender (2008) in that Compustat database might not be sufficiently rich to capture fundamental features of firm behavior regarding leverage decisions.
1.7 Consistency of the model

I study the consistency of leverage and dividend decisions of the dynamic firm model in the context of the trade-off theory and the empirical evidence. To this end, I use the simulated panel of firms described in Section 6.

1.7.1 Analysis of leverage decisions

I investigate the relationship between book leverage, \( \frac{d_{i,t+1}}{k_{i,t+1}} \), and the following variables: profitability or operating profits, \( \pi(k_{i,t}, z_{i,t})/k_{i,t} \), investment opportunities or market-to-book ratio, \( v_{i,t}/k_{i,t} \), lagged dividends, \( div_{i,t}/k_{i,t} \), and non-debt tax shields such as depreciation, \( \delta \).\(^{11}\) I present the results in Table 1.6. The large \( t \)-statistics for all estimated parameters imply that they are highly statistically significant. The results for market leverage are almost identical.

According to the trade-off theory, more profitable firms have more incentives to increase leverage because they have larger taxable income to shield. They also need more discipline from debt to reduce the agency problems generated by free cash flows. Furthermore, higher profitability implies lower expected bankruptcy costs, which induces firms to increase their leverage. However, Titman and Wessels (1988), Rajan and Zingales (1995), Hovakimian, Opler, and Titman (2001), Fama and French (2002) and Flannery and Rangan (2006) present empirical evidence suggesting that the relationship between leverage and profitability is, in fact, negative. Hennessy and Whited (2005) also find a negative relationship between leverage and profitability. Table 1.6 shows that, in agreement with the theory, the dynamic model generates a positive relationship between leverage and profitability. The regression coefficient for profitability is 0.2726 for the model specification in column (6). Thus, the sign of the

\(^{11}\)Dividends are calculated in the following way: \( div_{i,t} = 1[x_{i,t}^d < 0] \ast |x_{i,t}^d| \), where \( 1[x_{i,t}^d < 0] \) is an indicator function equal to one if \( x_{i,t}^d < 0 \) (i.e., when the firm is paying out money to shareholders). Thus, I define dividends in a broad sense, that is, they include cash dividends and share repurchases.
coefficient means that, in periods of high profitability, the firm increases leverage for the above causes.

The trade-off theory also suggests a negative relationship between leverage and investment opportunities. Firms with more growth options do not need so much debt to reduce the agency costs created by free cash flows. They also have stronger incentives to have less leverage to avoid the shareholder-bondholder agency problems that arise when investments are financed with risky debt, such as underinvestment, asset substitution and excessive dividends. The empirical evidence of Hovakimian, Opler, and Titman (2001), Fama and French (2002) and Lemmon, Roberts, and Zender (2008) is consistent with this prediction. In line with these findings, Hennessy and Whited (2005) report a negative relationship between leverage and investment opportunities. Results shown in Table 1.6 also support the trade-off theory. The model specification in column (6) shows a regression coefficient for the market-to-book ratio of -0.0193, which implies that firms with larger investment opportunities have lower leverage. This result is surprising because the model does not include agency costs. The reason for this behavior is the relationship between the profitability shock, investment opportunities, and leverage decisions. When there is a high profit shock, the market value of the firm increases immediately to reflect the high expected shocks in future periods. That is, investment opportunities grow very fast. However, leverage goes up more slowly as a consequence of the adjustment costs. If, later, lower profit shocks appear, the market value of the firm falls immediately even if leverage is still increasing. This different speed of adjustment of leverage and market value of the firm with respect to profit shocks creates the negative relationship between investment opportunities and leverage in the dynamic model.

Regarding the relationship between leverage and dividends, the trade-off theory predicts that it should be negative because they play substitute roles for controlling free cash flow problems. Fama and French (2001, 2002), and Lemmon, Roberts, and Zender (2008) present evidence consistent with this prediction, and Table 1.6 shows
that leverage and (lagged) dividends are, indeed, negatively related. Specifically, the
regression coefficient for lagged dividends is -0.2959 for the model specification in
column (6). In the model, this coefficient is negative because when the firm receives
high profit shocks, it issues equity to increase its size. That is, it pays negative
dividends. At the same time, it increases its leverage because it has larger taxable
income to shield and lower expected bankruptcy costs. The opposite is true when the
firm receives low or negative profit shocks.

The trade-off theory also argues that leverage should be negatively related to non-
debt tax shields, such as deductions for depreciation. Firms with higher depreciation
rates have fewer incentives to increase leverage because they have lower expected tax
rates (i.e., less taxable income to shield). The empirical evidence also supports this
prediction, as reported by Fama and French (2002), and Flannery and Rangan (2006).
A long simulation of the dynamic model for a single firm at the estimated values of the
parameters (i.e., at $\delta = 0.05728$) shows that the mean of the asymptotic distribution
of book leverage decisions is 44.18%. Increasing the depreciation parameter by 1%
to 0.06728, takes that mean value down to 40.21%. This result suggests a negative
relationship between leverage and non-debt tax shields (e.g., depreciation) around the
estimated values of the parameters.

1.7.2 Analysis of dividend payments

This subsection studies the relationship between dividends, $div_{i,t+1}/k_{i,t+1}$, and both
profitability or operating profits, $\pi(k_{i,t}, z_t)/k_{i,t}$, and investment opportunities or the
market-to-book ratio, $v_{i,t}/k_{i,t}$.

To this end, I estimate the following regression

$$
\frac{div_{i,t+1}}{k_{i,t+1}} = b_0 + b_1 \frac{div_{i,0}}{k_{i,0}} + b_2 \frac{\pi(k_{i,t}, z_t)}{k_{i,t}} + b_3 \frac{v_{i,t}}{k_{i,t}} + \varepsilon_{i,t+1}
$$

(1.27)

where $div_{i,0}/k_{i,0}$ is initial dividend. Results are presented in Table 1.8 and, as before,
all estimated parameters are highly statistically significant.

Footnote 12. The definition of dividends, $div_{i,t}$, is in footnote 11.
The trade-off theory predicts a positive relationship between dividends and profitability. Firms with higher profitability have higher dividends to limit the agency problems associated with large free cash flows. They also have higher dividends because they have a smaller chance of having to issue costly risky securities or forego good investments because of asymmetric information between managers and investors. The evidence of Fama and French (2001, 2002), and Lemmon, Roberts, and Zender (2008) also suggests that dividends are positively related to profitability. Table 1.8 shows results consistent with this prediction. The regression coefficient for operating profits is 0.0566 for the model specification in column (5).

According to the trade-off theory, firms with more investment opportunities have lower dividends because they have less need for the discipline of dividends to restrict the agency costs associated with free cash flows. They also have lower dividends to reduce the costs generated by asymmetric information problems between managers and investors. The empirical evidence presented by Fama and French (2001, 2002), and Lemmon, Roberts, and Zender (2008) also suggests that dividends are negatively related to growth options. Consistent with the predictions of the trade-off theory and the empirical evidence, the dynamic firm model generates a negative relationship between the variables. The model specification in column (5) of Table 1.8 exhibits a regression coefficient for investment opportunities of -0.0146.

Figure 1.22 exhibits the evolution of the dividend payout ratio over time for a single firm.

1.8 Sensitivity analysis: income tax payments

Another objective of this paper is to investigate the effect of changes in the income tax rate on the optimal policies generated by the firm model. One question I study is what happens to the income tax payments and leverage decisions made by firms when the tax rate changes. This type of analysis could help the government to estimate
the corporate tax income for different levels of corporate income tax rates. This kind of sensitivity policy analysis is another benefit of structural modeling.

Adam Smith in *The Wealth of Nations* (1776, ch. 2, p. 78) introduced the idea of a negative relationship between tax rates and tax revenue:

High taxes, sometimes by diminishing the consumption of the taxed commodities, and sometimes by encouraging smuggling, frequently afford a smaller revenue to government than what might be drawn from more modest taxes.

In 1974, Professor Arthur Laffer suggested the possibility of a bell-shape relationship between tax revenue and the tax rate, i.e., the Laffer curve. The upward-sloping segment is called the normal range, while the downward-sloping segment is called the prohibitive range. One of the main insights of this idea is that there exists a tax rate that maximizes total revenue for the government. When policymakers reduce tax rates in the prohibitive range, tax revenue increases. Figure 1.23 shows this relationship.

The economics literature has extensively studied the relationship between tax income and the tax rate. Fullerton (1982) investigates the relationship between labor income tax rates and government revenues. He finds that it is possible that the U.S. government is operating in the prohibitive range for labor income taxes. Lindsey (1987) studies the individual taxpayer response to the U.S. personal income tax rate cuts in the period 1982-1984. He suggests that a tax rate around 35% would have maximized federal income tax revenue, while a total tax rate (i.e., federal, state, and social security tax rates) of about 40% would have maximized total income tax revenue.

Hsing (1996) estimates the Laffer curve for personal income taxes during the period 1959-1991. The author specifies total personal income tax revenue as a quadratic function of the income tax rate and finds that the bell-shaped Laffer curve is statistically significant. Furthermore, he finds that a personal income tax rate between 33% and 35% would maximize government revenue. Clausing (2007) studies corporate in-
come tax revenues relative to GDP in OECD countries over the period 1979-2002. She finds a statistically significant bell-shaped Laffer curve and reports that a corporate income tax rate around 33% would have maximized corporate income tax revenue during the period.

The corporate finance literature has studied extensively the relationship between capital structure and taxes. However, results of empirical tests are inconclusive. Among the authors who fail to find plausible or significant tax effects on leverage, Titman and Wessels (1988) empirically study a wide range of determinants of capital structure and find that the effect of non-debt tax shields (e.g., depreciation, investment tax credits) on leverage is statistically insignificant. Fisher, Heinkel, and Zechner (1989) develop a dynamic capital structure choice model of the firm with recapitalization costs. They regress the leverage range (i.e., the difference between maximum and minimum leverage) on a set of explanatory variables and find that the corporate tax rate is not statistically significant. Bradley, Jarrell, and Kim (1984) develop a trade-off model of the firm and use it to test implications of the trade-off theory of capital structure. In contradiction with the theory, they find a positive relationship between the level of non-debt tax shields and leverage.

However, some authors find significant tax effects on financing decisions. Mackie-Mason (1990) analyzes incremental financing decisions using discrete choice analysis and finds strong evidence that the marginal tax rate affects the choice between issuing debt or equity. Givoly, Hayn, Ofer, and Sarig (1992) study the impact of the Tax Reform Act of 1986 on the relationship between tax-related variables and leverage. They find that both corporate taxes and non-debt tax shields are important determinants of capital structure. Gordon and Lee (2001) use the U.S. Statistics of Income (SOI) Corporate Income Tax Returns to study the impact of changes in corporate tax rates on firm leverage. They find that corporate tax rates have a large impact on leverage decisions of the smallest and largest firms in their sample, but a smaller impact on firms of intermediate size.
Graham (2000) estimates the value of the tax benefits of debt and finds that they represent around 10% of firm value. He also finds that large, profitable, and liquid firms use debt conservatively. Kemsley and Nissim (2002) estimate the value of debt tax shields by using reverse regressions, which mitigate the bias. They also report an estimated value of the tax benefits of debt of about 10% of firm value.

In this section, I study whether income tax payments would increase and by how much if the government raises the income tax rate. I also analyze the effects of changes in the income tax rate on leverage decisions made by firms. The economics papers previously mentioned construct the Laffer curve with parameters estimated using reduced form equations. The problem with this kind of sensitivity analysis is that reduced form parameters can change when the tax rate changes. This paper tackles this problem by estimating the bell-shaped curve with a structural model.

I start solving the model at the estimated values of the parameters, an income tax rate \( \tau_c \) of 40% and a discount factor \( \beta \) of 0.98. This is the base case used to analyze the optimal policies described in Section 6. I then generate a long simulation of optimal policies and calculate the mean income statement and mean leverage for this specification. Table 1.9 shows this mean income statement in column (4) under the heading “Base case”. All the values in this table (except the last row) have been normalized with respect to column (4) (i.e., the one with the tax rate at 40%), so that we can compare the different values when the tax rate changes. Furthermore, all the values in this table are the mean of a long simulation of the model.

Then I change the tax rate, solve the model again, generate a new long simulation of optimal policies and proceed to calculate the mean income statement and mean leverage for the new specification. For example, in column (5) I compute the income statement generated with a tax rate of 45%. Increasing the tax rate to 45% does not increase the amount of money the government receives as income tax, but in fact reduces it to 89.69% of the amount it raises when the tax rate is 40%. This seemingly contradictory finding is a consequence of the dynamic behavior of the firm, which
takes into account the new tax rate and its impact in the future states, and modifies its behavior accordingly. This result is confirmed when I increase the tax rate to 50% and 55% and observe that income tax payments decrease even further to 66.26% and 15.77%, respectively, of the value received by the government when the tax rate is 40%. It should be noted, though, that firm earnings also go down after the tax rate increases. For instance, they decrease to 73.08% of the pre-change amount when the tax rate goes up to 45%.

On the other hand, if I reduce the tax rate the income tax also decreases, but at a slower pace. For example, reducing the tax rate to 35% makes income tax payments go down to 97.25% of the pre-change amount. If I reduce the tax rate even further to 30% and 25%, income tax payments decrease to 90.42% and 79.65%, respectively, of the amount with the tax rate at 40%. However, in this case firm earnings go up after the tax rate decreases. For instance, they increase to 120.40% of the pre-change amount when the tax rate goes down to 35%.

Confirming the trade-off theory, mean leverage (reported in the last row of the table) increases monotonically as the tax rate increases, i.e., larger benefits of debt induce the firm to take on more debt. In column (4) the mean leverage is 44.18%, which I described in Section 6 as the mean of the asymptotic distribution of leverage decisions. The penultimate row shows mean net earnings for different income tax rates. As expected, mean net earnings fall steadily as the income tax rate increases.

Figure 1.24 shows graphically some of these results. It is clear that mean income tax payments have a bell-shaped form that is consistent with the results in Table 1.9. Income tax payments are maximized when the tax rate is around 40% and decline monotonically as we move away from that percentage. It is also apparent that they fall faster when the tax rate increases than when it decreases. It is noticeable that income tax payments are close to the maximum in the range 36%–40%. This interval includes the actual highest tax rates during sample period 1988–2009.

Figure 1.25 shows mean leverage as a function of the income tax rate. Consistent
with the trade-off theory, leverage increases monotonically as the tax rate increases. In particular, mean leverage goes from 26.90% when the income tax rate is 25% to 73.35% when the income tax rate is 55%. Furthermore, mean leverage seems to be a slightly convex function of the tax rate.

Figure 1.26 shows net earnings as a function of the income tax rate. I previously mentioned that net earnings diminish as the tax rate increases. Specifically, mean net earnings are 159.30% of the base case (i.e., column (4)) when the tax rate is 25%, but only 8.60% of the base case when the tax rate increases to 55%. Furthermore, mean net earnings seem to be a slightly concave function of the tax rate.

1.9 Sensitivity analysis: investment decisions

A further objective of the present article is to analyze the effects of financing constraints on the optimal policies generated by the firm model. In particular, I study the impact of financing constraints on firm investment. It is important to understand the effects of public policies on investment through their effect on internal capital. For example, reducing the corporate income tax rate increases the availability of internal funds, which reduces the financial constraints on firms and facilitates firm investment.

In perfect capital markets, investment decisions of firms are independent of their financial condition. That is, a firm’s capital structure is irrelevant to investment because internal and external capital are perfect substitutes. However, in imperfect capital markets, investment and financing decisions might be interdependent. Specifically, if external capital does not provide a perfect substitute for internal funds, then investment may depend on the firm’s financial condition, such as the availability of internal capital or the access to external funds.

The corporate finance literature has defined firms as financially constrained when they face a wedge between external and internal capital. The benefit of this definition is that it allows us to determine the degree to which a firm is financially constrained.
That is, a firm will be more financially constrained when it faces a larger wedge between its external and internal cost of funds. However, according to this definition all firms will be financially constrained to some extent. It would be sufficient that a firm faces a tiny cost of issuing debt or equity to be considered (slightly) financially constrained.

Fazzari, Hubbard, and Petersen (1988) study the sensitivity of investment to cash flow for a sample of firms classified by their dividend behavior. They argue that this classification according to the earnings retention practices is sufficient to identify firms more likely to face a larger wedge between the cost of internal and external capital. For instance, if the cost of external finance were small, then firms would use external capital to finance their investments when internal cash flows are low and the investment-cash flow sensitivity would be negligible. On the contrary, if the costs of issuing debt and equity were high, then firms that invest most of their cash flow (and pay low dividend) might decide not to raise external capital and the investment-cash flow sensitivity would be important.

Fazzari, Hubbard, and Petersen (1988) regress investment on cash flow (over assets) and report that the investment-cash flow sensitivity is positive and significant for all firms. Furthermore, they find that it is substantially greater for low-dividend firms. They interpret this greater investment-cash flow sensitivity as evidence that low-dividend firms are financially constrained. However, they suggest that the significant financial effect of cash flow on investment for mature, high dividend-paying firms is largely a puzzle.

More recent papers provide further support for the results of Fazzari, Hubbard, and Petersen (1988). For example, Hoshi, Kashyap, and Scharfstein (1991) study the investment-cash flow sensitivity of a sample of Japanese firms. They divide this sample on the basis of whether firms belong to a keiretsu, that is, an industrial group. These keiretsus include large banks that are both creditors and shareholders of group firms. This close bank relationship mitigates asymmetric information problems caused
by capital market imperfections. On the contrary, the Japanese firms that do not belong to a keiretsu have a weaker relationship to banks and are likely to suffer greater asymmetric information problems. Their main result is that firms that belong to an industrial group have lower investment-cash flow sensitivity and they conclude that a close bank relationship makes group firms less financially constrained. Schaller (1993) uses three tests for liquidity constraints on firms to analyze the investment-cash flow sensitivity. These tests are based on exogenous firm characteristics that are directly tied to asymmetric information problems, i.e., firm age, ownership dispersion, and specialization of assets. The author finds that young firms, firms with more dispersed ownership, and firms with more specialized assets exhibit higher investment-cash flow sensitivity.

However, Kaplan and Zingales (1997) challenge the previous literature and disagree with the interpretation that investment-cash flow sensitivity is higher for financially constrained firms. They use qualitative and quantitative information to classify firms from “Never financially constrained” to “ Likely financially constrained”. Contrary to Fazzari, Hubbard, and Petersen (1988), they find that investments of firms classified as “Never financially constrained” are significantly more sensitive to cash flow than investments of “ Likely financially constrained” firms. Therefore, they conclude that greater investment-cash flow sensitivities cannot be interpreted as proof of firms being more financially constrained.

The results of Kaplan and Zingales (1997) find further support from some more recent papers. For instance, Kadapakkam, Kumar, and Riddick (1998) study firm investment decisions in six OECD countries. They divide the sample according to firm size and find that investment-cash flow sensitivity is highest for large firms while it is smallest for small firms. Cleary (1999) uses a multivariate classification scheme to determine the creditworthiness of a large sample of U.S. firms. He finds that more creditworthy firms have greater investment-cash flow sensitivity than those classified as less creditworthy.
Other papers use the neoclassical model of the firm to study the investment-cash flow sensitivity controversy. Gomes (2001) and Alti (2003) find that the sensitivity of investment to liquidity exists even when firms do not face any financial constraint. This result is consistent with the puzzle described above about the observed significant investment-cash flow sensitivity of large, mature firms. Moyen (2004) reconciles the results of Fazzari, Hubbard, and Petersen (1988) with those of Kaplan and Zingales (1997). The author uses two different models of the firm: in one of them, firms are unconstrained and can freely raise external capital (debt and equity), and in the other, firms are constrained and can only use internal capital to fund new investments. In the latter model, the firm cannot access external capital markets and has an (initial) amount of debt that is constant over time. Because unconstrained firms can get external capital, they invest more aggressively in periods of favorable opportunities. This feature generates results consistent with those of Kaplan and Zingales (1997).

In addition, shareholders of constrained firms get larger dividends than those of unconstrained firms. Thus, the use of the level of dividends to identify firms more likely to face a larger wedge between the cost of internal and external capital replicates the results of Fazzari, Hubbard, and Petersen (1988).

Financially constrained firms face a wedge between external and internal capital, so that external and internal funds are not perfect substitutes. The natural way to introduce such a wedge into the dynamic model of the firm is through the costs of issuing debt and equity. I study the effects of varying financing constraints on optimal policies. In particular, I study the impact of financing constraints on firm investment.

I start solving the model for a single firm at the estimated values of the parameters, an income tax rate ($\tau_e$) of 40% and a discount factor ($\beta$) of 0.98. This is the base case used to analyze the optimal leverage decisions described in Section 6, and includes per-unit debt ($\lambda^d$) and equity ($\lambda^e$) issuance costs of 0.00523 and 0.10100, respectively. I then generate a long simulation of optimal policies and proceed to reproduce the regressions of Fazzari, Hubbard, and Petersen (1988). That is, I regress investment
decisions on internal cash flow and Q using the following model

\[ \frac{k_{i,t+1} - k_{i,t}}{k_{i,t}} = b_0 + b_1 \frac{CF_{i,t}}{k_{i,t}} + b_2 \frac{\nu_{i,t}}{k_{i,t}} + \varepsilon_{i,t+1} \]  

(1.28)

where \( CF_{i,t} \) is the internal cash flow (equation 6) available at the beginning of the period and \( \nu_{i,t}/k_{i,t} \) is the proxy for Q. I also calculate the mean ratio of dividends over profits for this specification. I set the simulation length at 4,000,000 observations and eliminate the first 1,000 observations to remove the effects of an arbitrary starting point. Table 1.10 shows the coefficients of the previous regression and mean dividends over profits in column (6) under heading “100%”.

Then I change both debt and equity issuance costs in the same proportion, solve the model again, generate a new long simulation of optimal policies and proceed to perform the previous regression and calculate mean dividends over profits for the new specification. For example, in column (7) I increase both adjustment costs by 20%. This increment reduces the coefficient of cash flow over assets from 1.7651 to 1.7363. In columns (8) through (11) I continue increasing both costs of adjustment and the coefficient of cash flow over assets falls even further.

On the contrary, if I reduce debt and equity issuance costs, the coefficient on cash flow over assets increases. For example, reducing both adjustment costs by 20% in column (5) makes that coefficient go up to 1.8208. I reduce the adjustment costs even further in columns (4) through (1) and the coefficient continues growing up to 2.1339.

The negative relationship between investment-cash flow sensitivity and adjustment costs (as a proxy for financial constraints) is due to the following: given the persistence of profitability, when the firm receives a positive shock, it expects that this will persist. Therefore, it increases its size to take advantage of the greater expected returns. However, the response to the profit shock (i.e., the increase in size) is stronger the lower the costs of debt and equity issuance. In other words, more financially constrained firms have lower investment-cash flow sensitivity. Figure 1.27 graphically shows these results. The cash flow sensitivity of investment decreases as the costs of
issuing debt and equity increase.

Regarding the mean of dividends over profits (the payout ratio), Table 1.10 shows it is negatively related to the costs of adjustment. First, it slightly falls as I increase the costs of issuing debt and equity. The mean payout ratio is 26.09% in the base case and falls steadily to 23.91% as the costs of adjustment increase to 200% of the base case. Second, it grows exponentially as I decrease the costs of debt and equity issuance. The mean of the payout ratio jumps to 64.93% when the costs of adjustment are zero.

The negative relationship between issuance costs and the mean payout ratio is intuitive. The wedge between internal and external funds decreases as adjustment costs become lower. Thus, the fact that the firm can obtain external funds more cheaply allows it to use more internal funds to pay higher dividends. Figure 1.28 shows mean dividends over profits as a function of debt and equity issuance costs. This line is clearly convex for adjustment costs below the base case, but it decreases quite linearly for adjustment costs greater than the base case.

To summarize, the results of the dynamic model in the present paper are consistent with Kaplan and Zingales (1997). Specifically, less financially constrained firms have higher investment-cash flow sensitivity. They also agree with Gomes (2001) and Alti (2003). That is, in the dynamic model, investment of financially unconstrained firms is significantly sensitive to liquidity. However, the results in the present article do not agree with Moyen (2004), mainly because constrained firms in her model are structurally different from those of my model. Specifically, in my model constrained firms have access to (costly) external capital and can change the amount of debt as desired.
1.10 Conclusion

I estimate a dynamic structural firm model based on the trade-off theory of capital structure that features endogenous investment, leverage, and payout decisions. The model explicitly includes the benefits (i.e., interest tax shields) as well as the costs (i.e., increased probability of financial distress) of debt, that is, the driving forces of the trade-off theory. I use the Efficient Method of Moments to estimate the structural parameters that best fit the characteristics of the data.

The dynamic model allows me to reconcile two apparently opposite positions in the corporate finance literature concerning the evolution of target leverage over time. On one hand, Hovakimian, Opler, and Titman (2001), Flannery and Rangan (2006), and Huang and Ritter (2009) suggest that target leverage varies over time as profitability, growth opportunities, and other firm characteristics change. On the other hand, Lemmon, Roberts, and Zender (2008) argue that target leverage is largely time-invariant, and that accounting for time-varying factors has a negligible effect on the determination of target leverage. Consistent with the first group, I find that target leverage varies over time as firm characteristics and market conditions change. I also find that leverage decisions are mean-reverting to a long-run constant value, which generates the leverage patterns reported by the second group. These results suggest that the seemingly conflicting empirical evidence presented by both positions might be reflecting different features of the same decision making process.

By reproducing the experiments of Lemmon, Roberts, and Zender (2008), I find that the dynamic model can reproduce their results. However, the dynamic model offers an alternative explanation for the observed convergence of leverage to more moderate ratios over time. The convergence of leverage emerges as a consequence of the fact that leverage decisions are mean-reverting because they are functions of a mean-reverting profit shock. This mean-reversion of leverage is mechanical because it is driven by the mean-reverting profit shocks, and not by the active efforts of the
firm to rebalance its leverage toward a largely constant level.

I analyze the persistence of leverage reported by Lemmon, Roberts, and Zender (2008) in the context of the dynamic model. I reproduce their studies controlling for all sources of firm heterogeneity and I find that persistence of leverage completely disappears. Therefore, the results from the dynamic model agree with Lemmon, Roberts, and Zender (2008) in that the Compustat database might not be rich enough to capture fundamental firm characteristics concerning debt decisions.

I also use the dynamic model to unify various existing results in the literature. I find that the dynamic model generates a positive relationship between profitability and leverage, and a negative relationship between leverage and investment opportunities, dividends and non-debt tax shields such as depreciation. In addition, the dynamic model creates a positive relationship between dividends and profitability, and a negative relationship between dividends and investment opportunities. All of these findings are consistent with the predictions of the trade-off theory.

The dynamic model also sheds light on the impact of different corporate income tax rates on tax payments by firms. The main result is that tax revenues are consistent with the idea of the Laffer curve. That is, tax payments to the government grow as the tax rate increases. However, at some point they reach a maximum and start to decrease. Tax payments achieve the maximum for a tax rate between 36% and 41%.

Finally, I investigate the predictions of the dynamic model regarding the debate between Fazzari, Hubbard, and Petersen (1988) and Kaplan and Zingales (1997). That is, I study the effects of financial constraints on investment decisions by firms. The results are consistent with Kaplan and Zingales (1997) in the sense that the sensitivity of investment to cash flow is higher for firms that are less financially constrained.

This article studies all firms in Compustat database and I use random effects in the structural estimation to address unobserved firm heterogeneity. However, firms in different industries have different characteristics that impact their speed of mean-
reversion of leverage as well as the long-run constant level around which leverage oscillates. By estimating the model for each industry, I could ascertain whether the relative speed of mean-reversion of leverage and the unconditional mean leverage generated by the model for each industry is consistent with those observed in Compustat database. Furthermore, the model generates a positive relationship between profits and leverage, which is consistent with the predictions of the trade-off theory but in conflict with the empirical evidence. A thorough investigation of this issue could shed some light on this apparent inconsistency. This suggests two natural and potentially productive extensions of this paper.
During period $t$, the firm operated with capital $k$ and had debt $d$. At the end of period $t$ (i.e., at moment $t$) the firm receives a realization of the profitability shock, $z$. This completes the state vector of the firm at moment $t$, namely, $(k, d, z, z^-)$. If the shock is bad enough to make firm value lower than the nominal value of the debt, then the firm goes into bankruptcy ($B$) and is liquidated ($L$), that is, the dynamic process ends. On the contrary, if the firm avoids bankruptcy ($No B$), it continues in the business and selects $x^d$ and $x^e$. These choices determine the capital $k'$ and debt $d'$ to be used during period $t + 1$. 

**Figure 1.1.** Dynamic behavior of the firm over time
Figure 1.2. Value of the firm

The model is parameterized at the values estimated in Section 4. This figure depicts an example of firm value generated by the dynamic model as a function of leverage, after the firm receives a profit shock. Firm value is normalized by the value of the all-equity firm.
Figure 1.3. Empirical distribution of investment ratio decisions

The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The histogram shows the empirical distribution of investment ratio decisions of the firms in the sample.
FIGURE 1.4. Empirical distribution of book leverage decisions

The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The histogram shows the empirical distribution of book leverage decisions of the firms in the sample.
The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The histogram shows the empirical distribution of investment ratio decisions of the firms in a subset of the sample that excludes observations with investment greater than 200%.
Figure 1.6. Empirical distribution of book leverage decisions

The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The histogram shows the empirical distribution of book leverage decisions of the firms in a subset of the sample that excludes observations with investment greater than 200%.
The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The histogram exhibits the unconditional distributions of investment ratio decisions generated by the dynamic model for a single firm.
The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The histogram exhibits the unconditional distributions of book leverage decisions generated by the dynamic model for a single firm.
The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure exhibits the evolution of profit shocks over time generated by equation (1).
The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The histogram exhibits the unconditional distributions of profit shocks generated by equation (1).
The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure exhibits the optimal book leverage policy generated by the dynamic model for a single firm. In each period, the firm receives a profitability shock and decides next-period optimal leverage. The model generates a target leverage that depends on the current state of the system (i.e., it varies over time). It also mean-reverts to a long-run constant level of 44.18% according to estimated parameters.
Figure 1.12. Simulation of investment decisions

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure exhibits the optimal investment policy generated by the dynamic model for a single firm. In each period, the firm receives a profitability shock and decides next-period optimal investment. The model generates an investment that depends on the current state of the system (i.e., it varies over time). It also mean-reverts to a long-run constant level of 5.73% per year according to estimated parameters.
Figure 1.13. Sensitivity analysis of the speed of mean-reversion of book leverage with respect to per-unit debt and equity issuance costs

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure shows the speed of mean-reversion of book leverage generated by the dynamic model for a single firm under different per-unit debt and equity issuance costs. The estimated costs of adjustment of debt ($\lambda^d = 0.00523$) and equity ($\lambda^e = 0.10100$) correspond to value 1 or 100% on the x-axis. The counter-factual adjustment costs of debt and equity are presented as proportions of the estimated ones.
The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure shows the mean of the unconditional distribution of book leverage decisions generated by the dynamic model for a single firm under different per-unit debt and equity issuance costs. The estimated costs of adjustment of debt ($\lambda^d = 0.00523$) and equity ($\lambda^e = 0.10100$) correspond to value 1 or 100% on the x-axis. The counter-factual adjustment costs of debt and equity are presented as proportions of the estimated ones.
Figure 1.15. Simulation of book leverage decisions with no adjustment costs

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure exhibits the optimal book leverage policy generated by the dynamic model for a single firm with zero debt and equity issuance costs. In each period, the firm receives a profitability shock and decides next-period optimal leverage. The model generates a target leverage that depends on the current state of the system (i.e., it varies over time). It also mean-reverts to a long-run constant level of 45.31% according to estimated parameters.
Each firm in the panel is simulated over 1,000 periods after discarding the first 1,000 observations to eliminate the influence of an arbitrary initial point. The panel is composed of 10 types of firms and there are 100 identical firms of each type. Each firm type is different in the conditional standard deviation of the profitability shock, $\sigma_e$. The simulation is parameterized at the values estimated in Section 4. The figure exhibits the optimal book leverage policy generated by the dynamic model for two different types of firms.
Figure 1.17. Actual book leverage portfolios

The figure displays the actual book leverage paths generated by the dynamic model. The four portfolios are constructed by replicating the procedure described by Lemmon, Roberts, and Zender (2008). Each line represents the average actual book leverage of each portfolio. The sample in this analysis includes 1,000 firms, which are simulated at the estimated values of the parameters. This sample is divided into 10 different types, each of which includes 100 identical firms. The types differ in parameter $\sigma_c$, which captures the conditional standard deviation of the profitability shock. Each firm is simulated over 1,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions.
The figure displays the net debt issuance activity of the unexpected book leverage portfolios generated by the dynamic model. The four portfolios are constructed by replicating the procedure described by Lemmon, Roberts, and Zender (2008). Each line represents the average net debt issuance activity (scaled by beginning-of-period book assets) for each of the four portfolios. The sample in this analysis includes 1,000 firms, which are simulated at the estimated values of the parameters. This sample is divided into 10 different types, each of which includes 100 identical firms. The types differ in parameter $\sigma_{\epsilon}$, which captures the conditional standard deviation of the profitability shock. Each firm is simulated over 1,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions.

**Figure 1.18. Net debt issuance activity**
The figure displays the net equity issuance activity of the unexpected book leverage portfolios generated by the dynamic model. The four portfolios are constructed by replicating the procedure described by Lemmon, Roberts, and Zender (2008). Each line represents the average net equity issuance activity (scaled by beginning-of-period book assets) for each of the four portfolios. The sample in this analysis includes 1,000 firms, which are simulated at the estimated values of the parameters. This sample is divided into 10 different types, each of which includes 100 identical firms. The types differ in parameter $\sigma_r$, which captures the conditional standard deviation of the profitability shock. Each firm is simulated over 1,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions.
The figure displays the unexpected book leverage paths generated by the dynamic model. The four portfolios are constructed by replicating the procedure described by Lemmon, Roberts, and Zender (2008). Each line represents the average unexpected book leverage of each portfolio. The sample in this analysis includes 1,000 firms, which are simulated at the estimated values of the parameters. This sample is divided into 10 different types, each of which includes 100 identical firms. The types differ in parameter $\sigma_c$, which captures the conditional standard deviation of the profitability shock. Each firm is simulated over 1,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions.
Figure 1.21. Unexpected book leverage portfolios controlling the conditional standard deviation of the profitability shock

The figure displays the unexpected book leverage paths generated by the dynamic model, after controlling for firm heterogeneity. The four portfolios are constructed by replicating the procedure described by Lemmon, Roberts, and Zender (2008). Each line represents the average unexpected book leverage of each portfolio. The sample in this analysis includes 1,000 firms, which are simulated at the estimated values of the parameters. This sample is divided into 10 different types, each of which includes 100 identical firms. The types differ in parameter $\sigma_\epsilon$, which captures the conditional standard deviation of the profitability shock. Each firm is simulated over 1,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions.
Figure 1.22. Simulation of payout decisions

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure shows the optimal dividend policy generated by the dynamic model for a single firm. In periods of high profitability, the firm pays dividends (cash dividends and/or share repurchases) to its shareholders. Dividends (scaled by profits) oscillate around a long-run mean payout ratio of 26.10% according to estimated parameters.
Figure 1.23. Laffer Curve

The “normal range” is the upward-sloping part of the bell-shaped curve, while the “prohibitive range” is the downward-sloping part of the Laffer curve.
Figure 1.24. Sensitivity analysis of income tax revenues with respect to income tax rate

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure shows the firm’s mean income tax payment generated by the firm model under different income tax rates. The mean income tax payments have been normalized with respect to column (4) in Table 1.7, i.e. the tax payments with the tax rate at 40%. Furthermore, all these values are the mean of a long simulation of the model.
Figure 1.25. Sensitivity analysis of book leverage with respect to income tax rate

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure shows the mean book leverage generated by the firm model under different income tax rates. These values are the mean of a long simulation of the model.
The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure shows the mean net earnings generated by the firm model under different income tax rates. These values are the mean of a long simulation of the model.
Figure 1.27. Sensitivity analysis of investment-cash flow relationship with respect to per-unit debt and equity issuance costs

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure shows the sensitivity of investment to cash flow generated by the dynamic model under different per-unit debt and equity issuance costs. The estimated costs of adjustment of debt ($\lambda_d = 0.00523$) and equity ($\lambda_e = 0.10100$) correspond to value 1 or 100% on the x-axis. The counter-factual adjustment costs of debt and equity are presented as proportions of the estimated ones.
Figure 1.28. Sensitivity analysis of dividends over profits with respect to per-unit debt and equity issuance costs

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The figure shows the mean of dividends over profits generated by the dynamic model under different per-unit debt and equity issuance costs. The estimated costs of adjustment of debt ($\lambda^d = 0.00523$) and equity ($\lambda^e = 0.10100$) correspond to value 1 or 100% on the x-axis. The counter-factual adjustment costs of debt and equity are presented as proportions of the estimated ones.
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| # Observations    | 111,944 |

**Table 1.1. Summary statistics of the sample**

The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The table presents the mean, median, standard deviation, minimum and maximum values, skewness, and kurtosis of the final sample. The table also contains the Jarque-Bera test statistic and its p-value. The Jarque-Bera (1987) test takes the sample skewness and kurtosis and produces a goodness-of-fit measure of departure from normality. The normal distribution has a skewness of 0 and a kurtosis of 3. The Jarque-Bera test statistic has an asymptotic chi-square distribution with two degrees of freedom. The 90% critical value for a $\chi^2_{2}$ random variable is 5.99. Investment ratio is $[kT - k(1 - \delta)]/[k(1 - \delta)]$, while book leverage is $dL/kT$. 


Table 1.2. Summary statistics of a subset of the sample

The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The table presents the mean, median, standard deviation, minimum and maximum values, skewness, and kurtosis of a subset of the sample that excludes observations with investment greater than 200%. The table also contains the Jarque-Bera test statistic and its $p$-value. The Jarque-Bera (1987) test takes the sample skewness and kurtosis and produces a goodness-of-fit measure of departure from normality. The normal distribution has a skewness of 0 and a kurtosis of 3. The Jarque-Bera test statistic has an asymptotic chi-square distribution with two degrees of freedom. The 90% critical value for a $\chi^2(2)$ random variable is 5.99. Investment ratio is $|k_t - k(1 - \delta)|/[k(1 - \delta)]$, while book leverage is $dt/k_t$. 

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment Ratio</td>
<td>0.11584</td>
<td>0.05630</td>
<td>0.36874</td>
<td>-0.99768</td>
<td>1.99984</td>
<td>1.61052</td>
<td>8.08297</td>
<td>164,535.61</td>
<td>0.00000</td>
</tr>
<tr>
<td>Book Leverage</td>
<td>0.46554</td>
<td>0.47069</td>
<td>0.23013</td>
<td>0.00000</td>
<td>1.00000</td>
<td>0.06291</td>
<td>2.22262</td>
<td>2.817.78</td>
<td>0.00000</td>
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<tr>
<td># Observations</td>
<td>109,049</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1.3. EMM estimates of model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.05728</td>
<td>0.00056</td>
<td>101.90</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.62957</td>
<td>0.00876</td>
<td>71.90</td>
</tr>
<tr>
<td>$c_z$</td>
<td>1.13021</td>
<td>0.01456</td>
<td>77.64</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.12562</td>
<td>0.00456</td>
<td>27.56</td>
</tr>
<tr>
<td>$c$</td>
<td>0.90827</td>
<td>0.01242</td>
<td>73.14</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.61616</td>
<td>0.01781</td>
<td>34.59</td>
</tr>
<tr>
<td>$\lambda^d$</td>
<td>0.00523</td>
<td>0.00053</td>
<td>9.79</td>
</tr>
<tr>
<td>$\lambda^e$</td>
<td>0.10100</td>
<td>0.00367</td>
<td>27.53</td>
</tr>
<tr>
<td>$\lambda^p$</td>
<td>0.05086</td>
<td>0.00214</td>
<td>23.77</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.15333</td>
<td>0.03426</td>
<td>4.48</td>
</tr>
</tbody>
</table>

The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The table presents Efficient Method of Moments (EMM) estimates, standard errors, and $t$-statistics of model parameters. These parameters are: the capital depreciation rate ($\delta$), the persistence parameter of the profitability process ($\rho_z$), the parameters that capture the scale of the profitability shock ($c_z, \sigma_v$), the constant coefficient of the profitability shock process ($c$), the concavity of the operating profit function ($\alpha$), the per-unit costs of issuing debt ($\lambda^d$) and equity ($\lambda^e$), the per-unit costs of partial reversibility of capital ($\lambda^p$), and the direct costs of bankruptcy ($\xi$).
<table>
<thead>
<tr>
<th>Book Leverage</th>
<th>Per-unit Costs of Debt and Equity Issuance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of M-R</td>
<td>0.5048 0.5079 0.4209 0.3518 0.3350 0.3164 0.3005 0.2933 0.3017 0.2811</td>
</tr>
<tr>
<td>Half-Life</td>
<td>0.9862 0.9776 1.2689 1.4902 1.5986 1.6991 1.8223 1.9393 1.9968 1.9302 2.1000</td>
</tr>
<tr>
<td>Mean</td>
<td>0.4531 0.4560 0.4505 0.4482 0.4448 0.4418 0.4406 0.4363 0.4348 0.4319 0.4300</td>
</tr>
<tr>
<td>Median</td>
<td>0.4516 0.4545 0.4500 0.4500 0.4444 0.4375 0.4375 0.4324 0.4324 0.4285 0.4242</td>
</tr>
<tr>
<td>St. Dev.</td>
<td>0.0382 0.0329 0.0338 0.0351 0.0359 0.0364 0.0366 0.0361 0.0362 0.0354</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.2500 0.2500 0.2500 0.2500 0.2500 0.2500 0.2500 0.2222 0.2222 0.2222</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.6315 0.6153 0.6315 0.6153 0.6486 0.6499 0.6499 0.6499 0.6666 0.6666</td>
</tr>
<tr>
<td>Range</td>
<td>0.3815 0.3653 0.3815 0.3653 0.3986 0.4000 0.4000 0.4277 0.4444 0.4444</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1348 -0.0435 0.0918 0.2877 0.5036 0.5609 0.6004 0.6712 0.6463 0.7125 0.5097</td>
</tr>
</tbody>
</table>

# Simulations 4,000,000

**Table 1.4.** Sensitivity analysis of the speed of mean-reversion of book leverage with respect to per-unit debt and equity issuance costs

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The table shows the speed of mean-reversion of book leverage, the corresponding half-life, and different properties of the asymptotic distribution of book leverage decisions generated by the dynamic model for a single firm under different per-unit debt and equity issuance costs. The estimated per-unit costs of adjustment of debt ($\lambda^d = 0.00523$) and equity ($\lambda^e = 0.10100$) are shown in column (6) under heading “100%”. The counter-factual per-unit adjustment costs of debt and equity are presented in the other columns as proportions of the estimated ones.
Table 1.5. Regressions to explain changes in book leverage

The observed panel consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The simulated panel is composed of 1,000 firms and is divided into 10 types of 100 identical firms. Each type is different in parameter $\sigma_e$, which ranges from 0.7 to 1.5. This parameter captures the conditional standard deviation of the profitability shock of each firm type. The simulation is parameterized at the values estimated in Section 4. The table shows the regression coefficients of book leverage, $d_{i,t+1}/k_{i,t+1}$, on initial leverage, $d_{i,0}/k_{i,0}$, mean leverage, $(\sum_{t=0}^{T-1} d_{i,t}/k_{i,t})/T$, the conditional standard deviation of profitability shocks, $\sigma_{i,t}$, profitability (operating profits), $\pi(k_{i,t}, z_{i,t})/k_{i,t}$, investments opportunities (market-to-book ratio), $v_{i,t}/k_{i,t}$, lagged dividends, $div_{i,t}/k_{i,t}$, and lagged book leverage, $d_{i,t}/k_{i,t}$. The numbers in parentheses are the $t$-statistics for the regression coefficients.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Compustat Data</th>
<th>Simulated Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.4814</td>
<td>-0.1009</td>
</tr>
<tr>
<td></td>
<td>(769.40)</td>
<td>(-34.46)</td>
</tr>
<tr>
<td>Initial Leverage</td>
<td>0.4844</td>
<td>0.0198</td>
</tr>
<tr>
<td></td>
<td>(199.35)</td>
<td>(22.02)</td>
</tr>
<tr>
<td>Mean Leverage</td>
<td>0.9565</td>
<td>0.9868</td>
</tr>
<tr>
<td></td>
<td>(269.88)</td>
<td>(869.07)</td>
</tr>
<tr>
<td>Conditional SD Profits</td>
<td>-0.3792</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-836.80)</td>
<td></td>
</tr>
<tr>
<td>Profitability</td>
<td>0.0192</td>
<td>0.2717</td>
</tr>
<tr>
<td></td>
<td>(11.71)</td>
<td></td>
</tr>
<tr>
<td>Market-to-Book</td>
<td>-0.0005</td>
<td>-0.0195</td>
</tr>
<tr>
<td></td>
<td>(-12.34)</td>
<td>(-5.90)</td>
</tr>
<tr>
<td>Lagged Dividends</td>
<td>-0.0503</td>
<td>-0.3005</td>
</tr>
<tr>
<td></td>
<td>(-6.12)</td>
<td>(-5.81)</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.00</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td># Observations</td>
<td>111,944</td>
<td>1,000,000</td>
</tr>
</tbody>
</table>

Table 1.6. Regressions to explain the level of book leverage

The observed panel consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The simulated panel is composed of 1,000 firms and is divided into 10 types of 100 identical firms. Each type is different in parameter \( \sigma_\epsilon \), which ranges from 0.7 to 1.5. This parameter captures the conditional standard deviation of the profitability shock of each firm type. Each firm is simulated over 1,000 periods after discarding the first 1,000 observations to eliminate the influence of an arbitrary initial point. The simulation is parameterized at the values estimated in Section 4. The table shows the regression coefficients of book leverage, \( d_{i,t+1}/k_{i,t+1} \), on initial leverage, \( d_{i,0}/k_{i,0} \), mean leverage, \( (\sum_{t=0}^{T-1} d_{i,t}/k_{i,t})/T \), the conditional standard deviation of profitability shocks, \( \sigma_\epsilon \), profitability (operating profits), \( \pi(k_{i,t}, z_{i,t})/k_{i,t} \), investment opportunities (market-to-book ratio), \( v_{i,t}/k_{i,t} \), and lagged dividends, \( div_{i,t}/k_{i,t} \). The numbers in parentheses are the \( t \)-statistics for the regression coefficients.
The observed panel consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The simulated panel is composed of 1,000 firms and it is divided into 10 types of 100 identical firms. Each type is different in parameter $\sigma_\epsilon$, which ranges from 0.7 to 1.5. This parameter captures the conditional standard deviation of the profitability shock of each firm type. Each firm is simulated over 1,000 periods after discarding the first 1,000 observations to eliminate the influence of an arbitrary initial point. The simulation is parameterized at the values estimated in Section 4. The table exhibits the variance decomposition for different model specifications for book leverage. The analysis of variance uses the Type III sum of squares for each effect in the model. The estimates are normalized to add up to one. Book leverage is $d_{i,t+1}/k_{i,t+1}$, initial leverage is $d_{i,0}/k_{i,0}$, mean leverage is $(\sum_{t=0}^{T-1} d_{i,t}/k_{i,t})/T$, the conditional standard deviation of profitability shocks is $\sigma_\epsilon$, profitability (operating profits) is $\pi(k_{i,t}, z_{i,t})/k_{i,t}$, investment opportunities (market-to-book ratio) is $v_{i,t}/k_{i,t}$, and lagged dividends is $div_{i,t}/k_{i,t}$.

### Table 1.7. Analysis of variance (ANOVA) for book leverage regressions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Compustat Data</th>
<th>Simulated Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Leverage</td>
<td>0.9974</td>
<td>0.0160</td>
</tr>
<tr>
<td>Mean Leverage</td>
<td>0.9821</td>
<td>0.8965</td>
</tr>
<tr>
<td>Conditional SD Profits</td>
<td>0.8901</td>
<td></td>
</tr>
<tr>
<td>Profitability</td>
<td>0.4194</td>
<td>0.0004</td>
</tr>
<tr>
<td>Market-to-Book</td>
<td>0.4660</td>
<td>0.0021</td>
</tr>
<tr>
<td>Lagged Dividends</td>
<td>0.1146</td>
<td>0.0001</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.00</td>
<td>0.23</td>
</tr>
<tr>
<td># Observations</td>
<td>111,944</td>
<td>1,000,000</td>
</tr>
</tbody>
</table>
The simulated panel is composed of 1,000 firms and it is divided into 10 types of 100 identical firms. Each type is different in parameter $\sigma_e$, which ranges from 0.7 to 1.5. This parameter captures the conditional standard deviation of the profitability shock of each firm type. Each firm is simulated over 1,000 periods after discarding the first 1,000 observations to eliminate the influence of an arbitrary initial point. The simulation is parameterized at the values estimated in Section 4. The table exhibits the regression coefficients of dividends, $\text{div}_{i,t+1}/k_{i,t+1}$, on initial dividend, $\text{div}_{i,0}/k_{i,0}$, mean dividend, $(\sum_{t=0}^{T-1} \text{div}_{i,t}/k_{i,t})/T$, profitability (operating profits), $\pi(k_{i,t}, z_{i,t})/k_{i,t}$, and investment opportunities (market-to-book ratio), $v_{i,t}/k_{i,t}$. The numbers in parentheses are the $t$-statistics for the regression coefficients.

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.0604</td>
<td>0.0171</td>
<td>0.0599</td>
<td>0.0000</td>
<td>0.0424</td>
</tr>
<tr>
<td></td>
<td>(430.38)</td>
<td>(670.00)</td>
<td>(424.84)</td>
<td>(0.03)</td>
<td>(207.86)</td>
</tr>
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<td>Initial Dividend</td>
<td>0.0139</td>
<td>0.0140</td>
<td>0.0019</td>
<td>0.0014</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(13.67)</td>
<td>(15.38)</td>
<td>(1.85)</td>
<td>(1.51)</td>
<td></td>
</tr>
<tr>
<td>Mean Dividend</td>
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<td></td>
<td></td>
<td>0.9979</td>
<td>1.0564</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(100.39)</td>
<td>(118.07)</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.0575</td>
<td>0.0580</td>
<td></td>
<td>0.0566</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(138.32)</td>
<td>(139.81)</td>
<td></td>
<td>(137.19)</td>
<td></td>
</tr>
<tr>
<td>Market-to-Book</td>
<td>-0.0146</td>
<td>-0.0145</td>
<td>-0.0146</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-398.45)</td>
<td>(-397.28)</td>
<td>(-403.49)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.19</td>
<td>0.00</td>
<td>0.19</td>
<td>0.01</td>
<td>0.20</td>
</tr>
<tr>
<td># Simulations</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.8. Regressions to explain the level of dividends
# Simulations 4,000,000

<table>
<thead>
<tr>
<th>Base Case</th>
<th>(1) 25.00%</th>
<th>(2) 30.00%</th>
<th>(3) 35.00%</th>
<th>(4) 40.00%</th>
<th>(5) 45.00%</th>
<th>(6) 50.00%</th>
<th>(7) 55.00%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income Tax Rate</td>
<td>0.9517</td>
<td>0.9644</td>
<td>0.9742</td>
<td>1.0000</td>
<td>1.0382</td>
<td>1.0859</td>
<td>1.1091</td>
</tr>
<tr>
<td>Operating Income</td>
<td>0.9196</td>
<td>0.9393</td>
<td>0.9563</td>
<td>1.0000</td>
<td>1.0676</td>
<td>1.1603</td>
<td>1.2199</td>
</tr>
<tr>
<td>Depreciation</td>
<td>0.9023</td>
<td>0.9363</td>
<td>0.9112</td>
<td>1.0000</td>
<td>0.9370</td>
<td>0.5305</td>
<td>0.2715</td>
</tr>
<tr>
<td>Debt Issuance Costs</td>
<td>0.6113</td>
<td>0.6819</td>
<td>0.8785</td>
<td>1.0000</td>
<td>1.4371</td>
<td>1.9934</td>
<td>2.8839</td>
</tr>
<tr>
<td>Equity Issuance Costs</td>
<td>1.4158</td>
<td>1.3144</td>
<td>1.2174</td>
<td>1.0000</td>
<td>0.7021</td>
<td>0.2032</td>
<td>0.0119</td>
</tr>
<tr>
<td>Partial Reversibility Costs</td>
<td>0.5234</td>
<td>0.6478</td>
<td>0.8023</td>
<td>1.0000</td>
<td>1.3413</td>
<td>1.7706</td>
<td>2.3855</td>
</tr>
<tr>
<td>Interest Payments</td>
<td>1.2744</td>
<td>1.2056</td>
<td>1.1114</td>
<td>1.0000</td>
<td>0.7972</td>
<td>0.5300</td>
<td>0.1147</td>
</tr>
<tr>
<td>Pre-Tax Income</td>
<td><strong>0.7965</strong></td>
<td><strong>0.9042</strong></td>
<td><strong>0.9725</strong></td>
<td><strong>1.0000</strong></td>
<td><strong>0.8969</strong></td>
<td><strong>0.6626</strong></td>
<td><strong>0.1577</strong></td>
</tr>
<tr>
<td>Income Tax Rate</td>
<td>1.5930</td>
<td>1.4065</td>
<td>1.2040</td>
<td>1.0000</td>
<td>0.7308</td>
<td>0.4417</td>
<td>0.0860</td>
</tr>
<tr>
<td>Book Leverage</td>
<td>26.90%</td>
<td>31.86%</td>
<td>37.77%</td>
<td>44.18%</td>
<td>52.69%</td>
<td>61.14%</td>
<td>73.35%</td>
</tr>
</tbody>
</table>

**Table 1.9. Sensitivity analysis of income tax revenues with respect to tax rates**

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The table shows the firm’s mean income statement and mean book leverage generated by the firm model under different income tax rates. The “Base case” is the firm’s mean income statement at the estimated values of the parameters, an income tax rate ($\tau_c$) of 40% and a discount factor ($\beta$) of 0.98. All the values in this table (except the last row) have been normalized with respect to column (4), i.e. the one with the tax rate at 40%. Furthermore, all these values are the mean of a long simulation of the model.
Table 1.10. Sensitivity analysis of investment-cash flow relationship with respect to the costs of issuing debt and equity

The model is simulated over 4,000,000 periods after removing the first 1,000 observations to eliminate the influence of the starting conditions. The simulation is parameterized at the values estimated in Section 4. The table shows the results of regressing investment decisions on cash flow and Q under different costs of issuing debt and equity. The table also displays mean dividends over profits. The “Base case” column shows the previous results at the estimated values of the parameters, an income tax rate ($\tau_c$) of 40% and a discount factor ($\beta$) of 0.98.
The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The table exhibits the family of density functions for the sample of investment and leverage decisions. Each density function (i.e., each row) is characterized by a set of tuning parameters, which are the following: \( L_\mu \) is the lag length of the location function, \( L_g \) is the lag length of the GARCH part of the scale function, \( L_r \) is the lag length of the ARCH part of the scale function. \( L_p \) is the lag length in the \( x_{t-1} \) component of \( P(z_t, x_{t-1}) \), \( K_z \) is the degree of the Hermite polynomial along the \( z \) dimension, \( K_x \) is the degree of the Hermite polynomial along the \( x \) dimension, and \( I_z \) and \( I_x \) filter out the cross product terms of the polynomial expansion. The remaining columns are: \( \text{dim}(\theta) \) is the dimension of the parameter vector of the SNP density function, \( s_n \) is the sample objective function, and \( \text{BIC} \) is the Schwarz Bayes Information Criterion. The bold row shows the BIC-preferred model, that is, the density function that best fits the data.

<table>
<thead>
<tr>
<th>( L_\mu )</th>
<th>( L_g )</th>
<th>( L_r )</th>
<th>( L_p )</th>
<th>( K_z )</th>
<th>( I_z )</th>
<th>( K_x )</th>
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Table 1.11. SNP density function selection
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**Table 1.12.** SNP density function estimation

The sample consists of all firms in the Compustat database from 1988 to 2009, except regulated, financial or public service firms. The table shows the estimates, standard errors, and $t$-statistics of the vector of parameters of the selected semi-nonparametric (SNP) density function, that is, the one that best fits the data. The preferred density function is a bivariate normal density function with a $VAR(1)$ structure for the mean (i.e., $L_{\mu} = 1$) and an $ARCH(3)$ structure for the variance (i.e., $L_{\nu} = 3$). Argument “$i$” refers to investments and argument “$l$” refers to leverage.
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**Table 1.13.** SNP density function estimation - continued
1.11 Appendix 1. Implementation of the model

The model in this paper cannot be solved in closed-form. However, its solution can be approximated numerically. The numerical solution of the dynamic model is obtained via value function iteration, as described by Judd (1998).

In order to implement the model, I need to discretize the state space of $k$, $d$, and $z$. I let the capital stock, $k$, belong to the set

$$\tilde{k} = \left[ \frac{1}{21}k, \frac{2}{21}k, ..., \frac{20}{21}k, k \right]$$

where $k$ satisfies equation (1.9). The stock of debt, $d$, lies in the set

$$\tilde{D} = \left[ 0, \frac{1}{20}k, ..., \frac{19}{20}k, k \right]$$

with $k$ also satisfying equation (1.9).

I transform the AR(1) process for the profitability shock, $z$, defined in (1.1) into a discrete-state Markov chain following the quadrature method of Tauchen (1986). I let $z$ have 11 points of support in the set

$$\tilde{Z} = \left[ c \frac{1}{1 - \rho_z} - 3 \frac{\sigma_z}{\sqrt{1 - \rho_z^2}}, c \frac{1}{1 - \rho_z} + 3 \frac{\sigma_z}{\sqrt{1 - \rho_z^2}} \right]$$

where $c/(1 - \rho_z)$ is the unconditional mean of $z$ and $\sigma_z/\sqrt{1 - \rho_z^2}$ is the unconditional standard deviation of the profitability shock. This implies that, in the numerical implementation of the model, $z$ can only take on 11 values in the interval $\pm 3$ unconditional standard deviations around the unconditional mean of the profitability shock.

1.12 Appendix 2. Construction of the Semi-Nonparametric (SNP) Density Function

Let $\{y_t\}$ denote the true stochastic process of investment rate and leverage. The data are assumed to be a realization of a stationary time series. Let the conditional
distribution of \( y_t \), \( p(y_t \mid y_{t-1}, y_{t-2}, \ldots) \), depend only on \( L \) lags of the data, and let \( x_{t-1} \) be the vector that collects the lags \( y_{t-j} \). Therefore, \( y_t \) is a vector of length 2, \( x_{t-1} = (y_{t-L'}, y_{t-L'+1}, \ldots, y_{t-v'})' \) is a vector of length \( 2L \), and the conditional distribution of \( y_t \) can be written as \( p(y_t \mid x_{t-1}) \). The objective is to closely approximate the true conditional distribution, \( p(y_t \mid x_{t-1}) \), using an SNP density function \( f(y_t \mid x_{t-1}, \theta) \).

This function has the following form

\[
f(y_t \mid x_{t-1}, \theta) = \frac{h \left[ R_{x_{t-1}}^{-1} (y_t - \mu_{x_{t-1}}) \right] | x_{t-1}}{\det (R_{x_{t-1}}^{-1})}
\]

where \( \mu_{x_{t-1}} \) is the location function, \( R_{x_{t-1}} \) is the scale function, and \( h(z_t \mid x_{t-1}) \) is the conditional density of innovation \( z_t \).

The location function is linear and has the form

\[
\mu_{x_{t-1}} = b_0 + B_1 y_{t-L} + B_2 y_{t-L+1} + \ldots + B_{L_L} y_{t-1}
= b_0 + B x_{t-1}
\]

where \( b_0 \) is a 2x1 vector and \( B \) is a 2x2\( L_L \) matrix.

The scale function is an upper triangular matrix that depends on \( x_{t-1} \) and can be constructed in the following way

\[
vech \left( R_{x_{t-1}} \right) = p_0 + \sum_{i=1}^{L_L} P_i \left| y_{t-1-L_i+1} - \mu_{x_{t-2-L_i+1}} \right| + \sum_{i=1}^{L_g} \text{diag} (G_i) \ vech \left( R_{x_{t-2-L_g+1}} \right)
\]

where \( vech \left( R_{x_{t-1}} \right) \) contains the elements of the upper triangle of matrix \( R_{x_{t-1}} \). That is, \( vech \left( R_{x_{t-1}} \right) \) is a 3x1 vector. Vector \( p_0 \) is 3x1, matrices \( P_i \) through \( P_{L_L} \) are 3x2, and vectors \( G_1 \) through \( G_{L_g} \) are 3x1. This is a GARCH(\( L_g \), \( L_r \)) specification of the scale function. Setting the \( G \) vectors to zero yields an ARCH(\( L_r \)) specification of the scale function. Additionally, \( \sum_{x_{t-1}} = R_{x_{t-1}} R_{x_{t-1}}' \).

The conditional density \( h(z_t \mid x_{t-1}) \) has the form of a Hermite polynomial

\[
h(z_t \mid x_{t-1}) = \frac{[P (z_t, x_{t-1})]^2 \phi (z_t)}{\int [P (u, x_{t-1})]^2 \phi (u) du}
\]
where \( P(z_t, x_{t-1}) \) is a polynomial in \((z_t, x_{t-1})\) of degree \( K \) and \( \phi(z_t) \) is a standard normal density function. In the present study, \( \phi(z_t) \) is bivariate.

Polynomial \( P(z_t, x_{t-1}) \) can be written in rectangular form as

\[
P(z_t, x_{t-1}) = \sum_{\alpha=1}^{K_z} \left( \sum_{\beta=1}^{K_x} a_{\alpha\beta} x_{t-1}^{\beta} \right) z_t^\alpha
\]

where \( \alpha \) and \( \beta \) are multi-indexes of maximal degrees \( K_z \) and \( K_x \), respectively, and \( K = K_z + K_x \).

Therefore, the parameter vector of the SNP density is

\[
\theta = \left( \{a_{\alpha\beta}\}', b_\theta', \vec{\{B_j\}_{j=1}^{L_\mu}'}, p_\theta', \vec{\{P_j\}_{j=1}^{L_r}'}, \{G_j\}_{j=1}^{L_{\theta}}' \right).
\] (1.37)

It is convenient now to discriminate among the different lag lengths that appear in the SNP density: \( L_\mu \) is the number of lags in \( \mu_{x_{t-1}} \), \( L_\mu + L_r \) is the number of lags in \( R_{x_{t-1}} \), and \( L_\mu \) is the number of lags in the \( x_{t-1} \) component of \( P(z_t, x_{t-1}) \). In addition, \( L = \max(L_\mu, L_\mu + L_r, L_\mu) \).

When multivariate stochastic processes require a high degree Hermite polynomial to fit them, several possibly unnecessary interactions or cross product terms appear. Therefore, two additional tuning parameters, \( I_z \) and \( I_x \), filter out these high order interactions. For example, a positive \( I_z \) implies that all interactions of order larger than \( k_z - I_z \) are eliminated. The same reasoning applies to \( I_x \).

Finally, the set of tuning parameters of the SNP density is

\[
(L_\mu, L_\mu, L_r, L_p, k_z, I_z, k_x, I_x).
\] (1.38)

The objective of this first step of the EMM procedure is to generate a family of density functions for the joint distribution of the data, namely, investment and leverage decisions, and to select that density function that achieves the optimal description of the data. This family is created by changing the tuning parameters of the SNP density. Different values of the tuning parameters imply different characterizations
of the process for $y_t$.$^{13}$

For a particular choice of the tuning parameters, I need to estimate $\theta$, the parameter vector of the SNP density, by minimizing the sample objective function

$$\tilde{\theta}_n = \arg \min_{\theta} s_n (\theta)$$

$$s_n (\theta) = -\frac{1}{n} \sum_{t=1}^{n} \log f (y_t | x_{t-1}, \theta)$$

where $n$ is the length of the observed time series.

After selecting different values for the tuning parameters and doing the corresponding minimization step, I have the family of density functions from which I can choose the one that best fits the data in the most parsimonious way. To search for the optimal model, Gallant and Tauchen (1998) suggest using the Schwarz Bayes Information Criterion (BIC) (Schwarz (1978)), which is computed as

$$BIC = s_n \left( \tilde{\theta}_n \right) + \frac{1}{2} \frac{\text{dim} (\theta)}{n} \log (n) .$$

Their suggestion is to move upward along an expansion path searching for low values of the criterion until a satisfactory model is found. The first term of the criterion rewards models that achieve good fits (i.e., low $s_n \left( \tilde{\theta}_n \right)$) and the second term penalizes models that are not parsimonious (i.e., large $\text{dim} (\theta)$).

Table 1.11 shows the family of density functions and the model selection strategy. I start expanding the number of lags of the location function, $L_\mu$, and find out that $L_\mu = 1$ achieves the lowest BIC value. Next, I expand the number of lags of the ARCH part of the scale function, $L_r$, and discover that $L_r = 3$ generates the lowest BIC value. Then I expand $K_z$, but the BIC values and the $t$-statistics of the Hermite polynomial suggest this expansion is not warranted. Finally, I expand the number of lags of the GARCH part of the scale function, $L_g$, but the BIC values and the $t$-statistics suggest this expansion is not acceptable. Therefore, the BIC-preferred

$^{13}$Gallant and Tauchen (2009) provide a complete description of the restrictions implied by different settings of the tuning parameters.
model is\textsuperscript{14}

\[(L_{\mu}, L_{\alpha}, L_{\tau}, L_{p}, k_{z}, I_{z}, k_{x}, I_{x}) = (1, 0, 3, 1, 0, 0, 0, 0). \quad (1.41)\]

This result implies that the SNP density function that best characterizes the stochastic process of investment and leverage decisions is a bivariate normal density function with a VAR(1) structure for the location function and an ARCH(3) structure for the scale function.

Tables 1.12 and 1.13 display the estimates, standard errors, and \(t\)-statistics of the vector of parameters of the selected SNP density function. All the coefficients are highly statistically significant.

\textsuperscript{14}L_{p} \geq 1 is a convention in the literature. It was adopted for programming convenience and it is irrelevant when \(K_{x} = 0\).
REFERENCES


