A STOCHASTIC MODELING FRAMEWORK FOR ENVIRONMENTAL
STRESS SCREENING OF MULTI-COMPONENT SYSTEMS

by

Edward August Pohl

A Dissertation Submitted to the Faculty of the
DEPARTMENT OF SYSTEMS AND INDUSTRIAL ENGINEERING
In Partial Fulfillment of the Requirements
For the degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

1995
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As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Edward August Pohl entitled A STOCHASTIC MODELING FRAMEWORK FOR ENVIRONMENTAL STRESS SCREENING OF MULTI-COMPONENT SYSTEMS and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

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SIGNED: [Signature]
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"All models are wrong,
but some are useful!"

George Box
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ABSTRACT

Environmental Stress Screening (ESS) is employed to reduce, if not eliminate, the occurrence of early field failures. In this dissertation, a general stochastic modeling framework is presented for multi-component systems. Environmental stress screening can be performed at one or more assembly levels for a system. Systems are modeled as a series-series collection of components and connections. Components and connections are assumed to come from good and substandard populations and their time to failure distributions are modeled with mixture distributions. ESS models currently found in the literature assume that time to failure distributions are mixtures of exponentials. This dissertation extends previous work by examining mixtures of Weibull distributions for both components and connections. The mixed Weibull distribution is used to examine how screening strategies change when wear-out mechanisms are present. A further generalization is made by modeling components and connections with mixtures of Phase-Type distributions.

Optimal screening strategies are developed using a variety of criteria. First, a life cycle cost model is developed for a general series-series multiple assembly level system. This is the first multi-component, multi-screening level cost model with imperfect failure detection to appear in the literature. Failure detection capability is shown to have a significant impact on the optimal screening strategy. Other criteria examined include system mean residual life and system mission reliability. Finally, the impact of a systems structure on optimal screening strategies is explored.
CHAPTER 1

INTRODUCTION

Companies spend a large amount of effort and money each year on measuring and improving product reliability. Much of the effort is devoted to identifying design risks, failure mechanisms, and evaluating design alternatives. Despite the extensive up-front efforts put forth by companies, many products fall short of their designed-in reliability goals. Since major business decisions impacting corporate profits and performance are made on the basis of the failure properties of their products, it is imperative that the products achieve their designed-in reliability. Environmental stress screening (ESS) is a tool that will help products achieve their reliability goals.

1.1 What is Environmental Stress Screening?

The concept of ESS is not new, although widespread acceptance and use has been slow in coming. As with many reliability and quality initiatives, the military sector has led the way in defining and directing the use of environmental stress screening. The concept of ESS can be traced back to World War II when environmental testing was first initiated. Environmental testing was used primarily during the developmental stages to verify design adequacy. Design adequacy was verified by exposing system components and assemblies...
to environments that simulated the operational limits. The logic was that components and assemblies capable of functioning at the operational limits in the laboratory would perform satisfactorily in the field. The most common environments simulated were temperature, vibration, humidity, and altitude [38].

As technology matured, the complexity of communication systems, aircraft and weaponry increased. Consequently, more time and money were devoted to the design process. Despite extensive initial design efforts put forth by designers, many systems fell short of their designed-in reliability when placed in the field. Fielded systems began to have significant increases in early life failures. Design adequacy was no longer sufficient evidence of system reliability.

The reason systems were falling short of their designed-in reliability was due to the fact that defects were being introduced into the product during the manufacturing process. As the complexity of the systems increased, so did the associated processing and assembly procedures. Analysis of the early life failure data, Figure 1.1, showed that manufacturing defects were the primary cause of these early system failures. The most common defects were attributed to faulty and/or marginal parts, poor workmanship, and insufficient manufacturing processes [39].
In the 1950's, in an effort to eliminate faulty and/or marginal parts the concept of functional testing was introduced. Components underwent a functional test before they were used in the manufacturing process. In the 1960's, burn-in was introduced in an attempt to isolate manufacturing defects by passing components and assemblies through infancy by operating them for an extended period in benign environments. Burn-in was effective in precipitating defective components, but did not significantly stress the assemblies to identify manufacturing defects at higher assembly levels [119].

The military realized that static testing processes did not provide adequate simulation of the environments encountered in the field. To address this problem they introduced the concept of mission profile testing. The goal of mission profile testing was to subject the assemblies to a combination of different environments as well as the cyclical changes within those environments. Early efforts focused on temperature cycling. Results indicated that mission profile testing was effective in isolating the types of failures that were being encountered in the field. Analysis showed that it was the temperature cycling
between extremes more so than the actual temperature extremes that precipitated many of
the failures. Additionally, it was discovered that temperature cycling precipitated failures
in a much shorter time than constant temperature. As a result of these efforts, the concept
of ESS was born [38].

Through the work of the military community, industry, and various technical
societies, ESS has evolved from a test to verify design adequacy into a screening
process. This process transforms latent defects into hard failures, through the application
of appropriate stresses, before the system is placed in the field. ESS may be applied to the
manufacturing process at various levels of assembly. Screening should take place at the
most cost effective point of assembly. Those components, assemblies, and systems which
fail during ESS will be scrapped or repaired and only those that survive the stress screen
will be moved to the next level of assembly or shipped to the customer. The goal is to
identify latent defects during the manufacturing process where the cost to repair and/or
replace the failed units is significantly less than the cost if done after the system leaves the
factory.

Today, all military acquisition programs are required by DOD Directive 5000.2 to
implement some form of an environmental stress screening program. As a result of this
requirement, the most comprehensive definition of ESS is found in the Tri-Service
Environmental Stress Screening Guidelines [119]. ESS is defined as:

*Environmental stress screening of a product is a process which involves the
application of one or more specific types of environmental stresses for the*
purpose of precipitating to hard failure, latent, intermittent, or incipient defects or flaws which would otherwise cause product failure in the use environment. The stress may be applied either in combination or in sequence on an accelerated basis but within the product design capabilities.

1.2 ESS and the Reliability Bathtub Curve

The preconditioning of an item to improve its reliability by eliminating early life failures implies the existence of an infant mortality period. During the infant mortality period the conditional probability of failure decreases with operating time. A variety of different types of systems have been found to have an infant mortality period. Electronic, mechanical, biological, as well as social systems have shown infant mortality periods [27,28].

It is widely documented within the reliability literature that hazard rates of many electronic, electro-mechanical, and electro-chemical devices follow a bathtub curve. The classical bathtub curve is shown in Figure 1.2. The classical bathtub curve is a composite of three separate curves. The first phase, infant mortality, is characterized by a decreasing failure rate (DFR). It represents early failures due to material and/or manufacturing defects. As weak parts are identified and removed from the population, either by ESS or through normal operation, the instantaneous failure rate decreases. The second phase, "useful life", is characterized by a near constant failure rate (CFR). The failures are random in nature and are randomly distributed with respect to time. These chance failures are usually caused by sudden stresses or unusually severe operating conditions. The final
phase, "wear-out", is typified by an increasing failure rate (IFR). During this period, parts begin to deteriorate as a result of the accumulated damage from stresses encountered during its lifetime.

Figure 1.2. Classical Reliability Bathtub Curve [42]

Jensen and Petersen [51] propose an alternate form of the bathtub curve, referred to in the literature as the generalized bathtub curve. The generalized bathtub curve is shown in Figure 1.3. This curve consists of four phases. The component population exhibits an increasing, then decreasing failure rate prior to entering its useful life and wear-out phases. The generalized bathtub curve is a result of using a bimodal distribution to model the time to failure of the component population (Figure 1.4). The failure
distribution for many components and systems is bimodal due to the existence of strong and weak sub-populations. Jensen and Petersen [51] document numerous examples where electronic and mechanical components exhibit a bimodal failure distribution.

A fundamental assumption for all ESS models is that the manufacturing process produces products that have a mixture of sub-populations; weak and strong. This assumption of a mixture of sub-populations and the resulting infant mortality period provide the theoretical foundation upon which ESS is built. The use of mixture distributions will play a key role in the development of quantitative ESS models.

Figure 1.3. Generalized Reliability Bathtub Curve [51].
1.3 ESS as a Quality Control Activity.

By definition, ESS is a screening *process* which is utilized during manufacture in order to maintain the designed-in reliability of a product. Its purpose is to force latent defects, introduced during the manufacturing process, into hard failures. ESS is not a test! Unlike a test, the principle intent of ESS is to precipitate as many latent defects to hard failures as is economically feasible. Once latent defects are identified, the causes of these defects must be investigated and corrective action taken. Like other quality control activities, ESS is a closed loop process which to be effective requires feedback.
Unlike many other quality control methods, ESS is not a sampling technique. In order to be effective, ESS must be applied to 100% of the products at the most cost-effective levels of assembly. Screening of 100% of the products is necessary for several reasons. First, the market life cycle for many products is short. Thus, the production lines for these products will not have time to completely mature. Second, workmanship variability will never be eliminated. Despite the increase in the amount of automated production facilities, many complex products require the use of humans in the manufacturing process. The more human involvement in the process, the greater the variability in the products coming of the production line. Finally, the cost of early failures for some products is so high that all efforts to reduce the probability of early failures must be made. Most military systems fall into this category.

By definition, ESS applies stress on an accelerated basis but within product design limits so as to minimize the amount of time necessary to precipitate latent defects into hard failures. The goal is to accelerate the normal processes that leads to failure in the field without changing the nature of the failure. As a quality control activity, it is important that ESS not over stress the good products and consume an unacceptable portion of their useful life. Additionally, ESS must minimize its impact on the length of time it takes to manufacture products.

On occasion, ESS will identify design flaws in a product. This usually occurs when developmental testing of the product was insufficient. While this is a very beneficial consequence of ESS, it should be emphasized that it is not its intent. It is because of this
that ESS is often confused with reliability growth. Figure 1.5 illustrates the distinct difference between reliability growth and ESS. ESS shifts the failure rate function of a product to the left in order to avoid an early high failure rate for products entering the field. ESS will not increase the reliability of products in the field beyond their designed-in value. Alternatively, reliability growth shifts the failure rate curve down by making changes to the design. Thus, it is changing the designed-in reliability of the product.

![Figure 1.5. Reliability Enhancement Through: (a) ESS and (b) Reliability Growth [51].](image)

As a quality control function, ESS need not be restricted to the manufacturing phase of a product life cycle. In fact, The Tri-Service Environmental Stress Screening
Guidelines [119] recommends that ESS be used during engineering and validation phases. Utilization of ESS on components and assemblies during development testing offers several advantages. First, by eliminating non-design related failures, test stoppages are minimized and schedule delays avoided. Ninety percent of the failures encountered during hardware development can be attributed to screenable defects [37,39]. Second, utilizing ESS during development will help ensure that an adequate ESS plan is in place for production. Finally, it will identify problem parts and vendors before the product is in production.

ESS can also be applied to repaired/refurbished systems. The process should be designed to screen out any workmanship defects introduced into the system as a result of the repair work. If the system does not pass the screen it will be returned for further repair. The repaired item should only be released upon successful completion of ESS. Screens for repaired items must be carefully designed so as to not put unnecessary wear on the elements of the system that have not been repaired or replaced. This will ensure that defect free systems are being placed back into the field.

1.4 The Benefits of Environmental Stress Screening

ESS provides many of the benefits that most quality control activities provide. Like other quality control activities, ESS costs are incurred up front, where the majority of
the benefits occur later. The early expenditure of resources must be balanced against the potential for savings in the future. The up front costs for ESS can be significant.

A major contributor to the cost of ESS is the fact that it is hardware unique. Since different environments stimulate different defect types, it may be necessary to employ different screens at different points of the manufacturing process [102]. One of the more significant costs of ESS is the purchase of equipment capable of providing the necessary stresses to stimulate latent defects. The two most common environmental stresses utilized in industry are temperature cycling and random vibration. In order to generate these stresses a manufacturer requires a thermal cycling chamber and an electrodynamic shaker. Recently these have been combined into one piece of equipment. In addition to the capital investment for equipment, recurring costs such as utilities and labor increase the overall cost of ESS.

Despite these significant up front costs many companies have begun to institute ESS programs for their products. For example, Bendix corporation instituted an ESS program for their electronic fuel injectors which reduced field failures due to manufacturing defects from 23.5% to 8%. Similarly, Hewlett Packard installed an ESS program on the production of its 9826A computer line. Using thermal cycling, they reduced their production screening time from five days for normal burn-in to a few hours. ESS also resulted in a 50% reduction in the number of repairs made under warranty. Hewlett Packard estimated that the ESS program would save them more than $1.5 Million dollars over five years [38].
The previous examples show that a properly implemented ESS program can result in substantial benefits. The most significant benefits as outlined in the IES ESS Guidelines [37,39], the ESS Handbook [38], and The Tri-Service ESS Guidelines [119] are:

1. Reduction in Rework Cost. The earlier defects are found in the manufacturing process the less the cost to repair or replace. Often times, the cost to repair/replace increases an order of magnitude for each increase in assembly level.

2. Reduced Field Repair Expenses. Products that meet their designed-in reliability result in fewer warranty claims.

3. Improved Production Efficiency. A properly implemented ESS program will determine the sources of flaws and when possible, take corrective action. By removing the sources of defects, the production process becomes more efficient.

4. Improved Customer Satisfaction. Products that meet their requirements result in satisfied customers. Customer satisfaction will have a positive effect on market share.
The end result of a properly implemented ESS program is reduced life cycle cost for the product being manufactured. Table 1.1 summarizes the benefits of ESS for the various phases of a product's life cycle.

<table>
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<tr>
<th>Development Phase</th>
<th>Benefits</th>
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<tr>
<td>Engineering &amp; Manufacturing</td>
<td>- Improves reliability growth testing</td>
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<tr>
<td>Development</td>
<td>- Assures readiness of production screens</td>
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<td></td>
<td>- Weeds out problem parts and vendors</td>
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<td></td>
<td>- Minimizes schedule delays</td>
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<tr>
<td>Production</td>
<td>- Reduces rework cost</td>
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<tr>
<td></td>
<td>- Precipitates failures at convenient levels</td>
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<td></td>
<td>- Improves productivity</td>
</tr>
<tr>
<td></td>
<td>- Products approach designed-in reliability</td>
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<tr>
<td>Field</td>
<td>- Higher field reliability</td>
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<td></td>
<td>- Lower support costs</td>
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<td></td>
<td>- Fewer warranty claims</td>
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<tr>
<td></td>
<td>- Improved customer satisfaction</td>
</tr>
<tr>
<td>End Result</td>
<td>Improved Life Cycle Cost</td>
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</table>

Table 1.1. ESS Benefits

1.5 Scope and Organization of the Dissertation.

Despite its lengthy evolution, and the recent mandates for use, ESS remains more of an art than a science. The majority of the literature on the subject is qualitative in nature. Very few quantitative models exist. Of those that do exist, most are limited to component level screens. The main purpose of this research is to explore quantitative approaches to modeling environmental stress screening for complex systems. These
models will be utilized to explore how ESS strategies can be employed to maximize system reliability and minimize life cycle costs.

This study has been carried out with the primary intention of developing a general modeling framework for environmental stress screening that can be used for a variety of systems. Some of the key issues explored are:

1. How do we model the assembly process?
2. Where in the assembly process should ESS be performed?
3. How long and at what stress levels should ESS be performed?
4. How do we model the effects of multiple stresses on components?
5. What type of failure distributions should we use?

The first step toward answering these questions was to survey the literature on ESS and related areas. Chapter 2 provides a comprehensive summary of the relevant literature and develops the theoretical foundation for this research. Special attention is paid to the related areas of burn-in and accelerated testing. Also, a variety of ESS optimization criteria are discussed.

Chapter 3 presents a generalized framework in which optimal ESS strategies for a variety of systems can be developed. The chapter begins with the development of a multi-level screening model. Using the multi-level screening model, a general cost model is developed. The cost model examines the life cycle costs associated with performing ESS
at multiple assembly levels for a multi-component system. The second criteria examined is reliability. Two reliability models are formulated for a series-series system. The first model allows one to examine the effects of ESS on a system's mean residual life. The second reliability model examines how ESS affects a system's mission reliability.

The general models developed in chapter 3 are dependent upon the use of mixture distributions as well as renewal theory. Chapter 4 examines the mixture distributions and associated renewal functions for probability distributions of interest. Specifically, the exponential, Weibull, and log-normal distribution are examined. The chapter discusses the specific implementation issues as well as the assumptions associated with using each distribution family in the various models. The chapter also includes a discussion of phase type distributions. The properties of phase type distributions are analyzed. Phase type distributions are shown to offer the advantage of mathematical tractability as well as flexibility in the types of failure distributions they can represent.

Chapter 5 examines several case studies which demonstrate the versatility of the general model developed in Chapter 3. First, a two-level screening model is examined for a variety of mixture distributions. The effect of imperfect failure detection during the screening process is examined. Next, a three-level multi-component screening model is examined. Screening strategies are developed using three different optimization criteria; life cycle cost, mission reliability, and mean residual life.

Chapter 6 summarizes the main contributions of the dissertation and discusses some suggestions for future research.
CHAPTER 2
LITERATURE REVIEW

The purpose of this chapter is to provide a summary of the relevant ESS models and techniques found in the literature. These models and techniques provide the foundation for the research contained in subsequent chapters. ESS is a technique used to increase the quality of components and systems delivered to the field by operating the item under accelerated environmental conditions prior to shipment. The concept of preconditioning components and systems to increase reliability by eliminating those with short lives is very similar to burn-in. ESS utilizes accelerated environmental conditions, often more severe than the operational environment, to stimulate failures. Whereas, burn-in is usually performed under ambient conditions and therefore takes a much longer period of time to stimulate failures. Despite these differences, modeling of the two processes can proceed along similar lines [102]. Therefore, the evolution of burn-in is also examined in this chapter. Since ESS utilizes accelerated environmental conditions, a thorough review of stress acceleration models is contained in this chapter.

2.1 Hazard Rates and the Possibility of Improvement

Hazard rate, also known as the force of mortality, is a very important function for determining the reliability characteristics of an item. Hazard rate is important because it
characterizes the change in failure rate over the lifetime of a population of items. Martz
and Waller [72] point out that hazard rate allows different distributions to be distinguished
on the basis of physical considerations. They suggest that it is often easier to select a
distribution based on the shape of its hazard rate rather than the shape of the probability
distribution function. Mathematically, hazard rate is defined as the limit of the failure rate
as the limit approaches zero:

\[
h(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t \cdot R(t)} = \frac{1}{R(t)} \left[ -\frac{d}{dt} R(t) \right] = \frac{f(t)}{R(t)} \quad (2.1)
\]

An important relationship between the hazard rate and reliability function is:

\[
\int_0^t h(x) dx = \int_0^t \frac{1}{R(x)} \left[ -\frac{d}{dx} R(x) \right] dx = -\ln R(x) \bigg|_0^t \quad (2.2)
\]

assuming \(R(0) = 1\), then:

\[
R(t) = \exp \left[ -\int_0^t h(x) dx \right] \quad (2.3)
\]

which means that
Table 2.1 summarizes the relationships between the reliability function $R(t)$, hazard rate $h(t)$, probability density function $f(t)$, and cumulative density function $Q(t)$.

Watson and Wells [121] are often credited with introducing the concept of burn-in. They developed a set of general conditions for which the mean remaining life of an item, operated for some fixed time $t$, is greater than the original mean life. They showed that if the item had a DFR (IFR), then the mean residual life is increased (decreased) with burn-in. The mean residual life function, $\delta(t)$, is given by:

$$\delta(t) = E[X - t | X > t] = \frac{\int \infty \int R(y)dy}{R(t)}.$$  \hspace{1cm} (2.5)

Watson and Wells [121] examined the Weibull, gamma, log-normal, and extreme value distributions in detail.

Ebrahimi [36] proposes an alternative to the mean residual life function for characterizing a life distribution. He investigates the use of Shannon’s information measure applied to the residual life time. In doing so, he attempts to quantify the expected uncertainty contained in the mean residual life about the predictability of the lifetime of a component.
Given that a component has survived up to time $t$, he proposes to measure the uncertainty about $T$, the life of the component, at time $t$ by:

$$H(f; t) = -\int_t^\infty \frac{f(x)}{R(t)} \log\left(\frac{f(x)}{R(t)}\right) dx$$

(2.6)

<table>
<thead>
<tr>
<th>$f(t)$</th>
<th>$h(t)$</th>
<th>$Q(t)$</th>
<th>$R(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(t) = \frac{f(t)}{h(t)\exp\left[-\int_0^t h(x)dx\right]} \frac{d}{dt}Q(t) - \frac{d}{dt}R(t)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h(t) = \frac{f(t)}{1-\int_0^t f(x)dx} h(t) \frac{1}{1-Q(t)} \frac{d}{dt}(Q(t)) - \frac{d}{dt}[\ln R(t)]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q(t) = \int_0^t f(x)dx 1-exp\left[-\int_0^t h(x)dx\right] Q(t) 1-R(t)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R(t) = 1-\int_0^t f(x)dx exp\left[-\int_0^t h(x)dx\right] 1-Q(t) R(t)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Relationships Between Various Reliability Functions

Ebrahimi [36] develops two new classes of life distributions based on the notion of uncertainty; decreasing uncertainty of residual life (DURL) and increasing uncertainty of residual life (IURL). If a component has a survival function that belongs to the class of
DURL, then as the component ages the conditional probability density function becomes more informative. He develops relationships between his measure and classical measures such as IFR and DFR.

The concept of a decreasing failure rate, or infant mortality period is key to the concept of ESS. Proschan [101] provided a theoretical explanation for the infant mortality period. Proschan examined the failure data for an air-conditioning system from a fleet of jet airplanes and found a decreasing failure rate. Closer examination revealed that the failure data for individual planes exhibited a CFR with each plane having a different value. Using a theorem developed by Barlow, Marshall, and Proschan [8], which shows that the mixture of distributions with non-increasing failure rates yields a distribution which has a non-increasing failure rate, Proschan [101] showed that a mixture of exponential distributions in fact yields a DFR. He attributed the variability of failure rates between airplanes to "inescapable manufacturing variability”. As shown by Proschan [101], mixed populations play a fundamental role in explaining the DFR. Thus, mixture distributions will play a key role in modeling the ESS process.

2.2 Interference Theory

Some recent reliability authors attempt to explain all failures from a physical point of view. The basic tenant is that an item fails when the combined effect of the stresses imposed on the item exceeds the strength of the item at that particular time. Figure 2.1
graphically illustrates the concept of stress strength interference. A general expression for the reliability of a component with probability density function $f_X(x)$ for the stress and probability density function $f_Y(y)$ for the strength is given as:

$$R = P(Y > X) = P(Y - X > 0).$$  \hfill (2.7)

The probability that the strength is greater than a certain stress is given by:

$$P(Y > x_0) = \int_{x_0}^{\infty} f_Y(y) \, dy. \hfill (2.8)$$

The probability that $x_0$ falls in a small interval of width $dx$ is given by:

$$f_X(x_0) \, dx. \hfill (2.9)$$

Assuming that the stress and strength distributions are independent, the probability that the strength is greater than the stress for all possible values of stress is given by:

$$R = \int_{-\infty}^{\infty} f_X(x) \left[ \int_{-\infty}^{\infty} f_Y(y) \, dy \right] \, dx = \int_{-\infty}^{\infty} f_X(x) [1 - Q_Y(x)] \, dx. \hfill (2.10)$$
Kapur and Lamberson [60] compute the reliability for components using a variety of stress and strength distributions, namely, normal, exponential, log-normal, gamma, Weibull, and extreme value distributions. They also develop reliability expressions when the stress and the strength distributions are functions of time. The practical motivation for this is that many items weaken with age due to the cumulative damage. Similarly, many items are subject to varying stresses that change with time. Dhillon [32,33] applies these same concepts to model mechanical component reliability for a variety of combinations of stress and strength distributions.

Figure 2.1 Stress-strength Interference
2.2.1 Populations with Varying Strength

Jensen and Petersen [51] use a bimodal distribution to model the strength of a component. They hypothesize that variability in the manufacturing process causes some components to be weaker than others which results in a bimodal, or in some cases, multimodal strength distribution. Although hard-defects may be detectable in the factory during product acceptance testing, weak items may exhibit failure only under certain stress conditions. This weak population of components is usually what causes early failures in the field. Jensen and Petersen [51] developed a model in which they assumed that component strength deteriorated with time, and weak components deteriorated faster than strong components. Their selection of the strength decay function appears to have been arbitrary. Figure 2.2 illustrates the transformation of the strength distribution to a time to failure distribution for a specific stress level. It should be pointed out that the resulting distribution is also bimodal. Given a bimodal time to failure distribution, they develop techniques for determining how long components should be burned in at that particular stress level.
Figure 2.2 Transformation of Strength Distribution to Time to Failure Distribution [51].

The selection of a strength decay function for an item is dependent upon the dominate failure mechanisms. Chan [16, 17, 18] categorizes failure mechanisms into three categories: threshold stress failures, cumulative stress failures, and combined threshold-cumulative stress failures. Threshold stress failures occur when a product's yield strength is exceeded. Examples of product yield strengths are the threshold voltage for an insulating fluid, the threshold temperature for a junction, or the threshold electric field for avalanche breakdown of an electronic device [16, 17, 18]. Products with low-threshold strength may fail during assembly, shipping, installation or field operation if a sufficient maximum stress is encountered. Good products will have high threshold strengths and should be resistant to the stresses encountered during manufacturing, shipping, installation
and normal operation. A screening process should sort out the weak products without damaging or putting significant wear on the good ones.

Cumulative stress failures occur when a product fails due to the repeated exposure of low level stresses. Each time the product is exposed to a particular stress, even very low levels, some irreversible damage is done to the product. Examples of cumulative stress failures are electromigration in electronic components, and fatigue failure of solder joints under thermal cycling [16, 17, 18]. Care must be taken when screening devices of this type because all products, good and weak deteriorate when subject to the stress. The cost of finding the weak components must be balanced against the costs associated with putting wear on the good components.

Finally, some products might have failure modes that are stimulated by a combination of threshold and cumulative stresses. An example might be a solder joint. A high maximum stress might initiate a crack in the joint that is later driven to failure by the cumulative effects of low level vibration encountered during normal operation. Another example is a cracked electronic package. Use of a high stress may cause the package to crack. This may not be immediately detectable. This package would now be more susceptible to corrosion and will probably fail very early in its design life. Great care must be taken when setting up stress screens. An understanding of the physical processes is necessary in order to choose the appropriate type of stresses.
2.2.2 The Distribution of Environmental Stresses

As discussed in Section 2.3.2, products must not only be capable of surviving the cumulative effects of nominal stress, but must also be capable of surviving random stresses of significant size in order to operate in a robust environment. Chan [15, 16, 17, 18] defines the threshold stress, \( X \), as the maximum physical stress that a product is expected to encounter during its design life. One modeling approach might be to form a "rare event" model. In the rare event model, the rate of occurrence for extreme loads is modeled with an appropriate probability distribution (Poisson) and then the magnitude of the load is modeled with another probability distribution (extreme value distribution).

In order to model the effect of cumulative stress, Chan [16, 17, 18] recommends redefining the stress parameter as "an effective cumulative stress that measures the effect a physical stress makes on a unit. Then \( X \) is the overall irreversible change that a physical stress has made on a unit [18]." Chan [18] and Jensen and Petersen [51] note that in general this effective cumulative stress are not necessarily linear with the magnitude of the stress. Yet, both authors point out that when the stress is approximately uniform over time, then the effective cumulative stress can be equivalent to a time stress (Figure 2.2).

For design testing, the stress distribution is usually defined by the operational environment. As mentioned earlier, for ESS the stress distribution is driven by the failure modes that one is trying to screen out of the system. As Dietrich and Reddy [102] point out, the two need not be identical.
Up to this point, stress has been treated in a generic sense. Figure 2.3 summarizes environmental stresses and their relative effectiveness in terms of screening defects. As shown in figure 2.3, the key stress environments are thermal cycling, random vibration, high temperature, electrical stress, and thermal shock.

![Figure 2.3 Relative Screening Effectiveness of Environments [2]](image)

All of these environments have different effects on different failure mechanisms in a product. For example, temperature cycling is highly effective at all levels of assembly.
It is effective at revealing part defects, printed circuit board defects, solder problems, bond separations, tolerance drifts, and thermal mismatches. Random vibration is primarily used at the unit and system levels. It is effective in identifying solder problems, part and printed circuit board defects, connector and contact problems, loose items, and structural problems. High temperature stress is relatively inexpensive and is used primarily to reveal time dependent defects. Electrical stresses (voltage/current), are used primarily at the component level, and are especially useful for detecting intermittent failures. Finally, thermal shock, used primarily at the component and unit levels, is good at identifying cracking, delamination, and electrical changes due to moisture or mechanical displacement [42].

Unfortunately, the most difficult task is grasping a complete understanding of the effects of these environments on the product. There is currently an extensive amount of research going on the "physics of failure." The goal of this research is to develop quantitative failure models which account for the effects of the environment. Until these models are developed, screens should be developed based upon estimates of cost and effectiveness, equipment design, manufacturing techniques, and prior experience with similar systems.
2.2.3 Populations with Varying Strength and Distributed Stress

Smith and Dietrich [106] expanded on Jensen and Petersen's [51] work by including a distributed stress. Figure 2.4 and 2.5 illustrate a bimodal strength distribution with a distributed stress.

Figure 2.4 Distributed Stress/Bimodal Strength [106].
Using interference theory the probability of failure for this situation is given as

\[ Q = \int_{0}^{\infty} f_Y(y) \int_{0}^{\infty} f_X(x) \, dx \, dy. \]  

(2.11)

which can be rewritten as

\[ Q = \int_{0}^{\infty} f_X(x) R_Y(x) \, dx. \]  

(2.12)
But, the strength distribution is a bimodal distribution comprised of a weak and main population. Therefore

\[ f_Y(y) = \pi_1 f_w(y) + \pi_2 f_m(y) \]  \hspace{1cm} (2.13)

where \( \pi_i \) is the mixing weight and \( \pi_1 + \pi_2 = 1 \).

Smith and Dietrich [106] define a function \( d(y_0, n) \) to represent strength decay as a function of initial strength and stress. They let \( D(y, n) \) represent the inverse of the strength decay function. Then the time to failure distribution as a function of stress is given by

\[ Q(n) = \int_{d(0,n)}^{\infty} g_T(y,n) \int_{\pi}^{\infty} f_X(x) dx dy. \]  \hspace{1cm} (2.14)

where

\[ g_T(y,n) = f_Y(D(y,n)) \left| \frac{d(D(y,n))}{dt} \right|. \]  \hspace{1cm} (2.15)

Smith and Dietrich [106] illustrated this concept using a fracture mechanics model in which Paris' crack propagation law was used as the strength decay function. Their results were very promising in that they were able to show that the entire reliability
bathtub curve can be explained "as a physical process of component strength deterioration governed by the operating environment and the original component quality." This concept is illustrated graphically by Chan [18] in Figure 2.6.

![Contour Map of Stress and Strength](image)

Figure 2.6 Contour Map of Stress and Strength [18].

In Figure 2.6, the population lying in the $Y<X$ region will fail in the field while those in the $Y>X$ region will not. Ideally, the units in the main population should possess enough of a design margin to withstand the expected stresses incurred in the field. But, in some cases, units from the main population, whose strength is on the low side of the distribution may wear out early. The effects of stress screening at a screening level, $X^{EST}$, are shown by the solid horizontal line in Figure 2.6. By carefully choosing a stress
screening level, the weak population of components can be eliminated. But, this may not prevent all early life failures. As shown in Figure 2.6, there will be some units from the main population that will fail early in their design life. These failures are due to the variance in the stress and strength distributions and could be eliminated by raising the screening stress level as shown by the dotted horizontal line. By raising the screening stress level, a significant portion of adequate parts from the main population would be screened out. Thus, there is a need for a cost analysis to trade-off the costs associated with units failing in the field versus decreasing the yield rate of the production process.

Smith and Dietrich [18], have shown that the application of one or more stresses not only weeds out the weak components but also causes a deterioration in the strength of the main population. Thus, the subject of aging and wear-out must be considered when using ESS. This is illustrated graphically in Figures 2.7 and 2.8. Figure 2.7 shows a product where the main population possess enough of a design margin to withstand the expected stresses incurred in the field. But, after screening, the effects of cumulative damage cause a shift in the strength of the main population (Figure 2.8). In practice, most items have multiple failure modes. All models discussed up to this point have assumed a single failure mode. It is quite possible that a stress designed to screen weak items for one failure mode is actually deteriorating the strength of the item for a different failure mode. Great care must be exercised when developing screening plans.
Figure 2.7 Product with High Strength and Low Scatter [18]

Figure 2.8. Effects of Cumulative Stress Testing [18].
2.3 Accelerated Degredation Models

Although screening is optimally performed under environmental conditions approximating the expected use conditions, this is not always possible, due to the amount of time required to screen out defects. Increasing the environmental stresses reduces the amount of time required to screen out defects. Environments are considered accelerated any time the operating conditions are more severe than normal operating conditions. As shown in Figure 2.3, environments that are commonly accelerated include temperature, vibration, voltage, and humidity.

The concept of accelerated aging may seem simple in theory, but, a rigorous scientific approach is sometimes difficult to put into practice. The selection of the environmental stress or stresses must be based on a knowledge of the failure mechanisms of the item being screened. The uncertainties involved in predicting failure mechanisms and the effects of increased stress drive the need for approximations and engineering judgment. ESS planners must have an understanding of the physics of the failure process in order to choose the environment (or environments) to be accelerated, the applicable acceleration function consistent with the physical model of the failure process, and the range of stresses for which the acceleration function is valid [12].

An underlying requirement of all ESS processes is that the failure mechanisms at the accelerated conditions are the same as those present at operational conditions. The
goal is to accelerate the normal processes which lead to failure without changing the nature of the failures. The most common approach to this problem is to assume that:

1. The life distributions at the specified stress levels come from the same family of distributions, but the parameters of the distribution vary as a function of the applied stress.

2. The relationship between the parameters of the life distribution and the stresses applied are known in advance. These relationships are known as acceleration functions or time transformation functions [71].

2.3.1 Linear Acceleration Factors

Tobias [118] states that any "well behaved, order preserving, continuous transformation could be used to model the acceleration from the normal environment to the accelerated environment." However, the assumption of a linear acceleration is often used. Under the assumption of a linear acceleration function, Tobias derives the following general linear acceleration relationships between use and stress conditions:

1. Time to Failure
   \[ t_u = AF \times t_s \]  \hspace{1cm} (2.16)

2. Failure Probability
   \[ Q_u(t) = Q_s\left(\frac{t}{AF}\right) \]  \hspace{1cm} (2.17)
3. Density Function

\[ f_u(t) = \left( \frac{1}{AF} \right) f_s \left( \frac{t}{AF} \right) \]  \hspace{1cm} (2.18)

4. Hazard Rate

\[ h_u(t) = \left( \frac{1}{AF} \right) h_s \left( \frac{t}{AF} \right) \]  \hspace{1cm} (2.19)

Two assumptions must hold for the above to be true. First, the failure distribution must be preserved from the normal usage environment to the increased stress environment. Secondly, the accelerated life failure times must always be shorter than their corresponding normal life failure times. The biggest difficulty in using accelerated life tests is in defining the acceleration function for different types of stresses.

2.3.2 Single Stress Degradation Models

When reading the literature one finds many different models that relate product life to different stresses. Most of these models attempt to characterize the physical characteristics of the failure process. This section briefly reviews the most common models for accelerated aging under constant stress. Nelson [79], in his monograph on accelerated testing, provides detailed coverage of the statistical treatment for these and other accelerated degradation models.
2.3.2.1 Arrhenius Reaction Rate Models

This model is commonly used when the accelerated environment is temperature. It is used to describe thermally activated mechanisms such as solid state diffusion, chemical reactions and many semi-conductor failure mechanisms. It relates the time rate of degradation as a function of the operating temperature. The theoretical relationship between reaction rate and temperature was initially proposed by Svante Arrhenius in 1889. The rationale for this model is that the strength degradation leading to component failure is governed by chemical and physical processes with a reaction rate of

\[ r = Ae^{-\frac{E_a}{kT}} \]  \hspace{1cm} (2.20)

where \( r \) is the process reaction rate, \( A \) is a constant, \( E_a \) is the activation energy, \( k \) is Boltzmann’s constant \((8.617 \times 10^{-5} \text{ eV/°K})\), and \( T \) is the reaction temperature, in °K. The activation energy is device and failure mode dependent. Common values for the activation energy of electronic components range from 0.3 to 1.5 electron volts. Table 2.2 list some activation energies for some semi-conductor device failure mechanisms. Small errors in the activation energy value can create large differences in the degradation factor. It is this factor that often creates controversy when the Arrhenius model is used.
<table>
<thead>
<tr>
<th>Failure Mechanism</th>
<th>$E_a$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal migration (contact degradation)</td>
<td>1.8</td>
</tr>
<tr>
<td>Charge injection</td>
<td>1.3</td>
</tr>
<tr>
<td>Ionic contamination</td>
<td>1.0-1.1</td>
</tr>
<tr>
<td>Gold-Aluminum intermetallic growth</td>
<td>1.0</td>
</tr>
<tr>
<td>Corrosion in humidity</td>
<td>0.8-1.0</td>
</tr>
<tr>
<td>Electromigration of aluminum</td>
<td>0.5</td>
</tr>
<tr>
<td>Electromigration of silicon in aluminum</td>
<td>0.9</td>
</tr>
<tr>
<td>Electrolytic corrosion</td>
<td>0.3-0.6</td>
</tr>
<tr>
<td>Surface charge accumulation</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 2.2 Activation Energies for Semiconductor Failure Mechanisms. [25]

The Arrhenius life temperature relationship states that the product or material is assumed to fail when some critical amount of the chemical reaction has taken place. Therefore, the nominal time to failure $\tau$, is inversely proportional to the chemical reaction rate, and is defined as follows:

$$
\tau = A e^{\frac{E_a}{kT}}
$$

(2.21)
The acceleration factor (AF) is the ratio of the nominal times to failure at the lower temperature to that at the higher temperature. It is expressed as follows:

\[
\frac{\tau_u}{\tau_t} = \exp \left[ \frac{E_a}{k} \left( \frac{1}{T_u} - \frac{1}{T_t} \right) \right]
\]

The Arrhenius model should be considered for items which have temperature dependent failure modes. Nelson [79] cites numerous references and applications where this model has been used. He also provides examples for using this model with each of the commonly used life distributions.

### 2.3.2.2 Inverse Power Law Model

The derivation of the inverse power law, or power model, is based on the principles of kinetic theory and activation energy, where the accelerated stress is applied voltage. The degradation rate is represented by a power function of the applied voltage \( V \), namely

\[
r = AV^r
\]
where \( A \) and \( \gamma \) are product specific parameters. This model has been applied to electrical insulating fluid [66], dielectric capacitors [71], and other electronic devices.

The inverse power law relationship has also been used to model other situations as well. For example the relationship has been used to model the wear of cutting tools where the applied stress is tool speed (Taylor's Model). Similarly, this same relationship has been used to model fatigue due to alternating stress. Examples include low-cycle fatigue due to thermal cycling (Coffin-Manson Model) and high-cycle fatigue due to mechanical vibration [25].

The nominal time to failure \( \tau \), is inversely proportional to the degradation rate, and is defined as follows:

\[
\tau = \frac{A}{\gamma Y}
\]  
\[2.23\]

The most common form of the inverse power law model is the Coffin-Manson law for fatigue testing. It is stated as:

\[
N_f = A \left( \frac{1}{\Delta \varepsilon_p} \right)^\gamma
\]  
\[2.24\]
where \( N_f \) is the number of cycles to failure, \( A \) and \( \gamma \) are material constants, and \( \Delta \varepsilon_p \) is the plastic strain range. A simplified acceleration factor for isothermal fatigue testing is:

\[
AF = \frac{N_{f_u}}{N_f} = \left( \frac{\Delta \varepsilon_f}{\Delta \varepsilon_u} \right)^\gamma
\]  \hspace{1cm} (2.25)

"The \( \Delta \varepsilon \)'s could be considered equal to displacement in bending, elongation in tension, or mechanical strains [25]." A similar acceleration factor for fatigue testing in thermal cycling is given by:

\[
AF = \frac{N_{f_u}}{N_f} = \left( \frac{\Delta T_f}{\Delta T_u} \right)^\gamma
\]  \hspace{1cm} (2.26)

where the \( \Delta T \)'s are the applied temperature cycling ranges. Table 2.3, taken from Condra [25], provides some approximate values for \( \gamma \) for various failure mechanisms. These values are based upon Condra's actual testing experience.

### 2.3.2.3 Eyring Single Stress Model

The Eyring model, like the Arrhenius model, describes the effect of constant operating temperature on the rate of degradation of an item. This model was developed
by Glasstone, Laidler and Eyring in 1941 and is based on quantum mechanics [79]. The degradation relationship is given by

\[
\tau = \left( \frac{1}{AT^\alpha} \right) e^{\left[ \frac{\Delta H}{kT} \right]}
\]

(2.27)

where A and \( \alpha \) are product specific parameters. \( \Delta H \) represents the amount of energy needed to move an electron to the state where the process chemical reaction can take place [118]. Comparing the Eyring model to the Arrhenius model one finds a lot of similarity. If one assumes \( \alpha \) to be small, which is often the case for small temperature ranges, then the two relationships are identical.

<table>
<thead>
<tr>
<th>Failure Mechanism</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metals</td>
<td>2-3</td>
</tr>
<tr>
<td>Electronic solder joints</td>
<td>2-3</td>
</tr>
<tr>
<td>Microelectronic plastic encapsulants</td>
<td>4-8</td>
</tr>
<tr>
<td>Microelectronic passivation layers</td>
<td>12</td>
</tr>
<tr>
<td>Cratering of microcircuits</td>
<td>7</td>
</tr>
<tr>
<td>Al-Au intermetallic fatigue</td>
<td>4-7</td>
</tr>
</tbody>
</table>

Table 2.3 \( \gamma \) Values for Various Failure Mechanisms [25].
2.3.2.4 Exponential Degradation Model

This model is most often used for characterizing degradation due to weathering variables such as humidity [79]. The degradation rate is an exponential function of the applied stress \( V \), and is given by

\[
r = Ae^{(\gamma V)}
\]  

(2.28)

where \( A \) and \( \gamma \) are product specific parameters.

2.3.3 Multiple Stress Degradation Models

When accelerating a single environmental stress to precipitate defective components within a reasonable time, it often requires stresses so high that the validity of the model becomes questionable. A potential solution is to accelerate more than one stress, thereby precipitating failures in a reasonable time with lower stress settings. From a practical point of view, most components are subject to multiple stresses in their operational environment. One problem/difficulty with using multiple stresses is how does one handle the interaction of stresses. In other words, what is the combined effect of the
stresses? For this reason, far fewer theoretical models exist for modeling the acceleration effects of multiple stresses. Two multiple stress models are presented below.

2.3.3.1 Generalized Eyring Model

This model is applicable to situations where the item undergoing screening is subject to a constant thermal stress and a constant non-thermal stress. The degradation relationship is given by

\[ r = \left( \frac{1}{A T^\alpha} \right) e^{\left[ \frac{\Delta H}{k_x T} \right]_e} e^{\left[ \frac{B + C}{T} \right] V} \] (2.29)

where T is the thermal stress and V is the non-thermal stress. A, B, C, \( \Delta H \), and \( \alpha \) are product and failure mode specific parameters. The term \( \left[ C/T \right] V \) defines the interactive effect of the thermal and non-thermal stresses on product degradation. This model can be extended to model a thermal stress and n additional non-thermal stresses. The relationship is given by

\[ r = \left( \frac{1}{A T^\alpha} \right) e^{\left[ \frac{\Delta H}{k_x T} \right]_e} e^{\left[ \frac{B + C}{T} \right] V_1} e^{\left[ \frac{D + E}{T} \right] V_2} ... e^{\left[ \frac{X + Y}{T} \right] V_n} \] (2.30)
2.3.3.2 Bazovsky Model

The Bazovsky model is based on the Arrhenius and inverse power law models. It models the acceleration of a thermal and non-thermal stress. The degradation rate is given by

\[ r = Ae^{-\left(\frac{E_a}{kT}\right)V^\gamma}. \] (2.31)

where T and V are the thermal and non-thermal stresses. All other parameters are product and failure mode specific. Gange [43] used this to model the degradation of capacitors, transistors, and diodes from thermal and electrical stresses. Peck [92] used this model for the temperature humidity relationship for electronic microcircuits. Specifically, the nominal time to failure is assumed to be inversely proportional to the rate of degradation and is given by:

\[ \tau = \frac{1}{A} (RH)^{-\gamma} e^{\left(\frac{E_a}{kT}\right)}. \] (2.32)

where A is a material constant, RH is the percent relative humidity, \( \gamma \) is a constant usually equal to 3.0 and \( E_a \) is assumed to equal 0.90. The acceleration factor for this model is:
Another variation of the Bazovsky model is Blacks equation for electromigration failures of microcircuit conductors [25]. The time to failure is given as:

\[ t_f = AJ^{-\gamma} e^{\frac{E_a}{kT}} \]  

(2.34)

where \( A \) is a material constant, \( J \) is the current density, \( \gamma \) is a material constant (approximately 2.0) and \( E_a \) is the activation energy (approximately 0.5).

2.3.4 Cumulative Damage Models

The cumulative damage model is used to model component degradation for non-constant stresses. Jensen and Petersen [51] use the cumulative damage model for burn-in. Jensen and Petersen [51] define three conditions that must hold to use this model. They are

1. Damage (strength degradation) accumulates continuously in the component.
2. Failure occurs when strength is degraded to the point of the imposed stress.
3. The rate of damage depends only on the current stress and cumulative damage and not on the previous stress history.

When these conditions are present, the component life is described by the generalized Miner's equation.

\[
\sum_{i=1}^{n} \left( \frac{t_i}{L_i} \right) = 1
\]

where \( t_i \) and \( L_i \) denote the actual time at stress \( i \), and the expected lifetime at this stress condition, respectively. Although there is evidence to support that the summation is not always equal to 1, this value is generally used in practice. Jensen and Petersen [51] point out that in some cases, a component's past history of stresses is significant in determining its failure properties.

Miner's rule has been effective in modeling a number of different types of items, including light bulbs, electric drills, ball bearings, and electric motors. This model has been used extensively in describing mechanical fatigue, random vibration effects on electronic equipment [69], the effects of high pressure on the life of refrigerator components [46], and cyclic wear on helicopter components [24].

Barker et al. [11] used a form of Miner's rule to model the combined effects of vibration and thermal cycling on solder joints. The equation is given as:
where \( R \) is the fraction of life exhausted, \( n_{th} \) is the number of thermal cycles applied, \( f_v / f_{th} \) is the ratio of the frequency of vibration cycles to thermal cycles, \( N_v \) is the number of vibration cycles to failure, and \( N_{th} \) is the number of thermal cycles to failure. Then the estimated solder joint fatigue life is

\[
N_f = \frac{1}{R}.
\] (2.37)

2.4 Optimization Criteria

Various criteria have been used in the ESS and burn-in literature for optimizing screen durations. Mi [75] classifies the burn-in literature into two groups according the criteria used: reliability and cost. Early on, optimal screen durations were determined based on maximizing some measure of reliability, usually the mean residual life function. Eventually, models that considered cost began to appear in the literature. The goal was to develop screens that minimized some particular cost function. These screen durations were usually shorter than those determined purely from a reliability perspective. This was
due to the fact that these models considered reliability in terms of the cost of failure and not as an independent objective. In general, any decision on ESS policies must involve a trade between cost and reliability. This section will briefly review the significant burn-in and ESS models found in the literature based upon the optimization criteria used.

2.4.2 Reliability Models

Guess, Walker, and Gallant [44] discuss the importance of knowing exactly what the end user of a product desires in terms of reliability before designing a burn-in strategy. They show that different reliability measures can give significantly different burn-in policies. The reliability measures examined were: mission reliability, hazard rate, and mean residual life. They point out that the hazard rate provides a "local" measure of reliability where as the mean residual life is a "long" term measure of reliability. Mission reliability is generally applicable as a measure for items in which it is almost impossible to replace the item such as satellites, or when the cost of failure is expensive, such as the space shuttle.

The reliability criterion most often utilized in the literature is the mean residual life. This is probably due to Watson and Wells [121]. Lawrence [67] developed sharp upper and lower bounds on burn-in time in order to achieve a specific mean residual life for items having a DFR. The development of these bounds require knowledge of the mean time to failure as well as a percentile of the failure distribution. No other
assumptions about the the life distribution were made. These bounds are noteworthy because, even with limited life test data, estimates of the mean and a percentile of the failure distribution can be made. The bounds are given by:

\[
\delta(t_b) = \begin{cases} 
\frac{tb}{1 + \epsilon_p / \log(1-p)} \left(1 - p\right)^{-\epsilon_p - \epsilon_p / \log(1-p)}, & t_b \leq \epsilon_p \\
\left(\log(1-p) + p\epsilon_p\right)/(1-p) \log(1-p), & t_b > \epsilon_p 
\end{cases}
\]

and

\[
\delta(t_b) = \sqrt{\alpha}, \quad t_b \leq \epsilon_p
\]

where \(\alpha\) is the unique solution of

\[
\alpha e^{-\alpha_\epsilon} = 1 - p
\]

and

\[
Q(\epsilon_p) = p.
\]

The only restriction in the development of these bounds is that \(\epsilon_p\) be less than the mean life which results in a lower bound on the mean residual life that is useful only up to \(\epsilon_p\).

Lawrence [1966] points out that this is a minor restriction since it is unlikely that an item would be burned-in for a time greater than its mean life.

Dishon and Weiss [33] examine burn-in strategies for repairable systems. Using aircraft engines as an example, they assume that an engine can be in one of three states:
good, poor or failed. Each operating state has its own reliability characteristics. The goal was to design a burn-in strategy to prevent poor engines from entering the field. Engines that failed during burn-in were repaired and the burn-in process repeated. If an engine failed from a good state, it always returned to a good state after repair. If it failed from a poor state, it can be repaired to either a good or poor state. The authors show that the mean residual life for engines is a function of reliability for both good and poor engines, proportion of good items, and the probability of incomplete repair. General conditions for the mean residual life of good and poor engines are developed for which burn-in is effective.

Chandrasekaran [18a] examines optimal burn-in strategies when the hazard function is strictly increasing, decreasing then increasing (bathtub shaped) and increasing then decreasing (hump shaped). Mi [77] performed a similar analysis for the classical and generalized bathtub curves. He developed some general conditions on the mean residual life for which burn-in is appropriate. Specifically, when an item has a strictly IFR then it is not optimal to burn-in. If an item has a classical bathtub curve where \( Q(t) \) is DFR for \( t < t_0 \) and IFR for \( t > t_0 \), then the optimal burn-in time is given by:

\[
t_b^* = \begin{cases} 
0 & \text{if } \delta'(0) < 0 \\
 t_0 & \text{if } \delta'(t_0) > 0 \\
 t \in [0, t_0] & \text{if } \delta'(0) \geq 0 \text{ and } \delta'(t_0) < 0
\end{cases}
\] (2.42)
When an item has a generalized bathtub curve where \( Q(t) \) has an IFR for \( t < t_0 \) and then a DFR, the optimal burn-in time is given as:

\[
  t_b^* = \begin{cases} 
    0 & \text{if } \delta'(t_0) < 0 \\
    t \in [0, t_0] \text{ satisfies } \delta(t_b^*) = \max[\delta(0), \delta(t)]
  \end{cases}
\] (2.43)

Park [91] showed that the burn-in time that minimizes the hazard rate does not necessarily maximize the mean residual life for systems with wear-out.

In the ESS literature, for performance critical items, ESS is conducted until a desired yield is achieved for a population of items. Perlstein et al. [96, 98] develop techniques for calculating screen durations to achieve a desired yield. They assume that components come from a population of mixed exponentials. This procedure is dependent on knowledge of the proportion of defective items present in the population prior to screening as well as the failure rates of good and bad components. The hazard rate for items from a mixed population of exponentially distributed items is:

\[
  h(t) = \lambda_m + \frac{\pi_w (\lambda_w - \lambda_m) e^{-\lambda_w t}}{\pi_g + \pi_w e^{-\lambda_w t}}
\] (2.43)

which can be redefined as
where $\varepsilon$ is defined as the screen residue, and is given by:

$$\varepsilon = \frac{1}{\lambda_m} \left( \frac{(\lambda_w - \lambda_m) \pi_w e^{-(\lambda_w - \lambda_m)t}}{\pi_m + \pi_w e^{-\lambda_w t}} \right)$$

Equation 2.45 allows the screen designer to control how close the screened population of components is to the main population. After some mathematical manipulation, the optimal screen duration is given by:

$$t^*_b = \frac{1}{(\lambda_w - \lambda_m)} \ln \left( \frac{(\pi_w/\pi_m)(\lambda_w/\lambda_m - 1) - (\pi_w/\pi_m)\varepsilon}{\varepsilon} \right)$$

Finally, a measure for the goodness of the screen is given by using a ratio of the actual screen results (difference between the hazard rates before and after screening) to the ideal screen in which all weak items are screened out. The power of the screen is given as:

$$PS = \frac{\pi_m(1 - e^{-(\lambda_w - \lambda_m)t})}{\pi_m(1 - e^{-(\lambda_w - \lambda_m)t}) + e^{-(\lambda_w - \lambda_m)t}}$$
Perlstein and Bazovsky [97], in another paper, show how the method of moments and historical data can be used to obtain estimates of the proportion of defective components, and the failure rates for both good and bad populations.

No review of the ESS literature would be complete without a discussion of the ESS model used in DOD-HDBK-344A [35]. This handbook is designed to provide uniform procedures, methods and techniques for planning, monitoring and controlling ESS programs for defense electronic equipment. The ESS program in DOD-HDBK-344A [35] is summarized in Figures 2.9 and 2.10. The relationship between defect types and product life is given in Figure 2.11. The math model used in DOD-HDBK-344A [35] is an empirical model which is based upon many hours of test data from a variety of defense electronic contractors. The general form of the math model is:

\[ D_{\text{Removed}} = DE \cdot D_{\text{PAT}} + DE \cdot D_{\text{LAT}}[1 - e^{-kt}] + DE \cdot CFR \cdot t \]  \hspace{1cm} (2.48)

\[ D_{\text{REMAINING}} = (1 - DE)D_{\text{PAT}} + (1 - DE)D_{\text{LAT}}(1 - e^{-kt}) + D_{\text{LAT}}e^{-kt} \]  \hspace{1cm} (2.49)

A latent defect, \( D_{\text{LAT}} \), is an inherent or induced weakness not detectable by ordinary means. It should be precipitated to failure with ESS. Without ESS it would fail in the intended use environment. A patent defect, \( D_{\text{PAT}} \), is an inherent or induced weakness which could be detected by inspection or functional testing without ESS. The
defect density is a measure of the average number of latent defects per item. \( D_{IN}, D_{OUT}, D_{REMAINING}, D_{REMOVED}, \) and \( D_O \) represent incoming, outgoing, remaining, removed and observed defect densities, respectively. The detection efficiency, \( DE \), is a measure of the probability of detecting a patent defect. The precipitation efficiency, \( (1 - e^{-k}) \), is a function of the stress duration, \( t \), and the stress constant \( k \), which is dependent upon the type of stress utilized, and is calculated as follows:

\[
\begin{align*}
\text{Temperature Cycling} & \quad k = 0.0017(\Delta T+.6)^{0.6}[\ln(rate + 2.718)]^3 \quad (2.50) \\
\text{Constant Temperature} & \quad k = 0.0017t(\Delta T+.6)^{0.6} \quad (2.51) \\
\text{Random Vibration} & \quad k = 0.0046(G_{RMS})^{1.71} \quad (2.52)
\end{align*}
\]

Finally, the yield for the screening process is given by:

\[
Yield = e^{-D_{REMOVED}} \quad (2.53)
\]
Figure 2.9 MIL-HDBK-344A ESS Program [35]

Figure 2.10 Stress Screening Variable Relationships [35].
The theoretical foundation for the empirical model used in DOD-HDBK-344A is the Chance Defect Exponential Model (CDE), developed by Fertig and Murthy [41]. The CDE model is based upon the assumption of a mixed population of parts, i.e., a main subpopulation of good parts and a much smaller subpopulation of weak parts. The model assumes that the failure distribution for the main and weak subpopulations is exponential. The model also assumes that the number of defectives in a product is independent and identically distributed with a binomial distribution with parameters \( N \), total number of parts in the product, and \( \bar{P} \), average part fraction defective. For large \( N \) and small \( \bar{P} \) the binomial distribution can be approximated by the Poisson distribution where the defect density is expressed as:
The unconditional survival probability for any number of defects $m$ is given by:

$$D = N\bar{P} = \sum_{i=1}^{\infty} n_i P_i$$

(2.54)

$$R(t) = R_m(t) \sum_{m=0}^{\infty} [R_w(t)]^m \left( \frac{D^m e^{-D}}{m!} \right) - R_m(t) \exp\left(-D(1-R_m(t))\right)$$

(2.55)

Assuming that the main and weak subpopulations have exponential failure distributions with average failure rates of $\lambda_m$ and $\lambda_w$ respectively. Then equation 2.55 can be written as:

$$R(t) = \exp\left[-(N-D)\lambda_m t - D \exp\left(-\lambda_w t\right) \right]$$

(2.56)

Using the CDE model, the reliability and hazard rate for a system that has not undergone ESS can be expressed as:

$$R(t) = \exp\left(- (N - D_m) \lambda_m t - D_m \left(1 - \exp\left(-\lambda_w t\right)\right) \right)$$

$$h(t) = (N - D_m) \lambda_m + D_m \lambda_w \exp(-\lambda_w t)$$

(2.57)
After ESS, the reliability and hazard rate would be given by equation 2.57 with $D_{IN}$ replaced by $D_{REMAINING}$. Figure 2.12 shows the failure rate function for a system with and without an ESS program.

![Field Failure Rate with and without ESS](image)

Figure 2.12 Field Failure Rate with and without ESS [35].

### 2.4.2 Economic Models

Only under rare circumstances is cost not a factor in the design and manufacture of products. The first burn-in cost model to appear in the literature was developed by Cozzolino [27,28]. He forms a sequential decision-making process where at each time point a decision is made as to whether to continue or stop burning in a unit. The decision is based on a reliability value function that considers the cost to repair a unit that fails during burn-in and the cost per unit test time. Using dynamic programming, Cozzolino
[27, 28] develops optimal burn-in strategies for several different repair policies. Cozzolino [27,28] concludes that the explicit modeling of the failure and repair process is highly important for burn-in testing of repairable equipment. Cozzolino [27,28] also extends his model to the case where multiple independent failure modes exist in the system.

Washburn [120] developed a burn-in cost model composed of three factors. The total cost is given by:

\[
C_i = C_1 t_b + C_2 NQ(t_b) + C_3 K [S_E(t_b)]^{-1}
\]  

(2.58)

The first cost factor, \(C_1\), represents the cost per unit time to burn-in the system. This factor includes all costs associated with operating a burn-in program, including equipment and labor costs. The second cost factor, \(C_2\), is the cost per unit failed during burn-in, where \(N\) is the total number of units exposed to burn-in. The final cost factor, \(C_3\), is the purchase price for units in the field. Washburn's model is unique for three reasons. First, it takes into consideration the minimal number of units required by the customer, \(K\), assuming that system effectiveness, \(S_E\), is 1. Second, system effectiveness is used to measure product performance. The system effectiveness parameter is a product of mission reliability, operational readiness, and design adequacy. This is one of the few models that considers mission reliability as part of the performance criteria, and the only model that directly examines the effect of burn-in on systems availability. Finally, unit life is modeled with the generalized gamma distribution. The generalized gamma distribution is a three
parameter distribution which has the exponential, gamma, Rayleigh, Weibull, and chi-squared distributions as special cases. The generalized gamma distribution is given as:

\[ f(t; a, \alpha, \beta) = \left[ \frac{\beta}{a^\alpha \Gamma(\alpha/\beta)} \right] t^{\alpha-1} e^{-\left[ \frac{\beta}{a} \right]^\alpha} \quad (2.59) \]

Stacy and Mihram [110] and Harter [47] derive procedures for estimating the parameters of the generalized gamma distribution.

Stewart and Johnson [112] use a life cycle cost model to determine optimum burn-in and replacement times for an item. The cost model has five cost elements: the cost of an item, the fixed burn-in cost per item, the cost of burn-in per unit time of burn-in, the cost of replacing an item that is in service and fails prior to the scheduled time of replacement, and the cost of a scheduled replacement. A piecewise linear hazard function is used to model the reliability bathtub curve from test data. Using Bayesian decision theory, the expected cost per unit time of service is minimized. This is the only model in the literature that examines burn-in strategies in conjunction with component replacement strategies.

Nguyen and Murthy [85] develop burn-in strategies which minimize costs for products sold under warranty for period T. The authors assume that the cost of failure during production is cheaper than the cost of failure in the field. The authors consider both repairable and non-repairable systems. Renewal theory is used to model the expected
number of replacements of non-repairable systems. The expected number of replacements after burn-in time \( t_b \) is given by:

\[
N_{NR}(T) = Q_n(T) + \int_0^T N_{NR}(T - t) \, dQ_n(t) \tag{2.60}
\]

where

\[
Q_n(t) = 1 - R(t|t_b) \tag{2.61}
\]

The expected number of repairs after burn-in is given by:

\[
N_R(T) = \int_0^T h(t + t_b) \, dt \tag{2.62}
\]

The cost model used by Nguyen and Murthy [85] includes two components: manufacturing and warranty. The manufacturing cost model contains four components: a.) the manufacturing cost per unit without burn-in, b.) the fixed set-up cost of burn-in per unit, c.) the cost per unit time of burn-in per unit, and d.) the cost to repair or replace units that fail during burn-in. For repairable products, the manufacturing cost model is:

\[
MC_R(t) = C_1 + C_2 + C_3 t_b + C_4 \int_0^{t_b} h(t) \, dt \tag{2.64}
\]
For non-repairable products, the manufacturing cost model is:

\[ MC_{NR}(t) = C_1 + C_2 + C_3 \int_0^t \frac{R(t)dt}{R(t_b)} \]  

(2.65)

The warranty cost model depends on the warranty policy. Both, failure free and rebate policies were investigated. The cost model for the failure free policy contains two components: the cost to repair or replace the unit, and the additional costs associated with repair or replacement in the field. The warranty cost models for repairable and non-repairable systems are:

\[ WFF_{R}(T, t_b) = (C_3 + C_4) \int_0^T h(t_b + t)dt \]  

(2.66)

\[ WFF_{NR}(T, t_b) = [MC_{NR}(t_b) + C_4]N_{NR}(T) \]

The warranty cost model for the rebate policy is used for non-repairable items and the amount of rebate is modeled as a function of the failure time. Mathematically, the model is given as:

\[ WRP(T, t_b) = kP \left[ T \cdot R(t_b) - (1 - \alpha)T \cdot R(T + t_b) - \alpha \int_0^T R(t + t)dt \right] / T \cdot R(t_b) \]  

(2.67)
where $P$ is the sales price and $k$ is the proportion refunded to the consumer if the product fails during the warranty period $T$. This model is the first to examine the difference between repairing and replacing units and their effects on burn-in policy. Also, this is the only model in the literature to use a mixed Weibull distribution to model unit lifetimes. Sultan [113, 114] uses a similar cost structure for a mixed exponential distribution.

The majority of the models found in the literature focus on individual components or a single system. Kuo [63a] examines burn-in strategies for a series-series configuration of components. He uses a cost optimization model from a systems viewpoint, with component burn-in times as the decision variables. The cost model is a function of the cost of components, burn-in time for components, cost of shop repair and field repair. Component failure rates are modeled as:

$$\lambda_{i,j}(t) = \alpha_{i,j} t^{-\beta_{ij}} \quad \text{for } t < t_{ij,L}$$

and

$$\lambda_{i,j}(t) = \lambda_{ij,L} \quad \text{for } t \geq t_{ij,L}$$

(2.68)

where

- $\lambda_{i,j}(t)$ is the failure rate for component $i$ in unit $j$
- $\alpha_{ij}, \beta_{ij}$ is the Weibull scale and shape parameters for component $i$ in unit $j$
- $\lambda_{ij,L}$ is the constant failure rate for component $i$ in unit $j$
\( t_{g,L} \) is the time at which \( \lambda_{g,L} \) is reached.

This model of the failure rate ignores the effects of wear-out. Kuo's paper is novel in that it views the burn-in of components from a systems viewpoint and is the first to consider reliability constraints for components and systems when optimizing the burn-in strategy.

The general form of the optimization model is given by:

\[
\begin{align*}
\text{Min } C_S(S, t_{ij,b}) \\
\text{S.T} \\
R_S(t_{ij,b}) \geq R_{S,\min}(t) \\
R_{ij}(\Delta t_{ij,b}) \geq R_{ij,\min}(t) \quad \forall \ i, j
\end{align*}
\]

(2.69)

where

- \( C_S \) is the total cost associated with burn-in for system configuration \( S \)
- \( t_{g,b} \) is the burn-in time for component \( i \) in unit \( j \)

Chi and Kuo [19] extended this model to include capacity constraints for the burn-in facility.

2.5 Component vs. System Level Analysis

Most of the early ESS and burn-in literature examine screening strategies for one level of assembly, be it component, sub-assembly, assembly or system level. Recently,
several researchers have proposed screening strategies for multiple levels of assembly.
Whitebeck and Leemis [124] present a method to find a burn-in policy at component and 

system level for a non-repairable system by maximizing mean residual life. They utilize a 
pseudo-component to represent assembly defects which affect system mean residual life.
This pseudo-component is placed in series with the prior assembly (Figure 2.12).
Component lifetimes were modeled with a two parameter exponential power distribution.
The pseudo-component was modeled using a Weibull distribution. Simulation was used to 
evaluate different burn-in policies.

![Diagram of pseudo component model](image)

**Figure 2.13 Psuedo Component Model [124.]**

Chien and Kuo [20, 21, 22] studied burn-in strategies for the complex system in 
Figure 2.13. Each sub-assembly is a series-series collection of components. They
examined the trade-offs associated with burning-in at component, sub-system, and system levels. Component failure rates are modeled as in Kuo [63a]. The Arrhenius reaction rate equation was utilized to model temperature acceleration during burn-in. A nonlinear optimization program was developed to determine optimal burn-in policy such that system reliability was maximized subject to system cost and sub-system reliability requirements. Chien and Kuo [21, 22] incorporate a redundancy allocation algorithm to this model. Using a mixed integer non-linear program they develop optimal burn-in times using redundancy allocation for extremely high reliability systems.

![Figure 2.14 Complex System [21]](image)

Reddy and Dietrich [102] developed a two-level ESS model. All components were assumed to be connected in series. Component lifetime was modeled using a bimodal exponential distribution. Connections between components were also modeled...
using a bimodal exponential distribution. The inverse power law was used to model stress acceleration at the component level. Renewal theory was used to calculate the expected number of component replacements. Connections were assumed to be minimally repaired. An optimal screening strategy was formed by minimizing the cost of the system while under warranty.

2.6 Bayesian Extensions

ESS and burn-in procedures are designed to purge populations of parts having hidden latent defects. By subjecting components to a stress designed to precipitate early failures, the population of parts entering the field will have a much smaller proportion of defectives. Most of the models in the existing literature for planning ESS and burn-in procedures are entirely non-Bayesian. They assume perfect knowledge of the proportion of weak components in the initial population. Perlstein and Welch [95] conducted a parametric study and showed that burn-in policies are extremely sensitive to the proportion of weak components in the sub-population.

One reason for considering a Bayesian approach is that in a time of rapidly changing technologies and production methods, the effects of production maturity will play a significant role on the proportion of defects in the population of products being produced. This effect is clearly demonstrated in Figure 2.4. Pantic [90] examined the benefits of burn-in at 125 °C for 168 hrs on integrated circuits over a period from 1979-
1983. The data revealed that the burn-in policy reduced the failure rate by 22% in 1979 and only 2.4% in 1983. These results suggest that the burn-in policies need to be periodically updated as the production process proceeds in time.

Figure 2.15 Effects of Production Maturity on Hazard Rate [90].

Perlstein [94] developed a Bayesian approach to burn-in. He utilized the mixed exponential and mixed Weibull distribution to characterize the time to failure distributions. The focus of his research effort was on modeling the proportion of weak components in the population. He used a beta distribution to model the uncertainty associated with the proportion of weak components. Using Bayesian analysis he constructed posterior estimates which can be used to reassess current burn-in policies.

Barlow, Bazovsky, and Wechsler [6], develop a Bayesian approach to ESS. They recognized that not only is there uncertainty associated with the degree of contamination, but there is uncertainty associated with the parameters of the life distributions. A mixed
exponential distribution is used to model the time to failure for the component population. Like Perlstein [94], they use a beta distribution to model the uncertainty associated with the mixture parameter. Assuming the mixture parameter is independent of the failure rates of the two populations, they develop a joint prior distribution given by

\[ f(\pi, \lambda_1, \lambda_2) = \theta \tau \left[ \frac{\Gamma(a + b)}{\Gamma(a) \Gamma(b)} \right] \pi_1^{a-1} \pi_2^{b-1} e^{-\lambda_2 (\theta - \tau)} e^{-\lambda_1 \tau} \]

(2.70)

where an exponential prior (\theta) is used to model the failure rate of good parts and a shifted exponential prior (\theta - \tau) is used for the failure rate of bad parts. They find the optimal screen duration using a rather simple cost model that uses the ratio of the cost of a bad part escaping the screen to the cost of a good part being destroyed by the screen. They also develop expressions for the yield, defined as the probability of having zero substandard components remaining in the population after the screen, remaining defect density, and screening strength.
Chapter 2 provided an extensive summary of the relevant ESS models and techniques found in the literature. Upon careful examination of the literature, one quickly discovers that there is no uniform framework in which ESS is modeled. Every model in the literature is unique in its own way and examines a very specific problem. Using the strengths and weaknesses of the models currently in the literature and expanding on some of their limitations, a general modeling framework for ESS is developed. The goal is to be able to take this general framework and apply it to a wide variety of specific problems. This will be demonstrated in subsequent chapters.

3.1 System Screening Model

In this section a general model for a multi-level stress screening process is developed. For notational convenience, a hypothetical electronics system will be studied. A generalization for all systems, is that they are nothing more than a collection of components and connections arranged in a specific fashion to perform a specific function. The system under study in this chapter consists of $i$ different types of components which are assembled into $l$ different types of printed circuit boards (PCBs) using $j$ different types
of board level connections. The \( l \) different types of PCBs are then assembled into \( m \) different types of line replaceable units (LRUs) using \( k \) different types of unit level connections. Finally, the \( m \) different types of units are assembled into a system using \( n \) types of system level connections. The term \textit{connection} is used in a general sense to refer to all defect prone elements introduced at that particular level of the assembly process.

Like many of the screening models discussed in Chapter 2, a mixed distribution will be used to model variability in components and \textit{connections}. The basic assumption is that at each level of the manufacturing and assembly process new defects may be introduced and latent defects may fail. Figure 3.1 shows the general screening and assembly process for the system studied in this chapter.

---

**Figure 3.1:** General screening and assembly process for the system studied in this chapter.

- **Component 1**
- **Component 2**
- **Component 3**
- **Component 4**
- **Component N**

- ** Defects Induced at Card Assembly **
- ** Defects Induced at Unit Assembly **
- ** Defects Induced at Final Assembly **

- **Screen**
- **Screen**
- **Screen**

- **Detected Defective Components**
- **Detected Defective PCBs**
- **Detected Defective LRU**
- **Detected Defective Systems**

- **Representative Repair Costs**
  - $1-5$
  - $25-50$
  - $250-500$
  - $500-1000$
  - $5000-20,000$
The screening model allows examination of stress screening strategies for four different levels of assembly; component, board, unit and system. Components are screened, and those passing the screen are used to assemble PCBs. The PCBs are then screened to detect defects introduced during their assembly process. Those PCBs that pass the screen are then used to assemble LRUs. The LRUs are subsequently screened to detect defects introduced during their assembly process. LRUs that pass their screens are then assembled into the final system. A final screen is performed at the system level in order to eliminate any defects introduced during final assembly.

Different types of stress screens can be used at each of the various assembly levels. Since it is often more economical to screen at stress levels higher than normal operational levels, it is assumed that the maximum stress at each level will not exceed the maximum allowable stress for any individual elements at the current assembly level. Finally, it is assumed that no new failure modes are introduced as a result of screening at higher than operational stress levels.

3.1.1 Component Level Screen

As shown in Figure 3.1, the purpose of component level screening is to weed out defective components by subjecting them to stresses that induce failures before they are used to assemble PCBs. A mixed distribution, corresponding to weak and good components, is used to characterize the population for each component type. Under most
circumstances, the fraction of weak components produced by a manufacturing process will be much smaller than the fraction of good components produced. Also, it is assumed that the weak components have much higher failure rates than the good components.

Components are screened for a specified duration. The length of the screen determines the probability that the component passing the screen is in fact a good component. The longer the screen, the higher the probability that the component comes from the good population of components.

Screening at the component level is usually done at stress levels that are much higher than operational stress levels. Using an appropriate degradation model, the equivalent operational time for components undergoing testing at higher stresses can be calculated. A linear acceleration factor is assumed to exist [118]. Thus, the equivalent operational time for a type $i$ component can be related to the time under increased stress by the relationship:

$$ t_{c_i} = K_{c_i} \cdot t_{ac} $$  \hspace{1cm} (3.1)

It is assumed that the value of $K_{c_i}$, which is dependent upon the type of accelerating stress, is the same for both weak and good component populations. Chapter 2 provides detailed coverage of accelerated testing models for a variety of stress environments.

For an unscreened type $i$ component, the probability density function and reliability function for the mixed distribution is:
\[ f_{q_i}(t) = p_{wc_i} \cdot f_{wc_i}(t) + p_{gc_i} \cdot f_{gc_i}(t) \]  
(3.2)

\[ R_{q_i}(t) = p_{wc_i} \cdot R_{wc_i}(t) + p_{gc_i} \cdot R_{gc_i}(t) \]  
(3.3)

where \( p_{wc_i} + p_{gc_i} = 1 \). The probability that a type \( i \) component which survives a screen of equivalent operational duration \( t_{c_i} \), is weak (good) is:

\[ p_{wc_i}(t_{c_i}) = \frac{p_{wc_i} \cdot R_{wc_i}(t_{c_i})}{R_{q_i}(t_{c_i})} \]  
(3.4)

\[ p_{gc_i}(t_{c_i}) = \frac{p_{gc_i} \cdot R_{gc_i}(t_{c_i})}{R_{q_i}(t_{c_i})} \]  
(3.5)

Therefore, the reliability and pdf for a type \( i \) component selected at random from the screened population are given by:

\[ R_{q_i}(t|t_{c_i}) = p_{wc_i}(t_{c_i}) \cdot \frac{R_{wc_i}(t + t_{c_i})}{R_{wc_i}(t_{c_i})} + p_{gc_i}(t_{c_i}) \cdot \frac{R_{gc_i}(t + t_{c_i})}{R_{gc_i}(t_{c_i})} \]  
(3.6)
which can be rewritten as:

\[
f_{c_i}(t|c_i) = p_{wc_i}(t_{c_i}) \cdot \frac{f_{wc_i}(t + t_{c_i})}{R_{wc_i}(t_{c_i})} + p_{gc_i}(t_{c_i}) \cdot \frac{f_{gc_i}(t + t_{c_i})}{R_{gc_i}(t_{c_i})}
\] (3.7)

which can be rewritten as:

\[
R_{c_i}(t|t_{c_i}) = \frac{p_{wc_i} \cdot R_{wc_i}(t + t_{c_i}) + p_{gc_i} \cdot R_{gc_i}(t + t_{c_i})}{R_{c_i}(t_{c_i})}
\] (3.8)

\[
f_{c_i}(t|t_{c_i}) = \frac{p_{wc_i} \cdot f_{wc_i}(t + t_{c_i}) + p_{gc_i} \cdot f_{gc_i}(t + t_{c_i})}{R_{c_i}(t_{c_i})}
\] (3.9)

Components are generally assumed to be nonrepairable and are usually discarded once they have failed. The probability that a type \(i\) component successfully passes the screen is given by \(R_{c_i}(t_{c_i})\).

### 3.1.2 Board Level Screen

The purpose of board level screening is to weed out manufacturing defects introduced during the assembly of the PCB as well as any other defects that may have
escaped the previous screen. As in Reddy and Dietrich [102] and Pohl and Dietrich [99, 100], it will be assumed that the connections are the primary source of defects introduced at this assembly level. The number of PCB connections of type $j$ on PCB type $l$ is given by $n p_{j,l}$.

Unlike components, connections are not screened separately. The assembled PCB is screened for a specified duration at a prescribed stress level and those PCB’s that survive the screen are used at the next level. The PCB’s may be screened at stress levels that are higher than operational levels. Therefore, an appropriate acceleration model is used to calculate the equivalent operational time for the connections as well as for the components on the PCB. The equivalent operational times for connections and components for type $l$ PCB’s are related to the time under increased stress by the following:

\begin{align}
    t_{b_{j,l}} &= K_{b_{j,l}} \cdot t_{ab_l} \\
    t_{bc_{i,l}} &= K_{bc_{i,l}} \cdot t_{ab_l} 
\end{align}  \quad (3.10)

The acceleration factors are dependent upon the type of stress applied during PCB level screening. Since stresses are chosen to induce specific defect types, it is possible that the stresses used at the PCB level might have no effect on a component lifetime. Under those circumstances, the acceleration factor for the components can be set to zero.
PCB connections are assumed to come from a mixed population of weak and good connections. The probability density function and reliability function for a screened type \( j \) connection on a type \( l \) PCB are found using the methods described in equations 3.2 - 3.9 and are given by:

\[
R_{b_j}(t|t_{b_{j,l}}) = \frac{p_{wb_j} \cdot R_{wb_j}(t + t_{b_{j,l}}) + p_{gb_j} \cdot R_{gb_j}(t + t_{b_{j,l}})}{R_{b_j}(t_{b_{j,l}})}
\]  
(3.11)

\[
f_{b_j}(t|t_{b_{j,l}}) = \frac{p_{wb_j} \cdot f_{wb_j}(t + t_{b_{j,l}}) + p_{gb_j} \cdot f_{gb_j}(t + t_{b_{j,l}})}{R_{b_j}(t_{b_{j,l}})}
\]  
(3.12)

In general, connections are assumed to be repairable. When a PCB fails due to a connection, that particular connection is minimally repaired. A component is minimally repaired when it is restored to the condition it was in just prior to failure. The expected number of repairs to a type \( j \) connection on PCB type \( l \) is found by integrating the connection failure rate. Using the following identity from Table 2.1:
Thus, the expected number of repairs to a type $j$ connection on a type $l$ PCB which has completed board level screening is given by:

\[ NB_j(t_{b,j,I}) = -\ln R_{b,j}(t_{b,j,I}) \]  \hspace{1cm} (3.14)

In some instances, certain connections may not be repairable and therefore must be replaced upon failure. Like nonrepairable components, nonrepairable connections are discarded once they have failed. The probability that a type $j$ connection on PCB type $l$ successfully passes the PCB level screen is given by $R_{b,j}(t_{b,j,I})$.

In addition to connections, an assembled PCB consists of screened components. Components are assumed to be nonrepairable. When a component fails during PCB screening, it is replaced with a stochastically identical component. In order to be stochastically identical, a component that fails during PCB screening must be replaced with a component taken from the inventory of screened components. The sequence of failures followed by replacement constitutes a renewal process, and the expected number of replacements in $[0,t]$ for a type $i$ component that has already undergone $t_{c,i}$ hours of screening is given by the following renewal equation [26]:

\[ N_{c,i}(t|t_{c,i}) = \int_{0}^{t} n_{c,i}(u|t_{c,i}) \cdot du \]  \hspace{1cm} (3.15)
where the renewal density is given by:

$$
n_{c_i}(t|\tau_{c_i}) = f_{c_i}(t|\tau_{c_i}) + \int_0^t n_{c_i}(t-u|\tau_{c_i}) \cdot f_{c_i}(u|\tau_{c_i}) \, du \quad (3.16)$$

Rather than solving the integral equation directly, Perlstein et al. [96, 98], Reddy and Dietrich [1102], and Pohl and Dietrich [99] all make use of Laplace transform theory to determine the expected number of renewals. The Laplace transform of the renewal density is:

$$
n^*(s|\tau_{c_i}) = \frac{f^*(s|\tau_{c_i})}{1 - f^*(s|\tau_{c_i})} \quad (3.17)$$

Unfortunately, the Laplace transforms for many distributions of interest (i.e. Weibull, lognormal) do not exist. Therefore, the integral equation must be solved numerically to compute the renewal density. Soland [108], Huang [49], Smeitink and Dekker [105], Baxter et al. [10], and Xie [126] all discuss numerical methods for solving the renewal integral equation (3.10). This topic will be discussed in detail in Chapter 4.
3.1.3 Unit Level Screen

The purpose of unit level screening is to weed out manufacturing defects introduced during unit assembly as well as any other defects that may have escaped earlier screens. Assume the system has $m$ types of units and there are $k$ different types of unit level connections. As in earlier sections, assume that connections are the primary source of defects introduced at this level. Let $n_{u_{k,m}}$ be the number of type $k$ connections on a type $m$ unit.

Like PCB connections, unit connections are not screened separately. The assembled LRU is screened for a specified duration at a prescribed stress level and once an LRU passes the screen it is sent to final assembly. The units' s may be screened at stress levels that are higher than operational levels, although screening at the unit level is usually performed at much lower stress levels than component or board level screening. An appropriate accelerated stress model is used to calculate the equivalent operational time for the unit connections, PCB connections, and components in the system. The equivalent operational times for the unit connections, PCB connections, and components in the system are related to the time under increased stress by the following:

$$t_{u_{k,m}} = K_{u_{k,m}} \cdot t_{a_{u,m}}$$
$$t_{ub_{j,m}} = K_{u_{j,m}} \cdot t_{a_{u,m}}$$
$$t_{uci,m} = K_{uci,m} \cdot t_{a_{u,m}}$$

(3.18)
Unit connections are also assumed to come from a mixed population of weak and good connections. The probability density function and reliability function for a screened type \( k \) connection in a type \( m \) unit are found using the methods described in equations 3.2-3.9 and are given by:

\[
R_{u_k}(t|t_{u_k,m}) = \frac{P_{wu_k} \cdot R_{wu_k}(t + t_{u_k,m}) + P_{gu_k} \cdot R_{gu_k}(t + t_{u_k,m})}{R_{u_k}(t_{u_k,m})} \tag{3.19}
\]

\[
f_{u_k}(t|t_{u_k,m}) = \frac{P_{wu_k} \cdot f_{wu_k}(t + t_{u_k,m}) + P_{gu_k} \cdot f_{gu_k}(t + t_{u_k,m})}{R_{u_k}(t_{u_k,m})} \tag{3.20}
\]

Like PCB connections, unit connections can be either be repairable or nonrepairable. When a repairable unit connection fails, that particular connection is minimally repaired. The expected number of repairs to a type \( k \) repairable connection in a type \( m \) unit is found by using equation 3.13. The expected number of repairs to a type \( k \) connection in a type \( m \) unit during unit level screening is:

\[
NU_k(t_{u_k,m}) = -\ln[R_{u_k}(t_{u_k,m})] \tag{3.21}
\]
In some instances, certain unit connections may not be repairable and therefore must be replaced upon failure. Like nonrepairable board connections, nonrepairable unit connections are discarded once they have failed. The probability that a type $k$ unit level connection in a type $m$ LRU successfully passes the unit level screen is given by

$$R_{uk}(t_{uk,m}).$$

In addition to unit connections, unit level screening also provides additional screening to components and PCB connections. If a component fails during unit level screening and is replaced by a component from the inventory of screened components, then the resulting stochastic process is a modified renewal process. This component will not be stochastically identical to the component it replaces because it will not have undergone PCB level screening. The renewal density for the modified renewal process for a type $i$ component that is installed on a type $l$ PCB is:

$$n_{ci}(t|tc_i + t_{bc_{ci,1}}) = f_{ci}(t|tc_i + t_{bc_{ci,1}}) + \int_0^t n_{ci}(t - u|tc_i + t_{bc_{ci,1}}) \cdot f_{ci}(u|tc_i) \cdot du$$  \hspace{1cm} (3.22)

The failed component can be replaced with a stochastically identical component by providing the equivalent amount of board level screening to it before it is installed in the unit. The renewal density for this situation is given by:
Similarly, PCB connections that fail during unit level screening must either be repaired or replaced. For repairable PCB connections, the expected number of repairs to a type $j$ connection on PCB type $l$ in a type $m$ LRU during unit level screening is:

$$nb_j(t|t_{b_{j,l}}) = f_{b_j}(t|t_{b_{j,l}}) + \int_0^t n_{c_i}(t-u|t_{c_{i,l}} + t_{b_{c_{i,l}}}) \cdot f_{c_i}(u|t_{c_{i,l}} + t_{b_{c_{i,l}}}) \cdot du$$ (3.23)

Nonrepairable board level connections that fail during unit level screening can be replaced with connections from the inventory. The resulting stochastic process is a modified renewal process. The renewal density for the modified renewal process for a nonrepairable type $j$ connection on a type $l$ PCB is:

$$NB_j(t_{ub_{j,m}}|t_{b_{j,l}}) = -\ln[R_j(t_{ub_{j,m}}|t_{b_{j,l}})]$$ (3.24)

The failed connection could also be replaced with a stochastically identical connection by providing an equivalent amount of board level screening to it before it is installed in the unit. The renewal density for this situation is given by:

$$n_{c_i}(t|t_{c_{i,l}} + t_{b_{c_{i,l}}}) = f_{c_i}(t|t_{c_{i,l}} + t_{b_{c_{i,l}}}) + \int_0^t n_{c_i}(t-u|t_{c_{i,l}} + t_{b_{c_{i,l}}}) \cdot f_{c_i}(u|t_{c_{i,l}} + t_{b_{c_{i,l}}}) \cdot du$$ (3.25)
System level screening provides the final barrier before a product is released to the field. The LRUs are assembled into a system using $n$ different types of system level connections. Let $n s_n$ represent the number of type $n$ connections used to assemble the system.

Like LRU, and PCB connections, system level connections are not screened separately. The assembled system is screened for a specified duration at a prescribed stress level and those systems that successfully pass the screen are released for use in the field. The units may be screened at higher than operational stress levels. An appropriate accelerated stress model is used to calculate the equivalent operational times for system, unit, and board level connections, as well as components. The equivalent operational times are related to the time under increased stress by the following:

\[
\begin{align*}
    t_{s_n} &= K_{s_n} \cdot t_{as} \\
    t_{s_k} &= K_{s_k} \cdot t_{as} \\
    t_{s_j} &= K_{s_j} \cdot t_{as} \\
    t_{s_i} &= K_{s_i} \cdot t_{as}
\end{align*}
\] (3.27)
System connections, like connections from previous assembly levels, are assumed to come from a mixed population of good and weak connections. The probability density function and reliability function for a screened type $n$ connection are given by:

$$R_{sn}(t|t_{sn}) = \frac{p_{ws_n} \cdot R_{ws_n}(t + t_{sn}) + p_{gs_n} \cdot R_{gs_n}(t + t_{sn})}{R_{sn}(t_{sn})} \quad (3.28)$$

$$f_{sn}(t|t_{sn}) = \frac{p_{ws_n} \cdot f_{ws_n}(t + t_{sn}) + p_{gs_n} \cdot f_{gs_n}(t + t_{sn})}{R_{sn}(t_{sn})} \quad (3.29)$$

Like LRU connections, system connections can be either be repairable or nonrepairable. When a repairable system connection fails, that particular connection is minimally repaired. The expected number of repairs to a type $n$ repairable connection during system level screening is given by:

$$NS_n(t_{sn}) = -\ln[R_{sn}(t_{sn})] \quad (3.30)$$

In some instances, certain system level connections may not be repairable and therefore must be replaced upon failure. Like nonrepairable LRU connections, nonrepairable system
connections are discarded once they have failed. The probability that a type \( k \) system level connection successfully passes the system level screen is given by \( R_{sn}(t_{sn}) \).

In addition to screening system connections, system level screening also provides additional screening to components, PCB connections, and LRU connections. If a component fails during system level screening and is replaced by a component from the inventory of screened components, then the resulting stochastic process is a modified renewal process. This component will not be stochastically identical to the component it replaces because it will not have undergone PCB and LRU level screening. The renewal density for the modified renewal process for a type \( i \) component that is installed on a type \( l \) PCB in a type \( m \) LRU is given by:

\[
\frac{n_{ci}}{t_{ci} + t_{bc_{i,l}} + t_{uc_{i,m}}} = \left[ f_{ci} \left( t_{ci} + t_{bc_{i,l}} + t_{uc_{i,m}} \right) + \right] \\
\left[ 0 \int_{t_{ci}}^{t} n_{ci} \left( t - u, c_{i} + t_{bc_{i,l}} + t_{uc_{i,m}} \right) \cdot f_{ci}(u) \cdot du \right]
\]  

(3.31)

The failed component can be replaced with a stochastically identical component by providing the equivalent amount of board and unit level screening to it before it is installed in the unit. The renewal density for this situation is given by:
Similarly, PCB and LRU connections that fail during system level screening must either be repaired or replaced. For repairable PCB connections, the expected number of repairs made to a type $j$ connection on a type $l$ PCB in a type $m$ LRU during system level screening is:

$$n_{c_l}(t) = f_{c_l}(t_{c_l} + t_{bc_{j,l}} + t_{uc_{j,m}}) + \int_{0}^{t} n_{c_l}(t-u) f_{c_l}(u_{c_l} + t_{bc_{j,l}} + t_{uc_{j,m}}) \cdot du$$ (3.32)

Similarly, the expected number of repairs to a type $k$ connection in a type $m$ LRU during system level screening is:

$$NB_j(t_{s_j}, t_{b_{j,l}} + t_{ub_{j,m}}) = -\ln \left[ R_{b_j}(t_{s_j}, t_{b_{j,l}} + t_{ub_{j,m}}) \right]$$ (3.33)

Similarly, the expected number of repairs to a type $k$ connection in a type $m$ LRU during system level screening is:

$$NU_k(t_{s_k} t_{u_{k,m}}) = -\ln \left[ R_{u_k}(t_{s_k} t_{u_{k,m}}) \right]$$ (3.34)

Nonrepairable board level connections that fail during system level screening can be replaced with connections from the inventory. The resulting stochastic process is a modified renewal process. The renewal density for the modified renewal process for a nonrepairable type $j$ connection on a type $l$ PCB in a type $m$ LRU is:
The failed board level connection could also be replaced with a stochastically identical connection by providing an equivalent amount of board and LRU level screening to it before it is installed in the system. The renewal density for this situation is given by:

\[
nb_j(t|t_{b,j,l} + t_{ub,j,m}) = \int_0^t nb_j(t - |t_{b,j,l} + t_{ub,j,m}) \cdot f_b_j(u|0) \cdot du
\]  

(3.36)

Like nonrepairable board connections, nonrepairable unit connections that fail during system level screening can be replaced with unit connections from the inventory. The resulting stochastic process is a modified renewal process. The renewal density for the modified renewal process for a nonrepairable type \( k \) connection in a type \( m \) LRU is:

\[
nu_k(t|t_{u_k,m}) = f_{u_k}(t|t_{u_k,m}) + \int_0^t nu_k(t - |t_{u_k,m}) \cdot f_{u_k}(u|0) \cdot du
\]  

(3.37)
The failed unit level connection could also be replaced with a stochastically identical connection by providing an equivalent amount of unit level screening to it before it is installed in the system. The renewal density for this situation is given by:

\[ n_{uk}(t|t_{uk,m}) = f_{uk}(t|t_{uk,m}) + \int_0^t n_{uk}(t-u|t_{uk,m}) \cdot f_{uk}(u|t_{uk,m}) \cdot du \]  (3.38)

### 3.1.5 Field Performance

The ultimate goal of ESS is to reduce the number of early life failures in the field. The expected number of failures in the field is a function of the amount of screening done to the components and connections during manufacture and assembly. Also, it will be a function of the type of components and connections used to repair the system should it fail in the field. At the component level, when a component fails in the field, and is replaced by a component from the inventory of screened components, the resulting modified renewal process for a type \( i \) component that is installed on a type \( l \) PCB in a type \( m \) LRU is:

\[
n_{ci}(t|t_{ci} + t_{bc_{i,l}} + t_{uc_{i,m}} + t_{si}) = \left[ f_{ci}(t|t_{ci} + t_{bc_{i,l}} + t_{uc_{i,m}} + t_{si}) + \int_0^t n_{ci}(t-u|t_{ci} + t_{bc_{i,l}} + t_{uc_{i,m}} + t_{si}) \cdot f_{ci}(u|t_{ci}) \cdot du \right]
\]  (3.39)
Similiarly, nonrepairable *connections* that fail in the field can be replaced with *connections* from the inventory. As with components, the resulting stochastic processes are modified renewal processes. The renewal density for the modified renewal processes for nonrepairable board, unit and system level connections are given by:

\[
nb_j \left( t | t_{b_{j,l}} + t_{ub_{j,m}} + t_{s_j} \right) = \left[ f_{b_j} \left( t | t_{b_{j,l}} + t_{ub_{j,m}} + t_{s_j} \right) + \right. \\
\left. \int_0^t nb_j \left( t - u | t_{b_{j,l}} + t_{ub_{j,m}} + t_{s_j} \right) \cdot f_{b_j} (u|0) \cdot du \right]
\]

\[
nu_k \left( t | t_{u_{k,m}} + t_{s_k} \right) = \left[ f_{u_k} \left( t | t_{u_{k,m}} + t_{s_k} \right) + \right. \\
\left. \int_0^t nu_k \left( t - u | t_{u_{k,m}} + t_{s_k} \right) \cdot f_{u_k} (u|0) \cdot du \right]
\]

\[
ns_n \left( t | t_{s_n} \right) = f_{s_n} \left( t | t_{s_n} \right) + \int_0^t ns_n \left( t - u | t_{s_n} \right) \cdot f_{s_n} (u|0) \cdot du
\]

Upon examining equations 3.39-3.42, it is evident that the amount of screening performed on spare components and connections will have a significant impact on the amount of repair work done on the system in the field. In all cases, the modified renewal process could be replaced with an ordinary renewal process if all spare components and
connections underwent screening equivalent to the amount they would have seen if they were in the original system. Thus, when considering the purchase of spares, the life cycle cost impacts of screening spares should be examined.

For repairable connections, the expected number of field repairs for board, unit and system level connections are found using equation 3.13. They are given by:

\[
NB_j(t|t_{b,j,l} + t_{ub,j,m} + t_{s,j}) = -\ln[R_{b,j}(t|t_{b,j,l} + t_{ub,j,m} + t_{s,j})]
\]

\[
NU_k(t|t_{u,k,m} + t_{s_k}) = -\ln[R_{u,k}(t|t_{u,k,m} + t_{s_k})]
\]

\[
NS_n(t|t_{s_n}) = -\ln[R_{s_n}(t|t_{s_n})]
\]

### 3.2 Life Cycle Cost Model Formulation

ESS is only effective if it is a value added process. There is a necessary tradeoff between cost of screening and cost of failure. This section develops a general life cycle cost model which can be used in developing cost effective ESS strategies for complex systems. Screening costs generally increase with each level of assembly. The test
chambers and associated equipment become more expensive as items become larger. Thus, system level screening is more expensive than unit screening which is more expensive than PCB screening which is slightly more expensive than component level screening. As the duration of the screens increase, the field expenses decrease but the screening costs also increase. Optimal screen durations at each screening level are those which minimize the systems life cycle costs. The general life cycle cost model consists of both fixed and variable costs. Fixed costs are incurred from purchasing and setting up the various screens. The variable costs are a function of the amount of screening as well as the number of failures. The general life cycle cost model can be separated into four elements; component costs, PCB costs, unit costs, and system costs.

\[
\text{LCC} = \text{CC} + \text{PCB} + \text{UNIT} + \text{SYS} \tag{3.46}
\]

3.2.1 Component Costs

Life cycle costs due to components are dependent upon how much screening is done at component, PCB, unit, and system levels. The component life cycle cost model consists of both fixed and variable costs. A fixed cost is incurred if component level screening is performed. Variable costs include the cost of screening components for a specified duration, cost of replacing a failed component during component, PCB, unit, and system level screening, and the costs associated with replacing a failed component in the
field. The variable costs for components are dependent upon the number and type of components used, what PCB they are placed in, what unit those boards are used in, as well as the ability to detect failures at the various screening levels. The component cost model is:

\[ CC = sc_i \cdot I(t_{ci}) + \sum_m nuts_m \cdot \sum_t npcb_{t,m} \cdot \sum_i nc_{i,l} \cdot (ct_i \cdot t_{ac_i} + C1 + B1 + U1 + S1 + F1) \]

(3.47)

where

\[ C1 = DC \cdot N_{c_i}(t_{ci}|0) \cdot crcc_i \]

\[ B1 = DB \cdot \left( N_{c_i}(t_{bc_i,l}|t_{ci}) + (1 - DC) \cdot N_{c_i}(t_{ci}|0) \right) \cdot crbc_{c_i,l} \]

\[ U1 = DU \cdot \left( N_{c_i}(t_{uc_i,m}|t_{bc_i,l} + t_{ci}) + (1 - DB) \cdot \left( N_{c_i}(t_{bc_i,l}|t_{ci}) + (1 - DC) \cdot N_{c_i}(t_{ci}|0) \right) \right) \cdot cruc_{c_i,l,m} \]

\[ S1 = DS \cdot \left( 1 - DU \right) \left( N_{c_i}(t_{uc_i,m}|t_{bc_i,l} + t_{ci}) + (1 - DB) \cdot \left( N_{c_i}(t_{bc_i,l}|t_{ci}) + (1 - DC) \cdot N_{c_i}(t_{ci}|0) \right) \right) \cdot crsc_{c_i,l,m} \]
\[
F_l = \left( N_{c_i} \left( t_{s_i} + t_{uc_{i,m}} + t_{bc_{i,l}} + t_{c_i} \right) + (1 - DS) \right) \cdot crf_{i,l,m} \\
\times \left( (1 - DU) \cdot \left( N_{c_i} \left( t_{uc_{i,m}} + t_{bc_{i,l}} + t_{c_i} \right) + (1 - DB) \cdot \left( N_{c_i} \left( t_{bc_{i,l}} + t_{c_i} \right) + (1 - DC) \cdot N_{c_i} \left( t_{c_i} \right) \right) \right) \right)
\] (3.48)

C1 is the cost associated with finding a weak component during component level screening. DC is the detection efficiency for the test equipment used to detect defects for components which have been precipitated to failure by the stress screen. Not all failures precipitated during component level screening will necessarily be found at this level. Some failed components may be passed on to the next assembly level. \( N_{c_i} \left( t_{c_i} \right) \) is the expected number of type \( i \) component replacements during component level screening. Finally, \( crcc_i \) is the cost of a type \( i \) component.

B1 is the cost associated with finding weak components during board level screening. DB is the detection efficiency for the test equipment used to detect board level defects which have been precipitated to failure by the stress screen. In general, the detection efficiency tends to increase with each assembly level (i.e. \( DB > DC \)).

\( N_{c_i} \left( t_{bc_{i,l}} + t_{c_i} \right) \) is the expected number of component replacements for type \( i \) components on type \( l \) boards during board level screening that have survived component level screening. Additionally, those components that fail during component level screening but
are not detected may be found during board level screening. \( crbc_{i,l} \) is the cost of replacing a type \( i \) component on a type \( l \) board when it fails during board level screening. This cost would ordinarily include the cost of the component, fault isolation costs, the costs of screening a replacement component, removing the defective component, and installing the new component on the board. Depending on the product, and the costs to replace components, a component failure during board level screening may result in the entire board being scraped, and thus the cost due to component failure on a type \( l \) board would be the cost of a type \( l \) board.

\( U_1 \) is the cost associated with finding weak components during unit level screening. \( DU \) is the detection efficiency for the test equipment used to detect unit level defects which have been precipitated to failure by the stress screen. \( N_{ci} \left( t_{uc_{i,m}} \left\lfloor t_{ci} + t_{bc_{i,l}} \right\rfloor \right) \) is the expected number of components replaced during unit level screening for type \( i \) components on type \( l \) boards in type \( m \) units that have survived component and board level screening. In addition, those components that fail during component and/or board level screening and are not detected may be found during unit level screening. \( cruc_{i,l,m} \) is the cost of replacing a type \( i \) component on a type \( l \) board in a type \( m \) unit when it fails during unit level screening. This cost would ordinarily include the cost of the component, fault isolation costs, the costs of screening a replacement component, removing the defective component, and installing the new component on the board and replacing the board in the unit. Depending on the product, and the costs to replace components, a component failure during unit level screening may result in a board that contains the
component being scraped. Thus, the cost due to component failure on a type $l$ board in a type $m$ unit would be the cost of a type $l$ board.

S1 is the cost associated with finding weak components during system level screening. $DS$ is the detection efficiency for the test equipment used to detect system level defects which have been precipitated to failure by the stress screen.

$$N_{c_i} \left( t_{s_i} + t_{bc_{i,l}} + t_{uc_{i,m}} \right)$$ is the expected number of components replaced during system level screening for type $i$ components on type $l$ boards in type $m$ units which have survived component, board, and unit level screening. Additionally, those components that fail during component, board, and unit level screening and are not detected at subsequent levels may be found during system level screening. Finally, $crsc_{i,l,m}$ is the cost of replacing a type $i$ component on a type $l$ board in a type $m$ unit which fails during system level screening. This cost would ordinarily include the cost of the component, fault isolation costs, the costs of screening a replacement component, removing the defective component, installing the new component on the board, replacing the board in the unit, replacing the unit in the system. Depending on the product, and the costs to replace components, a component failure during system level screening may result in the board that contains the component being scraped. Thus, the cost due to component failure on a type $l$ board in a type $m$ unit would be the cost of a type $l$ board.

Finally, $F1$ is the cost associated with having weak components fail in the field.

$$N_{c_i} \left( TW + t_{bc_{i,l}} + t_{uc_{i,m}} + t_{s_i} \right)$$ is the expected number of component replacements in the
field for type $i$ components on type $l$ boards in type $m$ units during warranty period $TW$.

Additionally, those components that fail during component, board, unit, and system level screening and are not detected before entering the field will need to be replaced in the field. $crsc_{i,l,m}$ is the cost of replacing a type $i$ component on a type $l$ board in a type $m$ unit in the field. This cost will be dependent upon the maintenance strategy employed by the manufacturer. In some cases, an actual component may be replaced in the field, more often, the board or unit with the defective component on it would be replaced in the field. Therefore this cost includes component, board, or unit replacement costs, whichever is appropriate, fault isolation costs, removing the defective component from the system, and installing the replacement component, board, or unit in the system.

### 3.2.2 PCB Costs

Similar to component costs, the costs associated with a PCB are dependent upon how much screening is done on PCB connections during PCB, unit, and system level screening. The PCB life cycle cost model consists of both fixed and variable costs. A fixed cost is incurred if PCB level screening is performed. Variable costs are dependent upon the amount of screening done at the various screening levels and include the cost of screening PCBs for a specified duration, cost to repair or replace a failed PCB connection during PCB, unit, and system level screening, and the costs associated with repairing or replacing a failed PCB connection in the field. The variable costs for PCB connections are
also affected by the number and type of connections used, what PCB they are on, what unit those boards are used in, and the ability to detect failures at the various screening levels. The PCB model is:

\[
PCB = sb_t \cdot I(t_{b_{j,l}}) + \sum_m \text{nuts}_m \cdot \sum_{l'} \text{nppcb}_{l,m} \cdot \sum_i \text{nb}_{i,j,l} \cdot (c_b \cdot t_{ab_t} + B_2 + U_2 + S_2 + F_2)
\]

(3.49)

where

\[
B_2 = DB \cdot NB_j(t_{b_{j,l}}|0) \cdot crbb_{j,l}
\]

\[
U_2 = DU \cdot \left( NB_j(t_{ub_{j,m}}|t_{b_{j,l}}) + (1 - DB) \cdot NB_j(t_{b_{j,l}}|0) \right) \cdot crub_{j,l,m}
\]

\[
S_2 = DS \cdot \left( NB_j(t_{s_{j}}|t_{ub_{j,m}} + t_{b_{j,l}}) + (1 - DU) \cdot \left( NB_j(t_{ub_{j,m}}|t_{b_{j,l}}) + \left(1 - DB\right) \cdot NB_j(t_{b_{j,l}}|0) \right) \right) \cdot crsb_{j,l,m}
\]

\[
F_2 = \frac{1}{\left(1 - DS\right) \cdot \left( NB_j(t_{s_{j}}|t_{ub_{j,m}} + t_{b_{j,l}}) + (1 - DU) \cdot \left( NB_j(t_{ub_{j,m}}|t_{b_{j,l}}) + \left(1 - DB\right) \cdot NB_j(t_{b_{j,l}}|0) \right) \right)} \cdot crfb_{j,l,m}
\]

(3.50)
B2 is the cost associated with finding weak PCB connections during board level screening. $DB$ is the detection efficiency for the test equipment used to detect board level defects which have been precipitated to failure by the stress screen. $NB_j(t_{b,j,l}, 0)$ is the expected number of PCB connection repairs (replacements) for type $j$ connections on type $l$ boards during board level screening. $crb_{j,l}$ is the cost of repairing (replacing) a type $j$ connection on a type $l$ board when it fails during board level screening. This cost would ordinarily include the cost to fault isolate and repair (replace) the connection. Depending on the product, and the costs to repair (replace) connections, a connection failure during board level screening may result in the entire board being scraped, and thus the cost due to connection failure on a type $l$ board would be the cost of a type $l$ board.

U2 is the cost associated with finding weak PCB connections during unit level screening. $DU$ is the detection efficiency for the test equipment used to detect unit level defects which have been precipitated to failure by the stress screen. $NB_j(t_{ub,j,m}, t_{b,j,l})$ is the expected number of PCB connection repairs (replacements) during unit level screening for type $j$ connections on type $l$ boards in type $m$ units that have survived board level screening. In addition, those PCB connections that fail during board level screening and are not detected may be found during unit level screening. $crub_{j,l,m}$ is the cost of repairing (replacing) a type $j$ connection on a type $l$ board in a type $m$ unit when it fails during unit level screening. This cost would ordinarily include fault isolation costs, the costs of
repairing (replacing) the *connection* on the board and replacing the board in the unit.

Depending on the product, and the costs to repair (replace) PCB *connections*, a *connection* failure during unit level screening may result in the board that contains the *connection* being scraped. In this case, the cost due to PCB *connection* failure on a type *l* board in a type *m* unit would be the cost of a type *l* board.

S2 is the cost associated with finding weak PCB *connections* during system level screening. *DS* is the detection efficiency for the test equipment used to detect system level defects which have been precipitated to failure by the stress screen. $NB_j \left( t_{sj} \left[ t_{bj,l} + t_{ubj,m} \right] \right)$ is the expected number of PCB *connection* repairs (replacements) during system level screening for type *j* *connections* on type *l* boards in type *m* units which have survived board, and unit level screening. Additionally, those *connections* that fail during board, and unit level screening and are not detected at subsequent levels may be found during system level screening. Finally, *crsbj,lm* is the cost of repairing (replacing) a type *j* *connection* on a type *l* board in a type *m* unit which fails during system level screening. This cost would ordinarily include fault isolation costs, the costs to repair (replace) the *connection* on the board, replacing the board in the unit, replacing the unit in the system. Depending on the product, and the costs to repair (replace) *connections*, a *connection* failure during system level screening may result in the board that contains that *connection* being scraped. Thus, the cost due to a PCB *connection* failure on a type *l* board in a type *m* unit would be the cost of a type *l* board.
Finally, $F_2$ is the cost associated with having weak PCB connections fail in the field. $NB_j(TW_t_{b_j,,l} + t_{ub_{j,,m}} + t_{s_j})$ is the expected number of connection repairs (replacements) in the field for type $j$ PCB connections on type $l$ boards in type $m$ units during warranty period $TW$. Additionally, those connections that fail during board, unit, and system level screening and are not detected before entering the field will need to be replaced in the field. $crf_{b_{j,,l}}$ is the cost of repairing (replacing) a type $j$ connection on a type $l$ board in a type $m$ unit in the field. This cost will be dependent upon the maintenance strategy employed by the manufacturer. In most cases, the board or unit with the defective PCB connection on it would be replaced in the field. Therefore this cost includes board, or unit replacement costs, whichever is appropriate, fault isolation costs, removing the defective board or unit from the system, and installing the replacement board or unit in the system.

### 3.2.3 Unit Costs

Similar to PCB costs, the costs associated with a unit are dependent upon how much screening is done on unit connections during unit, and system level screening. The unit life cycle cost model consists of both fixed and variable costs. A fixed cost is incurred if unit level screening is performed. Variable costs are dependent upon the amount of screening done at the various screening levels and include the cost of screening units for a specified duration, cost to repair (replace) a failed unit connection during unit and system
level screening, and the costs associated with repairing (replacing) a failed unit connection in the field. The variable costs for PCB connections are also affected by the number and type of connections used, what unit those connections are on, and the ability to detect failures at the various screening levels. The unit model is:

\[
\text{unit} = su_m \cdot I(t_{u,m}) + \sum_m nus_m \cdot \sum_k nu_{k,m} \cdot (cu_m \cdot t_{au_m} + U3 + S3 + F3) \quad (3.51)
\]

where

\[
U3 = DU \cdot NU_k(t_{u,k,m} \mid 0) \cdot cru_{u,k,m}
\]

\[
S3 = DS \cdot \left( NU_k(t_{s,k} \mid t_{u,k,m}) + (1 - DU) \cdot NU_k(t_{u,k,m} \mid 0) \right) \cdot crs_{u,k,m}
\]

\[
F3 = \left( NU_k(\text{TW} \mid t_{s,k} + t_{u,k,m}) + (1 - DS) \cdot \left( NU_k(t_{s,k} \mid t_{u,k,m}) + (1 - DU) \cdot NU_k(t_{u,k,m} \mid 0) \right) \right) \cdot crf_{u,k,m}
\]

(3.52)

U3 is the cost associated with finding weak unit connections during unit level screening. DU is the detection efficiency for the test equipment used to detect unit level
defects which have been precipitated to failure by the stress screen. $NU_k(t_{u_{k,m}} | 0)$ is the expected number of unit connection repairs (replacements) for type $k$ connections on type $m$ units during unit level screening. $cruu_{k,m}$ is the cost of repairing (replacing) a type $k$ connection on a type $m$ unit when it fails during unit level screening. This cost would ordinarily include the cost to fault isolate and repair (replace) the connection.

$S_3$ is the cost associated with finding weak unit connections during system level screening. $DS$ is the detection efficiency for the test equipment used to detect system level defects which have been precipitated to failure by the stress screen. $NU_k(t_{s_k} | t_{u_{k,m}})$ is the expected number of unit connection repairs (replacements) during system level screening for type $k$ connections on type $m$ units that have survived unit level screening. In addition, those unit connections that fail during unit level screening and are not detected may be found during system level screening. $crsu_{k,m}$ is the cost of repairing (replacing) a type $k$ connection in a type $m$ unit when it fails during system level screening. This cost includes fault isolation costs, the costs of repairing (replacing) the connection on the unit, and replacing the unit in the system. Depending on the product, and the costs to repair (replace) unit connections, a connection failure during system level screening may result in the unit that contains the connection being scraped. In this case, the cost due to unit connection failure in a type $m$ unit would be the cost of a type $m$ unit.

Finally, $F_3$ is the cost associated with having weak unit connections fail in the field. $NU_k(TW | t_{u_{k,m}} + t_{s_k})$ is the expected number of connection repairs (replacements)
in the field for type $k$ unit connections on in type $m$ units during warranty period $TW$.

Additionally, those connections that fail during unit, and system level screening and are not detected before entering the field will need to be replaced in the field. $crfu_{k,m}$ is the cost of repairing (replacing) a type $k$ connection in a type $m$ unit in the field. This cost will be dependent upon the maintenance strategy employed by the manufacturer. In some cases, unit with the defective unit connection on it would be replaced in the field. Therefore this cost includes unit repair or replacement costs, whichever is appropriate, fault isolation costs, removing the defective connection or unit from the system, and installing the replacement connection or unit in the system.

3.2.4 System Costs

Similar to unit costs, the costs associated with a system are dependent upon how much screening is done on system connections during system level screening. The unit life cycle cost model consists of both fixed and variable costs. A fixed cost is incurred if system level screening is performed. Variable costs are dependent upon the amount of system screening done and include the cost of screening systems for a specified duration, cost to repair (replace) a failed system connection during system level screening, and the costs associated with repairing (replacing) a failed system connection in the field. The variable costs for system connections are also affected by the number and type of
connections used and the ability to detect failures at the system screening level. The system model is:

\[
SYS = S_4 \cdot I(t_{as}) + \sum_k N_{s,n,k} \cdot (c_s \cdot t_{as} + S_4 + F_4)
\]  \hspace{1cm} (3.53)

where

\[
S_4 = DS \cdot N_{s,n}(t_{s,n,0}) \cdot crss_n
\]

\[
F_4 = \left(N_{s,n}(TW|t_{s,n}) + (1 - DS) \cdot N_{s,n}(t_{s,n,0})\right) \cdot crfs_n
\]  \hspace{1cm} (3.54)

$S_4$ is the cost associated with finding weak system connections during system level screening. $DS$ is the detection efficiency for the test equipment used to detect system level defects which have been precipitated to failure by the stress screen. $N_{s,n}(t_{s,n,0})$ is the expected number of system connection repairs (replacements) during system level screening for type $n$ connections in the system. $crss_n$ is the cost of repairing (replacing) a type $n$ connection when it fails during system level screening. This cost includes fault isolation costs, the costs of repairing (replacing) the connection in the system.

$F_4$ is the cost associated with having weak system connections fail in the field. $N_{s,n}(TW|t_{s,n})$ is the expected number of connection repairs (replacements) in the field for type $n$ system connections during warranty period $TW$. Additionally, those connections that fail during system level screening and are not detected before entering the field will
need to be replaced in the field. $c_{frs_n}$ is the cost of finding and repairing (replacing) a type $n$ connection in the field.

### 3.3 Reliability Model Formulation

As discussed in Chapter 2, several different reliability measures have been used in the literature as criterion for optimizing ESS strategies. The most popular reliability measure used in the literature is a systems mean residual life. ESS strategies should be chosen which maximize a systems mean residual life. An optimization model which uses mean residual life as its criteria is developed in section 3.3.1 for the system discussed in section 3.1. The major assumption is that all components and connections form a series-series system. The impact of this restriction will be examined in later chapters of the dissertation. A second optimization model is developed in section 3.3.2 which uses the systems mission reliability as its optimization criteria. The goal is to choose an ESS strategy which maximizes the systems mission reliability. Like the previous model, this model assumes that all components and connections form a series-series system.

#### 3.3.1 Mean Residual Life Model

Like the general life cycle cost model developed in section 3.2, the mean residual life model can be separated into four elements; component reliability's, PCB connection
reliability's, unit connection reliability's, and system connection reliability's. These four elements are combined to form the systems reliability model. First, the component conditional reliability's must be calculated. For the general system described in section 3.1, the conditional reliability for the collection of components under the assumption of a series-series system is given by:

\[
R_c(t) = \prod_{m} \prod_{l} \prod_{i} [R_{c_i}(t_s + t_{uc_{i,m}} + t_{bc_{i,l}} + t_{c_{i}})]^{n_{m} \cdot n_{pcb_{i,m}} \cdot n_{c_{i,l}}} \tag{3.55}
\]

Similarly, the conditional reliability for the collection of PCB connections, unit connections, and system connections are given by:

\[
R_b(t) = \prod_{m} \prod_{l} \prod_{j} [R_{b_j}(t_s + t_{ub_{j,m}} + t_{b_{j,l}})]^{n_{m} \cdot n_{pcb_{i,m}} \cdot n_{b_{j,l}}} \tag{3.56}
\]

\[
R_u(t) = \prod_{m} \prod_{k} [R_{u_k}(t_s + t_{uk_{m}})]^{n_{m} \cdot n_{u_{k,m}}} \tag{3.57}
\]

\[
R_s(t) = \prod_{n} [R_{s_n}(t_s)]^{n_{s_n}} \tag{3.58}
\]
Then, the systems mean residual life (SMRL) for systems composed of components and connections which survive their respective screens is calculated using the following expression:

\[
SMRL = \int_0^\infty Rc(y) \cdot Rb(y) \cdot Ru(y) \cdot Rs(y) dy
\]

(3.59)

### 3.3.2 System Mission Reliability Model

The system mission reliability model is identical in structure to the mean residual life model in that conditional reliability's are calculated for four distinct groups; components, PCB connections, unit connections, and system connections. The mission reliability model has an additional parameter, TM, which represents the required length of a mission for the system. Using the notation developed in section 3.3.1, the mission reliability (MR) for a system that has survived screening at the various levels is given as:

\[
MR = Rc(TM) \cdot Rb(TM) \cdot Ru(TM) \cdot Rs(TM)
\]

(5.60)

The models developed in this chapter will be utilized and explored in Chapter 5.
CHAPTER 4
MIXED DISTRIBUTIONS

The general ESS modeling framework discussed in Chapter 3 is dependent upon the use of mixed distributions. There is a large quantity of literature on methodology for and applications of finite mixture models in the applied sciences. Monographs by Titterington, Smith, and Makov [117], McLachland and Basford [73] and Everitt and Hand [40] offer a methodical treatment of the structure of finite mixture distributions, detailed descriptions of how to apply statistical techniques to mixture data, as well as a variety of examples. Early references to the use of mixed distributions in reliability modeling are found in Acheson and McElwee [1], Davies [29], Steen [111], Wilde [125].

Mendenhall and Hader [74] were the first to suggest the use of mixture distributions in relationship with instituting an aging process to eliminate early failures. Ninomiya and Harada [86] use mixed distributions in the development of a multi-layer screening process. Jensen and Petersen [51] use a bi-modal and in some cases tri-modal distribution to model the strength distribution of electronic, electro-mechanical, and mechanical components. The multi-modal distribution is designed to account for unavoidable manufacturing variations in materials and processing. Similar approaches are taken by Nguyen and Murthy [85], Holcomb and North [48], Sultan [113], Boukai [13], Clarotti and Spizzichino [23], Yuan and Shih [127], and Perlstein and Welch [95] when modeling component burn-in. The mixed distribution is also used in a similar fashion to
model the ESS process by Perlstein, Littlefield, and Bazovsky [98], Perlstein and Littlefield [96], Perlstein and Bazovsky [97], Barlow, Bazovsky, and Wechsler [6], and Reddy and Dietrich [102].

The purpose of this chapter is to explore the properties of relevant mixture distributions as they apply to ESS. This chapter begins with a discussion of the exponential distribution. Mixtures of exponentials are used almost exclusively in the burn-in and ESS literature. This is primarily due to their mathematical tractability. The major drawback with the mixed exponential distribution is that it does not allow for wear-out.

In order to explore the effects of wear-out on ESS strategies, mixtures of Weibulls are explored. The mixed Weibull distribution offers increased flexibility over the mixed exponential distribution. A mixed Weibull distribution enables one to model the entire reliability bathtub curve for a population of components. The added modeling flexibility created by using a mixed Weibull distribution does not come without a cost. Specifically, the renewal equation for the mixed Weibull distribution has no known closed form solution and therefore can only be solved numerically. The computational issues associated with using a mixed Weibull distribution will be examined in the chapter.

For completeness, mixtures of lognormal distributions are also discussed. Many of the failure modes excited with ESS are best modeled with a lognormal distribution. This is especially true for modeling fatigue cracks [71]. Like the mixed Weibull
distribution, the use of the mixed lognormal distribution increases the computational complexity of the problem.

The chapter concludes with a brief discussion of Phase-Type (PH) distributions and their associated properties. Introduced by Neuts [80], PH distributions have gained widespread acceptance in algorithmic methods for queuing theory. The use of PH distributions brings analytical tractability to the ESS problem. At the same time the PH distribution is flexible enough to realistically represent failure distributions arising in practice.

4.1 Exponential Distribution

The exponential distribution is one of the most common life distributions found in the reliability literature. Martz [72] states that its use in reliability rivals that of the normal distribution in other areas of statistics. Although popular in the reliability literature due to its mathematical tractability, the exponential distribution is often misused. In fact, Nelson [79] states that based on his experience, “the exponential distribution adequately describes only 10-15% of products in the lower tail of the distribution.” Great care must be taken when using the exponential distribution to ensure it adequately models the process under study. The probability density function for the exponential distribution is:

\[ f(t) = \lambda \cdot e^{-\lambda t} \quad (4.1) \]
and the reliability function for the exponential distribution is:

\[ R(t) = e^{-\lambda t} \]  \hspace{1cm} (4.2)

Figure 4.1 shows the pdf and reliability function for the exponential distributions.

The characteristic property of the exponential distribution is that it is the only distribution with a constant hazard rate. A key result of having a constant failure rate is the "lack of memory property." It does not age, wear-out or degrade with time or use. Thus, a component that has operated for 10,000 hrs would have the same probability of failure in the next 2,500 hrs as a component that has only operated for 5 hrs. A failure is said to be a "chance" or random event, which occurs at some constant rate. Traditionally, the exponential distribution has been used to model the long flat portion of the reliability bathtub curve (Figure 1.1).
4.1.1 Mixtures of Exponentials

Following the convention used by Jensen and Petersen [51], assume that there are two distinct subpopulations of components produced by a manufacturing process; a main subpopulation of components and a much smaller subpopulation of weak parts. Assume that both the weak and main subpopulations have constant failure rates $\lambda_w$ and $\lambda_g$ respectively. Also assume that the failure rate for the weak components is several
magnitudes greater than that of the main population. Then, the distribution of time to
failure for a specific component is represented by the following probability density
function:

\[ f(t) = p_g \lambda_g \cdot e^{-\lambda_g t} + p_w \lambda_w \cdot e^{-\lambda_w t} \]  \hspace{1cm} (4.3)

where \( p_g + p_w = 1 \) and represents the proportions of weak and good components
produced by the manufacturing process. Similarly, the reliability is defined as:

\[ R(t) = p_g \cdot e^{-\lambda_g t} + p_w \cdot e^{-\lambda_w t} \]  \hspace{1cm} (4.4)

Then the hazard function for the mixed distribution is given by:

\[ h(t) = \frac{f(t)}{R(t)} = \frac{p_g \lambda_g \cdot e^{-\lambda_g t} + p_w \lambda_w \cdot e^{-\lambda_w t}}{p_g \cdot e^{-\lambda_g t} + p_w \cdot e^{-\lambda_w t}} \]  \hspace{1cm} (4.5)

Since \( \lambda_g \leq \lambda_w \), it is easily shown that the hazard rate for the population is a decreasing
function. Its initial value at \( t=0 \) is:

\[ h(0) = p_g \lambda_g + p_w \lambda_w \]  \hspace{1cm} (4.6)
while its limiting value is given by

$$\lim_{t \to \infty} h(t) = \lambda_g$$  \hspace{1cm} (4.7)$$

Which is easily proven by rewriting Eq. (4.6) in the form

$$h(t) = \lambda_g + \frac{p_w(\lambda_w - \lambda_g)e^{-(\lambda_w - \lambda_g)t}}{p_g + p_w e^{-(\lambda_w - \lambda_g)t}}$$  \hspace{1cm} (4.8)$$

As $t$ goes to infinity, the second term in equation 4.8 goes to zero. Figure 4.2 is a plot of the hazard function vs. screening time for a component population whose distribution is modeled as a mixture of exponentials. This decreasing hazard function models the first two components of the reliability bathtub curve found in figure 1.1. Thus, the longer the component population is screened, the more likely it is that components from the screen come from the main population. One should remember that Eq (4.5) is the hazard rate for the population. An individual component will have either a failure rate of $\lambda_w$ or $\lambda_g$, depending on whether it is a weak or good component.

Assume components are screened for a duration of $t_c$, then the probability of passing the screen is given by
The probability that the component passing the screen is a weak part is given by:

\[
p_{w(t_c)} = \frac{p_w e^{-\lambda_w t_c}}{p_w e^{-\lambda_w t_c} + p_g e^{-\lambda_g t_c}}
\]  

(4.10)
then the reliability of a component from the screened population is given by

\[
R(t|t_c) = p_w(t_c) \frac{e^{-\lambda_w (t+t_c)}}{e^{-\lambda_w t_c}} + p_g(t_c) \frac{e^{-\lambda_g t}}{e^{-\lambda_g t_c}}
\]

\[
= p_w(t_c) \cdot e^{-\lambda_w t} + p_g(t_c) \cdot e^{-\lambda_g t}
\]

\[
= \frac{p_w e^{-\lambda_w (t+t_c)} + p_g e^{-\lambda_g (t+t_c)}}{p_w e^{-\lambda_w t_c} + p_g e^{-\lambda_g t_c}}
\]

(4.11)

Similarly, the probability density function for a component from the screened population is

\[
f(t|t_c) = p_w(t_c) \frac{\lambda_w e^{-\lambda_w (t+t_c)}}{e^{-\lambda_w t_c}} + p_g(t_c) \frac{\lambda_g e^{-\lambda_g (t+t_c)}}{e^{-\lambda_g t_c}}
\]

\[
= p_w(t_c) \lambda_w e^{-\lambda_w t} + p_g(t_c) \lambda_g e^{-\lambda_g t}
\]

(4.12)
4.1.2 Renewal Theory

The modeling framework developed in Chapter 3 makes extensive use of renewal theory to calculate the expected number of failures for nonrepairable items. According to renewal theory, the Laplace transform of the renewal rate is given by

\[ n^*(s) = \frac{f^*(s)}{1 - f^*(s)} \]  (4.13)

The probability density function of a screened component is given by Eq. (4.12). The Laplace transform of the pdf is:

\[ f^*(s|t_c) = p_w(t_c) \frac{\lambda_w}{\lambda_w + s} + p_g(t_c) \frac{\lambda_g}{\lambda_g + s} \]  (4.14)

The renewal density transform is found by substituting Eq. (4.14) into Eq. (4.13) and when simplified yields:
The inverse Laplace transform of Eq. (4.15) is the renewal density:

\[
\begin{align*}
n(t|t_c) &= \left[ \frac{\lambda_w \lambda_g}{(p_w(t_c) \lambda_g + p_g(t_c) \lambda_w)} + \left( \frac{\lambda_w \lambda_g}{(p_g(t_c) \lambda_g + p_w(t_c) \lambda_w)} - \frac{\lambda_w \lambda_g}{(p_w(t_c) \lambda_g + p_g(t_c) \lambda_w)} \right) \cdot \frac{\lambda_w \lambda_g}{(p_w(t_c) \lambda_g + p_g(t_c) \lambda_w)} \right] \\
&= \left[ \frac{\lambda_w \lambda_g}{(p_w(t_c) \lambda_g + p_g(t_c) \lambda_w)} + \left( \frac{\lambda_w \lambda_g}{(p_g(t_c) \lambda_g + p_w(t_c) \lambda_w)} - \frac{\lambda_w \lambda_g}{(p_w(t_c) \lambda_g + p_g(t_c) \lambda_w)} \right) \cdot \frac{\lambda_w \lambda_g}{(p_w(t_c) \lambda_g + p_g(t_c) \lambda_w)} \right]
\end{align*}
\]

(4.16)

The expected number of renewals is obtained by integrating Eq. (4.16) and is given by:
Although rather complex, the renewal function for the mixed exponential distribution does have a closed form analytic solution. This is the primary reason that many of the ESS papers in the literature use mixtures of exponentials. The mixed exponential also provides a reasonable estimate of the “infant mortality” and “useful” life portion of the bathtub curve. But, it ignores the possibility of “wear-out.” Thus, any ESS model that uses the mixed exponential distribution assumes that components never wear-out and that ESS does not affect the components “useful” life.

4.2 Weibull Distribution

Nelson [79] claims that the exponential distribution is often misused for products better described by the Weibull distribution. In fact he states that for “the majority of life data the Weibull distribution fits better than the exponential, normal and lognormal distributions.” The Weibull distribution is highly adaptable and is often used to model product life due to its flexability in modeling increasing as well as decreasing failure rates. It is also used to model product properties such as strength (electrical or mechanical) in
accelerated tests. Specifically, it has been used to model the life of bearings, electronic components, insulating fluids and ceramics [79]. Another feature of the Weibull distribution is that it is a type III extreme value distribution. It is often used to model the “weakest link” of a system of components. It is especially appropriate for modeling a system composed of a number of components where failure is due to the most severe defect of a large number of possible defects [72].

The probability density function for the Weibull distribution is:

\[ f(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} e^{-\left( \frac{t}{\eta} \right)^{\beta}} \]  \hspace{1cm} (4.18)

where \( \beta \) is the shape parameter and \( \eta \) is the scale parameter. Nelson [1990] states that for most products and materials \( \beta \) is in the range of 0.5 to 5.0. The Weibull reliability function is given by:

\[ R(t) = e^{-\left( \frac{t}{\eta} \right)^{\beta}} \]  \hspace{1cm} (4.19)

The hazard function for the Weibull distribution is:
The hazard function for the Weibull distribution is a power function of time that increases when $\beta > 1$, and decreases when $\beta < 1$. For $\beta = 1$, the failure rate is constant. Thus, the exponential distribution is just a special case of the Weibull distribution.

The Weibull distribution has many different forms, depending on how the parameters are defined. Figure 4.3 shows some typical density, reliability, hazard, and cumulative distribution functions when $\eta = 1$.

\[
h(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1}
\]  

Figure 4.3 Weibull Probability Density Function, Hazard Function, Reliability Function, and Cumulative Distribution.
4.4.1 Mixtures of Weibulls

The main reason for investigating the use of mixtures of Weibulls is to investigate how ESS strategies change when items are subject to "wear-out." Following the convention used in section 4.1.1, the distribution of time to failure for a component that comes from a mixed population of weak and good components is:

\[
f(t) = \begin{cases} 
    p_w \left( \frac{\beta_w}{\eta_w} \right)^{\beta_w} \left( \frac{t}{\eta_w} \right)^{\beta_w-1} e^{-\left( \frac{t}{\eta_w} \right)^{\beta_w}} & \text{if } \text{weak component} \\
    p_g \left( \frac{\beta_g}{\eta_g} \right)^{\beta_g} \left( \frac{t}{\eta_g} \right)^{\beta_g-1} e^{-\left( \frac{t}{\eta_g} \right)^{\beta_g}} & \text{if } \text{good component} 
\end{cases}
\]  

(4.21)

Similarly, the reliability function is given by:

\[
R(t) = p_w e^{-\left( \frac{t}{\eta_w} \right)^{\beta_w}} + p_g e^{-\left( \frac{t}{\eta_g} \right)^{\beta_g}}
\]  

(4.22)

Then, the resulting hazard function is:
Since components that are subject to wear-out are of interest, then it will be assumed that $\beta_x$ and $\beta_w$ are both greater than one. Also, it is assumed that $\eta_x$ is several orders of magnitude smaller than $\eta_g$.

Assume components from this population are screened for a duration of $t_c$, then the probability of passing the screen for a component drawn at random is:

$$R(t_c) = p_w e^{-\left(\frac{t_c}{\eta_w}\right)^{\beta_w}} + p_g e^{-\left(\frac{t_c}{\eta_g}\right)^{\beta_g}}$$  \hspace{1cm} (4.24) $$

The probability that the component passing the screen is a component from the main population is:

$$p_g(t_c) = \frac{p_g e^{-\left(\frac{t_c}{\eta_g}\right)^{\beta_g}}}{p_w e^{-\left(\frac{t_c}{\eta_w}\right)^{\beta_w}} + p_g e^{-\left(\frac{t_c}{\eta_g}\right)^{\beta_g}}}$$  \hspace{1cm} (4.25) $$

$$h(t) = \frac{f(t)}{R(t)} = \frac{p_w \left(\frac{\beta_w}{\eta_w}\right) \left(\frac{t}{\eta_w}\right)^{\beta_w - 1} \left(\frac{t}{\eta_w}\right)^{\beta_w} + p_g \left(\frac{\beta_g}{\eta_g}\right) \left(\frac{t}{\eta_g}\right)^{\beta_g - 1} \left(\frac{t}{\eta_g}\right)^{\beta_g}}{-p_w e^{-\left(\frac{t}{\eta_w}\right)^{\beta_w}} + p_g e^{-\left(\frac{t}{\eta_g}\right)^{\beta_g}}}$$  \hspace{1cm} (4.23) $$
The reliability of a component from the screened population is given by:

$$R(t|t_c) = p_w(t_c) \cdot \frac{\left(\begin{array}{c} \text{expression} \\ \frac{(t+t_c)^{\beta_w}}{\eta_w} \end{array}\right)}{e^{\frac{(t+t_c)}{\eta_w}}} + p_g(t_c) \cdot \frac{\left(\begin{array}{c} \text{expression} \\ \frac{(t+t_c)^{\beta_g}}{\eta_g} \end{array}\right)}{e^{\frac{(t+t_c)}{\eta_g}}}$$

(4.26)

Similarly, the probability density function for a component from the screened population is:

$$f(t|t_c) = p_w(t_c) \cdot \frac{\left(\begin{array}{c} \text{expression} \\ \frac{(t+t_c)^{\beta_w-1}}{\eta_w} \end{array}\right)}{e^{\frac{(t+t_c)}{\eta_w}}} + p_g(t_c) \cdot \frac{\left(\begin{array}{c} \text{expression} \\ \frac{(t+t_c)^{\beta_g-1}}{\eta_g} \end{array}\right)}{e^{\frac{(t+t_c)}{\eta_g}}}$$

(4.27)

### 4.2.2 Renewal Theory

Mixtures of Weibull distributions pose a rather difficult problem in terms of renewal theory. The methods discussed in Section 4.1.2 relied upon the use of Laplace transform theory. Unfortunately, the Laplace transform for the Weibull distribution cannot be obtained in closed form. Numerous authors, Bartholomew[9], Deligonul [30],
Jaquette [50], Lomnicki [70], Ross [103], Smith and Leadbetter [107], Spearman [109] and White [123], all investigate approximation methods for the renewal integral. Many of these methods use a power series expansion of $f$. White [123] tabulates values for the Weibull renewal function using the power series expansion. Spearman [109] develops an approximation method for use in developing maintenance strategies for systems with Weibull failure times. His approximation performs poorly for small values of time and therefore would be inappropriate for determining screening times.

For the mixed Weibull distribution, the integral equation must be solved directly to compute the renewal density. The renewal integral equation is [26]:

$$
N(t) = \int_0^t n(u) \cdot du \quad (4.28)
$$

where the renewal density is given by [26]:

$$
n(t) = f(t) + \int_0^t n(t-u) \cdot f(u) \cdot du \quad (4.29)
$$

Therefore the renewal integral equation can be written as [26]:
\[ N(t) = Q(t) + \int_0^t (N(t-u)f(u)du \]  

(4.30)

Xie [126] uses an alternate form of the renewal integral equation:

\[ N(t) = Q(t) + \int_0^t Q(t-u)dN(u) \]  

(4.31)

Eq. (4.30) is a Volterra integral equation of the second kind. Many numerical techniques exist for solving such equations [5]. Xie [126] states that using such methods would be wasteful because they do not take advantage of the special properties of the cumulative distribution function as well as the well known asymptotic properties of the renewal equation. Specifically:

\[ \lim_{t \to \infty} N(t) = \frac{t}{u} + \frac{\sigma^2 - u^2}{2u^2} \]  

(4.32)

Xie [126] solves Eq. (4.31) using Riemann-Stieltjes integration rather than Eq. (4.30) because it makes use of Eq. (4.32) for large \( t \). For a given \( t \), Xie [126] partitions the time into \( n \) equal intervals such that \( 0 = t_0 < t_1 < \ldots < t_n = t \). Then:
\[ N(t_i) = Q(t_i) + \int_0^{t_i} Q(t_i - u) dN(u) \] (4.33)

which Xie [126] rewrites as:

\[ N(t_i) = Q(t_i) + \sum_{j=1}^{i} Q(t_i - t_{j-5}) \left( N(t_j) - N(t_{j-1}) \right) \] (4.34)

where

\[ t_{j-5} = \frac{t_j - t_{j-1}}{2} \] (4.35)

Therefore, the expected number of renewals can be calculated recursively using the following relationship developed by Xie [126]:

\[ N(t_i) = \frac{Q(t_i) + \sum_{j=1}^{i-1} Q(t_i - t_{j-5}) \left( N(t_j) - N(t_{j-1}) \right) - Q(t_i - t_{i-5})N(t_{i-1})}{1 - Q(t_i - t_{i-5})} \] (4.37)

Appendix A contains the fortran code used to implement Xie’s algorithm on the mixed Weibull distribution.
Soland [108] and Huang [49] attacked the problem by solving the Volterra integral, Eq. (4.29), and then obtained the expected number of renewals using Eq. (4.28). They used 2, 3, 4, and 5 point Newton-Cotes integration formulas. Their approach was dependent upon $f(0)=0$, which is only true for the Weibull when $\beta > 1$. They tabulated results for the Weibull distribution for values of $\beta$ between 1.25 and 8. For values less than 1.25 they had severe numerical stability problems which they attributed to an infinite derivative of the pdf at time equal to zero. This method is more computationally intensive and at the same time less stable and flexible than the approach taken by Xie [126].

Szidarovszky [116] recommended using an n-point trapezoidal integration routine with a Richardson extrapolation on Eq (4.30). This approach, like Huang’s, is dependent on $\beta > 1$. This algorithm was coded in Fortran for the mixed Weibull and is found in Appendix B. It seemed to perform as well as those of Xie [126] and Huang [49]. Table 4.1 gives the renewal function for a a variety of shape parameters using Xie’s algorithm.
Table 4.1 Renewal Table for Weibull Distribution

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<th>( \beta=1.0 )</th>
<th>( \beta=2.0 )</th>
</tr>
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<td>0.0100</td>
</tr>
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<td>0.2000</td>
<td>0.0395</td>
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<td>0.7523</td>
<td>0.4000</td>
<td>0.1519</td>
</tr>
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<td>0.9564</td>
<td>0.6000</td>
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</tr>
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<td>0.8000</td>
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</tr>
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<td>2.3814</td>
<td>2.5000</td>
<td>2.4574</td>
</tr>
</tbody>
</table>

4.3 Lognormal Distribution

The use of the lognormal distribution in reliability has become increasingly widespread. As discussed earlier, the lognormal distribution has been used in the literature to model failure due to fatigue cracks [79]. Therefore, it is an ideal candidate distribution for modeling connection type failures. The lognormal probability density function is

\[
f(t) = \frac{1}{\sigma \cdot t \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln(t) - \mu}{\sigma} \right)^2 \right]
\]  

(4.37)
where $\mu$ is the mean of the log of life and $\sigma$ is the standard deviation of the log of life. The value of $\sigma$ determines the shape of the distribution and $\mu$ determines the 50% point and the spread in the life $t$ [79].

Because of the logarithmic relationship between the normal and lognormal distributions the reliability and hazard functions are both functions of the normal distribution. Specifically, the population surviving age $t$ is:

$$R(t) = 1 - \Phi\left(\frac{\ln(t) - \mu}{\sigma}\right)$$ \hspace{1cm} (4.38)

and the hazard function is given by:

$$h(t) = \frac{\phi\left(\frac{\ln(t) - \mu}{\sigma}\right)}{\sigma \cdot t \left(1 - \Phi\left(\frac{\ln(t) - \mu}{\sigma}\right)\right)}$$ \hspace{1cm} (4.39)

Figure 4.4 shows a variety of shapes that the lognormal probability density function, reliability function and hazard function can take for a variety of parameter settings.

The lognormal distribution, like the Weibull distribution, offers flexibility in the variety of situations it can model. The lognormal hazard function has a unique quality in that it starts at zero, initially increases over time, and then decreases, approaching zero as
time becomes large. Nelson [79] points out that the lognormal distribution fits life data of many products over most of its range, especially over the lower tail. Thus, many researchers in the literature use the lower tail of the lognormal distribution to model component wear-out.

Figure 4.4 Lognormal Probability Density Function, Hazard Function, Reliability Function, and Cumulative Distribution
4.3.1 Mixtures of Lognormals

Like the Weibull, the main point in investigating the use of mixtures of lognormals is to investigate how ESS strategies change when items are subject to "wear-out." The distribution of time to failure for a component that comes from a mixed population of weak and good components whose underlying distribution is lognormal is given by:

\[
f(t) = \frac{p_w}{\sigma_w \cdot t^{1/2}} \exp \left( -\frac{1}{2} \left( \frac{\ln(t) - u_w}{\sigma_w} \right)^2 \right) + \frac{p_g}{\sigma_g \cdot t^{1/2}} \exp \left( -\frac{1}{2} \left( \frac{\ln(t) - u_g}{\sigma_g} \right)^2 \right) \quad (4.40)\]

and the corresponding reliability function is given by:

\[
R(t) = p_w \left( 1 - \Phi \left( \frac{\ln(t) - u_w}{\sigma_w} \right) \right) + p_g \left( 1 - \Phi \left( \frac{\ln(t) - u_g}{\sigma_g} \right) \right) \quad (4.41)\]

Then, the resulting hazard function is:

\[
h(t) = \frac{f(t)}{R(t)} = \frac{\frac{p_w}{\sigma_w \cdot t^{1/2}} \exp \left( -\frac{1}{2} \left( \frac{\ln(t) - u_w}{\sigma_w} \right)^2 \right) + \frac{p_g}{\sigma_g \cdot t^{1/2}} \exp \left( -\frac{1}{2} \left( \frac{\ln(t) - u_g}{\sigma_g} \right)^2 \right)}{p_w \left( 1 - \Phi \left( \frac{\ln(t) - u_w}{\sigma_w} \right) \right) + p_g \left( 1 - \Phi \left( \frac{\ln(t) - u_g}{\sigma_g} \right) \right)} \quad (4.42)\]
Assume components from a mixed lognormal population are screened for a duration of \( t_c \), then the probability that a component passes the screen is:

\[
R(t_c) = p_w \left( 1 - \Phi \left( \frac{\ln(t_c) - u_w}{\sigma_w} \right) \right) + p_g \left( 1 - \Phi \left( \frac{\ln(t_c) - u_g}{\sigma_g} \right) \right)
\]

(4.43)

The probability that the component passing the screen is a component from the main population is:

\[
p_g(t_c) = \frac{p_g \left( 1 - \Phi \left( \frac{\ln(t_c) - u_g}{\sigma_g} \right) \right)}{p_w \left( 1 - \Phi \left( \frac{\ln(t_c) - u_w}{\sigma_w} \right) \right) + p_g \left( 1 - \Phi \left( \frac{\ln(t_c) - u_g}{\sigma_g} \right) \right)}
\]

(4.44)

then the reliability of a component from the screened population is given by:

\[
R(t|t_c) = p_w(t_c) \left[ \left( 1 - \Phi \left( \frac{\ln(t + t_c) - u_w}{\sigma_w} \right) \right) \right] + p_g(t_c) \left[ \left( 1 - \Phi \left( \frac{\ln(t) - u_g}{\sigma_g} \right) \right) \right]
\]

(4.45)
Similarly, the probability density function for a component from the screened population is:

\[
f(t|t_c) = \frac{p_w(t_c)}{\sigma_w \cdot t \cdot 2\pi} \exp\left( -\frac{1}{2} \left( \frac{\ln(t + t_c) - u_w}{\sigma_w} \right)^2 \right) \left( 1 - \Phi\left( \frac{\ln(t_c) - u_w}{\sigma_w} \right) \right) + \frac{p_s(t_c)}{\sigma_s \cdot t \cdot 2\pi} \exp\left( -\frac{1}{2} \left( \frac{\ln(t + t_c) - u_s}{\sigma_s} \right)^2 \right) \left( 1 - \Phi\left( \frac{\ln(t_c) - u_s}{\sigma_s} \right) \right)
\]

(4.46)

4.3.2 Renewal Theory

Like the Weibull distribution, the Laplace transform for the lognormal distribution can not be obtained in closed form. Thus, the renewal integral equation must be solved directly. The direct numerical Riemann-Stieljies integration approach developed by Xie[126] and summarized in Section 4.2.2 is also applicable for the lognormal distribution. Eq. (4.35), which is used recursively to calculate the expected number of renewals is easily modified for use with the lognormal distribution. The difficulty with using Eq. (4.35) directly is that cumulative distribution function for the lognormal distribution can not be obtained in closed form. But, the probability density function does have a closed form. When given \( f(t) \) instead of \( Q(t) \), Xie [126] calculates \( Q(t) \) using the following:
4.4 Phase Type Distribution

This class of distributions, developed by Neuts [80], was investigated for several reasons. First, they remain analytically and computationally tractable under a variety of operations encountered in the analysis of stochastic models. In many instances, operations performed on phase type distributions lead again to distributions of phase type. Assaf [4], shows that the class of phase type distributions is closed under three basic operations: finite convolutions, finite mixtures, and the formation of coherent systems of independent components. Second, they are versatile in their ability to approximate many of the lifetime distributions used in reliability, (e.g., gamma, Weibull, lognormal, generalized Erlang, and hyperexponential) to any desired degree of accuracy. O'Cinneide [87, 88], Johnson and Taffe [53, 54], and Kao [55] discuss the types of lifetime distributions that can be approximated by phase-type distributions as well as methods for doing so. Lang [65] examined statistical fitting procedures and methods for the approximation of known probability distribution functions by phase type distributions and investigated the computation issues associated with the various methods.

This section, provides a brief review of some of the properties of phase-type distributions that are pertinent to the ESS modeling framework. A more comprehensive discussion of their properties is in Neuts [82]. A phase-type probability distribution $F(*)$
on $[0,\infty)$, denoted by $PH(\beta,S)$, is the distribution of the time until absorption of a finite-state Markov process with a single absorbing state. Let a continuous time Markov chain with transient states $1,\ldots,m$ and absorbing state $m+1$ have an infinitesimal generator $Q$ of the form:

$$Q = \begin{bmatrix} S & S^* \\ 0 & 0 \end{bmatrix}$$

(4.48)

and initial probability row vector $[\beta, \beta_{m+1}]$. Let $S$ be the $m\times m$ submatrix of the transient states and $S^* = -Se$, where $e$ is an $m$-vector of ones. Matrix $S$ has $S_{ii} < 0$, for $1 \leq i \leq m$, and $S_{ij} \geq 0, \forall i, j$ such that $S^{-1}$ exists. As in Neuts [82], we will assume without loss of generality that $\beta_{m+1} = 0$ in this section. This implies that the time to absorption is always greater than zero. Then, the probability density function for the time until absorption is given by:

$$f(t) = \beta \exp(St)S^*$$

(4.49)

Integrating equation (4.49) gives the reliability function:

$$R(t) = \beta \exp(St)e$$

(4.50)
and therefore the hazard rate is given by:

\[ h(t) = \frac{f(t)}{R(t)} = \frac{\beta \exp\{St\}S^*}{\beta \exp\{St\}e} \]  \hspace{1cm} (4.51)

The \(i\)th noncentral moments of \(PH(\beta, S)\) are all finite and given by [82]

\[ u_i = (-1)^{i}i!\beta \cdot S^{-1}e \]  \hspace{1cm} (4.52)

Also, the Laplace-Stieltjes transform \(\tilde{\mathcal{Q}}(s)\) of \(\mathcal{Q}(\cdot)\) is given by [82]

\[ \tilde{\mathcal{Q}}(s) = \beta(sI - S)^{-1}S^* \]  \hspace{1cm} (4.53)

4.4.1 Phase Type Renewal Processes

A renewal process in which the distribution of the time between renewals is phase type is known as a phase type renewal process (PH-renewal) [81]. A PH-renewal process, with time until absorption distribution \(PH(\beta, S)\), is modeled by an \(m\)-state irreducible continuous time Markov chain with infinitesimal generator \(Q^* = S + S^*\beta\). The renewal density function is given by [81]:
The expected number of renewals in \((0,t)\), also known as the renewal function, is given by \([56, 57, 58, 59]\):

\[
N(t) = \frac{t}{u_1} + \frac{1}{u_1} \beta \left( I - \exp\{Q^* t\} \right) S^{-1} e
\]  

(4.55)

The variance of the number of renewals in \((0,t)\) is given by \([84]\):

\[
V(t) = \frac{u_2 - (u_1)^2}{(u_1)^2} t + 2 \beta \left[ I - \exp\{Q^* t\} \right] \left[ \frac{1}{(u_1)^2} S^{-2} e + \frac{1}{2} \frac{u_2}{(u_1)^3} S^{-1} e \right]
\]  

(4.56)

4.4.2 Closure properties

There are three closure properties that make phase type distributions particularly appealing for modeling component lifetimes. First, Neuts \([80]\) proved that the class of phase type distributions are closed under finite convolution. For example, if two random variables, \(Z_1 \& Z_2\), are phase type distributions with representations \(PH(\beta_1, S_1)\) and \(PH(\beta_2, S_2)\), then their convolution is also a phase type distribution with parameters \(\beta_3, S_3\), where \(\beta_3, S_3\) can be calculated from \(\beta_1, S_1, \beta_2, S_2\).
\( \text{PH}(\beta^2, S^{\alpha}) \), then the CDF of \( Y = Z_1 + Z_2 \) will be phase type with representation \( \text{PH}(\beta^y, S^y) \)

where:

\[
\beta^y = \begin{bmatrix} \beta^z_1, \beta^z_1 \beta^{z_2} \end{bmatrix}, \quad S^y = \begin{bmatrix} S^{z_1} & S^{z_1} \beta^{z_2} \\ 0 & S^{z_2} \end{bmatrix}
\]  \hspace{1cm} (4.57)

A second closure property of interest is that the class of phase type distributions are closed under finite mixtures [82]. Let \( (p_1, \ldots, p_k) \) represent the mixing density and \( \text{PH}(\beta(j), S(j)), 1 \leq j \leq k \), then the mixture is phase type with representation \( \text{PH}(\alpha, T) \)

where:

\[
\alpha = \begin{bmatrix} p_1 \beta(1), p_2 \beta(2), \ldots, p_k \beta(k) \end{bmatrix}, \quad T = \begin{bmatrix} S(1) & 0 & \cdots & 0 \\
0 & S(2) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & S(k) \end{bmatrix}
\]  \hspace{1cm} (4.58)

Finally, Assaf [4] showed that the class of phase type distributions is closed under the formation of coherent systems of independent components. All of these properties make the PH distribution an excellent candidate for use in the multi-level ESS model developed in Chapter 3, provided that PH distributions can adequately approximate mixtures of exponentials, mixtures of Weibulls, and mixtures of lognormals.
4.4.3 Approximation Methods

Asmussen [3] proved that the class of PH distributions is dense. For any cumulative distribution function \( Q(\cdot) \) on \((0, \infty)\), there exists a sequence of order-\(n\) phase-type distributions \(\{\text{PH}_n\}\) such that \(\text{PH}_n\) converges to \(Q(\cdot)\) as \(n \to \infty\) [64, 65]. Thus, any cumulative distribution function can be approximated to some specified precision by a PH distribution. The precision of the approximation is driven by the number of phases \(n\) used in the approximation. As \(n\) gets large, the size of the \(S\) matrix increases. As the dimension of \(S\) increases, the computational advantages PH distributions bring to the ESS problem decrease. It becomes a trade-off between solving integral equations or performing matrix operations on large matrices. Thus, the use of PH distributions will only be advantageous if adequate approximations for the distributions of interest can be attained with a reasonable number of phases.

The actual fitting problem poses several challenges. First, the fitting problem is highly nonlinear. Second, the number of parameters to be approximated can be large. Third, the relationship between the parameters and the shape of the PH distribution is non-trivial. Finally, the set of parameters for a PH distribution are not necessarily unique [82, 87]. Despite these difficulties, many researchers have developed methods for approximating arbitrary probability distributions with PH distributions [64].

Parameter estimation methods for PH distributions can be separated into two general classes: moment matching techniques and maximum likelihood methods. Lang
in his dissertation, develops appropriate performance measures for evaluating several parameter approximation methods for a variety of probability distribution functions which use moment matching as well as maximum likelihood based procedures. Most of the approaches for fitting PH distributions investigated by Lang [64, 65] required that the empirical distribution be fitted to certain subfamilies of PH distributions; such subfamilies include the Erlang, generalized Erlang, Hyperexponential, Acyclic PH distribution, and Coxian distribution (see Lang [64]). The moment matching methods investigated were MEFIT [52] and MEDA[104]. Both restrict approximations to mixtures of Erlang distributions. Two maximum likelihood based methods MLAPH [64] and EMPHT [45] were investigated. MLAPH [64] restricts representation of the approximation to the acyclic PH class. EMPHT [45] uses the EM algorithm to approximate the PH distribution for a variety of subfamilies.

Lang [65] showed that satisfactory low-order PH approximations could be obtained provided that the empirical distribution did not have very long tails, high variability, or sharp jumps in the density functions. This is evidenced in Figures 4.5 and 4.6 which compares the probability density function of the approximated PH distribution with the theoretical distribution for several Weibull and lognormal distributions using EMPHT. Note that as the number of phases increases, so does the precision of the estimate.
Figure 4.5 PH Approximations to the Weibull Distribution [65]
Figure 4.6 PH Approximations to the Lognormal Distribution [65].
Figures 4.5 and 4.6 both show that PH distributions provide reasonable fits to the Weibull and Lognormal distributions. A key question is how well do PH distributions approximate the expected number of renewals for these distributions. To study this, a PH distribution was used to approximate a Weibull distribution with a shape parameter of 2 and a scale parameter of 1. This distribution was chosen so that the published Weibull renewal tables of Baxter, Sheuer, McConologue and Blishke [10] could be compared with data from Xie's algorithm as well as the PH approximation. Table 4.2 contains the data for Baxter et al. [1981], Xie's algorithm, and a variety of PH approximations. The general PH structure and the Coxian structure were used to approximate the Weibull distribution.

Both four as well as six phase approximations were examined.

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<th>Time</th>
<th>Baxter et al.</th>
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<th>Cox (6)</th>
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Table 4.2 Expected Number of Renewals for Weibull (1,2).
As shown in Table 4.2, the PH approximations provide reasonable estimates for the expected number of renewals for the Weibull distribution. As expected, since the PH distribution is an approximation to the Weibull distribution, the expected number of renewals are not exact. Although, careful examination of Table 4.2 will show that in most cases the approximation is good to two and three decimal places. It should be noted that the type of PH structure as well as the number of phases used plays a significant role in how well the PH approximation models the Weibull distribution. The general PH structure is the most flexible but has the largest number of parameters to estimate when trying to model the Weibull distribution. The larger the number of phases, the larger the computational effort to fit the distribution. Lang [1994] in his dissertation spends considerable time discussing the various trade-offs one can make when fitting PH distributions.

### 4.4.4 Mixtures of Phase Type Distributions

The ESS model developed in Chapter 3 requires the use of mixture distributions. The mixed exponential distribution is nothing more than a special case of the PH distribution. Specifically, the PH representation of the mixed exponential distribution is \( \text{PH}(\beta, S) \) where:
The probability density function is given by:

\[ f(t) = \beta \exp\{St\}S^* \]

\[ = \begin{pmatrix} p_w & p_g \end{pmatrix} \exp\begin{bmatrix} -\lambda_w t & 0 \\ 0 & -\lambda_g t \end{bmatrix} \begin{bmatrix} \lambda_w \\ \lambda_g \end{bmatrix} \]

\[ = p_w \lambda_w \exp(-\lambda_w t) + p_g \lambda_g \exp(-\lambda_g t) \]  

Similarly, the reliability function is given by:

\[ R(t) = \beta \exp\{St\}e \]

\[ = \begin{pmatrix} p_w & p_g \end{pmatrix} \exp\begin{bmatrix} -\lambda_w t & 0 \\ 0 & -\lambda_g t \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \]

\[ = p_w \exp(-\lambda_w t) + p_g \exp(-\lambda_g t) \]  

Finally, mean time to failure is given by:
The mixed Weibull and mixed lognormal distributions do require approximations. One approach is to form an appropriate PH approximation using one of the methods discussed by Lang [1994] for each probability density function of the original mixture distribution. This yields a mixture of PH distributions of the following form:

\[
\begin{align*}
\alpha &= \left(p_w \beta_w, p_g \beta_g \right) \\
T &= \begin{bmatrix} S_w & 0 \\ 0 & S_g \end{bmatrix}
\end{align*}
\]  

(4.64)

Similarly, the reliability function is:

\[
f(t) = p_w \beta_w \exp\{S_w t\} S_w^* + p_g \beta_g \exp\{S_g t\} S_g^*
\]  

(4.63)
or alternatively,

$$ R(t) = \alpha \exp\{t\} e $$

(4.66)

Assuming components are screened for a duration of \( t_c \), then the probability of passing the screen is given by

$$ R(t_c) = \alpha \exp\{t t_c\} e $$

$$ = p_w \beta_w \exp\{S_w t_c\} e + p_g \beta_g \exp\{S_g t_c\} e $$

(4.67)

The probability that the component passing the screen is weak is given by:

$$ p_w(t_c) = \frac{p_w \beta_w \exp\{S_w t_c\} e}{p_w \beta_w \exp\{S_w t_c\} e + p_g \beta_g \exp\{S_g t_c\} e} $$

(4.68)

then the reliability of a component from the screened population is:

$$ R(t|t_c) = p_w(t_c) \left[ \frac{\beta_w \exp\{S_w (t + t_c)\} e}{\beta_w \exp\{S_w (t + t_c)\} e} \right] + p_g(t_c) \left[ \frac{\beta_g \exp\{S_g (t + t_c)\} e}{\beta_g \exp\{S_g (t + t_c)\} e} \right] $$

(4.69)
and similarly the probability density function of a screened component is:

\[ f(t|c) = p_w(t_c) \left[ \frac{\beta_w \exp\{S_w(t + t_c)\}S_w^*}{\beta_w \exp\{S_w(t_c)\}e} \right] + p_g(t_c) \left[ \frac{\beta_g \exp\{S_g(t + t_c)\}S_g^*}{\beta_g \exp\{S_g(t_c)\}e} \right] \quad (4.70) \]

which can be rewritten as:

\[ f(t|c) = \left( \frac{p_w\beta_w}{p_w\beta_w \exp\{S_w t_c\}e + p_g\beta_g \exp\{S_g t_c\}e} \right) \exp\{S_w(t + t_c)\}S_w^* + \]

\[ \left( \frac{p_g\beta_g}{p_w\beta_w \exp\{S_w t_c\}e + p_g\beta_g \exp\{S_g t_c\}e} \right) \exp\{S_g(t + t_c)\}S_g^* \quad (4.71) \]

Let's define the following parameters:

\[ \alpha_s = \left[ \frac{p_w\beta_w}{R(t_c)}, \frac{p_g\beta_g}{R(t_c)} \right] \quad (4.72) \]

\[ T_s = \left[ \begin{array}{cc} S_w & 0 \\ 0 & S_g \end{array} \right] \quad (4.73) \]
Then the conditional probability density function for the time until absorption can be written as:

\[
f(t|t_c) = \alpha_s \exp\left[T_s(t + t_c)\right]T_s^e \tag{4.74}
\]

The expected number of renewals for time \( t \), given that the component survives until time \( t_c \) is easily calculated using the following relationship:

\[
N(t|t_c) = N(t + t_c) - N(t_c) \tag{4.75}
\]

which can be solved by using equation 4.56. Since, these equations are analytically tractable, the calculation is very efficient.

An alternative approach to calculate the expected number of renewals for a screened component is to use the conditional probability density function to form the conditional renewal function directly. Using this approach, the conditional expected number of renewals is given by:

\[
N(t|t_c) = \frac{t + t_c}{u_i} + \frac{1}{u_i} \alpha_s \left[I - \exp\left\{Q(t + t_c)\right\}\right]T_s^{-1}e \tag{4.76}
\]

where
\[ Q^* = T_s + T_s^* \alpha_s \]  \hspace{1cm} (4.77)

\[ T_s^* = -T_s e \]  \hspace{1cm} (4.78)

\[ u_i' = (-1)\alpha_s T_s^{-1} e \]  \hspace{1cm} (4.79)
The purpose of this chapter is to apply the general modeling framework developed in Chapter 3 to a variety of problems to demonstrate its usefulness. The chapter begins with the investigation of a two-level screening model. The two level screening modeled studied in this chapter is an extension of the single component, single connection example studied by Reddy and Dietrich [102]. The examples studied here differ in that the screening process is no longer assumed to be perfect, and component and connection failure times are no longer restricted to the exponential distribution. A more realistic, multiple component, multiple connection example is also studied. Finally, a more general screening model which uses PH distributions to model component and connection failure distributions is examined.

Next, a series of three level screening models are studied. The three level screening models are used to demonstrate the versatility of the general modelling framework developed in chapter 3. In the first example, a series-series collection of components and connections is modeled using a mixed exponential distribution for component failure distributions and mixed Weibulls for board and system level connections. The mixed Weibull distributions are used to model wear-out characteristics at the board and system levels. Thermal stresses are used to accelerated defects to failure at all assembly levels. In the second three level example, a mixture of Weibull distributions is
used to model component and connection failure distributions at all assembly levels.

Additionally, a multiple stress environment is used to accelerate defects to failure. Unlike the first example, perfect failure detection will not be assumed. Three different system configurations for PCBs are explored. Screening strategies for each configuration are established using the following criteria; life cycle cost, mission reliability, mean residual life. Table 5.1 shows a layout of the examples studied in this chapter.

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<th>$f_b$</th>
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Table 5.1 Matrix of Examples in Chapter Five.
5.1 Two Level Models

The following four examples are provided to illustrate the viability of the screening model developed in chapter 3 and investigate how screening strategies change when wear-out mechanisms are present and failure detection capabilities are imperfect. First, the numerical example used by Reddy and Dietrich [102] is repeated in order to examine the effects of imperfect failure detection on screening strategies. Reddy and Dietrich [102] studied a system composed of 1000 components and 1000 connections in series, all of which have the same mixed exponential distribution. The second example is designed to examine how screening strategies change when wear-out mechanisms are present. The same 1000 component-1000 connection system is modeled with a mixture of Weibull distributions with a shape parameter greater than 1. The scale parameters are chosen so that examples one and two have the same mean time to first failure. Next, a more general example in which multiple component types, multiple connection types, and acceleration at both component and system levels are examined when components and connections have mixed Weibull failure distributions. Finally, the two-level model is generalized by using PH distributions to model component and connection failure distributions. Equivalent PH distributions are obtained for the mixed Weibull and the resulting screening strategies compared.
5.1.1 Mixed Exponential With Imperfect Failure Detection

Using the notation developed in chapter 3, let $i=1$, and $j=1$, $l=1$, $m=1$. Define the system parameters as:

\[
\begin{align*}
nuts_m &= 1; \quad npcb_{1,m} = 1; \quad nC_{i,l} = 1000; \quad nb_{j,l} = 1000; \end{align*}
\]

Since the exponential distribution is a special form of the Weibull distribution, assume that components and connections have failure distributions modeled by a mixture of two Weibull distributions. The distribution characteristics for the components and connections are:

\[
\begin{align*}
p_{wci} &= 0.01; \quad p_{gci} = 0.99; \quad n_{wci} = 10^3; \quad n_{gci} = 10^7; \quad \beta_{wci} = \beta_{gci} = 1; \\
p_{wb} &= 0.01; \quad p_{gb} = 0.99; \quad n_{wb} = 10^3; \quad n_{gb} = 10^7; \quad \beta_{wb} = \beta_{gb} = 1;
\end{align*}
\]

ESS is only effective if it is a value-added process. There is a tradeoff between the cost of screening and the cost of failure in the field. Screening costs generally increase with each level of assembly. As the duration increases, the field expenses decrease but the screening costs also increase. The optimal screen duration at each screening level are those which minimize the life cycle costs. The life cycle cost model used in this example is a subset of the general model developed in chapter 3.2. For the two level screening
model, the life cycle cost model can be separated into two elements; component costs, and PCB level costs.

\[ \text{LCC} = \text{CC} + \text{PCB} \]  

(5.1)

where

\[ \text{CC} = s_{ci} \cdot I(t_{ci}) + \sum_{m} n_{mts} \cdot \sum_{l} n_{pcbl,m} \cdot \sum_{i} n_{ci,l} \cdot \left( c_{ti} \cdot I_{acj} + C_1 + B_1 + \left( N_{ci} \left( t_{bcj,l} + t_{ci} \right) + (1 - DB) \cdot \left( N_{ci} \left( t_{bcj,l} + (1 - DC) \cdot N_{ci} \left( t_{ci}|0 \right) \right) \right) \right) \right) \]

(5.2)

\[ \text{PCB} = s_{bi} \cdot I(t_{bji,l}) + \sum_{m} n_{mts} \sum_{l} n_{pcbl,m} \cdot \sum_{i} n_{bji,l} \cdot \left( c_{bl} \cdot t_{abl} + B_2 + \left( N_{bj} \left( t_{bji,l} \right) + (1 - DB) \cdot N_{bj} \left( t_{bji,l}|0 \right) \right) \cdot \text{crfbj,l,m} \right) \]

(5.3)

The cost parameters for the cost model are:

- $crcc_i = \$50; \quad crbc_{i,l} = \$500; \quad crcf_{i,l,m} = \$5000; \quad crbb_{j,l} = \$500; \quad crfb_{j,l,m} = \$5000; \quad sc_i = \$200; \quad ct_i = \$0.01; \quad cb_l = \$10; \quad TW = 20000hrs; \]

The acceleration factors are assumed to come from an appropriate accelerated stress model such as the Arhenius or Inverse Power Law model which are discussed in detail in
chapter 2. In this example, like the example used by Reddy and Dietrich [102], system
level acceleration is not considered. Therefore, $K_{b_{j,t}} = K_{bc_{x,t}} = 1$. This restriction will be
relaxed in a subsequent example. The life cycle cost model is minimized using the Quasi-
Newton routine found in the IMSL library. Eleven cases are solved and presented in
Table 5.2. Table 5.2 shows the effect that imperfect failure detection has on determining
the optimal screening strategy. Cases 1,2,3 replicate the cases presented in Reddy and
Dietrich [102] where perfect failure detection was assumed. The results show that
component level screening is only cost effective when component stresses are accelerated.
Figures 5.1 and 5.2 show the response surface for the cost model for cases 3 and 8.

Cases 4,5,6, & 7 replicate case 2 with the exception that component and unit level
failure detection are no longer assumed to be perfect. Case 4 shows that, for the
parameters selected, component level screening is not cost effective when component
level failure detection does not exist. Cases 5, 6, & 7 show that as component level failure
detection capability increases so does the cost effectiveness of component level screening.
In cases 5, 6, & 7 the amount of component level screening is increased and the expected
life cycle costs are decreased as the component level failure detection capability increases.
As expected, in all cases, the life cycle costs associated with imperfect failure detection are
higher than when perfect failure detection capability exists.

Similarly, cases 8,9,10, & 11 replicate case 3 with imperfect failure detection at
both component and PCB levels. Case 8 shows that component level screening under
highly accelerated conditions can be cost effective even when component level failures are
not detectable. Cases 9, 10, & 11 show that as failure detection capability increases, component level screening becomes more cost effective and the system life cycle costs are decreased. As illustrated by this example, failure detection capability plays a significant role in determining the most cost effective screening strategy.

<table>
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<th>$K_{b_{j,l}}$</th>
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<th>DB</th>
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<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
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<td>59231.07</td>
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<td>.95</td>
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<td>1789.49</td>
<td>64758.96</td>
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<td>1</td>
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<td>.95</td>
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<td>.95</td>
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<td>1710.48</td>
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<td>62918.32</td>
</tr>
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<td>.95</td>
<td>495.26</td>
<td>1651.31</td>
<td>61866.81</td>
</tr>
</tbody>
</table>

Table 5.2. Mixed Exponential Case
Figure 5.1. Response Surface for Case 3; Mixed Exponential

Figure 5.2. Response Surface for Case 8; Mixed Exponential
5.1.2 Mixed Weibull With Imperfect Failure Detection

This example is identical to the first example except that the distribution characteristics for components and connections are changed to model systems that have wear-out characteristics. The parameters for the failure distributions for the components and connections used in this example are:

\[ P_{wc} = 0.01; \quad \beta_{wc} = 3.2; \quad \eta_{wc} = 1116.503; \quad \eta_{gc} = 11135358.32; \]
\[ P_{gc} = 0.99; \quad \beta_{gc} = 3.375; \quad \eta_{gb} = 11135358.32; \quad \beta_{gb} = 3.2; \]

The parameters were chosen so that components and connections in examples one and two have the same mean time to first failure. Table 5.3 shows the effects of wear-out and imperfect failure detection on determining the optimal screening strategy. Figures 5.3 and 5.4 show the response surface for cases 3 and 8.

Cases 1, 2, & 3 assume perfect failure detection capability at both component and unit levels. The effects of wear-out on determining a system screening strategy are illustrated by comparing the results of cases 1, 2, & 3 from tables 5.2 and 5.3. The results show that the screening policies are different. First, component level screening is only cost effective under highly accelerated conditions. Second, the amount of screening required when wear-out mechanisms are present is less. Thus, developing screening strategies under the assumption of constant failure rates may lead to excessive screening.
Table 5.3. Mixed Weibull Case

Table 5.4 shows the life cycle costs associated with using the screening strategies found under the assumption of a mixed exponential distribution when in fact the true failure distribution characteristics are Weibull. As expected, the cost of using a strategy based upon the assumption of a constant failure rate leads to excessive screening and
Figure 5.3 Response Surface for Case 3; Mixed Weibull

Figure 5.4 Response Surface for Case 8; Mixed Weibull
higher life cycle costs. This is an important point because all models found in the literature as well as DOD-HDBK-344 [35] assume mixed exponential distributions. Therefore, screening strategies developed using these models and their associated assumptions may in fact be causing excessive screening. Companies must ensure that the assumptions associated with using a mixed exponential distribution are met otherwise they could be wasting valuable resources.

<table>
<thead>
<tr>
<th>Case</th>
<th>$K_c$</th>
<th>$K_{b,j}$</th>
<th>DC</th>
<th>DB</th>
<th>$T_{ac}$</th>
<th>$T_{ab}$</th>
<th>Cost</th>
<th>Optimal Cost</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2197.18</td>
<td>32244.71</td>
<td>29086.41</td>
<td>10.86%</td>
</tr>
<tr>
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<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>420.73</td>
<td>1862.09</td>
<td>32056.47</td>
<td>29086.41</td>
<td>10.21%</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
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<td>1739.33</td>
<td>29272.85</td>
<td>29032.07</td>
<td>0.83%</td>
</tr>
</tbody>
</table>

Table 5.4. Cost of Using Exponential Plan for Mixed Weibull Case.

Cases 4,5,6, & 7 of table 5.3 replicate case 2 with the exception that component and PCB level failure detection are no longer assumed to be perfect. Since component screening was not cost effective when failure detection was assumed to be perfect, one would not expect it to be cost effective when failure detection is imperfect. As expected, table 5.3 shows that component screening is not cost effective. Cases 8,9,10, & 11 replicate case 3 with imperfect failure detection at both component and PCB levels. Cases 8 and 9 show that component level screening under highly accelerated conditions is not
cost effective when component level failure detection capability is less than 50%. This is very different from what was found in the first example. There, component level screening was cost effective even when component level failures were undetectable. As in example 1, Cases 10 and 11 show that as failure detection capability increases, component level screening becomes cost effective and the system life cycle costs are reduced.

5.1.3 Multi-Component Mixed Weibull Example

In this example, it is assumed that a manufacturer is under contract to produce $l$ different types of circuit boards for a computer company. Each PCB type $l$ is made up of $n_{c_{i,l}}$ components and $n_{b_{j,l}}$ connections where $i$ and $j$ represent the number of different types of components and connections. For illustration purposes let $l=2$, $i=5$, and $j=3$. Assume the manufacturer is under contract to produce the following number of PCBs:

$$np_{cb_i} = [1000 \hspace{1em} 750]$$

where each board consists of the following combinations of components and connections:

$$
\begin{align*}
 n_{c_{i,l}} &= \begin{bmatrix} 
 20 & 30 \\
 40 & 10 \\
 60 & 30 \\
 40 & 50 \\
 20 & 70 
\end{bmatrix} & n_{b_{j,l}} &= \begin{bmatrix} 
 40 & 60 \\
 80 & 60 \\
 40 & 40 
\end{bmatrix}
\end{align*}
$$

The parameters for the failure distributions of the components and PCB connections are:
In this example a thermal stress will be used as the accelerating stress. It is well documented in the literature that electronic components subjected to thermal stresses follow the Arrhenius model. The acceleration factors for each component and connection type are calculated using the following relationships:

\[ K_{ci} = \exp\left(\frac{E_{ai}}{K} \cdot \left(\frac{1}{T_o} - \frac{1}{T_{ca}}\right)\right) \]  

(5.4)
\[ K_{b_{j},l} = \exp \left( -\frac{E_{a_{j}}}{K} \left( \frac{1}{T_{o}} - \frac{1}{T_{b_{j}}} \right) \right) \]  \hspace{1cm} (5.5)

\[ K_{bc_{i,l}} = \exp \left( -\frac{E_{a_{i}}}{K} \left( \frac{1}{T_{o}} - \frac{1}{T_{cs}} \right) \right) \]  \hspace{1cm} (5.6)

where \( K \) is Boltzman's constant \((8.617 \times 10^{-5})\), \( E_{a_{i}} \) and \( E_{a_{j}} \) are the component and connection specific activation energies, and \( T_{o}, T_{cs}, \) and \( T_{b_{j}} \) are the normal system operating temperature, the component screening temperature, and the board screening temperatures respectively. The acceleration factors for each component and connection type are calculated using the following data:

\[
E_{a_{i}} = \begin{bmatrix}
0.7 \\
0.8 \\
0.9 \\
0.85 \\
0.75
\end{bmatrix}
E_{a_{j}} = \begin{bmatrix}
0.4 \\
0.45 \\
0.6
\end{bmatrix}
\]

The cost parameters are:

\[
crcc_{i} = \begin{bmatrix}
$3 \\
$3.5 \\
$2.5 \\
$4 \\
$1.5
\end{bmatrix}
\quad crcb_{i,l} = \begin{bmatrix}
$13 \\
$13.5 \\
$12.5 \\
$14 \\
$11.5
\end{bmatrix}
\quad crcf_{i,l,m} = \begin{bmatrix}
$113 \\
$113.5 \\
$112.5 \\
$114 \\
$111.5
\end{bmatrix}
\]

\[
crbb_{j,l} = \begin{bmatrix}
$10 \\
$12 \\
$14
\end{bmatrix}
\quad crfb_{j,l,m} = \begin{bmatrix}
$110 \\
$112 \\
$114
\end{bmatrix}
\]

\[
s_{c_{i}} = $2500 \quad c_{t_{i}} = $0.025 \quad s_{b_{i}} = $5000 \quad c_{b_{i}} = $0.15 \quad T_{w} = 50000hrs
\]
The parameters were chosen so that the system life cycle costs are about three times the production cost. It should be noted that the cost parameters are much lower than those used in examples one and two. If the parameters from examples one and two were used, life cycle costs would be on the order of 50 times the anticipated production costs. Thus, the cost of rework data found in DOD-HDBK-344 [35] appears to be high.

<table>
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<tr>
<th>Case</th>
<th>$T_{sc}$</th>
<th>$T_{sb1}$</th>
<th>$T_{sb2}$</th>
<th>DC</th>
<th>DB</th>
<th>$T_{ac}$</th>
<th>$T_{ab1}$</th>
<th>$T_{ab2}$</th>
<th>Cost</th>
</tr>
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<td>298</td>
<td>298</td>
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<td>.95</td>
<td>0</td>
<td>718.18</td>
<td>824.11</td>
<td>419203.31</td>
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<td>298</td>
<td>358</td>
<td>.85</td>
<td>.95</td>
<td>0</td>
<td>718.18</td>
<td>28.19</td>
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<tr>
<td>3</td>
<td>298</td>
<td>358</td>
<td>298</td>
<td>.85</td>
<td>.95</td>
<td>0</td>
<td>27.92</td>
<td>824.11</td>
<td>293693.10</td>
</tr>
<tr>
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<td>298</td>
<td>358</td>
<td>358</td>
<td>.85</td>
<td>.95</td>
<td>0</td>
<td>27.92</td>
<td>28.19</td>
<td>187410.08</td>
</tr>
<tr>
<td>5</td>
<td>358</td>
<td>298</td>
<td>298</td>
<td>.85</td>
<td>.95</td>
<td>0</td>
<td>718.18</td>
<td>824.11</td>
<td>419203.31</td>
</tr>
<tr>
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<tr>
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<td>27.92</td>
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<td>187410.08</td>
</tr>
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<td></td>
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<td>927299.12</td>
</tr>
</tbody>
</table>

Table 5.5. Multiple Component and Connection Mixed Weibull Results
Table 5.5 shows the effects of wear-out and imperfect failure detection on determining the optimal screening strategy for a multiple component, connection, and product manufacturing process. For the example studied, component level screening is not cost effective. The most cost effective approach is to accelerate board level screening for both product types as much as possible. Case 9 shows the expected life cycle cost when no screening is performed. As evidenced in table 5.5, significant savings are incurred when ESS is performed. In this example, the life cycle cost can be reduced by a factor of 4.95 when ESS is properly implemented.

5.1.4 Phase Type Distribution Example

In this example we generalize the earlier examples by using PH distributions to model component and connection failure distributions. As was shown in Chapter 4, the mixed exponential distribution is a special case of a PH distribution. This case is used as a check to ensure that the PH distribution has been properly implemented in the life cycle cost model. The PH parameters for components and connections are:

\[ \beta = (0.01, 0.99); \quad S = \begin{bmatrix} -1 \times 10^{-3} & 0 \\ 0 & -1 \times 10^{-7} \end{bmatrix} \]

The expected number of renewals during the warranty period for a component that has survived component and board level screening is found using the following relationship:

\[ N_{c_i} \left( TW | _{t_{bc,i}} + t_{c_i} \right) = N_{c_i} \left( TW + t_{bc,i} + t_{c_i} \right) - N_{c_i} \left( t_{bc,i} + t_{c_i} \right) \]  (5.7)
where $N(t)$ is found using equation (4.56). Using the system parameters and cost model developed in section 5.1.1, optimal screening strategies are found. Table 5.6 gives the life cycle cost when using screening strategies obtained with the PH implementation.

<table>
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<tr>
<th>Case</th>
<th>$K_c$</th>
<th>$K_{b,i}$</th>
<th>DC</th>
<th>DB</th>
<th>$T_{ac}$</th>
<th>$T_{ab}$</th>
<th>Cost $/system</th>
</tr>
</thead>
<tbody>
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<td>.95</td>
<td>353.67</td>
<td>1859.55</td>
<td>65736.09</td>
</tr>
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<td>.95</td>
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<td>.95</td>
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<td>.95</td>
<td>498.19</td>
<td>1659.28</td>
<td>61867.4</td>
</tr>
</tbody>
</table>

Table 5.6. Mixed PH Case
In examining the results from Table 5.6 with those of Table 5.2 one finds that the screening strategies are nearly identical. The slight differences are probably due to two factors. First, the models used in Table 5.2 were run on the VAX system using Xie’s algorithm [126] where as those found in Table 5.6 were run using MATLAB on a 486 PC. MATLAB was used for the PH model in order to take advantage of its matrix analysis procedures. Second, two different optimization routines were used. The models that were run on the VAX used a quasi-Newton optimization method from the IMSL library. The models run on the PC were optimized using a sequential quadratic programming routine found in the MATLAB optimization tool box. Despite the slight differences in numerical values, the general strategies did not change. More importantly, the strategies identified in the tables result in dramatic improvements in the life cycle cost when compared to not doing any screening ($121191.70) despite the relatively small weak population (.01).

Next, a two level screening model which uses mixed PH distributions to model component and connection failures is investigated. In this example, we let the percentage of weak components increases, and the differences between good and weak populations to decrease. The distribution characteristics for components and connections are:

\[
p_{wc_i} = 0.1; \quad p_{gc_i} = 0.9; \quad n_{wc_i} = 10^2; \quad n_{gc_i} = 10^4; \quad \beta_{wc_i} = 2.0; \quad \beta_{gc_i} = 1.5;
\]
\[
p_{wb_i} = 0.1; \quad p_{gb_i} = 0.9; \quad n_{wb_i} = 10^2; \quad n_{gb_i} = 10^4; \quad \beta_{wb_i} = 2.0; \quad \beta_{gb_i} = 1.5;
\]
The cost parameters for the cost model are:

\[ crcc_i = \$5; \quad crbc_{i,l} = \$50; \quad crcf_{i,l,m} = \$500; \quad crbb_{j,l} = \$50; \quad crfb_{j,l,m} = \$500; \]
\[ sc_i = ss_j = \$200; \quad ct_i = \$0.01; \quad cb_i = \$10; \quad TW = 300\text{hrs}; \]

The program EMPHT [45], which uses the EM algorithm to estimate the parameters \((\beta, S)\) of a PH distribution, was used to get estimates for each component of the mixed Weibull distribution. The four phase Coxian PH representation for the weak components is given by:

\[
\beta = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}, \quad S = \begin{bmatrix}
-.04242957 & .04241848 & 0 & 0 \\
0 & -.04553665 & .0399228 & 0 \\
0 & 0 & -.0406783 & .0406783 \\
0 & 0 & 0 & -.0406674
\end{bmatrix}
\]

Similarly, the four phase Coxian PH representation for the strong components is given by:

\[
\beta = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}, \quad S = \begin{bmatrix}
-.000221 & .000221 & 0 & 0 \\
0 & -.52291 & .514908 & 0 \\
0 & 0 & -.487855 & .4776926 \\
0 & 0 & 0 & -.000220
\end{bmatrix}
\]

Figures 5.5, 5.6 and 5.7 compare the exact and approximate pdf's, cdf's and hazard rates for the weak component population respectively. Similarly, Figures 5.8, 5.9, and 5.10 compare the exact and approximate pdf's, cdf's and hazard rates for the weak component population respectively. The solid line represents the exact distribution and the (\text{-\text{-}}) represents the PH approximation.
Figure 5.5. Cumulative Distribution, Weak Population.

Figure 5.6. Probability Density, Weak Population.
Figure 5.7. Hazard Function, Weak Population

Figure 5.8. Cumulative Distribution, Good Population.
Figure 5.9. Probability Density, Good Population.

Figure 5.10. Hazard Function, Good Population.
<table>
<thead>
<tr>
<th>Case</th>
<th>$K_c$</th>
<th>$K_{b_{i,l}}$</th>
<th>DC</th>
<th>DB</th>
<th>$T_{ac}$ hrs</th>
<th>$T_{ab}$ hrs</th>
<th>Cost $$/system</th>
</tr>
</thead>
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<td>1</td>
<td>1</td>
<td>1</td>
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<td>216.99</td>
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Table 5.7. PH Approximation
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<th>DB</th>
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<th>$T_{ab}$</th>
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Table 5.8. Exact Distribution
Table 5.9. Mixed Exponential Approximation

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Table 5.10. Percent Increase for Approximations

Case 1 corresponds to a multi-level burn in situation. In this case components and connections are screened in their normal operating environment (acceleration factors equal to 1). For this example, both component and unit level screening are cost effective. Cases 2, 3, 4, and 5 replicate case 1 with the exception that component and unit level failure detection are no longer assumed to be perfect. Case 2 shows that component level screening is not cost effective when component level failure detection does not exist. Cases 3 and 5 show that as component level failure detection capability increases so does
the cost effectiveness of component level screening. Case 4 examines the situation where component level detection capability is significantly better than unit level failure detection capability. In this situation, one would expect component level screening to be more desirable in order to eliminate the majority of component defects before they are sent to the next level. Case 4 shows that the level of component level screening is increased even beyond that of case 1 while unit level screening is slightly decreased. It should be noted that the overall life cycle cost in this situation is much higher than all of the previous cases. Therefore, the most cost effective screens are produced when the failure detection capability is high at the final assembly level. Cases 6, 7, 8, 9, and 10 replicate cases 1 thru 5 with the exception that component level screening is now performed at a higher stress level. As expected, in all cases the amount of component level screening and overall life cycle cost are decreased.

Table 5.10 summarizes the results of using PH approximations and mixed exponential approximations to develop screening strategies for the system. The PH results are very encouraging. They show that for this case study, the maximum increase in cost associated with using the PH screening strategy as opposed to the exact Weibull distribution is less than 1.6%. On the other hand, the mixed exponential approximation has a maximum increase in cost of 10% over the exact Weibull distribution. This example demonstrated that PH distributions can be used to establish cost effective screening strategies.
5.2 Three Level Models

The three level screening models are used to demonstrate the versatility of the general modeling framework developed in chapter 3. In the first example, a series-series collection of components and connections is modeled using a mixed exponential distribution for component failure distributions and mixed Weibulls for board and system level connections. The mixed Weibull distributions are used to model wear-out characteristics at the board and system levels. Thermal stresses are used to accelerated defects to failure at all assembly levels. In the second three level example, a mixture of Weibull distributions is used to model component and connection failure distributions at all assembly levels. Additionally, a multiple stress environment is used to accelerate defects to failure. Unlike the first example, perfect failure detection will not be assumed. Three different system configurations for PCBs are explored; series-series, parallel, and bridge networks.

5.2.1 Example One

In this example a general model for a multi-level stress screening process is developed for a hypothetical electronic system. The basic assumption is that at each level of the manufacturing and assembly process defects are introduced. The system under study consists of $i$ different types of components which are assembled into $l$ different
types of printed circuit boards (PCBs). The PCBs are modeled as a series-series collection of components and connections. There are \( j \) possible types of connections which can be used to assemble the printed circuit board. The \( l \) different types of PCBs are then assembled into a system using \( n \) different types of system level connections. Similar to PCBs, the system is modeled as a series-series combination of PCBs and connections.

The screening model examines stress screening strategies at three different levels of assembly; component, PCBs, and system. Components are screened, and only those passing the screen are used to assemble PCBs. The PCBs are then screened to detect defects introduced during the assembly process. PCBs passing a screen are then used to assemble the system. A screen is performed at the system level to eliminate any defects introduced during final assembly. The goal is to define optimal screen duration's by minimizing the life cycle costs.

A mixed exponential distribution is used to characterize the failure distribution at the component levels. The mixed exponential distribution is used for components in order to take advantage of the readily available MIL-HDBK-217F and Bellcore component failure rate databases. A mixed Weibull distribution is used to model the times-to-failure for the connections at each assembly level.

Different types of stress screens can be used at each of the various assembly levels. Since it is often more economical to screen at stress levels higher than normal operational levels, it is assumed that the maximum stress at each level will not exceed the maximum allowable stress for any of the individual elements at the current assembly level. Finally, it
is assumed that no new failure modes are introduced as a result of screening at higher than operational stresses.

Screening at the component level is usually done at stress levels that are much higher than operational stress levels. Using an appropriate accelerated stress model, the equivalent operational time for components undergoing testing at higher stresses can be calculated. A linear acceleration factor is assumed to exist. Thus, the equivalent operational time for a type \( i \) component can be related to the time under the increased stress by using equation (3.1). It is assumed that the value of \( K_{c_i} \) is the same for both substandard and good parts. In this example a thermal stress will be employed. It is well documented in the literature that electronic components subjected to thermal stresses follow the Arrhenius model [79], and \( K_{c_i} \) can be calculated using the following relationship:

\[
K_{c_i} = \exp \left( \frac{E_{a_i} \cdot \left( \frac{1}{T_o} - \frac{1}{T_{cs}} \right)}{K} \right)
\]

(5.7)

where \( K \) is Boltzman's constant \( (8.617 \times 10^{-5}) \), \( E_{a_i} \) is the component specific activation energy, \( T_o, T_{cs} \) are the normal system operating temperature and component screening temperatures respectively.

As with component level screening, a thermal stress \( T_{ba} \) will be used for type \( l \) board level screening. Using the Arrhenius model, the acceleration functions are:
\[ K_{b_{i,l}} = \exp\left( \frac{E_{a_{i,l}}}{K} \left( \frac{1}{T_0} - \frac{1}{T_{b_{l,i}}} \right) \right) \]  
\[ K_{b_{c,l,l}} = \exp\left( \frac{E_{a_{i,l}}}{K} \left( \frac{1}{T_0} - \frac{1}{T_{b_{c,l}}} \right) \right) \]  

Likewise, a thermal stress \( T_s \) will be used for system level screening. Using the Arrhenius model, the acceleration functions are:

\[ K_{s_k} = \exp\left( \frac{E_{a_k}}{K} \left( \frac{1}{T_0} - \frac{1}{T_{ss}} \right) \right) \]  
\[ K_{s_j} = \exp\left( \frac{E_{a_j}}{K} \left( \frac{1}{T_0} - \frac{1}{T_{ss}} \right) \right) \]  
\[ K_{s_i} = \exp\left( \frac{E_{a_i}}{K} \left( \frac{1}{T_0} - \frac{1}{T_{ss}} \right) \right) \]  

There is a tradeoff between cost of screening and cost of failure. This example uses a life cycle cost model that is a variant of the general model developed in chapter 3. Components can fail during component level screening, PCB level screening, and system level screening, or during operation in the field. The replacement costs of components is an increasing function of screening level. Component replacement is cheapest during component screening and highest when components are replaced in the field. The same is true for the repair costs of connections. Similarly, screening costs are also a function of screening level. Screening costs generally increase with each level of assembly. As the
duration of the screens increase, the field expenses decrease but the screening costs increase. Optimal screen durations at each screening level are those which minimize the life cycle costs. The life cycle cost model is given by:

\[
LCC = CC + PCBC + SYSC
\]  

(5.10)

where

\[
CC = sc \cdot I(t_{ac}) + np \cdot \sum \sum nc_{i,l} \cdot 
\left[
ct \cdot t_{ac} + NC_i(t_{ci}) \cdot crcc_{i,l} + NC_i(t_{bc,i,l}) \cdot crbc_{i,l} 
+ NC_i(T_w | t_{ci} + t_{bc,i,l} | t_{si}) \cdot crfi_{i,l} 
\right] 
\] 

(5.11)

\[
PCB = sb \cdot f(t_{ab,l}) + np \cdot \sum npcb_i \cdot cb \cdot t_{ab,l} + 
np \cdot \sum npci \cdot \sum nb_{i,l} \cdot 
\left[
NB_j(t_{b,i,l}) \cdot crbb_{j,i,l} + NB_j(t_{sb,i,l}) \cdot crsb_{j,i,l} + 
NB_j(T_w | t_{b,i,l} + t_{sb,i,l}) \cdot crfb_{j,i,l} 
\right] 
\] 

(5.12)

\[
SYSC = sc \cdot I(t_{as}) + np \cdot cs \cdot t_{as} + np \cdot \sum n_{s} \cdot 
\left[
NS_n(t_{s_i}) \cdot crss_n + NS_k(T_w | t_{sk}) \cdot crfs_n 
\right] 
\] 

(5.13)
In this example, assume that a manufacturer produces \( l \) different types of circuit boards and produces \( np \) of each type. These boards are used to build a variety of systems. Each PCB type \( l \) is made up of \( n_{c_{i,l}} \) components and \( n_{b_{j,l}} \) connections where \( i \) and \( j \) represent the different types of components and connections. The \( np_{c_l} \) PCBs are then connected using \( ns_n \) type \( n \) connections. In this example we let \( l = 2, i = 5, j = 3, n = 3 \). The system under study is defined by the following combination of components, connections and boards:

\[
\begin{align*}
np_{c_l} &= \begin{bmatrix} 4 \\ 2 \\ 8 \end{bmatrix}, & ns_k &= \begin{bmatrix} 6 \\ 4 \\ 8 \end{bmatrix}, & nc_{i,l} &= \begin{bmatrix} 20 & 30 \\ 40 & 10 \\ 60 & 30 \\ 40 & 50 \\ 20 & 70 \end{bmatrix}, & nb_{j,l} &= \begin{bmatrix} 40 & 60 \\ 80 & 60 \\ 40 & 40 \end{bmatrix}, & np &= [10,000]
\end{align*}
\]

The distribution characteristics for the components are:

\[
\begin{align*}
P_{w_i} &= \begin{bmatrix} .005 \\ .007 \\ .008 \\ .006 \\ .009 \end{bmatrix}, & P_{g_i} &= \begin{bmatrix} .995 \\ .993 \\ .992 \\ .994 \\ .991 \end{bmatrix}, & \lambda_{w_i} &= \begin{bmatrix} 5 \times 10^{-3} \\ 4 \times 10^{-3} \\ 3 \times 10^{-3} \\ 2 \times 10^{-3} \\ 1 \times 10^{-3} \end{bmatrix}, & \lambda_{g_i} &= \begin{bmatrix} 1 \times 10^{-7} \\ 3 \times 10^{-7} \\ 5 \times 10^{-7} \\ 7 \times 10^{-7} \\ 9 \times 10^{-7} \end{bmatrix}
\end{align*}
\]

The distribution characteristics for PCB connections are:
The distribution characteristics for system level connections are:

\[
P_{ws_n} = \begin{bmatrix} .002 \\ .003 \\ .004 \end{bmatrix}, \quad \eta_{ws_n} = \begin{bmatrix} 9.8 \times 10^2 \\ 1.05 \times 10^3 \\ 1.0 \times 10^3 \end{bmatrix}, \quad \beta_{ws_n} = \begin{bmatrix} 3.5 \\ 3.2 \\ 3.4 \end{bmatrix}
\]

\[
P_{gs_n} = \begin{bmatrix} .998 \\ .997 \\ .996 \end{bmatrix}, \quad \eta_{gs_n} = \begin{bmatrix} 8.5 \times 10^6 \\ 8.3 \times 10^6 \\ 9 \times 10^6 \end{bmatrix}, \quad \beta_{gs_n} = \begin{bmatrix} 3.2 \\ 3.5 \\ 3.3 \end{bmatrix}
\]

The acceleration factors for each component and connection type are calculated using the following data:
The cost parameters are given by:

\[
E_{a_i} = \begin{bmatrix} .7 \\ .8 \\ .9 \\ 1.0 \end{bmatrix} \quad E_{a_j} = \begin{bmatrix} .4 \end{bmatrix} \quad E_{a_k} = \begin{bmatrix} .65 \end{bmatrix}
\]

The cost parameters are given by:

\[
\begin{align*}
crcc_{i,l} &= \begin{bmatrix} $2 & $2 \\ $3 & $3 \\ $4 & $4 \\ $5 & $1 \end{bmatrix} \\
crbc_{i,l} &= \begin{bmatrix} $12 & $17 \\ $13 & $18 \\ $14 & $19 \\ $15 & $20 \end{bmatrix} \\
crsc_{i,l} &= \begin{bmatrix} $52 & $67 \\ $53 & $68 \\ $54 & $69 \\ $55 & $70 \end{bmatrix} \\
crcf_{i,l} &= \begin{bmatrix} $127 & $142 \\ $128 & $143 \\ $129 & $144 \\ $130 & $145 \end{bmatrix} \\
crbb_{j,l} &= \begin{bmatrix} $10 & $15 \\ $12 & $17 \\ $14 & $19 \end{bmatrix} \\
crsb_{j,l} &= \begin{bmatrix} $60 & $65 \\ $62 & $67 \\ $64 & $69 \end{bmatrix} \\
crbf_{j,l} &= \begin{bmatrix} $135 & $140 \\ $136 & $142 \\ $137 & $144 \end{bmatrix} \\
\end{align*}
\]

\[
\begin{align*}
\text{crss}_n &= \begin{bmatrix} $70 \\ $72 \end{bmatrix} \\
\text{crfs}_n &= \begin{bmatrix} $140 \\ $142 \end{bmatrix} \\
\end{align*}
\]

sc = $2000  \quad ct = $0.0025  \quad sb = $5000  \quad cb = $0.05  \\
ss = $7000  \quad cs = $0.15  \quad T_w = 50000hrs

Figures 5.11 & 5.12 illustrate the benefits of stress screening on the expected number of field failures due to component type i, and PCB connection type j respectively.
For the sake of illustration, it is assumed that screening was done at operational temperatures at all levels. Figure 5.11 illustrates that as the screen duration increases, the expected number of field failures decreases. But as the screen duration increases beyond 750 hr's, there is no significant reduction in field failures. Yet, screening costs continue to increase monotonically. Thus, there is a trade-off between component screening costs and component failure costs.

In figure 5.12 a different phenomena occurs. For a board screening duration of 2500hrs, the expected number of board connection repairs is initially increased. But, if the screening duration is increased to 5000 hrs a decrease in the expected number of board connection repairs occurs for a certain period of time. After this period, the expected number of repairs may exceed that of a system that has not undergone screening. This phenomena is caused by the fact that screened connections enter wear-out sooner, but at the same time early field failures are eliminated. Thus, a trade-off between cost of screening, cost of failure, and warranty period must be made.

The life cycle cost model is minimized using the quasi-Newton method. Sixteen cases are solved and presented in Table S.11. Table S.11 shows that for the parameters selected system screening is not cost effective, even under accelerated conditions. Component screening is only cost effective when performed under accelerated conditions. The most effective strategy is to perform component and board level screening under accelerated conditions.
Figure 5.11 Screening Effect on Expected Number of Replacements

Figure 5.12. Screening Effect on Expected Number of PCB Connection Repairs
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<th>Tab1 °K</th>
<th>Tab2 °K</th>
<th>Tab3 °K</th>
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Table 5.11. Screening Strategies for 50,000hr Warranty Period

Case 1 represents the classical burn-in situation. Components, boards, and the system are screened under operational conditions. The expected life cycle cost of a system is $4795.16 which is a 14.91% improvement over the expected life cycle cost of
$5636.05 if no screening is accomplished. Cases 8, and 16 represent the most cost effective screening strategies. The expected life cycle cost for a system that undergoes accelerated component and board level screening is $4384.76 which is a 8.55% cost savings per system over the traditional burn-in approach, and a 22.2% cost savings per system over not doing any screening.

![Table 5.12. Screening Strategies for 10,000hr Warranty Period](image)

Tables 5.12, 5.13 & 5.14 illustrate the robustness of the screening strategy for different warranty policies. Cases 1 through 8 are repeated for warranty periods of 5,000, 10,000 and 80,000 hr's respectively. Tables 5.12 and 5.13 show that the screening
strategies are similar to the strategies developed for a 50,000 hour warranty although the life cycle costs are substantially less. Similarly, Table 5.14 illustrates the robustness of the screening strategy to increases in warranty periods.

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_{sc}$</th>
<th>$T_{sb1}$</th>
<th>$T_{sb2}$</th>
<th>$T_{ss}$</th>
<th>$t_{ac}$</th>
<th>$t_{ab1}$</th>
<th>$t_{ab2}$</th>
<th>$t_{as}$</th>
<th>LCC $$/\text{system}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>$^\circ\text{K}$</td>
<td>$^\circ\text{K}$</td>
<td>$^\circ\text{K}$</td>
<td>hrs</td>
<td>hrs</td>
<td>hrs</td>
<td>hrs</td>
<td>$^\circ\text{K}$</td>
<td></td>
</tr>
<tr>
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<td>298</td>
<td>298</td>
<td>298</td>
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<td>1581</td>
<td>0</td>
<td>1493.28</td>
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<td>0</td>
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<td>298</td>
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<td>160.4</td>
<td>1610</td>
<td>0</td>
<td>1272.05</td>
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<td>348</td>
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Table 5.13. Screening Strategies for 5,000hr Warranty Period
Table 5.14. Screening Strategies for 80,000hr Warranty Period

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_{ac}$</th>
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<th>$T_{sb2}$</th>
<th>$T_{ss}$</th>
<th>$t_{ac}$</th>
<th>$t_{ab1}$</th>
<th>$t_{ab2}$</th>
<th>$t_{as}$</th>
<th>LCC $/$system</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>°K</td>
<td>°K</td>
<td>°K</td>
<td>°K</td>
<td>hrs</td>
<td>hrs</td>
<td>hrs</td>
<td>hrs</td>
<td></td>
</tr>
<tr>
<td>1</td>
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<td>1739</td>
<td>0</td>
<td>6952.47</td>
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<td>348</td>
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<td>298</td>
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<td>0</td>
<td>6902.30</td>
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<td>348</td>
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<td>1446</td>
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<td>0</td>
<td>7090.47</td>
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<td>348</td>
<td>298</td>
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<td>0</td>
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<td>0</td>
<td>6839.74</td>
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<td>348</td>
<td>348</td>
<td>298</td>
<td>5.88</td>
<td>160.4</td>
<td>120.2</td>
<td>0</td>
<td>6791.46</td>
</tr>
</tbody>
</table>

5.2.2 Example Two

In this section, the versatility of the general screening model developed in Chapter 3 is demonstrated by examining its use on a variety of system configurations. More specifically, a series-series system, a pure parallel system, and a bridge network are studied. In each example, mixtures of Weibull distributions are used to model component
and connection failure distributions at each assembly level. Unlike earlier examples, components and connections are screened in a multiple stress environment. Screening strategies for each configuration are developed using a variety of optimization criteria. Mission reliability, mean residual life and the life cycle cost model developed in chapter 3 are all used as optimization criteria.

In all three examples, it is assumed that a manufacturer is under contract to produce \( l \) different types of circuit boards which are then assembled into a system. Each PCB type is made up of a collection of components and connections. Let \( n_{c_{i,l}} \) represent the number of type \( i \) components on PCB type \( l \) and \( n_{b_{j,l}} \) represent the number of type \( j \) connections on board type \( l \). The PCB’s are assembled into a system using \( n \) different types of unit level connections. Let \( n_{s_{n}} \) represent the number of type \( n \) connections used to assemble the PCBs into a system. As in earlier examples, components are screened and then those components that successfully pass the screen are assembled into PCB’s. The PCBs are screened and those that fail the screen are reworked. If a component has failed, it is replaced with a stochastically identical component. Where as if a connection fails, it undergoes minimal repair. Successfully screened PCBs are then assembled into a system. Once assembled the system undergoes screening. Failed PCBs are replaced with stochastically similar boards and system level connections are minimally repaired. For illustration purposes let \( l=3, i=5, j=3, \) and \( n=1 \). Let each PCB be made up of the following combinations of components and connections:
The parameters for the failure distributions for the components, PCB connections, and system connections are as follows:

\[
\begin{bmatrix}
25 & 0 & 25 \\
0 & 25 & 25 \\
25 & 25 & 0 \\
0 & 25 & 0 \\
25 & 0 & 25
\end{bmatrix}
\]

\[
\begin{bmatrix}
25 & 0 & 25 \\
0 & 25 & 25 \\
25 & 25 & 0 \\
0 & 25 & 0 \\
25 & 0 & 25
\end{bmatrix}
\]

\[
\begin{bmatrix}
40 & 0 & 35 \\
35 & 40 & 0 \\
0 & 35 & 40
\end{bmatrix}
\]

\[
p_{wc_i} = \begin{bmatrix} .02 \\ .03 \\ .02 \end{bmatrix}, \quad \eta_{wc_i} = \begin{bmatrix} 3 \times 10^3 \\ 5 \times 10^3 \\ 1.1 \times 10^4 \end{bmatrix}, \quad \beta_{wc_i} = \begin{bmatrix} 1.9 \\ 2.3 \\ 3.5 \end{bmatrix}
\]

\[
p_{gc_j} = \begin{bmatrix} .98 \\ .97 \\ .98 \end{bmatrix}, \quad \eta_{gc_j} = \begin{bmatrix} 3 \times 10^8 \\ 5 \times 10^8 \\ 1.1 \times 10^9 \end{bmatrix}, \quad \beta_{gc_j} = \begin{bmatrix} 2.8 \\ 2.9 \\ 3.2 \end{bmatrix}
\]

\[
p_{wb_j} = \begin{bmatrix} .005 \\ .01 \\ .0075 \end{bmatrix}, \quad \eta_{wb_j} = \begin{bmatrix} 5 \times 10^2 \\ 7 \times 10^2 \\ 9 \times 10^2 \end{bmatrix}, \quad \beta_{wb_j} = \begin{bmatrix} 2.7 \\ 2.9 \\ 3.1 \end{bmatrix}
\]
\[ p_{gbj} = \begin{bmatrix} .995 \\ .99 \\ .9925 \end{bmatrix}, \quad \eta_{gbj} = \begin{bmatrix} 5 \times 10^6 \\ 7 \times 10^6 \\ 9 \times 10^6 \end{bmatrix}, \quad \beta_{gbj} = \begin{bmatrix} 2.6 \\ 2.6 \end{bmatrix} \]

\[ p_{w_a} = 0.0075; \quad \eta_{w_a} = 5 \times 10^3; \quad \beta_{w_a} = 2.2; \]

\[ p_{v_a} = 0.9925; \quad \eta_{v_a} = 5 \times 10^6; \quad \beta_{v_a} = 2.7; \]

In this example, components are screened in a combined environment where temperature and humidity are used as the accelerating stresses. The assembled printed circuit boards and systems are screened in a combined environment where temperature cycling and random vibration are used as the accelerating stresses. It is assumed that only the temperature cycling affects the individual components during board and system level screening. The acceleration factors for each component type at each level of screening are calculated using the following relationships:

\[ K_{c_i} = \left( \frac{RH_{use}}{RH_{cs}} \right)^{3.0} \exp \left[ \frac{E_{a_i}}{K} \left( \frac{1}{T_o} - \frac{1}{T_{cs}} \right) \right] \quad (5.14) \]

\[ K_{bc_{i,d}} = \left[ \frac{\Delta T_{bhj}}{\Delta T_{use}} \right]^{\gamma i} \quad (5.15) \]
\[ K_{s_i} = \left[ \frac{\Delta T_s}{\Delta T_{use}} \right]^{\gamma_i} \] (5.16)

where \( K \) is Boltzman’s constant \((8.617 \times 10^{-5})\), \( E_{a_i} \) is the component specific activation energy, \( RH_{use} \) & \( RH_{cs} \) are the percent relative humidity in the use and screening environments respectively, and \( T_o \) & \( T_{cs} \) are the normal system operating temperature and the component screening temperatures. At the board level, components are only susceptible to thermal stresses where \( \Delta T_{bs} \), \( \Delta T_s \), and \( \Delta T_{use} \) are the applied temperature cycling ranges used during board level screening, system level screening and the nominal temperature range for the use environment. \( \gamma_i \) are component specific thermal fatigue exponents for failure mechanisms of interest. Table 2.3 provides a summary of some common values.

The acceleration factors for board connections and system connections are found by using a combined effects model for random vibration and fatigue. The acceleration factors for each connection type at each level of screening are [25]:

\[ K_{b_{j,i}} = \left[ n_{lk} \left[ \frac{f_i f_{lk}}{G_{bs} G_{use}} \right]^{c_{lj}} + \frac{1}{(\Delta T_{bs}/\Delta T_{use})^{\gamma_{j,i}}} \right] \] (5.17)
where \( n_{th} \) is the number of thermal cycles applied, \( f_v/f_{th} \) is the ratio of the frequency of vibration cycles to thermal cycles, \( G_{bs}, G_s \) and \( G_{use} \) are the Grms vibration levels for the board level screen, system level screen and the normal use environment. The vibration fatigue exponents for the board and system level connections are given by \( C_j \) and \( c_n \). The acceleration factors for each component and connection type at their respective screening levels are calculated using the following parameters:

\[
K_{s_j} = \left( \frac{n_{th}}{\left( \frac{f_v}{f_{th}} \right)^{c_j} + \frac{1}{\left( \frac{\Delta T_s}{\Delta T_{use}} \right)^{\gamma_j}}} \right)^{-1} \quad (5.18)
\]

\[
K_{s_n} = \left( \frac{n_{th}}{\left( \frac{f_v}{f_{th}} \right)^{c_n} + \frac{1}{\left( \frac{\Delta T_s}{\Delta T_{use}} \right)^{\gamma_n}}} \right)^{-1} \quad (5.19)
\]

\( T_o = 298; \quad T_{cs} = 348; \quad RH_{use} = 60; \quad RH_{cs} = 85; \quad n_{th} = 10; \quad f_v/f_{th} = 5; \quad N_{vbs} = 6; \quad N_{vuse} = 1; \quad N_{vs} = 6; \quad \Delta T_{bs} = 120; \quad \Delta T_s = 90; \quad \Delta T_{use} = 30; \)

\[
\gamma_i = \begin{bmatrix} 2.1 \\ 2.2 \\ 2.3 \\ 2.2 \\ 2.1 \end{bmatrix} \quad E_{ai} = \begin{bmatrix} .75 \\ .9 \\ .65 \\ .85 \\ .95 \end{bmatrix} \quad c_j = \begin{bmatrix} 6.3 \\ 6.4 \end{bmatrix} \quad \gamma_j = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \quad c_n = 6.4 \quad \gamma_n = 4
\]
These parameter values were taken from DOD-HDBK-344 [35] and are representative of values one would use on actual electronic systems.

5.2.2.1 PCBs in Series

The first model examined is shown in Figure 5.7. In this case, three different types of PCBs are arranged in a series configuration. Components and connections on each PCB form a series-series system. Four system level connections are used to connect the three PCBs. The system level connections form a series-series system with the PCBs. The reliability model for the screened system is a subset of the model developed in section 3.3 and is given by:

\[
R_{\text{System}}(t) = R_1(t) \cdot R_2(t) \cdot R_3(t) \cdot \prod_n R_{S_n} \left( \frac{t}{t_{S_n}} \right)^{n_{S_n}}
\]

where

\[
R_1(t) = \prod_i \left[ R_{c_i} \left( t_{s_{i1}} + t_{bc_{i1}} + t_{c_i} \right) \right]^{n_{c_i,1}} \prod_j \left[ R_{b_j} \left( t_{s_{j,1}} + t_{b_{j,1}} \right) \right]^{n_{b_{j,1}}}
\]

\[
R_2(t) = \prod_i \left[ R_{c_i} \left( t_{s_{i1}} + t_{bc_{i2}} + t_{c_i} \right) \right]^{n_{c_i,2}} \prod_j \left[ R_{b_j} \left( t_{s_{j,2}} + t_{b_{j,2}} \right) \right]^{n_{b_{j,2}}}
\]

(5.20)
Then, the mean residual life and mission reliability for the screened system are given by:

\[
R_3(t) = \prod_i \left[ R_{c_i} \left( t_{c_i} + t_{bc_{i,3}} + t_{c_{i,3}} \right) \right]^{n_{c_i,3}} \cdot \prod_j \left[ R_{b_j} \left( t_{b_j} + t_{b_{j,3}} \right) \right]^{n_{b_j,3}} \tag{5.23}
\]

Then, the mean residual life and mission reliability for the screened system are given by:

\[
SMRL = \int_0^\infty R_{\text{system}}(t) \cdot dt \tag{5.24}
\]

\[
MR = R_{\text{system}}(TM) \tag{5.25}
\]

![Series-Series System](image)

Figure 5.13 Series-Series System

Like the mean residual life and mission reliability models, the life cycle cost model is a subset of the general life cycle cost model developed in section 3.2. The life cycle cost model for the three level screen of the system in Figure 5.13 is:
LCC = CC + PCB + SYS \quad (5.26)

where

CC = sc_i \cdot I(t_{c_i}) + \sum_{i} npcb_i \cdot \sum_{i} nc_{i,l} \cdot (ct_i \cdot t_{ac_i} + C1 + B1 + S1' + F1')

S1' = DS \cdot \left[ N_{c_i} \left( t_{s_i}, t_{bc_{i,l}} + t_{c_i} \right) + (1 - DB) \cdot \left( N_{c_i} \left( t_{bc_{i,l}} | t_{c_i} \right) + (1 - DC) \cdot N_{c_i} \left( t_{c_i} | 0 \right) \right) \right] \cdot crs_{c_{i,l}}

F1' = \left[ N_{c_i} \left( TW | t_{s_i} + t_{bc_{i,l}} + t_{c_i} \right) + (1 - DS) \cdot \left( N_{c_i} \left( t_{s_i} | t_{bc_{i,l}} + t_{c_i} \right) + (1 - DB) \cdot \left( N_{c_i} \left( t_{bc_{i,l}} | t_{c_i} \right) + (1 - DC) \cdot N_{c_i} \left( t_{c_i} | 0 \right) \right) \right) \right] \cdot crf_{c_{i,l}} \quad (5.27)

PCB = sb_j \cdot I(t_{b_{j,l}}) + \sum_{l} npcb_i \cdot \sum_{j} nb_{j,l} \cdot (cb_j \cdot t_{ab_j} + B2 + S2' + F2')

S2 = DS \cdot \left[ N_{b_j} \left( t_{s_j} | t_{b_{j,l}} \right) + (1 - DB) \cdot N_{b_j} \left( t_{b_{j,l}} | 0 \right) \right] \cdot crs_{b_{j,l}}

F2' = \left[ N_{b_j} \left( TW | t_{s_j} + t_{b_{j,l}} \right) + (1 - DS) \cdot \left( N_{b_j} \left( t_{s_j} | t_{b_{j,l}} \right) + (1 - DB) \cdot N_{b_j} \left( t_{b_{j,l}} | 0 \right) \right) \right] \cdot crf_{b_{j,l}} \quad (5.28)

SYS = ss \cdot I(t_{as}) + \sum_{n} ns_n \cdot (cs \cdot t_{as} + S4 + F4) \quad (5.29)
The parameters used in the life cycle cost model are:

\[
\begin{align*}
\text{crcc}_i & = \begin{bmatrix} $3 \\ $4 \\ $5 \\ $4 \end{bmatrix}, \\
\text{crbc}_{i,l} & = \begin{bmatrix} $13 & $13 & $13 \\ $14 & $14 & $14 \\ $15 & $15 & $15 \\ $13 & $13 & $13 \end{bmatrix}, \\
\text{crcc}_{i,l} & = \begin{bmatrix} $63 & $83 & $73 \\ $64 & $84 & $74 \\ $65 & $85 & $75 \\ $63 & $83 & $73 \end{bmatrix}.
\end{align*}
\]

\[
\begin{align*}
\text{crf}_{i,l} & = \begin{bmatrix} $113 & $113 & $113 \\ $114 & $114 & $114 \\ $115 & $115 & $115 \\ $113 & $113 & $113 \end{bmatrix}, \\
\text{crbb}_{j,l} & = \begin{bmatrix} $15 & $15 & $15 \\ $16 & $16 & $16 \\ $17 & $17 & $17 \end{bmatrix}, \\
\text{crsj}_{i,l} & = \begin{bmatrix} $15 & $85 & $75 \\ $16 & $96 & $76 \end{bmatrix}, \\
\text{crf}_{j,l} & = \begin{bmatrix} $16 & $16 & $16 \\ $17 & $17 & $17 \end{bmatrix}, \\
\text{crss}_{n} & = $25. \\
\text{sbs} & = $5000, \\
\text{ss} & = $7000, \\
\text{cc}_{i} & = \begin{bmatrix} $0.025 \\ $0.025 \\ $0.025 \end{bmatrix}, \\
\text{cb}_{i} & = \begin{bmatrix} $0.15 \\ $0.15 \end{bmatrix}, \\
\text{sc}_{i} & = \begin{bmatrix} $2500 \\ $2500 \end{bmatrix}.
\end{align*}
\]

\[
\begin{align*}
\text{TW} & = 60,000 \text{hrs}, \\
\text{TM} & = 30,000 \text{hrs}, \\
\text{DC} & = 0.80, \\
\text{DB} & = 0.95, \\
\text{DS} & = 1.0.
\end{align*}
\]
Screening strategies which maximize equations (5.24), (5.25) and minimize (5.26) were found by using the direct search complex algorithm (DPCOL), and the Quasi-Newton algorithm (DBCONF) from the IMSL library. As a check, a simulated annealing algorithm [43a] was also used for the mission reliability and mean residual life functions and the results compared. The annealing algorithm provided the same screening strategy as the complex search and quasi-Newton algorithms. Table 5.15 summarizes the optimization results.

<table>
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<tr>
<th>Criteria</th>
<th>t_a</th>
<th>t_b1</th>
<th>t_b2</th>
<th>t_b3</th>
<th>t_z</th>
<th>LCC</th>
<th>MR</th>
<th>SMRL</th>
</tr>
</thead>
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<td>815</td>
<td>1087</td>
<td>0</td>
<td>33942</td>
<td>0.9685</td>
<td>678304</td>
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<tr>
<td>LCC</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MR</td>
<td>182</td>
<td>0</td>
<td>319</td>
<td>319</td>
<td>197</td>
<td>17693</td>
<td>0.9988</td>
<td>706411</td>
</tr>
<tr>
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<td>175</td>
<td>0</td>
<td>284</td>
<td>287</td>
<td>172</td>
<td>16971</td>
<td>0.9987</td>
<td>706617</td>
</tr>
<tr>
<td>No Screening</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>151852</td>
<td>0.000313</td>
<td>222</td>
</tr>
</tbody>
</table>

Table 5.15 Optimization Results Series-Series Model

As shown in table 5.15, ESS has a dramatic impact on a systems life cycle cost, mission reliability, and mean residual life. For the case where no screening is performed, the mean residual life and mission reliability of the system are unacceptably low. In fact the mission reliability for the system is improved by a factor of 3094 if the most cost
effective screen is employed. Table 5.15 also shows that in order to maximize mission reliability and mean residual life, screening must be accomplished at all levels, with a significant amount of screening being accomplished at the system level. This should be expected since it is the last place in the production cycle to eliminate defects before the product is placed into the field. Another point of interest is that the marginal increase in mission reliability (0.968532 to 0.998811) costs $1398081. Thus, there is a significant cost associated with optimizing a screen based on a systems mission reliability and mean residual life. Only in cases where failure is catastrophic should the screening strategy be based on mission reliability or mean residual life.

The impact of field failure cost was studied by increasing the field failure costs by a factor of 100 and 1000. As expected, table 5.16 illustrates that the minimum cost screening strategy begins to resemble the screening strategies that maximize reliability and mean residual life in that system level screening is now cost effective. Also, the mission reliability and mean residual life begin to approach their maximum values.

<table>
<thead>
<tr>
<th>Field Cost Multiplier</th>
<th>$t_{ac}$</th>
<th>$t_{b1}$</th>
<th>$t_{b2}$</th>
<th>$t_{b3}$</th>
<th>$t_{s}$</th>
<th>LCC $</th>
<th>MR</th>
<th>SMRL hrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
<td>1168</td>
<td>853</td>
<td>1106</td>
<td>161</td>
<td>350019</td>
<td>0.998353</td>
<td>697977</td>
</tr>
<tr>
<td>1000</td>
<td>0</td>
<td>3892</td>
<td>851</td>
<td>1140</td>
<td>182</td>
<td>654425</td>
<td>0.997465</td>
<td>685235</td>
</tr>
</tbody>
</table>

Table 5.16 Field Failure Cost Increases
5.2.2.2 PCBs in Parallel

The second model examined is shown in Figure 5.14. In this case, three different types of PCBs are arranged in a parallel configuration. Components and connections on each PCB form a series-series system. Eight system level connections are used to connect the three PCBs as shown in Figure 5.14. The reliability model for the screened system is given by:

\[
R_{\text{system}}(t) = \left[ 1 - \left( 1 - R_1(t) \cdot R_{s_1}(t|s_1)^3 \right) \cdot \left( 1 - R_2(t) \cdot R_{s_1}(t|s_1)^2 \right) \right] 
\cdot R_{s_1}(t|s_1)^2
\]  
(5.30)

Figure 5.14 Parallel System
The mean residual life, and mission reliability for the system are found using equations 5.24, and 5.25. Optimal screening strategies are found for each criteria using the direct search complex algorithm (DPCOL) from the IMSL library. As a check, a simulated annealing algorithm [43a] was used on the mission reliability and mean residual life models and the results compared. The annealing algorithm provided nearly identical results.

The cost model in equation 5.26 was used to estimate the life cycle costs. In order to use this model for the parallel system two assumptions must be made. First, all component and connection failures are assumed to be repaired or replaced immediately, even if the system has not failed. Second, the cost to repair or replace is assumed to be the same whether or not the system has failed. If these assumptions are not true, then a new cost model that takes into consideration the structure of the system must be developed. Table 5.17 summarizes the optimization results.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>( t_{ae} )</th>
<th>( t_{b1} )</th>
<th>( t_{b2} )</th>
<th>( t_{b3} )</th>
<th>( t_s )</th>
<th>LCC $</th>
<th>MR</th>
<th>SMRL hrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCC</td>
<td>0</td>
<td>1089</td>
<td>815</td>
<td>1087</td>
<td>0</td>
<td>345094</td>
<td>0.985040</td>
<td>1466440</td>
</tr>
<tr>
<td>MR</td>
<td>182</td>
<td>0</td>
<td>319</td>
<td>319</td>
<td>199</td>
<td>1779933</td>
<td>0.999986</td>
<td>1502085</td>
</tr>
<tr>
<td>SMRL</td>
<td>162</td>
<td>0</td>
<td>259</td>
<td>268</td>
<td>158</td>
<td>1581319</td>
<td>0.999980</td>
<td>1502642</td>
</tr>
<tr>
<td>No Screening</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1524179</td>
<td>0.200571</td>
<td>212609</td>
</tr>
</tbody>
</table>

Table 5.17 Optimization Results for Parallel System
The first notable discovery when examining Table 5.17 is that the screening strategies are nearly identical to the series-series system. The screening strategies for optimizing mission reliability and mean residual life are nearly identical to those found in table 5.16. As expected the mission reliability and mean residual life for the parallel system is significantly higher than those of the series-series system. The optimal life cycle cost is a little higher, but the screening strategy is identical. These results indicate that the general practice of assuming that a system is a series-series system in order to develop a screening strategy is reasonable. The results appear to be relatively robust to the actual configuration of the system. Of course, if the assumptions on the cost model are relaxed and failures aren’t repaired immediately one would expect to find that the optimal screening strategy may differ significantly for the parallel system.

5.2.2.3 PCBs in a Bridge Network

The third model examined is shown in Figure 5.15. In this case, three different types of PCBs are arranged in a bridge network. Components and connections on each PCB form a series-series system. Twelve system level connections are used to connect the three PCBs as shown in Figure 5.15. The reliability model for the screened system is given by:
The mean residual life, and mission reliability for the system are found using equations 5.24, and 5.25. Optimal screening strategies are found for each criteria using the direct search complex algorithm (DPCOL) from the IMSL library. As a check, a simulated annealing algorithm [43a] was used on the mission reliability and mean residual life models and the results compared. The annealing algorithm provided screening strategies that were nearly identical to those found using the complex search algorithm. The cost model of equation 5.26 was used to estimate the life cycle costs. As in section
5.2.2.2, in order to use this model two assumptions must be made. First, all component and connection failures are assumed to be repaired or replaced immediately, even if the system has not failed. Second, the cost to repair or replace is assumed to be the same whether or not the system has failed. Table 5.18 summarizes the optimization results.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$t_{ac}$</th>
<th>$t_{b1}$</th>
<th>$t_{b2}$</th>
<th>$t_{b3}$</th>
<th>$t_{s}$</th>
<th>LCC</th>
<th>MR</th>
<th>SMRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCC</td>
<td>0</td>
<td>1089</td>
<td>815</td>
<td>1087</td>
<td>0</td>
<td>350745</td>
<td>0.985041</td>
<td>1010938</td>
</tr>
<tr>
<td>MR</td>
<td>191</td>
<td>0</td>
<td>319</td>
<td>319</td>
<td>199</td>
<td>1848579</td>
<td>0.999986</td>
<td>1041629</td>
</tr>
<tr>
<td>SMRL</td>
<td>174</td>
<td>0</td>
<td>269</td>
<td>274</td>
<td>161</td>
<td>1685244</td>
<td>0.999819</td>
<td>1041916</td>
</tr>
<tr>
<td>No Screening</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1529831</td>
<td>0.015774</td>
<td>11874</td>
</tr>
</tbody>
</table>

Table 5.18 Optimization Results for Bridge Network

Once again, the screening strategies for optimal mission reliability and mean residual life are very similar to those found in tables 5.16 and 5.18. Life cycle costs for the optimal mission reliability and mean residual life are slightly higher than those found for the parallel system and the series system. This is due to the increased number of components and connections found in the bridge network. The main reason for exploring the parallel structure and bridge network was to examine what effect, if any, the system structure has on the screening strategy. The case studies in this section indicate that the optimal screening strategy for mission reliability and mean residual life are robust to the
system structure. Initial indications are that minimal cost screening strategies may also be robust to the system structure. Additional work in this area needs to be done where by the assumptions used in sections 5.2.2.2 and 5.2.2.3 need to be relaxed and their effect on the most cost effective screening strategy examined.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

This dissertation develops a generalized modeling framework which can be used by reliability and maintainability engineers to develop environmental stress screening strategies for multi-component systems. Specifically, the model can be used to determine where in the assembly process ESS should be performed, how long and at what stress levels it is cost effective, as well as what happens to the screening strategy when the emphasis is shifted from cost to reliability.

6.1 Summary and Contributions of Work

This dissertation was developed in a series of incremental steps with the major contribution being a general modeling framework for environmental stress screening of multi-level, multi-component systems. During the course of development several significant contributions were made, each of which is illustrated in the examples in Chapter 5. Initially, two-level screening models were investigated. Example 5.1.1 demonstrates the importance of failure detection capability in determining the most cost effective screening strategy for a two-level screening model. It is shown that the capability to detect failures at component or system level significantly affects the screening strategy in terms of length and level of screening. Example 5.1.2 uses a mixture of Weibull distributions to model a component that has wear-out characteristics. This is the first multi-level ESS model to use a mixed Weibull distribution to model the component population. The major obstacle encountered with the use of mixed Weibull distributions was the calculation of the renewal function. Several methods were investigated and discussed in Chapter 4. In this example, the Weibull parameters were chosen so that the
component population had the same mean time to first failure as the component population in example 5.1.1. Optimal screening strategies that minimized the systems life cycle cost were found and compared to the system life cycle cost when the optimal strategies from example 5.1.1 were used. As expected, the cost of using a strategy based upon the assumption of a constant failure rate leads to excessive screening and higher life cycle costs. This is significant because all ESS models found in the literature as well as DOD-HDBK-344 assume mixed exponential distributions for component populations. Thus, screening strategies developed with these models and their associated assumptions may in fact be causing excessive screening. Example 5.1.3 generalizes example 5.1.2 to a multi-component system where the synergistic effects of screening individual components and the entire system are investigated. The Arrhenius model was used to model the impact of increased temperature on component and connection lifetimes. This is the first two level model to examine the effects of increased environmental stress at both component and system levels. Finally, example 5.1.4. generalizes the previous models by exploring the use of PH type distributions to model components and connections. PH distributions were investigated because they remain analytically and computationally tractable under a variety of mathematical operations encountered in the analysis of stochastic models, and they are versatile in their ability to approximate many of the lifetime distributions used in reliability. Several examples are worked and PH distributions are shown to be effective in modeling component and connection failure distributions.

Next, several three level models were examined. Example 5.2.1 is the only three level, multi-component ESS model found in the literature. Component populations were modeled with mixtures of exponential distributions and connection populations were modeled with mixtures of Weibull distributions. Temperature was used as the environmental stress at all three levels. Subsequent examples in the chapter explore the impact of system structure and multiple stresses on ESS strategies. All of the previous
multi-component models assumed that all components and connections formed a series-series system. Examples in section 5.2.2 explore the differences in strategies for series-series, series-parallel, and complex system configurations. Temperature and humidity are used as the environmental stress for components and temperature and vibration are used as the environmental stress at the unit and system levels. This is the first multi-level, multi-component, multiple stress model to appear in the literature. Screening strategies which minimize system life cycle cost for each of these systems were found using the general model found in Chapter 3. These screening strategies were compared to strategies which minimize the systems mean residual life, and the systems mission reliability. Strategies that optimized mean residual life and mission reliability were found to be extremely costly when compared to the strategies found using the life cycle cost model. The extra cost resulted in increases of reliability of less than .04. It was also shown that as the cost of failure increases, strategies which minimize life cycle cost begin to resemble strategies which maximize mean residual life and mission reliability.

The final contribution of this dissertation is the comprehensive survey of the relevant literature on ESS found in Chapters 1, 2 and 4. Chapter 1 reviews the history of ESS and establishes its importance to the system development process. Chapter 2 contains a comprehensive review of the ESS models found in the literature, as well as the applicable literature from the related disciplines of burn-in, and accelerated testing. Finally, Chapter 4 explores the basics of renewal theory for a variety of mixture distributions relevant to the reliability literature.

6.2 Recommendations for Future Study

The general stochastic modeling framework developed in this dissertation can be extended in several areas. First, the impact of screening on warranty policies for multi-
level, multi-component systems can be investigated. Work by Nguyen and Murthy [85] on optimal burn-in time to minimize cost for a product sold under warranty can be used as a stepping stone for analysis of warranty policies for screened systems. Recent work by Kao and Smith [56, 59] on the cost analysis of warranty policies for PH type distributions can be utilized to develop models for a variety of warranty policies.

Second, the impact of ESS on maintenance policies can be explored. The goal of this research would be to combine an ESS strategy with a maintenance strategy such that life cycle cost is minimized. Spearman [109] developed optimal maintenance policies for multi-component systems with Weibull failure times. Using Spearman's work [109], preliminary work has already been accomplished to extend the general modeling framework developed in this dissertation to study the effects of ESS on failure replacement, block replacement, and age replacement maintenance policies. Additional areas for future research include adding constraints to the optimization models used in the dissertation. The impact of constraints like cost, sub-system reliability and capacity can be added and optimal screening strategies developed. Similarly, additional constraints on stress level and stress types for various assemblies can be explored.

Third, investigate the development of dynamic ESS strategies. A properly implemented ESS program is a closed loop process. Manufacturing defects found during the screening process are supposed to be analyzed and when possible, corrective action taken. This corrective action will change the proportion of weak components found in the population after the change has been made. As the proportion of defects changes, so will the optimal screening strategy. The use of reliability growth to model changes in the manufacturing process could be investigated and dynamic ESS strategies developed.

Fourth, another area that is worth examining is the development of robust screening strategies using robust design techniques. Preliminary work has been begun on using Taguchi methods to develop robust screens for systems that have uncertainty in
them. Specifically, screens that are robust to uncertainty in distribution parameters and acceleration functions are being explored.

Finally, investigate using the general modeling framework developed in this dissertation to model software development and testing processes for large software systems. In conclusion, it is hoped that the general stochastic modeling framework presented in this dissertation will help motivate continued research efforts into the development of highly reliable, cost effective, quality systems.
APPENDIX A
XIE’S ALGORITHM

PROGRAM MIXED_WEIBULL (Xie’s Algorithm)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION Q(1001),XM(1001)
EXTERNAL RMW
OPEN(UNIT=10,FILE='MXDATA.OUT',STATUS='NEW')
X=2.0
SL=0.0
N=100
H=X/N
QQ=1-RMW((0.5*H),SL)
DO 10 K=1,N
 Q(K)=1-RMW((h*(K+0.5)),SL)
 X(M(K)=1-RMW((K*H),SL)
10 CONTINUE
DO 20 K=1,N+1
 DO 30 L=1,K-1
 X(M(K)=X(M(K)+Q(K-L)*(XM(L)-XM(L-1))
30 CONTINUE
 XM(K)=(XM(K)-QQ*XM(K-1))/(1-QQ)
WRITE(10, *) K*H,XM(K)
20 CONTINUE
CLOSE(10)
END

FUNCTION RMW(HK,SL)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
ETAB=1, BETAB=1
ETAG=1.0
BETAG=2.0
PB=0.0
PG=1-PB
PBS=(PB*DEXP(-(SL/ETAB)**BETAB))/(PB*DEXP(-(SL/ETAB)**BETAB)+PG*D
+EXP(-(SL/ETAG)**BETAG))
PGS=1-PBS
RMW=PBS*DEXP(-
(((HK+SL)/ETAB)**BETAB)+((SL/ETAB)**BETAB))+PGS*DE
+XP-(((SL+HK)/ETAG)**BETAG)+((SL/ETAG)**BETAG))
RETURN
END
APPENDIX B

SZIDAROVSZKY'S ALGORITHM

PROGRAM MIXED_WEIBULL (Szidarovszky’s Approach)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION HCAP(10001),PHI(20002),PHID(20002),H2CAP(20002),HCCAP(10001)
EXTERNAL RMW,PDFMW
OPEN(UNIT=10,FILE='MXDATA.OUT',STATUS='NEW~
N=500
X=3
H=X/N
SL=0.0
DO 10 K=0,N
HK=K*H
PHI(K)=I-RMW(HK,SL)
PHID(K)=PDFMW(HK,SL)
c WRITE(*,*) PHI(K),PHID(K)
10 CONTINUE
C HCAP(0)=PHI(0)
DO 20 K=1,N
SHCAP=0.0
IF (K.EQ.1) THEN
SHCAP=SHCAP
ELSE
DO 21 L=1,K-1
SHCAP=SHCAP+HCAP(L)*PHID(K-L)
21 CONTINUE
ENDIF
HCAP(K)=(1/(1-0.5*H*PHID(0)))*(PHI(K)+H*SHCAP)
20 CONTINUE
HH=X/(2*N)
DO 25 K=0,2*N
HHK=K*HH
PHI(K)=1-RMW(HHK,SL)
PHID(K)=PDFMW(HHK,SL)
25 CONTINUE
H2CAP(0)=PHI(0)
DO 30 K=1,2*N
SH2CAP=0.0
IF (K.EQ.1) THEN
SH2CAP=SH2CAP
ELSE
   DO 31 L=1,K-1
   SH2CAP=SH2CAP+H2CAP(L)*PHID(K-L)
31   CONTINUE
ENDIF
H2CAP(K)=(1/(1-0.5*HH*PHID(0)))*(PHI(K)+HH*SH2CAP)
30 CONTINUE
DO 40 L=0,N
TM=L*H
HCCAP(L)=(4*H2CAP(2*L)-HCCAP(L))/3
WRITE(10,50) TM,HCCAP(L),H2CAP(2*L),HCCAP(L)
40 CONTINUE
50 FORMAT (F12.4,2X,F20.1S,2X,F20.1S,2X,F20.1S)
CLOSE(10)
C
FUNCTION RMW(HK,SL)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
ETAB=1
BETAB=1
ETAG=1
BETAG=1
PB=0.00
PG=1-PB
PBS=(PB*DEXP(-(SL/ETAB)**BETAB))/(PB*DEXP(-
(SL/ETAB)**BETAB)+PG*D
+EXP(-(SL/ETAG)**BETAG))
PGS=1-PBS
RMW=PBS*DEXP(-
(((HK+SL)/ETAB)**BETAB)+((SL/ETAB)**BETAB)+PGS*DE
+XP(-(((SL+HK)/ETAG)**BETAG)+((SL/ETAG)**BETAG))
RETURN
END
C
REAL*8 FUNCTION PDFMW(HK,SL)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
ETCW=1
BETCW=1
ETCG=1
BETCG=1
PBC=0.00
C
C
PBS = (PBC * DEXP(-(((SL/ETCW)**BETCW)))/(PBC * DEXP(-(((SL/ETCW)**BETCW))+(1-PBC) * DEXP(-(((SL/ETCG)**BETCG)))))
PGS = 1 - PBS
IF (BETCW .EQ. 1 .AND. BETCG .EQ. 1) THEN
  PDFMW = PBS * (1/ETCW) * DEXP(-(((HK+SL)/ETCW)**BETCW) + (((SL/ETCW)**BETCW)) + PGS * (1/ETCG) * DEXP(-(((HK+SL)/ETCG)**BETCG))
ELSE IF (BETCW .EQ. 1 .AND. BETCG .NE. 1) THEN
  PDFMW = PBS * (1/ETCW) * DEXP(-(((HK+SL)/ETCW)**BETCW) + (((SL/ETCW)**BETCW)) + PGS * (((SL+HK)/ETCG)**(BETCG-1)) * DEXP(-(((HK+SL)/ETCG)**BETCG)) + ((SL/ETCG)**BETCG)
ELSE IF (BETCW .NE. 1 .AND. BETCG .EQ. 1) THEN
  PDFMW = PBS * (((SL+HK)/ETCW)**(BETCW-1)) * DEXP(-(((HK+SL)/ETCW)**BETCW) + (((SL/ETCW)**BETCW)) * (BETCW/ETCW) + PGS * (1/ETCG) * DEXP(-(((HK+SL)/ETCG)**BETCG) + ((SL/ETCG)**BETCG)
ELSE
  PDFMW = PBS * (((SL+HK)/ETCW)**(BETCW-1)) * DEXP(-(((HK+SL)/ETCW)**BETCW) + (((SL/ETCW)**BETCW)) * (BETCW/ETCW) + PGS * (((SL+HK)/ETCG)**(BETCG-1)) * DEXP(-(((HK+SL)/ETCG)**BETCG) + ((SL/ETCG)**BETCG))
END IF
RETURN
END
REFERENCES


[37] *Environmental Stress Screening Guidelines for Parts*, 1986, Mount Prospect Illinois, Institute of Environmental Sciences.


