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Reading mathematics: Mathematics teachers’ beliefs and practices

Lehmann, Jane Nedine, Ph.D.
The University of Arizona, 1993
READING MATHEMATICS: MATHEMATICS TEACHERS' 
BELIEFS AND PRACTICES

by

Jane Nedine Lehmann

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A Dissertation Submitted to the Faculty of the 
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In the Graduate College
THE UNIVERSITY OF ARIZONA

1993
As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Jane Nedine Lehmann entitled Reading Mathematics: Mathematics Teachers' Beliefs and Practices and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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Date: April 9, 1993
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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copy of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

Dissertation Director
Dr. Patricia L. Anders

Date: April 9, 1993
STATEMENT BY AUTHOR

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DEDICATION

For

Ira and Helen Lehmann
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1 A grid for framing the problem of "reading mathematics"

2 A grid for framing the problem of "reading mathematics":
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This study explores the relationship between university mathematics teachers' beliefs about the nature of reading mathematics and their practices regarding reading mathematics. It is a response to the calls for reform in mathematics education, particularly to the assertion made by the National Council of Teachers of Mathematics in 1989 that not all students can read mathematical exposition effectively and that all students need instruction in how to read mathematics textbooks. It presupposes a collaboration between reading and mathematics teachers to help students learn to read mathematics.

The objectives were 1) to examine mathematics teachers' beliefs and practices regarding reading, mathematics, and thereby, reading mathematics; 2) to determine whether the theoretical perspectives implicit in those beliefs and practices could be characterized vis-a-vis the theoretical orientations that inform Siegel, Borasi, and Smith's (1989) synthesis of mathematics and reading; and 3) to determine the relationship, if any, that exists between mathematics teachers' beliefs about reading mathematics and their practices regarding reading mathematics. The synthesis presents dichotomous views of both mathematics and reading: Mathematics is characterized as either a body of facts and techniques or
a way of knowing; reading, as either a set of skills for extracting information from text, or a mode of learning. The latter view, in each case, can be characterized as constructivist.

The researcher was a participant observer in a university summer program. The primary participants were fourteen mathematics instructors. Interviews were conducted using a heuristic elicitation technique (Black & Metzger, 1969). Field notes were taken during observations of classroom activities and other non-academic summer program activities. The data were coded using a constant comparative method (Glaser & Strauss, 1967) comparative method.

Twelve instructors held conceptions of reading that were consistent with their conceptions of mathematics. Of those twelve, two held conceptions that could be characterized as constructivist; ten held conceptions that were not constructivist. Two instructors held conceptions of reading that were not consistent with their conceptions of mathematics. Of those two, one held a constructivist conception of reading but not of mathematics; one held a constructivist conception of mathematics but not of reading. Teachers' practices reflected their theoretical orientations.

The study has implications for teacher education: If teachers' beliefs are related to their practices, then teacher education programs should 1) acknowledge
the teachers' existing beliefs and 2) address the theoretical orientations implicit in various aspects of pedagogy.
CHAPTER 1

INTRODUCTION: "I DON'T UNDERSTAND THE BOOK"

Jack, a student in a university mathematics class, told me one day, I don't understand the book. The examples don't go with what they say. Sometimes it kinda does, but then they skip steps and then I get lost. It would be better if I read it, but it makes no sense because they skip steps in the examples, so when I get to class I still can't do it. . . . See, the reason he gives us that [reading assignment] is so we will be prepared the next day when he goes over it. . . .

With some people it helps, you know, but me, I can't understand the book. I mean, I can and I can't. (In. 136-182)

Jack's instructors, too, realized that reading mathematics could be difficult. It was sometimes difficult for them. One mathematics instructor, Deborah, said, regarding reading mathematics,

I don't know what makes it difficult, but I, myself, as a student have read sentences over and over again and thought to myself, "I know what every single one of these words means, but I don't know what the sentence is saying." I guess you just have to absorb it. (In. 762-769)

1Citations refer to lines from the transcript of a recorded interview or an observation.
Gene, another mathematics instructor, concurred:

I know that sometimes when I read a math book, I have no clue
what they're talking about. And sometimes it's just real easy. I
always thought it was the author's fault and not mine. I don't know
if that's true or not. (In. 429-435)

Jack and Deborah and Gene seem to agree that it can be difficult to read
mathematics even though they may not agree on what it is that makes it difficult.
Is it the author's fault? Can the difficulty be minimized if authors would only put
in all the steps? Is it that readers need to be able to "absorb" information?
Whatever it is, Jack needed help.

This study was undertaken in an effort to help Jack and other students like
him. If reading mathematics is difficult, perhaps reading teachers could
 collaborate with mathematics teachers to find ways to help students read
mathematics more effectively. The first step in that process was seen to be an
investigation of mathematics teachers' beliefs about reading mathematics.

Following a statement of the purpose of the present study is a brief
discussion of the problem and the rationale for conducting this research. The
third section presents the conceptual framework and the fourth gives an overview
of the methods used.
Purpose

The purpose of this study is threefold:

1) To examine mathematics teachers' beliefs and practices regarding reading, mathematics, and thereby, reading mathematics;
2) To determine whether the theoretical perspectives implicit in those beliefs and practices can be characterized vis-a-vis the theoretical orientations that inform the Siegel, Borasi, and Smith (1989) synthesis of mathematics and reading;
3) To determine the relationship, if any, that exists between mathematics teachers' beliefs about the nature of mathematics, about the nature of reading, and thereby, about reading mathematics and the teachers' practices regarding reading mathematics.

Statement of the Problem

In 1983, the National Commission on Excellence in Education characterized the nation as being at risk because of the failures of the public schools. One of those failures occurred in mathematics education. During that decade, criticisms of mathematics education and subsequent calls for reform were also heard from The National Council of Teachers of Mathematics (NCTM) (1980), the National Science Board Commission on Pre-College Education in
Mathematics, Science, and Technology (1983), and the National Research Council (1989). The calls for reform were not limited to the elementary and secondary schools; colleges and universities were also called to make sweeping changes in mathematics education to parallel those changes called for in the schools (National Research Council, 1991).

In 1991, the National Assessment Governing Board reported that, regarding mathematics education, little progress had been made. Jones (1988-89), summarizing findings from reports focusing on mathematics achievement maintained that "over the most recent 20 years in the United States, there is no credible evidence for large systematic change in average mathematics achievement at any level of school through high school" (p. 314).

While it may seem that there is a prima facie argument for attempting to solve this problem, the NCTM (1989) makes the argument explicit: The society needs mathematically literate workers and workers whose mathematical education will enable them to respond to the changes in the demands of the work-place that will surely occur throughout the workers' lives; women and minorities are "seriously underrepresented in careers using science and technology" (p. 4) and mathematics education must be such that it affords equal opportunity; furthermore, the complexity of the issues facing the citizens in a democratic society requires that citizens be able to think through highly complex
technological issues, and mathematical literacy is a requisite for such an endeavor (p. 5).

It is clear that we need to solve the problem of mathematics education. It is equally clear to the NCTM (1989) that helping students learn to read mathematics exposition will contribute to the solution. They tell us that as students progress through the grades they "become increasingly self-directed and dependent on textual materials" (p. 142). Unfortunately, they also assert that "it cannot be assumed that even students who are skilled readers can read mathematical exposition effectively. All students will need specific instructions on how to read mathematical textbooks with understanding and how to use textbooks as valuable resources" (p. 142).

Reading educators had apparently come to a similar conclusion much earlier and, through research, were attempting to address the problem. Nolan (1984), in his review of the literature from 1957-1981 on the topic of reading in the content area of mathematics, reports that "there is a significantly large body of research and literature which is available in the area of reading math, most notably in the areas of general problems, vocabulary or concept development, and problem solving of word problems" (p. 36). Earp (1970), after reviewing the literature on the relationship between arithmetic and vocabulary knowledge and word problem solving, concluded that "reading comprehension and arithmetic
achievement tend to be positively related" (p. 531) and called for reading educators to give more attention to "teaching skills pertinent to areas such as mathematics . . . in courses in the teaching of reading" (p. 531).

Conceptual Framework

To say that we should help students to learn mathematics by helping them learn to read mathematics, however, raises some pertinent questions: What is the nature of mathematics? What, if any, implications for instruction follow from a particular conception of mathematics? What is the nature of reading? What, if any, implications for instruction follow from a particular conception of reading? What does it mean to read mathematics? It would seem to follow that particular answers to those questions are presupposed in the research we design or the instructional strategies we suggest for teachers and learners.

The Nature of Mathematics

The NCTM (1989), in its response to the call for reform in mathematics education, addresses the first two questions. It begins by recognizing that mathematics is not a timeless certainty, but rather a discipline "more than half of . . . [which has] been invented since World War II" (p. 8-9). And following Dewey's (1916, cited in NCTM, 1989) distinction between "knowledge" and the "record of knowledge" (1989, p. 7) they declare that "knowing mathematics is 'doing'
mathematics" (p. 7). Being able to simply identify the basic concepts and procedures is no longer adequate:

A person gathers, discovers, or creates knowledge in the course of some activity having a purpose. This active process is different from mastering concepts and procedures. We do not assert that informational knowledge has no value, only that its value lies in the extent to which it is useful in the course of some purposeful activity. (p. 7)

To ensure more purposeful activities in mathematics classes, NCTM (1989) created new curriculum standards for mathematics and outlined five goals for students:

(1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically. (p. 5)

The new curriculum standards, according to the NCTM (1989), require that students "study much of the same mathematics that is currently taught, but with quite a different emphasis" (p. 5).

The new emphasis is one that recognizes, for example, that skill in computation is likely to develop as students create and solve genuine problems, so
drill in computation need not, necessarily, always precede word problems. Furthermore, learning is not conceived of as a process of passively absorbing, storing, and later retrieving information, but rather as a process in which learners bring their own experiences and knowledge to a learning task, "assimilate new information and construct their own meanings" (p. 10). What it means to learn mathematics, then, has undergone a shift in the thinking of mathematics professionals from a process focused on learning about concepts and procedures and acquiring the facts and techniques necessary for computations to one in which learners actively construct their own meanings.

The Nature of Reading

Siegel et al. (1989) report that a similar shift of focus has occurred among reading educators. They claim that reading educators, when thinking about the nature of reading, are less likely today than in the past to think of reading from the perspective of a skills-based model (Gough, 1972; LaBerge & Samuels, 1974). Such a model presents a notion of reading in which meaning is found primarily--almost exclusively--in the text and a set of skills is applied in order to transfer meaning from the text to the reader (Harste, 1985). Today, reading educators, according to Siegel et al., are more likely to use the perspective of a comprehension-based model (Goodman, 1967, 1972, in press; Goodman & Goodman, 1978) on which to base their research and their pedagogy. This model
views reading as a transaction that occurs between a reader and a text in which both contribute to the construction of meaning (Goodman, 1984; Rosenblatt, 1978). Here the learner is an active participant in the process of constructing meaning and brings a full range of experiences, both linguistic and non-linguistic, including knowledge of the reading context, to bear on the reading situation (Carey & Harste, 1987; Goodman, 1984; Goodman & Goodman, 1990; Rosenblatt, 1978). Reading, like mathematics, is no longer seen as simply applying a set of skills to a text, but rather as a way of knowing, a way of constructing one’s own meaning.

**The Nature of Reading Mathematics**

Siegel et al. (1989) suggest that four alternative approaches to reading mathematics result from the juxtaposition of these dichotomous views of both the nature of reading and the nature of mathematics. They represent these alternative views of both reading and mathematics, and thereby, of reading mathematics, with a grid as a way of "framing the problem of 'reading mathematics'" (p. 272). This grid is reproduced as Figure 1.
Reading mathematics, as characterized by "Box I" (p. 272), for example, follows from a conception of both reading and mathematics that is skills-based. According to this view, reading skills can not only be enumerated, they can be arranged hierarchically. Mastering the individual skills, for example, knowing the alphabet, being able to produce the sounds associated with individual letters and groups of letters in isolation from whole words, being able to identify and say words in isolation from a meaningful context, is a condition that must be met before a student progresses to reading meaningful, connected discourse. Likewise, the view of mathematics is one that requires students to have mastered certain computations and to have learned certain rules and algorithms before attempting to deal significantly with mathematical understanding, problem solving, or reasoning. To develop proficiency at reading mathematics, from a Box I
perspective, would be to develop the "skills" of reading mathematics, for example, learning the meanings of the non-alphabetic symbols used in mathematics, learning the specialized vocabulary of mathematics, and eventually applying those skills toward reading exercises created by the text authors.

Readers operating from a "Box II" (p. 273) perspective, on the other hand, while maintaining a skills and computational orientation toward learning mathematics, would, nevertheless, view reading as a "mode of learning" (p. 272). In this conception of reading, meaning is not extracted from the text by applying skills; rather, meaning is constructed by the reader using the text, the reader's knowledge, and the pragmatics of the reading situation. Reading is not seen as a process whereby one applies a set of skills to a text, much as one applies the skills of riding a bicycle to any bicycle be it a Schwinn or a Raleigh; rather, reading is seen as a way of knowing, a way of constructing one's own meaning through a transaction with a text. Becoming more proficient at reading mathematics from a Box II perspective might still focus on exercises in which students could practice and test their mathematical computational ability or their ability to apply algorithms to representative problems, but, according to Siegel et al. (1989), students might be reading nontraditional mathematics textbooks or finding examples of mathematics problems in newspaper stories.
Readers operating from a "Box III" (p. 274) perspective might be engaged in reading mathematics that would take them beyond computations. Here, Siegel et al. (1989) suggest students may be reading essays, stories, or other nontraditional mathematical texts as they begin to move beyond the computational aspects of mathematics and away from the notion that mathematics problems exist only in mathematics textbooks. But if mathematics educators have a view of reading that is skills-driven, Siegel et al. fear that such teachers may not exploit fully the potential of such texts.

To read mathematics from a "Box IV" (p. 275) perspective is to view both reading and mathematics as a way to construct meaning rather than extract meaning from text. The mathematics that would come from such a view would focus on problem solving and mathematical understanding. Students could set up and solve problems that had real significance for them rather than focusing solely on textbook exercises. The reading done to facilitate such activities could come from nontraditional, "rich" (p. 274) mathematical texts, or even texts written by the readers themselves.

The NCTM (1989), in its new curriculum standards, espouses a view of mathematics as a way of knowing and no longer as simply a body of facts and techniques to be mastered. If one were to operate within the guidelines of the NCTM curriculum standards, one would hope to find research and classroom
instruction regarding reading mathematics informed by a view of mathematics commensurate with either Box III or Box IV rather than by Box I or Box II.

There is no document set forth by reading educators that is comparable to the NCTM (1989) curriculum standards. There clearly are dichotomous views within the profession regarding the reading process and, therefore, what constitutes responsible reading instruction. (See, for example, Adams, 1990; Chall, 1967; Harste, 1985; Smith, 1992, 1993.) Nevertheless, Siegel et al. (1989) contend that a shift has occurred among reading educators that parallels the shift that has occurred among mathematics educators. Spivey's (1987) contention that "constructivist findings permeate" (p. 182) the report from the Commission on Reading (Anderson, Hiebert, Scott, & Wilkinson, 1985) and that the report "clearly presents reading from a constructivist perspective" (Spivey, p. 182) would seem to lend support to the notion that such a shift has occurred.

Despite the fact that the authors of that report continue to support traditional, that is, less constructivist, instructional approaches, they nevertheless make a clear statement that reading is now seen as a "process of constructing meaning from written texts" (Anderson et al., p. 6), that "performing the subskills [of reading] one at a time does not constitute reading" (p. 7), and that the common view of reading as a "process in which the pronunciation of words gives access to their meanings; the meanings of the words add together to form the
meaning of clauses and sentences; and the meanings of sentences combine to produce the meanings of paragraphs. . . . is only partly correct" (p. 8). This "only partly correct" view of reading is characterized by Box I and Box III; therefore, Box IV offers the best opportunity for reading mathematics so as to meet the goals set forth by the NCTM (1989).

Helping students read to learn mathematics, since it entails a view of reading that may be unfamiliar to mathematics teachers and a view of learning mathematics that may be unfamiliar to reading teachers, is best approached as a joint venture between reading and mathematics teachers who can both operate from a Box IV perspective. The reading teacher's role in such a venture would be to work with mathematics teachers to create reading strategies and choose appropriate texts commensurate with a Box IV view of reading mathematics. Together with mathematics teachers who view mathematics as a way of knowing, reading teachers could provide assistance for mathematics teachers ready to change their views of reading from a set of skills to a mode of learning and their views of mathematics from a body of facts and techniques to a way of knowing, that is, make the shift to a Box IV perspective.

It becomes important, then, to make explicit the implicit theoretical frameworks from which teachers might be operating for at least two reasons: Teachers operate from a theoretical framework even if they do not or cannot
articulate that framework easily (Harste & Burke, 1977; Lerman, 1983); teachers implement change in their classrooms from the perspective of their own theoretical frameworks, stated or not (Cooney, 1988; Richardson, 1990; Russell, 1980). The implication of the second point is that teachers might engage in activities that look like activities that would be commensurate with a constructivist conception of reading mathematics, but if their theoretical frameworks were something other than a constructivist framework, the activities would differ significantly from the same activities directed by teachers whose theoretical frameworks were more consistent with the activities.

Methodology

An ethnographic study was conducted. The researcher was a participant-observer (Spradley, 1980) for six weeks in the mathematics component of a summer program for minority students at a large research university in the Southwestern United States. The primary participants were the fourteen mathematics instructors who were teaching in the program. Information regarding mathematics teachers' beliefs and practices as they related to reading and mathematics was gathered via interviews, a survey, and classroom observations. Field notes were taken during classroom observations. Information regarding the context in which the instructors were operating was gathered through interviews with other summer program participants: students, tutors, administrators, and
staff members. In addition, the researcher was a participant-observer in both formal and informal faculty meetings, faculty in-service activities, tutor in-service activities, student orientation activities, tutoring sessions, and the mathematics teachers' volleyball team.

The data from all these sources were coded and analyzed. Instructors' statements about the nature of reading and about the nature of mathematics and about teaching and learning reading and mathematics were analyzed to determine their implicit theoretical orientations for both reading and mathematics. Those orientations were further analyzed in reference to the theoretical orientations that inform the Siegel et al. (1989) synthesis of mathematics and reading. Finally, instructors' practices regarding reading mathematics were noted, analyzed vis-a-vis these theoretical orientations, and compared to instructors' theoretical orientations regarding reading mathematics.

Organization of the Report

Chapter Two discusses teachers' beliefs, in general, and teachers' beliefs in relationship to their practice, in particular. Specifically, the relationship between teachers' practices and their beliefs about the nature of reading and between teachers' practices and their beliefs about the nature of mathematics is examined. The theoretical orientations for the conceptions of reading and mathematics that inform the Siegel et al. (1989) synthesis of mathematics and reading and the
instructional implications for those orientations are also addressed. Chapter Three discusses the research setting and methods. Chapter Four presents the findings and Chapter Five presents the conclusions and discusses some of the implications of the study.
CHAPTER 2

REVIEW OF LITERATURE: "FIXED PLANS OR MENTAL MAPS?"

This chapter reports first on the literature on teachers' beliefs, in general, then on the literature on teachers' beliefs in relationship to their practice. Specifically, the relationship between mathematics teachers' beliefs and their practices and the relationship between reading teachers' beliefs and their practices are examined. Next, two theoretical orientations of both mathematics and reading are discussed along with the instructional implications for each perspective.

Teachers' Beliefs

Since the 1970's, there has been a growing recognition by researchers in education of the importance of the relationship between teachers' beliefs and their practice (Clark & Peterson, 1986; Isenberg, 1990; Nespor, 1987; Shavelson, 1983; Shavelson & Stern, 1981). This recognition can be seen in the shift from an almost exclusive emphasis on process-product studies that examined teachers' behaviors and their effects on students' behavior and achievement to studies that focused more on the role played by teachers' thought processes during instruction, that is, planning, decision making, theories, and beliefs (Clark & Peterson, p. 257).

Despite the interest in teachers' beliefs, Thompson (1992) maintains that the discussion of teachers' beliefs suffers from the unwillingness or inability of
researchers to adequately provide a definition of belief, a complaint echoed by Eisenhart, Shrum, Harding, and Cuthbert (1988). Particularly, this is a problem, Thompson (1992) maintains, because belief and knowledge are often erroneously used as though they were synonymous. Thompson (1992) acknowledges the difficulty of making the distinction and concedes that the value of making such distinctions for the purposes of educational research may be debatable; nevertheless, she urges the reader to look to both philosophy and psychology for help in clarifying the nature, function, and structure of beliefs.

Knowledge has traditionally been defined by epistemologists as justified, true, belief (Lehrer, 1990). Knowledge claims must be believed, so belief is a necessary condition of knowledge; but it is not sufficient. In addition to being true, knowledge claims must also be justified or warranted, that is, they must meet accepted criteria for what constitute justification and truthfulness even though those criteria may change over time. Beliefs need meet no such criteria for warrantability (Green, 1971). Furthermore, beliefs, according to Green, "always occur in sets or groups. They take their place always in belief systems, never in isolation. . . . never in complete independence of one another. In general, whenever we can say, 'A believes that Q' we shall also be able to say, 'A believes that R' where R is a different belief from Q but related to Q in such a way that if A believes Q he must also believe R" (p. 41-42). It would seem to follow then,
that when we speak of teachers' beliefs, we are, in fact, speaking of their belief systems. This becomes important when we speak of acquiring or changing beliefs:

When beliefs are acquired, they will be acquired as parts of a belief system, and when they are modified, they will be modified as parts of a belief system. Therefore, . . . it may be . . . important in the philosophy of education to explore the nature of sets of beliefs or belief systems. (Green, p. 42)

This study uses the definition that a belief is a proposition that is affirmed; a proposition is defined as the meaning of a declarative sentence and need be neither true nor false nor have good evidence to support it.

The significance of teacher beliefs for this study is argued, following Munby (1984) and Richardson, Anders, Tidwell and Lloyd (1991), from the standpoint of instructional change. There are two related points: First, it is accepted that teachers' beliefs are "vital components of their practice" (Nespor, 1987, p. 317). Nespor suggests that beliefs, rather than research-based knowledge or academic theory, best fit the contexts, environments, and the "ill-defined and deeply entangled" (p. 324) problems teachers encounter and that may explain why beliefs play such an important role in teachers' practice. Clark and Peterson (1986) maintain that teachers' theories and beliefs represent "the rich store of knowledge that teachers have that affects their planning and their interactive thoughts and
decisions" (p. 258), and that "teachers' actions are in a large part caused by teachers' thought processes" (p. 258).

Second, in order to change present practice, regardless of whether the proposed innovations are initiated by teachers, themselves, or by others, for example, curriculum developers, researchers, or reformers, the proposals must be made in terms of the teachers' present beliefs. Alternatively, if the proposals are not made in terms of the teachers' present beliefs, the teachers must change their beliefs to accommodate the proposals (Cooney, 1988; Hersh, 1986; Richardson, 1990). Russell (1980), too, maintains that teachers' beliefs are important in regard to any changes they may make regarding their practices. He is critical of the "scientific paradigm" (p. 118) that informs much research in education. Such research, he claims, ignores the convictions and beliefs of teachers and expects that teachers will change their practices simply on the basis of the "logical force of research findings" (p. 118):

The theory of change embedded within the scientific paradigm is indicated by one of its most common research strategies, comparing two means to the same end to see which is the more effective. When statistically significant differences are obtained, the more effective means is regarded as the appropriate one for future work. Practitioners working toward the end studied are expected to bring
their practices into agreement with research findings, having recognized the logical force of these findings. (p. 118, italics in the original)

Teachers, according to Russell (1980), do not find such research persuasive, for it "can provide evidence for the existence of relationships but little insight into the nature of those relationships" (p. 118). Research that fails to provide the kind of evidence that teachers find meaningful and, therefore, persuasive, does not get implemented as the researchers had hoped or intended. Rather, teachers continue to make instructional decisions on the basis of evidence they do trust, namely, "personal convictions" (p. 118).

Russell (1980) suggests that alternative research paradigms, specifically anthropological or philosophical paradigms, offer more opportunities for teachers to reflect on their own theories and, therefore, are better suited for research that is intended to result in teacher change. He cites Elliott (1976) as support for his point:

The fundamental problem of curriculum reform lies in the clash between the theories of the reformers and those implicit, often unconsciously, in the practice of teachers. Reformers fail to realize that fundamental changes in classroom practice can be brought
about only if teachers become conscious of the latter theories and are able
to reflect critically about them. (p. 2)

Roberts (1980) also speaks to the point of the importance of teachers' beliefs regarding instructional change. He points out that curriculum often is not implemented as a developer, who is not part of the classroom activities, intended because teachers interpret the curriculum in terms of their own beliefs about what teaching is, how it should be conducted, the nature of knowledge, and so forth. Olson (1981) found that teachers had either "ignored or redefined" (p. 259) certain elements of a proposed curricular innovation when the teachers' roles, as envisioned by the curriculum developers, differed from the teachers' own beliefs about their roles. His findings suggest that teachers' beliefs are a factor in the implementation of curricular change and instructional reform and called into question the effectiveness of "centralized curriculum projects remote from the practical problems of schools" (p. 259).

While it may be generally accepted that teachers' beliefs are a factor in their practice (Nespor, 1987), one must be aware that there are any number of things about which teachers may have beliefs: beliefs about how students learn; beliefs about the nature of the discipline; beliefs about how to teach; and beliefs about environmental and cultural influences and constraints on their teaching. The present study investigated teachers' implicit theoretical orientations regarding
the nature of mathematics and reading and, thereby, the nature of reading mathematics and the relationship, if any, that such orientations had to the teachers' practices regarding reading mathematics.

Relationship Between Teachers' Beliefs About Mathematics and Their Practice

Dossey (1992) holds that "the understanding of different conceptions of mathematics is as important to the development and successful implementation of programs in school mathematics as it is to the conduct and interpretation of research studies" (p. 39). Brown, Cooney, and Jones (1990) seemingly agree, for they point out that different conceptions of mathematics "lead to quite different instructional responsibilities" (p. 647). Lerman (1983) suggests that "many controversies in mathematics teaching will only be resolved when we recognize the consequences of commitment to particular attitudes towards the nature of mathematical knowledge" (p. 60). Schoenfeld (1992) adds that "a teacher's sense of the mathematical enterprise determines the nature of the classroom environment that the teacher creates" (p. 359).

Skemp (1978) claims that when teachers have different conceptions of mathematics, there will be definite instructional implications. He sees profound differences depending on whether teachers think in terms of "relational
understanding" (p. 9), which he refers to as "knowing both what to do and why" (p. 9), and "instrumental understanding" (p. 9), that is, "rules without reasons" (p. 9):

For we are not talking about better and worse teaching of the same kind of mathematics. . . . I used to think that maths teachers were all teaching the same subject, some doing it better than others. I now believe that there are two effectively different subjects being taught under the same name, "mathematics." (p. 11, italics in the original)

But like the research in reading on the relationship of teachers' beliefs to their practices, the research in mathematics reveals, according to Thompson (1992), "varying degrees of consistency between teachers' professed beliefs about the nature of mathematics and the teachers' instructional practices" (p. 134).

Thompson (1984) found high degrees of consistency between teachers' beliefs and their practices. She reports that "teachers' beliefs, views, and preferences about mathematics and its teaching played a significant, albeit subtle, role in shaping their instructional behavior" (p. 105). McGalliard's (1983) observations of teachers revealed a high degree of consistency between the teachers' conceptions of mathematics and their practice. Wood, Cobb, and Yackel (1990) report a case study of a teacher whose beliefs and subsequent instruction changed as she implemented the instructional activities suggested by a
research project investigating children's learning of mathematics. Kesler (1985) also reports that teachers' conceptions of mathematics were related to their practice; however, he reported varying degrees of consistency regarding that relationship. The relationship was consistent for two of the teachers in his study and inconsistent for the other two. Underhill (1988), in his review of literature on mathematics teachers' beliefs, reports that Cooney (1983, 1985) found inconsistencies between teachers' beliefs about mathematics, teaching and learning, and classroom practice. Jones, Henderson, and Cooney (1986) reported similar findings.

Underhill (1988) offers some explanations for inconsistencies between beliefs and practice. He suggests that beliefs may represent ideals and that there is often a discrepancy between our ideals and our behavior: We may not have the knowledge or skills required to enact some of our beliefs; alternatively, beliefs may be held hierarchically and when there are conflicts between sets of beliefs we may make compromises between "importance in our personal beliefs hierarchies and ease of operationalizing" (p. 54). But, as we will see in the research in reading, results do not negate the notion that there is a relationship between beliefs and practice.
Relationship Between Teachers' Beliefs About Reading and Their Practice

The interest in the relationship between teachers' beliefs and their practice shown by the education research community, in general, is also apparent in research investigating reading. In the research in reading, as in the research in other disciplines, there is a wide spectrum of teacher beliefs to be considered: how pre-service teachers' beliefs about reading differ from in-service teachers' beliefs (Borko, 1982); how beliefs are formed and how they might be changed (Stansell, Moss, & Robeck, 1982); and what beliefs teachers hold concerning how students learn and how best to teach (Borko, Shavelson & Stern, 1981). There is also an interest in investigating the relationship between teachers' practices and their beliefs about the nature of reading (Gove, 1983; Stern & Shavelson, 1983).

Goodman and Watson (1977) argue that before teachers adopt a new reading program, they should be "able to articulate the program's theoretical base as well as to describe the activities found in it" (p. 868). Kamil and Pearson (1979) make a similar point when they assert that "every teacher operates with at least an implicit model of reading" (p. 10). They are less sanguine, however, regarding the relationship of those beliefs to practice: "[Those models] may or may not conform to [the teacher's] instructional goals" (p. 10). Kamil and Pearson concede that teachers may not be consciously aware of these models, but
suggest that since different models "dictate different (and sometimes opposing) instructional methods" (p. 10), teachers should be aware of their implicit models at least to the point where they can recognize where their goals and the means they use to achieve those goals are theoretically inconsistent.

Some researchers who have investigated the relationship between teachers' beliefs about reading--specifically their theoretical orientations--and their practice have concluded that there is a relationship between teachers' beliefs and their practice. Based on observations of teachers working with students in the classroom, Harste and Burke (1977) concluded that "despite atheoretical statements, teachers are theoretical in their instructional approach to reading" (p. 32). Mitchell (1980), in investigating the responses teachers made to students' oral reading errors, found differences in those responses that she relates to differences in theoretical orientations. Gove (1981) reports that teachers hold theories of reading that reflect theories found in the literature and that the teachers' behaviors often reflect those views. Borko, Shavelson, and Stern (1981) report on four studies that looked at teachers' decisions regarding grouping for reading instruction. Using Barr's (1975) study, they are able to report that teachers' conceptions of reading are among the factors that influence their decisions about forming reading groups. Borko (1982) also found that teachers' conceptions of reading influenced their practice. Richardson et al. (1991) found
that teachers' beliefs about reading relate to their practices; the researchers were able to predict classroom behavior on the basis of information elicited from teachers, via interviews, about those beliefs.

Not all researchers, however, are able to find the kind of strong relationship between theoretical orientation and practice that Kamil and Pearson (1979) are so confident exists. Martoncik (1981), for example, reports that teachers' use of verbal cues during reading instruction do not reflect the teachers' theoretical orientations as measured by DeFord's (1979) instrument, the Theoretical Orientation to Reading Profile (TORP). Kinzer and Carrick (1986) found that teachers do have conceptions of reading, but that their practices are more likely to reflect their conceptions of how reading is best taught and learned than reflect their conceptions of the reading process.

In other cases, however, findings do not necessarily negate the assertion that teachers operate from theoretical frameworks. Hoffman and Kugle (1982), for example, in their study of the relationship of theoretical orientation to verbal feedback, report that beliefs and behaviors are "situation specific" (p. 2). But they also report "clear areas of relationship between teacher beliefs and feedback particularly with respect to timing and form of sustaining feedback" (p. 6). They suggest that their focused interviews yielded "far more enlightening" (p. 6) data than the paper-pencil instruments they used--DeFord's (1979) TORP and the
Propositions About Reading Instruction Inventory (PRI) (Duffy & Metheny, 1979), thus raising some issues regarding methodology in attempting to elicit teachers' beliefs.

Bawden, Buike, and Duffy (1979) report that the teachers in their study held conceptions of reading even though those conceptions did not match the theoretical categories often found in the literature. Furthermore, they found that those conceptions were "reflected" (p. 10) in instructional practices. They conclude, however, that "it is not possible to state that instructional decision making in reading is exclusively [italics added] guided by reading conceptions" (p. 12). Duffy and Anderson (1982), referring to the same research, report that "approximately 80 percent of teachers' reading beliefs, as expressed in the interviews, were predicted from the time utilization data. This suggests a fairly close relationship between conceptions and instructional practice, at least when practice is measured as time use" (p. 27).

But teachers in the study did not attribute their practices to their beliefs about reading and, in that regard, seem similar to the teachers Harste and Burke (1977) observed and whose statements Harste and Burke claimed were atheoretical. Duffy and Anderson's (1982) teachers attributed their practice to the nature of their students, the commercial reading materials, and to the need to keep things moving. Apparently, their beliefs about reading represented only one
set of beliefs among many sets of beliefs about teaching, or only one aspect of their set of beliefs about teaching, and according to Duffy (1981), beliefs other than those about the nature of reading took precedence in their decision-making.

The conclusion from this research that the relationship between beliefs and practice is not a simple, linear relationship and that teachers' conceptions of reading do not exclusively determine their practice seems warranted. Nevertheless, this research does not negate the notion that beliefs about reading influence practice even though they may not be the sole cause of such practice. Duffy (1981) does not speak of inconsistencies between beliefs and practices; he says teachers do not attribute their practices to their beliefs. That teachers attribute their practices to something other than beliefs, however, does not lead necessarily to the conclusion that there is no relationship between beliefs and practices nor to the conclusion that beliefs do not in some way guide practice. Rather, we might conclude, as Duffy (1981) does, that beliefs about reading are but one set of beliefs among other beliefs that teachers use to guide their practice, but we should not deny that theoretical orientation is one aspect of that set of beliefs.

We might also conclude that explicit talk about students, classroom management, and reading materials is much more common among teachers than explicit talk about theoretical orientations. Talk about students, classroom
management, and reading materials may very well be talk about theoretical orientations, but those theoretical orientations must be inferred from the talk.

Students, classroom management, and reading materials may simply become the more obvious focus for the attribution of the influences on practice. So while it may be an overstatement to say, based on this research, that teachers base their practice solely on their theoretical orientations, it also seems to be an overstatement to imply that those theoretical orientations have no effect on practice.

Perspectives on Mathematics

Siegel et al. (1989) offer two perspectives on mathematics that parallel their perspectives on reading: mathematics as a body of facts and techniques and mathematics as a way of knowing. Like their counterparts in reading, these perspectives represent different philosophical and theoretical viewpoints.

Mathematics as a Body of Facts and Techniques

The conception of mathematics as a body of facts and techniques seems to have its origins in the ideas of Plato. Regarding the origins of mathematics, Plato held that "objects of mathematics had an existence of their own, beyond the mind, in the external world" (Dossey, 1992, p. 40). This notion of mathematics is based on a theory of an "external, independent, unobservable body of knowledge" (p. 40). Davis and Hersh (1981) explain that in this view, mathematical objects are
real whether or not we have knowledge of them. They are neither physical nor material, existing outside the spatial and temporal boundaries of physical existence. These objects are unchangeable and will not disappear. Meaningful questions have a definite answer. The mathematician, according to the Platonic view, "cannot invent anything, because it is all there already. All he can do is discover" (p. 318). According to Lerman (1983), such a conception presents mathematics as "a steadily accumulated body of knowledge, linear or hierarchical, dependable, reliable and value-free" (p. 62).

But the view of mathematics as a body of facts and techniques owes a debt to Aristotle as well. It seems to be closely related to the conception of mathematics known as formalism, a view usually regarded as Aristotelian (Dossey, 1992, p. 41). It differs, according to Davis and Hersh (1981), from the Platonic view in terms of what exists and what is real, but not in terms of "what principles of reasoning would be permissible in mathematical practice" (p. 320). For formalists, there are only "axioms, definitions and theorems--in other words, formulas" (p. 319). These formulas have no meaning, no truth value; they are not "about anything" (p. 319, italics in the original). The formalists, according to Davis and Hersh (1981), begin with assumptions (axioms) not "self-evident truths" (p. 341) and deduce their way to theorems. Theorems are neither true nor false since they have no content, but neither is there any possibility for doubt or error
"because the rigorous proof and deduction leaves no gaps or loopholes" (p. 340). These formulas, then, offer a body of facts and a collection of techniques that can be applied in various situations.

The seemingly contradictory positions of Platonism and formalism, at least in terms of their metaphysics, are reconciled by a typical working mathematician, according to Davis and Hersh (1981), by the mathematician's operating as a Platonist on weekdays and a formalist on Sundays. That is, when [the working mathematician] is doing mathematics, he is convinced that he is dealing with an objective reality whose properties he is attempting to determine. But then, when challenged to give a philosophical account of this reality, he finds it easiest to pretend that he does not believe in it at all. (p. 321)

These conceptions of mathematics are supported and sustained by research of a particular nature. Researchers operating from these conceptions of mathematics treat mathematics as an "existing, established body of concepts, facts, principles, and skills available in syllabi and curricular materials" (Dossey, 1992, p. 43) and focus on finding ways for teachers to be more successful in transferring this information to students (Dossey, p. 43). This research, according to Dossey, focuses on teachers' actions and methods, not on the conception of the mathematics being taught or how students may be learning it. Studies, in general,
he claims, are experimental, and may focus, for example, on how teachers sequence instruction or how expert teachers differ from novices. Romberg (1992) claims that the roots of such research are clearly in behavioral psychology.

**Instructional Implications**

For Lerman (1983), there are clear pedagogical implications related to one's view of mathematics. To adopt a formalist approach, he claims, is to teach in such a way as to help students see the deductive nature of mathematics. This means, he says, that correct methods of deduction must be applied, and that provided the questions are set correctly, the desired result will be achieved. There is no 'purpose' in the sense of a particular problem to be solved. It is the method that is of central significance and provided that is thoroughly learnt and tested by repeated exercises, mathematics has been successfully conveyed (p. 62). Furthermore, he claims that in this view, one must learn methods first and "understand uses, applications or relevance afterwards" (p. 62).

According to Copes (1979, cited in Thompson, 1992), this perspective on mathematics could be communicated to students through an emphasis on "transmission of mathematical facts, right versus wrong answers and procedures, and single approaches to the solutions of problems" (Thompson, 1992, p. 133). Thompson (1984) suggests that teachers who view mathematics as a body of facts and techniques may see that their role is to present clear demonstrations of
procedures, explain rules clearly, and allow students time to practice the procedures they have seen demonstrated. Mathematics instruction is thus seen as transferring information from the teacher to the student.

This view of mathematics, according to Kuhs and Ball (1986, cited in Thompson, 1992), leads to a view of instruction that they called the "content-focused view with emphasis on performance" (Thompson, 1992, p. 136). Kuhs and Ball hold that the central premises of this view are:

- Rules are the basic building blocks of all mathematical knowledge and all mathematical behavior is rule-governed.
- Knowledge of mathematics is being able to get answers and do problems using the rules that have been learned.
- Computational procedures should be "automatized."
- It is not necessary to understand the source or reason for student errors; further instruction on the correct way to do things will result in appropriate learning.
- In school, knowing mathematics means being able to demonstrate mastery of the skills described by instructional objectives.

(Thompson, 1992, p. 136)

Skemp (1978) maintained that such instruction was analogous to giving a person "a set of fixed plans" (p. 14) to use to find his or her way around town:
"Turn right out of the door, go straight on past the church" (p. 14). Rather, he said, what is needed is a mental map of the town . . . from which he can guide his steps from any starting point to any finishing point, provided only that both can be imagined on his mental map. And if he does take a wrong turn, he will still know where he is, and thereby be able to correct his mistake without getting lost; even perhaps to learn from it. (p. 14)

Mathematics as a Way of Knowing

The perspective of mathematics as a way of knowing can look to Lakatos (1976) for a philosophical basis for the nature of mathematical knowledge. Mathematics, for Lakatos, is, according to Davis and Hersh (1981), fallible, not indubitable; it [like the natural sciences] grows by the criticism and correction of theories which are never entirely free of ambiguity or the possibility of error or oversight. Starting from a problem or a conjecture, there is a simultaneous search for proofs and counterexamples. . . . "Proof" in this context . . . does not mean a mechanical procedure which carries truth in an unbreakable chain from assumptions to conclusions. Rather, it means explanations, justifications, elaborations which make the conjecture more plausible, more convincing, while it is being made more detailed and
accurate under the pressure of the counterexamples. (p. 347)

Such a view of the nature of mathematics is in stark contrast to the absolutist, infallible views of the Platonist and the formalist. And unlike the mathematics of the Platonist, which is discovered, the mathematics, in this case, is created or constructed.

The school of epistemology, or post-epistemology (Noddings, 1990), that seems to explain the perspective of mathematics as a way of knowing is constructivism. Constructivism, particularly radical constructivism (von Glasersfeld, 1990, 1991), does not evaluate knowledge from a traditional epistemological perspective, hence the designation by Noddings (1990) of the term post-epistemology to this school of thought. In fact, according to von Glasersfeld (1990), the principles of constructivism are "incompatible" (p. 23) with the traditional notions of epistemology regarding truth and objectivity.

The principles of constructivism, according to von Glasersfeld (1990) are:

1. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject.

2. a. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;
b. Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality. (p. 22-23)

Such principles presuppose, he claims, not a world that is "an inaccessible realm beyond perception and cognition, [but] the experiential world we actually live in. This world is not an unchanging independent structure, but the result of distinctions that generate a physical and a social environment to which, in turn, we adapt as best we can" (p. 23).

Researchers in mathematics and mathematics education operating from a constructivist conception of mathematics focus on mathematics as personally constructed knowledge and use research strategies and methodologies that reflect that conception. Dossey (1992) claims that according to this conception of mathematics, "knowing mathematics is equated with doing mathematics. Research in this tradition focuses on examining the features of a given context that promotes the 'doing'" (p. 44). Studies, then, often reflect a field-study or ethnographic approach to research (Romberg & Carpenter, 1986).

Instructional Implications

Perhaps the most important instructional implication of viewing mathematics as a way of knowing, that is, from a constructivist perspective, is that "one must reject the assumption that one can simply pass on information to a set
of learners and expect that understanding will result" (Confrey, 1990, p. 109).
Ernest (1992) suggests other pedagogical emphases that he sees as being strongly suggested by a constructivist perspective on teaching and learning mathematics, including focusing on students' and teachers' beliefs about mathematics and learning, and emphasizing "discussion, collaboration, negotiation and shared meaning" (p. 11-12).

Steffe (1991), too, claims that to accept a constructivist view of mathematics has clear instructional implications. Among those implications are that teachers use the mathematics of children rather than the conventional school mathematics as the basis for instruction; that teachers determine the mathematics "for children through interactive communication" (p. 182, italics in the original); that teachers should interpret children's mathematical activity through interactions with the child, making qualitative distinctions, not simply by looking at the result of their activity; and, in general, "legitimatize children's mathematical knowledge" (p. 188).

Schoenfeld (1992) suggests that given the goals for instruction outlined by the calls for reform in mathematics education and given the epistemological perspective on which such goals rest, it becomes a "pedagogical imperative" (p. 345) that teachers create classrooms that are "communities in which mathematical sense-making . . . is practiced" (p. 345). Confrey (in press) concurs and maintains
that these communities should be characterized by more talk by students and "close listening" (p. 16) by teachers:

Close listening involves an act of decentering by an adult, or possibly a peer, in order to imagine what the view of the child might be like. It includes repeated requests for a child to explain what the problem is they are addressing, what they see themselves doing, and how they feel about their progress. It requires one to ask for elaboration from them, about what, where, how, and why. The questioner should repeat [the student's] language as much as possible and invent tests of whether one's own interpretation is consistent with the statements and actions of the child. (p. 16)

Borasi (1990) seemingly agrees and suggests that some of this talk be students' reflecting on their own conceptions of mathematics; she suggests journal writing (Borasi & Rose, 1989) as another way to encourage such reflections.

Lerman (1987) claims that such a view of mathematics leads to an emphasis on process over content in mathematics instruction and, following Freire (1972, cited in Lerman, 1987), an emphasis on problem-posing not merely problem-solving. He proposes that "enabling students to examine situations and to pose problems for themselves . . . places a powerful tool in the hands of people
to examine what is happening to their lives, and provide[s] them with the possibility of changing it" (Lerman, p. 54).

Perspectives on Reading

Siegel, Borasi, and Smith’s (1989) synthesis of mathematics and reading includes two perspectives on the reading process: reading as a set of skills for extracting information from text and reading as a way of knowing. Each perspective represents a different theoretical orientation to the reading process.

Reading as a Set of Skills for Extracting Information From Text

The perspective of reading as a set of skills is akin to a "bottom-up" (Kamil & Pearson, 1979) or "information transfer" (Harste, 1985) model of reading. Following a model proposed by Gough (1972), the reader is thought to progress through a text left to right (in English text), letter by letter, and word by word. Readers do not go directly from print to meaning; rather they go from print to a "systematic phonemic" (p. 337) representation of a word to the meaning of the word. Gough concedes that "solving the decoding problem does not automatically make a child a reader" (p. 353), distinguishing between readers who "bark at print" (p. 353) and those who are able to "read connected discourse" (p. 353, italics in the original).

The difference, according to this model, relates to the speed with which a reader can determine what a particular word may be. When a reader locates a
word in his or her "lexical search" (p. 338), it must be deposited somewhere until every word in the sentence has been similarly located. That place, according to this model, is primary memory. If a reader takes too long to locate subsequent words, words previously located may be lost from memory and the reader's ability to comprehend a sentence will have been affected. Thus, readers must be able to identify words rapidly. Oral reading errors, in this model, are signs that a reader has not decoded the word fast enough to read "normally" (p. 354).

La BERGE and SAMUELS (1974) also offer a model that supports the perspective of reading as a set of skills and that focuses on the speed with which skills and sub-skills can be performed:

During the execution of a complex skill, it is necessary to coordinate many component processes within a very short period of time. If each component process requires attention, performance of the complex skill will be impossible, because the capacity of attention will be exceeded. But if enough of the components and their coordinations can be processed automatically, then the load on attention will be within tolerable limits and the skill can be successfully performed. Therefore, one of the prime issues in the study of a complex skill such as reading is to determine how the processing of component subskills becomes automatic. (p. 293)
The LaBerge and Samuels model focuses on "automaticity" (p. 314) as being important in "performance of fluent reading" (p. 314). It is not clear whether "fluent reading" (p. 314) implies a comprehending reader. LaBerge and Samuels define the fluent reader as one who "has presumably mastered each of the subskills at the automatic level" (p. 318). The authors of the model concede that the "complexity of the comprehension operation appears to be as enormous as that of thinking in general" (p. 320); nevertheless, the model does not address the issue of comprehension in a substantive manner. It is left to Harste (1985) to summarize the characteristics of reading comprehension that adhere in such a model:

- Meaning is in text
- Reading is a process of transferring meaning from text to reader
- Key variables in this process are reading skills
- Good readers transfer more information than poor readers
- The criterion for judging reading success is how much information was transferred (p. 12:3)

Even though these models do little to explain how readers make sense of what they read, the conception of comprehending text that follows from such models is closely aligned with a particular view of literary criticism. The idea that information can be extracted from text follows from the view of the New Critics.
This group, according to Rosenblatt (1991), formed in the 1930's and was concerned that the idea of literature as an art was being neglected in the study of literature because of the focus on the personality of the author and the historical period which produced the literature. According to Rosenblatt (1991),

The New Critics urged an impersonal "intrinsic" analysis of "the poem itself," as an autonomous entity, instead of study of "extrinsic" biographical or historical materials. . . . The poem was presented as an autonomous entity embodying its meaning and existing in its own right as a unified system whose workings could be objectively studied and analyzed. . . . The New Critics viewed the text as an entity embodying a determinate message or meaning. (p. 58)

The influence of the behaviorist school of psychology on these reading models is evident. The research upon which these theories and models are based used an experimental design, laboratory conditions, and often used nonsense words or syllables or artificial alphabets of nonsense figures.

**Instructional Implications**

La Berge and Samuels (1974) discuss the implications of their model for reading instruction and state clearly that "pedagogically, we favor the approach which singles out these skills for testing and training and then attempts to sequence them in appropriate ways" (p. 318). By "these skills" they refer to the
skills and subskills they believe are necessary for "decoding" (p. 307) words. They would begin with students learning to discriminate letters, then learning letter-sound correspondences, and, once students had learned letter-sound correspondences, they could move on to learning to blend sounds into syllables or words.

La Berge and Samuels (1974) make a distinction between comprehension and word meaning. Word meaning refers to individual words; comprehension refers to the "organization of these word meanings" (p. 319). Since comprehension relies on knowing the meanings of individual words, word meanings, in this model need to be "automatically processed" (p. 320), thus one can infer an instructional emphasis on learning vocabulary. Also, since reading acquisition is the result of applying a sequence of skills, meaningful text would not be required for the beginning stages of this process. Letters, groups of letters, and words in isolation would necessarily have to be used for instruction before words combined into meaningful units could be used. Since meaning follows the acquisition of these skills, nonsense materials could be used for instruction in the beginning.
Reading as a Mode of Learning

Siegel et al. (1989) contrast the "skills-based" (p. 271) perspective of reading to a "comprehension-based" (p. 271) perspective. The comprehension-based perspective is represented by the Goodman model of reading (Goodman, 1967, 1972, in press; Goodman & Goodman, 1978), which defines reading as "a receptive language process" (Goodman & Goodman, 1978, p. 2-1) in which the reader seeks to construct meaning; "it is a psycholinguistic process in that it starts with a linguistic surface representation encoded by a writer and ends with meaning which the reader constructs" (p. 2-1). The notion that readers construct meaning rather than extract meaning from text is an essential component of this model and, in that regard, is supported by the constructivist notion of schema theory investigated by cognitive psychologists and other reading researchers (Anderson, 1977; Anderson & Pearson, 1984; Bartlett, 1932; Rumelhart, 1980; Rumelhart & Ortony, 1977; Spiro, 1980).

Readers, according to the Goodman model, use three kinds of information from language to make sense of what they are reading: grapho-phonemic, syntactic, and semantic. The relationship between these sources of information is one of interdependence and is not hierarchical. Furthermore, this model recognizes that since reading is language, it "operates in a social context" (Goodman & Goodman, 1978, p. 2-13) and that readers depend on their own
experiences and conceptual development (Goodman, 1976a, p. 58) to construct meaning. Unlike the skills-based model, which is hierarchical and focuses attention on letters and words as the most important units of reading, the Goodman model identifies the clause as an essential unit since it is through an understanding of clauses and their interrelationships, according to Goodman, that meaning can be constructed by the reader.

The Goodman model does not represent a reading process in which comprehension follows "decoding," a process, Goodman points out, more accurately labeled "recoding" (1971, p. 462). This model is one in which comprehension is integral to the process. A reader need not master the skills of identifying letters and naming words in isolation before reading a comprehensible text. In fact, Goodman points out, comprehension, not identifying words, is the ultimate concern of the reader (1970, p. 129). "The reader does not use context simply to identify words, rather he uses all cues in relationship to all others to reconstruct the message" (1970, p. 129).

The meaning that a reader makes of a text, according to this model, "is constructed through transactions with the text" (Goodman, 1984, p. 80). Since readers use their linguistic knowledge, as well as their "schemata, or knowledge of the world" (Anderson & Pearson, 1984, p. 255) and of the reading situation, they play an active, not a passive, role in the creation of meaning from the text. The
meaning is found neither in the text nor in the reader, but rather "involves both the author's text and what the reader brings to it" (Rosenblatt, 1978, p. 14). Rosenblatt (1978), drawing on the work of John Dewey and Arthur Bentley (1949, cited in Rosenblatt), explains that "transaction" (Rosenblatt, 1978, p. 16) describes the process better than "interaction" (p. 17). An interaction, she points out, implies "separate, self-contained, and already defined entities acting on one another" (p. 17), but a "transaction' designates . . . an ongoing process in which the elements or factors are, one might say, aspects of a total situation, each conditioned by and conditioning the other" (p. 17). There is thus a transaction between the reader and the writer's text and the result is the reader's construction of the reader's text--a text that will differ from reader to reader. It is this reader's text, which has been constructed by the reader through the transaction with the writer's text, that the "reader comprehends and on which any reader's later account of what was read is based" (Goodman, 1984, p. 97).

Goodman's work does not follow behaviorist psychology as the models previously discussed do; rather it is more compatible with the ideas of Piaget, Vygotsky, and other constructivists or social-constructivists. Furthermore, his work clearly shows the influence of linguistics, he being among the first reading researchers to recognize fully the notion that reading is language. The research that led to the Goodman model was not experimental in design, nor was it
conducted in laboratories with artificial texts. Goodman observed children reading stories orally and independently and noted the instances when what they actually read differed from what was expected.

Through the analysis of those differences, which Goodman termed miscues, he concluded that children were using the information that they possessed as users of language to construct meaning from text. Miscues could not be explained simply as "careless or imprecise identification of letters" (Goodman, 1967, p. 128). Rather, readers were relying on their syntactic and semantic knowledge as well as their phonic-graphemic knowledge as they read. Miscues were not errors. In fact, Goodman explains that "the term error is a misnomer . . . since it implies an undesirable occurrence" (1976b, p. 15). Miscues did not always interrupt or distort the reader's search for meaning; rather, they served as "windows on the reading process" (1973, p. 3), allowing teachers and researchers a means by which they could more fully understand how readers were making sense of what they were reading.

**Instructional Implications**

The Goodman model is a model of the reading process, not a theory of reading instruction; furthermore, Goodman says, a theory of the reading process cannot be translated directly into instructional practice (1972, p. 143). He calls for a theory of reading instruction, suggesting what he calls the "essentials" (p.
that such a theory would contain. Those essentials include the following:
meaning is always the goal in reading; language is "indivisible" (p. 158) and,
therefore, instruction should not focus on parts of language such as words or word
parts; language only "exists in the process of its use. . . . There is no possible
sequencing of skills" (p. 158, italics in the original); children's primary resource for
learning to read is their existing competence as language users; children will find
it easier to read what they need to understand--the focus should be on
"communicative need" (p. 158, italics in the original); children need to learn
strategies for "predicting, sampling and selecting information, guessing, confirming
or rejecting guesses, correcting and reprocessing" (p. 158) and for reading special
forms of language; and selections chosen for instructional purposes need to be
chosen with an awareness that "meaning is both input and output in reading" (p.
158), and that selections will be understood only insofar as readers bring the
"prerequisite concepts and experiences" (p. 158) for understanding.

Goodman and Watson (1977) suggest that an instructional program that is
consistent with this view of reading would provide students with authentic learning
experiences so that students could read for purposes important to them and which
draw on meaningful, not nonsensical, materials. In addition, they suggest that
readers become writers, a idea that has been explored by others working from a
constructivist perspective regarding the relationships between reading and writing
Perhaps one of the best sources of information about the instructional implications of such a model of reading is the work being done in the whole-language movement. (See, for example, Edelsky, Altwerger, & Flores, 1991; Goodman, K., 1986, 1989; Goodman, Bird, & Goodman, 1990; Goodman & Goodman, 1981; Goodman, Y., 1989.) Edelsky et al. (1991) call whole language a "professional theory, an explicit theory in practice" (p. 7, italics in the original) and claim that this perspective developed "out of research into the reading process" and cite Goodman (1968, 1969) and Smith (1971) in regard to that research. The whole-language "position on learning in general and on curriculum--"Learning is a social process. . . . Learning is best achieved through direct engagement and experience. . . . Learners' purposes and intentions are what drives learning. . . . Learning involves hypothesis testing" (Edelsky et al., pp. 23-26)--follows closely from the essentials of an instructional theory suggested by Goodman (1972, p. 155-159).

That learning is social leads to an emphasis in the classroom as a learning community, and recognizes, for example, following Vygotsky (1978), that learners can be assisted through their zones of proximal development through their interactions with other students as well as with their teachers. That learning
should be direct engagement and experience follows Dewey (1963, cited in Edelsky et al., 1991) and Piaget (1967, cited in Edelsky et al.) and leads to "doing science or history" (Edelsky et al., p. 25; italics in the original) not simply reading about science or history. It is learners' purposes, not teachers' purposes that become important and that preclude the use of "empty exercises" (p. 25) in favor of authentic projects or experiences. Finally, hypothesis testing suggests predicting, sampling, confirming, or rejecting on the way to constructing one's own meaning rather than practicing, memorizing, and receiving information.

Summary

Both the education community, in general, and the reading and mathematics communities, in particular, recognize the importance of exploring the relationship between teachers' beliefs about the nature of reading and of mathematics and their practice. Neither the reading researchers nor the mathematics researchers find that teachers' beliefs exclusively influence their practice, but neither do they find that beliefs are not a factor in influencing practice. Furthermore, both recognize that different instructional implications seem to follow logically, if not always in practice, depending on one's conception of the nature of the subject.

This study examines teachers' beliefs about reading mathematics by determining their beliefs about the nature of reading and of mathematics and by
characterizing those beliefs vis-a-vis the theoretical orientations that inform the Siegel et al. (1989) synthesis of mathematics and reading. If, as the literature suggests, teachers' beliefs, including beliefs about the nature of the subjects they teach, are vital components of their practice, and if proposed changes in practice must be related to those beliefs, such information becomes important in terms of proposing and implementing instructional and curricular change.
CHAPTER 3

SETTING AND METHODS: "A COLLABORATIVE, DYNAMIC PROCESS"

The present study was undertaken to examine mathematics teachers' beliefs and practices regarding reading mathematics. Specifically, mathematics teachers' beliefs about the nature of mathematics and the nature of reading were examined. The theoretical orientations implicit in those beliefs were characterized vis-a-vis the theoretical orientations that inform Siegel, Borasi, and Smith's (1989) synthesis of mathematics and reading. Instructors' practices were also examined to determine what, if any, relationship existed between their beliefs about the nature of mathematics and reading and their practices regarding reading mathematics. Following a statement of the underlying assumptions, this chapter describes the setting and environment in which the instructors were working and then explains the methodology used in the study.

Assumptions

Following Magoon (1977), DeFord (1985), and Richardson, Anders, Tidwell, and Lloyd (1991), this study takes a constructivist perspective. Such a perspective, according to Magoon, operates from at least three underlying assumptions: that the "subjects" (p. 651) being studied are "knowing beings" (p. 652), and their knowledge "has important consequences for how behavior or
actions are interpreted" (p. 652); that subjects have control over their own behavior even though there may be constraints on that behavior as a result of social norms; and that people have a highly developed capacity for dealing rapidly with complexity, attending to more than surface meaning in communications and taking on and constructing complex social roles (p. 652). Given these assumptions, Magoon claims, the phenomena that educational researchers deal with is sufficiently sophisticated and complex to require a methodology, such as a social anthropological methodology, that can begin to make sense of and explain the complexity.

Setting

The research was conducted at a land grant university in the Southwestern United States during a summer program designed for minority students who would be enrolled at the university in the fall. The focus of the research was the instructors teaching in the mathematics component of that summer program. While the mathematics classes taught during the summer program were the same classes taught during the regular fall and spring terms, there was an important difference. There was a prescribed instructional approach during the summer program, whereas during the regular terms instructors were free to design their own approaches. A description of the summer program, in general, and the
The instructional approach required by the mathematics component, in particular, follows.

**The Summer Program**

The summer program was established, originally, to aid in the recruitment and retention of high risk minority students. At the time of the study, the primary focus was still on minority students, but the admissions criteria had been expanded to allow for the participation of non-minority students. Any student who had been accepted to attend the university in the fall, who had a minimum high school cumulative grade point average of 2.700, and who demonstrated financial need or was an ethnic minority was eligible to participate in the program.

There was both an orientation and an academic component to the summer program. The goal of the orientation component was to assist in the transition a student would need to make between high school and college. To that end, students were assigned to groups based on their declared majors and met with those groups daily to learn about the university's policies and procedures. A group leader, usually an undergraduate student, was in charge of the group. In addition to helping students learn about the university, the group leaders organized various social activities. Group leaders functioned as combination
social directors, social workers, discussion leaders, and friends in order to help students make the adjustment to university life.

The academic component consisted of a credit class and tutoring. In the mathematics program of the academic component, tutors were either undergraduate or graduate students and were responsible for three separate tutoring activities: classroom tutoring, group tutoring, and drop-in tutoring. With the exception of the single section of Math 125a, which had three tutors assigned to it, each of the other twelve sections of mathematics classes was assigned one tutor who was present in the classroom during the second hour of each class. In addition, the tutor met with the students from his or her assigned class during group tutoring, which was held after class each day from 9:15 a.m. until 10:45 a.m. Students were required to attend four of the five sessions each week. Drop-in tutoring was available for students in the afternoons and one evening a week. Drop-in sessions were staffed by tutors and faculty, faculty being required to serve three hours per week in either drop-in or group tutoring.

The Mathematics Component of the Summer Program

During the summer program, there were thirteen sections of four mathematics classes offered. Students were placed into the classes based on an examination. All classes met daily from 7:00 a.m. until 9:00 a.m. If it was determined that students had been placed into classes that were inappropriate,
they could be reassigned to different classes with no disruption to their schedules. Table 1 presents the mathematics classes offered during the summer program.

<table>
<thead>
<tr>
<th>Course Title</th>
<th>Course Number</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intro to College Algebra</td>
<td>Math 116</td>
<td>6</td>
</tr>
<tr>
<td>College Algebra</td>
<td>Math 117</td>
<td>4</td>
</tr>
<tr>
<td>Pre-Calculus</td>
<td>Math 120</td>
<td>2</td>
</tr>
<tr>
<td>Calculus</td>
<td>Math 125a</td>
<td>1</td>
</tr>
</tbody>
</table>

The summer program administrators had not wanted to duplicate the students' high school mathematics experiences. The mathematics component of the summer program, therefore, had begun as a pilot program nine years before this research was undertaken. The pilot focused on problem solving, did not offer credit, used non-lecture methods and evolved, two years after it was begun, into a program in which credit classes--the same credit classes offered by the Mathematics Department during the fall and spring semesters--were offered.

The summer program had, in the words of the Director of the summer program always been very "intrusive" (In. 51) regarding the mathematics component. In addition to required textbooks and a department syllabus and
class outline, there were prescribed methods to follow—the Jo Kemper Method. Jo Kemper had been involved in the pilot program and her job, as she explained, had been to "coordinate what happened in the mathematics classes" (In. 673). She had undertaken to develop an instructional approach to mathematics that would contribute to students' success. The goal had been to have students actively involved in the classroom.

The Jo Kemper Method, as it was referred to by the teachers in the program—at least the teachers who had been participating in the summer program for several years—remained as the basis for instruction in the mathematics classrooms in the summer program. Jo Kemper, herself, was no longer involved, so the method had been translated to new faculty by others and individual faculty members had, in some cases, put their personal stamps on the method, modifying it to suit their needs. Even with those modifications, however, enough of the original method remained to justify continuing to refer to it as the Jo Kemper Method. What follows is a discussion of the method as it was explained to me by its originator.

The Jo Kemper Method

Jo Kemper's notion of what should occur in mathematics classrooms was founded on the premise that students should be "actively involved" (In. 678). In

\footnote{All proper names used in this dissertation are pseudonyms.}
terms of the teacher's methods, that premise translated, for her, into not lecturing. If the teacher is lecturing, she maintained, "the only one actively involved is the one giving the [lecture]" (In. 1827).

Instead of using a lecture as the means of presenting the concepts of mathematics, Jo Kemper tried, as she put it, "to make the text central to the mathematics--to the learning process. Because one of the things that college-level students have to be able to do is to deal with technical texts" (In. 1257-1262). And, she pointed out, not only do students need to be able to deal with technical texts, they need to be able to do that on their own, without much help from their teachers. She said, "The idea was that you could use the text to talk about something rather than just the teacher telling the student something" (In. 1441). In other words, tutors and teachers were to refer students to the text and ask them to read the explanations or work through sample problems rather than tell the students about the concepts involved without the students having to have read anything.

Ms. Kemper was quick to say, however, that requiring a student to read the text, while perhaps a necessary condition to helping students learn to deal with technical texts, was not sufficient:

Kids have to be taught how to read the text. . . . Then it gets reinforced, but they first have to be taught it. It usually takes one or two class periods
to do that. But unless you teach a kid how you want the text dealt with, they will never look at it, not a math text. They'll only look at the back to look up the problems and that's it. In fact, they don't even look at the examples. . . . You're not going to send students home--at least I would not send students home--to read a textbook on their own without a lot of work, as a class or as individual students, on how you deal with text and how you deal with formulas. I mean, I'm not going to send students home to read a text on their own. . . . There is a difference between requiring [students] to read a text and teaching them how to read the text. (In. 1091-1618)

One of the techniques Jo Kemper taught students to use to read the text was previewing. She taught students to preview by using 3x5 cards to cover up the headings. Students were to predict what they thought might be coming next, then write that down or write down things that puzzled them. She said she would then walk around the classroom to make sure that everyone had written something down. A short discussion would follow. "The idea is just to get them to work with language and to think about what might be coming next. Usually there are a couple of sentences, so we just discuss those sentences and the terms that are in those sentences and what they might mean" (In. 1038-1045).

She used the same technique with the examples: Display the problem statement of the example, but cover the solution. She then asked students to try
to solve the problem. Again, she circulated through the classroom to see that everyone was participating. She explained the rationale for this procedure:

The point is not necessarily that they get through the problem. The point is that they think about what the next step might be. They can be totally confused; it really doesn't matter. The point is to get them involved, not sitting back passively and watching a teacher put the next step on the board, but for them to actually start to think about what the next step might be. And I run around that classroom watching them put that next step down and I see who can. Then I let them uncover that next step and take a look at it. And then there's discussion about what they thought it was and what it is. And then they will go to the next step. (In. 1052-1075)

Once the students have been taught to read the text, however, Jo Kemper expected them to "wrestle with the problems before they come into class" (In. 896). "You want them to come back with questions. You want them to be aware at a metacognitive level what it is they don't understand" (In. 1425).

If the instructor is not going to lecture and if the textbook is central to the learning process, then it becomes imperative that students read the text. But simply teaching them how to read the text is no assurance that they will read it. The Jo Kemper Method confronted that potential problem by requiring a quiz on
the reading at the beginning of every class period. Not only did the quiz give
students an incentive to read the text--Jo Kemper referred to the quiz as a
"reward" (In. 840) for reading the text--but, according to Kemper, the results of
the quiz gave the instructor some insights into what it was students did not
understand.

After the quiz, and the discussion of the quiz, the class moved on to
homework problems--problems based on the reading they had been quizzed on
the day before. As students came into class, they wrote the numbers of
troublesome homework problems on the board. If more than one student
struggled with a particular problem, the number would appear with a series of
check marks beside it, one check for each student who wanted to indicate some
confusions about that problem. Instructors need look no further than the list of
numbers and check marks to get a quick assessment of the ability of the class to
handle the concepts for that day. They could then organize the class discussion
around those problems. As instructors went over those problems, however, they
were not to stand at the board and demonstrate how to do each problem. Either
students were to be at the board, or, if the instructor put the problem on the
board, he or she was to ask students to supply each step as the instructor wrote it
and to ask the students to explain why that step was necessary.
Getting students actively involved in the classroom was also manifested in the "no-pencil rule" (In. 540-550). Teachers were not to use the pencil when they helped a student try to understand a problem. The student was in charge of the pencil and, as a result, the pace of the problem solving.

The mathematics faculty were expected to use the Jo Kemper Method. Specifically, they were expected to require students to read the text before coming to class; to ask students to write the numbers of problems they wanted help with on the board as they arrived in the classroom; to begin class each morning with a quiz on the previous night's reading assignment; not to lecture; to make the text central to the learning process; and to require that students supply the explanations for work done at the board.

With the exception of administering the quiz first thing every morning before the concepts had been discussed, the instructors generally agreed with and tried to implement these requirements. Two instructors submitted a quiz to the tutors to be administered during the tutoring session, and at least one instructor said that she discussed the concepts before administering the quiz because not to do so resulted in students' not doing well on the quizzes. Since the quiz scores counted toward the students' grades, albeit seemingly insignificantly from the instructors' points of view, some students were concerned about the effect the quiz scores would have on their grades and this instructor thought that stress was
unnecessary. All instructors supported the requirement that students be required to read the text before coming to class. One of the instructors, Robert, could have been speaking for all of them when he said, "If there's anything about the [summer program] that is really top notch, [it's that it] make[s] them read the book. [Students] are paying a price for not being able to read the book well" (In. 983-989).

As stated, instructors asked students to put the numbers of troublesome exercises on the board as the students came into class in the morning. All the instructors were aware that they were not to lecture and seemed to agreed with that rule, in principle, even if it was sometimes difficult to comply with it in practice. One instructor, Duane, complained, good naturedly, that he had a "little beef" (In. 637) with the program:

I always say, 'Do you mean I can't get up there and do my thing?' and they say, 'Well, no.' [But] I need that. Until I establish things the way I want them, I don't think I can get anything done. And I need to talk to my class. You've got to give me a chance to talk to my class, and then, we'll do this stuff. But I want to be able to talk to my class. (In. 637-653)

One of the new instructors, Janelle, found the no-lecturing requirement challenging, but a challenge she was willing to accept because she thought that she had to "constantly change or I don't stay alive" (In. 774). She said of the no-
lecture requirement, "I'm not as comfortable asking questions instead of just lecturing because I know that my lecturing technique is a good one. I know that I'm successful; I've seen it. It's successful, maybe for 78%. Well, maybe [not lecturing] could be successful for the others" (ln. 816).

In-service Workshop

The prescribed methods were communicated to faculty through the in-service workshop prior to the beginning of the summer term. Some years, a video tape, prepared by Jo Kemper, was shown in order to demonstrate how students, not the instructors, should be actively solving the problems, using the pencil, controlling the pace of the instruction. Missing from the tape, however, was any discussion of how to help students learn by using the textbook. This year the tape was not shown. The two instructors who were new to the program had never seen the video; some of the instructors who were returning to the program had seen it in previous years' in-service sessions. This year's in-service was conducted by neither Jo Kemper nor anyone whom she had trained.

The aspect of the Jo Kemper Method that she claimed had been most difficult to get into place initially--"getting teachers to teach them how to read the text" (ln. 1122)--seemed to be deemphasized during the in-service workshop. Reading the text was still seen to be important, and teachers helping students learn to read the text was also seen to be important, but the in-service workshop
for the mathematics teachers did not focus in any significant way on helping instructors help students read their textbooks. Jo Kemper was not surprised. "Well, I can see that dropping out. That was hard for me, very difficult for me, to get into place" (In. 1114-1117). By "that" she was referring to "the part of getting the teachers to teach them how to read the text" (In. 1122).

At the beginning of the workshop, instructors were asked to talk about the "goals of the summer program" (wksp., 6/10/91, p. 1). Instructors volunteered several statements of goals, including, "Try to make students independent learners" (p. 1) and "Help students read the textbook" (p. 1). There were two specific workshop activities that could be construed as having been designed to help instructors help students learn how to read their textbooks. One consisted of a discussion about the required quiz. The quiz was required, in large part, to encourage students to read the textbook before coming to class. Part of the discussion centered on whether or not it was a problem for students to be quizzed on the material before it had been discussed in class. Despite some teachers' contentions that students had experienced some difficulty with the quiz the previous summer, there seemed to be a general consensus that "[students] should be able to answer the questions on the quiz if [they] have read the material" (p. 4). Further, there was the contention that students might "not necessarily be able to work a problem, but [they] should be able to answer quiz questions" (p. 4).
The other activity that could be construed as helping instructors help students read the text was the discussion led by various instructors about the textbook, itself. Each chapter had been assigned to an instructor and that instructor had examined the chapter looking for inconsistencies or other difficulties of which instructors should be aware. They pointed out, for example, where explanations given for solving problems were confusing or not as helpful as alternative methods of solving those problems; where the textbook gave contradictory information; where the order of presenting a concept was questionable. They discussed what ambiguous terms meant: "What does relationship mean here?" "What do we mean by standard form?" (p.7). They pointed out that the textbook and the study guide, at times, used different notation and different terminology to refer to the same concepts. And they discussed the difficulty of there not being enough examples in the textbook to teach a particular concept. The implication was that once the instructors were aware of these features of the textbook, they could help students use the textbook more effectively.

The instructors were implementing the Jo Kemper Method as they understood it. The aspects that were more concrete, such as not using the pencil, administering a quiz the first thing in the morning, asking students to write the numbers of exercises on the board, were implemented, seemingly, easily. Those
aspects that had to do with helping students learn to read the textbook were observed less consistently and less frequently. (These observations will be discussed in more detail in Chapter Four.) Jo Kemper, herself, was quick to acknowledge, however, that, "Things change when you're not around. . . . All of it is a collaborative and a dynamic process and it is always changing and it changes with different people around" (In. 639).

Methods

This study used an ethnographic approach to examine teachers' beliefs and their practices regarding reading mathematics. For six weeks, the researcher was a participant-observer (Spradley, 1980) in the mathematics component of a university summer program.

Participants

The primary participants in the study were the fourteen mathematics instructors who were teaching in the summer program. The instructors' experience, the process by which they were selected, and their attitudes toward the summer program are described below. Other participants informed the study. Those participants included students, program administrators, tutors, and program staff members.

Instructors

The primary participants of the study were the fourteen mathematics
instructors who were teaching in the summer program. All fourteen instructors were experienced teachers, and, except for two instructors, all had taught in the summer program before. In addition to their assignments in the summer program, all of the instructors had taught at the University in the Mathematics Department Entry Level Program as adjunct lecturers or as graduate assistants. Two of the instructors taught full-time in the Entry Level Program during the regular school year; twelve taught in the Entry Level Program on a part-time basis; five of the twelve who taught part-time also taught full-time in local high schools. See Table 2 for specific information about instructor's degrees and experience.

Selection criteria. Instructors in the program had to be approved, first, by the Mathematics Department. But while approval by the Mathematics Department was a necessary condition, it was not sufficient. The Director of the summer program and the summer program Mathematics Faculty interviewed the applicants and made the final selections. According to the Faculty Coordinator, one of the qualities they were looking for in the applicants was a "genuine caring" (ln. 541). "We need enthusiastic people, people who haven't been in a job where they just feel they are performing because they have to. There's none of that in this group at all. [That enthusiasm] shows. It is not something I have trouble identifying. I think it just shows" (ln. 564-571). At least one student, Cynthia,
Table 2
Instructor Information

<table>
<thead>
<tr>
<th>Name</th>
<th>Degree</th>
<th>Teaching Experience</th>
<th>Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaime</td>
<td>B.A., Math</td>
<td>13 years</td>
<td>Math 116</td>
</tr>
<tr>
<td>Sharon</td>
<td>M.Ed., Math Educ.</td>
<td>10 years</td>
<td>Math 116</td>
</tr>
<tr>
<td>Janelle</td>
<td>M.Ed., Counseling</td>
<td>16 years</td>
<td>Math 116</td>
</tr>
<tr>
<td>Duane</td>
<td>M.S., Phys. Educ.</td>
<td>15 years</td>
<td>Math 116</td>
</tr>
<tr>
<td>Dennis</td>
<td>M.S., Applied Math</td>
<td>5 years</td>
<td>Math 116</td>
</tr>
<tr>
<td>Deborah</td>
<td>M.Ed., Math Educ.</td>
<td>4 years</td>
<td>Math 116</td>
</tr>
<tr>
<td>Catherine</td>
<td>M.B.A., Accounting</td>
<td>12 years</td>
<td>Math 117</td>
</tr>
<tr>
<td>Walt</td>
<td>M.A., Math</td>
<td>33 years</td>
<td>Math 117</td>
</tr>
<tr>
<td>Gene</td>
<td>M.Ed., Math Educ.</td>
<td>11 years</td>
<td>Math 117</td>
</tr>
<tr>
<td>Robert</td>
<td>M.Ed., Math Educ.</td>
<td>18 years</td>
<td>Math 117</td>
</tr>
<tr>
<td>Anne</td>
<td>M.A., Math</td>
<td>19 years</td>
<td>Math 120</td>
</tr>
<tr>
<td>Jeff</td>
<td>M.S., Math</td>
<td>11 years</td>
<td>Math 120</td>
</tr>
<tr>
<td>Michael</td>
<td>Ph.D., Math</td>
<td>13 years</td>
<td>Math 125a</td>
</tr>
<tr>
<td>Rashied</td>
<td>Ph.D., Applied Math</td>
<td>10 years</td>
<td>Math 125a</td>
</tr>
</tbody>
</table>

cconcurred. She pointed out that, "[The instructors] have a special caring even though they are college teachers" (In. 957-959).

Attitudes toward the summer program. Not only did the instructors care about their students, they also spoke of their admiration and respect for the goals of the summer program and of the many advantages of such programs for students. They also spoke of the advantages of the summer program for them as
teachers. They appreciated the smaller classes, the support of a tutor in the classroom, the requirement that students meet with a tutor at least four days a week, and the provision for drop-in tutoring for those students who wanted additional help. They liked the opportunities to get to know their students, to be able to talk with them about how mathematics would fit into their lives and careers--opportunities made possible by the intensity of a six-week session, smaller classes, and the structured activities that brought teachers and students together outside of the classroom. Michael, who had taught in the summer program the year before, said, "I came out of last summer feeling really revitalized about teaching and really enthusiastic. I love this program. I'm really gung ho for it. It really works" (In. 1526-1527).

One of those structured activities, the volley ball tournament, served as a rallying point for the instructors and a way to participate in the summer program in a non-academic manner. Initially the faculty were barred from participating in the tournament, seemingly due to the concerns of the group leaders organizing the activity that a team other than a student team might take home the trophies. This was not an unfounded concern; the previous year the mathematics faculty team had done very well. This year's team had the potential to do equally well. There were two volley ball coaches, one former member of the university women's volley ball team, and a semi-professional volley ball player in the line-up. Perhaps more
important, however, was the enthusiasm and the seemingly genuine desire of some of the faculty to participate in the tournament as a way to interact with their students. Eight of the fourteen faculty and the Faculty Coordinator participated in this Saturday morning activity.

The faculty persevered and, after a meeting with the organizers and the Director of the summer program, the faculty team was allowed to compete. They chose "Michael and the Concepts," as their team name, a reference to the many discussions that had occurred during the summer about the relative importance of emphasizing concepts or skills and techniques in the mathematics classroom. Each faculty team member wore a red t-shirt that sported a mathematical concept on the back to identify the players. The concepts displayed on the backs of jerseys included $e^0$, $\pi i$, and -17. I had been invited to play with the faculty team. Since I was not a mathematics teacher, I suggested that the concept on my jersey should be "odd." Also, I asked if I could be characterized as "real" but with a touch of imagination, that is, an imaginary number. As a result of my requests, I was assigned to wear the square root of $7 + 3i$, a concept I was told could be characterized as odd, radical, prime, real, complex, imaginary, and . . . irrational.

The faculty won every game, but the compromise that had been struck, which allowed the faculty to participate, had been that only student teams could advance to the finals, a compromise that the faculty accepted with grace.
In addition to the increased interaction with their students that the summer program format afforded, faculty members seemed to enjoy and appreciate the opportunities to meet with each other to share ideas--opportunities that were rare or even non-existent during the regular school year. All the mathematics classes met from 7:00 a.m. until 9:00 a.m. The previous year, faculty members tended to congregate at their mail boxes after class. The daily gatherings had provided such a convenient way to keep faculty informed of summer program news and events, that the Mathematics Faculty Coordinator asked the faculty at their first meeting if they would continue that practice this summer. So every day, the faculty gathered at their mail boxes in Old Main, sat in the lounge for a few minutes, and talked with one another about their classes, the content of up-coming tests, and other happenings in the summer program.

The faculty did not seem to be meeting this year because they had been asked to meet, but rather for the same reason they had met the year before: It was a time to touch base with one another and to share ideas. Janelle remarked, when asked if there was a collegial atmosphere among the instructors, "I think even more so than during the school year because there's more time to sit around and share" (In. 2263-2266). She also said, "I like to hang out with teachers because they always challenge my thought processes" (In. 536-538). The summer program schedule made it easy, if not to "hang out" with each other, certainly to
have more contact with each other than they might have had during a fall or spring semester.

At the informal faculty meetings that occurred each day after class, conversations among the faculty were often about their teaching. Michael said that he had been helped by those informal meetings as he had tried to accommodate the summer program requirements to his own approach to teaching: "One thing that I did find very helpful [the previous year] was we'd meet informally after every class at 9:00 a.m. and . . . we found a lot of good information got swapped" (In. 1725-1731).

Other Participants

Other participants in the study include students, program administrators, tutors, and program staff members. In some cases, comments from participants other than instructors could be used to corroborate a judgment made about an instructor's theoretical orientation. For the most part, however, interviews with participants other than instructors were used to understand the context in which the instructors were working.

Data Collection

There were three means by which data were collected: interviews, a survey completed by the faculty, and observations of the summer program, particularly the mathematics classrooms.
Interviews

Interviews were conducted with each instructor using an adaptation of an elicitation technique used by anthropologists, the heuristic elicitation technique described by Black and Metzger (1969). This method uses open-ended questions to elicit initial responses from participants and closed-ended questions to clarify the researcher's understanding of the responses. The interview guide is included as Appendix A. Students, program administrators, and program staff members were also interviewed in order to gain some understanding of the context in which instructors were working.

The researcher conducted all of the interviews. Each interview was audio-taped and transcribed. Interviews typically lasted 45 minutes to an hour; four interviews extended to an hour and a half. One instructor was interviewed at the beginning of the summer program and again on the last day. Interviews were conducted with only the interviewer and the interviewee present in order to create an atmosphere that was free from distractions and one in which the interviewee could express him or herself freely.

Survey

A survey was distributed to all the mathematics instructors asking them to comment on what, if anything, they had done on the first day of class to help students use their textbooks. The survey is included as Appendix B.
Observations

Classroom observations were conducted in each of the thirteen classes (one class was team taught) for a minimum of two hours per class; five instructors were observed at least twice. Field notes were taken during the observations using a laptop computer. The sequence of events was noted. Of particular importance were any references to reading or reading instruction made by instructors. Verbatim notes of the instructors' comments were taken where possible. Classes were also tape recorded. In addition to classroom observations, observations were made and field notes were taken at both formal and informal faculty meetings, faculty in-service activities, formal and informal tutor meetings, tutor in-service activities, tutoring sessions, and social activities.

Data Analysis

The interviews were coded using a constant comparative method (Glaser & Strauss, 1967) to develop coding categories. Those categories are included as Appendix C. The coded material was analyzed four times: first, to determine teachers' beliefs about the nature of mathematics; second, to determine teachers' beliefs about the nature of reading; third, to determine if those beliefs about the nature of mathematics and the nature of reading could be characterized according to the theoretical perspectives that inform the Siegel et al. (1989) synthesis of mathematics and reading. Finally, they were analyzed in terms of instructors'
practices regarding reading mathematics. The Ethnograph (Seidel, Kjolseth, & Seymour, 1988), a set of computer programs designed to aid in data management, was used during the process of analyzing the data.

The following categories emerged from the initial analysis regarding teachers' beliefs about mathematics: teachers' metacognition; language; what should be emphasized in mathematics class; why students should study mathematics; purpose [of mathematics class]; what makes mathematics difficult; factors important for student success. The following categories emerged during the second analysis regarding teachers' beliefs about reading: teachers' metacognition; language; what makes reading mathematics difficult; teaching students to read mathematics; prior knowledge.

In the third analysis, teachers' statements considered to implicitly state their theoretical conceptions of mathematics and of reading were analyzed vis-a-vis the theoretical perspectives that inform the Siegel et al. (1989) synthesis of mathematics and reading. The theoretical perspectives of mathematics as a way of knowing and reading as a mode of learning were characterized, for the purposes of data analysis, as "constructivist" perspectives; reading as a set of skills and mathematics as facts and techniques were characterized as "not constructivist." The teachers' statements were then coded using the constant comparative method (Glaser & Strauss, 1967) as either constructivist or not constructivist. If not all of
a teacher's statements were coded as either constructivist or not constructivist, the instructor was placed on the Siegel et al. (1989) grid according to the predominant category.

In the fourth analysis, classroom observations, instructor interviews, and the surveys were analyzed in terms of instructors' practices regarding teaching students to read mathematics. References to reading or reading instruction found in the field notes, interviews, or surveys were coded as "teaching reading mathematics" (TRM). For example, if, in class, an instructor referred a student to the text for any reason—to find an example, to read directions—that was coded, during the analysis, as TRM. Likewise, if instructors asked students to translate a mathematical expression into English, pointed out a meaning of a word, or made a suggestion for ways to make sense of the text such as suggesting that students write in their books, those instances were also coded as TRM. In the interviews, the instructors' comments about anything they did or said to help students read the textbook were coded using the same designation.

When these references were analyzed, four categories emerged from the data. The first included those references that underscored the notion that reading was emphasized; the other three included, respectively, references that could be categorized as strategies that students could use before they read, while they read, and after they read. The items in each of the last three categories were further
analyzed as to whether they were more consistent with a conception of reading as a set of skills or as a mode of learning. The first category was excluded from that analysis because it seemed not useful as a way in which to discriminate between the two conceptions of reading. Finally, instructors' conceptions of reading mathematics were compared to the conception of reading that seemed to be associated with each of the particular practices the instructors employed.

Summary

An ethnographic study was conducted to explore the mathematics component of the summer program. The primary participants were the mathematics instructors who were teaching in the program. Data were collected via interviews, a survey, and observations of the summer program, in general, and the mathematics classrooms, in particular. These data were analyzed using a constant comparative method (Glaser & Strauss, 1967). The Ethnograph (Seidel, Kjolseth, & Seymour, 1988) was used to assist in managing the data.

Unlike the mathematics classes conducted during the regular terms, the mathematics classes of the summer program were expected to be conducted in a particular manner. There was an official expectation that instructors would follow the Jo Kemper Method. They were expected to follow a particular order of instruction and to strive to help students to become independent learners. They were not to lecture and the textbook was to be central to the instruction. An in-
service workshop was conducted for the mathematics instructors to help them to become familiar with those expectations.

Chapter Four discusses the findings based on the analyses of the interview, survey, and observation data. Instructors' conceptions of the nature of mathematics and of their conceptions of the nature of reading are determined, then characterized vis-a-vis the theoretical orientations that inform the Siegel et al. (1989) synthesis of mathematics and reading. Finally, teachers' practices regarding teaching mathematics are compared to their theoretical orientations.
CHAPTER 4

FINDINGS: "IT'S NOT LIKE READING A NOVEL. OR IS IT?"

This chapter discusses mathematics instructors' theoretical orientations regarding reading mathematics as characterized by the theoretical orientations represented in the Siegel, Borasi, and Smith (1989) synthesis of mathematics and reading. In addition, it presents the findings regarding the relationship between the instructors' practices surrounding reading mathematics and those theoretical orientations. The first section of the chapter discusses the mathematics instructors' conceptions of mathematics; the second section discusses their conceptions of reading. The third section presents those conceptions of reading and of mathematics in terms of the Siegel et al. grid. Finally, instructors' practices regarding reading mathematics are discussed.

Conceptions of Mathematics

Instructors' conceptions of the nature of mathematics were characterized as being compatible with the conception of mathematics as a set of facts and techniques or mathematics as a way of knowing. Eleven of the fourteen instructors in this study were categorized as holding a conception of mathematics as a set of facts and techniques. Comments from selected instructors are presented in the next section to illustrate that conception. Three instructors were
categorized as holding a conception of mathematics as a way of knowing and are discussed in the section, Mathematics as a Way of Knowing.

When instructors' comments were coded, no instructor's comments were coded exclusively into one category. Instructors were categorized on the basis of the code that predominated in the analysis. Two instructors, Walt and Catherine, had an equal number of comments coded in each category. They are counted among the eleven instructors in the mathematics as a set of facts and techniques category because of the nature of their comments regarding mathematics as a way of knowing. Those comments, while not compatible with mathematics as a set of facts and techniques, were not sufficiently indicative of mathematics as a way of knowing to warrant their inclusion in that category.

Mathematics as a Set of Facts and Techniques

Instructors whose comments led to an inference of a conception of mathematics as a set of facts and techniques tended to focus on "tools" and "skills." All instructors said they wanted students to understand the concepts underlying the formulas and techniques they were learning, but in at least one case, understanding was not seen as always obtainable or even necessary. Other instructors focused on learning the techniques as a way to know the concepts rather than using deeper understanding of the concept to help them learn the skills. Also, these instructors thought it important that students practice problems,
partly to increase their speed in performing the operations, but also so that they would recognize similar problems when they encountered them again. Focusing on "practice" rather than on "gaining more experience" suggests that there may be a finite number of kinds of problems to be solved and that mathematical learning can take place outside of any meaningful context. At least three instructors contrasted reading mathematics to reading a novel; it was unclear, however, in what ways they thought it was different.

The metaphors that instructors in this category used included giving students "tools" (Dennis, In. 198), offering them "cookbook steps" (Gene, In. 145), and expecting students to "absorb" (Deborah, In. 795) the ideas and concepts. Instructors in this category were also more likely than the other three instructors to talk in terms of "giving" (Anne, In. 80) information to students. One described herself as an "imparter of knowledge" (Sharon, In. 19). Brief descriptions and comments from instructors whose conceptions of mathematics seemed to fit the category of facts and techniques follow.

Jeff's comments suggested that he is a teacher who sees mathematics as a set of facts and techniques. Jeff was teaching the pre-calculus class. When asked what should be emphasized in a mathematics class, he claimed to be "torn between two things" (In. 239), conceptual understanding and procedural skill. As
he talked, however, it became clear that skills were more important than understanding:

I would like to emphasize concepts [so that] students truly understand, but if they understand but can't do it, don't have the skill, then I don't know. . . . I feel like I'm kind of ultimately responsible, skill-wise [to see that] the students can do the things they have to do. And if I can get them to really understand what they're doing also, that's all the better. . . . If I can get them through on skills and master those things, that's great. And then try to push the more deeper conceptual things as I go along, as I can. (In. 241-356)

Gene also emphasized the skills as almost a prerequisite for understanding the concepts:

What we want them to catch at the end is the concept that we're trying to teach, the actual reason behind what you're doing. In the course of that, there are cookbook steps that you have to show them. If you show them the cookbook steps they can do these steps and get the answer. . . . After we can learn the cookbook steps then we see the reason behind the steps. (In. 143-172)
Dennis used the familiar metaphor of the tool box when he talked about categorizing problems:

I want them to try and categorize things into certain types of problems. It's like having a tool box sitting there and if I know whenever I see a bolt that I want to use a wrench, then I'll always pull out that wrench. If it has a little Phillips head on it, I'll pull out that and use that. Sometimes, though, maybe you run into something that's different; maybe you need a slightly different tool. Or maybe you can adapt another tool to use that's in your tool box.

(In. 197-216)

One instructor's conception of mathematics was categorized as a set of facts and techniques based on his comments regarding the possibility of "interpretation" in mathematics. In discussing the possibility of different interpretations of a mathematical problem, he simply denied that could be the case. Using two hypothetical Ph. D. candidates in mathematics as the participants in order to suggest mature students of mathematics rather than beginners, I asked Rashied if when they read a mathematical problem would it "say" the same thing to each of them. He replied that the "main idea may be the same, but they may have different ways to look at it and to come to the same point. . . . A few
problems have different ways of solving the problem. The final [solution] will be
the same" (In. 1109-1145).

I used an analogy of the story of Jack and the Bean Stalk to explain what I
meant by interpretation. I suggested that one reader may decide that Jack was
lucky and another may decide that Jack was successful not because of luck but
because he was clever and had planned well. I suggested that both interpreta-
tions may be supported by the story. Rashied's response was,

Yes, I understand, but mathematics is not like that. You cannot read it
that [way] . . . . The story means you can interpret different ways, you can
see different ways, and you can interpret your own different things. But in
mathematics, you cannot do it that way . . . . The solution is the same. So
even if you come this way and this way, you come to the same solution.
(In. 1231-1269)

Anne wanted a class in which she could focus on "critical thinking," but
quickly added, "But then I have the syllabus I'm supposed to cover" (In. 160). She
talked about her responsibility as one of "giving them the tools" (In. 166) they
needed in order to succeed in the next class, but ended up "wishing I had
somehow taught them to be more independent about it and more thoughtful" (In.
170). She said, "My class is more of a class that is a tool, rather than a vehicle to
say, 'Okay, now. We're really going to think" (In. 310). Mathematics is difficult,
she claims, because, "It's different from a class that lets you discuss things and think about the implications into real life" (In. 372).

Walt's and Catherine's comments were equally divided between the two categories, but were included here because their comments coded as mathematics as a way of knowing were not deemed to be significant enough to tip the balance in that direction. Walt, for example, saw himself as both a "facilitator" (In. 13), a term coded as compatible with the way of knowing designation, and an "instructor" (In. 14), which connoted, for Walt, presenting material to the group and was coded as compatible with the facts and techniques category. Catherine saw herself as a "facilitator" (In. 9), but also seemed to focus on breaking the material down into manageable pieces (In. 58) and on there being clear instructions in the textbook (In. 493), thus suggesting that the textbook, not the students, was the source of mathematical knowledge.

Mathematics as a set of facts and techniques was unambiguous, free from uncertainty and doubt, and not open to discussion. Students could be "given" mathematical knowledge. The reason often stated for why students might study a particular class was that it was necessary as preparation for the next class. Students were being helped to become independent, not in terms of constructing their own mathematical knowledge, but in terms of being able to reproduce the mathematical information presented in the text on their own.
Mathematics as a Way of Knowing

Three instructors, Robert, Michael, and Janelle, were categorized as holding conceptions of mathematics that were compatible with viewing the subject as a way of knowing. Two of the three acknowledged the possibility of considering mathematics from a constructivist viewpoint in terms of the nature of mathematics, that is, whether mathematics was true and absolute, or fallible and open to interpretation. The third instructor did not comment on the nature of mathematics in those terms, but was categorized here on the basis of the strength of her statements regarding how students construct mathematical knowledge.

These instructors used metaphors that differed from their colleagues. They spoke of its being "my job to see if I can pull [ideas] out of [students]" (Robert, ln. 334), of the fact that "kids really do take the ball" (Robert, ln. 301), of students "owning" (Janelle, ln. 1393) the problems, and of trying to "hone in on [students'] majors" (Michael, ln. 332) as a way to contextualize mathematical problems. They were also aware that from their perspectives the line between teacher and learner was blurred. Brief descriptions and comments to illustrate their conceptions follow.

When asked about the nature of mathematics, Robert said it was both a tool and a philosophy depending on whom you asked:
Math to an engineer is a tool; math to a doctor is a tool; to a researcher, it's a tool. Math to a teacher, to a mathematician, is not a tool at all. It's a language. It's a philosophy. . . . There's a certain amount of mathematics that are concepts, that are at higher levels of thinking and are for philosophy. But other parts of mathematics are building blocks of things that we just do every day and that we need. We need it, and if no one ever taught us mathematics, we would have it anyway. We would develop our own; and, eventually, we'd probably develop [these concepts] all the same. (In. 1763-1805)

Mathematics, then, is a human creation, but it seems to be universal. In fact, the language of mathematics, Robert says, is universal:

It's absolutely universal. It exceeds all boundaries, culture, and everything. It was meant that way. When you think about the people who put mathematics together, they were very diverse.

Different worlds. And they all could speak to one another without the language of their native tongues. (In. 504)

But what is meant when we say that the language of mathematics is universal? It may mean, simply, that mathematicians are working with the same "alphabet," the same "syntax," and, in some cases, unambiguous meanings for symbols. Using the
same alphabet and using symbols according to conventionally agreed upon meanings, however, does not necessarily result in a freedom from ambiguity. Despite the "universality" of the language, mathematicians can, nevertheless, look at the same mathematical concepts from different perspectives, thus removing, or reducing, the universal quality from mathematics. But, according to Robert, such mathematical thinking occurs only among mature mathematicians:

There are mathematicians who [look at mathematics] philosophically--same concepts of mathematics, but present them in a completely different way. . . . It only happens after you understand the innate parts of mathematics, the tools of mathematics and the philosophy of mathematics. Then you talk like that, where you can look at mathematics from a completely different perspective. But you can't do that until you have this real solid base where you understand it all. (In. 1829)

As the conversation continued, however, he began to consider how such thinking did not have to be limited to mature mathematicians. He speculated about how one might consider such thinking as occurring within the minds of novice learners and offered an example of a first grader who might come to the conclusion that 1 plus 1 is 15:
You might look at the way he came to an answer that's totally bizarre and wrong. One plus one is fifteen. [The teacher may ask], "How did you arrive at that?" [The student may say,] "Well, I took your little rods and I put 15 little ones together and I made one [bunch of 7 rods] and I made another one [of 8 rods]." You don't want to stifle that idea. He's attempting to do something based on what he learned before. . . . [You can't say], "Oh, no. You can't do it like that." You've got to be careful because what is he really doing? Well, he's really learning how to problem solve here. He's [saying], "He gave me the problem and I'm giving him a different answer." Are we more interested in the answer or more interested in the problem? (In. 2507-2531)

Robert went on to say that he was open to the possibility that the child's answer might be acceptable:

[My] answer (2) is right. I trust my answer. Your answer is 15. And we can't trust your answer because no one's ever said that before, so the onus is on you. We can go back and look at what you did and maybe find some flaws. Or [maybe we'll find] some positive things about what you said and maybe I'll change my mind. Maybe I'll trust you and not the book. But not yet. . . .
I think that's where interpretation comes in. I think there has to be some open-mindedness about that. When you grade exams with older kids, you have to be very careful. That's why you have to require them to show you every step that they take. If [students] don't show [their] work, then you should just mark it wrong. And there should be no beef from [the students]. None. Because you're not communicating with me. I will accept your answer if you'll communicate with me in a reasonable and rational manner. . . .

First, I talked about coming into a problem from two different ways and I don't know that that's interpretation other than just different processes. But I think when you have this conflict—interpretation comes when there's a conflict. That is interpretation. Then the kid has to have the wherewithal to say, "Hey, this is how I got it." And then we'd better look [at it]. We're going to look at each one of those things. And that can be very exciting. Real exciting. (In. 2531-2584)

Robert was also excited by the discussions that could ensue when some students saw an alternative way of solving the problem from the way presented either by the teacher or by the solutions guide. The dilemma for Robert was that
while the discussion might interest him and the few students who were following it, he feared that the rest of the class would become bored:

A younger kid, who is just getting started [studying mathematics], . . . could interpret something based on the experience that we've shared . . . and could look at something in mathematics and say, "But I remember that and I want to apply it here. And you just want to go straight." And that happens all the time. . . . It happens every day in geometry. . . . You present them with a problem in geometry and you show how you think that they should solve that problem. Nine times out of ten, I'd get one kid who says, "But can't we . . .?" I get this all the time. You know what happens? That stops the class and then we get to discuss it. But what happens for the kids who are weak? They just fade away. They don't want to hear about it. So as a teacher you have to be real careful. You'd love to perpetuate that, but you want them all to be involved in this conversation. (In. 1929-1968)

Robert was quite aware that students seem to receive mixed messages from teachers. On the one hand, teachers may tell students that the process of finding the answer to a mathematical problem is what is important; on the other
hand, the students know that the answer is what counts as to whether or not the problem is "correct":

The answer is important, there's no doubt about it. But it's not everything. Winning isn't everything. We profess that it is; we want to win. But there are some achievements in the ups and downs of getting there even if you don't [win]. That's how you have to relate mathematics to them, too. It's not cut and dried. Although in mathematics it looks cut and dried. (In. 2256-2270)

Robert wanted to focus on students' strengths, rather than to "dwell on what they don't know" (In. 920). When students do not pick up an idea immediately, he claimed that teachers sometimes gave students "worksheet after worksheet after worksheet until [they] get it" (In. 858). Such an instructional technique, he claimed, kept students stuck in the same place unnecessarily. He advocated, instead, "presenting it in a thousand different ways . . . " (In. 924) and suggested that teachers should "just go on. They'll get it" (In. 922). He also seemed confident that the teacher was also a learner and the student was also a teacher:

You are not the god and they are not the lowly little kids out there that don't have any ideas and don't know more than you know. You have to be willing to let them speak to you, too, because you can
learn from them. You have to be learning, too. (In 930-938)

Michael also seemed to acknowledge two views of mathematics--one for mature mathematicians and one for novice learners. In our discussion of the nature of mathematics, he said,

This is going back to the old debate that keeps raging, and that is whether mathematics was discovered or invented. You know, discovered in the sense that math is already set; it's out there in the universe. Does the universe determine it or is it constructed of ideas in the human mind? Virtually all my students that I've ever talked to about it, are amazed that people actually invent new math or do new math that's never been done before. To them it's absolutely inconceivable. And I like to point out, "Well, calculus was new math that somebody came up with." At this stage, I think [students] don't discover it or invent it. They practice it more than anything. (In. 927b-962b)

He thought that mathematics could be open to interpretation, but mainly at the graduate and post-graduate level. And while he conceded that it was possible at lower levels, he maintained that it did not happen often:

You'd have to refer to it [different views of a problem] as a conjecture. That means all the evidence points to this idea or this
result, but you're not positive. You're missing a step of reasoning. You can't find a bridge from here to get to that result, but since the evidence is so overwhelmingly pointing in that direction, you can't use it to prove other things because it's unproving itself. But people can argue over conjectures. (In. 1158b-1175b)

When I asked him about interpretation, using my Jack-in-the-bean-stalk analogy, he replied,

No, I would not expect that type of interpretation to be going on unless you get to the 500 level courses and above. Then I think you might be able to interpret results or interpret conjectures in different fashions. (In. 1247b-1254b)

When I asked how a student got to the point of knowing that conjecture was even a possibility in mathematics if his or her entire experience with mathematics had been free of uncertainty and doubt, he began to consider that perhaps such thinking could go on at levels below the graduate level:

There's something referred to as mathematical maturity and that's the more math you take, the easier it gets to learn it overall. I think that the conjecturing ability grows with that. You begin to think about implications of what this theorem means, what this technique means. At the lower end of the scale, [Math] 116, [Math] 117, even
through calculus, I think it's possible to bring it into the classroom via the teacher, the teacher's guidance, and possibly through textbooks, but it's not done often. It should be, I imagine, when I think about it. It's an interesting idea, but... (In. 1266b-1286b)

It's an interesting idea, but... Michael seemed doubtful that students at this level can see the ambiguity that may be inherent in mathematics or do much more than practice mathematics that has been invented by someone else. Yet, in his own classroom, he seemed to be able to create a mathematical community in which students could do some inventing of their own. He encouraged students to work together, claiming with a smile, "When two or three work together, more often than not the individual pieces of knowledge that they have almost makes the whole" (In. 1642-1646). Unlike his colleague who claimed that mathematics classes did not lend themselves to discussions, he tried "to keep [discussions] as frequent as possible" (In. 1044).

He, too, called himself a "facilitator" (In. 1324) and was comfortable relinquishing the role of the authority in the classroom, saying, "I don't like a stern, authoritative structure. I don't think that is conducive to anything" (In. 1335-1338). I commented to him that I had observed that he responded to students' questions with questions of his own and students then seemed to answer their own questions. He replied, "If they don't know it, somebody sitting next to
them or down the table will offer a suggestion" (In. 1318-1320).

Sunil, one of the students in his class, corroborated that observation:

It's almost like we are creating calculus as we go along. . . . It's almost like we're teaching ourselves and he's like asking us questions. He's helping us self-teach ourselves. . . . In Michael's style, I think you get a better understanding of what you're doing. Because you're doing it; you're creating it, basically. . . . So I understand more. I'm creating the procedure of how to do this, how to solve this problem, what the problem is asking for. I know, because I'm doing it. . . . With Michael, you're creating the problem and solving it. With [other teachers, they are] giving you the problem; [they are] telling you. (In. 987-1092)

Janelle did not comment on the origins of mathematics knowledge, nor its nature in terms of its fallibility, but her comments regarding how students come to know mathematics suggested a conception of mathematics as a way of knowing. She stressed the need for "interaction" (In. 2434) with her students, pointing out that in order to analyze students' mistakes, students "have to be willing to interact with you" (In. 2434). Simply getting a response to a question that she had posed, however, was not interaction. She preferred "talk" to "responses," thereby
rejecting the typical teacher-student interchange of teacher question, student response, teacher evaluation (Mehan, 1979).

I ask them questions; they give me some response. I talk a little bit about that response. But there's no interactive talking back. . . .

What I would like to do is write something on the board, and say, "This is what it says," because they can all tell me what it says. They can all "typewriter" real well. . . . [And then say], "What do you suppose that means?" And then get their response from that. And with that response, [ask another question]. . . . It keeps them talking back and forth. (In. 2581-2613)

Talking back and forth was what Janelle was after. Even though she was administering the quiz as the program prescribed, she thought that it was not the best way to focus students' attention on their reading because it focused rather arbitrarily on a necessarily limited number of ideas from the chapter; furthermore, it stifled conversation:

I don't like the idea of giving five different questions and then going back and picking up five different concepts, or critical points. . . . I think [it would be better] if I would ask one [question] and [the students] all thought about it for awhile and we all shared on it for awhile. And maybe I'd have to ask another question to get you to
gain insight. [The teacher] gets a one-shot deal; that's why I don't like it. I put the question up there [on the board] and if you don't have a clue, [the quiz] comes in blank. Now if you don't have a clue, maybe I didn't ask the question right. Give me another chance. Let me see how can I rephrase that and still get at the same idea. (In. 3067-3097)

Janelle, like Robert and Michael, was comfortable being a fellow learner in the classroom. She explained her notion of her role in the classroom:

I always thought that a teacher was someone who led you to learn on your own, but shared insights. I grew up in a kind of question-answer environment where the question was valued as much as the answer. And the process for arriving at the answer was the valuable part, not necessarily the answer. It's pretty hard in that kind of cycle--where the question is as valued as the answer--to separate roles as to who is the teacher and who is the learner. (In 381-397)

Conceptions of Reading

Instructors' conceptions of reading were characterized as either reading as a set of skills or reading as a mode of learning. Eleven of the fourteen instructors were categorized as having a conception of reading that was more akin to reading as a set of skills than to reading as a mode of learning. One of the eleven had no
predominant code, but was placed in the reading as a set of skills category based
on the nature of those comments. Illustrative comments from those instructors
are included in the following section. Three instructors were categorized as
holding a conception of reading as a mode of learning and their comments are
included in the section, Reading as a Mode of Learning.

**Reading as a Set of Skills for Extracting Information From Text**

Instructors whose comments were coded predominantly as revealing a
conception of reading as a set of skills, tended to focus on learning vocabulary, on
learning to verbalize mathematical expressions correctly and being able to
translate them into English, on reading slowly and carefully. Since the meaning
was primarily in the text, instructors looked for text features to aid the process of
extracting that meaning: bold face, important ideas highlighted or placed into
boxes, and clear examples. Instructors did not attempt to activate students' prior
knowledge partly, perhaps, because of their conceptions of reading. For some
instructors, however, to talk about the ideas before students read was seen as
tantamount to telling them what was in the text and that ran counter to one of the
goals of the summer program, which was to help make students independent
learners. Instructors also focused on the non-alphabetic symbols of mathematics
as being a major stumbling block for students trying to read mathematics texts.
Janelle was one instructor whose conception was that reading was a set of skills for extracting information from text. She said that one factor that made reading mathematics textbooks difficult was the lack of redundancy, a concern shared by one of her colleagues, Jaime. He pointed out, "When [you are] reading a math book, if you find fewer words, it is almost perfect. . . . If there is one extra word, you can erase it" (In. 960-986). But the problem of lack of redundancy could be overcome by careful reading. Janelle gave an example of concise writing that students were sure to find difficult:

We don't dress up anything that needs to be said. It is just said.

The statement is there. Here is one--I love to give this example.

The statement is: "The logarithm is the exponent." Four words. . . .

And that's how quick it comes: "The logarithm is the exponent."

And then it starts giving examples. . . . If you read all the examples that followed that and you did not comprehend what they said in those four words, there's no way any of those examples would make sense. (In. 1585-1611)

In other words, the examples explained the statement, but one had to understand the statement in order to make sense of the examples. Confronted with that bit of circularity, Janelle, nevertheless, maintained that the statement
would make sense, "If you read it. If you, in fact, know how to read it" (In. 1644-1645).

Knowing "how to read it" began, for Janelle, with knowing how to pronounce mathematical expressions correctly. She said, "You have to learn how the pronunciation goes with that symbolism just like you did when you learned the alphabet of another language" (In. 1803-1807). She referred to the way some students read mathematics as "typewriter" (In. 1861) reading. Such reading was akin to "spelling" (In. 1866) the mathematical expression because of the students' apparent inability to correctly translate the mathematical expression into English. The number, one million, five-hundred, thirty-seven thousand, six hundred, forty-two (1, 537, 642) would be verbalized by students, Janelle claimed, as "one, five, three, seven, six, four, two" (In. 1943-1944). Likewise, the expression, $a(x-y)$, which should be verbalized as "$a$ times the quantity $x$ minus $y$" (In. 1751), will be pronounced as "$a$ parenthesis $x$ minus $y$ parenthesis" (In. 1753-1754).

But the emphasis on pronunciation raised the question of whether we could assume a student understood what an expression meant simply by knowing that they could pronounce it correctly. Janelle replied:

That parenthesis has a piece of information. The piece of information is that you must do everything inside there first. . . . You read it "the
quantity." But it means you have to do what's in there first. So we have reading and we have meaning. (In. 1823-1835)

We have reading and we have meaning. Even though we cannot assume that students know what something means even if they can pronounce it correctly, and even though Janelle recognized that there was a difference between reading and meaning, nevertheless, it seemed that Janelle advocated beginning with correct pronunciation.

When asked what made one textbook better than another in terms of its being easy to read, Janelle said it depended on, "How much they use bold face and how clear their examples are" (In. 2469-2471).

When asked what made reading mathematics difficult, Gene said it was the vocabulary. But it was also a matter of explanations not being clear enough, of not progressing smoothly from easy problems to more difficult ones:

They [should] start off with an easy example and then show all the steps that they do to get from the top to the bottom. Then go to a harder one with a similar concept. Some books start with an easy one, but then the next problem they do is a hard one that's not even related to [the first] one. Some books just have three hard ones. (In. 447-458)
Catherine, too, focused on learning vocabulary and the non-alphabetic mathematical symbols as the factors that make reading mathematics difficult. She said, "I think about half of learning math is learning terminology and the symbols that are used. Math is full of symbols representing words and concepts and that makes it difficult" (In. 544-550). Catherine also commented on the lack of cohesion in the text: "Sometimes you'll be reading through the text and it will change topics and it takes a paragraph before you realize that they've changed" (In. 773-780). Students sometimes asked her, "How did they get from here to here?" (In. 799-800). She would respond, "Well, they are really on a new topic here. This doesn't relate to what's above it" (In. 801-806). She found that aspect of the textbook a distraction, but she could deal with it and thought that students could, too, given enough time: "I've gotten used to it. But it's going to take the students a lot longer to get used to it" (In. 789-791).

Deborah mentioned the mathematical symbols in response to the question of why it is sometimes difficult to read mathematics. She suspected that students have difficulty with the symbols because of their inexperience with reading them. But her suggestion to her students regarding what they could do to overcome that difficulty was to slow down and reread:

You just have to have patience and read things slowly and sometimes [with] the repetition of reading things, eventually it will
sink in. . . . I really don't know any better way of telling them how
to read a textbook than to take it slowly and reread things. (In. 873-877)

Jeff did not have a predominant category. On the one hand, he seemed to
think of the meaning as being in the text: "When students ask about things,
particularly when they bring up things I know are right there in the book and
they're spelled out very well, we try to refer them to the book" (In. 736-741). He
added, "If they really read it well, I shouldn't have to talk about anything other
than a few minor questions" (In. 812-815). On the other hand, he wanted students
to understand that "math is [not] something you do by yourself" (In 596-597) and
that includes reading mathematics:

If they are working all together, the hope would be that even when
they're reading an assignment, they're not doing it alone. Ideally, it
would be nice if they were sitting down [with] a friend reading the
same section. [If] they got stuck, [they'd say] "Hey, I just read this.
I don't know what the heck they're talking about. Do you know
what they're talking about?" And between them, they might be able
to work something out. (In 788-798).

Ironically, one of Jeff's students had the same idea. Martha, a student in Jeff's
class, was reading with her friends. Perhaps because she had thought of that
approach on her own, she characterized it, not as the way to read mathematics, but as "weird" (Martha, In. 231). She agreed with Jeff, though; she thought it was a good idea:

I have some friends that come over. We all do our work together. We all read together, too. It helps a lot. We read it and then talk about it. It's kind of weird. Different. (In. 216-231)

Read slowly; reread; learn the vocabulary; learn the symbols; follow the examples carefully. Certainly not bad advice. But advice that presupposes a particular conception of reading. Meaning for these instructors seems to be in the text, almost exclusively, and they suggest, therefore, that students reread, or read more slowly in order to understand more thoroughly. Also, there seems to be the assumption that vocabulary and symbols can be learned successfully outside of any context and outside of any meaningful purpose for learning them.

Reading as a Mode of Learning

Three instructors, Anne, Robert, and Michael, were categorized as holding a conception of reading as a mode of learning. Anne and Robert seemed more able than their colleagues to acknowledge the role of the reader in making sense of the text. Michael required his students to write an outline as a response to their reading, preferring that activity to the quiz as a way to ensure that students would read the textbook before coming to class.
Anne seemed to recognize the contribution of the reader as she struggled with the question of how much prior knowledge an author should presuppose. She recognized the burden placed on the reader if the writer presupposes too much, but also recognized the burden placed on the writer if he or she cannot presuppose some things. Initially, she thought this was just one more way in which reading mathematics was different from reading in other disciplines, but she corrected herself as she talked: [Students] don't realize that there is a step left out. I usually assume that they know there are steps left out, and I tell them to try to fill them in. But sometimes they don't know what that means either. I think that's a real source of frustration. That's another way [a mathematics text] differs from other books. Well, that's not true, you know? You read history or something, there's a lot of stuff left out. There's a lot of background knowledge that's assumed, and you pick up a sort of advanced text or you pick up something that's from the source and you can't understand it either. And so you need your teacher to fill in that background information. In a way, the math book is exactly like that. The steps that they skip, in a good text, are things that they say, "Well, you know them anyway. That's background information, so why should I put it in?" I mean, you
would be writing huge, huge volumes if you put in every single step of the logic along the way. (In. 611-648)

Anne also sometimes asked her students to "see if they can draw a picture or something that makes sense to them" (In. 1133-1135) in response to their reading. And she attempted to discourage students from trying to memorize something or "doing something else besides putting it into their brain's logic. I just try to point out how you have to be able to say it so that you know that it makes sense to you" (In. 1145-1153).

Robert, like some of his colleagues, urged his students to reread, but he seemed to have a different understanding of why rereading might be helpful. Unlike Jeff, who urged rereading because he suspected that students simply were not thinking while they read a passage the first time--"They don't stop to think about it at all. They just want to go right through a whole page at a time and then . . . they haven't a clue" (In. 887-892)--and unlike Deborah, who appeared to consider the need to reread unique to reading mathematics, Robert wanted students to reread as a way to activate their prior knowledge. He did not see it as a strategy that was unique to reading mathematics:

One of the strategies that you teach a kid in mathematics is that when he's reading a problem, to read it more than once. Is that so different from [rereading] a newspaper article or a novel? I don't
think so. When someone asks you a question, "Did Moby Dick die?" you have to go read to know and it might take you a while and you might have to read it a couple of times. And the same thing happens in mathematics. I mean, "How do you find the square root of 2?" You have to go back and read that and bring with you the whole idea, first of all, of what is a square root and what is the value of 2 and how do I put these together. (In. 755-785)

Students, Robert claims, have a difficult time bringing their past experiences to bear on a new mathematical situation. They see things as unrelated and do not consider that they may already know something that could help them to solve a new problem:

It's so hard for kids to understand that it's all related. They think about this book as being Section 6-4 and Section 6-5 and all they need to concentrate on now is 6-5, not thinking that all the things they've learned along the way here are going to be important. . . . That's why word problems are always so scary for everybody because word problems ask you to go back and bring your experiences forward and to attach what you already know to the words. (In. 565-593)
Robert said that he sometimes suggested that students read an easier book or reread a book they had used for a previous class when they had difficulty understanding a particular concept or procedure in the textbook.

Michael said he did not do too much to help students read the textbook, but he seemed to acknowledge that it was not just text features that made the reading difficult; the reader, too, played a role in making sense of the text. In order to help his students make sense of the text, he had modified the prescribed procedure for the summer program classes, particularly, the idea that a quiz was to be the first item of business each morning. Michael rejected that idea, calling it a "waste of class time" (In. 940) and "a silly idea to test [students] on something that hasn't been covered adequately" (In. 941-943). But he, too, wanted students to read the text before they came to class. His modification was to submit a quiz to the tutors and ask them to administer it during the tutoring session after he and the students had had a chance to discuss the assignment and work some practice exercises.

His solution to the problem of getting students to read before coming to class was to ask them to outline the chapter after they had read it, make up their own examples for an idea or a concept or a formula, and then to apply the idea or formula to their examples. Writing the outline had some advantages over the quiz, he thought:
When it comes time to study for the final, they have a complete synopsis of the course that they did themselves, in their own words, in their own thoughts. So they're not trying to interpret notes that they might have gotten from me. (In. 967-974)

When I asked Michael what made the text difficult for students to read, he did not focus on bold print or highlighted concepts. He said, "Most just aren't very interesting" (In. 848). Particularly, authors seem not to be able to provide interesting examples:

The emphasis [in a mathematics textbook] really does need to be on story problems, on interesting story problems. Instead of learning this for its own sake, they should give examples--real world examples--of why this might be interesting to know or what you can do with this information you just learned. (In. 482-490)

But instead of interesting examples, textbooks give students contrived examples. Michael said a text might include a problem in which students are asked to take the outside temperature every hour on the hour and set up a correspondence between time and temperature. "But," Michael said, "What's the point? Who wants to do that" (In. 910-911)?

For these instructors, the student is a factor in making sense of a mathematics textbook. Students can be helped with the process by acknowledging
what they already know and showing them how to activate that knowledge before they read, by giving them interesting, rather than contrived, material to read, and by asking them to respond to their reading by writing or, perhaps, by creating a diagram, model, or sketch.

Reading Mathematics

Using the Siegel et al. (1989) grid for framing the question of reading mathematics, ten of the instructors would be placed in Box I. One instructor is placed in Box II, one in Box III, and two in Box IV. See Figure 2. Since the grid

<table>
<thead>
<tr>
<th>Reading as a set of skills for extracting information from text</th>
<th>Mathematics as a body of facts and techniques</th>
<th>Mathematics as a way of knowing</th>
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<tbody>
<tr>
<td>Reading as a mode of learning</td>
<td>II</td>
<td>IV</td>
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<tr>
<td></td>
<td>Rashied Jaime Deborah Dennis Gene Walt Duane Catherine Sharon Jeff</td>
<td>Anne Robert Michael Janelle</td>
</tr>
</tbody>
</table>

Figure 2. A grid for framing the problem of "reading mathematics": Summer program mathematics teachers.
is composed of boxes, rather than a more open continuum, the instructors who had no predominant category are slightly misrepresented. A more accurate representation might be to place Jeff on the line between Boxes I and II and to place Walt and Catherine on the line between Boxes I and III. The theoretical orientations that inform Box IV are those that are most compatible with the NCTM (1989) proposed standards.

Practices Regarding Reading Mathematics

There was an official expectation that instructors would require students to read the text before coming to class and that the text was to be central to the instruction, but there was seemingly no similar expectation of what instructors were to do or say to help students learn how to read the text. Even so, instructors' approaches to teaching students to read mathematics were very similar and are discussed in the first section below. If their approaches were similar, however, the suggestions instructors reported that they offered, or were observed offering, varied. The frequency and nature of those suggestions are discussed in the next two sections.

Teaching Students to Read Mathematics

Michael seemed to put the responsibility for the student reading the textbook squarely on the teacher when he said, "Students who aren't pushed, rarely read a textbook. They go right to the homework exercises . . . which is a
shame because it really does make a difference when they do read it" (In. 453-462). The rest of the faculty agreed with Michael's statement and supported the Jo Kemper Method prescriptions that students be required to read the textbook before coming to class and that the textbook be central to the instruction. But as Jo Kemper pointed out, there is a difference between requiring students to read and helping them learn how to read.

The approach instructors seemed to take regarding helping students learn to read the textbook was to mention strategies that students might find useful. I did not observe anyone teaching students how to employ a particular strategy, although two instructors reported that they modeled for their students the reading behaviors they expected of them. Dennis told me:

I sat down and said, "When I am reading a book, this is what I do."

I try to make myself a bit of a role model. And we sat down and we read a couple of sentences. . . . We went to a random spot in the book, where I knew they couldn't know everything in this sentence. And I get to this and I say, "OK, What's a polynomial? This word sitting here. What is it?" And so I said, "How are we going to figure out what this is?" And they said, "Well, we could look for it in the book." (In. 1141-1166)
Janelle said that she also sometimes modeled the behavior she was seeking. She wrote mathematical expressions on the board and read them to her students or asked students to read them to her. Students confronted with the mathematical expression, \( \log_b x = y \), will read it, she claimed, "\( \log_b x \) equals \( y \" (In. 1717). But, she pointed out, "It says, 'the log base \( b \) of \( x \) equals \( y \'). Now you have to know that that says 'base \( b \) because that's an inherent part of that problem" (In. 1719-1723). Hearing someone read the expression who could supply "base \( b \)" from their prior knowledge helped students to verbalize that expression correctly.

**Frequency of References to Reading**

During the interviews, all of the instructors reported that they did something to help students read the textbook; 17 different ideas were noted. In response to the written survey, 11 of 14 instructors reported that they had offered suggestions on the first day of class on how to read the textbook. A total of nine ideas were offered, four of which were offered by more than one instructor.

During a total of 38 hours of classroom observations--each instructor was observed for at least two hours and five instructors were observed for at least four hours--I noted 35 instances where instructors referred to reading or reading instruction. (Those 35 instances do not include references to outlining in Math 125a, to the quiz in all classes, nor to the requirement that the text be read before class; those were course requirements and were referred to daily.) The 35
instances I refer to occurred during 24 hours of observation; no references were noted during 14 hours of observation. It should be noted, however, that most instructors worked individually with students during the second hour of class and it was not always possible to hear comments they made to individual students. Fourteen ideas were observed, nine of which were observed as being practiced by more than one instructor.

The interviews, survey, and observations together yielded 20 suggestions regarding reading instruction. It should be noted that instructors may have engaged in reading-related instruction at times other than when I was present in the classroom; furthermore, they may not have mentioned during the interviews or on the surveys, some of the practices they employed.

Nature of References to Reading

The nature of the references to reading either reported by instructors or noted during observations is reported in Table 3. A brief discussion of the categories in the Table and the items within each category follows. The categories emerged from the analysis of the data. I did not observe any instructor refer to the suggestions from that perspective, nor did anyone mention such categories during the interviews or on the survey.
<table>
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<th>Survey</th>
<th>Observation</th>
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<td>Reading is required</td>
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<td>Reading is important</td>
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<td>The text is central</td>
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<td>Activities Before Reading</td>
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<td>Set purpose</td>
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<td>Read for main ideas</td>
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<td>Read technical texts</td>
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<td>Clarify text inconsistencies</td>
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<td>Read line-by-line</td>
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<td>Use examples</td>
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<td>Translate math into English</td>
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<td>Use text features</td>
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<td>Use Solution Guide</td>
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<td>Emphasize vocabulary</td>
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<td>Write in the book</td>
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### Activities After Reading

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<tr>
<td>Recite</td>
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<td>Take the quiz</td>
<td>7</td>
<td>14</td>
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<tr>
<td>Outline the chapter</td>
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**Note.** "Interview" and "Survey" column totals indicate number of faculty reporting each item.

*a* Observed in every class.

*b* Included in the "Text is central" category.

*c* Reading as a "mode of learning" references.

### Category I: Reading is Emphasized

References to reading that stressed the importance of reading the text were grouped into Category I. Requiring that students read the text before coming to class and making the text central to the instruction were both basic tenets of the Jo Kemper Method.

**Reading is required.** Three instructors, during the interview, and one instructor, via the survey, suggested that what they did to help students read was to require that they read. For one instructor, that was the extent of the help that he offered. As noted earlier, requiring students to read the text is necessary, but probably not sufficient in terms of helping students learn how to read.
Reading is important. At least two instructors, during the observations, made explicit comments about the importance of reading. Sharon said, for example, "You're going to have to read; [even] if you don't like to read, you're going to have to read" (In. 290, ob.).

The text is central. The faculty emphasized the importance of the text in ways other than simply requiring that it be read. Seven instructors either reported that they referred students to the text or were observed doing so. When students asked questions, they were referred to the text to find the answer rather than the instructor simply giving them the answer. When students, for example, encountered problems that they claimed did not make sense, Anne reported that she told them, "Find me an example in the book where it doesn't make sense" (In. 675-677). She continued, "I like the students to feel that they can get something out of a math book" (In. 693-696). Duane took the same approach, but thought it was easier to apply with the textbook used the year before, which had exercises in the margins to help students work through problems:
[Students] would ask me a question in class . . . and I would say, "Let me see your margin exercises." And they hadn't done them yet, so I'd say, "Well, you do that, and I'll come back in a minute. See what it says in the book." It kind of forces them to read the book. . . I never leave them hanging, [but] I want to make sure that they've given the book every possibility before they come to me. (In. 543-557)

Dennis also wanted to be sure that his students read the book, although he may have been more willing than some of his colleagues to locate a suitable example for his students:

If I have a student say, "I don't know what I'm doing here; I don't understand this," usually the first thing I do is go to an example that's just like it and say, "Read this and I'll come back." (In. 532-539)

Category II: Activities Before Reading

The items included in each of the following categories were analyzed in terms of which conception of reading they represented. The items, in and of themselves, were not thought of as representing one conception of reading instead of another; rather, it is the context in which those activities were suggested that
led to the determination that they were more closely aligned with reading as a set of skills or with reading as a mode of learning.

Two of the activities suggested for students as before reading activities could be characterized as activities consistent with a conception of reading as a mode of learning. Those two activities, activating background knowledge via either an alternative text or instructor comments, were observed or reported by only two instructors.

The remaining items, in the context in which they were used, were determined to represent reading as a set of skills. Students were not, for example, encouraged to set purposes that could be construed to be personal; rather, they were encouraged, for example, to be attentive to the instructors' quizzes and use that knowledge to help them determine purposes. Also, when instructors clarified inconsistencies in the text, it was done, not before students read, where it might have been construed as a way to activate prior knowledge, but after students had read.

**Activate background knowledge--instructor.** Only one instructor, Anne, reported that she sometimes talked with students about the ideas they were going to be reading about in order to help them read more effectively. She was also observed providing that assistance. At one point, she said, "Then we're going to
work in groups; I'll have to stop you and talk about what is the one-to-one function, so you'll know what the book is talking about" (In. 221-226).

**Activate background knowledge--an easier text.** One instructor, Robert, reported that he sometimes suggested that students reread the text from the previous class in order to review--or, perhaps, learn for the first time--concepts presupposed by the present class.

**Set Purpose.** Five instructors indicated that they helped students to set purposes before they read. At least two instructors suggested that students be attentive to the types of questions that they asked on the quizzes and use that information to guide their reading. One said that he told his students:

First, you have to learn me. You have to learn who I am to figure out what kind of questions I'm going to ask. And then when you do that, then you'll have to go back to read that book and you'll have to read the book with the idea of, "Is that the type of question he's going to ask?" (Robert, ln. 1014-1022)

Janelle emphasized that students should ask questions before reading as the questions would "lead to a more detailed reading" (survey).

**Read for main ideas.** One instructor told his students to read for the main points; another wrote the section headings on the board when he made the
reading assignment in order to help students recognize the main ideas, but he did so without comment.

**Read technical texts.** Four instructors reported that they talked with their students about how to read mathematics or how to read technical texts.

**Clarify text inconsistencies.** Two instructors were observed alerting their students to problems in the text. One instructor said, "We'll find that this book is a bit contradictory; I'll try to point out those places" (Sharon, In. 92b, ob.) She said, however, that she usually pointed out those problems after students read the text.

**Category III: Activities While Reading**

Instructors made some suggestions concerning what students could do while they read. Some of those ideas—reread, read slowly, read line-by-line—suggest that instructors thought that the meaning was to be found more in the text than as a result of a transaction between the reader and the text. Even references to using the examples, which seemingly suggest that students are actively engaged in making sense of the text, seem to reflect the emphasis on reading slowly and carefully rather than an emphasis on constructing one's own meaning.

Robert's use of the suggestion to reread, however, seemed to be offered in the spirit of reading as a mode of learning. He suggested that students reread so
that they could have more time to "bring with you the whole idea" (In. 781) of the ideas represented. Also, two instructors suggested that students write in their books as a way to be engaged with the ideas in the text.

**Reread.** At least five instructors told their students that once was not enough. They reminded them that they would sometimes have to read the material more than once before it made sense. Catherine said, "I tell them to read through it to get the idea of what it's about, what it is that they're trying to do in this section. Then I tell them to go back and reread it. And this time reread it line-by-line" (In. 445-450).

**Read line-by-line.** At least three instructors reported that they told students that they could not skip parts of a mathematics text. They needed to read line-by-line and to think about the significance of each word.

**Use examples.** A frequently mentioned suggestion was for students to pay particularly close attention to the examples. Instructors reported that they told students to work through the examples with a pencil and paper. Catherine said that she told students to,

> Work through the examples, not just read them. Actually work through them with a piece of paper and a pencil. Make sure that they understand each step of the way, what was done, and how the
explanation applies to the example. We did a section recently where there were many very good examples. If they work through those examples step-by-step they couldn't help but learn what it was they were to learn from that section. (In 451-462)

Duane reported that he told students about a specific strategy. He used the 3x5 card technique, suggesting that students cover up portions of the example with the card and try to work through the problem on their own. This was one of the Jo Kemper strategies. Duane had worked in the summer program for at least six years and had seen the Jo Kemper video tape.

**Translate mathematical expressions into English.** At least four instructors were observed asking their students, "How would you say that in English?"

Janelle wrote mathematical expressions on the board and asked students to read them to her since, as she said, "There is not a say-after-me section in any math book. . . . Nor is there anywhere where it teaches you how to read this stuff" (In. 1732).

**Use Text Features.** Four instructors mentioned that they alerted their students to the various features of the text that could help students read more efficiently. For example, they reminded students to pay attention to information
set apart from the rest of the text in green-shaded blocks, or highlighted by bold print. One instructor reminded his students to use the index.

**Use Solution Guide.** Three instructors reported that they talked to students about the importance of using the *Solution Guide* as they were reading and working through the problems. The *Solution Guide*, as the name suggests, included the answers to the problems as well as a step-by-step explanation of how the authors arrived at those answers.

**Emphasize Vocabulary.** The reference to reading most frequently observed was instructors' clarifying a term or reminding students to be more precise in their usage of a term. These discussions usually came after students had read an assignment, so could not be construed as teaching the vocabulary before students read, but more as reminders of the need for clarity and precision while one reads. For example, Dennis said, "Inverse and invert don't always mean the same thing, so be careful" (In. 73-74, ob.).

**Write in the book.** At least two instructors urged students to write in the book. Janelle acknowledged that they had probably always been told not to write in their books, but, she said, "I know you've been told all through your life, 'Don't write in your books.' This is your chance to get revenge on the last twelve years
of school. A good thing to write in your book is, 'I have no idea what they are doing" (ln. 141-146, ob.).

Category IV: Activities After Reading

The after reading activity that every instructor used was the required quiz. Some students complained about not being able to do well on the quizzes because the quiz preceded any discussion of the material, but the consensus among the instructors seemed to be that if the students read the book carefully, they should be able to do well on the quizzes. Questions were not posed in the form of problems to be solved; rather, the questions focused on definitions or material that had been highlighted and the questions generally assessed what is sometimes referred to as "literal" comprehension. This activity seemed to reflect the view of reading as a set of skills.

Asking students to be able to summarize what they have read, to put it into their own words, or to create an outline of the chapter, creating their own examples, seems to reflect more the view of reading as a mode of learning.

Recite. Four instructors reported that they told their students that they needed to be able to verbalize what they had read when they finished reading. One instructor suggested that they be able to summarize what they had read.
Take the quiz. All instructors administered a quiz on the reading. Twelve of the fourteen instructors administered it at the beginning of the class period. Michael and Rashied submitted a quiz to the tutors to be administered during the tutoring session.

Outline the chapter. Michael and Rashied required their students to outline the reading assignment as an after reading activity.

Relationship Between Teachers' Beliefs and Their Practices

Instructors' conceptions of reading were predominantly that of reading as a set of skills. Likewise, their references to reading were predominantly to activities that could also be characterized as being informed by that conception of reading. Excluding the references to reading included in Category I, Reading is Emphasized, there were a total of 79 references to reading. Of those, 12 could be considered to represent a view of reading as a mode of learning. Those 12 references are noted in Table 3.

Of those 12 references to reading, half of them were made by three instructors who acknowledged the role of the reader. Anne thought that a student's prior knowledge was important and she said she sometimes talked about concepts before sending students home to read in order that they would have
some background before they read. She was also observed explaining a concept before students were to read about it. Robert also thought that a student's prior knowledge affected his or her ability to read the text and he said that he sometimes suggested that students seek out an easier text to read. Michael urged his students to write their understandings of what they had read rather than simply testing them on it. On average, the instructors whose conceptions of reading were more akin to reading as a mode of learning were twice as likely to engage in reading practices that reflected that view than were instructors whose conceptions were more akin to reading as a set of skills.

The remaining six references were distributed among six instructors: Janelle and Dennis urged students to write in their books; Jaime told students to summarize what they had read, and Catherine and Jeff said they told students to recite what they had read when they had finished. Rashied, who acknowledged that what students already know is important, nevertheless, limited his help with reading mathematics to telling his students to read the book. Even though he team-taught with Michael and they required students to write the outline after reading, the outline was initiated by Michael, not by Rashied.
Summary

The conception of mathematics held by the instructors in this study was, predominantly, mathematics as facts and techniques. Three instructors, Robert, Michael, and Janelle, held conceptions of mathematics as a way of knowing. They were included in that category on the basis of their views of how students construct mathematical knowledge more than on their notions of the origins and nature of mathematics. Robert and Michael can talk about mathematics as a constructivist conception and their practice reflects that conception; however, it was not clear that they held that conception consistently.

The conceptions of reading held by the instructors in this study were predominantly that of reading as a set of skills to be used to extract meaning from text. Three instructors, Robert, Michael, and Anne, acknowledged that the reader contributed to making sense out of the text and they were categorized as having a conception of reading as a mode of learning.

Using the Siegel et al. (1989) grid for framing the problem of reading mathematics, ten of the instructors would be placed into Box I, one into Box II, one into Box III, and two into Box IV. Box IV represents the theoretical orientations presupposed by the NCTM (1989) standards for reform and by the view of reading represented by the Goodman model.
Even though it was assumed that instructors would require students to read the text, and even though it was emphasized that reading should be central to the mathematics courses, there was no prescription for instructors to follow regarding what they should do to help students read their mathematics textbooks. There was seemingly no expectation that instructors would give particular information regarding reading with the exception of telling students that they would be required to read the text before class. Nevertheless, almost all instructors reported that on the first day of class they discussed reading the textbook. They offered that advice that they felt was most pertinent to mathematics students. After the first day, any further instruction or advice regarding reading was offered on an ad hoc basis.
CHAPTER 5
IMPLICATIONS: CONNECTING "SEPARATE" WORLDS

This chapter presents a brief summary of the study followed by a statement of the conclusions. Finally, it offers some comments regarding implications.

The Study

The present study explored teachers' beliefs about the nature of reading and about the nature of mathematics. Using the theoretical orientations implicit in teachers' beliefs about reading and about mathematics, those orientations were characterized vis-a-vis the theoretical orientations that inform the Siegel et al. (1989) synthesis for mathematics and reading. Instructors' beliefs about reading mathematics were, thereby, examined. Instructors' practices regarding reading mathematics were noted and examined to determine the relationship that adhered between instructors' beliefs about reading mathematics and their practices.

The Siegel et al. (1989) synthesis of mathematics and reading is represented by a grid for framing the problem of reading mathematics. Both reading and mathematics are represented by dichotomous views. Views of mathematics are characterized as either mathematics as a body of facts and techniques or mathematics as a way of knowing. The former is a Platonic
viewpoint; the latter is a constructivist view. Views of reading are characterized as either reading as a set of skills for extracting information from text or reading as a mode of learning. The former is represented by the LaBerge and Samuels model and the Gough model, the latter by the Goodman model.

An ethnographic study was conducted. I was a participant-observer (Spradley, 1980) in a six-week summer program for ethnic minority students at a large research university in the Southwestern United States. The primary participants were the fourteen mathematics instructors who were teaching in the program. Data were gathered via interviews using an adaptation of the heuristic elicitation technique, a survey, classroom observations, and observations of the summer program, in general. Data from these sources were analyzed using Glaser and Strauss' (1967) constant comparative method.

Twelve instructors held conceptions of reading that were consistent with their conceptions of mathematics. Of those twelve, two held conceptions that could be characterized as constructivist; ten held conceptions that were not constructivist. Two instructors held conceptions of reading that were not consistent with their conceptions of mathematics. Of those two, one held a constructivist conception of reading but not of mathematics; one held a constructivist conception of mathematics but not of reading.
Teachers' practices reflected their theoretical orientations. Just as the majority of the instructors held a view of reading as a set of skills, the majority of the references to reading reflected that view. Only twelve of the seventy-nine references to reading observed during classroom observations or noted in the interviews or surveys seemed to reflect the view of reading as a mode of learning. Instructors whose beliefs about reading were more akin to reading as a mode of learning were more likely to engage in reading practices that reflected that view than instructors whose view of reading was reading as a set of skills.

Conclusions

The following conclusions were drawn from this study:

*Instructors hold beliefs about mathematics that can be characterized vis-a-vis the theoretical orientations that inform the Siegel et al. (1989) synthesis of mathematics and reading.

*Instructors hold beliefs about reading that can be characterized vis-a-vis the theoretical orientations that inform the Siegel et al. (1989) synthesis of mathematics and reading.

*Instructors' practices regarding reading mathematics were related to their beliefs about mathematics and about reading.
Implications

Various implications can be drawn from this study. Following are implications for theory, research, and practice.

**Implications for Theory**

The study has implications for the Goodman (1967, 1972, in press) model of reading and for the Siegel et al. (1989) synthesis of mathematics and reading as represented by the grid for framing the question of reading mathematics.

**The Goodman Model**

For a model of reading to be accepted as an adequate model, it must meet certain criteria. Such a model, according to Goodman and Goodman (1978) must be applicable to the following factors of reading: all characteristics of text, reader, syntax and grammar, semantic systems, memory and perception as they involve language and cognition, orthography, and conditions of reading (pp. 2:13-14). The question arises as to whether or not reading mathematics is "reading" in the same sense as "reading" is used in the Goodman model and, therefore, if the Goodman model is applicable to reading mathematics. That question was not explored directly in this study; rather, the present study assumed the applicability of the Goodman model and nothing in the analysis of the data disconfirmed that assumption.
The question, however, remains an important one if reading teachers are to work with mathematics teachers in helping them to develop ways to help students learn to read mathematics. If reading mathematics is, in fact, qualitatively different from reading in other disciplines, that is, the theoretical orientation that informs the Goodman model can not be extended to reading mathematics, then a new model, or a revised model, is necessary. If reading mathematics is not, in fact, qualitatively different from reading in other disciplines, an assertion I am willing to posit based on the lack of evidence from the present study to the contrary, but mathematics teachers think that it is, then efforts will have to be made to help mathematics teachers understand reading mathematics vis-a-vis some theoretical conception of reading. As has been argued in Chapter One, the theoretical orientation regarding reading that supports the view of reading mathematics called for in the NCTM (1989) standards for reform in mathematics education is the Goodman model.

That mathematics instructors could entertain the notion that reading mathematics is qualitatively different from other kinds of reading may be suggested by the study. Three instructors in this study told me that reading mathematics was not like reading a novel. One possible interpretation of that statement is in terms of Rosenblatt's (1978) notion of stance. Readers reading a
novel may be more likely than readers reading mathematical exposition to adopt an "aesthetic" (p. 27) stance; readers reading mathematics may be more likely to adopt an "efferent" (p. 27) stance. In terms of the stance adopted, then, reading mathematics is not like reading a novel, but is not qualitatively different from other kinds of reading; therefore, the present models apply to reading mathematics as they apply to reading other disciplines.

Another interpretation is also possible. Mathematics instructors may be claiming that reading mathematics is different from reading any other discipline and, therefore, the present models do not apply. If the latter interpretation is found to more accurately reflect what some mathematics teachers mean when they talk about reading mathematics, then efforts will need to be made to address those understandings.

The Siegel, Borasi, and Smith Synthesis of Mathematics and Reading

The Siegel et al. (1989) grid for framing the question of reading mathematics is a useful device for examining their proposed synthesis of mathematics and reading. It could, perhaps, be adapted to address disciplines other than mathematics. A limitation, however, is the implication, perhaps unintended, that instructors would clearly hold one view or the other regarding reading and mathematics. Such an implication can possibly be inferred by their
use of the term box to distinguish the four possible conceptions of reading mathematics. Often we think of things as being either in or out of a box. Granted, we can think of things as sticking out of a box, spilling over the edges of a box, pushing the top off a box, and, thereby, imply that something is not all the way into the box. A box, nonetheless, particularly as it is represented in the grid, is a fixed container that does not suggest fluidity or flexibility. As Green (1971) points out, however, it is commonplace for people to hold contradictory beliefs. The theoretical orientations may be represented adequately by a model that suggests a dichotomy rather than a continuum; however, the complex, and sometimes contradictory, nature of many instructors' beliefs mitigates against such a simplification.

A more helpful representation may be Richardson, Anders, Tidwell, and Lloyd's (1991) quadrants. Their construction is similar to Siegel et al. (1989), but Richardson et al. speak of teachers' beliefs in terms of a position on a continuum, thus suggesting more of the complexity that one is likely to find when speaking of beliefs and belief systems. Their term has the added advantage of suggesting a connection to mathematics.
Implications for Research

Implications for research include both additional questions to pursue and implications for research methods.

Additional Questions to Pursue

Research often raises as many questions as it answers. This study, likewise, raises some questions that may elucidate aspects of some of the issues surrounding teachers' beliefs and their practices. Those questions follow:

* Can the Siegel et al. (1989) synthesis of mathematics and reading, represented by the grid for framing the problem of reading mathematics, be adapted for other disciplines?

* Are theoretical conceptions of reading mathematics related to the teacher's preparation, particularly whether he or she prepared for service in an elementary, secondary, or post-secondary setting or has not had formal preparation in either mathematics or education? This question has been addressed in relation to conceptions of mathematics, but not reading mathematics, for example, by Rector and Ferrini-Mundy (1986) and Jones, Henderson, and Cooney (1986).

* To what extent are teachers' beliefs and practices regarding
reading mathematics affected by the context in which they work?
This question has been addressed by, for example, Cooney, (1985) regarding beliefs about mathematics, Richardson et al. (1991) regarding reading, and Wood et al. (1990) regarding beliefs about mathematics and reading.

* To what extent do teachers' beliefs about the nature of reading mathematics influence students' beliefs about the nature of reading mathematics? Harste and Burke (1977), for example, found that students' "predisposition to apply one theoretical model over another will be influenced by the instructional environment, i.e., the teacher's theoretical orientation" (p. 33). Lerman (1983) takes a similar position regarding mathematics. Borasi (1990) has investigated students' conceptions of mathematics and says that helping students overcome "inappropriate conceptions of mathematics" (p. 181) should be a "priority" (p. 181) for mathematics instruction.

Implications for Research Methodology

This study has implications for research methodology. Limitations can be imposed by the researcher's understanding and by time constraints. On the other
hand, it confirmed, for the researcher, Hoffman and Kugle's (1982) contention that interviews reveal more enlightening data about teachers' beliefs than paper and pencil methods of gathering the same information.

The researcher. Black and Metzger (1969) claim that the eliciting heuristic might begin with categories that may be outside the culture under investigation, but that those categories can be discarded once there is an initial set of responses and "from then on everything [the researcher] does depends on the last thing he did. The boundaries of the system he explores are revealed as he proceeds" (p. 138). They also warn, however, that it is sometimes difficult to detect those boundaries. No real information is obtained, they claim, until the researcher knows what question is being answered by a particular response. The dilemma is that "the ethnographer is greeted, in the field, with an array of responses. He needs to know what question people are answering in their every act" (p. 141, italics in the original).

These caveats are well founded. Since I was a reading teacher, not a mathematics teacher, I was not always aware of what questions were being answered regarding mathematics. Opportunities were surely missed because I sometimes did not fully appreciate what I was hearing when I was hearing it; later, as my own conception of mathematics broadened as a result of my
conversations with the mathematics instructors, I could see, when reading the transcript of an interview, where a different follow-up question, a more pointed probe could possibly have elicited a more enlightening response than the one elicited. Also, I carried with me some of the language of the culture of literature, language that was seemingly foreign to the culture of mathematics. For example, I used the term, interpretation, regarding mathematics, a concept for which we did not always have a shared meaning.

Time. Another limitation that affects the interpretation of this study is time. The summer program was conducted over a six-week period. My interpretations must be viewed in light of the fact that they are based on the number of possible interactions and conversations that can occur in that amount of time. Also, time limitations prevented me from presenting my findings to the participants. My findings have been neither confirmed nor denied by the participants; rather, they represent my interpretations without benefit of their instructive comments.

Sources of data. Despite these limitations, however, I concur with Hoffman and Kugle (1982) that interviews reveal enlightening data about teachers' beliefs and that that source of data reveals more information about teachers' beliefs and the frameworks in which they may hold those beliefs than
can be obtained in a non-interactive manner. For example, I submitted a survey to the instructors asking them to comment on their activities on the first day of classes since, presumably, that was when a substantial portion of any direct instruction regarding reading mathematics would take place. Obviously, I could not observe every teacher on the first day. While the surveys contained information that was useful, it became more useful when I was able to discuss it with the instructors during the interviews.

**Implications for Practice**

This study has implications for practice, both the practice of teachers, which is discussed in the first section, and the practice of teacher educators, which is discussed in the second section.

**Teachers' Practice**

This study has implications for teachers' practice. It suggests the need for increased opportunities for collaboration between mathematics teachers and reading teachers. It also suggests that the practices presupposed by the Siegel, Borasi, and Smith (1989) Box IV perspective take time. Finally, it suggests that alternative means of assessment and a wider variety of reading material are required.
Increased opportunities for collaboration. Reading teachers and mathematics teachers could seemingly benefit from increased opportunities for collaboration. There is a danger in ignoring the commonality between the theoretical orientations that inform the two disciplines. To do so could lead to situations similar to the one described by Wood et al. (1990) in which an elementary teacher participating in a constructivist oriented mathematics research project "dramatically changed her practice and beliefs about teaching mathematics . . . [but] this change did not influence her teaching of reading" (p. 497). It was as if, according to the researchers, she "seemed to live in two separate worlds" (p. 510).

But the two worlds need not be so separate. The parallels between the conceptions of reading and of the conceptions of mathematics and their respective instructional implications are striking. Confrey (in press), for example, referring to a constructivist mathematics research project, described the teachers' approaches to mathematics in terms usually associated with a constructivist approach to reading; she referred to them as "connected to whole language approaches" (p. 5). Her notion of "close listening," despite her contention that it is analogous to "close reading" (p. 16), seems to have more in common with miscue analysis (Goodman, Watson, & Burke, 1988) and kid watching (Goodman,
Learning letters and mastering letter-sound correspondences before reading meaningful materials is akin to learning computational facts in mathematics before attempting story problems. Seeing mathematics as static and fixed is akin to seeing only one possible interpretation in a story. Relying on authority of either axioms, the textbook, or the teacher in learning mathematics, rather than encouraging students to create their own mathematical strategies for solving problems, is akin to assessing a reader's comprehension of a story, not by listening to him or her talk about the story, but by forcing the reader's comments into answers to multiple choice questions that someone else has created.

Time. Teaching by listening closely, watching carefully, and interacting thoughtfully takes time. As Janelle pointed out, "It seems like the inquiry method is not the quickest" (In. 1225). She made that observation in the context of covering the material: "We have to cover the objectives that are set for [Math] 116 for a normal credit at the university" (In. 1232). Anne, too, felt those constraints when she wished that she could focus on critical thinking, but realized she had a "syllabus I'm supposed to cover" (In. 157). Teachers choosing to teach
from a more constructivist perspective must often choose to do so in an environment that seems not to support such an approach.

**Assessment.** Calls for assessing student progress in mathematics accompany the calls for reform. The National Research Council (1989) reports that "governors and political leaders in all fifty states are advocating assessment in order to raise expectations and evaluate programs" (p. 67). Often the kind of assessment that is advocated, that is, standardized, multiple-choice tests, are contrary to the objectives being sought by the NCTM (1989) reforms. The National Research Council (1989) suggests that "to assess development of a student's mathematical power, a teacher needs to use a mixture of means: essays, homework, projects, short answers, quizzes, blackboard work, journals, oral interviews, and group projects" (p. 70).

Michael, in fact, had administered an essay test for a makeup test in his calculus class once with "surprising" (In. 161) results:

I found that my students who did better on the first test, which was basically computational in nature, didn't do as well as the poor students did on the essay test. For some reason, the poor students could write a detailed, organized essay, everything they knew about it and they knew the information. But when presented with an
actual problem to apply it to, they got tripped up somewhere. I was very surprised. It made me think that maybe we are emphasizing the wrong things. (In. 177-195)

Emphasizing the wrong things in class and then assessing those things with inappropriate means of assessment will not help students learn to read mathematics more effectively.

**Textbooks.** Another implication for teachers' practice concerns the textbooks they use. The National Research Council (1989) quotes Harriet Tyson-Bernstein as saying,

> According to virtually all studies of the matter, textbooks have become the de facto curriculum of the public schools. . . . It is, therefore, critical that textbooks stimulate rather than deaden students' curiosity and that teacher manuals encourage rather than squelch teachers' initiative and flexibility.

(p. 67)

Janelle echoes that lament: "I hate it in math because where you're supposed to be is always defined by the text" (In. 437). Michael had complained that the books were not interesting and that the problems and examples were contrived. Robert complained that textbooks repeated "addition, subtraction, multiplication, fractions, decimals and whole numbers" (In. 1269) throughout
elementary school and into high school and, as a result, students were never exposed to the language and concepts of algebra or geometry or trigonometry. Textbooks, he said, did not give him the "ammunition" (In. 1304) that he needed:

A kid tests into high school and they put him into something called General Math. Guess what General Math is. Addition, subtraction, multiplication, division. Then he gets a D in General Math. He's not going to take Algebra I; he's not going to take Geometry; he's not going to take Algebra II; he's not going to take Trigonometry. He's going to take Consumer Math and guess what Consumer Math is. Addition, subtraction, multiplication and division, but this time with taxes. It's horrible. It's so hard to find a textbook. As a teacher you need ammunition. Your job is to deliver. But these textbook companies, their job is to give you the ammunition. And they don't give you much at all. They give you garbage. (In. 1269-1306)

If teachers begin to view reading mathematics from the perspective of the theoretical orientations that inform Box IV of the Siegel et al. (1989) synthesis of mathematics and reading, they may recognize the "power of stories" (Borasi, Sheedy, & Siegel, 1990) in learning mathematics and begin to request "rich
mathematical texts" (Siegel et al., 1989, p. 275), that is, "essays, dialogues, stories, even poems" (p. 275) to use as supplements or replacements for traditional mathematics textbooks. Textbook companies can either support or thwart such requests; if thwarted, teachers will be forced to gather and create their own materials.

Implications for Teacher Educators

Thomas Green (1971) asserts that teaching might be described as "the unending effort to reconstitute the structure of our ways of believing--the effort to reorganize our systems of belief" (p. 51-52). Elsewhere he says that it could be argued that, "Teaching is an activity aimed at developing belief systems of a particular kind" (p. 52, italics in the original). Given this characterization of teaching, it would seem to hold for any teaching situation--whether the students are college students trying to understand and apply quadratic equations or other teachers trying to understand and apply the curriculum standards offered by a professional organization.

When I asked Michael if he did anything to help students get ready to read a chapter, he replied, "That's interesting, but no. At the most I might sketch out what we're going to be talking about the next day, but actually, I guess I pretty much abandon them to reading it on their own" (ln. 1032-1039). Michael might
have been speaking about some teacher education programs. They, too, can abandon their students, forcing them to come to some understandings on their own. Pre-service and in-service teachers can be abandoned to think through on their own what they believe about the nature of their discipline, the nature of teaching, the nature of learning. They can be abandoned if their educational experiences focus on what to do at the expense of exploring or developing an understanding of how that particular activity or approach to instruction fits with their existing beliefs. It is particularly important that an understanding of the theoretical orientations presupposed by those activities be explored and developed. If teachers are abandoned to struggle with those issues by themselves, what may be abandoned, ultimately, is the proposed instructional reform.

The standards proposed by the NCTM (1989) arise from a particular conception of the nature of mathematics and the nature of learning mathematics. But like the teacher Wood et al. (1990) describe, researchers and teachers may be operating in "separate" worlds. That view of mathematics may be the prevailing view among mathematicians and researchers; it may not be the prevailing view among mathematics teachers. It was not the prevailing view among the teachers in the present study. Most of the mathematics teachers in the present study seemed to hold beliefs about the nature of mathematics and about mathematics
learning that differed from the beliefs inherent in the standards.

Unfortunately, the guidelines offered to teachers to help them implement the standards do not address the issue of the discrepancy that may exist between the beliefs inherent in the standards and the beliefs held by teachers, even though the NCTM (1991) acknowledges that the "beliefs and dispositions that both students and teachers bring to the mathematics classroom" (p. 2) can serve as "obstacles to making significant changes in mathematics teaching and learning in schools" (p. 1).

In the section, "Education of Teachers," in one of the responses to the national call for reform submitted by the National Research Council (1989), the issue of teachers' beliefs about mathematics and how those beliefs may affect their instruction is not addressed directly. It is addressed indirectly, insofar as it is acknowledged that teachers will teach as they are taught. Teachers need, according to that publication, to learn contemporary mathematics appropriate to the grades they will teach, in a style consistent with the way in which they will be expected to teach. They also need to learn how students learn. . . . They need to learn science . . . so that they can teach mathematics in the context where it arises most naturally. . . . And they need to learn the history of mathematics and its impact on society. (p. 64)
I found no evidence that the writers thought that prospective mathematics teachers also needed to address the issue of what they believed about the nature of mathematics and about the particular view of mathematics inherent in the proposed reforms in any manner other than participating in classes taught from a particular orientation to mathematics. Similarly, the Mathematics Association of America (1991) and the NCTM (1991) seem to assume that teachers will adopt a particular conception of mathematics if their teachers teach in accordance with the theoretical orientations implied by that conception. Certainly, that may be a necessary condition; whether it is sufficient is open to question. It seems at least possible, that prospective teachers and in-service teachers could focus on the activities involved in such teaching and learning without fully considering the theoretical orientations that inform those activities. Such a focus could help to explain why Jones (1988-89) reports no change in mathematics achievement over the past two decades.

But teacher beliefs are not ignored only by professional organizations writing for a vast audience. The teachers in the summer program were told what to do--they were to follow the Jo Kemper Method. There was an attempt made to justify the prescriptions--the need to help students to become independent learners, the need to get students to be active, rather than passive, in the
classroom—but seemingly little attempt to explore how those activities fit with the teachers' existing beliefs about mathematics and about reading mathematics. Such was not always the case. Jo Kemper characterizes the process by which those activities became part of the pilot program as one of "negotiation" (In. 619) between the other instructors and herself: "I have a vision in my mind of what I think math should be like in classrooms, but it is a negotiation between the math teachers and myself as to how far you can go with any of those ideas" (In. 614-621).

That negotiation seemingly takes place now in the mind of the instructor. Duane decides that he will lecture now and again because he believes he can be more effective with a lecture than without; Michael replaces the quiz with an outline because he believes that for students to write in response to their reading helps them learn mathematics more than the quiz does; Janelle wants to—but does not--replace the quiz with a conversation because she believes the quiz focuses too narrowly on too few topics. But those decisions were personal, private, and idiosyncratic. The prescribed approach was still prescribed, but without accounting for individual teacher's beliefs about mathematics or about how those particular activities fit with those beliefs.
A starting point for teacher education programs, including professional development experiences for in-service teachers, and especially regarding any attempt at instructional reform, would seem to be an acknowledgment of the beliefs inherent in those enterprises. At the same time, one would hope for an acknowledgment of the existing beliefs of the teachers. Starting with such acknowledgments may help us to move more effectively toward the attempt to reconstitute our belief systems, thus affecting curricular and instructional reform.
APPENDIX A

INSTRUCTOR INTERVIEW PROTOCOL
Instructor Interview Protocol

The following questions were used to elicit information about instructors' beliefs and practices regarding reading and study skills instruction.

1. How would you describe the summer programs?
2. What are your perceptions of the programs' goals?
3. Describe the advantages and disadvantages of the program.
4. What experiences have you had with programs similar to this one?
5. How would you describe yourself as an instructor? What is your role?
6. What do you think should be emphasized in a math class?
7. Why is it important for students to study mathematics? How do you share that information with students?
8. What do you think is the most important factor(s) in a student's success (or lack thereof) in mathematics? How do you encourage (or compensate for the absence of) the development of those factors?
9. What do you think makes mathematics hard?
10. What skills and knowledge do you assume of students when they enter this course?
11. How do you determine if those skills and knowledge are present?
12. Do you use any teaching practices in the summer program that are different from the practices you ordinarily engage in? If so, what are they?
13. What study skills do you think are important for students learning mathematics? What do you do to help students learn these skills?
14. How important is students' prior knowledge for success in college? For success in your class? What can an instructor do to compensate for a lack of background knowledge? How would an instructor do that? Do you do that? Why or why not?
15. What do you think makes this textbook easy or difficult to use?
16. If you could change this textbook, how would you do so?
17. What kinds of reading assignments are made in your class?
18. When you make a reading assignment do you typically give students any tips on ways to go about reading the assignment? If so, what are they?
19. What is it that makes reading mathematics hard?
20. Do you help students prepare for exams? How?
APPENDIX B

FACULTY SURVEY
Faculty Survey

1. Did you use an ice breaker or some other specific activity to help you and your students get to know each other better? If so, please describe it briefly.

2. Did you discuss study habits or study skills? If so, please note topics covered.

3. Did you give your students any tips about how to use their textbooks? If so, please note topics covered.
APPENDIX C

CODING CATEGORIES
Coding Categories

Teachers' metacognition
role
teaching style
teaching methods

Language
vocabulary
mathematics as a language

In-service
faculty

Jo Kemper Method
quiz
no pencil
no lecture
textbook
numbers on board

What makes reading mathematics hard
no redundancy
textbook features
steps left out
different language (see also language)
students do not know it is different from reading other things
more than one way to get right answer
an abstraction from real world
not risk takers
authors do not anticipate student problems
not like a novel
interpretation

Teaching students to read mathematics
emphasize importance of using book
making reading assignments
quiz
attention to details
Prior knowledge
(Background knowledge)

What makes mathematics hard
  confidence
  no real life applications
  do not know how to study for it
  see no purpose for it
  practice
  lack of prior knowledge (see also prior knowledge)
  bad teachers
  society does not value
  more than one way to solve problems/say something in mathematics

Factors for students' success (see also what makes mathematics hard)
  [focuses on what students do; what teachers do is coded with teachers' metacognition]
  patience
  practice
  prior knowledge
  confidence/attitude
  prior success
  family support

What should be emphasized in mathematics classes
  concepts
  practice
  skills
  tools
  procedures
  problem solving
  real world applications
  story problems
  calculations
  determined by textbook

Why mathematics?
Purpose of a particular class
   (big picture)

Independent learners

Summer program
   publications
   volley ball
   mathematics scholarship
   parents' day
   retention
   success of summer program
   expectations of students
   history
   personnel
   students
   relationship to mathematics department
   planning
   instructor background
   advantages/disadvantages
   room problems
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