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AN INTERFEROMETRIC APPROACH TO SUPPRESSION
OF SCATTERED RADIANT ENERGY

by

Christopher John Campbell Thompson

A Dissertation Submitted to the Faculty of the
COMMITTEE ON OPTICAL SCIENCES (GRADUATE)
In Partial Fulfillment of the Requirements
For the Degree of
DOCTOR OF PHILOSOPHY
In the Graduate College
THE UNIVERSITY OF ARIZONA

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The University of Arizona
Graduate College

As members of the Final Examination Committee, we certify that we have read the dissertation prepared by Christopher John Campbell Thompson entitled An Interferometric Approach to Suppression of Scattered Radiant Energy and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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SIGNED: Christopher J. C. Thompson
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# TABLE OF CONTENTS

## LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>vi</td>
</tr>
</tbody>
</table>

## LIST OF TABLES

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>xiii</td>
</tr>
</tbody>
</table>

## ABSTRACT

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>xiv</td>
</tr>
</tbody>
</table>

## 1. INTRODUCTION

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

## 2. BASIC THEORETICAL APPROACH

- Concepts .......................... 4
- Image of the Distant Point Source - A Diffraction Approach .... 8
- Three Dimensional Iso-Intensity Contour Plots ................. 15
- Optical Axis Intensity Profiles and the $4\pi$ Anomaly .... 22
- Appraisal of Point Image characteristics .................. 49
- Nulling of Scatter Sources ....................... 51
- An Elementary Scatter Nulling Interferometer System ..... 56
- Signal-to-Noise Considerations ...................... 61
- Factors Affecting Signal-to-Noise Ratio ................. 70

## 3. EXPERIMENT

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
</tr>
</tbody>
</table>

- Objectives and Constraints ....................... 72
- Interferometer Design Considerations ............ 73
- Experiment Environment .......................... 78
- Instrumentation and Procedure ................... 80
- Observations and Results ........................ 83

## 4. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>86</td>
</tr>
</tbody>
</table>

- Summary ..................................... 86
- Conclusions .................................. 88
- Recommendations ............................. 89

## APPENDIX A: Evaluation of Complex Amplitude in the Neighborhood of Focus of a Converging Spherical Wave for Four Special Cases

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>93</td>
</tr>
</tbody>
</table>

## APPENDIX B: Analytic Solution for the Intensity in Terms of Lommel Functions

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>97</td>
</tr>
<tr>
<td>APPENDIX C:</td>
</tr>
<tr>
<td>APPENDIX D:</td>
</tr>
<tr>
<td>APPENDIX E:</td>
</tr>
<tr>
<td>APPENDIX G:</td>
</tr>
<tr>
<td>APPENDIX H:</td>
</tr>
<tr>
<td>APPENDIX I:</td>
</tr>
<tr>
<td>REFERENCES</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Illustration of the Basic Conceptual Scheme</td>
<td>5</td>
</tr>
<tr>
<td>2.</td>
<td>Geometrical Superposition of Two Axially Shifted</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>On-Axis Convergent Spherical Waves, with Phase Difference of ( \pi )</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Notation and Definition of Variables for Diffraction of a Converging</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Spherical Wave at a Circular Aperture</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Geometrical Location of Shadow and Illuminated Regions for a Convergent</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Spherical Wave</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Iso-Intensity Contours Near Focus of an Error-Free Spherical Wave</td>
<td>19</td>
</tr>
<tr>
<td>6.</td>
<td>Iso-phantes (Lines of Equal Intensity) Near the Focus</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>of an Aberration-Free Pencil Without Central Obstruction</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Iso-Intensity Contour Plot Near Focus:</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>Two Wave Superposition with ( \pi ) Phase Shift; Focal Separation ( \Delta u = 0.4\pi ), F/10</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Iso-Intensity Contour Plot Near Focus:</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>Two Wave Superposition with ( \pi ) Phase Shift; Focal Separation ( \Delta u = 2.0\pi ), F/10</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Iso-Intensity Contour Plot Near Focus:</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Two Wave Superposition with ( \pi ) Phase Shift; Focal Separation ( \Delta u = 3.0\pi ), F/10</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Iso-Intensity Contour Plot Near Focus:</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Two Wave Superposition with ( \pi ) Phase Shift; Focal Separation ( \Delta u = 3.8\pi ), F/10</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>Iso-Intensity Contour Plot Near Focus:</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Two Wave Superposition with ( \pi ) Phase Shift; Focal Separation ( \Delta u = 4.0\pi ), F/10</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS—Continued

12. Iso-Intensity Contour Plot Near Focus:
   Two Wave Superposition with \( \pi \) Phase Shift;
   Focal Separation \( \Delta u = 8.0\pi, F/10 \) .... 28

13. Intensity Profile Along Transverse Plane of
    Symmetry: Two Wave Superposition with
    \( \pi \) Phase Shift; Focal Separation \( \Delta u = 0.4\pi, F/10 \) .... 29

14. Intensity Profile Along Transverse Plane of
    Symmetry: Two Wave Superposition with
    \( \pi \) Phase Shift; Focal Separation \( \Delta u = 2.0\pi, F/10 \) .... 30

15. Intensity Profile Along Transverse Plane of
    Symmetry: Two Wave Superposition with
    \( \pi \) Phase Shift; Focal Separation \( \Delta u = 3.8\pi, F/10 \) .... 31

16. Intensity Profile Along Transverse Plane of
    Symmetry: Two Wave Superposition with
    \( \pi \) Phase Shift; Focal Separation \( \Delta u = 3.81\pi, F/10 \) .... 32

17. Intensity Profile Along Transverse Plane of
    Symmetry: Two Wave Superposition with
    \( \pi \) Phase Shift; Focal Separation \( \Delta u = 4.0\pi, F/10 \) .... 33

18. Intensity Profile Along Transverse Plane of
    Symmetry: Two Wave Superposition with
    \( \pi \) Phase Shift; Focal Separation \( \Delta u = 8.0\pi, F/10 \) .... 34

19. Modulus of the Complex Amplitude Along the Optical
    Axis for a Single Converging Spherical Wave
    in the Vicinity of Focus .... 37

20. Phase of the Complex Amplitude of an F/10
    Converging Spherical Wave as a Function of \( u \) Along
    the Optical Axis .... 38

21. Intensity Profile Along the Optical Axis:
    Two Wave Superposition with \( \pi \) Phase Shift;
    Focal Separation \( \Delta u = 0.4\pi, F/10 \) .... 39
<table>
<thead>
<tr>
<th>List of Illustrations--Continued</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Page</strong></td>
</tr>
<tr>
<td>22. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 1.0\pi$, F/10</td>
</tr>
<tr>
<td>23. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 2.0\pi$, F/10</td>
</tr>
<tr>
<td>24. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 3.8\pi$, F/10</td>
</tr>
<tr>
<td>25. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 4.0\pi$, F/10</td>
</tr>
<tr>
<td>26. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 8.0\pi$, F/10</td>
</tr>
<tr>
<td>27. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 12.0\pi$, F/10</td>
</tr>
<tr>
<td>28. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 16.0\pi$, F/10</td>
</tr>
<tr>
<td>29. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 52.0\pi$, F/10</td>
</tr>
<tr>
<td>30. Converging Wavefronts Emerging from the Interferometer System</td>
</tr>
<tr>
<td>Figure</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>31</td>
</tr>
<tr>
<td>32a</td>
</tr>
<tr>
<td>32b</td>
</tr>
<tr>
<td>33a</td>
</tr>
<tr>
<td>33b</td>
</tr>
<tr>
<td>34a</td>
</tr>
<tr>
<td>34b</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>37</td>
</tr>
<tr>
<td>38</td>
</tr>
<tr>
<td>39</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>Illustration</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>41.</td>
</tr>
<tr>
<td>42.</td>
</tr>
<tr>
<td>43.</td>
</tr>
<tr>
<td>44.</td>
</tr>
<tr>
<td>45.</td>
</tr>
<tr>
<td>46.</td>
</tr>
<tr>
<td>47.</td>
</tr>
<tr>
<td>48.</td>
</tr>
<tr>
<td>49.</td>
</tr>
<tr>
<td>50.</td>
</tr>
<tr>
<td>Page</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>51. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation Δu = 3.0π, F/10</td>
</tr>
<tr>
<td>52. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation Δu = 3.6π, F/10</td>
</tr>
<tr>
<td>53. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation Δu = 4.2π, F/10</td>
</tr>
<tr>
<td>54. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation Δu = 20.0π, F/10</td>
</tr>
<tr>
<td>55. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation Δu = 50.0π, F/10</td>
</tr>
<tr>
<td>56. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation Δu = 70.0π, F/10</td>
</tr>
<tr>
<td>57. Coaxially Symmetric Converging Spherical Waves with Foci Separation Δ, and Their Common Coordinate Axis u'</td>
</tr>
<tr>
<td>58. Cosine Factor in the Expression for Intensity at the Origin of the Symmetry Planes</td>
</tr>
<tr>
<td>59. Intensity at the Origin of the Symmetry Plane as a Function of Separation Δ, at F/10</td>
</tr>
<tr>
<td>60. Notation and Definition of Variable s for a Spherical Wave Expanding from Point P1</td>
</tr>
<tr>
<td>61. Iso-Intensity Contour Plot: Determination of Fractional Energy in the V Plane</td>
</tr>
<tr>
<td>Illustration</td>
</tr>
<tr>
<td>--------------</td>
</tr>
<tr>
<td>62</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>63</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>64</td>
</tr>
<tr>
<td>65</td>
</tr>
<tr>
<td>66</td>
</tr>
<tr>
<td>67</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Gain in Signal-to-Noise Ratio with the Introduction of a Scatter Nulling Interferometer, for a Single On-Axis Point Scatter Source</td>
<td>70</td>
</tr>
</tbody>
</table>

xiii
ABSTRACT

The fundamental validity of using an interferometric process to reduce scattered light in optical systems has been examined from a physical optics point of view, and an experiment to illustrate the basic concepts of a particular scheme is described. Attention is focused on the important case of a telescope viewing a distant quasi-monochromatic point source, whose image has been degraded by scattering from dust particles on the primary lens.

A basic conceptual scheme is described, whereby a modified Twyman-Green interferometer is introduced into the optical train of an elementary telescope to null the scattered energy while preserving the image intensity associated with the distant point object, thereby enhancing the signal-to-noise ratio. Special characteristics of the interferometer fundamental to the scheme include amplitude division, and the introduction of a \( \pi \) phase shift and a focal power difference between the arms. The effect of an interferometer with these characteristics on the image of a distant point object and on a single on-axis scattering source is examined in detail. The three-dimensional intensity distribution in the neighborhood of focus of two converging coaxial spherical waves has been calculated for various focal separations and a \( \pi \) phase difference between the waves. Computer techniques were used to solve Lommel function based series representations of the diffraction
integral, giving the complex amplitude field distributions. Subsequent superposition of the two wave fields provided the intensity distributions which are presented as a series of iso-intensity contour maps. The results indicate that a distant point object could be reimaged successfully, and further analytic evaluations reveal specific operating conditions for the interferometer that optimize the peak intensity at the focal plane.

A wavefront model was used to derive the radially dependent intensity distribution at the focal plane from the single on-axis scattering point. The results are presented graphically.

To evaluate the effectiveness of the scatter nulling interferometer technique, an analytic expression for the gain in signal-to-noise ratio was developed. Utilizing elementary models of a reimaging system with and without the inclusion of a nulling interferometer, expressions for signal and noise power collected by a detector of arbitrary radius at the focal plane were derived. Example calculations are presented and the results tabulated for gain as a function of various system parameters such as wavelength and detector diameter. Gains up to a factor of $10^8$ were observed with the assumption of a reasonable set of system parameters and for the limited set of conditions employed. Results imply that the interferometric approach could be successful and effective under restricted conditions.

A description is given of the design and construction of an interferometer system that clearly illustrates the basic conceptual scheme. Imagery from the experimental arrangement is presented that
shows a projection of the concentrated distant source image along with the broad null field from the scattering source.
One of the most challenging problems in optical design, and one too often neglected, is that of reducing scattered light within an optical system. The earliest systems that man devised paid little attention to the problem, but inevitably, as demands were made for more sensitive systems, designers were faced with the task of inventing techniques to get rid of unwanted noise in the form of scattered radiant energy arriving at the detector. Initial approaches to the solution included the use of absorptive black paints on interior structural elements, and baffles to physically block the path of spurious light rays heading for the detector. Other generalized approaches evolved as designers and designs became more sophisticated, such as temporal techniques involving synchronous detection, and cryogenic applications in the case of infrared systems where structural elements emitted unwanted thermal energy. Most of the progress in these diverse areas has been accomplished in the past three decades; however, detector technology has advanced to the point where most high-sensitivity detector subsystems are background limited. Thus, there is a reemphasis on improving background reduction processes.

Recently, a radical new approach to reduction of scattered light in optical systems was proposed by Hill et al. (1977). This unique
approach, in a category by itself, suggested the use of a radial shearing interferometer to phase-null unwanted scattered energy. An interferometer, included as part of the optical train of a system such as a telescope, was designed to provide destructive interference at the focal plane for scattered light originating from a given plane exterior to the main optical system. If the object of interest, such as a star, was located at a distance which differed from that of the noise sources, it would have a distinctly different wavefront curvature at the entrance pupil of the interferometer. This would allow the interferometer subsystem to selectively focus the object within the null region at the focal plane, thereby enhancing signal-to-noise ratio. It was suggested that noise reduction factors of 50 to 500 were possible with this approach; a significant gain indeed.

If valid, this technique has dramatic significance and deserves further development. However, since 1977 when the method was first proposed, no definitive experiment has been conducted to demonstrate its effectiveness, and no further publications on the subject have appeared in the literature.

The major objectives of this study are to investigate the feasibility of the interferometric approach to scattered light suppression, and to probe the effectiveness of the scheme. To this end, the basic premises and assumptions are examined first. Notably, Figoski (1977) in his Master's thesis dealing with the problem used a geometrical theory approach, whereas the present study uses diffraction theory to
characterize signal and noise. Hill et al. (1977) proposed a form of radial shearing interferometer, while in this study a modified form of a Twyman-Green interferometer is used as a basis from which to study the problem. A particular design is presented that lends itself to a simple test of the theory's validity.

Extensive computer studies were conducted of the three-dimensional distribution of signal intensities about the region of the focal plane. The results were crucial to predicting effectiveness of the technique.

Finally, an experiment to demonstrate the basic validity of the theoretical approach is described. Conclusions and proposed extensions to this study are included in the final chapter.
CHAPTER 2

BASIC THEORETICAL APPROACH

Concepts

The fundamental scheme is illustrated in Figure 1. Let us consider the interferometer system as a black box inserted in the optical train ahead of the object's primary image and the final focal plane or detector plane of an optical system. Several assumptions are made here that serve to limit and circumscribe the problem. To begin with, the object of interest is assumed to be an on-axis quasimonochromatic point source, which is to be imaged at the final detector plane of a telescope system. Scattering sources are constrained to a single surface, for example, the exterior surface of the first element of the optical system; in practice an extremely important special case. The interferometer system is to be a two-beam amplitude division type.

Hill's concept suggests that an interferometer system could possibly be designed to intercept the diverging scattered wavefronts and operate on them to provide a null region about the optical axis at the detector plane. This could be accomplished by amplitude division in two legs of the interferometer, phase shifting by adjusting the path lengths to differ by a multiple of \( \pi \) or a halfwave, and recombining the wavefronts at the focal plane to provide destructive interference, hence
FROM DISTANT STAR SOURCE

PRIMARY LENS

CONVERGING WAVES

AMPLITUDE DIVISION INTERFEROMETER SYSTEM WITH FOCAL POWER AND $\pi$ PHASE DIFFERENCE

SCATTERING SOURCES

BROAD NULL FIELD AT FOCAL PLANE

Fig. 1. Illustration of the Basic Conceptual Scheme.
a null region about the optical axis. At the same time, according to Figoski (1977), if unequal path lengths are designed into the interferometer, the star's image would appear as a pair of longitudinally separated point images located within the null region at the focal plane. Since the star's wavefronts would be nominally spherical, bull's eye fringes would be centered on-axis and concentrated within the null fringe associated with the scatterer sources. Of course, the star's signal would also undergo amplitude division and phase shifting. It was predicted, however, that this would result in a center fringe null for the bull's eye pattern, which overall would be a highly condensed series of concentric circles.

The above concept of signal formation is questionable from both a geometrical and physical optics point of view. It is apparent that a simple geometric approach predicts a region of signal enhancement between the two foci, as illustrated in Figure 2. However, geometric theory is inadequate in predicting the detailed nature of the image outside illuminated regions, or, for example, in the region where caustics of individual foci overlap. Thus a physical optics viewpoint must be taken, and a three-dimensional diffraction calculation is necessary to correctly characterize the star image. If indeed a viable image of the star at the focal plane can be generated with the introduction of an interferometer, it remains to be demonstrated that the resulting signal-to-noise ratio represents an effective improvement on the status quo.
Fig. 2. Geometrical Superposition of Two Axially Shifted On-Axis Convergent Spherical Waves, with Relative Phase Difference of $\pi$. 
Considering the quasimonochromatic case initially, the following fundamental questions are identified and explored:

1. What is the intensity distribution about the focal plane for the case of two superposed amplitude divided, $\pi$ phase shifted, convergent wavefronts whose geometrical focal points are axially separated?

2. What is the nature of the scatter source's null field at the focal plane?

3. What general form of interferometer system can successfully accomplish the task of reimaging a distant point source within a scatterer null field at an optical system's focal plane?

4. What improvement in terms of signal-to-noise ratio is possible by the introduction of a scatter nulling interferometer?

**Image of the Distant Point Source--A Diffraction Approach**

Intensity distributions near the focus of a single converging spherical wave that has been diffracted at a circular aperture have been extensively investigated both theoretically and experimentally in the visible (Struve, 1886; Debye, 1909; Berek, 1926; Linfoot and Wolf, 1953, 1956; Taylor and Thompson, 1958; Hufford and Davis, 1962; Boivin and Wolf, 1965; Born and Wolf, 1975) and microwave domains (Mathews and Cullen, 1956; Boivin, Dion, and Koenig, 1956; Bachynski and Bekefi, 1957). The theoretical investigation is extended here to the case of two such coherent converging spherical waves, whose foci are longitudinally separated along the optical axis and whose relative phase is varied.
In this study, the term field intensity $I$ is used to denote the quadratic time average of a dimensionless field quantity $E$, where $E$ is a scalar complex amplitude

$$I = \langle E^2 \rangle .$$

(1)

The use of intensity in the nonradiometric sense is common throughout the optics literature, especially in theoretical expositions, and should not be confused with the radiant intensity whose units are watts per steradian. With the introduction of a suitable constant ahead of $\langle E^2 \rangle$ in Eq. (1), the field intensity could be transformed into its analogous radiometric equivalent, irradiance, whose units are watts per centimeter squared ($W \text{ cm}^{-2}$).

The use of Lommel functions (Lommel, 1886; Watson, 1944) in deriving the distribution of the intensity in the neighborhood of the focus of a single error-free convergent spherical wave diffracted at a circular aperture has been discussed by Born and Wolf (1975). The theoretical results first derived by Linfoot and Wolf (1953) have been confirmed through experimental observation by Taylor and Thompson (1958). For ease of comparison and analytic tractability, the approach and assumptions of Born and Wolf are followed in extending the method to determine the complex amplitude field distribution in the neighborhood of focus of two converging spherical waves whose foci are longitudinally separated along the optical axis, and whose relative phase is a variable.
Figure 3 shows a circular aperture $W$ of radius $a$, from which a coherent train of spherical waves with focal length $f$, and wavelength $\lambda$, converges to a focus at $O$. By application of the Huygens-Fresnel principle, the disturbance at $P$ may be expressed as

$$E(P) = \frac{A}{\lambda f} \exp \left[ -i \left( \frac{2\pi f}{\lambda} + \frac{s}{2} \right) \right] \int \int_W \frac{e^{iks}}{s} \, dS,$$

(2)

where $A/f$ is the amplitude at $Q$ of the incident wave, $k$ is $2\pi/\lambda$, $s$ is the distance from $P$ to $Q$, and $dS$ is an element of the wavefront subtending a solid angle $d\Omega$ at $O$. The basic assumptions involved are that the distance $OP$, and the radius of the aperture $W$ are small compared to the focal length, $f$; the wavelength $\lambda \ll a$, and with small angles involved, the inclination factor over the wavefront has been neglected.

It is convenient to introduce dimensionless variables $u$ and $v$:

$$u = k \left[ \frac{a}{f} \right]^2 z = \left[ \frac{2\pi}{\lambda} \right] \left[ \frac{1}{4(\Pi - \Omega)^2} \right] z$$

(3)

$$v = k \left( \frac{a^2}{f^2} \right) \sqrt{x^2 + y^2} = \left[ \frac{2\pi}{\lambda} \right] \left[ \frac{1}{2(\Pi - \Omega)} \right] \sqrt{x^2 + y^2}$$

(4)

whose physical significance is that $u/4\pi$ is the number of waves of defocus, and $v/\pi$ the number of waves of lateral displacement of $P$ from the optical axis (Taylor and Thompson, 1958). With the introduction of
Fig. 3. Notation and Definition of Variables for Diffraction of a Converging Spherical Wave at a Circular Aperture.
polar coordinates at \(O(r, \psi)\) and at \(C(\rho, \theta)\), it can be shown (Born and Wolf, 1975) that

\[
E(\rho) = \frac{\pi a^2 A}{f^2} \exp\left[i\left(\frac{f^2}{a} u - \frac{\pi}{2}\right)\right] \left[2 \int_0^1 J_0(\nu \rho) \exp(-i\nu^2/2) \, d\rho\right]
\]  
(5)

Using Euler's relationship on the exponential within the integral, the real and imaginary parts of the integral can be separated into two terms, such that

\[
2 \int_0^1 J_0(\nu \rho) \exp(-i\nu^2/2) \, d\rho = 2 \int_0^1 J_0(\nu \rho) \cos(\nu^2/2) \, d\rho
\]  
(6)

\[
- i \int_0^1 J_0(\nu \rho) \sin(\nu^2/2) \, d\rho.
\]

Let

\[
C(u, v) = 2 \int_0^1 J_0(\nu \rho) \cos(\nu^2/2) \, d\rho
\]  
(7)

\[
S(u, v) = 2 \int_0^1 J_0(\nu \rho) \sin(\nu^2/2) \, d\rho.
\]  
(8)
The expression for the complex field reduces to

\[ E(u,v) = \frac{\pi a^2 A}{r^2} \exp \left[ i \left( \frac{r}{2a} u - \frac{x}{2} \right) \right] [C(u,v) - iS(u,v)]. \]  

(9)

Note that Eqs. (7) and (8) may be integrated by parts (Born and Wolf, 1975) and evaluated using the Lommel functions \( U_n(u,v) \) and \( V_n(u,v) \), where \( J_{n+2p}(v) \) are Bessel functions of order \( n+2p \), and

\[ U_n(u,v) = \sum_{p=0}^{\infty} (-1)^p \frac{u^{n+2p}}{v^p} J_{n+2p}(v) \]  

(10)

\[ V_n(u,v) = \sum_{p=0}^{\infty} (-1)^p \frac{v^{n+2p}}{u^p} J_{n+2p}(v). \]  

(11)

Two sets of equations are required to evaluate \( C-iS \): one set applying when \( P \) lies in the shadow region of the geometric cone of rays; the other set applying when \( P \) lies in the illuminated region as shown in Figure 4. The equivalent dual sets assure swift series convergence in their respective zones, and are described in Appendix A. Computational considerations dictate four special cases for evaluation of \( E(P) \): at the origin \( E(0,0) \), at the focal plane \( E(0,v) \), along the optical axis \( E(u,0) \), and along the line \( u=v \). The appropriate equations are given in Appendix A.
Fig. 4. Geometrical Location of Shadow and Illuminated Regions for a Convergent Spherical Wave
The expression for complex amplitude, Eq. (9), can be separated into pure phase-related and amplitude-related terms if one considers the polar form of $C(u,v) - iS(u,v)$,

$$C(u,v) - iS(u,v) = \alpha \exp[-iX(u,v)] = \alpha [\cos X - i \sin X] \quad (12)$$

$$C(u,v) = \alpha \cos X; \quad S(u,v) = \alpha \sin X; \quad \alpha = \sqrt{C^2(u,v) + S^2(u,v)} \quad (13)$$

which yields

$$C(u,v) - iS(u,v) = \sqrt{C^2(u,v) + S^2(u,v)} \exp[-iX(u,v)] \quad (14)$$

where

$$\cos X = \frac{C}{\sqrt{C^2 + S^2}}; \quad \sin X = \frac{S}{\sqrt{C^2 + S^2}} \quad (15)$$

$$X = \tan^{-1} \left[ \frac{S(u,v)}{C(u,v)} \right] \quad (16)$$

which leads to

$$E(u,v) = \frac{\pi^2 A}{\chi f^2} \sqrt{C^2(u,v) + S^2(u,v)}$$

$$\times \exp \left[ i \left( \frac{f}{a} \right)^2 u - \frac{f}{2} - X(u,v) \right] \quad (17)$$

Three-Dimensional Iso-Intensity Contour Plots

It is significant that the phase depends on the F-number of the convergent cone of rays, where $(f/a)^2 = 4(F-No.)^2$. For our purposes, we
consider an additional phase factor $\eta = k\Delta z$, where $\Delta z$ is the optical path length difference between two amplitude-divided wavefronts of common origin, as may be encountered at the output of an unequal path interferometer, and ahead of the circular diffracting aperture.

Thus, allowing the phase factor intrinsic to $C(u,v) - iS(u,v)$ to remain in that expression, the computer was programmed to calculate a $101 \times 51$ element intensity array over the quadrant $u = 0$ to $10\pi$, $v = 0$ to $5\pi$, using an amplitude normalized to 1, in the form of

$$E(u,v) = \exp\left[i\left(4(\pi-N\sigma)u - \frac{\pi}{2} + \eta\right)\right] [C(u,v) - iS(u,v)].$$

A basic non-phase-shifted array was calculated with $\eta = 0$. Subsequently, a set of arrays with $\eta = \pi$ was calculated and stored. Superposition of the complex amplitude in phase-shifted arrays was effected by first introducing an axial shift $\Delta u$ between appropriate pairs of elements in each array with values of $\Delta u$ ranging from $0.2\pi$ to $8\pi$. If $E_1(u_1,v_1)$ is the complex field at a nonphase-shifted point, and $E_2(u_1,v_1)$ is a corresponding superposed point in a phase-shifted and axially shifted array, and $(u_1,v_1)$ are the coordinates in image space, then the total field is given by

$$E_{total} = E_1(u_1,v_1) + E_2(u_1,v_1)$$

(19)
and the intensity, \( I(u_1, v_1) \) is given by

\[
I(u_1, v_1) = \langle E_{\text{total}}^* \cdot E_{\text{total}} \rangle = \langle (E_1 + E_2)^* \cdot (E_1 + E_2) \rangle
\]

\[
I(u_1, v_1) = \langle E_1^* \cdot E_1 \rangle + \langle E_2^* \cdot E_2 \rangle + \langle E_1^* \cdot E_2 \rangle + \langle E_2^* \cdot E_1 \rangle.
\]

(20)

An analytic solution for \( I(u,v) \) is given in Appendix B. However, a simpler, more efficient method of solution is available on the computer. Since

\[
I(u_1, v_1) = |E_1 + E_2|^2
\]

(21)

the computer program can simply sum the real and imaginary parts of the complex field elements for each pair of superposed points, then sum the square of each pair to arrive at the intensity. The generating programs for complex amplitudes and succeeding superposition intensity arrays are given in Appendix C.

The basis for Lommel function calculations lies in generating Bessel series, the terms of which are Bessel functions of ascending order. Bessel functions were computed by a subroutine (Appendix C) using a recurrence-relation technique described by Goldstein and Thaler (1959), and Stegun and Abramowitz (1957). The Bessel series constituting the Lommel functions were assured of convergence by requiring final value stabilization at the 10th decimal place.
A contour plotting program, SURFACE II (Sampson, 1978), was employed to map the intensity arrays over the positive quadrant. This provided necessary and sufficient information to characterize the intensity distribution over all four quadrants since there was symmetry about both the optical axis and focal plane (Born and Wolf, 1975).

The validity of the overall computer program was first checked by two separate steps. Initially, an isointensity plot for a single converging spherical wave near focus was generated. The resulting plot, Figure 5, showing a single quadrant, duplicated that published by Linfoot and Wolf (1956) as shown in Figure 6. Intensity values in both parts were normalized to unity. Next, the contour plot for two identical superposed convergent spherical waves with no relative phase shift or axial displacement was generated. The plot was essentially identical to those of Figures 5 and 6. However, the maximum intensity at the origin was 4.0 not 1.0. This was to be expected since the modulus of the complex amplitude of each wavefront was chosen to be 1. One may recall that $I = |E_1 + E_2|^2$. Consequently, in the consideration of all subsequent superposition plots, the intensities are to be considered relative to a maximum of 4. In addition, for superposition plots generated from axially displaced complex amplitude arrays, the focal plane is replaced by an axis of symmetry equidistant between the two geometric focal points as shown in Figure 2 and distances along the abscissa and ordinate are given in terms of the dimensionless variables $u$ and $v$ respectively.
Fig. 5. Iso-Intensity Contours Near Focus of an Error Free Convergent Spherical Wave
Fig. 6. Isophotes (Lines of Equal Intensity) Near the Focus of an Aberration-Free Pencil Without Central Obstruction.

The values of axial shifts along the optical axis are dependent on the chosen F-number and wavelength. Recall the definition of the dimensionless variable $u$ in Eq. (3). For arrays shifted axially by an amount $\Delta u$, the axial separation of the two point images is given by

$$\Delta z = \frac{4(F\text{-No.})^2\lambda}{2} \Delta u. \quad (22)$$

Of special interest are axial shifts less than the depth of focus of a single convergent wave, since geometric intuition fails in this case. If the Rayleigh criterion is used for allowable defocus ($W_{\text{22}} = \lambda/4$), the depth of focus $\delta_Z$ then is given by

$$\delta_Z = 8(F\text{-No.})^2 W_{\text{22}} = \pm 2(F\text{-No.})^2\lambda. \quad (23)$$

As an example, using an F/10 cone, $\delta_Z = \pm 200\lambda$ and shifts less than $\Delta u = \pi$ will result in interference of two converging waves within the boundaries of each wave's caustic region. Also recall that the physical significance of the dimensionless variable $u$ was that $u/4\pi$ is the number of waves of defocus. Thus, $u = \pi$ is a significant point where we have $\pm 1/4$ wave of defocus, commensurate with the Rayleigh criterion.

After validity of the calculation procedure was established, iso-intensity or isophote contour plots were generated for two f/10 convergent waves with a relative $\pi$ phase shift, and for a series of axial geometric focus displacements ranging from $\Delta u = 0.2\pi$ to $\pi$ at $0.2\pi$
intervals, and at $\Delta u = 2\pi, 3\pi, 3.8\pi, 4\pi,$ and $8\pi$. It should be understood that a focus displacement of $\Delta u$ actually means the relative displacement along the \( u \) axis of the origins of the two-dimensional complex amplitude array. Figures 7 through 12 show the isophote plots for $\Delta u = 0.4\pi, 2\pi, 3\pi, 3.8\pi, 4.0\pi,$ and $8\pi$ as examples of superpositions occurring for shifts less than the depth of focus and greater than the depth of focus of a single convergent wave. The full series of plots is relegated to Appendix D. Each contour plot shows only the right, positive quadrant of the focal region. Rotational symmetry exists about the \( u \) axis, and the \( v \) axis lies in the orthogonal symmetry plane dividing right- and left-hand regions.

Further insight to the nature of the diffraction image can be gained by plotting the transverse intensity profiles at the symmetry plane and the optical axis intensity profiles for the isophote plots of Figures 7 through 12. The corresponding transverse plane plots are presented in Figures 13 through 18 with the \( v \) axis ranging from $v = 0$ to $5\pi$. For $f/10$ convergent cones, the range would be 50 wavelengths. Note that the transverse intensity profile for the $\Delta u = 8\pi$ case is zero at the optical axis, a result that differs from all other cases presented. To understand this behavior it is necessary to examine the optical axis intensity profiles both from a physical and analytic viewpoint.

**Optical Axis Intensity Profiles and the $4\pi$ Anomaly**

It has been observed that the complex amplitude can in general be expressed as a combination of two terms, the modulus $|E|$ and the phase $\phi$
Fig. 7. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 0.4\pi \), F/10.
Fig. 8. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 2.0\pi \), F/10.
Fig. 9. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 3.0\pi \), F/10.
Fig. 10. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 3.8\pi$, F/10.
Fig. 11. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 4.0\pi$, $F/10$.  

OPTICAL AXIS, $u$, UNITS X PI

TRANSVERSE SYMMETRY PLANE, $v$, UNITS X PI
Fig. 12. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with π Phase Shift; Focal Separation ΔU = 8.0π, F/10.
Fig. 13: Intensity Profile Along Transverse Plane of Symmetry: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 0.4\pi$, $F/10$. 
Fig. 14. Intensity Profile Along Transverse Plane of Symmetry: Two Wave Superposition with π Phase Shift; Focal Separation ΔU = 2.0π, F/10.
Fig. 15. Intensity Profile Along Transverse Plane of Symmetry: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 3.0\pi$, F/10.
Fig. 16. Intensity Profile Along Transverse Plane of Symmetry: Two-Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 3.8\pi \), F/10.
Fig. 17. Intensity Profile Along Transverse Plane of Symmetry: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 4.0\pi$, F/10.
Fig. 18. Intensity Profile Along Transverse Plane of Symmetry: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 8.0\pi$, F/10.
as expressed in Eq. (17), where the modulus normalized to the constant factor \( \frac{4a^2 A}{\lambda f^2} \) is given by

\[
|E| = \left[ C^2(u,v) + S^2(u,v) \right]^{1/2}, \tag{24}
\]

and the phase is given by

\[
\phi = 4F u - \frac{\pi}{2} - \tan^{-1} \left[ \frac{S(u,v)}{C(u,v)} \right]. \tag{25}
\]

Along the optical axis \( v=0 \), and the Lommel functions \( C(u,0) \) and \( S(u,0) \) as derived in Appendix A, given by Eq. (A.22), lead to the expression for the modulus in terms of the sinc function by using the identity

\[
1 - \cos^2 x = 2 \sin^2 x
\]

\[
|E| = \left| \frac{\sin(u/4)}{u/4} \right|. \tag{26}
\]

The term \( \tan^{-1}[S(u,0)/C(u,0)] \) reduces to \( u/4 \) by appropriate use of the trigonometric identities \( 1 - \cos 2x = 2 \sin^2 x \) and \( \sin 2x = 2 \sin x \cos x \), which leads to the phase along the optical axis expressed as

\[
\phi(u) = 4F^2 u - \frac{u}{4} - \frac{\pi}{2}. \tag{27}
\]
One may note at this point that along the optical axis the complex amplitude is not an even function. Symmetry conditions show that

\[ C(-u) = C(u) \]  \hspace{1cm} (28)
\[ S(-u) = -S(u) \]  \hspace{1cm} (29)
\[ \phi(-u) = -\phi(u) - \pi . \]  \hspace{1cm} (30)

Thus

\[ E(-u) = \left[ C^2(u) + S^2(u) \right]^{1/2} \left[ -e^{-i\phi(u)} \right] . \]  \hspace{1cm} (31)

However, the modulus is an even symmetric function, and it is shown plotted in normalized form in Figure 19, while the phase \( \phi(u) \) is shown in Figure 20. The plot of the modulus plays an important part in understanding the physical basis for the results obtained in the optical axis intensity profiles.

A modified form of the Fortran program included in Appendix C that produced the isophote plots was used to obtain the optical axis intensity profiles for the same series of focus displacements \( \Delta u \). However, the axial range was extended from \( \Delta u = 0 \) to 1000\( \pi \), and the results plotted on log-log format. Figures 21 through 26 show the results for \( \Delta u = 0.4\pi, \pi, 2\pi, 3.8\pi, 4.0\pi, \) and \( 8.0\pi \). In addition, Figures 27 through 29 give the intensity profiles for \( u = 12.0\pi, 16.0\pi, \) and \( 52.0\pi \).
Fig. 19. Modulus of the Complex Amplitude Along the Optical Axis for a Single Converging Spherical Wave in the Vicinity of Focus.
Fig. 20. Phase of the Complex Amplitude of an F/10 Converging Spherical Wave as a Function of u Along the Optical Axis.
Fig. 21. Intensity Profile Along the Optical Axis: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 0.4\pi \), F/10.
Fig. 22. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation ΔU = 1.0π, F/10.
Fig. 23. Intensity Profile Along the Optical Axis: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta u = 2.0\pi \), F/10.
Fig. 24. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 3.8w$, F/10.
Fig. 25. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation ΔU = 4.0π, F/10.
Fig. 26. Intensity Profile Along the Optical Axis: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 8.0\pi \), F/10.
Fig. 27. Intensity Profile Along the Optical Axis: Two Wave Superpositions with $\pi$ Phase Shift; Focal Separation $\Delta U = 12.0\pi$, F/10.
Fig. 28. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 16.0 \pi$, F/10.
Fig. 29. Intensity Profile Along the Optical Axis: Two Wave Superposition with π Phase Shift; Focal Separation ΔU = 52.0π, E/10.
Three especially notable results are apparent from a study of the axial intensity plots. First, at all multiples of \( \Delta u = 4\pi \), the envelope of the intensity function for \( \Delta u > 2\pi \) falls off with a slope of \(-4\), that is, an intensity falloff proportional to the inverse fourth power of the axial distance. This anomalous behavior degenerates rapidly for values close to \( 4n\pi \) as evidenced in Figure 24, where \( \Delta u = 3.8\pi \). For all values other than \( 4n\pi \), the slope becomes \(-2\) and the intensity falloff follows the inverse square law. Second, only at \( \Delta u = 4n\pi \) do the minima fall to a theoretical zero level, although the intensity range of the oscillating function does increase upon approaching the anomalous region. The third notable result is that for even values of \( n \) in \( 4n\pi \), where \( \Delta u = 8\pi, 16\pi, 24\pi, \ldots \), the intensity falls to zero at the symmetry plane, in effect providing a double peaked intensity function. For all odd values of \( n \), where \( \Delta u = 4\pi, 12\pi, 20\pi, \ldots \), the intensity peaks at the symmetry plane and suggests a cylindrical or oblate cylindrical form for the central concentration of intensity.

The above results are compatible with the analytic development given in Appendix E. Nevertheless, it is more satisfying to relate to physical reasoning in explaining these plots. The modulus of the complex amplitude as presented in Figure 19 can be used as a tool in understanding the above results. If a duplicate of the modulus function is used as an overlay, the two graphs can be relatively spaced by a factor \( \Delta u = 4n\pi \). At this point it must be remembered that in the region between the separated vertical axes, the wavefronts are in phase and the
moduli can be summed, and the sum can be squared to obtain the intensity. Outside this region, the wavefronts are $\pi$ out of phase and a subtraction process reduces the intensity. One may easily derive the gross features of the above results from this graphical process.

**Appraisal of Point Image Characteristics**

Two general results stand out from a study of the iso-intensity contour plots and their associated transverse and axial profile plots that bear on the theoretical feasibility of the interferometric scheme presented at the beginning of this chapter. Foremost is the fact that the energy from a distant point source after traversing an interferometer, which amplitude divides incoming wavefronts, introduces a phase shift and nominal focal separation, can be brought to a focus producing an image whose general characteristics are not unlike the Airy pattern for certain specific conditions of the interferometer. Although a nominal loss of 50% of the energy results from the reflections of the interferometer, a concentration of energy can exist in a central "cylinder" about the optical axis.

For the special focal separation condition of $\Delta u = 4\pi$, an optimum image condition ensues with flux density concentrated within the central maxima, and an inverse fourth power falloff of intensity along the optical axis. The transverse plane distribution of intensity appears as a bull's eye pattern with central maximum at the focal plane. As the transverse plane viewed moves along the optical axis, the central region alternates between a null and bright circular spot, with the latter's
flux density rapidly reduced by the inverse fourth power condition. It is apparent that the above model holds for relatively small distances about the symmetry plane and along the optical axis.

An estimation of total power collected by a particular detector of interest can be made by interpreting the iso-intensity plots in terms of total energy lying in a circle of radius $V_o$ about the optical axis and in a given plane $U_0$, or an equivalent square area. Appendix G outlines how this can be done.

The "far-field" intensity profile along the optical axis for the $4\pi$ case shows the extraordinary inverse fourth power falloff, with a promise of exceptionally low intensity levels when located at a null point on axis. However, one must be cautious not to jump to the conclusion that the intensity levels off-axis match this falloff rate. It should be apparent that energy conservation dictates a redistribution of energy over the wavefront, and the average energy distribution may be falling off at the inverse square law rate. Thus, a wavefront with uniform amplitude distribution may be a more reasonable model in the far field.

Also, the absolute zeros in intensity promised in theory at null points for the $4\pi$ condition have doubtful utility in practice. A practical example of an $f/10$ system with visible wavelength of 0.5 $\mu$m shows that the width of the null region is 27 $\mu$m for a reduction of intensity level by two orders of magnitude from the peaks.
**Nulling of Scatter Sources**

Consider that dust or other particulate matter settles on the front surface of the primary lens or mirror of a telescope system. This situation represents a common but major problem encountered by high sensitivity systems, notwithstanding the highest quality of surface smoothness achieved for the optical elements (Thomas, 1982). Let the problem be confined to dealing only with these scattering sources.

A simplified analytic approach may be taken to gain insight concerning the interference pattern one may expect at the detector plane due to the scatter source on the primary. The following model assumes no focal power difference between the interferometer arms, thus the wavefronts have identical curvatures as they exit the interferometer. When a focal power differential is introduced, the major features of the resulting interference pattern are not expected to change substantially, except for a reduction in the extent of the null region. Figure 30 shows slightly converging wavefronts emerging from the interferometer system and becoming incident on the detector plane. For a single scattering center located on axis at the primary, each emitted quasimonochromatic wavefront will be amplitude divided, phase shifted by introducing an optical path difference of \( \Delta \) between the two legs of the interferometer, and superposed at the detector plane. Figure 30 shows that the phase-shifted wavefronts will have a pair of foci at some very large distance, \( Z_{12} \), from the detector plane, separated by \( \Delta \). It is shown in Appendix F
Fig. 30. Converging Wavefronts Emerging from the Interferometer.
that the normalized irradiance interference pattern for this single scattering source located on axis is of the form

\[ I = 2 \left[ 1 + \cos \left( \frac{\pi}{\lambda} (k r_1^2 - 2 \Delta \lambda^2) \right) \right] = 2[1 + \cos \xi], \quad (32) \]

where

\[ r_1^2 = x_1^2 + y_1^2 \quad (33) \]
\[ r_1/z_{12} = \tan \theta \quad (34) \]
\[ k = 2\pi/\lambda. \quad (35) \]

For a null on axis:

\[ \cos \xi = -1 \quad (36) \]

therefore the condition of \( \xi \) for a null is

\[ \xi = (2n-1)\pi, \quad n = 0, \pm 1, \pm 2, \ldots \quad (37) \]
\[ \xi = \frac{k\Delta}{2} \frac{x_1^2 + y_1^2}{z_{12}^2} - k\Delta = (2n-1)\pi. \quad (38) \]

Since \( x_1 = y_1 = 0 \) on axis,
\[ \xi = -k \Delta = (2n-1)\pi = -\frac{2\pi \Delta}{\lambda} \] (39)

down therefore,

\[ \Delta = \frac{\lambda}{2} (2n-1). \] (40)

This implies that the optical path difference must be an odd multiple of \( \lambda/2 \) for a null to occur on axis, or alternatively, that the phase difference be an odd multiple of \( \pi \). Consider that \( z_{12} \) is the focal length \( f \) of the wavefront emerging from the interferometer system at the plane \((x_2, y_2)\), and the clear aperture radius is given by \( r_{\text{max}} \); the F-number, \( F \), is given by \( f/2r_{\text{max}} \). The radial distribution of intensity can be expressed as a function of \( r \) normalized by the F-number and clear aperture radius \( r_{\text{max}} \), where the intensity is

\[
I = 2 \left[ 1 + \cos \left( \frac{\pi}{2} (2n-1)(\tan \theta)^2 + \pi(2n-1) \right) \right] \]
\[
= 2 \left[ 1 - \cos \left( \frac{\pi}{2} (2n-1) \frac{1}{4F^2} \left( \frac{r}{r_{\text{max}}} \right)^2 \right) \right]
\] (41)

which is plotted in Figure 31.

Each off-axis scatterer contributes an intensity pattern at the detector plane identical to the on-axis pattern but laterally shifted a distance proportional to the off-axis position of the scattering center. Thus, the null area decreases in relation to the angular extent of the
Fig. 31. Interference Pattern from Two Converging Spherical Waves Originating from a Single On-Axis Point Scattering Source.
off-axis scatterers. Analytically, the intensity distribution is found by solving

\[ I = 2 \int_{\delta^{'}} 1 - \cos \left( \frac{\pi}{2} \tan(\theta - \theta') \right) d\theta'. \]  

(42)

Thus, the optimum approach would be to collimate the scatterer wavefronts emerging from the interferometer system, and achieve a null region of maximum extent at the detector plane. One may question where the energy from the scatter sources exists if it is nulled at the focal plane. Figure 31 shows theoretically that the energy is "pushed" out to the outermost fringes of the pattern. Energy is also reflected back toward the source through the beamsplitter.

An Elementary Scatter Nulling Interferometer System

So far, the scatter nulling interferometer has been characterized as a black box inserted within the optical train of, for example, a telescope system. It is convenient at this time to summarize the basic requirements of a practical system and achieve a perspective of the bridge between theory and instrumentation. The theoretical aspects of distant point source and nearby scatter source imaging have just been discussed. From this one can infer that a scatter nulling interferometer must satisfy the following:

1. The interferometer must reimage a distant point source at a given focal plane.
2. The interferometer must null the field contributions of nearby scatter sources at the given focal plane.

3. The interferometer must accomplish the above while providing a significant increase in signal-to-noise ratio.

An elementary optical system is depicted in Figure 32a. A scatter point source P resides on the surface of lens L1 contributing unwanted radiant energy at the focal plane. One simple solution for the form of the interferometer, which at least could satisfy criteria 1 and 2, is shown in Figure 32b. Observe that lens L2 intercepts the converging beam from L1 and collimates it. The beam is amplitude divided by a beamsplitter BS and recombined by virtue of reflection at mirrors M1 and M2. Note that while M1 is a flat, M2 has slight concavity, introducing focal power into one arm of the interferometer. This will provide a predetermined longitudinal separation of the focal points along the optical axis at the focal plane of imaging lens L3.

At the same time, L3 may be located such that its back focal distance is, for at least one arm of the interferometer, coincident with the distance of the scatter point P from L3. This assures a collimation of the scatter source and a broad based null pattern at the focal plane. Optical path length in the interferometer must of course be adjusted for a \( \pi \) phase difference to assure that an optimum null is produced.

A slightly more complex telescope system is schematicized in Figure 33a. Here, the commonly encountered intermediate focal plane
Fig. 32a. Lens L1 focusing distant point object, with scatter point P contributing irradiance at the focal plane.

Fig. 32b. Introduction of interferometer between L1 and focal plane.
Fig. 33a. Optical System with Intermediate Focal Plane

Fig. 33b. Introduction of an Interferometer into an Intermediate Focal Plane System
system is depicted, with scatter source P located on the objective L1. Figure 33b depicts the introduction of a simple interferometer, virtually identical to that shown in Figure 32b, into the optical train. In this case, P is collimated at the final focal plane by adjusting L3 as before, but with the added power of L2 taken into account. Again, criteria 1 and 2 may be satisfied, at least for the simplified case of a single scatter point on axis.

A significant point can be made: for large aperture telescope systems with intermediate focal planes, it is not necessary to build a correspondingly large interferometer. Van Heel and Simons (1967) have shown that a compact, unified, perhaps even one-piece device could be constructed, with obvious advantages in cost, weight, and optical quality.
Signal-to-Noise Considerations

Ultimately, the effectiveness of the scatter nulling interferometer scheme must be measured in terms of its improvement with respect to the signal-to-noise ratio. Consider the advantage gained to be $G$, where

$$G = \frac{S_I/N_I}{S_O/N_O}$$

$S_I = \text{Total flux (watts) from the distant point source collected by the detector, with the interferometer in the system.}$

$N_I = \text{Total flux (watts) from the scatter source collected by the detector, with the interferometer in the system.}$

$S_O = \text{Total flux (watts) from the distant point source collected by the detector without the interferometer present.}$

$N_O = \text{Total flux (watts) from the scatter source collected by the detector without the interferometer present.}$

Consider the elementary optical system to be that depicted in Figure 34a. With the interferometer introduced, the equivalent optical layout could appear as in Figure 34b. To maximize the gain $G$, an important assumption is made about the operating condition of the interferometer. It is assumed that geometric focal separation of $\Delta u = 4\pi$ is maintained, and that the $\pi$ phase difference between the paths is a constant. This allows an optimum image condition for the distant point source, with a concentration of flux incident on the detector. Further,
Fig. 34a. Elementary Optical System without Interferometer.

Fig. 34b. Equivalent Optical Circuit with Interferometer Included.
a convenient assumption is made about the size of the detector, whose shape is circular with radius $y_D$. The area of the detector, $A_D$, would be optimized in a practical situation with respect to achieving maximum signal-to-noise ratio with the interferometer in the system, and it would encompass at least the central maximum of the focal plane image. In comparison, without the interferometer present, the same detector would encompass at least the central maximum of an Airy pattern produced by the diffraction-limited lens $L_1$.

The signal power $S_0$ (watts) collected by the detector can be expressed as

$$S_0 = P_I \tau_{L_1} \beta_1 \text{ (watts)},$$  \hspace{1cm} (44)$$

where

- $P_I$ = Total flux from distant point source collected by lens $L_1$ (watts).
- $\tau_{L_1}$ = Transmission factor of lens $L_1$.
- $\beta_1$ = Fraction of total flux from distant point source arriving at detector plane, collected by detector.

Similarly, the signal power $S_I$ (watts) collected by the detector with the interferometer in the system can be expressed as

$$S_I = P_I \tau_{L_1} \tau_{I_1} \beta_2 \text{ (watts)},$$  \hspace{1cm} (45)$$
where

\[ T_I = \text{Transmission factor of interferometer.} \]

\[ \beta_2 = \text{Fraction of total flux from distant point source arriving at detector plane, collected by detector}. \]

For a specific case, \( \beta_2 \) could be estimated by the method detailed in Appendix G.

The noise power \( N_o \) (watts) collected by the detector can be expressed as

\[ N_o = \frac{T'_p y^2 D}{4f^2} \text{ (watts)} \]

where \( T'_p \) = Radiant intensity of the scatter point source \( P \) on axis (watts/steradian).

\( y_D \) = Radius of circular detector (cm).

\( f \) = Rear focal length of lens \( L3 \) and half the focal length of lens \( L1 \) (cm).

To determine the final element \( N_I \) in the gain equation, it is necessary to develop a physical model on which the calculation can be based. Recall that the \( 4\pi \) anomalous behavior of intensity along the optical axis suggested an inverse fourth power falloff of the intensity maxima, while the minima in theory approached zero. However, more pertinent to the problem is a definition of the intensity distribution in the transverse plane at the detector. Figure 35 schematicizes two representative wavefronts, \( E_1 \) emanating from the scatter point \( P \), and \( E_2 \)
Fig. 35. Wavefront Relationships within the Interferometer and at the Focal Plane.
generated by the interferometer mirror M2, whose focal power is determined by the separation $\Delta = 4\pi$ required for an optimum operating condition. Note that the wavefronts would interfere at the second beamsplitter plane, and because of the $\pi$ phase difference built into the interferometer path lengths, there would be destructive interference, hence a central null at the optical axis. The radius of the null can be determined by considering the separation $\varepsilon$ between wavefronts near the marginal edge of lens L3. Murty (1978) has shown that for small $\Delta$, the expression for $\varepsilon$ is accurately represented by

$$\varepsilon = (1 - \cos \theta) \Delta.$$

For the case of $\Delta u = 4\pi$, the actual separation $\Delta$ in terms of wavelength is given from Eq. (3) by $\Delta = 8F^2\lambda$; and $\tan \theta = 1/2F$, where $F$ is the F-number of the beam. Thus

$$\varepsilon = \left[ 1 - \cos [\tan^{-1}(1/2F)] \right] 8F^2\lambda .$$

For an F/10 system, $\varepsilon \sim 1\lambda$. This implies that a central null exists with one bright fringe in the field. The peak of the bright fringe lies at approximately 0.7 the distance to the edge of the field falling off to zero at the edge of the field. Figure 31 shows the general form of the intensity pattern taking only the first maximum and zero into account.
Whereas L3 collimates the wavefront from scatter point source P, the wavefront \( E_2 \) from the virtual scatter point \( P' \) is made slightly convergent. If the wavefronts \( E_1 \) and \( E_2 \) are considered to have uniform amplitude and the beamsplitter has provided a 50:50 amplitude division, which would be the ideal condition, one could observe that the intensity as a function of the radius \( y \) depended on the phase separation \((2\pi/\lambda)\varepsilon'\). In this case the intensity can be expressed in the form of Eq. (B.6), where \( I_{\text{total}} = |E_1|^2 = 2I_0 = 1 \). Thus,

\[
I = 2I_0 + 2|E_1| |E_2| \cos \delta. \tag{49}
\]

In this case, since the incident wavefronts are \( \pi \) out of phase, the phase is given by \( \delta = \pi + (2\pi/\lambda)\varepsilon' \). For each complex amplitude, the moduli are normalized to 1 and the intensity becomes

\[
I = 2I_0 \left[ 1 + \cos \left( \pi + \frac{2\pi}{\lambda} \varepsilon' \right) \right] \\
= 2I_0 \left[ 1 - \cos \left( \frac{2\pi}{\lambda} \varepsilon' \right) \right], \text{watts/cm}^2. \tag{50}
\]

An expression for \( \varepsilon' \) in terms of \( y \) and the curvature \( C_2 \) of the spherical wavefront \( E_2 \) can be derived from the sag formula. For small values of \( y \), the sag \( \varepsilon' \) may be given by
\[ e' = \frac{C_2 y^2}{1 + (1 - C^2 y^2)^{3/2}} = \frac{C_2 y^2}{2}. \]  

If we let \( u = \pi C_2/\lambda \), the intensity takes the form of

\[ I = 2I_0 (1 - \cos ay^2). \]  

Further, if the intensity for the \( u = 4\pi \) case is taken to be zero at the optical axis, the noise flux on the circular detector with radius \( Y_D \) can be found by integrating the intensity over the area of the detector. This gives

\[ N_I = 2I_0 \int_0^{2\pi} \int_0^{Y_D} (1 - \cos ay^2) \, y \, dy \, d\theta. \]  

It is convenient to expand \( \cos ay^2 \) in a power series and include the first two terms as in \( \cos x = 1 - x^2/2! \). The integral becomes

\[ N_I = 4\pi I_0 \int_0^{Y_D} \left[ \frac{a^2 y^4}{2} \right] \, y \, dy \]  

\[ N_I = \frac{I_0 \pi^3 C_2^2 y_D^6}{3\lambda^2}. \]
The gain can now be expressed as

\[ G = \frac{3L_{1}B_{2}^{2}\lambda^{2}}{4\beta_{1}r^{2}C_{2}^{2}y_{D}^{4}} \tag{56} \]

Recall that an assumption of unit amplitude was made for the wavefronts arriving at the focal plane in the development of the expression for \( N_{I} \). This implies that intensity \( I_{BS} \) (watts/cm\(^{2}\)) incident on the beamsplitter prior to amplitude division would be 16. Consider that incident on the beamsplitter \( |E_{1}| = |E_{2}| = 2 \). If the beamsplitter was assumed to have a 50:50 transmission factor and mirror transmissions were equal and no losses were experienced by the lenses, the amplitudes at the focal plane would be unity. Consequently, the intensity at the beamsplitter would be the square of the sum of the moduli. Hence, the radiant intensity would be

\[ I'_{p} \text{ (watts/steradian)} = \frac{I_{BS}A_{BS}}{\Omega} \tag{57} \]

where

\[ A_{BS} = \text{Projected area of beamsplitter (cm}^{2}\text{)} \]

\[ \Omega = \text{Solid angle subtended by the projected area of the beamsplitter at the scatter point (steradian).} \]

With reference to Figure 34b, \( \Omega = A_{BS}/d^{2} \), and \( I'_{p} = 16d^{2} \). Further simplifications are possible if one allows the reasonable approximations.
of \( \beta_1 = \beta_2, \tau_I = 0.5, \) and \( d = f/2. \) One may now express the gain as

\[
G = \frac{3\lambda^2}{2\pi^2 C^2 \gamma_D^4}.
\]

(58)

If some typical values for \( \lambda \) and curvature \( C \) are chosen, one can observe the range of gain as a function of detector radius. For example, Table 1 lists the values of gain for a visible and for an infrared wavelength, where the curvature is taken as \( 2 \times 10^{-3} \text{ mm}^{-1} \).

**Table 1. Gain in Signal-to-Noise Ratio with the Introduction of a Scatter Nulling Interferometer for a Single On-Axis Point Scatter Source.**

<table>
<thead>
<tr>
<th>( \lambda (\mu\text{m}) )</th>
<th>( \gamma_D (\mu\text{m}) )</th>
<th>( G )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>10</td>
<td>( 9.5 \times 10^5 )</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>( 2.4 \times 10^4 )</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>( 1.5 \times 10^3 )</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>( 9.5 \times 10^1 )</td>
</tr>
<tr>
<td>10.0</td>
<td>10</td>
<td>( 3.8 \times 10^8 )</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>( 9.8 \times 10^6 )</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>( 6.0 \times 10^5 )</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>( 3.8 \times 10^4 )</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>3.8</td>
</tr>
</tbody>
</table>

**Factors Affecting Signal-to-Noise Ratio**

For this initial study, the conceptual approach has been examined from the most elementary point of view, where a quasimonochromatic point source is reimaged within the null field of a single quasimonochromatic on-axis point scatterer. As the real world's complexities are introduced, the signal-to-noise ratio advantage of the method will
probably be reduced. These factors will have to be evaluated carefully to determine them quantitatively.

The first degree of complexity to be considered is the noise contribution from off-axis scatterers. Equation (42) illustrates one method of determining the extent of the null region for a continuum of scatterers over a given angular field of view. Obviously, the greater the angular extent of the scatterers, the smaller the null field. With a noncontinuum field of scatterers composed of dust particles for example, the density and size of particles become a factor.

Spectral bandwidth is a critical factor in determining the irradiance in the null region. Recall from Eq. (17) that the phase of the complex amplitude of a converging wave depends on both F-number and the dimensionless variables u and v. Both of these variables involve the wavelength factor \( k = \frac{2\pi}{\lambda} \), and as wavelength varies, the phase will vary and degrade the null field irradiance.

The degree of temporal coherence of the energy incident on a scatter surface will be an additional major factor in determining effectiveness of the null field.

Finally, the presence of the interferometer itself will add some scattering factor by virtue of the number of surfaces added to the original optical system. In addition, the spectral transmission and throughput of the interferometer should be taken into account.
CHAPTER 3

EXPERIMENT

Objectives and Constraints

As a step along the path to a more practical system, an experiment was undertaken to demonstrate the basic concepts of a scatter reduction interferometer. It should be emphasized that no parametric evaluation of the experimental scheme was intended at this stage; that was to be left for a future effort.

Two major objectives were identified. First, to simulate in an elementary way a practical test situation whereby a star or point source would be reimaged by an optical system, such as a telescope, within whose optical train a scatter nulIling interferometer would be imbedded. Scattering sources were to be located on a chosen surface of the reimaging system. Second, to observe the irradiance distributions associated with the point source images and scattering sources with the interferometer set up for a null condition.

Economy and time dictated that the mechanical and optical elements be readily available off the shelf. However, stringent requirements for preservation of wavefront structure with minimum aberrations suggested that optimum quality elements should be acquired. Simplicity dictated two principal constraints. The star was to be simulated by a quasimonochromatic helium-neon laser source instead of a
broad spectrum source, and the scatter sources were to be limited to a single point source on axis.

In essence, the basic objective was to show, for the limited set of conditions described above, that the energy from a far-field source such as a star could indeed be preserved at the image plane after traversing the dual paths of an interferometer set up for the null condition, while simultaneously the energy from a point scattering source in the near field would be nulled at the focal plane.

**Interferometer Design Considerations**

The major design question involved the type of interferometer to use. Hill et al. (1977) had suggested that a plane of scatterers would be nulled over a large field of view and, that the null field at the focal plane from scatterers off-axis would exhibit stationarity. For the purpose at hand, it seemed sufficient to demonstrate the theoretical principles with a simple modified version of a Twyman-Green interferometer as depicted in Figure 36. The key modification to the unequal path interferometer and the critical item that allows this scheme to work was the introduction of unequal focal power in each of the arms of the interferometer. Unequal power allowed the point source to be reimaged as two axially separated images, and as theory predicted, the energy from the source would be preserved to some extent near the focal plane, even though the interferometer paths differed in optical path length by a multiple of $\pi$. 
Fig. 36. Experimental Layout: Modified Version of Twyman-Green Interferometer.
A method was required that allowed a continuum of focal power variations in order to study the point image distributions at various axial separations. A low power concave mirror in one of the arms was one possibility that would have allowed a collimated beam input to the beamsplitter, thereby minimizing aberrations introduced by the parallel plate. Again, expediency suggested a compromise, where the combination of a slightly convergent input beam and variable path length for the movable mirror produced the necessary power variation in one arm. Because of this choice, an ACCOS-V study was conducted to determine the aberrations introduced by the convergent beam's multiple passes through the plate beamsplitter, and to determine the combination of path lengths and amount of beam convergence necessary to match point image separations as considered in the theoretical study. Figures 37 and 38 illustrate ray trace paths through the interferometer for the point source and on-axis scatter source respectively.

Movable mirror path length was influenced in part by the multimode laser being used. It is well known that the theoretical visibility in an interferometer, when several longitudinal modes are present, is a function of optical path difference (OPD) (Collier, Burkhardt, and Lin, 1971). To have good contrast in the fringes, the OPD on axis should be close to an integral multiple of 2L, where L is the cavity length of the laser. In this case the laser had at least two modes, and satisfaction of the contrast requirement constrained the movable mirror position to two possible locations on the table.
Fig. 37. Ray Trace of Point (Star) Source through Scatter Nulling Interferometer.
Fig. 38. Ray Trace of Scatter Source through Nulling Interferometer.
Phase adjustment to provide the 180 degree phase difference between the interferometer legs was accomplished with a piezoelectric-driven motor mount supplied by Burleigh Instruments, Inc. The model PZ-90 PZT aligner/translator was driven by an RC-44 ramp generator in DC mode, and was capable of a total translation of 6 µm, with a minimum incremental motion of $7.64 \times 10^{-6}$ mm as determined by calibration procedures described in Appendix H. The limiting minimum motion was dictated by the ramp generator power supply, which could be set in 1-volt increments. This corresponded to 0.012 wavelengths of HeNe laser light.

Alignment of the system was a carefully considered task, understandably because of the critical need to preserve wavefront quality. The procedure is described in greater detail in Appendix I.

**Experiment Environment**

Several general problem areas exist in experimental interferometry. Mechanical and acoustical vibration, thermal stability of mechanical supports and elements, thermal convection in the environment, and the ubiquitous dust particles in the air and on optical elements all serve to degrade the quality of imagery. Cognizant of this, appropriate steps were taken to minimize these effects on the experimental setup. For example, a tent enclosure was constructed from two layers of plastic sheeting to fully enclose and isolate the setup from room convection currents. It also minimized dust settlement and
reduced stray light in the system. The optical elements were mounted on a Newport Research Corporation honeycomb-core table designed for static and dynamic rigidity. The table was shock mounted and isolated from vibration by layers of foam pads, and an air bag was independently placed under each leg. Individual optical elements were mounted on interferometer-quality mounts supplied by Newport Research Corporation and fastened to the table by heavy-duty magnetic bases or bolted down securely.

In spite of all these precautions, the resulting interference images were subject to instabilities from the sources which were difficult to overcome. An individual's body heat disturbed the thermal equilibrium inside the plastic tent and tended to cause slow drift. Acoustic shock waves generated by building air conditioners, doors closing in the immediate vicinity of the chamber, and even speech tended to cause damped vibrations of the image. Finally, manual adjustment of micrometers in the measuring process caused temporary vibration and blurring of the image. It should be noted that the laser source, when first switched on, contributed to a constant image phase drift due to the progression of the cavity temperature toward an equilibrium point. It required an overnight period to thermally stabilize the laser source. Ultimately, an image sensing electronic feedback system is necessary for precise measurements on resulting interference patterns. Expediency dictated a less sophisticated approach in this experiment. A point detector sampled the center of the pattern exiting the interferometer.
and triggered an audio signal when a null was present. This allowed measurements to be made on the point image over short periods in the drift cycle when the null was present.

Instrumentation and Procedure

With reference to Figure 36, the schematic of the experimental arrangement, it may help to consider the setup as a combination of four parts. (1) The point source, provided by the laser and spatial filter, (2) the reimaging system, comprising the stops, converging lens, window, and imaging lens, (3) the interferometer, made up of the beamsplitter and two mirrors, and (4) the projection system, as represented by the translating microscope assembly and projection screen.

Energy provided by a Hughes model 3224H-PC linearly polarized HeNe laser was formed into a point source by a spatial filter comprised of a 20X microscope objective and 15-μm diameter precision pinhole. The spatial filter, in effect, provided diverging coherent wavefronts free from spurious diffraction effects. An entrance pupil stop S1, ahead of L1 on an f/8, 31.5-mm diameter doublet, ensured a clean beam entering L1. The doublet was placed at a distance greater than its 250-mm back focal length from the point source, thus forming a slightly convergent beam. An aperture stop S2, placed ahead of the interferometer, limited the beam diameter to ensure clear aperture passages through the beamsplitter, and it was adjusted to provide an f/20 cone at the final focused beam. A flat glass window was placed ahead of the beamsplitter. The window served as a convenient locating surface for a point scatter
source. The front surface of the window was judiciously placed at the rear focal point of the imaging lens L2, with respect to the fixed mirror path through the interferometer. This was done to ensure collimation of the energy from an on-axis scatter source and to provide the broadest possible null field at the image plane.

Several characteristics of the beamsplitter are worthy of note. A flat glass plate was chosen primarily for the superior optical figure that it offered compared to a cube type or pelicle beamsplitter. Each surface was measured to better than 1/20 wave flatness peak to peak across the 50-mm diameter. Because of the additive effect of wavefront distortions due to multiple passes of the wavefronts through the beamsplitter, it was critical to use the best off-the-shelf element available. A 50-mm diameter fused quartz blank, 9-mm thick with a wedge of less than 1 second of arc was available from the Oriel Corporation. The blank was specially coated with a multilayer thin film that gave a 50:50 transmission split for a 0.6328-μm beam incident at the Brewster angle to the plate. Using the beamsplitter at Brewster's angle, in conjunction with a vertically p-polarized incident HeNe laser beam, provided a means to avoid the generation of spurious images due to multiple internal reflections within the beamsplitter.

Each path of the interferometer terminated in a high-quality, 50-mm diameter, flat first surface mirror, with a measured figure better than 1/20 wave peak-to-peak across its diameter. The fixed path length
M1 was shorter than that of the adjustable mirror M2.

The movable mirror M2 was mounted on a piezoelectric transducer (PZT) driven mount provided as previously described. Three axially located PZT elements in the mirror mount could be simultaneously elongated or shortened depending on an applied DC voltage. The Burleigh power supply allowed a stabilized precise increment of voltage to be applied to the PZTs, which adjusted the axial position of the movable mirror with a least increment of 1/100 wavelength. M2 was the mirror used to adjust phase difference between the interferometer arms. The movable mirror mount was itself based on a translation stage driven by a differential micrometer capable of 0.5-μm increments. This offered a manual adjustment option.

The combined output beams from the interferometer were collected and brought to focus by the imaging lens L2, an f/10, 400-mm focal length doublet. Considering the combination of input converging beam and unequal paths in the interferometer, the imaging lens provided a double image of the point source, with longitudinal separation on axis. Image separation distance depended on the degree of beam convergence and the difference in path lengths between the interferometer arms.

Point images were examined and projected onto a viewing screen by a 10X microscope objective L3 mounted on a micrometer-driven translation stage. In addition, the far-field intensity distribution at a plane near the imaging lens was projected onto the viewing screen along the point image. This was accomplished by inserting a pelicle
beamsplitter BS2 as close as possible to the rear of the imaging lens and reflecting the diverted beam with an appropriately placed flat mirror M3. The arrangement allowed simultaneous viewing of the point image and adjustment of the interferometer path length for the 180 degree phase difference which could be viewed as a null on the far-field projected pattern.

A small phototransistor, D1, sampled the irradiance distribution within a few millimeters of the front surface of the reimaging lens. Its signal was fed to an electronic circuit that determined if a null was present at the center of the interference pattern. An audio oscillator was enabled if a null appeared, and a tone generated from a small loudspeaker during the period the null, or intensity minimum, existed. This device was a convenient tool during measurements of point image parameters because it was difficult otherwise to know the measurements were being made while a null condition of the interferometer existed.

Observations and Results

With the system aperture stop set to provide an f/20 cone from the imaging lens, and interferometer path lengths adjusted to give a 180 degree phase difference between the combined wavefronts, the point source image appears as in Figure 39. The null condition present in the far field as sampled by the pelicle beamsplitter is conveniently imaged side by side with the point image. It is apparent that the point image diffraction pattern fits well within the null region. That the null
Fig. 39. Projected Images of Point Source and Far Field Null Pattern.
region persists uniformly along the optical axis was confirmed by moving the pelicle beamsplitter along the axis and noting the non-localization of the central null area. Furthermore, removal of the microscope projection lens and insertion of a viewing screen completed the check as the far field pattern was viewed downstream from the focal plane with the null region constant along the axis. As a final check, the far-field pattern was viewed along the path back toward the source, and indeed the inverse of the null pattern was observed as expected.
CHAPTER 4

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The concept of using an interferometric process to reduce scattered light in optical systems is unique and tantalizing in its enormously promising suggestion of significant improvement in signal-to-noise ratio, especially for telescope systems observing distant point objects. A first step has been taken in this study to establish the fundamental validity of the concept.

A basic scenario was defined, wherein the image of a distant point object was negatively influenced by scattered light originating from a nearby plane. An analogy may be drawn with a star source being imaged by an optical system with dust on an exterior element; an important special case. Simplifying assumptions were made including monochromaticity of the source and reduction of the scatterers to a single on-axis scatter source. The fundamental objective was to introduce an interferometer system between the scatterer and focal plane that would null the scattered energy arriving at the focal plane while simultaneously preserving the focused energy from the distant point object.

A modified form of a Twyman-Green interferometer was devised to satisfy the objective. Key elements were the provision of unequal focal...
power between the arms of the interferometer and a 180-degree phase difference between superposed exiting wavefronts. Theoretical studies ensued that investigated the image structure in the vicinity of the focal plane from a diffraction point of view. Three-dimensional contour plots of the focal region were produced for various power and phase combinations of the interferometer. The plots showed the presence of a circular fringe pattern at the focal plane, with an enhanced central intensity maxima in spite of the 180-degree out-of-phase condition of the interferometer.

Far-field axial intensity distributions were investigated for the 180-degree out-of-phase condition, and for a range of focal power differences. A surprising result surfaced. For specific focal separations, the intensity modulation peaks along the optical axis were found to decrease as the inverse fourth power, rather than to follow the expected inverse square law. The unique condition appeared when the focal separation $\Delta u$ was any integer multiple of $4\pi$. Because of the sudden appearance of the effect, highly localized about the $4\pi$ values, the phenomenon was dubbed the "4$\pi$ anomaly."

A wavefront model was applied to determine the theoretical gain in signal-to-noise ratio with the introduction of the interferometer into a simple optical imaging system, with a single on-axis scatter source located on its objective lens. It was discovered that gains of up to eight orders of magnitude could be realized under certain conditions chiefly dependent on wavelength, F-number, and detector size.
An experiment was undertaken to illustrate the principles examined in the theoretical study. A modified Twyman-Green interferometer was set up to reimage a point source provided by a HeNe laser and spatial filter. Both the focal plane image and the far-field image were viewed simultaneously for the 180-degree out-of-phase condition of the interferometer. It was clearly shown that a null region existed about the optical axis for the far-field image, while the energy from the focused point image fell well within the boundary of the null region.

Conclusions

The implications of the theoretical study are clear for the simplified case considered. An interferometer of the modified Twyman-Green type described, set up in the 180-degree out-of-phase or null condition, can be introduced into a distant point reimaging system and effectively null a nearby point scatterer located on axis, while simultaneously preserving the distant point image at its focal plane. Theoretical results show that while the maximum intensity of the distant point source images is reduced by at least a factor of two, the scattered energy at the focal plane can be reduced by a margin representing a significant gain in signal-to-noise ratio of several orders of magnitude. This is made possible by a unique operating condition of the interferometer relying on a specific focal power differential between the arms. It remains to be determined as to how effective the interferometric approach is for multiple scatter sources.
With respect to the experimental demonstration, the pattern of the focused point image is shown to be contained well within the diameter of the null region, suggesting that for the simplified case of a single on-axis scatter point the technique certainly is successful, and that some latitude exists for accommodating a larger field of scatterers before the null region is diminished. It remains for an extension of the investigation to observe and measure on a parametric basis the limits of effectiveness of the technique as additional effects are introduced.

**Recommendations**

To translate the initial conceptual success of the interferometric approach into a more practical assessment of the technique, the following tasks should be undertaken in the experimental and theoretical domains:

1. The first step should be to stabilize the image at the output of the interferometer. Notwithstanding the enclosure and vibration isolation tactics employed for the experimental demonstration, interference pattern drift is experienced because of thermal convection currents induced by the experimenter's body heat and even his speech. In addition, mechanical vibration from micrometer or support adjustments during measurements cause temporary oscillations of the image which frustrate precise monitoring and measurements. Stabilization could be
accomplished by an active feedback control system linked to the piezoelectric-driven mirror in one arm of the interferometer. A null stabilizing system would require a test signal injection procedure to overcome the ambiguity inherent in this situation. A small detector sampling the intensity of the null region at the output of the imaging lens could inform the feedback electronics circuits only of the fact that there was a drift away from minimum intensity. However, the correct direction of mirror motion to compensate and correct the drift would remain unknown. The injection of a test voltage pulse of known polarity by the PZT controller would enable determination of the correct direction to drive the mirror, hence allowing the closing of the feedback loop.

(2) A crucial step to be taken is the introduction of a simulated scatter source into the experimental setup. This is easily accomplished by simply reimaging the existing point source onto the front surface of the window, which in turn could be placed at the rear focal distance from the imaging lens. The effect of lateral displacement of this single scatter source on the interference pattern generated for a null condition of the interferometer should be measured, and an estimate of null effectiveness as a function of scatterer field-of-view derived.
(3) An attempt should be made to introduce real scatter centers onto the surface of the window with the distant point source present in the system. The scatter centers should be embedded in a nonscattering matrix that could be easily attached to the window. A range of scatter center densities and matrix diameters should be considered. At this stage, the scattering surface could be illuminated by an independent source whose irradiance was made a variable. Signal-to-noise ratios should be measured at the focal plane both for the interferometer set to the null condition and for the in-phase condition. These practical advances in the experimental study should enhance the credibility of the scatter nulling scheme and serve as a transition stage for the development of a prototype device to be used in a specific situation.

(4) Experimentally observe the inverse fourth power falloff of intensity along the axis for the specific operating condition of \( \Delta = 4\pi \), as predicted in theoretical developments and illustrated in Figure 25.

(5) An attempt should be made to identify a practical goal and to devise a prototype instrument to satisfy its requirements.

(6) Two areas of concern remain to be investigated. First, the effects of incoherent objects with finite area in
reduction of the scheme's effectiveness. And second, the variation of spectral bandwidth of the source and scattering points.

(7) Investigate the self-scatter contribution of the interferometer within a telescope optical train. This may be accomplished through an APART study (Breault, 1979).
APPENDIX A

EVALUATION OF COMPLEX AMPLITUDE IN THE NEIGHBORHOOD OF FOCUS OF A CONVERGING SPHERICAL WAVE FOR FOUR SPECIAL CASES

In the geometrical shadow zone (see Figure 4) the following equations apply for the solution of $C(u,v) - iS(u,v)$ as derived in Born and Wolf (1975)

\[ C(u,v) = \frac{\cos u/2}{u/2} \cdot U_1 + \frac{\sin u/2}{u/2} \cdot U_2 \quad (|u/v| < 1) \quad (A.1) \]

\[ S(u,v) = \frac{\sin u/2}{u/2} \cdot U_1 - \frac{\cos u/2}{u/2} \cdot U_2 \]

\[ U_1 = \sum_{P=0}^{\infty} (-1)^P \left[ \frac{u}{v} \right]^{1+2P} J_{(1+2P)}(v) \quad (A.2) \]

\[ U_2 = \sum_{P=0}^{\infty} (-1)^P \left[ \frac{u}{v} \right]^{2P+2} J_{(2P+2)}(v) \quad (A.3) \]

In the illuminated region, the following equations apply

\[ C(u,v) = \frac{2}{u} \sin \frac{v^2}{2u} + \frac{\sin u/2}{u/2} + \frac{\sin u/2}{u/2} \cdot V_0 - \frac{\cos u/2}{u/2} \cdot V_1 \quad (|u/v| > 1) \quad (A.4) \]
\[ S(u,v) = \frac{2}{u} \cos \frac{v}{2u} - \cos \frac{u}{2} \frac{v}{u} - \sin \frac{u}{2} \frac{v}{u} \]

\[ V_0 = \sum_{p=0}^{\infty} (-1)^p \left( \frac{v}{u} \right)^{2p} J_{2p}(v) \quad (A.5) \]

\[ V_1 = \sum_{p=0}^{\infty} (-1)^p \left( \frac{v}{u} \right)^{1+2p} J_{(1+2p)}(v) \quad (A.6) \]

There are four special cases: (1) at the origin \( u = v = 0 \), (2) on the focal plane \( u = 0 \), (3) on the optical axis \( v = 0 \), and (4) on the line \( u = v \). For the geometric shadow region, where \(|u/v| < 1\)

\[ \lim_{u \to 0} C(u,v) = \left[ \lim_{u \to 0} \cos \frac{u}{2} \right] \left[ \lim_{u \to 0} \frac{V_1}{u} \right] + \left[ \lim_{u \to 0} \sin \frac{u}{2} \right] \left[ \lim_{u \to 0} \frac{U_2}{u} \right] \]

\[(A.7)\]

\[ \lim_{u \to 0} C(u,v) = 2 \frac{J_1(v)}{v} \quad (A.8) \]

\[ \lim_{u \to 0} S(u,v) = \left[ \lim_{u \to 0} \sin \frac{u}{2} \right] \left[ \lim_{u \to 0} \frac{V_1}{u} \right] - \left[ \lim_{u \to 0} \cos \frac{u}{2} \right] \left[ \lim_{u \to 0} \frac{U_2}{u} \right] \]

\[(A.9)\]

\[ \lim_{u \to 0} S(u,v) = 0 \quad (A.10) \]

For the illuminated region, where \(|u/v| > 1\)
\[
\lim_{v \to 0} C(u,v) = \lim_{v \to 0} \left[ \frac{2}{u} \sin \frac{v}{2u} \right] + \sin \frac{\nu}{2} \left[ \frac{\lim_{v \to 0} \nu_0}{u/2} \right] - \cos \frac{\nu}{2} \left[ \frac{\lim_{v \to 0} \nu_1}{u/2} \right]
\]

(A.11)

\[
\lim_{v \to 0} C(u,v) = \frac{\sin \frac{\nu}{2}}{u/2}
\]

(A.12)

\[
\lim_{v \to 0} S(u,v) = \lim_{v \to 0} \left[ \frac{2}{u} \cos \frac{v}{2u} \right] - \cos \frac{\nu}{2} \left[ \frac{\lim_{v \to 0} \nu_0}{u/2} \right] - \sin \frac{\nu}{2} \left[ \frac{\lim_{v \to 0} \nu_1}{u/2} \right]
\]

(A.13)

\[
\lim_{v \to 0} S(u,v) = \frac{2}{u} - \frac{1 - \cos \frac{\nu}{2}}{u/2}
\]

(A.14)

Using the results in Eqs. (A.8), (A.10), (A.12), and (A.14) in the four cases, where amplitude is normalized to unity, the special solutions are

1. At the origin \( u = v = 0 \)

\[
\lim_{u \to 0} C(u,v) = \lim_{u \to 0} \left[ \frac{\sin \frac{\nu}{2}}{u/2} \right]
\]

(A.15)

Using L'Hospital's rule

\[
\lim_{v \to 0} C(u,v) = \lim_{u \to 0} \left[ \frac{\sin \frac{\nu}{2}}{u/2} \right] = \lim_{u \to 0} \left[ \frac{1}{2} \cos \frac{\nu}{2} \right] = 1
\]

(A.16)

\[
\lim_{v \to 0} S(u,v) = \lim_{u \to 0} \left[ \frac{2}{u} - \frac{1 - \cos \frac{\nu}{2}}{u/2} \right]
\]

(A.17)

Using L'Hospital's rule
\[ \lim_{v \to 0} S(u,v) = \lim_{u \to 0} \left[ \frac{\sin u/2}{1/2} \right] = 0 \quad (A.18) \]

\[ E(0,0) = \exp\left[-i \frac{\pi}{2}\right] = -i \quad (A.19) \]

2. On the focal plane, \( u = 0, v \neq 0 \)

\[ C(0,v) = \frac{2J_1(v)}{v} ; \quad S(0,v) = 0 \quad (A.20) \]

\[ E(0,v) = \frac{i2J_1(v)}{v} \quad (A.21) \]

3. On the optical axis, \( v = 0, u \neq 0 \)

\[ C(u,0) = \frac{\sin u/2}{u/2} ; \quad S(u,0) = \frac{2}{u} - \frac{\cos u/2}{u/2} \quad (A.22) \]

\[ E(u,0) = \exp\left[i\left[\frac{\pi}{2} - i\right] u - \frac{\pi}{2}\right] \left[ \frac{\sin u/2}{u/2} - i\left[\frac{1 - \cos u/2}{u/2}\right] \right] \quad (A.23) \]

4. On the boundary of the geometric shadow, \( u = \pm v \)

\[ U1(u,u) = \frac{1}{2} \sin u \quad (A.24) \]

\[ U2(u,u) = \frac{1}{2} [J_3(u) - \cos u] \quad (A.25) \]

\[ V0(u,u) = \frac{1}{2} [J_3(u) + \cos u] \quad (A.26) \]

\[ V1(u,u) = \frac{1}{2} \sin u. \quad (A.27) \]

Substituting Eqs. (A.24) through (A.27) in Eq. (A.1) allows the solution of \( E(u,u) = E(v,v) \).
APPENDIX B

ANALYTIC SOLUTION FOR THE INTENSITY IN TERMS OF LOMMEL FUNCTIONS

A general analytic solution for the intensity $I$ generated from two superposed spherical waves with complex amplitudes $E_1$ and $E_2$ and phases $\phi_1$ and $\phi_2$ can be developed as follows:

\[ I = |E|^2 = (E_1 + E_2)(E_1^* + E_2^*) \quad (B.1) \]
\[ I = |E_1|^2 + |E_2|^2 + E_1E_2^* + E_2E_1^* \quad (B.2) \]

Let

\[ I_{\text{Total}} = I_1 + I_2 = |E_1|^2 + |E_2|^2 \quad (B.3) \]
\[ E_1 = |E_1| \exp(i\phi_1); \quad E_2 = |E_2| \exp(i\phi_2). \quad (B.4) \]

Thus

\[ I = I_1 + I_2 + |E_1| |E_2|\exp[i(\phi_1 - \phi_2)] \]
\[ + |E_1| |E_2|\exp[-i(\phi_1 - \phi_2)] \quad (B.5) \]
\[ I = I_{\text{Total}} + 2|E_1| |E_2| \cos(\phi_1 - \phi_2) \quad (B.6) \]
\[ I = I_{\text{Total}} \left[ 1 + \frac{2(I_1I_2)^{1/2}}{I_{\text{Total}}} \cos(\phi_1 - \phi_2) \right] \quad (B.7) \]

Introducing the Lommel function form for the complex amplitude as in Eq. (18), and noting that the detailed form of $C(u,v)$ and $S(u,v)$ are
given in Appendix A, let the phase be

\[ \phi = 4(F - No)^2 u - \frac{\pi}{2} + \eta \]  
(B.8)

\[ I = \left[ \exp(i\phi_1)(C_1 - iS_1) + \exp(i\phi_2)(C_2 - iS_2) \right] \]
\[ \times \left[ \exp(-i\phi_1)(C_1 + iS_1) + \exp(-i\phi_2)(C_2 + iS_2) \right] \]  
(B.9)

\[ I = [C_1^2 + S_1^2 + C_2^2 + S_2^2] + 2[C_1C_2 + S_1S_2] \cos(\phi_1 - \phi_2) \]
\[ + 2[C_2S_1 - C_1S_2] \sin(\phi_1 - \phi_2) \]  
(B.10)

Let

\[ I_{\text{Total}} = [C_1^2 + S_1^2 + C_2^2 + S_2^2] \]  
(B.11)

\[ a = C_1C_2 + S_1S_2 \]  
(B.12)

\[ b = C_2S_1 - C_1S_2 \]  
(B.13)

\[ x = \phi_1 - \phi_2 \]  
(B.14)

\[ y = \tan^{-1} \left[ \frac{b}{a} \right] \].  
(B.15)

The manipulation that results in Eq. (B.15) is as follows. Let

\[ a^2 + b^2 = a^2 + b^2(\cos^2 y + \sin^2 y). \]  
(B.16)

Then

\[ a = (a^2 + b^2)^{1/2} \cos y \]  
(B.17)

\[ b = (a^2 + b^2)^{1/2} \sin y \]  
(B.18)
\[ \tan y = \frac{b}{a}. \] 

Finally

\[ I = I_{\text{Total}} \left[ 1 + \frac{2(a^2 + b^2)^{1/2}}{I_{\text{Total}}} \cos(y - x) \right]. \]
APPENDIX C

COMPUTER PROGRAMS FOR THE GENERATION OF COMPLEX AMPLITUDE AND INTENSITY ARRAYS OF TWO SUPERPOSED CONVERGING SPHERICAL WAVES

C

C PROGRAM IN1

C PURPOSE:
C THIS PROGRAM PRODUCES A MAP OF THE INTERFERENCE PATTERN GENERATED BY THE SUPERPOSITION OF TWO CONVERGING SPHERICAL WAVES OF THE SAME F-NUMBER, WHOSE GEOMETRICAL FOCAL POINTS ALMOST COINCIDE ON THE OPTICAL AXIS. THE MAP CONSISTS OF A SECTION OF A MERIDIONAL PLANE NEAR THE SEPARATED FOCAL POINTS. THE ARRAY INT(51,101) CONTAINS THE FIELD INTENSITY AT EACH POINT ON A GRID WITHIN THE 10PI BY 5PI MAP AREA, WHERE THE DIMENSIONLESS COORDINATES ARE GIVEN BY U ALONG THE OPTICAL AXIS, AND V ALONG THE TRANSVERSE AXIS.

C FOCAL POINT SEPARATION "SEP" IS ENTERED AS A DATA ENTRY AS A MULTIPLE OF THE LEAST DIVISION ALONG EITHER AXIS 0.1PI.

C

C PROGRAM IN1(INPUT,OUTPUT,CT,FOC,TAPE6=CT,TAPE7=FOC,TAPE10=/1150)

C REAL INT1(51,101)
DIMENSION VV(51),UU(101),V(51),FOCAL(51)
COMPLEX E,E1,E2, SUP
INTEGER DEL,R,C,A,B,D
DATA SEP/3.0/,FNUM/10.0/,PI/3.1415926535898/
DATA DELU/0.1/,DELV/0.1/,R,C/51,101/,A,B,0/0,1,2/
DATA ACC,CON/1.0E-4,1.0E-8/
ETA1=0.0
ETA2=PI

LEAST INTERVAL BETWEEN POINTS ON THE U AND V
AXIS IS 0.1PI.

\[
\begin{align*}
DV &= DEL*PI \\
DU &= DELU*PI \\
\end{align*}
\]

INITIALIZE AND PLACE COORDINATE VALUES OF V IN ARRAY V(I).

```
DO 10 I=1,R
DO 11 J=1,C
11 INIT(I,J)=0.0
10 CONTINUE
UU(1)=0.0
U=0.0
VI=0.0
UU1=0.0
UU2=0.0
W=0.0
X=0.0
VV(1)=0.0
V(1)=0.0
DO 20 I=2,R
20 V(I)=V(I-1)+DV
E=CMPLX(0.0,0.0)
E1=CMPLX(0.0,0.0)
E2=CMPLX(0.0,0.0)
SUP=CMPLX(0.0,0.0)
DO 21 I=1,R
21 VV(I)=V(I)/PI
DO 22 J=2,C
22 UU(J)=UU(J-1)+DELU
DO 23 I=1,R
23 FOCAL(I)=0.0
```

CHECK WHETHER THE ODD OR EVEN ALGORITHM APPLIES

```
w=SEP/(DELU+.00001)
n=IFIX(w)
x=(FLOAT(N))/2.0
DEL=IFIX(x)
CC=x-FLOAT(DEL)
IF(CC.EQ.0.0)30,31
```

THIS IS THE "ODD" ALGORITHM; IT APPLIES ONLY IF
DELU=0.2PI.

```
31 L=DEL+1
DO 140 J=1,C
```
CHECK IF U2 IS OUT OF THE NEGATIVE REGION

IF(L.EQ.0)150,151
151 UU1=(SEP/2.0+(J-1)*DELU)*PI
        UU2=-(SEP/2.0-(J-1)*DELU)*PI
        DO 141 I=1,R

FIND THE COMPLEX AMPLITUDES E1 AND E2

        VI=V(I)
        U=UU1
        ETA=ETA1
        CALL ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
        E1=E
        U=UU2
        ETA=ETA2
        CALL ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
        E2=E
        SUP=E1+E2
        INT1(I,J)=(CABS(SUP)**2.0

IF POINT LIES ON SYMMETRY PLANE, THEN STORE INTENSITY IN THE ARRAY FOCAL(I).

IF(J.EQ.1)2001,141
2001 FOCAL(I)=INT1(I,J)
141 CONTINUE
        L=L-1
        GO TO 140

FIND THE COMPLEX AMPLITUDES E1 AND E2

        VI=V(I)
        U=UU1
        ETA=ETA1
        CALL ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
        E1=E
        U=UU2
        ETA=ETA2
        CALL ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
        E2=E
        SUP=E1+E2
        INT1(I,J)=(CABS(SUP)**2.0
CONTINUE
GO TO 900

THIS IS THE "EVEN" ALGORITHM. IF DELU=0.1, THE
PROGRAM WILL ALWAYS USE THIS BRANCH.

30 L=DEL+1
DO 40 J=1,C

CHECK IF U2 IS OUT OF THE NEGATIVE REGION.

IF(L.EQ.0)50,51
51 UU1=(SEP/2.0+(J-1)*DELU)*PI
UU2=-(SEP/2.0-(J-1)*DELU)*PI

DO 41 I=1,R

FIND THE COMPLEX AMPLITUDES E1 AND E2

VI=V(I)
U=UU1
ETA=ETA1
CALL ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
E1=E
U=UU2
ETA=ETA2
CALL ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
E2=E
SUP=E1+E2
INT1(I,J)=(CABS(SUP))**2.0

IF POINT LIES ON THE SYMMETRY PLANE, THEN
STORE INTENSITY IN THE ARRAY FOCAL(51).

IF(J.EQ.1)1001,41
1001 FOCAL(I)=INT1(I,J)

41 CONTINUE
L=L-1
GO TO 40

50 UU1=(SEP/2.0+(J-1)*DELU)*PI
UU2=(J-(DEL+1))*DELU*PI

DO 42 I=1,R

FIND COMPLEX AMPLITUDES E1 AND E2
C

VI=V(I)
U=UU1
ETA=ETA1
CALL ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
E1=E
U=UU2
ETA=ETA2
CALL ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
E2=E
SUP=E1+E2
INTI(I,J)=(CABS(SUP))**2.0
42 CONTINUE
40 CONTINUE

900 DO 100 I=1,R
101 J=1,C
100 CONTINUE

99 WRITE(10,102)INTI(I,J)
102 FORMAT(101(I,X,E10.4))

99 WRITE(7,400)FOCAL
400 FORMAT(lH ,5C5X,E10.4)

300 FORMAT(lH ,F4.2,51X,19HVALUES OF INTENSITY,/,48X,
130HPOR FOCAL POINT SEPARATION OF, F4.2,2HPI,/,52X,
227HAND PHASE SHIFT 180 DEGREES,/,52X,27(1H*))
K=4
DO 201 J=1,C
JJ=K-4
IF(JJ.EQ.0)202,203
202 WRITE(6,204)
204 FORMAT(lH ,)
WRITE(6,205)J,UU(J)
205 FORMAT(1H ,I3,6H:U=,F5.2,///4X,4HV/PI,///4X,
14(1H*))
DO 206 I=1,46,5
206 WRITE(6,207)VV(I),INTI(I,J),INTI(I+1,J),
INTI(I+2,J),INTI(I+3,J),INTI(I+4,J)
207 FORMAT(lH ,3X,F4.1,5(5H *=,E10.4))
WRITE(6,208)VV(R),INTI(R,J)
208 FORMAT(lH ,3X,F4.1,5H *=,E10.4,///4X,80(1H*))
K=1
GO TO 201
203 WRITE(6,205)J,UU(J)
DO 209 I=1,46,5
209 WRITE(6,207)VV(I),INTI(I,J),INTI(I+1,J),
INT1(I+2,J), INT1(I+3,J), INT1(I+4,J)
WRITE(6,208)VV(R), INT1(R,J)
K=K+1
201 CONTINUE
STOP
END

---

SUBROUTINE ECALC(U,VI,ETA,PI,ACC,CON,A,B,D,FNUM,E)
---

C THIS SUBROUTINE CALCULATES THE COMPLEX AMPLITUDE E AT A GIVEN POINT (U,VI) FOR SPECIFIED F-NUMBER FNUM, AND PHASE FACTOR ETA.
C
COMPLEX E,E1,E2,SUP,P,Z,TEMPC1
SI(X)=(SIN(X/2.0))/(X/2.0)
CO(X)=(COS(X/2.0))/(X/2.0)
PHI(X)=4.0*X*FNUM**2.0-(PI/2.0)

TO CALCULATE THE LOMMEL FUNCTIONS (U1,U2,V0,V1) WE MUST FIRST TEST FOR THE LOCATION OF POINT (U,VI) IN ONE OF THREE REGIONS: REGION A(ABS(U/VI)<1) IS THE SHADOW REGION, REGION B(ABS(U/VI)>1), AND THE LINE U=V.

BJ=0.0
IER=0
UUU=ABS(U)
IF(UUU-VI)102,103,104

CHECK TO SEE IF POINT LIES ON FOCAL PLANE LINE U=V

102 IF(U.EQ.0.0)105,106

USE E(0,VI)=C(0,VI)*EXP[i*(0-PI/2 +ETA)]=-IM(2J1(V)/V)

105 CALL BESJ(VI,1,BJ,ACC,IER)
TEMP=2.0*BJ/VI
PSI=PI/2.0+ETA
TEMPC1=CEXP(CMPLX(0.0,PSI))
E=TEMPC1*TEMP
GO TO 98
C CALCULATE LOMMEL FUNCTIONS U1, U2 AND FUNCTIONS C(U, V), S(U, V) LEADING TO THE DETERMINATION OF E(U, V) = EXP[IM(\Phi + \Theta)]/[C + IM(S)]

C CALCULATE U1 AND TEST FOR CONVERGENCE

106 X = U/VI
CALL BESJ(VI, 1, BJ, ACC, IER)
U1 = X * BJ
K = 0
DO 107 N = 3, 100, 2
K = K + 1
BJ = 0.0
IER = 0
Z1 = X**N
CALL BESJ(VI, N, BJ, ACC, IER)
W1 = Z1 * BJ
UTEMP1 = U1 + W1 * ((-1)**K)

C CHECK FOR CONVERGENCE OF LOMMEL FUNCTION U1

DELTA = ABS(U1 - UTEMP1)
IF(DELTA.GT.CON) 108, 109
108 U1 = UTEMP1
107 CONTINUE
WRITE(6, 900) B, U1, VI
900 FORMAT(1H0, 12HSERIES FOR U, I1, 16HHAS NOT CONVERGE 122D; CALCULATION EXCEEDS 19HTHE NUMBER OF TERMS
223HALLOWED BY THE DO LOOP., 6HLAST U, I1, 3H = , E20.14, 3H/2H U, I1, 3H = , E20.14, 3H V(I2, 2H) = , E20.14)
STOP
109 U1 = UTEMP1

C CALCULATE U2 AND TEST FOR CONVERGENCE

BJ = 0.0
IER = 0
CALL BESJ(VI, 2, BJ, ACC, IER)
U2 = (X**2) * BJ
K = 0
DO 110 N = 3, 100, 2
K = K + 1
BJ = 0.0
IER = 0
Z2 = X**N
CALL BESJ(VI, N, BJ, ACC, IER)
W2 = Z2 * BJ
UTEMP2 = U2 + W2 * ((-1)**K)

CHECK FOR CONVERGENCE OF LOMMEL FUNCTION U2

DELTA = ABS(U2 - UTEMP2)
IF(DELTA.GT.CON)CONTINUE
111 U2 = UTEMP2
110 CONTINUE
WRITE(6,900) O, D, U2, U, I, VI
STOP
112 U2 = UTEMP2

CALCULATE AMPLITUDE E(U, Y) = EXP[I*(PHI+ETA)]*(C-I*S)

PHASE = PHI(U) + ETA
P = CEXP(CMPLX(0.0, PHASE))
S = SI(U)*U1 - CO(U)*U2
C1 = CO(U)*U1 + SI(U)*U2
Z = CMPLX(C1, -S)
E = P*Z
GO TO 96
103 IF(U.EQ.0.0) 600, 601

CALCULATE E(0, 0) = EXP[I*(PHI+ETA)]

600 PSI = PI/2.0 + ETA
E = CEXP(CMPLX(0.0, PSI))
GO TO 98

FOR U=V AND NEITHER EQUAL TO ZERO WE CALCULATE E FROM: E = EXP[I*(PHI+ETA)]*(C-I*S), WHERE WE USE U1 = 0.5*SIN(U) AND U2 = 0.5*(JO(ABS(U)) - COS(U)) IN THE EXPRESSION FOR S AND C.

601 BJ = 0.0
IER = 0
U1 = (SIN(U))/2.0
U = -U
CALL BESJ(U, 0, BJ, ACC, IER)
U = -U
U2 = (BJ - COS(U))/2.0
PHASE = PHI(U) + ETA
P = CEXP(CMPLX(0.0, PHASE))
C1 = CO(U)*U1 + SI(U)*U2
S = SI(U)*U1 - CO(U)*U2
Z = CMPLX(C1, -S)
E = P*Z
GO TO 98
104 IF(VI.EQ.0.0)113,114
113 WHERE C=EXP[(PHI+ETA)]*[C-IM(S)]
WHEN C=EXP[(U-0.5*U)/U] AND
S=(2.0/U)-(COS(0.5*U))/(0.5*U)
114 WHERE C=EXP[(PHI+ETA)]*[C-IM(S)]
115 WHERE C=EXP[(PHI+ETA)]*[C-IM(S)]
116 WHERE C=EXP[(PHI+ETA)]*[C-IM(S)]
117 WHERE C=EXP[(PHI+ETA)]*[C-IM(S)]
117 VO=VTEMPO

CALCULATE V1 AND TEST FOR CONVERGENCE

BJ=0.0
IER=0
CALL BESJ(VI,1,BJ,ACC,IER)
Vl=V*BJ
K=0
DO 118 N=3,100,2
K=K+1
SJ=0.0
IER=0
AI=Y**N
CALL BESJ(VI,N,BJ,ACC,IER)
81=AI*BJ
VTEMPl=Vl+81*((-1)**K)

CHECK FOR CONVERGENCE OF LOMMEL FUNCTION VI

DELTA=ABS(VI-VTEMPl)
IF(Delta.GT.CON)119,120
119 Vl=VTEMPl
118 CONTINUE
WRITE(6,901)B,B,Vl,U,I,VI
STOP
120 Vl=VTEMPl

CALCULATE AMPLITUDE E=EXP[i*(PHI+ETA)]*[C-i*S]

PHASE*PHI(U)+ETA
P=EXP(CMPLX(0.0,PHASE))
QSI=(2.0/U)*SIN(VI**2.0/(2.0*U))
QCO=(2.0/U)*COS(VI**2.0/(2.0*U))
Cl=QSI+SI(U)*VO-CO(U)*Vl
S=QCO-CO(U)*VO-SI(U)*Vl
Z=CMPLX(Cl,-S)
E=P*Z
98 T=SECOND(CP)
IF(T.GE.200.00)99,100
99 STOP
100 RETURN
END
SUBROUTINE BESJ(X,N,BJ,D,IER)


BJ=0.
IF(N)10,20,20
10 IER=1
RETURN
20 IF(X)30,30,31
30 IER=2
RETURN
31 IF(X-15.)32,32,34
32 NTEST=20.+10.*X-X**2/3
GO TO 36
34 NTEST=90.+.X/2
36 IF(N-NTEST)40,38,38
38 IER=4
RETURN
40 IER=0
N1=N+1
BPREV=.O

COMPUTE STARTING VALUE OF M

IF(X-5.)50,60,60
50 MA=X+6.
GO TO 70
60 MA=1.4*X+60./X
70 MB=N+1*FIX(X)/4+2
MZERO=MAXO(MA,MB)

SET UPPER LIMIT OF M

MMAX=NTEST
100 GO 190 M=MZERO,MMAX,3

SET F(M),F(M-1)
FM1=1.0E-28
FM=.0
ALPHA=.0
IF(M-(M/2)*2)120,110,120
110 JT=-1
GO TO 130
120 JT=1
130 M2=M-2
DO 160 K=1,M2
MK=M-K
BMK=2.*FLOAT(MK)*FM1/X-FM
FM=FM1
FM1=BMK
IF(MK-N-1)150,140,150
140 BJ=BMK
150 JT=-JT
S=1+JT
160 ALPHA=ALPHA+BMK*S
BMK=2.*FM1/X-FM
IF(N)180,170,160
170 BJ=BMK
180 ALPHA=ALPHA+BMK
BJ=BJ/ALPHA
IF(ABS(BJ-BPREV)-ABS(D*BJ))200,190,200
190 BPREV=BJ
IER=3
200 RETURN
END
APPENDIX D

SUPPLEMENTARY ISO-INTENSITY CONTOUR PLOTS
WITH RELATED CROSS-SECTIONS AND AXIAL PLOTS
Fig. 40. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition In-Phase; No Focal Separation, F/10.
Fig. 41. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 0.2\pi$, F/10.
Fig. 42. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 0.6\pi$, F/10.
Fig. 43. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 0.8\pi \), F/10.
Fig. 44. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with π Phase Shift; Focal Separation ΔU = 1.0π, F/10.
Fig. 45. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 3.2\pi$, F/10.
Fig. 46. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 3.4\pi \), F/10.
Fig. 47. Iso-Intensity Contour Plot Near Focus: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 3.6\pi$, F/10.
Fig. 48. Intensity Profile Along the Optical Axis: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 0.2\pi \), F/10.
Fig. 49. Intensity Profile Along the Optical Axis: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 0.6\pi \), F/10.
Fig. 50. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 0.8\pi$, F/10.
Fig. 51. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 3.0\pi$, F/10.
Fig. 52. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 3.6\pi$, F/10.
Fig. 53. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta u = 4.2\pi$, F/10.
Fig. 54. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 20.0\pi$, F/10.
Fig. 55. Intensity Profile Along the Optical Axis: Two Wave Superposition with $\pi$ Phase Shift; Focal Separation $\Delta U = 50.0\pi$, F/10.
Fig. 56. Intensity Profile Along the Optical Axis: Two Wave Superposition with \( \pi \) Phase Shift; Focal Separation \( \Delta U = 70.0\pi \), F/10.
APPENDIX E

ANALYTIC SOLUTION FOR THE INTENSITY ALONG THE OPTICAL AXIS OF TWO SUPERPOSED COHERENT SPHERICAL WAVES

With reference to Figure 57, one can observe two converging axially symmetric spherical waves of common origin with complex amplitude $E_1$ and $E_2$ respectively, with geometrical focal points on axis of $0_1$ and $0_2$ separated by a dimensionless distance $\Delta$. If $z$ is a dimensioned distance along the optical axis, then referring to Eq. (3) $u$ is a dimensionless coordinate given by

$$u = \left[ \frac{2\pi}{\lambda} \right] \left[ \frac{1}{4F^2} \right] z,$$

(E.1)

where $\lambda$ is the wavelength, and $F$ the F-number of the wave as depicted in Figure 3. If $u_1$ is the coordinate of the wave $E_1$ with origin at $0_1$, and $u_2$ is the coordinate of the wave $E_2$ with origin at $0_2$, we can define a common coordinate system along the optical axis with origin at $0$, midway between $0_1$ and $0_2$, where

$$u_1 = u' + \frac{\Delta}{2}$$

(E.2)

$$u_2 = u' - \frac{\Delta}{2}$$

(E.3)
Figure 57. Co-axially Symmetric Converging Spherical Waves with Foci Separation $\Delta$, and their Common Coordinate Axis $u'$
Because $E_1$ and $E_2$ have a common origin, from the coherent point source in an interferometer, it is assumed as a special case in the following derivations that a fixed phase difference of $\pi$ has been engineered between the waves at the output of the interferometer. This is represented by the phase factors $\eta_1 = 0$ and $\eta_2 = \pi$. Recall from Appendix B that

\[ E = (C - iS) \exp(i\phi) \] (E.5)

and

\[ \phi = 4F^2u - \frac{\pi}{2} + \eta, \] (E.6)

thus

\[ x = \phi_1 - \phi_2 = 4F^2\Delta - \pi. \] (E.7)

The general solution for the intensity due to two superposed spherical waves has already been presented in Appendix B as

\[ I = I_{\text{Total}} \left[ 1 + \frac{2(a^2 + b^2)}{I_{\text{Total}}} \cos(y - x) \right]. \] (E.8)

Note that

\[ (I_1I_2)^{1/2} = (a^2 + b^2)^{1/2}. \] (E.9)

Recall that

\[ u_2 = u' - \frac{\Delta}{2} \] (E.3)

\[ \Delta = u_1 - u_2. \] (E.4)
\[ I_1 = |E_1|^2 \]  
(E.10)

\[ I_1 = (C_1 - iS_1)(C_1 + iS_1) = C_1^2 + S_1^2 \]  
(E.11)

Similarly,

\[ I_2 = (C_2 - iS_2)(C_2 + iS_2) = C_2^2 + S_2^2 \]  
(E.12)

\[ I_{\text{Total}} = I_1 + I_1 \]  
(E.13)

And \( a, b, \) and \( y \) are as in Eqs. (B.12), (B.13), and (B.15), respectively.

For this specific case along the optical axis, the functions \( C(u,0) \) and \( V(u,0) \) are given by (refer to Appendix A)

\[ C_1 = \frac{\sin(u_1/2)}{u_1/2} ; \quad C_2 = \frac{\sin(u_2/2)}{u_2/2} \]  
(E.14)

\[ S_1 = \frac{1 - \cos(u_1/2)}{u_1/2} ; \quad S_2 = \frac{1 - \cos(u_2/2)}{u_2/2} . \]  
(E.15)

With appropriate transformations to the \( u' \) coordinate system, the intensity is completely defined in terms of \( u' \) and \( \Delta \).

The log-log plots of intensity as a function of \( u' \) for various values of \( \Delta \) are presented in Figures 21 to 29. A remarkable feature is observed at \( \Delta = 4n\pi \), where \( n \) is any positive integer. The slope of the envelope of the intensity function is seen to increase dramatically and rapidly from -2 for all other values of \( \Delta \), to -4 for the region \( u' > 2\pi \). This increased intensity falloff as \( 1/u' \) for the specific focal point separation condition has significant implication for the reduction of scatter source intensity within the null region as discussed in Chapter 2.
In the following, the intensity function is examined analytically for the special case of \( \Delta = 4n\pi \).

**The Cosine Term**

It is convenient to focus first on the cosine term to see if simplifications are possible upon expansion in terms of \( u' \) and \( \Delta \)

\[
\cos(y - x) = \cos\left[\tan^{-1}\left(\frac{b}{a}\right) - (4F^2\Delta - \pi)\right]. \tag{E.16}
\]

Let

\[
Q = \frac{u' + \Delta/2}{2} = \frac{u_1}{2} \tag{E.17}
\]

\[
P = \frac{u' - \Delta/2}{2} = \frac{u_2}{2} \tag{E.18}
\]

\[
Q - P = \frac{\Delta}{2}; \quad P = Q - \frac{\Delta}{2} \tag{E.19}
\]

Then

\[
C_1 = \frac{\sin Q}{Q}; \quad C_2 = \frac{\sin P}{P} \tag{E.20}
\]

\[
S_1 = \frac{1 - \cos Q}{Q}; \quad S_2 = \frac{1 - \cos P}{P} \tag{E.21}
\]

Recall from Eq. (B.12) and (B.13)

\[
a = C_1C_2 + S_1S_2
\]

\[
= \frac{1}{PQ} [\sin Q \sin P + \cos Q \cos P + 1 - \cos Q - \cos P]
\]

\[
= \frac{1}{PQ} [\cos(Q - P) + 1 - \cos Q - \cos P] \tag{E.22}
\]
\[ b = C_2S_1 - C_1S_2 \]
\[ = \frac{1}{PQ} [\sin Q \cos P - \sin P \cos Q + \sin P - \sin Q] \]
\[ = \frac{1}{PQ} [\sin(Q - P) + \sin P - \sin Q]. \quad \text{(E.23)} \]

In terms of \( \Delta \) and \( u' \)
\[
(y - x) = \tan^{-1} \left[ \frac{\sin \left[ \frac{\Delta}{2} \right] + \sin \left[ \frac{u'}{2} - \frac{\Delta}{4} \right] - \sin \left[ \frac{u'}{2} + \frac{\Delta}{4} \right]}{1 + \cos \left[ \frac{\Delta}{2} \right] - \cos \left[ \frac{u'}{2} + \frac{\Delta}{4} \right] - \cos \left[ \frac{u'}{2} - \frac{\Delta}{4} \right]} \right] - [4F^2\Delta - \pi]. \quad \text{(E.24)}
\]

For the special case of \( \Delta = 4n\pi, n = 1, 2, 3, \ldots \)
\[
(y - x) = \tan^{-1} \left[ \frac{\sin(2n\pi) + \sin \left[ \frac{u'}{2} - n\pi \right] - \sin \left[ \frac{u'}{2} + n\pi \right]}{1 + \cos(2n\pi) - \cos \left[ \frac{u'}{2} + n\pi \right] - \cos \left[ \frac{u'}{2} - n\pi \right]} \right] - [16F^2n\pi - \pi]. \quad \text{(E.25)}
\]
\[
(y - x) = \tan^{-1} \left[ \frac{\sin \left[ \frac{u'}{2} - n\pi \right] - \sin \left[ \frac{u'}{2} + n\pi \right]}{2 - \cos \left[ \frac{u'}{2} + n\pi \right] - \cos \left[ \frac{u'}{2} - n\pi \right]} \right] - [(16F^2n - 1)\pi]. \quad \text{(E.26)}
\]

For \( n \) an odd integer, let \( n = (2m - 1) \), with \( m = 1, 2, 3, \ldots \)
\[
(y - x) = \tan^{-1} \left[ \frac{\sin \left( \frac{\mu'}{2} - (2m - 1)\pi \right) - \sin \left( \frac{\mu'}{2} + (2m - 1)\pi \right)}{2 - \cos \left( \frac{\mu'}{2} + (2m - 1)\pi \right) - \cos \left( \frac{\mu'}{2} - (2m - 1)\pi \right)} \right]
- [16F^2(2m - 1)\pi - \pi]. \quad (E.27)
\]

\[
= \tan^{-1} \left[ \frac{-\sin \frac{\mu'}{2} + \sin \frac{\mu'}{2}}{2 \left[ 1 + \cos \frac{\mu'}{2} \right]} \right] - [16F^2(2m - 1)\pi - \pi] \quad (E.28)
\]

\[
= -\pi [16F^2(2m - 1) - 1] = -\pi \xi. \quad (E.29)
\]

Note that if \( F \) is an integer, \( \xi \) is always an odd integer. Thus, for the special case of \( \Delta = 4n\pi, n=1,3,5,\ldots(2m-1) \)

\[
\cos(y - x) = \cos(-2\pi) = \cos(n\pi) = -1. \quad (E.30)
\]

For \( n \) an even integer, let \( n = 2m \), with \( m = 1, 2, 3, \ldots \)

\[
(y - x) = \tan^{-1} \left[ \frac{\sin \left( \frac{\mu'}{2} - 2m\pi \right) - \sin \left( \frac{\mu'}{2} + 2m\pi \right)}{2 - \cos \left( \frac{\mu'}{2} + 2m\pi \right) - \cos \left( \frac{\mu'}{2} - 2m\pi \right)} \right]
- [16F^2(2m)\pi - \pi] \quad (E.31)
\]

\[
(y - x) = \tan^{-1} \left[ \frac{\sin \frac{\mu'}{2} - \sin \frac{\mu'}{2}}{2 \left[ 1 - \cos \frac{\mu'}{2} \right]} \right] - [(16F^2(2m) - 1)\pi] \quad (E.32)
\]
\[(y - x) = -\pi[16F^2(2m) - 1]. \quad (E.33)\]

And similarly with \(F\) an integer, \([16F^2(2m) - 1]\) will always be an odd integer. Thus, for \(\Delta = 4n\pi\), \(n = 2, 4, 6 \ldots (2m)\)

\[
\cos(y - x) = \cos[-\pi(16F^2n - 1)] = -1 \quad \text{(for } F \text{ an integer)}. \quad (E.34)
\]

**The Intensity Function at \(\Delta = 4n\pi\)**

Under the condition of Eq. (E.34), the intensity is

\[
I = I_1 + I_2 + 2(a^2 + b^2)^{1/2} \cos[-\pi(16F^2n - 1)]
\]

\[
= I_1 + I_2 - 2(I_1I_2)^{1/2}
\]

\[
= (\sqrt{I_1} - \sqrt{I_2})^2. \quad (E.35)
\]

Recall from Eqs. (E.8) through (E.12), (E.20) and (E.21)

\[
I = \frac{2(1-\cos Q)}{Q^2} + \frac{2(1-\cos P)}{P^2} + 2 \sqrt{\frac{2(1-\cos Q)(1-\cos P)}{Q^2P^2}} \cdot \cos(y-x). \quad (E.36)
\]

In an alternative form, using the relationship \(1 - \cos 2x = 2 \sin^2 x\)

\[
I = \sin^2 \frac{Q}{2} + \sin^2 \frac{P}{2} + 2 \sin \frac{Q}{2} \sin \frac{P}{2} \cdot \cos(y-x), \quad (E.37)
\]
or in an expanded form, including \( \cos(y-x) = -1 \)

\[
I = \frac{\sin^2\frac{u' + \Delta}{8}}{\frac{u' + \Delta}{8}} + \frac{\sin^2\frac{u' - \Delta}{8}}{\frac{u' - \Delta}{8}}
- \frac{2 \sin\frac{u' + \Delta}{8}}{\frac{u' + \Delta}{8}} \cdot \frac{\sin\frac{u' - \Delta}{8}}{\frac{u' - \Delta}{8}}
\]

(E.38)

Examining \( I \) at some key values of \( u' \) for \( \Delta = 4n\pi \), let \( u' = 0 \). Then

\[
I\bigg|_{u'=0} = \frac{\sin^2\frac{n\pi}{2}}{\frac{n\pi}{2}^2} + \frac{\sin^2\frac{-n\pi}{2}}{\frac{-n\pi}{2}^2}
- \frac{2 \sin\frac{n\pi}{2} \cdot \sin\frac{-n\pi}{2}}{-\frac{n\pi}{2}^2}
\]

(E.39)

For \( n \) an odd integer, \( \sin\left(\frac{n\pi}{2}\right) = 1 = \sin\left(\frac{-n\pi}{2}\right) \)

\[
I\bigg|_{u'=0} = \frac{4}{(n\pi)^2} + \frac{4}{(n\pi)^2} + \frac{2 \cdot 4}{(n\pi)^2} = 1.61
\]

(E.40)

For \( n \) an even integer, \( \sin\left(\frac{n\pi}{2}\right) = \sin(-n\pi) = 0 \), and
Let $u' = 2\pi$. Then

$$I_{u'=0} = 0.$$  \hspace{1cm} (E.41)

$$I_{u'=2\pi} = \frac{2[1 - \cos(\pi + n\pi)]}{(\pi + n\pi)^2} + \frac{2[1 - \cos(\pi - n\pi)]}{(\pi - n\pi)^2} - \frac{4[1 - \cos(\pi + n\pi)][1 - \cos(\pi - n\pi)]^{1/2}}{[(\pi + n\pi)^2(\pi - n\pi)^2]^{1/2}}. \hspace{1cm} (E.42)$$

For $n = 1$, the first term is zero, but the second and third terms are indeterminate as zero divided by zero. Therefore, L'Hospital's rule is invoked as follows:

$$I_{u'=2\pi} = \lim_{u' \to 2\pi} \frac{2\alpha [1 - \cos\left(\frac{u'}{2} - \pi\right)]}{\alpha\left[\frac{u'}{2} - \pi\right]^2} - \lim_{u' \to 2\pi} \frac{4\alpha [1 - \cos\left(\frac{u'}{2} + \pi\right)][1 - \cos\left(\frac{u'}{2} - \pi\right)]^{1/2}}{\alpha\left[\frac{u'}{2} + \pi\right]^2\left[\frac{u'}{2} - \pi\right]^2]^{1/2}}.$$  \hspace{1cm} (E.43)

$$= \lim_{u' \to 2\pi} \frac{2\alpha [1 + \cos\left(\frac{u'}{2}\right)]}{\alpha\left[\frac{u'}{2} - \pi\right]^2} - \lim_{u' \to 2\pi} \frac{4\alpha [1 + \cos\left(\frac{u'}{2}\right)]}{\alpha\left[\frac{u'^2}{4} - \pi^2\right]}.$$  \hspace{1cm} (E.44)
\[ \lim_{u' \to 2\pi} \frac{-\sin \left( \frac{u'}{2} \right)}{u' - \pi} - \lim_{u' \to 2\pi} \frac{-\frac{1}{2} \sin \left( \frac{u'}{2} \right)}{u' + i} = \left( E.45 \right) \]

\[ = \lim_{u' \to 2\pi} \frac{-\frac{1}{2} \cos \left( \frac{u'}{2} \right)}{1/2} = 1. \quad \left( E.46 \right) \]

It can be shown that for \( n > 1 \) and \( n \) an even integer, where \( n = 2m, \ m = 1, 2, 3, \ldots \)

\[ I \bigg|_{u' = 2\pi} = \left[ \frac{4}{(2m + 1)^2 \pi^2} \right]^2. \quad \left( E.47 \right) \]

However, for \( n > 1 \) and \( n \) an odd integer, where \( n = (2m - 1), \ m = 1, 2, 3, \ldots \)

\[ I \bigg|_{u' = 2\pi} = 0. \quad \left( E.48 \right) \]

A more compact form of the intensity equation allows further insights at \( \Delta = 4n\pi \) and \( u' > 2\pi \)

\[ I = \frac{(2m - 1)^2 8\pi^2 \left[ 1 + \cos \frac{u'}{2} \right]}{\left[ \frac{u'^2}{4} - (2m - 1)^2 \pi^2 \right]^2}. \quad \left( E.49 \right) \]

For \( n \) an even integer, or \( n = 2m, \ m = 1, 2, 3, \ldots \), and \( u' > 2\pi \)
With reference to Figure 25, and Eqs. (E.49) and (E.50), the envelope of the intensity function in the region \( u' > 2\pi \) is given by \( g(u') \), where

\[
g(u') = \frac{128(\pi)^2}{u'^* - 8(\pi)^2u'^2 + 16(\pi)^4}.
\]  

(E.51)

If one examines the slope of \( g(u') \) for \( u' > 2\pi \) with respect to the log-log scales in Figures 21 to 29, let the slope be \( S \), where

\[
S = \left. \frac{\partial \log g(u')}{\partial \log u'} \right|_{u' = u'^*} = \frac{3(\log 128 - \log u'^*) - 8(\pi)^2u'^2 + 16(\pi)^4)}{3 \log u'}.
\]

(E.52)

For \( u' > 2\pi \) the terms in \( u'^2 \) and \( 16(\pi)^4 \) become negligible, and the slope reduces to

\[
S = \left. \frac{\partial \log g(u')}{\partial \log u'} \right|_{u' = u'^*} = -4.
\]

(E.53)

The slope of the envelope of intensity implies a falloff of the maxima
in intensity away from the origin as the inverse of the fourth power of distance. In comparison, the intensity falloff from a point source follows the inverse square law.

For the region $u' < 2\pi$ in the intensity function, let us examine Eq. (E.36) for the specific case where $n = 1$, corresponding to $A = 8\pi$ as in Figure 26

$\begin{align*}
I &= \frac{2\left[1 + \cos \frac{u'}{2}\right]}{\left[\frac{u'}{2} + \pi\right]^2} + \frac{2\left[1 + \cos \frac{u'}{2}\right]}{\left[\frac{u'}{2} - \pi\right]^2} \\
&\quad - \frac{4\left[1 + \cos \frac{u'}{2}\right]^{1/2}}{\left[\frac{u'}{2} + \pi\right]^2\left[\frac{u'}{2} - \pi\right]^{1/2}}.
\end{align*}$

(E.54)

Since $u' < 2\pi$

$\begin{align*}
I &= 2\left[1 + \cos \frac{u'}{2}\right] \\
&\times \left[\frac{1}{\left[\frac{u'}{2} + \pi\right]^2} + \frac{1}{\left[\frac{u'}{2} - \pi\right]^2} + \frac{2}{\left[\frac{u'}{2} + \pi\right] \left[\frac{u'}{2} - \pi\right]}\right],
\end{align*}$

(E.55)

which reduces to

$\begin{align*}
I &= \frac{\left[2\left[1 + \cos \frac{u'}{2}\right]u' \right]^2}{\left[u'\frac{4}{2} - \pi^2\right]}
\end{align*}$

(E.56)
Expanding $\cos u'/2$ in a power series, as

$$
\cos \frac{u'}{2} = 1 - \frac{(u'/2)^2}{2!} + \frac{(u'/2)^4}{4!} \cdots
$$

(E.57)

and neglecting terms in $u'^{n}$ and higher powers

$$
I = \frac{3u'^2}{2\pi^2 - \pi^2u'^2}.
$$

(E.58)

Examining the slope $S$ on the log-log scale

$$
S = \frac{3[(\log 8 + 2 \log u') - \log(2\pi^2 - \pi^2u'^2)]}{3 \log u'}
$$

(E.59)

Again, for $u' \ll 2\pi$ the $\pi^2u'^2$ term may be neglected, which results in

$$
S = \frac{3[\log 8 + 2 \log u' - \log(2\pi^2)]}{3 \log u'} = 2.
$$

(E.60)

From Figures 21 to 29, it can be easily seen that at all values of $\Delta$ other than $\Delta = 4n\pi$, the limiting slope for $u' > 2\pi$ is $-2$.

Maxima and minima for the region of the intensity function $u' > 2\pi$ and for the special case of $\Delta = 4n\pi$ are separated in all cases $n = 1, 2, 3, \ldots$ by $4\pi$ along the optical axis. This can be seen by taking the derivative of Eq. (E.49) and (E.50) and setting the result equal to zero.
The result for Eq. (E.49) is

$$\frac{1}{2} \sin \frac{u'}{2} \left[ \frac{u'^2}{4} - (2m - 1)^2 \pi^2 \right]^3 = -u \left[ 1 + \cos \frac{u'}{2} \right] \left[ \frac{u'^2}{4} - (2m - 1)^2 \pi^2 \right]^3, \quad (E.61)$$

and for Eq. (E.50)

$$\frac{1}{2} \sin \frac{u'}{2} \left[ \frac{u'^2}{4} - (2m)^2 \pi^2 \right]^3 = u \left[ 1 - \cos \frac{u'}{2} \right] \left[ \frac{u'^2}{4} - (2m)^2 \pi^2 \right]^3. \quad (E.62)$$

A solution is found where

$$\frac{u'^2}{4} = (2m - 1)^2 \pi^2, \quad \text{for } n \text{ as an odd integer} \quad (E.63)$$

and

$$\frac{u'^2}{4} = (2m)^2 \pi^2, \quad \text{for } n \text{ as an even integer}. \quad (E.64)$$

Thus, for $\Delta = 4n\pi$, where $n$ is an odd integer, maxima and minima exist at

$$u' = (4m - 2)\pi, \quad m = 1, 2, 3, \ldots \quad \text{and at } u' = 4m\pi, \quad m = 1, 2, 3, \ldots \quad \text{for } n \text{ is an even integer.}$$

The maxima and minima may be specifically identified by solving for the sign of the second derivative of intensity with respect to $u'$ at the particular value of $u'$.
Intensity at the Origin of the Symmetry Plane as a Function of Separation \( \Delta \)

At the origin of the symmetry plane \( u'' = 0 \), and referring to Eq. (E.37), the intensity at that point can be expressed as

\[
I|_{u''=0} = \frac{\sin^2(\Delta/8)}{(\Delta/8)^2} + \frac{\sin^2(-\Delta/8)}{(-\Delta/8)^2} + \frac{2\sin(\Delta/8) \cdot \sin(-\Delta/8)}{(\Delta/8)(-\Delta/8)} \cdot \cos(y-x)
\]  

(E.65)

From Eq. (E.24), the argument of the cosine can be expressed as

\[
(y-x) = \tan^{-1} \left[ \frac{\sin(\Delta/2) - \sin(\Delta/4)}{1 + \cos(\Delta/2) - \cos(\Delta/4)} \right] - [4F^2\Delta - \pi].
\]  

(E.66)

Using the identities \( \sin 2\theta = 2 \sin \theta \cos \theta \) and \( \cos 2\theta = 2 \cos^2 \theta - 1 \), Eq. (E.66) reduces to

\[
(y-x) = -[(4F^2 - 1/4)\Delta - \pi].
\]  

(E.67)

Since the \( \cos(-\theta) = \cos \theta \), the intensity becomes

\[
I|_{u''=0} = \frac{2\sin^2(\Delta/8)}{(\Delta/8)^2} \left[ 1 + \cos[(4F^2-1/4)\Delta - \pi] \right].
\]  

(E.68)

The cosine function is plotted in Figure 58 and the intensity shown in Figure 59 for a range of values between \( \Delta = 0 \) and \( 6\pi \). Note the peak intensity at approximately \( \Delta = 3\pi \).
Fig. 58. Cosine Factor in the Expression for Intensity at the Origin of the Symmetry Plane.
Fig. 59. Intensity at the Origin of the Symmetry Plane as a Function of Separation, $\Delta$, at F/10.
APPENDIX F

DERIVATION OF THE EXPRESSION FOR NORMALIZED IRRADIANCE IN THE INTERFERENCE PATTERN GENERATED BY A SINGLE SCATTERING SOURCE LOCATED ON AXIS EXTERIOR TO THE SCATTER NULLING INTERFEROMETER

With respect to Figure 60, consider initially a point source located in the x,y plane at \((x_1,y_1,z_1)\), with a spherical wave emanating from \(P_1\). The expression for the field in the \(x_2,y_2\) plane is of the form (Born and Wolf, 1975)

\[
U(x_2,y_2,z_2) = \frac{A_{12}}{i\lambda} \frac{e^{ikR}}{R} \cos \beta
\]

where it is assumed that \(R \gg \lambda\), the wavelength. The obliquity factor is \(\cos \beta\), and the complex constant \(A/\lambda\) represents the amplitude of the wave.

Expressing \(U\) as an explicit function of \(x\), \(y\), and \(z\):

\[
\cos \beta = \frac{z_{12}}{R} \quad (F.2)
\]

\[
R = [z_{12}^2 + \delta^2]^{1/2} \quad (F.3)
\]
Fig. 60. Notation and Definition of Variables for a Spherical Wave Expanding from Point $P_1$. 
\[ \delta = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2} \]  

(P.4)

\[ R = z_{12} \left[ 1 + \left( \frac{x_2 - x_1}{z_{12}} \right)^2 + \left( \frac{y_2 - y_1}{z_{12}} \right)^2 \right]^{1/2} \]  

(P.5)

\[ U_3(x_3, y_3, z_3) = \frac{A_{12}z_{12}}{4\pi R^3} e^{ikR} \]  

(P.6)

\[ U_4(x_4, y_4, z_4) = \frac{A_{12}}{i\lambda} \left[ \exp \left[ ikz_{12} \left( \frac{x_2 - x_1}{z_{12}} \right)^2 + \left( \frac{y_2 - y_1}{z_{12}} \right)^2 \right] \right] \times \left[ \frac{z_{12} \left( \frac{x_2 - x_1}{z_{12}} \right)^2 + \left( \frac{y_2 - y_1}{z_{12}} \right)^2}{1 + \left( \frac{x_2 - x_1}{z_{12}} \right)^2 + \left( \frac{y_2 - y_1}{z_{12}} \right)^2} \right] \]  

(P.7)

If the point source at \( P_1 \) is now considered to be on axis at \( P_1(x_1 = 0, y_1 = 0, z_1) \).
To simulate the situation encountered in the scatter nulling interferometer, it is necessary to view the field at the $x_2,y_2$ plane as a combination of two mutually coherent spherical waves propagating toward the $x_1,y_1$ and $x_2,y_2$ planes, and in fact converging separately to the on-axis point $(0,0,z_1)$ and $(0,0,z_2)$. To change the direction of propagation of the field described in Eq. (F.8), a minus sign is introduced in front of every imaginary quantity $i$. Thus, $U_2$ becomes

$$U_2(x_2,y_2,z_2) = \frac{-A_{12}}{i\lambda}$$

$$\exp(-ik|z_1| \left[ 1 + \left( \frac{x_2}{z_{12}} \right)^2 + \left( \frac{y_2}{z_{12}} \right)^2 \right]^{1/2})$$

If a small region about the axial point $(0,0,z_2)$ is considered, then the
angle $\beta$ is small and the obliquity factor $\cos \beta = 1$, and $R = z_{12}$.

Furthermore, if one uses the quadratic approximation

$$$(1 + \theta)^{1/2} = 1 + \frac{1}{2} \theta - \frac{1}{8} \theta^2 + \ldots$$$

neglecting terms beyond the second, $U_z$ becomes

$$$
U_z(x_2, y_2, z_2) = \frac{-A_{12}}{i\lambda |z_{12}|} \exp \left[ -ik|z_{12}| \left[ 1 + \frac{1}{2} \left( \frac{x_2}{z_{12}} \right)^2 + \frac{1}{2} \left( \frac{y_2}{z_{12}} \right)^2 \right] \right] \times \exp \left[ -ik|z_{12}| \left[ 1 + \frac{1}{2} \left( \frac{x_2}{z_{12}} \right)^2 + \frac{1}{2} \left( \frac{y_2}{z_{12}} \right)^2 \right] \right].$$$

Introducing the field at the plane $(x_2, y_2, z_2)$ associated with the wave converging to the point $(0, 0, z_0)$

$$$
U_z(x_2, y_2, z_2) = \frac{-A_{12}}{i\lambda |z_{12}|} \exp \left[ -ik|z_{12}| \left[ 1 + \frac{1}{2} \left( \frac{x_2}{z_{12}} \right)^2 + \frac{1}{2} \left( \frac{y_2}{z_{12}} \right)^2 \right] \right] - \frac{A_{32}}{i\lambda |z_{32}|} \exp \left[ -ik|z_{32}| \left[ 1 + \frac{1}{2} \left( \frac{x_3}{z_{32}} \right)^2 + \frac{1}{2} \left( \frac{y_3}{z_{32}} \right)^2 \right] \right]$$$

Substituting for $z_{32}$ in terms of $z_{12}$ and $\Delta$, where

$$$
|z_{32}| = |z_{12}| + |\Delta|$$
\[ U_2(x_2, y_2, z_2) = \frac{-A_{12}}{i\lambda |z_{12}|} \exp(ik|z_{12}|) \]

\[ \times \exp \left[ \frac{-ik}{2|z_{12}|} (x_2^2 + y_2^2) \right] \]

\[ \frac{-A_{22}}{i\lambda(|z_{12}| + |\Delta|)} \exp(ik(z_{12} + \Delta)) \exp \left[ \frac{-ik(x_2^2 + y_2^2)}{2(z_{12} + \Delta)} \right] \] (E.14)

Let

\[ K_1 = \frac{-A_{12}}{i\lambda |z_{12}|}; \quad K_2 = \frac{-A_{22}}{i\lambda(|z_{12}| + |\Delta|)}; \]

\[ r_{2z}^2 = x_2^2 + y_2^2 \] (E.15)

Note that

\[ K_1 K_2 = K_2 K_1; \quad \text{and} \quad K_1^2 = K_1^2. \] (E.16)

The intensity \( I(x_2, y_2, z_2) \) is expressed as

\[ I(x_2, y_2, z_2) = U_2^* U_2 = K_1^* K_1 + K_2^* K_2 \]

\[ \quad + K_2^* K_1 \exp \left[ -ik|z_{12}| + ik|z_{12}| + ik\Delta - \frac{ikr_{2z}^2}{2|z_{12}|} + \frac{ikr_{2z}^2}{2(|z_{12}| + |\Delta|)} \right] \]
\[ + K_1 K_2 \exp \left[ -ik|z_{12}| - ik\Delta - \frac{ikr_z^2}{2(|z_{12}| + |\Delta|)} + \frac{ikr_z^2}{2|z_{12}|} + ik|z_{12}| \right] \]

\[ I(x_{21}y_{21}z_{21}) = K_1^2 + K_2^2 + K_1 K_2 \]

\[ \left[ \exp \left[ ik\left[ \Delta + \frac{r_z^2}{2} \left[ \frac{1}{|z_{12}| + |\Delta|} - \frac{1}{|z_{12}|} \right] \right] \right] \]

\[ + \exp \left[ -ik\left[ \Delta + \frac{r_z^2}{2} \left[ \frac{1}{|z_{12}| + |\Delta|} - \frac{1}{|z_{12}|} \right] \right] \right] \]

Let

\[ K_1 = K_2 = 1; \text{ and } \psi = \frac{k}{2} r_z^2 \left[ \frac{1}{|z_{12}|} - \frac{1}{|z_{12}| + |\Delta|} \right] \]

Then

\[ I(x_{21}y_{21}z_{21}) = 2 + \exp[i(\psi - k\Delta)] + \exp[-i(\psi - k\Delta)] \]

\[ I(x_{21}y_{21}z_{21}) = 2[1 + \cos(\psi - k\Delta)]. \]

Examining the expression for \( \psi \) and consolidating

\[ \psi = \frac{\pi r_z^2}{\lambda} \frac{\Delta}{|z_{12}|^2 + |\Delta||z_{12}|}. \]
The term $|\Delta| z_{12}$ can be neglected, as it is very small compared to $z_{12}^2$. This leads to

$$\psi = \frac{\pi}{\lambda} \left( \frac{r_{12}}{z_{12}} \right)^2 \Delta$$ \hspace{1cm} (E.23)$$

and

$$I(x_1, y_2, z_2) = 2 \left[ 1 + \cos \left( \frac{\pi}{\lambda} \left( \frac{r_{12}}{z_{12}} \right)^2 \Delta - \frac{2\pi\Delta}{\lambda} \right) \right].$$ \hspace{1cm} (E.24)$$
APPENDIX G

INTERPRETATION OF ISOINTENSITY PLOTS IN TERMS OF ENERGY COLLECTED BY A DETECTOR OF RADIUS $V_0$ AT A GIVEN RECEIVING PLANE $U_0$

In estimating signal-to-noise ratio at a given detector receiving the image of a distant point source (as generated by transfer through a scatter nulling interferometer set up for the null case), the calculation of total energy falling on a detector of radius $V_0$ is required. Better yet, it would be advantageous to determine the plane $U_0$ which would optimize total energy collected by a circular detector of given radius. Consider for example the iso-intensity plot in Figure 61. The objective is to find the fraction of the total energy lying in the circle with radius $V_0$ about the axial point $(U_0, 0)$, where $U_0 = \text{constant}$ is the receiving plane.

1. Determine the total power, $P_{\text{TOTAL}}$, over the volume created by the quadrant plot rotated about the $u$-axis

   \[ P_{\text{TOTAL}} = \int_{0}^{u_{\text{max}}} \int_{0}^{2\pi} \int_{0}^{v_{\text{max}}} I(u,v)vdv \theta du. \quad (G.1) \]

2. Find the fraction $P_F$ of the total power $P_{\text{TOTAL}}$ lying
Fig. 61. Iso-Intensity Contour Plot: Determination of Fractional Energy in the $U_0$ Plane.
within the circle of radius \( v_0 \) about an axial point \((u,0)\) at \( u = \) constant (a receiving plane); for each array point \((u,v)\) from \( u = 0 \) to \( u_{\text{max}} \) and \( v_{\text{max}} \). For example, for one array point \((u_0, v_0)\), using the irradiance profile at \( u_0 = \) constant,

\[
PV(u_0, v_0) = \frac{1}{P_{\text{TOTAL}}} \int_{\theta=0}^{\theta=2\pi} \int_{v=0}^{v=v_0} I(u_0, v) \, vd\theta.
\]  

Thus, \( P_F \) can be determined as a two-dimensional array with a 1:1 correspondence with the associated two-dimensional intensity array depicted by Figure 61.

(3) Contour plot the \( P_F \) array to determine contours for \( P_{0.1}, P_{0.2}, \ldots, P_{0.9} \) over the map \( v = 0 \) to \( v_{\text{max}}, u = 0 \) to \( u_{\text{max}} \), for each iso-intensity plot of interest, as in Fig. 62.

(4) The \( P_F \) contour plots will allow an immediate determination of the position of the optimum receiving plane \((u_0 = \) constant) for a given detector radius \( v_0 \), for a specific system F-number and wavelength.
Fig. 62. Example of a Contour Plot Depicting the Fraction of Total Energy Falling within Circles of Radius $v$ Centered on the Optical Axis for any Detector Plane $u = \text{Constant}$.
APPENDIX H

CALIBRATION PROCEDURE TO DETERMINE TRANSLATION AS A FUNCTION OF APPLIED VOLTAGE FOR THE PIEZOELECTRIC DRIVEN INTERFEROMETER MIRROR

The phase adjusting mirror, M2, as depicted in the scatter nulling interferometer schematic, Figure 36, is mounted on a Burleigh Instruments, Inc., PZ-90 PZT aligner/translator that allows a total linear travel of 6 µm, or approximately 9.5 wavelengths for the 0.6328-µm HeNe laser beam. Translator motion is activated and controlled by a highly linear dc power supply, the Burleigh Instruments, Inc., RC-44 digital ramp generator, whose output voltage ranges from 0 to 500 V dc, in minimum steps of 1 volt. The translator displacement was measured and plotted as a function of the applied voltage over the full range of available power supply voltage.

To provide a calibrated measure of the PZT driven mirror displacement, a Hewlett-Packard HP5526A laser/display measurement system was utilized in an interferometer arrangement as depicted in Figure 63. The HP5500C laser head incorporates a low-power HeNe-based Zeeman laser that emits two slightly different optical frequencies of opposite circular polarizations. After conversion to orthogonal linear polarizations, p and s, the beam is expanded, collimated, and directed through a sampling reference beamsplitter before exiting the head. The
Fig. 63. Schematic of Calibration System for the Interferometer Phase Control Mirror
dual polarized beam is directed to an interferometer composed of a polarizing beamsplitter and two arms, each with a 1/4 wave plate and their respective mirrors M1 and M2. The beam exiting the laser head is split at the polarizing beamsplitter, with one frequency reflected to M1 as an s-polarized beam, and the other transmitted toward the PZT controlled mirror M2 as a p-polarized beam. The 1/4 wave plates rotate the polarization to their opposite sense in each arm for the returning beams, which allow their recombination and redirection by mirror M3 into the receiving port of the laser head. Both frequencies are reflected back along a common path to a photodetector internal to the head. Since their polarizations are orthogonal, no interference fringes are formed until the beam arrives at a demodulating polarizer mounted in front of the photodetector. Relative motion between the two mirrors M1 and M2 causes a difference in the Doppler shifts in the return frequencies, thus creating a difference between the frequency detected by the return beam photodetector, and the frequency seen by the reference photodetector. This difference is converted by a fringe-count register and multiplier to a display reading in millimeters or inches.

The smallest digital count on the HP5505A display is for a displacement of 1 x 10^{-5} \, \text{mm}, or 0.01 \, \mu\text{m}. With the entire interferometer system situated on a vibration-isolated table, and enclosed within a plastic sheet, optimum operating stability was achieved whereby the display stabilized to within plus or minus the least count of 0.01 \, \mu\text{m} for each reading taken. The RC-44 power supply voltage was stepped from 0
to 500 V in 25-V increments, and the HP5505A display readings were plotted for each resulting position of mirror M2 as shown in Figure 64. The most linear region of the plot, between 100 and 250 V was chosen as the operating region for mirror M2 in the scatter nulling interferometer arrangement. Consequently, that portion of the curve was considered for a specification of the average calibration factor. This turned out to be 7.647 x 10^-6 mm/V. Since the power supply could be set to a stable value within 1 V, the least possible displacement of mirror M2 was equal to 0.012 wavelengths of HeNe laser energy.
Fig. 64. PZT Translated Mirror Calibration Curve.
APPENDIX I

ALIGNMENT PROCEDURE FOR THE SCATTER NULLING INTERFEROMETER

The following is a detailed alignment procedure for the experimental arrangement shown in Figure 36. With the vibration isolated table initially clear of all elements, the procedure starts with the laser source polarization orientation and continues with a step-by-step addition of each element in the optical train.

Alignment Procedure for Object Point Source Utility

1. The laser in its mount is nominally leveled with respect to the table using a bubble level. A polarizer mounted in an angularly indexed rotatable mount is placed in front of the laser, with the p axis of the polarizer vertically aligned. The laser is rotated in its mount until the beam is nulled after passage through the polarizer. This assures that the p axis of the laser beam is horizontally aligned.

2. Using a fine wire crosshairs set mounted on a vertically adjustable post, the laser beam is leveled accurately with respect to the table by an iterative procedure. The crosshairs diffraction pattern is viewed on a screen placed behind the crosshairs. Initially placed close to the laser, the crosshairs are adjusted in height for a symmetrical diffraction image. The crosshairs set is then moved
to the most distance point on the table in alignment with the laser beam, and the laser is adjusted in elevation to compensate for any discrepancy in level with respect to the table. Repeating the process several times allows one to establish the correct laser beam level parallel to the main table surface. The crosshairs are then locked in at the beam height. The diffraction pattern observed on a screen behind the crosshairs is observed to be symmetrical with the main crosshairs shadow. The crosshairs can henceforth be used as a reference for the optical axis height and location.

3. Several stages in the alignment procedure rely on retroreflection from an element back to the laser exit head. To more precisely define the position of the retroreflected beam on the laser head, a cap with a white reflecting surface drilled with a hole matching the diameter of the laser beam is placed over the laser exit head.

4. The beamsplitter's rotationally adjustable table is introduced and leveled with a precision bubble level with respect to the main table. The beamsplitter in its tipable mount is then placed on the rotational table and adjusted to retroreflect the laser beam back on itself. Since the beamsplitter is to be used at the Brewster's angle to the incident beam, it is rotated over that precise angle while the level of the beam is checked by the crosshairs at each of the extreme positions. The exact angular position of the beamsplitter is decided by observing the vanishing point of the multiple reflections as the element is rotated.
5. The fixed reference mirror M1 is introduced and adjusted to retroreflect the laser beam back on itself.

6. With the M1 path temporarily blocked, the movable mirror M2 is introduced and similarly adjusted to retroreflect the laser beam back to the laser head. A double check of M1 and M2 alignment is possible by locating the crosshairs set at the edge of the main table as shown in Figure 65. Here it can remain as a fixed alignment reference throughout the following procedure. A screen placed behind it allows the diffraction pattern formed by the incident beam to be observed for symmetry with respect to the crosshairs principal shadow, as each successive element is introduced into the optical train.

7. A screen S1 is introduced as close as possible to the beamsplitter BS as shown in Figure 65. The diffusely reflecting screen has a pinhole whose diameter is a clear aperture to the laser beam. The screen mount should allow adjustment of the X-Y position of the pinhole as well as tip and tilt of the pinhole plane. A flat mirror is held against the front surface of S1, with M2's path blocked, and S1 adjusted for orthogonality to the incident beam by observing the retroreflected image on the laser head. The pinhole position is adjusted in X and Y while observing the crosshairs screen to optimize intensity throughput and center the beam on the crosshairs.

8. Lens L2, the final imaging lens, is introduced as closely as possible to screen S1. The lens is adjusted with respect to tip, tilt, and
Fig. 65. Initial Alignment Layout for the Object Point Source.
decentration to correctly align it. Three separate checks are made:
(1) the crosshairs diffraction pattern must be symmetrical, (2) the
retroreflected beam from the front of L2 onto the rear surface of
S1 must be symmetrical, and (3) a most sensitive alignment check is
provided by observing the retroreflected beam from the front
surface of L2 on the temporary screen S2 located as in Figure 46.
After L2 is adjusted, screen S1 is removed.

9. Introduce window WI at a location ahead of the beamsplitter such
that the front surface of WI (that surface closest to the laser
head) lies at a back focal distance from L2 for the M1 path. Note
that screen S2 remains in place to block the M2 path. WI is
adjusted in tip and tilt to make it orthogonal to the incident laser
beam by observing the retroreflection on the laser head, and by
observing the symmetry of the crosshairs diffraction pattern.

10. Iris diaphragms A1 and A2 are introduced and sequentially aligned in
the laser beam. First A1 is stopped down to just allow passage of
the laser beam. Its orthogonality to the laser beam is assured by
retroreflection back to the laser head off a small mirror held
against the diaphragm. The X-Y position of A1 is adjusted until the
aperture is symmetrical to the incident laser beam, and the
retroreflection off M1 is symmetrical with the rear side aperture
of A1. Iris diaphragm A2 is deployed in the same manner, while A1
is opened up.

11. Pinhole screen S1 is introduced ahead of the iris diaphragm A1 and
is aligned in the laser beam as described in step 7. The spatial filter SF is introduced, and the precision pinhole and microscope objective are adjusted to provide a clean, symmetrical beam pattern. The retroreflected image from the rear surface of the microscope objective onto the laser head is used to correctly locate the spatial filter and provide an output beam symmetrical about the pinhole in S1. Tip, tilt, and X-Y adjustment of the spatial filter combination may be necessary to assure beam symmetry at the S1 pinhole.

12. Collimating lens L1 is introduced, as in Figure 66 with its rear surface as close as possible to its back focal distance from the spatial filter's pinhole. This initially is a tape rule measurement. L1 is aligned at first by observing the retroreflection from its rear surface onto screen S1. Tip, tilt, and X-Y adjustment may be necessary to achieve a symmetrical reflected beam pattern about the pinhole in S1. A shear plate, SP, is introduced at a slight angle to the optical axis as shown in Figure 66 and the resulting interference pattern observed on screen S3. L1 is adjusted in the z direction until the shear plate's interference pattern indicates a collimated beam output from L1. The shear plate is removed. The symmetry of the retroreflected beam from the rear surface of L1 onto S1's pinhole is rechecked, and tip, tilt, and X-Y position are adjusted as necessary to assure correct alignment of L1. At this stage, S1 and S2 may be removed, and the interference pattern
Fig. 66. Final Alignment Layout for the Object Point Source.
observed on the crosshairs screen. Fine adjustment of mirror M2 may be necessary to provide a circularly symmetrical bullseye pattern of fringes. L1 is adjusted by a micrometer stage along the optical axis to provide the necessary convergent beam.

Alignment Procedure for On-Axis Scatter Source Utility

If an on-axis scatterer point source is to be simulated, the following gives the procedure for providing it. It is assumed that steps 1 through 12 have been completed at this point, and that a collimated beam emerges from lens L1. The objectives are to introduce a lens L3 into the collimated beam and ensure that the resulting point image lies on axis and precisely on the rear surface of window W1. At the same time, the rear surface of W1 is to be located at the back focal point (for the M1 path) of lens L2, which assures collimation of the simulated scatter source for at least one path through the interferometer.

1. With reference to Figure 67 in the following procedure, screen S2 is placed in the path of M2 to block it temporarily. S1, the screen with a pinhole in it is introduced and aligned such that it is orthogonal to the incident collimated beam, and on-axis with reference to the crosshair's image as previously described in step 7.

2. Lens L3 is introduced in the collimated beam emerging from S1's pinhole, and placed as close as possible to the rear focal point on the M1 path from the lens L2. L3 is aligned by observing the retroreflected beam of the rear surface of L3 at S1, and adjusting
Fig. 67. Alignment Layout for the Simulated Scatter Source.
tip, tilt, and X-Y position to provide a symmetrical image at Si about its pinhole. Si is then removed.

3. In addition to screen S2, a temporary screen is placed between BS and M1. Shear plate SPI is located ahead of L3 and the fringe pattern is observed at screen S3. This pattern will be faint because the uncoated rear surface of WI will have about a 4% reflection factor. However, the interference pattern irradiance is sufficient to allow adjustment of L3 along the optical axis and observe the collimation condition. Thus, L3 will be located at its focal distance away from WI's rear surface. The temporary screen between BS and M1 and the shear plate are then removed.

4. With shear plate SPI in position, the fringes produced by the plate on screen S3' are observed. These fringes originate from the on-axis simulated scatter source at WI's rear surface after collimation by L2. Lens L2's axial position may be adjusted to provide accurate collimation by observing the shear plate's fringe pattern. At this point, SPI and the temporary screen S2 may be removed. This facilitates viewing the simulated scatter source's interference pattern on the crosshairs screen.
REFERENCES


