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Efficiency of the T-bill futures market

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EFFICIENCY OF THE T-BILL FUTURES MARKET

by

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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

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SIGNED: James Dorthaig Li

To the memory of my Father

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ABSTRACT

Part I of this dissertation examines the effect of financing costs on the efficiency of the T-bill futures market. The cost-of-carry model is used and three types of financing costs are selected as proxies for RP (repurchase agreement) rates. The results suggest that the cost-of-carry model assuming a constant RP rate is unreliable in explaining the pricing of T-bill futures. A search for "true" financing costs shows that such financing costs could be a nonlinearly weighted rate of the term RP rate (or the 90-day-maturity T-bill rate) and the federal funds rate. Theoretically implied RP rates in the year of 1983 are also generated for comparisons. Part II examines the impact of inflation uncertainty on the futures-forward rate differential. The cost-of-carry model assuming a constant RP rate ignores the future fluctuations of financing costs. A "risk premium" could arise due to inflation uncertainty. This part provides evidence that there exists a systematic relationship between the daily futures-forward rate differences and the inflation rate. Part III provides a theoretical treatment of the optimal arbitrage investment under uncertainty and of equilibrium pricing in the T-bill futures market. A dynamic stochastic programming model shows

that a "myopic" property exists in the T-bill futures market in the sense that expectations of the future one-period price movements do not exert an impact on the current optimal arbitrage investment decision under uncertainty. It shows, however, that such a "myopic" property is not pure in that expectations of financing costs in the next period affect the investment decision in the current period. Equilibrium pricing of the T-bill futures is obtained under arbitrage arguments. It shows that an equilibrium price is achieved at the point where the expected current one-period arbitrage profits are zero when cost-of-carry is required, even in a multi-period setting.

PART I

TESTING THE EFFICIENCY OF THE T-BILL FUTURES MARKET: THE ROLE OF FINANCING COSTS

Part I of this dissertation examines the effect of financing costs on the efficiency of the T-bill futures market. The cost-of-carry model is used and three types of financing costs are selected as proxies for RP (repurchase agreement) rates. The results from both the mean equality test and the regression approach test (a joint test) show that the T-bill futures market may have experienced some inefficiencies on some days in 1983. It is suggested that the cost-of-carry model assuming a constant RP rate is unreliable in explaining the pricing of T-bill futures. A search for "true" financing costs which may explain the futures-forward rate differential shows that such financing costs could be a nonlinearly weighted rate of the term RP rate (or the 90-day-maturity T-bill rate) and the federal funds rate. Theoretically implied RP rates in the year of 1983 are also generated for comparisons.

CHAPTER 1

INTRODUCTION TO PART I

On January 6, 1976, the International Monetary Market (IMM) of the Chicago Mercantile Exchange began trading the Treasury bill (T-bill) futures contract. The IMM trades T-bill futures contracts for delivery of a $1,000,000 par value, 90-day U.S. T-bill, but 91- or 92-day bills are substitutable. Initially, delivery on each contract was made on the three business days beginning with the day of the 3-month (13-week) Treasury bill auction of the third week of the delivery month. Effective June 1983, the delivery of a futures contract was made in the three successive business days beginning with the first day of the delivery month on which a 3-month Treasury bill is issued. There are eight contract maturities currently traded and the delivery months are March, June, September, and December.[1]

The efficiency of the T-bill futures market has been studied extensively. Most previous studies used the mean equality test to test whether futures rates are significantly different from forward rates as implied by the

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[1]For details about the Treasury futures market, see Appendix 1.

term structure of spot market T-bills. In most cases, the hypothesis that these forward and futures rates are equal is rejected, and that the efficiency of the T-bill futures market is thereby rejected. Explanations of these inefficiencies include bars to entry and a liquidation premium (Puglisi, 1978), expectations and segmentation (Branch, 1978), the default risk premium of futures rates as compared to forward rates (Lang and Rasche, 1978), and the effects of daily resettlement (Chow and Brophy, 1978 and Morgan, 1981). Several related empirical studies are summarized in Table 1.

Applying Working's theory of storage costs, Vignola and Dale (1980) used the overnight cost-of-carry model to explain the pricing of Treasury bill futures. Kawaller and Koch (1984) also examined the impact of financing costs on the efficiency of the T-bill futures market. Working (1949) developed the cost-of-carry model in which the difference between spot and futures prices of a commodity is explained by carrying charges that include transportation, warehousing, insurance and interest costs. In the T-bill futures market interest costs are the only relevant carrying costs. Vignola and Dale (1980) specify the overnight repo rate as the overnight carrying cost and they use the federal funds rate as a proxy. After accounting for the carrying costs, they show that the mean price differences between

Table 1

Summary of the Related Empirical Studies

Authors	Cost Treatments[a]	Time Span of Data	Efficiency	Test Method
Branch	No	June 76-July 78	No	Mean Equality
Capozza & Cornell	No	Mar. 76-June 78	No[b]	Mean Equality
Chow & Brophy	TC[c]	Jan. 76-Jan. 78	No[d]	Regression Model
Kawaller & Koch	FC[c]	Sep. 77-June 82	Yes[e]	Mean Equality
Lang & Rasche	No	Mar. 76-Mar. 78	No	Mean Equality
Poole	TC	Jan. 76-June 77	Yes[f]	Mean Equality
Puglisi	TC	Mar. 76-Sep. 77	No	Mean Equality
Rendleman & Carabini	With/No TC	Jan. 76-Mar. 78	No[g]	Autoregression Model
Vignola & Dale (1979)	TC	Mar. 76-Dec. 77	No	Mean Equality
Vignola & Dale (1980)	FC	Mar. 76-Dec. 77	NA[h]	Difference by Sign

[a]This applies to the named authors' obtaining summary statistics. It does not necessarily indicate that transactions costs, financing costs and other factors which may affect the results are not discussed in the corresponding papers.

[b]Except for the near term contract.

[c]FC ≡ Financing costs; TC ≡ Transactions costs which are $60

Table 1, Continued

per Treasury bill futures contract (one million). All those that follow are in the same amount. Note that such round-trip costs may be estimated differently. For example, Figlewski (1986) uses $20 for transactions costs.

[d]This is their inference which serves as one of the possibilities to explain why they reject the validity of three conventional term structure hypotheses---pure expectations, market segmentation and liquidity preference.

[e]On the other hand, they find that the T-bill futures market is inefficient when the short-term T-bill rate is used as a proxy for the term RP rate.

[f]Poole concludes that transactions costs in trading bill futures are very small so that the extra transactions costs from selling long-term T-bills which are relatively larger cause futures rates to be slightly lower than forward rates.

[g]They find, however, that there is no consistent divergence between T-bill and futures prices.

[h]No decisive conclusion is made. But, they indicate that fluctuations of mean differences around zero are common.

futures prices and forward prices constructed from the cost-of-carry model fluctuate around zero. They then concluded that "the overnight cost-of-carry model is better for explaining futures price..." than other explanations including institutional practices, risk premia, transactions costs, and other market imperfections. Similarly, when RP rates were used, Kawaller and Koch found that the T-bill futures market is efficient when such compounded overnight financing rates are used to calculate the implied forward rates. They therefore indicated that "one need not appeal to such considerations as transaction costs, variation margin uncertainty, tax treatment, etc. in order to explain futures prices."

This study extends the previous work showing the effect of financing costs, based on the cost-of-carry model, on the efficiency of the Treasury bill futures market. Since there can be various financing costs (RP rates) in either overnight RPs or term RPs for different borrowers, the "appropriate financing charges" used by arbitrageurs must be carefully chosen when testing the efficiency of the T-bill futures market based on the cost-of-carry model. In this study, we do not specify a "true" RP rate. Rather, the federal funds rate is selected as the higher proxy for the level of the overnight RP rates and the 90-day-maturity T-bill rate as the lower proxy for the level of the overnight RP rates. That is, although both can be regarded to be "close" to the "true" overnight RP rates in practice, we allow a range for overnight financing rates in testing the efficiency of the T-bill futures market based on the cost-of-carry model. The actual overnight financing costs for most traders is likely to be in this range. For the third type of financing rate, the term RP rate, the short-term T-bill rate is used as its proxy.[2] This T-bill rate is in practice called the discount yield or the banker's yield (or bank discount rate, see Appendix 1). We performed a mean equality test as Kawaller and Koch (1984) on daily data for the year 1983 to attempt to explain the futures-forward rate differential in the form of financing costs. In addition, we also ran a regression to compare the results obtained from the mean equality test. In particular, our results from both the mean equality test and the regression test show that the cost-of-carry model utilizing the noted proxy borrowing rates cannot explain the pricing of T-bill futures. Our findings clearly conflict with those found by Vignola and Dale (1980) and Kawaller and Koch (1984). The findings are consistent, however, with most of

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[2]The use of the short-term T-bill rate as a proxy for the term RP rate was also adopted by Kawaller and Koch. The corresponding constructed forward rate is essentially derived from the term structure of T-bill yields. For the implications of the overnight RP and the term RP, see Kawaller and Koch (1984).

the previous studies, even after financing costs are considered. The remainder of Part I is organized as follows. In Chapter 2 we elaborate on how pure arbitrage profits arise and describe the resulting equilibrium condition. Chapter 3 gives the data specification and methodology. Chapter 4 provides the empirical evidence. Chapter 5 summaries the work in Part I.

CHAPTER 2

PURE ARBITRAGE OPPORTUNITIES

Pure arbitrage consists of risk-free profits on a zero net investment.[3] Pure arbitrage in the nearby T-bill futures market involves a short-term T-bill that matures T days from now (time t) and a long-term T-bill that matures T+91 days from now. Suppose that an investor sells a long-term T-bill short at time t and uses the proceeds to buy a short-term T-bill. Simultaneously the investor buys a T-bill futures contract that expires at time T so that he can return the T-bill to the lender at time T. The cash flows from these trades follow the pattern:

	Sell Cash In	Buy Cash Out
t = t	$\$P_{t,T+91}$	$\$P_{t,T} \cdot (P_{t,T+91}/P_{t,T})$
t = T	$\$100 \cdot (P_{t,T+91}/P_{t,T})$	$\$F_{T,T+91}$

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[3]The variation in margin requirements of the futures contract and the appreciation or depreciation in the collateral value are ignored. It is also assumed that investors expect the overnight RP rates to remain constant, which follows Kawaller and Koch (1984).

The prices of all securities here represent their market prices. The asked price (purchased price) is conventionally referred to as the market price. As a result, the so-called "market price" ignores the bid-asked spread. The bid-asked spread can be a reflection of the existence of transaction costs, which is regarded as minimal as opposed to financing costs (see Capozza and Cornell, 1979). With this in mind when using the cost-of-carry model, we shall use the asked price to test the efficiency of the T-bill futures market. For simplicity, the prices are expressed as a percent of face value, \$100. The spot prices of long-term and short-term T-bills are $\$P_{t,T+91}$ and $\$P_{t,T}$ respectively. The futures price is $\$F_{T,T+91}$. It will be paid at time T for the underlying T-bill that matures 91 days from time T. The net cash flow at time t is zero so that there is no initial investment. Since the total funds available for the investor at time t are $\$P_{t,T+91}$, he can only buy $(P_{t,T+91}/P_{t,T})$ fractional short-term T-bills maturing T days from time t, where T days is defined as the number of days from time t to T. The assumption of infinite divisibility of securities is made here for convenience. In practice, the number of units traded in pure arbitrage is so large that this assumption which places the analysis on a per dollar basis is of no consequence. At time T, the investor has a cash inflow of $\$100 \cdot (P_{t,T+91}/P_{t,T})$ because each unit of T-bill has a par value of \$100 at maturity. There is a cash outflow of $\$F_{T,T+91}$ at time T. This is the settlement price on the futures contract at time t to be paid at time T. In order that no risk-free arbitrage profits can result, the market equilibrium condition requires that

$$\$100 \cdot (P_{t,T+91}/P_{t,T}) = \$F_{T,T+91}. \tag{2.1}$$

This equilibrium condition can be maintained over time only if the futures and cash markets are perfect (such as no transactions costs, liquidation premiums, etc.) and efficient. The equilibrium condition can also be expressed as

$$(100/P_{t,T+91}) = (100/P_{t,T}) \cdot (100/F_{T,T+91}). \tag{2.2}$$

The expression $(100/F_{T,T+91})$ is usually referred to as one plus the implied forward rate. Two investment strategies can thus be evaluated directly from equation (2.2). The left-side is the return per dollar from the long-term T-bill cash market, while the right-side is the return per dollar from an investment of $\$P_{t,T}$ in the short-term T-bill cash market followed by re-investment in a 91 day T-bill at a futures rate given now. The investor can earn a risk-free profit if $\$100 \cdot (P_{t,T+91}/P_{t,T})$ is greater than $\$F_{T,T+91}$ in equation (2.1), or equivalently if $(100/P_{t,T+91})$ is greater than $(100/P_{t,T}) \cdot (100/F_{T,T+91})$ from equation (2.2). If

$\$100 \cdot (P_{t,T+91}/P_{t,T})$ is less than $\$F_{T,T+91}$, the investor simply reverses his position to engage in pure arbitrage. The cash flows differ, however. The investor sells a short-term T-bill short, and uses the proceeds to buy $(P_{t,T}/P_{t,T+91})$ long-term T-bills in the cash market. Simultaneously he sells a futures contract of consisting of the $(P_{t,T}/P_{t,T+91})$ long-term T-bills. The cash flows then have the following pattern:

	Sell Cash In	Buy Cash Out
t = t	$\$P_{t,T}$	$\$P_{t,T+91} \cdot (P_{t,T}/P_{t,T+91})$
t = T	$\$F_{T,T+91} \cdot (P_{t,T}/P_{t,T+91})$	$100

We can see that the funds available for the investor at time T are $\$P_{t,T}$ so that he can buy $(P_{t,T}/P_{t,T+91})$ units of the long-term T-bill which will be delivered at time T to fulfill obligations on the futures contract. Consequently, the cash inflow at time T is $\$F_{T,T+91} \cdot (P_{t,T}/P_{t,T+91})$.[4] On the other hand, he must return $100 to the lender of the borrowed short-term T-bill at time T because at the maturity date T the value of it is equal to the face value of $100.

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[4]Note that the investor must sell $(P_{t,T}/P_{t,T+91})$ units of the T-bill futures contract, since he holds as many units as $(P_{t,T}/P_{t,T+91})$ of the long-term T-bill.

The net cash flow at time t is zero so that there is no initial investment. The investor can reap a risk-free profit if $\$F_{T,T+91}\cdot(P_{t,T}/P_{t,T+91})$ is greater than \$100. It is clear that these risk free profits disappear if either equation (2.1) or (2.2) holds. Thus, the equation (2.1) or (2.2) represents a statement of equilibrium condition.

Each of the above trading processes fully satisfies the definition of a pure arbitrage. In practice, however, transactions costs must be taken into account. In view of that, the processes, above, may not be the cheapest ways to reap an arbitrage profit because three transactions costs are involved, two for the cash market and one for the T-bill futures market. In practice, such costs must be shared by investors. As a matter of fact, one of the transactions costs from the cash market can be eliminated by a pure arbitrageur. We shall introduce an alternative pure arbitrage operation that proceeds as follows. A banker's discount rate construction is used in this alternative statement of pure arbitrage.

Let $R_{t,T+91}$ be the annual banking discount yield for a T-bill with \$1 received at the maturing date, i.e., T+91 days from now, so that its current price $\$\{1-R_{t,T+91}\cdot[(T+91)/360]\}$ based on theis banker's discount rate construction. Similarly, the current price for a T-bill maturing T days from now is $\$[1-R_{t,T}\cdot(T/360)]$. The implied forward rate, FO, derived from the term structure of Treasury bill yields is given by

$$[1 - FO_{T,T+91} \cdot (91/360)]$$
$$= \{1 - R_{t,T+91} \cdot [(T+91)/360]\}/[1 - R_{t,T} \cdot (T/360)]. \qquad (2.3)$$

Equilibrium in the T-bill futures market now requires that the futures rate (FU) equal the forward rate (FO) from T to T+91 as derived from equation (2.3) when FU is computed on a banker's discount basis as well. When the cost-of-carry model is used, however, the financing rate will not be $R_{t,T}$ except that it is used as a proxy for the term RP rate as adopted by Kawaller and Koch (1980). As a result, the implied forward rate based on the cost-of-carry model will be

$$[1 - FO^{*}_{T,T+91} \cdot (91/360)]$$
$$= \{1 - R_{t,T+91} \cdot [(T+91)/360]\}/[1 - CRP^{*} \cdot (T/360)], \qquad (2.4)$$

where, CRP^{*} represents the compounded overnight financing rate of either the overnight federal funds rate or the 90-day-maturity T-bill rate.[5] The comparison between the T-bill futures rates and the forward rates derived from equation (2.4) will determine whether a pure arbitrage opportunity exists. It will be clear from the following pure

[5]The compounded overnight RP rate, CRP^{*}, is calculated as: $CRP^{*} = (360/T) \cdot [1 - (1 - RP/360)^{T}]$.

arbitrage operation. Note that the transaction of short-term T-bill will be eliminated.

Suppose that an investor decides to sell short a long-term T-bill and simultaneously buy a futures contract. The cash flow pattern will be as follows:[6]

	Sell Short Long-term Cash Bill	Buy Futures
	Cash In	Cash Out
t = t	$\$\{1-R_{t,T+91}\cdot[(T+91)/360]\}$	0
t = T	0	$\$[1-FU_{T,T+91}\cdot[(91/360)]$

Since no short-term T-bill is traded, cash-out and cash-in at t=t and t=T are zero, respectively. However, the investor can lend out the amount of $\$\{1-R_{t,T+91}\cdot[(T+91)/360]\}$ at time t and earn a CRP^* rate through the RP market.[7] Note that if t > 1 day, then CRP^* represents a continuous rolling over of the RP at rates not known at t=T. The profit can be

[6]It should be clear that the corresponding sell-and-buy decision to reap profits, if any, can be made a priori depending on the sign of NP in equation (2.5). Also note that, for illustration of no requirement in trading short-term T-bill, the following cash flow pattern does not include the cash flows resulting from transactions in the RP market, which will appear in the next trading strategy.

[7]For consistency, the overnight RP rate is transformed to the compounded overnight RP rate, CRP^*. That is, the interest rate used in evaluating potential profits resulting from sell-and-buy decisions is on the continuous overnight basis.

evaluated by the time value of money either at time t or time T.[8] For consistency, based on the banker's discount rate construction, we will evaluate the resultant profit at time t. That is,

$$NP = \{1 - R_{t,T+91}\cdot[(T+91)/360]\}$$
$$-[1 - CRP^{*}(T/360)]\cdot[1 - FU_{T,T+91}\cdot(91/360)]. \quad (2.5)$$

Or,

$$NP = \{1 - R_{t,T+91}\cdot[(T+91)/360]\}$$
$$-[1 - R_{t,T}\cdot(T/360)]\cdot[1 - FU_{T,T+91}\cdot(91/360)]. \quad (2.5a)$$

It can be seen that equation (2.5) is comparable to equation (2.4) and equation (2.5a) is comparable to equation (2.3) for the relationship between forward rates and futures rates. The value of NP depends on which financing rate (CRP^{*} or $R_{t,T}$) is used. It can be easily shown that the sign of NP is determined by the difference between $FU_{T,T+91}$ and $FO^{*}_{T,T+91}$ in the equations (2.5) and (2.4), or by the difference between $FU_{T,T+91}$ and $FO_{T,T+91}$ in the equations (2.5a) and (2.3). If NP is greater than zero, the investor will gain a profit and it is a pure arbitrage profit because there is no initial investment and it is a risk-free profit. What he needs to do is simply to return the borrowed long-term T-bill to its lender at time t by taking the

[8]When evaluated at time T, it is still in the world with certainty, because the cash-out at time T resulting from the purchased T-bill futures is certain.

delivery of the purchased T-bill futures.

Should NP in equation (2.5) be negative, the investor simply reverses his position to reap the profit. The cash flow pattern, however, will be different in order to satisfy the pure arbitrage conditions. The cash flow is[9]

	Sell Futures	Buy long-term Cash bill
	Cash In	Cash Out
t = t	$\$\{[1 - CRP^{*}\cdot(T/360)]\cdot[1 - FU_{T,T+91}\cdot(91/360)]\}$	$\$\{1 - R_{t,T+91}\cdot[(T+91)/360]\}$
t = T	$\$[1 - FU_{T,T+91}\cdot(91/360)]$	$\$[1 - FU_{T,T+91}\cdot(91/360)]$

The cash flow pattern shown is more complicated. Note that, first, the investor must agree in advance to use the purchased long-term T-bill as collateral so that he can be entitled to borrow money from the RP market to purchase the underlying long-term T-bill. By selling a futures contract, the funds available to the investor at time T are only $\$[1 - FU_{T,T+91}\cdot(91/360)]$ which is the money value at time t of \$1 paid (or received) at time T+91 based on the banker's discount rate construction. Consequently, he cannot borrow as much as $\$[1 - CRP^{*}\cdot(T/360)]$ at time t because it is the

[9]The trading strategy is as follows: borrowing funds from the RP market (resulting in cash-in at time t), buying long-term T-bill, and simultaneously selling T-bill futures. This cash flow pattern assumes that the investor uses an overnight financing rate. In case of a term RP rate being used, simply change CRP^{*} to $R_{t,T}$ for Cash-In at t=t.

present value of \$1 paid at time T. What he should borrow at time t and return at time T is $\$[1 - CRP^{*}\cdot(T/360)]\cdot[1 - FU_{T,T+91}\cdot(91/360)]$, which is the present value of $\$[1 - FU_{T,T+91}\cdot(91/360)]$ that is exactly available to him at time T. It thus can be seen that the process for a pure arbitrage profit is completed so long as the profit is greater than zero.

Following the cost-of-carry model and using the banker's discount rate construction, we will use equation (2.3) and (2.4) to calculate the implied forward rates constructed by three proxies of financing rates, respectively. Note that CRP^{*} is used in equation (2.4) because the overnight compounding is taken into account as investors generally finance their positions on a continuous overnight basis. By using the mean equality test for the mean values of the T-bill futures rates and the calculated implied forward rates, we will be able to test the efficiency of the T-bill futures market. To compare our findings with those of Vignola and Dale, Kawaller and Koch, we will replicate Kawaller and Koch's work by means of the mean equality test.[10] In addition, a regression approach will be presented to compare with the results obtained from the mean equality test.

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[10]Transactions costs and other market imperfection factors are ignored.

CHAPTER 3

DATA SPECIFICATION AND METHODOLOGY

The relationships between the T-bill futures rates of the nearby contracts and the forward rates derived from different financing rates will be empirically examined. Equation (2.3) will be used to obtain the corresponding forward rates. The derived forward rates are calculated from the term structure of Treasury bill yields and $R_{t,T}$ is referred to as the term RP financing costs. Equation (2.4) will be used to calculate the corresponding forward rates when the compounded overnight financing costs are considered. This is because investors often enter the repurchase market (RP market) in order to finance their positions. Repurchase agreements are most frequently made for one business day (overnight). When it is made for longer than one day, it is called a term RP. In the RP market the borrower pays interest on the funds at a rate that is negotiated with the lender. Consequently, there can be various RP rates, or repo rates in the RP market. For example, larger and better known dealers are often able to borrow in the RP market at more favorable rates than some smaller dealers and corporations. This diversity raises a problem in choosing the appropriate financing charges for testing the efficiency of the T-bill futures market based on the cost-of- carry model. To circumvent this problem, we selected the overnight RP rates in two ways. The two rates representing financing costs are the 90-day-maturity T-bill rate (repo rate1 in the corresponding Tables) and the federal funds rate (repo rate2). Since the federal funds rate is the rate on unsecured overnight loans (Simpson, 1979), it will generally be slightly higher than the "normal" RP rate.[11] The 90-day-maturity T-bill rate can be expected to be lower than the federal funds rate because it is secured by the U.S. Treasury. After these financing costs have been taken into account, we can test whether the T-bill futures market is efficient based on the cost-of-carry model.

The Data

The daily data covers the period extending from January 3 to December 20 of 1983.[12] Daily T-bill rates are collected from the asked yields of U.S. Treasury bills

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[11]The "normal" RP rate can be thought of as the "average" RP rate in the RP market.

[12]Except for June 1983 because, unfortunately, this author's school library had lost the microfilm of Wall Street Journal for this month at the time this study was conducted. For those missing data in June, 6 out of 35 observations are in Group 1 and 15 out of 84 are in Group 5.

reported in the Wall Street Journal.[13] The corresponding T-bill futures rates, the 90-day-maturity T-bill rates (repo rate1), and the federal funds rates (repo rate2) are collected from the 1983 yearbook published by the International Monetary Market. Both the 90-day-maturity T-bill rates and the overnight federal funds rates are close rates. The T-bill futures rates are settlement discount rates.[14] There are four corresponding nearby T-bill futures expiration dates in 1983---March 24, June 9, September 1, and December 22. Four data sets are accordingly collected to match each of the four corresponding nearby futures' expiration dates. Because of the effect of time span (the number of days to the delivery of the futures contract) on

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[13]The bond-equivalent yields were collected first. We then computed the asked prices for short-term and long-term T-bills by the formula: Bond Equivalent Yield = [(Face Value - Price)·365]/[(Days to Maturity)·Price]. An annualized forward rate can be accordingly computed. When the overnight financing costs are used, the (compounded) overnight federal funds rates and the (compounded) 90-day-maturity T-bill rates are used as substitutes for the short-term T-bill rates. For computations, see equations (2.3) and (2.4).

[14]The available data makes it extremely difficult to match perfectly (in terms of the investment timing) the spot T-bill rates and T-bill futures rates in an intra-day. We also doubt whether one should make effort to do so. For one thing, investors do not necessarily buy-and-sell T-bill and T-bill futures simultaneously in their intra-day investment decisions. What we need in testing the efficiency of the T-bill futures market is that investors must complete their arbitrage trades during the day because we calculate the corresponding interest rates by days.

the divergence between the futures rates and the implied forward rates derived under different methods of financing the carrying costs, each set of observations is divided into five categories (groups) according to the time span. These five categories consist of fewer than 15, 15 to 29, 30 to 44, 45 to 59, and greater than 59 days to expiration. The category of the time span to the expiration date constitutes the event. The observations in the four data sets are pooled together for those cases where they belong to the same time span category.

The Methodology

For the entire sample and each group we perform both the mean equality test and the regression approach test to examine the efficiency of the T-bill futures market based on the cost-of-carry model. By mean equality test, the null hypothesis for efficiency is H_0: $D = \mu_{fu} - \mu_{fo} = 0$, where $\mu_{fu} \equiv$ the mean of futures rates; $\mu_{fo} \equiv$ the mean of those derived forward rates under different financing costs. The t-statistic is $t = (D - 0) / (SD/\sqrt{N})$, where SD is the standard deviation of the N corresponding observations of daily futures-forward rate differences. By the regression approach test, we regress forward rates on futures rates. The model is

$$\text{Forward Rate} = \alpha + \beta\ (\text{Futures Rate}) + \epsilon. \tag{3.1}$$

In equation (3.1), all Ordinary Least Squares (OLS) standard assumptions are made. The null hypothesis for efficiency is a joint test: H_0: $\hat{\alpha} = 0$ and $\hat{\beta} = 1$. The joint test which requires that $\hat{\alpha} = 0$ is based on the reason that the financing costs have been implicitly embodied in the calculated forward rates. The bid-asked spread is not considered, however. Thus, the joint test also assumes that

(1) Bid-asked spreads are presumably to account for transactions costs which are ignored since they are minimal relative to financing costs;[15]

(2) Other factors affecting futures rates and forward rates (such as liquidity preference and interest

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[15]As mentioned by Capozza and Cornell (1979), potential arbitrage costs can be broken down into (1) the costs of opening and closing a futures position, (2) the costs of buying and selling spot bills, and (3) the costs of taking a short position in spot bills (short-selling costs). They pointed out that the first two costs are minimal, but the third cost (a premium of 50 basis points per annum is required on the borrowed bill) is significant which is a function of time because the borrower is forced to pay the premium as long as he maintains the short position. Note that the cost-of-carry model involves operations of the RP market and thus the short-selling costs in the spot T-bill market is similar to the financing costs as characterized by the cost-of-carry model. The similarity between them, however, is by no means that the amount of the financing costs is necessarily identical because the financing costs in the cost-of-carry model is the RP rate.

rate expectations, etc.) are assumed to be the same.[16]

It should be clear that the above reason and assumptions follow those assumptions in performing the mean-equality test using the cost-of-carry model advocated by Vignola and Dale (1980) and Kawaller and Koch (1984) in explaining the pricing of T-bill futures and testing the efficiency of the T-bill futures market.

[16]This assumption was also made by Lang and Rasche (1978).

CHAPTER 4

EMPIRICAL EVIDENCE

Mean Equality Test

Table 2 presents the summary statistics for arithmetic differences between futures rates and forward rates which are calculated by equation (2.3). Note again that $R_{t,T}$ in the equation is used as a proxy for the term RP rate. From Table 2, the first group ($K < 15$) and the second group ($15 \leq K \leq 29$) have insignificant t values at the 1% level by the two-tail test; where, K is the number of days to the delivery of the (nearby) futures contracts. The t values in the rest of groups and the entire sample are significantly different from zero. The findings are consistent with previous studies that indicated evidence of the inefficiencies of the T-bill futures market, when the term structure of T-bill yields is used to derive the corresponding forward rates. These empirical results show that financing costs, when the forward rates derived from the term structure are used for the proxy for the term RP rates, cannot explain the futures-forward rate differential. Except for the first group, it can be observed that the mean differences (in absolute value) increase with the number of

Table 2

Summay Statistics for Arithmetic Differences: Futures Rates Minus Forward Rates Calculated Using the Term RP Rates[a]

Statistics[b]	Entire Sample	Group 1 <15	Group 2 15-29	Group 3 30-44	Group 4 45-59	Group 5 ≥60
		Number of Days to Delivery of Futures Contract				
Maximum	0.973	0.973	0.130	0.227	0.026	0.311
Minimum	-0.412	-0.082	-0.227	-0.252	-0.265	-0.412
Mean	-0.090	0.031	-0.019	-0.102	-0.113	-0.168
Standard Deviation(SD)	0.131	0.186	0.080	0.099	0.060	0.122
Number of Observations	218	29	42	41	43	63
t-value[c]	-10.14[d]	-0.90[e]	-1.54[e]	-6.60[d]	-12.35[d]	-10.93[d]

[a]Daily Data for January 1983 through December 1983.The short-term T-bill rate from t to T is used as a proxy for the term RP rate; where, t is the event day and T is the days to the delivery of futures contract.

[b]All variables are measured in percentages.

[c]$t = (D - 0)/(SD/\sqrt{N})$.

[d]Significantly different from zero at the 1% level, two-tail test.

[e]Insignificatly different from zero at the 1% level, two-tail test.

days to the delivery of the (nearby) futures contract. It is also of interest to observe that the minimum value (in absolute value) of mean difference between futures rates and forward rates increases with the time to expiration of the T-bill futures contract. The consistent tendency of this behavior reveals that the gap between the T-bill futures rates and forward rates derived by the term structure of T-bill yields is reduced as the time to the delivery date of the futures contract approaches zero. Although the term structure of T-bill yields is observed to be ascending in each of the observation days of 1983, the short-term T-bill rate, $R_{t,T}$, does not necessarily increase with the decrease of t.[17] As a matter of fact, it fluctuates through time in the entire data sample. We may conclude that this consistent tendency suggests that the efficiency of the T-bill futures market could be a function of the time to expiration of the contract.

Tables 3 and 4 present the summary statistics for arithmetic differences between futures rates and forward rates derived from different financing rates. Table 3 shows

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[17]When t is decreasing, i.e., closer to the delivery date of the futures contract, the corresponding calendar date is increasing within each of categories (groups). If $R_{t,T}$ were increasing relatively more than $R_{t,T+91}$ as time approaching to the delivery date of the futures contract the corresponding forward rate will be decreasing, which can be seen from equation (2.3).

Table 3

Summay Statistics for Arithmetic Differences: Futures Rates Minus Forward Rates Calculated Using the Compounded Overnight Repo Rate1[a]

	Number of Days to Delivery of Futures Contract					
Statistics[b]	Entire Sample	Group 1 <15	Group 2 15-29	Group 3 30-44	Group 4 45-59	Group 5 $\geq$60
Maximum	0.994	0.994	0.190	0.225	0.161	0.366
Minimum	-0.376	-0.051	-0.089	-0.156	-0.252	-0.376
Mean	-0.038	0.055	0.049	0.011	-0.035	-0.174
Standard Deviation(SD)	0.151	0.184	0.064	0.101	0.096	0.138
Number of Observations	218	29	42	41	43	63
t-value[c]	-3.72[d]	1.61[e]	4.96[d]	0.70[e]	-2.39[e]	-10.01[d]

[a]Daily Data for January 1983 through December 1983. The 90-day-maturity T-bill rate (repo rate1) is used as a proxy for the overnight RP rate.

[b]All variables are measured in percentages.

[c]$t = (D - 0)/(SD/\sqrt{N})$.

[d]Significantly different from zero at the 1% level, two-tail test.

[e]Insignificantly different from zero at the 1% level, two-tail test.

Table 4

Summay Statistics for Arithmetic Differences: Futures Rates Minus Forward Rates Calculated Using the Compounded Overnight Repo Rate2[a]

	Number of Days to Delivery of Futures Contract					
Statistics[b]	Entire Sample	Group 1 <15	Group 2 15-29	Group 3 30-44	Group 4 45-59	Group 5 $\geq$60
Maximum	1.352	1.010	0.323	0.360	0.627	1.352
Minimum	-0.475	-0.049	-0.042	-0.058	-0.124	-0.475
Mean	0.193	0.080	0.131	0.166	0.172	0.318
Standard Deviation(SD)	0.226	0.184	0.091	0.102	0.173	0.325
Number of Observations	218	29	42	41	43	63
t-value[c]	12.61[d]	2.34[e]	9.33[d]	10.42[d]	6.52[d]	7.77[d]

[a]Daily Data for January 1983 through December 1983. The overnight federal funds rate (repo rate2) is used as a proxy for the overnight RP rate.

[b]All variables are measured in percentages.

[c]$t = (D - 0)/(SD/\sqrt{N})$.

[d]Significantly different from zero at the 1% level, two-tail test.

[e]Insignificantly different from zero at the 1% level, two-tail test.

the results when the lower level of the overnight RP rates (the 90-day-maturity T-bill rates, repo rate1) is used; while the results in Table 4 are obtained by using the higher level of overnight RP rates (the federal funds rates, repo rate2). For the entire sample in Tables 3 and 4, the results from using both types of financing rates show evidence that Kawaller and Koch's findings cannot be supported. The t values in Table 4 show that the mean differences between futures rates and forward rates constructed by the federal funds rate are significantly different from zero at the 1% level by the two-tail test for the entire sample and four groups except for the first group. However, the repo rate1 (the 90-day-maturity T-bill rate) seems to give a "better proxy" for the overnight financing costs because the first, third and fourth groups are insignificant in t values. The systematic tendency of the (absolute) mean differences across groups in Table 3 begins at Group 3. Such a tendency in Table 4, however, consistently increases with time to the delivery date. The (absolute) minimum values reveal the similar pattern as that in Table 2. The evidence presented in Tables 3 and 4 also indicates that the cost-of-carry model cannot explain the futures-forward rate differential. Our findings would clearly conflict with those found by Vignola and Dale (1980) and Kawaller and Koch (1984).

Search for the "True" Financing Costs

Viewed from the mean differences in the entire sample for Tables 2, 3 and 4, we see that the Term RP gives the smallest mean difference and the federal funds rate gives the largest. This means that, from overall point of view, there must be a relationship among these three types of financing costs such that Term RP rate < 90-day-maturity T-bill rate < federal funds rate. Since both the Term RP rate and the 90-day-maturity T-bill rate give a negative mean difference whereas the federal funds rate gives a positive mean difference, it is conceivable that the "true" financing costs which may lead to the efficiency of the T-bill futures market could be somewhere between the Term RP rate and the federal funds rate. Two sets of the financing rates are generated by linearly weighting two different (collected market-observed) financing rates. The weights are chosen so that the averaged rate either approaches the Term RP rate or the 90-day-maturity T-bill rate when the federal funds rate is the other rate used in the average. The reason is that the federal funds rate appears to give relatively high mean differences.[18] The statistical results based on these generated financing rates are presented in Appendix 2.

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[18] We also generate financing rates obtained by increasing the weight of the federal funds rate. They cannot explain the futures-forward rate differential better after the first weighting (equally weighted), however.

It is found that the linear-weighting cannot provide a "true" financing rate, which could be a "real" financing cost obtained by practitioners in the market, to explain the futures-forward rate differential. It is therefore conjectured that either the "true" financing costs are nonlinearly weighted between these proxies for financing costs, or that the T-bill futures market is inefficient in the year of 1983 even if the cost-of-carry model is used.

Theoretically, a "true" financing rate, y, can be found by using the following formula

$$(1 - FU_{T,T+91} \cdot 91/360)$$
$$= [1 - R_{t,T+91} \cdot (T+91)/360]/(1 - y \cdot T/360), \qquad (4.1)$$

where y is the implied (constant) RP rate. We used three interest rates as proxies for financing costs, which are the short-term spot T-bill rate, the daily compounded federal funds rate and the daily compounded 90-day-maturity T-bill rate. They are all assumed to be constant financing costs when a pure arbitrage position is engineered. To see the "degree" of efficiency of the T-bill futures market, we may look into the frequency of such implied RP rates which is outside the boundary of

$$\mathrm{Min}(STR,\ FEDR,\ 90R) < IMRP < \mathrm{Max}(STR,\ FEDR,\ 90R) \qquad (4.2)$$

where, STR ≡ the short-term T-bill rate; FEDR ≡ the (compounded) federal funds rate; 90R ≡ the (compounded) 90-day-maturity T-bill rate; and IMRP ≡ the implied RP rate computed by equation (4.1). The results are reported in Table 5. There are 74 (out of 218 total observations) times giving IMRPs which are outside the boundary of equation (4.2). That means there are about 34% of the total observations in the year of 1983 which give an "inefficient" (not statistically) market of the T-bill futures, provided that practitioners in the market do use these proxies as their financing costs. Note that it is shown in Table 5 that Group 1 and 2 give higher frequencies. Noting that we statistically look at the mean difference and t-value in Tables 2, 3, and 4, it can be seen that part of the out-of-boundary observations in Group 1 and 2 could be statistically insignificant (recall that all t values of Group 1 in Tables 2, 3, and 4 are insignificant). This implies that the out-of-boundary "inefficient" observations in the entire sample could be statistically lower than 34%.

Regression Approach Test

In addition to doing the conventional mean equality test on the T-bill futures and forward rates, we also performed a regression for comparison. Forward rates derived from the three different financing rates are regressed on futures rates, respectively. For each case in Table 6, there

Table 5

Frequency of the Implied RP Rate which is outside Equation (4.2)[a]

	Number of Days to Delivery of Futures Contract				
Entire Sample	Group 1 <15	Group 2 15-29	Group 3 30-44	Group 4 45-59	Group 5 ≥60
74	19	22	12	13	8

[a]Equation (4.2) is

Min(STR,FEDR,90R) < IMRP < MAX(STR,FEDR,90R). STR ≡ the short-term T-bill rate; FEDR ≡ the (compounded) federal funds rate; 90R ≡ the (compounded) 90-day-maturity T-bill rate; IMRP ≡ the implied RP rate.

Table 6

Regression Results: Joint Test[a]

	Number of Days to Delivery of Futures Contract					
	Entire Sample	Group 1 <15	Group 2 15-29	Group 3 30-44	Group 4 45-59	Group 5 ≥60
Case I: Forward Rate Calculated Using the Term RP Rate[b]						
$\hat{\alpha}$	0.632%	0.248	0.219	0.350	0.690	0.943
$\hat{\beta}$	0.938	0.968	0.977	0.971	0.933	0.911
R^2	0.953	0.795	0.986	0.974	0.991	0.971
N	218	29	42	41	43	63
F Value	69.307	0.567	2.067	21.672	124.907	89.501
Critical Value	4.70	5.49	5.18	5.20	5.17	4.97
Efficiency	No*	Yes	Yes	No*	No*	No*
Case II: Forward Rate Calculated Using the Repo Rate1[c]						
$\hat{\alpha}$	0.753%	0.213	0.435	1.170	1.058	0.648
$\hat{\beta}$	0.918	0.970	0.944	0.863	0.882	0.945
R^2	0.938	0.797	0.993	0.989	0.978	0.957
N	218	29	42	41	43	63
F Value	21.819	1.199	29.609	43.864	21.693	52.959
Efficiency	No*	Yes	No*	No*	No*	No*

Table 6, Continued

Case III: Forward Rate Calculated Using the Repo Rate2[d]

$\hat{\alpha}$	0.029%	0.160	-0.041	0.730	0.418	-0.400
$\hat{\beta}$	0.974	0.973	0.990	0.896	0.932	1.009
R2	0.972	0.799	0.982	0.979	0.901	0.812
N	218	29	42	41	43	63
F Value	83.179	2.634	40.623	98.498	22.586	30.362
Efficiency	No*	Yes	No*	No*	No*	No*

[a]H_0: $\alpha = 0$ and $\beta = 1$. (Forward Rate) = $\hat{\alpha} + \hat{\beta}$ (Futures Ra ·e). The unit is percentage; Daily data for January 1983 through December 1983.

[b]The short-term T-bill is used as a proxy for the term-RP rate.

[c]The 90-day-maturity is used as a proxy for Repo Rate1.

[d]The federal funds rate is used as a proxy for Repo Rate2.

*The hypothesis of joint test is rejected at the 1% level.

are six regressions containing the entire sample and the five groups categorized according to the number of days to the delivery of (nearby) futures contract. The joint test for the estimated parameters in the regressions[19], $\hat{\alpha} = 0$ and $\hat{\beta} = 1$, gives the same results as the mean-equality test, except for the third and fourth groups in Case II where the 90-day-maturity T-bill rate is used as a proxy for the overnight financing costs. These findings seems to reveal that the mean equality test is not necessarily equivalent to the regression approach (joint test), when the value of constant term is relatively high. A common interpretation of the constant term is that some explanatory variables are excluded from the model (misspecification) and it measures the impact of these "omitted" variables. These findings therefore suggest that two statistical methods may not be equivalent when the other factors (in addition to financing costs) exert strong impact on the existence of the futures-forward rate differential.

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[19]The test statistic is obtained as follows. If the joint test H_0: $\hat{\alpha} = 0$ and $\hat{\beta} = 1$ is true, then $[Q/(2\hat{\sigma}^2)]$ is distributed as an F distribution with 2 and (N-2) degrees of freedom, ie., F(2,N-2); where, $Q = N(\hat{\alpha} - \alpha)^2 + 2N\mu_{FU}(\hat{\alpha} - \alpha)(\hat{\beta} - \beta) + \Sigma_1^N FU_i^2(\hat{\beta} - \beta)^2$, μ_{FU} ≡ the mean of futures rates and N ≡ the number of observations.

CHAPTER 5

SUMMARY OF PART I

Most of the previous empirical tests rejected the hypothesis that the T-bill futures market is efficient when financing costs are ignored. Arguing that financing costs can play an important role in explaining the pricing of T-bill futures and that the cost-of-carry model will lead to efficiencies of the T-bill futures market, Vignola and Dale (1980) and Kawaller and Koch (1984) have presented some evidence on this issue. We, however, are not able to support their findings. After elaborating on the search for pure arbitrage strategies, we employ three types of financing costs to examine whether a risk-free profit is possible without an initial invetment, i.e., whether a pure arbitrage profit exists. The empirical results show evidence that the T-bill futures market is inefficient, at least for the year of 1983, based on the cost-of-carry model. Two approaches, the mean equality test and the regression approach test, are performed and both reveal the similar results. We conclude that the reliability of the cost-of-carry model in explaining that the T-bill futures market is efficient is suspect, although it may hold validity. We should point out that the validity of the cost-of-carry model cannot give us a license to say that it is reliable. The reason is that Kawaller and Koch (1984) who used the 1977--1982 data were able to show that the T-bill futures market is efficient based on the cost-of-carry model, but we cannot support their findings by using the 1983 data although the methodology is identical. If one is able to demostrate that the findings are different for the same time period, it could be most likely due to the choice of financing rates. As it is known that a repurchase agreement (RP) rate is negotiated by individual borrower and lender, it could not be a uniform interest rate at any point in time. Thus, the choice of suitable RP rates in testing the efficiency of the T-bill futures market with the cost-of-carry model would be critical. For this reason, we feel that it would be more appropriate to choose a range of financing rates as proxies for the borrowing (or lending) rates in the RP market. They are the federal funds rate which serves as the higher level of the overnight RP rates, and the 90-day-maturity T-bill rate which serves as the lower level of the overnight RP rates. The third set of financing rates is adopted from the term structure of T-bill yields, which is the conventional way of obtaining the implied forward rate, and the corresponding short-term T-bill rate serves as the term RP rate for the cost-of-carry model. The use of each of the three financing rates, however, rejects the hypothesis of efficiency of the T-bill futures market. Moreover, all of them identically reveal a systematic tendency of the time pattern regarding the differences between futures rates and the corresponding constructed forward rates. Namely, the differences are reduced as the time approaches the expiration date of the T-bill futures contract.

Now that our test period is different, in addition to our different treatments of financing costs, it is therefore suggested that the reliability of the cost-of-carry model in demonstrating the efficiency of the T-bill futures market is suspect. We feel that the choice of time periods in doing empirical testing by using historical data may result in different estimates of the parameters, and the findings may be accordingly statistically different from others'. There are two implications relevant to the time periods chosen for an empirical study. First, the effect of the macro-environment of the economy may be different across time.[20] Second, the results may be affected

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[20]An example for the impact of the macro-environment can be referred to the work by Kawaller and Koch (1984). Part of their results changed (from significance to insignificance so that their findings became stronger) after they restricted the sample to the period from January 1980 to June 1982 when Federal Reserve policy targeted bank reserves and permitted greater interest rate volatility than previously. Strictly speaking, this is the

by the length of the time-interval chosen for the empirical test. The second implication is related to the question of the statistical significance of a sample size in terms of the time horizon selected for an empirical study. Sometimes, the difference in the time horizon may result in controversial conclusions, even if the effect of the macro-environment of the economy is held constant. It should be noted that, in Table 1, the time length in most of the studies is two years except for Kawaller and Koch's, which is five years, while our daily data is one year.[21] Viewed from the findings presented in this study, we recommend that those conventional considerations, mentioned in the introductory section of this study, not be ignored in explaining the pricing and testing of the efficiency of the T-bill futures market, even if the cost-of-carry model is used. Additionally, we conjecture that bid-asked spreads and differences between the quoted price and the transaction price could partly explain the futures-forward rate differential. The assumption of constant financing cost through the life of a given T-bill futures contract could be

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combined effects of the macro-environment and the time-interval for making such comparison. However, it will be a pure macro-environment effect, holding other things equal, that one selects such time-interval for the comparison as, for example, from January 1977 to June 1979.

[21]Some of them in Table 1 used weekly or monthly data.

too strong in practice since fluctuations in the future repo rates are unavoidable in the cost-of-carry model. This study also suggests that a "true" financing cost which may explain the futures-forward rate differential could be a nonlinearly weighted rate of the term RP rate (or the 90-day-maturity T-bill rate) and the federal funds rate.

PART II

THE COST-OF-CARRY MODEL, INFLATION UNCERTANITY, AND THE EFFICIENCY OF THE T-BILL FUTURES MARKET

Part II examines the impact of inflation uncertainty on the futures-forward rate differential. The cost-of-carry model commonly assumes a constant RP rate in testing the efficiency of the T-bill futures market. That ignores the future fluctuations of financing costs. Because pure arbitrageurs are forced to finance continuously their positions until the expiration date of the T-bill futures contract, a "risk premium" could arise due to inflation uncertainty. The movement of inflation rate is used as a surrogate measure of inflation uncertainty. This study provides evidence that there exists a systematic relationship between the daily futures-forward rate differences and the inflation rate.

CHAPTER 6

INTRODUCTION TO PART II

The empirical findings in Part I indicate that the Treasury bill (T-bill) futures market may have experienced some inefficiencies on some days after financing costs had been taken into account in a cost-of-carry model. In particular, Part I shows that, provided that practitioners in the market do use the short-term T-bill, the 90-day-maturity T-bill or the federal funds rates as proxies for financing costs, the futures-forward rate differential exists in some periods of 1983. The primary focus of Part I was a reexamination of previous findings generated by the cost-of-carry model. The findings in Part I are just the opposite of those arising from some previous work (e.g., Vignola and Dale, 1980; Kawaller and Koch, 1984). The findings also do not support the arguments for the existence of a default risk premium for futures contracts (Lang and Rasche, 1978; Kane, 1980) or of a greater volatility cost in futures prices (Morgan, 1981). From the latter arguments, we would expect the T-bill futures rate to be statistically higher than the forward rate. The results in Part I indicate that it was not always the case for the 1983 period.

Inflation uncertainty is proposed in an attempt to explain this puzzling phenomenon. That is motivated by the various findings in Part I where three levels of interest rates were used as proxies for financing costs in the cost-of-carry model. It is conjectured here that inflation uncertainty plays an important role in the efficiency of the T-bill futures market because there seems to exist systematic movements in the futures-forward rate differential. As reported by Fama (1976), his study supports the hypothesis about the relationships between expected premiums and uncertainty about future expected rates of change in purchasing power. Fama (1975) also demonstrated that expected real returns on bills are constant through time. As such, the expected premiums in bills, which mature in a short period such as within 6-month so that a liquidity premium and a maturity risk premium are presumably zero, reduced to the expected inflation rates over time. It is also known that the uncertainty about future expected rates of change in purchasing power is what the inflation uncertainty is commonly referred to. We therefore will adopt Fama's findings and use the movement of the inflation rate as a proxy for the measure of inflation uncertainty which concerns investors. Of course, there were some debates about Fama's findings (e.g., see Nelson and Schwert, 1977). It is not the interest of the current essay, however, to join such a debate. We shall focus on the issue of why inflation uncertainty arises when cost-of-carry is of concern to investors who perform locked-in arbitrage positions in the T-bill futures market. In a cost-of-carry model, a pure-arbitrage investor needs to finance his positions either daily or in a short period of time (i.e., with a term-RP). Thus, in no way can the financing costs be constant through the life of a T-bill futures contract. Some investors may anticipate that the inflation rate will rise whereas some may expect a downward movement of the inflation rate. However, all investors are concerned with changes in the rates of inflation because the future financing costs (for both those who are in repurchase agreements and in reverse repurchase agreements) are accordingly affected. Such a mechanism creates inflation uncertainty for arbitrage investment in a cost-of-carry model. It is clearly that, for a risk-averse investor, some "risk premium" needs to be considered due to the inflation uncertainty associated with the requirements of continuously financing his positions. The impact of inflation uncertainty on the futures-forward rate differential can be measured in the following way. Since the market efficiency is (statistically) measured by the mean difference between futures rates and forward rates, we may regress the daily futures-forward rate differences on the inflation rates. This procedure accordingly allows us to measure the impact of inflation uncertainty on the efficiency of the T-bill futures market. If inflation uncertainty plays a role in explaining the futures-forward rate differential (the futures rate was lower than the forward rate in the entire sample as reported in Part I when two lower levels of financing costs were used),[1] then such regressions should show a tendency that gives,

(1) From the overall (entire sample) point of view, $\hat{\beta}$ must be significant and the correlation between the futures-forward rate differences and the inflation rate must be high.

(2) Since the degree of inflation uncertainty should be a function of the length of time (i.e., number of days to the delivery of the futures contract), if the hypothesis were true then $\hat{\beta}$ must be insignificant and the correlation be extremely low in those contracts which are close to the expiration date of the T-bill futures.

If the tendency of (1) and (2) exists, then inflation uncertainty will really matter in the market-observed "inefficiency" of the T-bill futures market. In turn, it means that, if such a "risk premium" were added to the

[1]The implied repo rates in 1983 were also reported in Part I. They are the theoretical rates, not the market-observed rates. We thus do not use them to run regressions in this part.

financing costs the futures-forward rate differential could disappear, because the derived forward rate will be lower than otherwise based on the cost-of-carry model. On the other hand, if a higher level of interest rate like the federal funds rate has been used as a proxy for the overnight financing costs, then the derived forward rate becomes relatively low. It could be even statistically lower than the futures rate as also reported in Part I. Under this case, the regression should show a tendency that gives

(3) The correlation between the futures-forward rate differences and the inflation rate is relatively low and $\hat{\beta}$ must be insignificant in both the entire sample and the subsamples (i.e., groups of observations according to the number of days to the expiration date of the T-bill futures).

(3) means that the financing costs using a high level of interest rate as a proxy have taken whatever risk premiums into account. These premiums giving a relatively high rate of interest could include any possible premiums (for example, recall that the federal funds rate is the rate on unsecured overnight loans). Such high premiums are the mixture of the premiums required by investors in the market. If (3) were true, it can be interpreted as that there is no need to add any kind of risk premiums to such a high financing cost (the federal funds rate).

The procedures and their implications above are to test the hypothesis that inflation uncertainty could give rise to some "risk premium" which attaches to the fluctuations in the overnight financing rate (overnight RP rate) or in the short-term financing rate (term RP rate).[2] As mentioned earlier, if the relationships between inflation uncertainty and the movement of inflation rate exist, a systematic relationship between the daily futures-forward rate differences and inflation uncertainty can be in turn investigated by using the inflation rates as surrogate measures of inflation uncertainty. In other words, the empirical test in Part II for the impact of inflation uncertainty on the futures-forward rate differential is a "qualitative" test rather than a "quantitative" test. It is because the "risk premium" of inflation uncertainty in the T-bill futures market has not been successfully quantified in the current state of art. Put differently, what we

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[2]A repurchase agreement (RP), or "repo," is the sale of securities on a temporary basis (overnight or longer; the latter case is called term RP). A repo involves the seller's agreement to repurchase the same or similar securities at a later date. The other party has a corresponding obligation to sell them back. A reverse repo is simply the same transaction viewed from the perspective of the other party. It is clear that every repurchase agreement comprises a repo on one side and a reverse repo on the other. Repurchase agreements have been used extensively in the money market in recent years by investors (or institutions) who either seek to earn a return on idle cash or need to raise funds on a temporary basis.

investigate is to see whether there exists a risk premium resulting from inflation uncertainty when continuous financing is required to reap pure arbitrage. It is by no means that how much the "risk premium" is known. The fundamental argument of the impact of inflation uncertainty on the efficiency of the T-bill futures market is that the implied forward rate derived from a long-term T-bill and an overnight RP rate (or a term RP rate) does not contain the same risk premium of inflation uncertainty as does the futures rate. The inflation uncertainty hypothesis implies that there exists a "markup" of the futures price over the forward price because of the uncertainty associated with the future financing rates.[3] The inflation uncertainty hypothesis is aimed to search for new candidate(s) of market imperfection to explain the futures-forward rate differential. An equilibrium and efficient condition in the T-bill futures market under the assumptions of perfect markets implies that T-bills and T-bill futures can be utilized in perfectly substitutable trading strategies (see Appendix 3). Therefore, it is clear that the existence of the T-bill futures market must reflect "imperfections" of

[3]By "forward price," it actually means "implied forward price" because it is a derived price, not a price of a real forward contract. We will write "implied forward price (rate)" as "forward price (rate)" in the paper when there can be no confusion as to its real meaning.

the market such as the existence of transaction costs, financing costs, marking to the market, default risk premium, liquidity preference, taxes treatment, etc., on which an equilibrium and no-arbitrage condition can be based.[4] Some previous work has shown that the T-bill futures market is inefficient after one or a combination of the above-mentioned factors of imperfection have been considered. It is conjectured that a "risk premium" of inflation uncertainty resulting from continuous financing could account for the phenomenon of futures-forward rate differential. The empirical results are consistent with the hypothesis. Because the cost-of-carry model assumes a constant RP rate through the life of a T-bill futures contract, an ignorance of such a "risk premium" resulting from the impact of inflation uncertainty on the future RP rates could be a factor to cause "market inefficiency."

Part II is organized as follows. Chapter 7 show why factors of market imperfection could exist to affect the measure of the efficiency of the T-bill futures market and how arbitrage can arise when there exists a futures-forward rate differential. In Chapter 8, we describe the data and methodology used in the study. Chapter 9 presents the empirical evidence. We conclude Part II in Chapter 10.

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[4]Note that some of the mentioned factors of imperfection could also prevail in the cash market.

CHAPTER 7

MARKET IMPERFECTION AND ARBITRAGE OPPORTUNITY

A forward price in the Treasury bill market is constructed with two spot T-bills, a long-term T-bill and a short-term T-bill. Suppose a long-term T-bill matures (T+91) days from now (time t) and a short-term T-bill matures T days from now. Further, let a T-bill futures contract expire T days from now so that the underlying T-bill for delivery should mature 91 days from the delivery date of the futures contract, denoted as time T, according to the IMM (International Monetary Market) rules.[5] By using the banker's discount rate method, a forward price ($PO1_{T,T+91}$) is

$$
\begin{aligned}
PO1_{T,\,T+91} &= (1 - FO1_{T,\,T+91} \cdot 91/360) \\
&= [1 - R_{t,\,T+91} \cdot (T+91)/360]/[1 - R_{t,\,T} \cdot (T/360)], \qquad (7.1)
\end{aligned}
$$

where, the notation of (T,T+91) in $PO1_{T,T+91}$ represents that the implied "forward contract" matures at time (T+91) from time T; $[1 - R_{t,T+91} \cdot (T+91)/360]$ is the spot price of a

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[5]There should be no confusion as to "T days" and "time T." The date of "time T" is the day on which a T-bill futures contract expires. "T days" is the number of days from time t to T and it decreases as t approaches T.

long-term T-bill with par value of \$1; and $[1-R_{t,T}\cdot(T/360)]$ is the spot price of a short-term T-bill with a par value of \$1. By the cost-of-carry model, an investor uses a <u>constant</u> RP (or "repo") rate in the RP (Repurchase Agreement) market to construct a forward price in the T-bill market so that equation (1) becomes,

$$PO2_{T,T+91} = (1 - FO2_{T,T+91}\cdot 91/360)$$
$$= [1 - R_{t,T+91}\cdot(T+91)/360]/[1 - CRP^{*}\cdot(T/360)], \quad (7.2)$$

where, $CRP^{*} = (360/T)\cdot[1 - (1 - RP/360)^{T}]$. That is, RP is the current annual repurchase agreement rate and CRP^{*} is the current annual repurchase agreement rate using daily compounding. Since a futures contract requires that the underlying financial instrument mature 91 days from the delivery date (time T), a T-bill futures price ($PU_{T,T+91}$) is

$$PU_{T,T+91} = (1 - FU_{T,T+91}\cdot 91/360). \quad (7.3)$$

Equations (7.1), (7.2) and (7.3) say that there are three "financial instruments" (recall that two of them are the "implied" forward contracts) which identically run from time T to time T+91 in their maturities. If the markets (T-bill, RP, and T-bill futures markets) are perfect and perfectly competitive,[6] we must have

$$PO1_{T,T+91} = PU_{T,T+91}, \tag{7.4}$$

$$\text{or, } PO2_{T,T+91} = PU_{T,T+91}. \tag{7.5}$$

Or, equivalently,

$$FO1_{T,T+91} = FU_{T,T+91}, \tag{7.4a}$$

$$\text{or, } FO2_{T,T+91} = FU_{T,T+91}. \tag{7.5a}$$

Note that quation (7.4a) is a "conventional" model which involves two financial markets---T-bill (contains a long-term T-bill and a short-term T-bill) and T-bill futures markets;[7] equation (7.5a) is so-called the cost-of-carry model which involves three financial markets---T-bill (contains a long-term T-bill), RP and T-bill futures

---

[6]A "perfect market" is sometimes defined as one that is frictionless and operated in a certainty world. By "perfect competitive" we mean that a market is sufficiently large that no individual or group of individuals acting together can affect market prices by changes in the amounts they are willing to buy or sell. Narrowly speaking, for example, each investor at any point in time should obtain a unique repo rate in the RP market regardless of the size of institutions or the credibility of investors.

[7]This "conventional" model considered almost exclusively transaction costs only. It was predominantly used, since the International Monetary Market (IMM) of the Chicago Mercantile Exchange began trading the Treasury bill futures contract on January 6, 1976, until an entirely different focus on the futures-forward rate relationship (so-called the cost-of-carry model) was provided by Vignola and Dale (1980). Kawaller and Koch (1984) pursued it in more detail.

markets. These identities comprise the rationale on which most work in testing the efficiency (i.e., no arbitrage opportunity) of the T-bill futures market is based, subject to a variety of relaxed perfect market assumptions involving consideration of transaction costs, financing costs, marking to the market, default risk premium, liquidity preference, or taxes treatment, etc. Among these factors of imperfection which have been taken either empirically or theoretically into account in testing for inefficiencies of the T-bill futures market, there apparently is no research that gives attention to inflation uncertainty. However, it is known that uncertainty about the rate of inflation may affect investors' investment decisions and, from a macro-point of view, inflation can distort the allocation of resources in the economy (Barro, 1986; Fama and Gibbons, 1982; Fischer, 1975; Keynes, 1930; and Roll, 1972; among others). Probably one of the reasons that the previous work in dealing with the efficiency of the T-bill futures market did not give a weight to inflation uncertainty is that a cost-of-carry model was not widely used in this financial market. When financing costs are considered in calculating arbitrage profits from trading financial instruments in the futures market against those in the spot market, the cost-of-carry model is used.[8] However, a constant RP (either the overnight RP or the term RP) rate is commonly assumed in using the cost-of-carry model. This procedure ignores the uncertainty of future financing costs which a locked-in investor must face because he needs to continuously finance his pure arbitrage positions. As a consequence, we conjecture that a "risk premium" associated with inflation uncertainty, which is due to the requirement of continuous financing for arbitrageurs, could be a factor to account for the futures-forward rate differential as observed by using the cost-of-carry model with constant RP rates. As reported in Part I, there exist unexplained factors of market imperfection which cause futures rates to be less than forward rates except that a higher level of interest rate (the federal funds rate) is used as a proxy for financing costs. If inflation uncertainty gives rise to a premium in compensation for continuously financing arbitrage positions, this premium will lower down the forward rates in the cost-of-carry model. As a result, inflation uncertainty can explain, at least in part, the puzzling phenomenon of the futures-forward rate differential. To qualitatively demonstrate the existence of "risk premium" of inflation uncertainty as a

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[8]The spot markets here also include the RP market. A RP is equivalent to a short-term T-bill when the cost-of-carry model is used.

new candidate to account for "inefficiency" of the T-bill futures market, we need to show that there exists a systematic relationship between the daily futures-forward rate differences and the inflation rate. As mentioned in Chapter 6, the correlation between the daily futures-forward rate differences and the inflation rate provides evidence of an uncertain inflation premium.

CHAPTER 8

DATA AND METHODOLOGY

This chapter will describe the available data sets, methodologies to interpolate the unavailable daily CPI data, and the methods used in testing for the impact of inflation uncertainty on the futures-forward rate differential. The tests employ the derived daily inflation rates and the constructed implied forward rates using different financing rates. Because the weekly and daily inflation rates are unavailable, we will first utilize a regression interpolation method to obtain weekly inflation rates; secondly, Lagrange interpolating polynomial functions are derived to calculate daily inflation rates. The reasons to use different financing rates as proxies for RP rates are explained in section of Test Method. It is clear that the derived daily inflation rates are the proxies for the "true" daily inflation rates. The necessity of using proxies for the daily inflation rates seems unavoidable, that is similar to the choice of proxies for the "true" repurchase agreement rates (either the overnight RP rates or the term RP rates) in the RP market to account for financing costs.

Data Description

Ten data sets were collected for testing for the impact of inflation uncertainty on the futures-forward rate differential. The data sets are divided into five broad categories: (1) the T-bill market (long-term T-bill); (2) the T-bill futures market; (3) the RP market;[9] (4) measures of the money stocks (M1, M2, M3 and L);[10] (5) the CPI (Consumer Price Index). The data sets of (1), (2) and (3) comprise the daily data of year 1983.[11] There are two kinds of money stocks data. The first consists of the monthly data of M1, M2, M3 and L from 1980 to 1985 and the second consists of the weekly M1 data in 1983. The CPI data are monthly data from 1980 to 1985. All daily data sets used in the study cover from January 3 to December 20 of 1983. This period matches the four corresponding nearby T-bill futures' expiration dates of 1983 which are March 24, June 9, September 1, and December 22. The long- and short-term daily

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[9]The overnight federal funds rate and the 90-day-maturity T-bill rate are used as proxies for the overnight RP rates. The short-term T-bill rate is used as a proxy for a term RP rate.

[10]For the definitions of M1, M2, M3 and L, see Table 1A.

[11]Except for those data in June 1983, the library lost them when this study was conducted. For those missing data in June, 6 (out of 35) observations fall in Group 1 ($K<15$) and 15 (out of 84) fall in Group 5 ($K \geq 60$), where K represents the number of days to the delivery of a futures contract.

data are collected from the yields of U. S. Treasury bills reported in the Wall Street Journal.[12] The corresponding T-bill futures rates, the overnight federal funds rates, and the 90-day-maturity T-bill rates are collected from the 1983 yearbook provided by International Monetary Market (IMM). Both the 90-day-maturity T-bill rates and the overnight federal funds rates are close rates. The T-bill futures rates are settlement discount rates. Monthly money stock data (M1, M2, M3 and L) from 1980-1984 are collected from Annual Statistical Digest of the Board of Governors of the Federal Reserve System. The 1985 monthly money stock data are collected from Survey of Current Business, because Annual Statistical Digest is available only up to 1984 at the present. The weekly M1 data of 1983 are collected from the U. S. Financial Data by the Federal Reserve Bank of St. Louis. Finally, the CPI data are collected from the annual Economic Report of the President.[13]

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[12]The bond-equivalent yields were collected first. We then computed the asked prices for short-term and long-term T-bills by the formula: Bond Equivalent = [(Face Value-Price)·365]/[(Days to Maturity)·Price]. An annualized forward rate can be accordingly computed. When the overnight financing costs are used, the overnight federal funds rates and the 90-day-maturity T-bill rates are used as substitutes for the short-term T-bill rates.

[13]The Economic Report of the President is published annually by United States Government Printing Office, Washington, D. C. It also lists money stock data but some of L data are missing.

Because there are four delivery dates in the T-bill futures market in 1983, four categories of data in terms of calendar time are collected to match the corresponding futures' delivery dates. It is conjectured that the effect of the "time span" (the number of days to the delivery date), which symbolizes the resolution of information about inflation uncertainty, could be important to the market efficiency. Each category is thus divided into five groups according to the number of days to the delivery of the corresponding nearby T-bill futures. The number of days is specified as fewer than 15, 15 to 29, 30 to 44, 45 to 59, and greater than 59. The final step is to assemble those observations in the four categories which belong to the same "time span." When a term RP rate is used as a financing rate, a forward rate is calculated by equation (7.1). When an overnight RP rate is used as a financing rate, a forward rate is obtained from equation (7.2). The daily inflation rates are derived from the daily CPIs. Because the CPI data is available on a monthly basis, an interpolation procedure is needed. There are no daily money stock data so that it is impossible to use the parameters obtained from regressing (monthly) CPI on (monthly) money stock to procure daily CPIs. On the other hand, it is impracticable to use the Lagrange Interpolation Polynomial (LIP) to obtain daily CPIs because it would be a polynomial of degree 364 for 365 days a year.[14] Thus, a two-step interpolation procedure is used to obtain daily CPIs. Namely, a regression interpolation method and a Lagrange Interpolation Polynomial (LIP) are used for this purpose. The details are described as follows.

Methodology

Regression Interpolation with Highly Related Time Series[15]

Before using the regression interpolation it is necessary to find an interpolator which is highly correlated with CPI. As it is well known that money supply is intimately connected with CPI, M1, M2, M3 and L are chosen as the candidates for interpolation. The correlations of M1, M2, M3 and L with CPI from 1980 to 1985 are shown in Table 1A. We find that M3 has the highest correlation with CPI, whereas M1 has the lowest. Unfortunately, among these four money stocks, M1 is the only one with weekly-reported data available. The correlations ranging from 0.951 to 0.976,

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[14]Linear interpolation is ruled out, although it is much simpler than the LIP. The weekly CPIs derived from a linear regression method are not monotonically increasing (or decreasing). Presumably, the LIP will thus better estimate daily fluctuations in the CPI than will the linear interpolation method.

[15]The relationship between the movements of money supply and CPI is well acknowledged. Therefore, from cost-effective point of view and by Occam's razor, there is no point to using subtler regression methods. For a more complex regression method to fit particular needs, the interested readers may refer to Appendeix 2.

Table 1A

Correlations of M1, M2, M3, and L with CPI:[a]

	M1	M2	M3	L
CPI	0.951	0.967	0.976	0.970

a

1. Monthly data from 1980 to 1985. The following definitions of M1, M2, M3, and L are adopted from the annual Economic Report of the President.
2. M1 is sum of currency, demand deposits, travelers' checks, and other checkable deposits (OCDs).
3. M2 comprises M1 plus overnight RPs and Eurodollars, MMMF (Money Market Mutual Funds) balances (general purpose and broker/dealer), MMDAs (Money Market Deposit Accounts), and savings and small time deposits.
4. M3 comprises M2 plus large time deposits, term RPs, and institution-only MMMF balances.
5. L comprises M3 plus other liquid assets.

Table 1B

Regression of CPI on M1: (monthly data from 1980 to 1985)

$$\underset{}{CPI} = \underset{(16.805)}{115.33525} + \underset{(25.693)}{0.35670}\ M1$$

$R^2 = 0.90413$; t statistics in parenthesis.

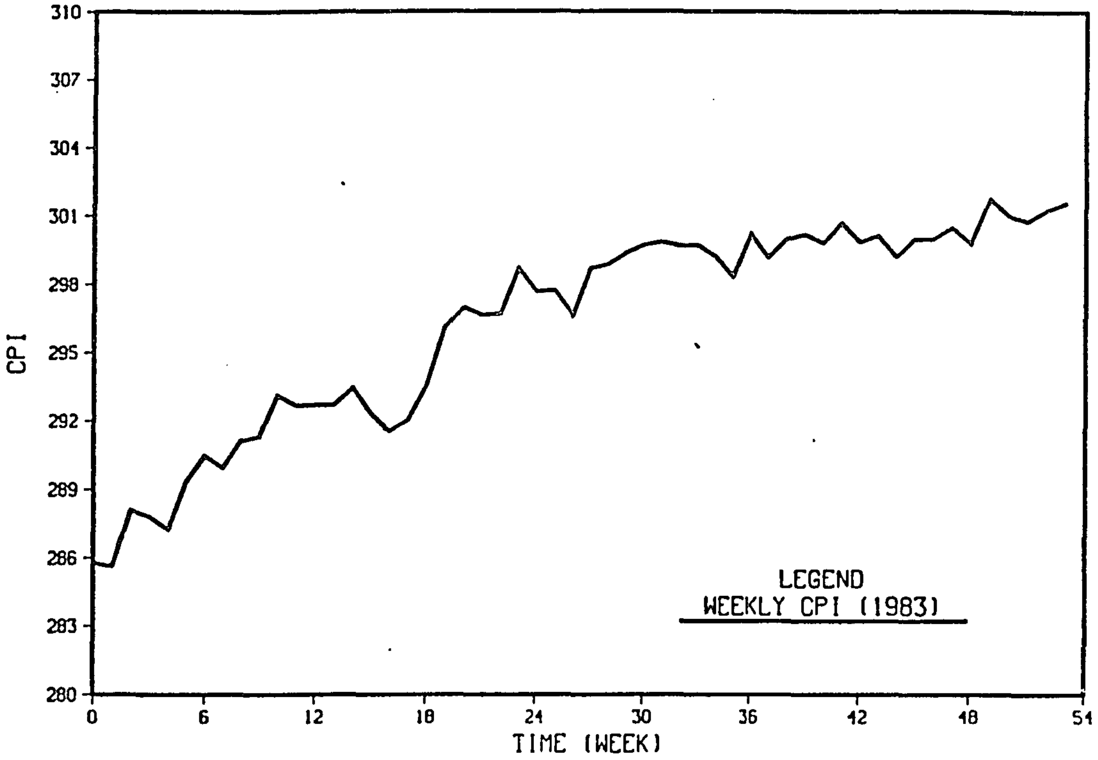

FIGURE 1. WEEKLY CPI GENERATED BY THE REGRESSION METHOD

however, suggest that the differences can be conceivably treated as trivial. We therefore use the monthly M1 and CPI data from 1980 to 1985 to run a regression and obtain parameters as shown in Table 1B. By using the regression parameters and the 1983 weekly M1 data, the weekly CPIs of 1983 are estimated and plotted in Figure 1.

Interpolation with a Lagrange Interpolating Polynomial (LIP)

LIP is an approximating polynomial that determines a function based on certain points on the plane through which the function must pass. The properties and derivation of LIP are presented in Appendix 3. After weekly CPIs have been generated, we have 54 certain points on the weekly basis including the last week in 1982 and the first week in 1984. Because the first weekly M1 datum is for January 5, 1983, the last weekly M1 datum of 1982 must be included so that the daily data from January 1 to 4 can be interpolated. For similar reasons the first weekly M1 datum of 1984 is included. Even though the degree of the LIP has been reduced to 53 from 364, it is still impracticable to estimate daily CPI data from it. Instead 12 LIP functions were generated by using a month as an interval. As a result, there are overlaps in the ending and starting points among these monthly intervals. That is, for those points around the end of a month and the beginning of its succeeding month, they serve as an ending point in one interval and, at the same time, also as a starting point in the succeeding interval. A property of the LIP guarantees, fortunately, that the consecutive functions must pass through these points no matter whether they are used as ending or starting points. We thus chose a weekly-CPI date as the ending point of an interval so long as it is closer to the last day of a particular month than its nearby month. This ending point of the interval will be the starting point of the next interval. By circularity, the starting and ending points for each interval can be chosen so that 12 LIP functions are generated as provided in Table 2. The daily CPI data are accordingly approximated and plotted in Figure 2. Note that the daily-intervals in each of 12 LIP functions are measured by 1/7 when interpolating them based on their respective LIP functions. After inspecting Figure 1 and Figure 2, we see that Lagrange Interpolating Polynomial fits the (weekly) CPI trend surprisingly well. These estimated CPIs will be used as proxies for the "true" daily CPIs in 1983. After they have been trnasformed into the calendar-date basis, they are grouped in a way identical to the other data sets. Therefore, an <u>ex post</u> inflation rate is computed by

$$\pi_{t,T} = \frac{CPI_{T,T} - CPI_{t,T}}{CPI_{t,T}}. \tag{8.1}$$

Table 2

Lagrange Interpolating Polynomial (LIP) Functions:[a]

1.	$P_1(m)$ =	$-0.0623m^5+1.2629m^4-9.4759m^3+32.1901m^2-47.3949m+309.247.$
2.	$P_2(m)$ =	$-0.2530m^4+3.0902m^3-13.0405m^2+22.3993m+227.165.$
3.	$P_3(m)$ =	$-0.1382m^4+1.8459m^3-8.7623m^2+17.2586m + 281.091.$
4.	$P_4(m)$ =	$-0.0596m^4+0.9767m^3-5.3164m^2+10.7563m+286.329.$
5.	$P_5(m)$ =	$-0.0146m^5+0.3447m^4-2.9408m^3+10.8368m^2-15.1102m+298.928.$
6.	$P_6(m)$ =	$-0.2677m^4+3.3671m^3-15.0263m^2+27.5229m+281.121.$
7.	$P_7(m)$ =	$0.0256m^5-0.4947m^4+3.6550m^3-12.8073m^2+21.5345m+284.733.$
8.	$P_8(m)$ =	$0.0372m^4-0.4967m^3+2.1583m^2-3.7348m + 301.928.$
9.	$P_9(m)$ =	$-0.3019m^4+3.8158m^3-16.8106m^2+30.1408m+281.514.$
10.	$P_{10}(m)$ =	$-0.0930m^5+1.6423m^4-10.8808m^3+33.2443m^2-45.7096m+322.046.$
11.	$P_{11}(m)$ =	$-0.1248m^4+1.4623m^3-6.0457m^2+10.5582m+293.400.$
12.	$P_{12}(m)$ =	$0.0136m^5-0.3232m^4+2.8690m^3-11.6988m^2+21.4003m+287.559.$

[a]1983 is divided into 12 intervals, and 12 corresponding LIP functions are generated accordingly. The unit in generating a LIP function is a week.

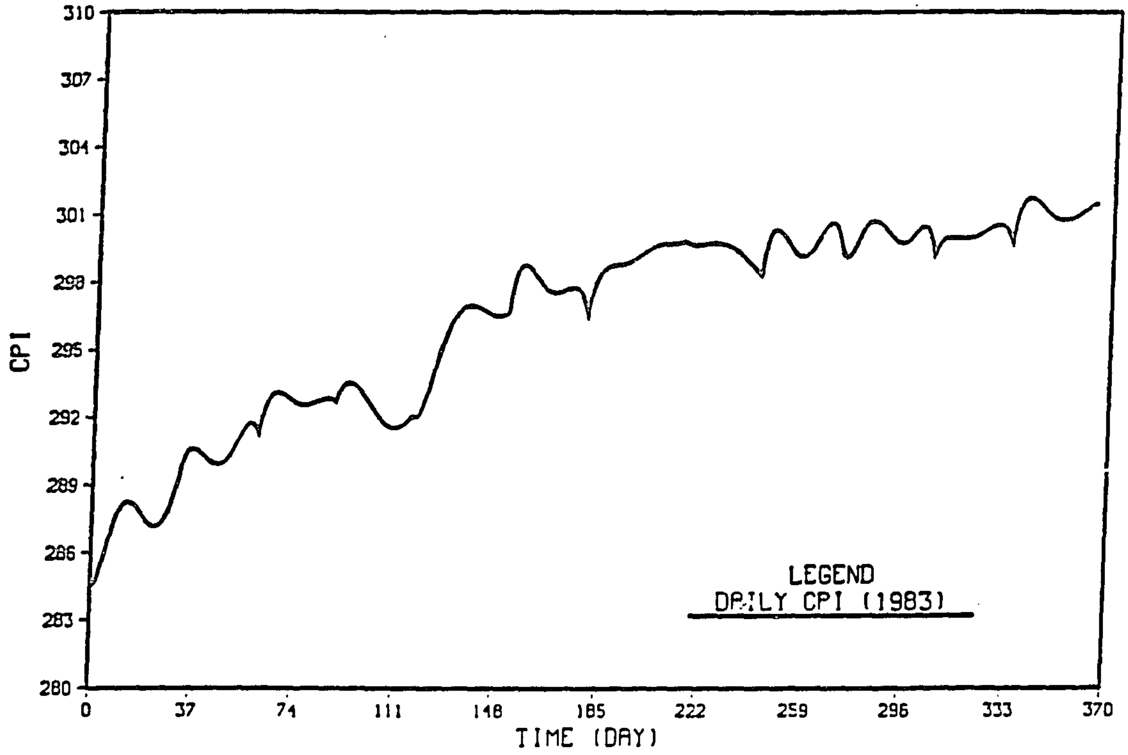

FIGURE 2. DAILY CPI GENERATED BY THE LIP METHOD

Where, $CPI_{T,T}$ is the CPI in the delivery date of a T-bill futures contract; $CPI_{t,T}$ is the CPI at time t. Note that t is approaching T when time passes so that the number of days to the delivery date is decreasing. It is clear that there are four Ts because there are four expiration dates of the T-bill futures contracts in 1983. However, there should be no confusion that the time from t to T is equivalent to the number of days to the delivery of the T-bill futures contracts so that the choice of t relative to T in calculating $\pi_{t,T}$ is based on the "time span" as previously described rather than on the calendar time when testing for the impact of inflation uncertainty on the efficiency of the T-bill futures. That is, there will be multiple "T days" in a "time span."

Test Method

A regression approach test is performed to test whether inflation uncertainty can explain the market-observed differentials of the futures and forward rates in the year of 1983. Daily futures-forward rate differences of 1983 are regressed on the daily inflation rates computed by equation (8.1). There are three types of financing costs which are selected as proxies for the "true" financing costs in the RP market. The corresponding forward rates from time T to T+91 are derived from these proxies. The choice of proxies is deemed to be necessary to account for the differentials in the RP rates among different borrowers. As it is known that the borrower pays interest on the funds at a rate that is negotiated with the lender in the RP market, there can be biases by specifying a unique RP rate at any particular point in time. That can be easily seen from the fact that larger and better known dealers are often able to borrow in the RP market at more favorable rates than some dealers and corporations (see Bowsher, 1979). To circumvent this problem, we use the federal funds rate and the 90-day-maturity T-bill rate as proxies for the overnight RP rates. Since the federal funds rate is the rate on unsecured overnight loans and the 90-day-maturity T-bill is secured by the U. S. Treasury, the former can be expected to be higher than the latter. As a consequence, a range in financing costs is allowed to derive the implied forward rates using the cost-of-carry model. For the term RP rate, a short-term spot T-bill rate is used as its proxy. All Ordinary Least Square (OLS) statnard assumptions are made in the regression. If inflation uncertainty resulting from the requirement of continuously financing aribitrageurs' positions do exert impact, the empirical results should show tendencies of (1), (2) and (3) as described in Chapter 6.

CHAPTER 9

EMPIRICAL EVIDENCE

The empirical results presented in Tables 3, 4 and 5 are consistent with the implications of inflation uncertainty. Tables 3 and 4 are the results using the lower level of interest rates as proxies for financing costs, namely, the short-term spot T-bill rates and the 90-day-maturity T-bill rates. Both two Tables show that the correlations between the daily futures-forward rate differences and the inflation rate are relatively high, except for Group 1 where the number of days to the delivery of futures contract is fewer than 15. The t values of $\hat{\beta}$ are significant in the entire sample in both Table 3 and Table 4. Moreover, $\hat{\beta}$ is insignificant in the first three groups and significant in the last two groups of Table 3. In Table 4, $\hat{\beta}$ is insignificant in Groups 1 and 5 and significant in Groups 2, 3 and 4. These findings show that inflation uncertainty which arises from the requirement of continuous financing plays a role in explaining the phenomenon of futures-forward rate differential. The insignificance of $\hat{\beta}$ in those groups which have short length of time to the expiration date of the T-bill futures

Table 3

Futures-Forward Rate Difference
as a Function of Inflation Rate

CASE I: The Term RP Rate Used as a Financing Cost[a]

	Entire Sample	Group 1 $K<15$	Group 2 $15\le K\le 29$	Group 3 $30\le K\le 44$	Group 4 $45\le K\le 59$	Group 5 $K\ge 60$[b]
Correlation[c]	-.408	-.086	-.256	-.139	-.497	-.316
$\hat{\alpha}$	-.047	.033	-.009	-.089	-.079	-.125
$\hat{\beta}$	-.066	-.074	-.057	-.017	-.032	-.046
	(-6.568)*	(-.451)	(-1.678)	(-.875)	(-3.666)*	(-2.60)**
Cases	218	29	42	41	43	63

[a]The short-term spot T-bill rate is used as a proxy for the term-RP rate.

[b]$K \equiv$ the number of days to the delivery of the T-bill futures market.

[c]The correlation between the daily futures-forward rate difference and the inflation rate.

*t-value which is significantly different from zero at the 1% level, two-tail test.

**t-value which is significantly different from zero at the 2% level, two-tail test.

Table 4

Futures-Forward Rate Difference
as a Function of Inflation Rate

CASE II: The 90-day-maturity T-bill Rate Used as a Financing Cost[a]

	Entire Sample	Group 1 $K<15$	Group 2 $15 \le K \le 29$	Group 3 $30 \le K \le 44$	Group 4 $45 \le K \le 59$	Group 5 $K \ge 60$[b]
Correlation[c]	-.435	-.001	-.403	-.765	-.654	-.072
$\hat{\alpha}$	.014	.055	.062	.079	.036	-.162
$\hat{\beta}$	-.080	-.001	-.072	-.097	-.067	-.012
	(-7.104)*	(-.004)	(-2.783)*	(-7.413)*	(-5.533)*	(-.564)
Cases	218	29	42	41	43	63

[a]The 90-day-maturity T-bill rate is used as a proxy for the overnight RP rate.

[b]$K \equiv$ the number of days to the delivery of the T-bill futures market.

[c]The correlation between the daily futures-forward rate difference and the inflation rate.

*t-value which is significantly different from zero at the 1% level, two-tail test.

contract is consistent with the resolution of inflation uncertainty through time. The correlation and β is Group 5 of Table 4 is the only exception. In Table 5, a relative high level of interest rate, i.e. the federal funds rate, is used as a proxy for the financing costs. The correlations are relatively low and βs are insignificant in both the entire sample and all five groups. Such an interesting result could be interpreted as that the premiums (a combination of all possible risks) contained in the federal funds rate are so high that the risk premium of inflation uncertainty is overridden. As evidenced by the empirical findings presented in Tables 3, 4 and 5, there exists a systematic relationship between the futures-forward rate differential and inflation uncertainty, at least for the 1983 period, when the cost-of-carry model is used to test the efficiency of the T-bill futures market.

Table 5

Futures-Forward Rate Difference
as a Function of Inflation Rate

CASE III: The Federal Funds Rate Used as a Financing Cost[a]

	Entire Sample	Group 1 $K<15$	Group 2 $15 \le K \le 29$	Group 3 $30 \le K \le 44$	Group 4 $45 \le K \le 59$	Group 5 $K \ge 60$[b]
Correlation[c]	.063	-.054	.026	-.371	-.360	.059
$\hat{\alpha}$	.181	.080	.130	.199	.242	.296
$\hat{\beta}$	.018	-.046	.007	-.048	-.067	.023
	(.934)	(-.280)	(.166)	(-2.495)	(-2.472)	(.464)
Cases	218	29	42	41	43	63

[a]The federal funds rate is used as a proxy for the overnight RP rate.

[b]$K \equiv$ the number of days to the delivery of the T-bill futures market.

[c]The correlation between the daily futures-forward rate difference and the inflation rate.

CHAPTER 10

SUMMARY OF PART II

This Part investigates the impact of inflation uncertainty on the futures-forward rate differential. The cost-of-carry model commonly assumes a constant RP rate as a financing cost through the life of a locked-in arbitrage position. That is a natural outcome of the data availability. The disadvantage is, however, that the future fluctuations of financing costs are empirically ignored. Any assertion that the T-bill futures market is efficient or inefficient is therefore only valid under those assumptions imposed in the studies. Since a pure arbitrageur is forced to continuously finace his positions until the delivery date of the T-bill futures contract, a risk premium of inflation uncertainty could arise. As reported in Part I, there exists a futures-forward rate differential even if the cost-of-carry model is used to test for the efficiency of the T-bill futures market in the year of 1983. There are two possibilities to explain the findings. First, the T-bill futures market is inefficient in the cost-of-carry model. Second, there are some factors of market imperfection which have not been discovered. The second possibility motivates this study to search for new candidates of factors of imperfection which may exist when investors trade financial instruments in the futures market against the spot markets. The existence of transaction costs, financing costs, marking to the market (or daily resettlement), default risk premium, liquidity preference, and taxes treatment has been extensively studied as the imperfect market's factors to explain the existence of the futures-forward rate differential. This study proposes that "risk premium" of inflation uncertainty be a factor to account for the puzzling phenomenon of the futures-forward rate differential. The empirical findings are consistent with the implications of inflation uncertainty which results from the requirement of continuous financing for arbitrageurs. This study is, however, not a "quantitative" investigation into the amount of risk premium of inflation uncertainty which is required by the investors (or arbitrageurs) in compensation for continuously financing their positions. Rather, the study is to qualitatively show the existence of a systematic relationship between the daily futures-forward rate differences and the inflation rate.

The empirical findings give evidence that inflation uncertainty can be one of factors of imperfection in the T-bill futures market. However, whether the T-bill futures market is efficient or not in the cost-of-carry model, at least as the findings reported in Part I, remains in question because risk premiums (any kinds) in this financial area have not been quantified (or successfully theorized) in the current state of art. "Inefficiencies" of the T-bill futures market have been widely reported (see Table 1 and the findings of Part I). Among others, Elton, Gruber, and Rentzler (1984) concluded that the T-bill futures market is not perfectly efficient; Jarrow and Oldfield (1981) argued that forward prices need not equal futures price unless default free rates are deterministic. Hicks (1946), when he spoke of the pure "Spot Economy" and the pure "Futures Economy," said that "...this can have no claim to be a good approximation to reality, for it would be in a world where uncertainty was absent and all expectations definite, that everything could be fixed up in advance." All of this seems to favor the notion that the T-bill futures market may have days in which there exists a non-zero futures-forward rate differential. That most of studies show inefficiency of the T-bill futures market is puzzling and mind-boggling because, on the other hand, everyone recognizes that the T-bill futures market is so large that arbitrage profits should not exist or exist for very short periods. Nevertheless, through the findings of this study, an alternative could arise to answer such an enigma: we may not completely take "practitioners' concerns" in the form of risk premiums into account when undertaking empirical tests for the efficiency of the T-bill futures market.

PART III

OPTIMAL ARBITRAGE INVESTMENT UNDER UNCERTAINTY AND EQUILIBRIUM PRICING IN THE T-BILL FUTURES MARKET: A DYNAMIC STOCHASTIC PROGRAMMING MODEL

This part of the dissertation provides a theoretical treatment of the optimal arbitrage investment under uncertainty and of equilibrium pricing in the T-bill futures market. A dynamic stochastic programming model shows that a "myopic" property exists in the T-bill futures market in the sense that expectations of the future one-period price movements do not exert an impact on the current optimal arbitrage investment decision under uncertainty. It shows, however, that such a "myopic" property is not pure in that expectations of financing costs in the next period affect the investment decision in the current period. Equilibrium pricing of the T-bill futures is obtained under arbitrage arguments in the framework of the dynamic stochastic programming model. It shows that an equilibrium price is achieved at the point where the expected current one-period arbitrage profits are zero when cost-of-carry is required, even in a multi-period setting.

CHAPTER 11

INTRODUCTION TO PART III

Most of the empirical tests on the efficiency of the Treasury bill futures market are carried out in a certainty framework. Consequently, the concept of equilibrium in the T-bill futures market is based on arbitrage arguments under certainty. That is, an equilibrium price in the T-bill futures market at any point in time under certainty is achieved so that there are no arbitrage profits available to investors by taking positions in the T-bill futures, T-bill and/or RP (repurchase agreement) markets. If arbitrage opportunities exist, investors can lock-in a positive return on a zero initial net investment assuming no transaction costs. With free entry and costless information, an equilibrium T-bill futures price will obtain such that arbitrage-trading stragetegies cannot result in profits.

Investors do not face a world of certainty when reaping arbitrage profits, however. When investors undertake arbitrage trades, there are usually two well-known uncertainty factors: fluctuations in the margin requirements and in the future financing rates such as the repo rates. As documented in Part II, inflation uncertainty is also of concern to investors. The premium due to inflation uncertainty results from the fact that a locked-in investor is forced to finance continuously his arbitrage positions with a cost-of-carry model until the expiration date of the T-bill futures contract. Other uncertainties contained in their arbitrage operations include default risk, interest rate uncertainty, uncertain future personal income tax brackets, unstable liquidity preferences, changeable consumption plans, wild card options,[1] general economic conditions, etc. All of these uncertainty factors which may exist through the life of a locked-in arbitrage position may exert constraints on the investor's current arbitrage decisions. As a result, "arbitrage" in a practical setting cannot be risk-free. Because speculative arbitrageurs are exposed to uncertainty, this form of arbitrage is called "risk arbitrage." In this part, we develop a dynamic stochastic programming model to determine the optimal arbitrage investment under uncertainty in the T-bill futures market. The equilibrium price of T-bill futures is then

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[1]The International Monetary Market (IMM) of the Chicago Mercantile Exchange allows the short position in the T-bill futures contract a delivery option of three business days in the delivery month. This creates a put option for the short position in the T-bill futures, which is similar to the put options for short position in the T-bond futures contract of the Chicago Board of Trade which allows the short position to choose any business day in the delivery month to make delivery. Such a put option for the short position has been dubbed the "wild card option."

obtained by arbitrage arguments. In the framework of a dynamic stochastic programming model, an investor can close out his positions in each period before the expiration date of a T-bill futures contract; new positions are then reestablished for the next period. The movements of T-bill and T-bill futures prices will determine his arbitrage profits at the end of each period. By pure arbitrage, it is clear that the investor starts with zero initial wealth if no other types of costs are involved in pure arbitrage. However, there will be some net wealth realized at the end of each period when the investor closes out his positions. This net wealth could be positive or negative, or even zero again. Because closeout and reestablishment of positions in each period are allowed, the above-mentioned uncertainties which may exist through the life of a given T-bill futures contract is considered <u>in each period</u> by the dynamic stochastic programming model. As a result, most of those described uncertainty factors can be conceivably assumed away in the model, except for the intrinsic uncertainty of future price movements in the T-bill and T-bill futures markets.

Part III is organized as follows. Chapter 12 describes assumptions of the model and investment decisions in each period in the framework of pure arbitrage. In Chapter 13, we introduce a dynamic stochastic programming model for an arbitraging investment in the T-bill futures market. Chapter 14 develops the optimal arbitrage investment under uncertainty and the equilibrium pricing of the T-bill futures market via the model under arbitrage arguments; it also describes some implications of the results derived by the model. We summarize Part III in Chapter 15.

CHAPTER 12

BASIC ASSUMPTIONS AND INVESTMENT STRATEGIES

The dynamic stochastic programming model used in this part assumes that a pure-arbitrage investment continues for two periods. The investor liquidates his positions at the end of period 2. We denote the starting time point in making a pure-arbitrage investment decision as time 0. It is assumed that the investor will close out his positions at the end of period 1 which is defined as the period from time 0 to time 1. Simultaneously, the investor will reestablish new pure-arbitrage positions at the end of period 1 according to the prices of T-bill and T-bill futures at that time. It is assumed in the model that "the end of period 1" is equivalent to "the beginning of period 2." Therefore, the investor's net wealth at the beginning of period 2 will equal the realized arbitrage profits, denoted as W_1, from the investment in the first period. This net wealth is not necessarily equal to his initial wealth. Because the investor reestablishes new positions at the beginning of period 2, we assume that W_1 will be included in the funds needed for the new positions. If transaction costs are zero, any change in positions at the beginning of period 2 can be viewed as closing out previous positions and reestablishing new ones. We assume that the investor maximizes his expected utility of the net wealth at the end of period 2 via such a sequence of pure-arbitrage investments. An optimal investment at the beginning of period 2 is obtained assuming a level of wealth equal to W_1 at that time. The optimal investment at time 0 can be accordingly obtained using dynamic stochastic programming. As a result, the equilibrium prices in the T-bill futures market under arbitrage arguments can be determined in a two-period dynamic stochastic programming model.

Our model involves the RP market, in addition to the T-bill and T-bill futures markets. A margin requirement is required in the T-bill futures market because of daily resettlement, or marking to the market. The one-period cost-of-carry is incorporated into the model so as to take into account financing costs. Since an investor reestablishes his arbitrage position in each period, he is not in a locked-in arbitrage position through the life of the T-bill futures contract. Thus, a new but known current-period financing rate will be taken into account in the positions of each period. It is assumed that there are no transaction costs. We assume, for simplicity, that one period is one day in the model.

To construct a pure arbitrage position at time 0, an investor borrows $P_{r0}x_0$ dollars in the RP market, where P_{r0} is the market price at time 0 of a bill used in the repurchase agreement which expires at time 1 and x_0 is the number of bills. He purchases $P_{10}y_0$ dollars worth of long-term T-bills, where P_{10} is the market price at time 0 of a long-term T-bill which can be used for the delivery of the futures contract (but it may not be necessary to make delivery at time 1), and y_0 is the number of long-term T-bills acquired. Simultaneously, the investor sells $P_{f0}y_0$ dollars worth of T-bill futures contracts, where P_{f0} is the market price of a T-bill futures at time 0 and y_0 is the number of T-bill futures contracts sold. In the model, y_0 represents the <u>net</u> positions in T-bill and T-bill futures and its sign is not predetermined. Namely, if y_0 is negative it means that the investor sells $P_{10}|y_0|$ dollars worth of long-term T-bills and purchases $P_{f0}|y_0|$ dollars worth of T-bill futures. For the same reasons, x_0 can be either positive or negative. When x_0 is negative it means that the investor is in a reverse repurchase agreement position, i.e., he lends $P_{r0}|x_0|$ dollars to a fund-raiser in the RP market. Because the number of long-term T-bills and T-bill futures is identical (i.e., y_0), the obligation of delivery (or taking the delivery) can be fulfilled at time 2, provided that time 2 is the expiration date of the given T-bill futures contract. Note that the investor will simply close out his positions if time 2 is not the expiration date and, if that were the case, time 1 cannot be the delivery date too. By pure arbitrage, it requires that $P_{r0}x_0$ be equal to $P_{10}y_0$ so that the investor performs arbitrage with zero investments, provided that no other types of costs are needed. As a consequence, the investor starts out pure-arbitrage investments at time 0 on a one-day position. After the investor closes out his positions at the end of the first period, his net wealth is W_1 which is the realized arbitrage profits. Thus, W_1 is the increment to his wealth over the first period. Such an increment in his wealth allows the investor to reduce the amounts of funds, which are to be borrowed from the RP market, in order to engage in arbitrage in period 2.[2]

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[2]W_1 could turn out negative and it is then a decrement to the investor's wealth. However, if that were the case, an investor holds the option of not closing his positions in practice even if <u>after</u> he had started out a period-by-period closing-position plan at the beginning of a multi-period investment as assumed in the dynamic stochastic programming model, except that the beginning of the next period happens to be the delivery date of the T-bill futures contract.

CHAPTER 13

THE DYNAMIC STOCHASTIC PROGRAMMING MODEL OF ARBITRAGE

A discrete-time two-period dynamic stochastic programming model to maximize the expected utility of an investor's arbitrage profits in each period is formulated as follows.

At time 1, the maximization is

$$J_1(W_1) = \max_{y_1} E_1\{U[W_2(y_1)] \mid \Omega_1, W_1\}$$

$$= E_1\{U[W_2(\hat{y}_1(W_1))] \mid \Omega_1, W_1\} \qquad (13.1)$$

where, $\hat{y}_1(W_1)$ means that $\hat{y}_1$ is a function of the initial wealth W_1 at time 1, i.e., the beginning of the last period (period 2); and W_2 which is a function of y_1 is the random wealth level at the end of period 2. At time 0, we then regard W_1 as a random one-period profit. We have W_0 as initial wealth level and wish to determine y_0. Ω_1 is the information set at time 1 which affects investors' subjective probability distribution of W_2; Ω_1 can be dropped without affecting the results when homogeneous expectations are assumed in the market.

We first let

$$J_1(W_1) = E_1\{U[W_2(\hat{Y}_1(W_1))]|\Omega_1, W_1\} = U(W_1) \tag{13.2}$$

be the implicit utility function. So, the maximization at time 0 is

$$J_0(W_0) = \underset{y_0}{\text{Max}}\ E_0[U(W_1)\,|\Omega_0, W_0]. \tag{13.3}$$

Each maximization problem is subject to the following constraints,

(1) budget constraint:

$$P_{rt}x_t + W_t - P_{lt}y_t = 0, \text{ for } t=1; \text{ and}$$

$$P_{rt}x_t - P_{lt}y_t = 0, \text{ for } t=0$$

(2) wealth increment after closing out one-day positions:

$$W_{t+1} = P_{l,t+1}y_t - x_t - P_{f,t+1}y_t + P_{ft}y_t, \quad t=0,1$$

where,

P_{rt} ≡ the market price of a one-day repurchase agreement from time t to t+1, with a par value of \$1.

P_{lt} ≡ the market price of a long-term T-bill at time t; the underlying long-term T-bill can be used for the delivery of the T-bill futures contract;

P_{ft} ≡ the market price of a T-bill futures at time t;

x_t ≡ the number of repurchase agreements at time t;

y_t ≡ the number of long-term T-bills or T-bill futures at time t.

Note that the budget constraint at time 0 says that the investment is a pure-arbitrage investment because of $W_0 = 0$ under the provisions of no other types of costs. It can be seen that the value of $(-P_{f,t+1}y_t + P_{ft}y_t)$ in the wealth increment after closing out the (pure arbitrage) one-day position is equivalent to the cash flow into (or out of) the margin account due to marking to the market (daily resettlement) in the case where pure arbitrage is performed in a locked-in position. From the budget constraint and the specification of wealth increment, we see that the wealth increment (W_{t+1}) in each period is rolled over into the next budget constraint, and so on if the model is specified as running more than two periods. Note that the optimal value of the decision variable y is defined to represent the net positions in the buy-and-sell of T-bills and T-bill futures.

By the two-period model to determine the optimal investment under uncertainty in the T-bill futures market, there are two planning periods. As it is often the case in dynamic stochastic programming, we begin working on the optimal investment decision at the beginning of the last period. This brings us to the well-known one-period investment decision problem. Thus, after the optimal value $J_1(W_1)$ in equation (13.1) has been obtained the optimal solution for equation (13.3) can be accordingly determined. From the constraint (1),

$$x_1 = \frac{P_{11}Y_1}{P_{r1}} - \frac{W_1}{P_{r1}}. \tag{13.4}$$

Substituting (13.4) into the constraint (2), we have the random wealth level, W_2, at time 2 (the end of period 2) as

$$W_2 = (P_{12} - P_{11}/P_{r1} - P_{f2} + P_{f1})Y_1 + W_1/P_{r1}. \tag{13.5}$$

It is clear that W_1 is the realized arbitrage profits resulting from the positions being constructed at time 0. Because W_1 is rolled over into the arbitrage investment at period 2 as shown by the budget constraint, its value grows to W_1/P_{r1} at time 2. Note that the financing costs have been implicitly built into the growth of W_1 since $(1 - P_{r1})$ is the one-day financing rate from the RP market. That is, the market value of a repurchase agreement maturing in one day with a par value of \$1 is equal to P_{r1} = (1 - one-day financing rate) based on the banker's discount rate construction. Because it is a par value of \$1, such a market value can be treated as a discount function (which is used for W_2, not for W_1). P_{12} and P_{f2} are the random prices for the long-term T-bill and T-bill futures respectively when the investor closes his arbitrage positions at time 2. They determine the wealth increment per unit of Y_1. We define the term of $(P_{12} - P_{11}/P_{r1} - P_{f2} + P_{f1})$ in (13.5) as Θ_2 which is the random wealth increment per unit of arbitrage position (or per unit of y_1) being engineered at time 1. As a result, $W_2 = \Theta_2 y_1 + W_1/P_{r1}$ and we rewrite (13.1) as

$$J_1(W_1) = \max_{y_1} E_1[U(\Theta_2 y_1 + W_1/P_{r1}) | \Omega_1, W_1] \tag{13.6}$$

Equation (13.6) allows us to obtain the optimal arbitrage positions of y_1 in the T-bill futures market. The optimal value, $J_1(W_1)$, can be accordingly known. By solving the dynamic stochastic programming recursively by working backward, we can determine the optimal arbitrage positions of y_0 at time 0. The equilibrium pricing of the T-bill futures can be consequently determined by arbitrage arguments.

CHAPTER 14

THE OPTIMAL ARBITRAGE INVESTMENT AND EQUILIBRIUM PRICING

In this chapter we derive the optimal arbitrage investment in each period and describe its implications. The equilibrium pricing in the T-bill futures market will be accordingly obtained under arbitrage arguments. We first maximize (13.6) with respect to y_1 to get,

$$\frac{dJ_1(W_1)}{dy_1} = E_1\left[\frac{\partial\{[U(\Theta_2 y_1 + W_1/P_{r1})]\,|\,\Omega_1, W_1\}}{\partial W_2}\;\frac{\partial W_2}{\partial y_1}\right]$$

$$= E_1\left[\{[U_{W_2}(W_2)]\,|\,\Omega_1, W_1\}\Theta_2\right] = 0. \qquad (14.1)$$

We have ignored the diversities of subjective probability distributions and utility functions among investors in that homogeneous expectations and utility functions are assumed in the model. We also assume that such a homogeneous utility function among investors is a type of quadratic utility function. For a quadratic utility function, the (random) utility at time 2 is

$$U(W_2) = a + bW_2 + cW_2^2$$

$$= a + b(\Theta_2 y_1 + W_1/P_{r1}) + c(\Theta_2 y_1 + W_1/P_{r1})^2. \qquad (14.2)$$

Thus, (14.1) becomes

$$E_1\{[b + 2c(\theta_2 y_1 + W_1/P_{r1})]\theta_2\} = 0, \tag{14.3}$$

and (14.3) gives

$$[b + 2c(W_1/P_{r1})]\mu_2 + 2cy_1(\sigma_2^2 + \mu_2^2) = 0, \tag{14.4}$$

where, $\mu_2 \equiv E_1[\theta_2]$ and $(\sigma_2^2 + \mu_2^2) \equiv E_1[\theta_2^2]$.

The optimal positions the investor will hold given the wealth level of W_1 at time 1 is that

$$\hat{y}_1(W_1) = \frac{-[b + 2c(W_1/P_{r1})]\mu_2}{2c(\sigma_2^2 + \mu_2^2)}. \tag{14.5}$$

Since the initial wealth at time 1 is W_1 and $U(W_1) = a + bW_1 + cW_1^2$ as of time 1, a property of the quadratic utility function, gives the absolute risk aversion (ARA) at time 1 as

$$ARA_1 = \frac{1}{-b/2c - W_1} \tag{14.6}$$

so that

$$\frac{-b}{2c} = \frac{1}{ARA_1} + W_1. \tag{14.7}$$

Decomposing (14.5) and using (14.7),

$$\hat{Y}_1(W_1) = \frac{(1/ARA_1 + W_1)\mu_2}{(\sigma_2^2 + \mu_2^2)} - \frac{(W_1/P_{r1})\mu_2}{(\sigma_2^2 + \mu_2^2)}$$

$$= \frac{\mu_2}{ARA_1(\sigma_2^2 + \mu_2^2)} - \frac{(W_1\mu_2)(1/P_{r1} - 1)}{(\sigma_2^2 + \mu_2^2)}. \tag{14.8}$$

This implies that the optimal arbitrage decision is a decreasing function of the investor's absolute risk aversion at time 1, <u>ceteris</u> <u>paribus</u>. The greater the measure of absolute risk aversion, the more unwilling is the investor to take an arbitrage position in the T-bill futures market. Thus, the solution is consistent with the well-acknowledged behavioral characteristic of investors. If P_{r1} approaches 1 the second term of the right-hand side in (14.8) disappears so that

$$\hat{Y}_1(W_1) = \frac{\mu_2}{ARA_1(\sigma_2^2 + \mu_2^2)}$$

$$= \frac{W_1}{RRA_1} \frac{\mu_2}{(\sigma_2^2 + \mu_2^2)}. \tag{14.9}$$

The optimal arbitrage positions at time 1 in (14.9) is thus the product of two ratios. The first one is the ratio of the wealth level at time 1 (i.e., the realized arbitrage profits at time 1) to RRA_1. The second ratio reflects the relationship between the expected dollar return per unit of arbitrage position and its variation. This relationship is reflected as the ratio of the expected dollar return per unit of arbitrage position to the second moment of this random arbitrage profit per unit of arbitrage position, Θ_2.

Given the optimal arbitrage investments at time 1, we can then determine the optimal solution at time 0. Equation (13.3) can be rewritten as

$$J_0(W_0) = \underset{y_0}{\mathrm{Max}}\ E_0[J_1(W_1)|\Omega_0, W_0]. \tag{14.10}$$

W_1 here is the random wealth viewed from time 0 when the first arbitrage positions are set up. Substituting (13.1) into (14.10) and using (14.2) one has

$$E_0[J_1(W_1)|\Omega_0, W_0] = E_0[E_1\{U[W_2(\hat{y}_1(W_1))]|\Omega_1, W_1\}|\Omega_0, W_0]$$

$$= E_0[E_1[a + b(\Theta_2\hat{y}_1 + W_1/P_{r1}) + c(\Theta_2\hat{y}_1 + W_1/P_{r1})^2]].$$

$$= E_0[[a + b(\Theta_2\hat{y}_1 + W_1/P_{r1}) + c(\Theta_2\hat{y}_1 + W_1/P_{r1})^2]]. \tag{14.11}$$

Equation (14.11) assumes that investors hold consistent assessment of probabilities over time. By consistent assessment of probabilities, we mean that for any random variable $\tilde{x}_{t+k+\tau}$, $E_{t-1}[E_{t+k}(\tilde{x}_{t+k+\tau})] = E_{t-1}(\tilde{x}_{t+k+\tau})$, $\tau>0$ and $k\geq 0$ (see, for example, Samuelson, 1965). To maximize $J_0(W_0)$, we differentiate (14.11) with respect to y_0,

$$0 = \frac{dJ_0(W_0)}{dy_0}$$

$$= E_0\left[\frac{\partial J_1(W_1)}{\partial W_1} \frac{\partial W_1}{\partial y_0}\right]$$

$$= E_0\left[b\left[\theta_2 \frac{\partial \hat{y}_1}{\partial W_1} \frac{\partial W_1}{\partial y_0} + \frac{\partial W_1/P_{r1}}{\partial y_0}\right]\right.$$

$$\left. + 2c(\theta_2\hat{y}_1 + W_1/P_{r1})\left[\theta_2 \frac{\partial \hat{y}_1}{\partial W_1} \frac{\partial W_1}{\partial y_0} + \frac{\partial W_1/P_{r1}}{\partial y_0}\right]\right] \quad (14.12)$$

As viewed from time 0, the random wealth of W_1 at time 1 is

$$W_1 = (P_{11} - P_{10}/P_{r0} - P_{f1} + P_{f0})y_0$$

$$\equiv \theta_1 y_0, \quad (14.13)$$

where, θ_1 is defined as the random arbitrage profits per unit of position being engineered at time 0 in a one-day holding period. Using the optimal value, $\hat{y}_1$, obtained in (14.5) to solve (14.12) and substituting $\theta_1 y_0$ for W_1,

$$0 = \frac{dJ_0(W_0)}{dy_0}$$

$$= E_0\left[\frac{-b\theta_1\theta_2\mu_2/P_{r1}}{\mu_2^2 + \sigma_2^2} + b\theta_1/P_{r1}\right.$$

$$\left. - \frac{2cy_0\theta_1^2\theta_2\mu_2/P_{r1}^2}{\mu_2^2 + \sigma_2^2} + 2c\theta_1^2 y_0/P_{r1}^2\right] \quad (14.14)$$

It is assumed that the arbitrage profit (i.e., the dollar return per unit of arbitrage position) in each period is a random walk so that Θ_1 and Θ_2 are independent. Thus, (14.14) is simplified as

$$\frac{Y_0(2cE_0[\Theta_1^2])}{E_0[P_{r1}^2]}\left[1 - \frac{\mu_2^2}{\mu_2^2 + \sigma_2^2}\right] = \frac{bE_0[\Theta_1]}{E_0[P_{r1}]}\left[\frac{\mu_2^2}{\mu_2^2 + \sigma_2^2} - 1\right]. \quad (14.15)$$

Therefore, the optimal arbitrage positions the investor will hold at time 0 is

$$\hat{Y}_0 = \frac{-b}{2c} \frac{\mu_1}{(\mu_1^2 + \sigma_1^2)} \frac{(\mu_{r1}^2 + \sigma_{r1}^2)}{\mu_{r1}} \quad (14.16)$$

where, $\mu_1 \equiv E_0[\Theta_1]$; $(\mu_1^2 + \sigma_1^2) \equiv E_0[\Theta_1^2]$; $\mu_{r1} \equiv E_0[P_{r1}]$; and $(\mu_{r1}^2 + \sigma_{r1}^2) \equiv E_0[P_{r1}^2]$. Equation (14.16) shows that the optimal holding at the beginning of the arbitrage horizon is the product of three ratios. The first ratio, (-b/2c), is the measure of investor's risk tolerance. The second ratio is the relationship between the expected dollar return per unit of arbitrage position established at time 0 and its variation. The third ratio shows the similar relationship (but reversed) for the second-period financing costs which <u>starts at time 1</u> and, clearly, is not known at time 0. The variations of the first-period random arbitrage profits and the second-period random financing costs are captured by their respective second moments which implicitly involve the variances of the respective random variables. Examining

(14.16), we see that changes in the optimal value $\hat{y}_0$ tends to be offset by the variances of the first-period random arbitrage profits and the second-period random financing costs if their variances move in the same direction, holding other factors (i.e., their expected values and the investor's risk aversion) equal. It is of interest to observe that the expected second-period arbitrage profits disappear in (14.16). It therefore shows that a "myopic" property of the model exists to the extent that enables investors in the market to disregard the arbitrage profits they may reap in the second period. However, this "myopic" property is not pure in that expectations of financing costs at the start of the second period affects the current optimal investment decision. The explanation of this result obtained by the dynamic stochastic programming model is quite simple, however. The investor's arbitrage positions in each period in the T-bill futures market depend on the financing costs at the beginning of the period in which an arbitrage position can be feasibly engineered. And, recall from footnote 2 in Chapter 12 that an investor holds the option of not closing his positions <u>in practice</u> if future price movements in T-bill and T-bill futures are adverse to position closing. As a result, an investor only cares about the financing costs one period ahead in a multi-period arbitrage decision. Applying the Pratt-Arrow risk aversion measure again, equation (14.16) can be rewritten as

$$\hat{y}_0 = \left[\frac{W_0}{RRA_0} + W_0\right] \frac{\mu_1}{(\mu_1^2 + \sigma_1^2)} \frac{(\mu_{r1}^2 + \sigma_{r1}^2)}{\mu_{r1}}.$$

$$= 0, \tag{14.17}$$

if $W_0 = 0$, i.e., if the investment is pure arbitrage and provided no other types of costs are involved. That the optimal arbitrage positions to maximize the investor's utility are always zero is somewhat a strange result; but it is just the reflection of the intrinsic problem of the risk aversion measured by a quadratic utility function when the initial wealth is zero. Strictly speaking, (14.17) is undefined instead of 0 because $(W_0/RRA_0) = (W_0/W_0ARA_0)$ so that $RRA_0 = 0$. As a matter of fact, such a result simply indicates a market-observed fact that in practice a "perfect" pure arbitrage in the T-bill futures market is impossible. Among other things, the existence of transaction costs or the intangible (but real) wealth such as human capital (e.g., investor's labor time) can easily show that an investor somehow must possess some initial wealth at time 0 in order to perform pure arbitrage. Thus, $W_0 = 0$ is generically non-existent even for pure arbitrage so that $(W_0/RRA_0 + W_0)$ in (14.17) cannot be zero (or will not be undefined).

However, arbitrage arguments tell us that arbitrage

<u>positions must be zero in equilibrium</u>. Namely, if the market is in equilibrium, then it occurs at the point where $\Sigma_i \hat{y}_{i0} = 0$, where i represents investor i. Because we assumed that investors are homogeneous in those parameters used in the model, we have $\hat{y}_{i0} = 0$ for each i. With this in mind it is sufficient to describe $\hat{y}_0 = 0$ as the market equilibrium point. As mentioned previously in the model, $\hat{y}_0$ is the <u>net</u> positions in arbitrage of buying-and-selling T-bill futures. Therefore, $\hat{y}_0 = 0$ implies that there are neither buying nor selling activities of arbitrage in the T-bill futures market, and equilibrium is so defined.

If $\hat{y}_0 = 0$ then

$$-b\mu_1(\mu_{r1}^2 + \sigma_{r1}^2) = 0, \qquad (14.18)$$

or equivalently,

$$-b(E_0[P_{11}]-P_{10}/P_{r0}-E_0[P_{f1}]+P_{f0})(\mu_{r1}^2 + \sigma_{r1}^2) = 0, \qquad (14.19)$$

by equations (14.13) and (14.17). By a quadratic utility function, b is presumably not zero. $(\mu_{r1}^2 + \sigma_{r1}^2)$ is simply the second moment of P_{r1}, i.e., $E_0[P_{r1}^2]$; but $E_0[P_{r1}^2]$ can be zero only when $P_{r1,j} = 0$ for each probability j, which is conceivably non-existent. Consequently, we have $(E_0[P_{11}] - P_{10}/P_{r0} - E_0[P_{f1}] + P_{f0}) = 0$ and the equilibrium pricing of the T-bill futures is

$$P_{f0}^* = P_{10}/P_{r0} + E_0[P_{f1}] - E_0[P_{11}]. \qquad (14.20)$$

Part I of the dissertation descriptively showed that $P_t^* = P_l/P_s$ in equilibrium for a locked-in arbitrage position through the life of a T-bill futures contract; where P_t^* is the equilibrium market price of T-bill futures at time t; P_l is the market price of long-term T-bill at time t which matures at 91 days after the delivery date of the T-bill futures contract; P_s is the market price at time t of a "rollover" repurchase agreement from time t to the delivery date. A special case of (14.20) is that time 1 is the delivery date so that $E_0[P_{f1}]$ must be equal to $E_0[P_{l1}]$ and we have $P_{t0}^* = P_{l0}/P_{r0}$ which is exactly the same as that described in Part I. The equilibrium pricing of the T-bill futures as shown in equation (14.20) by the dynamic stochastic programming model is intuitively simple: the equilibrium T-bill futures price emerges in the market where the expected one-period arbitrage profits are zero when financing costs at time 0 are required. Such a statement is conventionally made in a descriptive manner. The theoretical developments in our model show that it is also true when investors try to manage a multi-period arbitrage investment.

CHAPTER 15

SUMMARY OF PART III

Arbitrage in the T-bill futures market cannot be completely risk-free. If arbitrage is engineered in a locked-in position through the life of a T-bill futures contract, it is at least subject to the risks of fluctuations in the future financing rates and in the margin requirements until the delivery date. On the other hand, if it is engineered in a period-by-period position, it is subject to the risk of the one-period price movements of T-bill and T-bill futures. A dynamic stochastic programming model is used to provide a theoretical framework for the optimal arbitrage investment under the uncertainty raised in the second case. By arbitrage arguments, the equilibrium in the T-bill futures market emerges at the point where the arbitrage activity disappears in the market. The dynamic stochastic programming model enables us to find the equilibrium pricing of the T-bill futures by working backward from the end of the time horizon of a multi-period investment plan. In the framework of the model developed in this part, we see that a "myopic" property of the model exists in the sense that expectations of the future one-period price movements of T-bill and T-bill futures do not exert an impact on the current optimal arbitrage investment decision. This "myopic" property is not pure, however, because expectations of financing costs in the next period, which determine the feasibility of arbitrage in that period, affect the current optimal level of arbitrage positions. The equilibrium pricing of the T-bill futures in a multi-period setting, however, depends only on expectations of the prices of T-bill and T-bill futures at the end of the first period. The price of T-bill futures in equilibrium is obtained by the dynamic stochastic programming model in the case where expected one-period arbitrage profits are zero when cost-of-carry is required. Such a result shows that the equilibrium pricing of the T-bill futures remains identical regardless of whether investors perform a one-period arbitrage investment or a multi-period one. It also shows that equilibrium pricing in the case of a locked-in position is a special case of that in a period-by-period position. That is not surprising because a locked-in position is simply performed in a way where an investor treats the whole life of a given T-bill futures as one "period." This demonstrates further the "myopic" property of the equilibrium pricing in the T-bill futures market.

APPENDIX 1

AN INTRODUCTION TO THE T-BILL FUTURES MARKET[1]

The International Monetary Market (IMM) of the Chicago Mercantile Exchange inaugurated trading in Treasury-bill (T-bill) on January 6, 1976. Each futures contract is for 3-month (13-week) U.S. Treasury bills having a face value at maturity of $1,000,000. There are eight contract maturities currently traded. The trading months are March, June, September, and December. The trading hours are 8:00 a.m. to 2:00 p.m. Delivery shall be made to a Chicago or New York bank, registered with the Exchange and a member of the Federal Reserve System, specified by the buyer's clearing member. Before June 1983, delivery was made on the three business days beginning with the day of issue of 13-week T-bills in the third week of the spot month. The "third week of the spot month" means the week commencing on the third Monday of the spot month. For those futures contracts effective June 1983, delivery shall be made on three successive business days. The first delivery day shall

[1]For complete descriptions of the T-bill futures market, the interested readers may refer to Figlewski, 1986;International Monetary Market, 1983 Yearbook; Kolb, 1985; Rebell, Gordon and Platnick, 1984; and Schwarz, Hill and Schneeweis, 1986.

be on the first day of the spot month on which a 13-week T-bill is issued and a one-year T-bill has 13 weeks remaining to maturity. In practice, however, 90-, 91- or 92-day T-bills are substitutable. T-bill prices are quoted in terms of the IMM index. For example, A T-bill yield of 5.20 shall be quoted as 94.80. Thus, the IMM index is the difference between the actual T-bill yield and 100.00. A T-bill yield, or bank discount rate, is the difference between the face value of a bill and its market value on an annualized basis. Minimum price fluctuations of the IMM index is in multiple of one basis point, referred to as an "01" The minimum price change for bills is thus set at $25 (= $1,000,000·.0001 90/360). According to IMM rule, there is a limit to daily price fluctuations. There shall be no trading at a price more than .60 (60 basis points) above or below the preceding day's settlement price except when the expanded daily price limit schedule goes into effect, and on the last day of trading when there is no limit. The settlement price may be outside of the day's High/Low range, and/or may be different from the midpoint of the Closing Range.

The settlement price is related to the issue of margin requirements. There are two kinds of margin requirements--initial margin and variation margin. Initial margin for each contract purchased or sold must be established. This margin is in effect a good faith deposit to ensure that the client fulfills all obligations. Initial margin requirements vary among investors. Retail rates of initial margin are typically the highest, while institutional investors can post lower rates. Hedgers generally post less than those engaging speculative trades. Initial margins may be deposited either in cash or in Treasury bills. The interest on initial margins accrues to the investor's account. Variation-margin requirements is due to "mark to the market," or daily resettlement. This is one of the risks facing the T-bill futures' traders. Variation-margin deposits represent the gains or losses from changes in the price of a contract. Unlike initial margins, variation margins may be met only in cash. There will be written margin notices when an account falls below margin requirements. Normally, margin notices are communicated daily by telephone, referred to as "margin calls." If there are gains in a day, investors can realize them by the favorable variation margin move.

The cash-futures relationship is represented by the basis which is the difference between the futures price and the cash (spot) market price. If the basis exceeds the financing cost of acquiring the financial instruments in the cash market, then an arbitrage opportunity exists. Suppose that an investor purchases a T-bill, then the cost of storing it equals the interest rate on the funds being used to buy the bill minus the yield on the bill. Therefore, the cost of storing an asset in the futures market like T-bill futures can be sometimes negative. Namely, there can be arbitrage profits provided that uncertainty, such as variation margins, is absent. Following is an example. In the morning of January 21, 1983, one observed a T-bill futures price under a contract expiring on March 24, 1984. He also learned at that morning the yields on the two spot T-bills which respectively matured at March 24, 1984 and June 23, 1984. He could also notice a federal funds rate at that day. With this information at hand he could calculate pure arbitrage profits after taking financing costs into account. This example is precisely the way by which we collected and constructed the data in this study.

APPENDIX 2

THE MEAN DIFFERENCE FROM THE GENERATED FINANCING RATES (SET I)

	Number of Days to Delivery of Futures Contract					
	Entrire Sample	Group 1 <15	Group 2 15-29	Group 3 30-44	Group 4 45-59	Group 5 ≥60
			(FEDR* + 90R*)/2			
Mean	.077	.068	.090	.089	.068	.072
SD	.139	.184	.068	.091	.111	.188
t	8.179	1.990	8.577	6.262	4.017	3.040
			(FEDR + 2*90R)/3			
Mean	.039	.063	.076	.063	.034	-.010
SD	.129	.184	.064	.092	.098	.155
t	4.464	1.844	7.696	4.385	2.275	.512
			(FEDR + 3*90R)/4			
Mean	.019	.061	.069	.050	.016	-.051
SD	.129	.184	.063	.094	.095	.143
t	2.175	1.785	7.098	3.406	.104	-2.831
			(FEDR + 4*90R)/5			
Mean	.008	.060	.065	.042	.006	-.075
SD	.131	.184	.063	.095	.093	.138
t	.902	1.756	6.686	2.831	.423	-4.313

(FEDR + 5*90R)/6

Mean	.000	.059	.062	.037	-.001	-.092
SD	.133	.184	.063	.096	.093	.136
t	0	1.727	6.378	2.468	-.071	-5.369

(FEDR + 6*90R)/7

Mean	-.005	.059	.060	.034	-.006	-.104
SD	.135	.184	.063	.096	.093	.135
t	-.547	1.727	6.172	2.268	.423	-6.115

(FEDR + 7*90R)/8

Mean	-.009	.058	.059	.031	-.009	-.112
SD	.136	.184	.063	.097	.093	.135
t	.977	1.697	6.069	2.046	.635	-6.585

(FEDR + 8*90R)/9

Mean	-.013	.058	.058	.029	-.012	-.119
SD	.138	.184	.063	.097	.093	.134
t	1.391	1.697	5.967	1.914	.846	7.049

(FEDR + 9*90R)/10

Mean	-.015	.058	.057	.027	-.015	-.125
SD	.139	.184	.063	.098	.093	.134
t	1.593	1.697	5.864	1.764	1.058	-7.404

APPENDIX 2, Continued

THE MEAN DIFFERENCE FROM
THE GENERATED FINANCING RATES (SET II)

	Number of Days to Delivery of Futures Contract					
	Entire Sample	Group 1 <15	Group 2 15-29	Group 3 30-44	Group 4 45-59	Group 5 ≥60
			(FEDR + STR*)/2			
Mean	.037	.055	.054	.025	.015	.039
SD	.133	.184	.073	.073	.100	.178
t	4.108	1.610	4.794	2.193	0.984	1.739
			(FEDR + 2*STR)/3			
Mean	-.015	.047	.028	-.022	-.037	-.054
SD	.121	.184	.073	.076	.080	.140
t	-1.830	1.376	2.486	-1.854	-3.033	-3.062
			(FEDR + 3*STR)/4			
Mean	-.041	.043	.015	-.045	-.063	-.100
SD	.122	.184	.074	.080	.072	.126
t	-4.962	1.258	1.314	-3.602	-5.738	-6.299
			(FEDR + 4*STR)/5			
Mean	-.057	.040	.007	-.059	-.079	-.128
SD	.124	.185	.074	.083	.068	.121
t	-6.787	1.164	.613	-4.552	-7.618	-8.396

			(FEDR + 5*STR)/6			
Mean	-.067	.039	.002	-.069	-.089	-.147
SD	.126	.185	.075	.086	.065	.118
t	-7.978	1.135	.173	-5.137	-8.979	-9.888
			(FEDR + 6*STR)/7			
Mean	-.075	.037	-.002	-.075	-.097	-.160
SD	.128	.185	.076	.087	.064	.116
t	-8.651	1.077	-.171	-5.520	-9.939	-10.948
			(FEDR + 7*STR)/8			
Mean	-.080	.037	-.004	-.080	-.102	-.170
SD	.130	.185	.076	.089	.063	.116
t	-9.086	1.077	-.341	-5.756	-10.617	-11.632
			(FEDR + 8*STR)/9			
Mean	-.085	.036	-.006	-.084	-.106	-.178
SD	.132	.185	.076	.090	.063	.116
t	-9.508	1.048	-.506	-6.120	-10.904	-12.180
			(FEDR + 9*STR)/10			
Mean	-.088	.035	-.008	-.087	-.110	-.184
SD	.133	.185	.077	.091	.062	.115
t	-9.769	1.019	.665	-6.269	-11.498	-12.670

*FEDR ≡ the (compounded) federal funds rate; 90R ≡ the (compounded) 90-day-maturity T-bill rate; STR ≡ the short-term-t-bill rate.

APPENDIX 3

PERFECT SUBSTITUTABILITY BETWEEN T-BILL AND T-BILL FUTURES IN A PERFECT MARKET

In a perfect and perfectly competitive market, there should be no arbitrage profits, because otherwise a market cannot be in equilibrium. The fact that T-bills and T-bill futures are perfect substitutes in a perfect market can be demonstrated by the following trading strategies.

1. Pure Investment Case:

   A. Buy a long-term T-bill + Sell a T-bill futures
      = Buy a short-term T-bill.

   B. Buy a short-term T-bill + Buy a T-bill futures
      = Buy a long-term T-bill.

Therefore the T-bill futures is, as matter of fact, redundant because either A or B gives the same return.

2. Quasi-Arbitrage Case:

   A. When holding a short-term T-bill:

      Sell the short-term T-bill and buy a long-term T-bill and simultaneously sell a T-bill futures, if the return from buying a long-term T-bill is greater than that from the short-term T-bill held and a T-bill futures.

B. When holding a long-term T-bill:

Sell the long-term T-bill and buy a short-term T-bill and simultaneously buy a T-bill futures, if the return from buying a short-term T-bill and a T-bill futures is greater than that from the long-term T-bill held.

However, one will not bother to complicate his portfolio holdings because there are no excess returns by doing so. In other words, the two markets are perfect substitutes.

3. Pure Arbitrage Case: (No initial holdings)

A. Sell short a short-term T-bill in 2.A and keep the rest unchanged.

B. Sell short a long-term T-bill in 2.B and keep the rest unchanged.

Again, investors will not be engaged in such tradings because the arbitrage profits are zero. That implies that T-bills and T-bill futures are perfect substitutes and investors may hold whatever portfolios they prefer.

APPENDIX 4

THE FRIEDMAN INTERPOLATION METHOD

Let X be the series to be interpolated and Y be the related series to be used in interpolation. Suppose that x_i, x_j, and x_K are known consecutive monthly values in X. The weekly values in X are unknown, however. Further, let their corresponding monthly values in Y are y_i, y_j, and y_K which are all known values. Moreover, the weekly values in Y are known. The Friedman method is to use a straight-line interpolation first to obtain the trend values of x_j and y_j, denoted as x_j^* and y_j^* respectively. These trend values are chosen as

$$x_j^* = [(1 - a_j)x_i + a_j x_K], \qquad \text{(A2. 1)}$$

$$\text{and,} \quad y_j^* = [(1 - a_j)y_i + a_j y_K], \qquad \text{(A2. 2)}$$

where, a_j is the relative weight attached to the terminal value in computing the straight-line trend. For example, a_j is 1/2 when i, j and K are expressed in unit of (consecutive) month. The deviation of x_j from its trend value x_j^* denoted as u_j and the deviation of y_j from its trend value y_j^* denoted as v_j are then used to run a regression, provided that the two series are highly correlated. Finally, the weekly values of X are interpolated according to equations (A2.1) and (A2.2). For example, let x_{ik} whose value is to be interpolated represent the first-week value between month i and k (note that the months of i and k here now represent the consecutive months). Similarly, let y_{ik} be the first-week value which is known. Then a_j will be 1/4 and the deviation of y_{ik} from its trend value y^*_{ik} denoted as v_{ik} can be calculated. By using the regression line, u_{ik} is obtained so that the interpolated value x_{ik} is acquired (since its trend value x^*_{ik} is known by using the fact that $a_j = 1/4$).

By using deviation form the higher the corelation between $\{u_j\}$ and $\{v_j\}$ is, the better the use of the related series than the use of linear interpolation. For proof see Friedman (1962).

APPENDIX 5

DERIVATION OF THE LAGRANGE INTERPOLATING POLYNOMIAL

Applying the Lagrange Interpolating Polynomial (LIP) Theorem (Burden, et al. 1981), a LIP function to interpolate daily CPIs by using weekly data can be stated as follows. If there are (N+1) distinct numbers of weekly data and f is a function whose values are given at these numbers, then there exists a unique polynomial P of degree at most N with the property that

$$f(m_K) = P(m_K) \qquad \text{for each } K = 0,\ 1,\ 2,\ \ldots,\ N.$$

This polynomial is given by

$$P(m) = f(m_0)L_{N,0}(m) + \ldots + f(m_N)L_{N,N}(m)$$

$$= \sum_{K=0}^{N} f(m_K)L_{N,K}(m),$$

where,

$$L_{N,K}(m) = \frac{(m-m_0)(m-m_1)\ldots(m-m_{K-1})(m-m_{K+1})\ldots(m-m_N)}{(m_K-m_0)(m_K-m_1)\ldots(m_K-m_{K-1})(m_K-m_{K+1})\ldots(m_K-m_N)}$$

$$= \prod_{\substack{i=0 \\ i\neq K}}^{N} \frac{(m - m_i)}{(m_K - m_i)} \qquad \text{for each } K = 0,\ 1,\ 2,\ \ldots,\ N.$$

Note that $P(m_K) = f(m_K)$, i.e., the polynomial values coincide with those observed values reported weekly. That can be seen from the fact that $L_{N,K}(m_j) = 1$ for $j = K$ and $L_{N,K}(m_j) = 0$ for $j \neq K$, $j = 0, 1, 2, ..., N$. The time points where their values will be interpolated is between the consecutive integers from 0 to N. For daily data to be interpolated, the interval of a daily datum between two consecutive integers is 1/7 when k is in unit of week.

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