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EMPIRICAL TESTS OF SOME EQUILIBRIUM STOCHASTIC PROCESSES OF THE TERM STRUCTURE OF INTEREST RATES

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Allen B. Atkins	Date 06/06/91
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To my Parents and the memory of my Grandparents

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ABSTRACT

In duration-based immunization models, the measure of duration is dependent on the assumed stochastic process of the term structure of interest rates. Most tests of immunization have been based on single factor duration models. However, specifications of term structure motions in earlier models permit the possibility of riskless arbitrage profits, as demonstrated by Ingersoll, Skelton and Weil(1978).

This paper draws upon the work of Bierwag (1987b) and presents some discrete equilibrium two-state one-factor model of the return generating process. These models are based on processes that give rise to some familiar measures of duration, namely the Fisher-Weil(FW) duration from the Fisher-Weil Equilibrium Process(FWEP), the special additive duration from the Special Additive Process(SAP), and the additive duration measure from the Additive Discrete Equilibrium Stochastic Process(ADEP). All of durations were initially based on disequilibrium processes. Therefore, measures of duration associated with disequilibrium processes can also be associated with discrete equilibrium stochastic processes and hence the criticism of these duration measures as being inconsistent with equilibrium is without merit in

this discrete time framework.

The focus in this work is to test empirically discrete equilibrium stochastic processes which give rise to the FW, SAP, and ADEP durations. The empirical approach consists of directly testing the implications of the models using Treasury Bill data and not the immunization efficacy approach utilized in most studies of term structure modelling.

Initial tests indicate no support for the SAP and the ADEP and partial support for the FWEP. Tests also indicate that the tax treatment of Treasury Bills has some effect on the sensitivity measures and may partly explain from initial regression tests the weak support for the FWEP model. The support for the FWEP varies with the choice of independent variables and errors in variables as a possible explanation is explored. Various techniques, including two non-parametric techniques, which attempt to overcome the bias induced by errors in variables are explored. Overall, additional tests using these techniques strongly support the FWEP.

Chapter I

Introduction and Literature Review

1. INTRODUCTION AND LITERATURE REVIEW:

Investors in default- and option-free fixed-income securities face two types of interest rate risks: reinvestment risk and price risk. A change in yield over the planning horizon affects the reinvestment rate on intermediate cash flows, if any, received from fixed-income securities. The uncertainty associated with reinvestment return on intermediate cash flows is called the reinvestment risk. A change in the yield also affects the value of a fixed-income security, and this immediate effect is called price risk.

A given fixed-income security may not have any reinvestment risk if it does not offer any intermediate cash flows before its maturity. A zero coupon bond is an example of a fixed income security with no reinvestment risk. Investors in fixed-income securities with intermediate cash flows face reinvestment risk. The price risk of a fixed-income security is related to the planning period of the investor. If the maturity of the security is equal to the planning period then there is no price risk since the price of security at maturity is equal to the promised face value. If the maturity of the security is greater than the planning period then the investor faces price risk as the price of the security at the end of the planning period is a function of the market yield at that time.

Over time price and reinvestment rate risk have offsetting tendencies. A rise in the yield will decrease the value of a bond but will provide an opportunity to invest coupon payments received in the future at the higher rate. The reverse is true when the yield declines. The time required for the reinvestment cash flows to offset any initial capital gain or loss is of utmost importance in the management of interest rate risk.

Investors, in principle, can eliminate price and reinvestment risk if they are able to purchase zero coupon securities with maturities equal to their planning periods. An investor with a specific time pattern of planned outflows can buy a portfolio of zero coupon bonds with payments matching the time and size of his planned outflows. In recent times, the availability of zero coupon bonds with a wide range of maturities makes possible the avoidance of interest rate risk for some investors. These zero coupon bonds are created by investment banking firms by "stripping" coupon bonds issued by the Treasury. The zero coupon bonds so created can be considered default-free given that the payments are funded by the underlying Treasury coupon bonds. Some of these zeros created by investment banking firms are known as CATS, LIONS, and TIGERS. However, the variety of zeroes available is not yet sufficient to match the planning periods for all investors. Moreover, the term structure of the zero coupon bonds may be different from the one for coupon bonds. This may induce certain investors to consider the coupon bonds if their yields are more attractive relative to the zeroes and if they can devise strategies to manage interest rate risk.

Many techniques have been developed to protect investments in default- and option-free bonds against interest rate risk. The concept of "duration" arises in the development of these techniques. Duration is measured in units of time. Duration, originally defined by Macaulay(1938), was originally used as a measure of the average life of an income stream. Hicks (1939), Samuelson (1945) and Redington (1952) independently developed similar measures of the average life and found new uses for the concept. Redington coined the term "immunization" to capture the notion of hedging a balance sheet by equating the Macaulay duration of the assets with the duration of liabilities. He suggested that hedging could enable institutions, with yield sensitive assets and liabilities, to immunize their net worth against interest rate risk. All of these measures of duration discussed above were based on a flat term structure that was assumed to remain flat.

Immunization, broadly speaking, is the protection of the value of a fixed-income portfolio at a specified date in the future, against changes in value resulting from interest rate changes. The objective of immunization is to balance off the price risk and reinvestment risk of a portfolio so that its yield over the planning period is not less than its promised yield. This, in principle, can be achieved by equating the the duration of the cash flows of a portfolio to be immunized to the planning period. The measure of duration specified is a function of the assumed nature of the term structure shifts. In this context the duration of a portfolio is equal to the time it takes for the price risk and the reinvestment risk to offset each other.

Fisher and Weil(1971) (henceforth FW) were the first to test empirically the effectiveness of duration in the management of interest rate risk. They were also the first not to assume a flat term structure and the first to acknowledge some of the problems in building models that describe shifts in the term structure of interest rates over time. FW assume a single source of uncertainty that affects bond returns. Their model is set in a continuous time, continuous compounding framework and additive random fluctuations in the term structure of interest rates is the source of uncertainty in their model.

However, the measure of duration derived in their paper can also be derived within a discrete time framework. The nature of the stochastic shock is different in the discrete time framework. Bierwag (1987a) has a derivation of the FW duration in a discrete time, non-continuous compounding framework, and the shifts in interest rates are a combination of additive and multiplicative shifts. However, both shifts are related to a single source and, therefore, the model is of the one-factor variety.

Fisher and Weil(1971) establish that a portfolio with duration equal to the planning period guarantees a return that is at minimum the return that can be earned on a zero coupon bond with maturity equal to the planning period. Recall that a zero coupon bond with maturity equal to the planning has no price risk and it also has no reinvestment risk since there are no intermediate cash flows. The Fisher-Weil duration measure, in discrete terms, is similar to the Macaulay duration except that it allows for different discount rates in computing discount factors for each cash flow.

To examine empirically the usefulness of immunization in managing interest rate risk, Fisher and Weil conduct simulations over planning periods of five, ten and fifteen years. Hypothetical bonds are constructed based on default-free term structure estimated from low risk corporate bond data. Immunized portfolios for each planning period contain two bonds, one with maturity equal to the planning period and the other with the largest possible duration. The duration of an immunized portfolio is set equal to the planning period and the portfolios are re-balanced every period so that the duration is equal to the length of the time remaining in the planning period.

The performance of these immunized portfolios is compared to that of a naive strategy of rolling over twenty year bonds, and another of choosing a bond whose maturity is equal to the length of the planning period. The performance measure used is the wealth ratio. The wealth ratio is constructed by dividing the value of the portfolio for each strategy at the end of the planning period to the value initially invested. This ratio for each strategy is then compared to the expected wealth ratio which is the value of the portfolio at the end of the planning period which has accumulated according to the forward rates in the initial term structure divided by the value initially invested. This is done for all three planning periods.

The effectiveness of each strategy is determined by computing the standard deviation of the difference between the actual wealth ratio and the expected wealth ratio. A completely successful strategy should produce a zero standard deviation. The naive strategy has the highest standard deviation, followed by the maturity strategy. The duration strategy has the lowest standard deviation. This is so for all three planning periods. They conclude that the duration strategy, though not completely successful, is more successful than the strategy of maturity matching in managing interest rate risk. They also consider the higher transaction costs, for the duration strategy, necessitated by the need for re-balancing. The results stated earlier are not materially affected by the transaction costs.

Tests of some duration-based immunization models require a knowledge of term structure of interest rates for default- and option-free fixed-income securities. In practice, one needs to measure the term structure as zero coupon bonds beyond maturity of one year, which are not generally issued by the Treasury. The term structure is estimated using coupon bond data. Techniques to measure the zero coupon term structure using coupon bonds have been developed by McCulloch(1971,1975), Carleton and Cooper(1976), and Houglet(1980), and many others.

In general the duration measure derived in immunization models depends on the stochastic process assumed for the term structure of interest rates. Definitions of duration under the assumption of other stochastic processes have followed since the work by Fisher and Weil. Bierwag (1977) specifies an additive stochastic process where the instantaneous shift in the term structure is additive in nature. The new term structure is given by adding a random variable to the current term structure. He also specifies a multiplicative stochastic process where the new term structure is the product of a random variable and the current term structure. The value of this random variable and the slope of the term structure determine the behavior of short-term and long-term interest rates in this process. The multiplicative model can account for the observed greater variability of short-term rates. He also notes that some stochastic processes will permit immunization and some will not.

Khang (1979) specifies two stochastic processes, log-additive and log-multiplicative, which allow instantaneous movements in short-term rates to exceed long-term rates. The instantaneous movements in these two processes are term dependent and therefore the movement in the term structure at various points may be related to the term to maturity at those points. The formulation of measures of duration for the stochastic processes discussed above is more difficult. Bierwag, Kaufman, and Toevs(1983) contains the expressions for durations related to the above mentioned stochastic processes. It also contains expressions for the duration of a portfolio and for the empirical evaluation of the immunization efficacy of the stochastic processes discussed above.

The stochastic processes discussed above can be used to construct bond portfolios to immunize against interest rate risk. The portfolio compositions are dependent on the underlying stochastic process specified and therefore for the same planning period, the portfolio composition generally would be different for each of the stochastic processes discussed above. The success of the immunization strategy depends on the validity of the underlying stochastic process. A mis-specified stochastic process can fail to immunize against interest rate risk. The mis-specification of the stochastic process is referred to as stochastic process risk.

Immunization oriented tests have been carried out for the above stochastic processes using data for common time periods. Bierwag, Kaufman, Toevs, and Schweitzer (1981) conduct tests on the additive, multiplicative, FW, log-multiplicative, Macaulay processes and then compare the results with maturity, roll over and long bond strategies. Their results show that , in general, the immunization strategies generate returns that are closer to the promised yield than the maturity strategy and other non-duration based strategies. The Macaulay duration strategy workes as well as or better than sophisticated strategies like the additive and the FW strategies. All immunization strategies fell short of the promised rate of return on many occasions suggesting that the true stochastic process generating the movement in the term structure of interest rates is not consistent with the stochastic processes used for immunization in their paper.

Babbel (1983) discusses one of the uses of duration, immunization, which attempts to "lock in" prevailing market yields over the planning period irrespective of the future course of interest rates. Although the success of an immunizing strategy, in "locking in" yields does not require correct forecast of interest rates, the success of an immunizing strategy does depend on selecting the appropriate immunizing duration. The appropriate immunizing duration requires some knowledge of the form of the movement of the term structure of interest rates over time. This movement of spot rates over time may be related to a specific factor or factors. Babbel considers a factor that he calls the term structure of interest rate volatility (TSIRV). He notes that in deriving duration measures customarily some simplifying assumptions on TSIRV are employed. He further notes that most duration measures are developed on the basis of mathematical tractability and that the TSRIV implicit in the development has little to do with thè TSIRV in reality.

He develops some generalized duration measures which do not restrict TSRIV to any particular shape. These measures permit the possibility of long rates, moving in a direction that is the opposite of the one-year spot rate, or long rates that stay stable given changes in one-year spot rate. However, the relative volatility between one-year spot rate and other spot rates is deterministic in Babbel's development.

Employing McCulloch's (1975) procedure for estimating the term structure of interest rates, Babbel computes the relative volatility of a t-period spot rate to the one period spot rate. The relative volatility is the slope of the regression of the percentage change in the t-period spot rate on the percentage change in the one period spot rate. One set of relative volatilities is computed utilizing generalized least square regressions and the other set of relative volatilities utilizing ARIMA procedure. However, the relative volatilities used in his paper are based on the technique of generalized least squares.

The bond price data used are from January 1947 -January 1980. There are two broad categories of tests, denoted as "no look-ahead" and "look-ahead". In the "no look-ahead" category, the immunizing duration is set equal to the remaining length of the planning period. The "look-ahead" category sets the immunizing duration such that the expected duration one period from now will be equal to the remaining length of the planning period. The "no look-ahead" strategy is widely employed in the literature and he dubs it as "closing the barn door after the fox has eaten the chicken."

For each category, the strategies are maturity, Old duration(here the relative volatility between spot rates is set equal to 1), and the new duration which incorporate relative volatilities gleaned from the past data. Tests show a ten percent failure rate in achieving the promised yield for all three strategies. This is true for both the "look-ahead" and the "no-look ahead" categories of tests. However, the failures for the maturity strategy are on an average ten times bigger than the duration strategies. This is also reflected in the semi-standard error. The new duration strategy does not outperform the old duration strategy. Babell speculates that these results could change if more sophisticated methods were employed to model the TSIRV.

Ingersoll, Skelton, and Weil(1978) have noted that an immunized portfolio is constructed in the same manner as a hedged portfolio in option theory. Therefore, the problem of immunization is also the problem of asset replication. They also note, along with Cox, Ingersoll, and Ross(1979), that stochastic processes specified in most single-factor duration models are inconsistent with general equilibrium. Typically, in these disequilibrium models the return on an immunized bond portfolio is greater than a zero coupon target bond when there is a shift in interest rates and it is equal to the zero coupon bond return when there is no shift in interest rates. This creates opportunities for riskless arbitrage profits. This can be done by issuing zero coupon bonds or selling them short and using the proceeds to construct an immunized portfolio. Although some of the processes may be inconsistent with equilibrium conditions, the durations derived from these processes and used to formulate particular bond portfolio strategies can also be derived from equilibrium processes, as shown in Bierwag(1987b). Thus, in practice, the criticism that the durations utilized come from disequilibrium processes is empty.

Nelson and Schafer(1983), in proposing a model of the bond-return generating process, follow the asset replication approach. They develop an equilibrium model where the term structure of interest rates may be modeled by a multi-factor process. The price of each bond is sensitive to these factors. The factors employed are interest rates of particular maturities. To test their model, they employ the "tracking" approach. In the tracking approach the return on a portfolio of bonds should be equal to the return on the portfolio it replicates for each period of time, if the factor sensitivities of the two portfolios are equal.

In conventional tests of immunization models, one compares the return on an immunizing portfolio of bonds to a target zero coupon bond, for a given planning period. Nelson and Schafer take a different approach. They choose a bond on which raw price data is available and then attempt to replicate the bond. This overcomes the problem of term structure estimation error inherent in conventional tests of immunization, which require knowledge of term structure estimates in order to re-balance the portfolios during the planning period. Furthermore, they note that since the prices of bonds are observable, it is possible to measure the differences in value between the target bond and the immunized portfolios. They contend that this should lead to more reliable measure of the errors associated with different strategies.

Note that they do utilize term structure estimates to measure sensitivity of bonds to factors. They utilize Schafer's (1981) linear programming approach specific to a zero tax bracket for estimating the term structure for the period 1921-1979. They also use McCulooch's(1975) tax adjusted term structure for the period 1946-1971. Both sets of term structures utilize data provided by Center for Research in Security Prices(CRSP). All tracking tests are conducted using the sensitivity measures derived from Schafer's term structure estimates. They report that factor sensitivities are similar for both sets of term structure estimates.

In the spirit of Fisher and Weil(1971), they specify five, ten, and fifteen year bonds as target bonds. As noted above, these are not hypothetical pure discount bonds and prices are observable for these target bonds. The tests are conducted over the period, 1930-1979.

The five strategies employed are: 1. Maturity strategy 2. FW duration 3. Short rate 4. Long rate 5. Long rate/spread. The short rate strategy is a single factor strategy where the factor sensitivities of different bonds are calculated with respect to the one-year rate. In the long rate strategy, the sensitivities are calculated with respect to the thirteen-year rate. In the long rate/spread strategy, the sensitivities are calculated with respect to the thirteen-year rate and the spread between the thirteen- and five-year rates.

Unlike conventional tests, the target bond's maturity is kept constant in the simulations. Every period, a new target bond is chosen with maturity equal to the target period and the immunizing portfolios are reconstructed such that characteristics match those of a target bond.

Broadly speaking, there is very little difference in the performance of any of the strategies, including the maturity strategy, except that of the short rate strategy. The short rate strategy performs poorly for all three target maturities over the fifty-year period. The standard deviation of the difference between the returns for the target bond and the immunizing portfolios is used as the performance measure. Overall, the conventional strategy does better than the long rate strategy. They also explore the effect of the holding period length on the performance of various strategies. Two holding periods are considered, one- and six-month. The six-month holding period reduces the estimated variance of the difference by an order of about 50% for all strategies. They ascribe the better results under the six-month holding period to less noise, because the measurement error in the CRSP data has a smaller impact when the period over which returns are computed increase.

The two-factor strategy, tested only for the ten-year planning period, has a standard deviation slightly higher than the single-factor long rate strategy. This is surprising since, as they note, two-factor models are better at explaining term structure movements, as documented in their paper, than the one factor models. This indicates that even if multi-factor models can better describe the term structure of interest rate movements, it does not follow that multi-factor models will do a better job in tracking the target bond.

Brennan and Schwartz (1983) note that duration theory starts from an a priori definition of the stochastic process governing the term structure of interest rates and no check is made as to whether the stochastic process is consistent with equilibrium. They propose a two-factor equilibrium mode in a continuous time framework. Their objective is to compare the results from immunizing a portfolio using the Fisher-Weil duration to immunity (hedging) according to a two-factor model.

Their approach is similar to the one taken by Nelson and Schafer. They measure the sensitivity of a bond to two factors and use these sensitivity measures to replicate a target bond. Noting that portfolios of bonds with similar factor-sensitivity should generate similar returns, they simulate a hedging strategy.

They use the CRSP Data for December 1958 - December 1979. The two-factors are the return on a 30-day Treasury Bill and the return on a highest yielding Treasury Bond with maturity exceeding twenty years.

Two distinct types of tests are carried out for both the FW strategy and the two-factor strategy. In one of the tests, a hypothetical zero coupon, five-year bond is constructed and the wealth ratios from both the duration and two-factor hedging strategies are computed. The wealth ratio used is the accumulated investment over a planning period divided by the initial investment. The hedging strategy provides a lower root mean square error of the wealth ratios indicating that the hedging approach is able to get closer to the expected wealth ratio more often than the immunization approach. For these types of tests, monthly term structures are estimated using the technique of fitting a cubic spline, as suggested in McCulloch (1971).

In the second type of test, a real portfolio of coupon bonds, which they call the basic portfolio and for which prices are available, is replicated by other bonds for both the duration and hedging strategy. Tests are conducted over different maturities for the basic portfolio, where maturities range from 1 year to 20 years. The difference in the return for the replicating portfolios are computed for each month. The standard deviation of the differences in monthly returns is used as the performance measure. Note that this approach does not require term structure estimates and therefore avoids the estimation error when coupon bonds are used to infer the zero coupon term structure. The standard deviation of the returns for both strategies are very similar and the superiority of two-factor model over the duration method developed earlier, when zero coupon target bond is used, does not hold.

Ingersoll(1983) utilizes CRSP data in tests of three immunization strategies, namely the bullet, barbell, and ladder. These strategies are compared to the maturity strategy, and the naive strategies of investing in a two period bond and investing in a 10 period bond. There are two broad categories of tests. One category utilizes the yield on a five-year zero coupon bond as the promised yield, constructed using the term structure estimates generated by following the methodology of Houglet(1980). The other category of tests takes the expected yield on a bond having a five-year Macaulay duration. These tests are over the period 1950-1979.

Overall the maturity strategy immunizes as well as the duration based immunization strategies for both categories of tests. The performance measure utilized is the Root Mean Square Difference(RMSD) of the difference between the actual yield and the target yield. The unusual result has been attributed, by some, to the term structure estimation technique utilized in the paper. Others have noted that it is one of the few papers where the immunizing portfolios did not include the maturity bond as one of the bonds.

Ingersoll also tests a two-factor model and achieves lower RMSDs of the difference between the actual yield and the target yield. The two factors are the instantaneous spot rate and a continuously compounded forward rate. This, he concludes, indicates that multi-factor models may provide a practical solution to immunization. Ingersoll notes that his and the Nelson-Schafer two-factor models are ad-hoc models and allow for the possibility of riskless arbitrage profits. He further notes that this criticism does not apply to the two-factor model developed by Brennan and Schwartz. He also notes that only his two-factor model substantially improves upon the one-factor model and attributes it to the use of artificial bonds constructed from term structure estimates. Nelson-Schafer and Brennan-Schwartz utilize real prices in their tests.

Bierwag, Kaufman, Toevs and Schweitzer(henceforth BKTS) (1981) utilize Durand data for the period 1925-1978 to test the efficacy of five duration based passive strategies and some non-duration active strategies for interest rate risk management. This study uses annual interest rates and transaction costs and taxes are assumed to be zero. A ten-year planning horizon is utilized for all tests.

The five duration based strategies utilized are the Macaulay(ID1), Additive Stochastic Process(ID2), Fisher-Weil(ID3), Log-multiplicative process with α=0.1(ID4,α=0.1) and log-multiplicative process with α=1(ID4,α=1). All five duration strategies contain two bonds, a twenty year bond and a bond with maturity equal to the remaining length of the planning period. The five passive strategies will assign different weights to these two bonds so that the immunizing duration of each strategy is equal to the remaining planning period. The portfolios are re-balanced annually. Note that duration in general declines less than the maturity of a bond.

The three active strategies are: 1. a roll over strategy, where all funds are invested in a one year bond and the proceeds are rolled over every year into a one-year bond, 2. a long-bond strategy, where all the funds are invested in a twenty year bond and intermediate cash flows are reinvested in the same bond, 3. a maturity strategy, where the funds are invested in a 10 year bond and the intermediate cash flows are invested in the same bond.

Tests are conducted over the period, 1925-1978 as well as over the sub-periods 1925-1949, 1940-1963, and 1954-1978. The tests over sub-periods reveal some properties of robustness for different strategies. For the overall period, ID1-ID4(0.1) strategies returns are closer to the promised yield than the returns of the maturity strategy about 85% of the time. The ID4(1.0) does not do well since it gets closer to the promised yield, relative to the maturity strategy, only about 50% of the time. It appears that the simple Macaulay strategy performs as well as the more sophisticated strategies, ID2-ID4(α=0.1). The roll-over and the long-bond strategies perform very poorly relative to all other strategies.

They show that the results are sensitive to the composition of the portfolios. If one- and twenty-year bonds are utilized for immunization strategies, then all duration strategies do worse than the maturity strategy in getting closer to the promised yields. This is an interesting result since, in principle, one would expect different portfolios with the same duration to do equally well. Note that the better results from the immunizing strategies are obtained when the portfolios include a bond with a maturity equal to the remaining period. This issue is addressed later.

Bierwag, Kaufman, Toevs(1982b) also test the strategies utilized in the previous study. However, the term structure estimates from Babell(1981), which use CRSP data, are utilized. They also present results based on the Durand data for comparison. Further, a five-year planning period is used.

Tests show that the duration based strategies, with the exception of ID4(α=1) do better than the maturity strategy in getting closer to the promised yield for both the CRSP data and Durand data. This is in contrast to results reported by Ingersoll(1983), where the maturity strategy does as well as the duration strategies using CRSP data. Therefore the inference by Ingersoll that the differences in the results between his study and that of Fisher-Weil is due to the use of Durand data is not supported. However, the difference in the results of BKT(1982b) and Ingersoll(1983), both utilizing CRSP data, may be due to the different techniques utilized to estimate the term structure of interest rates.

Bierwag, Kaufman, Toevs(1983) test three new duration based strategies along with the other strategies discussed in BKTS(1981). The three additional strategies are the generally additive stochastic process(GASP), denoted IDG, as developed by Bierwag, Kaufman, Toevs(1982a), and the log multiplicative process where the immunizing portfolio is constructed with a twenty-year long bond and a ten-year bond and is denoted as ID4*. and where α for this strategy is estimated empirically. The third strategy is similar to ID4* except that the immunizing portfolio contains a one year bond and a twenty year bond and is denoted as ID3*. Note that the IDG also requires knowledge of α, a parameter in the process, as does ID4* and is consistent with general equilibrium.

The important result from these tests is that the IDG immunizes better than the maturity and the other duration based strategies. Further, IDG* does better than the ID4(α=0.1) and ID5(α=1.0), suggesting that the use of an empirically estimated volatility of short-term rates relative to the long-term rate helps in getting closer to the promised yield, compared to a priori specifications. The poor performance of ID3*, which has a barbell portfolio composition, indicates the importance of including the maturity bond in the immunizing portfolios.

They further explore the impact of portfolio composition on immunization strategies by simulating an additive stochastic process and assuming the Khang process with α =1.0. The strategies with the maturity bond in the immunizing portfolios do better than the strategies without the maturity bond. This indicates the importance of portfolio composition in the immunization strategies given the possibility of stochastic process risk.

Bierwag, Kaufman, Toevs(1982a) is an early development of a generally additive stochastic process consistent with equilibrium. This is set in a discrete time framework and addresses one of the deficiencies of Single Factor Duration

Models(SFDM). This paper provides a theoretic base for SFDM which do well in immunization simulations.

Bierwag, Kaufman, Latta(1987) note that immunizing durations are a function of the assumed stochastic process and failure to identify correctly the stochastic process results in stochastic process risk. Stochastic process mis-specification arises when the number of factors are not correctly identified or when the stochastic process itself is mis-specified or both. They conduct tests of the SFDM(Fisher-Weil), two-factor duration models and the maturity strategy using Bank of America U.S. Treasury security data maintained by DRI/McGraw Hill for the period 1975-1985.

Overall, the SFDM outperforms the maturity strategy. For the five-year planning period the SFDM outperforms the maturity strategy 100% of the time. For the two and one half-year planning period the maturity constrained SFDM outperforms the maturity strategy 100% of the time. Overall,the two-factor duration models outperform SFDM for both planning periods. They conclude that the success of the immunizing models can be enhanced by increasing the number of factors, but the improvement is at a decreasing rate and must be weighed against the cost of implementing these strategies.

Bierwag and Roberts(1990) conduct tests on an equilibrium single factor model, namely the Fisher-Weil

Equilibrium Process(FWEP), on Canadian bond data for the period, January 1963-August 1986. The model generates theoretical values and these are compared to empirical estimates for the model. The overall results support the FWEP and the results are robust with respect to changes in the reference security, although there is evidence that the coefficients of the model shifted significantly during the 23 year period.

Chambers, Carleton, and McEnally(1988) utilize a duration vector to immunize against interest rate risk. The duration vector approach is a multi-factor approach. The duration vector approach expresses the term structure as a function of uncertain factors. This approach also gives the Macaulay duration when only one-factor is specified. The model is tested using data on default-free Treasury Notes, Bonds, and Bills collected from Wall Street Journal. Overall, nine strategies are tested. One of the strategies is to constrain the duration vector to be a scalar and is the traditional Fisher-Weil duration strategy. This is denoted as D1. The other immunizing strategies simply add higher order durations. The immunizing duration vector with two durations is denoted as D1-D2, and with three durations as D1-D3 and so on up to D1-D7. These seven duration strategies along with the maturity strategy and the naive strategy are tested.

Overall, the tests show that all duration strategies get closer to the promised yield than the maturity and the naive strategies. The sum of the squared deviations measure for the duration strategies tends to decrease as more durations are added. The sum of squared deviation for the maturity strategy is 0.00169 and for the D1-D7 strategy is 0.00001, which is a substantial improvement over the maturity strategy. They note that not much is to be gained by having a duration vector with more than four elements since for D1-D4, the sum of squared deviation is 0.0003 and is not much greater than for D1-D7. These models may have a limited practical use because to satisfy some of the higher order duration conditions for immunization it is necessary that some bonds be sold short and the capacity to do that in today's market is very limited.

As observed earlier, portfolio design plays an important role in the success of the duration-based strategies over the maturity strategy. Fong and Vacicek(1983,1984) have addressed the role of portfolio design in the success of immunization strategies. They argue that two different portfolios with the same immunizing duration will protect against interest rate risk only if the stochastic process assumed in deriving immunization conditions is consistent with the actual process. If the assumed stochastic process is inconsistent with the actual process, then these two portfolios may provide different degree of success in immunizing against interest rate risk.

They derive a measure called M^2 and denote it as a measure of immunization risk. M^2 is similar to the measure of duration and is the weighted time-to-payment variance around the horizon date. They state that portfolios of deep discount bonds with maturities close to the horizon date(bullet portfolio) are likely to have M^2 lower than portfolios with maturities and cash flows that are farther away from the horizon date(barbell). Further, M^2 is non-negative and attains its lowest value of zero if and only if the portfolio consists of one zero coupon bond with maturity equal to the length of the planning period.

Fooladi and Roberts(1988) explore the role played by portfolio design in the success of duration-based strategies. They note that the widely cited study of Ingersoll(1983), where it is found that the maturity strategy works as well as the duration strategies, is the only study where the immunizing portfolios did not include a maturity bond. They further note that based on Fong and Vasicek's work, one would expect the immunizing bullet portfolios with the lower M^2 to perform better than immunized barbell portfolios and immunized ladder portfolios with or without the maturity bond. By performing better the immunizing bullet portfolio would more closely achieve the returns for a zero coupon bond for the period tested. They also note that the empirical work for the most part would suggest that portfolios with the maturity bond do better than the immunizing bullet portfolios without the maturity bond. This is in contrast to the prediction by Fong and Vasicek.

Fooladi and Roberts conduct tests of various strategies discussed above utilizing Canadian government bond data for the period, January 1963-August 1986. Tests are conducted using both actual bond prices and using zero coupon prices from fitted term structure. Their major conclusion is that the immunized portfolios with maturity bond, maturity barbell and maturity bullet outperform the bullet strategy in getting closer to the promised yield, in contrast to the theoretical predictions implicit in the work of Fong and Vasicek.

Bierwag, Fooladi and Roberts(1988) show in an analytical framework that the minimum M^2 portfolio is not necessarily the bullet portfolio as suggested by Fong and Vasicek. Further, minimizing M^2 may not be appropriate in dealing with stochastic process risk if a certain technical property of M^2 with respect to duration fails to hold. Empirical tests show that the minimum M^2 portfolio, which need not include the maturity bond, fails to outperform the bullet and barbell strategies constrained to contain the maturity bond. This calls into question the usefulness of M^2 in dealing with stochastic process risk.

Duration can also be used to approximate the change in price due to a term structure shift. Bierwag (1987b) uses duration to approximate price changes in bonds given term structure shocks. Using simulated data, it is shown that Macaulay duration works as well as other durations based on more elaborate specification of term structure movements. The use of more sophisticated modelling of the term structure movements has, on an average, not led to better bond price approximations. However, in most empirical tests the a priori stochastic process is assumed to apply to all time periods of the study and no allowance is made for the specification changing through time.

Ingersoll, Skelton, and Weil (1978) were the first to demonstrate that the Fisher-Weil and other stochastic processes developed thus far were disequilibrium processes. If the above processes were in effect, it would be possible to acquire riskless profits by engaging in arbitrage. Bierwag (1987b) has recently shown that a duration measure derived from a disequilibrium process,could also be derived from an equilibrium process. There does not exist a one to one correspondence between duration measures and the underlying stochastic processes, and therefore the Ingersoll-Skelton-Weil result does not hold in general.

Many have questioned the need for focusing on an immunization strategy since it may imply that investors are strongly risk-averse. As Nelson and Schafer(1983) state

> "One of the deficiencies of the immunization literature is the absence of a convincing explanation of why investors should wish to immunize. It is almost entirely a "how-to-do-it" literature. In the case of a consumer, immunization would be optimal only if he displayed zero risk tolerance".

and quoting Sharpe(1983)

> "A serious aspect of the immunization literature is its basic premise. In most case the approach is consistent with the maximization of a utility function having as an argument the net worth of an organization or individual at a single time. Moreover this utility function in effect exhibits zero risk tolerance(infinite risk aversion).

and quoting Jozsa(1983), a practitioner,

> "While I would not advocate a maturity strategy(except in the case of a computer failure), my point is that we may be splitting hairs in an effort to improve upon the preciseness of duration, particularly for all government portfolios. I personally doubt the significance of improving the predictive value from 18 basis points to 7 basis points."

The criticisms quoted above are addressed in Bierwag(1987a). For the purpose of this thesis, the goal of immunization is taken as a given.

Duration also plays a role in the concept of contingent immunization where the portfolio manager is expected to provide a return at least equal to some base but is free to take some risk so as to attempt to achieve a return greater than the promised yield on a target maturity. Another application of duration is in the are of "Duration Gap" management, which is essentially protecting the net worth of institutions with interest rate sensitive assets and liabilities. Both these topics are discussed in detail in Bierwag(1987a).

Chapter II

Discrete Time Single Factor Models

2.DISCRETE TIME SINGLE FACTOR MODELS:

In discrete time single factor models, there is only one source of uncertainty that affects the returns (or prices) on each security. The "complete" versions of these models are a one period two state models in which only two possible outcomes can result one period later. These outcomes can be thought of as term structures or as discount functions or prices to arise one period later.

Let $P(0,t)$, $t = 1,2,....$, be the currently observed price on a security which promises one dollar at date t. The 0 in the notation above denotes the current time at which the price of a t period security is observed. By restricting the payoff on these securities to $1 at the end of the period, these prices are equivalent to discount functions. The term structure can be derived from these given prices. Forward discount functions or prices are denoted by appropriately specifying the first number in the parenthesis of P( ). As an example, $P(1,t)$ denotes the forward price or the discount function, implied by the currently observed term structure, for a t period security at date 1, when there are t-1 time periods left to maturity.

Let $P_s(1,t)$ be the price of a t period security, one period later, if state s occurs, where $s = 1,2$. This captures the two state assumption of the model. Furthermore, assume that $P_s(1,1) = 1$ for both states. The one period security thus pays \$1 in each state and is consequently a risk free security.

The return on a t-period security is then

$$R_{st} = \frac{P_s(1,t)}{P(0,t)} \quad . \tag{1}$$

Here, R_{st} is the one period return per dollar and is equivalent to

$$R_{st} = (1+r_{st}) \tag{1.1}$$

where r_{st} is the one period rate of return on the t-period security. A loss on the investment will be reflected by a negative rate, $r_{st,}$ which means that a loss in terms of R_{st} is reflected by a number less than 1. A zero rate of return is reflected by $R_{st} = 1$ and a positive rate of return is reflected by a value of $R_{st} > 1$.

The price $P_s(1,t)$ for a t-month security one period later can be expressed in terms of the current forward price of a t-month security that is subject to a stochastic movement. Thus,

$$P_s(1,t) \;=\; P(1,t) * g(s,t) \qquad (2)$$

$$t = 1,2,\ldots\ldots, \quad s = 1,2$$

where P(1,t) is the forward price or the discount function implied by the currently observed prices, and g(s,t) is a stochastic variable whose value depends on the state to materialize and the maturity of the security under consideration. In this expression, g(s,1) = 1 for s=1,2, and P(1,1) = 1. In this expression P(1,t) is the forward price which would be observed one period later if a version of the pure expectation hypothesis were to hold. The forward price P(1,t) can be expressed in terms of the currently observed prices as

$$P(1,t) = \frac{P(0,t)}{P(0,1)} \quad . \qquad (2.1)$$

Let h(1,t) be the forward rate in a continuous compounding context and h(0,1) be the current rate for a one period security and h(0,t) for a t-period security. This permits prices or the discount functions in terms of the rates.

In the equations that follow, e is the exponential function. Re-writing components of equation (2.1) in terms of discount function yields:

$$P(1,t) = e^{-h(1,t)(t-1)} \tag{3.0}$$

$$P(0,t) = e^{-h(0,t)t} \tag{3.1}$$

$$P(0,1) = e^{-h(0,1)} \tag{3.2}$$

and

$$P(1,t) = e^{-h(1,t)(t-1)} = \frac{e^{-h(0,t)}}{e^{-h(0,1)}} = \frac{P(0,t)}{P(0,1)} \tag{3.3}$$

where P(1,t) is the price on a t-period security one period later if the current forward rate h(1,t) turns out to be the actual rate in the future. Rearranging (3.3) we have

$$e^{-h(1,t)(t-1)} = e^{-h(0,t)} \; e^{h(0,1)} \tag{3.4}$$

and taking natural log and simplifying,

$$h(1,t) = \frac{h(0,t) - h(0,1)}{(t-1)} \tag{3.5}$$

The forward rate for a t-period security, one period later, can be readily computed, given the current rates on the t-period and the one-period securities. For the one-period security to be riskfree g(s,1) must equal 1 for both states. Substituting equation (2) into (1) , the return for a t period security can be expressed as follows:

$$R_{st} = \frac{P(1,t)\ g(s,t)}{P(0,t)} = \frac{P(1,t)}{P(0,t)} * g(s,t) \qquad (4)$$

$$t = 1,2,......, \quad s = 1,2.$$

and using equations (3.3) yields

$$\frac{p(1,t)}{P(0,t)} = \frac{1}{P(0,1)}$$

where $\frac{1}{P(0,1)}$ is the risk free return, R_f over the period when g(s,1) = 1 and therefore

$$R_{st} = \frac{1}{P(0,1)} * g(s,t) = R_f * g(s,t) \qquad (4.1)$$

$$t = 1,2,....., \quad s = 1,2.$$

Markets in this two-state, one-period model are complete if there exist two securities with independent returns across the states. The return matrix for two independent securities with maturities q and k can be expressed as

$$R = \begin{pmatrix} R_{1q} & R_{2q} \\ R_{1k} & R_{2k} \end{pmatrix} . \tag{5}$$

To ensure that there are no arbitrage opportunities available, it is assumed that no security dominates the other in the sense that the returns per dollar is larger in both states than for the other security. This is achieved by requiring that the stochastic term $g(s,t)$ for the two states satisfy the condition $[g(1,t)-1][g(2,t)-1]<0$. If $g(1,t)>1$, then it follows that $g(2,t)<1$, and when $g(1,t)<1$ then it follows that $g(2,t)>1$, and thus non-dominance is assured. If not, either the t period security will dominate the one period security or be dominated by the one period security.

The non-dominance condition above does not rule out dominance of one risky security by another if short selling is permitted. To overcome this, we need the following condition

$$\theta = [g(2,t)-1]/[g(1,t)-1] = [g(2,q)-1]/[g(1,q)-1]$$

where t is any risky security and q is any arbitrarily chosen reference security. Here, θ is independent of states s and of t and q. A change in the stochastic process will bring about a change in θ.

With complete markets, the returns on a t-period security can be expressed in terms of the two reference securities, q and k, as

$$R_{st} = (1-\beta_t)R_{sq} + \beta_t R_{sk} \qquad (6)$$

$$t = 1,2,\ldots\ldots, \quad s = 1,2.$$

where β_t is the proportion invested in security k and $(1-\beta_t)$ is the proportion invested in security q. In other words, the return on security t is replicated by a portfolio of the reference securities, q and k.

Consistency implies $\beta_q = 0$ and $\beta_k = 1$. Equation (6) can be re-written as:

$$R_{st} - R_{sq} = \beta_t(R_{sk} - R_{sq}) \qquad (7)$$

$$t = 1,2,\ldots\ldots, \quad s = 1,2.$$

Here, $(R_{st} - R_{sq})$ and $(R_{st} - R_{sq})$ are excess returns. The excess return between two securities is a linear homogeneous function of the excess returns on the two reference securities, k and q and β_t is a sensitivity factor which measures excess return on a t-period security as a function of excess returns on the k-period security.

In the two-state, one-period model above, the returns on all risky securities are pair-wise perfectly correlated. Let the qth security be the one period risk free security, then $R_{sq} = R_f$, and rewriting equation (6) and taking expectation yields

$$E(R_{st}) = (1 - \beta_t)R_f + \beta_t E(R_{sk}) \qquad (7.05)$$

and subtracting equation (7.05) from equation (6) gives

$$R_{st} - E(R_{st}) = \beta_t[R_{sk} - E(R_{sk})] \qquad (7.1)$$

and multiplying both sides by $R_{st} - E(R_{st})$ and taking expectations gives

$$Var(R_{st}) = \beta_t[Cov(R_{st}, R_{sk})] \qquad (7.2)$$

which upon re-arrangement gives

$$Cov(R_{st}, R_{sk}) = Var(R_{st})/\beta_t \quad . \qquad (7.3)$$

Next, multiply both sides of equation (7.1) by $[R_{sk} - E(R_{sk})]$

and take expectations so that

$$\mathrm{Cov}(R_{st},R_{sk}) = \beta_t[\mathrm{Var}(R_{sk})] \quad . \tag{7.4}$$

Dividing equation (7.3) by (7.4) gives

$$1 = \mathrm{Var}(R_{st})/(\beta_t)^2\ \mathrm{Var}(R_{sk}) \quad . \tag{7.5}$$

Note that $\beta_t = \mathrm{Cov}(R_{st},R_{sk})/\mathrm{Var}(R_{sk})$ and substituting it into equation (7.5) yields:

$$1 = \mathrm{Var}(R_{st})\mathrm{Var}(R_{sk}) \ / \ \mathrm{Cov}(R_{st},R_{sk})^2 \tag{7.6}$$

and taking the inverse first and then the square root yields

$$1 = \mathrm{Cov}(R_{st},R_{sk}) \ / \ [\mathrm{Var}(R_{st})\mathrm{Var}(R_{sk})]^{1/2} \tag{7.7}$$

Note that the expression on the right hand side of equation (7.7) defines the correlation coefficient between the securities t and k and is equal to 1. Therefore all risky securities are pair wise perfectly correlated.

Using equations (1) and (2) we can express R_{sq}, R_{sk} and R_{st} as

$$R_{sq} = \frac{1}{P(0,1)} * g(s,q) \tag{8}$$

$$R_{sk} = \frac{1}{P(0,1)} * g(s,k) \tag{8.1}$$

$$R_{st} = \frac{1}{P(0,1)} * g(s,t) \tag{8.2}$$

The model developed so far has dealt with zero coupon securities. A similar model can be developed to express the return on a coupon bond or a portfolio of coupon bonds as a function of the two reference securities, q and k. Let the face value of a coupon bond be denoted by F, its time to maturity denoted by N, and its coupon rate be denoted by c. The cash flows, S_t, where t denotes the time period, are then

$$S_t = cF. \quad t = 1,2,\ldots,N-1, \text{ and}$$

$$S_t = (1+c)F. \quad t = N. \tag{9}$$

The value of a coupon bond at time 0 is given as the aggregation of the present value of all coupon and maturity flows. The value of a coupon bond is then

$$\sum_{t} = P(0,t)S_t \qquad (10)$$

Since R_{st} is the one period dollar return per dollar invested in a zero coupon bond of maturity t, the one period dollar return on a t period cash flow is given by

$$P(0,t)S_tR_{st} \qquad (10.1)$$

where $P(0,t)S_t$ is the present value of S_t dollars to be received in t periods.

The aggregate dollar return on the coupon bond to be received in one period then is

$$\sum_{t} P(0,t)S_tR_{st} \quad . \qquad (10.2)$$

The return per dollar on the investment in the coupon$, bond over a one period is

$$\frac{\sum_t P(0,t) S_t R_{st}}{P} = \sum_t \omega_t R_{st} = R_s \tag{11}$$

where

$$\omega_t = P(0,t)S_t/P \ , \ \sum_t \omega_t = 1$$

and P is the price of the coupon bond at date 0,and ω_t is the proportion of the portfolio invested in the t period cash flow. One can view the sum of the excess return on the coupon bond as the excess returns on the t period cash flows weighted by the proportions ω_t. Multiplying equation (7) by ω_t and summing over t gives

$$R_s - R_q = \left(\sum_t \beta_t \ \omega_t \right)(R_{sk} - R_{sq}) = \beta(R_{sk} - R_{sq}) \tag{12}$$

$$s = 1,2.$$

where $\Sigma \ \beta_t w_t = \beta$ is the aggregate sensitivity coefficient for the coupon bond.

Now using equations (4),(8),(8.1), and (8.2), and substituting in (7) and simplifying gives

$$g(s,t) - g(s,q) = \beta_t[g(s,k) - g(s,q)] \tag{13}$$

and isolating β_t, we can write

$$\beta_t = \left[\frac{g(s,t) - g(s,q)}{g(s,k) - g(s,q)}\right], \tag{14}$$

and similarly one can isolate $\sum_t \beta_t \omega_t$. The beta coefficient for the coupon bond is

$$\sum_t \beta_t\omega_t = \frac{\sum_t g(s,t)\omega_t - g(s,q)}{g(s,k) - g(s,q)} \tag{15}$$

which is found by multiplying (14) by ω_t and summing. If the coupon bond is equivalent to a zero coupon bond with maturity d, then equating right hand sides of (14) evaluated at the point when t = d, and (15), and simplifying gives

$$\sum_t g(s,t)\omega_t = g(s,d) \tag{16}$$

and dividing both sides by g(s,d), we have

$$[\Sigma_t\ g(s,t)w_t\ /\ g(s,d)] = 1 \qquad (16.1)$$

Equations (15) and (16) relate the duration of a portfolio of bonds to its beta coefficient with respect to arbitrarily chosen reference securities. Equation (16) gives an implicit definition of d, the duration of the coupon bond. The one factor model permits the use of duration in explaining the motion of excess returns relative to the excess returns on a reference security. Furthermore, Equation (16) shows that all portfolios of bonds having the same duration must have the same beta, regardless of the composition of the portfolios.

Note that β_t is a function of the stochastic process, g(s,t), and therefore the duration is also a function of the stochastic process. An implication of the one-factor duration model is that the return on every portfolio over any time period is either equal to the return on a zero coupon bond or is equal to the return on a portfolio of two zero coupon bonds with adjacent maturities. The returns on all bonds or portfolios of bonds having the same duration or beta are expected to be identical each period. Furthermore, excess returns each period, on a portfolio of bonds will be proportional, with beta being the proportionality factor, to the excess return on an arbitrarily chosen reference security.

Duration and beta are a function of the stochastic process, as represented by g(s,t), generating the term structure fluctuations. Different stochastic processes give rise to different measures of duration and beta. Three different stochastic processes are considered below. Each process gives rise to a measure of duration which also can arise in a disequilibrium context. Two of the three betas associated with these stochastic processes are not "stable" and are a function of the shape and the position of the term structure of interest rates.

Fisher-Weil Equilibrium Process(FWEP):

An example is the stochastic process

$$g(s,t) = 1 + K_s(t-1), \quad s = 1,2; \quad t = 1,2,3... \tag{16.1}$$

where K_s is a parameter in the process. To ensure that the above specification of g(s,t) implies an equilibrium stochastic process, $K_1K_2<0$ and $[g(s,\tau) - 1]/[g(s,t) - 1]$ must be independent of s for $t \neq \tau$.

Furthermore the ratio $\phi = [g(s,q) - 1]/[g(s,t) - 1]$ must be equal for both states for all risky securities t and an arbitrarily chosen reference security q. This ensures that there are no opportunities for riskless arbitrage profits. Note that the value of g(s,t) is 1 when t = 1 and thus the one period security is riskfree for the specification above. Substitution of equation (16.2) into equation (16.1) and simplifying gives the duration for this process in the implicit relationship below

$$\sum_t \frac{1 + K_s(t-1)}{1 + K_s(d-1)} \omega_t = 1 \qquad (17)$$

which upon rearranging becomes

$$\sum_t t\omega_t = d \qquad (18)$$

where the left hand side of equation (18) is the definition for the Fisher-Weil duration measure. The Fisher-Weil duration measure which was criticized as arising from a disequilibrium process can thus also arise in an equilibrium discrete two-state, one-period model.

This paper will focus on the empirical implications of the equilibrium process generating the Fisher-Weil duration measure. If the reference security q in equation (14) is specified as the risk free security, and if the stochastic process specified above in equation (16.2) expresses the sensitivity measure β_t, in terms of the maturity of the reference security and the t period security as;

$$\beta_t = \frac{g(s,t) - 1}{g(s,k) - 1} \tag{19}$$

then upon substituting equation (16.2) into (19), we have

$$\beta_t = (t-1) / (k-1) \quad . \tag{20}$$

The formulation in equation (20) shows beta of a t-period security as a function of the maturity of the t period security, and the maturity of the chosen reference security. Note that β_t in the above formulation derives from the FW process for an equilibrium two state, one period model. As can be seen, β_t is linear in t and increases monotonically with t, and $\beta_k = 1$ and $\beta_1 = 0$. More will be said about the formulation above in the methodology section of this paper.

The Special Additive Process(SAP):

Another duration measure which arises in a disequilibrium context is the special additive process(henceforth SAP). A variant of this process is used to simulate immunization strategies in Bierwag, Kaufman, and Toevs(1982b). The SAP is also discussed in Bierwag and Gordon(1990) in an equilibrium context. There the stochastic element, g(s,t), enters the return generating process as an additive variable whereas it enters as a multiplicative factor in the model developed here. The g(s,t) for SAP in an equilibrium context is defined as

$$g(s,t) = 1 + [\ K_s(\Omega^{t-1} - 1)/P(0,t)\ ] \qquad (20.1)$$

where K_s and Ω are parameters in the process. To ensure that the above specification of g(s,t) implies an equilibrium stochastic process, $K_1K_2<0$ and $[g(s,\tau) - 1]/[g(s,t) - 1]$ must be independent of s for $t \neq \tau$. Furthermore the ratio $\phi = [g(s,q) - 1]/[g(s,t) - 1]$ must be equal for both states for all risky securities t and an arbitrarily chosen reference security q. This ensures that there are no opportunities for riskless arbitrage profits. Note that the value of g(s,t) is 1 when t = 1 and thus the one period security is risk free for the specification above. Substitution of equation (20.1) into equation (16) and simplifying gives the duration for this process in the implicit relationship below

$$\Sigma_1^n\ w_t[(\Omega^{t-1} - 1)/P(0,t)] = [(\Omega^{d-1} - 1)/P(0,d)] \qquad (20.2)$$

Therefore the implicit duration arising in a disequilibrium context also can arise in an equilibrium context. Substituting equation (20.1) into (14) and simplifying and letting q = 1 yields

$$\beta_t = [P(0,K)(\Omega^{t-1} - 1)]/[P(0,t)(\Omega^{k-1} - 1)] \qquad (20.3)$$

Here β_t is not a function of K_s and therefore is independent of the states. Any change in K_s over time will have no effect on β_t. Unlike the FW equilibrium model discussed earlier β_t depends on the term structure of interest rates since the right hand side of equation (20.3) includes prices of the t period and the k period security observed at time period zero. Therefore the SAP permits the β to change over time as the term structure changes and thus has built in "beta instability".

The Additive Discrete Equilibrium Process(ADEP):

Another duration measure that arises in a disequilibrium context is the discrete additive duration. In a disequilibrium context it arises when the term structure moves up or down by a random amount. Bierwag, Kaufman, Schweitzer, and Toevs(1982) use it to simulate immunization strategies. The additive discrete equilibrium process (henceforth ADEP) can also be derived in an equilibrium context by specifying

$$g(s,t) = 1 + K_s[tP(0,t)^{1/t} - P(0,1)] \qquad (20.4)$$

where K_s is a parameter in the process. To ensure that the above specification of g(s,t) implies an equilibrium stochastic process, $K_1K_2<0$ and $[g(s,\tau) - 1]/[g(s,t) - 1]$ must be independent of s for $t \neq \tau$. Furthermore the ratio $\phi = [g(s,q) - 1]/[g(s,t) - 1]$ must be equal for both states for all risky securities t and an arbitrarily chosen reference security q. This ensures that there are no opportunities for riskless arbitrage profits. Note that the value of g(s,t) is 1 when t = 1 and thus the one period security is risk free for the specification above. Substitution of equation (20.4) into equation (16) and simplifying gives the duration for this process in the implicit relationship below

$$\Sigma_1^n w_t tP(0,t)^{1/t} = DP(0,D)^{1/D} \qquad (20.5)$$

Here, also the implicit duration arising in a disequilibrium context also can arise in an equilibrium context. Substituting equation (20.5) into (14) and simplifying and letting q = 1 yields

$$\beta_t = [tP(0,t)^{1/t} - p(0,q)]/[kP(0,K)^{1/k} - P(0,q)] \quad (20.6)$$

Here β_t is not a function of K_s and thus is independent of the states. Any change in K_s over time will have no effect on β_t. Unlike the FW equilibrium model discussed earlier β_t depends on the term structure of interest rates since the right hand side of equation (20.5) includes prices of the t period, q period and the k period securities observed at time period zero. Therefore the ADEP permits the β to change over time as the term structure changes and thus has built in "beta instability".

Bierwag(1987a) has shown that a two-state equilibrium model as developed in this paper can be generalized to allow for the specification of an infinitely large number of states in the next period. However, the markets are incomplete in the new specification, but the single factor duration model still obtains. Furthermore, it is also shown that the model could be revised to accommodate an n period horizon, where n need not be restricted to 1. The focus in this paper is on the two-state, one-period model as developed in this paper.

The two-state, one-period model provides some interesting implications about the liquidity premium hypothesis and the expectation hypothesis. Let us assign probabilities p_1 and p_2 to states 1 and 2 respectively. Corresponding to these two states are the possible term structures, $h_1(1,t)$ and $h_2(1,t)$, that can arise one period later. Starting with equation 4.1 and applying the expectations operator, we get

$$E(R_{st}) = \{ p_1[P_1(1,t)/P(0,t)] + p_2[P_2(2,t)/P(0,t)] \} \quad (21)$$

which upon further manipulations yields

$$E(R_{st}) = [p_1 g(1,t) + p_2 g(2,t)] * R_f \quad (21.1)$$

which in a continuous compounding framework,is equivalent to

$$E(R_{st}) = \frac{p_1 e^{-h_1(1,t)(t-1)} + p_2 e^{-h_2(1,t)(t-1)}}{P(0,t)} \quad (22)$$

A version of the pure expectation hypothesis is that

$$E[g(s,t)] = 1 \quad (23)$$

which suggests that the next period's prices are those evaluated at the forward rates given by today's term structure. This implies that in equation (23), all the bonds have the same expected return over the next period, regardless of maturity. In equation (22) this implies that

$$p_1 e^{-h_1(1,t)(t-1)} + p_2 e^{-h_2(1,t)(t-1)} = e^{-h((1,t)(t-1)} \qquad (23.1)$$

If this is true, then it follows that

$$p_1 h_1(1,t) + p_2 h_2(1,t) \neq h(1,t) \qquad (24)$$

except in the special case of perfect certainty when s = 1. Equation (24) indicates that there exists a term premium as a result of applying the expectation hypothesis to the prices. Tests of the betas against the theoretical counterparts would shed light on the existence of term premium. Thus if the excess returns on the t period security are as described in the model, then the betas would be increasing in t. This would suggest the existence of term premium but still would be consistent with the notion of expectation hypothesis holding on prices.

Bierwag(1987a) extends the one-period model to three states, resulting in a two-factor one-period model. The work in his paper is based on specifying a model with an additive stochastic variable, g(s,t), as compared to a multiplicative framework discussed in this paper. It should be noted that a multiplicative stochastic specification as developed in this paper leads to the same implications reached by Bierwag with an additive model. We do not derive a two-factor model in a framework where the stochastic variable is multiplicative but the result derived in Bierwag can also be derived under the multiplicative model. The important point to note is that not only a duration measure derived from a disequilibrium model can also be derived from an equilibrium process but it can be derived from many different equilibrium processes.

The three-state model as discussed above will give rise to a two-factor model in which the excess return on a t-period security can be expressed as a function of the excess return on two reference securities, q and k. Betas in the two factor model are also a function of the stochastic process and can be stable or unstable. The specification which gives the FWEP in a one factor model can also be used for a two factor model(see Bierwag(1987b)and the expressions for beta measures, β_{tk} and β_{tq} are

$$\beta_{tk} = \frac{(q-t)\ (t-1)}{(k-1)\ (q-k)}$$

$$\beta_{tq} = \frac{(t-k)\ (t-1)}{(q-k)\ (q-1)}$$

where β_{tk} is the sensitivity of the t period security to the reference security k, and β_{tq} is the sensitivity of the t period security to the reference security q. The beta measures shown above are related to the Fisher-Weil duration and another measure of duration, Inertia, defined as $\Sigma_t\ w_t t^2$.

Chapter III

Empirical Methodology, Data, and Results

3. EMPIRICAL METHODOLOGY

To test indirectly the FW equilibrium process, one can empirically estimate betas for each t and carry out a joint test by comparing the empirical betas to theoretical betas given by the equation(9). For a given choice of q, the reference security, the return on a t-period Treasury Bill in excess of the certain one-period Treasury Bill return can be regressed on the excess return of the chosen q period Treasury Bill over the one-period Treasury Bill. As an example, the theoretical beta for a 3 month Treasury Bill, with a 6 month reference security is given by (20) as

$$\beta_3 = (3-1)/(6-1) = 0.4.$$

The empirical estimation of the beta of a t month Treasury Bill would involve estimation using the following specification:

$$R_{ti} - R_{fi} = a + \beta_t (R_{qi} - R_{fi}) + e_i \quad i=1,2,..,n \quad t=1,2,....12$$

where R_{ti} is the one month return on a t-month Treasury Bill in month i, R_{fi} is the risk free return on a one month bill for month i, R_{qi} is the one month return on the q month reference treasury bill in month i. A 6-month Treasury Bill will be chosen as the reference security. Here, β_1 and β_6, a priori are 0 and 6 respectively. There will be 10 empirically estimable betas for the other 10 months in a year.

In the SAP model, the beta is a function of the observed prices at time t. The expression for beta under SAP is repeated below.

$$\beta_{ti} = [P_i(0,K)(\Omega^{t-1} - 1)]/[P_i(0,t)(\Omega^{k-1} - 1)] \qquad (20.3)$$

where Ω is a parameter in the process. Note that there exists a beta for each value of the ratio P(0,K)/P(0,t). However to conduct a test of this process, the data is ordered in an ascending order of the ratio P(0,K)/P(0,t) and then divided into two equal groups. If the SAP describes the motion of the term structure, then the betas for these two groups should be statistically different. A finer test is provided by dividing the data into four groups using the ordering criteria for the two groups discussed above. Both the F test for structural shifts and the dummy variable method are utilized to test for the instability of the betas. Johnston(1984) provides a detailed discussion on the use of the F test and the dummy variable methodology.

No specific tests are carried out for the ADEP, but the tests of SAP indicate the likelihood of the ADEP process being in effect. The expression of beta for the ADEP is repeated, here, as

$$\beta_t = [tP(0,t)^{1/t} - p(0,q)]/[kP(0,K)^{1/k} - P(0,q)] \qquad (20.6)$$

In the ADEP like the SAP, beta is a monotonic function of the initial term structure prices. Furthermore beta is a function of the ratio P(0,t)/P(0,k) which is the inverse of the ratio used to order the data for testing SAP. Therefore, the grouping used to test the SAP can also be used to test the ADEP. Both models suggest beta instability. If the tests support the instability of beta, then further tests can be carried out to see which one of the instability models discussed in this paper are consistent with the results. As a prelude to discussions of the tests later, there does not exist much support for beta instability models.

The null hypothesis for the FWEP model, H_0 is

$$b_t = \beta_t \qquad t= 1,2,\ldots, 12$$

where β_t is the theoretical value given by equation (20) and b_t is the estimated value from regressing the excess returns of a t-period Treasury Bill on the excess returns on the reference Treasury Bill.

For the two-factor model, the estimation of β_{tq} and β_{tk} involves the estimation using the following specification.

$$R_{ti} - R_{fi} = a + \beta_{tk}(R_{ki} - R_{fi}) + \beta_{tq}(Rqi - R_{fi}) + e_i$$
$$i = 1,2,\ldots\ldots 480$$
$$t = 1,2,\ldots\ldots\ldots 12$$

A nine-month bill is chosen as the reference security q, and a five-month bill is chosen as the reference security k. In the model, the choice of the reference security can be arbitrary. The selection of the five- and nine-month Treasury Bills as reference securities provides an equal number of Treasury Bills having maturities less than five months, between five and nine months, and over nine months. As an example, given our choices for the reference securities, the betas of a three-month bill with respect to the reference securities are

$$\beta_{35} = (9-3)(3-1)/(5-1)(9-5) = 3/4$$
$$\beta_{39} = (3-5)(3-1)/(9-5)(9-1) = -4/32$$

where the reference securities are k=5 and q=9 respectively.

The theoretical betas, along with the estimated betas, for the other maturities of Treasury Bills are given in Table 11. The theoretical beta with respect to the five- and nine-month reference securities are denoted as "Theoretical Beta1" and "Theoretical Beta2" respectively in Table 12 for all maturities. The theoretical betas for the FWEP model are given in Table 2.

DATA:

The data set of Treasury Bill returns is provided by the Center for Research in Security Prices (CRSP) at the University of Chicago. This data set is referred to as the Fama Treasury Bill term structure file in the CRSP documentation. There are two versions of this file available, one based on data provided by the Salomon Brothers and the other based on data provided by the Federal Reserve. The CRSP documentation states that the data based on the Federal Reserve Quotes is more accurate, and therefore it is used in this study. The Fama Treasury Bill one-month holding period returns based on average prices(average of the bid and ask quotes) are used. The Fama data covers 274 months in the period 1965-1987.

In the Fama Treasury Bill data set, the one month Treasury Bill holding period return is calculated using the following equation,

$$\ln\left[\frac{P_{t,t+1}}{P_{t,t}}\right]\left[\frac{30.4}{N_{t,t} - N_{t,t+1}}\right] \qquad (25) \ , \ t = 1,2..$$

where $P_{t,t}$ and $P_{t,t+1}$ are the prices of a t period bill observed at time t and t+1 respectively, and $N_{t,t}$ and $N_{t,t+1}$ are the days to maturity for a t period treasury bill at dates t and t+1 respectively. The above process standardizes the holding period return to 30.4 day basis and the returns across different months are rendered comparable.

The one month holding period returns for the twelve-month Treasury Bill are coded as zero for 46 observations in the CRSP data. A systematic check of other maturities revealed only non-zero holding period returns for all 275 observations. A telephone conversation with a CRSP researcher revealed that for many quote dates, bills with maturities sufficiently close to 12 months were not available and a zero was recorded for these dates. Therefore a zero return for the twelve-month security represents missing values. To correct for this, all observations with zero return for the twelve-month security were discarded. This left 229 usable observations for the twelve-month security. For all other securities, all 275 observations were usable.

4. RESULTS

If the discrete time FWEP model as developed above fits the data well, then the excess returns per dollar for all maturity classes should be perfectly correlated with excess returns per dollar for the reference security and the estimated slopes of the relationship should increase monotonically with maturity. The relationship between the excess returns for all maturities and the excess return on the reference security, 6 month Treasury Bill, is plotted and presented in Figures 1 through 10. The plots indicate monotonically increasing betas. If the correlation was perfect than all observations for each plot would fall on a straight line, the slope of the relationship.

The correlation is less than perfect for all maturities as indicated by deviations of the observations from any slope fitted to the observations. The correlations are lower for securities with maturities farther away from the reference security. As an example, the deviations of observations from any fitted slope is higher for the two-month Treasury Bill than it is for the five-month Treasury Bill. Further, some of the observations for all maturities fall in the negative-positive and positive-negative quadrants of the plots, contrary to expectations of the model which call for all observations falling in the positive-positive and negative-negative quadrants.

Table 1 presents the results of regressions using the Fama data. For the Fama data, the correlation ranged from 0.33 to 0.97. Except for the two- and twelve- month bills, the correlations are in excess of 0.82. The estimated betas are all significantly different from zero. The estimated betas increase monotonically with maturity.

If the model fits well, then we would expect the intercept term, Alpha, not to be significantly different from zero. However this is not the case. The intercept terms for all the regressions are significantly different from zero at the 99% confidence level. The intercept terms are positive for regressions involving maturities less than the reference security, but negative with respect to regressions involving maturities greater than the reference security.

Table 2 presents regression results when the intercept term is suppressed. Betas for maturity categories less than the reference category of the six-month bill show an upward revision and the betas for maturity categories above the reference category show a downward revision. Overall, suppression of the intercept does not result in a better fit of the model.

Next the data is split into two groups, one covering the period 1965-1977 and the other covering the period 1978-1987. Regressions for these two periods are presented in Tables 3 and 4 respectively. The correlation, except for the twelve-month maturity, is higher for all regressions for the period 1978-1987 as compared to the period 1965-1977. The estimated betas are significantly closer to the theoretical values for all regressions except the 12 month bill for the latter period. The differences in beta across different time periods suggest instability and that there may be other factors contributing to the differences. The latter period is known to be more volatile than the earlier period for interest rates.

Valid use of OLS estimation and inferences from it requires satisfaction of some assumptions. One of these assumptions is the homoscedasticity of the error term. In the presence of heteroscedastic error terms the OLS estimates are unbiased but inefficient and this results in invalid inference procedures. Tests to check heteroscedasticity of error terms are carried out using the Goldfeld-Quandt and Grejler methods. Both methods are discussed in detail in Johnston(1984). The results are not reported here but Grejler method tests do not indicate any heteroscedasticity of error term for all regressions. The Goldfeld-Quandt method gives similar results, with the exception of the regression similar results, with the exception of the regression involving M3 for which the hetroscedasticity of the error terms can not be rejected. This indicates that hetroscedasticity is not a problem in the regression tests discussed earlier.

Autocorrelated error terms result in unbiased but inefficient estimates. Autocorrelation is indicated if the underlying error terms are autocorrelated, or if relevant explanatory variables are omitted, or if the true relationship is non-linear. The results are not reported here, but a check of autocorrelation for the error terms up to 5 lag periods indicates autocorrelation for most regressions. However, when the estimated autocorrelation are utilized to re-estimate the slopes utilizing the Yule-Walker method, none of the earlier inferences for the FWEP model change. This indicates that autocorrelation, though present, is not significant enough to reverse inferences. The Yule-Walker procedure is discussed in the SAS/ETS User's Guide under the Autoreg family of procedures.

Tables 5 and 6 present the regression results for observations where the excess return for the reference security is positive and negative respectively. The correlation is higher for data when the excess return is positive. Strangely enough, the betas are higher for positive excess return data, for maturity categories below the reference maturity, and lower for maturity categories below the reference category.

Out of the 11 regressions under Table 1, three of the estimated betas are statistically not different from the respective theoretical value at 99% confidence level. To check if the results are sensitive to the choice of the reference security, a ten-month Treasury Bill was chosen as the reference security. The Fama data is utilized for these regressions and the results are presented in Table 7. The intercept terms, except for the eleven-month bill, are all significantly different from zero at 99% confidence level. The intercept terms are positive for maturity categories less than the reference maturity of ten months and positive for maturities greater than the ten-month bill. This was also observed for regressions with the six-month bill as the reference security.

The estimated betas all increase monotonically with respect to maturity except for the twelve-month bill. However, the estimated values are statistically equivalent to the theoretical values for eight maturity categories as opposed to only three for the regressions with six-month bill as the reference security. Except for the two- and twelve-month maturities the estimated betas are statistically equivalent to the respective theoretical values. It is not clear as to why the model has more support with a different reference security. This issue is further explored in chapter 4.

The model can also be tested by forming portfolios of different maturity bills and regressing the portfolio returns on a single bill which the portfolio emulates. One of the implication of the model is that the portfolio returns would be equal to the return on a Treasury Bill whose maturity is the duration of the portfolio. Four portfolios with a duration of 6 months are formed and the regression results are presented in Table 8. One would expect the intercept terms to be insignificantly different from zero and that is the case for three of the four regressions. The correlation ranges from 0.92 to 0.98 suggesting a good explanation of the return variation. However, the betas are all less than 1 and statistically different from 1 at the 99% confidence level.

Looking back at Tables 1 and 7 it is clear that not all estimated betas are statistically equivalent to the respective theoretical values. It is possible that the true relationship is as specified by the FWEP model on an average but may deviate from it in a non-systematic fashion for some periods. If this is true than the estimated betas when regressed on the theoretical betas should be expected to generate a slope of 1 and an intercept equal to zero. These regressions are reported in Table 9. For the regressions with six-month Treasury Bill as the reference security, the beta is equal to 0.94 and is significantly different from 1. For the regressions with ten-month Treasury Bill the estimated beta is equal to 0.999 and is insignificantly different from 1. This supports the FWEP model. However the intercept term is significantly different from zero. The results here are sensitive to the choice of reference security as before.

To check if the relationship between excess returns on various maturities and the excess returns on the reference security is non-linear, the dependent variables are regressed on the reference security and the square of the reference security. The results are presented in Tables 10 and 11. Overall, the coefficients of the squared independent variables are significant. However, the coefficients are extremely close to zero and further the adjusted R^2 shows very little improvement. This indicates that the linear specification is not misspecified.

The regressions for the two-factor model are presented in Table 12. The adjusted correlation, as expected shows an improvement over the one factor regressions for all maturity categories. All of the estimated betas have the same sign as the theoretical betas but the estimated values are all significantly different from the theoretical values at the 99% confidence level. This rejects the particular specification of the two factor model but does not rule out the possibility that some other stochastic process in a two factor framework can explain the term structure motions at the short end. However, the low improvement in adjusted correlation for the two-factor model in relation to the one-factor model suggests that one factor need not be discarded in favor of the more complex two-factor model.

Park and Reinganum(1986) report that the yield curve on Treasury Bills exhibits a kink as it crosses calendar months and is most pronounced as the yield curve passes between calendar years. They caution academic researchers to use care when using time series of Treasury Bill returns. They consider the tax treatment of interest income on Treasury Bills as a possible reason for the kink. Under current tax laws, bills which mature in the next calendar year escape taxes on the implied interest earned if the individual uses cash basis for reporting income. This is so for most individuals and would effectively postpone taxes for one year for many of them. For corporations and individuals who make estimated tax payments,the postponement of taxes is about 90 days.

An attempt is made to recognize the role of taxes in pricing of Treasury Bills and its impact on the excess returns used in this study to test some one-factor models. The data can be conceptually divided into four groups. The first group will have observations where both the dependent variable and the independent variable mature in the same calendar year as the quote date of the returns. The second group has the dependent and the independent variable maturing in the next calendar year. The third group has the dependent variable maturing in the same year as the quote date and the independent variable maturing in the next calendar year. The fourth group has an independent variable maturing in the same calendar year as the quote date and the dependent variable maturing in the next calendar year.

Note that for the case where the maturity of the dependent variable is less than that of the independent variable, there are no observations in the fourth group. For the case where the maturity of the dependent variable is greater than that of the independent variable, there are no observations in the third group. Therefore, for both cases, there are three distinct groups.

It is possible that the calendar year in which a Treasury Bill matures may affect the returns from it. As an example, Investors may prefer a six-month bill over a five-month bill if it matures in the next calendar year given that taxes on the return from the six-month bill can be postponed upto a year in some cases. This could lead to a higher price and therefore a lower holding period return for the six month bill relative to what it would be if it did not mature in the next calendar year. This should affect the estimated relationship of the regressions if tax effects exist. The beta for regressions for observations where the reference bill matures next year and the dependent variable matures in the current year, should be higher than the case for which both mature in the same year. Similarly, if a bill matures in the next year, and the reference bill matures this year, then the relationship should be different, i.e. the beta should be lower relative to the case when both mature on the same year. When both the bill and the reference security mature next year, the effect on beta is unclear.

To summarize, for the top panel of Tables 13 through 22, α and β are the estimated intercept and slope for the group when both Treasury Bills in the regressions mature in the same year (group 1). Here, α_1 and β_1 are the difference between intercepts and slopes of group 2 (when both bills mature in the next year) and group 1 respectively. α_2 and β_2 are the differences between intercepts and slopes of group 3 and group 1 respectively. For dependent variable with maturities less than that of the reference bill, group 3 contains observations where the dependent variable matures in the current calendar year and the independent variable matures in the next calendar year. For dependent variable with maturities greater than the independent variable, group 3 contains observations where the dependent variable matures in the next calendar year and the independent variable matures in the current calendar year.

Based on our previous discussion about tax effects, the null hypothesis is that $\beta_1 = 0$ and the alternative hypothesis is, β_1 not equal to zero, and further, $\beta_2 = 0$, and $\beta_2 > 0$ for cases where the dependent bill's maturity is less than that of the independent bill's maturity, and $\beta_2 < 0$ for cases where the dependent variable's maturity is greater than that of the independent variable's maturity.

To test for any tax related effect the dummy variable method (discussed later) is utilized. Results of the tests are reported in Tables 13 through 22. The top panel of each table presents the results based on the complete data and the bottom panel presents the results based on division of data into pre-1978 and post-1978 groups. The grouping in the bottom panel is arbitrary and is done to capture any time-related effect compounding the tax-related effect.

For the top panel β represents the coefficient for the group 1 and β_1 and β_2 represent the difference between the group specific coefficient and the group 1 coefficient, for groups 2 and 3 respectively when the dependent variable's maturity is less than that of the independent variable, and for groups 2 and 4 respectively when the dependent variable's maturity is greater than that of the independent variable. If there is no difference in the betas across different groups then the expected values for β_1 and β_2 should be insignificantly different from zero.

For the bottom panel the same setup as in the top panel holds but now the data is divided by both time and groups discussed earlier. The first subscript for β denotes the time period and the second corresponds to grouping. For the time subscript 1 denotes pre-1978 data and 2 denotes post-1978 data. The second subscript for grouping carries the same meaning as discussed in the preceding paragraph.

The results from the regression tests utilizing dummy variables are discussed first for cases where $t<q$, where t and q are the maturities of the dependent variable and the reference treasury bills respectively. The reference security chosen for these tests is the six-month Treasury Bill. For two maturities, three-, and four-month bills, the subscripted betas are insignificantly different from zero indicating no tax effects. For the two-month bill, β_1 and β_{12} are negative and significantly different from zero. The signs of the

significant coefficients are contrary to expectations of tax effects reasoning. For the five-month bill, β_{22} is positive and significantly different from zero and again the sign is negative, contrary to expectations.

For cases where t>q, only the ten-month bill has insignificant betas for groups 2 and 3. Many of the group 3 betas, β_2, β_{12}, β_{22}, are significant and negative. This is in line with expectations and provides some support for tax effects. However, the significant values for betas may be due to the tax effect groupings acting as proxies for unidentified factors.

It is interesting to note that the estimated betas, for 7 out of 10 regressions(3,4,5,7,8,9,and 10 month Treasury Bills) for the first group (where both bills mature in the same calendar year as the quote date) are insignificantly different from the respective theoretical values. For the complete data, only the estimated betas for 3,4, and 5 month bills are insignificantly different than their respective theoretical values, as reported in Table 2. The stronger support for the FWEP model when only the data for the first group is utilized indicates that the tax effects may partly explain the weaker support for the FWEP seen in Table 2.

Let us turn our attention to tests of SAP and ADEP. Recall that these two specifications give rise to betas which are a function of the observed term structure and therefore are unstable. Tables 23 through 32 provide the results of tests of beta instability models. Each table provides tests of stability for a specific maturity of Treasury Bill. The independent variable or the reference security is the excess return on a six month Treasury Bill for each table.

The F test and the dummy variable methods are utilized to test the SAP and ADEP models. The F test method requires computation of residual sum of squares for a restricted and an unrestricted model. In the restricted model it is assumed that betas are stable and are not a function of term structure observations. The regression is run with no division of data and generates the intercept, slope and the residual sum of squares. The restricted model residual sum of squares are denoted as RSS_r in the Tables 23 through 32. The degrees of freedom for the restricted regression is 273(275-2) where 275 is the number of observations and 2 is the number of parameters estimated. In the unrestricted model, the intercepts and the slopes are allowed to vary. The number of the parameters to be estimated would depend on the number of different regimes assumed for the relationship.

For the top panel of each table, the data is divided into two groups based on the ratio, P(0,t)/P(0,q). Therefore the number of parameters to be estimated is 4 and the degrees of freedom is 271(275-4). The residual sum of squares for this unrestricted regression is denoted as RSS_u in the table. The difference between the RSSs is denoted as RSS_d in the tables and its degree of freedom is the difference between the degrees of freedom for the restricted and the unrestricted regressions.. If the relationship across different groups is not significantly different than RSS_d is likely to have a low value. The F value is the ratio,

$$(RSS_d/\text{degrees of freedom})/(RSS_u/\text{degrees of freedom}).$$

This value is then compared to critical values of F the appropriate degrees of freedom to check significance. The critical values are given in each table. For the bottom panel of each table, the data is divided into four groups for test of beta instability using the F values.

Each panel also presents tests of the beta instability models utilizing the dummy variable method. The regression specification for the beta instability model with two groups is

$$y = \alpha + (\alpha_2 - \alpha)D_2 + \beta X + (\beta_2 - \beta)D_1X$$

where D_1 is the dummy variable and is assigned a value of 1 if the observation belongs to group 2, if not it is assigned a value of zero. Note that α and β are the intercept and the slope for the group 1 data and α_2 and β_2 are the intercept and the slope for the group 2 data. The difference $(\alpha_2 - \alpha)$ and $(\beta_2 - \beta)$ are denoted as α_1 and β_1 in the tables. Under the null hypothesis, α_1 and β_1 are both expected to equal zero. If β_1 is significantly different than zero, it would indicate support for the beta instabilty models. The dummy variable enables one to pinpoint the instability, if any, to a specific grouping of data and is therefore likely to reveal any specific pattern of instability. The dummy variable method is similarly utilized for the case with division of data into four groups. Detailed description of the F test and the dummy variable methods can be found in Johnston(1984).

Each table has two panels and the top panel provides results based on division of data into two groups. The lower panel provides results based on division of data into four groups. The criteria for division of data is discussed in the methodology section. For each panel, tests are conducted based on both the F test method and the dummy variable method.

The results based on F test, for both panels of each table, indicate instability for three out of ten regressions. The maturities where the beta is unstable are the three-, four-, and the five-month bills. The results based on the dummy variable method are identical. Only in the case of the five-month bill are the betas β_1, β_2, and β_3 significantly different from zero. For the three- and the four-month bills , only β_2 is significantly different from zero. This suggests very little support for the beta instability models, ADEP and SAP. However, this does not rule out other models where beta is unstable but the grouping of data for testing of these models may be a function of criteria other than the one used for testing ADEP and SAP.

5. CONCLUSIONS:

One-factor models have been extensively used in studies dealing with immunization of bond returns against interest rate movements. The Fisher-Weil model and the duration used in it is an example of a one-factor approach to the analysis of rate movements that is widely used in research. However it has been shown that the Fisher-Weil model is a disequilibrium model in some contexts. An equilibrium model of the term structure movement is developed and it is shown that the Fisher-Weil duration measure can also arise in a two state, one period context. Therefore, use of one-factor models can be justified on the grounds that a equilibrium process can also give rise to a duration measure derived in a disequilibrium context.

Treasury Bill return data is used to test the empirical validity of some equilibrium models. The results are mixed. Regressions using the Fama data show a high correlation suggesting that a one-factor model explains Treasury Bill return data well. However, the intercept terms are all non zero at 99% confidence level, which is contrary to the predictions of the model. More importantly the coefficients of the regressions for the most part are significantly different from the theoretical values generated by the model.

This suggests an overall support for the one-factor model of Treasury return data but not for the FWEP one-factor model developed in this thesis.

Strangely enough, the support for the FWEP model increases considerably when the model is tested with the ten-month bill as the reference security. Out of the 10 regressions only the two- and twelve-month bill return regressions give rise to betas not significantly equivalent to their respective theoretical values. This discrepancy is explored in more detail in the next chapter. It is interesting to note that even though all the estimated betas are not statistically equivalent to their theoretical values, they seem to be in agreement on sign and the monotonic increase suggested by the model.

The model seems to fit the data better for the latter period, 1978-1987. Regressions of the estimated betas, with the ten-month bill as the reference security, on the theoretical betas generate a slope close to 1 and the intercept terms are not significantly different from zero. Thus even though all the individual regressions do not support the model, the regression of estimated betas on the theoretical values generate a slope equal to 1, indicating support for the FWEP model.

The F tests and Dummy variable tests indicate no support for the SAP and ADEP models. Dummy variable tests indicate some support for tax effects and these tests may partly explain the weak support for the FWEP model.

Tests of the two-factor model on the Treasury Bill return data does not provide any support for model. There is a slight improvement in the correlation for the individual regressions but the estimated betas are not meaningfully related to the theoretical values. Only the signs of the estimated and the theoretical values seem to be in agreement. In conclusion, the one-factor model developed so far in this paper has partial support. The two-factor model does not significantly improve upon the one-factor model and has very little support.

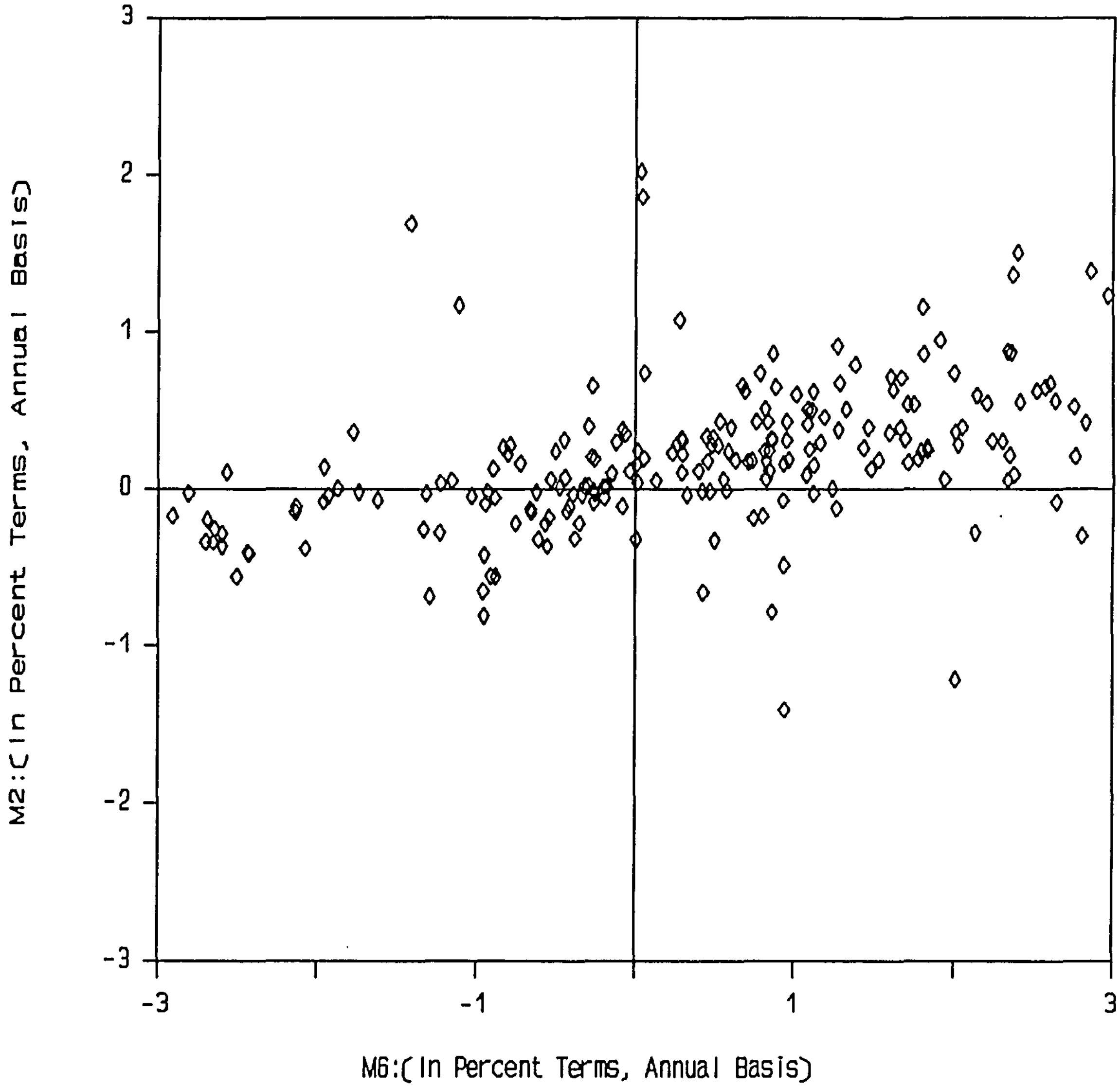


Figure 1: Plot of M2 Against M6. Both Variables Measure one Period(one month) Excess Return Over the one Month Bill.

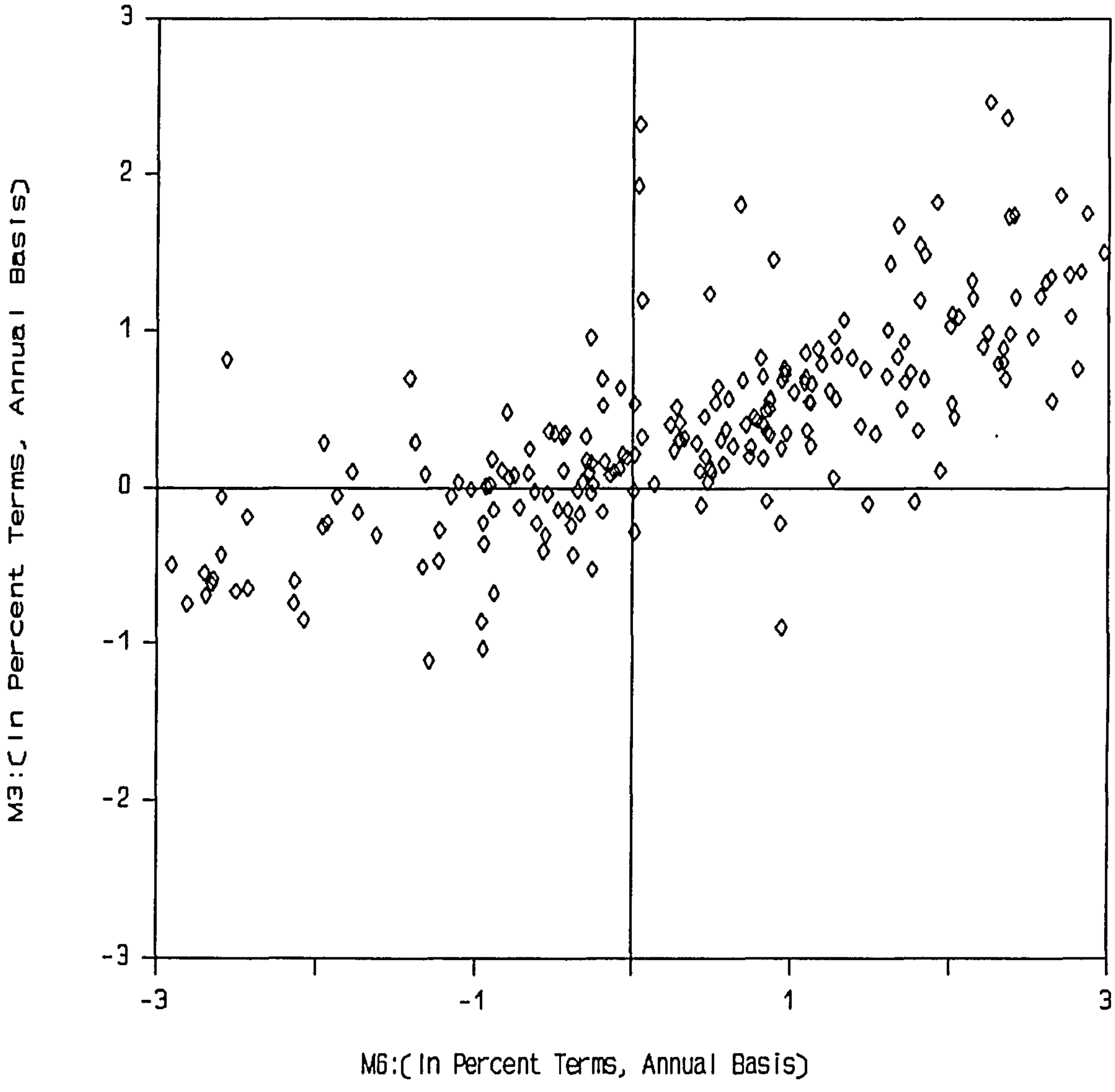


Figure 2: Plot of M3 Against M6. Both Variables Measure one Period(1 month) Excess Return Over the one Month Bill.

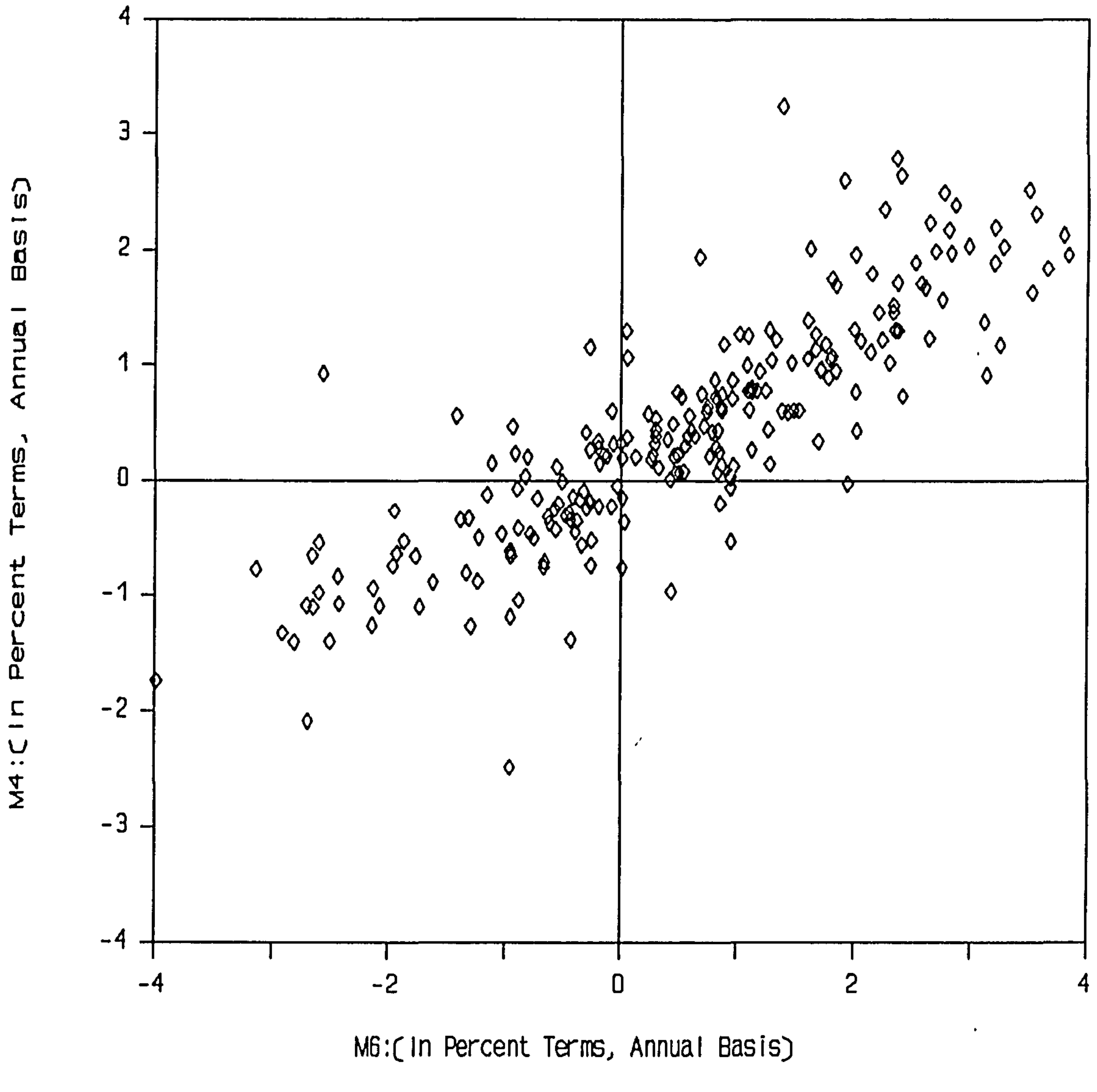


Figure 3: Plot of M4 Against M6. Both Variables Measure one Period Return(1 month) over the one Month Bill.

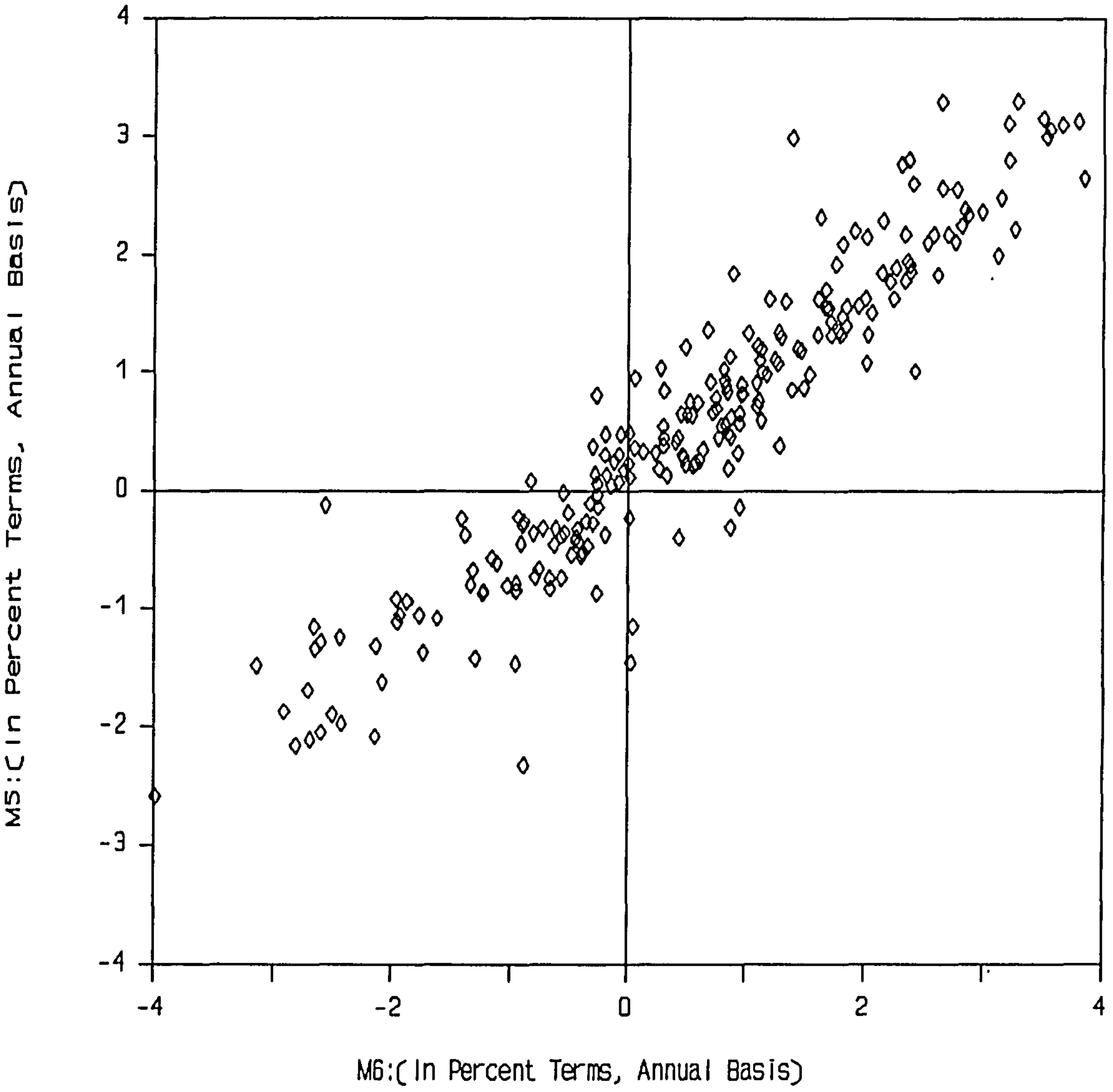


Figure 4: Plot of M5 Against m6. Both Variables Measure one Period(1 month) Excess Return over the one Month Bill.

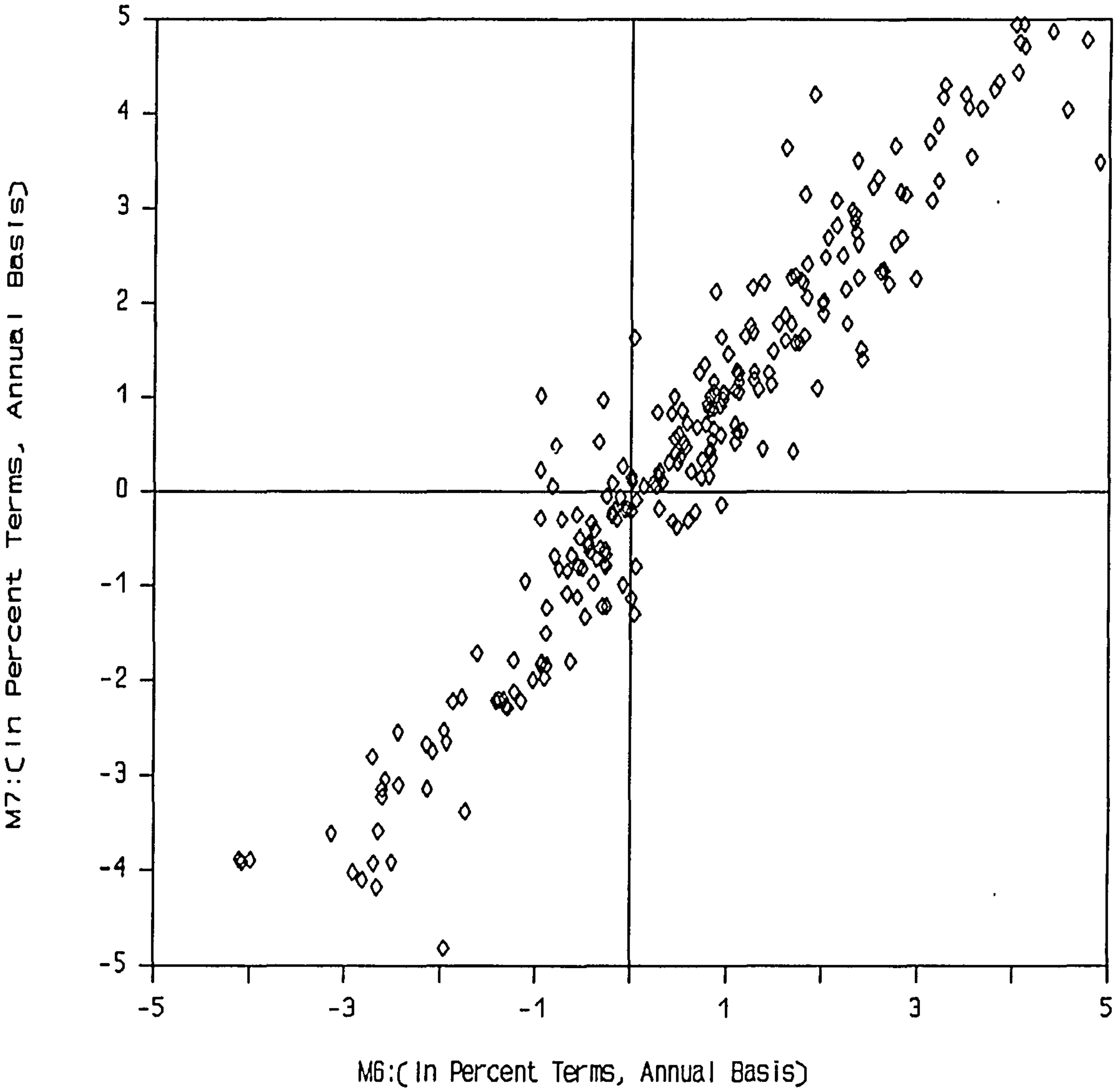


Figure 5: Plot of M7 Against m6. Both Variables Measure one Period(1 month) Excess Return over the one Month Bill.

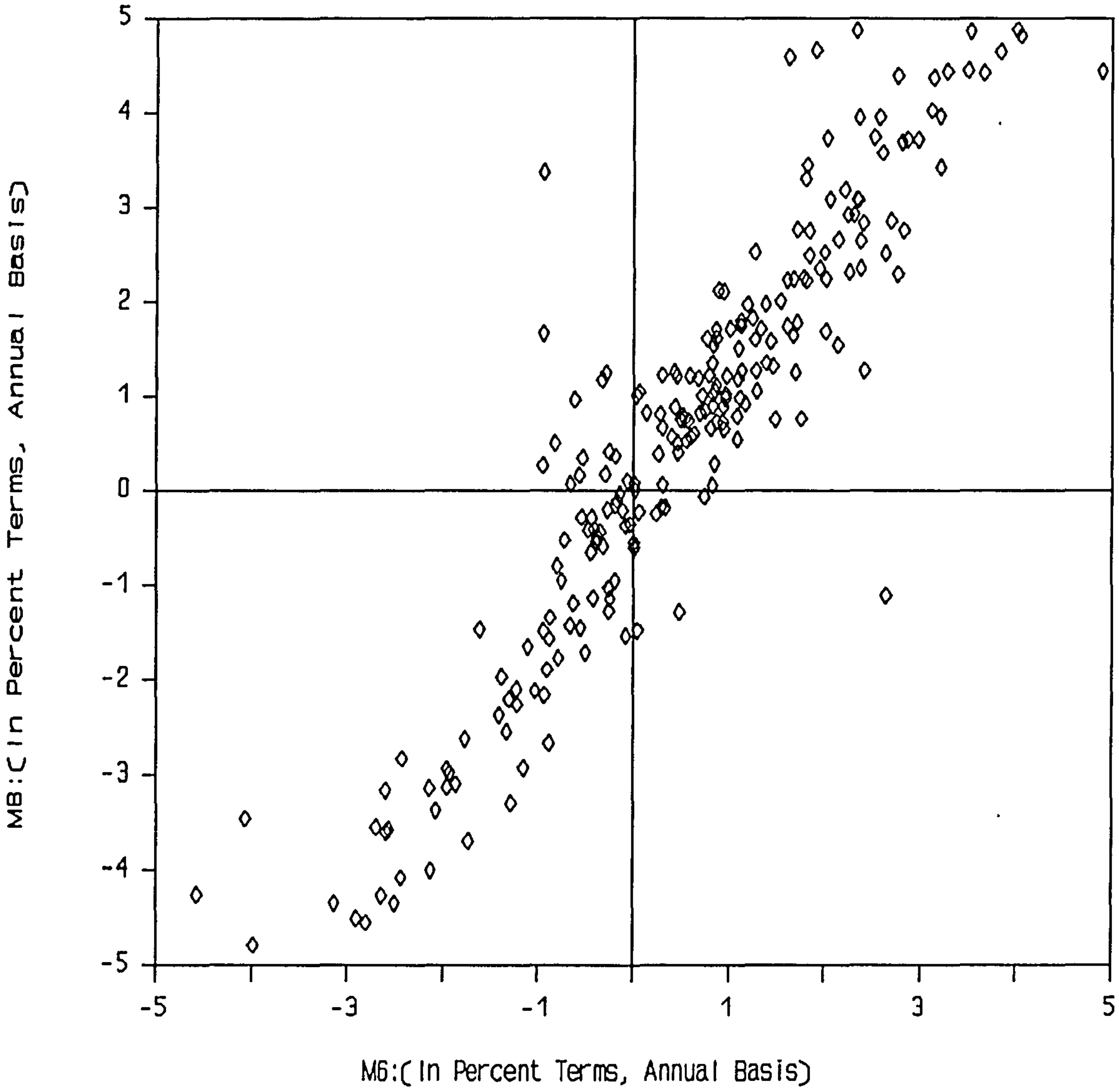
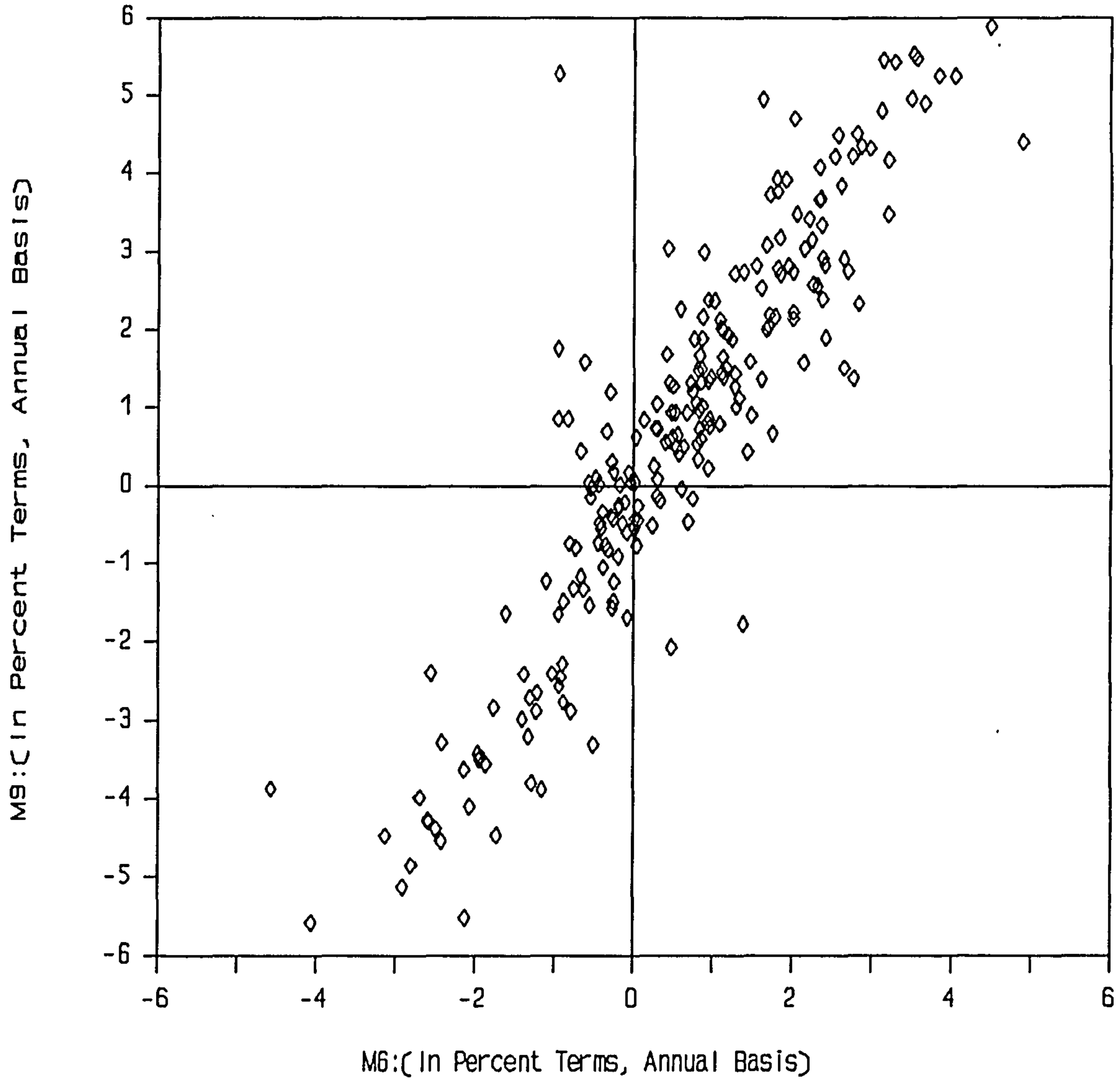
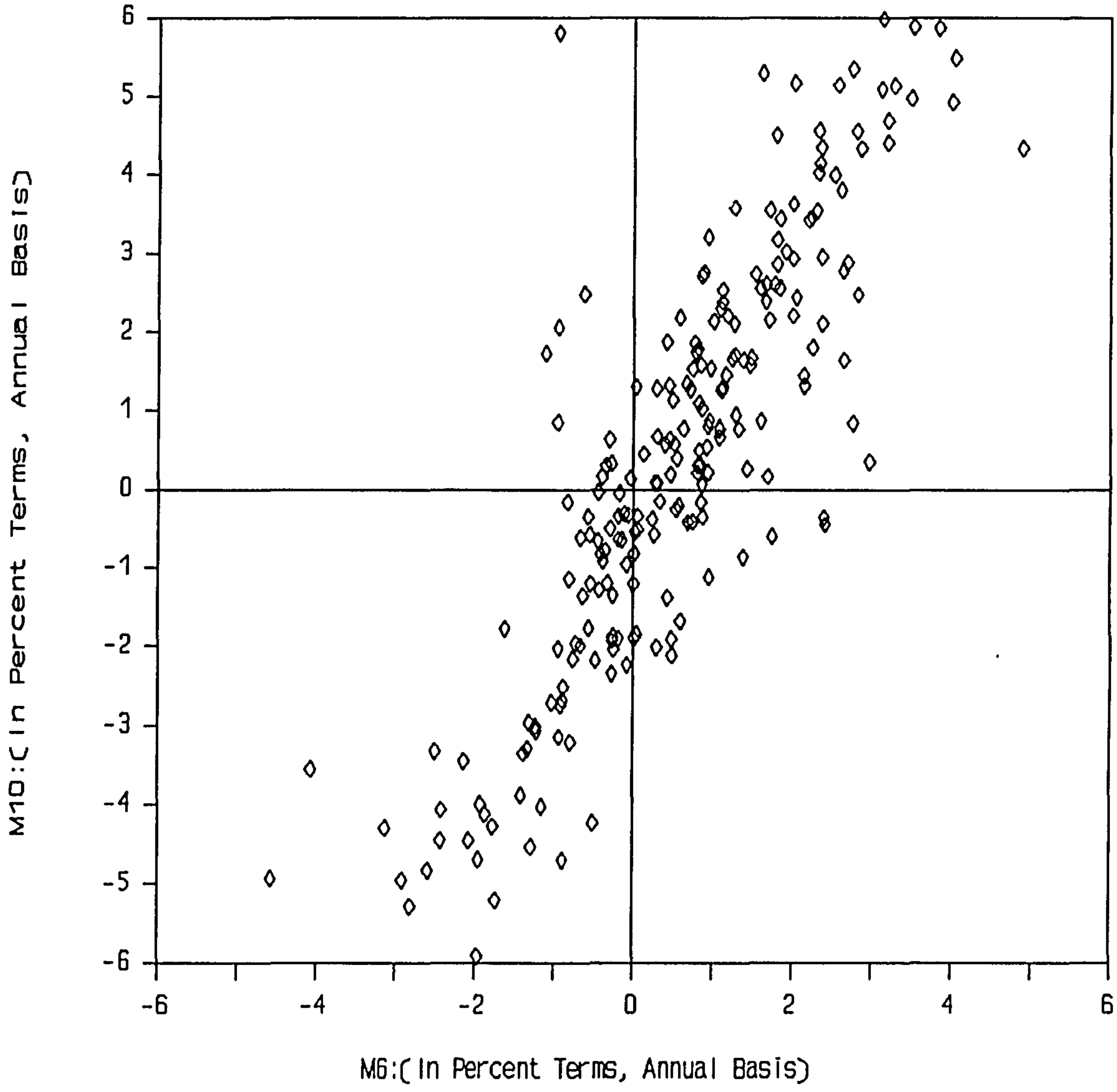


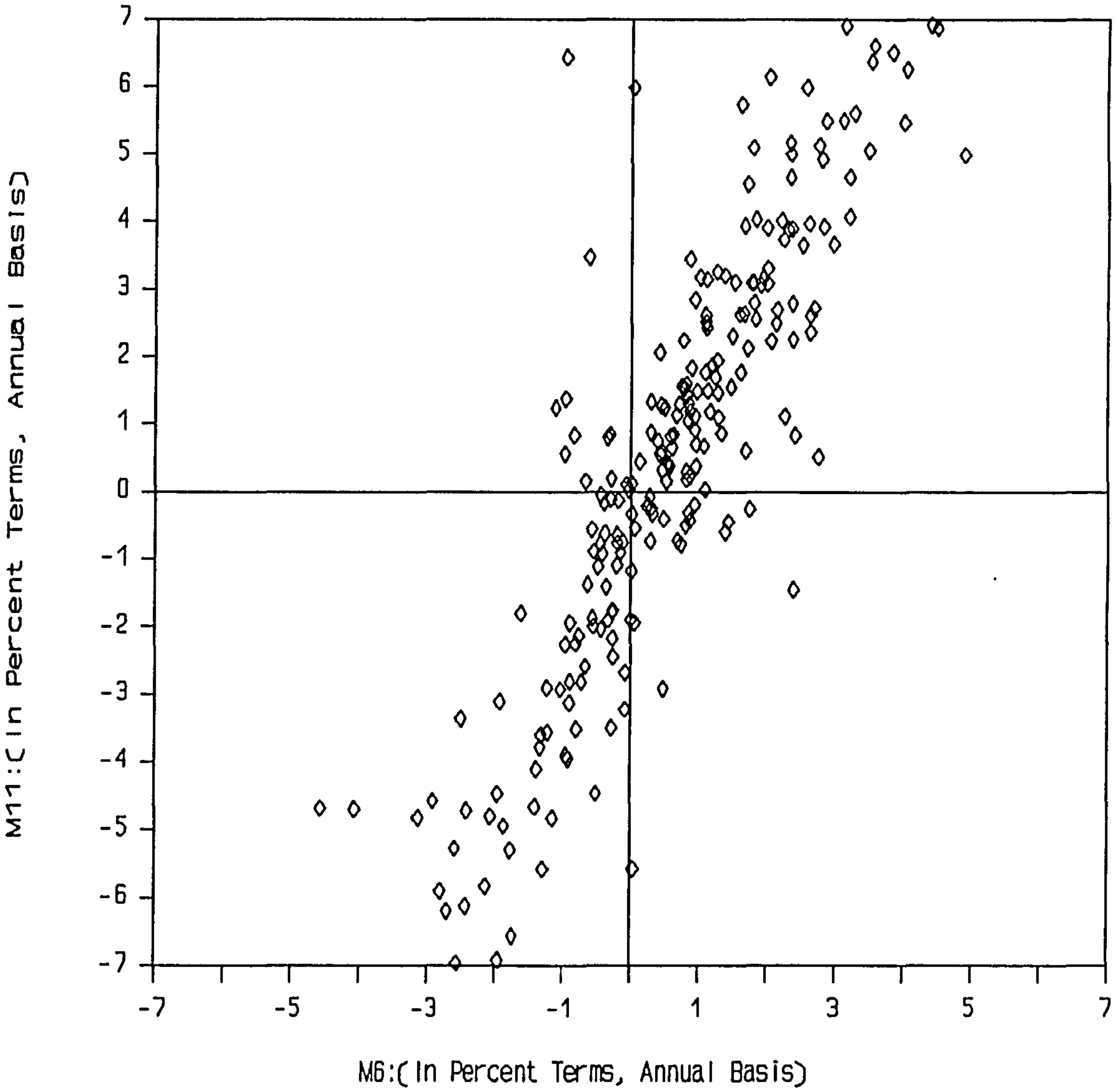
Figure 6: Plot of M8 Against M6. Both Variables Measure one Period(1 month) Excess Return over the one Month Bill.



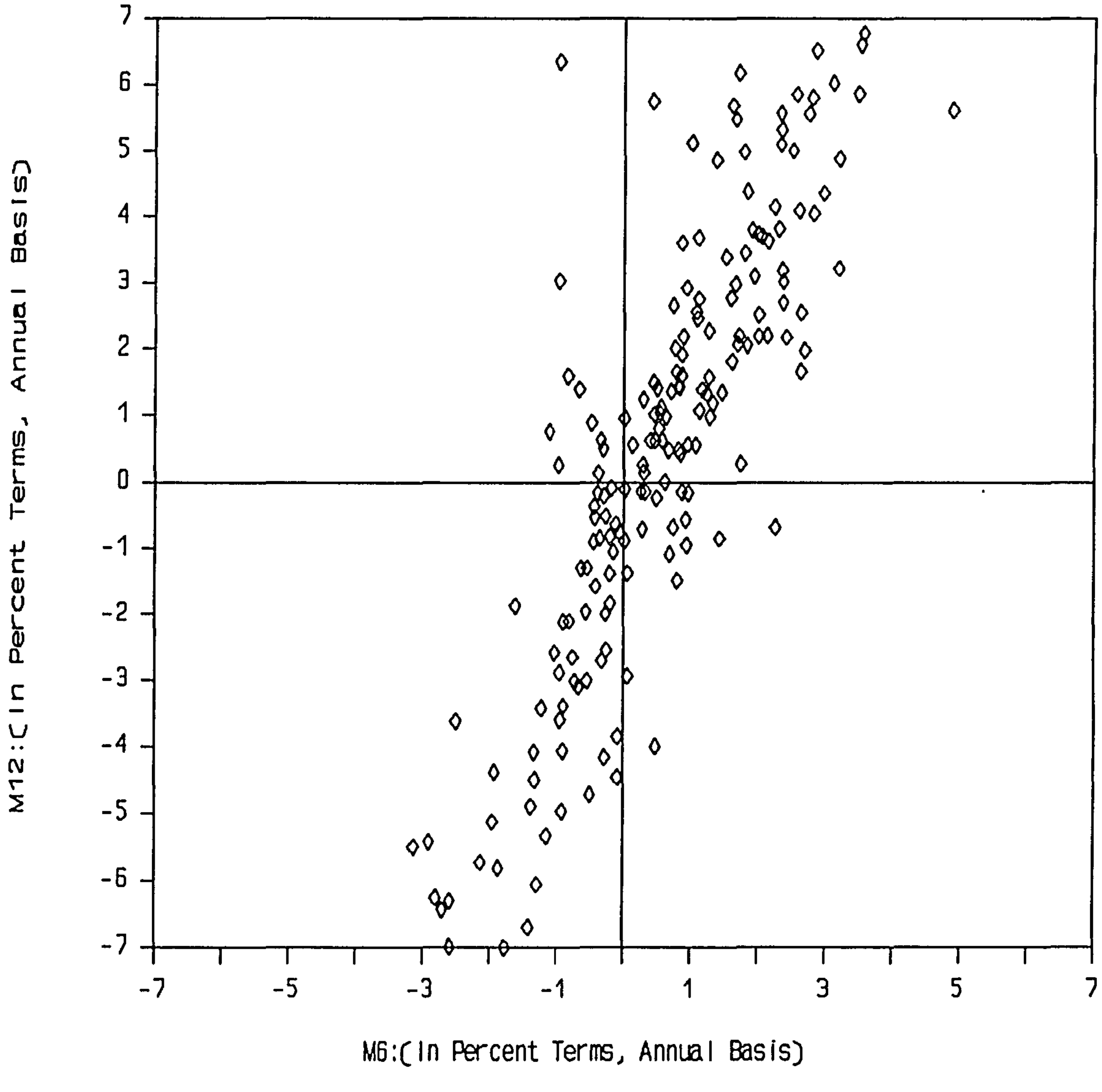
<u>Figure 7:</u> Plot of M9 Against M6. Both Variables Measure one Period(Excess Return) over the one Month Bill.



<u>Figure 8:</u> Plot of M10 Against M6. Both Variables Measure one Period(1 month) Excess Return over the one Month Bill.



<u>Figure 9:</u> Plot of M11 Against M6. Both Variables Measure(1 month) Excess Return over the one Month Bill.



<u>Figure 10:</u> Plot of M12 Against m6. Both Variables Measure one period(1 month) Excess Return over the one Month Bill.

TABLE 1

Regression of Excess Returns for Various Maturity Categories on Excess Returns of 6 Month Treasury Bill, 1965-1987, (t-statistics in parentheses)
Fama Data

Maturity in Months	Theoretical Beta	Estimated Beta	Estimated Alpha	R^2
2	0.2	0.163	0.17	.55
		(18.45)	(5.58)	
3	0.4	0.380[ab]	0.34	.82
		(36.26)	(8.95)	
4	0.6	0.589[ab]	0.173	.91
		(54.32)	(4.42)	
5	0.8	0.814[ab]	0.154	.97
		(94.9)	(4.96)	
7	1.2	1.145	-0.199	.97
		(96.44)	(-4.63)	
8	1.4	1.335	-0.135	.95
		(75.44)	(-2.11)	
9	1.6	1.538	-0.252	.94
		(68.70)	(-3.10)	
10	1.8	1.704	-0.705	.91
		(54.83)	(-6.25)	
11	2.0	1.861	-0.799	.89
		(48.19)	(-5.70)	
12	2.2	1.885	-0.806	.87
		(39.52)	(-5.22)	

[a]theoretical beta with in the 95% confidence interval.
[b]theoretical beta with in the 99% confidence interval.

TABLE 2

Regression of Excess Returns for Various Maturity Categories on Excess Returns of 6 Month Treasury Bill, 1965-1987, No intercept Term is Used, (t-stats in parenthesis). Fama Data

Maturity in Months	Theoretical Beta	Estimated Beta	R^2
2	0.2	0.175 (19.51)	.58
3	0.4	0.404[ab] (35.12)	.81
4	0.6	0.601[ab] (55.72)	.91
5	0.8	0.82[ab] (95.44)	.95
7	1.2	1.13 (95.04)	.97
8	1.4	1.32 (76.99)	.95
9	1.6	1.52 (69.16)	.94
10	1.8	1.65 (51.59)	.90
11	2.0	1.80 (45.77)	.88
12	2.2	1.82 (38.15)	.86

[a]theoretical beta with in the 95% confidence interval.
[b]theoretical beta with in the 99% confidence interval.

TABLE 3

Regression of Excess Returns for Various Maturity Categories on Excess Returns of 6 month Treasury Bill, 1965-1977, (t-statistics in parenthesis), Fama Data.

Maturity in Months	Theoretical Beta	Estimated Beta	Estimated Alpha	Adj R^2
2	0.2	0.167 (14.17)	0.101 (3.55)	0.56
3	0.4	0.347 (25.38)	0.255 (7.67)	0.80
4	0.6	0.567[ab] (31.46)	0.144 (3.29)	0.86
5	0.8	0.784[ab] (50.63)	0.155 (4.12)	0.94
7	1.2	1.130 (53.35)	-0.096 (-1.87)	0.94
8	1.4	1.267 (39.31)	0.003 (0.05)	0.90
9	1.6	1.464 (35.07)	-0.113 (-1.12)	0.88
10	1.8	1.656 (26.89)	-0.775 (-5.18)	0.82
11	2.0	1.744 (24.28)	-0.640 (-3.67)	0.79
12	2.2	2.02 (28.32)	-0.569 (-3.26)	0.85

[a]theoretical beta with in the 95% confidence interval.
[b]theoretical beta with in the 99% confidence interval.

TABLE 4

Regressions of Excess Returns for Various Maturity Categories on Excess Returns of 6 month Treasury Bill, 1978-1987, (t-statistics in parenthesis), Fama Data.

Maturity in Months	Theoretical Beta	Estimated Beta	Estimated Alpha	Adj R^2
2	0.2	0.160 (12.07)	0.28 (4.42)	0.55
3	0.4	0.388[ab] (24.96)	0.47 (6.43)	0.84
4	0.6	0.595[ab] (40.25)	0.22 (3.22)	0.93
5	0.8	0.824[b] (74.64)	0.17 (3.31)	0.97
7	1.2	1.15 (75.31)	-0.32 (-4.49)	0.97
8	1.4	1.359[ab] (60.80)	-0.27 (-2.60)	0.96
9	1.6	1.564[ab] (56.00)	-0.38 (-2.91)	0.96
10	1.8	1.717 (46.73)	-0.580 (-3.31)	0.94
11	2.0	1.90[b] (39.11)	-0.93 (-4.03)	0.92
12	2.2	1.81 (28.20)	-1.28 (-4.71)	0.90

[a]theoretical beta with in the 95% confidence interval.
[b]theoretical beta with in the 99% confidence interval.

TABLE 5

Regression of Excess Returns for Various Maturity Categories on Excess Returns of 6 month Treasury Bill, Restricted to Observations Where Excess Returns on the Reference Security is Greater than Zero,1965-1987, (t-statistics in parenthesis), Fama Data.

Maturity in Months	Theoretical Beta	Estimated Beta	Estimated Alpha	Adj R^2
2	0.2	0.183[ab] (13.28)	0.117 (2.14)	0.50
3	0.4	0.445 (29.69)	0.117 (1.96)	0.83
4	0.6	0.635[b] (42.42)	0.007 (0.12)	0.91
5	0.8	0.852 (69.68)	0.005 (0.12)	0.91
7	1.2	1.115 (69.16)	-0.096 (-1.49)	0.96
8	1.4	1.315 (53.86)	-0.088 (-0.90)	0.94
9	1.6	1.510 (50.49)	-0.209 (-1.76)	0.93
10	1.8	1.664 (39.22)	-0.639 (-3.78)	0.89
11	2.0	1.777 (32.69)	-0.578 (-2.67)	0.86
12	2.2	2.068 (19.14)	-1.014 (-3.40)	0.81

[a]theoretical beta with in the 95% confidence interval.
[b]theoretical beta with in the 99% confidence interval.

TABLE 6

Regression of Excess Returns for various Maturity Categories on Excess Returns of 6 month Treasury Bill, where the Excess Returns on the Reference Security are Negative, 1965-1987, (t-stats in parenthesis), Fama data.

Maturity in Months	Theoretical Beta	Estimated Beta	Estimated Alpha	Adj R^2
2	0.2	0.112 (6.08)	0.10 (1.86)	0.27
3	0.4	0.252 (12.01)	0.19 (3.21)	0.59
4	0.6	0.510 (18.50)	0.10 (1.34)	0.77
5	0.8	0.773[ab] (36.80)	0.17 (2.77)	0.93
7	1.2	1.200[ab] (36.03)	-0.14 (-1.48)	0.92
8	1.4	1.412[ab] (28.59)	0.01 (0.12)	0.89
9	1.6	1.678[ab] (26.12)	0.06 (0.35)	0.87
10	1.8	1.890[ab] (21.78)	-0.29 (-1.14)	0.82
11	2.0	2.12[ab] (20.96)	-0.30 (-1.03)	0.81
12	2.2	1.62 (24.44)	0.03 (0.17)	0.80

[a]theoretical beta with in the 95% confidence interval.
[b]theoretical beta with in the 99% confidence interval.

TABLE 7

Regression of Excess Returns for Various Maturity Categories on Excess Returns of 10 month Treasury Bill, 1965-1987, (t-statistics in parenthesis)
FAMA DATA

Maturity in Months	Theoretical Beta	Estimated Beta	Estimated Alpha	Adj R^2
2	0.11	0.084 (15.64)	0.256 (7.52)	0.47
3	0.22	0.199[b] (26.46)	0.519 (10.94)	0.71
4	0.33	0.311[b] (34.09)	0.447 (7.77)	0.80
5	0.44	0.434[ab] (43.72)	0.530 (8.45)	0.87
6	0.55	0.537[ab] (54.83)	0.456 (7.38)	0.91
7	0.66	0.635[b] (69.50)	0.306 (5.32)	0.94
8	0.77	0.753[ab] (83.46)	0.443 (7.78)	0.96
9	0.88	0.874ab (89.66)	0.409 (6.65)	0.96
11	1.11	1.079[b] (74.70)	-0.018 (-0.19)	0.95
12	1.22	1.12 (59.14)	0.039 (0.38)	0.93

[a]theoretical beta with in the 95% confidence interval.
[b]theoretical beta with in the 99% confidence interval.

TABLE 8

Regression of Excess Returns for Various Portfolio Maturity Categories on Excess Returns of 6 month Treasury Bill, 1965-1987, (t-stats in parenthesis), Fama Data.

Maturity of Securities in months	Theoretical Beta	Estimated Beta	Estimated Alpha	Adj R^2
2 and 10	1	0.93 (59.94)	-0.26 (-4.65)	0.92
3 and 9	1	0.96 (84.01)	0.04 (1.06)	0.96
4 and 8	1	0.96 (99.10)	0.03 (0.53)	0.97
5 and 7	1	0.98 (138.4)	-0.02 (-0.87)	0.98

NOTE: None of the theoretical betas are with in the 95% or the 99% confidence interval.

TABLE 9

Regression of Estimated Betas on their Respective Theoretical Values with the 6-month and the 10-month Treasury Bills as Reference Securities.

Reference Security: 6 month bill

Adjusted R^2 = 0.998

	Estimate	Std. Error	T stat
Intercept	0.013	0.0196	0.68
Slope	0.94	0.0155	60.49

Reference Security: 10 month bill

Adjusted R^2 = 0.999

	Estimate	Std. Error	T stat
Intercept	-0.017	0.0064	-2.76
Slope	0.999[ab]	0.0101	98.76

[a]theoretical beta with in the 95% confidence interval.
[b]theoretical beta with in the 99% confidence interval.

Table 10

Regression of Excess Return for Various Maturity Categories on Excess Return and Excess Return Squared on Reference Security, 6 Month Bill, 1965-1987, (t-statistics in parentheses), Fama Data.

Maturity in Months	Estimated Beta1	Estimated Beta2	Adjusted R^2	Change in R^2
2	0.143 (13.8)	0.003 (3.59)	0.5720	0.022
3	0.334 (29.2)	0.007 (7.36)	0.8556	0.035
4	0.558 (44.86)	0.004 (4.57)	0.9213	0.011
5	0.793 (79.4)	0.003 (3.88)	0.9718	0.001
7	1.160 (82.3)	-0.002 (-1.94)	0.9716	0.001
8	1.339 (63.3)	-0.001 (-0.36)	0.9539	0.003
9	1.554 (57.7)	-0.001 (-0.44)	0.9450	0.005
10	1.726 (46.6)	-0.003 (-1.12)	0.9166	0.006
11	1.894 (41.1)	-0.005 (1.33)	0.8947	0.004
12	1.976 (39.4)	-0.026 (-4.51)	0.8826	0.012

Table 11

Regression of Excess Return for Various Maturity Categories on Excess Return and Excess Return Squared on Reference Security, 10 Month Bill, 1965-1987, (t-statistics in parentheses), Fama Data.

Maturity in Months	Estimated Beta1	Estimated Beta2	Adjusted R^2	Change in R^2
2	0.074 (13.1)	0.001 (4.22)	0.5016	0.031
3	0.179 (23.4)	0.002 (6.40)	0.7545	0.044
4	0.294 (30.55)	0.002 (4.49)	0.8217	0.021
5	0.418 (39.5)	0.002 (3.91)	0.8808	0.010
6	0.526 (49.8)	0.001 (2.77)	0.9185	0.008
7	0.628 (63.2)	0.001 (1.67)	0.9467	0.006
8	0.741 (76.5)	0.001 (3.15)	0.9633	0.003
9	0.861 (82.2)	0.001 (3.01)	0.9680	0.008
11	1.078 (68.3)	0.000 (0.09)	0.9530	0.003
12	1.130 (55.9)	-0.002 (-1.67)	0.9335	0.003

TABLE 12

Regression of Excess Returns for Various Maturity Categories on Excess Returns of 5 and 9 Month Treasury Bills, 1965-1987, (t-stats in parentheses), Fama Data.

Mat. in Mo.	Theoretical Beta1	Est. Beta1	Theoretical Beta2	Est. Beta2	Est Alp	Adj R^2
2	0.43	0.29 (8.84)	-0.09	-0.04 (-2.77)	0.12 (3.8)	0.60
3	0.75	0.57 (16.0)	-0.12	-0.05 (-3.0)	0.23 (6.8)	0.87
4	0.93	0.81 (25.4)	-0.09	-0.04 (-2.78)	0.03 (1.10)	0.95
6	0.93	0.76 (25.0)	0.15	0.23 (14.54)	-0.04 (-1.5)	0.98
7	0.75	0.50 (12.2)	0.37	0.47 (21.78)	-0.15 (-3.7)	0.97
8	0.43	0.21 (6.47)	0.65	0.75 (42.90)	0.02 (0.71)	0.98
10	-0.56	-0.01 (-0.07)	1.40	1.10 (27.6)	-0.42 (-5.6)	0.96
11	-1.25	-0.30 (-3.02)	1.87	1.36 (25.9)	-0.39 (-4.0)	0.95
12	-2.06	-0.48 (-5.52)	2.40	1.51 (32.1)	-0.39 (-4.8)	0.96

NOTE: None of the theoretical betas are with in the 95% or the 99% confidence interval.

Table 13

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M2 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	0.146[b]	3.36	0.0009
α_1	0.335[b]	3.39	0.0008
α_2	-0.025	-0.36	0.7155
β	0.168[b]	14.32	0.0001
β_1	-0.080[a]	-2.13	0.0335
β_2	-0.002	-0.16	0.8727

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	0.146[b]	3.41	0.0007
α_{11}	0.086	0.64	0.5176
α_{12}	0.563[b]	4.39	0.0001
α_{21}	-0.143	-1.59	0.1118
α_{22}	0.095	1.06	0.2870
β	0.168[b]	14.53	0.0001
β_{11}	-0.030	-0.66	0.5062
β_{12}	-0.123[a]	-2.02	0.0442
β_{21}	0.007	0.25	0.7984
β_{22}	-0.007	-0.37	0.7052

[a]significant at 5% level   [b]significant at 1% level

Table 14

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M3 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	0.350[b]	6.65	0.0001
α_1	0.100	1.02	0.3085
α_2	-0.099	-1.07	0.2854
β	0.389[b]	27.52	0.0001
β_1	-0.045	-1.42	0.1555
β_2	-0.009	-0.41	0.6764

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	0.350[b]	6.73	0.0001
α_{11}	-0.012	-0.09	0.9237
α_{12}	0.242	1.91	0.0569
α_{21}	-0.268[a]	-2.20	0.0280
α_{22}	0.078	0.65	0.5132
β	0.389[b]	27.84	0.0001
β_{11}	-0.067	-1.43	0.1537
β_{12}	-0.027	-0.70	0.4818
β_{21}	0.003	0.07	0.9389
β_{22}	-0.012	-0.47	0.6379

[a]significant at 5% level [b]significant at 1% level

Table 15

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M4 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	0.249[b]	4.62	0.0001
α_1	-0.102	-1.13	0.2559
α_2	-0.204	-1.87	0.0615
β	0.602[b]	41.56	0.0001
β_1	-0.039	-1.55	0.1209
β_2	-0.005	-0.19	0.8461

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	0.249[b]	4.64	0.0001
α_{11}	-0.145	-1.18	0.2378
α_{12}	-0.013	-0.11	0.9066
α_{21}	-0.305[a]	-2.14	0.0330
α_{22}	-0.121	-0.83	0.4071
β	0.602[b]	41.76	0.0001
β_{11}	-0.074	-1.69	0.0916
β_{12}	-0.026	-0.93	0.3524
β_{21}	0.053	1.16	0.2446
β_{22}	-0.035	-1.03	0.3004

[a]significant at 5% level   [b]significant at 1% level

Table 16

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M5 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	0.225[b]	5.28	0.0001
α_1	-0.118	-1.78	0.0747
α_2	-0.276[a]	-2.45	0.0146
β	0.807[b]	70.28	0.0001
β_1	0.010	0.54	0.5849
β_2	0.044	1.54	0.1245

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	0.225[b]	5.39	0.0001
α_{11}	-0.137	-1.59	0.1117
α_{12}	-0.115	-1.42	0.1542
α_{21}	-0.489[b]	-3.33	0.0010
α_{22}	-0.049	-0.32	0.7471
β	0.807[b]	71.73	0.0001
β_{11}	0.025	0.85	0.3917
β_{12}	0.004	0.20	0.8391
β_{21}	-0.085	-1.68	0.0941
β_{22}	0.083[b]	2.61	0.0095

[a]significant at 5% level   [b]significant at 1% level

Table 17

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M7 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	-0.133[a]	-2.09	0.0371
α_1	-0.034	-0.38	0.7007
α_2	-0.470[b]	-2.99	0.0030
β	1.175[b]	68.51	0.0001
β_1	-0.054[a]	-2.22	0.0270
β_2	-0.061	-1.45	0.1473

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	-0.133[a]	-2.13	0.0341
α_{11}	0.119	1.09	0.2737
α_{12}	-0.179	-1.68	0.0931
α_{21}	-0.169	-0.84	0.3990
α_{22}	-0.790[b]	-3.54	0.0005
β	1.175[b]	69.66	0.0001
β_{11}	-0.079[a]	-2.19	0.0293
β_{12}	-0.045	-1.71	0.0883
β_{21}	-0.215[a]	-2.44	0.0151
β_{22}	-0.019	-0.42	0.6685

[a]significant at 5% level [b]significant at 1% level

Table 18

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M8 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	-0.081	-0.76	0.4481
α_1	-0.062	-0.44	0.6570
α_2	-0.092	-0.50	0.6142
β	1.404[b]	51.87	0.0001
β_1	-0.096[a]	-2.56	0.0109
β_2	-0.184[b]	-3.39	0.0008

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	-0.081	-0.76	0.4425
α_{11}	0.217	1.28	0.2016
α_{12}	-0.274	-1.64	0.1016
α_{21}	0.028	0.13	0.8965
α_{22}	-0.025	-1.00	0.3174
β	1.404[b]	52.52	0.0001
β_{11}	-0.198[b]	-3.62	0.0003
β_{12}	-0.055	-1.37	0.1714
β_{21}	-0.149	-1.64	0.1007
β_{22}	-0.192[b]	-3.12	0.0020

[a]significant at 5% level   [b]significant at 1% level

Table 19

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M9 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	-0.306[a]	-1.96	0.0500
α_1	0.084	0.43	0.6634
α_2	0.153	0.69	0.4890
β	1.655[b]	42.92	0.0001
β_1	-0.171[b]	-3.37	0.0008
β_2	-0.174[b]	-2.88	0.0042

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	-0.306[a]	-1.97	0.0489
α_{11}	0.376	1.63	0.1028
α_{12}	-0.157	-0.69	0.4883
α_{21}	0.111	0.43	0.6655
α_{22}	0.204	0.71	0.4754
β	1.655[b]	43.12	0.0001
β_{11}	-0.254[b]	-3.53	0.0005
β_{12}	-0.138	-2.53	0.0118
β_{21}	-0.249[a]	-2.35	0.0194
β_{22}	-0.156[a]	-2.38	0.0178

[a]significant at 5% level   [b]significant at 1% level

Table 20

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M10 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	-0.637[a]	-2.36	0.0187
α_1	-0.011	-0.03	0.9717
α_2	-0.144	-0.43	0.6621
β	1.784[b]	19.25	0.0001
β_1	-0.118	-1.13	0.2572
β_2	-0.066	-0.64	0.5213

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	-0.637[a]	-2.35	0.0192
α_{11}	0.031	0.08	0.9301
α_{12}	0.005	0.01	0.9875
α_{21}	-0.316	-0.85	0.3941
α_{22}	0.092	0.22	0.8188
β	1.786[b]	19.18	0.0001
β_{11}	-0.185	-1.45	0.1469
β_{12}	-0.090	-0.82	0.4077
β_{21}	-0.066	-0.43	0.6675
β_{22}	-0.076	-0.71	0.4745

[a]significant at 5% level [b]significant at 1% level

Table 21

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M11 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	-1.018[a]	-2.22	0.0273
α_1	0.245	0.48	0.6249
α_2	0.312	0.62	0.5351
β	2.508[b]	14.99	0.0001
β_1	-0.700[b]	-3.96	0.0001
β_2	-0.665[b]	-3.78	0.0002

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	-1.018[a]	-2.21	0.0278
α_{11}	0.482	0.88	0.3779
α_{12}	0.103	0.19	0.8492
α_{21}	0.288	0.53	0.5904
α_{22}	0.345	0.61	0.5393
β	2.508[b]	14.94	0.0001
β_{11}	-0.826[b]	-4.16	0.0001
β_{12}	-0.650[b]	-3.59	0.0004
β_{21}	-0.693[b]	-3.28	0.0012
β_{22}	-0.660[b]	-3.70	0.0003

[a]significant at 5% level [b]significant at 1% level

Table 22

Tests of Tax Effect Using Dummy Variables. The Top Panel Present Results for the Complete Data and the Bottom Panel Present Results With Division of Data into Two Groups: Pre 1978 and Post 1978. Dep Var:M12 Ind Var:M6

No Division of Data

Variable	Estimate	T Score	Prob>|T|
α	-0.854^{b}	-3.99	0.0001
α_1	0.113	0.36	0.7164
β	1.918^{b}	28.74	0.0001
β_1	-0.070	-0.73	0.4632

Division of Data into Two Groups

Variable	Estimate	T Score	Prob>|T|
α	-0.854^{b}	-4.06	0.0001
α_{11}	0.378	1.08	0.2774
α_{12}	-0.780	-1.73	0.0842
β	1.918^{b}	29.28	0.0001
β_{11}	0.030	0.25	0.8014
β_{12}	-0.175^{b}	-1.57	0.0004

[a]significant at 5% level   [b]significant at 1% level

Table 23
Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M2 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	$RSS_r = 72.36$	273	0.265
Unrestricted	$RSS_u = 71.90$	271	0.265
Difference	$RSS_d = 0.46$	2	0.230

$F = [(RSS_d/2)]/[(RSS_u/271)] = 0.86$
Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	0.221^b	4.86	0.0001
α_1	-0.084	-1.29	0.1958
β	0.161^b	16.85	0.0001
β_1	0.005	0.19	0.8431

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	$RSS_r = 72.36$	273	0.265
Unrestricted	$RSS_u = 70.60$	267	0.264
Difference	$RSS_d = 1.76$	6	0.293

$F = [(RSS_d/6)]/[(RSS_u/267)] = 1.11$
Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	0.286^b	4.41	0.0001
α_1	-0.114	-1.24	0.2129
α_2	-0.142	-1.54	0.1244
α_3	-0.155	-1.69	0.0912
β	0.166^b	15.62	0.0001
β_1	-0.035	-1.45	0.1467
β_2	0.003	0.01	0.9903
β_3	-0.001	-0.04	0.9641

[a]significant at 5% level [b]significant at 1% level.

Table 24

Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M3 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =101.68	273	0.372
Unrestricted	RSS_u = 95.88	271	0.353
Difference	RSS_d = 5.80	2	.900

$F = [(RSS_d/2)]/[(RSS_u/271)] = 8.21^b$
Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

<u>Dummy Variable Method:</u>

Variable	Estimate	T Score	Prob> |T|
α	0.430^b	8.18	0.0001
α_1	-0.143	-1.90	0.0573
β	0.389^b	35.22	0.0001
β_1	-0.085^b	-2.89	0.0041

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =101.68	273	0.372
Unrestricted	RSS_u = 88.90	267	0.332
Difference	RSS_d = 12.78	6	2.130

$F = [(RSS_d/6)]/[(RSS_u/267)] = 6.41^b$
Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

<u>Dummy Variable Method:</u>

Variable	Estimate	T Score	Prob>|T|
α	0.628^b	8.69	0.0001
α_1	-0.371^b	-3.59	0.0004
α_2	-0.256^a	-2.48	0.0137
α_3	-0.425^b	-4.14	0.0001
β	0.391^b	33.22	0.0001
β_1	-0.025	-0.88	0.3746
β_2	-0.120^b	-3.40	0.0008
β_3	-0.034	-0.76	0.4452

[a]significant at 5% level [b]significant at 1% level.

Table 25
Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M4 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =107.99	273	0.395
Unrestricted	RSS_u =103.28	271	0.381
Difference	RSS_d = 4.71	2	2.355

$F = [(RSS_d/2)]/[(RSS_u/271)] = 6.18^b$
Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	0.232^b	4.24	0.0001
α_1	-0.080	-1.03	0.3025
β	0.600^b	52.25	0.0001
β_1	-0.090^b	-2.93	0.0036

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =107.99	273	0.395
Unrestricted	RSS_u =101.63	267	0.380
Difference	RSS_d = 6.36	6	1.060

$F = [(RSS_d/6)]/[(RSS_u/267)] = 2.78^a$
Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	0.311^b	4.01	0.0001
α_1	-0.144	-1.31	0.1900
α_2	-0.128	-1.15	0.2481
α_3	-0.192	-1.75	0.0806
β	0.601^b	47.74	0.0001
β_1	-0.019	-0.62	0.5348
β_2	-0.118^b	-3.12	0.0020
β_3	-0.047	-0.97	0.3300

[a]significant at 5% level [b]significant at 1% level.

Table 26

Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.

Dependent Variable:M5 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r = 68.14	273	0.249
Unrestricted	RSS_u = 65.94	271	0.243
Difference	RSS_d = 2.20	2	1.100

$F = [(RSS_d/2)]/[(RSS_u/271)] = 4.52^a$

Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	0.157^b	3.60	0.0004
α_1	0.023	0.37	0.7104
β	0.824^b	89.84	0.0001
β_1	-0.072^b	-2.96	0.0033

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r = 68.14	273	0.249
Unrestricted	RSS_u = 61.58	267	0.230
Difference	RSS_d = 6.56	6	1.093

$F = [(RSS_d/6)]/[(RSS_u/267)] = 4.75^b$

Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	0.263^b	4.36	0.0001
α_1	-0.175^a	-2.05	0.0410
α_2	-0.125	-1.45	0.1466
α_3	-0.042	-0.49	0.6232
β	0.834^b	84.48	0.0001
β_1	-0.070^b	-2.99	0.0030
β_2	-0.074^a	-2.51	0.0125
β_3	-0.093^a	-2.48	0.0136

[a]significant at 5% level [b]significant at 1% level.

Table 27

Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M7 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =130.46	273	0.477
Unrestricted	RSS_u =128.74	271	0.475
Difference	RSS_d = 1.72	2	0.860

$F = [(RSS_d/2)]/[(RSS_u/271)] = 1.81$
Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	-0.218[b]	-3.53	0.0005
α_1	0.015	0.17	0.8610
β	1.199[b]	38.52	0.0001
β_1	-0.063	-1.87	0.0623

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =130.46	273	0.477
Unrestricted	RSS_u =128.02	267	0.479
Difference	RSS_d = 2.44	6	0.406

$F = [(RSS_d/6)]/[(RSS_u/267)] = 0.84$
Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	-0.289[b]	-3.33	0.0010
α_1	0.146	1.18	0.2392
α_2	0.099	0.80	0.4245
α_3	0.074	0.61	0.5418
β	1.218[b]	23.43	0.0001
β_1	-0.033	-0.51	0.6052
β_2	-0.082	-1.34	0.1792
β_3	-0.081	-1.50	0.1333

[a]significant at 5% level [b]significant at 1% level.

Table 28

Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M8 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =289.82	273	1.061
Unrestricted	RSS_u =289.62	271	1.068
Difference	RSS_d = 0.20	2	0.100

$F = [(RSS_d/2)]/[(RSS_u/271)] = 0.09$
Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	-0.163	-1.75	0.0804
α_1	0.052	0.39	0.6902
β	1.345[b]	28.85	0.0001
β_1	-0.012	-0.24	0.8037

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =289.82	273	1.061
Unrestricted	RSS_u =286.01	267	1.071
Difference	RSS_d = 3.81	6	0.635

$F = [(RSS_d/6)]/[(RSS_u/267)] = 0.59$
Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	-0.277[a]	-2.13	0.0341
α_1	0.232	1.24	0.2153
α_2	0.181	0.98	0.3263
α_3	0.134	0.73	0.4648
β	1.441[b]	17.38	0.0001
β_1	-0.148	-1.47	0.1411
β_2	-0.083	-0.87	0.3818
β_3	-0.112	-1.31	0.1890

[a]significant at 5% level [b]significant at 1% level.

Table 29

Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M9 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =463.54	273	1.697
Unrestricted	RSS_u =462.03	271	1.704
Difference	RSS_d = 1.51	2	0.755

$F = [(RSS_d/2)]/[(RSS_u/271)] = 0.44$
Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	-0.320^b	-2.70	0.0073
α_1	0.141	0.86	0.3905
β	1.531^b	26.42	0.0001
β_1	0.006	0.10	0.9139

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =463.54	273	1.697
Unrestricted	RSS_u =449.53	267	1.683
Difference	RSS_d = 14.01	6	2.335

$F = [(RSS_d/6)]/[(RSS_u/267)] = 1.38$
Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	-0.461^a	-2.81	0.0052
α_1	0.290	1.23	0.2198
α_2	0.461^a	2.01	0.0454
α_3	0.113	0.49	0.6218
β	1.698^b	16.64	0.0001
β_1	-0.254^a	-2.05	0.0411
β_2	-0.212	-1.81	0.0708
β_3	-0.145	-1.37	0.1699

[a]significant at 5% level [b]significant at 1% level.

Table 30
Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M10 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =892.97	273	3.270
Unrestricted	RSS_u =878.68	271	3.242
Difference	RSS_d = 14.29	2	7.145

$F = [(RSS_d/2)]/[(RSS_u/271)] = 2.20$
Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	-0.942[b]	-5.76	0.0001
α_1	0.470[a]	2.06	0.0397
β	1.718[b]	21.74	0.0001
β_1	-0.021	-0.25	0.7998

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =892.97	273	3.270
Unrestricted	RSS_u =872.49	267	3.267
Difference	RSS_d = 20.48	6	3.413

$F = [(RSS_d/6)]/[(RSS_u/267)] = 1.04$
Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	-1.132[b]	-4.84	0.0001
α_1	0.362	1.10	0.2725
α_2	0.726[a]	2.24	0.0256
α_3	0.586	1.80	0.0729
β	1.831[b]	12.57	0.0001
β_1	-0.116	-0.95	0.3400
β_2	-0.115	-0.69	0.4892
β_3	-0.135	-0.89	0.3692

[a]significant at 5% level [b]significant at 1% level.

Table 31

Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M11 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =1379.57	273	5.053
Unrestricted	RSS_u =1371.21	271	5.059
Difference	RSS_d = 8.36	2	4.180

$F = [(RSS_d/2)]/[(RSS_u/271)] = 0.82$
Critical Values: $F_{0.95}(2,271) = 3.03$ and $F_{0.99}(2,271) = 4.69$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	-0.768^b	-3.78	0.0002
α_1	-0.011	-0.04	0.9675
β	1.751^b	18.28	0.0001
β_1	0.130	1.24	0.2148

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =1379.57	273	5.053
Unrestricted	RSS_u =1359.45	267	5.091
Difference	RSS_d = 20.12	6	3.353

$F = [(RSS_d/6)]/[(RSS_u/267)] = 0.658$
Critical Values: $F_{0.95}(6,267) = 2.13$ and $F_{0.99}(6,267) = 2.88$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	-0.950^b	-3.30	0.0011
α_1	0.377	0.92	0.3556
α_2	0.335	0.83	0.4039
α_3	-0.006	-0.01	0.9866
β	1.681^b	10.27	0.0001
β_1	0.094	0.46	0.6416
β_2	0.208	1.07	0.2837
β_3	0.203	1.19	0.2334

[a]significant at 5% level [b]significant at 1% level.

Table 32

Tests of Beta Instability Model Using Both Restricted Regression(F Test) and Dummy Variable(T test) Methods.
Dependent Variable:M12 Independent Variable:M6

Two Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =1164.71	227	5.130
Unrestricted	RSS_u =1151.66	225	5.118
Difference	RSS_d = 13.05	2	6.525

$F = [(RSS_d/2)]/[(RSS_u/225)] = 1.27$
Critical Values: $F_{0.95}(2,225) = 3.04$ and $F_{0.99}(2,225) = 4.70$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob> |T|
α	-0.777[b]	-3.34	0.0010
α_1	-0.146	-0.46	0.6441
β	2.109[b]	17.64	0.0001
β_1	-0.162	-1.29	0.1972

Four Group Tests

Model	Residual Sum of Squares	Degrees of Freedom	RSS/DF
Restricted	RSS_r =1164.71	227	5.130
Unrestricted	RSS_u =1123.76	221	5.084
Difference	RSS_d = 40.95	6	6.825

$F = [(RSS_d/6)]/[(RSS_u/221)] = 1.342$
Critical Values: $F_{0.95}(6,221) = 2.14$ and $F_{0.99}(6,221) = 2.90$.

Dummy Variable Method:

Variable	Estimate	T Score	Prob>|T|
α	-1.255[b]	-3.84	0.0002
α_1	0.912[a]	1.98	0.0488
α_2	0.589	1.27	0.2053
α_3	0.101	0.22	0.8189
β	2.071[b]	12.37	0.0001
β_1	-0.091	-0.40	0.6892
β_2	-0.207	-1.06	0.2888
β_3	-0.234	-1.31	0.1912

[a]significant at 5% level [b]significant at 1% level.

Chapter IV

Errors-in-Variables and a Re-Examination of the FWEP

4. Errors-in-Variables and the FWEP:

The empirical tests of some one-factor equilibrium models of term structure of interest rates provide partial support for FWEP but not the SAP and ADEP. The two-factor model tests do not generate a substantial increas in the explanatory power over the one-factor models.

In this chapter, we will focus on the FWEP model. There is the possibility that the earlier results may be biased if there are errors in measuring the variables. The econometric problems in the presence of measurement errors are discussed first and various suggested methods to deal with it are explored.

Let us explore the problems in estimating the coefficient of the simple linear regression model where we have one independent variable. Consider the simple linear regression model

$$y_t = \alpha + \beta x_t + \delta_t \ . \tag{1}$$

If the standard assumptions of a linear regression model are met, then the regression provides an unbiased estimate of the true ß. One of the standard assumptions is that the independent variable x is non stochastic and independent of the error term. Consider the case where both y_t and x_t are observed with error. The observed variables are

$$y_t' = y_t + v_t$$
$$x_t' = x_t + u_t \tag{2}$$

and upon rearranging equation (2)

$$y_t = y_t' - v_t$$
$$x_t = x_t' - u_t \tag{2a}$$

and substituting equation (2a) into (1)

$$y_t' - v_t = \alpha + \beta(x_t' - u_t) + \delta_t \tag{3}$$

and simplifying equation (3) gives

$$y_t' = \alpha + \beta x_t' + (\delta_t + v_t - \beta u_t) \tag{4}$$

If δ_t, v_t, u_t and x_t are independent of each other and are independently and identically distributed as $N(0,\sigma_\delta^2)$, $N(0,\sigma_v^2)$, $N(0,\sigma_u^2)$ and $N(\mu,\sigma_x^2)$ respectively, then the ordinary least squares estimate β' is inconsistent as long as σ_u^2 is not equal to zero and the probability limit of the estimate is

$$\beta' = \beta(\sigma_x^2)/(\sigma_x^2 + \sigma_u^2) \quad \leq \beta \tag{5}$$

and therefore in the presence of measurement error the least square estimate of the slope is biased downward. It is unbiased when σ_u^2 is zero. The measurement error biases the intercept term upward.

Note that the inconsistency of the least square estimator comes about due to the fact that the observed independent variable is stochastic in the presence of measurement error and is negatively correlated to the error term, as can be seen in equation (4a). This biases the least squares estimator downwards. It should be noted that the measurement error in the dependent variable, σ_v^2, causes no special problem as the error is incorporated into the disturbance term, σ_δ^2. Thus, measurement error in the dependent variable poses no problem, but measurement error in the independent variable renders the least squares estimate inconsistent. As can be seen from equation (5), the downward bias is directly related to the magnitude of the variance of the error in measuring the independent variable.

Textbook exposition on estimation in the presence of measurement errors in the variables and a proof of equation (5) can be found in Johnston(1984), Kmenta(1971), and Theil(1971). The discussion so far has been confined to the case where there is only one independent variable. Levi(1973) has explored the effect on estimates when there are more than one independent variable and only one of them is measured with error. It is shown that the coefficient of the variable measured with error is biased downward and the bias of the coefficients on the other independent variables, measured without error, can be determined. Levi(1977) explores bounds on the coefficient of the independent variables where only one of them is measured with error.

Klepper and Leamer(1984) explore the case where all independent variables are measured with error. They establish conditions under which bounds can be found for all coefficients. Leamer(1987) explores errors in variables in the context of systems of equations. Klepper(1988) develops a series of diagnostics to explore the effect on the coefficient of a variable when all the variables are measured with error. The above mentioned work is useful for models with more than one independent variable. The model studied in this chapter has only one independent variable.

As Leser(1974) has pointed out, errors of measurement are generally assumed to be negligible in comparison to errors in equation. Otherwise, ordinary least squares would have to be rejected in most cases.

Let us explore some of the methods available in measuring the coefficient of ordinary least squares in the presence of measurement error. Let us note that without any additional information on the magnitude of σ_u^2, σ_v^2 or the ratio, $\tau = \sigma_u^2/\sigma_v^2$, we can not hope to find an unbiased estimator as the parameters are not all identified. The proof on the above statement can be found in Johnston(1984) or Schmidt(1976). An excellent text book exposition can be found in Leser(1974) where a numerical example is used.

As already noted before, $\sigma_u^2 = 0$ would imply that no estimation bias exists and OLS estimate would be consistent even in the presence of measurement error in the dependent variable. If $\sigma_v^2 = 0$ and $\sigma_u^2 > 0$, then the inverse of the slope found by regressing x on y would provide an unbiased estimate. This procedure is referred to as inverse least squares estimation and also as reverse regression.

In most applications, these extreme situations are unlikely to exist. It is more likely the case that measurement errors will exist in both the dependent and the independent variables. In these situations, the unbiased estimate requires knowledge of σ_u^2, or σ_v^2, or the ratio τ. The ratio τ will range from zero to infinity. An expression for finding the slope given knowledge of τ can be found in Schmidt(1976) and Johnston(1984). The estimate based on knowledge of τ lies between the estimates given by OLS and Reverse regressions.

In the event that this ratio is 1, then the estimate is given by orthogonal regression, where sum of the squared errors is minimized along a line perpendicular to the estimating line. In the event that the ratio is equal to the ratio of the variance of the dependent and the independent variables, then the estimate is given by diagonal regression. Here again the sum of the squared errors are minimized along a line perpendicular to the regression line but the independent and the dependent variables have been standardized such that they have the same variance. The diagonal regression estimate is a geometric mean of OLS and inverse least square estimates. The diagonal regression estimator, unlike the orthogonal regression estimator is not sensitive to units of measurement.

Unless one has prior knowledge of the variances of measurement errors or their ratio, one faces serious difficulty in estimating the coefficient of a simple linear regression. All that can be said is that the OLS and the reverse regression estimates will bound the true value of the regression coefficient. In some applications this knowledge may suffice and in others it may not.

One method which receives attention in most econometrics text (see Johnston(1984), Theil(1971)) is the Instrumental Variable method, and is commonly referred to as the IV method. The IV method calls for replacing the independent variable, which is measured with error, with another variable which is closely related to it and is not measured with error. This method provides a consistent estimate of the slope but the variance is likely to be high. It is usually difficult to find a variable which is both closely related and is not measured with error.

Another variant of the IV approach is to arrange the original independent variable with the corresponding dependent variable in an ascending order and then partition the data into two or three groups. If three groups are used the middle group of observations are discarded. The slope of the relationship is then given by the line joining the means of the two groups. This method requires that the groups based on the observed independent variable,one measured with error, be identical to, in rank, the groups based on the true variable which can not be observed. This requires assumption of some very strange distribution of the independent variable and the measurement error. Furthermore, it also calls for very small measurement errors. These conditions are unlikely to be satisfied in most cases(see Madansky(1959), Schmidt(1976),and Theil(1971)).

Let us look at an application of OLS, Inverse, Orthogonal and diagonal estimators to a set of data provided by the O.E.C.D. This is adopted from Leser(1974) and the reader is referred to it for a complete discussion. The data for thirteen countries on growth in GNP(independent variable), changes in unemployment rate(dependent variable), and gross fixed investment(instrumental variable) is utilized for the example. The results of applying the five estimators follow.

$$b_x = 0.3067 \text{ (OLS)}$$

$$b_y = 0.7212 \text{ (Inverse)}$$

$$b_o = 0.3465 \text{ (Orthogonal)}$$

$$b_d = 0.4703 \text{ (Diagonal)}$$

$$b_z = 0.2725 \text{ (IV)}$$

The above example suggests considerable uncertainty regarding the value of true coefficient if measurement error is suspected, but the researcher has no knowledge of its magnitude. Without prior information all that can be said is that the slopes computed from the forward and reverse regressions ,b_x and b_y respectively, will bound the true value.

Theil(1971) suggests a plausible reformulation of the model specification so that the problems due to measurement errors are avoided. Suppose the relationship in equation (1) is re-specified as

$$y_t = \beta x_t \exp(\delta_t) \tag{6}$$

This specification assumes that the relationship is proportional except for a multiplicative error term. Furthermore the observed variable, x_t', is assumed to be related to the true variable xt as

$$x_t' = x_t(1+\Gamma_t) \tag{7}$$

and taking the natural log on equation (6), gives

$$\log y_t = \log \beta + \log x_t + \delta_t \tag{8}$$

and taking the natural log of equation 7, gives

$$\log x_t' = \log x_t + \epsilon_t \tag{9}$$

where ϵ_t is equal to log $(1 + \Gamma_t)$. Combining equations (8) and

(9) yields

$$\log (y_t/x_t') = \log \beta + (\delta_t - \epsilon_t) \qquad (10)$$

If δ and ϵ are assumed to have zero means, constant variances, constant contemporaneous covariance and uncorrelated over time, than the standard linear regression model can be applied to equation (10).

The least squares estimator of log β is the geometric mean of the log of the ratio, (y_t/x_t). Taking the anti-log of the estimate gives the coefficient of the relationship specified in equation (7). Confidence intervals can than be computed since the ratio on the left hand side of equation (10) is distributed with the same mean and variance.

We have not attempted to explain how the data may be measured with error. At this time a selected set of articles which deal with errors in variable model applications in finance will be discussed. A possible reason for the measurement errors in the data employed for this study are then explored.

Scholes and Williams(1977) study nonsynchronous trading and its impact on estimating the betas. It is shown that given nonsynchronous trading in securities market, the returns based on transaction prices are measured with error. This biases the estimates of the intercept and slope in the simple market model. Much research in finance calls for a correctly measured beta and therefore the problem can be severe. It is shown that the problem is particularly severe when daily data is used. They document the bias for securities with differing levels of trading frequency and suggest estimators which are consistent. They also show that the problem of nonsynchronous trading of securities has for most common stocks little impact on returns when monthly returns are utilized. The Treasury Bill returns used in this study utilize monthly returns and are not likely to be affected by the problem of nonsynchronous trading.

Booth and Smith(1985) reexamine the much noted small firm effect given nonsynchronous trading and the potential error in choosing the correct market index. They use estimates from forward and reverse regressions to bound the true value of the coefficient. It is concluded that the small firm effect cannot be attributed to errors in variables. The model for Treasury Bill returns developed in this paper does not call for a specific index. One is free to choose the maturity of the independent variable unlike the capital asset pricing model where the index used is the market portfolio.

Blume and Stambaugh(1983) also examine the size effect and conclude that use of closing transaction prices explains about half of the observed size effect. The closing transaction price will deviate from the true price if the transaction reflects a market order. The use of closing prices bias the computed daily returns upward. They also conclude that the bias induced by nonsynchronous trading is 1000 times smaller than the one induced by the bid-ask effect. However, the use of continuously compounded returns overcomes the bias induced by nonsynchronous trading and the bid-ask effect. Treasury Bill returns employed in this study are continuously compounded.

Keim(1989) attributes the observed high returns for stocks surrounding the two days at the turn of the calendar year to the bid-ask effect. Thomson(1989) also concludes that the bid-ask effect may explain the observed turn of the year effect in many studies.

In studies employing stock market data, use of closing prices tends to bias the computed return upward. These causes severe econometric problems, especially if daily returns are utilized, as noted in the discussion earlier. The Treasury Bill data employed in this study are continuously compounded returns. These returns are in the Fama Treasury Bill term structure files, which are part of the bond files provided by Center for Research in Security Prices, University of Chicago.

It should be noted that the computed returns in the Fama files are based on an average of the bid-ask prices, for which the Federal Reserve Bank of New York is the source. Based on our earlier discussion on stock market studies utilizing daily closing prices and the concomitant econometrics problems, one may erroneously conclude that use of monthly and average prices will get around the problems of bias and thus all the discussion on errors in variable problem is not relevant to the present study on Treasury Bill data. It may be true that bias in computed returns may not exist but measurement error in computed returns still exists even if the average of the bid-ask is an unbiased estimate of the true price but is measured with some error. This is true because there is no reason to believe that the true price is the average price in all cases.

Let us make this point clear through an example. Suppose one third of the returns are measured with no error, another one third measured with 5% error on the positive side and another one third measured with 5% error on the negative side. It is easily seen that no systematic bias, upward or downward, in overall returns exists in the data. The errors-in-variable econometric problems are caused not only by systematic bias but also by measurement errors. The data in the above example has no systematic bias but is measured with some error and thus would impart a downward bias to the slope of a regression which utilizes these returns as an explanatory variable.

Let us look at the problems induced by measurement error in a theoretical framework. The measurement errors in returns are a function of the measurement error in prices. In cases where the measured returns are not continuously compounded, the returns are biased if the prices are measured with error. The latter situation is applicable for studies which utilize closing prices to measure non-continuously compounded returns. Blume and Stambaugh(1983) show that the measured returns are biased upward and provide approximate lower bounds for the bias, and the bias is shown to be a function of the variance of the measurement error on prices.

The theoretical framework presented below is drawn from Blume and Stambaugh(1983). Let the true return for a security for the period t-n to t be denoted as

$$r_{t-n,t} = (P_t/P_{t-n}) - 1 \tag{6}$$

where P_t is the true price at time t, and P_{t-n} is the true price at time t-n. Let the return based on observed prices be

$$r'_{t-n,t} = (P'_t/ P'_{t-n}) - 1 \tag{7}$$

The observed price for any period t is modeled as

$$P_t' = (1+u_t)P_t \tag{8}$$

where E(u) = 0. Combining (6), (7), and (8) yields

$$r_{t-n,t}' = [(1+u_t)/(1+u_{t-n})](1+r_{t-n,t}) - 1 \tag{9}$$

and noting that the true prices are independent of contemporaneous and non-contemporaneous measurement errors and taking expectation of both sides of (9) gives

$$E(r_{t-n,t}') = E[(1+u_t)/(1+u_{t-n})]\ [1+E(r_{t-n,t})] - 1 \tag{10}$$

and invoking Jensen's inequality, $E[(1+u_t)/(1+u_{t-n})] > 1$ and therefore the expectation of return based on observed prices, $E(r_{t-n,t}')$, is greater than the expectation of return based on true prices, $E(r_{t-1,t})$. Blume and Stambaugh(1983) show that half of the size related effect is due to the bias in measured return as discussed above. Note that the computed returns are not continuously compounded.

Recall our earlier assertion that the bias in applying OLS procedures to the regression models does not exist if the computed returns are measured on a continuously compounded basis. Rewrite equation (9) for continuous compounding framework as

$$\ln(1+r'_{t-n,t}) = \ln[(1+u_t)/(1+u_{t-n})] + \ln(1+r_{t-n,t}) \quad (11)$$

The expectation of $\ln[(1+u_t)/(1+u_{t-n})]$, i.e., $\ln(1+u_t) - \ln(1+u_{t-1})$, is equal to zero if it is assumed that the measurement errors, u, are identically and independently distributed over time. Therefore, the expectation of the measured returns are equal to the expectation of the true returns when continuously compounded returns are used and thus no systemic bias exists for the measured returns. However, the measured returns involve errors and the errors in variable problem remains.

This development for continuously compounded return opens up for handling the errors in variable problem. If we lengthen the period over which the returns are computed, the variance of the measurement error as a proportion of the variance of the measured return gets smaller. The variance of the measured return for the period, t-n to t, is

$$\mathrm{var}[\ln(1+r'_{t-n,t})] = \mathrm{var}[\ln(1+r_{t-n,t})] + 2\mathrm{var}[\ln(1+u)] \quad (12)$$

because

$$var[ln(1+u_t) - ln(1+u_{t-n})] = var[ln(1+u_t)] + var[ln(1+u_{t-n})] \quad (13)$$

and given that the error term, u, is drawn from the same process for each period, the variance of the error term for any return interval will be 2var[ln(1+u)] where the time subscript has been dropped. An n-period continuously compounded return is a sum of n one period continuously compounded returns. Note that if returns are assumed to be independently and identically distributed, then

$$var[ln(1+r_{t-n,t})] = n\ var[ln(1+r)] \quad (14)$$

where var[ln(1+r)] is the variance for one period and time subscripts have been dropped. Thus, the variance of an n period return is the sum of n one period variances. Combining (12) and (14) yields

$$var[ln(1+r'_{t-n,t}) = n\ var[ln(1+r)] + 2var[ln(1+u)]. \quad (15)$$

As the measurement interval is increased, the variance of the measurement error decreases as a proportion of the variance of the true but unobservable return, i.e., $var[ln(1+u)]/var[ln(1+r'_{t-n,t}]$ decreases as n increases. Recall that the bias due to measurement error when using OLS is

$$1/[1+(\sigma_u^2/\sigma_x^2)] \tag{16}$$

where σ_x^2 and σ_u^2 are variances of the explanatory variable and the variance of the measurement error respectively. Given our modeling of the measurement error, the bias is reduced as the measurement interval is increased. Tests of the single factor model based on excess returns computed over a two month period are discussed later.

The bias term in (16) should also be sensitive to the choice of the explanatory variable used to test the model. The variance of the measurement error is invariant to the choice of the explanatory variable. The explanatory variable with higher variance will impart a smaller bias to the OLS estimates. Tests based on the six-month and the ten-month bill excess returns as explanatory variables are discussed below.

The testing of the FWEP model utilized both the six-month bill and the ten-month bill. The results are in Table 1 and 7 respectively. Note that at 99% confidence level only three of the 10 regressions validate the FWEP model when the excess return on 6-month Bill is utilized as the independent variable and 8 of the 10 regressions validate the model when the excess return on a 10-month Bill is utilized as the independent variable. Recall that the choice of maturity of the independent variable is not specified in the model.

Let us look at the results of forward and reverse regressions in Table 33 where M6 is chosen as the reference security. The forward and reverse regressions bound the theoretical value for seven of the 10 regressions. Thus, if measurement error is suspected, the lack of validation of the model based on just the forward regression in Table 1 needs to be viewed with caution. The validation of the model is strong when both forward and reverse estimates are used to test the model. The validation of FWEP with forward and reverse estimates is even stronger when M10 is used as the reference security. In nine of the ten regressions, the forward and reverse estimates bound the theoretical values. The results are presented in Table 34.

The results based on just the forward estimates are much stronger when M10 is used as the reference security. The differences can either be ascribed to statistical artifact or explained by positing that the choice of the 10 month Bill is more appropriate given the possibility of measurement error. Recall that the bias of the OLS estimate is given by the following term:

$$\sigma_x^2/(\sigma_x^2 + \sigma_u^2)$$

and rearranging,

$$1/(1 + (\sigma_u^2/\sigma_x^2))$$

where σ_u^2 and σ_x^2 denote the variance of measurement error and the variance of the true variable respectively. Note that, for a given value of σ_u^2, the bias induced by measurement error is smaller, the larger the variance of the true value. The observed variance of the true variable includes both the true variance and the measurement variance. Thus, the observed variance should be a good proxy for the true variance.

The mean and the variance of the bid-ask spread of Treasury bill prices may be related to measurement errors in monthly returns. However, it is not possible to reconstruct prices from the Treasury bill return file and thus gauge the differences in measurement error, if any, across various maturities of bills. There is no reason to believe that the returns for a 10-month Bill are measured with much greater error than for a 6 month Treasury Bill. Thus, the bias would be related to the observed variance of the 6-month and the 10-month Treasury Bills. The greater the observed variance for the explanatory variable, the lower the bias. The sum of squares for the 10-month Bill is 10,729.57 and for the 6-month Bill it is 3387. Unless the variance of the measurement error for the 10-month Bill is three times as high the variance for the 6-month Bill, using the 10-month Bill would impart a smaller bias to the OLS estimate. Thus, the stronger results provided by the use of the 10-month Bill can be attributed to the fact that the errors in variable problem is less severe with the use of 10 month Bill.

In conclusion, the use of forward and reverse estimates or the use of forward estimates with M10 as the reference security provide much stronger support for the single factor beta stability model and this casts doubt on the earlier conclusion of weak support for the FWEP model based on M6 as the reference security.

Tests of the model where the excess return is measured over a two-month period are presented in Tables 35 and 36. Table 38 presents regression results where the explanatory variable is the two-month excess return on a six-month bill. For six of the nine regressions, the theoretical values are within the 99% confidence interval. This compares very favorably to results presented in Table 1 when the measurement interval used is one month. The results in Table 39 also provide strong support for the model as eight of the nine regression confidence intervals include the theoretical value.

It is interesting to note that the variance for the explanatory variable is about twice as large for a two-month measurement interval than for a one-month measurement interval. This is the case for both the six- and the ten-month Bills. This is consistent with the expectations based on assumptions about the return generating process. The stronger validation of the model based on the two-month period excess returns indicates that the suspicion of the measurement error, is perhaps, correct. It is apparent that measurement error induced bias can be partly overcome either by increasing the measurement interval or by choosing an explanatory variable with a higher variance, if possible. Fortunately, testing of the single-factor model imposes no restriction on the measurement interval and the choice of the explanatory variable.

Let us turn our attention to the results based on the log approach due to Theil, which was discussed earlier. Under this approach, the anti-log of the geometric mean of the log of the ratio, y_t'/x_t', is the slope of the regression. Recall that these ratios are distributed with the same mean and variance. Given that we have a large sample,and invoking the central limit theorem(see DeGroot(1975), it is reasonable to assume that the log of the ratios are normally distributed. This enables us to construct confidence intervals for the point estimates.

Table 37 presents point estimates and confidence intervals at 95% and 99% level. The result is strongly in favor of the single factor beta stability model and in contrast to the result based on the OLS test. All confidence intervals bound the theoretical values implied by the model. However, one must view these strong results with some caution as the log based specification is not as efficient as the OLS specification, but it does correct for the errors in variable problem by providing consistent estimates. The large confidence intervals in Table 43 reflect the loss of efficiency.

Furthermore, not all data points could be utilized for the results presented in Table 43. The logs of negative ratios are undefined and so negative ratios were excluded. The number of data points excluded ranged from 19 to 55. Recall that the sample size is 274 for all regressions except the 12 month bill for which the sample size is 229. However, it is not unreasonable to expect that the excluded negative ratios fell equally between the negative-positive and positive-negative quadrants. If this is the case the results in Table 43 need not be biased due to the exclusion of negative ratios. Furthermore, the results based on absolute values and positive ratios should not be different statistically if the excluded negative ratios are indeed divided into the two quadrants as discussed earlier. Note that the theoretical relationship calls for all data points falling in the negative-negative and positive-positive quadrants.

The results are not presented here, but a means test shows that the results based on positive ratios and absolute value ratios are statistically not different. The test used assumes equal population variances for both types of ratios and is discussed in DeGroot(1975). A test under the possibility of unequal population variances was also carried out and the results were identical. This test is discussed in Mendenhall, Renmuth, Beaver, and Durhan(1986). In conclusion, the log-based approach provides strong support for the single factor beta stability model.

At this point let us explore non-parametric methods to handle the errors in variable problem. In particular, the focus is on two non-parametric estimators, both due to Theil(1971). One involves the computation of n/2 slopes, and the other n(n-1)/2 slopes, where n is the number of observations, to measure point estimates and confidence intervals for the slope of the relationship. Denote these two estimators as TH_1 and TH_2 respectively.

The TH_1 estimator is based on the following conditions: (1) the independent variable is fixed in repeated samples and

(2) the error terms are either (2a) distributed independently, identically and continuously or (2b) distributed independently, symmetrically and continuously. Note that TH_1 allows the error terms to have unequal variances and, thus, it can handle hetroscedasticity.

TH_2 is also based on the conditions listed above except that (2b) is not applicable. Furthermore both estimators can handle error in measuring the independent variable as long as the errors do not change the ranking with probability 1. Let us briefly describe these two estimators. Given n observations, arrange them in an ascending order,and delete the middle one if n is odd. Divide the observations into two groups of m observations each, where m = n/2. Now compute m slopes as follows:

$$b_i = (y_i - y_{m+i})/(x_i - x_{m+i}) \qquad i = 1,\ldots m$$

These m slopes contain the median ß. The confidence interval is given by the following expression:

$$P[b_r \leq \beta \leq b_{m-r+1}] = 1 - 2^{1-m}\, \Sigma_i\, {}_mC_i$$

where i runs from zero to r-1 and ${}_mC_i$ denotes combinations of i objects taken at a time from a set of m. The farther r is away from m, the greater is the term on the right hand side, thus giving a larger confidence coefficient. Theil(1976) contains an application of this approach to Dutch data on a Consumption-Income relationship.

The TH_2 estimator utilizes all possible slopes. Here the slope is given as follows:

$$\beta_{ij} = (y_i - y_j)/(x_i - x_j) \qquad i<j \text{ and } i,j = 1..n.$$

Given n observations this estimator computes n(n-1)/2 slopes. The median value of these slopes is the estimate of the slope of the relationship.

For a large sample, construction of a confidence interval for the slope at a confidence coefficient, $1-\alpha$, where α is the significance level for a two tailed test, involves the following steps. First, the slopes need to be arranged in an ascending order. The lower limit of the confidence interval β_L is the kth value of the slope counting from the smallest value of the array of the slopes. The upper limit β_U is the kth value of the slopes counting backwards from the largest value of the array of slopes. Here, k is defined as

$$k = (N - C_{\alpha/2})/2$$

where N is the total number of possible slopes given n observations and

$$C_{\alpha/2} \approx Z_{\alpha/2}\ [n(n-1)(2n+5)/18]^{1/2}$$

where $Z_{\alpha/2}$ is the value of z from a standard normal table for which $\alpha/2$ of the area under the curve is to its right. For all tests except where M12 is the dependent variable, the number of observations, n , is 275. The number of all possible slopes, N, is 37,675. The median value is 18,837 and the value for k is 16,875 for 99% confidence level and 17,343 for 95% confidence level.

Hollander and Wolfe(1973) and Daniel(1978) provide a detailed discussion on constructing the confidence interval for the TH_2 estimator. Daniel(1978) also provides a table for values of k for small sample sizes. Sen(1968) has a more advanced discussion on the relative efficiency of the TH_2 estimator to the OLS estimator for non-normal distributions. It also addresses the sensitivity of the TH_2 estimator to the presence of measurement error in the explanatory variable. It suggests that if the errors are finite and sufficiently small, and the ranking is unaffected, then the estimator is consistent.

Note that the OLS estimate is a weighted average of all slopes computed under TH_2. Given that the TH_2 estimate is the median of these slopes, it is likely to be more robust than the OLS estimate in the presence of large errors in the data. It also is more robust than the OLS estimator in the presence of departure from normality. As stated previously TH_1 and TH_2 may be less affected by the errors in variable than the OLS which is clearly inconsistent in the presence of measurement errors. Note that the non-parametric estimators are non-linear. A linear estimator is one where the estimate can be expressed as a linear combination of the dependent variable. The best linear unbiased estimator is given by OLS and it is the best unbiased(both linear and nonlinear) if it is further assumed that the error terms are normally distributed.

Let us illustrate the effect of measurement error on TH_1, TH_2 and OLS estimators through the following set of data. Assume that the following represents the true values of the independent and the dependent variables.

Y:	10	20	30	40	50	60
X:	1	2	3	4	5	6

In the regression of Y on X, the TH_1 estimate, TH_2 estimate, and the OLS estimates are, respectively:

$TH_1 = 10$ $TH_2 = 10$ $OLS = 10$

Recall that it is the measurement error in the explanatory variable that causes the bias when OLS is used. Let us explore the impact of measurement error in the independent variable by inserting an error factor of 0.1. Three of the explanatory variables are measured with error on the positive side and the other three with error on the negative side. The revised data set is as follows:

Y:	10	20	30	40	50	60
X:	0.9	2.1	2.9	3.9	5.1	6.1

The new estimates are:

$TH_1 = 10$ $TH_2 = 10$ $OLS = 9.698$

The non-parametric estimates, TH_1 and TH_2, are not affected by the presence of measurement error, whereas the OLS estimate is clearly biased downward. Recall that the non-parametric estimators remain consistent in the presence of measurement error as long as the errors do not change the natural ranking of the observed independent variable. This is so in the example used above. The TH_1 estimation process generates 3 slopes, which range from 9.375 to 10, and whose median is 10. The TH_2 estimator generates 15 slopes, which range from 8.33 to 12.5, and the median value of these slopes is 10.

Let us explore the robustness of the non-parametric estimators to the presence of gross outliers. The data set is as follows:

Y:	10	20	30	40	50	200
X:	1	2	3	4	5	6

The revised data set is different from the original data set only with respect to the last element of Y, which is now 200 as opposed to 60 before. A re-estimation yields:

$TH_1 = 10$ $\qquad$ $TH_2 = 10$ $\qquad$ $OLS = 29.31$

The non-parametric estimates are unaffected by the presence of a gross outlier. The TH_1 estimator generates three slopes which range from 10 to 53.125, and the median is 10. The TH_2 estimator generates 15 slopes which range from 8.33 to 150 and the median value is 10. Although the examples used may seem contrived, it does demonstrate the point that the non-parametric estimators may be more suitable in the presence of measurement error and gross outliers.

The assumption of normality is clearly made for convenience. Notwithstanding the central limit theorem in many cases non-normality of the error term is documented and so casts doubt on conclusions drawn from tests which assume normality. Blattberg and Gonedes(1974) document the non-normality of the error term for regressions with stock market returns. Presence of fat-tailed error distributions renders invalid the use of test statistics based on OLS regressions. In some cases the problem may be severe enough to reverse inferences.

D'Agostino and Stephens(1986) note that the OLS residuals tend to be "super normal", meaning that the OLS residuals are usually normal even when the true residuals are non-normal. They suggest that detection of non-normal OLS residuals would indicate non-normality of population error term. Theil(1971) discusses the impact of non-normal error terms on testing and inferences. The robustness of these tests seem to depend on the normality of the explanatory variable. A more advanced discussion can be found in Kendall and Stuart(1961).

Tests are conducted to check for the normality of the error term, the excess return on Treasury Bills which also serves as the explanatory variable, and the raw return on bills. There are many methods available to test for normality. D'Agostino and Stephens(1986) provide perhaps the most comprehensive list of techniques available to test for normality. Many statistical packages include some of these tests.

One method to test for normality of a variable is to measure the statistics, skewness, and kurtosis. The skewness and kurtosis are 0 and 3 respectively for a normal distribution. The test employed in this paper is documented in SAS Basics(1985). The definitions of skewness and kurtosis are

$$\text{Skewness} = [n(n-1)(n-2)]\ \Sigma_i(x_i - x)^3/s^3$$

$$\text{Kurtosis} = \{\{[n(n+1)(n-1)(n-2)(n-3))]\ \Sigma_i(x_i - x)^4/s^4 - [3(n-1)(n-1)/(n-2)(n-3)]$$

where

n : number of observations

x_i: a variable where i ranges from 1 to n.

x : average of variable set x_i

s : standard deviation of the variable x_i

The skewness detects the tendency of deviations from mean to be larger in one direction than in the other. In a normal distribution, deviations are of the same magnitude in both direction. Thus for a normal distribution, the expected value for the sample statistic is zero. Note that the skewness measure is invariant to change in scale since it is divided by the cube of sample standard deviation.

Kurtosis measures the heaviness of the tails of a distribution. This statistic is also invariant to scale. For a normal distribution, the expected value of the kurtosis measure is 3. However the SAS procedure subtracts 3 from this statistic as can be seen in the definition above. Therefore, for a normal distribution, the expected SAS defined kurtosis measure is 0. For a large sample size, the variance of the sample coefficients of skewness and kurtosis are given by 6/n and 24/n respectively, where n is the sample size(see Stuart and Ord(1987). Dividing the sample coefficients, skewness and kurtosis, by the respective standard deviation yields a Z value. One can than use a standard normal probability table to test for deviations from normality. Aggarwal, Rao, and Hiraki(1989) document skewness and kurtosis for the Japanese equity returns using the test statistics discussed in the preceding paragraphs.

Skewness and Kurtosis statistics on OLS residuals from Treasury Bill regressions are presented in Table 38. Except for the residuals from the regression with the eight-month bill, the skewness measure is statistically different from zero. The sign of the skewness statistics is almost equally divided between positive and negative. Kurtosis values are all significantly different from zero and positive. This provides strong support for non-normal error terms for the regressions used in this study.

As noted earlier, departure of the error term from normality would affect testing and inferences. The effect is dependent on the distribution of the explanatory variable. A test of normality for the explanatory variables M6 and M10, along with the dependent variables is presented in Table 39. The skewness and kurtosis measures are all significantly different from zero. The skewness statistics are all positive except for the 12 month Bill. This indicates a positively skewed distribution with fat tails. For completeness, results of normality tests for raw monthly returns are presented in Table 40. The skewness statistics are all positive and significantly different from zero. The kurtosis measures are all significantly different from zero. The Treasury Bill returns are characterized, like the stock market returns, with a fat tailed distribution.

In a recent paper by Connolly(1989) the much documented day of the week and the weekend effect are shown to depend on the estimation and the testing method employed. When estimation and testing procedures which account for non-normality are used, the results are much weaker.

Corrado(1988) documents the TH_2 to be about 12% more efficient than the OLS for computation of regression slopes with student-t distributions, which is fat-tailed relative to the normal. In a related paper Corrado(1989) also documents the non-normality of the abnormal returns utilized in many studies and shows that non-parametric tests are better specified than the parametric t-test.

Tables 41 through 44 present the tests of the single-factor model using non-parametric testing procedures. The test of the model based on TH_1 estimator is presented in Table 41. The independent variable for these tests is M6. In 9 out of 10 regressions, the theoretical value is within the 99% confidence interval. Recall that only 3 of the 10 regressions using the OLS estimator bound the theoretical value for a 99% confidence interval.

The results based on the TH_2 estimator, where the reference security is the six-month Treasury Bill, are presented in Table 42. Recall that the TH_2 estimator is computationally more demanding than the TH_1 estimator. Given 275 observations, the TH_1 estimator computes 137 slopes, whereas the TH_2 estimator computes 37,675 slopes. Six of the ten regressions bound the theoretical values within the 99% confidence interval. The non-parametric estimators, TH_1 and TH_2, both provide strong support for the single factor model, in contrast to the OLS results which provide very weak support for the model. This suggests that the measurement errors are present but affect the OLS and non-parametric estimators differently. Recall that the measurement errors do not bias the non-parametric estimates if the errors do not change the natural ranking of the explanatory variable.

Tests utilizing the TH_1 and TH_2 are also conducted with the ten-month Bill as the reference security. The results are presented in Tables 43 and 44 and support the FWEP model. The degree of support is very similar to the case when the six-month Bill is utilized as the reference security. This is in sharp contrast to the results based on OLS tests, where the support for the FWEP model,is weak when the six-month Treasury Bill is chosen as the reference security, and is strong when the ten-month Treasury Bill is chosen as the reference security.

Given documented non-normal errors of the OLS residuals, and thepossibility of measurement error, non-parametric tests are better specified than the OLS tests. The results based on non-parametric tests indicate strong support for the single factor model for Treasury Bill returns. We are compelled to revise our earlier conclusion of very weak support for the FWEP model based on OLS tests to strong support for the FWEP model based on alternative tests.

CONCLUSIONS:

In conclusion, many attempts have been made to test the FWEP model with alternative methods given the possibility of measurement error. Tests of the model using estimates from forward and reverse regressions, suggest that the FWEP model can not be rejected. Lengthening the period from 1 to 2 months over which the excess returns are measured and then testing the model with OLS also provides strong support for the model. Test of the model based on the log approach provide strong support for the single factor model. This is also true when two non-parametric tests due to Theil are utilized. This leads us to revise our earlier conclusions of weak support for the FWEP model based on OLS tests in favor of strong support based on alternative tests which better account for errors in variables.

TABLE 33

Test of one-factor model using both forward and reverse regressions. The estimates provide a bound on the theoretical value for each regression. Reference Security:M6

Dep Var	Forward Estimate	Theoretical Value	Reverse Estimate
M2[a]	0.163	0.20	0.290
M3[a]	0.380	0.40	0.460
M4[a]	0.589	0.60	0.645
M5	0.814	0.80	0.840
M7	1.145	1.20	1.190
M8[a]	1.335	1.40	1.400
M9[a]	1.335	1.60	1.630
M10[a]	1.700	1.80	1.880
M11[a]	1.861	2.00	2.080
M12	1.885	2.20	2.150

[a]thoeoretical value within the bounds provided by forward and reverse regressions.

TABLE 34

Test of one-factor model using both forward and reverse regressions. The estimates provide a bound on the theoretical value for each regression. Reference Security:M10

Dep Var	Forward Estimate	Theoretical Value	Reverse Estimate
M2[a]	0.084	0.11	0.179
M3[a]	0.199	0.22	0.276
M4[a]	0.311	0.33	0.384
M5[a]	0.430	0.44	0.490
M6[a]	0.530	0.55	0.586
M7[a]	0.635	0.66	0.671
M8[a]	0.753	0.77	0.783
M9[a]	0.874	0.88	0.904
M11[a]	1.079	1.11	1.132
M12	1.120	1.22	1.204

[a]thoeoretical value within the bounds provided by forward and reverse regressions.

Table 35

Regression of Excess Returns for Various Maturity Categories on Excess Returns of 6 month Treasury Bill, where the Excess Returns are Computed over a two month Period,1965-1987, (t- statistics in parentheses). Fama Data.

Maturity in Months	Theoretical Beta	Estimated Beta	Estimated Intercept	R^2
3	0.25	0.238[b] (27.0)	0.243 (5.65)	0.84
4	0.5	0.490[b] (42.4)	0.277 (4.93)	0.93
5	0.75	0.757[b] (73.0)	0.046 (0.91)	0.97
7	1.25	1.221[b] (87.9)	-0.334 (-4.98)	0.98
8	1.5	1.418 (67.1)	-0.561 (-5.43)	0.97
9	1.75	1.679[b] (60.71)	-0.629 (-4.67)	0.96
10	2.00	1.901[b] (48.18)	-1.209 (-6.28)	0.94
11	2.25	2.107 (45.10)	-1.59 (-7.01)	0.93
12	2.50	2.073 (24.11)	-1.83 (-5.51)	0.83

[b]theoretical value within the 99% confidence interval.

Table 36

Regression of Excess Returns for Various Maturity Categories on Excess Returns of 10 month Treasury Bill, where the Excess Returns are Computed over a two month Period, 1965-1987, (t- statistics in parentheses). Fama Data.

Maturity in Months	Theoretical Beta	Estimated Beta	Estimated Intercept	R^2
3	0.125	0.117[b] (21.9)	0.408 (8.25)	0.78
4	0.250	0.240[b] (28.1)	0.616 (7.83)	0.85
5	0.375	0.374[b] (37.5)	0.565 (6.13)	0.91
6	0.500	0.496[b] (48.1)	0.681 (7.15)	0.94
7	0.625	0.611[b] (59.1)	0.472 (4.95)	0.96
8	0.750	0.726[b] (70.78)	0.371 (3.92)	0.97
9	0.875	0.865[b] (86.13)	0.464 (5.00)	0.98
11	1.125	1.040[b] (94.52)	-0.251 (-2.32)	0.98
12	1.250	1.145 (38.03)	-0.460 (-2.17)	0.92

[b]theoretical value within the 99% confidence interval.

TABLE 37

Test of one-factor model using a log based approach due to Theil, which Provides a Point Estimate and a Confidence Interval. For Each Dependent Variable the First Line Gives Values For 99% Confidence Interval and the Second Line for 95% Confidence Interval. Independent Var:M6.

Dep Var	Lower Limit	Point Estimate	Upper Limit
M2[ab]	0.170	0.225	0.299
	0.186	0.225	0.272
M3[ab]	0.358	0.442	0.544
	0.384	0.442	0.508
M4[ab]	0.517	0.611	0.722
	0.546	0.611	0.683
M5[ab]	0.736	0.836	0.950
	0.768	0.836	0.911
M7[ab]	1.042	1.165	1.302
	1.082	1.165	1.125
M8[ab]	1.250	1.410	1.577
	1.308	1.410	1.519
M9[ab]	1.394	1.563	1.752
	1.448	1.563	1.686
M10[ab]	1.421	1.660	1.930
	1.497	1.660	1.841
M11[ab]	1.614	1.895	2.220
	1.703	1.895	2.110
M12[ab]	1.814	2.206	2.680
	1.936	2.206	2.513

[a]thoeoretical value within the 95% confidence interval.
[b]thoeoretical value within the 99% confidence interval.

Table 38

Skewness and Kurtosis measures of error term for various maturity categories with z statistic in parenthesis. Reference Security is the 6 month bill.

Maturity in Months	Skewness	Kurtosis
2	1.401 (64.2)[b]	7.432 (85.1)[b]
3	1.843 (84.4)[b]	7.188 (82.3)[b]
4	1.061 (48.6)[b]	3.993 (45.7)[b]
5	0.324 (14.8)[b]	3.009 (34.4)[b]
7	-0.383 (17.5)[b]	3.735 (42.7)[b]
8	-0.008 (0.36)	4.599 (52.6)[b]
9	0.244 (11.1)[b]	4.666 (53.4)[b]
10	-0.743 (34.0)[b]	5.480 (62.7)[b]
11	-1.841 (84.3)[b]	13.544 (155.1)[b]
12	-0.049 (0.46)[b]	3.344 (31.9)[b]

[a]significant at 95% confidence level.
[b]significant at 99% confidence level.

Table 39

Skewness and Kurtosis measures of excess return for various maturity categories with z statistic in parenthesis.

Maturity in Months	Skewness	Kurtosis
2	1.997 (91.5)[b]	10.007 (114.6)[b]
3	2.567 (117.6)[b]	12.904 (147.8)[b]
4	1.864 (85.4)[b]	10.182 (116.6)[b]
5	1.620 (74.2)[b]	9.171 (105.0)[b]
6	1.322 (60.59)[b]	7.946 (91.0)[b]
7	1.104 (50.6)[b]	7.594 (87.0)[b]
8	1.237 (56.6)[b]	8.669 (99.3)[b]
9	1.216 (55.7)[b]	9.423 (107.9)[b]
10	1.014 (46.4)[b]	7.995 (91.6)[b]
11	0.924 (42.3)[b]	8.991 (103.0)[b]
12	-0.043 (1.64)[a]	2.457 (23.4)[b]

[a]significant at 95% confidence level.
[b]significant at 99% confidence level.

Table 40

Skewness and Kurtosis measures of monthly return for various maturity categories with z scores in parenthesis.

Maturity in Months	Skewness	Kurtosis
2	1.223 (56.0)[b]	1.229 (14.0)[b]
3	1.409 (64.5)[b]	2.288 (26.2)[b]
4	2.071 (94.9)[b]	6.448 (73.8)[b]
5	2.256 (103.4)[b]	8.032 (92.0)[b]
6	2.253 (103.2)[b]	8.505 (92.0)[b]
7	2.179 (99.8)[b]	8.830 (101.1)[b]
8	2.221 (101.7)[b]	9.761 (111.8)[b]
9	2.173 (99.5)[b]	10.499 (120.3)[b]
10	1.893 (86.7)[b]	8.930 (102.3)[b]
11	1.888 (86.53)[b]	10.150 (116.3)[b]
12	0.790 (30.1)[b]	2.277 (26.0)[b]

[a]significant at 95% confidence level.
[b]significant at 99% confidence level.

TABLE 41

Test of Beta Stability Model Using a Non-parametric Estimator Due to Theil. This Estimator Computes n/2 Slopes to Provide a Point Estimate and a Confidence Interval, where n is the no. of Observations. For Each Dep Var the First Line Gives Values For 99% Confidence Interval and the Second Line for 95% Confidence Interval. Independent Variable:M6

Dep Var	Lower Limit	Median	Upper Limit
M2[b]	0.123	0.166	0.201
	0.130	0.166	0.189
M3	0.307	0.336	0.383
	0.312	0.336	0.370
M4[b]	0.515	0.556	0.608
	0.530	0.556	0.595
M5[ab]	0.738	0.791	0.824
	0.766	0.791	0.819
M7[ab]	1.125	1.192	1.237
	1.147	1.192	1.227
M8[ab]	1.247	1.345	1.431
	1.267	1.345	1.408
M9[ab]	1.403	1.537	1.651
	1.454	1.537	1.635
M10[ab]	1.610	1.687	1.842
	1.632	1.687	1.819
M11[ab]	1.763	1.922	2.015
	1.799	1.922	2.010
M12[ab]	1.863	2.133	2.307
	1.925	2.133	2.257

[a]thoeoretical value within the 95% confidence interval.
[b]thoeoretical value within the 99% confidence interval.

TABLE 42

Test of Beta Stability Model Using a Non-parametric Estimator Due to Theil. This Estimator Computes n(n-1)/2 Slopes to Provides a Point Estimate and a Confidence Interval, where n is the no. of Observations. For Each Dependent Variable the First Line Gives Values For 99% Confidence Interval and the Second Line for 95% Confidence Interval. Independent Var:M6.

Dep Var	Lower Limit	Median	Upper Limit
M2	0.130	0.154	0.177
	0.137	0.154	0.172
M3	0.314	0.339	0.369
	0.318	0.339	0.361
M4[b]	0.526	0.558	0.605
	0.534	0.558	0.583
M5[ab]	0.762	0.787	0.810
	0.768	0.787	0.805
M7[b]	1.139	1.171	1.205
	1.147	1.171	1.197
M8[b]	1.300	1.354	1.403
	1.314	1.354	1.390
M9[b]	1.463	1.532	1.601
	1.480	1.532	1.584
M10[b]	1.638	1.727	1.812
	1.660	1.727	1.792
M11	1.796	1.893	1.989
	1.821	1.893	1.965
M12	1.906	2.040	2.179
	1.937	2.040	2.141

[a]thoeoretical value within the 95% confidence interval.
[b]thoeoretical value within the 99% confidence interval.

TABLE 43

Test of Beta Stability Model Using a Non-parametric Estimator Due to Theil. This Estimator Computes n/2 Slopes to Provide a Point Estimate and a Confidence Interval, where n is the no. of Observations. For Each Dep Var the First Line Gives Values For 99% Confidence Interval and the Second Line for 95% Confidence Interval. Independent Variable:M10

Dep Var	Lower Limit	Median	Upper Limit
M2	0.050	0.067	0.098
	0.052	0.067	0.094
M3	0.136	0.174	0.206
	0.154	0.174	0.195
M4[ab]	0.256	0.300	0.343
	0.265	0.300	0.337
M5[b]	0.347	0.415	0.446
	0.366	0.415	0.432
M6[b]	0.451	0.513	0.567
	0.463	0.513	0.548
M7[ab]	0.578	0.621	0.674
	0.594	0.621	0.669
M8[b]	0.698	0.729	0.779
	0.700	0.729	0.768
M9[b]	0.816	0.852	0.891
	0.826	0.852	0.877
M11[b]	1.066	1.090	1.119
	1.069	1.090	1.105
M12[b]	1.104	1.182	1.235
	1.136	1.182	1.219

[a]thoeoretical value within the 95% confidence interval.
[b]thoeoretical value within the 99% confidence interval.

TABLE 44

Test of Beta Stability Model Using a Non-parametric Estimator Due to Theil. This Estimator Computes n(n-1)/2 Slopes to Provide a Point Estimate and a Confidence Interval, where n is the no. of Observations. For Each Dependent Variable the First Line Gives Values For 99% Confidence Interval and the Second Line for 95% Confidence Interval. Independent Var:M10

Dep Var	Lower Limit	Median	Upper Limit
M2	0.060	0.076	0.091
	0.064	0.076	0.088
M3	0.148	0.171	0.194
	0.153	0.171	0.188
M4	0.260	0.290	0.316
	0.268	0.290	0.310
M5[b]	0.382	0.411	0.446
	0.389	0.411	0.433
M6[b]	0.492	0.521	0.557
	0.499	0.521	0.541
M7[b]	0.608	0.636	0.668
	0.614	0.636	0.657
M8[b]	0.729	0.752	0.779
	0.735	0.752	0.769
M9[b]	0.850	0.871	0.891
	0.855	0.871	0.886
M11	1.064	1.086	1.107
	1.069	1.086	1.105
M12	1.118	1.160	1.203
	1.128	1.160	1.190

[a]thoeoretical value within the 95% confidence interval.
[b]thoeoretical value within the 99% confidence interval.

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