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The optical design of far infrared detector arrays

Milner, Thomas Edward, Ph.D.
The University of Arizona, 1991
The Optical Design of
Far Infrared Detector Arrays

by

Thomas Edward Milner

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A Dissertation Submitted to the Faculty of the
COMMITTEE ON OPTICAL SCIENCES
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For the Degree of
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1991
As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Thomas Edward Milner entitled The Optical Design of Far Infrared Detector Arrays and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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11/12/91

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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

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Dissertation Director

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11/12/91
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SIGNED: Thomas Edward Milner
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Abstract

The imaging properties of far infrared detector arrays are analyzed. An arbitrary optical system imaging partially coherent light is reviewed. The imaging and detection process is correlated to the coherence properties of the imaged light. A selection set of optical flux concentrator arrays is set forth with reference to a literature review. A selection procedure for the concentrators is outlined which includes a derived performance function.

Transverse and longitudinal detector geometries are frequently considered in far infrared imaging problems. A ray model is constructed to describe the optical-transverse detector system. The absorbed photon density in a transverse detector is computed with a Monte Carlo simulation. The subsequent transport of the photogenerated holes is evaluated by solving the steady state diffusion equation. With the evaluation of the steady state current density, transfer functions, point spread functions and diffusive cross talk are determined.

With a Boltzmann transport equation approach, the response of a longitudinal detector array is analyzed. Signal equations are derived which relate the signal current density to the absorbed photon distribution and other relevant parameters. The various parameters derive from scattering and recombination of the photogenerated charges. Each parameter is qualitatively discussed, mathematically
analyzed and algebraically modeled. The absorbed photon distribution is computed with the aid of an optical multilayer model. The absorptive efficiency and the spatial distribution of the absorbed photons is computed for various layers of the longitudinal detector. The imaging response of the detector array is evaluated from the derived signal equations. An analytical expression is derived for the transfer function of an arbitrary optical-longitudinal detector array system. The derived transfer function includes the effects of diffusion and recombination of the photogenerated charges. Alternate measures of the detector's imaging response are derived; the transfer function, the point response function, the diffusive cross talk and the responsivity are computed as a function of various detector operating and design parameters. Conclusions which relate the detector's imaging performance to several operating and design parameters are made.
Chapter 1 Introduction

Far infrared detector arrays hold tremendous potential for many future scientific and technical endeavors. Many astronomical phenomenon are uniquely observed in the far infrared region of the electromagnetic spectrum. Examples include star formation, superluminous galaxies at the edge of the universe, and the existence of planets of neighboring stars. The reality of space-based astronomical observations (e.g., IRAS) greatly enhances various observational possibilities; therefore, the development of far infrared detector arrays has been enthusiastically undertaken by the space sciences community. In addition to astronomical and space sciences, far infrared detector arrays promise to enhance medical imaging, remote sensing of the earth together with various other scientific and technological activities.

Given an object to image, a model of far infrared detector arrays should predict the imaging response of the detector array. The model should describe the detector and its operation in terms of various operating and design parameters specific to the detector array. The model should predict the detector response to a given object as a function of the various operating and design parameters of the detector array.

A mathematical model of far infrared detector arrays can expedite their development. The development of high speed digital computers has expanded the possibilities for model evaluation and device simulation. Development of a reliable
model that is capable of predicting the performance of a far infrared detector array has many uses. First, the model serves as a reference for making initial detector array design decisions. Second, it guides the technological development of the detector array. Third, a detector array model advances image simulation and processing studies. The topic of this discussion is the development of mathematical models of extrinsic photoconductive far infrared detector arrays.

Only extrinsic photoconductive detector materials are considered. A photoconductive detector material relies on a change in resistivity when the material is exposed to optical illumination. An extrinsic material is a type IV semiconductor which is doped with shallow acceptors or donors. The shallow dopants are either acceptors (p-type) or donors (n-type). The dopants and the impurities are of primary consideration in determining the optical and electronic properties of the semiconductor. For shallow n-type donors, the n-type species have electrons which energetically occupy states just below the conduction band of the host semiconductor as illustrated in figure 1-1. The host semiconductor can also be doped with shallow acceptors. The shallow acceptors or p-type dopants have an electron vacancy or a hole which energetically lies just above the valence band as illustrated in figure 1-1. Invariably the host semiconductor contains residual impurities which can be either p- or n-type. The residual impurities may occupy deep levels as illustrated in figure 1-1.

Photoexcitation of the charge carriers in far infrared extrinsic semiconductor
Figure 1-1: Energy level diagram of an extrinsic semiconductor.

Materials can occur through two types of excitation. First, an electron from a shallow n-type dopant can be excited into the conduction band of the host semiconductor. The process leaves a hole bound to the n-type dopant and a free electron in the conduction band. With application of an electric field, the free electron is collected and recorded as photo-generated charge. Alternatively, an electron in the valence band of the host semiconductor can be excited to a electron-vacancy associated with a shallow p-type dopant. The process results in an electron bound to an acceptor and a free hole in the valence band of the host semiconductor. The photogenerated hole is free to move under the influence of an applied electric field. As with an electron, a photogenerated hole is collected and recorded as a photogenerated charge.
In the process of migrating toward the collecting contacts, photogenerated charges diffuse and recombine. Both recombination and diffusion are taken into account in the detector array model. Both recombination and diffusion of the photogenerated charges degrade the imaging performance of the detector.

A model of a far infrared detector array predicts the response of the detector array as a function of its design and operating parameters. The detector transfer function, the point spread function, the diffusive cross talk and the responsive quantum efficiency are all measures used to evaluate the detector array.

Figure 1-2: A generic optical-detector system.

In addition, a number of associated issues are considered. In practice, optoelectronic detector arrays are but a single component in an optical instrument. A generic optical instrument is illustrated in figure 1-2. The instrument frequently
has an optical system forming an image on the detector and an electronic system which reads the detector array. The detector array serves as an important link in the conversion of optical information into an electronic form. The following discussion limits its focus to the optical-detector system; the instrument section outlined in figure 1-2. The electronic readout system which frequently accompanies the detector is not considered. A complete description of the instrument design, however, should include the optical, detector and the electronic readout systems.

![Diagram of transverse and longitudinal detector geometries.](image)

**Figure 1-3:** Transverse and longitudinal detector geometries.

Two detector geometries are considered in the detector array analysis. Both geometries, transverse and longitudinal, are illustrated in figure 1-3. In the transverse geometry the detector array is constructed by stacking linear array elements on top of each other. In the longitudinal geometry the detector is fabricated as an optical multilayer structure. The longitudinal geometry will also be referred to
as a front plane illuminated array or FPIA. This discussion explores the tradeoffs between the two geometries as well as their respective optimization.

The optical and electronic properties of the detector material also have an important influence on the design of the detector array. The complex dielectric constant or alternatively the complex refractive index of the infrared sensitive material directly affects the distribution density of the absorbed photons. The scattering properties of the photogenerated charge carriers also are important in determining various properties of the detector array. Finally, the use of flux concentrators influences the optical design of the detector array. Each of these factors is considered in detail; their impact on the design of the detector array is analyzed.

This dissertation is not an engineering analysis of a specific detector type or design; nor is it a purely numerical study of a system of equations. A number of results of a general nature are derived. The results, expressed as mathematical equations, are limited in application only to the extent of the assumptions and/or approximations made to derive them. When any assumption or approximation is made its application and range of validity is discussed.

The following discussion is arranged into six chapters. The organization follows the information flow through the optical detector-system. The light emitted by the source is collected at the entrance pupil of the optical system. The optical system forms an image of the source on the focal plane of the detector. The
performance of the optical-detector system can often be enhanced with an optical flux concentrator array positioned between the optical system and the detector array. The detector array converts the image formed by the optical system into an electronic signal. The electronic signal is collected, stored and made available for future analysis.

A mathematical viewpoint is taken to describe the optical system. Many of the mathematical concepts used to describe the optical system recur in the subsequent analysis. The mathematical concepts of linear vector spaces and linear system theory form the mathematical basis for not only the analysis of optical systems but also for many of the mathematical derivations in this dissertation.

In addition to the mathematical methodology discussed in chapter two, some basic characteristics of imaging optical systems are reviewed. The discussion of imaging optical systems highlights the basic considerations of coherence and transfer functions. The discussion assumes an optical system with a centrally obscured pupil; a case frequently encountered with astronomical telescopes. The size of the circular pupil obscuration is variable; thus, the case of a clear aperture is a specific example of the general case considered here. Computations of coherent and incoherent point response functions are presented. Finally, examples of important distinctions between coherent and incoherent imaging and detection are given.

In some cases an optical flux concentrator is used in conjunction with a
detector array; a topic discussed chapter three. A flux concentrator, as its name suggests, concentrates the incoming optical flux onto a smaller area on the detector array. In space applications, flux concentrators can be used to reduce the cosmic ray hit rate. With a literature review, various types of flux concentrators are described, along with their qualitative and quantitative selection criteria. A two-step selection process is described. In step one, a candidate set of concentrators is determined. In step two, a performance criteria evaluates specific designs.

![Diagram of a flux concentrator on a detector array](image)

**Figure 1-4: Wedged flux concentrators on linear transverse arrays.**

Flux concentrators can be used in both longitudinal and transverse detector geometries. The performance of transverse arrays is especially improved with use of a flux concentrator. The optimum thickness of a linear array element in transverse designs is frequently less than the inter-array spacing. The optimum thickness of a transverse linear element is dependent on the recombination time of the
photogenerated charge. The spacing between successive transverse linear arrays is determined by the Nyquist sampling interval of the optical system. In a two-dimensional transverse array, the disparity in spacing and thickness results in large gaps between the linear arrays elements. A flux concentrator positioned on the front facets of the linear arrays can substantially increase the effective fill factor of the array. A wedge concentrator in optical contact with a transverse array is illustrated in figure 1-4.

A simplified approach is assumed in the analysis of transverse arrays. A ray model is used to evaluate the optical transverse detector response. Rays are traced through the optical system into the detector. A Monte Carlo simulation is used to evaluate the distribution of absorbed photons in the detector. The position at which each ray is absorbed is determined from a randomly generated absorption length.

The subsequent transport of the photogenerated charges is evaluated with the two dimensional steady state diffusion equation. The diffusion equation is solved numerically with a finite difference method. The numerical solution for the steady state current density allows the computation of various imaging measures. The transfer function, the point spread function and the diffusive cross talk are all evaluated. Finally, the transverse array model is used to explain some experimental data. Experimental data is best fitted by varying the model diffusion length. Conclusions and recommendations are made.
The simplified model of the transverse detector arrays describes the important phenomenon occurring within a detector array. A more rigorous approach is taken in chapters 5-7 to describe a longitudinal or FPIA. The basic physics of the important phenomenon are described. The approach gives a detailed account of the operation of a FPIA.

A derivation for the photogenerated current density in a longitudinal detector material is presented in chapter 5. With a Boltzmann transport equation approach, a time dependent partial differential equation is derived for the drifting photogenerated charge. The derived partial differential equation is solved and an integral expression for the photogenerated current density is derived. The integral expression involves a Green's function corresponding to absorption of photons at a single point within the detector material.

The photogenerated current density equations derived in chapter 5 are dependent on a number of optical and electronic parameters. Electronic parameters include: electric mobility, diffusion length and recombination time of the photogenerated charges. The mobility and diffusion of the photogenerated charges depends on various scattering mechanisms the charges encounter when migrating toward a collecting contact. The primary scattering mechanisms include acoustic phonon scattering, ionized impurity scattering, and neutral impurity scattering. Each scattering mechanism is examined and a total scattering rate is derived. Computations for the electric mobility and diffusion length follow directly from the
total scattering rate. The recombination time of photogenerated charges is an important parameter. The associated capture cross section of the photogenerated charges is modeled and recent work is discussed.

Computation of the absorbed photon distribution in the detector is necessary to link the detector response to the current density equations derived in chapter 5. The distribution of the absorbed photons in a longitudinal or FPIA geometry is discussed in chapter 6. Computation of the absorbed photon distribution relies on an optical multilayer model of the FPIA. Use of the optical multilayer model requires the evaluation of the optical constants of each detector material. The optical constants include the thickness and complex dielectric constant or alternatively the complex refractive index of each layer in the detector material. Each parameter is discussed and the process to evaluate them is outlined with examples. The optical multilayer model allows an evaluation of a number of optical properties of the detector array. The absorbed photon density in various layers of the detector is evaluated as a function of various optical and detector array design parameters.

The imaging performance of an FPIA is analyzed in chapter 7. Beginning with the photogenerated current density equations derived in chapter 5 and the optical multilayer model described in chapter 6, a general expression is derived for the longitudinal-detector transfer function. The derived longitudinal detector transfer function accounts for the finite pixel size, the photoconductive gain, diffusion and recombination of the photogenerated charges. Based on the transfer function,
expressions are derived for the point spread function, the diffusive cross talk, and the responsive quantum efficiency. Examples of each detector imaging measure are presented for various combinations of optical and detector design parameters.
Chapter 2 Analysis of Optical Systems

The analysis of optical systems is necessary to understand the operation of optical imaging detectors. A precise and symbolic description of a physical system is often done in the language of mathematics. This section highlights some concepts used to analyze imaging optical systems. The optical system is described as a linear system. The concepts of plane waves and coherence are highlighted. The presentation is by no means rigorous; however, it is intended to point out meaningful concepts.

Fourier Optics provides a unifying viewpoint for the linear analysis of optical systems. Linear theory has played a central role in the classical analysis of physical systems. The discussion begins with a review of the basic mathematical concepts of linear system theory. In particular, emphasis is placed on eigenfunction expansions and Green's functions. The optical system is examined as a particular example of a linear system. It will be demonstrated that the point spread function (PSF) of an optical system is the Green's function for the linear system. The discussion assumes partial coherent illumination; of which, coherent and incoherent illumination are specific cases. The optical transfer function (OTF) is the eigenvalue spectrum of a linear operator representative of the optical system. Systems imaging coherent and incoherent light are examined. Specific examples of coherent and incoherent image formation are computed in an optical system with a centrally obscured pupil. Finally, the importance of coherence in the detection process is discussed.
2.1 Linear Vector Spaces

The mathematical analysis of physical systems often relies on linear vector space theory. Most of the major equations of mathematical physics are linear equations (Morse and Feshbach, 1953). There are many examples of linear descriptions of the natural world. Classical mechanics, Maxwell's electrodynamics, Schrödinger's and Heisenberg's quantum mechanics, Einstein's relativity theory and Dirac's theory of the electron are examples. In systems that require a nonlinear description, linear theory still plays an important role. It is helpful, therefore, to review some of the basic mathematical concepts of linear theory.

Modern mathematical analysis assumes the existence of a space of points or a set of primitive objects. In system theory the state of a physical system is represented by a point in a mathematical space. Sets of points in the space satisfy various relationships with respect to each other. The most general spaces are subject to the fewest defining relationships; more specific spaces satisfy stricter criteria. The most general class of mathematical spaces are the topological spaces (Abraham et. al., 1983). In order of sophistication, other mathematical spaces are as follows: metric spaces, Banach spaces, and linear vector spaces. Linear vector spaces are highly sophisticated mathematical spaces. The classical description of most physical systems is done within the framework of a linear vector space - which here we call $V$. 
Mathematicians have described a wide range of linear vector spaces. A linear vector space contains elements or vectors; denoted by lower case greek letters (e.g., \( \alpha, \beta \)). The vectors can be multiplied by elements of a field, called \( F \). The result of the scaler-vector operation, denoted by \( a\alpha \), is another vector in \( V \); that is \( a\alpha \in V \), where \( \in \) represents "an element of". The field \( F \) is commonly the real numbers, denoted by \( \mathbb{R} \), or the complex numbers, denoted by \( \mathbb{C} \).

Common to all vector spaces is the concept of a basis set; which here we call \( B=\{\alpha_1, \alpha_2, \alpha_3, \ldots\} \). From the basis set, any element in the space can be resolved as a superposition of basis elements. That is, \( \forall \beta \in V, \exists c_1, c_2, c_3, \ldots \in F, \) such that \( \beta = \sum c_i \alpha_i \); here we have introduced the notation \( \forall \) "for every", and \( \exists \) "there exists at least one". The basis set is said to be linearly independent if any vector in the basis set \( B \) can not be resolved from the remaining elements in the basis set. Stated concisely: \( B=\{\alpha_1, \alpha_2, \alpha_3, \ldots\} \) is a basis set if and only if \( \forall \alpha_i \in B, \exists c_1, c_2, c_3, \ldots \in F, \) such that \( \alpha_i = \sum c_k \alpha_k \), where the sum excludes the index \( i \) and \( \exists \) is "there does not exist any". If the vector space \( V \) contains a linear independent basis set with a finite number of elements, \( V \) is called a finite dimensional vector space. The number of linear independent vectors in a finite basis set of \( V \), is the dimension of \( V \). If the space contains a linear independent set with an infinite number of elements, the space is an infinite dimensional vector space.

Another important concept is the dual space of a vector space. The dual space, denoted by \( V' \), of a linear vector space \( V \), is the space of all linear functions on \( V \),
which map into the field $F$. That is, for $\alpha, \beta \in V$ and $\lambda' \in V$, $\lambda'(a\alpha + b\beta) = a\lambda'(\alpha) + b\lambda'(\beta) \in F$.

It can easily be shown the space $V$ is also a vector space; $V$ is often called the dual space or the space of linear functionals. If the vector space $V$ is finite dimensional the dual space $V'$ is isomorphic to the original vector space $V$; that is, there is a one-one correspondence between the elements of $V$ and $V'$ which preserves all the operations of the vector space. If the vector space $V$ is infinite dimensional, then the space of linear functionals on $V$, that is $V'$ can contain elements which have no counterparts in $V$.

The concept of the dual space can be motivated from an alternate point of view. An inner product space is a linear vector space which has an inner product function. The inner product function is a mapping $V \times V \rightarrow F$; here $\rightarrow$ is the mapping and $F$ is either the field of real ($\mathbb{R}$) or complex ($\mathbb{C}$) numbers. The concept of an inner product is motivated from the well known inner product in $\mathbb{R}^2$ or $\mathbb{R}^3$. The inner product is denoted by $(\alpha, \beta) \in F$, where $\alpha, \beta \in V$. If $F=\mathbb{R}$, then the inner product is linear in both arguments; that is, $(a\alpha + b\beta, \gamma) = a(\alpha, \gamma) + b(\beta, \gamma)$ and $(\alpha, b\beta + c\gamma) = b(\alpha, \beta) + c(\alpha, \gamma)$. For $F=\mathbb{C}$, the inner product is linear in the second argument and conjugate-linear in the first argument; that is, $(a\alpha + b\beta, \gamma) = a^*(\alpha, \gamma) + b^*(\beta, \gamma)$, where $a^*$ and $b^*$ are respectively the complex conjugates of $a$ and $b$. If the norm, $\|\alpha\| = (\alpha, \alpha)$, generated by the inner product is complete, then the linear vector space is known as a Hilbert Space.

For infinite dimensional spaces, the vector space $V$ is defined as a well behaved set of test functions. The dual space, $V'$, is called the distribution space. A physical
system is represented by a point in the distribution space $V$. A number of test function spaces have been constructed for various applications. Generally, the smoother test function spaces can admit oscillatory and more exotic distribution spaces.

A well known example of an element of a distribution space is Dirac’s delta distribution. The delta function, $\delta_\alpha$, has the sifting property described by equation 2.1-1.

$$\delta_\alpha(\phi) = \phi(\alpha)$$  \hspace{1cm} (2.1-1)

Here, $\delta_\alpha \in V$ and $\phi \in V$, where $V$ is the test function space. Dirac’s delta function is, in a mathematical sense, a distribution. The distribution-function terminology will not be distinguished in this discussion; the term "function" will typically refer to a distribution.

2.2 Linear System Theory

The mathematical analysis of linear systems is done within the framework of a linear vector space. The basic concepts of linear system theory are highlighted in this section. The treatment is beyond a qualitative level; however, it is not mathematically rigorous. Two methods of analyzing linear systems are discussed - eigenfunction expansions and Green’s functions.
Often the response of a physical system is the solution to a linear equation in distribution space. Within the framework of the theory, the response of a specific system is the solution to equation 2.2-1.

\[ L\alpha' = \gamma' \quad (2.2-1) \]

Here, \( L \) is a linear operator on the distribution space \( V \), \( \alpha' \in V \) is the unknown system response, and \( \gamma' \in V \) is a known forcing function. The solution of the problem 2.2-1 can be found by solving the eigenvalue problem for the linear operator \( L \). The eigenvalue problem is written as \( L\beta' = \lambda\beta' \); \( L \) is the linear operator, \( \beta' \) is an eigenvector and \( \lambda \) is an eigenvalue. In an infinite dimensional vector space a linear operator usually has an infinite number of eigenvalues and eigenvectors. The entire set of eigenvalues is the eigenvalue spectrum.

The mathematical study of the eigenvalue spectrum of linear operators on a Hilbert space is known as spectral theory. A fundamental result of spectral theory is used to solve equation 2.2-1. When the operator \( L \) is self adjoint, the action of \( L \) can be constructed as a superposition of eigenvectors as denoted in equation 2.2-2.

\[ L(\alpha) = \sum \lambda(\xi_\lambda, \alpha)\xi_\lambda \quad (2.2-2) \]

In equation 2.2-2, \( \lambda \in \mathbb{C} \) is an eigenvalue of \( L \); \( \xi_\lambda \in V \) is the eigenvector corresponding to \( \lambda \); \( (\xi_\lambda, \alpha) \in \mathbb{C} \) is the inner product of \( \xi_\lambda \) and \( \alpha \); the summation is taken over all the eigenvectors \( \xi_\lambda \). When the eigenvalue spectrum is continuous the summation turns into an integral over the continuum of eigenvalues.
The solution to the eigenvalue problem is used to solve equation 2.2-1. Expansion of the forcing vector in terms of the eigenvectors $\gamma = \sum (\xi, \gamma) \xi$, the solution $\alpha$ is written in equation 2.2-3.

$$\alpha' = \text{ker}(L) + \sum \lambda^{-1}(\xi, \gamma) \xi$$  \hspace{1cm} (2.2-3)

Here, ker$(L)$ is the kernel of the operator $L$. The kernel of the linear operator $L$ is the space of eigenvectors with a zero eigenvalue. The summation extends over all eigenvectors with a nonzero eigenvalue. Determination of the eigenvalues and eigenfunctions of a linear operator $L$ is sufficient to solve a wide range of problems.

Green's function theory is an alternative approach to describe a linear system. Green's functions are quite general in their applicability. Green's functions are formulated for physical systems which are represented by a linear self adjoint operator. The simplest and most basic concepts of Green's function theory are presented here; a modern rigorous treatment is found in Mizohata (1973).

The concept of a Green's function is at the heart of linear theory. The response of a physical system is determined by the forcing function(vector), $\gamma \in V$ in equation 2.2-1. The forcing function can be represented as a linear superposition of orthogonal basis vectors in the distribution space $V$. If the basis set is chosen such that the system response for each basis vector is known (or can be determined), then the system response to a general $\gamma \in V$ can be constructed.
The eigenvector expansion in 2.2-2 is one such example. Here, the eigenvectors of a linear self adjoint operator form an orthogonal basis set in the distribution space \( V \). The system response to an eigen-forcing function is simply a multiple of the eigenfunction itself. The system response to a general forcing function is formulated as a linear superposition of the basis vector responses (ie. the eigenfunctions), as in equation 2.2-3.

The conceptual framework of Green's function theory is similar to the eigenvector expansion method. In Green's function theory the basis set is composed of Dirac delta distributions. A general forcing vector \( \gamma \) is represented in equation 2.2-4, as a superposition of delta distributions.

\[
\gamma'(x) = \int \delta_x \gamma d\Omega
\]  

(2.2-4)

Here, \( \Omega \) is the domain of interest and \( \delta_x \) is the Dirac distribution corresponding to \( x \in \Omega \). A solution to equation 2.2-2 is constructed by finding the system response to a delta distribution; or equivalently, solving for \( g_x \) in equation 2.2-5. Here, \( g_x \in V \) is the system response to a delta distribution at \( x' \). The solution to 2.2-2 is constructed as a superposition of responses to the basis distributions \( \delta_x \); the solution so constructed is presented in equation 2.2-6.

\[
Lg_x(x) = \delta_{x'}
\]  

(2.2-5)

Since the forcing distribution \( \gamma(x') \) is given, once the Green's distribution \( g_x(x) \) is
known the solution $\alpha'(x)$ is easily constructed according to equation 2.2-6.

Green's function theory and eigenfunction expansions are two methods used to predict the response of a physical system. Both methods rely on superpositions of distributions in a Hilbert space. The superpositions are formed from distributional solutions of a self adjoint linear operator equation. In the eigenvector method, the superposition is formed from an orthonormal basis set of the eigenvectors of the self adjoint linear operator. In Green's function theory, the basis set is composed of Dirac delta distributions. The system response is constructed as a linear superposition of distributional responses (ie. the Green's function) to the basis set of delta distributions. Both the eigenfunction expansion and the Green's function methods are applied to imaging optical systems and the analysis of the detector response.

2.3 Optical Imaging Systems

Linear system theory is of central importance to the analysis of optical imaging systems. Within linear system theory, the concepts of plane wave expansions and coherence are basic to the description of optical systems. Plane waves are mathematical idealizations which have great practical utility. Their utility derives from unique mathematical properties-as exemplified by the Fourier transformation.
pair. Coherence is a statistical property of light. Coherence is a measure of the random variation of the relative phase of two oscillators. When the relative phase of two oscillators has a small variation, the two oscillators are said to have a large degree of coherence. If the relative phase between the two oscillators varies a great deal, there is little coherence between them. These are qualitative descriptions; precise mathematical statements will clarify the concepts.

The coherence of the light illuminating the entrance pupil partially determines the image response. The coherence of the imaged light can affect a number of optical methods. These include the testing of an optical system, the imaging properties under various source-observer arrangements and the detection process within the detector.

In many optical imaging instruments the optical source is assumed to be incoherent. A number of exceptional cases exist, however, which affect the design-testing process. At far infrared wavelengths the spatial extent of the source can determine its coherence properties. Since any "incoherent" source has coherence over a transverse spatial extent of one wavelength (Wolf, 1978), a source with a total spatial extent of approximately one wavelength is coherent. Such a coherent source can be a thermal or black body emitter with lateral dimension(s) on the order of a wavelength of the imaged light. A fine wire heated by ohmic losses and placed in a far infrared test jig is an example of such a source.

Coherence also affects imaging properties of an optical system. The
relationship between coherence and the source-observer geometry is embodied in the well known van Cittert-Zernike theorem, which will be discussed shortly. The longitudinal and the transverse coherence of the imaged light on the detector array will effect the details of detection process; a topic which will be examined later in this chapter.

Plane waves are a very useful mathematical idealization. The utility of plane waves derives from the ability to express a wide range of optical wave fields as a superposition of plane waves. Knowledge of the optical wave field in image space is tantamount to predicting the response of the optical system. The classical (ie. when the fields are not quantized) optical wave field satisfies the Helmholtz wave equation, denoted by equation 2.3-1.

\[ ( \nabla^2 + n^2 k_0^2 ) u(\mathbf{r}) = 0 \]  \hspace{1cm} (2.3-1)

Here, \( u(\mathbf{r}) \) is the optical wavefield and \( n \) is the refractive index of the material at \( \mathbf{r} \). In this discussion the dielectric medium is assumed to be linear, isotropic and homogenous. The medium can be absorptive - in which case the refractive index will be a complex number. The free-space wavevector is denoted by \( k_0 = 2\pi/\lambda \). Equation 2.3-1 is an example of a self adjoint linear operator problem discussed in the previous section. Hence the methods of eigenfunction expansion and Green’s function theory are directly applicable.

Plane waves form a complete orthonormal basis of distribution functions for the
Helmholtz equation as specified in equation 2.3-1. A large family of wavefields can be expressed as superpositions of plane waves. The general expression for a plane wave is denoted in equation 2.3-2.

\[ u(x,y,z) = A e^{ikz} \] (2.3-2)

Here \( k = k_r + ik_i \); \( k_r \) is the propagation constant and \( k_i \) is the amplitude absorption constant. Both vectors, \( k_r \) and \( k_i \) are elements of \( \mathbb{R}^3 \) and in general may be antiparallel. The constant \( A \) in equation 2.3-2 represents the complex amplitude of the wave. Expressed in polar form \( A = A_o e^{i\phi} \), \( A_o \) is the real amplitude and \( \phi \) is the phase of the wave when \( r = 0 \); that is, at the origin of the chosen coordinate system.

The utility of plane waves stems from the ability to express a wide range of optical fields as superpositions of plane waves. Mathematically, the superposition is expressed in a Fourier transform pair, as defined by equations 2.3-3, 4. The wave field in real space, (equation 2.3-3) is represented as an integral superposition of plane waves with complex amplitude \( U(k) \). Alternatively, given the real space wave field, \( u(r) \), the complex amplitude \( U(k) \), corresponding to the plane wave with wavevector \( k \), is defined in equation 2.3-4.

\[ u(r) = (1/2\pi)^3 \int_{-\infty}^{\infty} U(k) e^{ikr} dk \] (2.3-3)

\[ U(k) = \int_{-\infty}^{\infty} u(r) e^{-ikr} dr \] (2.3-4)

Restrictions on the field \( u(r) \) are defined by the Riemann-Lesbegue theorem.
The limits of integration of the complex wavevector $\mathbf{k}$ in equation 2.3-3 partially depends on the optical media the waves are propagating in. At the arbitrary point $\mathbf{r}$, we must have $\mathbf{k} \cdot \mathbf{k} = n^2 k^2$; where equality is for both the real and imaginary parts. In effect, this constrains $k_x$ and $k_y$ to lie on quartic surfaces in $\mathbb{R}^2$.

In optical imaging problems the field on a two dimensional surface is of interest. Conceptually, the function of an optical imaging system is to transfer a field distribution from a given surface, the object field, to a second surface, the image field. The fields on the object and image surfaces are often expressed as a superposition of plane waves. When the waves freely propagate, the imaginary wavevector is zero; $k_z = 0$. The field amplitude on a plane surface is expressed in equations 2.3-5 and 2.3-6.

\[
\begin{align*}
    u(x,y) &= (1/2\pi)^2 \int U(k_x,k_y) e^{i(k_x x + k_y y)} \, dk_x \, dk_y \quad (2.3-5) \\
    U(k_x,k_y) &= \int u(x,y) e^{-i(k_x x + k_y y)} \, dx \, dy \quad (2.3-6)
\end{align*}
\]

The Fourier representation of the optical field on the object or image planes is fundamental to the linear system interpretation of optical systems. In equations 2.3-5 and 6, each $(k_x, k_y)$ component represents a plane wave with wavevector $\mathbf{k} = (k_x, k_y, (n^2 k^2 - k_x^2 - k_y^2)^{1/2})$ and amplitude $U(k_x, k_y)$. Small $(k_x, k_y)$ represent waves at near normal incidence to the plane. These waves contain the coarse or low frequency information in an image. Larger $(k_x, k_y)$ represent waves leaving the plane at steeper angles.
These wave components provide the sharpness or high frequency information in an image.

The most general linear relation relating the object and image plane wave spectrums is given by equation 2.3-7 (Walther, 1968). Here, the plane wave spectrum of the object and the image fields are respectively given by \( U_o(k) \) and \( U_i(k') \). The transverse wavevector is defined by, \( k = k_x \hat{x} + k_y \hat{y} \); here \( \hat{x} \) and \( \hat{y} \) are unit vectors in the \( x \) and \( y \) directions respectively, primes denote the image plane, while the unprimed quantities denote the object plane. The kernel function, \( K(k',k_x,k_y) \), indicates the relative amplitude of the plane wave \( (k',k_y) \) in the image plane resulting from the plane wave \( (k_x,k_y) \) leaving the object plane. The amplitude of the plane wave \( (k',k_y) \) in image space is a superposition of the contributions from all object waves \( (k_x,k_y) \). The kernel function determines the imaging properties of the optical instrument.

\[
U_i(k') = \int K(k',k_x,k_y) U_o(k) dk_x
\]  
\( (2.3-7) \)

Once the plane wave spectrum of the object field and the kernel function are known, the field distribution on the image plane can be determined.

An expression for the field amplitude on the image plane can be constructed from the above equations. Equation 2.3-5 combined with equation 2.3-7 relates the field amplitude on the image plane in terms of the kernel function and the plane wave spectrum of the object field. Equation 2.3-6 is used to express the object field plane.
wave spectrum in terms of the field amplitude in the object plane. Combining these equations, the field amplitude in the image plane is expressed in equation 2.3-8 in terms of the object field amplitude;

\[ u(x') = \int u(x) g(x, x') dx \]  

(2.3-8)

\( g(x', x) \) is defined in equation 2.3-9.

\[ g(x', x) = \left( \frac{1}{2\pi} \right)^2 \int \exp[i(k'_x x' - k_x x)] K(k'_x, k_x) dk'_x dk_x \]  

(2.3-9)

Equation 2.3-8 has a simple interpretation. The function \( g(x', x) \) is the Green’s function or the amplitude PSF for the optical system. The field amplitude at any point in the object plane is a superposition of Green’s functions; each Green’s function is centered at an image point and weighted by the amplitude of object point being imaged.

Equations 2.3-8 and 2.3-9 can be simplified further under certain approximations. First, in some cases the optical system is replaced by an equivalent thin lens system with the stop at the lens. Second, the image plane can be scaled by the geometric magnification, \( M \), of the optical system. Scaling has the effect of normalizing the image plane coordinates. Third, the stationary phase approximation (Born and Wolf, 1980) applied in the object space is equivalent to the Fourier optics approximation (Walther, 1968). With these assumptions, the amplitude point spread
function takes the form (Goodman, 1968) of equation 2.3-10.

\[ g(r, r') = \frac{1}{z_0 z_i} \exp\left(-\frac{-ikr^2}{2z_0}\right) \exp\left(-\frac{-ikr'^2}{2z_i}\right) P(Mr/\lambda z) \]  

(2.3-10)

Here: \( z_0 \) and \( z_i \) are the distances from the lens to the object and image plane respectively; \( P(k) \) is the Fourier transformation of the pupil function. Points on the object and image planes are denoted by \( r \) and \( r' \) respectively. Thus, except for the quadratic phase factors, the amplitude point spread function is the Fourier transform of a scaled version of the pupil function \( P \).

In some cases the quadratic phase factors in equation 2.3-10 are neglected (Goodman, 1968). For these cases, the kernel function is given by equation 2.3-11.

\[ K(k', k) = p(\lambda z) \delta(k' - k) \]  

(2.3-11)

This case has a particularly simple linear system interpretation. The eigenfunctions of the linear system are the plane waves exiting the object plane and arriving at the scaled image plane. The eigenvalues corresponding to each plane wave component are given directly from a scaled version of the complex amplitude transmittance of the pupil function. The scaled pupil transmittance function is the continuous eigenvalue spectrum of the linear system. In the imaging process, a spectrum of plane waves leaving the object plane, is multiplied by a continuous eigenvalue spectrum, given by the pupil transmittance function, which gives the image plane wave spectrum. Alternatively, the scaled pupil transmittance function is a band pass filter for the object.
plane signal. Although these results are only strictly applicable to the simple thin lens case with the aperture stop at the lens, similar results exist for more general systems (Gaskill, 1978).

The formation of the field on the image plane is complicated by the coherence properties of the light being imaged. Coherence is described by the mutual coherence function. The mutual coherence function relates the optical field at two space-time points and is defined in equation 2.3-12.

\[ \Gamma(P_1, P_2; t_2 - t_1) = \langle u(P_1, t_1) u^*(P_2, t_2) \rangle \]  

(2.3-12)

The angular brackets represent an ensemble average over the random process which describes the optical field in space-time. Assuming quasi-monochromatic light, \( \Delta \nu < \nu_c \) (Goodman, 1985), and expressing the field amplitudes by their associated analytic signal representations (Papoulis, 1991), the mutual coherence function is defined in equation 2.3-13. Here, \( \tau = t_2 - t_1 \) is the separation in time and \( \nu_c \) is the mean spectral frequency of the light. When \( \tau = 0 \), the mutual coherence function becomes the mutual intensity function, denoted by \( J(r_1, r_2) \).

\[ \Gamma(r_1, r_2; \tau) = \langle u(r_1) u^*(r_2) \rangle e^{-i2\pi \nu_c \tau} \]  

(2.3-13)

\[ J(r_1, r_2) = \langle u(r_1) u^*(r_2) \rangle \]  

(2.3-14)

In imaging problems where the light is partially coherent, the mutual intensity
function replaces the field intensity or amplitude. The mutual intensity function simplifies to the intensity when $r_1 = r_2 = r$, then $I(r) = J(r, r)$. Thus the intensity function, $I(r)$, can always be deduced from the mutual intensity $J(r_1, r_2)$. The mutual intensity function enables a uniform and consistent treatment of image formation by coherent, incoherent and arbitrary partially coherent light.

The propagation of the mutual intensity function from the object plane to the image plane was originally investigated by H.H. Hopkins (1953, 1955). Subsequently many workers have investigated the topic. The basic equations are derived with the results presented here. Using equation 2.3-8 for the field amplitude on the image plane, the mutual intensity function on the image plane takes the form of equation 2.3-15.

$$J(\mathbf{r}', \mathbf{r}'') = \langle u(\mathbf{r}_1)g(\mathbf{r}_1, \mathbf{r}_2)\mathbf{d}r_1 \rangle$$

If the random processes $u(\mathbf{r}_1)$ and $u^*(\mathbf{r}_2)$ are spatially independent the averaging is taken inside the integral. The integrals are rearranged to give equation 2.3-16.

$$J(\mathbf{r}', \mathbf{r}'') = \int \int J(r_1, r_2)g(\mathbf{r}_1, \mathbf{r}_2)g^*(\mathbf{r}_1, \mathbf{r}_2)\mathbf{d}r_1 \mathbf{d}r_2$$

This is a standard equation: it expresses the transfer of the mutual intensity function from the object plane to the image plane (Goodman, 1985). When the Green's function, $g(\mathbf{r}, \mathbf{r})$, is the Fourier transform of the scaled pupil transmittance function, then the Fourier transform of the mutual intensity function $(\mathcal{F}J)$ simplifies to equation...
2.3-17.

\[
(\mathcal{F}J)(k_1,k_2) = M(k_1,k_2) = (\mathcal{F}J)(k_1,k_2)
\]  

(2.3-17)

Here, \((\mathcal{F}J)\) is the Fourier transform of the mutual intensity function and \(M(k_1,k_2)\) is given in equation 2.3-18.

\[
M(k_1,k_2) = p(\lambda z_k/M)p^*(-\lambda z_k/M)
\]  

(2.3-18)

The transfer of the mutual intensity function between the object and the image plane is also a linear system. Here, however, the mathematics is complicated since the eigenfunctions are in a four dimensional wavevector space.

The cases of coherent and incoherent imaging are addressed. For coherent light, the mutual intensity function in equation 2.3-16 becomes, \(J(x_1,x_2) = u(x_1)u^*(x_2)\).

The two integrals over the object plane separate into the product of two integrals, as in equation 2.3-19. The intensity on the image plane is evaluated by setting \(x_1' = x_2' = x\) in equation 2.3-19. The expression for the intensity on the image plane simplifies to equation 2.3-20.

\[
J(x_1',x_2') = \left(\int u_0(x_1)g(x_1',x_1)dx_1\right) \left(\int u_0(x_2)g(x_2',x_2)dx_2\right)^*
\]  

(2.3-19)

\[
I(x) = \left|\int u_0(x)g(x',x)dx\right|^2
\]  

(2.3-20)

For the case of incoherent imaging, the intensity on the image plane is also
derived from equation 2.3-16. For an ideally incoherent source, the mutual intensity simplifies to equation 2.3-21.

\[ J(r_1, r_2) = I_0 \delta(r_1 - r_2) \]  

(2.3-21)

Actually, equation 2.3-21 is an idealization of an incoherent source; all real sources have coherence lengths of the order of the wavelength of light (Wolf, 1978). Substitution of equation 2.3-21 into equation 2.3-16, and setting \( r_1 = r_2 = r' \) gives an expression, equation 2.3-22, for the incoherent image intensity.

\[ I(x') = \left| I_0(x) \right|^2 dx \]  

(2.3-22)

Thus for incoherent imaging, the intensity point spread function is simply the square of the amplitude point spread function. In cases where the point spread function is the Fourier transform of a scaled pupil function, the intensity optical transfer function, that is the Fourier transform of the intensity point spread function, is simply the complex autocorrelation of the scaled pupil transmittance function. The intensity optical transfer function for a thin lens is given in equation 2.3-23. Here \( G(k_x) \) is the intensity optical transfer function, or the Fourier transform of the intensity point spread function and \( \star \) represents the autocorrelation operation.

\[ G(k_x) = \exp(i \lambda z k_x / M) \star p^*(\lambda z k_x / M) \]  

(2.3-23)

The above results demonstrate the differences that can result when the optical system is imaging coherent, incoherent, and partial coherent light. When imaging
coherent light, interference effects arise because the intensity is formed from a coherent superposition of amplitudes. With incoherent light, interference effects are washed out. The intensity at an image point is formed from a superposition of intensity point responses. The differences between coherent and incoherent imaging will be discussed by way of example in the following section.

### 2.4 Examples of Optical Imaging

The mathematical prediction of the response of various optical systems has become a refined discipline. The research of various workers and the advent of the high speed digital computer have provided important advances. This subchapter points out some of the differences that can arise when imaging coherent and incoherent light. Many astronomical telescope designs have a centrally obscured pupil; hence, imaging systems with a centrally obscured pupil are considered throughout the discussion. The case of an unobscured pupil or clear aperture is a specific example of the general case considered here.

Light collected from an incoherent astronomical source is coherent under various observing geometries. The transverse coherence properties of light emitted by an incoherent source are mathematically defined by the van Cittert-Zernike theorem. The geometry for the van Cittert-Zernike theorem is illustrated in figure 2-1. The theorem (Goodman, 1985) defines the complex coherence factor between two
arbitrary observation points in space according to equation 2.4-1. The complex coherence factor is the normalized mutual intensity function; it is defined by

\[ \mu(r_2, r_1) = \frac{e^{-i\phi} \int [I(\alpha) \exp(ik_0(r_2-r_1) \cdot \alpha)d\alpha]}{\int I(\alpha)d\alpha} \]  

(2.4-1)

\[ \phi = \frac{\pi(r_2^2 - r_1^2)}{2z_o} \]  

\[ k_0 = \frac{2\pi}{\lambda} \]  

\[ \alpha = (\alpha_x, \alpha_y) \]  

\[ I(\alpha) \]  

**Observation Plane**

**Source Plane**

**Figure 2-1:** Geometry of the van Cittert-Zernike theorem.

The phase angle is defined by \( \psi = \frac{\pi(r_2^2 - r_1^2)}{2z_o} \). \( z_o \) is the distance between the object and image planes; \( k_0 = \frac{2\pi}{\lambda} \) is the free space wavevector; \( \alpha = (\alpha_x, \alpha_y) \) is the two-dimensional vector defining a direction toward the source; \( I(\alpha) \) is the intensity of the source in direction \( \alpha \).

The van Cittert-Zernike theorem is used to predict the transverse coherence of observed star light. With a uniform source with an angular diameter \( \alpha_o \), the
complex coherence factor is computed in equation 2.4-2.

\[
\mu(\Delta r=|r_2-r_1|) = 2 J_1\left(\frac{\pi \alpha_x \Delta r}{\lambda}\right)(\frac{\pi \alpha_x \Delta r}{\lambda})
\]  

(2.4-2)

Here, \( J_1(x) \) is the first order Bessel function. The phase \( \psi \) in equation 2.4-1 is approximately zero when the distance from the source to observation plane, denoted by \( z \), satisfies \( z > r^2 / \lambda \). The complex coherent factor is plotted versus the normalized observing plane coordinate separation in figure 2-2 for three source diameters.

---

**Figure 2-2:** The magnitude of the complex coherence factor.

Since the first zero of the first order Bessel function \( J_1(x) \) is at \( x = 1.22 \), the
The transverse coherence length is $\Delta r = 1.22\lambda/\pi a$. The result indicates that smaller sources have greater coherence lengths. Also, the transverse coherence length scales with the wavelength; hence coherence effects become more pronounced at longer wavelengths.

The importance of coherence effects in astronomical imaging systems is ascertained by computing the complex coherence factor over the entrance pupil for a typical source diameter at the wavelength(s) of interest.

Many astronomical telescopes have a centrally obscured pupil. The obscuration is often due to a circular hole in the center portion of the primary mirror. For such a system, it is necessary to compute the optical transfer function for both coherent and incoherent light to ascertain the imaging performance. The following discussion assumes that the optical transfer function can be determined from a scaled version of the exit pupil transmittance function. This approximation (Gaskill, 1978) is valid for an entire class of optical systems.

For coherent illumination, the optical transfer function is given directly from the exit pupil function as in equation 2.4-3.

$$G(k) = p(z_i/k_{\lambda})$$  \hspace{1cm} (2.4-3)

In equation 2.4-3: $p(r)$ is the complex amplitude transmittance of the exit pupil function at the transverse position $r$, $k_i$ is the transverse wavevector in the image plane, $z_i$ is the distance from the exit pupil to the image plane, and $k_\lambda$ is the free space wavevector. For a centrally obscured pupil, the optical transfer function $G(k_\lambda)$ is a
binary function (i.e. it takes on only the values of 1 or 0). With a pupil diameter of $D_p$ and an obscuration diameter of $fD_p$, the amplitude optical transfer function takes the simple form of equation 2.4-4.

$$G(k) = \begin{cases} 
0 & |k_z| < \frac{fD}{2} \\
1 & \frac{Dk_z}{2z} < |k| < \frac{fDk_z}{2}
\end{cases}$$  \hspace{1cm} (2.4-4)

The case of an incoherent illuminated, centrally obscured pupil is slightly more complicated. The intensity optical transfer function is derived from equation 2.3-23. With a magnification $M=1$, the expression for the intensity transfer function is given by equation 2.4-5.

$$G(k) = (p \star \tau^*)(z/k)$$  \hspace{1cm} (2.4-5)

In equation 2.4-5: $p(r)$ is the complex amplitude transmittance of the exit pupil, $\star$ is the autocorrelation operation, $z$ is the distance from the exit pupil to the image plane and $k_z$ and $k$ are as defined previously. When there is no phase variation within the pupil, determination of the intensity optical transfer function, $G(k_z)$, reduces to the computation of the autocorrelation of the pupil function. Since the pupil function has circular symmetry and can only take on values of 1 or 0, the autocorrelation operation reduces to the computation of overlap area of two identical but displaced pupil functions.
The geometrical computation of the autocorrelation function is diagrammed in figure 2-3. Here, $\Delta$ is the offset between the two pupils, $f$ is the diameter of the circular obscuration, and the outer diameter of the pupil is unity.

![Figure 2-3: Geometry for autocorrelation function computation.](image)

The overlap of the two pupil functions depends on the size of the central obscuration $f$ and the relative offset $\Delta$. A perspective of all the geometrical possibilities is gained by plotting the separation $\Delta$ versus the obscuration diameter for three concurrent point relationships. The three concurrent point relationships correspond to three pupil offsets. First, when point $Y$ of the offset pupil (the rightmost pupil in figure 2-3) is concurrent with point $B$. Second, when point $X$ of the offset
pupil is concurrent with point A. Third, when point X of the offset pupil is concurrent with point B. Each possibility is indicated in figure 2-4.

Figure 2-4: $\Delta$ vs. $f$ relationships. Three concurrent point relationships, Y-B, X-A, and X-B.

Examination of figure 2-4 indicates that for any fixed $f$ there are four regions of $\Delta$ with distinct overlap patterns. For $0 \leq f \leq 1/3$ (case I), there are four regions of $\Delta$. In region 1, $0 \leq \Delta \leq f$; for region 2, $f \leq \Delta \leq (1-f)/2$; in region 3, $(1-f)/2 \leq \Delta \leq (1+f)/2$; and finally for region 4, $(1+f)/2 \leq \Delta \leq 1$. For $1/3 \leq f \leq 1$ (case II), the four regions of $\Delta$ correspond to: region 1, $0 \leq \Delta \leq (1-f)/2$; region 2, $(1-f)/2 \leq \Delta \leq f$; region 3, $f \leq \Delta \leq (1+f)/2$; and region 4, $(1+f)/2 \leq \Delta \leq 1$. Totally, there are two intervals of $f$, which correspond to cases I and II, to each of which there correspond four regions of $\Delta$. 
The area of the overlap patterns can be evaluated by the addition or subtraction of two area types-diagrammed in figure 2-5. The overlap area in figure 2-5a, denoted by $A_1(f)$, is evaluated in equation 2.4-6. The overlap area diagrammed in figure 2-5b is given by equation 2.4-7.

\[
A_1(\Delta f) = \left(\frac{f^2}{2}\right) \cos^{-1}(\Delta f) - \left(\frac{\Delta}{2}\right) \sqrt{f^2 - \Delta^2}
\]  
(2.4-6)

\[
A_2(\Delta f) = \frac{1}{4} \left[ \cos^{-1}(\delta_+) + f^2 \cos^{-1}(\delta_+ f) - \right. \\
\left. 2\Delta (1 - \delta_+^2)^{1/2} \right]
\]  
(2.4-7)

Here, $\delta_+$ and $\delta_-$ are respectively defined as $\delta_+ = \Delta + (1-f^2)/4\Delta$ and $\delta_- = \Delta - (1-f^2)/4\Delta$. With the areas of the two basic overlap patterns known, the autocorrelation function is evaluated.

\[a)\]

\[b)\]

**Figure 2-5:** Two overlap areas for the autocorrelation computation.
The result of the autocorrelation function for the two ranges of $f$, $0 \leq f \leq (1/3)$ and $(1/3) \leq f \leq 1$, are tabulated in tables 2-1 and 2-2 respectively. The modulation transfer function is evaluated by substituting $\Delta = |z_k(k_xD_p)|$ into the expressions in tables 2-1 and 2. The modulation transfer function is plotted in figure 2-6 for three fractional obscuration diameters. In contrast to the coherent transfer function, the incoherent transfer function has a significant low frequency component. In addition, when the obscuration approaches unity there is a range of high frequencies which are passed with a greater amplitude than a clear aperture. With a knowledge of the coherent and incoherent transfer functions, the resulting image of a given object can be computed.
<table>
<thead>
<tr>
<th>Δ Range</th>
<th>Autocorrelation Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \Delta \leq f$</td>
<td>$\frac{2}{\pi(1-f)} \left[ \cos^{-1}(\Delta) - \Delta (1-\Delta^2)^{1/2} - \pi f^2 + f^2 \cos^{-1}(\frac{\Delta}{f}) - \Delta (f^2 - \Delta^2) \right]$</td>
</tr>
<tr>
<td>$f \leq \Delta \leq \frac{1-f}{2}$</td>
<td>$\frac{2}{\pi(1-f)} \left[ \cos^{-1}(\Delta) - \Delta (1-\Delta^2)^{1/2} - \pi f^2 \right]$</td>
</tr>
<tr>
<td>$\frac{1-f}{2} \leq \Delta \leq \frac{1+f}{2}$</td>
<td>$\frac{2}{\pi(1-f)} \left[ \cos^{-1}(\Delta) - \Delta (1-\Delta^2)^{1/2} + \pi f^2 \right]$</td>
</tr>
<tr>
<td>$\frac{1+f}{2} \leq \Delta \leq 1$</td>
<td>$\frac{2}{\pi(1-f)} \left[ \cos^{-1}(\Delta) - \Delta (1-\Delta^2)^{1/2} \right]$</td>
</tr>
</tbody>
</table>

Table 2-1: The autocorrelation function for $0 \leq f \leq (1/3)$. 
<table>
<thead>
<tr>
<th>Δ Range</th>
<th>Autocorrelation Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \Delta \leq \frac{1-f}{2}$</td>
<td>$\frac{2}{\pi(1-f^2)} \left[ \cos^{-1}(\Delta) - \Delta(1-\Delta^2)^{1/2} - \pi f^2 \right.$ $+ f^2 \cos^{-1}(\frac{\Delta}{f}) - \Delta(f^2-\Delta^2)^{1/2} \left. \right]$</td>
</tr>
<tr>
<td>$\frac{1-f}{2} \leq \Delta \leq f$</td>
<td>$\frac{2}{\pi(1-f^2)} \left[ \cos^{-1}(\Delta) - \Delta(1-\Delta^2)^{1/2} + f^2 \cos^{-1}(\frac{\Delta}{f}) - \Delta(f^2-\Delta^2)^{1/2} - 4A_3 \right]$</td>
</tr>
<tr>
<td>$f \leq \Delta \leq \frac{1+f}{2}$</td>
<td>$\frac{2}{\pi(1-f^2)} \left[ \cos^{-1}(\Delta) - \Delta(1-\Delta^2)^{1/2} - 4A_3 \right]$</td>
</tr>
<tr>
<td>$\frac{1+f}{2} \leq \Delta \leq 1$</td>
<td>$\frac{2}{\pi(1-f^2)} \left[ \cos^{-1}(\Delta) - \Delta(1-\Delta^2)^{1/2} \right]$</td>
</tr>
</tbody>
</table>

Table 2.2: The autocorrelation function for $(1/3) \leq f \leq 1$.

The point spread function for coherent illumination can be computed. A single dimension is considered in order to simplify the computations. We assume a point source centered in the field. The image plane is scaled to give unit magnification. The obscuration of the pupil has a diameter of $fD_p$, where $D_p$ is the exit pupil.
The coherent frequency cutoff, $k_c = k_y/(2f\#)$, of the optical system is $k_c = .5$, measured in arbitrary inverse length units. The coherent frequency cutoff of .5 is assumed for comparison with the incoherent case, which assumes $k_c = 1$. The intensity of the coherent point spread function is illustrated in figure 2-7.

The point spread function in figure 2-7 clearly shows the effects of the coherent illumination. The central obscuration removes some of the low frequency components. The low frequency attenuation results in an energy transfer from the central maximum to the first order maximum. With a larger obscuration, the effect becomes more pronounced as evidenced by the $f=.5$ case.
In a similar manner, the incoherent case is considered. For comparison with the coherent case, the f# of the optical system is left unchanged. For incoherent imaging the cutoff frequency is \( k_c = k_0 / f# \). For the system considered here \( k_c = 1 \), measured in arbitrary inverse length units. As in the coherent case, the image plane is scaled to give unit magnification, and the object is taken as a centrally placed point source in the center of the field. Two obscuration diameters were considered: \( f = 0.25 \), and \( f = 0.75 \). The image formed is illustrated in figure 2-8. A similar effect, compared to the coherent case, is evident. There is less energy in the central maximum and the first order side lobes are larger when the obscuration increases. The transfer of energy into the secondary lobes is less pronounced than in the incoherent case.
The examples presented illustrate some of the differences between coherent and incoherent imaging. The differences can be accentuated by a centrally obscured pupil. The detailed nature of the differences depends on the spatial frequency spectrum of the imaged object. For coherent illumination, relatively more energy goes into the secondary maximum and the image is susceptible to ringing.

Coherence also effects the detection process in the detector material. For the purpose of this discussion, transverse coherence, as described by the van Cittert-Zernike theorem, is distinguished from longitudinal coherence. Longitudinal coherence is similar to transverse coherence; the two spatial points under consideration are positioned along the line of propagation in the longitudinal case and
perpendicular in the transverse case.

Longitudinal coherence is determined by the bandwidth of the imaged light. Under the quasi-monochromatic approximation, $\Delta \nu \ll \nu_0$, the light has a longitudinal coherence length of $\sim c/\Delta \nu$, which can be appreciable at optical frequencies. The quasi-monochromatic approximation and hence longitudinal coherence is assumed throughout the remainder of this dissertation. Longitudinal coherence is an important factor in computing the density of absorbed photons in longitudinal or front plane illuminated detector arrays.

The transverse spatial coherence of the imaged light affects the detection process in transverse detector geometries. As the transverse dimension of the linear detector element decreases, the detector begins to behave as an optical waveguide. If the transverse dimension is reduced far enough, the incoming light will have a high degree of coherence over the launching facet of the transverse detector element. Under these circumstances, the absorption of light inside the detector must be considered as a coherent superposition.

The primary components in an imaging optical instrument are the optical elements and the detector array. Oftentimes, a nonimaging optical element is positioned between the imaging optics and a transverse detector array. Such nonimaging optical elements can be found in many naturally occurring optical systems (eg. the insect eye). An array of nonimaging optical elements - or alternatively, a flux
concentrator array - often enhances the performance of the optical-detector system.
Chapter 3 Optical Flux Concentrator Arrays

An optical flux concentrator array can enhance the performance of an optical detector array. Each concentrator is a nonimaging optical element which concentrates the incoming optical flux onto a corresponding pixel element of the detector array. With a concentrator array, each pixel element of the detector array is made considerably smaller; thereby isolating adjacent pixel elements and decreasing the size of each pixel without sacrificing the effective fill factor of the detector array. The reduction in pixel size can decrease the cross talk between the pixels and, in space imaging applications, reduce the effect of stray cosmic ray strikes.

This chapter has a two fold objective. First, a literature search of candidate concentrator designs will be reviewed. Second, general selection criteria will be defined and outlined. The selection criteria consists of a zero-order selection algorithm followed by a performance analysis. Application of the selection criteria to a specific optical-detector system will not be done in this chapter. A description of the dynamic operation of the detector array is a prerequisite to the application of the performance analysis.

This chapter primarily deals with the choice and selection of an optical flux concentrator array. Concentrators are frequently used with transverse arrays. If the thickness of a transverse detector is smaller than the inter-array spacing, a
A simplified treatment of the detection and transport process in a transverse detector is given in chapter 4. A more complete discussion of the longitudinal detector geometry is covered in chapters 5-7. The arguments in this chapter and the next do not assume a full physical optics model. A performance analysis derives selection criteria by following a geometrical ray optics argument. A full physical optics analysis of a concentrator array is beyond the scope of this discussion. The problem is essentially that of waveguide coupling (Marcatili, 1985). The concentrator couples the radiant energy at the image plan into the detector which is viewed as a pseudo waveguide. Finally, a general account of non-imaging concentrators is not given; rather, reference to the text by Welford and Winston (1978) is made.

3.1 Choice of a Flux Concentrator

The choice of a flux concentrator array can be resolved into a two stage process—a zero order selection and a performance analysis. First, a general set of concentrator arrays appropriate to a given application is identified. Based on the general set of concentrator arrays, a zero-order selection is made. The zero-order selection results in a set of candidate concentrator arrays. The zero-order selection process first defines the parameter space of the detector array design. Second, the applicability of each concentrator array in the general set to the detector array's parametric space is determined. The result of the zero-order selection is a set of candidate concentrator
arrays that subsequently undergo a performance-based selection criteria.

The general set of concentrator arrays includes a variety of optical devices. The general set can be classified according to whether the concentrator element relies on reflection, refraction, diffraction, or a combination of these. Reflective concentrators rely on total internal or specular reflection from a dielectric or metallic surface, respectively. The operation of refractive concentrator array elements relies on the bending of light. The light bending is due to a surface gradient (eg. a lens), a refractive index gradient, or a combination of the two. Diffractive or holographic concentrators use a diffraction grating pattern on the surface of a material. Hybrid diffractive elements utilize holographic and conventional components.

Reflective concentrator array elements rely on reflection from either a metallic surface or total internal reflection within a dielectric. Total internal reflecting concentrators (Ning et. al., 1987) require the concentrator material to have an index greater than that of the exterior media. The classical cone (Schmidt-Kloiber and Schoefmann, 1986) and the Winston cone (Hildebrand, 1983) are two common reflective geometries. The Winston cone or parabolic optical concentrator has been analyzed by Barnett (1980). Hybrid elements which use both reflection and refraction have been analyzed (Ning et. al. 1987); a reflective-refractive concentrator is sketched in figure 3-1. O'Gallagher et. al.(1987) have designed a "flow line concentrator" which is a combination of a thin lens and a hyperbolic condensing cone. The flow-line concentrator resembles a trumpet horn with a positive lens positioned at its mouth.
The hyperbolic condensing cone is a reflective hyperbolic surface of revolution. The flow line concentrator achieves concentrations arbitrarily close to the theoretical optimum - as determined from the étendue invariant (Welford, and Winston, 1978). The efficiency of nonimaging concentrators is discussed within a physical optics framework by Winston and Welford (1982).

![Figure 3-1: A hybrid reflective-refractive concentrator.](image)

For transverse detector geometries, one-dimensional concentrator arrays are required. A simple design which relies on total internal reflection is diagrammed in figure 1-2. The wedge concentrator relies on total internal reflection. The incoming light is not deviated in the transverse direction. The ray path through a wedge concentrator is illustrated in figure 3-2.
Of practical importance is a process developed by Cowan (1985). Cowan developed a method of fabricating hexagonal and rectangular arrays of concentrating elements. In his technique, a photoresist is exposed to the interference pattern formed by three (hexagonal array) or four (rectangular array) coherent laser beams. A sketch of the interference pattern is illustrated in figure 3-3. The exposed photoresist is etched to produce an orderly array of optical elements. The arrays thus produced have a striking resemblance to various insect eyes.

The resemblance between Cowan’s arrays and various insect eyes is more than coincidental. The spatial form of various body parts of many biological organisms (e.g., the coats of mammals) have recently been explained in terms of a reaction-diffusion mechanism (Murray, 1989). In fact, under certain circumstances the linear combination of symmetrically placed plane waves is a solution to the reaction-diffusion
Figure 3-3: Hexagonal interference pattern.

Cowan’s optical generation of arrays in the eye of a fly is an analog procedure which corresponds to various solutions of the reaction-diffusion equations of mathematical biology.

Conventional lenses, gradient index lenses, and hybrids are examples of refractive-based concentrator elements. Conventional lenses can be fabricated on a material substrate in an array pattern. Linear arrays of cylindrical lenses etched onto silicon (Si) (Erhardt et. al., 1984) have been tested (Wiemokly et. al., 1985). A cylindrical lens array achieves concentrations in one dimension only; the set up is illustrated in figure 3-4. Arrays of spherical micro lenslets have been etched on GaAs by Ostermayer et. al.(1983). Lenslet arrays are routinely manufactured for optical detector arrays and other applications (Corning, 1988). Arrays of gradient index
(GRIN) lenses have been fabricated by Oikawa, et. al. (1984). An array of gradient index lenses acts analogously to an array of conventional refracting lenses. Hybrids of conventional and gradient-index lenses have been fabricated by Küppers and Schelhas (1984). The hybrid designs have the benefit of utilizing the refractive power of both the lens surface and the graded index.

![Figure 3-4: A cylindrical lens array.](image)

Diffractive or holographic optical elements are utilized as flux concentrators. The fabrication technology for these devices has made significant improvements in recent years (Arnold, 1989). The binary optics group at the MIT Lincoln laboratory pioneered the development of binary or diffractive optics. The primary breakthroughs (Veldkamp and Swanson, 1983) that have fueled the technical advances are electron beam lithography and a precise mathematical treatment of the electromagnetic
properties of diffraction gratings. A notable property of diffractive elements is the large amount of dispersion. The focal length of diffractive lenses varies inversely as the wavelength of the incident light; thus, diffractive elements have large amounts of chromatic aberration. An example of this array type is sketched in figure 3-5. Arrays of diffractive lenses have been demonstrated by Shiono et al. (1987). Hybrid configurations, diffractive elements combined with conventional lenses, have been analyzed by Stone and George (1988). Single elements for the infrared have been fabricated by Fritz and Cox (1989) and Hasman et al. (1989). The hybrid elements can be constructed as achromats; thereby reducing the severity of the large dispersion of purely diffractive elements.

Figure 3-5: A diffractive lens array.

For a given detector array, the zero-order selection process considers each type
of flux concentrator array. The detector array design is described by a point in the
detector parameter space. In the design stage, the description of the detector array
parameters will take the form of mathematical inequalities. In this case, the detector
parameter space is characterized by a volume. A number of detector array parameters
are considered. The mechanical dimensions of the detector array (figure 3-1), the
spectral range of the detectable radiation, and detector operating parameters such as
the temperature are important. Other factors, such as the detector manufacturing
cost and resource availability are also considered.

![Figure 3-6: Geometry of the detector array.](image)

Diffraction effects must also be considered. The qualitative effects of diffraction
can be understood with a plane-wave interpretation of diffraction (Goodman, 1968).
The Fourier transform of the amplitude distribution across the collecting aperture can
be viewed as a distribution of plane waves in wavevector space. For a given \((k_x, k_y)\), the complex value of the Fourier transform is the relative amplitude and phase of the plane-wave with propagation vector given in equation 3.1-1.

\[ k = (k_x, k_y) \sqrt{n^2 k_x^2 - k_y^2} \]  

(3.1-1)

The parameters include \(k_x\), the free space wavenumber; \(n\), the refractive index of the optical media. Also, it is assumed that the aperture is in the \(x,y\) plane and that the \(z\)-direction is normal to the aperture. The size of the elemental collecting aperture of each concentrator element and the optical spectrum of the incident light determine the angular distribution of plane waves.

\[ \text{Yes--Concentrator Accepted} \]

\[ \text{Concentrator} \]

\[ \text{Parameter Space Occupied?} \]

\[ \text{No--Concentrator Rejected} \]

**Figure 3-7: Zero-order selection process.**

The zero-order selection limits the general set of concentrator arrays to a subset of candidate concentrator arrays. The selection process is diagrammed in...
figure 3-7. The process consists of considering each type of concentrator array element and the region in parameter space defined by the detector array. Each concentrator array under consideration is accepted or rejected based on its applicability to the parameter space of the detector array. If for a given concentrator array type, a particular detector array parameter can't be realized, that concentrator type is rejected. If all detector array parameters are realized by a given concentrator array, that concentrator array is included in the subset of candidate concentrator arrays.

3.2 Performance Analysis

The performance analysis further limits the subset of candidate concentrator arrays. The performance of each concentrator array element in the zero-order selection set is analyzed based on performance criteria. This section derives a performance function that can be applied to any concentrator-detector array pair.

In space applications, background cosmic rays striking the detector array are a major concern. Not only disrupting the short term operation of the detector array, cosmic ray hits can cause cumulative radiation damage in the detector material. The performance function is derived by considering the noise equivalent photon irradiance of the detector-concentrator array. Two sources of radiation arise in a measured signal at the detector: photon signal flux from the imaged source and unwanted particle flux primarily due to noisy energetic background sources. The noise
equivalent irradiance of the concentrator-detector array is defined as the photon source irradiance, at the detector array, which gives rise to a signal amplitude that equals the erroneous signal amplitude due to energetic particle background sources. The performance function $P$ is proportional to inverse noise equivalent photon irradiance of the detector-concentrator array.

An expression for $P$ is derived by initially equating the photon source signal with the background particle noise signal as in equation 3.2-1.

$$I_s = I_n$$  \hspace{1cm} (3.2-1)

The photon source signal is rewritten in equation 3.2-2.

$$I_s = E_s A_c \eta_d$$  \hspace{1cm} (3.2-2)

There are three terms in equation 3.2-2: $E_s$ is the source irradiance, $A_c$ is the area on the entrance pupil of the concentrator array element and $\eta_d$ is the quantum efficiency of the detector. The quantum efficiency is defined as the number of detected charges for each incident photon. Likewise, the noise signal $I_n$ is defined in equation 3.2-3.

$$I_n = E_A A_d \eta_b$$  \hspace{1cm} (3.2-3)

The three terms in equation 3.2-3 are likewise defined: $E_A$ is the irradiance of the background cosmic ray flux, $A_d$ is the area of the detector pixel and $\eta_b$ is the number of detected charges resulting from a background particle. In practice, a cosmic ray hit causes a cascade of charges; hence, $\eta_b > 1$. 
Substitution of equations 3.2-2 and 3.2-3 into equation 3.2-1 results in a solution for the inverse noise irradiance in equation 3.2-4.

\[
\frac{1}{E_n} = \frac{A_c}{A_d} \frac{P_d}{E p_b}
\]  

(3.2-4)

The ratio \( \frac{A_c}{A_d} \) is the concentration C. The probability of photon detection, \( \eta_d \), is defined in equation 3.2-5.

\[
\eta_d = \int p(k) f(k) dk
\]

(3.2-5)

The integral contains three terms: \( f(k) dk \) is the fraction of photons in the element \( dk \) of wavevector space; \( p(k) \) is the probability that a ray with direction \( k \) is detected. Substitution of equation 3.2-5 into equation 3.2-4 results in an expression for the inverse irradiance (equation 3.2-6).

\[
\frac{1}{E_n} = (E_n \eta_d)^{-1} \int p(k) f(k) dk
\]

(3.2-6)

The performance function \( P \) is defined by considering the detector-concentrator terms in equation 3.2-6 and is defined in equation 3.2-7.

\[
P = C \int p(k) f(k) dk
\]

(3.2-7)

In words, the performance function is the product of the concentration and the fraction of photons in the detector material that are detected. The optical properties
of the concentrator element will determine the distribution function \( f(k) \). The probability function \( p(k) \) will be determined by the geometry of the detector array and the nature of the photon detection process in the detector array. Qualitatively, the performance function will be large when the concentration is large and the two functions \( p(k) \) and \( f(k) \) are peaked in the same region of \( k \)-space; alternatively, \( P \) is large when most of the rays entering the detector array have a large probability of being detected.

The performance analysis is completed by evaluating the performance function \( P \) for each candidate concentrator-detector array identified in the zero-order selection. Upon evaluation and consideration of other relevant factors (e.g., ease and cost of manufacture), a concentrator array is chosen. The evaluation of \( p(k) \), the probability of detection of a ray with wavevector \( k \), is discussed for transverse arrays in chapter 4 and longitudinal arrays in chapter 6.

Flux concentrator arrays are often interfaced with detector arrays to improve the system performance. The detector array can be either a monolithic semiconductor material in a longitudinal geometry or a stack of linear detectors in the transverse geometry. Concentrators are frequently used in conjunction with transverse detector arrays. The imaging performance of a linear transverse detector array is discussed in the next chapter.
Chapter 4 Transverse Detectors

Two detector geometries will be examined. Longitudinal arrays are analyzed in depth in chapters 5-7; this chapter studies the transverse geometry. In practice, a two-dimensional transverse imaging array is constructed by stacking identical one-dimensional arrays; a sketch of the construction is illustrated in figure 4-1. Transverse arrays are used when the detector design is constrained by detector materials with small absorption coefficients or long absorption lengths. The transverse geometry accommodates materials with a significant smaller absorption coefficients.

Figure 4-1: Construction of a transverse array.

There are a number of similarities and differences that exist between longitudinal and transverse detector arrays. The basic physics of the transport process
is the same in both detector geometries; diffusion and recombination of the photogenerated charges are the dominant mechanisms. As with longitudinal arrays, transverse arrays are modeled in the steady state. The primary difference is in the mathematical description of the transport process. The difference arises in the boundary conditions which are assumed in solving the equation for the current density. In a longitudinal array, the current density will be taken as zero on the boundary; in a transverse array mixed conditions are assumed.

The optical absorption in the two detector types is modeled differently. Since transverse detectors normally have a longer optical path, defocus of the detected light must be considered. The defocus can be alternatively viewed as photon cross talk between adjacent pixels. In longitudinal detector geometries the width of the optical point spread function is the sole contribution to the optical or photon cross talk. In transverse detectors both the width of the optical point spread function and the beam defocus in the detector contribute optical cross talk. Both types of cross talk result in degradation of the detector transfer function.

As with a longitudinal detector array model, the goal of a transverse detector array model is prediction of the imaging response as a function of various detector design and operating parameters. A model of the detector array can be quite useful. The model often serves as an integral part of simulation and image processing studies and aids in making design and construction decisions.
This chapter presents a simplified method of modeling transverse far infrared detector arrays. First, the optical absorption in the detector is described with a ray model. The ray model predicts a spatial distribution of absorbed photons in the transverse detector material. A number of assumptions are made in the describing the transport and diffusion of the photogenerated charges. A steady state diffusion equation is used to model the signal current density in the detector. The imaging performance of the detector is quantified with computations of the detector transfer function, point spread function, and the diffusive cross talk. Finally, the model is used to analyze and evaluate some experimental results.

4.1 The Optical Detector Model

The first component in a transverse detector array model is a description of the optical system. A number of assumptions are made in the optical model; each assumption brings a simplification to the analysis. The optical model must interface with the description of the optical absorption in the detector. Since a ray model will be used to describe the absorption process, a ray model is also used to describe the optical system.

We assume an optical system which is circularly symmetric, aberration free with a central obscuration in the pupil. A diagram of the optical-detector system is illustrated in figure 4-2. For transverse detectors we assume geometric imaging on the
detector array. Diffraction effects will be accounted for when the detector transfer function is constructed.

The pupil of the optical system is uniformly filled and aberration free. The image on the detector array is formed by the bundle of rays extending to each image point from each point on the exit pupil. Each ray forming the geometric image is traced through the transverse detector. An illustration of a ray entering the transverse detector is illustrated in figure 4-3.

Upon entering the detector a ray can be absorbed, multiply reflected or exit the facet it entered. A flux concentrator array may be in optical contact with the detector material as discussed in chapter 3. We assume the Fresnel relations to describe the
reflection and refraction amplitudes of a given ray. The intensity of photons, after entering the detector, is described by equation 4.1-1.

\[ I(r) = I_0 e^{-\alpha(\Phi)z} \]  \hspace{1cm} (4.1-1)

Here: \( I(r) \) is the number of photons per unit area at the point \( r \); \( I_0 \) is the number of photons incident on the entrance facet of the detector; \( \alpha(\Phi) \) is the absorption coefficient which depends on the incident angle \( \Phi \) and is measured in inverse length units; \( z \) is the z-coordinate of the point \( r \). The dependence of the absorption coefficient follows from the boundary conditions at the vacuum-detector interface.
The absorption of rays entering the detector is modeled with a Monte Carlo simulation. To compute the effects of defocus and diffusion a field-centered point source is assumed. Since geometric imaging is presumed, the point image is formed with a bundle of rays extending from the pupil to the detector. For each ray entering the detector a random number $x$ is generated from a uniform distribution on $[0,1]$; an absorption length $z_a$ is then computed according to equation 4.1-2.

$$z_a = -\frac{1}{\alpha} \ln(1-x) \quad (4.1-2)$$

The function in equation 4.1-2 is derived from the photon absorption distribution density, $p(x) = \alpha e^{-\alpha x}$. The numerical generation of arbitrary probability laws is discussed by Frieden (1983).

From each absorption length $z_a$, a position and number fraction is computed. The position $(y_n,z_n)$ of the absorbed photon is determined from the incident angle $\theta_d$ of the ray in the detector as described by equation 4.1-3.

$$x_n = z_n \tan(\theta_d) \quad (4.1-3)$$

If the computed position lies outside the detector boundaries, the ray is folded back into the detector. The folding accounts for the reflections that can occur from the facets of the detector material. Since reflection losses occur, a less-than-unity number is added to absorbed distribution at point $(y_n,z_n)$. An example absorbed photon distribution generated in the way just described is computed and displayed in figure 4-4.
4.2 The Transport Model

With a knowledge of the absorbed photon distribution the transport of the photogenerated charges may be considered. The objective in modeling the transport process is to compute the signal current density generated in the detector. A considerably simplified approach, relative to the longitudinal transport, is taken in this chapter.

A number of assumptions are made to simplify the description. In effect, the simplifications render a signal current density given by the solution of the time independent diffusion equation. First, we assume the recombination time of the
charges satisfies equation 4.2-1.

\[ \tau_r < t_d \]  \hspace{1cm} (4.2-1)

Here \( \tau_r \) is the recombination time and \( t_d \) is the drift time of a photogenerated charge across the thickness of the detector. In other words, multiple injections of the photogenerated charges are rare; most charges recombine in the first or second pass through the detector. The assumption in equation 4.2-1 is often satisfied. Second, we assume the detector operates in the steady state; the current density in the detector has equilibrated. This assumption is also often realized in practice. A diagram of the detection and subsequent transport is illustrated in figure 4-5.

![Diagram of detection and transport process](image)

**Figure 4-5:** The absorption and transport process.

With the assumptions made, the diffusion of the photogenerated charges can
be modeled with a mean diffusion length $L_d$. The current density is described by the steady state diffusion equation, written in equation 4.2-2.

$$\frac{\partial J_o}{\partial y} + \frac{\partial J_o}{\partial z} - \frac{J_o}{L_d^2} + s(y,z) = 0$$  \hspace{1cm} (4.2-2)

Here, $J_o(y,z)$ is the steady state current density; $L_d$ is the diffusion length; $s(y,z)$ is the photon source term. The source distribution, $s(y,z)$, is determined from the Monte Carlo simulation of the rays entering the detector; an example source distribution is illustrated in figure 4-4.

The steady state current density is obtained by solving equation 4.2-2. A finite difference numerical approach is used to solve the steady state diffusion equation. In the finite difference method, the spatial domain under consideration is divided into a finite set of discrete points. For a transverse detector, the spatial domain is the rectangle corresponding to the detector material. A mesh is illustrated in figure 4-6.

The equation in 4.2-2 is numerically approximated at each point. At the $(i,j)$th lattice point, the second derivatives in equation 4.2-2 are approximated according to equation 4.2-3. The interval $\Delta y$ is the spacing between successive grid points in the $y$-direction. A similar expression can be written for the second derivative with respect to $z$. The discrete approximations are written at each of the grid points illustrated in figure 4-6.
Figure 4-6: A mesh for the transverse detector geometry.

\[ \frac{\partial^2 J_x}{\partial y^2} = \frac{J_x(i+1,j)-2J_x(i,j)+J_x(i-1,j)}{\Delta y^2} \quad (4.2-3) \]

A complication arises at the boundary points. Since the approximation for the second partial derivative in equation 4.2-3 requires functional values outside the boundary, a boundary condition must be assumed. Two boundary conditions are supposed for the transverse geometry. At the sides of the detector the boundary condition in equation 4.2-4 is presumed. Here \( L_y \) is the length of the detector in the y-direction; the right and left boundaries are positioned at \( y = \pm L_y / 2 \), respectively. Since a centered point source is assumed, the vanishing current density at the sides is satisfied.

At the front and rear surfaces of the detector the boundary conditions in
In equation 4.2-5, \( z^* \) represents a derivative taken in the plus or minus \( z \) direction. Stated otherwise, equation 4.2-5 requires the current density at the front and rear surfaces to be identical to the value just inside each respective surface. With the boundary conditions in equations 4.2-4 and 4.2-5, the steady state diffusion equation can be discretized at each point.

The steady state diffusion equation can be transformed into a finite dimensional matrix equation. The unknown steady state current density is transformed into an undetermined column vector according to equation 4.2-6.

\[
x(n)=J_o(i,j), \quad n=jN_y+i
\]

(4.2-6)

The source term, \( s(y,z) \), becomes a known column vector given in equation 4.2-7.

\[
b(m)=s(i,j), \quad m=jN_y+i
\]

(4.2-7)

Combination of the each of the discretizations gives the matrix equation in equation 4.2-8. Solution of equation 4.2-8 for the vector \( x_n \) yields the numerical solution for the unknown steady state current density \( J_o \). An example current density corresponding
to the detected photon distribution is illustrated in figure 4-7.

\[ A_n x_n = b_n \]  

(4.2-8)

Figure 4-7: An example steady state current density.

4.3 The Imaging Response

Ultimately we wish to determine the imaging response of the detector. The imaging response is measured with respect to transfer functions, point spread functions and cross talk. Computation of the image response is quite useful for image simulation and processing studies. With a knowledge of the detector image response, appropriate image reconstruction algorithms can be designed. The image
reconstruction algorithms can take into account the degradation in the detector response. The problem is to interface the result for the steady state current density with a measure of the imaging response.

The steady state current density computed in the last section reveals the degree of diffusive spreading and photon cross talk in a transverse detector. To compute the steady state diffusive response, geometrical point imaging was assumed. In real optical systems, however, the image is never a delta-like function. In real optical systems, the point response is broadened due to diffraction out of the exit pupil. Thus, there are three degradations in the response: the diffraction of the optical system, the defocus of the beam in the detector and the diffusion of the photogenerated charges. We will characterize how each of these contributes to the degradation in the imaging response.

We assume the steady state current is a Green's function for the defocus and the diffusion. The detector point response is given by equation 4.3-1.

\[
I(x) = L_v \text{rect}(\frac{x}{L_v}) \text{comb}(\frac{x}{L_s}) \text{rect}(\frac{x}{L_p}) \ast [J_o(x) \ast \text{PSF}_o(x)]
\]  

(4.3-1)

In equation 4.3-1, \(L_v\) is the length of the linear transverse array; \(L_s\) is the spacing between the pixels; \(L_p\) is the length of the collecting pad; \(J_o(x)\) is the steady state current computed numerically in section 4.2; \(\text{PSF}_o(x)\) is the optical point spread...
function; the symbols ★ and * are the autocorrelation and the convolution, respectively.

The validity of equation 4.3-1 is easily established. First, consider the correlation and convolution term is given by equation 4.3-2.

\[
    i_p(x) = \text{rect}(\frac{x}{L_y}) \ast \left[ J_p(x) \ast \text{PSF}_p(x) \right] 
\]

(4.3-2)

The autocorrelation of the rect(\_) function merely integrates the detector current in the detector over the finite pixel size. The current arises from two terms - the point response of the optical system and the point response of the detector. The net current response is formed by the convolution of the optical point response with the point response of the detector.

The remaining terms in equation 4.3-1 take into account the finite size of the detector and the discrete spacing of the pixel elements. The \text{comb(\_)} function is defined in equation 4.3-3.

\[
    \text{comb}(\frac{x}{L_y}) = \frac{1}{L_y} \sum_{n=-\infty}^{\infty} \delta(x-nL_y) 
\]

(4.3-3)

The function \(i_p(x) \delta(x-nL_y)\) is the pixel current in the nth pixel centered at \(x=nL_y\). The delta function, \(\delta(x-nL_y)\), is zero everywhere except at \(x=nL_y\). At \(x=nL_y\), the delta function is multiplied by the current \(i_p\), collected in the nth pixel. The final function, \text{rect}(\frac{x}{L_y})\), truncates the \text{comb(\_)} function at boundaries of the linear detector.
The detector transfer function can easily be determined. We define the detector transfer function as the Fourier transform of the detected current when imaging a point source. The Fourier transform of equation 4.3-1 is computed in equation 4.3-4.

$$DTF(k) = [L_s \text{sinc}(\frac{k}{k_p}) * \text{comb}(\frac{k}{k_s})] *$$ $$L_s \text{sinc}(\frac{k}{k_p}) * \mathcal{F}_s(k) * MTF(k)$$

(4.3-4)

In equation 4.3-4, $k_p = 2\pi/L_p$, $k_s = 2\pi/L_s$, $k_p = 2\pi/L_p$, $\mathcal{F}$ is the Fourier transform and $MTF(k)$ is the modulation transfer function of the optical system.

The expression in equation 4.3-4 may be simplified with some approximations. In most detectors, the size of the detector array is much larger than the spacing between the individual detector pixels; that is $L_s \gg L_p$. When this approximation holds, the $\text{sinc}(\frac{k}{k_p})$ function is very narrow and behaves as a delta function. The expression in equation 4.3-4 then simplifies to equation 4.3-5.

$$DTF(k) = \frac{L_s}{L_p} \text{sinc}(\frac{k}{k_p}) * \mathcal{F}_s(k) * MTF(k)$$ $$* \sum_{n=-\infty}^{\infty} \delta(k-nk_p)$$

(4.3-5)

The transfer function is the $MTF(k) * \mathcal{F}_s(k)$ term repeated on a one-dimensional lattice in frequency space.

Aliasing is the overlap which can occur between adjacent lattice points in
frequency space. When aliasing is small, the high frequency lattice point contributions can be neglected. Only the low frequency component, \( n=0 \), is considered. The expression for the transfer function simplifies to equation 4.3-6.

\[
DTF(k) = \frac{k_p}{k} \times \mathcal{F}_o(k) \times MTF(k) 
\]

Equation 4.3-6 can be interpreted. The sinc(\( \frac{k}{k_p} \)) function contains the finite size of the pixel. As the pixel size becomes smaller, keeping the spacing constant, the value \( k_p \) gets larger; thus reducing the size of the pixel broadens the sinc(\( \frac{k}{k_p} \)) function and extends the frequency response. The \( \mathcal{F}_o(k) \) term represents the degradation due to defocus and diffusive spreading of the photogenerated charges. The diffractive spreading of the optical system is contained in the MTF(k) term. The MTF of the optical system was discussed in chapter 2.

The expression for the detector transfer function can be evaluated. Each of the terms in equation 4.3-6 can be computed. The sinc(\( \frac{k}{k_p} \)), \( \mathcal{F}_o \) and MTF terms can be computed. The results for the transfer function and other detector measures will be presented and discussed in the next section.

Other measures of the detector response are of interest. The point spread function and the diffusive cross talk are two measures of interest. The detector point spread function is defined as the inverse Fourier transform of the detector transfer function. In equation 4.3-7, PSF\(_D\)(x) is the point spread function of the detector, and DTF is the detector transfer function computed in equation 4.3-6.
The diffusive cross talk is a measure of the diffusive spreading of the photogenerated charges. We define the diffusive cross talk as the fractional signal drop of the diffused point spread function relative to the optical point spread function, normalized by the central maximum of the optical point spread function; the definition is made in equation 4.3-8.

\[ \eta_d(x) = \frac{|\text{PSF}_d(x) - \text{PSF}_o(x)|}{\text{PSF}_o(0)} \]  

(4.3-8)

The terms in equation 4.3-8 are defined: \( \text{PSF}_d(x) \) is the detector point spread function at position \( x \); \( \text{PSF}_o(x) \) is the optical point spread function at \( x \).

Each measure of the detector response can be evaluated. The detector transfer function, the point response and the diffusive cross talk are valuable in determining the response of the detector. Each measure will be evaluated numerically in the next section.
4.4 Results

Various measures of the imaging response of a transverse detector will be computed. The detector transfer function, the point spread function and the diffusive cross talk are common measures of the detector's imaging response. In addition, the validity of the model will be tested with experimental results.

The detector transfer function is useful for image processing and simulation studies. Knowledge of the transfer function enables real-time acquired images to be deconvoluted. The detector point spread function is useful for both qualitative and quantitative prediction of the imaging response. The diffusive cross talk measures the diffusive coupling between adjacent pixels of the detector. It is a common measure used to describe the degradation of the detector array.

Each of the imaging measures will be computed. The results will indicate the severity of the diffusive degradation of the detector array. In the computations, the diffusion length is normalized by the full width half maximum (FWHM) of the optical point spread function, \( D_{\text{PSF}} = 1.02 \lambda f\# \). Two cases are considered: When the diffusion length is varied and the optical system remains fixed, the \( L_d/D_{\text{PSF}} \) ratio is used. If the diffusion length is held fixed and the optical system is varies, the \( D_{\text{PSF}}/L_d \) ratio is assumed. One might expect the imaging performance will improve as the \( L_d/D_{\text{PSF}} \) ratio decreases or alternatively, the \( D_{\text{PSF}}/L_d \) ratio increases; the results will confirm this.
The detector transfer function is computed in equation 4.3-6. The computation of each term has been discussed. The \( \text{sinc}(\cdot) \) function accounts for the finite size of the pixel; \( \mathcal{J}_o \) is the Fourier transform of the point response steady state current density; the optical system is accounted for in the MTF(k) term.

\[ \mathcal{J}_o \]

**Figure 4-8:** The FFT approximation of \( \mathcal{J}_o \).

The numerical computation of \( \mathcal{J}_o \) is complicated by the discrete and finite representation of data. The Fourier transform is numerically approximated by the fast Fourier transform (FFT) algorithm. The FFT of a delta function is susceptible to appreciable error at large frequencies. Ideally, the Fourier transform of a delta function is unity for all spatial frequencies. The FFT of a numerical approximation to a delta function suffers from roll-off at large frequencies; the effect is illustrated in figure 4-8. The plot depicts the FFT amplitude of a 256 element binary array. The
binary array ($\Delta_i$) is zero everywhere except at elements 127 and 128, where the values are unity. The spatial frequency is normalized by $k_p = 2\pi/\Delta y$; $\Delta y$ is the distance between successive array elements.

To compensate for the roll-off effect, a normalization function is introduced. The normalization function, equation 4.4-1, is used to compensate for the roll-off error inherent in the FFT algorithm.

$$N(k) = |FFT(\Delta_i)(k)|$$

Here, $\Delta_i$ is the binary array that approximates the delta function; $k$ is the spatial frequency; $N(k)$ is the normalization function. The effect of normalization is small. The cutoff frequency of the optical system satisfies, $k_c \approx 0.005 k_p$. The small normalization compensates for the small discrete error.

The transfer function for various $L/D_{psf}$ ratios is plotted in figure 4-9. The optical system is fixed and the diffusion length varies. The optical system has a centrally obscured pupil with a fraction obscuration $f_-0.33$ and an $f#=30$. The plot illustrates the transfer function of the optical system and the degradation due to the finite pixel size, defocus, and diffusion.

Conclusions can be drawn from the plot in figure 4-9. First, the effect of defocus is negligible. The large $f#$ and the high refractive index of the detector material result in nearly collimated light in the detector; hence, defocus is small. Second, increasing
diffusion results in larger degradations to the transfer function; this is expected.

The Detector Transfer Function

![Detector Transfer Function Diagram]

**Figure 4-9:** The detector transfer function for various $L_D/D_{PSF}$.

The effect of varying the optical system, while keeping the diffusion length constant is illustrated in figure 4-10. For reference, the optical and Nyquist-sampled transfer functions are illustrated. The degradation for four increasingly smaller f#’s is shown. As one would expect, the effect of diffusion becomes worse as the f# is decreased. With smaller f#’s, the FWHM of the point spread function is smaller and the effect of diffusion is more severe.

The detector point spread function is computed according to equation 4.3-7. The plot in figure 4-11 illustrates the point spread function for two diffusion lengths. The
Figure 4-10: The detector transfer function for various $D_{PSF}/L_D$.

optical system is held fixed, with $f\# = 30$. The effect of diffusion is evident. For a $L_D/D_{PSF}$ ratio of .07 the first order lobes are clearly visible. At a ratio of .16, the first order minimum is blurred and the amplitude is less. Both effects are characteristic of diffusive spreading.

The diffusive cross talk is plotted in figure 4-12. The cross talk is measured one pixel away from the central maximum of the detector point spread function. Nyquist sampling is assumed. The optical system is held fixed and the diffusion length varied. The result indicates for increasing diffusion, the diffusive cross talk becomes worse. The size of the cross talk is small. At ratios near 2., the diffusive cross talk is less than 4%.
The transverse detector model can be tested experimentally. A gallium (Ga) doped germanium (Ge) far infrared transverse detector array was tested. The 32-element linear detector array is operated at 2(K). The GeGa detector material absorbs in the [55,110](pm) range. The pixels are uniformly spaced at 750(pm). The optical system consists of a 4(mm$^2$) square black body source. The source is heated by ohmic losses to emit in the far infrared. A single concave mirror images the source onto the detector with a beam f# = 5.5. Two linear slits are imaged; the slits have widths of 500(µm) and 1.5(mm).

The measured and the modeled data for a 250(µm) slit is illustrated in figure 4-13. The data is modeled by best-fitting the experimental data with the diffusion length as a free parameter in the modeled data. In addition, electronic cross talk is
Figure 4-12: The diffusive cross talk.

added to the experimental data. The electronic cross talk is assumed to be 7% between neighboring pixels. A plot of the absolute value deviation, between the experimental and modeled data, is illustrated in figure 4-14. The plot indicates the best fit occurs for a diffusion length of \(-307(\mu m)\); the modeled data in figure 4-13 corresponds to \(L_D=300(\mu m)\).

A 1.5(mm) slit was also imaged. The fitted data and the measured data are illustrated in figure 4-15. The best fit for the 1.5(mm) data corresponds to \(L_D=250(\mu m)\). The fit is good but noticeably worse than for the 500(\mu m) slit.

The best fitted diffusion lengths differ by \(-18\%\). Diffraction effects were not taken into account in the modeled data. The wavelength of the imaged light in the
Figure 4-13: Image of a 500(μm) slit.

Figure 4-14: The measured fit for a 500(μm) slit.

detector is ~25(μm); hence, the slit is approximately 20 wavelengths thick. Diffraction effects, although small, cannot be neglected. The smaller diffusion length for the
Figure 4-15: The image of a 1.5(mm) slit.

500(µm) slit suggests that diffraction might be a factor. Diffraction from the 500(µm) slit could partially account for the larger diffusion predicted for the 500(µm) aperture.

The detector transfer function, the point spread function and the diffusive cross talk are useful measures of the detector's imaging response. Each measure clearly indicates the degradation that occurs with diffusion of the photogenerated charges. The computed results are useful in ascertaining general trends in transverse detector's response when various operating parameters are varied. The model falls short, however, in predicting the image response in terms of the basic physical processes that occur in the detector. A basic analysis of the basic physics of the transport process leads to a precise prediction of the detector response - which is the topic of chapter 5.
Chapter 5 *The Longitudinal Detector Response*

This chapter analyzes the response of a front plane illuminated detector array (FPIA). Many of the arguments also hold for transverse detector geometries. The discussion is focused on a longitudinal geometry in the remaining three chapters. The transverse detector analysis in chapter 4 is useful for general information - diffusion effects and absorbed photon distributions. A more rigorous approach, however, is needed to extract detailed predictions. This chapter lays the ground work for a basic description of longitudinal far infrared detector arrays.

The goal of a semiconductor detector array model is to predict the imaging response of the array as a function of its operating and design parameters. There are a number of basic physical processes that contribute to the operation of an extrinsic photoconductive far infrared detector array. The optical system creates a distribution of photogenerated charges within the detector material. The photogenerated charges are swept toward a collecting contact by an applied electric field. The mean speed at which the photogenerated charges travel and the degree to which they diffuse are important parameters in predicting the detector response. In moving toward the collecting contact, the photogenerated charges can recombine with opposite charged impurity centers. Under some conditions, neutral acceptor atoms can trap photogenerated charges giving rise to overcharged centers. Each of these physical processes affects the response of the detector array, and will be considered in
constructing a device model.

The model presented here is applicable to the typical detector operating parameter space, which will be defined shortly. The various nonlinear effects (Westervelt and Teitsworth, 1985) will not be dealt with here. Initially, a function relating the steady state detector signal current density to the material and detector array operating parameters will be derived. The primary material processes effecting the detector imaging response are the transport properties (i.e., the electric mobility), diffusion and recombination of the photogenerated charges. Each process is individually examined in the remaining sections.

For simplicity, the following analysis will assume a nondegenerate, p-doped, extrinsic germanium (Ge) photoconductive detector material. The analysis for a n-doped material and/or a different host follows analogously with simple modifications. In a p-doped extrinsic semiconductor, holes in the valence band are the predominant charge carriers. The p-type dopant which supplies the photogenerated charges is called the primary acceptor or primary dopant; it is denoted by $A$. We assume $A$ is a double acceptor; single acceptors can be similarly handled. In any semiconductor residual impurities are present and consist of both shallow acceptors and shallow donors. In practice, to avoid thermal excitation of charge carriers, either p or n-type, the detector material is cooled. The temperature required for cooling satisfies

$$N_A e^{-E_A/k_B T} < \left( \frac{\nu_d}{\hbar} \right) \tau;$$

here $N_A$ is the primary dopant density; $E_A$ is the activation energy.
of an electron from the valence band to a bound level of the primary dopant; \( k_B \) is Boltzmann's constant; \( \mathbf{S} \) is the Poynting vector; \( \tau \) is the recombination time of the photogenerated charge. In other words, the thermally generated density of holes must be much less than the photogenerated hole density. In practice for long wavelength applications (ie. \( \lambda > 30(\mu m) \)) the detector material must be cooled to liquid helium temperatures (ie. \( T \approx 4(K) \)).

### 5.1 The Detector Signal Equations

The basic operation of p-type extrinsic photoconductive longitudinal detectors will be considered. An integral expression is derived for the photogenerated current density representative of the signal amplitude indicated by the detector. The signal equation is derived by considering some of the physical processes and material properties of the detector material. The signal equation is the basis for the subsequent analysis of the longitudinal or FPIA imaging response.

In an p-type extrinsic photoconductor negatively ionized p-type acceptors are created or destroyed through three different mechanisms: photo and/or impact ionization of neutral acceptors, and recombination of a hole at a negatively ionized acceptor site. In photoionization, an electron in the semiconductor valence band is excited to a p-type primary acceptor level, through the absorption of a far infrared photon. The absorption process leaves a hole (absence of an electron) in the valence
band, and negatively ionizes the p-type primary acceptor. With an applied electric field, the free photogenerated hole drifts toward a negatively biased collecting ohmic contact. If the hole arrives at the negative biased contact, without recombining, a second hole is injected from the opposite positive biased contact. The second hole is reinjected to conserve space charge neutrality in the bulk detector material (Haegel and White, 1989a). Negative ionized p-type acceptors can also be created by impact ionization (Reggiani and Mitin, 1989). In this process, an accelerated hole collides with a neutral acceptor and ionizes a hole. The process creates a hole in the valence band and a negative charged acceptor. Negative ionized acceptors can be destroyed when a hole recombines at a negative ionized site. In this process, a hole and a negative ionized acceptor initially form a weakly bound exciton. Through a series of phonon emissions (Lax, 1960, Abakumov et. al., 1978), the hole can drop into the ground state of the exciton. The process results in the loss of both a hole and a negative ionized acceptor.

The three processes discussed above are mathematically described by equation 5.1-1. Here: $N_{A^-}$ is the negative ionized primary acceptor concentration; $N_{A^0}$ is the neutral primary acceptor concentration; $S^\omega$ is the Poynting vector in the spectral range corresponding to photon absorption by a neutral acceptor; $p$ is the free hole density; $r'(E)$ is the electric field dependent recombination coefficient; $\kappa'(E)$ is the field dependent impact ionization coefficient; $\omega$ is the frequency of the absorbed light and $\hbar$ is Planck's constant divided by $2\pi$. 
Alternatively, positive ionized acceptors can also exist in an extrinsic semiconductor (Shiff, 1982 and Haller et al., 1983). An overcharged positive ionized acceptor in a p-doped extrinsic photoconductor can be both created and destroyed. First, a positive charged acceptor can be created by trapping a hole. A positive ionized acceptor can be neutralized by photo, thermal or impact ionization. The thermal ionization is negligible when \( k_b T < E_b(A^+) \)-which is assumed here; \( E_b(A^+) \) is the binding energy of the hole, \( k_b \) is Boltzmann's constant and \( T \) is the temperature of the photogenerated charges. Hence, only photoionization and impact ionization of the positive ionized acceptors are considered here. The balance is described by equation 5.1-2. Here, \( r^* \) is the recombination coefficient of a hole with a neutral acceptor, \( \overline{v}S^*/h\omega \) is the rate of photoionization of \( A^+ \), and \( \kappa^*(E) \) is the field dependent impact ionization coefficient.

\[
\frac{\partial N_A^+}{\partial t} = -\frac{\overline{v}S^*}{h\omega} - pN_A^+ r^*(E) + pN_A^+ \kappa^*(E) \tag{5.1-1}
\]

For reference the detector geometry illustrated in figure 5-1 will be assumed.

A third equation relating the space-time dependence of the free hole density, \( p \), and the ionized acceptor densities, \( N_A^- \) and \( N_A^+ \), can be derived (Teitsworth, 1986) from the Boltzmann transport equation. Assuming the number current density is as
Figure 5.1: Coordinate geometry for the detector analysis.

given by equation 5.1-3, the Boltzmann transport equation reduces to the equation 5.1-4. Here, $D_p$ and $D_\perp$ are respectively the diffusion coefficients in the direction parallel and perpendicular to the applied field.

$$J = (p \nu_d - D_p \frac{\partial p}{\partial z} - D_\perp \frac{\partial p}{\partial y} - D_\perp \frac{\partial p}{\partial x})$$  \hspace{1cm} (5.1-3)

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial z} (p \nu_d (E) - D_p \frac{\partial p}{\partial z} - D_\perp \frac{\partial p}{\partial y} - D_\perp \frac{\partial p}{\partial x}) = \frac{\partial N_A^+}{\partial t} - \frac{\partial N_A^-}{\partial t}$$  \hspace{1cm} (5.1-4)

A fourth equation can be derived from Poisson's equation, $V \cdot D = p$. Assuming the dielectric media is linear and homogenous, and that the electric field has a $z$-
component only, the divergence of the field simplifies to \( \nabla \cdot \mathbf{D} = \varepsilon \frac{\partial E}{\partial z} \); where \( \varepsilon \) is the static dielectric permittivity. The total space charge is given by \( \rho = p + N_d + N_A^+ - N_A^- \); \( N_d \) is the impurity donor density, \( N_A^+ \) is the positive ionized primary acceptor density and \( N_A^- \) is the negative ionized acceptor density. Substituting the results into Poisson's equation, equation 5.1-5 is obtained.

\[
\frac{\partial E}{\partial z} = \frac{\varepsilon(p + N_d + N_A^+ - N_A^-)}{\varepsilon} \tag{5.1-5}
\]

The above equations are combined into a single equation. Expansion of the \( \frac{\partial \rho D}{\partial z} \) term in equation 5.1-4, definition of the differential mobility as \( \mu = \frac{\partial v}{\partial E} \), and use of Poisson's equation 5.1-5, results in \( \frac{\partial (\rho v)}{\partial z} = \frac{\varepsilon(p + N_d + N_A^+ - N_A^-)}{\varepsilon} + v_d \frac{\partial \rho}{\partial z} \). With spatially homogenous diffusion coefficients, the expressions in equations 5.1-1 and 5.1-2 for \( \frac{\partial N_A^-}{\partial t} \) and \( \frac{\partial N_A^+}{\partial t} \) are substituted into equation 5.1-3 to arrive at equation 5.1-6.

\[
\frac{\partial p}{\partial t} + v_d \frac{\partial (\rho + p \mu v)}{\partial z} \frac{\partial E}{\partial z} - D_1 \frac{\partial^2 p}{\partial x^2} - D_1 \left( \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right) = \frac{-\nabla S^*}{\hbar \omega} + pN_A^+ \kappa(E) - pN_A^- r^*(E) - \frac{\nabla S^*}{\hbar \omega} - r^*(E) pN_A^+ + \kappa(E) pN_A^- \tag{5.1-6}
\]
With some additional assumptions equation 5.1-6 is simplified further. First, in the bulk detector the space charge is zero (Haegel and White, 1989a); hence 

\[ p + N_d + N_A^- = N_A^- = 0 \]

Second, the detectors are normally operated below the impact ionization threshold for the neutral A\(^+\) centers; thus \( \kappa(E) = 0 \). Third, the free hole density \( p \), primarily due to photogeneration, is much less than the impurity donor density, \( N_d \), in low light level applications; a assumption satisfied in many astronomical applications. The zero space charge assumption gives \( N_A^- = N_d + N_A^+ \). With these four assumptions, equation 5.1-6 reduces to equation 5.1-7.

\[
\frac{\partial p}{\partial t} = D_1 \frac{\partial^2 p}{\partial z^2} + D_1 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) - r_p N_d + N_A^- - \nu_d \frac{\partial p}{\partial z} - \frac{\nabla \cdot S^*}{\hbar} - \frac{\nabla \cdot S^+}{\hbar} \tag{5.1-7}
\]

Each of the terms in equation 5.1-7 has a physical interpretation. The equation expresses charge conservation in a differential volume element, \( dV \), at any point in the detector material. The first term on the right, \( D_1 \frac{\partial^2 p}{\partial z^2} + D_1 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) \), is the diffusion of holes into \( dV \). The second term, \( -r_p N_d + N_A^- \), is the loss of holes due to recombination at negative ionized impurity centers. The third term, \( -\nu_d \frac{\partial p}{\partial z} \), is the drift of holes out of the element \( dV \) due to the application of the electric field. The
fourth term, \( \frac{-\nabla \phi}{\hbar \omega} \), is the creation of holes in the volume element \( dV \) due to photoionization of the neutral primary acceptors \( A^o \); properly, this term is a spectral average over the appropriate region of the observed spectrum. The fifth term, \( -r^o(E)pN_A^o \), is the trapping of holes at neutral \( A^o \) centers. The remaining two terms, \( \kappa^o(E)pN_A - \frac{-\nabla \phi}{\hbar \omega} \), are the impact and the photo ionization of overcharged \( A^+ \) centers.

The trapping of holes by neutral primary acceptor atoms is important in understanding the response of p-type extrinsic photoconductive detectors. Neutral \( A^o \) centers can readily form a bound state with a hole, forming an overcharged \( A^+ \) complex. The \( A^+ \) complex has been observed spectroscopically (Haller et al., 1983) for a large number of p-type dopants in Ge.

The ionization energy of the various bound holes have been measured and determined (Kalia et al., 1984) by a Green's function Monte Carlo computation. The Pauli spin-exchange interaction between the bound holes is nonexistent due to the double degeneracy of the valence band. The bound holes of the acceptor are viewed as a spin 3/2 system; each hole occupies a ground state s-type orbital.

The existence of the overcharged \( A^+ \) state has important implications for the
detector's transient response. Initially, after exposure to incoming light photogenerated holes are swept away toward the negative collecting contact of the detector by the applied electric field. Many of the holes, however, are trapped at the neutral \( A^o \) centers. The trapped holes can be freed by either impact or photoionization. The trapping of photogenerated holes continues until an equilibrium is reached. At equilibrium, the net rate of impact and photoionization equals the rate of trapping by the \( A^o \) centers; that is, \( \frac{\partial N^*_A}{\partial t} = 0 \).

In this discussion the operation of the detector is considered over times after which the density of positive ionized acceptors has equilibrated; that is, a steady state is reached. In a steady state condition, the time variation of the positive ionized acceptor density is zero, \( \frac{\partial N^*_A}{\partial t} = 0 \). This assumption removes the last three terms in equation 5.1-7.

The relative size of the diffusion terms in equation 5.1-7 can be estimated. The transverse and the longitudinal diffusion coefficients can differ at low fields, ie. \( E = 100 \text{(V/m)} \). With a Monte Carlo simulation, Mitin and van Vliet (1990) predict that, for p-type Ge at low temperatures, the longitudinal diffusion coefficient, \( D_L \), can be up to 50% smaller than the transverse coefficient, \( D_T \). Within the detector, the hole
density will approximately follow \( p(x,z) = e^{-\alpha z} e^{-x^2 / \sigma^2} \). The relative sizes of the \( z \) and \( x \) dependent diffusion (ie. \( \frac{\partial^2 p}{\partial z^2} (\frac{\partial}{\partial x})^2 \)), is \( \alpha^2 L^2 \). Typically, in an extrinsic photoconductive detector \( \alpha = 6 \text{(cm}^{-1}\) and \( L = 0.002 \text{(cm)} \) or \( \alpha^2 L^2 = 2.4 \times 10^{-5} \). Thus the combined effects of a reduced longitudinal diffusion coefficient and smaller \( z \)-concentration gradients, result in a \( z \)-component of diffusion which is negligible compared to the diffusion in the transverse direction; therefore \( \frac{\partial^2 p}{\partial z^2} = 0 \).

With the steady state and diffusion assumptions, equation 5.1-7 simplifies to equation 5.1-8.

\[
\frac{\partial p}{\partial t} = D_z \left( \frac{\partial^2 p}{\partial z^2} + \frac{\partial^2 p}{\partial y^2} \right) - r p (N_d + N_A) - v_d \frac{\partial p}{\partial z} - \frac{\nabla \cdot \mathbf{J}}{\hbar \omega} \quad (5.1-8)
\]

The drift term, \( -v_d \frac{\partial p}{\partial z} \), in equation 5.1-8 can be removed by transferring into a coordinate frame moving along the \( z \)-axis at the hole drift velocity, \( v_d \). The \( z \) coordinate in the moving system will be referred to by \( z' \). We assume the hole density can be factored according to equation 5.1-9. Here, \( \Delta p(x,y,t) \) is the two-dimensional time dependent hole density in a charge layer with thickness \( \Delta z' \). The function \( f \) is interpreted as the hole density in the \( z \)-direction at the position \( z' \) and goes as \( \frac{1}{\Delta z'} \).
Δz' is the thickness of a photogenerated charge layer moving at velocity v_d, positioned at z' = z - v_d t in the moving coordinate frame. Substitution of equation 5.1-9 into equation 5.1-8 results in equation 5.1-10. Equation 5.1-10 describes the space-time evolution of the number density of the moving charge layer at position z' in the moving coordinate frame.

\[ p(x,y,z',t) = \Delta p(x,y,t)f(z-v_d t) \]  \hspace{1cm} (5.1-9)

Equation 5.1-10 is the time dependent diffusion equation with recombination (\(-p_r N_d\)) and generation (\(-\frac{\nabla \cdot \mathbf{D}}{\hbar} \Delta z\)). Solutions to equation 5.1-10 can be computed by solving for its Green's function. By definition, the Green's function of equation 5.1-10 is the solution of equation 5.1-11. Once the Green's function, \(g_0(x,y,t;x',y',t')\), is determined, the hole density in the two-dimensional charge layer can be determined according to equation 5.1-12. Here, \(\Delta^2 p\) is the second order charge density generated in time \(\Delta t\) at \(z'\) and contained in the layer with thickness \(\Delta z'\).
An initial equation for the signal current density can be derived by summing all the photogenerated charge layers which arrive at the collecting plane. At a given time $t$, the photogenerated charge layer generated at $z_g$ and at time $t_g = t - t_d(z_g)$, will be collected at the contacts; here, $t_d(z) = v_d(d-z_g)$ is the time for a hole to drift from $z = z_g$ to $z = d$. The hole density collected, in one sweep, at the contact in a time $\Delta t$ is the sum of all the photogenerated charge layers generated at $z_g \in [0, d]$ at time $t - t_d(z_g)$, as given by equation 5.1-13.

$$J_1(x, y) = \lim_{\Delta t \to 0} \frac{\Delta p(x, y)}{\Delta t}$$

Here, $J_1(x, y)$ is the number current density; i.e., the number of holes collected per unit time in a unit area at the point $(x, y)$ on the plane of the collecting contacts. Evaluation of the Green’s function, $g_0(x-x', y-y', t-t')$, allows evaluation of the detector signal.

The Green's function, $g_0$, as defined in equation 5.1-11 is incomplete without
the specification of boundary conditions. The boundary conditions are the
distinguishing point between the analysis of longitudinal and the transverse detectors.
Up to now, the arguments have been valid for both geometries. With the boundary
effects assumed in equation 5.1-14 the analysis applies to longitudinal geometries.

$$\lim_{x,y \rightarrow \pm \infty} g_e(x,y,t) = 0$$ (5.1-14)

The approximation in equation 5.1-14 is valid if the smallest areas under
consideration are greater than a couple of diffusion lengths from the detector edge.

The Green's function problem, (equation 5.1-11), together with the boundary
conditions, (equation 5.1-14), constitutes a well defined mathematical problem. The
linear partial differential equation can be solved with well known techniques from
linear theory. Fourier transforming equation 5.1-11 into the frequency domain,
followed by solving the resulting first order linear differential equation in time, and
then Fourier transforming back into the spatial domain, yields the solution as
indicated in equation 5.1-15.

$$g_e(x-x',y-y',t-t') = \begin{cases} 0 & t < t' \\ \frac{1}{4\pi D_e(t-t')} e^{-\frac{(x-x')^2+(y-y')^2}{4D_e(t-t')}} & t > t' \end{cases}$$ (5.1-15)

The solution, equation 5.1-15, checks with a similar problem treated be Morse and
Feshbach (1953).
The various terms in the Green's function solution, equation 5.1-15, have a physical interpretation. For times t<t' the response is zero, which is expected for a causal system. Following the absorption at t=t', a concentrated packet of holes diffuses and recombines. The first exponential term indicates recombination of the holes and brings about an exponential decay with a time constant given by $\tau_r = \frac{1}{(r-N_d)}$. The second exponential describes diffusion of the charge carriers in space with a diffusion length given by $L_d = 2\sqrt{\frac{d}{D_d}}$. Knowledge of the Green's function enables the explicit form of equation 5.1-13 to be determined.

Substitution of the 2-D Green's function, equation 5.1-15, into equation 5.1-13, results in an explicit form for the first-pass current density in equation 5.1-16.

$$J_1(x,y) = \int_0^d dz' \int_{x_0}^{x_0 + d} dx' \int_{y_0}^{y_0 + d} dy' \frac{1}{4\pi D D_d(z')} e^{-\frac{z'-z}{L_d}} e^{\frac{-x'^2 - y'^2}{4\mu_e \tau}} e^{\frac{-x'^2 - y'^2}{4\mu_h \tau}} \left( \frac{\nabla \delta(x', y', z')}{\hbar \omega} \right)$$

(5.1-16)

Equation 5.1-16 is the steady state signal due to a single sweep of holes through the detector. The signal due to successive sweeps of holes through the detector must be added to the result of equation 5.1-16. Since the bulk detector retains space charge neutrality, the external circuit injects a hole into the detector material from the positive contact when a hole leaves the detector material and is collected at the negative contact, giving rise to successive sweeps.
Each successive sweep of holes through the detector results from the reinjection of holes from the positive biased ohmic contact which were collected in the previous pass at the negative biased collecting contact. The photocurrent due to the nth pass of holes is according to equation 5.1-17. Each $J_n(x,y)$ may, in principle, be evaluated recursively once $J_1(x,y)$ is known. The total steady state current density, $J_{\text{net}}(x,y)$, is found by summing all the current densities, according to equation 5.1-18.

\[ J_n(x,y) = \frac{e^{-\frac{\gamma d}{D_t \tau}}}{4\pi t_h D_t} \int_{x,y} J_{n-1}(x',y') e^{-\frac{(x-x')^2+(y-y')^2}{4D_t \tau}} \]  

(5.1-17)

\[ J_{\text{net}}(x,y) = \sum_{n=1}^{\infty} J_n(x,y) \]  

(5.1-18)

Equations 5.1-16, 17 and 18 form the basis of the analysis for longitudinal detectors. Application of the current density results require evaluation of each of the parameters in the equations. The transverse diffusion coefficient $D_t$, the recombination coefficient $r'$, the drift time $t_d$, and the photon absorption density $\frac{\nabla n}{\hbar \omega}$ need to be determined as a function of the operating parameters of the detector array.
5.2 Transport Properties

Application of equations 5.1-16, 17 and 18 requires the evaluation of the various transport parameters as a function of the operating parameters of the detector. The recombination time, \( \tau_r = \frac{1}{(r-N)\tau} \), the diffusion length, \( L_d = 2\sqrt{D\tau} \), and the drift time \( \tau_d \), each need to be evaluated in terms of the operating parameters of the detector. In this discussion, the detector temperature, the detector thickness, the applied electric field or bias, the pixel size, and the shallow impurity donor doping level are the design and operating parameters considered. The hole photogeneration rate, \( \frac{\nabla \cdot \mathbf{S}}{h\omega} \), for a longitudinal or FPIA will be considered in chapter 6.

The transport properties of the detector material are primarily determined by the hole scattering mechanisms present. When a photogenerated hole is accelerated toward a collecting contact it can be scattered through both elastic and inelastic collisions. The primary scattering mechanisms considered are optical phonon, acoustic phonon, ionized impurity and neutral impurity scattering. Phonon scattering is generally an inelastic process and is responsible for the energy loss from the charge carriers. Ionized and neutral impurity scattering are quasi-elastic processes. Scattering from impurities is an inelastic process when the energy of the charge carriers is sufficient to impact-ionize the dopants and/or the impurities. In this
discussion, the energy of the charge carriers is assumed to be sufficiently low for the impurity scattering to be an is effective elastic process. The nature of the charge transport is a function of the applied electric field, the temperature, and the type and density of the dopants and impurities.

Hole transport in p-type Ge has been modeled with a Monte Carlo method (Mitin and van Vliet, 1990; Reggiani et. al. 1977) and investigated experimentally by many workers. In terms of the applied electric field, the transport can be viewed as falling into three regimes. First, for low fields ($E<1-5(V/cm)$) the motion is of the diffusion-drift type with the dominant scattering mechanisms being absorption and emission of acoustic phonons, ionized and neutral impurity scattering. For larger fields, $10(V/cm)<E<50(V/cm)$, a fraction of the charge carriers begin to emit optical phonons, while the remaining carriers are in a diffusion-drift mode. For the carriers that emit optical phonons, a "streaming" motion is characterized by acceleration to a velocity $v_o = \left( \frac{2\hbar\omega_{LO}}{m_h} \right) n$ ($\omega_{LO}$ is the longitudinal optical phonon frequency), deceleration by emission of an optical phonon, and acceleration back to $v_o$ (Peeters et. al., 1985). At still higher fields ($E>100(V/cm)$), most of the carriers are streaming and emit optical phonons. At this point, the mean velocity of the carriers saturates at $v_s = v_o/2$.

We assume the detector is operating in the diffusion-drift regime. At higher fields, when streaming motion begins, impact ionization and breakdown begin to
occur. Various nonlinear effects (Westervelt and Teitsworth, 1985) begin with the onset of impact ionization. In the diffusion-drift regime three scattering mechanisms are present: acoustic phonon absorption and emission, ionized and neutral impurity scattering. The distribution of the charge carriers is approximated as a drifted Maxwellian distribution according to equation 5.2-1.

\[ f_d(v) = \left( \frac{\beta m^*}{2\pi} \right)^{1/2} e^{-\beta(m^*v/^2kT) - v^2} \]  

(5.2.1)

Here \( \beta = \frac{1}{k_bT} \) and \( m^* \) is the effective mass of the hole. To evaluate the distribution and the transport properties it is necessary to determine the drift velocity \( v_d \).

The material parameter that determines the drift velocity is the electric mobility as defined in equation 5.2-2.

\[ v_d = \mu E \]  

(5.2-2)

Here, \( \mu \) is the electric mobility and \( E \) is the electric field. When a number of scattering mechanisms are present the mobilities due to each scattering mechanism generally can not be added in a simple algebraic way as in Matthiesson's rule (Ashcroft and Mermin, 1976). Instead, the scattering rates are summed as in equation 5.2-3.

\[ \tau_s = (\tau_{AC}^{-1} + \tau_{T}^{-1} + \tau_{N}^{-1})^{-1} \]  

(5.2-3)

Here, \( \tau_s(E_k) \), \( \tau_{AC}(E_k) \), \( \tau_T(E_k) \), and \( \tau_N(E_k) \) are the total, acoustic, ionized impurity, and the neutral impurity momentum scattering times, respectively; each is a function of the
hole energy $E_k$. The mobility can be determined according to equation 5.2-4 (Ridley, 1988) if the total scattering time $\tau_r(E_k)$ is known.

$$\mu = \frac{e <E_k \tau_r(E_k)>}{m^* <E_k>} \quad (5.2-4)$$

Substitution of $x = E_n/k_n T$ in equation 5.2-4 gives an integral expression for the mobility in equation 5.2-5.

$$\mu(T) = \frac{4e}{3\sqrt{\pi m^*}} \int_0^\infty x^{3/2} \tau_f(x, T) e^{-x} dx \quad (5.2-5)$$

To evaluate the mobility, it is necessary to determine the scattering time $\tau_r(E_k)$, which is dependent on each of the individual scattering mechanisms as in equation 5.2-3.

Emission and absorption of acoustic phonons is responsible for the ohmic heating in the detector material. Brown and Bray (1962) investigated the scattering of holes in Ge from 7-300(K). They obtained good results for hole electric mobility given by equation 5.2-6.

$$\mu_{ac} = 3.37 \times 10^7 \left( \frac{cm^2K^{3/2}}{Vs} \right) \frac{1}{T^{3/2}} \quad (5.2-6)$$

The result has been subsequently used by a number of researchers. Reggiani et al. (1977) investigated the hole velocity in Ge both experimentally and theoretically. They measured the hole drift velocity in very pure Ge samples (eg. $N_i = 10^{10}(cm^{-3})$) between 8-220(K) with a time of flight technique. Theoretically, they modeled the drift velocity with a Monte Carlo simulation. They measured up to a 38% anisotropy
between transport in the <100> and <111> directions. In addition, for an applied field of 1(V/cm) they measured a drift velocity of $1.3 \times 10^6$(cm/s). The Brown and Bray result (equation 5.2-6), at $T=8$(K), gives $v_d=1.5 \times 10^6$(cm/s) with reasonable agreement.

The acoustic phonon scattering rate depends on the lattice temperature. At liquid He temperatures the elastic equipartition assumption is not valid (Jacoboni and Reggiani, 1983). In such cases an inelastic analysis must be considered. For holes scattering off acoustic phonons Jacoboni and Reggiani have derived an expression, equation 5.2-7, for the scattering rate due to acoustic phonon absorption and emission.

\[
\tau^{-1}_{ac}(E_k) = \frac{\mathcal{E}^2(m^\star)^{1/2}(k_B T)^3}{2^{2/3} \pi \rho c \hbar^4} \frac{1}{E_k^{1/2}} \begin{pmatrix} F_3(x) \cdot \left( \frac{k_B T}{E_k} \right) F_4(x) \\ G_3(x) \cdot \left( \frac{k_B T}{E_k} \right) G_4(x) \end{pmatrix}
\] (5.2-7)

The upper and lower equations represent absorption and emission, respectively, of acoustic phonons. Also, $\mathcal{E}$ is a deformation potential; $m^\star$ is the effective mass of the hole; $k_B$ is Boltzmann's constant; $T$ is the lattice temperature; $\rho$ is the material density; $c$ is a mean sound velocity defined by $c = \frac{1}{3}(2c_t + c_l)$, where $c_t$ and $c_l$ are the transverse and longitudinal sound velocities; $\hbar$ is Planck's constant divided by $2\pi$. The functions $F_3$, $F_4$, $G_3$, and $G_4$ are defined in terms of the Einstein-Bose distribution function applicable to acoustic phonons. The dimensionless argument $x$, is defined by

\[
x = \frac{2(m^\star)^{1/2}cE_k^{1/2}}{k_B T}.
\]
Substitution of the parameters in equation 5.2-7 allows the scattering rate due to acoustic phonons to be evaluated. A plot of the scattering rate due to acoustic phonons versus hole energy with a lattice temperature of 4.0(K) is shown in figure 5-2.

![Acoustic Phonon Scattering Rate](image)

**Figure 5-2:** Acoustic phonon scattering rate, T=4(K).

As the temperature of the device is lowered into the liquid He range, scattering by ionized and neutral impurities becomes more frequent. Ionized impurities can result from two processes. First, a donor impurity ionizes and the free electrons bind to an acceptor impurity; a positive ionized donor and a negative ionized acceptor result. Second, when a photogenerated hole is trapped at a neutral A⁺ site, A⁺ and A⁻ charged sites result. The effect of both processes is an equal concentration of positive and negative ionized centers.
At very low temperatures the positive and negative ionized impurities begin to pair up; the opposite charged impurities become correlated in space. In a p-type semiconductor, the number of holes detached or uncorrelated from the negative ionized acceptors is \( n = N_A e^{-\frac{E_a}{k_B T}} \) (Shklovskii and Yanchev, 1975). Here \( N_A \) is the acceptor concentration and \( E_A \) is the activation energy associated with hopping conduction. As before \( k_B \) is Boltzmann's constant. For Ge the temperature at which the impurities become decorrelated is \( T > 1.2(K) \). Thus, at temperatures below 1.2(K) dipole condensation of the positive and negative ionized impurities begins. With dipole condensation the scattering is quite different than that from decoupled Coulomb centers. We assume the temperature of the device is above 1.2(K) and the charged impurities act as Coulomb scattering centers.

Ionized impurity scattering in semiconductors was originally described by Conwell and Weisskopf (1950). The Conwell-Weisskopf result assumes a cutoff distance \( b = (N_l)^{-1/2} \) for ionized impurity scattering. Later, Brooks and Herring derived a result which considered a screened potential surrounding the charged impurities. Falicov and Cuevas (1967) considered the case when opposite charged ionized impurities become correlated in space - as with dipole condensation discussed earlier. Ionized impurity scattering has been reviewed by Chattopadhyay and Queisser (1981).

Ridley (1977) has reconciled the Conwell-Weisskopf and Brooks-Herring
approaches. Ridley introduced a parameter $\eta = \frac{4ZN_i^{1n}}{q_o} \frac{R^*}{E_k}$: here, $Z$ is the ionic charge, $N_i$ is the ionized impurity density, $q_o$ is the Debye-Huckel screening wavenumber, $R^*$ is the effective Rydberg, and $E_k$ is the hole energy. Ridley showed that the Conwell-Weisskopf and the Brooks-Herring results correspond to the cases of $\eta > 1$ and $\eta < 1$ respectively. Evaluation of the parameter $\eta$ for holes in Ge for a double acceptor $A$ at typical detector operating conditions gives $\eta = 10^2$. In general, however, the value of $\eta$ should be checked for the detector material under study. We assume the Conwell-Weisskopf approximation sufficient to describe the ionized impurity scattering.

The scattering rate due to ionized impurities, in the Conwell-Weisskopf approximation, is given in equation 5.2-8.

$$\tau_f^{-1} = \frac{Z^2 e^4 N_i}{2^{3/2} \pi \varepsilon^2 (m^{*1/2}) E_k^{3/2}} \log \left(1 + \frac{4 \pi e E_k}{Z^2 N_i^{1/2}}\right)$$ (5.2-8)

Evaluation of the parameters in equation 5.2-8 for p-type Ge gives a numerical expression for the scattering rate in equation 5.2-9.

$$\tau_f^{-1} = 3.54 \times 10^{-15} \left( \frac{J^{12} m^2 s^{-1}}{E_k^{12}} \right) \frac{N^{2a}}{E_k^{12}}$$ (5.2-9)

In a detector material with a trap density $[A^+] = 0.001 [A^+]$, the scattering rate due to ionized impurities is $\sim 8 \times 10^9 (s^{-1})$. If the $A^+$ formation is inhibited, as sometimes is
accomplished by stressing the detector material, the scattering rate is typically \(3 \times 10^8 \text{s}^{-1}\).

The final scattering mechanism considered is neutral impurity scattering. Neutral impurity scattering has been identified in polar (e.g., GaAs) semiconductors at low temperatures. For covalent semiconductors (e.g., Ge and Si), definitive identification has been somewhat elusive (Blakemore, 1980). The effects of spin polarization of free carriers has been demonstrated by Maxwell and Honig (1966). Erginsoy (1950) deduced the scattering by hydrogen-like impurities by considering the low energy electron scattering off neutral hydrogen atoms as derived by Massey and Moiseiwitsch (1950). Sclar (1956) considered the case of neutral impurity scattering when the charge carrier can form a bound state with the neutral impurity. A general discussion of neutral impurity scattering can be found in Ridley (1988).

Two cases of neutral impurity scattering are considered. First, consider the case when the hole can form a bound state with neutral acceptor atoms. The result derived by Sclar (1956) describes this case. Second, if formation of the overcharged state is inhibited, as with stressing the detector material, an analogy with low energy electron scattering by neutral He atoms is made. Helium (He) atoms are assumed since A is taken as a double acceptor. For single acceptors analogy with low energy scattering by neutral hydrogen atoms can be made. In this case, analogy with Erginsoy (1950), or a similar analysis is appropriate.
A basic consideration is the short range electric potential of the scattering center. Sclar (1956) modelled the potential of the neutral impurity as a square well in which a bound state can form. The existence of the bound state results in a resonant-like scattering. Sclar assumed the bound state is very shallow compared to the depth of the well; he derived a result for the scattering rate according to equation 5.2-10. Here, $N_n$ is the density of the neutral impurity atoms which can bind a hole with energy $E_0$. Substitution of the appropriate parameters for p-type Ge into equation 5.2-10 evaluates the neutral impurity scattering rate, equation 5.2-11, when formation of the overcharged A$^+$ state is possible.

$$\tau_N^{-1}=\frac{2\pi^2\hbar^2N_n}{m^*\gamma^2} \left(\frac{E_k+E_b}{E_k^{1/2}}\right)$$  \hspace{1cm} (5.2-10)

$$\tau_N^{-1}=5.50\times10^{-22}(m^3s^{-1})N_n\frac{E_k^{1/2}}{E_k+E_b}$$  \hspace{1cm} (5.2-11)

At low trap densities (ie. [A$^+]<1[A^+]$) the scattering rate is typically $\sim 4\times10^9(s^{-1})$.

If binding of the hole to the neutral acceptor is inhibited, as occurs with the application of stress, the scattering is analogous to the scattering of low energy electrons by neutral He. Since A is a double acceptor and has two bound holes in its neutral state the analogy with He is appropriate. For the case of a single acceptor scattering by neutral hydrogen is appropriate. The momentum scattering cross section, $\sigma_m(E)$, of low energy electrons scattering off neutral He atoms has been
measured experimentally (Crompton et. al., 1970) and computed theoretically (Nesbet, 1979) to an accuracy better than \(-1\%\). By appropriate scaling of the momentum cross section the scattering rate can be estimated.

The scattering rate is computed from the momentum scattering cross section according to equation 5.2-12. As mentioned above, the momentum scattering cross section has been worked out to a reasonable degree of accuracy for the low energy electron scattering from neutral He. With appropriate scaling of the momentum cross section the scattering rate is computed numerically from equation 5.2-12. For neutral acceptor concentrations on the order of \(N_A = 7.5 \times 10^{14} \text{(cm}^3\text{)}\), the hole scattering rate is typically \(\tau_n^{-1} = 2 \times 10^9 \text{(s}^{-1}\text{)}\).

\[
\tau_n^{-1} = \sqrt{\frac{2}{m^*} E_k^{1/2} \sigma_{n}(E_k) N_n}
\]  
(5.2-12)

With analytical and/or numerical descriptions of all the scattering mechanisms, an evaluation of the total scattering rate is made according to equation 5.2-3. With the relative complexity of the expression for \(\tau_r(E_k)\), a numerical evaluation is appropriate. The result of the numerical evaluation of the total scattering rate, equation 5.2-3, is illustrated in figure 5-3. In figure 5-3, \(N_A = 7.5 \times 10^{14} \text{(cm}^3\text{)}\) for both inhibited and uninhibited detector materials. The ionized impurity concentrations are \(N_i = 2 \times 10^{12} \text{(cm}^3\text{)}\), for the uninhibited (eg. unstressed) material, and \(N_i = 4 \times 10^{11} \text{(cm}^3\text{)}\) for the inhibited (eg. stressed) material. The scattering rates, for both material types, are
dominated by neutral impurity scattering. As one might expect, the existence of the overcharged acceptor state, $A^*$, gives a larger scattering rate in the inhibited (e.g. unstressed) material.

The mobility versus temperature is evaluated in equation 5.2-5. The mobility in an uninhibited (unstressed) material corresponding to three [$A^*$] trap densities is shown in figure 5-4. If formation of the $A^*$ overcharged state is inhibited, the computed mobility is as illustrated in figure 5-5. The mobility in the inhibited material is greater than the uninhibited material. The inhibited material does not form the overcharged acceptor state; therefore ionized impurity scattering is less in the inhibited material, thereby resulting in a greater mobility.
Figure 5-4: Mobility vs temperature, $[A^+] = 7.5 \times 10^{14} \text{(cm}^{-3}).$

### 5.3 Diffusive Properties

The diffusive properties of the detector material are a primary consideration in describing the imaging properties of the detector array. The regime of hole transport has a major effect on the diffusive properties of the detector. As the transport passes from the diffusion-drift to the drift-streaming regime Monte Carlo simulations (Mitin and van Vliet, 1990) indicate the diffusive effects begin to "freeze" out. The diffusion coefficient decreases throughout the drift-streaming region. Throughout the pure streaming regime the longitudinal diffusion coefficient has been measured (Nava et. al., 1979) and computed to decrease with increasing electric field...
Figure 5-5: The mobility vs temperature for an inhibited material.

We assume the detector is operated at a bias corresponding to charge transport at the top end of the drift-diffusion region; just prior to the point when some holes begin streaming motion with the accompanying emission of an optical phonon. In such a regime, the Einstein relation, equation 5.3-1, is approximately valid (Jacoboni and Reggiani, 1983). With the mobility computations in section 5.2, the temperature dependence of the diffusion coefficient is easily determined.

\[ D = \frac{\mu k_B T}{e} \]  

(5.3-1)

We wish to compute a mean measure of the diffusion length. For a hole
photogenerated a distance $z$ away from the collecting contact the distance it diffuses is $L(z) = 2\sqrt{D t(z)}$. The time, $t(z)$, from when the hole is generated to being collected, is given by $t(z) = \frac{z}{\mu E}$. With the Einstein relation, equation 5.3-1, the diffusion length is given by equation 5.3-2. A mean diffusion length can be computed by averaging $L(z)$ over the detector thickness (i.e., the interval $[0,d]$). With the averaging, a mean diffusion length is computed in equation 5.3-3. A plot of the diffusion length for three different electric field strengths is shown in figure 5-6. The thickness of the detector was fixed at $d \alpha_{\text{ave}} = 4$, where $\alpha_{\text{ave}}$ is the average photon absorption coefficient and will be discussed in chapter 6.

\[ L(z) = 2\left(\frac{kT}{eE}\right)^{1/2}\sqrt{z} \]  
(5.3-2)

\[ \bar{L} = \frac{4}{3} \left(\frac{kT \mu d}{eE}\right)^{1/2} \]  
(5.3-3)

5.4 Hole Recombination

The recombination of holes in the detector material has at least three effects on the performance of the detector array. First, recombination limits the responsivity of the detector; longer recombination times result in increased responsivities. Second,
recombination partially limits the amount of diffusive cross talk between adjacent pixels in the detector. The diffusive cross talk in a single pass of photogenerated charges is also limited by the drift time of the hole, $\tau_d$. A shorter recombination time results in less diffusive cross talk. Hence, the diffusive cross talk and the responsivity are in opposition in regards to the recombination time. Third, recombination contributes directly to generation-recombination noise. Generation-recombination noise arises from the fluctuations due to trapping and de-trapping of photogenerated charges at recombination centers.

Measurement of the recombination time was traditionally done by a noise measurement (Darken, 1991). An alternating bias was applied to the material and the noise was monitored. The frequency of the applied bias was increased until the
generation-recombination noise disappeared. The frequency of the alternating bias at which the noise dropped was taken as the measured inverse recombination time or recombination rate.

Capacitive transient spectroscopy is an alternative means of experimentally determining the recombination time (Darken, 1990). In this procedure, the test material is implemented in a p-n diode configuration. A $1-10\mu s$ pulse is applied to the material, the transient response is measured, and the recombination time deduced.

The commonly accepted theory of free carrier capture in semiconductors was originally formulated by Lax (1960). He proposed a capture process in which a free carrier (e.g., a hole) is initially trapped in the attractive screened Coulomb field of an ionized impurity. The trapped carrier occupies an excited state of a hydrogen-like exciton. Through a series of phonon emissions the bound carrier falls into the potential well of the charged impurity. If the trapped carrier drops to a bound energy level deeper than $k_B T$, the probability of ejection is small and the carrier most often recombines. Alternatively, the trapped carrier can absorb thermal energy and move out of the trap. This process is responsible for generation-recombination noise.

Abakumov et. al. (1978) have substantially extended and corrected Lax's work. Abakumov et. al. have identified two temperature regimes for the capture cross section. The low and high temperature regimes are defined by $k_B T < m^* v_s^2$ and
\[ k_B T > m^* v_s^2, \] where \( v_s \) is the velocity of sound in the material. For holes in Ge, \[ \frac{m^* v_s^2}{k_B} = 0.73(K). \] The physical mechanism giving rise to the two temperature regimes is the spatial correlation of the positive and negative ionized impurities discussed earlier. At very low temperatures, the negative charged donors begin to pair with positive charged acceptors and form dipoles. The recombination physics of a hole with a dipole center is fundamentally different from recombination with a negative-charged Coulomb center. In the low temperature range, dipole recombination dominates Coulomb recombination and the capture cross section has a \( T^{-1} \) temperature dependence. Above the transition temperature, holes recombine primarily with negative charged acceptors. Coulomb recombination gives a \( T^{-3} \) temperature dependence for the cross section. For temperatures near the transition point, the capture cross section is not defined by either regime. An interpolation formula, equation 5.4-1 (Abakumov et. al., 1978), gives the correct high and low temperature behavior. Equation 5.4-1 compares well with experimental data compiled by Abakumov et. al.(1978) for a wide range of impurities in both Ge and Si. For the detectors considered here, the temperature is above the dipole saturation temperature (~0.7(K)). Hence, the recombination centers will be totally comprised of negative charged Coulomb centers. With this assumption, the capture cross section is given by equation 5.4-2.
Reggiani and Mitin (1989) have reviewed recombination and impact ionization of semiconductor impurities. They have constructed Monte Carlo simulations of the cascade process. In addition, they have generated results for current-voltage relationships and noise in various semiconductor materials.

\[
\sigma = \frac{1}{48\pi^2\varepsilon_0^3\lambda} \frac{e^2}{ek_BT} \left( \frac{e^2}{ek_BT + \sqrt{\frac{15}{2}m^*\nu_e^2}} \right)^2 \tag{5.4-1}
\]

\[
\sigma = (1.11 \times 10^{-13} \varepsilon^3 m^2) \left( \frac{1}{T} \right)^3 \tag{5.4-2}
\]

None of the data compiled by Abakumov et. al. was collected with the capacitive transient spectroscopic technique. Recently Darken et. al. (1990) have reported experimental results which call into question not only Abakumov’s result, but also the single phonon cascade picture. With capacitance transient spectroscopy, Darken et. al. have measured recombination times substantially different than those predicted by Abakumov’s result. Darken et. al. have determined a phenomenological relationship which fits their data well over a wide range of temperatures and impurity center types.

For photogenerated holes in Ge, Darken’s phenomenological relationship for the inverse recombination time is expressed in equation 5.4-3. For typical detector operating parameters, the recombination rates predicted from the cascade model and the phenomenological result are respectively \( \tau_{r,c}^{-1} = 1 \times 10^7 (s^{-1}) \), \( 4 \times 10^5 (s^{-1}) \). The Darken
result for $\tau_r^{-1}$ is substantially smaller than that predicted from the cascade model. The empirically deduced diffusion length in chapter 4 is consistent with the Darken result. For typical detector parameters the Darken recombination time gives a diffusion length $L_n=325(\mu m)$. The Darken et. al. results call for further experimental and/or theoretical clarification.

$$\frac{1}{\tau_r} = 7.25 \times 10^{-11} (m^{-2} s^{-1} K^{1/2}) \frac{P}{T^{1/2}}$$

The application of stress to the detector material may effect the recombination time. The application of a stress to the detector material will introduce a deformation strain potential. The strain potential will alter the Coulomb potential of the negative charged impurities. The altered potential will affect the recombination process.

The application of stress has been demonstrated to increase the responsivity of Ge:Be and Ge:Ga detectors (Haegel, 1989b). Laboratory data at the University of Arizona for Ge:Be materials indicate the overcharged Be center, $Be^+$, has a concentration of $[Be^+] < 0.05[Be^0]$, in an unstressed material. Thus with a relatively small fraction of $Be^+$ centers overcharged, the observed increase in responsivity, with the application of stress, is likely connected to a recombination effect. Again, experimental and theoretical work along these lines is needed.
Chapter 6 The Optical Model

Before the longitudinal detector signal equations can be evaluated, we must predict the spatial distribution of the absorbed photons in the detector material. The spatial distribution of the absorbed photons in the detector material is required to evaluate the detector current density signal equations (ie. eqs. 5.1-16, 17, and 18) derived in the last chapter. In addition, evaluation of the signal current density, and hence the spatial distribution of the absorbed photons, are prerequisites for the computation of the various measures of a longitudinal detector’s imaging performance discussed in chapter 7.

The objective of the optical model is to ascertain some of the optical design parameters of a longitudinal or front plane illuminated array (FPIA). The optical model is based on Maxwell’s electromagnetic equations, therefore it gives a fundamental description of the optical properties of the FPIA. Before applying the model, a description of the model and its assumptions is presented.

The optical response of an FPIA is modeled as a thin-film optical multilayer. The structure of the FPIA is composed of a finite sequence of dielectric/metallic films. Light incident on a FPIA is reflected, transmitted and absorbed in subsequent layers. Some of the layers typically have thicknesses on the order of or less than a single wavelength of the incident light. Interference effects can predominate in such
multilayer structures; thus, any complete description of the optical properties of a FPIA should account for interference effects.

The analytic properties of multilayer films have been worked out for a number of years (Macleod, 1986). The computational aspects have been improved by various workers. M. Mansuripur (1990) at the Optical Sciences Center/University of Arizona has formulated a computationally-efficient method which completely describes the optical properties of an arbitrary multilayer structure. Mansuripur's method uses a 2X2 complex number matrix formalism and can handle birefringent and/or absorptive films. Each layer of the multilayer structure is completely described by its thickness and complex dielectric tensor.

The details of Mansuripur's method is not be presented here. The method is used, however, to understand the optical properties of the FPIA. In particular, a number of quantities are determined; the Poynting vector and its divergence (ie. $\mathbf{V}_0$), and the absorptive efficiency of the infrared absorbing layer are computed. For purpose of example, the FPIA is assumed to be a four layer thin-film structure. The detector construction is illustrated in figure 6-1. Other multilayer geometries can be similarly analyzed.
6.1 Optical Constants of a FPIA

The top layer of the FPIA is a quarter-wave ($\lambda/4$) anti-reflective (AR) coating satisfying the quarter-wave rule. The quarter-wave rule specifies that a perfect AR coating, at a given wavelength, on an infinite substrate with refractive index $n_s$, can be achieved by depositing a quarter-wave thickness of material with refractive index of $n_s^{1/2}$ on top of the substrate.

A candidate material for the AR coating on detectors utilizing a Ge substrate is diamond like carbon (DLC). DLC has many attractive physical properties. DLC is very hard and resistant to scratching and mechanical failure. Gatesman et al. (1990) have measured the far infrared spectral properties of CVD diamond films. They deduced an empirical relationship for the complex refractive index according to
equations 6.1-1a,b. Here, $\lambda$ is the free space wavelength and is measured in cm. Since Ge has a refractive index of $-4$, DLC, which has an index of $\approx 2.5$, is well matched to Ge to satisfy the quarter-wave rule.

$$n = 2.49 - 1.54\lambda$$
$$k = 2.77 \times 10^{-3} + 0.831\lambda$$ \hspace{1cm} (6.1-1a,b)

CVD diamond films can have quite rough surfaces due to crystal formation. The rough crystalline surfaces can result in significant scattering losses at various wavelengths. Wang et. al. (1990) have successfully applied a polishing process that significantly reduces the scattering losses.

Diamond films have very attractive electrical, thermal, mechanical and optical properties for various applications. With the keen technological interest, the fabrication procedures, characterization, and availability of diamond films will surely make advances in the future.

The second layer of the FPIA is a transparent ohmic contact. The determination of its optical constants is considered in section 6.2.

The remaining layers in the detector are an active infrared absorbing layer and an array of ohmic conductive collecting contacts. The infrared absorbing material is assumed to be a $p$-doped extrinsic semiconductor material as discussed in chapter 5.
In the following example computations, the p-type dopant will be the double acceptor Be. Other shallow dopants can be handled analogously.

The photoionization cross section of shallow level impurities in semiconductors is discussed by Ridley (1988). The derived photoionization cross section, equation 6.1-2, depends on a number of parameters: \( \alpha \) is the fine structure constant; \( a_B \) and \( a_B^* \) are the Bohr and the effective Bohr radii, respectively; \( n \) is the refractive index; \( m^* \) the effective mass of the photogenerated charge; \( R_H \) and \( R_H^* \) are the Rydberg and effective Rydberg energies, respectively; \( \omega \) is the angular frequency of the incident light; \( E_k \) is the kinetic energy imparted to the hole by photoionization and is given by \( E_k = \hbar \omega - R_H^* \).

\[
\sigma_p = \frac{512 \pi}{3} a_B^2 \left( \frac{R_H}{\hbar \omega} \right) \frac{1}{n} \frac{m}{m^*} \left( \frac{2m^*}{\eta^2} \right) \frac{\pi a_B^* \left( \frac{R_H^*}{E_k} \right)^{1/2}}{(1 + \frac{E_k}{R_H}) E_k^{3/2}} \]

The computed spectral dependence of the absorption coefficient is deduced and plotted in figure 6-2. Two normalization parameters were introduced in figure 6-2. The cutoff wavelength \( \lambda_c \) is the minimum wavelength for photon absorption. The absorptance is normalized by \( \alpha_{\text{ave}} \). The average absorptance, \( \alpha_{\text{ave}} \), is the mean spectral absorptance over \([\lambda_c, \infty]\). In all the following computations the derived model absorption, from equation 6.1-2, was scaled by the peak of the experimentally derived absorption (e.g. for Be, and Ga see Wang et. al, 1986). With scaling, the peak of the
model absorptance was made equal to the experimentally measured peak. Typically, the spectral dependence of the absorption in equation 6.1-2 was reduced by ≈30%.

![Absorption by Shallow Impurity](image)

**Figure 6-2:** The model absorption constant vs wavelength.

The back wall of the detector array is composed of an array of ohmic contacts. In practice, the back wall contacts on the array are ion-implanted with gold(Au) or aluminum(Al) metallization. The contacts are close enough together that they optically act as a single uniform conducting plane. The closeness of the ohmic contacts is in part determined by the transverse diffusion length of the free holes as was discussed in chapters 4 and 5. The assumption of a uniform conducting sheet in place of an array of conducting contacts is necessary to apply the optical multilayer model. In this discussion, the array of back wall ohmic contacts are modeled by a 1(µm) thick gold(Au) film which is typical in a actual device. The ion-implantation of the back wall
contacts is not included in the multilayer model. This assumption is not of physical consequence since the optical absorption in the real contact and the modeled contact are quite small. A single gold film accounts for most of the optical reflection and absorption that occurs. The optical constants for Au were interpolated from data compiled by Hagemann et. al.(1974).

6.2 The Transparent Contact

The transparent contact, as its name suggests, has a two-fold function. First, it must be optically transparent in the appropriate spectral range. Second, it must be electrically conductive to inject photogenerated holes back into the detector material. The transparent contact is positioned between the active infrared absorbing layer and the AR coating; therefore, incoming light propagates through the contact and transparency is needed. Often, the two attributes, transparency and conductivity, are conflicting in that good conductive properties usually accompany strong optical absorption or reflection. In practice, the transparent layer is realized by reducing its thickness to a point where its conductive properties remain intact while its optical absorption becomes insignificant due to the very short optical path length through the contact.

The proper design of the transparent contact is an important aspect of the general detector array design. Surface science has made some great strides in the last
A number of experimental methods have been refined which have enabled researchers to view the dynamics of metal-semiconductor interfaces on an atomic level. A number of common probes are used to study metal-semiconductor interfaces: examples are synchrotron-radiation photoelectron spectroscopy, polar-angle resolved X-ray photoemission spectroscopy, Auger electron spectroscopy, surface photovoltage spectroscopy, and various other vacuum techniques. Surface scientists have found a wide variety of phenomenon occurring at the metal-semiconductor interface. These phenomenon include phase transitions, formation of new compounds and other effects. Most of the experimental tools have been applied to III-V semiconductors (Shapira, 1989) such as GaAs. Much work has also been done to understand the Metal-Si interface (Rhoderick, and Williams, 1988). For cleaved Ge surfaces, little has been reported on surface metallization. Thanailakis and Northrop (1973) investigated the Schottky barriers that result when various metals are evaporated onto etched Ge surfaces.

P-type ion implanted low temperature ohmic contacts have been fabricated on Ge by Wu et. al. (1991) and Hadek et. al. (1985) and studied by K. Jones (1986). When considering electrical contacts on lightly doped semiconductors one often distinguishes between ohmic and blooding contacts (Haller, 1991). In a blooding contact, a potential barrier exists between the contact and the semiconductor. A p-type ohmic contact is realized by constructing a reservoir of holes (Haller, 1991) that can easily flow in the valence band of the bulk semiconductor.
To appreciate the design of an ion-implanted ohmic contact it is helpful to look at some of the related physics. One basic consideration is the doping concentration of donors and/or acceptors in the semiconductor material under consideration. For the most pure semiconductors, impurities have concentrations of $\sim 10^{10}$ (cm$^{-3}$); the most heavily doped semiconductors can have doping levels up to $10^{21}$ (cm$^{-3}$). In a doped semiconductor, the effective Bohr radius of a hole (p-doped) or an electron (n-doped) is lengthened due to the reduced effective mass and dielectric screening. For Ge, the dielectric constant is $\varepsilon = 16$. The effective mass of a hole is $m^* = 0.35m_e$, where $m_e$ is the electron rest mass. The effective Bohr radius (Ashcroft and Mermin, 1976) is given in equation 6.2-1.

$$a^* = \frac{\varepsilon}{(m^*/m_a)}$$  \hspace{1cm} (6.2-1)

Here, $a_a$ is the Bohr radius in the hydrogen atom. For boron, a common implanted ion in Ge with an ionization energy of $\sim 10.5$ (meV), the effective Bohr radius becomes $a^* = 45.3 \AA$, which is almost ninety times that of neutral hydrogen. For Ge, where the nearest neighbor distance is $r = 2.5 \AA$, doping levels of $10^{17}$-$10^{18}$ cm$^{-3}$ can result in significant overlap of the hole wavefunctions; the result being a material with metallic properties (Mahan, 1981).

The metal-non metal transition in semiconductors, sometimes called the Anderson transition, has been an active area of experimental and theoretical research. Today, its solution remains an outstanding problem in modern physics (Kramer and Schon, 1990). Mott (1961) postulated a simple relationship, equation 6.2-2, which
specifies the critical doping concentration at which the metal-nonmetal transition occurs.

\[ N_e^{1/3}a_e^{*} = 0.26 \pm 0.05 \]  \hspace{1cm} (6.2-2)

Here, \( N_e \) is the critical doping concentration at which the metal-non metal transition occurs and \( a_e^{*} \) is the Bohr radius of the semiconductor dopant. The relationship in equation 6.2-2 has been verified over a range of \( 10^{10} \) in critical concentration and 600\( \text{Å} \) in Bohr radii (Edward and Sienko, 1979). In addition, the relation has been verified for shallow n-type (Sb, P, and As) and a p-type dopant (Ga) in Ge by Fritzsch (1958, 1962).

The value of the conductivity at the metal-non metal transition, \( \sigma_c \), has been found (Mott, 1972) to follow (Fritzsch, 1980) equation 6.2-3. Here, \( e \) is the unit of electronic charge, \( \hbar \) is Planck's constant divided by \( 2\pi \) and \( d_e \) is the mean distance between the dopant atoms at the critical concentration. Both equations 6.2-2 and 6.2-3 are valid at liquid He temperatures.

\[ \sigma_c = \frac{0.026e^2}{\hbar d_e} \]  \hspace{1cm} (6.2-3)

Fritzsch (1980) has emphasized the importance of compensation effects on the value of the critical concentration at which the metal-non metal transition occurs. When the compensation, \( K \), exceeds \( K > 0.4 \), equations 6.2-2 and 6.2-3 are not strictly
followed. For larger compensations, the metal-non metal transition occurs at critical densities in excess of those predicted by equation 6.2-2. In practice, far infrared detectors are constructed from high purity semiconductors, in which case compensation effects will not be a factor.

For p-doped Ge, equations 6.2-2 and 6.2-3 yield useful estimates of the critical concentration and the conductivity of the transparent contact. In boron doped Ge, the ionization energy is ~10.5(meV). Extrapolation of a Bohr radius from the ionization energy followed by substitution into equation 6.2-2, results in a critical concentration of \( N_c(B) = 1.88 \times 10^{17} \text{cm}^{-3} \). Similarly for the conductivity, equation 6.2-3 yields a critical conductivity of \( \sigma_c = 5.84 \text{cm}^{-1} \Omega^{-1} \).

Optical modeling of ion implanted transparent contacts in Ge has received little attention. Schroeder et. al.(1978) have modeled ion implanted ohmic contacts in Si with a simple Drude model (see equation 6.2-4). The Drude model, however, must be applied with discretion (Mahan, 1981). Strictly, it gives the correct asymptotic behavior only in the low and high frequency limits (i.e. \( \omega \to 0, \infty \)). When not in the asymptotic limit, many body effects should be taken into account (Mahan, 1981). Schroder et. al.(1978), in modelling B-ion implanted contacts, assume \( (\omega \tau)^2 > 1 \) in equation 6.2-4; however, the validity of this assumption is dubious. Sample calculations indicate \( \omega \tau = 0.53 \) for metallic SiB at liquid He temperatures and \( \lambda = 10 \mu \text{m} \). The simple model constructed by Schroder et. al. has good agreement with experimental data. Perhaps the approximation \( (\omega \tau)^2 > 1 \), or alternatively neglecting the
1 in the denominator of equation 6.2-4, compensates for the presence of hole scattering mechanisms not included in the model.

Initial considerations in modelling the transparent contact should account for its quasi two-dimensional structure. The B-ion implanted contact confines free holes to a layer ~1500Å thick; therefore, the hole wavefunction is confined in a potential well in the z-direction. With a potential well, the hole energy is quantized in the z-direction. The energy quantization results in discrete sub-bands. Such a system is an example of a mesoscopic quantum structure.

Heitmann and Ensslin(1991) have characterized the spectroscopic properties of 2-D mesoscopic MOS structures on Si. The optical absorption between the sub-bands of n-type inversion layers on Ge(111) with surface state densities $N_s=10^{12}$$(cm^{-2})$, have been observed by Scholz and Koch(1980) and modeled by Ando(1977). The experiment by Scholz and Koch(1980) required tilting the film; the tilt provided a sufficiently large component of the optical electric field perpendicular to the film to excite sub-band resonances. In imaging astronomical detectors, the light incident on the detector is often nearly collimated. The component of the optical electric field perpendicular to the transparent contact is very small compared to the transverse component; hence, direct optical excitation into the sub-bands is minimal.

Despite its shortcomings, the Drude theory is used to give a first order description of the optical absorption in the transparent contact. The Drude expression
for the imaginary part of the dielectric constant is given in equation 6.2-4.

\[ \varepsilon_i = 2nk\frac{n_h e^2}{\omega \varepsilon_0 m^* \frac{\tau}{1 + \omega^2 \tau^2}} \]  

(6.2-4)

In equation 6.2-4: \( n_h \) is the hole density; \( \omega \) is the angular frequency of the light; \( \tau \) is a phenomenological parameter known as the relaxation time of the free charge carriers.

Precisely, \( \tau \) is the momentum scattering time. In a qualitative sense, \( \tau \) is approximately the time between successive scattering events. The parameter \( \tau \) is determined by the relevant scattering mechanisms of the charge carriers in the material. Due to the relatively large hole density in the transparent contact, the scattering time \( \tau \) is much shorter in the transparent contact than in the bulk material. A complete discussion of \( \tau \) and its theoretical evaluation can be found in Mahan(1981).

With large free hole densities in the transparent contact, the Coulomb potential is screened. Since holes in the transparent contact follow Fermi-Dirac statistics\( (k_B T = E_F / 30) \), Thomas-Fermi screening is applicable. Computation of the Thomas-Fermi screening length, equation 6.2-5, yields \( L_{TF} = 1.5 \text{Å} \); thus, the ionized impurity potential is screened out. Other scattering mechanisms, such as phonon scattering due to diffuse hole reflection off the potential walls are candidates for the dominate scattering mechanism(s). Theoretical work, which determines the relevant scattering mechanisms and predicts the optical properties of the implanted contact
based on a Green's function or RPA (Mahan, 1981) approach is needed.

\[ L_{T-F} = \frac{1}{2\pi} \sqrt{\frac{\epsilon_0 E_f}{6\pi e^2 n_h}} \]  

(6.2-5)

The primary quantities of optical interest can be derived from a quasi-empirical knowledge of the spatial distribution of the implanted ions and the scattering time \( \tau \). With knowledge of the implanted ion distribution or concentration and the scattering time \( \tau \), it is possible to deduce the optical absorption of the implanted contact from equation 6.2-4.

In an ion-implanted transparent contact, the carrier density, \( n(z) \), is often modeled as a Pearson distribution (Selberherr, 1984). The Pearson distribution is Gaussian-like, with a range and standard deviation which depend on the implanted ion energy. In this discussion, the implanted contact is approximated by a uniform layer with spatially optical properties. The spatial extent of the model transparent contact is defined as those depths where the implanted density exceeds the metal-non metal critical density. This definition typically results in a connected spatially homogenous layer, extending from the outside surface of the implant to the \( z \)-position within the material where the implant density drops just below the critical metal-non metal density. For a typical B-ion implant (Beeman, 1991) in Ge, corresponding to an ion energy \( E_0 = 25(\text{keV}) \) and dose \( d = 2 \times 10^{13}(\text{cm}^{-2}) \), the approximated layer has a thickness of \( \sim 156(\text{nm}) \). The concentration of the free holes used in equation 6.2-4 is
equal to the mean concentration of the density profile over the extent of the layer.

To evaluate the imaginary index, $\varepsilon_2$, in equation 6.2-4, the phenomenological relaxation time $\tau$ must be determined. As mentioned above, $\tau$ is determined by the relevant scattering mechanisms in the material. A phenomenological time $\tau$ can be determined, equation 6.2-6, from the conductivity at the metal-non metal transition, equation 6.2-3.

$$\tau = \frac{0.042m^*}{\hbar N_e^{23}}$$

(6.2-6)

For a typical B-ion implanted contact in Ge, the phenomenological scattering time evaluates to $\tau = 4 \times 10^{-14}$ (sec). The imaginary part of the dielectric constant, equation 6.2-4, is evaluated with knowledge of the phenomenological time $\tau$. The imaginary part of the refractive index at $\lambda = 40 \, \mu m$ of a B-ion implanted contact in Ge, is plotted versus the ion dose for three implant energies in figure 6-3. With approximations to both the real and the imaginary parts of the refractive index for the transparent contact, its optical behavior can be modeled.
Figure 6-3: The imaginary refractive index of the transparent contact.

6.3 Results of the Optical Model

With the evaluation of the optical constants in each layer of the detector, the optical response of the detector array can be described. The primary application of the optical model is the evaluation of the spatial distribution of the absorbed photons; that is to compute $\frac{\nabla \cdot S(x',y',z')}{\hbar \omega}$ in equation 5.1-16. The optical absorption in various layers of the detector is also of interest. The analysis of the longitudinal detector's imaging response will be discussed in chapter 7.
As discussed in chapter two, the light incident on the detector is incoherent in the transverse dimension and confined to a narrow spectral band. The absorption length in the infrared absorbing layer, $\alpha^{-1}$, is typically much less than the depth of focus of the optical system; that is, $\alpha^{-1} < \lambda f^2 / n$. Here, $f^\#$ is the f-number of the optical system in the detector image space and $n$ is the real part of the refractive index of the detector material. Also when the $f^\#$ of the optical system is large, most of the light is at normal incidence or collimated on the FPIA. When these conditions are satisfied, the Poynting vector, $S(x,y,z)$, is approximated according to equation 6.3-1.

$$S(x,y,z) = S_0(x,y) f(z)$$

(6.3-1)

In equation 6.3-1, $S_0(x,y)$ is the magnitude of the Poynting vector incident on the detector array and $f(z)$ is the $z$-dependence of the Poynting vector in the detector material. The function $f(z)$ is determined from the multilayer model.

The optical multilayer model correctly describes the longitudinal spatial distribution of absorbed photons. The function $f(z)$ in equation 6.3-1 is directly computed from the multilayer model. The difference between a simple exponential dependence, corresponding to an infinitely thick detector, and the prediction of the multilayer model is illustrated in figure 6-4. The plot clearly shows the differences that can exist between the two models. Assumed in generating the plots in figure 6-4 are $N_A = 7.5 \times 10^{14}$ (cm$^{-3}$), $d = 2.0 \times 10^{13}$ (cm$^{-2}$), and $E_{\text{ion}} = 25$ (keV). Also, the results were spectrally averaged over the absorption spectrum illustrated in figure 6-2.
The fractional absorption in the transparent contact may be computed. The fractional absorption, denoted by $\eta$, is defined as the number of photons absorbed in the transparent contact per photon incident on the detector array. The fractional absorption in the transparent contact is plotted versus the implanted ion dose, for three implant energies in figure 6-5. The results were averaged over the absorption spectral range illustrated in figure 6-2.

Meaningful conclusions can be derived from the plot in figure 6-5. First, the optical absorption in the transparent contact at any particular dose is relatively independent of the implant energy. The independence can be interpreted as a balance between the implant thickness and the free hole density. At constant dose, larger
Implant energies yield thicker layers with smaller mean concentrations. Smaller concentrations compensate for the thicker layers. The result, as the plot displays, is a fractional absorption that is independent of the implant energy, over the given range of doses. Since the electrical conductivity properties of the layer are improved with larger free carrier concentrations, contacts fabricated at low ion implant energies have good conductivity and small optical absorption.

A second conclusion can be drawn from the plot in figure 6-5. For the given parameters, the fractional absorption in the transparent contact is small. Even at the largest doses considered (i.e., $10^{14}$ cm$^{-2}$), the total absorption is less than 8%. At a typical dose level, $2\times10^{13}$ cm$^{-2}$, the total absorption is 1%-2%. With the relatively small absorptive losses, it is not expected that the dose of the implant is a critical
parameter in the detector array design.

In view of the above results, an initial understanding of the optical considerations for a transparent contact design are drawn. First, since the optical absorption in the transparent contact is relatively independent of the implant energy, better conductive properties can be attained by fabricating thinner contacts, with low energy ion beams. A similar conclusion was reached by Schroder et. al.(1978) for boron implants in Si. Second, the magnitude of the fractional absorption is less than 2% for typical doses. At such low absorption levels, the ion dose is not expected to be a critical parameter in the transparent contact design.

The fractional absorption in the infrared absorbing layer is also relevant. The absorptive efficiency as a function of both the absorbing layer thickness and the wavelength of the incident light are two relationships of interest. The absorptive efficiency of the infrared absorbing layer is plotted versus the wavelength of the incoming light for three layer thicknesses in figure 6-6. The primary dopant assumed to generate figure 6-6 has an absorption spectrum illustrated in figure 6-2.

A notable feature of the plot in figure 6-6 is the large amplitude oscillations. The oscillations are due to Fabry-Perot resonances in the detector. Such resonances have been noted by Farhoomand and McMurray (1991). In spectrometry instruments, the oscillations are an important consideration. In photometric instruments, however, they do not have the same impact on the instrument design. The results depicted in
Figure 6-6: The fractional absorption in the infrared layer.

Figure 6-6 assume parallel and optically flat surfaces between the layers of the detector. This idealization will not be met in practice in which case the resonant structure of the absorption spectrum will be broadened. The degree of broadening will depend on the roughness of the surfaces and their relative misalignment. An investigation of the broadening due to the surface roughness and their misalignment should be undertaken.

Wedging the layers has been suggested as a possible solution to the oscillation problem. It is dubious, however, that wedging the detector will remove the oscillations. It is well known that a dielectric wedge exhibits interference effects. Furthermore, wedging the layers will likely introduce optical cross talk between the pixel elements which degrades the array's imaging performance.
The general shape of the absorption spectrum of the primary dopant, figure 6-2, is also evident in figure 6-6. Qualitatively, the shape of the absorptive efficiency curves can be viewed as being similar to the primary acceptor absorption curves with Fabry-Perot oscillations superposed.

Finally, the thickness of the infrared absorbing layer has an important influence on the absorptive efficiency versus wavelength curves. For thinner detectors, the absorptive efficiency is less and oscillations are more prevalent. The amplitude of the oscillations diminishes as the thickness of the infrared absorbing layer is increased. Also for thicker detectors, the absorption begins to saturate at the shorter wavelengths.

The absorptive efficiency of the detector as a function of the thickness of infrared absorbing layer is plotted in figure 6-7. The spectral dependence of the primary dopant is given by equation 6.1-2 and plotted in figure 6-2. The absorption efficiency in figure 6-7 was averaged over the absorption spectrum illustrated in figure 6-2. The absorptive efficiency curve resembles a saturated exponential, \( \xi = 1 - e^{-d/d_0} \), where \( d \) is the thickness of the detector and \( d_0 \) is a constant. The small kinks in the curve are likely a result of the discrete nature of the calculation and the spectral oscillations discussed earlier.

With the optical multilayer model of the detector array described, the imaging performance of the detector array can be evaluated. The optical multilayer model
Figure 6-7: The absorptive efficiency vs infrared layer thickness.

allows the evaluation of the absorbed photon distribution in the detector material. The absorbed photon distribution is a necessary component in the calculation of the steady state current density in equations 5.1-16, 17 and 18. These equations are the basis for the subsequent evaluation of the imaging performance of the detector array in chapter 7.
Chapter 7 Analysis of the Longitudinal Detector Response

The focus of this chapter is the imaging response of a front plane illuminated far infrared detector array. Analysis of the imaging response interfaces the longitudinal detector signal equations (i.e., equations 5.1-16, 17, and 18) with many detector design considerations. The predicted detector imaging response is a basic input for image simulation and processing algorithms. In addition, the predicted imaging response serves as a guide for the design, construction and testing of far infrared detector arrays. The central issue addressed in this chapter is the translation of the longitudinal detector response equations (5.1-16, 17 and 18) into a form that will facilitate image processing studies, detector design, construction, and testing issues. Analytical results are derived which translate the longitudinal detector signal equations into appropriate forms for further analysis.

A number of specific forms of the detector response will be derived and discussed. A particularly useful concept is the contrast transfer function of the optical-detector system and its associated point spread function. The detector transfer function (DTF) enables prediction of the image response to an arbitrary object. The predicted imaging response can serve as a basic input for image simulation and processing studies. The cross talk between detector array pixels is another useful measure of the detector array response. The diffusive cross talk serves as a useful input into the mechanical design of the detector array. Finally, the responsivity is an
important performance parameter for integration times and many associated issues. The following results, which are mathematically derived, follow logically from the longitudinal detector response equations derived in chapter 5; therefore the results are limited by the assumptions and approximations implicit in the derivation of the signal equations.

7.1 The Detector Transfer Function

The detector transfer function is a particularly useful concept. The transfer function (Goodman, 1968) is essentially the eigenvalue spectrum of the optical-detector system. The transfer function measures the relative attenuation of a plane wave (the eigenfunction) of the optical-detector system. Ideally, the value of the transfer function, for a given plane wave, is a complex number with an amplitude and phase. The amplitude represents the relative attenuation. The phase represents the relative phase shift referenced to the origin of the coordinate system. For systems considered here, the relative phase shift is zero and the transfer function takes on only real values.

The transfer function for the optical-detector system can be derived from the longitudinal detector signal equations 5.1-16, 17 and 18 derived in chapter 5. The response of the detector array is represented by equation 7.1-1. Here, \( i(x,y) \) is the detector current at position \((x,y)\) on the detector array; \( i_p(x,y) \) is the current from the
The two-dimensional rect(\_\_) or rectangle function (Gaskill, 1978) is defined in equation 7.1-3.

\[
rect\left(\frac{x-x_i}{L_{px}}, \frac{y-y_j}{L_{py}}\right) = \begin{cases} 
1 & \text{if } \left|\frac{x-x_i}{L_{px}}\right| \text{ and } \left|\frac{y-y_j}{L_{py}}\right| \leq \frac{1}{2} \\
0 & \text{otherwise}
\end{cases}
\] (7.1-3)

Equations 7.1-1, 2 and 3 can be written in a more compact form that allows a more transparent derivation of the detector transfer function. The detector current is expressed in equation 7.1-4 in terms of \(J_{net}\), rect(\_), and comb(\_) functions. The comb(\_) function is defined in equation 7.1-5; the remaining quantities, \(L_x\), \(L_y\), \(L_{px}\), \(L_{py}\), and \(L_{py}\) are illustrated in figure 7-1.

Equation 7.1-4 is entirely equivalent to equations 7.1-1, 2, and 3. The autocorrelation of the rect(\_\_) function with \(J_{net}\) gives the current measured by a pixel.
i(x,y) = \frac{1}{L_x L_y} \text{rect}\left(\frac{x}{L_x}, \frac{y}{L_y}\right) \text{comb}\left(\frac{x}{L_x}, \frac{y}{L_y}\right) \\
\left[ \text{rect}\left(\frac{x}{L_x}, \frac{y}{L_y}\right) \ast \ast J_{\text{net}}(x,y) \right] \\
(7.1-4)

comb\left(\frac{x}{L_x}, \frac{y}{L_y}\right) = \text{comb}\left(\frac{x}{L_x}\right) \text{comb}\left(\frac{y}{L_y}\right), \text{ where } \\
comb\left(\frac{x}{L_x}\right) = |L| \sum_{n=-\infty}^{\infty} \delta(x-nL) \\
(7.1-5)

centered at position (x,y); or i_p(x,y) = \text{rect}\left(\frac{x}{L_x}, \frac{y}{L_y}\right) \ast \ast J_{\text{net}}(x,y), \text{ where } i_p(x,y) \text{ is the current in a pixel centered at (x,y). In equation 7.1-4, the current in the pixel centered at (x,y), } i_p(x,y), \text{ multiplies the comb(\_\_) function. The comb(\_\_) function is zero everywhere except at the lattice points, (x,y) = (nL_x, mL_y), with n, m \in I}; \text{ where I is the set of integers. The product of the comb(\_\_) and } i_p(x,y) \text{ functions is zero everywhere except at the lattice points. The final multiplication by the rect(\_\_) function truncates the comb(\_\_) function at the outer dimensions of the array. Thus we are left with an expression equivalent to equation 7.1-1; the summation of delta functions, each centered at a detector pixel with a coefficient representing the current in the respective pixel. }

The detector transfer function, or DTF, can be derived from the detector response, i(x,y), defined in equation 7.1-4. By definition, the transfer function of an optical system is the Fourier transform of the point response function or the Green's function of the optical system as discussed in chapter 2. To evaluate the Fourier
Figure 7-1: The pixel geometry of a FPIA.

Transform of equation 7.1-4, the transform of the cross correlation of two arbitrary functions is defined in equation 7.1-6; the function \( g_x = g(-x) \). The transfer function, equation 7.1-7, is directly obtained by Fourier transforming equation 7.1-4. The Fourier transforms of the \( \text{rect}(\_\_) \) and \( \text{comb}(\_\_) \) functions in equation 7.1-4, can be directly evaluated or referenced in Gaskill(1978).

\[
\mathcal{F}(f \ast g) = \mathcal{F}(f) \cdot \mathcal{F}(g_x) \tag{7.1-6}
\]

\[
\text{DTF}(k) = [L_{ax}L_{ay}\text{sinc}(\frac{k_x}{k_{ax}})\ast\ast\text{comb}(\frac{k_x}{k_{ax}}, \frac{k_y}{k_{ay}})]\ast\ast

[\Phi\text{sinc}(\frac{k_x}{k_{px}}, \frac{k_y}{k_{py}})(\mathcal{F}_{\text{psd}})(k_x, k_y)] \tag{7.1-7}
\]

In equation 7.1-7, the various terms are defined as follows: \( k_{ax,y} = 2\pi/L_{ax,y} \), \( k_{ax,y} = 2\pi/L_{ax,y} \), \( k_{px,y} = 2\pi/L_{px,y} \), \( A_p = L_{px}L_{py} \) and \( J_{\text{psd}} \) is the total detector current density when the detector
is imaging a point source centered in the field of view. The \( \text{sinc}(\cdot) \) function is defined in equation 7.1-8.

\[
\text{sinc}(x,y) = \frac{\sin(\pi x) \cdot \sin(\pi y)}{\pi x \cdot \pi y} \quad \text{(7.1-8)}
\]

In addition, the inversion symmetry of \( \mathcal{J}_{p,\text{set}} \) was utilized in equation 7.1-7. The function \( \text{DTF}(k_x,k_y) \) is the detector transfer function for the plane wave corresponding to wavevector \( k = (k_x,k_y) \). Its value is the relative attenuation of the \( (k_x,k_y) \) plane wave amplitude by the optical-detector system.

The expression in equation 7.1-7 can be simplified. When the dimensions of the detector array are much larger than the individual pixel spacing, that is \( L_{\text{ax,y}} \gg L_{\text{sx,y}} \), the approximation in equation 7.1-9 is valid. Stated otherwise, a narrow \( \text{sinc}(\cdot) \) function approximates a delta function. With the approximation of equation 7.1-9, the expression for the transfer function simplifies to equation 7.1-10.

\[
\frac{L_{\text{ax,y}} \cdot \text{sinc}(\frac{k_x}{k_{\text{ax}}}, \frac{k_y}{k_{\text{ay}}}) ** \text{comb}(\frac{k_x}{k_{\text{ax}}}, \frac{k_y}{k_{\text{ay}}})}{A_2} = \sum_{n,m=-\infty}^{\infty} \delta(x-nL_{\text{ax}}) \delta(y-mL_{\text{ay}}) \quad \text{(7.1-9)}
\]

\[
\text{DTF}(k_x,k_y) = \frac{A_2}{A_z} \left[ \text{sinc}(\frac{k_x}{k_{\text{az}}}, \frac{k_y}{k_{\text{az}}}) \left( \mathcal{J}_{p,\text{set}}(k_x,k_y) \right) \right] ** \sum_{n,m=-\infty}^{\infty} \delta(k_x-nk_{\text{az}}) \delta(k_y-mk_{\text{az}}) \quad \text{(7.1-10)}
\]

The quantity \( A_z/A_x \) is the ratio of the conductor size to the area occupied by the pixel; it is the fill factor of the detector array pixels. Since the fill factor was implicitly taken
as unity in the derivation of the current density signal equations in chapter 5, it will also be set to unity here. Examination of equation 7.1-10 reveals that the detector transfer function is formed by the $\text{sinc}(\frac{k_x}{k_{p0}}) <\mathcal{F}_{J_p,\text{net}}(k_x, k_y) >$ function duplicated on an infinite 2-D lattice. The infinite 2-D lattice, $L_0$, in frequency space is defined by $L_0 = \{ (nk_x, mk_y) \in \mathbb{R}^2 : n, m \in \mathbb{I} \}$, where $\mathbb{I}$ represents the set of integers.

In image processing or signal recovery applications the high frequency components of equation 7.1-10 are removed by filtering. If aliasing effects are negligible, the transfer function, equation 7.1-11, is that of equation 7.1-10 retaining only the $n=m=0$ component.

$$\text{DTF}(k_x, k_y) = \text{sinc}(\frac{k_x}{k_{p0}}) <\mathcal{F}_{J_p,\text{net}}(k_x, k_y) >$$ (7.1-11)

Hence, with the determination of $\mathcal{F}_{J_p,\text{net}}$ the detector transfer function can be evaluated.

The detector current density is generally defined in equations 5.1-16, 17 and 18. Since the Fourier transform is a linear operation, the transform of $J_{p,\text{net}}$, equation 7.1-12, can be expressed as a linear superposition. Here, $J_{p,l}$ is the detector current density corresponding to $l$th pass of the photogenerated holes when the detector is imaging a point source. The current densities, $J_{p,l}$, are defined by equation 5.1-16 for...
i=1, and by equation 5.1-17 for i\geq 2.

\[
(\mathcal{F}_p)_n^{(i)} = \sum_{i=1}^{\infty} (\mathcal{F}_p)_n^{(i)}
\]  

(7.1-12)

For the case i\geq 2, the current density \( J_{p,i} \) is defined recursively according to equation 5.1-17. The expression for \( J_{p,i}(x,y) \) can be put in an alternate form, equation 7.1-13, that accommodates further simplification.

\[
J(x,y) = \frac{e^{-r_0 x^2}}{4\pi t_s(d)D_k} \mathcal{F}_s^{(i-1)}(x,y)^* * gaus(x, y) 
\]

(7.1-13)

Here, the Gaussian function \( gaus(x, y) \) is defined in equation 7.1-14, ** is the 2-D convolution operation, and the remaining quantities are as defined earlier.

\[
gaus(x, y) = e^{-\frac{x^2}{a^2} - \frac{y^2}{b^2}}
\]

(7.1-14)

The Fourier transform of equation 7.1-13 is computed in equation 7.1-15.

\[
(\mathcal{F}_p)_n^{(i)}(k_x, k_y) = e^{-r_0 \sqrt{2\pi} k_x} \mathcal{F}_s^{(i-1)}(k_x, k_y) 
\]

(7.1-15)

\[
gaus(2\sqrt{\pi} t_s(d)D_k k_x, 2\sqrt{\pi} t_s(d)D_k k_y)
\]

With the recursive relation between the \( \mathcal{F}_p \), equation 7.1-15, the Fourier transform of the point response current density, \( \mathcal{F}_p \), will complete the evaluation the DTF in equation 7.1-11.
Before computing $\mathcal{F} J_{p,1}$, an approximation made in equation 6.3-1 is recalled. To compute the DTF in equation 7.1-11 the absorbed photon density, $\frac{\nabla S}{\hbar \omega}$, must be computed. With the approximation made in equation 6.3-1, the distribution of the absorbed photons, equation 7.1-16, is easily computed. In equation 7.1-16, $S_\omega(x,y)$ is the number magnitude of the Poynting vector incident on the detector array and $f(z)$ is the $z$-dependence of the Poynting vector in the detector material. The function $f(z)$ was determined from the multilayer model in chapter 6.

$$\frac{\nabla S}{\hbar \omega} = S_\omega(x,y)f(z) \quad (7.1-16)$$

With the approximation for the absorbed photon distribution, equation 7.1-16, the Fourier transform of $J_{p,1}$ can be evaluated. Substitution of equation 7.1-16 into equation 5.1-16 and rearrangement of the terms results in equation 7.1-17.

$$J_{p,1}(x,y) = \int_0^\infty e^{-jy} \int_0^\infty S_\omega(x,y) f(z') \frac{4\pi f(z')D_\perp}{\sigma_{gauss}(\frac{x}{2\sqrt{\pi f(z') D_\perp}}, \frac{y}{2\sqrt{\pi f(z') D_\perp}})} dz' \quad (7.1-17)$$

In equation 7.1-17, $S_{op}(x,y)$ is the point response number flux on the front surface of the detector array. $S_{op}(x,y)$ is determined from the optical system and is proportional to the point spread function of the optical system. The remaining terms in equation 7.1-17 were defined earlier.
With the expression for \( J_{p,l}(x,y) \), the Fourier transform can be directly evaluated. Since the Fourier transform of a convolution of two functions is the algebraic product of the transformed functions, the Fourier transform of \( J_{p,l}(x,y) \) is determined in equation 7.1-18.

\[
(\mathcal{F}J_{p,l})(k_x,k_y) = -\text{MTF}(k_x,k_y) \int_0^d \frac{e^{-i(k_x x + k_y y) - N_2 f(z')}}{\text{gaus}(2\sqrt{\pi}l(z')D_x k_x, 2\sqrt{\pi}l(z')D_y k_y) dz'}
\]  

(7.1-18)

Here, \( \text{MTF}(k_x,k_y) \) is the modulation transfer function of the optical system and the \( \text{gaus(} \) function is defined in equation 7.1-4. With the definition of a characteristic wavevector due to diffusion, equation 7.1-19, the result can be simplified further. Substitution of the definition for \( k_d(z') \) into equation 7.1-18 results in a simplified result for \( \mathcal{F}J_{p,l} \) in equation 7.1-20. Equation 7.1-20 is the seed term which enables \( \mathcal{F}J_{p,\text{net}} \) in equation 7.1-11 to be evaluated.

\[
k_d(z') = \frac{1}{2\pi \sqrt{k_d(z')D_z}}
\]  

(7.1-19)

\[
(\mathcal{F}J_{p,l})(k_x,k_y) = -\text{MTF}(k_x,k_y) \int_0^d \frac{e^{-i(k_x x + k_y y) - N_2 f(z')}}{\text{gaus}(2\sqrt{\pi}l(z')D_x k_x, 2\sqrt{\pi}l(z')D_y k_y) dz'}
\]  

(7.1-20)

The result in equation 7.1-20 has a simple physical interpretation. \( \mathcal{F}J_{p,l} \) is the transfer function for the first sweep of holes through the detector. The first pass transfer function is degraded from the optical MTF by diffusion in both amplitude and
bandwidth. The $-df(z')e^{-t/\tau}N_d$ term in the integral degrades the amplitude of the first pass transfer function. The degradation is expected since this term corresponds to the fractional optical absorption and subsequent recombination of photogenerated holes, both of which behave as damped exponentials. The exponential term, $e^{-\alpha z^2}$, degrades the spatial frequency response of the detector. This is also expected, since this term corresponds to the transverse diffusive spreading of photogenerated holes. Thus, fractional absorption and recombination of photogenerated hole limits the amplitude of the transfer function, while transverse diffusive spreading limits the bandwidth.

An expression for the detector transfer function can be derived. The Fourier transform of the point current density $J_{p,net}$ may be evaluated. Combination of the defining expression for $J_{p,net}$ (eq. 7.1-12) with the recursive definition for $J_p$ (eq. 7.1-15) and the just derived expression for $J_{p,1}$ (eq. 7.1-16) results in an expression (eq. 7.1-21) for $J_{p,net}$.

\[
\{J_p(k)\} = MTF(k)(1 + r(k) + r(k)^2 + \ldots) \int_d df(z') e^{-t/\tau}N_d e^{i^2 k}\]

(7.1-21)

with $r(k) = e^{-t/\tau}N_d e^{\frac{k^2}{2\alpha^2}}$, $k^2 = k_\alpha^2 + k_\beta^2$

The infinite geometric series in equation 7.1-21 is summed in equation 7.1-22. With
the definition of the detector transfer function in equation 7.1-11, and equations 7.1-
21, and 7.1-22, an expression for the DTF is computed in equation 7.1-23. Equation
7.1-23 is the final expression for the detector transfer function. It is the central result
for the analysis of a front plane illuminated detector array’s imaging response.

\[
(1 + r + r^2 + \cdots) = \frac{1}{1 - e^{-2 \Delta x N_e \text{Pe}^{-2 \Delta y^2 \text{Pe}}}}
\]  
(7.1-22)

\[
\text{DTF}(k) = -\text{MTF}(k) \cdot \text{sinc}(\frac{k_{px} L_x}{k_{py}}) \left\{ \frac{1}{1 - e^{-2 \Delta x N_e \text{Pe}^{-2 \Delta y^2 \text{Pe}}} \text{Pe}^{-2 \Delta y^2 \text{Pe}}} \right\}
\]  
\]  
(7.1-23)

Each of the terms in equation 7.1-23 can be interpreted on physical grounds.
The detector transfer function (DTF) is the modulation transfer function (MTF)
multiplied by three factors. Each of the three factors degrade the amplitude and/or the
bandwidth of the MTF. First, the finite pixel size is accounted for by the \text{sinc}(\_)
function. Smaller pixels or equivalently smaller \( L_x \) and \( L_y \) values, while keeping the
pixel spacing constant, extend the \text{sinc}(\_)
function in the frequency domain. The
extended \text{sinc}(\_)
function broadens the system frequency response. Second, the term
in large braces accounts for the conceivable multiple passes of photogenerated charges
through the detector. This term increases the amplitude response but decreases the
bandwidth. This is expected, since multiple passes of photogenerated charges increase
the photoconductive gain but limit the bandwidth through diffusion. Finally, the
integral term represents the optical absorption and subsequent first pass of the
photogenerated holes through the detector. This term decreases both the signal amplitude and the bandwidth. The signal amplitude is attenuated by hole recombination discussed in chapter 5 and by the less than unity absorptive quantum efficiency discussed in chapter 6. The exponential term in the integral degrades the bandwidth as discussed previously.

The detector transfer function is simply a degraded version of the modulation transfer function. The degradation is due to three effects: the finite pixel size and both recombination and diffusion of photogenerated charges. Each of the effects is represented in equation 7.1-23.

7.2 Alternate Measures of the Detector Response

The detector transfer function is a useful measure of the detector array's imaging response. Alternate measures, however, exist that give additional insight into the response of the detector array. The detector point spread function, the diffusive cross talk between adjacent pixels, and the responsivity all provide quantitative descriptions of the detector response. Each measure, the detector point spread function, the diffusive cross talk, and the responsivity will be discussed below.

The detector point spread function, \( i_p(x,y) \), can be defined directly from the DTF. Since the DTF is the Fourier transform of the detector point spread function, the
inverse relation defines the detector point spread function in terms of the DTF. Hence
the detector point spread function, equation 7.2-1, may be deduced by computing the
inverse Fourier transform of the DTF.

\[ j_y(x,y) = (F^{-1}DTF)(x,y) \]  
(7.2-1)

Here \( F^{-1} \) is the inverse Fourier transform and all other quantities are as before.

The cross talk is a measure of the signal that spills over into an adjoining pixel
from a given source pixel. Cross talk can arise from a number of sources. Three
familiar sources are optical, diffusive and electrical cross talk. Electrical cross talk is
due to the capacitive coupling between the collecting ohmic contacts. Of the three
types of cross talk, only optical and diffusive are considered here. Optical cross talk
arises from the transverse extent of the point spread function; its amplitude and
extent follow from the assumption made in equation 7.1-16. Diffusive cross talk arises
from the transverse spreading of the photogenerated charges.

The diffusive cross talk can be defined at a given point on the detector surface.
It is assumed that the optical system is imaging a point source. At position \((x,y)\) the
diffusive cross talk, \( \eta_d(x,y) \), is defined in equation 7.2-2.

\[ \eta_d(x,y) = \frac{|J_{p,net}(x,y) - J_{p,d}(x,y)|}{J_{p,d}(0,0)} \]  
(7.2-2)

In equation 7.2-2, \( J_{p,net}(x,y) \) is the current density when imaging a point source.
\( J_{p,d}(x,y) \) is an idealized current density which would exist if diffusion did not occur.
The responsivity is another important measure of the detector response. An expression for the responsivity will be derived. The responsivity, denoted by \( R \), is defined as the number of holes detected per photon incident on the detector. This definition differs from the conventional definition of responsivity. The conventional definition is measured in amperes of photocurrent per watt of incident optical power. The conventional responsivity, \( R_e \), is related to \( R \) in equation 7.2-3. Here, \( \lambda \) is the wavelength of the incident light; \( e \) is the electronic charge; \( \hbar \) is Planck's constant divided by \( 2\pi \); and \( c \) is the speed of light.

\[
R_e = \frac{\lambda e}{2\pi \hbar c} \quad (7.2-3)
\]

The responsivity is determined by three effects: first, the recombination of the photogenerated holes degrades the responsivity; second, the less than unity absorptive quantum efficiency degrades the responsivity; third, the possible multiple passes of photogenerated charges increases the responsivity.

Beginning with equations 5.1-16 17, and 18 an expression for the responsivity can be derived. The responsivity is defined in equation 7.2-4. Here \( i_{\text{net}} \) is the net current collected over the entire detector array; it is measured in number of charges collected per unit time.

\[
R = \frac{i_{\text{net}}}{s_{\text{net}}} \quad (7.2-4)
\]

\( s_{\text{net}} \) is the total number of photons incident on the detector array in a unit time. The
total number of collected charges in a single pass per unit time, denoted by \( i_1 \), is given in equation 7.2-5.

\[
i_1 = \int_x \int_y J_{1,p}(x,y) \, dx \, dy
\]  

(7.2-5)

\( J_{1,p}(x,y) \) is given by equation 5.1-16. Substitution of equation 5.1-16 into equation 7.2-5, and changing the order of the \((x,y)\) and \((x',y')\) integration results in a new expression for \( i_1 \) in equation 7.2-6. The function \( s_o(x,y) \) is the number of photons per unit area incident on the detector in a unit time at the position \( (x,y) \). The term \( s_{\text{tot}} \) in equation 7.2-4 is simply the total number of photons incident on the entire detector array and is given by equation 7.2-7.

\[
i_1 = -\int_o \mathcal{d}(x') e^{-\int_{x'}^\infty} \int_{y'}^\infty s_o(x',y') \, dx' \, dy'
\]  

(7.2-6)

\[
s_{\text{tot}} = \int_{x'} \int_{y'} s_o(x',y') \, dx' \, dy'
\]  

(7.2-7)

The currents due to successive passes, \( i_n, n \geq 2 \), can also be determined. Integration of both sides of equation 5.1-17 over the detector surface gives a recursion relation, equation 7.2-8, for the currents \( i_n, n \geq 2 \).

\[
i_n = e^{-\int_{\delta}^{\delta'} \mathcal{N}} i_{n-1}
\]  

(7.2-8)

Combination of equations 7.2-4, 5, 6, 7 and 8, gives an expression for the responsivity in equation 7.2-9.
Each of the three mechanisms mentioned above is reflected in equation 7.2-9. First, the integral contains the effects of the absorptive quantum efficiency in the \( df(z) \) term. Second, recombination is accounted for in the \( e^{-\mu_{dr}N_d} \) term. Third, the increase in the responsivity, due to the charge neutrality requirement, is contained in the denominator of equation 7.2-9.

It is useful to look at the responsivity as a function of the thickness of the infrared absorbing layer. The integral in the numerator of equation 7.2-9 can be rewritten, equation 7.2-10, utilizing the mean value theorem of integral calculus.

\[
R = \frac{-\int_0^d df(z) e^{-\mu_{dr}N_d}}{1 - e^{-\mu_{dr}N_d}}
\]

(7.2-9)

In equation 7.2-10 \( z_\epsilon \in (0,d) \) and its existence is guaranteed by the mean value theorem. The integral expression in the right hand side of equation 7.2-10 is the absorptive quantum efficiency of the detector array and is defined in equation 7.2-11. The absorptive quantum efficiency is plotted versus the wavelength of the incoming light and the thickness of the infrared absorbing layer in figures 6-6 and 6-7 respectively.

\[
\xi(d) = -\int_0^d df(z)
\]

(7.2-11)

Thus the responsivity can be written, equation 7.2-12, in terms of the absorptive quantum efficiency.
Two limiting cases of the responsivity will be investigated. First, as \( d \to \infty \) the drift time \( t_d(d) \to \infty \) and the denominator goes to unity. For the numerator, the absorptive quantum efficiency approaches a finite value \( \xi \). The point \( z_0 \) is unbounded, therefore, the exponential term, \( e^{-\frac{d}{d_0}N_d} \), goes to zero. Thus, when \( d \to \infty \), \( R \to 0 \).

Second, when \( d \to 0 \) \( z_0 \) also goes to zero. The exponential term in the numerator of equation 7.1-12 goes to unity. For the absorptive quantum efficiency \( \xi \), the plot in figure 6-7 indicates that as \( d \to 0 \), \( \xi \to \xi d/d_0 \), to first order in \( d \). The denominator of equation 7.1-12 will go as \( t_d(d)r^{-N_d} - (t_d(d)r^{-N_d})^2 + \ldots \). If the electric field is held constant as \( d \to 0 \), the responsivity goes according to equation 7.2-13.

\[
R(d=0) = \frac{\nu_d e_N}{d_0}
\]  

(7.2-13)

The responsivity has an interesting asymptotic behavior. In the limit where most of the incident photons are absorbed (i.e. \( d \to \infty \)) the responsivity goes to zero. Although most of the photons are absorbed, at large \( d \), they can not migrate to the collecting contacts before recombining. The result is a vanishing small responsivity. In the opposite extreme (i.e. \( d \to 0 \)), negligibly few photons are absorbed. In this case, the photoconductive gain increases as \( d/d_0 \) and the fractional absorption goes as \( d/d_0 \). The result is a nonzero finite responsivity given in equation 7.2-13.
The nonvanishing responsivity predicted in equation 7.2-13 will break down as the detector is thinned indefinitely. The photogenerated charges have a finite relaxation time to the electric field. The relaxation time, t_r, can be due to various mechanisms; dielectric relaxation is a possible mechanism. The result in equation 7.2-13 requires the photogenerated charges to respond much faster than d/v_d. The relaxation time must be much smaller than the time to drift across the detector; that is equation 7.2-13 is valid when t_r < d/v_d.

The detector point spread function, the diffusive cross talk, and the responsivity provide useful measures of the detector's imaging response. Algebraic expressions for each have been derived. With appropriate expressions for the detector response formulated, quantitative performance measures can be ascertained.

7.3 Detector Array Performance Results

In this section the detector response measures derived in the last section are evaluated and interpreted. This section contains the fruit of the study from which design and testing decisions can be made. Each of the performance measures is examined in this section. Each measure discussed in sections 7.1 and 7.2 will be evaluated as a function of detector array operating and/or design parameters. The presentation parallels that of sections 7.1 and 7.2; quantitative results for the detector transfer function, detector point spread function, diffusive cross talk between
pixels, and the responsivity are presented.

A number of definitions and assumptions are made in evaluating the various measures of the detector response. The diffusion length used in the following results is defined in equation 7.3-1. The terms are defined: \( k_b \) is the Boltzmann constant; \( T \) is the lattice temperature; \( e \) is the electronic charge; \( V_b \) is the applied electrical bias; \( d \) is the thickness of the infrared layer.

\[
L_D = \sqrt{\frac{k_b T}{e V_b d}}
\]  

(7.3-1)

The diameter of the optical system point spread function is defined as its full width half max (FWHM) as \( D_{psf} = 1.02 \lambda_f \). The fractional size of the obscuration in the pupil of the optical system is approximately \( f = 1/3 \). In all the results the pixel spacing is the Nyquist sampling interval of the optical system. As in chapters 5 and 6, the detector material is GeBe; results for related materials can be similarly attained. Finally, the detector is biased so the drift velocity is approximately one-twentieth the longitudinal optical phonon velocity, \( v_d = 0.05v_{LO} \). The bias assumption places the operation of the detector in the drift-diffusion regime discussed in chapter 5.

### 7.3.1 Results

The detector transfer function is the central result for use in image simulation and processing algorithms. As derived in section 7.1, equation 7.1-23, the detector
transfer function is the product of four terms. Four physical effects degrade the
detector transfer function from the optical transfer function. First, the finite pixel size,
represented in the sinc(·) function, limits the frequency response of the transfer
function. Second, the requirement of charge conservation increases the amplitude and
narrows the frequency response of the transfer function. Third, recombination of
photogenerated charges limits the amplitude of the transfer function. Fourth, the
transverse diffusion of the photogenerated charges limits the frequency response of
the transfer function.

Each of the computed detector transfer functions is normalized at k=0 to unity.
The normalization conceals the potentially significant drop due to less than unity
responsivity; however, the normalization does highlight the effects of diffusion.

The detector transfer function for various L_o/D_{PSF} ratios is plotted in figure 7-2.
The optical system was held fixed with a f#=30 and the diffusion length was varied.
The transfer function is plotted versus the normalized spatial frequency of the optical
system. The plot portrays the optical system MTF, the degradation due to Nyquist
sampling, and the combined effects for three L_o/D_{PSF} ratios. The plot illustrates the
significant degradation that can occur when the ratio L_o/D_{PSF} exceeds 4×10^2.

The effect of temperature on the detector transfer function is shown in figure
7-3. As in the previous plot, the optical system was held fixed and only the
temperature was varied. The optical MTF and its degradation due to Nyquist
Figure 7-2: The detector transfer function for various $L_D/D_{PSF}$ ratios. sampling are shown for reference. As the plot indicates, the operating temperature of the detector array can degrade the imaging performance of the detector array. For temperatures near 1.5(K) the degradation due to diffusion is small. For larger temperatures, $T=2.5(K)$, the degradation begins to become significant. At still larger temperatures there is significant degradation to the transfer function.

A second case of interest is obtained by varying the f# of the optical system while holding the diffusion length constant. Figure 7-4 illustrates the detector transfer function for various $D_{PSF}/L_D$ ratios, plotted versus the normalized spatial frequency. The spatial frequency is normalized by the diffusive spatial frequency defined in equation 7.1-19. The diffusion length was held fixed at $L_D=.66\lambda$, where $\lambda$ is the free space wavelength of the incident light.
Figure 7.3: The detector transfer function for three temperatures.

As the f# of the optical system decreases the cutoff frequency of the optical system increases and the spot size on the detector array decreases. One might intuitively expect that the image degradation due to diffusion will become worse as the f# and the spot size are decreased; figure 7-5 verifies this. As the f# decreases, the effects due to diffusion become more severe. At smaller $D_{psf}/L_D$ ratios a larger proportion of the transfer function is degraded. At ratios less than 25 a significant portion of the transfer function is degraded.

The effect of a variable optical cutoff frequency is normalized out in figure 7-5. The plot illustrated in figure 7-5 is that of 7-4 with each DTF curve scaled to the cutoff frequency of the optical system assumed to generate the curve. The optical and Nyquist sampled transfer functions are indicated for reference. The effects of diffusion
are clearly evident. As the f# decreases the transfer function becomes significantly degraded. For ratios less than 50, the optical-detector system loses much of its ability to image spatial frequencies above half the cutoff frequency.

The effect of the infrared absorbing layer thickness on the detector transfer function is illustrated in figure 7-6. Each of the transfer functions has been normalized at k=0 to unity. Thus, the plots do not evidence an amplitude degradation due to responsivity. The plot displayed in figure 7-6 is not surprising when the dynamic operation of the detector is considered. As discussed in section 5.1, the photogenerated charges diffuse while drifting toward the collecting contacts. The distance the charges diffuse is determined by the recombination time and not the thickness of the infrared absorbing layer. When the detector becomes very thin, the
collected charges continue to diffuse after they are re-injected by the positively biased transparent contact. Hence, the thickness of the infrared absorbing layer has no effect on the degradation of the normalized transfer function. The amplitude of the un-

**Figure 7-5**: Transfer functions with normalized spatial frequencies.

**Figure 7-6**: The detector transfer function for three detector thicknesses.
normalized transfer function is affected by the infrared absorbing layer thickness. The amplitude effect has been normalized out of the transfer function and will be discussed later with the responsivity results.

Figure 7-7: The point spread function vs $L_D/D_{PSF}$ and $x/D_{PSF}$.

The point spread function is an alternative measure of the imaging performance of the detector array. A sequence of detector array point spread functions is illustrated in figure 7-7. The optical system was held fixed and the diffusion length was varied. The axes shown are the $L_D/D_{PSF}$ ratio and the normalized position on the array. For small ratios, the effects of diffusion are small and the point spread function does not deviate significantly from the optical point spread function. As the ratio increases the effects of diffusion become evident—the point spread function amplitude decreases and the width increases characteristic of diffusive spreading. The first
minimum of the optical PSF is filled in at approximately $L_D/D_{PSF}=5.10^2$. For larger $L_D/D_{PSF}$ ratios the point spread function is seriously degraded.

![Detector Point Spread Function](image)

**Figure 7-8:** The detector point spread function for three temperatures.

The temperature of the detector array also affects its imaging properties. The detector PSF for three array temperatures is shown in a staggered fashion in figure 7-8. The optical system was held fixed and only the temperature was varied in generating each curve. Each function can be viewed as a slice of figure 7-7, corresponding to a specific pair of $T$ and $L_D/D_{PSF}$. At low temperatures, $T=1.5(K)$, the detector PSF is slightly degraded, with the first order diffraction ring still clearly visible. At $T=2.5(K)$, the first minimum is filled in, the amplitude is less and the width is broadened. At $T=4(K)$, the first order structure of the PSF has disappeared completely.
Cross talk between pixels also degrades the imaging performance of the detector array. Cross talk can originate from at least three sources: the optical system, diffusion in the detector, and capacitive coupling between the conductive ohmic contacts or electrical cross talk. Only optical and diffusive cross talk are considered here. To ascertain the effects of diffusion, the diffusive cross talk is defined, equation 7.2-2, as the absolute difference between the optical current density, that is the current density generated in the absence of diffusion, and the total current density including the effects of diffusion. Point source imaging is assumed in all the cross talk computations.

\[
\text{Figure 7-9: The diffusive cross talk vs } \frac{L_D}{D_{PSF}} \text{ and } \frac{x}{D_{PSF}}.
\]
The cross talk across a radial diameter of the field versus the $L_\theta/D_{\text{PSF}}$ ratio is shown in figure 7-9. As the $L_\theta/D_{\text{PSF}}$ ratio increases, the diffusive cross talk becomes greater, as one might expect. The diffusive cross talk is concentrated in the center area of the detector array. The spatial concentration follows from the central concentration of photogenerated holes when observing a centered point source. The structure of the optical PSF is clearly visible in figure 7-9. The center lobe in figure 7-9 corresponds to the migration of photogenerated holes out of the central maximum of the optical PSF. The first order subsidiary lobes correspond to the in-filling of the first order minimum in the optical PSF. At large $L_\theta/D_{\text{PSF}}$ ratios, second order lobes begin to form. The second order lobes occur when the photogenerated holes have diffused past the first order maximum of the optical PSF.

![Cross Talk vs Temperature](image)

**Figure 7-10:** Diffusive cross talk vs detector array temperature.
As the temperature of the array increases, the diffusive cross talk increases. The plot in figure 7-10 shows the diffusive cross talk, in either of the two pixels adjacent to the central two pixels. As before, centered point source imaging with Nyquist sampling is assumed. The diffusive cross talk increases from 2% to just over 8% as the temperature increases from 1.5(K) to 4.(K). The results in figure 7-10 do not reflect the severity of the imaging degradation at higher temperatures. The transfer function plotted in figure 7-3 indicates an accelerated thermal degradation which is not evident in figure 7-10.

![Figure 7-11: The responsivity vs wavelength.](image)

The responsivity is a measure of a detector's ability to convert photons to charges. The responsivity was discussed in section 7.2 and expressed in equation 7.2-9. The responsivity is plotted versus the normalized wavelength of the incoming light.
for three thicknesses of the infrared absorbing layer in figure 7-11. The Fabry-Perot oscillations that appeared in the absorptive quantum efficiency of the infrared absorbing layer in figure 6-6 are clearly evident in each of the plots. The oscillations in the thinner layers are of a relatively larger amplitude than for the thicker layers. Also evident in the plots is the spectral absorption of the primary dopant illustrated in figure 6-2. Each responsivity curve is qualitatively similar in shape to the primary dopant absorption curve with Fabry-Perot oscillations superposed.

The responsivity is dependent on the infrared absorbing layer thickness. The responsivity begins to saturate for thicker layers. With a thicker absorbing layer the relative responsivity at the shorter wavelengths is increased compared to thinner layers as one might expect. Also, the responsivity for thinner layers is, on the average, greater than that for thicker layers.

A plot of the responsivity versus the infrared absorbing layer thickness is shown in figure 7-12. The responsivity was spectrally averaged over the \([.55\lambda_c, \lambda_c]\) interval, where \(\lambda_c\) is the cutoff wavelength of the absorption. The plot in figure 7-12 illustrates that increasing the thickness of the infrared absorbing layer actually decreases the responsivity. The effect is in contrast to the dependence of the absorptive efficiency of the absorbing layer on \(\alpha d\). As the absorbing layer becomes thicker fewer photogenerated holes arrive at the collecting contact due to recombination. The qualitative shape of the curve in figure 7-12 is a simple
Figure 7.12: The responsivity vs the absorbing layer thickness.

Exponential, $R = R_e e^{-d/\alpha}$, with $R_e = .33$ and $\alpha = .8$. The small kinks in the curve are likely due to the discrete spectral averaging process. The asymptotic behavior of the responsivity is as predicted in section 7.2. For large values of $d$, the responsivity goes to zero. As $d \to 0$, the responsivity attains a predicted finite value given by equation 7.2-13. In practice, the responsivity will go to zero for thin layers. As discussed in section 7.2, the photogenerated charges have a finite response time. As the detector is thinned, the charges can not respond fast enough, and the responsivity will drop for thin layers.
7.3.2 Summary of Results

The detector transfer and point spread function results give useful information for the design and operating parameters of extrinsic photoconductive detector arrays. The transfer and point spread functions are direct measures of the imaging performance of the detector array. With a fixed optical system, higher temperatures and longer diffusion lengths degrade the imaging performance of the detector array. At T=1.5(K), the degradation in the amplitude of the normalized transfer function is small. At T=2.5(K), the degradation due to diffusion can be as much as ~25%, at selected spatial frequencies. Thus, to retain a satisfactory level imaging performance across the optical bandwidth the temperature of the detector array needs to be kept at approximately 2(K) or less.

Other important design parameters for the detector array are the pixel size and the thickness of the infrared absorbing layer. The pixel size is determined by the Nyquist sampling interval. As the f# decreases, the Nyquist sampling interval also decreases. With a smaller sampling interval the effects of diffusion increasingly degrade the imaging performance. The degradation in the point spread function for a diffusion length $L_0=0.66\lambda$, where $\lambda$ is the free-space wavelength, was investigated by varying the $D_{ps}/L_0$ ratio. For ratio's as large as 100 the degradation in the transfer function was small. For a ratio of 50 the transfer function degradation was as large as ~35%, at selected spatial frequencies. For smaller ratios the degradation was
significantly larger.

The point spread function is degraded at large $L_p/D_{psf}$ values and/or high temperatures. At temperatures below 2(K) the PSF retains its first order maximum. At 2.5(K), the effects of diffusion fill in the first order minimum, obscuring the first order maximum. At 4(K), the first order lobes disappear and the point spread function is degraded in both amplitude and width.

The diffusive cross talk becomes worse for larger $L_p/D_{psf}$ ratios. With Nyquist sampling and an f# = 30, the cross talk in the next to center pixel varied from 2% to 8% as the temperature increased from 1.5(K) to 4(K). The diffusive cross talk, however, does not fully indicate the severity of the image degradation at higher temperatures.

The responsivity was evaluated as a function of the optical wavelength of the incoming light and the infrared absorbing layer thickness. The responsivity exhibits Fabry-Perot oscillations that were observed in the absorptive efficiency plot, figure 5-6 in chapter 5. The shape of the responsivity versus wavelength curves is similar to the spectral absorption curve of the primary dopant.

The responsivity was found to decrease as the thickness of the infrared absorbing layer increased. The shape of the curve is qualitatively similar to a simple exponential, $R=R_e^{-\alpha d}$. For a typical detector array, $R_e = 3.3$ and $\alpha = 0.8$. From the
responsivity versus d curve, one might naively expect the performance of the detector can be optimized by making the absorbing layer as thin as possible. As the absorbing layer is made thinner, however, the photogenerated charges can not respond fast enough to the applied electric field. In addition, charge fluctuations become greater and noise effects increasingly degrade the performance of the detector array (Sclar, 1984). Thus completion of the detector array evaluation will require a study of the various noise sources present in the operation of the detector array.
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