ANALYSIS AND APPLICATION OF A PASSIVE ELECTRONIC ANALOG MODEL TO THE HYDROLOGIC REGIME OF A WATERSHED

BY

Richard McGee Tinlin

A Dissertation Submitted to the Faculty of the DEPARTMENT OF WATERSHED MANAGEMENT In Partial Fulfillment of the Requirements For the Degree of DOCTOR OF PHILOSOPHY In the Graduate College THE UNIVERSITY OF ARIZONA

1972
THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction by Richard McGee Tinlin entitled Analysis and Application of a Passive Electronic Analog Model to the Hydrologic Regime of a Watershed be accepted as fulfilling the dissertation requirement of the degree of Doctor of Philosophy.

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Dissertation Director

Date

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ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. John L. Thames, Professor of Watershed Management, The University of Arizona, under whose direction this study was completed. His willingness to put aside his own work and spend the time necessary to overcome some of the obstacles encountered in this study is gratefully acknowledged.

To Dr. John H. Ehrenreich, a very special thanks for his extra efforts which provided my family with a sufficient income and the needed medical insurance to survive what could have been a financially crippling experience, due to the illness of my youngest son.

To my wife, Margaret, and four sons, Craig, James, Bryan and Daniel, my gratitude for being so understanding over the past four years.
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ABSTRACT

A digitally simulated electronic watershed analog has been developed for the analysis of the hydrologic regime of a watershed. Individual electrical circuits were designed to synthesize the physical characteristics of the hydrologic components of a watershed: interception, surface storage, runoff, infiltration, and subsurface storage. These circuits were related to pertinent empirical studies of significance to each component. Electrical circuit analogies, despite advantages inherent in their direct physical correspondence to hydrologic systems, have fallen into disuse due to the inflexibility of fixed component networks. A digital simulation program developed by the electrical engineering profession to provide flexibility in the design of electronic circuitry has been adapted for the simulation of the electronic watershed analog. The typical digital circuit analysis program is "canned" and the user need not understand its intricacies. Input is in the form of circuit parameters on punched cards. The output is in numeric or graphic form. Using digital simulation methodology, the electronic watershed analog has been used to analyze a 1.63 acre forested watershed.
CHAPTER 1

INTRODUCTION

This dissertation describes a direct passive electric analog model which is used as a means of analysing the interrelationships of hydrologic processes on a watershed.

Watershed modelers are faced with two conflicting requirements in selecting a model design for the simulation of watershed hydrologic processes. The first is to select a design which is as useful and descriptive as possible, and the second is to avoid over-complexity. The direct passive electric watershed analog described in this report provides an excellent compromise between these two requirements. The flow of electrical charge on one hand and the flow of water on the other provides the analogous link between the two systems. The electrical circuit serves as a mathematical shorthand to transform an otherwise highly complex set of integro-differential equations to a much lower level of difficulty.

The complexity of natural watershed systems makes it unlikely that there will ever be a model capable of taking into consideration all the spatial variability encountered in parameters significant to the hydrologic regime of a watershed. On the other hand, the physical system under investigation is such that micro-variations of these parameters are integrated naturally and obscured by macro-variations.
This works to the advantage of the investigator since a watershed may then be subdivided according to its dominant hydrologic characteristics. Each of these subdivisions is hereafter referred to as a descriptive unit.

The following assumptions were made about the descriptive unit: (1) the physical characteristics of the unit are time invariant; (2) the unit has essentially the characteristics of a unit-source area, defined by Amerman (1965, p. 499) "as having single cover, a single soil type, and otherwise physically homogeneous." If a number of these units were to be operated together they would then form a complex or heterogeneous area for the integration of the spatial variability of watershed physical characteristics.

The initial objective of this research was the development of an electrical circuit analog of the descriptive unit. The direct passive analog was built to synthesize step by step the hydrologic processes on a watershed as presently understood. Empiricisms developed over the years have been related to the electrical circuitry of the passive analog, and as a result insight into the problems of watershed modeling has been gained.

Analyses of the passive direct electric analog were made using ECAP, a versatile digital electric circuit network analysis program developed by IBM. The technical and theoretical capability of electrical circuit analysis methods undoubtedly exceeds our mastery of watershed systems analysis. Once a transformation to the electrical format has been made, the analogous watershed system can be subjected to parameter modification tests on the digital computer.
Particular attention is paid to how accurately the analogous electrical circuit describes the physical system in the following discussions. The validity of the modeling technique is demonstrated by its application to a 1.63 acre forested watershed in the Ouachita Mountains of central Arkansas.
A direct passive electrical analog is defined as the simulation of the physical behavior of a prototype system by an analogous physical system (Karplus, 1958). The term "passive" is used to describe the behavior of a system containing no internal energy sources. In the strict sense of the definition the analog described in this dissertation would be classified as "active," since internal energy sources are used as linkages for current transfer between the circuit elements. Since the purpose of these energy sources is to link passive subcircuits representing the various subcomponents of the hydrologic cycle, the term passive has been retained in the description of the analog.

The use of electrical circuits in a computing mode is a classical approach to analog simulation (McCann, 1956). Currently the use of passive electrical circuits as a simulation tool, except in the case of large R-C groundwater electrical networks, has become almost nonexistant due to the inflexability of fixed component networks.

In hydrology, the applications of direct passive electronic analog devices to date have dealt with flood routing, (Harder, 1963), and groundwater studies, (Bermes, 1960). Rosa (1963) and Beyers (1962), attempted to simulate certain watershed characteristics using cascaded resistance-capacitive combinations. Shen (1963) made application of
direct analog techniques, using the operational amplifier as a voltage transferring device between electrical components to analyze flood runoff.

The development of the solid state operational amplifier resulted in the rapid evolution of the differential analyzer, or indirect analog, which assumed a predominance over the direct analog in the simulation field. The differential analyzer is characterized by the use of feedback circuits to perform such functions as addition, multiplication by a constant, and integration. Voltages at the output terminals of the operational amplifiers represent dependent variables and their derivatives. Indirect analogs are highly suited to the solution of mathematically describable systems. Some of the most notable watershed simulation studies utilizing the indirect analog have been conducted at the Utah Water Research Laboratory (Riley, 1967).

A parallel development to analog simulation methodology has been the adaptation of the digital computer to the solution of numerical models describing hydrologic systems. Noteworthy digital models are the Stanford Model, (Crawford and Linsley, 1966), and the mathematical model of Huggins and Monke (1966).

The apparent dichotomy between modelers using the differential analyzer and digital methods appears to have been breached with the development of the hybrid computer. The hybrid computer combines the major advantages of both.

Due to continuing technological advances there are strong indications that the digital computer using simulation techniques and
competing at equal computing speeds, will surpass the hybrid computer on both economic and on human engineering grounds (Korn, 1969). One significant development in digital simulation has been in the area of electrical circuit analysis (Jensen and Lieberman, 1968). As a result, a powerful tool now exists for the simulation of electronic analog models by digital circuit analysis techniques.

Vemuri (1969) suggests that passive electrical network analogs are not flexible enough to treat nonlinear partial differential equations and time varying hydrologic field problems. However, with the advent of digital simulation methods for electrical circuit analysis, these nonlinear equations and transient phenomena can be readily investigated.

The typical digital circuit analysis program is "canned" and the user need not understand its intricacies. Input is in the form of circuit parameters on punches cards. Output is in numeric or graphic form. Several modes of circuit analysis exist within these programs including DC, AC, and transient analysis. The programs include the capacity to automatically vary parameter values, as well as provide sensitivity coefficient, worst case analysis, and standard deviation calculations for the analog networks.

This modeling approach should result in an upsurge in the application of electrical circuit analogies to many facets of watershed process simulation.
CHAPTER 3

MODEL DEVELOPMENT

The aim of this research has been to develop a general purpose hydrologic model of a watershed. All of the major hydrologic landphase components were included, interception, surface depression and litter layer storage, runoff, infiltration, near surface soil moisture storage, interflow, loss to groundwater and evapotranspiration losses. A descriptive model was used as a reference framework for the basic model concept. A forested watershed was selected as a basic unit of study, however the principles derived may be readily applied to a variety of other watershed types.

The Descriptive Model

The basic descriptive unit is shown in Figure 1. The total rainfall component is subdivided into intercepted rain and direct throughfall. Intercepted rain is defined as all that precipitation which is momentarily or permanently stored in the vegetative canopy. There are three interception subcomponents, indirect throughfall, stemflow, and subtransient leaf storage. Subtransient leaf storage is the precipitation that remains in storage to be later evaporated. Evaporation from the storage zones are controlled externally by the energy available to stimulate the process, ceasing only when water storage no longer exists. Indirect throughfall and stemflow were
Figure 1. The Descriptive Model
combined in the model. Direct throughfall is the component of total rainfall which passes directly through the canopy or open spaces to the ground with no interference. Direct throughfall combined with indirect throughfall and stemflow form the total rainfall input at the forest floor after modification due to storage lags and evaporation losses.

The precipitation reaching the forest floor was divided into five subcomponents: infiltration, litter storage, surface depression storage, surface runoff, and evaporation from litter layer storage. The infiltration component is subdivided into near surface soil moisture storage and interflow. Near surface soil moisture storage is subject to transpiration and groundwater recharge. The magnitude of the groundwater component is a function of near surface antecedent soil moisture.

The Hydraulic Model

The interaction between the major components of the descriptive model can be illustrated with a hydraulic model composed of storage reservoirs (Figure 2). Flow between reservoirs follows the same paths as those in the descriptive model, but gives a clearer view of how the system operates. Time lags are built into the model, as a determinable time interval is required to fill the storage reservoirs.

A portion of the total precipitation passes directly to the surface reservoir as direct throughfall. The remainder is diverted into canopy storage. A specified volume is retained by the canopy before transient storage (due to plant surface friction and adhesion) develops, which results in indirect throughfall and stemflow.
The surface of a forested watershed usually has a porous litter layer with a transient water holding capacity, and a small residual or hygroscopic component. The transient water holding capacity can be as large as two times the litter weight per unit area, and the residual component about one-half the litter weight per unit area (Helvey, 1967). The reservoirs representing surface depression storage and litter storage are stacked in the sense that litter storage must be satisfied before surface depression storage can fill. Litter storage must also be satisfied before infiltration occurs.

Infiltration commences when the litter layer has become sufficiently wetted, and continues throughout the event. After the event ends the surface depression volume will continue to infiltrate into near surface storage until it is depleted. The constriction in the infiltration arrow signifies a resistance or conductivity factor in series with a soil moisture capacity zone. The transpiration arrow indicates that a portion of this storage is allowed to bleed off, by an amount proportional to an external transpiration function.

Once surface soil moisture storage is raised to field capacity, lateral flow (interflow) will commence. Vertical and lateral resistance controls are included to proportion the amount of flow in each direction. Interflow in the lateral direction combines with surface flow while vertical flow passes on to groundwater storage.

Depending on the particular case under study, a groundwater system may be readily added. If this is done, groundwater storage would be subdivided into two components, much in the same manner as near
Figure 2. The Hydraulic Model
surface soil moisture storage. After the time lag associated with raising the groundwater level enough to develop transient storage, baseflow would commence through a second lateral flow resistance. A deep seepage component could also be added.

When surface depression storage requirements have been met, surface runoff commences. Downslope friction to overland flow causes transient storage to build up over the surface depression storage area. This transient storage decays off as surface runoff at the end of the storm event, and is routed through channel storage.

The hydraulic model results in a lumped system closely related to the actual physical system of interest. The model has been designed as a building block for which parallel-series combinations can provide a technique for complex watershed problem analysis. The two model outputs, surface runoff and interflow, may be routed separately into adjoining units or combined in a single unit. Interaction between these units will provide a realistic approach to a direct analogy of the physical system under investigation.
CHAPTER 4

ANALYTICAL METHOD AND APPLICATION

Analysis of the passive electronic watershed analog circuit was accomplished using a modified version of IBM's ECAP (ECAMP) which is available on the University of Arizona campus. ECAP stands for "Electronic Circuit Analysis Program." The modification of ECAP is for computer compatibility and does not effect the program language; therefore, the code names ECAP and ECAMP can be used interchangeably. A detailed discussion of ECAP capabilities is provided by Jensen and Lieberman (1968).

Briefly, ECAP is an integrated system of four programs developed to aid in the design and analysis of electronic circuits. The four programs consist of: an input language, a DC analysis program, an AC analysis program, and a transient analysis program. The input language and transient analysis programs were used exclusively in this analysis. The input language is user-oriented and allows complex circuits to be easily coded for computer analysis. The transient analysis program provides the time response solution of linear or nonlinear electrical networks subject to arbitrary user-specified driving functions.

ECAP, in addition to being an effective electronic circuit simulator, can also be used in analyzing other physical systems that have corresponding electrical network analogs. Circuit analysis
techniques, such as ECAP, can be powerful tools for modeling many of the physical processes of interest to hydrologists.

The use of digital simulation techniques allows nearly complete freedom in the selection of network component values for the electrical analog. In actual practice, a one farad capacitor is physically unrealistic, but presents no problem using digital simulation methods since the capacitor value is defined numerically. Expensive electronic hardware such as function generators can be simulated with a few punched cards.

The primary components of the passive electronic watershed analog are resistors, capacitors, and diodes. The resistors are used to restrict flow, and in other instances are combined with capacitors to determine decay rates for recession curves. The capacitors are used to simulate storage. Diodes serve as electrical check valves directing charge flow and holding charge on the capacitors. To provide driving functions both dependent and independent current sources are used. Current transfer between the passive circuit elements and the current sources is accomplished by using transfer function or T-cards, a useful feature of ECAP.

Before the application of the model can be accomplished, the analogy between fluid flow and electrical flow must be established. The following conventions were selected.
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<td>Available Storage</td>
<td>Capacitance (C)</td>
</tr>
<tr>
<td>Friction</td>
<td>Resistance (R)</td>
</tr>
<tr>
<td>Water in Storage</td>
<td>Charge (q)</td>
</tr>
<tr>
<td>Head</td>
<td>Voltage (V)</td>
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</table>

The units of charge flow are coulombs per second, which is a volume rate. A common mistake is to speak of "current flow;" in reality charge flows, not current.

The scaling factors selected for this model were designed to be applicable to conventional watershed data. The first scaling relationship is between area and capacitance. The unit of capacitance is the farad or coulomb per volt. If one micro-coulomb is set equivalent to one cubic foot, and one volt to one foot of head, then the following relationship exists:

\[ 1 \text{ ft.}^3/1 \text{ ft.} = 1 \text{ micro-coulomb/volt} = 1 \text{ micro-farad} \]

The equation

\[ q = CV \]

establishes the relationship between charge \( q \), capacitance \( C \), and voltage \( V \). For example, one acre inch of storage in the analog would be 43,560 micro-farad \( \times \) 1/12 volt = 3630 micro-coulombs, which is equivalent to the number of cubic feet in an acre inch of water. The rainfall/discharge scaling relationship is that one ft.\(^3\)/sec. equals one micro-ampere or micro-coulomb/sec. A rainfall rate of one inch per hour per acre will then be one micro-ampere.
The units for time could be the same in the model as in the physical system, however, small current leakages from storage through the internal resistances of the model current sources can cause error in relation to the groundwater component. The groundwater component on an actual watershed is of a small magnitude, a few thousandths of an inch per hour, and without a reduction in these leakage losses the simulated groundwater component is not significantly larger than the combined leakage losses through the current sources. For this reason the time scale of the model is shortened in relation to real time by three orders of magnitude, since the magnitude of the charge loss is a function of time.

A reduction in the model time scale of one order of magnitude requires a corresponding reduction in all the model R-C time constants. This is accomplished by reducing the appropriate resistance values by one order of magnitude and increasing the current by a factor of ten for each order of magnitude of reduction in the time scale.

The watershed analog was tested using data collected by the U. S. Forest Service on a 1.63 acre forested watershed located in central Arkansas (Rogerson, 1971). The vegetation on the watershed consists of a shortleaf pine overstory and a mixed hardwood understory. Pine-hardwood litter covers the forest floor to a depth of about 2 inches. The soils are shallow (2 1/2 to 3 feet deep).

The area is characterized by hot summers and short cold winters. Annual precipitation on the study area averaged 52.5 inches during the 1961-69 gaging period. Precipitation is evenly distributed throughout
the year except for the month of October, which usually has less than 3 inches. A few light snowfalls occur each year.

The scaling relationships per acre for the Arkansas watershed are tabulated below:

<table>
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<th>Watershed</th>
<th>Watershed Analog</th>
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<tbody>
<tr>
<td>Rainfall/Discharge 1&quot;/hour</td>
<td>Current 1x10^-3 ampere</td>
</tr>
<tr>
<td>Available Storage 1 ft.³/ft.</td>
<td>Capacitance 1 micro-coulomb/volt</td>
</tr>
<tr>
<td>Water in Storage 1 ft.³</td>
<td>1 micro-coulomb</td>
</tr>
<tr>
<td>Elevation Potential 1 ft.</td>
<td>1 volt</td>
</tr>
<tr>
<td>Head 1 ft.</td>
<td>1 volt</td>
</tr>
<tr>
<td>Friction</td>
<td>Resistance ohms</td>
</tr>
<tr>
<td>Time 1 sec.</td>
<td>.001 sec.</td>
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</table>

The concepts behind the modeling approach will be outlined in the following chapters. Each component of the descriptive model will be reviewed. This will be followed by an explanation of the analogous electrical circuit. To demonstrate the correspondence between the physical system and the analog, an application will first be made to an empirical study and then to the Arkansas watershed.

The Interception Component

Of the total precipitation that falls on a forested watershed a portion of the input falls directly on the vegetative surface and accumulates as a surface storage. The remaining portion, identified as throughfall in the descriptive model of Figure 1, passes through voids or breaks in the canopy to the forest floor. When leaf storage
requirements have been satisfied, any excess rainfall of the intercepted portion will pass on to the ground as leafdrip and stemflow.

A detailed illustration of this process is shown in Figure 3. A step function or square wave input corresponding to a constant rainfall with a time dimension of \( t_0 \) to \( T_0 \) is hypothesized. At time \( t_0 \) the initial rainfall input \( (I_0) \) is divided into direct throughfall and canopy inflow, for illustrative purposes assumed to be 15 and 85 percent of \( I_0 \) respectively. The area at the center-right of the figure encompassing indirect throughfall and active stemflow is subject to a time of concentration lag \( T_l \). Indirect throughfall \( (I_t) \) in this instance is differentiated from direct throughfall \( (I_d) \) as precipitation which has actually passed through a transient storage lag and thus has been appreciably delayed.

The area above direct throughfall and prior in time to the dashed line extended vertically at time \( t_c \) represents the storage volume available in the canopy. This volume of precipitation is retained as an interception loss component \( (I_L) \), which will eventually be lost to evaporation. To the upper right of the vertical dashed line the area identified as a transient storage volume \( (I_c) \) represents the overstorage of precipitation known to occur on vegetative surfaces. This volume is expected to be approximately equivalent to the area under the trailing exponential decay curve identified as passive stemflow and leafdrip. Any difference would be attributed to evaporation during the storm interval.
Figure 3. Time Distribution of Interception Components
The sum of direct throughfall, indirect throughfall and active stemflow, along with passive stemflow and leafdrip, make up the total throughfall component. The interception loss volume determines how soon, if at all, stemflow and indirect throughfall occur. At low rainfall intensities it is conceivable that the canopy storage requirements will not be met, given sufficient evaporation rates, regardless of storm duration.

The Interception Analog

The electrical circuit for interception is shown in Figure 4. By relating to the basic model concept for interception discussed earlier, the interception circuit can be analyzed. The input to this circuit may be a constant current pulse or a time varying function which simulates an actual storm. The rainfall generator has been designed to provide either step function outputs or time varying waveforms.

To generate a step function output branch B1 is used with independent current source I1 to operate switch S1. This switch is used to control a dependent current source I2 in branch B2, and cause an instantaneous constant current through branch B3.

For the generation of a time varying function switch S1 is modified to control itself and not branch B2. Branch B2 will then function as the rainfall generator independently of branch B1. Its output will pass through branch B3.

Transfer card T7 is used to transfer a portion (85%) of the simulated rainfall passing through branch B3 into the canopy segment through dependent current source I7. A constant evaporation loss can be
Figure 4. The Interception Analog
removed at this point. If a 5% loss is assumed, then the transfer value of T7 would be 80%. The remaining 15% is transferred by T-card T8 to dependent current source I9 as direct throughfall.

The charge injected into capacitor $C_1$ (canopy storage) via T-card T7 is effectively trapped between the high internal impedance of dependent current source I7 and a simulated zener diode in branch B16. The diode is modeled using battery E1 to reverse bias switch S3. Switch S3 has two states, off and on, which have corresponding resistance values of 10 megohms and a model parameter $R_1$. Initially the reverse current in branch B16 will cause switch S3 to be off and the branch resistance to be 10 megohms.

When the potential on capacitor $C_1$ reaches that of the zener diode breakdown potential, subtransient leaf storage requirements are met. The voltage at which the zener diode is set to breakdown corresponds to a storage depth in feet. Note that when using ECAP it is a simple matter to simulate a zener diode with any desired breakdown voltage. A significant feature is the time delay created by the time required to charge the capacitor up to the zener diode breakdown potential. The delay concept is important in all subsequent stages since it provides electronically for the control of time lags associated with storages.

Simultaneously with the diode breakdown the internal resistance of branch B16 will drop from 10 megohms to the value of resistor value $R_1$ through the action of switch S3. This resistance shapes the charge flow out of leaf storage at the canopy saturation point. Charge flow
through the resistor elevates the voltage on the capacitor so that an overstorage continues to build up, although at a slower rate. At the end of the simulated rainfall event, the overcharge decays back through the resistance simulating post storm leafdrip and stemflow.

The indirect and direct throughfall components are combined in a summing resistance, branch B19. The value of this branch is 0.01 ohms and therefore so small that its influence on the circuit is negligible. This summed component is the total throughfall as modified by the forest canopy.

A main evapotranspiration (ET) function generator consisting of independent current source I3 and branches B4 and B5 can be programmed to generate a function approximating potential ET. T-cards T1, T3, and T5 control, as a percentage of the main ET function, three separate dependent current sources used to extract charge or by analogy moisture from interception storage, litter storage, and subsurface storage.

For interception storage, T-card T2 is used to draw charge out of storage via dependent current source I8 as a function of dependent current source I4, the interception evaporation driving mechanism. Otherwise the charge trapped on capacitor $C_1$ after the end of the simulated rainfall event would remain indefinitely. A storage sensing switch S2 is operated by branch B15 to detect a no charge condition on capacitor $C_1$. If this condition exists, dependent current source I4 of branch B6 will be shorted out. Litter storage evaporation is simulated by a second dependent current source I5. The current transferred into current source I5 from the main ET generator is transferred via T-card
T4 to dependent current source I12 in the surface storage analog to remove (dry out) the charge held in storage there.

A third dependent current source is included to provide a means for simulating transpiration from subsurface soil moisture storage. The driving mechanism is from the main ET generator through T-card T5, through dependent current source I6. The output of current source I6 is then used to drive dependent current source I15 which is tied into subsurface storage. Charge will be withdrawn by this current source at a rate proportional to estimated transpiration rates.

It would be a simple matter to transfer the interception evaporation potential to the transpiration circuit of branch B10 upon depletion of intercepted moisture if so desired. There seems to be evidence to warrant this energy transfer (Thorud, 1967). Unfortunately an adequate evapotranspiration relationship has not been developed which will provide a suitable function to operate the ET generator on an individual storm basis. However, one important modeling objective is to provide insight into how the field system will respond to external energy stresses, and for this reason the ET function generation routine is useful.

Integration of any of the branch currents can readily be accomplished using the circuit formed by current source I10, and branches B20 and B21. Since digitally simulated component values are ideal, the capacitor of branch B21 serves as an excellent integrator. Integration can be accomplished over long periods without errors due to component drift as is the case with the differential analyzer. Proper
selection of the capacitor size permits cumulative values to be obtained in either cubic feet or watershed inches. Selection of these values will be demonstrated in the model application to the field problem.

Validation of the Interception Model

Substantiation of the model was made with the empirical relationship developed by Horton (1919). Horton derived an equation of the form

\[ L = S + RET \]  

(4.1)

where

- \( L \) is interception loss in inches depth over the projected area of the canopy.
- \( S \) is the water stored on the vegetation in inches depth over the projected area of the canopy.
- \( R \) is the ratio of vegetation surface area to the projected area of the canopy.
- \( E \) is the evaporation rate in inches depth per hour during the storm.
- \( T \) is the duration of the storm in hours.

Linsley, Kohler, and Paulhus (1949) note that equation (4.1) yields a value of interception which is independent of the amount of precipitation, and assumes that the rainfall in each storm completely fills interception storage. They conclude that for light rainfalls, computed values of interception may exceed observed precipitation.
A modification of Horton's equation was proposed as follows:

\[ L = (S + RET) \left(1 - e^{-KP}\right) \]  \hspace{1cm} (4.2)

where \( P \) is the amount of rainfall, \( K \) is a constant, and \( e \) is the base of the Naperian logarithms. The equation suggests that total interception loss approaches the term \((S + RET)\) exponentially as the amount of rainfall is increased from zero to some higher value. The following development takes these factors into consideration.

Mathematically the circuit of the interception electric analog can be described in a manner compatible with the empirical developments. The two equations describing the charge and discharge of the interception storage capacitor are:

\[ q = CV_c(1 - e^{-t/RC}) \hspace{1cm} (4.3) \]

\[ q = C(V_c - V_{ZP})e^{-t/RC} + CV_{ZP} \hspace{1cm} (4.4) \]

where \( q \) is charge at any time \( t \)

\( R \) is resistance
\( C \) is capacitance
\( t \) is time

\( T_0 \) is equilibrium time at the end of the charging cycle

\( V_c \) is voltage on the capacitor at equilibrium

\( CV_c \) is the charge in storage at equilibrium

\( V_{ZP} \) is the zener diode breakdown potential

\( CV_{ZP} \) is the charge corresponding to the interception loss volume
Differentiation of equations (4.3) and (4.4) with respect to time puts these equations, respectively, in terms of rates:

\[ I_c = \frac{V_c e^{-t/RC}}{R} \]  

(4.5)

and

\[ I_s = \frac{-(V_c - V_{zp})}{R} e^{-\frac{(t-T_0)}{RC}} \]  

(4.6)

where \( I_c \) is defined as the transient storage inflow component and \( I_s \) is the transient storage outflow component or by analogy leafdrip and stemflow after the storm. The \( V_{zp} \) term of equation (4.6) is the boundary condition (storage depth) to which the transient storage head decays with increasing time after the end of the event.

To illustrate how the modification of the canopy input is related to the mathematical development just made, a simplified version of the interception analog circuit, Figure 5, has been devised. Initially the charge, or by analogy, rainfall input into interception storage, point A of the figure, proceeds at a constant rate since a constant current step function with an amplitude of 85% \( I_0 \) has been hypothesized.

During the interception storage lag interval \( \tau_1 \), the transient storage inflow component \( I_c \) is zero. All charge input during this interval is used to satisfy subtransient interception storage requirements. A waveform illustrating this is shown at point B of Figure 5.

A further clarification of this is apparent upon examination of equation (4.5). Prior to time \( t_c \) the exponential term of the equation...
Figure 5. A Simplified Version of the Interception Analog
is very small and therefore $I_c = V_c/R$. Since the value of $R$ is very large, the resulting value of $I_c$ is negligible. When the smaller resistance $R_1$ is switched in, the RC time constant of equation (4.5) is greatly reduced and indirect throughfall and transient storage inflow commence. At this time transient storage inflow is at its maximum value.

A mathematical representation for the active portion of indirect throughfall is

$$I_t = 0.85I_0 - I_c$$

(4.7)

where $I_c = V_c/R \ e^{-t/RC}$

At equilibrium $I_c = 0$ and outflow (indirect throughfall) is equal to inflow ($0.85 \ I_0$). Since the passive or post storm component of indirect throughfall is described by equation (4.6), equations (4.7) and (4.6) together completely describe indirect throughfall. The indirect throughfall component is illustrated by the waveform at point C of Figure 5. The combined direct and indirect throughfall component is shown at point D of the figure.

Grah and Wilson (1944) conducted interception studies on selected specimens of Monterey pine and Baccharis (an evergreen shrub). The plants were suspended from a balance in a sheet metal chamber and sprayed with water. Changes in surface detention were determined at one-minute intervals. At the end of the simulated rain the excess water was allowed to drip from the foliage and stems, resulting in a second
value which may be compared to the maximum interception under field
conditions. The result of this experiment is shown in Figure 6.

In the same experiment Grab and Wilson noted that the resulting
curve is similar to the exponential relationship characterized by the
behavior of many natural phenomena. The dashed plot superimposed on
Figure 6 was computed using equations (4.3) and (4.4) and the observed
data from the experiment. Evaluation of the RC time constant was
accomplished graphically on Figure 6 by constructing a line tangent to
the curve, originating at \( t = 0 \), and extended to intercept a horizontal
line passing through equilibrium. A vertical line was then dropped from
this intersection to the time axis. The point of interception on the
time axis defines an RC time constant of 8 minutes.

The boundary condition, \( CV_{zp} \), to which transient storage decays
could not be determined from the observed data of Grah and Wilson
because they stopped their experiment short of reaching this value.
Equation (4.4) was used to compute a value for \( CV_{zp} \). A value of 118
grams was determined for \( q \) at \( t = 40 \) minutes from the observed data.
The equilibrium charge, \( CV_c \), was 160 grams and the RC time constant
evaluated to be 8 minutes.

Using these values, equation (4.4) was solved and \( CV_{zp} \)
determined to be 108 grams. With the determination of \( CV_{zp} \) it was then
possible to evaluate equation (4.4) for \( q \) at any time. The accuracy
with which the computed values fit the observed values of Grah and
Wilson is evident in Figure 6.
Figure 6. The Rise and Fall of Surface Detention and Its Subdivision Into Storage Types. (After Grah and Wilson, 1944)
The physical relationship between the analog model and a classic experiment used to analyze the buildup of detention storage on plant surfaces has been shown to be mathematically describable. It is apparent that the mechanisms governing the interception process have a close analogy to the physical laws governing the behavior of the interception analog.

Canopy storage has a well defined analogy in capacitance and plant surface friction a less well defined analogy in resistance. Resistance can be further defined as

\[ R = \rho \frac{L}{A} \]

where \( \rho \) is resistivity

L is the flow path length

A is the cross-sectional area normal to the flow path

The value for \( \rho \) represents a friction factor related to leaf and bark surfaces. The flow path L could be the cumulative length of the leaf surfaces and flow paths over the bark surfaces. The cross sectional area A is fixed for a given cover in width but depth will vary with rainfall in an undefined manner. It is likely that this depth will become reasonably constant for a given species of tree or shrub, given sufficient precipitation to satisfy subtransient interception storage. The overall behavior of the canopy can be likened to many parallel-series RC combinations distributed in space.
Field Application of the Interception Analog

The interception analog was applied to the Arkansas watershed. The source of the model parameters are reports by Lawson (1967) and Rogerson (1971).

The estimated canopy area is approximately 60,352 ft.\(^2\) or 85% of the total 1.63 acres. The equivalent capacitance is 60,352 microfarads. Note that at zero storage (zero volts), capacitance is equivalent to area.

Based on this estimation, 85% of the gross rainfall is diverted through the canopy. This was further reduced by 5%, which is a canopy evaporation constant derived by Lawson. The remaining 15% of the gross rainfall was routed directly to the ground surface as direct throughfall.

Lawson (1967) estimated dormant season interception to be 7.3% of the gross rainfall. The total rainfall for the storm modeled, (storm of 12/26/68), here-after referred to as the calibration storm, was 2.40 inches, resulting in a 1,038 ft.\(^3\) interception loss. Using the charge storage relationship \(q = CV\) the corresponding diode breakdown voltage is 1.723x10\(^{-2}\) volts, which limits the interception loss by analogy to 1,038 micro-coulombs. Storage buildup in excess of this value is termed "transient storage" and this overstorage is eventually decayed out through the internal diode model resistance, selected to provide a 5.0 minute transient storage time lag.

The lag times resulting from interception storages for actual watershed canopies are unknown but there is evidence that they do exist, e.g., in the work of Grah and Wilson (1944). Realistic values are
probably on the order of a few minutes. Under dense canopy conditions it is possible that these lags are large enough to affect the time distribution of the rainfall prior to reaching the forest floor. The result could be a significant modification of the runoff hydrograph.

A plot of the rainfall and throughfall obtained by an ECAP analysis of the calibration storm is shown in Figure 7, and is illustrative of the modification in the time distribution of rainfall due to the vegetative canopy. The leading portion of the rainfall event is reduced due to initial interception losses.

In Figure 8 total throughfall is plotted as a percentage of the gross rainfall passing through the vegetative canopy. The computations for this figure were also obtained by an ECAP analysis of the calibration storm. Approximately 0.2 inch of rain must fall before indirect throughfall commences. Initially there is a 15% direct throughfall component until the interception loss requirements are met; at that point indirect throughfall increases rapidly and levels off at about 87.7%. This is because of a 7.3% interception component and a 5% evaporation component.

A data listing of the electrical interception analog in ECAP format is provided in Appendix A. The function of the circuit components are explained by comment cards interspersed with the circuit parameter information.
Figure 7. Modification of the Rainfall Interception Component by the Forest Canopy
Figure 8. Throughfall as a Percentage of Gross Precipitation
Surface Storage and Runoff

Rainfall prior to reaching the forest floor is modified by interception storage. To facilitate description of the surface storage and runoff component of the model, this modification will be ignored and a step function rainfall input will again be hypothesized. For the same reason infiltration will not be considered at this time.

Figure 9 illustrates the time distribution of surface storage and runoff. Surface storage is defined as that portion of the rainfall input required to fill up depression storage on the land surface, as well as wet the highly porous forest litter layer. As illustrated in the descriptive model (Figure 1) and the hydraulic model (Figure 2) a specified depression storage volume must be supplied before surface runoff commences in response to a rainfall input. Initially the input from time $t_0$ to $t_s$ is at the maximum rate ($I_0$) with no outflow, resulting in a time lag $T_2$ defined as time of storage.

Overland flow encounters surface friction from grass, twigs, and other surface features, resulting in a transient surface storage buildup (surface detention). Transient storage inflow will start at the maximum rainfall rate and decrease toward zero as equilibrium is approached. Rainfall in excess of surface storage and transient storage becomes surface runoff. At the end of the hypothesized step function storm, transient overland flow stored in the system will be released. The area labeled transient overland storage ($I_{ts}$) in Figure 9 should equal the area under the recession curve but will not necessarily be distributed equally in time.
Figure 9. Time Distribution of Surface Storage and Runoff
The Surface Storage and Runoff Analog

The circuit used to describe the runoff phase of the model is in principle much the same as the interception circuit. Figure 10 shows the electrical component arrangement in ECAP format. Branch B20 is a dependent current source controlled by the output from the interception analog through a T-card. Note that this model component starts with branch B20. The integrators have been dropped from the interception circuit, since they can be included conveniently at the end of the program. A dummy branch, B21, has been included in the circuit to monitor surface inflow. Branch B22 is a dependent current source controlled by the surface evaporation circuit, branch B9 of the interception analog. Branch B23 is a dependent current source controlled by the infiltration analog.

Capacitor $C_2$ of branch B24 will charge at a rate determined by its size and by the rate at which electrical charge flows into it. The size of this storage element is 71,000 micro-farads, representing an area of 1.63 acres or 71,000 ft.$^2$. This capacitor sets on a battery, $(E_2)$ of branch B25 which is used to position surface storage relative to subsurface storage. The voltage differential between battery $E_2$ and that present on a capacitor representing subsurface storage will later be shown to establish a potential gradient to drive the infiltration process.

When the electrical potential on capacitor $C_2$ reaches that of the initial breakdown voltage of nonlinear diode model, $Z_2$, surface
Figure 10. Surface Storage and Runoff Analog
storage requirements have been met. Charge will then flow out through resistor $R_2$ of branch B29, causing capacitor $C_2$ to develop an overcharge.

The overcharge or overstorage now develops at a slower rate until the next three resistances $R_3$, $R_4$, and $R_5$ of branches B30, B31, and B32 are sequentially switched parallel with $R_2$. The result is a nonlinear surface friction component in opposition to overland flow.

The voltages (storage depths in feet) at which these resistances switch in are determined by biasing batteries $E_6$, $E_7$, $E_8$, and $E_9$ of the nonlinear diode model. The method for determining the nonlinear nature of the surface friction component will be discussed in the next section.

Three sensing switches are included in the circuit of the surface analog; they are branches B26, B27, and B28. These sense switches are controlled by batteries $E_3$, $E_4$, and $E_5$ which act as storage depth detectors. The sense switch of branch B26 indicates when litter storage requirements have been met. When they have, the switch activates the infiltration circuit. The sense switch of branch B27 shorts out the surface evaporation generator, branch B8 of the interception circuit, when litter layer storage has been depleted. The third sense switch in branch B28 senses when surface depression storage exists. If it does exist, the switch causes infiltration to continue at its capacity rate. A detailed explanation of how this is accomplished is given in the discussion of the infiltration analog.

The overland flow component passes to ground through the summing resistance of branch B33 and in turn is transferred to dependent current source 113 of branch B34. This current source is used to charge
capacitor $C_3$ which is analogous to the available channel storage. The size of capacitor $C_3$ combined with the reverse biased battery $E_{10}$ determines how much channel storage exists. When channel storage requirements are met switch $S_{ll}$ of diode model $Z_3$ changes state and switches resistor $R_6$ into the outflow path, simulating channel roughness.

The diode model ($Z_3$) of Figure 10 would realistically be non-linear but there is not sufficient information to determine the nature of its nonlinearity. The reasons for including a channel segment are to illustrate the technique and provide insight into the routing scheme for overland flow. This scheme is to proportion the overland flow, using T-cards, into channel storage and onto adjacent elements where topographical conditions warrant a subdivision in overland flow.

Validation of the Surface Storage and Runoff Model

In design the hydraulic model (Figure 2) suggests that surface depression storage can be considered a linear reservoir. Chow (1964) describes a linear reservoir as a fictitious reservoir in which the storage $S$ is directly proportional to the outflow $Q$, or

$$S = KQ \quad (4.8)$$

where $K$ is a reservoir constant called the storage coefficient. Since the difference between inflow $I$ and outflow $Q$ is the rate of change in storage, the continuity equation is written as

$$I - Q = \frac{ds}{dt} \quad (4.9)$$
Differentiating equation (4.8) and substituting the differential into equation (4.9) gives

\[ K \frac{dQ}{dt} + Q = I \]  

(4.10)

Upon obtaining the complementary function and the particular integral for equation (4.10) and setting the initial conditions that \( Q = 0 \) when \( t = 0 \), the following equation for outflow can be derived:

\[ Q = I (1 - e^{-t/K}) \]  

(4.11)

In the above equation, as \( t \) approaches \( \infty \), inflow \( I \) approaches outflow \( Q \). When the inflow terminates, a similar derivation gives the outflow \( Q \) at time \( t \) in terms of equilibrium discharge \( Q_0 \) as

\[ Q = -Q_0 e^{-T/K} \]  

(4.12)

where \( T = t - T_0 \), which is equal to the elapsed time after inflow termination.

Shen (1963), using an electrical analog, demonstrated that the linear reservoir constant \( K \) is equivalent to resistance times capacitance, \((RC)\). The equilibrium discharge term \( Q_0 \) of equation (4.12) has its equivalent in the \( (V_c - V_{zp})/R \) term of equation (4.6). The latter term defines the ratio between the head differential \( (V_c - V_{zp}) \) and surface friction for transient storage.
By substituting $I_0$ for $I$, and $RC$ for $K$, in equation (4.11), equation (4.13) which describes the active portion of surface runoff in terms of electrical theory was derived:

$$Q = I_0(1-e^{-t/RC})$$

(4.13)

Substitution in equation (4.12) of $RC$ for $K$ and $(V_c-V_{zp})/R$ for $Q_0$ results in equation (4.14), which describes the passive or post storm transient decay:

$$Q = \frac{-(V_c-V_{zp})}{R} e^{-T/RC}$$

(4.14)

The two equations when combined define by analogy the transient response of a linear reservoir to a step function input. Equations (4.13) and (4.14) are of identically the same form as equations (4.7) and (4.6), used to describe the interception storage process.

Attempts to fit the above linear storage derivation to the experimental overland flow data of Izzard (1946) resulted in a poor match. This was expected since Izzard's study and that of Mitchell (1962) demonstrated that surface runoff is characterized by a non-linear storage-outflow relationship. The source of this nonlinearity, assuming no infiltration, is the manner in which surface friction or viscous drag varies in response to a number of influencing variables such as surface slope, surface channelization, and surface storage characteristics.

There is no suitable overland flow equation which is applicable to a forested watershed having the confounding characteristics
described above. On the other hand, a hydrograph under certain ideal conditions, namely an abrupt beginning or end to a spatially uniform high intensity storm, contains a signature, unique to a given watershed, which can be used to define the nonlinear manner in which surface friction varies.

The analog circuit of Figure 11 was devised to reconstruct the current vs. voltage relationship or by analogy outflow vs. water depth on Izzard's (1946) test plot. Izzard's data, Table 1, was used to evaluate a nonlinear surface friction value unique for the actual initial conditions of his experiment. The function of the circuit is to input the simulated rainfall into a capacitor simulating the area of the test plot and extract an outflow equal to that observed during the experiment. The result is that a voltage will build up on the storage element equivalent to the water depth on the test plot at corresponding times during the experiment. The scaling values used can be found within the data listing in Appendix B.

Since the depth of water on Izzard's plot at any time is analogous to voltage (V) and the outflow (Q) is analogous to current (I), the I vs. V relationship was developed (Figure 12). A piecewise linear approximation consisting of four connected straight-line segments was superimposed on the rising limb of the I-V relationship derived above. Jensen and Lieberman (1968) give a detailed discussion of piecewise approximation methods.

No significant runoff occurred for approximately two minutes in Izzard's experiment. For this reason the first breakpoint on the
Figure 11. Analog Circuit to Determine Depth of Storage on Izzard's Test Plot From Inflow-Outflow Considerations
Figure 12. Piecewise Approximation to I-V Curve for Izzard's Experiment
Table 1. Izzard's Experimental Data for Overland Flow on a Turf Plot, With a Length of 72 ft., and Slope of 4%

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<th>Time (Min)</th>
<th>Run A I₀ = 1.89 in./hr.</th>
<th>Run B I₀ = 3.60 in./hr.</th>
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<td>(Rise)</td>
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voltage axis is designed to provide a storage equivalent to that which would accumulate in two minutes at an intensity of 3.60 inches per hour. Table 2 provides the necessary information derived from the piecewise approximation to construct a nonlinear diode model for the rising limb of Izzard's experiment. The segment slope values are given in mhos, the reciprocal of resistance, to simplify computation.

Figure 13 shows the analog circuit, with the determined nonlinear diode model in place, used to evaluate Izzard's experiment. The ECAP data listing for this circuit is given in Appendix C. The analog output for two rainfall intensities, 3.60 in./hr. and 1.89 in./hr. is plotted against Izzard's observed values in Figure 14a. The higher intensity event fits well on the rising limb and poorly on the recession limb. The lower intensity event fits poorly on both limbs of the hydrograph. Figure 14b shows the result of fitting the diode model to the recession limb of Izzard's hydrograph. At the higher intensity the rising limb fits moderately well but there is a perfect fit on the recession limb. A better fit also exists for the lower intensity event.

To get a better fit for the entire hydrograph it would be possible to construct a two-mode diode model which would operate selectively on the rising and falling hydrograph limbs. This would be much the same as the procedure used by Golany and Larson (1971) who used variable Manning's n values on both the rising and falling hydrograph limbs.

The anomaly between the rising and falling limbs is normally attributed to turbulent flow during the runoff event. It has been
### Table 2. Slope Values for Piecewise Approximation to I-V Curve

<table>
<thead>
<tr>
<th>Line</th>
<th>Slope</th>
<th>Line</th>
<th>Slope</th>
<th>Breakpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mohs)</td>
<td>(mohs)</td>
<td></td>
<td>(volts)</td>
<td></td>
</tr>
<tr>
<td><strong>Rising Limb</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>2.44x10^{-5}</td>
<td>A</td>
<td>2.44x10^{-5}</td>
<td>.60x10^{-2}</td>
</tr>
<tr>
<td>2-3</td>
<td>11.90x10^{-5}</td>
<td>B</td>
<td>9.46x10^{-5}</td>
<td>1.85x10^{-2}</td>
</tr>
<tr>
<td>3-4</td>
<td>40.25x10^{-5}</td>
<td>C</td>
<td>28.35x10^{-5}</td>
<td>3.37x10^{-2}</td>
</tr>
<tr>
<td>4-5</td>
<td>103.20x10^{-5}</td>
<td>D</td>
<td>62.95x10^{-5}</td>
<td>3.75x10^{-2}</td>
</tr>
<tr>
<td><strong>Recession Limb</strong></td>
<td>a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>2.44x10^{-5}</td>
<td>A</td>
<td>2.44x10^{-5}</td>
<td>E=-0.60x10^{-2}</td>
</tr>
<tr>
<td>2-3</td>
<td>11.20x10^{-5}</td>
<td>B</td>
<td>8.76x10^{-5}</td>
<td>E=-1.47x10^{-2}</td>
</tr>
<tr>
<td>3-4</td>
<td>20.50x10^{-5}</td>
<td>C</td>
<td>9.30x10^{-5}</td>
<td>E=-1.90x10^{-2}</td>
</tr>
<tr>
<td>4-5</td>
<td>32.10x10^{-5}</td>
<td>D</td>
<td>11.60x10^{-5}</td>
<td>E=-3.05x10^{-2}</td>
</tr>
</tbody>
</table>

a. Only the slope segments for rising limb of Izzard's (1946) experimental hydrograph are shown on Figure 12.
Figure 13. Surface Storage and Runoff Analog Used to Simulate Izzard's Experiment - Values Derived From the I-V Curve of Figure 12.
Figure 14. Izzard's (1946) Hydrograph Computed Using Surface Storage and Runoff Analog
suggested by Riley et al. (1967) that the effects of momentum are negligible for a watershed system unless relatively high velocities are encountered such as in channel flow. It may well be that inertial considerations are the actual cause of the discrepancy between the rising and falling limbs of Izzard's hydrograph. An interesting test of this would be to include an inductance in the output of the circuit of Figure 13, since inductance has a hydraulic equivalent in inertia. The result of this experiment would be to retard outflow during the active portion of the event and to accelerate it during the relaxation portion of the event.

Surface Storage and Runoff Field Application

The storm selected for calibration of the interception analog was also used to calibrate the surface storage and runoff analog. A modified version of the surface storage and runoff analog was used to determine an outflow versus storage depth relationship, the I-V curve of Figure 15, for the calibration storm.

The modification to the analog consisted of replacing diode model Z2 with a dependent current source. This current source is driven by an independent current source programmed to generate a time varying function describing the observed outflow from the calibration storm. The depth of storage on the surface storage element was computed using ECAP as was done for Izzard's experiment. The observed outflow data were then plotted against the computed storage depths (Figure 15).

The I-V curve representing the rising limb of the hydrograph represents the nonlinear characteristics of the Arkansas watershed, and
Figure 15. Piecewise Approximation to the I-V Curve of the Calibration Storm
for this reason was fitted with a piecewise approximation. The I-V curve for the recession limb was disrupted by short bursts of rainfall and as a result was not as suitable to fit with a piecewise approximation. The dashed line segments through portions of the recession limb show how the curve of the rising limb will overlay the portions of the recession limb where rainfall input was zero. This alignment is an apparent departure from the experiment conducted using Izzard's data where the shape of the rising and falling limbs of the hydrograph were different.

The depth to which surface storage accumulates is indicated on the voltage axis of Figure 15 by the initiation of outflow. This depth includes water which would normally infiltrate, but at this stage in the model development, infiltration losses are not taken into consideration.

Voltage breakpoints for the diode model were determined at the intersections of the straight line segments used in the piecewise approximation of the I-V curve. A vertical line dropped from each intersection point to the voltage axis defines a corresponding breakpoint voltage. All breakpoint voltages were referenced to the intersection of the I-V curve and the voltage axis.

A lack of knowledge concerning the actual amount of surface depression and litter storage complicates the exact location of the breakpoint voltages. Helvey (1967) suggested a value of 0.1 inches for the latter. An additional 0.1 inches was added to account for surface depression storage. Table 3 lists the slope and breakpoint values determined from the above considerations. A constant 3 volts was added
to each breakpoint value to compensate for the elevated position of surface storage due to battery \( E_2 \) of the surface storage and runoff analog.

Table 3. Slope Values for Piecewise Approximation to Hydrograph of the Calibration Storm

<table>
<thead>
<tr>
<th>Line</th>
<th>Slope (mhos)</th>
<th>Line</th>
<th>Slope (mhos)</th>
<th>Breakpoint (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>8.44x10^{-3}</td>
<td>A</td>
<td>8.44x10^{-3}</td>
<td>3.02080</td>
</tr>
<tr>
<td>2-3</td>
<td>13.60x10^{-3}</td>
<td>B</td>
<td>5.16x10^{-3}</td>
<td>3.03100</td>
</tr>
<tr>
<td>3-4</td>
<td>20.30x10^{-3}</td>
<td>C</td>
<td>6.70x10^{-3}</td>
<td>3.04480</td>
</tr>
<tr>
<td>4-5</td>
<td>21.30x10^{-3}</td>
<td>D</td>
<td>11.00x10^{-3}</td>
<td>3.05780</td>
</tr>
</tbody>
</table>

The data from Table 3 were incorporated in nonlinear diode model, \( Z_2 \), of the surface storage and runoff analog. Then the combined interception and surface runoff analogs were used to analyze the calibration storm. With the exception of surface depression storage no parameter adjustment was necessary to obtain the computed hydrograph of Figure 16. The computed hydrograph is larger than the observed since at this point infiltration has not been included. Nevertheless a close similarity in shape between the simulated and observed hydrographs exists. The initial high intensity rainfall of the calibration storm generated no runoff because interception losses absorbed this input.
Figure 16. Comparison of Observed Hydrograph of the Calibration Storm With Computed Hydrograph, Infiltration Excluded
Infiltration and Subsurface Storage

Empirical evidence by Horton (1933) and Holtan (1961) indicates that the infiltration rate is a function of the soil moisture deficit, assuming that the supply rate exceeds the potential or instantaneous infiltration rate. Infiltration and how it varies with time and antecedent soil moisture conditions is one of the most important processes within a watershed. The significance of infiltration in relation to runoff can readily be appreciated since the magnitude of surface runoff is highly dependent on infiltration losses.

The infiltration process moves water through the surface into subsurface storage. The water in subsurface storage is redistributed by leakage losses to groundwater storage and as interflow if the soil water content is high. External energy stresses on the forest vegetation causes plants to transpire water which puts an additional drain on the water in subsurface storage.

Infiltration is dependent upon a number of factors; e.g., surface conditions such as vegetal cover, slope, the water transmission characteristics of the soil, and antecedent soil moisture. The mechanics of the infiltration process for a forested watershed are not fully understood and certainly not the interactions of the above named infiltration modifying parameters. Field study of infiltration has proven difficult, primarily because it is difficult to monitor the process without disrupting the system, and the system is continually changing in a dynamic manner.
An illustration of the infiltration process as presently understood is shown in Figure 17. It is assumed that the rainfall available for infiltration falls at a constant rate. Two conditions will exist depending on whether the applied rainfall rate \( (I_0) \) is less than or greater than the initial instantaneous infiltration rate \( (f_0) \). If the rainfall rate is less than \( f_0 \), infiltration will proceed at the rainfall rate \( (I_0) \) until the infiltration equals \( I_0 \) due to a buildup of soil water. At this point the infiltration rate tends to decay exponentially toward a final constant value \( (f_c) \). At the end of the rainfall event, as shown in the illustration, infiltration will continue at the same decay rate until surface depression storage is depleted.

Should the rate of applied rainfall exceed or be equivalent to the instantaneous infiltration rate, the exponential rate of decay will commence immediately. In this case the infiltration curve remains the same for all rainfall rates in excess of \( f_0 \). On the other hand all infiltration curves where \( I_0 \) is less than \( f_0 \) will be separated. Convergence at the final \( f_c \) values will eventually take place but only when the system is approaching an equilibrium condition. This is because the infiltration rate has been made a function of the soil moisture deficit, and for the lower rainfall rates the soil moisture level will be charging at proportionately lower rates, resulting in different infiltration rates at any given time.
Figure 17. Infiltration Process Model
The Infiltration and Subsurface Storage Analog Circuit

To fully demonstrate the infiltration analog it was necessary to combine it with the surface storage and runoff analog (Figure 18). A complete ECAP data listing for the combined analogs can be found in Appendix D. The analog deviates to some extent from a truly passive electrical circuit, yet for the most part, the descriptive advantages of a passive circuit are maintained.

The infiltration analog has been designed to compare the infiltration source, rainfall or surface depression storage, with the potential infiltration rate. When the comparison has been made, the smaller value of the two is used to drive the infiltration process. The following discussion will describe how this is accomplished.

The simulated rainfall, as modified by the forest canopy, can be monitored in either branch B19 or B21. Since under certain conditions small reverse charges will flow through branch B21, branch B19, the output of the interception analog, is used exclusively in the process of controlling other branches, e.g., the control of the comparator circuit. Branch B21 is a suitable point to monitor the surface charge inflow for transfer to an integrator. The charge passing through branch B21 is stored on capacitor C2, of branch B24, which is analogous to a reservoir consisting of litter and depression storage.

The simulated rainfall input monitored in branch B19 is compared with the potential infiltration rate determined by the potential difference between the top of battery E2 (node 12) and a hypothetical soil moisture level on the soil moisture storage element of branch B35.
to determine which is larger. The potential difference is monitored across the 10 megohm resistance of branch B39. The resistance value is large so that no charge will flow from surface storage into subsurface storage, capacitor C₃.

The initial charge condition of capacitor C₃ determines the antecedent condition of the subsurface storage and thereby the potential infiltration rate. Initial charge conditions are easily set in the ECAP data listing. The capacitor can also be proportionately adjusted in size to simulate other porosity values; e.g., reducing the size of capacitor C₃ will result in a correspondingly faster rising potential at node 14, and thus during an infiltration cycle a more rapid decrease in the potential difference between nodes 12 and 14.

The potential difference is converted to drive the dependent current source I₁₆ of branch B₄₆ using a selected conversion factor. For example, if the potential difference between nodes 12 and 14 is 0.5 volts and the voltage to current conversion factor is 1.0x10⁻³, dependent current source I₁₆ would be driven at a rate of 5x10⁻⁴ amperes. The conversion is accomplished by means of transfer card T₁₆. The conversion factor is electrically identified as transconductance (GM) and has an analogy to the permeability of the soil system. The dependent current source of branch B₄₆ forms one leg of the comparator bridge. The simulated rainfall input is transferred from branch B₁ₙ to the other leg of the bridge, dependent current source I₁₇ of branch B₄₈. This current transfer is accomplished with T-card T₁₇.
The comparator circuit develops voltages at nodes 18 and 19 which correspond respectively to the magnitude of the potential infiltration rate and the rainfall rate. Branch B50 of this circuit is a switch (S12) which responds to the voltage differential and in turn controls the potential infiltration (PI) generator and the rainfall transfer function (RTF) generator.

The PI generator consists of branches B40, 41, and 42. Branch B40 is dependent current source controlled also by the potential difference across branch B39 via T-card T10, a transconductance. Branch B40 is initially shorted out internally as is branch B41, so that the voltage across branch B42 will be insignificant. Branch B42 is a dummy branch used as a "from" branch for T-cards T11 and T12. These T-cards will perform a voltage to current transfer from the PI generator to dependent current sources in branches B23 and B34. Current is drawn out of surface storage by current source I12 to ground and equally introduced into subsurface storage through current source I13 at the PI rate providing that the PI generator has been enabled.

The RTF generator is controlled by a current transfer from branch B19 via T-card T13, to dependent current source I15, which is also initially shorted. Branch B44 is initially open. Branch B45 serves the same function as the dummy branch B42 of the PI generator, and that is to control the dependent infiltration current sources, I12 and I13, of branches B23 and B34, via T-cards T14 and T15.
The control sequence of the PI and RTF generators is dependent on the state of three sense switches, S4, S12, and S6. Sense switch S4 of branch B26 is reverse biased by battery $E_3$ to turn on when surface litter storage requirements have been met; this in turn switches on the dependent current sources in the PI and RTF generators, branches B40 and B43. The RTF generator will now cause infiltration to proceed at the rainfall rate. At the same time a comparison of the PI rate against the rainfall rate is being made. If rainfall exceeds the PI rate, switch S12 is enabled and shorts out branch B44 of the RTF generator and opens branch B41 of the PI generator causing infiltration to take place at the PI rate. The third sense switch S6 monitors the development of surface depression storage, in excess of litter layer storage. If an excess develops, the switch will be activated and short out branch B46 of the comparator. This will cause the PI generator to stay on even though the rainfall rate drops below the PI rate. When surface depression storage is depleted, the infiltration rate will again be reduced to the rainfall rate.

The subsurface storage element, capacitor $C_3$, is subject to a constant leakage component through branch B36 which represents a groundwater or deep seepage loss component. The magnitude of this loss is only a few thousandths of an inch per hour. It becomes significant when long term considerations are being made, i.e., over a period of a week at a rate of 0.002 inches per hour, 0.336 inches of soil moisture could be lost. A constant transpiration component can also be included in this branch if so desired.
An interflow provision is included in the analog consisting of one linear resistance, branch B37, and a biased switch, S11. The switch is biased to turn on when some specified soil moisture condition is reached, perhaps near field capacity.

Validation of the Infiltration Analog

Horton (1939) proposed the following infiltration equation:

\[ f = (f_0 - f_c)e^{-kt} + f_c \]  
\[(4.16)\]

where \( f \) = the infiltration rate at time \( t \),
\( f_0 \) = the initial infiltration rate,
\( f_c \) = the final infiltration rate,
\( e \) = base of natural logarithms,
\( k \) = a constant.

Equation (4.16) is an empirical relationship used by Horton to describe the observed phenomenon that the infiltration rate decreases exponentially toward some final value as long as the supply rate is greater than the infiltration rate.

Mathematically the infiltration analog has an identical form to Horton's infiltration equation as follows:

\[ i = (i_0 - i_c)e^{-t/RC} + i_c \]  
\[(4.17)\]

where \( i \) = rate of flow of charge at time \( t \),
\( i_0 \) = initial charge inflow,
\( i_c \) = final rate of charge flow,
\text{e} = \text{base of natural logarithms},
\text{R} = \text{resistance},
\text{C} = \text{capacitance}.

A simplified version of the infiltration analog circuit is presented in Figure 19 to physically illustrate the relationship between equations (4.16) and (4.17). Battery E2 of the figure serves to elevate the surface storage held on capacitor C_1 with respect to subsurface storage. A potential difference exists across resistor R_1 resulting from the voltage of battery E2 and that on the ungrounded side of capacitor C_2, subsurface storage. Charge will flow through resistor R_1, but the magnitude of this charge flow will be insignificant, amounting to only a few ten-thousandths of an inch per hour at a maximum, because R_1 is large, 10 megohms.

The transconductance used to transfer the potential difference across resistor R_1 is essentially a current multiplier, through which the small current in resistor R_1 is amplified and used to control charge withdrawal from surface storage through dependent current source I_1 to ground. This is accomplished via T-card T_1. A second T-card, T_2, is used to simultaneously reintroduce this loss into subsurface storage.

Since transconductance is a function of the potential across R_1, as capacitor C_2 charges up, the simulated infiltration value (i) will decrease exponentially from its initial value i_0. It is necessary, of course, to shut down the circuit if no charge exists to be infiltrated from capacitor C_1. Outflow (i_c) at any time is a function of the voltage on capacitor C_2 divided by the value of resistance R_2. The
Figure 19. A Simplified Version of the Infiltration Analog
circuit will eventually stabilize during a charging cycle at a point where inflow is equivalent to the outflow through resistor R2.

The initial infiltration component \( i_0 \) has been shown to be equal to the voltage across resistor R1 times a transconductance or GM factor,

\[
i_0 = (E2 - VC2)GM
\]

The final infiltration value \( i_c \) can be written as

\[
i_c = \frac{VC2}{R2}
\]

Insertion of these two terms into equation (4.17) gives:

\[
i = [(E2 - VC2)GM - \frac{VC2}{R2}]e^{-t/RC2} + \frac{VC2}{R2}
\]  \hspace{1cm} (4.18)

The only difference between the equations (4.16) and (4.17) is in the \( k \) term. Setting \( k = 1/RC2 \) makes the two identical, where \( R = k1/GM \). In the electrical system GM is transconductance and \( C2 \) represents capacity, while \( k1 \) is a constant representing the current amplification factor. Philip (1956) has suggested that Horton's \( k \) has no physical significance but in the electrical analogy the apparent relationship is that \( k \) is equivalent to the ratio of conductivity to capacitance or by analogy permeability to porosity.

To demonstrate how the infiltration analog responds to increasing rainfall intensities and changes in antecedent soil moisture, a number of tests were conducted using the analog. To conduct these tests, the scaling applicable to the Arkansas watershed was used. The rainfall generation circuit of the interception analog was modified
slightly to generate a step function output to simulate abruptly starting and stopping rainfall.

The circuit modification consisted of changing switch S1 to control branch B2. Branch B2 in turn has a double valued dependent current source, the state of which is controlled by switch S1. The state of switch S1 is controlled by the direction of the current from independent current source I1 programmed to have nearly instantaneous rise and fall times. A data listing illustrating this modification can be found in Appendix B.

The manner by which infiltration on the watershed responds to step increases in simulated rainfall is illustrated in Figure 20. The soil porosity was arbitrarily considered to be 25% of the 36 inch soil layer. Thus the capacitor representing the watershed subsurface storage area was scaled down by setting its value to 17,750 micro-farads, or one-fourth of the original value 71,000 micro-farads, which represented the total watershed area. The scaled down capacitance is equivalent to a storage capacity of 9 inches.

An antecedent moisture condition of 50% or 1.5 volts was initially put on the capacitor, leaving a moisture deficit of 4.5 inches. The initial infiltration value \( f_0 \) was set at 0.90 inches/hour, with a final rate of 0.004 inches/hour. At a simulated 0.60 inch/hour rainfall rate, infiltration proceeded at the same rate until the soil water level built up sufficiently to cause the infiltration rate to start decreasing exponentially. Although the simulated rainfall event ends abruptly, infiltration continues until surface detention and
Figure 20. Infiltration Rate Response to Simulated Step Increases in Rainfall Intensity
depression storage are depleted. At higher rainfall intensities there are greater rainfall excesses and as a result more time is required to deplete surface storage.

The 1.20 and 1.80 inch/hour events exceed the maximum potential infiltration rate. They therefore have a common infiltration curve except that at the 1.80 inch/hour rate, surface depression depletion takes longer. This curve is different from the infiltration rate curve for the 0.60 inch/hour event because at the higher intensities, soil storage is being filled at its maximum rate.

Figure 21 illustrates the effect of antecedent charge on the infiltration rate. The rainfall intensity was held at 1.80 inches/hour for 5 hours for each simulated antecedent charge. The initial infiltration rate for zero antecedent charge, 1.80 inch/hour, is the result of a $6.13 \times 10^{-4}$ transconductance and a 3 volt or foot soil potential. With higher antecedent charge conditions and correspondingly smaller infiltration rates, larger surface storages accumulate which require greater post-event infiltration periods.

In addition to the above, other experiments may be conducted by varying model parameters. For example, the transconductance factor will change the value representing soil conductivity, and thereby the slope of the infiltration curve.

The Field Application of the Infiltration Analog

Data provided by Rogerson (1971) indicated that the soil mantle of the Arkansas watershed averages about three feet in depth, with a maximum water holding capacity of 13.2 inches. This corresponds to a
Figure 21. Infiltration Rate Response to Antecedent Moisture
porosity of 35.8%. Thus the capacitance representing soil moisture storage is 35.8% of 71,000 micro-farads or 26,050 micro-farads. The average soil moisture content on the date of the calibration storm was determined from soil water data to be about 11.8 inches.

Since the maximum water holding capacity corresponds to a 3 volt potential on the subsurface storage element, the initial voltage (Va) needed to establish the correct antecedent charge on capacitor $C_2$ can be determined from the following relationship:

$$Va = \frac{3.0 \times 11.8}{13.2}$$

The value computed for $Va$ was 2.68 volts.

It was necessary to determine the initial infiltration rate by trial since no infiltration data was available. This was accomplished by the trial selection of a transconductance value. The value selected was $3.5 \times 10^{-4}$, which corresponds to an initial infiltration rate of about $1.11 \times 10^{-4}$ amperes or about 0.068 inches per hour.

The calibration storm was previously tested on the interception and surface components of the analog model. To evaluate the effect of the infiltration analog on the hydrograph of the calibration storm, all three of the analog circuits were operated together.

Since analysis of the hydrograph of the calibration storm gave no indication that interflow existed, the interflow path was blocked by a 10 megohm resistance. The groundwater leakage value was roughly estimated during periods of no rain from soil water depletion curves to be about 0.002 inches per hour. To account for transpiration this value
was doubled. The combined groundwater and transpiration components were not significant for the period of the storm. However, over longer periods, particularly between storms, the steady loss attributed to these factors does become an important consideration. The fact that this model has the capability to adjust dynamically to these losses is demonstrated in the next chapter where a month's water balance is computed for the Arkansas watershed based on the analog as fitted to the calibration storm.

The result of adding the infiltration circuit to the interception and surface analog circuits is shown in Figure 22. The observed and computed hydrographs show close agreement in both shape and timing. The total volume of runoff for the computed hydrograph was also very close to that of the observed hydrograph.
Figure 22. The Observed and Fitted Hydrographs for the Calibration Storm With the Infiltration Analog in Operation
CHAPTER 5

THE APPLICATION OF THE INTEGRATED ANALOG CIRCUIT TO THE ARKANSAS WATERSHED

To test the uniqueness of the parameters derived during the calibration of the watershed analog to the Arkansas watershed, an independent storm which occurred five days earlier on the same watershed was analyzed. This storm is herein identified as the test storm. The only parameter adjustments made were to adjust the antecedent storages to coincide with values determined from field data.

The interception and surface antecedent storages were set at zero and 0.02 inches respectively. The surface value of 0.02 inches was used on the assumption that the litter layer will not completely dry out and it also serves to electrically pad switch S5 which monitors surface depression storage. Although the soil moisture deficit as computed from field data on the date of the test storm was 1.33 inches, a correction to 1.59 inches was made to correspond with the 0.26 inch adjustment required to get a good fit between the observed and computed hydrographs for the calibration storm.

Once these changes were implemented the rainfall data for the test storm was input to the model and an ECAP analysis of the analog performed. The closeness with which the analog model predicted the runoff resulting from the test storm is illustrated in Figure 23. The
Figure 23. The Observed and Computed Hydrographs for the Test Storm Used to Evaluate the Uniqueness of the Parameters Derived for the Arkansas Watershed
recession curve on the computed hydrograph of the calibration storm was nearly a perfect match with the observed recession curve, while for the test storm considerable separation can be observed at the end of the recession curves of both of the storm peaks.

Subsequent analysis of this discrepancy revealed that reducing the soil moisture deficit corrected the separation in the recession curves but resulted in even a larger error on the leading edge of the hydrograph. This led to a reexamination of the calibration storm where it was found that increasing surface depression storage or increasing the initial infiltration rate had the same effect on the leading edge of the hydrograph. The higher initial infiltration rates charge up the subsurface early, resulting in lower rates toward the end of rainfall events, lengthening the recession curves. The above conclusions were tested by increasing the initial infiltration rate for the analysis of the test storm, resulting in a near perfect hydrograph when compared with the observed hydrograph for the test storm.

To demonstrate the dynamic manner in which the integrated analog circuits operate relative to the physical system they describe, rainfall data for the month of December, 1968, for the Arkansas watershed were input using the parameter values derived from fitting the calibration storm. The antecedent moisture conditions were adjusted to those observed at the beginning of the month. The selection of a winter month was primarily done for two reasons, (1) antecedent moisture conditions are such that runoff occurs, and (2) evapotranspiration rates are thought to be nominal.
Table 4 was devised to provide a summary of the twenty-one model variables. Comments associated with each variable explain its source and derivation. A complete ECAP data listing for the analog model can be obtained by combining Appendicies A and D. The listing contains the parameter settings and storm data used to evaluate the calibration storm.

Since an adequate evapotranspiration function could not be defined, the model variables associated with the ET generator were replaced with constant values. The main ET generator was set for a constant 0.001 inch per hour evaporation rate which dropped to zero during periods of rainfall. This same rate was used for the evaporation of post-storm litter layer storage. Interception storage was evaporated at a rate 10 times greater than the value selected for litter storage evaporation.

The interflow provision in the analog was not used. Of the remaining sixteen variables only two, surface depression storage and the soil conductivity factor, required repetitive trial substitutions, to fit the calibration storm. If infiltration data had been available for the watershed, the soil conductivity factor could have been evaluated as discussed earlier in the section on the infiltration analog. An initial estimate based on the assessment of the field data provided a good starting point for these two values. The remaining parameter values were derived directly or estimated from the watershed field data.

The water balance computed for the month of December, 1968, on the Arkansas watershed is plotted as a continuous event on Figure 24.
### Table 4. Parameter Evaluation List

<table>
<thead>
<tr>
<th>Hydrologic Component</th>
<th>Circuit Variable</th>
<th>How Derived</th>
<th>Parameter Value Used</th>
<th>Corresponding Physical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Rainfall</td>
<td>I2</td>
<td>Field data</td>
<td>Variable</td>
<td>Rainfall in/hr</td>
</tr>
<tr>
<td>(2) Evapotranspiration</td>
<td>I4</td>
<td>Estimated ET constant</td>
<td>1.63x10^{-6} amp during periods of no rainfall</td>
<td>0.001 in/hr</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Interception</td>
<td>T2 -- I8</td>
<td>Estimated fractional</td>
<td>10 times the ET constant</td>
<td>0.010 in/hr</td>
</tr>
<tr>
<td>evaporation</td>
<td></td>
<td>part of ET function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Surface evaporation</td>
<td>T4 -- I4</td>
<td>Same as ET constant</td>
<td></td>
<td>0.001 in/hr</td>
</tr>
<tr>
<td>c. Transpiration</td>
<td>T6 -- I13</td>
<td>Included in groundwater</td>
<td></td>
<td>About 0.002 in/hr</td>
</tr>
<tr>
<td>loss</td>
<td></td>
<td>component</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) Interception</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Canopy input</td>
<td>T7 -- I7</td>
<td>Estimated 85% canopy area</td>
<td>85% of gross rainfall</td>
<td></td>
</tr>
<tr>
<td>b. Canopy evaporation</td>
<td>T7 -- I7</td>
<td>Field data 5% total</td>
<td>BETA = 0.80</td>
<td>5% loss in gross rainfall</td>
</tr>
<tr>
<td>constant</td>
<td></td>
<td>rainfall</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. Canopy storage</td>
<td>C1</td>
<td>Field data</td>
<td>6.0352x10^{-2}</td>
<td>60,382 ft.²</td>
</tr>
<tr>
<td>Hydrologic Component</td>
<td>Circuit Variable</td>
<td>How Derived</td>
<td>Parameter Value Used</td>
<td>Corresponding Physical Value</td>
</tr>
<tr>
<td>----------------------</td>
<td>-----------------</td>
<td>-------------</td>
<td>----------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>d. Canopy storage volume control</td>
<td>E1</td>
<td>Field data 7.3% rainfall total</td>
<td>E1 = -0.0172300 volts</td>
<td>1040 ft.³</td>
</tr>
<tr>
<td>e. Direct throughfall</td>
<td>T8 -- I9</td>
<td>15% of total rainfall</td>
<td>BETA = 0.15</td>
<td>15% of gross rainfall</td>
</tr>
<tr>
<td>f. Indirect throughfall time lag</td>
<td>R1</td>
<td>Estimated</td>
<td>5.0 ohms</td>
<td>A 5 min. lag in time for an instantaneous drop in rainfall to 37% of the initial value</td>
</tr>
<tr>
<td>(4) Surface storage</td>
<td>C2</td>
<td>Watershed area</td>
<td>$7.1 \times 10^{-2}$ micro-farads</td>
<td>71,000 ft.²</td>
</tr>
<tr>
<td>a. Litter storage depth</td>
<td>E3</td>
<td>Estimated from literature</td>
<td>E = -0.00834000 volts</td>
<td>0.1 inches</td>
</tr>
<tr>
<td>b. Depression storage depth</td>
<td>E6-E3</td>
<td>Trial</td>
<td>E = -0.00834000 volts</td>
<td>0.1 inches</td>
</tr>
<tr>
<td>c. Surface friction</td>
<td>B29, 30, 31, 32, G1, G2, G3, G4, G6, E7, E8, E9</td>
<td>Outflow vs. depth relationship from field data</td>
<td>G1 = 8.44 x 10⁻³ mhos, G2 = 5.16 x 10⁻³ mhos, G3 = 6.70 x 10⁻³ mhos, G4 = 11.00 x 10⁻³ mhos</td>
<td>Nonlinear surface friction</td>
</tr>
<tr>
<td>Hydrologic Component</td>
<td>Circuit Variable</td>
<td>How Derived</td>
<td>Parameter Value Used</td>
<td>Corresponding Physical Value</td>
</tr>
<tr>
<td>----------------------</td>
<td>------------------</td>
<td>-------------</td>
<td>----------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>(5) Soil moisture</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Soil depth</td>
<td>E2</td>
<td>Field data</td>
<td>E = 3.0 volts</td>
<td>Average soil depth of 3 feet</td>
</tr>
<tr>
<td>b. Storage</td>
<td>C4</td>
<td>Field data</td>
<td>2.605x10^{-2}</td>
<td>Maximum storage capacity of 13.21 inches</td>
</tr>
<tr>
<td>c. Antecedent storage</td>
<td>C4</td>
<td>Field data</td>
<td>E=-2.68 volts</td>
<td>Antecedent storage 11.81 inches</td>
</tr>
<tr>
<td>d. Groundwater loss</td>
<td>B36</td>
<td>Estimated from field data soil moisture depletion curves</td>
<td>R=4.60x10^5</td>
<td>0.004 in/hr</td>
</tr>
<tr>
<td>and constant</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>transpiration loss</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. Interflow</td>
<td>B37 -- E10</td>
<td>Estimation based on field capacity</td>
<td>No indication that interflow existed for the calibration storm</td>
<td>---</td>
</tr>
<tr>
<td>f. Soil water</td>
<td>T-10 &amp; T-16</td>
<td>Trial (if infiltration data existing can be evaluated)</td>
<td>GM=3.5x10^{-4}</td>
<td>Initial infiltration rate will be 0.06 in/hr</td>
</tr>
</tbody>
</table>
This prediction is based on the assumed uniqueness of the parameter values derived during the process of fitting the model to the calibration storm. It is evident from the closeness between the values computed for the soil moisture deficit against those observed in the field that a close physical analogy between the watershed analog and the watershed hydrologic regime exists. The dynamic behavior of the watershed analog during storm intervals is indicated by the readjustment of the antecedent charge condition in the various storage areas.

The first event of the month occurred on the 12th of December with no runoff or soil moisture addition in evidence. The bulk of the rainfall for this event was lost to interception and litter layer storages. These storages were depleted after the rainfall event at constant rates, 0.01 inch per hour for interception storage and 0.001 for litter layer storage.

The observed soil moisture deficit on the 2nd of December, 1968, was 0.74 inches. The estimated soil moisture and transpiration depletion constant of 0.004 inch per hour proved consistent with the observed soil moisture deficit of a week later. Table 5 lists the soil moisture data as computed from field data for the month. To calculate the soil moisture deficit the average maximum soil water for the 6 inch to bedrock profile was calculated from 5 neutron access wells for May 15, 1968. This value was 10.9 inches. The average maximum for the 6 to 12 inch profile was 2.3 inches. This value was assumed to be the same for the 0 to 6 inch profile which was not measured, therefore the average maximum soil moisture was considered to be 13.2 inches. Based
on this value, weekly soil moisture deficit values for the month were computed.

The simulated rainfall event of December 12, 1968, resulted in a recharge of subsurface storage and also 0.05 inches of runoff while during the actual event no runoff was observed. This was the result of an initial infiltration value which was too small, as discussed earlier. The event of December 18th resulted in a similar addition to soil moisture storage and a small runoff component. The first major storm, the test storm, occurred on December 21st. This storm resulted in a 1 inch decrease in the soil moisture deficit. The runoff component was only slightly in error from that observed. (See Figure 23.) After the end of the event it was observed in Figure 24 that the soil moisture depletion curve passes close to the observed field data point for December 23rd.

The second major storm of the month occurred on December 26th. This storm has been previously identified as the calibration storm. At the time this storm began the soil moisture had depleted to only 0.84 inches. This was a smaller value than was determined when the model was fitted to this storm. As a result there was a significant error in runoff for the storm when analyzed using this low soil moisture deficit. The total runoff for the month was in error by about 24% of which 19% occurred during the last event.

Assuming that the field soil moisture data are correct, a possible explanation can be that at high soil moisture contents inter-flow occurs. The result would be a steepening of the depletion curve
Table 5. Computed Weekly Soil Moisture Deficit for Month of December, 1968

<table>
<thead>
<tr>
<th></th>
<th>12/2/68</th>
<th>12/9/68</th>
<th>12/17/68</th>
<th>12/23/68</th>
<th>12/30/68</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Soil Water for 6&quot; to Bedrock Profile</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>10.49</td>
<td>10.01</td>
<td>10.31</td>
<td>10.48</td>
<td>10.47</td>
</tr>
<tr>
<td>Deficit</td>
<td>0.42</td>
<td>0.90</td>
<td>0.60</td>
<td>0.43</td>
<td>0.44</td>
</tr>
<tr>
<td><strong>Average Soil Water for 6&quot; to 12&quot; Profile Assumed for 0 to 6&quot; Profile</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1.98</td>
<td>1.87</td>
<td>1.85</td>
<td>2.03</td>
<td>1.92</td>
</tr>
<tr>
<td>Deficit</td>
<td>0.32</td>
<td>0.43</td>
<td>0.45</td>
<td>0.27</td>
<td>0.38</td>
</tr>
<tr>
<td>Total Deficit</td>
<td>0.74</td>
<td>1.33</td>
<td>1.05</td>
<td>0.70</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Maximum Soil Water Average Measured 6" to Bedrock, Profile May 15, 1968: 10.91
Maximum Soil Water Measured 6" to 12", Profile May 15, 1968: 2.30
between the two major events of the 21st and the 26th. There was no obvious surface expression of an interflow component but this does not rule out the likelihood of the loss of soil water as interflow along the downgradient edge of the watershed.

The model has been designed with a built in step function rainfall generator. This feature was used to subject the field system by analogy to hypothetical inputs to examine the Arkansas watershed non-linear response characteristics, such as was done by Amorocho (1963) using a mechanical rainfall simulation device. In Amorocho's experiment a basin consisting of a shallow rectangular metal channel, 120 cm. long by 10 cm. wide, with a slope of about 2 1/2 percent, and containing a thin layer of gravel was used.

One of the tests performed by Amorocho was with a succession of three simulated square wave rainfall inputs for the purpose of observing the magnitude of the resulting departures from linearity in the output. The square wave inputs had a constant amplitude with a 100-second duration and a 50-second interval. It was observed that the three resulting output peaks of the test differed markedly in shape from one another, and that a linear superposition based on the response to a single pulse underestimated the output during the latter part of the sequence.

The nonlinear response in runoff of the Arkansas watershed is quite evident in Figure 25 in response to a series of simulated step function inputs. The interval of separation between each square wave input was made large enough so the successive outputs would not overlap.
Figure 25. The Nonlinear Response of the Arkansas Watershed To Simulated Step-Function Rainfall Inputs
The simulated runoff totals for each of the three peaks were 1.49, 2.11, and 2.48 inches, respectively. A very small interflow component developed during the middle of the second hydrograph. The recession curves for the simulated hydrographs were influenced by the lower infiltration rates toward the end of the test. The indentation in the leading edge of the first hydrograph, creating the characteristic S-shaped hydrograph, is the result of the high initial infiltration rate which decreases with time.

It should be apparent that many problems of hydrologic significance to the Arkansas watershed can now be investigated using the calibrated model. As the season progresses it is expected that prediction attempts would have greater errors because of changes in the physical nature of some variables, e.g., interception storage due to leaf loss or evapotranspiration characteristics.
CHAPTER 6

CONCLUSION

The major intent of this research endeavor has been to develop an electronic watershed analog with a direct physical correspondence to the hydrologic regime of a watershed, capable of synthesizing the hydrologic characteristics of a watershed. The use of digital network simulation methodology has made this possible. In addition many avenues for further model improvement have been opened.

The basic analog unit developed in this dissertation is readily amenable to use as a basic unit in larger lumped-discretized models describing large watershed systems. More powerful and efficient digital network simulation methods, e.g., SCEPTRE (Bowers and Sedore, 1971) are available and have the capacity to treat the basic analog unit in iterative fashion allowing a watershed to be subdivided to compensate for the spatial variability apparent in watershed parameters.

A limited number of the basic analog units can be tied together using ECAP, to compensate for watershed spatial variability, but ECAP is limited in the number of branches, nodes, current sources, and switches. For this reason only two or three descriptive units can be built and tied together. The interaction between units would be informative, since, for example, subsurface storage in adjacent units would interact in a manner analogous to the watershed system. The cost
involved in terms of computer time for analyzing such a large electrical network could prove to be prohibitive. This disadvantage would be offset by the fact that most of the model parameter values can be determined from field data requiring only limited optimization to calibrate the watershed analog to the watershed system under study.

Optimization techniques are available for electrical circuit design which can be used to facilitate calibration of rainfall events for evaluation of undetermined parameter values. An example of such a program is GOSPEL, a General Optimization Software Package for Electrical network design (Huelsman, 1968). The worth of a model may be described as being inversely related to the number of parameters which require optimization to evaluate the model. Using this criterion the analog model described in this dissertation is worthy. The direct physical correspondence shown to exist by this study between the watershed physical system and passive electronic watershed analog is the reason for the good correlation between the watershed empiricisms and the physical laws on which electrical circuit theory is based.

The accuracy of the model makes it excellent for analyzing subtle changes in the hydrologic regime of watersheds. Experimentation related to evapotranspiration effects could be conducted. Also, capability for handling precipitation in the form of snow could be built into the model. In addition, it is possible to define the electrical circuit in mathematical terms, and thereby define the watershed system mathematically.
One of the more exciting areas of application can be in the classroom. The direct analogy and scaling can provide students the opportunity to study watershed interrelationships in a realistic manner. The use of ECAP requires no special training in computer programming. Students need not necessarily understand electronics to use this model but only understand how the parameters of Table 4 effect the model's response. These parameters are directly describable in terms of the physical system.
APPENDIX A

THE INTERCEPTION ANALOG ECAP DATA LISTING

C A PASSIVE ELECTRONIC WATERSHED ANALOG MODEL
C
C ARKANSAS WATERSHED NO. 1, STORM 521, DECEMBER 26, 1968
C
C ECAP TRANSIENT ANALYSIS
C
C
C BRANCH B1 COMBINED WITH INDEPENDENT CURRENT SOURCE I1
C CONSTITUTE A MECHANISM TO GENERATE SINGLE PULSE STEP
C FUNCTIONS OR PERIODIC WAVE TRAINS AS INPUTS TO THE MODEL.
C SWITCH S1 IS USED TO CONTROL A DEPENDENT CURRENT SOURCE
C INSERTED IN PLACE OF THE TIME VARYING FUNCTION OF
C BRANCH B2.
C
B1 N(1,0),R=1
I1 P(15),1,1,1,1,1
S1 B=1,(1),ON

C BRANCH B2 AND I2 CONSTITUTE A CURRENT SOURCE WHICH
C SERVES AS A RAINFALL GENERATOR, BRANCH B3 IS A DUMMY
C LOAD USED AS A CONTROL SOURCE. 0.001 AMPERE IS EQUIVA-
C LENT TO 1 IN./HR./ACRE OF RAINFALL.
C
B2 N(2,0),R=10E6
I2 (15), 0, 2.65E-4, 2.65E-4, 5.87E-4, 3.92E-4,
*1.042E-3, 3.26E-4, 1.30E-4, 6.5E-5, 6.5E-5, 6.5E-5,
*1.30E-4, 6.5E-5, 6.5E-5, 6.5E-5, 0, 0,
*0, 6.5E-5, 6.5E-5, 6.5E-5, 6.5E-5, 6.5E-5,
*1.95E-4, 1.95E-4, 1.30E-4, 6.5E-5, 6.5E-5, 6.5E-5,
*1.3E-4, 1.3E-4, 1.3E-4, 6.5E-5, 6.5E-5, 6.5E-5,
*6.5E-5, 1.3E-4, 6.5E-5, 6.5E-5, 6.5E-5, 6.5E-5,
*0, 1.3E-4, 6.5E-5, 6.5E-5, 6.5E-5, 6.5E-5,
*6.5E-5, 1.3E-4, 1.3E-4, 6.5E-5, 6.5E-5, 6.5E-5,
*6.5E-5, 1.3E-4, 1.76E-3, 1.24E-3, 1.63E-3, 1.892E-3,
*5.22E-4, 6.5E-5, 0, 6.5E-5, 0, 1.95E-4,
*2.61E-4, 3.92E-4, 4.56E-4, 6.5E-5, 0, 0,
BRANCH B4 AND I4 ARE USED TO GENERATE THE MAIN EVAPOTRANSPIRATION FUNCTION. BRANCH B5 IS A DUMMY LOAD USED AS A CONTROL SOURCE FOR THE FOLLOWING EVAPOTRANSPIRATION COMPONENTS.

BRANCHES B6 AND B7 FORM THE DEPENDENT CURRENT SOURCE USED TO EVAPORATE OR REMOVE CHARGE FROM THE CANOPY. THE CONTROL IS FROM THE MAIN ET FUNCTION GENERATOR VIA T-CARD T1 AND CARD T2 WHICH CAUSE AN OUTFLOW THROUGH BRANCH B13. A SENSING SWITCH (S2) CONTROLLED BY BRANCH B15 WILL SHORT OUT THE DEPENDENT CURRENT SOURCE OF BRANCH B6 IF NO CHARGE IS MONITORED IN CANOPY STORAGE.

BRANCHES B8 AND B9 ALONG WITH T-CARDS T3 AND T4 CONSTITUTE THE CIRCUIT FOR SURFACE EVAPORATION CONTROL.
T3 B(5,8), BETA=1
B8 N(0,5), R=(0.01, 10E6)
B9 N(5,0), R=10E2
T4 B(9,22), BETA=1
C
C BRANCHES B10 AND B11 ALONG WITH T-CARDS T5 AND T6 FORM
C THE CIRCUIT TO INITIATE TRANSPIRATION LOSSES AS A Func-
C TION OF THE MAIN ET GENERATOR.
C
T5 B(5,10), BETA=1
B10 N(0,6), R=(10E6, 0.01)
B11 N(6,0), R=10E2
T6 B(11,34), BETA=0
C
C THE INTERCEPTION CIRCUIT. BRANCH B12 IS A DEPENDENT
C CURRENT SOURCE CONTROLLED THROUGH T-CARD T7 AS 80 PERCENT
C OF B3, TOTAL RAINFALL, AND INCLUDING A CONSTANT 5 PERCENT
C EVAPORATION LOSS. IT IS ASSUMED THAT AN 85 PERCENT
C CANOPY COVER EXISTS ON THE WATERSHED. BRANCH B18 IS THE
C DIRECT THROUGHFALL INPUT POINT, VIA T-CARD T8, IN THIS
C CASE 15 PERCENT OF TOTAL RAINFALL. BRANCH B13 IS THE
C INTERCEPTION EVAPORATION EXIT, CONTROLLED BY T-CARD T2 TO
C SIMULATE AN EVAPORATION LOSS OF 0.01 INCHES/HOUR FROM
C INTERCEPTION STORAGE DURING PERIODS OF NO RAINFALL. THIS
C IS 10 TIMES THE 0.001 CONSTANT USED FOR THE MAIN ET
C EVAPORATION RATE. BRANCH B13 IS THE INTERCEPTION CAPACITANCE.
C BRANCH B16 COMBINED WITH SWITCH S2 CONTROLS THE DEPTH
C TO WHICH CHARGE, OR BY ANALOGY STORAGE DEVELOPS ON THE
C ELEMENT OF BRANCH B14. INCLUDED IN THIS BRANCH IS A RE-
C SISTANCE SIMULATING LEAF AND BARK SURFACE FRICTION TO
C TRANSIENT FLOW. SWITCH S3 THROWS THE RESISTANCE INTO
C OPERATION WHEN STORAGE REQUIREMENTS ARE MET. BRANCH
C B17 IS A DUMMY BRANCH RESISTOR USED TO MONITOR INDIRECT
C THROUGHFALL.
C
T7 B(3,12), BETA=0.80
T8 B(3,18), BETA=0.15
B12 N(0,7), R=10E6
B13 N(7,0), R=10E6
B14 N(7,0), C=6.0352E-2
B15 N(7,0), R=(10E6, 10E6), E=-0.00008340
S2 B=15, (6), OFF
S2 B=(16, 16), OFF
S2 B=16, (16), OFF
S2 B=16, (16), OFF
B17 N(8,9), R=0.01
B18 N(0,9), R=10E6
C
C BRANCH B19 SUMS INDIRECT AND DIRECT THROUGHFALL AND IN-
C PUTS THIS TO THE GROUND SURFACE VIA T-CARD T9 TO A DE-
C PENDENT CURRENT SOURCE.
C
B19 N(9,0),R=0.01
C
C TOTAL RAINFALL INTEGRATOR. IF THE CUMULATIVE TOTAL IS DESIRED IN
C CUBIC FEET, C=1.0, IF IN INCHES C=0.001 X CUBIC FEET/WATERSHED INCH.
C THE ANSWER WILL BE AT VOLTAGE NODE 10.
C
T9 B(3,20),BETA=1.0
B20 N(0,10),R=10E6
B21 N(10,0),C=5.917E-3
C
C CANOPY INPUT INTEGRATOR.
C
T10 B(3,22),BETA=0.8
B22 N(0,11),R=10E6
B23 N(11,0),C=5.917E-3
C
C CANOPY INDIRECT THROUGHFALL INTEGRATOR.
C
T11 B(17,24),BETA=1.0
B24 N(0,12),R=10E6
B25 N(12,0),C=5.917E-3
C
C TOTAL THROUGHFALL INTEGRATOR.
C
T12 B(19,26),BETA=1
B26 N(0,13),R=10E6
B27 N(13,0),C=5.917E-3
C
C SINCE 1 SECOND = 0.001 SECOND IN THE MODEL, A 0.06 SECOND TIME STEP
C IS EQUIVALENT TO 1 MINUTE. THE TOTAL STORM DURATION IS 25 HOURS.
C
TIME STEP=0.06
OUTPUT INTERVAL=15
FINISH TIME=90
PRINT,NV,CA
EXECUTE
APPENDIX B

ECAP DATA LISTING USED TO EVALUATE IZZARD'S INFLOW-OUTFLOW RELATIONSHIP

TR
B1 N(1,0),R=1
S1 B=1,(2),ON
I1 (10),0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,-1,-1,-1,-1,-1,-1,-1,-1,-1,
  *-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,
  *-1,-1,-1,-1
B2 N(2,0),R=10E6,I=(5.95E-6,0)
B3 N(2,0),R=10
T1 B(3,4),BETA=1
B4 N(0,3),R=10E6
B5 N(3,0),C=7.2E-2
B6 N(3,0),R=10E6
B7 N(0,4),R=10E6
I7 (10),0,1.65E-8,6.6E-8,1.65E-7,3.3E-7,1.12E-6,1.78E-6,2.05E-6,
  *2.78E-6,3.64E-6,4.23E-6,5.03E-6,5.56E-6,5.82E-6,5.92E-6,5.95E-6,
  *5.95E-6,5.95E-6,5.95E-6,5.95E-6,5.95E-6,
  *4.66E-6,3.59E-6,2.91E-6,2.35E-6,1.90E-6,1.54E-6,1.34E-6,
  *1.14E-6,9.60E-7,8.11E-7,6.62E-7,6.12E-7,5.62E-7,5.04E-7,4.46E-7,
  *4.05E-7,3.64E-7,3.24E-7,2.80E-7,2.40E-7,
B8 N(4,0),R=10E1
T2 B(8,6),BETA=-1
TIME STEP=6
OUTPUT INTERVAL=10
FINISH TIME=3000
PRINT,NV,CA
EXECUTE
APPENDIX C

ECAP DATA LISTING USED TO EVALUATE IZZARD'S EXPERIMENT WITH THE NONLINEAR DIODE MODEL IN PLACE

TR
B1  N(1,0),R=1
S1  B=1,(2),ON
I1  (10),0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,-1,-1,-1,-1,-1,-1,-1,
    *-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,
    *-1,-1,-1,-1,
B2  N(2,0),R=10E6,I=(5.95E-6,0)
B3  N(2,0),R=10
T1  B(3,4),BETA=1
B4  N(0,3),R=10E6
B5  N(3,0),C=7.2E-2
B6  N(3,4),G=(.4E-9,2.44E-5),E=-0.60E-2
S2  B=6,(6),OFF
B7  N(3,4),G=(.4E-9,9.46E-5),E=-1.85E-2
S3  B=7,(7),OFF
B8  N(3,4),G=(.4E-9,28.35E-5),E=-3.37E-2
S4  B=8,(8),OFF
B9  N(3,4),G=(.4E-9,62.95E-5),E=-3.75E-2
S5  B=9,(9),OFF
B10 N(4,0),R=0.01
    TIME STEP=6
    OUTPUT INTERVAL=10
    FINISH TIME=3000
    PRINT,NV,CA
    EXECUTE
APPENDIX D

ECAP DATA LISTING FOR THE COMBINED SURFACE STORAGE AND RUNOFF, INFILTRATION ANALOGS

C BRANCH B20 IS THE INPUT POINT FOR THE TOTAL CANOPY THROUGHFALL COMPONENT. THIS DEPENDENT CURRENT SOURCE IS CONTROLLED FROM BRANCH B19 VIA T-CARD T9.

C T9 B(19,20), GM=100
B20 N(0,10), R=10E6
C
C BRANCH B21 IS A DUMMY RESISTOR USED AS A POINT TO MONITOR THE INPUT TO THE FOREST FLOOR.

C B21 N(10,11), R=0.01, E=3.0
C
C BRANCH B22 IS A DEPENDENT CURRENT SOURCE CONTROLLED BY THE SURFACE EVAPORATION SECTION OF THE ET GENERATOR, BRANCH B9, VIA T-CARD T4. IT IS SET TO EVAPORATE A CONSTANT 0.001 INCHES/HOUR FROM SURFACE STORAGE DURING PERIODS OF NO RAINFALL.

C B22 N(11,0), R=10E6
C
C BRANCH B23 IS A DEPENDENT CURRENT SOURCE CONTROLLED BY THE INFILTRATION ANALOG TO EXTRACT SURFACE WATER FROM STORAGE ON THE CAPACITOR OF BRANCH B24.

C B23 N(0,11), R=10E6
B24 N(11,12), C=7.1E-2, E=-0.00166800
C
C THE 3 VOLT BATTERY OF BRANCH B25 REPRESENTS THE AVERAGE 3 FOOT SOIL DEPTH OVER THE WATERSHED. THE CAPACITOR OF BRANCH B24 SETS ON TOP OF THIS BATTERY TO POSITION SURFACE STORAGE RELATIVE TO SUBSURFACE STORAGE.

C B25 N(12,0), R=0.01, E=-3.00000000
C

100
SWITCH S4 is controlled by branch B26 and determines when litter layer storage requirements are met. S4 then activates the current sources of branches B40 and 43, the potential infiltration (PI) and rainfall transfer function (RFT) generators.

Branch B26 will turn on the surface evaporation generator through switch S5 signifying moisture as being available for evaporation at the ground surface.

Branch B28 is used to indicate the presence of surface depression storage. If it exists the comparator bridge is unbalanced by the action of switch S6 which disables the current source of branch B46 forcing infiltration at the PI rate.

The nonlinear surface friction component is the result of the diode model formed by branches B29, B30, B31, and B32. The current direction in these branches controls switches S7, S8, S9, and S10.

Branch B33 is the surface outflow point.

Branch B34 is the infiltration input point for subsurface storage. This branch is controlled in the same manner as the dependent current of branch B23.
SUBSURFACE STORAGE ACCUMULATES ON BRANCH B35. THE CAPACITOR OF THIS BRANCH IS PROPORTIONATELY REDUCED IN SIZE TO SIMULATE A SOIL SYSTEM OF A SPECIFIED POROSITY. THE ANTECEDENT CHARGE IS SET FOLLOWING THE CAPACITOR VALUE, WHERE \( E = \text{ANTECEDENT CHARGE}/\text{CAPACITANCE} = \text{VOLTS} \).

\[
\begin{align*}
B35 & \quad N(14,0), C=2.605E-2, E=-2.65934000 \\
B36 & \quad N(14,0), R=4.60E5 \\
B37 & \quad N(14,15), R=(10E6,10E6), E=-2.83000000 \\
S11 & \quad B=37,(37), OFF \\
B38 & \quad N(15,0), R=0.01 \\
B39 & \quad N(12,14), R=10E6 \\
T10 & \quad B(39,40), GM=3.5E-4 \\
B40 & \quad N(0,16), R=(0.01,10E6)
\end{align*}
\]
BRANCH B41 SERVES TO DISABLE THE PI FUNCTION GENERATOR. IT CAN SHORT OUT THE GENERATOR BY THE ACTION OF SWITCH S12. THIS SWITCH IS IN TURN PART OF A COMPARATOR CIRCUIT WHICH IS DESIGNED TO COMPARE THE RAINFALL RATE AGAINST THE PI RATE AT ANY GIVEN TIME. IF THE PI RATE EXCEEDS THE RAINFALL RATE IT WILL SHORT OUT THE PI GENERATOR. IT IS INITIALLY SHORTED SINCE RAINFALL WILL INITIALLY BE LESS THAN THE PI RATE. IF NOT THE PI GENERATOR WILL BE ACTIVATED.

B41 \( N(16,0), R=(0.01, 10E6) \)

WHEN BRANCHES B40 AND B41 ARE NOT SHORTED OUT THE CURRENT SIMULATING PI PASSES THROUGH BRANCH B34.

B42 \( N(16,0), R=10E2 \)

THE CURRENT PASSING THROUGH B42 IS TRANSFERRED BY T-CARDS T11 AND T12 TO THE DEPENDENT CURRENT SOURCES OF BRANCHES B23 AND B34. BRANCH B23 WITHDRAWS CHARGE FROM SURFACE STORAGE AT THE PI RATE AND THIS CHARGE IS EQUALLY INPUT INTO SUBSURFACE STORAGE THROUGH BRANCH B34.

T11 \( B(42,23), GM=-0.001 \)
T12 \( B(42,34), GM=0.001 \)
T-CARD T13 TRANSFERS THE SIMULATED RAINFALL INPUT MONITORED IN BRANCH B19 AT THE LAND SURFACE TO THE DEPENDENT CURRENT SOURCE OF BRANCH B43. BRANCH B43 IS A RAINFALL TRANSFER FUNCTION (RTF) GENERATOR, AND IS ALSO INITIALLY SHORTED OUT BY SWITCH S4, THE LITTER LAYER STORAGE SENSE SWITCH.

T13 \( B(19,43), GM=100 \)
B43 \( N(0,17), R=(0.01, 10E6) \)
BRANCH B44 SERVES TO DISABLE THE RTF GENERATOR. IT IS OPERATED BY SWITCH S12 OF THE COMPARATOR IN THE SAME MANNER AS BRANCH B41, ONLY THE SWITCHING SEQUENCE IS REVERSED, IT IS INITIALLY OPEN. THE COMPARATOR DETERMINES WHICH IS GREATER PI OR RAINFALL AND THEN ENABLES THE CORRECT GENERATOR.

B44 \( N(17,0), R=(10E6, 0.01) \)
WHEN BRANCHES B43 AND B44 ARE NOT SHORTED OUT THE CURRENT SIMULATING THE RTF PASSES THROUGH BRANCH B45.
THE CURRENT PASSING THROUGH B45 IS TRANSFERRED BY T-CARDS T14 AND T15 TO THE DEPENDENT CURRENT SOURCES OF BRANCHES B23 AND B34 WHICH CONTROL INFILTRATION AT THE RAINFALL RATE.

T14  B(45,23),GM=-0.001
T15  B(45,34),GM=0.001


T16  B(39,46),GM=3.5E-4
B46  N(0,18),R=(10E6,0.01)
B47  N(18,0),R=10E2
B48  N(0,19),R=10E6
T17  B(19,48),GM=100
B49  N(19,0),R=10E2
B50  N(19,18),R=(10E6,10E6)
S12  B=50,(41,44),OFF
T18  B(39,48),GM=1.0E-7

TOTAL RAINFALL INTEGRATOR. VALUE IN WATERSHED INCHES AT VOLTAGE NODE 20.
T19  B(3,51),BETA=1.0
B51  N(0,20),R=10E6
B52  N(20,0),C=5.917E-3
C  C TOTAL THROUGHFALL INTEGRATOR. VALUE IN WATERSHED INCHES
C AT VOLTAGE NODE 21.
C
T20  B(19,53),BETA=1.0
B53  N(0,21),R=10E6
B54  N(21,0),C=5.917E-3
C  C TOTAL RUNOFF INTEGRATOR. VALUE IN WATERSHED INCHES AT
C VOLTAGE NODE 22.
C
T21  B(38,55),BETA=1.0
B55  N(0,22),R=10E6
B56  N(22,0),C=5.917E-3
C  C INTERCEPTION STORAGE IN WATERSHED INCHES RELATIVE TO
C CANOPY AREA. VALUE AT VOLTAGE NODE 23.
C
T22  B(14,57),BETA=1.0
B57  N(0,23),R=10E6
B58  N(23,0),C=5.029E-3
C  C SURFACE STORAGE. VALUE IN WATERSHED INCHES AT VOLTAGE
C NODE 24.
C
T23  B(24,59),BETA=1
B59  N(0,24),R=10E6
B60  N(24,0),C=5.917E-3
C  C GROUNDWATER LOSS INTEGRATOR. VALUE IN WATERSHED INCHES
C AT VOLTAGE NODE 26.
C
T25  B(36,63),BETA=1.0
B63  N(0,26),R=10E6
B64  N(26,0),C=5.917E-3
TIME STEP=0.06
OUTPUT INTERVAL=15
FINISH TIME=90
2ERROR=0.2
PRINT,NV,CA
EXECUTE
REFERENCES

Amerman, C. R.

Amorocho, J.

and W. E. Hart

Bermes, B. J.

Beyers, L. A.

Bowers, J. C. and S. R. Sedore
1971 SCEPTRE: A computer program for circuit and system analysis. Prentice-Hall, Inc.

Chow, V. T.

Crawford, N. H. and R. K. Linsley

Golany, P. and C. L. Larson
1971 Effects of channel characteristics on time parameters for small watershed runoff hydrographs. Water Resources Res. Center, University of Minnesota, Bul. 31.

Grah, R. F. and C. C. Wilson

106
Harder, J. A.

Helvey, J. D.

Holtan, H. N.
1961 A concept for infiltration estimates in watershed engineering. USDA ARS 41-51.

Horton, R. E.
1933 The role of infiltration in the hydrologic cycle. Trans. AGU 14:446-460.

Huelsman, L. P.

Huggins, L. F. and E. J. Monke

Izzard, C. F.

Jensen, R. W. and M. D. Lieberman
1968 IBM Electronic circuit analysis program. Prentice-Hall.

Karplus, W. J.

Korn, G. A.
Lawson, R. L.  

Linsley, R. K., M. A. Kohler and J. L. Paulhus  

McCann, G. D.  

Mitchell, W. D.  

Philip, J. R.  

Riley, J. P., D. G. Chadwick and E. K. Israelson  

Rogerson, T. L.  

Rosa, J. M.  

Shen, J.  

Thorud, D. B.  
Vemuri, V.  
1969  Computers in the research and teaching of hydrology.  
The progress of hydrology. Proceeding of the first  
International Seminar for Hydrology Professors.  
University of Illinois, Urbana.  Vol. I.  

Wisler, C. O. and E. F. Brater  