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HIERARCHICAL PREFERENCES AND CONSUMER CHOICE

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As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Don Lawton Coursey

entitled Hierarchical Preferences and Consumer Choice

and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

ABSTRACT

This study considers the problem of the consumer in light of work presented by classical economists who discussed consumption. Richer assumptions about the tasks of an individual consumer and technology of consumption activities are used to develop a static model of consumer behavior. This model is extended through the introduction of opponent-process theory to develop a dynamic model which includes habit formation.

Particular emphasis is placed in Chapter 2 upon the psychological underpinnings of consumption activities and the allocation of time aspect of these activities. It is assumed that a consumption activity is defined as a production function combining commodity and time inputs to produce satisfaction.

Chapter 3 presents the framework over which preferences about different activities are defined. Preference relationships are assumed to be rational, transitive, and constant over time and location. In addition, satiation in a particular consumption activity is assumed to exist and the ranking over satiation states is defined.

Chapter 4 deals with the behavior of a time and income constrained consumer who seeks to choose an optimal bundle of commodity and time inputs over the ordered activity set. The solution to this problem is characterized by affordable allocation of resources from the

highest ranked down to the lowest ranked activity. Comparative statics results associated with this solution are considered for non-labor income, wage rate, and price changes. It is shown that besides the production substitution effects brought about by changes in the wage rate and in commodity prices, the net effect of changes in economic variables is predominantly at the lower end of the preference ordering.

Chapter 5 presents both a psychological version of opponent-process theory and an economic interpretation of this theory which is used to describe habit dynamics.

Chapter 6 combines the static consumer problem and the dynamic description of activity productions under habit formation to present an extended problem of a dynamic consumer behavior.

CHAPTER 1

INTRODUCTION

Economic analysis of consumer behavior has traditionally emphasized expenditure on commodities. Utility functions are assumed to have goods and services as direct arguments and in this type of analysis, economists view consumer satisfaction equivalently with the acquisition of these goods and services. Nothing is usually said about what is done with the commodities. The goal is to explain consumption, but the end result is only a theory of expenditure.

Consumption is explained as an instantaneous act of transforming commodities into utility. Through a maximization of utility, demand functions for these commodities are readily derived. This is the neoclassical approach to economic behavior of individuals taken by Slutsky[1] and Hicks.[2] A consumer maximizes utility subject to a limited budget constraint and the arguments of the utility function are the actual commodities themselves. Given rational assumptions about the shape of the utility function, the solution to the consumer's problem yields consumer demand functions which relate prices of commodities and consumer income to quantities of commodities demanded.

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1. E. E. Slutsky, "On the Theory of the Budget of the Consumer," Giornale degli Economisti, Vol. 51, (1915), pp. 1-26.

2. J. R. Hicks, Value and Capital, Second Edition, (Oxford Clarendon Press, 1946).

By presenting the consumer with hypothetical price - income situations and observing resultant choices, consumer reaction to price and income effects can be analyzed in terms of so-called substitution and income effects. In such a fashion a schedule of expenditure on commodities is constructed.

Such treatment of consumer behavior as a solution to an extremum problem has become the basis of consumer analysis. This is the cornerstone of Samuelson's[3] Foundations of Economic Analysis. However, original economic discussion of the act of behavior emphasized consumption, as opposed to the act of expenditure, and included richer assumptions about the technology of consumption acts. Consumers were assumed to face the possibility of satiation in consumption acts. After a certain period of a particular consumption activity, the consumer was assumed to become saturated in the activity. Presumably then, his interest might turn toward other economic activities. Another fact pertinent to traditional consumption theory was the observation that consumer activities are not all equally important. In particular, the satisfaction of certain basic wants was assumed to supercede satisfaction of more luxurious or cultural activities. Consumers were assumed to have a ranking over consumption activities which reflected this importance scale.

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3. Paul A. Samuelson, Foundations of Economic Analysis (Cambridge: Harvard University Press, 1948).

When allocation of time was added to the economists' model of consumer behavior, it was also treated as a direct argument in the utility function. However, people do not obtain satisfaction from abstract time, but are satisfied only when they put time to use in consumption acts. Time allocation is explained well by the economist, but how this time is actually used is left as another unanswered question.[4]

The first goal of this research is to reconsider the behavior of economic agents when these agents perform as classical consumers. Consumption behavior will be defined as the manner through which consumers actually use time and commodities to produce satisfaction. Operationally, the model of expenditure developed by the neoclassical economists is expanded by adding the possibility of satiation and subordinality. When the use of time as an input <u>and</u> how this time is used are included, the neoclassical model is enriched to yield a new model of actual consumption behavior. From this model of consumption an implied model of expenditure is derived providing a more satisfying explanation of empirical observations about consumer behavior.

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4. The exception is Gary S. Becker in "A Theory of the Allocation of Time," <u>Economic Journal</u>, (September, 1965), pp. 493-517. Becker develops a theory of consumption of "z goods." The "z goods" have as arguments time and commodities. However, nothing is said about how the time is <u>used</u> with the commodities to produce satisfaction. For instance, the model leaves open the question of whether eating an orange at one sitting is equivalent to eating the same orange in a series of sittings. Therefore, the distinction of this research is that time is viewed as blocks of input rather than a divisible and fungable resource.

Another fact pertinent to consumption analysis, and often overlooked by traditional economic analysis, is that consumption behavior is heavily controlled by habit.[5] Most agents in an economy take the comforts of everyday life for granted and only notice discomfort when forced to behave differently from routine. Habit plays a powerful role in the explanation and prediction of economic behavior. Perhaps the reason economics has greatly underplayed habit explanations of behavior is the fact that habit is psychological in nature. However, through the introduction of some basic psychological constructs describing habit formation, the simple model of consumption developed as the first part of this research, can be enriched to explain dynamic consumption behavior.

What has been lacking in traditional dynamic explanations of consumer behavior is an underlying theory of how habits are developed, grow and decay, and change. The economic community now has at its disposal, thanks to psychologists, quite a powerful model of habit dynamics. Solomon's[6] model of opponent-processes provides a framework

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5. Here the exception is the economic-psychological state adjustment model proposed by H. S. Houthakker and Lester D. Taylor in Consumer Demand in the United States: Analysis and Projections (Cambridge: Harvard University Press, 1970). This model does little, however, to explain how consumption habits are formed, and simply assumes that habits may exist.

6. Richard L. Solomon, "The Opponent-Process Theory of Acquired Motivation, The Costs of Pleasure and Benefits of Pain," American Psychologist, Vol. 35, No. 8 (August, 1980).

for the systematic treatment of how different types of habits develop for different types of consumer behavior. The second goal of this research is to expand the initial model of consumption, by applying the psychological constructs which explain habit, into a mode of dynamic consumption. Such a model of consumption will include the classical treatment of satiation and subordinality of preferences, and at the same time will answer questions about the nature of the satiation effects of consumption activities and how subordinality of preference is a natural consequence of personal behavior. By answering questions such as these, a more detailed statement about the psychology of expenditure, as studied by Houthakker and Taylor,[7] can be made. The combination of psychology and economic demand analysis is brought into focus in their study; the research in this paper helps clarify and expand this meeting of doctrine and the implications thereof.

The organization of the research is as follows. In Chapter 2 of the paper, the economic and psychological definitions relevant to consumption theory are developed. Consumption activities are formally defined in a manner consistent with both economic and psychological concepts of arousal and production. In the next chapter, the classical approach to consumption theory is developed into a formal axiomatic model. Five assumptions of consumer choice, expanding on the work of the neoclassical economists, are used to prove the existence of a completely ordered preference relation of an individual consuming agent.

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7. Houthakker and Taylor, Consumer Demand in the United States: Analysis and Projections.

These five assumptions include classical discussion of consumption acts as opposed to expenditure assumptions used in the past. Chapter 4 examines the behavior of an agent with limited resources when choosing optimal consumption plan. Derived expenditure consistent with this model is then developed and compared with neoclassical results concerning expenditure. Psychological interpretations of arousal production and habit formation are considered in qualitative form in Chapter 5. Here the Solomon model is formalized and interpreted for the first time in a manner which provides answers to traditional economic questions about behavior. Quantification of the psychological model allows consideration of a dynamic consumption model in Chapter 6. Finally, the expenditure implications of the dynamic model are exposed and the applicability of the model to complex behavioral situations traditionally ignored in the past by economics is considered.

CHAPTER 2

DEFINITIONS

This chapter presents the psychological and economic definitions and concepts which are used to develop the axiomatic foundations for the model of consumer behavior. The concepts of commodities and actions are made precise in order to define consumption activation levels consistent with psychological explanations of motivation and stimulation. A price system containing both commodity prices, wages, and wealth is combined with time constraints to define a consumer's resource constraints.

Dates and Locations

Time[8] is divided into a finite number of compact elementary intervals which cover the real numbers. The units of measurement for time may be weeks, minutes, or any unit small enough to define economic activity. Similarly, physical space may be divided into elementary regions small enough to define economic activity. Thus, dates and locations can be defined as points on a real line and a real three dimensional space respectively.

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8. See Gerard Debreu, Theory of Value (New Haven: Yale University Press, 1959).

Commodities

The concept of a commodity in the framework formally involves a complete specification of all of the inherent qualities or inherent physical characteristics of the commodity, the time that the commodity is to be available for use, and the location at which the use of this commodity will occur. A simple example of an economic commodity is a good such as butter. A complete description of butter would involve a particular grade of butter at a particular point in time at a particular location. The quantitity of this so defined commodity butter can be expressed as any non-negative real number of pounds available at the fixed quality, time, and location specified in the definition of butter. Another example of a commodity is the economic service of a haircut. A particular type of haircut performed on Monday at 10:00 A.M. at location A is a different commodity than the same type of haircut performed on Monday at 10:00 A.M. at location B. Thus, as soon as one of the three factors defining a good or service changes, a different good or service results. Formally define a commodity vector and associated commodity space as follows:

Definition - A commodity q_n is a good or service completely specified physically, temporally, and spatially. It is assumed that there exist a finite number of distinguishable commodities. Call this number N. A commodity vector is the N-tuple $q = (q_1, q_2, \ldots, q_N)$, and can be represented by a point in the non-negative space R^N defined by a product of N sets of real numbers. Call the so defined space the commodity space.

Consumption Activities

Having defined both the measure of time over which economic activity takes place and the concept of a commodity, definitions of a psychological nature can be constructed. Consider psychological explanations of behavior. All such behavior may be described in two dimensions. First, the choice of one activity over another activity constitutes variation in the direction of behavior. So, for example, an agent may read a book or go to a sporting event. Secondly, the intensity of a particular activity can be varied. An agent, after choosing say the activity of reading a book, may become greatly excited by a particular book or bored with the book. Or similarly, the agent may become highly aroused at the sporting event or only be casually interested in the course of the play. The first dimension describing behavior can be measured by looking at all possible forms of behavior and noting whether an agent participates in that activity. It is thus a binomial choice problem. An agent either participates or he does not participate. The second dimension describing behavior can be measured by the psychological level of activity inherent in a particular form of behavior. Borrowing a psychological definition of arousal,[9] define arousal on activity as: "[Arousal] is the intensity aspect of behavior which has been variously referred to as the degree or excitation, arousal, activation, or energy mobilization."

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9. The definition used is the accepted psychological definition stated by Elizabeth Duffy in "The Psychological Significance of the Concept of "Arousal" or "Activation", The Psychological Review, Vol. 64, No. 5 (September, 1957) p. 265.

In summary, then, behavior at a particular time for an agent can be described by a (probably very large if not infinite) vector of 1's and 0's defining the direction of behavior in terms of behavior states of the agent and an associated vector of activity states defining the level of those activated states. More precise physiological underpinnings of the role of arousal and its measurement in this model can be found in Hebb,[10] but for the purposes of this study all that need be assumed is that the activity levels can somehow be compared by an agent through the use of some measure.

Following Scitovsky,[11] the particular level of activity associated with a particular action can be described as the sum of exteroceptive stimulation, enteroceptive stimulation, and cerebral stimulation associated with the activity. For example, the arousal due to participation in an activity such as running in a park can be described as the additive effects of sensual stimulation of the run through the park in terms of sounds, sights, smells etc., muscle and internal organ stimulation caused by the mechanics of running, and mind or brain stimulation caused by the emotions and thoughts of the run. Arousal can thus be modified by the application of changes in sensual arousal, muscle and internal organ arousal, and cerebral arousal. Examination of each of these three arousal sources for a given specific

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10. See D. O. Hebb, "Drives and the Conceptual Nervous System," The Psychological Review, Vol 62. No. 4 (July, 1955) for an interesting discussion on actually measuring levels of stimulation of an organism's central nervous system.

11. Tibor Scitovsky, The Joyless Economy (London: Oxford University Press, 1976), pp. 23-24.

arousal state leads to a definition of psychological/economic activity. Sensual stimulation is derived mainly through the combination of commodities and time. In the above running example the commodity may be the scenery of the run. Combined with time spent looking at this scenery, this commodity may produce aural arousal for the runner. Muscular or internal organ stimulation also is a result of commodity and time inputs. Internal muscular stimulation during the run may be produced by the intake of a commodity such as food and the use of the associated time taken for this intake, providing arousal during the run. Finally, time may be spent thinking about an interesting aspect of the run. Cerebral stimulation is mainly a function of the time spent in the thinking processes of the agent. Activity can, in general, then be defined over all its inputs which will be the commodities $q_1, q_2, \ldots, q_N$ used by the agent and the time spent using these commodities or simply the time spent for cerebral stimulation associated with the arousal state. Assume that the number of these activities is large but finite. Formally, arousal states or consumption activity levels can be defined. Consumption is introduced for the first time as the technology associated with combining commodities and time inputs. The assumptions on arousal are summarized in Figure 1.

Definition - Define a consumption activity or consumption state A_i as a function of inputs of time τ_i to the activity and as a function of commodity inputs $q = (q_1, q_2, q_3, \ldots, q_N)$. $A_i = A_i(q, \tau_i)$. Assume that there exist a finite number of distinquishable activities.

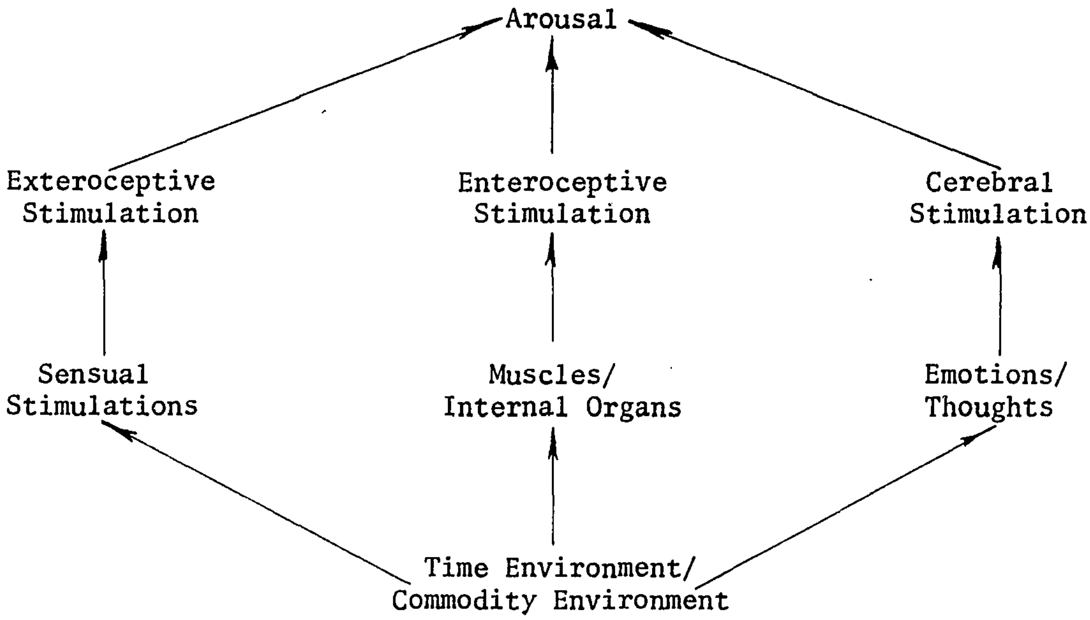

Figure 1: The components of arousal. Arousal is composed of three types of stimulation all of which are dependent upon the time and commodity environment of an individual.

Call this number I. The activity state of an agent can be described by the activity vector $A = (A_1, A_2, \ldots, A_I)$ and is represented by a point in the space R^I defined by a product of I sets of real numbers. Call the so defined space the activity space.

It should be noted that the definition of A_i appears similar to Becker's[12] "z goods" in that A_i combines commodity inputs with time to produce activity. The reader is cautioned at this point to beware of equating the two constructs. It will be evident in the next chapter that there exist critical differences in interpretation of the time inputs τ_i used here and the use of time by Becker. At this point it is also assumed that, in general, the activity level associated with a particular state is a function of all commodity inputs of the agent. For example, it is assumed that if the runner eats steak for lunch before his run in the park that his activity level associated with the run may be different from a case where he ate shrimp. While these types of cross effects affecting consumption will be later simplified, there is evidence that for most activity states combinations or bundles of goods and services are relevant as inputs. Also note the A_i is not limited to be non-negative. It will be convenient to assume that for some activity states a binomial decision not to participate in the arousal of that state may yield a default value or a zero level of activity for that state, while for other states, such as the state of respiratory activity brought upon by regular breathing, non-participation

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12. Becker, "A Theory of the Allocation of Time," pp. 493-517.

could lead to a net negative measure of activity. The choice of zero level of activity as a measuring benchmark is merely for convenience in the analysis. Finally, note that while both the number of commodities, N, and the number of activity states I are both assumed to be large, but finite, it is intuitive to expect that I, which measures the number of ways an agent might combine commodities to produce activity, would be greater than N, which measures the number of these commodities.

Prices and Incomes

Assume that there exists a price p_n for each commodity q_n which is the amount paid by an economic agent for each unit of the commodity q_n which will be made available to him. This price can be positive for a scarce commodity such as butter, zero for a free commodity such as air, or negative for a noxious commodity such as unpleasant labor. Assume that there exists only one such noxious commodity called labor[13] and call its specific price the price of time or wage rate w. Note that the price of a commodity is not an intrinsic property of the commodity but is a function of the prevailing technology, resources, and tastes of all agents in the economy.

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13. Labor is assumed to have neutral arousal effects on the agent in the analysis.

Definitions for prices and wage rate follow:

Definition - The price system is the N-tuple $p = (p_1, p_2, \ldots, p_N)$ associated with the commodity vector $q = (q_1, q_2, \ldots, q_N)$. Thus, p_n is the price of q_n, $n = 1, 2, \ldots, N$. The vector p can be represented by a point in the non-negative space R^N as the product of N sets of real numbers.

Definition - The wage rate is the non-negative real number w describing the amount paid to an agent for each unit period of time of labor worked by the agent.

Finally, define non-labor income as a flow of commodities available to an agent corresponding to the debts owed to the agent in terms of the N commodities. Compute the value of this income by summing over the products of price p_n and a fixed flow of commodities $\hat{q}_n$ in each period.

Definition - The non-labor income Y of an agent is the sum of all values of commodities paid to an agent. Thus $Y = p \cdot \hat{q}$.

Horizons

The view taken in this paper[14] will be that each consumption activity A_i has an associated time horizon T_i of which τ_i is a subinterval. For example, the time horizon from which an agent extracts

14. Lester D. Taylor, "Some Notes on a New Model of Household Consumption and Saving" (unpublished discussion paper, University of Michigan, 1975). Taylor looked at a formal model of expenditure where choice decisions over groups of commodities are organized into decreasingly smaller planning periods. The model of time horizons in this paper is the logical extension of Taylor's model in that every arousal state can have an associated time horizon.

time input τ_i to read a book may be different than the time horizon from which an agent extracts time input τ_j to go on an extended vacation. If T_i is the same for a group of consumption activities $i = 1, 2, \ldots, k$, then an elementary horizon may be considered for a subset of the totality of consumption activities. Formally:

Definition - For each activity level A_i, $i = 1, 2, 3, \ldots, I$ there exists an associated time horizon T_i from which τ_i, the time input for A_i is extracted. If $T_i = T_j$ for all $i,j : i,j \propto K \propto I$, that is the time horizon is the same for a subclass of consumption activities. Then call $T = T_i = T_j$ an elementary horizon for the activities $i,j \propto K$.

In Chapter 6 the concept of a time horizon will be expanded to become an explanatory variable associated with consumer choice activity. Through the end of Chapter 4 the analysis will follow more classical treatment of the time horizon in assuming that T is a relevant time horizon for all goods or that there exists only a few different horizons T_i for all activities A_i.

CHAPTER 3

AXIOMATIC FOUNDATIONS FOR A THEORY OF CONSUMPTION

This chapter studies a class of economic agents and their behavior in the consideration of activity vectors. The main focus of the chapter will be to axiomatize the economic principles used in modeling a psychological/economic choice institution. The consequences of this model will be explored for a static consumption plan in Chapter 4 and a dynamic consumption plan in Chapter 6. Consumption as a form of production activity will be assumed to be derived from a structure of wants discussed in seperate but unconnected fashion by neoclassical economists and later summarized by Georgescu-Roegen.[15] By making precise The Principle of Satiable Wants and The Principle of the Subordination of Wants discussed by these classical economists and incorporating these principles into a complete model of choice, the concept of a preference ordering over activity vectors can be derived,

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15. Nicholas Georgescu-Roegen, "The Pure Theory of Consumer's Behavior," Quarterly Journal of Economics, L (1936), pp. 545-593 and "Choice Expectations and Measurability," Quarterly Journal of Economics, LXVIII (1954), pp. 503-534.

providing a model of a consuming agent. This model expands and enriches the Becker model of consumption technology. This chapter also makes the assumptions concerning consumption activation vectors precise and provides a proof for the existence of an ordering over activity states which is a consequence of these assumptions of choice.

Assumptions on Activity Vectors

The model developed for a consuming agent borrows three assumptions from the common work of neoclassic economists: the definition of preference over activity states is borrowed from the definition of commodity preferences axiomatized by Pareto,[16] the transitivity of these preferences follows Marshall's transitivity of preference,[17] and the constant nature of these preferences is borrowed from both Marshall and Pareto.

The first assumption makes the economic agent a perfect chooser over activity vectors associated with combining time and commodity inputs. It allows the agent to compare any two activity vectors:

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16. Vilfredo Pareto, Manuel d'economie Politique (Paris, 1927).

17. Alfred Marshall, Principles of Economics, 8th Edition (New York: The Macmillan Company, 1949).

Assumption 1: Preference Over Activity States

Given any two activity vectors which are elements of the activity space, say

$$[A]_i = [A_{1i}[q, \tau_{1i}], A_{2i}[q, \tau_{2i}], \ldots A_{Ii}[q, \tau_{Ii}]]$$

and $[A]_j = [A_{1j}[q, \tau_{1j}], A_{2j}[q, \tau_{2j}], \ldots A_{Ij}[q, \tau_{Ij}]]$

then an agent will either prefer one of the activity vectors to the other or regard the two activity vectors as indifferent. For preference write $A_i P A_j$ or $A_j P A_i$; for indifference write $A_i I A_j$. Indifference is a symmetric relationship between vectors while preference is not.

For example, an agent can always compare the activity level from eating an orange or eating an apple. Transitivity over multiple activity vectors is also assumed, allowing the agent to compare sets or strings of arousal choices:

Assumption 2: Transitivity of Preference

Given any three activity vectors, $[A]_i$, $[A]_j$, $[A]_k$, then if $A_i P A_j$ and $A_j P A_k$, then $A_i P A_k$. Also if $A_i I A_k$ and $A_j I A_k$, then $A_i I A_k$.

For example, if the agent prefers eating an orange over taking a walk and taking a walk over reading a book, then the agent prefers eating an orange over reading a book.

Finally, the preferences of the agent will be assumed to be constant over time and location:

> Assumption 3: Preference Over Time and Space
>
> The preference of an agent is the same in every time and every location when confronted with a choice between two activity vectors $[A]_i$ and $[A]_j$.

Tastes cannot develop over time in this model nor can they be influenced by physical or geographic location.

Neither of these first three assumptions differ from the traditional neoclassical approach to choice theory. The postulate of rationality described by Assumption 1, merely requires that the consumer can rank activities in order of preference. As in this traditional analysis, it need not be assumed that the consumer can assign numbers which measure the activity or satisfaction associated with an activity state, but merely that the activities can be compared. Assumption 2 allows such a comparison over multiple sets of vectors and Assumption 3 allows the analysis to disregard the problem of taste changes. In the final chapter of the paper the problem of dynamic framework and Assumption 3 will be partially relaxed.

The following two assumptions combine some psychological observations concerning optimal arousal levels with early economic observations concerning the satisfaction of wants. These two assumptions will be the point of departure for the analysis. Among other things, they provide the meeting place for traditional economic choice analysis, optimal arousal analysis as studied by psychologists, and the theory of the allocation of time. Activity states will be assumed to be completely satiable. For each state it will be assumed that more activity is preferred to less activity up to a critical point, after this point is reached the agent prefers less activity for that state. The existence of such optimal activity levels is derived from both psychologists and early economists studying consumption theory. Noting Scitovsky:[18]

> "Such unpleasant states are the extreme consequences of extremely high and extremely low levels of total stimulation; there is a wide intermediate range between them. But if very high and very low levels of stimulation are painful to the point that they cause major disturbances and pathological symptoms, less deviations from the normal are also unpleasant. Indeed, psychologists postulate the existence of an optimum level of total stimulation and arousal, one which is optimal in the sense that it gives rise to a feeling of comfort and well-being. This, probably, is not stable over time, but varies with the wakefulness cycle. Deviations from the optimum level of total stimulation are believed to give rise to feelings of strain, fatigue or anxiety when it is above, and these unpleasant feelings seem the greater the longer the duration of the divergence and the greater its extent. They are believed, therefore, to constitute the inducement to bring back arousal to its optimal level. In short, psychologists picture the organism as striving to maintain its arousal level at or near its optimum."

Speaking slightly differently in terms of consumption activation levels as opposed to arousal, Georgescu-Roegen states:[19]

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18. Scitovsky, The Joyless Economy, pp. 23-24.

19. Georgescu-Roegen, "The Pure Theory of Consumer's Behavior," p. 513.

". . . all of our wants are finally satiable. It is true that one may speak of satiety being reached after a continuous decrease in the intensity of the corresponding want, but the only thing of which one can be sure in general, is that satiety exists. It is that state of mind where any addition of the object of previous desire is no longer wanted."

This principle, called The Principle of Satiable Wants is formalized in the following mathematical assumption:

<u>Assumption 4: Satiability of Activity States</u>

Each activity state is capable of complete satiability. That is, there exists a critical level of activity A_i for each activity state $\hat{A}_i$ such that for all small positive additions ε to activity state A_i,

i) If $A_i < \hat{A}_i$, then

$$[A_1, A_2, \ldots, A_i + \varepsilon, \ldots, A_I]P[A_1, A_2, \ldots, A_i, \ldots, A_I]$$

ii) If $A_i > \hat{A}_i$, then

$$[A_1, A_2, \ldots, A_i, \ldots, A_I]P[A_1, A_2, \ldots, A_i + \varepsilon, \ldots, A_I]$$

This is true for each element A_i of an activity vector $A = [A_1, A_2, \ldots, A_I]$.

An example of the workings of this assumption is the consumption activity of eating an orange. By combining time and a commodity called oranges an agent may arouse his "consuming oranges" activity state. This activity state may increase in intensity over the course of consuming many oranges, but there exists a level of activity after which the further combination of time and oranges causes a reduction in the level of the activity state associated with "consuming oranges."

The dynamics of this reduction are the topic of Chapter 4. At this time all that is required for the model is a recognition of the fact that an optimal activity level exists for each arousal state.

It has often been pointed out that the wants of consumers are hierarchized[20] so that the uses for particular commodities as inputs to consumption activities are arranged in a particular order of use. For example, an agent may use a commodity such as wood in a specific order such as for heating, to provide shelter, for household furnishings, and finally in the production of recreational items. The concept of scaling all wants is clarified in detail by Banfield:[21]

> "The first proposition of the theory of consumption is that the satisfaction of every lower want in scale creates a desire of a higher character. . . . The removal of a primary want commonly awakens a sense of more than one secondary privation. Thus, a full supply or ordinary food not only excites to the delicacy in eating, but awakens attention to clothing. The highest grade in the scale of wants, that of the pleasure derived from the beauties of nature or art, is usually confined to men who are exempted from all the lower privations. . . . It is the constancy of a relative value in objects of desire, and the fixed order of successtion in which this value arises, that makes the satisfaction of our wants a matter of scientific calculation."

Insightful hints concerning the possible dynamic forces driving the activity levels associated with the satisfaction of wants are provided by Jevons. The urgency associated with satisfying wants and

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20. Plato, Republic, II, 3690.

21. Banfield, Four Lectures on the Organization of Industry, pp. 11-21.

the degree of this urgency will be an important factor in the dynamics of consumer choice. Following Jevons' discussion from his section on The Theory of Wants:[22]

> "Man . . . is surrounded with wants which are renewed every day; some of them are so imperious and indispensable that he is forced to satisfy them under pain and suffering and death; others are less urgent, although very necessary; there remain some which are matters merely of convenience and enjoyment, so that he only thinks of satisfying them when he is at rest concerning the previous ones."

In terms of the terminology developed so far, each activity state A_i can be assumed to be comparable by an agent to all of the other arousal states A_j. Before stating this result as an assumption, a final clarification on the nature of the subordinization of preferences must be made. Quoting again from Jevons'[23] ". . . the satisfaction of a lower want . . . merely permits the higher want to manifest itself."

Not only then are wants subordinated, but any addition to an activity state will not be preferrred unless all higher or more important activity states are completely satisfied or satiated. A consumption activation level must reach complete satiation in the model before another level can manifest itself. Thus, The Principle of the Subordinality of Wants may be developed into two parts:

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22. Jevons, The Principles of Economics, p. 7.

23. Ibid, p. 8.

<u>Assumption 5: Subordinality of Activity States</u>

1. The satiation of certain activity states allows other activity states to be activated. In particular, there exist an activity state A_1 and associated satiation level $\hat{A}_1$ such that

$$[0, 0, \ldots, A_1, \ldots, 0, \ldots, 0]P[0, 0, \ldots, 0, \ldots, \hat{A}_j, \ldots, 0]$$

for all A_j associated satiation levels $\hat{A}_j$ with $1 \neq j$. There also exists an activity state A_2 and associated satiation level $\hat{A}_2$ such that if $A_1 \geq \hat{A}_1$, then

$$[0, 0, \ldots, \hat{A}_2, \ldots, \hat{A}_1, \ldots, 0, \ldots, 0]$$

$$P$$

$$[0, 0, \ldots, A_2, \ldots, A_1, \ldots, A_j, \ldots, 0]$$

for all A_j and associated satiation level $\hat{A}_j$ with $1 \neq j$, $2 \neq j$. Similarly, for each activity state A_i there exists an associated satiation level $\hat{A}_k$ such that an activity vector containing the elements 0, A_k, and $\hat{A}_{k-1}$, $\hat{A}_{k-2}$, ..., $\hat{A}_2$, $\hat{A}_1$ is preferred over any activity vector with $\hat{A}_k = 0$ and $A_j = \hat{A}_j$ with $j \neq 1, 2, \ldots, k-1$.

2. Also, for each of these activity states A_i, if $0 < A_i < \hat{A}_i$, then an ε addition of activity to state A_i is preferred to any $\varepsilon << \infty$ addition to state A_j with A_{k-1}, A_{k-2}, ..., A_2, A_1 held constant. This is true regardless of the level of the state A_j.

The five assumptions of activity can be stated in the following mathematical summary:

<u>Assumption 1</u>: For all $[A]_i$, $[A]_j$, $A_i P A_j$ or $A_j P A_k$ or $A_i I A_j$

<u>Assumption 2</u>: For all $[A]_i$, $[A]_j$, $[A]_k$, $[A_i P A_j$ and $A_j P A_k]$ imply $[A_i P A_k]$ and $[A_i I A_j$ and $A_j I A_k]$ imply $[A_i I A_k]$

<u>Assumption 3</u>: Assumptions 1 and 2 are invariant over time and location.

<u>Assumption 4</u>: For all A_i there exists an $\hat{A}_i$ such that for $0 < \varepsilon << \infty$

$[A_i < \hat{A}_i]$ implies $[[A_1, A_2, \ldots, A_i + \varepsilon, \ldots, A_I]$

$$P$$

$$[A_1, A_2, \ldots, A_i, \ldots, A_I]]$$

$[A_i > \hat{A}_i]$ implies $[[A_1, A_2, \ldots, A_i, \ldots, A_I]$

$$P$$

$$[A_1, A_2, \ldots, A_i + \varepsilon, \ldots, A_I]]$$

<u>Assumption 5</u>: 1. For all A_i, there exists an integer k with $1 \leq k \leq I$ such that if $A_j \geq \hat{A}_j = \hat{A}_\ell$ for all $\ell = 1, 2, \ldots, k\text{-}1$, then $[\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_{k-1}, \hat{A}_i, 0, \ldots, 0, \ldots, 0, \ldots, 0]$

$$P$$

$$[\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_{k-1}, \ldots, 0, \ldots, \hat{A}_m, \ldots, 0]$$

for all A_m, $m \neq 1, 2, \ldots, k\text{-}1$.

2. In addition, if $0 < A_i < \hat{A}_i$, then

$$[\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_{k-1}, \ldots, 0, \underbrace{\varepsilon}_{A_i}, 0, 0, \ldots, 0]$$

$$P$$

$$[\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_{k-1}, \ldots, 0, \underbrace{\varepsilon}_{A_m}, 0, 0, \ldots, 0]$$

for all A_m and all values of A_m with $\varepsilon << \infty$.

The Ordering of Activity States

The existence of an ordered vector describing the ranks an agent places upon the satisfaction of activity states is a natural extension of the preceding five assumptions concerning activity. This existence will provide the basis for all of the treatment of the behavior of the economic agent as a consumer in the following chapters. This existence is stated formally and then proven, and some relevant terminology concerning the ranked activity vector is then defined.

Theorem: Ordering of Activity States[24]

Assumptions 1, 2, 3, 4 and 5 imply that there exists an ordering $\mathcal{O}(A)$ over possible activity vectors A such that for each activity state A_i, if $A_j = \hat{A}_j$ for all $j < i$, then for all $j \geq i$ and for small additions of activity ε,

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24. Commonly referred to as lexicographical ordering in utility theory. See Debreu, Theory of Value, pp. 72-73. Here satiation is added.

$$[\hat{A}_1^R, \hat{A}_2^R, \ldots, \hat{A}_{i-1}^R, \underbrace{\varepsilon}_{A_i^R}, 0, 0, 0, \ldots, 0]$$

$$P$$

$$[\hat{A}_1^R, \hat{A}_2^R, \ldots, \hat{A}_{i-1}^R, 0, \ldots, 0, \underbrace{\varepsilon}_{A_j^R}, 0, \ldots, 0]$$

where R denotes the ordering over activity states.

PROOF: Assumptions 1, 2 and 3 provide the rational time/space taste framework defining preferences over activity vectors A; Assumption 4 ensures the existence of satiated levels of activity from which the ordering will work; Assumption 5 is used to prove the theorem. Thus, let A_1^R, the first ranked activity state, be A_1. From Assumption 5 it can be concluded that small ε additions to A_1 will be preferred by an agent to all similar additions ε to any other activity state A_j. Thus:

$$[\varepsilon, 0, 0, \ldots, 0] \; P \; [0, 0, \ldots, \underbrace{\varepsilon}_{A_j}, \ldots, 0]$$

for $j \neq 1$. Similarly, given $A_1^R = \hat{A}_1$, let $A_2^R = A_2$. Applying Assumption 5 once again, notice that small ε additions to A_2 will be preferred to similar additions of ε to any other activity state A_j with A_1^R being equal to $\hat{A}_1^R$. Thus:

$$[\hat{A}_1^R, \underbrace{\varepsilon}_{A_2}, 0, 0, \ldots, 0] \; P \; [\hat{A}_1^R, 0, \ldots, \underbrace{\varepsilon}_{A_j}, \ldots, 0]$$

for all $j \neq 1, 2$. To generalize the proof, assume that $A_k^R = A_k$ with $A_\ell^R = \hat{A}_\ell$ for $\ell < k$. Then Assumption 5 implies that small additions to A_k will be preferred to similar additions of ε to any other activity state A_j with j counting those activity states $a_{k+1}, a_{k+2}, \ldots, a_I$. Thus, for each activity state A_k if $A_j = \hat{A}_j$ for all $j < i$, then for all $j \geq i$ and for small additions of activity ε

$$[\hat{A}_1^R, \hat{A}_2^R, \ldots, \hat{A}_{k-1}^R, \underbrace{\varepsilon}_{A_k}, 0, 0, \ldots, 0]$$

$$P$$

$$[\hat{A}_1^R, \hat{A}_2^R, \ldots, \hat{A}_{k-1}^R, 0, 0, \ldots, \underbrace{\varepsilon}_{A_j}, \ldots, 0].$$

Therefore an ordering, and in particular the ordering $A_i^R = A_i$ for $i = 1, 2, \ldots, I$, exists for the activity vector.

CHAPTER 4

THE CONSUMER'S PROBLEM: STATIC CASE

This chapter discusses the static consumption choice problem of the agent. In order to simplify the complex activity functions defined in Chapter 2, an assumption about the inputs to activity levels is made, and the functional form of the activity levels is made precise in terms of a production function. Income and time are combined in budget and time constraints respectively and the problem of consumption choice is then investigated. The choice of optimal activity levels, given the theorem on ordered preferences proved in the last chapter, turns out to be simple in that agents spend all of their income and time to satisfy themselves down to the lowest ranked activity level possible. However, the allocation of time aspect and the resultant commodity demand functions which result from this consumption plan are more complex than in traditional economic theory. In particular, careful attention must be paid to the comparative statics implications for commodity demand resulting from the consumption decisions. By introducing the concept of work, the consuming agent becomes a psychological, consuming, working agent. It is also shown that the hours of work decision is a simultaneous decision concurrent with the total consumption plan.

Arousal Functions and Budget Constraints

Recall that in Chapter 2 consumption activation states were introduced as functions of all commodity inputs and own time inputs. While it is certainly true that most consumption activities are dependent upon the use of other commmodities, it will be assumed that each arousal level A_i has associated with it a single commodity q_i which is A_i's most important commodity input. In particular, it will be assumed that if an agent were trying to raise the activity level of A_i from zero and if he were given a small quantity of units of income to spend on any good q_n, $n = 1, 2, \ldots, N$, then q_i would be the good which would increase A_i the most. For example, the input of oranges will be assumed to be the most important and only commodity input used in raising the activity level associated with eating oranges. Also assume that there is an activity state A_i for which each q_n, $n = 1, 2, \ldots, N$ is the most important input. Thus, the model is limited to activity states associated with commodity and time inputs and it will be assumed that $I = N$.

The functional form of the activity functions will be assumed to be neoclassical such that: $A_i = A_i(q_i, \tau_i)$ with

$$\partial A_i/\partial q_i > 0, \ \partial A_i/\partial \tau_i > 0, \ \partial^2 A_i/\partial q_i^2 < 0,$$

$$\partial^2 A_i/\partial \tau_i^2 < 0, \text{ and } \partial^2 A_i/\partial q_i^2 < 0,$$

$$\partial^2 A_i/\partial \tau_i^2 - \left[\frac{\partial A_i}{\partial q_i \partial \tau_i}\right]^2 > 0.$$

Also assume the boundary conditions $A_i(0, \tau_i) = A(q_i, 0) = 0$ hold.[25] This function is illustrated in Figure 2.

Recall from Chapter 2 that there exists for each activity level A_i an associated time horizon T_i from which τ_i, the time input for A_i is extracted. Assume in this chapter that there exists an elementary time horizon T for all of the I = N activity levels. The agent's time in the model will be dichotomized into time spent in a labor market working for wage rate w; call this hours of work h, and the time spent on consumption activities which is the sum of the τ_i over all activities participated in by the agent. Obviously then, time must be spent either working or consuming and:

$$h + \sum_{i=1}^{I} \tau_i = T$$

over the course of an elementary time horizon. This is the time constraint facing the agent over the horizon T.

The income constraint relevant to the horizon T is obtained by summing the sources of income of the agent and equating this sum to his expenditure on time and commodities associated with activity levels. Thus:

$$wT + Y = \sum_{i=1}^{I} (w\tau_i + p_i q_i).$$

It is assumed that the value of time in the market, w, is also the shadow price of time spent on arousal levels τ_i. The preceding equation is then the income or budget constraint relevant to the agent over horizon T.

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25. Arousal cannot be produced by time input alone, nor can it be produced merely through the acquisition of commodities.

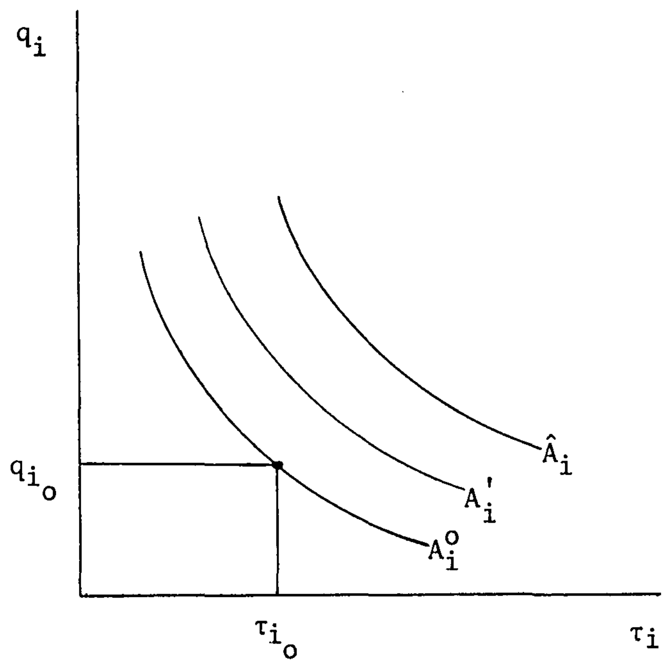

Figure 2: The technology of arousal production for activity i. Isoquants are neoclassical and do not intersect the axis. Production level A_i^o is less than A_i' which is less than $\hat{A}_i$, the satiation level of activity i. Combination of q_{i_o} and τ_{i_o} would result in production of A_i^o units of activity A_i.

The Static Problem of the Consumer

The static problem of the agent is to optimize, that is choose a most preferred vector, from his ranked activity ordering subject to the fact that activity may only be produced according to a specified technology and subject to this time and income constraints over the elementary time horizon in question. Formally, write the static consumption problem:[26]

$$\text{Optimize:} \quad \underset{q,\ \tau,\ h}{\mathcal{O}[A_1^R,\ A_2^R,\ \ldots,\ A_I^R]}$$

$$\text{Subject to:} \quad A_i = A_i(q_i,\ \tau_i)$$

$$i = 1,\ 2,\ \ldots,\ I$$

$$h + \sum_{i=1}^{I} \tau_i = T$$

$$wT + T = \sum_{i=1}^{I} (w\tau_i + p_i q_i)$$

$$h \geq 0,\ q_i \geq 0,\ \tau \geq 0$$

$$i = 1,\ 2,\ \ldots,\ N$$

where $\mathcal{O}[A^R]$ is the ordering A^R derived from Assumptions 1 through 5.

The solution to this problem is the topic of the next section.

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26. The terminology "optimize" is used since there is no specific maximand. As will be apparent, the solution to the problem involves a plan as opposed to the usual neoclassical set of differential equations.

Solution to the Static Consumption Problem

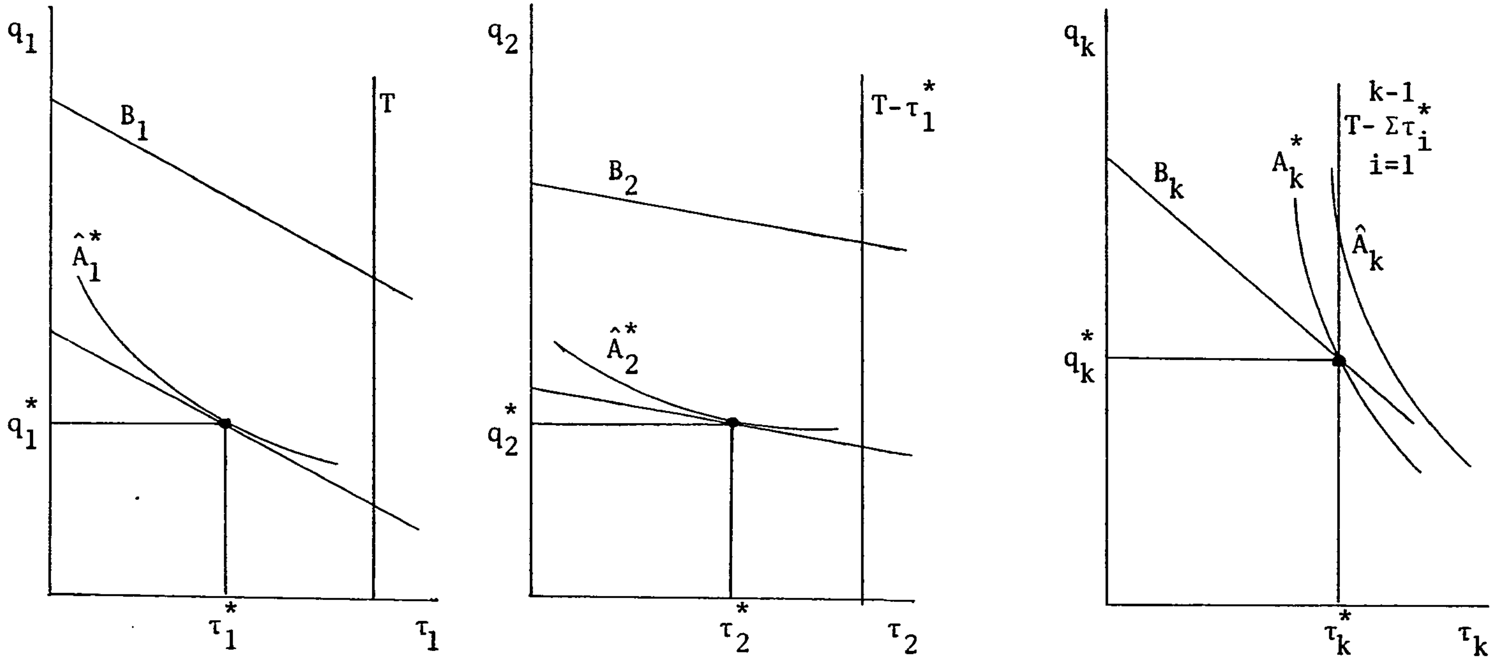
It is assumed that the production of each activity state is subject to cost minimization. That is, the consumer chooses the level of activity desired for the state and then allocates his limited time and income to producing that level of activity in such a manner that the cost of reaching the optimal state is minimal. The solution is then a consumption program consisting of a series of interrelated productions of consumption activities.

In particular, assume that a consumer considers his ranked activity vector A^R. Then from the Theorem on the Ordering of Activity States it was shown that any activity should first be allotted to ranked activity state A_1, then the residual allocated to ranked activity A_2, and so forth until all activity was allocated in some final ranked activity state A_k. Now, if the consumer is faced with limited time and income constraints over a time horizon T, then the activity associated with allocation is the combination of commodity inputs and time to produce activity along the ranked states. The agents consumption problem for state A_1 is to choose commodity input q_i and time input τ_i such that he reaches a level which is either the satiation level $\hat{A}_1$ or as close to $\hat{A}_1$ as possible. The agent is also faced with the total income constraint $wT + Y \geq w\tau_1 + p_1q_1$ since all of his income $wT + Y$ is available to be spent on state A_1. The agent is faced with the time constraint $\tau_1 \leq T$, since all of his time is available for state A_1. Note that both of these constraints are linear in τ_1 and q_1. If both of these constraints are non-binding in the sense that $\hat{A}_1$ is affordable, then the selection of q_1^* and τ_1^* will be the selection of q_1 and τ_1 such that the isoquant describing the production of $\hat{A}_1$ is tangent to a linear expenditure curve parallel to the budget line $wT + Y = w\tau_1 + p_1q_1$. Figure 3 illustrates the nature of the solution pair (q_1^*, τ_1^*) for state A_1.

Now, since it was assumed that the agent did not spend all of his time or income producing the satiation level of activity $\hat{A}_1$, the next problem is the production of state A_2. The new income constraint facing the agent is his total income net of what he spent producing A_1^* or $wT + Y - w\tau_1^* - p_1q_1^* \geq w\tau_2 + p_2q_2$. His new time constraint is similarly his total non-labor time available net of what time he spent producing arousal state $\hat{A}_1^*$ or $\tau_2 \leq T - \tau_1^*$. Minimizing the cost of $\hat{A}_2$ involves the selection of a q_2^* and τ_2^* such that the isoquant describing $\hat{A}_2$ is tangent to a linear expenditure curve parallel to $wT + Y - w\tau_1^* - p_1q_1^* = w\tau_2 + p_2q_2$. This part of the solution is also illustrated in Figure 3.

This production of satiated levels of activity would continue until a state A_k is reached where the agent runs out of time and income. At this point he will reach as high a level of activity as possible state A_k subject to the fact that he spent income and time satiating states A_1, A_2, A_3, ..., A_{k-1}. His income constraint is

$$wT + Y - \sum_{i=1}^{k-1}(w\tau_i + p_iq_i) \geq w\tau_k + p_kq_k$$



Activity 1 Activity 2 Activity k

Figure 3: The solution for quantities and times are plotted for the static consumer problem. B_1 is the locus $wT + Y = w\tau_1 + p_1q_1$, B_2 is the locus $wT + Y - w\tau_1^* - p_1q_1^* = w\tau_2 + p_2q_2$ and B_k is the locus $wT + Y - \Sigma(w\tau_i^* + p_iq_i^*) = w\tau_k + p_kq_k$, where the sum runs from $i=1$ to $i = k-1$.

and his time constraint is

$$T - \sum_{i=1}^{k-1} \tau_i^* \geq \tau_k .$$

The selection of q_k^* and τ_k^* is such that the highest level of activity is reached. Note that at q_k^*, τ_k^* in Figure 3, all income and time is spent.

In summary, the solution vector of activity for the agent will consist of a series of satiated levels of activity $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_3$, $\hat{A}_{k-1}$, a partially or fully satiated arousal state A_k, and zero levels of activity for states A_{k+1}, A_{k+2}, ..., A_I. Thus:

$$A^* = [\hat{A}_1^*, \hat{A}_2^*, \ldots, \hat{A}_{k-1}^*, A_k^*, 0^*, 0^*, \ldots, 0^*].$$

An agent does not participate in all forms of consumption even though they all can potentially please him and he does not buy all commodities even though they could cause higher activity levels.

The commodity demands are functions of the residual incomes allocated to their associated states, the wage rate, and own price of the commodity. Thus:

$$q_1^* = q_1^*(wT + Y, w, p_1)$$

$$q_2^* = q_2^*(wT + Y - w\tau_1^* - p_1 q_1^*, w, p_2)$$

$$\vdots$$

$$q_i^* = q_i^*(wT + Y - \sum_{j=1}^{i-1} (w\tau_j^* + p_j q_j^*), w, p_i)$$

$$\vdots$$

Note that commodity demands are not functions of all commodity prices, but functions of all prices of commodities used for higher ranked activity and the own price of the commodity. Similarly, the time inputs to activity states are functions of the residual times allocated to activity states, the wage rate, and the own price of the associated commodity. Thus:

$$\tau_1^* = \tau_1^*(wT + Y,\ w,\ p_1)$$

$$\tau_2^* = \tau_2^*(wT + Y - w\tau_1^* - p_1q_1^*,\ w,\ p_2)$$

$$\vdots$$

$$\tau_i^* = \tau_i^*(wT + Y - \sum_{j=1}^{i-1}(w\tau_j^* + p_jq_j^*),\ w,\ p_i)$$

$$\vdots$$

The consumer's demand functions for commodities are derived as a product of the optimal consumption plan. The demand functions state quantities of input demanded as a function of residual income allocated to the activity, wages, and own price. Given the neoclassical development of activity production functions, it is apparent that the marginal solution of equating w/p_j to $-Aq_j/A\tau_j$ in each activity state implies a single valued demand function. Also, since a constant proportional change in wage rate, income, and prices leaves the budget lines unaffected in each activity state, it is also clear that the demand functions associated with the solution to the static problem are homogenous of degree zero in wage rate, income, and prices. Both of these results are consistent with traditional utility maximization behavior of consumers.

The hours of work or labor decision for the agent is now considered. As noted, the optimal solution for production in the final or marginal arousal state involves an adjustment of τ_k^* so that the marginal activity lost due to working more hours is equal to the marginal gain in consumption due to the increased purchasing power of the agent. Given that hours of work are defined by:

$$h^* = T - \sum_{i=1}^{k} \tau_i^*$$

then:

$$h^* = h^*(T, w, p_1, p_2, \ldots, p_k, Y)$$

since each τ_i^* is a function of T, w, p, and Y. The comparative statics results associated with this result will be considered later in this chapter.

Consider now some of the comparative statics results associated with commodity demands in this model and the testable hypothesis which the model implies through these comparative statics results.

Some Comparative Statics Results for the Static Consumption Problem

Consider first the reaction of an agent, initially in equilibrium, to a change in the wage rate. In particular, assume, without loss of generality, that the wage rate w increases. Also, assume that the agent was initially in an equilibrium position which allowed him to produce activity out to the k^{th} ranked state A_k^R. The wage rate increase will have two effects on activity 1's budget constraint. First, the increase in the wage rate will cause the budget line to shift outward, remaining parallel to the old budget line. This is because the agent is now wealthier in terms of total time income. Secondly, the increase in the wage rate will cause the budget line to rotate upward since the slope, defined as the ratio of w to p_1, will increase. The net effect of both of these shifts will be that the agent can now afford to produce $\hat{A}_1^R$ even more readily than before. New production equilibrium will occur using a larger amount of commodity input q_1 and smaller amount of time input τ_1 since time is now relatively more valuable to the agent than income. Figure 4 illustrates the movement in equilibrium for this agent.

For activity 2, the budget constraint also shifts outward and becomes more steeply sloped. However, since some time was freed up due to a more commodity intensive production of activity 1, the time constraint $T - \tau_1^*$ will also shift outward. Equilibrium in state A_2^R will be at a point where less time τ_2 and more commodity q_2 is used in the production of $\hat{A}_2^R$. Similarly, for states A_3^R, A_4^R, ..., A_{k-1}^R, production will occur at equilibrium points where more commodity input relative to time input is used to satiate these states.

Equilibrium in state A_k^R, assuming A_k^R remains the marginal arousal state,[27] will be subject to the combined increase in the time freed up by the other k-1 states and the outward shift in the budget line for that state. Without question, both of these increases will

27. Spillover effects into state A_{k+1} simply imply income effects so great that the consumer can afford to produce arousal for the next lower ranked activity.

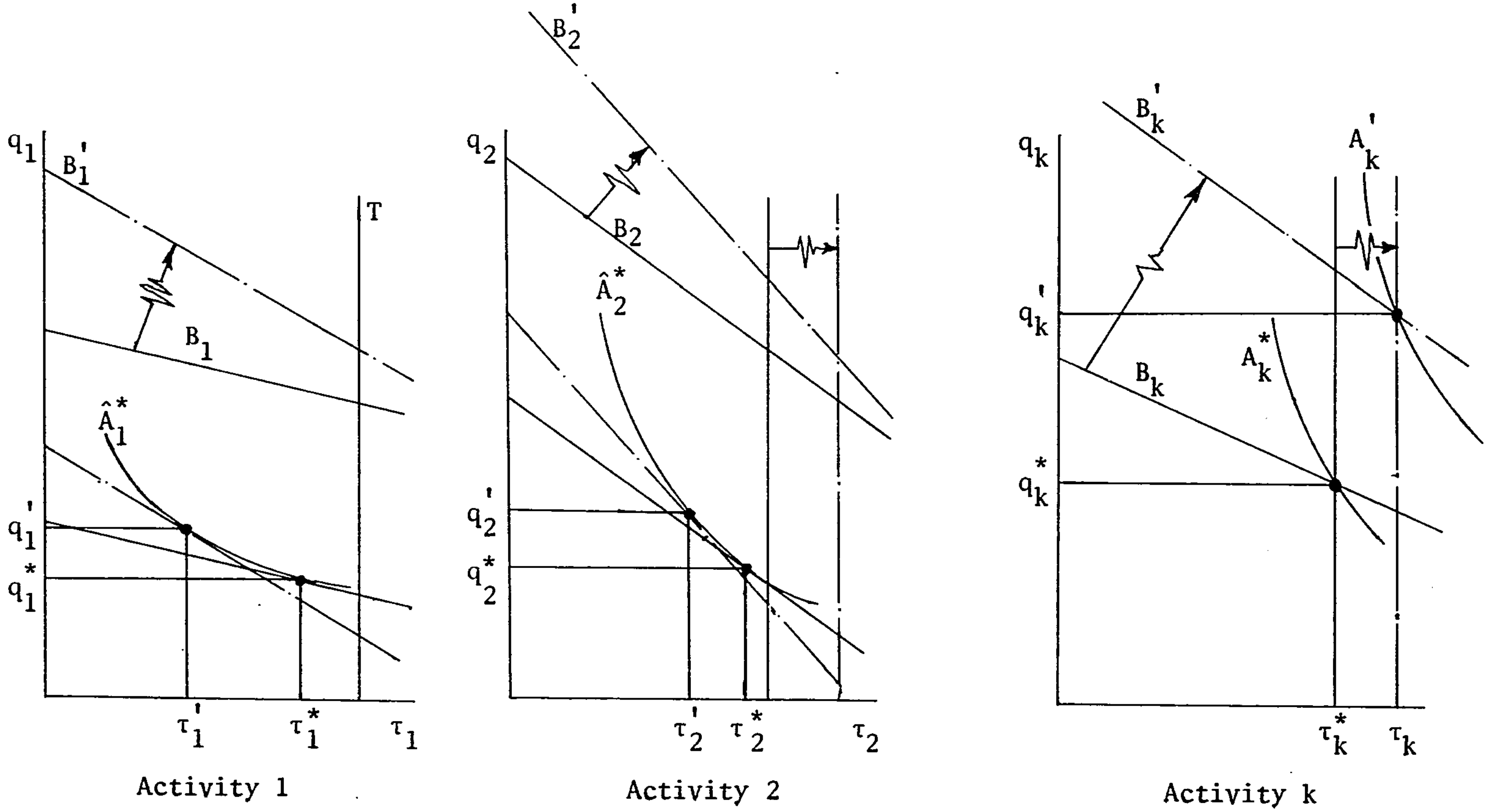
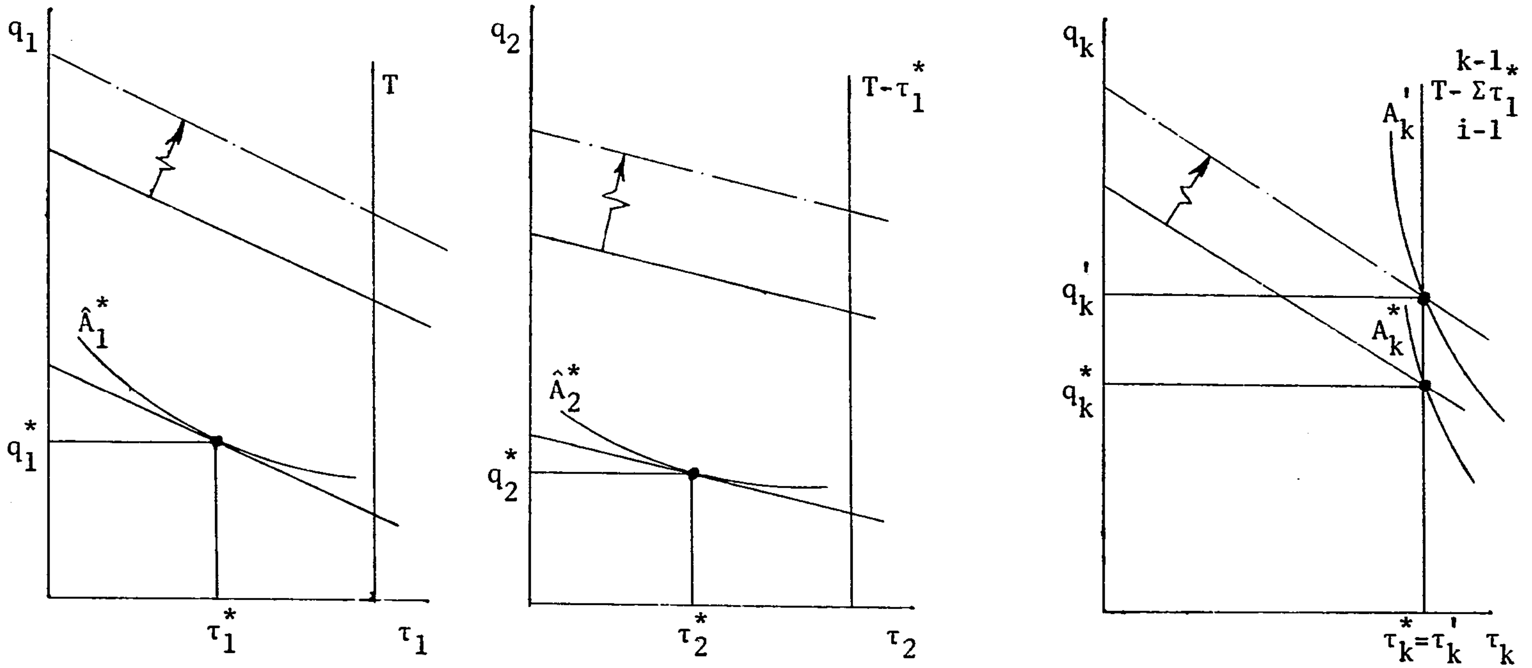


Figure 4: The solution change for a change in the wage rate is plotted. It is assumed that the wage rate increases. The new budget and time constraints are plotted in the broken lines.

combine to allow the agent to reach a higher level of activity for state k than reached initially. Wage increases will cause substitution effects of commodities for time in all the first k-1 initially satiated states and an income effect in the final activity A_k^R.

Wage effects cause only substitution effects for the interior states since they only affect the ratio of w to p_j, thus optimality dictates movement toward a new ratio of $-A_{q_j}$ to A_{τ_j}. Since satiation already exists for these states the isoquant of optimal production remains unchanged. In the final state the combined freeing up of time causes the time constraint to move out and the budget constraint to move out. The net effect is a windfall for that state. Optimal adjustment is reached when all time and income are used up as illustrated. Without doubt, more q_k will be purchased. In summary then, $\partial q_i/\partial w$ will be positive for all states i that production occurs in. This is a result consistent with the Becker approach to consumption activities.

Next consider a change from initial equilibrium when non-labor income Y increases. This change and associated effects are illustrated in Figures 5 and 6. Since non-labor income increases for all periods, the budget line relevant to each of the k activities will shift outward parallel to the original budget lines. This will cause satiated levels $\hat{A}_1^R$, $\hat{A}_2^R$, ..., $\hat{A}_{k-1}^R$ to be more readily affordable, but will not affect the manner of their production since optimal production will continue to occur at the same ratio of commodity to time input.



Activity 1 Activity 2 Activity k

Figure 5: The solution change for a change in non-labor income Y is plotted. It is assumed that Y increases. The resultant new budget constraints are plotted in broken lines.

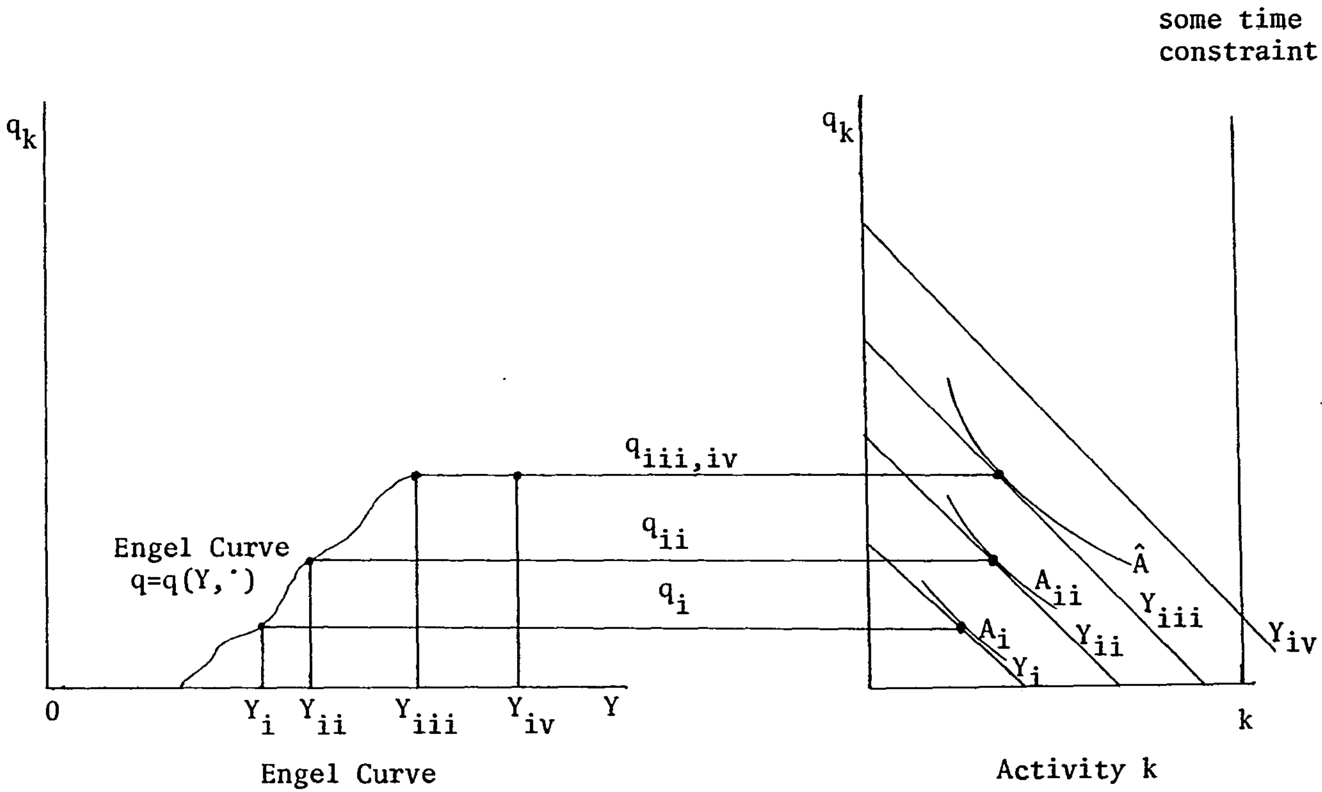


Figure 6: Derivation of the Engel Curve for an activity state. It is assumed that income Y increases causing increasing expenditure of q_k until satiation is reached.

The outward shift in the budget line will, however, manifest itself in the marginal state A_k^R, where again it is assumed that A_k^R remains the marginal consumption activity. For this state the outward shift in the budget line allows more activity to be produced through the purchase of commodity input q_k. Thus, the net effect of a change in the flow of non-labor income is entirely in the marginal state. The economic agent uses the entire increase in purchasing power to move to a higher level of consumption in this final state. Again, it is assumed that activity k remains the marginal activity.

This result is different than the result of a change in non-labor income applied to the traditional neoclassical model of behavior. In that theory, the change in income is allocated to <u>all</u> activities until the marginal effects of the addition are equal. The static model developed here states that the totality of effect occurs at the marginal state. A testable implication of the model is that an agent will spend all such windfall in the marginal state or he will satiate that state and move to a new, lower ranked activity and take up production if some time can be found by adjusting h. A casual observation of a field situation may exemplify such behavior. Consider a college student who is satiated in all higher ranked activities and that his marginal activity is consumption of music. An unexpected flow of income may be observed to result entirely in purchases of musical recordings. Such allocation of transitory income to marginal activities is quite common in occurrence and explained well by the model. Such an explanation also explains well estimates of income elasticities associated with luxury commodities. In summary then, $\partial q_i/\partial Y$ is zero for all $i < k$ and $\partial q_k/\partial Y > 0$.

Finally, consider the movement in equilibrium caused by a change in a commodity price. The most general results can be obtained by assuming that the particular price that changes is an interior price in the ranked scheme, that is, between p_2 and p_{k-1}. Assume, for example then, that p_2 increases.

Production of the satiation level of arousal for state 1 will not be affected since the budget constraint for that state is a function only of own price p_1 and wage rate w. Equilibrium choices of q_1 and τ_1 will not change. This is illustrated in Figure 7.

Production of optimal activity for state 2 will be affected by a decrease in the slope of the budget constraint caused by the increase in p_2. Similar to the changes brought about by wage increases, the increase in p_2 will not affect the level of production of arousal but will cause the optimal arousal level to be produced using more time input τ_2 and less commodity input q_2. The net effect for production in activity 2 is that substitution from relatively more expensive commodity intensive production to relatively cheaper time intensive production occurs.

Production of states 3 through k-1 will now be subject to two possible changes in the time and budget constraints. Since more time input τ_2 will be used to produce $\hat{A}_2$ there will be less time available to produce $\hat{A}_3$, $\hat{A}_4$, ..., $\hat{A}_{k-1}$. Also, there may be shifts in the budget

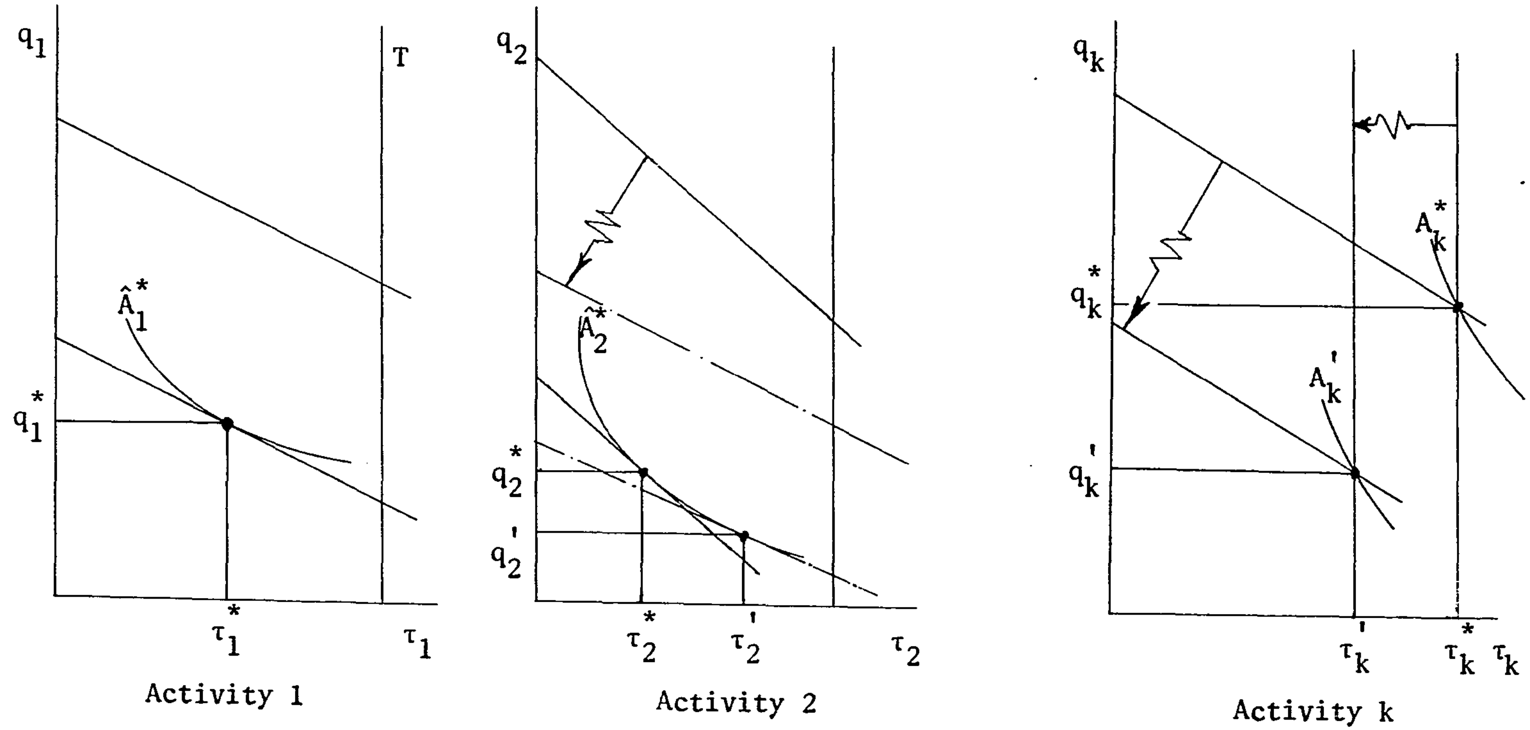


Figure 7: The solution change for a change in an interior commodity price is plotted. It is assumed that p_2 increases. The resultant new budget constraints are plotted in broken lines.

constraints for these states if the net change in income due to the change in commodity cost $p_2'q_2'$ is different than the net change in time cost $w\tau_2'$. If, as shall be assumed, these changes have no or little effect on the production of these states, then the entire income and time constraint changes will be filtered down to the last marginal state k.

Production in state k due to the change in price p_2 will be subject to the decrease in time available and the decrease in budget available for production of A_k. As illustrated, both of these changes will cause optimal production of an unambiguously lower level of production of state k. Again, it is assumed k remains the marginal state.

To summarize, if a price p_j changes then there will be no effect on production for states A_1^R, A_2^R, A_3^R, ..., A_{j-1}^R, there will be a substitution effect to a more intensive production of A_j, there may be income effects for states A_{j+1}, A_{j+2}, ..., A_{k-1}, and there will most certainly be an income effect for state A_k in that production of arousal will shrink due to loss of available consumption time and residual income.

Own price effects in the model are always negative and demand curves always slope downward. The commonly discussed notion of a Giffen good with upward sloping demand would never occur in the model. The reason all demand curves must slope downward is that the ratio w/p_j must be set equal to $-A_{q_j}/A_{\tau_j}$. Neoclassical restrictions about A_j dictate a pure substitution effect over q_j and τ_j. Cross price effects are certainly not equal in any sense in the model. Given that two price changes occur, the price associated with the higher ranked activity may affect the lower ranked activity only through an income effect. A change in a price of a lower ranked activity will have no effect upon the higher ranked activity. Such ranked effect of price changes also is observed as behavior of economic agents. For example, large price changes of gasoline affect transportation commodity demands through substitution effects and may also have unexpected income effects in strange places. A consumer may substitute a slower rate of automotive travel for lower fuel costs and at the same time decide against the purchase of a marginal commodity such as a tennis racket. The effect of the gasoline price rise may have strong effects upon tennis rackets even though the substitutability - complimentary relationship is unclear for that person. In summary, $\partial q_i/\partial p_i$ is less than zero for all i. A price change in state i has no affect on states $j < i$ and possible income effects on states $j > i < k$. These results are also testable implications of the model.

The solution set for commodity inputs and associated comparative statics results may thus be characterized by the following set of inequalities:

$$q_1^* = q_1^* \ (wT + Y_1, w_1, p_1).$$

The solution set again is:

$$q_2^* = q_2^*(wT + Y - w\tau_1^* - p_1q_1^*, w, p_2)$$

$$\vdots$$

$$q_k^* = q_k(wT + Y - \sum_{j<k}(w\tau_j^* + p_jq_j^*), w, p_k).$$

Now the signs of the derivatives relating to changes in w, p, and Y are:

$$\partial q_i/\partial w > 0 \qquad \forall i$$

$$\partial q_i/\partial p_i < 0 \qquad \forall i$$

$$\partial q_i/\partial p_j \begin{cases} = 0 & i < j \\ < 0 & i = j \\ \leq 0 & i < j < k \end{cases}$$

$$\partial q_i/\partial Y \begin{cases} = 0 & i < k \\ > 0 & i \geq k. \end{cases}$$

The Work Decision and the Static Consumption Problem

Recall that the hours of work decision h^* is a function of T_1, w_1, p_1, p_2, ..., p_h, and Y is derived endogenously as the residual of total time minus consumption time. It will be convenient to write h^* as the sum of its components and consider the changes in the components resultant from changes in exogenous variables. Write:

$$h^* = T - \sum_{i=1}^{k} \tau_i^* .$$

Or equivalently:

$$h^* = T - \sum_{i=1}^{k-1} \tau_i^* - \tau_k^*$$

where h^* is now decomposed into total time minus inferior state time minus marginal state k's time.

Begin with a consideration of changes in the wage rate upon h^*. As noted previously, an increase in the wage rate will cause substitution effects in the k-1 activity states away from time and towards commodities. The effect of the increase in the wage rate upon the marginal k^{th} state is an income effect; more A_k is produced. Therefore, the effect of the wage change is the positive substitution of work for consumption time as wages increase net of the income effect from the marginal state. Formally,

$$\partial h^*/\partial w = -\sum_{i=1}^{k-1}(\partial \tau_i^*/\partial w) - \partial \tau_k^*/\partial w$$

where it is noted that $\partial \tau_i^*/\partial w < 0$, $\forall i < k$ and $\partial \tau_k^*/\partial w > 0$. In general, the net effect of the substitution and income effect associated with a change in the wage rate is indeterminant. It can be concluded that only if the marginal activity is a highly normal type of activity, that is $|\partial \tau_k^*/\partial w|$ is large relative to the sum of the other $|\partial \tau_i^*/\partial w|$, will the agent fit a model of so called "backward bending" supply of labor. Otherwise, the use of less time in the first k-1 states will cause labor supplied to increase and the normal positive relationship between w and h will exist.

Non-labor income effects upon h^* can also be considered by examining the effects of changes in Y upon the individual consumption times τ_i^*. Recall that the entire effects of in increase in non-labor income are transmitted to the final state A_k^R. The net effect at that state is an unambiguous increase in activity. However, depending on the nature of production of this arousal, either more or less time input τ_k^* will be used. Formally:

$$\partial h^*/\partial Y = -\sum_{i=1}^{k-1}(\partial \tau_i^*/\tau Y) - \partial \tau_k^*/\partial Y.$$

And since $\partial\tau_i^*/\partial Y = 0$, $\forall < k$, then $\partial h^*/\partial Y$ is simply:

$$\partial h^*/\partial Y = -\partial\tau_k^*/\partial Y$$

which is indeterminant given the assumptions made. Activity in state k will increase, but whether more or less τ_k^* is used is indeterminant.

Finally, consider the change of an interior price and the reaction of the agent through changes in hours worked. Assume that the price of the commodity input to an interior state A_j increases. The increase in this p_j will not affect production of all states i < j in any manner; thus, τ_j^* for i < j will remain unaffected. The increase in p_j will, however, cause a substitution effect away from q_j^* and towards τ_j^*. States A_{j+1}^R, A_{j+2}^R, ..., A_{k-1}^R and associated τ_{j+1}^*, τ_{j+2}^*, ..., τ_{k-1}^* are also unaffected since the residual income and time constraint changes caused in state A_j^R are passed down to the marginal state A_k^R. Write $\partial h^*/\partial p_j$ as the sum of all components and then decompose as before:

$$\partial h^*/\partial p_j = -\sum_{i=1}^{k}(\partial\tau_i^*/\partial p_j)$$

$$\partial h^*/\ p_j = -\sum_{i<j}(\partial\tau_i^*/\partial p_j) - \partial\tau_j^*/\partial p_j - \sum_{j+1\ <\ i<k}(\partial\tau_i^*/\partial p_j) - \partial\tau_k^*/\partial p_j)$$

and since all the individual derivative terms in both the summations terms are zero,

$$\partial h^*/\partial p_j = -\partial\tau_j^*/\partial p_j - \partial\tau_k^*/\partial p_j.$$

Since $\partial\tau_j^*/\partial p_j > 0$ and $\partial\tau_k^*/\partial p_j < 0$, the net effect of the increase in

p_j is the hours of work lost due to increase in consumption time for state j net of the hours of work increased due to decrease in consumption time for state k. The net effect is again a matter of the specific functional form of the arousal functions involved and is indeterminant.

CHAPTER 5

OPPONENT-PROCESS THEORY AND THE PRODUCTION OF ACTIVITY

This chapter reconsiders the production of activity and how this production may change over time because of dynamic psychological forces. In particular, by introducing the psychological model of opponent-processes into the static consumption model, the optimal satiation levels for each activity state are shown to be a function of past consumption behavior. Opponent-process theory is first reviewed in its psychological form and then translated into a mathematical form. This mathematical model will form the basis for the underlying production dynamics used in the dynamic consumption problem of the agent.

The Psychological Form of the Opponent-Process Theory

Recall that when preference in the static consumption problem was considered, it was found that the tastes of an agent can be completely specified by the order of activity states and the corresponding satiation levels associated with the states. Presumably, this ordering and associated levels are adopted by the consumer because they have contributed to the well being of the agent or perhaps conform to other agent's consumption habits. If the agent is forced away from this adopted consumption plan, he then experiences the dissatisfaction of breaking the engrained habits. Consumption habit can be defined[28] as any increase in future probability of a consumption act associated with past activation of the particular corresponding activity state. Any increase in the production of a particular activity implies greater probability of this production in the future; thus, habits may strengthen. In general, the strength of a particular consumption habit increases as a function of past consumption.

In addition to the positive reinforcement of consumption acts, there exists for each activity a negative reinforcement mechanism driven by the absence or cessation of production. For example, if an agent attempts to stop breathing, he will experience strong physical and psychological forces caused by this respiratory interruption. As another example, an agent may experience physical or psychological discomfort if the size of his daily consumption of food is changed dramatically.

The explanation of the interaction of positive and negative reinforcement in the formation of habits is the thrust of psychologists' opponent-process theory. This theory postulates that for each activity there is a secondary opponent process whose hedonic sign is opposite to that of the primary process. For instance, participation in a sporting activity may be followed by feeling of letdown upon the activity's termination. This feeling will linger long after the primary activity occurred and will decay slowly.

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28. See Tibor Scitovsky, The Joyless Economy, p. 125.

The theory also explains how habits are formed. As the production of activity for a particular state is repeated, the level of primary reaction remains unchanged, but the opponent-process starts faster and becomes more severe. The aftereffects of activity are enhanced. In the limit then, when these aftereffects are strong enough, addition will occur. A simple example of building up of a habit is an addictive drug. Initial experiences with such a drug are usually entirely pleasant, repeated usage increases the severity of aftereffects, and finally redosage becomes imperative to the agent. This is an extreme example of the workings of opponent-process theory. However, the model forms a basis upon which to build a general model of activity and habit formation. Returning to the sporting example, if the agent participates more frequently in the sporting activity and in so doing increases the strength of the negative effects associated with non-participation, then he may reach a new equilibrium of participation at a more frequent level of indulgence in the sport.

Quantitatively, opponent-process theory is an empirical explanation of various biological organism's (man, monkey, duckling, and dog are all experimental subjects used by Solomon and others) response to a stimulus and the change of this response after repeated stimulus. This is summarized by Solomon and Corbit:[29]

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29. Richard L. Solomon and John D. Corbit, "An Opponent-Process Theory of Motivation: I. Temporal Dynamics of Affect," Psychological Review, Vol. 81, No. 2 (1974), p. 125.

> "the sudden onset of some new stimulus aroused an affect of hedonic state not present prior to onset. The state terminated when the stimulus terminated. Then, a new state appeared, qualitatively unlike either the pre-stimulation state or the state produced by the onset and maintenance of the stimulus. Finally, this new post-stimulus state persisted for a while and died out. The baseline state eventually returned. In none of the examples did the subject's affective state return directly to baseline upon cessation of stimulation. Baseline was regained via some new state which became manifest at stimulus termination, and then slowly died away.
>
> Secondly, in some cases the states changed in their quality and intensity with successive repeated stimulations. Whenever this occurred, the a states became weaker and the b states stronger and longer lasting.
>
> The two phenomena, the dynamic hedonic response pattern and its modification with repeated experience, were seen as whether the a state was pleasurable or aversive."

Solomon uses the letters a and b to refer to the hedonic states associated with the primary and opponent-process levels respectively. The a process for a given state is caused by the production activity of combining commodities and time. After this production, the agent remains aroused at a steady level of satiation for a period of time. Then the secondary b process dominates the agent. This process has a negative effect on the agent's activity level and takes time to decay back to a level of neutral arousal. The strength of the b process is increased by use and decreased by disuse.

The steady level of the a process associated with increasing repitition of production may also decrease if novelty effects are present. The combination of a fixed amount of commodity and time input may arouse an agent for initial exposures, but less after many repititions.[30] The intensity of the a process for some activities

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30. See Richard Solomon, "The Opponent-Process Theory of Acquired Motivation, The Costs of Pleasure and Benefits of Pain."

decreases with use. As an extreme example, this steady level may approach zero. An agent may use production to offset the negative b process even though there is no large psychological arousal associated with the positive a process.[31] For example, all living economic agents breathe to offset discomfort. These dynamic effects upon hedonic states a and b are summarized as follows:[32]

> ". . . three phenomena are corollaries of the use postulate. First, the peak of a will be less intense because the latency of the b process is decreased and intensity is increased. Second, the steady level of a . . . during maintained stimulation will be close to baseline and perhaps even below it in some cases. Third, the peak of b should be intense and longer lasting, compared to what it was during early stimulation."

Graphically, opponent-process theory is illustrated in Figure 8.[33] In this figure the hedonic paths through time of arousal are plotted for two points in time. The left part of the figure corresponds to the case of arousal evolution for the first few stimulations of a state. The right part of the figure illustrates the same evolution after many stimulations. Note that both the intensity of the a process decreases and the depth of the b process increases after repeated stimulation.

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31. Howard S. Hoffman and Richard L. Solomon, "An Opponent-Process Theory of Motivation: III. Some Affective Dynamics in Imprinting," Learning and Motivation, No. 5, (1974), pp. 149-164.

32. Richard L. Solomon and John D. Corbit, "An Opponent-Process Theory of Motivation: I. Temporal Dynamics of Affect," p. 130.

33. This figure is adopted from Solomon and Corbit, p. 128.

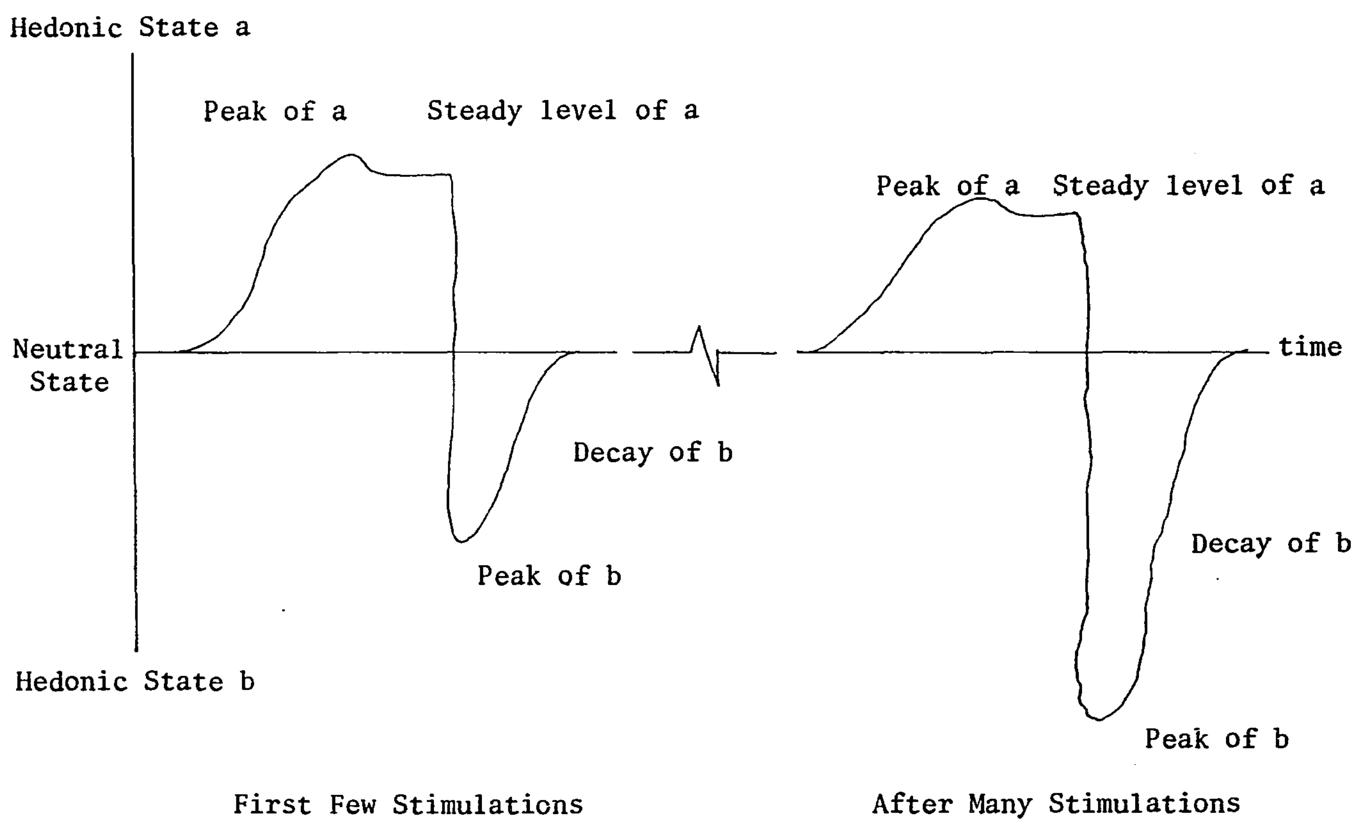

Figure 8: The Psychological Form of The Opponent-Process Theory.

Finally, each of the b processes has a critical evolution level above which levels of the b state are relatively insignificant to the agent and below which cause the agent to experience discomfort of dissatisfaction. Again drawing from Solomon and Corbit:[34]

> "The opponent-process is a slave process. It is activated indirectly via the activation of the a process. Presumably, the slave process has an evocation threshold, a latency, a recruitment or augmentation time, and a decay function, all characteristic of a given opponent process system."

As an example, as the satiation from the consumption activity of eating wears off, an agent will not immediately experience the desire for food, but will gradually become more and more hungry until he again eats. For a period of time before the threshold level of activity is reached, the agent is not hungry. The concept of the critical activity level will be important in the next section where it is used to define signal activity levels associated with activity states. These signal levels will form the basis of production decisions made by the agent in a hierarchical dynamic environment.

A Mathematical Model of Opponent-Processes

The qualitative opponent-process model is simplified by assuming that the production and satisfaction associated with activity is separated into three periods of time. Refer to Figure 9 during this discussion. This figure replicates Figure 8 and incorporates the simplifying assumptions used in the mathematical formulation.

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34. Richard L. Solomon and John D. Corbit, "An Opponent-Process Theory of Motivation: I. Temporal Dynamics of Affect," p. 127.

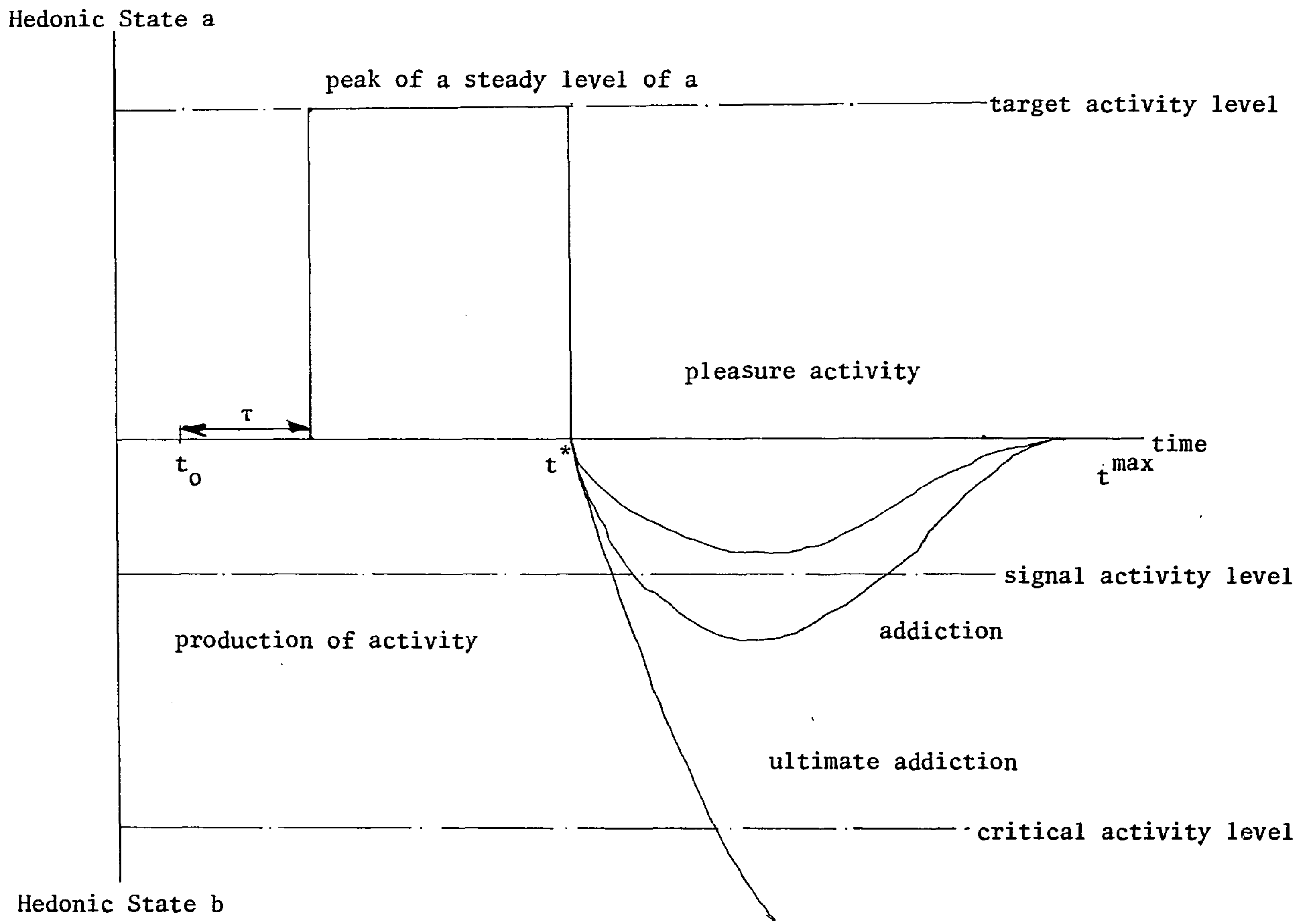

Figure 9: The Mathematical Form of Opponent-Process Theory

The first distinct period of time in the model will be the time used to produce the activity. It is assumed that at t_o, τ units of time are used by the agent to produce the target or satiation level of activity for the state. At time $t_o + \tau$, when the production process is completed, activity assumes its satiation level. During the second period, activity is at its target level and remains constant until t^*. The time t^* is a function of the particular activity and is the time period of latency of satiation of hedonic state a. This is the period of time after production in which the state remains satiated. Once this time passes, the b process dominates. The third period is dominated by the growth and decay of the b process. Three possible cases of the evolution of a particular b process are possible. For a simple pleasure activity such as listening to music, the b process may never reach the signal level which would cause the agent to feel discomfort. The b process simply decays until neutrality is reached at time t^{max}. For an addictive activity such as sport participation, the b process follows a similar U-shaped curve and decays to neutrality at time t^{max}. However, in this case, the plunge of the b state below the signal level means that the agent will experience a period of discomfort. The ultimate form of addiction, such as breathing, will be associated with a b process that falls below the signal level to a critical level defined as a level which causes severe psychological or physiological damage to the agent. For example, if the agent ceases to redose breathing, he will be signaled to redose and breathe again. If this is not done, the agent may experience bodily damage or ultimately expire. This is the case of the ultimate addiction.

The simplified opponent-process model, therefore, involves a starting time t_0, production of arousal for τ units of time, satiation at steady target level until time t^*, and finally growth and decay of the b process until neutrality is reached at time t^{max}.

Consider now how an agent acquires a new habit. Assume that the agent tries a new commodity, uses a certain amount of time τ, and produces activity associated with this commodity. The process being described is illustrated in Figure 10. The first combination of commodities and time will cause some level of activity to be produced. After this level is produced, the agent will remain satiated for an interval. Finally, a mild b process will occur and the agent will return to a state of equilibrium. Now assume that the agent again experiments with the commodity. This time, given fixed time and commodity inputs, less activity level will be produced since novelty effects will weaken the a process. The b process associated with the second production period will be deeper, but as illustrated, will not reach the signal level. The habit is formed in this example during the third production period. This time even less arousal is produced for given input. Now the b process increases enough to signal the agent to redose. Redosing is assumed to start at the time this b process signals. After redosing, another cycle of satiation followed by a b process occurs. Equilibrium in the model is reached when the cycle of a process, b process, signal becomes symmetric from the third production period onward. Such cyclical production occurs for quite a few activity states; in particular it occurs for those states associated with bodily maintenance: breathing, eating, sleeping.

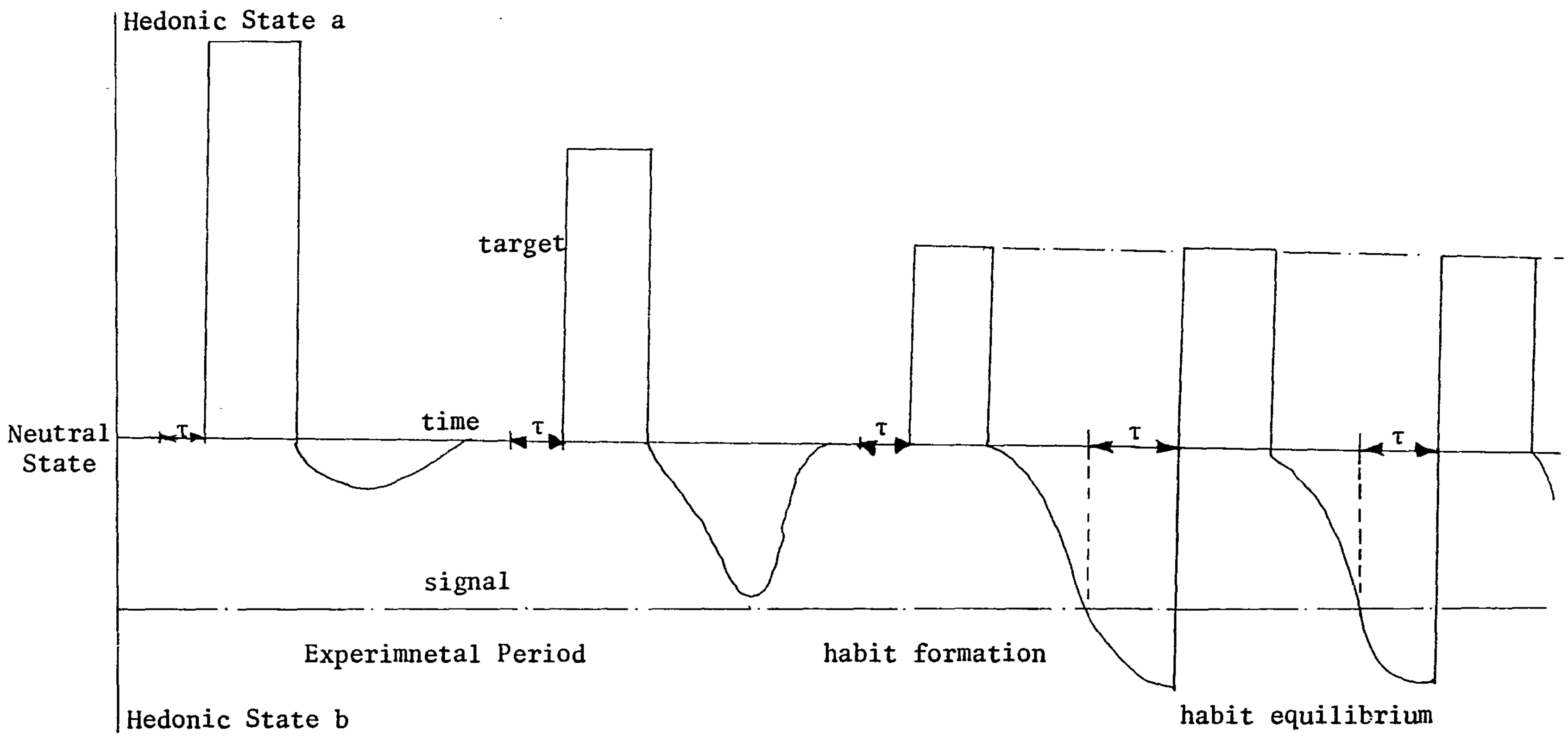

Figure 10: Acquisition of a habit. After an experimental period the b process causes the need for redosage. Cyclical habit equilibrium then results.

Now consider starting from a cyclical habit pattern and breaking or overcoming the habit. There are two cases: gradual withdrawal and the cold turkey case. Gradual withdrawal begins from a cyclical habit equilibrium pattern. It is assumed that the agent gradually decreases his time input for the succeeding production periods. This process is illustrated in Figure 11. Since the b process is weakened by disuse, the depth of the b, though, is lower for the second production period. In the third production period, even less time input is used and the resultant b process does not reach the signal level. The b process decays back to the neutral state and the habit is broken. An example of this type of withdrawal behavior is breaking a tobacco habit through increasingly smaller doses of tobacco input.

The cold turkey case involves breaking a habit cycle in one production period. The aftereffects of the b processes are left to run their course. Refer to Figure 12 for the two possible subcases of this type of behavior. Case 1 involves an agent experiencing a period of discomfort after which the habit is broken by normal decay of the b process. Case 2 is the extreme extension of Case 1 when the failure to redose causes physiological or psychological damage to the agent. It is less clear if and how these effects decay and thus the broken decay line for Case 2 is illustrated. Examples of these types of withdrawals are common. Case 1 is exemplified by the tobacco consumer who simply stops smoking all at once, experiences discomfort, then is rid of the habit. Case 2 models an agent who tries to stop sleeping or breathing. Eventually the agent will die or experience permanent damage.

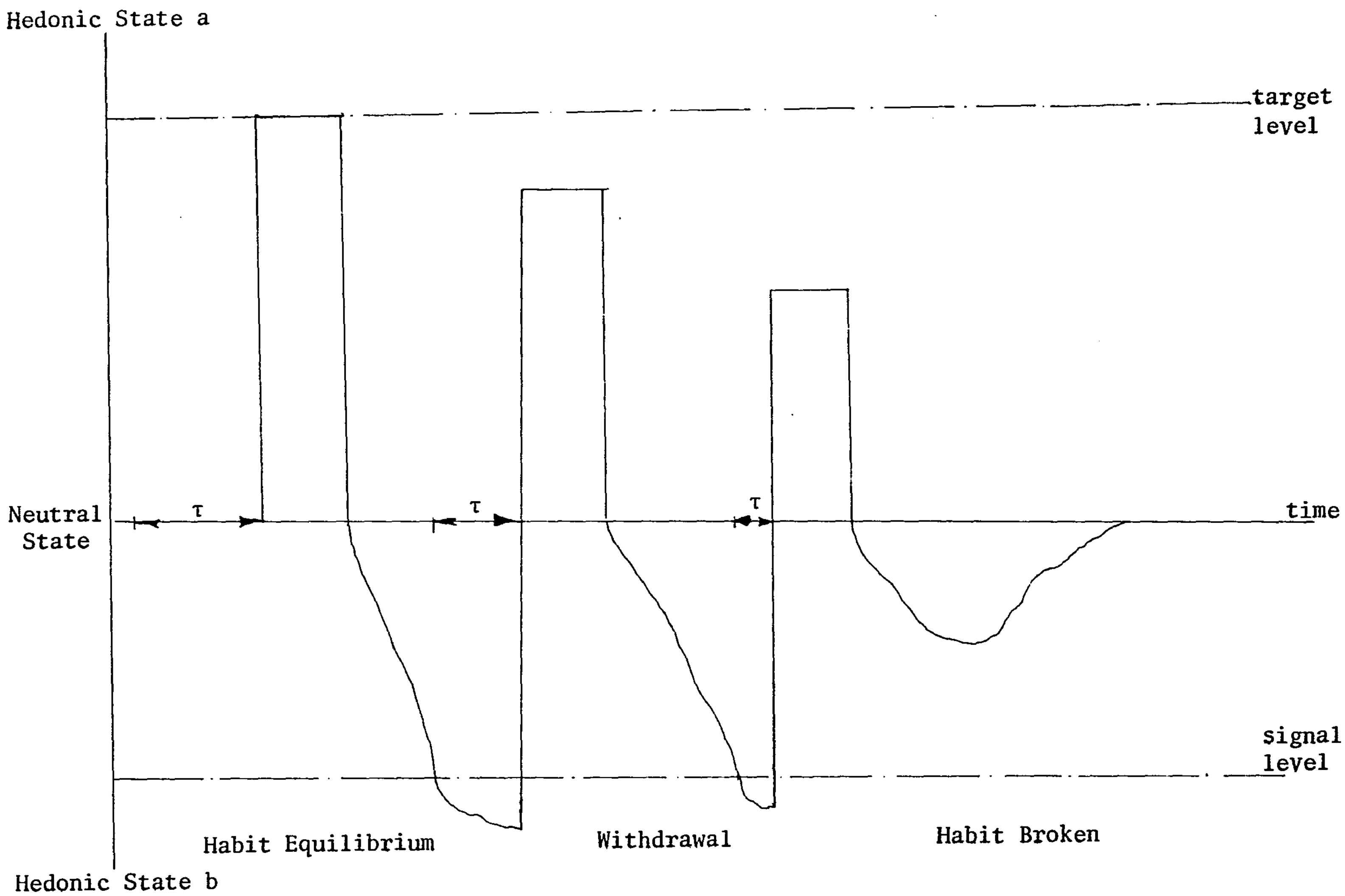

Figure 11: Breaking a habit: The gradual withdrawal case.

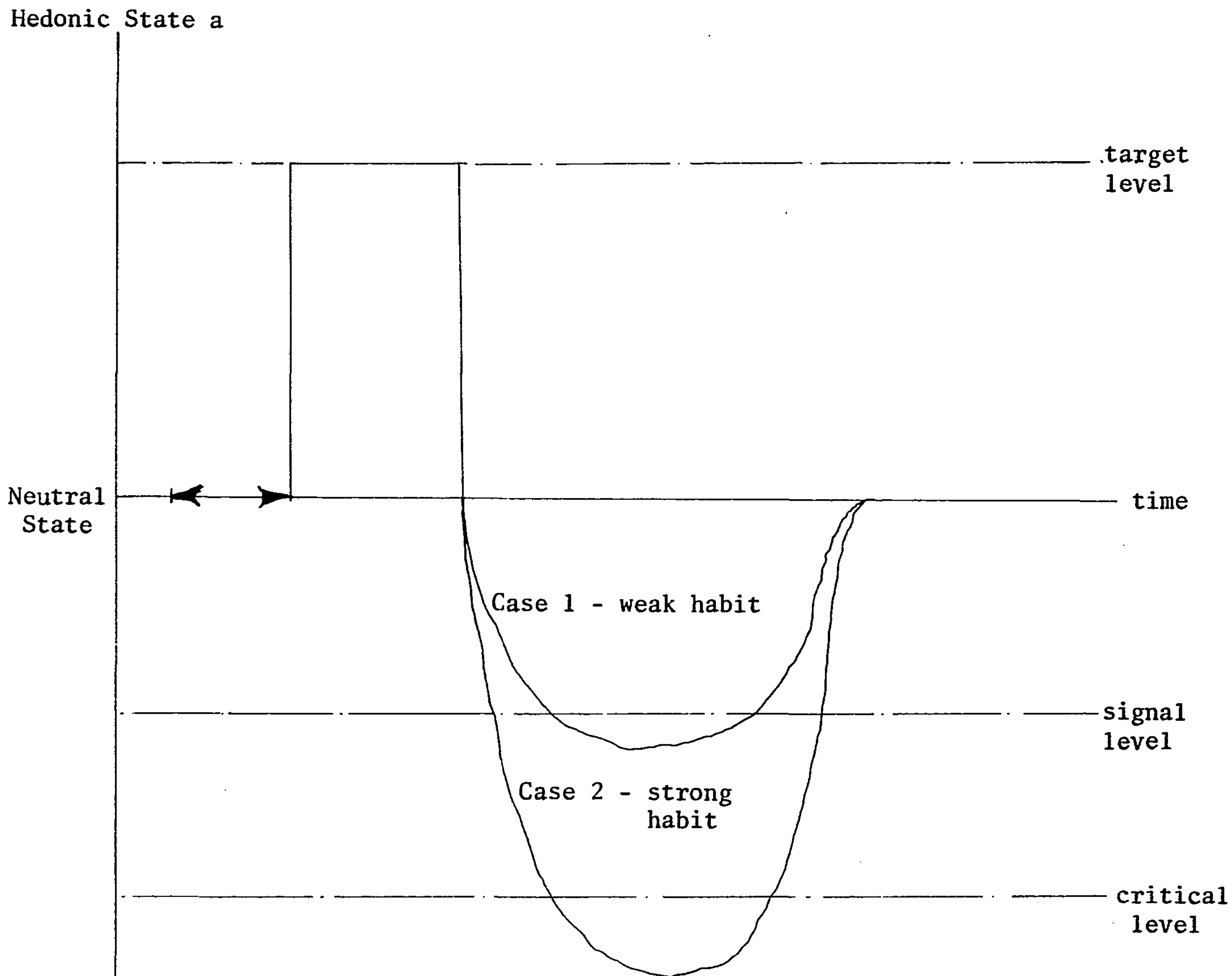

Figure 12: Breaking a habit: The cold turkey cases.

The simplification developed for the opponent-process theory presented in this section can be formalized mathematically. Recall that each activity state A_i is assumed to be a function of single commodity input q_i and time input τ_i: $A_i = A_i(q_i, \tau_i)$. If q_i and τ_i are now considered as flow inputs to dynamic production of activity, then $q_i = q_i(t)$, $\tau_i(t)$ and $A_i = A_i(q_i(t))$ or $A_i = A_i(t)$. Activity production is then simply a function of time. Opponent-process theory states that this function is the sum of the a process and the b process for state A_i: $A_i = a_1(t) + b_i(t)$.

Consider now a and b separately. The a process is a function of both the time and commodity inputs and the novelty associated with repeated production. Write $F_i(q_i, \tau_i)$ as the production function for a_i. Assume F_i satisfies the same neoclassical production assumption as the state A_i function did:

$$\partial F_i/\partial q_i > 0,\ \partial F_i/\partial \tau_i > 0,\ \partial^2 F_i/\partial^2 q_i < 0,\ \partial^2 F_i/\partial \tau_i{}^2 < 0 \text{ and}$$

$$\partial^2 F_i/\partial \tau_i{}^2 - \left[\frac{\partial F_i}{\partial q_i \partial \tau_i}\right]^2 > 0,$$

with $F(0, \tau_i) = F(q_i, 0) = 0$.

Define the novelty function as a function of time, $N_i(t)$, such that $N_i(t)$ is weighted by past consumption history. The weighting function $W_i(F_i)$ will be assumed to decrease as more production in the state is undertaken. Therefore: $\partial W/\partial F < 0$. In general, the novelty effects are accumulated over time back to some relative past horizon $\tilde{t}_i$

defined as the forgetting horizon for state i:

$$N_i(t) = \int_{t-\tilde{t}_i}^{t} W_i(F_i(q_i(\xi), \tau_i(\xi)))d\xi.$$

Finally, a_i is assumed to be zero during production and after satiation. Define a step variable:

$$\text{dum}(t_i^* - t) = \begin{cases} 1 & t_i - t > 0 \\ 0 & \text{otherwise} \end{cases}$$

to reflect this constraint. From the preceding discussion:

$$a_i(t) = F_i(q_i(t), \tau_i(t)) \text{ dum } (t_i^* - t) \; N_i(t).$$

The b process is a function only of the habits accrued through the use and disuse of the arousal state. Define:

$$b_i(t) = -H_i(t) \text{ dum } (t - t_i^*)$$

where $H_i(t)$ incorporates the entire stock of habits accumulated and dum is another step function with argument $t - t_i^*$.

Total habit is written as follows:

$$H_i(t) = h_i(t) \; (t - t_i^*) \; (t - t_i^* - t_i^{max})$$

where the last two factors reflect the fact that $b_i(t)$ must be zero at both time $t = t_i^*$ and time $t = t_i^{max}$. The term $h_i(t)$ counts past consumption. $h_i(t)$ weights and depreciates exponentially the influence of past production of A_i on habit:

$$h_i(t) = \int_{t-\tilde{t}_i}^{t} e^{-\delta_i \xi} F_i(q_i(\xi), \tau_i(\xi))d\xi$$

where δ_i is a constant depreciation rate.

CHAPTER 6

THE CONSUMER'S PROBLEM: DYNAMIC CASE

Models of activity production derived from opponent-process theory can be combined within the static consumption framework. When resource constraints are introduced, a model of dynamic consumption behavior is then formed. This model is developed and stated formally in this chapter. It is assumed that there are a finite set of planning periods for the consumer and that consumption in one period affects consumption in later periods. The solution to the dynamic problem is then similar to multiple replicates of the solution for the static problem. The complicating addition of the dynamic model is the habit mechanism and its associated signaling mechanism. This mechanism is studied for a simple example. The solution for the commodity demand functions resultant from the new optimal consumption plan are presented and their relationship with state adjustment demand models exposed. Finally, some of the extensions of the consumption models are presented as suggestions for further research.

The Dynamic Consumption Problem: Statement

Again, assume that I = N, that is, the number of arousal states is equal to the number of commodity inputs. Also assume that there exists an elementary time horizon T for all of the I = N arousal levels. The relative time constraint for each period is:

$$h + \sum_{i=1}^{N} \tau_i = T$$

over the course of the time horizon. The income constraint over each time horizon T is also the same as that used in the static model:

$$wT + Y = \sum_{i=1}^{N} (w\tau_i + p_i q_i).$$

Each arousal state is assumed to be the sum of its a and b states. Write: $A_i = a_i + b_i$ for all the N states. Here a_i and b_i are defined in the mathematical description in the preceding opponent-process section.

If there are Z periods for which the time horizon T applies to the agent, then his dynamic consumption problem may be written:

$$\text{Optimize:} \quad \sum_{i=1}^{Z} \left[\int_0^T O(A(\xi))d\xi\right]$$

$$q(t),\ \tau(t),\ h(t)$$

$$\text{Subject to:} \quad A_i = a_i + b_i$$

$$i = 1, 2, \ldots, N$$

$$a_i(t) = F_i(q_i(t),\ \tau_i(t))\ \text{dum}\ (t_i^* - t)\ N_i(t)$$

$$b_i(t) = -H_i(t)\ \text{dum}\ (t - t_i^*)$$

$$h(t) + \sum_{i=1}^{N} \tau_i(t) \leq T$$

$$wT + Y = \sum_{i=1}^{N} (w\tau_i + p_i q_i)$$

$$\tau_i \geq 0,\ h \geq 0,\ q_i \geq 0,\ w > 0,\ p_i > 0$$

$$i = 1, 2, \ldots, N$$

where z counts from 1 to Z the number of planning horizons and a_i and b_i are as defined in the last section.

Solution to the Dynamic Consumption Problem

Again, assume that the production of each activity state is subject to cost minimization. The solution to the problem is a dynamic plan or program consisting of a series of doses and redoses of activities. Activity is assumed to signal for redosage when the b process signals. Redosage will occur when higher ranked activity states are satiated or not signaling.

Figure 13 illustrates the solution plan for a simplified example where only three activity states are reached during the time interval T. At time t = 0 the agent uses time input τ_1^* to produce $\hat{A}_1^*$. From time τ_1^* to $\tau_1^* + t^*$ he is satiated in state A_1. Therefore, at time τ_1^* he may produce a satiated level of A_2^* using τ_2^* units of time. After state A_2 is satiated, it is assumed that the agent has time τ_3' to produce some A_3, but not enough time to satiate state A_3 before the b_1 process signals the consumer to redose state A_1. The agent returns to resatiate state A_1. Once this is complete, states A_1 and A_2 are satiated and the agent can produce more A_3 and uses τ_3' units of time in this production. Again, state A_3 cannot be satiated because state A_2 signals for redosage. At time horizon T, the consumer is satiated in A_1, redosing in A_2, and partially satiated in state A_3. These become the initial conditions for the next period's production plan and the production process begins anew.

More complex examples of the solution set may be considered using more than three states. Once again, the consumer's solution vector of maintenance levels of arousal will consist of a series of generally satiated levels of activity $\hat{A}_1$, $\hat{A}_2$, ..., $\hat{A}_{k-1}$, a possibly changing

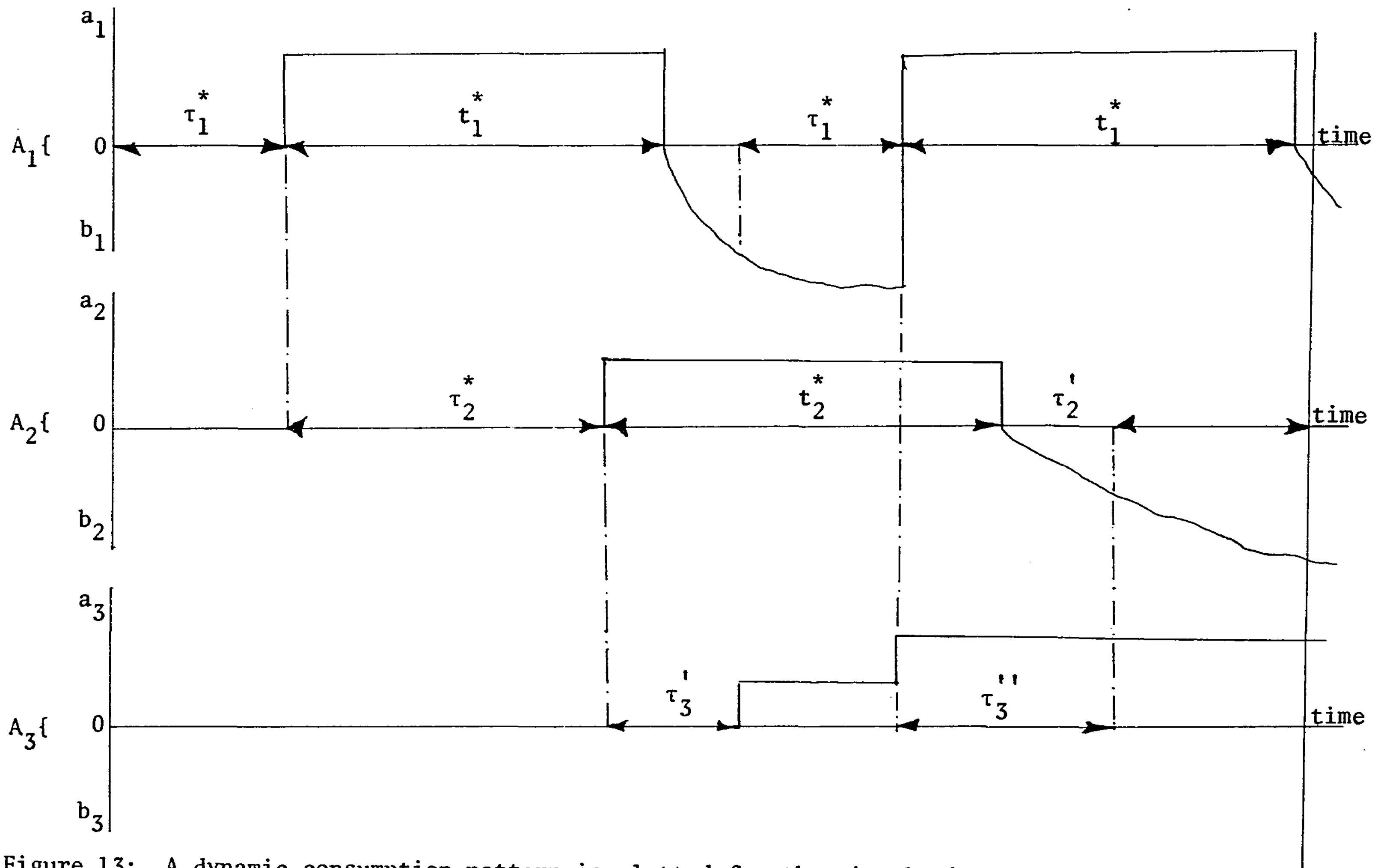

Figure 13: A dynamic consumption pattern is plotted for the time horizon T. The three highest ranked activities are considered.

marginal state A_k, and the remainder of states with zero levels of activity. The difference between this model's solution and the solution for the static case is that satiation levels may decay within a time horizon and they must be repeatedly redosed.

Commodity demand for a given period T and a given arousal state i is the sum of the uses of the commodity for redosage of activity during the period. Write:

$$q_i\Big|_T = \sum_m q_i(t)$$

where $q_i\Big|_T$ is the demand for q_i during period T and where m counts the number of times that q_i is used to produce A_i in the period 0 to T. If it is assumed that each of the productions are equivalent, then $q_i\Big|_T$ can be written:

$$q_i\Big|_T = mq_i(t)$$

where $q_i(t)$ is the amount of input used in each satiation production period. The counter function m is a function of the relative size of the satiation period t_i^* to the entire period T. If satiation period t_i^* is large relative to T, then redosage and usage of $q_i(t)$ will be less frequent. If t_i^* is small relative to T, then redosage will be more frequent. The counter function m is also a function of the strength of the habit function h_i during the occurrence of state A_i's b process. If habit is strong, that is, if past production levels of A_i were large, then h_i will be stronger. Physical production during each dosage period will remain the same as for the static model. Production, given the decision to redose, will occur exactly as production in the static model.

Combining the discussions about m and $q_i(t)$, write $q_i\big|_T$ as:

$$q_i^*\big|_T = m(t_i^*/T,\ h_i)\ q_i(wT + Y - \sum_{j<i} (w\tau_j^* + p_j q_j^*),\ w,\ p_i)$$

where t_i^*/T measure the satiation to entire period ratio. Here the counter function m is expressed as a function of satiation period to total period and the strength of habit. Note that q_i is exactly the same as for the static model; the function m simply counts the number of productions of q_i. Also, by similar discussion:

$$\tau_i^*\big|_T = m(t_i^*/T,\ h_i)\ q_i(wT + Y - \sum_{j<i} (w\tau_j^* + p_j q_j^*),\ w,\ p_i).$$

Choice of hours of work for the dynamic consumption problem involves an adjustment in the average marginal state so that on average, the average marginal activity lost from working more time is equal to the average marginal gain in activity due to the increased purchasing power of the agent. Again, writing:

$$h^* = T - \sum_{i=1}^{k} \tau_i^*$$

then, since each τ_i^* is a function of h_i,

$$h^* = h^*(T,\ w,\ p_1,\ p_2,\ \ldots,\ p_k,\ Y,\ h_1,\ h_2,\ \ldots,\ h_k).$$

Note that the hours of work decision is dependent upon <u>all</u> of the consumption habits of the agent.

Derived commodity demand equations incorporate all of the static aspects of arousal production and the repetitive aspects of production caused by habit. If the commodity demand equation is written in a collapsed form:

$$q_i\Big|_T = q_i(t_i^*/T,\ h_i,\ wT + Y - \sum_{j<i}(w\tau_j^* - p_j q_j^*),\ w,\ p_i),$$

then the psychological state variable studied by Houthakker and Taylor is identified as h_i. The horizontal aspects of observing consumption are also classified; if one wishes to study the demand for a particular commodity q_i with satiation period t_i^*, then it is necessary to choose a period of observation T sufficient to observe multiple redosages of q_i. For example, one should not choose T to be a day if q measures household purchasing decisions which occur over months or years of planning.

Comparative statics results for changes in w, Y, and p are identical in form to those studied during the analysis of the static problem. The commodity demand solutions for the dynamic case are multiples of the static solution. Therefore, the monotonicity of comparative statics results follows. Wage increases will cause substitution effects away from time intensive production and towards commodity intensive production. Non-labor income changes will have concentrated effects upon the purchases of the set of marginally aroused states. Price changes of commodities will have no effects upon the higher ranked states, substitution effects in the state whose price changed, and possible income effects in lower ranked states. Refer to the discussion of the static solution set's comparative statics results for a detailed discussion.

Some Extensions of the Model

One aspect of consumption ignored in the analysis is the fact that work effects may not be totally neutral. Different forms of work may produce positive or negative levels of satisfaction. A more general model of consumption can be developed if ranking of the job's activity can be determined relative to the ranking of consumption activities. Also, given that for each agent there is a selection of available jobs, a quantal decision rule may be used by the agent in selecting a job. An agent may trade off disutility of a job against higher pay, for example. Thus, a model of occupational choice using the framework of this paper can be developed. As an example, an agent who enjoys the activity of music may choose a lower paying job as a musician over a higher paying job as an accountant if music is high in his ranked preference set. This addition to the theory of occupational choice is left for further research.

The dynamic model of consumption developed also has applications in marketing theory. Recall that the dynamic solution involves intermittent productions of marginal activity. If a marketer wishes to increase purchases of products during periods of wage or non-labor income changes, then the model tells him that most of the action in terms of new purchasing will occur at these marginal activities. A theory of the psychology of buying can be developed by an examination of the growth of the b processes of the marginal state. Also, if there are novelty effects associated with marginal consumption activities, a marketer deciding to advertise in a period of consumption growth may wish to closely examine the underlying a processes of the marginal states where the consumption growth will be most intense.

Throughout this paper it has been assumed that the tastes, defined as the ranking over activity states, was fixed. If this assumption is relaxed, a theory of preference formation may be developed. In particular, if it is assumed that states are ranked according to the depth or severity of their b states,[35] then preference changes may occur as habits strengthen. For example, a marginal heroin user may redose and increase the intensity of his heroin activity state. As this intensity increases, heroin consumption may rise in importance over the other states, reaching a high level of rank as the agent becomes an addict. The process of drug rehabilitation involves either a gradual withdrawal or lessening of the power of the b process or a one period of disruption of the b process. Either of these causes drug activity to fall in rank and drug consumption to decrease.

A more common example of this form of preference change is that of automobile consumption. As automobile usage increased in the early part of this century, consumer's ranking of this activity was raised. Habits were formed. Today, many consumers have inherited automobile consumption habits which continue this pattern of high automotive ranking. Habits about automobile usage have been deeply engrained. Changes away from these patterns involve high psychological withdrawal costs as noted in this country during the late 1970's. A detailed analysis of such behavior in terms of changes in ranking is within the scope of the models developed in this paper.

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35. This observation is not the author's, but was a comment made by Lester D. Taylor about an earlier draft of the work.

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