

THERMAL GRADIENTS AND WATER TRANSFER  
IN UNSATURATED SOIL

by

John Merrill Tromble

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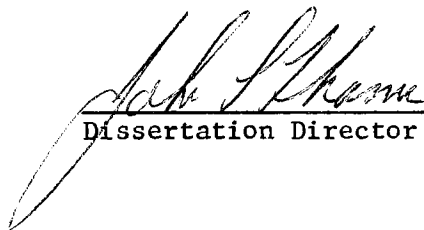
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Dissertation Director

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|                        |               |
|------------------------|---------------|
| <u>David B. Thornd</u> | <u>4-3-73</u> |
| <u>Samuel A. Evans</u> | <u>4-3-73</u> |
| <u>L. Gruber</u>       | <u>4-3-73</u> |
| <u>J. Klemm</u>        | <u>4-9-73</u> |

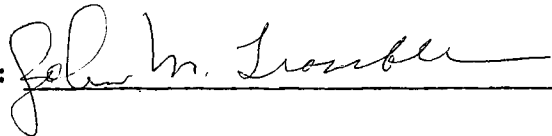
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DEDICATION

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## ABSTRACT

An investigation into the flux of soil water under the influence of a thermal gradient was conducted in closed soil systems in the laboratory. A thermal gradient was imposed across the sandy loam soil columns and measured continuously for the duration of the experiment. The movement of soil water was monitored periodically using a gamma ray attenuation device until the columns reached an apparent steady state condition with no net flow. Imposition of boundary conditions enabled delineation and evaluation of the system parameters. Values of net water flux in soil columns were analyzed using the Taylor-Cary irreversible thermodynamic and the Philip-de Vries theory of water movement.

Application of the Taylor-Cary equation to describe the flow reveals that for initial time periods the flow is slightly over-estimated, however, this small difference may be within the realm of experimental error. The flow predicted by the Taylor-Cary equation for the succeeding time periods greatly exceeded the measured flow rates.

The Philip-de Vries theory of soil-water movement predicted greater net water movement than was observed in soil columns with a temperature gradient of 2.67 °C/cm and with an average soil water content of 10.5 to 11.5 cm<sup>3</sup>/cm<sup>3</sup>.

The following conclusions were reached after analyzing the data for the sandy loam soil material.

Water content and temperature influence the diffusion transfer coefficient,  $\beta^*$ , in a closed soil system. Thus the transfer coefficient is not an independent entity.

Hysteresis is present in the wetter part of the system, although the magnitude of hysteresis involved is unknown. The  $\beta^*$  coefficient can be evaluated only in regions where hysteresis is not present.

The transfer of soil water was greater in a leached soil with no air gap than in an unleached soil with no air gap. There was supporting evidence that liquid water continuity did not exist throughout the column since there was no appreciable solute transport.

The observed change in soil water content distribution at  $18.0 \text{ cm}^3/\text{cm}^3$  to the imposed temperature gradient was not significant for the sandy loam soil material.

The observed soil water flux increased in response to the imposed temperature gradient as soil water content decreased from  $18.0$  to  $10.5 \text{ cm}^3/\text{cm}^3$ .

No analytical procedure is presently available that will describe thermally induced flow under all conditions. The approach of Philip and de Vries requires that the physical properties of the soil must be known accurately so that correct estimates can be made of the individual diffusivities. This approach has been found to predict the flow with some success on relatively dry soils, however, it possibly would be in error when estimating thermally induced flow in regions where liquid continuity exists and up through to a saturated system.

The Taylor and Cary equation may be adequate for describing water flow to predict trends or obtain comparative values, however, much additional work needs to be done before it will adequately describe the flow for transient conditions or steady state conditions where nonuniform water content distributions are present.

## INTRODUCTION

A well-known characteristic of arid-land soils is that the temperature undergoes a wide diurnal and annual fluctuation. These fluctuations in temperature cause thermal gradients in the surface soil. It is well known that these conditions exist, but the effect on the processes that occur in the soil is not as well known. Thus, interest has developed in recent years on temperature influences on water movement in soils. Early investigations into this problem indicated a net water flux from warm to cold regions in soils subjected to a thermal gradient. These studies were concerned with whether the water movement was in a vapor or liquid phase. Some investigations supported the vapor flow theory, while others supported the theory of liquid flow. Gurr, Marshall and Hutton (1952) developed a technique in which they demonstrated the net flux of vapor from warm to cool regions with a return flow of liquid from cool to warm regions in a closed and sealed soil system. However, they obtained a vapor diffusion coefficient that was many times greater than was predicted by diffusion theory. Another study (Taylor and Cavazza, 1954) supported these findings, giving a vapor diffusion coefficient ten times greater than that predicted on the basis of simple diffusion laws. Research based on the simultaneous flow of heat and water have been made to explain this phenomenon (Philip and de Vries, 1957; de Vries, 1958; Derjaquin and Melnikova, 1958).

Taylor and Cary (1960) demonstrated a coupled flow of heat and water through a saturated soil sample using the theory developed by

Henry (1939) for describing the movement of heat and water in cotton bales. Cary (1961) showed also that a coupling occurred for both liquid and vapor flow through unsaturated soils. The most productive approaches appear to be those of Philip and de Vries (1957) and those of Taylor and Cary (1964). The theoretical approach of Philip and de Vries is based on classical concepts of water flow. This theory takes into account the thermal gradient influence on both the vapor and liquid flow, and recognizes the simultaneous heat flow that accompanies water flow. A more general approach to the problem of thermally induced flow was proposed by Taylor and Cary (1960) and developed more thoroughly since then by them (Taylor and Cary, 1964, 1965; Cary, 1961, 1963, 1965, 1966; Taylor, 1962, 1963). Their approach was based on the general theory of irreversible thermodynamics, and was used to describe certain types of water flow that could occur under both isothermal and nonisothermal conditions. Their theory is not limited to heat and water flow but may be extended to include the flow of any constituent of the system.

The equations resulting from the approaches of Philip and de Vries, and Taylor and Cary are superficially similar. However, when the Taylor and Cary equations are used, fewer approximations are necessary and coefficients are obtained more easily.

The purpose of this study is to evaluate the effects of thermal gradients on water transfer coefficients through an investigation of the mechanisms of water transport and analysis of the flow and to examine the applicability of the Taylor-Cary equation and the Philip-de Vries equation for use as prediction equations for water flow.

## LITERATURE REVIEW

### Basis for Water Flow Under Isothermal and Nonisothermal Conditions

#### General Theory

The flow of water through a porous medium, such as a soil, may be described by a general flow equation

$$q_i = \sum_{k=1}^n L_{ik} X_k \quad (k = 1, 2, \dots, n) \quad (1)$$

where

$q_i$  = flow rate or water flux

$n$  = number of driving forces

$L_{ik}$  = transmission coefficient,  $i$ th flux,  $k$ th driving force

$X_k$  = driving force of the  $k$ th component

Upon choosing the proper transmission coefficients and driving forces this equation may be used to describe liquid, vapor, or absorbed phase water flow. The soil-water system is a highly complex, variable entity and a large volume of work has been done towards finding and describing driving forces and fluxes.

Under isothermal conditions the potential energy gradients are the driving forces for water flow in a system. Potential energy gradients in the soil-water system are caused by position differences of the water molecules in the force fields. Potential energy gradients may arise from forces caused by electricity, solute-water interactions, surface tension, gravitation and other force fields. Heat and mass

flow may result from kinetic energy gradients that arise from non-uniform temperature distribution under nonisothermal conditions. In the case of concentration gradients, flow may be expressed using diffusion theory and regarding the gradient as an index of the driving force.

Each driving force has an associated transmission coefficient for expressing the water flow rates through a system in response to a unit strength driving force. Thus, in a system where water is the only flowing component, the total driving force is an expression of the potential energy gradient and the transmission coefficient is expressed as a single term. Although the nature of the flow path, density, and viscosity are included in the transmission coefficient, under unsaturated flow conditions the flow path changes and thus the transmission coefficient would change. The transmission coefficient is expressed as a concentration dependent diffusivity when expressing the treatment in diffusion theory terms.

#### Isothermal Flow

Saturated Conditions. In the case of saturated conditions where the pores of the soil system are completely filled with water and the matrix remains constant the transmission coefficient will remain constant. The transmission coefficient then depends principally on the viscosity and density of the water and the nature of the porous media. The negative of the potential gradient is the driving force.

The rate of water flow through soil or other porous material is usually given by Darcy's law as:

$$q = -K\nabla\phi \quad (2)$$

where

$q$  = volumetric water flux

$K$  = hydraulic conductivity

$\nabla\phi$  = hydraulic head gradient, which includes pressure and elevation head gradients.

Except for the saturated condition at constant temperatures, Darcy's law as originally postulated does not apply precisely to water flow through soil. Darcy's law for describing flow through porous materials has been used since 1856 with little reservation until questions recently have been raised about its validity in describing flow (Gee, 1966).

Unsaturated Flow. When a soil contains both a liquid (water) and a gaseous (vapor) phase, the most frequently used modification involves substituting the gradient of the matric suction,  $\nabla\Psi$ , for the pressure gradient as the driving force. Under these conditions (including isothermal conditions), the proportionality factor is no longer a constant but is a function of the potential which gives an equation for horizontal flow

$$q = K(\Psi)\nabla\Psi \quad (3)$$

where

$q$  = volumetric water flux

$K(\Psi)$  = transmission coefficient (unsaturated conductivity)

$\nabla\Psi$  = matric suction gradient

The matric suction gradient is the driving force.

A more generalized form of Equation 3 may be written as

$$q = K(\Psi)\nabla\phi \quad (4)$$

where

$q$  = volumetric water flux

$K(\Psi)$  = transmission coefficient

$\nabla\phi$  = hydraulic head gradient.

The driving force may be defined as  $\nabla\phi$ , which accounts for suction gradients and gravity. The water potential is the amount of work that must be done per unit quantity of soil water in order to transport reversibly and isothermally an infinitesimal quantity of water from a pool of soil water, at a specified elevation at atmospheric pressure to the soil water at the point under consideration (Gee, 1966).

In an unsaturated soil the assumption of uniform water content may no longer be adequate. This necessitates finding an appropriate relationship between the water content and the matric suction. Introduction of the expression for the conservation of mass, called the equation of continuity, will account for the change in water content at any point in the system.

Klute (1951) derived the equation of flow through a porous medium by using the equation of continuity and Darcy's law. His development is as follows when assuming that the fluid is incompressible. The continuity equation for flow through a porous medium, is (see Figure 1)

$$\frac{\partial \rho^t}{\partial t} = - \frac{\partial \rho q_x}{\partial x} - \frac{\partial \rho q_y}{\partial y} - \frac{\partial \rho q_z}{\partial z} \quad (5)$$

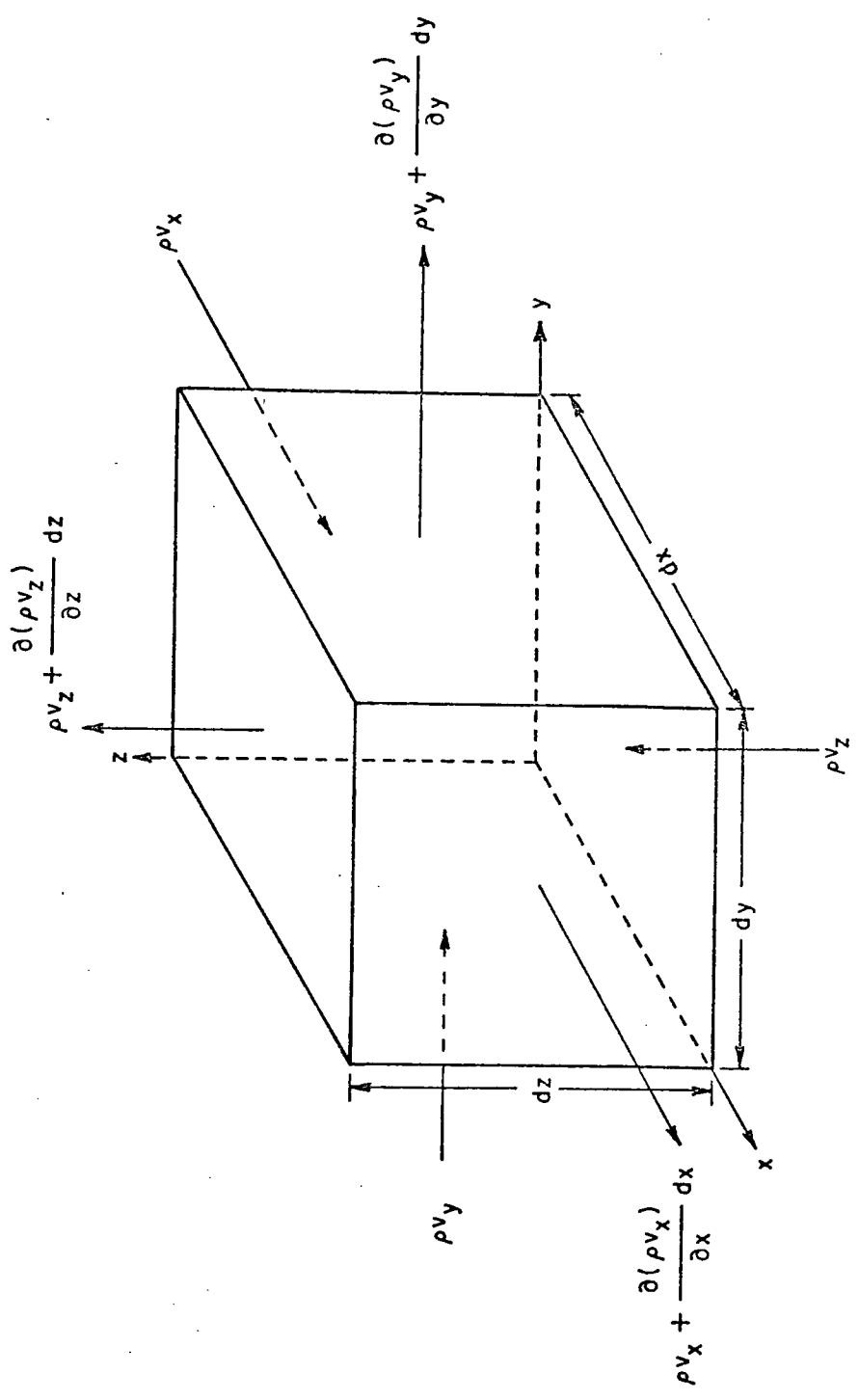


Figure 1. Flow into and out of each face of an infinitesimal parallelepiped of the soil medium.

where

$\rho'$  = mass of fluid per unit volume of medium

$t$  = time

$q$  = volumetric water flux

$\rho$  = fluid density

Equation 5 written in vectorial form is

$$\frac{\partial \rho'}{\partial t} = -\text{div}(\rho q) = -\nabla \cdot (\rho q) \quad (6)$$

Substituting equation 4 into Equation 6 gives

$$\frac{\partial \rho'}{\partial t} = \nabla \cdot (K(\Psi) \nabla \Phi) \quad (7)$$

For a medium with constant water content,  $\rho'$  is constant and  $\rho$  is constant (assuming water is incompressible). Then Equation 7 can be expressed as

$$\nabla \cdot (K(\Psi) \nabla \Phi) = 0 \quad (8)$$

Since all the void space in an unsaturated medium is not filled with water, Equation 7 can be written as

$$\frac{\partial (\rho \theta)}{\partial t} = \frac{\partial (\rho_s \theta_s)}{\partial t} = \nabla \cdot (\rho K \nabla \Phi) \quad (9)$$

where

$\theta$  = water content

$\rho_s$  = bulk density on a dry weight basis

$\theta_s$  = water content on a dry weight basis

It can easily be seen from this that

$$\rho' = (\rho_s \theta_s) = \rho \theta \quad (10)$$

If  $\rho$  is a constant, Equation 10 becomes

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K(\Psi) \nabla \Phi) \quad (11)$$

This is the equation for the general case of water flow through an unsaturated porous medium. This equation was originally developed by Richards (1931). Klute (1952) divided  $\phi$  into two components:

$$\phi = \psi + z \quad (12)$$

where

$\psi$  = matric suction, and

$z$  = gravitational head

for flow in a vertical direction in the medium. Flow in a vertical direction may be expressed by

$$\frac{\partial \theta}{\partial t} = \partial (K(\psi) \frac{\partial \psi}{\partial z} + z) = \frac{\partial (K(\psi) \frac{\partial \psi}{\partial z})}{\partial z} + \frac{\partial K(\psi)}{\partial z} \quad (13)$$

Assuming  $\psi$  is a unique single valued function of  $\theta$ , Equation 13 may be written

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (K(\psi) \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial z}) + \frac{\partial K(\psi)}{\partial z} \quad (14)$$

$K(\psi) \frac{\partial \psi}{\partial \theta} = D(\psi)$  which is defined as the diffusion coefficient or diffusivity, Equation 14 is the differential equation that expresses water movement in a vertical column in the medium and may be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} (D(\theta) \frac{\partial \theta}{\partial z}) + \frac{\partial K}{\partial z} \quad (15)$$

Considering diffusivity as a constant and disregarding the gravity term Equation 15 becomes

$$\frac{\partial \theta}{\partial t} = D(\theta) \frac{\partial^2 \theta}{\partial z^2} \quad (16)$$

Flow rates change as the soil becomes unsaturated. The water potential gradient varies with changes in the flow rate. Under unsaturated conditions the attraction force of the water molecule to the soil particle becomes important and is reflected in a decrease in the

water potential. With a decrease in water content in the soil pores, the capillary potential decreases (increases negatively) as a result of the increasing effect of the absorptive forces of the soil. Thus, the energy state of the soil water is reduced by the soil absorptive forces. Direct measurement of absorptive forces are not made; however, these forces are reflected in the surface adsorption of water and the capillary phenomenon. In very dry soils the capillary potential can attain very great negative values and for most cases concerning water flow in the unsaturated regions above the water table the capillary potential gradient is the predominant motive factor included in the water potential.

As the soil water content decreases, flow rates are altered because the effective size of the water conducting channels is reduced. The larger soil pores are the first to drain, thus the ability of the system to transmit water decreases rapidly. This change is reflected in the transmission coefficient ( $K(\Psi)$ ); therefore, for unsaturated flow  $K(\Psi)$  reflects the water content of the pores and the nature of the pore system.

#### Simultaneous Flow

The preceding discussion was directed to the case where the only substance moving in the system is water. The isothermal theory has limited application in nature since there are few water flow problems that occur in an isothermal manner. Heat is evolved at the wetting front when water infiltrates a dry soil (Anderson and Linville, 1960). When water is evaporated from soil, the evaporating site is cooled by

the evaporation process (Wiegand and Taylor, 1962). Also, radiation at the soil surface and chemical or biological activity within the soil may cause temperature gradients. Thus, an equation is needed that will incorporate the temperature gradient into the flow equation in order to describe and explain such natural processes in a precise way.

If a soil system, initially at a uniform water content with a constant water potential and constant temperature throughout, is subjected to a temperature gradient, water will flow from the warm region to the cool region (Gurr, Marshall, and Hutton, 1952; Taylor and Cavazza, 1954; Woodside and Cliffe, 1959). When the soil has been saturated or near saturated, the flow occurs primarily in the liquid phase (Taylor and Cary, 1960). Water movement occurs primarily in the vapor phase when the soil is relatively dry. At intermediate soil water contents, movement is by a combination of liquid and vapor water flow (Cary and Taylor, 1962a; Cary and Taylor, 1962b; and Taylor and Cary, 1964).

The energy state of the water molecules in soil changes when a temperature gradient is imposed in the system, and water will move in response to the energy gradient. For the duration of the imposed temperature gradient, an irreversible and spontaneous flow of heat occurs. Thus, both flow processes will interact with each other and cannot be considered as independent processes but must be considered together in terms of the interaction. Where there is simultaneous movement of more than one component in the system, isothermal flow equations and simple equilibrium thermodynamics cannot be used to predict or describe the flow.

Analysis of Simultaneous Water and Heat Flow

Two approaches have been developed in an attempt to describe the phenomenon of simultaneous flow where water is moved by a thermal gradient. In the first, theory and equations were developed (to describe heat and water transfer in porous medium) in terms of the classical mechanisms of vapor diffusion and liquid movement by capillarity and further defined to account for heat flow. The second approach used irreversible thermodynamic theory to develop general equations for the simultaneous flow of water and heat in the soil system.

Philip and de Vries Theory

In an attempt to expand on the previous theory of water vapor diffusion in a porous medium under temperature gradients, Philip and de Vries (1957) were apparently the first to include the interaction of vapor, liquid and solid phases, and the difference between average temperature gradients in the air-filled pores and in the soil as a whole. Taking these factors into account, an approximate analysis was developed which would predict orders of magnitude and general behavior that was in satisfactory agreement with the experimental data of Gurr, Marshall, and Hutton (1952); Rollins, Sprangler, and Kirkham (1954); Smith (1943) and Jones and Kohnke (1952); Staples and Lehane (1954).

Philip and de Vries (1957) have taken the "simple" theory of vapor diffusion which may be shown by the equation (Penman, 1940; van Bavel, 1952; Rollins and others, 1954)

$$q_v = -D_{atm} \nu \alpha a \nabla \rho_v \quad (17)$$

where

- $q_v$  = vapor flux
- $D_{atm}$  = diffusion coefficient of water vapor in air
- $\alpha$  = dimensionless factor depending on the structure
- $a$  = volumetric air content of the medium
- $\nabla \rho_v$  = water vapor density gradient
- $v$  = "mass flow factor" introduced to allow for the mass flow arising from the differences in boundary conditions governing the air and vapor components of the diffusion systems

and extended it to (a) separate the isothermal and thermal components of vapor transfer and (b) to show the effect of relative humidity on the transfer. As they pointed out the developments depend only on the proportionality of the vapor flux to the vapor-density gradient.

By the introduction of the thermodynamic relationship as given by Edlefsen and Anderson (1943) and considering the influence of temperature on relative humidity for a constant value of  $\theta$ . Philip and de Vries (1957) were able to show that the density of saturated water vapor was a function of temperature only and that the relative humidity was a function of  $\theta$  only. They were able then to separate the flux into two components, that due to the temperature gradient and that due to the moisture gradient and illustrate it by equation

$$\frac{q_v \rho}{\rho} = -D_{Tv} \nabla T - D_{\theta v} \nabla \theta \quad (18)$$

where

- $\rho$  = density of water = 1
- $D_{tv}$  = thermal vapor diffusivity
- $D_{\theta v}$  = isothermal vapor diffusivity

In order to account for the liquid flow of water due to temperature and water gradients Darcy's law for liquid flow in unsaturated media was combined with Equation 12.

In the water content range where liquid transfer occurs ( $K > 0$ ) water pressure is determined by capillarity. Philip and de Vries (1957) show that

$$\frac{\partial \psi}{\partial T} = \frac{\Psi}{\sigma} \frac{d\sigma}{dT} = \gamma \psi \quad (19)$$

where

$\gamma$  = temperature coefficients of surface tension of water,

$\sigma$  = surface tension.

Using Equations 19 and 13 in Darcy's law gives

$$\frac{q_{liq}}{\rho_w} = -k\gamma\psi\nabla T - k \frac{\partial \psi}{\partial \theta} \Delta \theta - ki \quad (20)$$

where  $i$  = unit vector in a positive  $z$  direction and  $k$  is the unsaturated hydraulic conductivity.

Thus, the liquid flux was separated into three components, that caused by temperature gradients, that caused by the water gradients and that caused by gravity.

Thus, if  $\theta = f(\psi)$  and is single valued then it is easily seen that Equation 20 is of the form

$$\frac{q_{liq}}{\rho_w} = -D_{Tliq} \nabla T - D_{\theta liq} \nabla \theta - ki \quad (21)$$

In describing the interaction of vapor and liquid phases where liquid continuity no longer exists there still remains water molecules adsorbed on the surface of the matrix of the soil. These water

molecules form bridges between the isolated pores of wedges of liquid water. Thus, if a vapor pressure gradient due to a temperature gradient produces a vapor flux, the water vapor condenses on one side of the liquid wedges and is transported by capillarity to the other side where evaporation would take place. The liquid wedge would be at "equilibrium" when the rate of condensation equals the rate of evaporation. These regions would permit the rapid transport of water since the changes produced in the curvature of the menisci result in essentially no resistance to liquid flow through the wedge.

In a system where liquid continuity no longer exists, rapid transport of water through the liquid wedges would react positively to speed the overall water transfer. Thus, water transfer due to temperature gradients was regarded as a series-parallel process of flow through regions of vapor and liquid. This would involve a detailed analysis of the fine structure of the vapor and temperature fields in the soil pore system. Since this analysis is very complicated and virtually impossible to evaluate, an alternate method using the thermal properties of the system is used for approximating the parameters. The thermal properties are denoted by the ratio of the average temperature gradient in the air-filled pore spaces to the overall temperature gradient. Large differences in thermal conductivity of the liquid, vapor, and solid phases were noted by Philip and de Vries (1957) which could cause temperature changes in the solid phase to occur more rapidly than those in the pores. With the increased thermal conductivity due to the evaporation-condensation process, the overall differences in thermal conductivity of the liquid, vapor, and solid phases

were such that the average pore temperature gradient may be three times the overall temperature gradient of the system.

Evaluation of flow in a closed system was done by Philip and de Vries (1957) realizing that the observations were of net water transfer and not of total water transfer. In the case where the average water content is so low that no liquid continuity exists, the net water transfer would be a measure of the total water flow because the water transfer was in vapor phase. The vapor phase transfer would increase rapidly with increased water content up to the point where liquid continuity is established. Then a liquid phase return flow produced by the water gradient will tend to balance the thermal vapor phase flow and reduce the net water transfer. It then follows that maximum net transfer occurs at some point below where liquid continuity is established and that net water transfer will gradually decrease above the liquid continuity point until at some given point it will essentially become zero. It is apparent that a static equilibrium cannot exist under these conditions but that a cyclic flow of water in the liquid and vapor phase is accompanied by a transfer of heat.

The pressure component and the general influence of temperature on the flow system were accounted for in the diffusivity terms. Then for describing the water and heat transfer an equation was developed. Using Equation 18 for vapor phase transfer and redescribing the diffusibilities in terms of the Philip and de Vries (1957) theory gave

$$D_{\theta \text{ vap}} = \frac{D_{\text{atm}} v_{\text{ag}} \rho}{\rho_w RT} \frac{\partial \psi}{\partial \theta} \quad (22)$$

The thermal transfer mechanism, however, failed in this case since the vapor pressure gradient is due solely to the water gradient and very small changes in the curvature of the meniscus would cause a reversal in the direction of water vapor movement. Using the combination of Equations 18 and 21 yields

$$\frac{q}{\rho_w} = - D_T \nabla T - D_\theta \nabla \theta - ki \quad (23)$$

where  $q$  is the quantity that describes the total water flux and

$$D_T = D_{Tliq} + D_{Tvap} \quad (24)$$

and

$$D_\theta = D_{\theta liq} + D_{\theta vap} \quad (25)$$

where  $D_T$  = thermal water diffusivity

$D_\theta$  = isothermal water diffusivity.

Differentiation of Equation 23 and combining with the equation of continuity yields an equation for describing water transfer under the combined influence of water and temperature gradients.

$$\frac{\partial \theta}{\partial t} = \nabla \cdot \left( D_T \nabla T \right) + \nabla \cdot \left( D_\theta \nabla \theta \right) + \frac{\partial k}{\partial z} \quad (26)$$

The existence of hysteresis in  $\psi(\theta)$  which could occur if a portion of the system was decreasing in water content while another portion was increasing could invalidate the use of Equations 23 and 26.

Equation 23 neglecting gravity is (Philip and deVries, 1957)

$$q = - D_T \nabla T - D_\theta \nabla \theta \quad (27)$$

where

$$D_T = D_{Tv} + D_{Tliq} = \frac{[a + f(a) \cdot \theta] D_o v h \partial \rho_o}{\rho_w} \frac{(\nabla T)_a}{\partial T} - k \gamma \psi$$

$$D_\theta = D_{\theta v} + D_{\theta liq} = \frac{D_o \alpha a}{\rho_w} \frac{\partial \rho}{\partial \theta} + k \frac{\partial \psi}{\partial \theta}$$

$D_T$  = thermal moisture diffusivity

$D_\theta$  = isothermal moisture diffusivity

$D_{Tv}$  = thermal vapor diffusivity

$D_{Tliq}$  = thermal liquid diffusivity

$D_{\theta v}$  = isothermal vapor diffusivity

$D_{\theta liq}$  = isothermal liquid diffusivity

$\theta$  = volumetric water content

$a$  = volumetric air content

$f(a)$  =  $a/a_k$ , for  $\theta \leq a \leq a_k$  as a first approximation;

$f(a) = 1$  for  $a \geq a_k$

$a_k$  = value of  $a$  at which "liquid continuity" no longer exists

$D_o$  = molecular diffusivity of water vapor in air

$v$  =  $P/(P-p)$  = mass flow factor (approximately 1 for most soil systems) where  $P$  = air pressure;  $p$  = soil water vapor pressure

$h$  =  $p/p_o$  = relative vapor pressure of the soil air

$\rho_\theta$  = density of saturated water vapor

$(\nabla T)_a$  = average temperature gradient in the air filled pores

$\nabla T$  = overall temperature gradient

$\gamma$  =  $1/\sigma \, d\sigma/dT$  = temperature coefficient of the surface tension of water,  $\sigma$

- $k$  = unsaturated hydraulic conductivity  
 $\alpha$  = tortuosity factor allowing for the extra path length  
 $\rho$  = density of soil water vapor  
 $\rho_w$  = density of liquid water  
 $\psi$  = capillary potential

An extensive analysis of the soil water characteristics must be made for the evaluation of Equation 27. The water content at which liquid continuity no longer exists must be estimated as well as the water potential and the diffusivity as a function of water content in the water content range of interest. The thermal properties as influenced by the water in the system must be estimated to evaluate the  $\frac{\Delta T}{\Delta T} a$  term. Values for this term for water contents and different porosities were presented by Philip and de Vries (1957). An elaborate analysis pertaining to the thermal conductivities of the gas, solid, and liquid phases of a hypothetical porous system was used for these data. Thus, the values only held in the 10-30°C range and were regarded only as first approximations.

#### Irreversible Thermodynamics

Taylor and Cary (1964) developed a theory for describing water flow caused by a thermal gradient. Their general approach of describing flow was based on thermodynamic considerations using nonequilibrium systems. The standard works of Prigogine (1961); de Groot (1959); and de Groot and Mazur (1962) review the general theory of nonequilibrium thermodynamics. A discussion follows that gives the general theory and

a systematic guide for setting up interrelated equations that describe the transport of one or more components through the soil-water system.

Consider a soil which remains unchanged in the x-y plane though moisture, heat, salt and air fluxes may occur in the z plane. Thermal, osmotic, attractive forces between soil water and soil solids, and gravitational force fields may cause these gradients. If this is so the continuity equation may be written as

$$\rho \frac{dC_k}{dt} = - \frac{dJ_k}{dz} \quad (28)$$

where

$\rho$  = total mass of material in a unit bulk volume

$C_k = \frac{\rho_k}{\rho} = \frac{M_k}{M}$  = mass of component k per unit total mass of the system (concentration)

$\rho_k$  = mass of component k in a unit bulk volume

$J_k$  = flow of component k

$t$  = time

Equation 28 shows that the total mass per unit volume multiplied by the rate of change with time of the unit mass of a component is equal to the rate of change of the flow component (k) of the mass with respect to distance.

Taylor and Cary (1964) have written the equations for the conservation of energy per unit mass given by de Groot (1951) as

$$\rho \frac{d(\frac{1}{2} V^2 + u)}{dt} = \rho V \frac{dV}{dt} + \rho \frac{du}{dt} = - \frac{d}{dz} (PV + J_u) + \sum_{k=1}^n F_k V_k \rho_k \quad (29)$$

$V$  = velocity of the center of mass of the system

$J_u$  = flow of energy

$F_k$  = external force acting on the component k

$V_k$  = velocity of component k

Choosing the fluxes with respect to the center of mass, the velocity of the center of mass is zero. If it is assumed that the acceleration of the center of mass ( $\frac{dv}{dt}$ ) is insignificant when compared to the acceleration due to the gravitational force in the term  $F_k$  then the kinetic energy term and the flow energy terms are very small and approximate zero. Thus Equation 29 may be simplified to

$$\rho \frac{du}{dt} = - \frac{dj_u}{dz} + \sum_{k=1}^n F_k V_k \rho_k \quad (30)$$

Application of this theory to the flow problem of either open or closed systems can be started with the Gibbs equation of equilibrium thermodynamics which states

$$TdS = dU + PdY - \sum_k^n \mu_k dM_k \quad (31)$$

where

T = absolute temperature.

S = internal entropy

U = internal energy

P = pressure

Y = volume

$M_k$  = mass of k at a point in the system

n = number of components.

$\mu_k$  = chemical potential of the kth component

The instantaneous thermodynamic state at any point in the system can be described by Equation 31. The assumption is made in theory that the molecules only deviate slightly from equilibrium, even though the system is not in an equilibrium condition. The internal entropy is produced when the molecules return to the equilibrium condition. The basis for the determination of the fluxes in the phenomenological laws that describe the flow system is the entropy production.

Fourier's law used for heat flow, Darcy's law used for saturated water flow, and Ohm's law used for electrical current flow show that flow may be described as being proportional to the driving force. Such laws which describe irreversible processes are sometimes termed phenomenological laws.

The theory of irreversible thermodynamics assumes that a linear equation will describe the flow as a summation of the individual driving forces, so

$$J_i = L_i X_i + \dots + L_{in} X_n = \sum_{k=1}^n L_{ik} X_k \quad (32)$$

where

$J_i$  = flux of the i-th component

$L$  = phenomenological coefficient

$X$  = driving force

$n$  = number of driving forces

This equation, if applicable to flow in porous media, resolves the difficulty of interference through the assumption that the driving forces are independent and linearly related to the flux. Thus, Equation 32 is general and will describe the flow of any component as a

linear relationship between the flow of that component and the driving forces involved. Therefore, mass transfer could be evaluated with this analysis.

An understanding of the interrelationship between the phenomenological coefficients (proportionality constants) is necessary to solve the equations which describe this interaction of forces.

Onsager's reciprocal relations

$$L_{ik} = L_{ki}$$

are based on the concept of microscopic reversibility under close to equilibrium conditions and requires that any molecular process and the reverse of that process will on the average take place at the same rate. In the case at hand of heat and water flow, the reciprocal relationship provides the information that the coefficient of water flow resulting from a unit water content gradient is equal to the coefficient for water flow resulting from a unit thermal gradient.

In an irreversible flow system the general relationship for entropy production is

$$T \frac{\partial s}{\partial t} = \sum_{i=1}^n J_i X_i \quad (33)$$

where

T = absolute temperature

$\frac{\partial s}{\partial t}$  = rate of entropy

Combining the laws of conservation of mass, energy, and momentum with Equation 31 the entropy production of a system can be calculated.

Entropy production depends upon the heat flow at a given temperature.

Heat flow is a function of the temperature gradient  $(\frac{\Delta T}{\Delta z})$ , the temperature at which the heat flow occurs, and the thermal conductivity ( $\lambda$ ) of the material.

Writing the equation for entropy production, or the irreversible component in the notation of Taylor and Cary gives

$$T\sigma = -J_u \frac{d \ln T}{dz} + \sum_{k=1}^n J_k F_k - T \frac{d\mu_{k/T}}{dT} \quad (34)$$

where  $\sigma$  is the entropy produced and other terms are as previously defined.

A linear combination of the forces will describe a first approximation of the fluxes in Equation 34. Then upon application of Onsager's theory the phenomenological equations take the form

$$J_i = \sum_{k=1}^n L_{ik} F_k - T \frac{d(\mu_{k/T})}{dz} - L_{iu} \frac{d \ln T}{dz} \quad (35)$$

$$J_u = \sum_{k=1}^n L_{uk} F_k - T \frac{d(\mu_{k/T})}{dz} - L_{uu} \frac{d \ln T}{dz} \quad (36)$$

where the Onsager reciprocal relations (phenomenological coefficients) are

$$L_{ik} = L_{ki}$$

and

$$L_{iu} = L_{ui}$$

The subscript  $i$  refers to any of the  $n$  components of the system that contain  $k$  kinds of materials. Taylor and Cary (1964) have demonstrated that this theory can be applied to a porous medium and show that generalized flow equations can be written for a soil system where fluxes of water, heat, salt or other components may exist.

For a porous system in which the simultaneous flow of heat and only one component, water, are considered, the appropriate equations may be written

$$q = L_{ww} \left[ F_w - \left( \frac{d\mu_w}{dz} \right)_T \right] - L_{wq} \frac{d\ln T}{dz} \quad (37)$$

and

$$V_q = L_{qw} \left[ F_w - \left( \frac{d\mu_w}{dz} \right)_T \right] - L_{qq} \frac{d\ln T}{dz} \quad (38)$$

where

$q$  = water flux

$V_q$  = calorimetric heat flux

$L_{ww}$  = phenomenological coefficient for water flux

$L_{wq} = L_{qw}$  = interaction coefficients

$L_{qq}$  = phenomenological coefficient for heat flux

Equation 38 expresses the flux in terms of calorimetric heat flow rather than in the thermodynamic energy flux as given in Equation 36.

Taylor (1958) and Babcock and Overstreet (1955) have shown that the soil water potential can be defined as the difference in potential between soil water and pure water, thus

$$\Phi = \mu_w - \mu_w^o \quad (39)$$

Since the potential for pure water is a constant then

$$\frac{d\Phi}{dz} = \frac{d\mu_w}{dz} \quad (40)$$

assuming that external forces do not influence the flow.

Consider a closed horizontal system that is a mixture of air, water, and soil which is subjected to a constant temperature gradient

and where only heat is allowed to flow in and out of the system.

Where the potential distributions vary in only one direction, the water and heat flux equations will reduce to

$$q = - L_{ww} \frac{d\phi}{dx} - L_{wq} \frac{d \ln T}{dx} \quad (41)$$

and

$$J_q = - L_{qw} \frac{d\phi}{dx} - \frac{L_{qq}}{T} \left( \frac{dT}{dx} \right) \quad (42)$$

where

$L_{ww}$  = unsaturated conductivity

$L_{qq}$  = Fourier heat conductivity coefficient

Cary (1963) was successful in testing the validity of the Onsager relationship for vapor diffusion in air under thermal gradients. Apparently it is also applicable for water movement in soil (Taylor and Cary, 1965; Cary, 1965). This can be demonstrated by an independent measurement of the heat flux and the two conductivities (heat and capillary).

Placement of a constant thermal gradient across a horizontal column of soil will cause heat to flow. This will induce water flow which will continue until a steady state condition has been reached. At this point  $q = 0$  and Equation 41 becomes

$$q = 0 = - L_{ww} \frac{d\phi}{dx} - L_{wq} \frac{d \ln T}{dx} \quad (43)$$

which leads to the relation

$$\frac{d\phi}{d \ln T} = - \frac{L_{wq}}{L_{ww}} = - \frac{L_{qw}}{L_{ww}} = - \beta \quad (44)$$

Then  $\beta$ , the apparent heat of transfer, can be obtained by plotting  $\Phi$  against the  $\ln T$  and evaluating the slope of the curve. If  $\beta$  can be found, it is then possible to solve the flow equation.

If it is more desirable to measure soil water content ( $\theta$ ) than water potential ( $\Phi$ ) as the independent variable then Equation 43 becomes

$$q = -D \frac{d\theta}{dx} - L_{wq} \frac{d \ln T}{dx} \quad (45)$$

where

$$D = L_{ww} \frac{d\Phi}{d\theta} = \text{coefficient of diffusivity}$$

Expressing Equation 45 for a steady state ( $q = 0$ ) when no flux is occurring

$$q = 0 = -D \frac{d\theta}{dx} - L_{wq} \frac{d \ln T}{dx} \quad (46)$$

which upon rearrangement becomes

$$\frac{d\theta}{d \ln T} = \frac{L_{wq}}{D} = -\beta^* \quad (47)$$

where  $\beta^*$  is defined as  $\beta \frac{d\theta}{d\Phi}$  at constant temperature, pressure, and composition or sometimes as the diffusion transfer coefficient.

Applying Equation 47 to Equation 45 and rearranging gives

$$q = -D \frac{d\theta}{dx} + \beta^* \frac{d \ln T}{dx} \quad (48)$$

and also using Equation 42 and 47, and remembering the phenomenological coefficients leads to

$$q = -D\beta \frac{d\theta}{dx} - L_{qq} \frac{d \ln T}{dx} \quad (49)$$

which gives a set of simultaneous equations that can be solved after making appropriate measurements.

This development suggests that two coefficients,  $\beta^*$  and  $D\beta$ , should be related by the slope of the  $\Phi$  vs  $\theta$  curve as a consequence of the phenomenological coefficients. This same system may be more readily described by Equations 41 and 42 with the use of only three coefficients. It may be possible to expand the commonly used flow equations to account for the influence of temperature gradients simply by measuring the temperature and evaluating the ratio,  $\beta$  or  $\beta^*$  which may prove to be constant over rather wide ranges of water contents (Taylor and Cary, 1964). If  $\beta$  and  $\beta^*$  are independent of the applied temperature and constant over a significant portion of the water content range, then for conditions where a constant thermal gradient exists, it would be possible to account for the thermally induced water flow by adding a constant term to the usual flow equations.

The conductivities and coefficients which have been used in the above equations represent the sum of the individual vapor, liquid and in some instances the absorbed phase of water flow. The flow equations assume that the driving forces for flow in each individual phase was the same or at least directly proportional. It is also well to keep in mind the following conditions which must be met, in addition to the general assumption inherent with the general theory of irreversible thermodynamics.

- a. The soil is inert and no chemical reactions take place in it.
- b. The system is never far from equilibrium.
- c. The flow can be adequately described by linear equations.

- d. All fluxes of matter and energy are specifically accounted for.
- e. If the "diffusion" form is used, it must further be assumed that the water potential is a unique function of water content.

The conductivities and coefficients represent a composite of the individual conductivities, therefore mechanisms for flow are masked. A more detailed analysis must be made to evaluate contributions of the individual phases to total flow. Jackson (1965) reported separation of vapor and liquid flow components in near air dry soil and the temperature and pressure dependence of the diffusivities measured. Cary (1965) has been able to separate the liquid and vapor flow in a loam soil under suctions from 5 to 34 cm of Hg.

Additional analyses of this nature are needed on the entire water content range for a more adequate understanding of the flow mechanisms and their dependence upon water content and temperature.

Groenevelt and Bolt (1969) reviewed the thermodynamic transport process assumptions and stated that use of an equation for describing entropy production that implies recognition of one "overall" flux of solution through a system appears to be an oversimplification. Only after careful examination of the experimental behavior of such systems could one conclude on the merits of a simplified equation. Application of the Onsager relations to the set of equations derived from this equation needs a careful theoretical study of certain models and/or experimental verification before being acceptable.

## MATERIALS AND METHODS

### Soil Bulk Density and Water Content Measurement By Gamma Radiation Attenuation

#### Theory

The intensity  $I$  of a narrow beam of gamma radiation of one given energy is decreased proportionately by the quantity  $-\Delta I$  when it is passed through a thin section of material of a thickness  $\Delta x$ :

$$-\Delta I = \mu_0 I \Delta x \quad (50)$$

where  $\mu_0$  is the proportionality constant or more appropriately the linear absorption coefficient. The value depends upon the nature of the absorbing material and represents that portion of the incident beam absorbed per unit thickness per unit area of material. In the limit Equation 50 becomes

$$-dI = \mu_0 I dx \quad (51)$$

Absorption of a radiation beam is governed by the density and thickness of material through which it travels. Thus, absorber thickness can be appropriately expressed on a mass basis in grams per square centimeter. Therefore, the attenuation coefficient is expressed in square centimeters per gram and labeled the mass absorption coefficient. The mass absorption coefficient  $\mu$  is defined as

$$\mu = \frac{\mu_0}{\rho} \quad (52)$$

where  $\rho$  is the density of the material.

Solving the differential equation with boundary conditions

$I = I_0$  at  $x = 0$  produces

$$I = I_0 \exp(-\rho\mu x) \quad . \quad (53)$$

The value of  $I$  decreases when  $\rho$  (density) and/or  $x$  (thickness) increases. If a number of absorbers are placed between a source and detector fixed a given distance apart their absorptive ability is independent and additive. Thus, for a moist soil and a container positioned between source and detector, neglecting air and assuming 1 as the density of water, Equation 53 may be written

$$I_m = I_0 \exp [-(\mu_s \rho_s + \mu_w \theta)x_s - \mu_c \rho_c x_c] \quad (54)$$

where  $\theta$  is the soil water content in  $\text{gm/cm}^3$ ,  $x_s$  and  $x_c$  refer to the thickness of the soil and the container walls,  $\rho_s$  and  $\rho_c$  refer to the density of the soil and the container and  $\mu_s$ ,  $\mu_w$  and  $\mu_c$  refer to the mass absorption coefficients of the soil, water, and container, respectively.

The corresponding equation for a dry soil can be shown as

$$I_d = I_0 \exp [-\mu_s \rho_s x_s - \mu_c \rho_c x_c] \quad (55)$$

Then, dividing Equation 54 by Equation 55 and solving for the soil water content gives

$$\theta = \frac{-\ln(I_m/I_d)}{\mu_w x_s} \quad (56)$$

Therefore, if  $\mu_w$  is known, the water content for a given thickness of soil can be determined from the ratio of the count rate at the water content to be determined to that at a known water content. However, Equation 56 is applicable only to systems where the change in mass per unit volume is due to a change in water content.

## Gamma Radiation Apparatus

The gamma radiation apparatus was designed (Thames, 1966) and constructed to allow measurements to be taken on stationary vertical soil columns up to 130 cm in length and 25 cm in diameter. A horizontal unit permitted scanning a movable soil column up to 2 meters in length and 25 cm in diameter. The main components of the system are a source of gamma radiation, gamma scintillation detector, pulse height analyzer, rate meter, scaler, and the elevator apparatus for scanning the soil column (Figure 2).

The gamma radiation source is 201 mc of Cesium<sup>137</sup> sealed in a capsule 1.59 cm in length and 0.64 cm in diameter. Three principal reasons for the use of Cs<sup>137</sup> were; (1) the greater part of the radiation has an energy of 0.66 mev which facilitates the use of Equation 56, (2) the low energy is desirable since water and soil are poor absorbers of high energy radiation, and (3) the half-life is 30 years, thus corrections for decay during normal use are negligible.

The source capsule is centrally located in a steel encased lead cylinder 14.0 cm in length and 21.5 cm in diameter (Figure 3). Radiation dose at the surface of the holder is less than 0.5 mr/hr. The detector is mounted in a cylinder of the same construction 10.0 cm in length and 21.5 cm in diameter. The shield and holder are mounted to the elevator platform on steel rails which maintain alignment and allow adjustment of the distance between the source and the detector.

The gamma beam is collimated through a pin-hole in lead blocks clamped firmly between two gates 3.8 cm thick and 5.0 cm wide positioned in the end of the holder and the shield. A screw adjustment permits

1. Scaler
2. Ratemeter
3. Recorder
4. Detector
5. Lift Screw
6. Frame
7. Platform
8. Track
9. Source Holder
10. Lift Drive
11. Stage
12. Soil Column
13. Heat Exchanger
14. Collimator
15. Collimator Gate

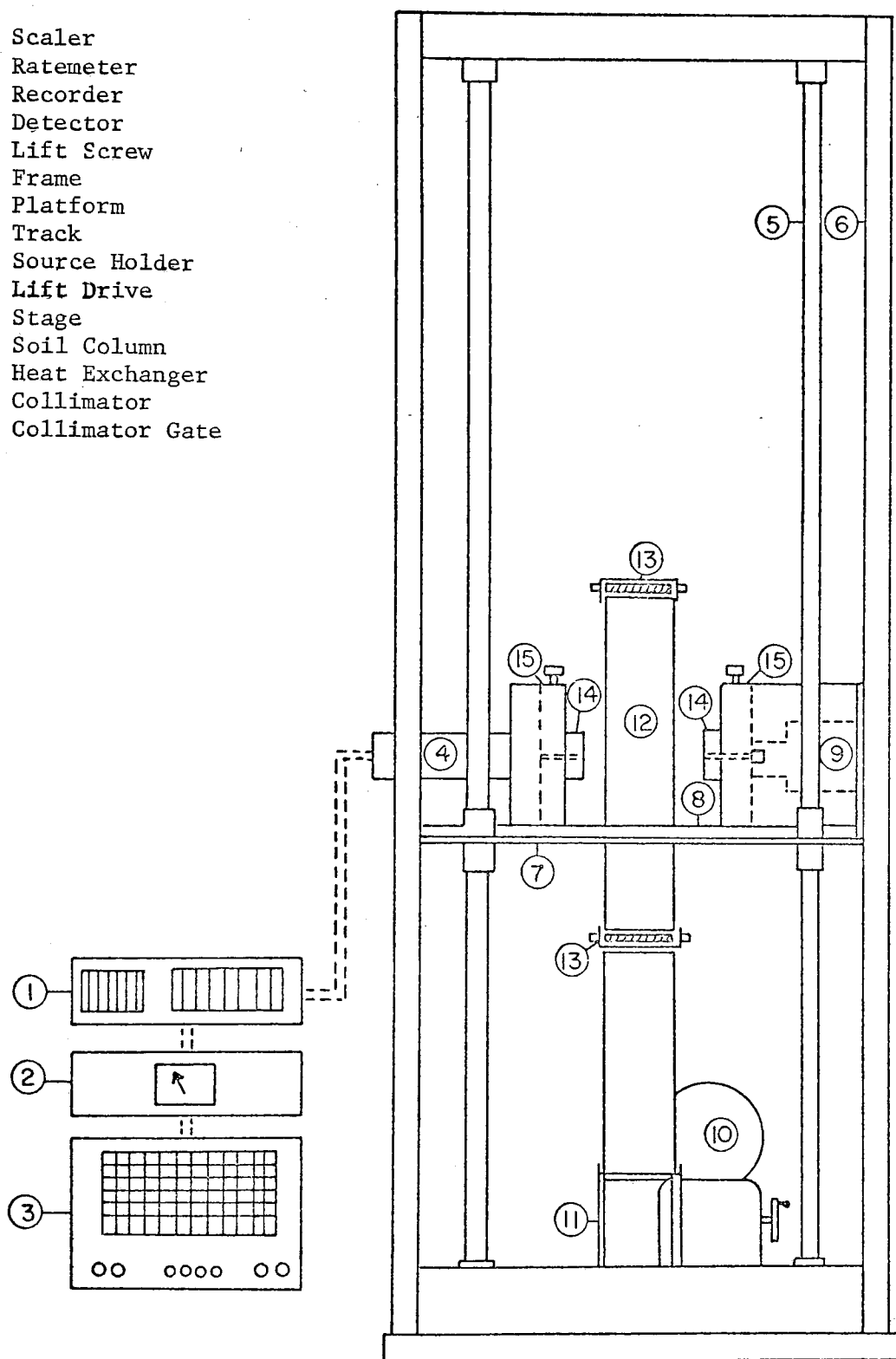


Figure 2. Gamma radiation apparatus.

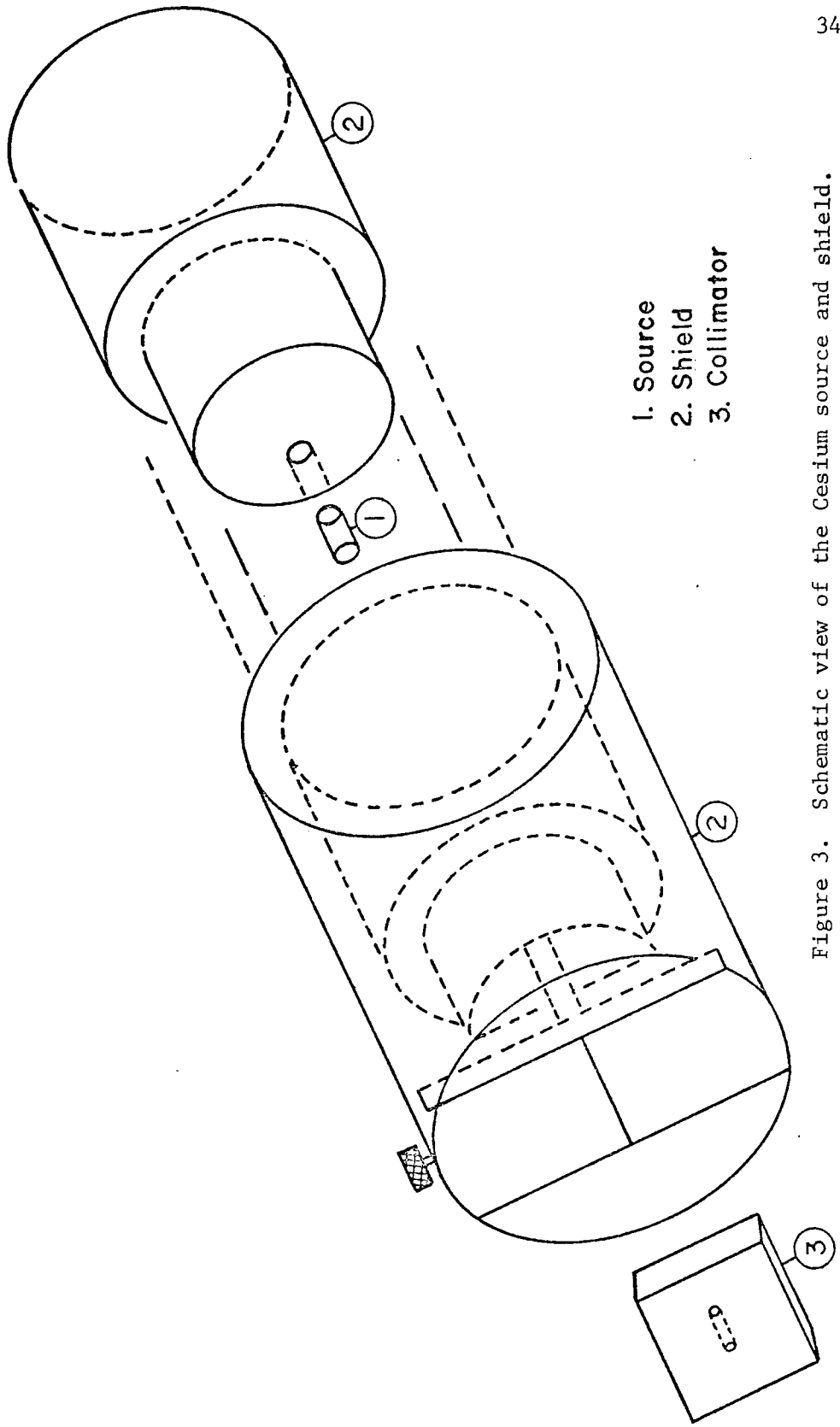


Figure 3. Schematic view of the Cesium source and shield.

the gates to be moved apart vertically to facilitate changing collimation blocks.

The elevator assembly consists of a framework 61.0 cm by 61.0 cm by 213.0 cm in height of 5.08 cm square steel tubing, an elevator platform of 1.2 cm thick steel, and a variable speed, reversible power unit (Sterling Model R61D035). The elevator platform is raised and lowered within the framework by four 2.54 cm diameter, acme-threaded rods connected by a chain drive assembly through a gear box (Model VL 113-40) to the power unit. The platform is supported and stabilized on the threaded rods by 8.0 cm long bronze nuts. Tracking speeds can be varied from 13.5 to 33.5 cm/min. Magnetic disc brakes on the power unit allows the platform to be stopped at any height without overriding. Height of the platform can be measured directly from a meter stick attached to the frame.

The detector is a scintillation system consisting of a 2.54 cm long by 2.54 cm diameter, thallium activated, sodium iodide crystal, photomultiplier, and preamplifier (Nuclear Chicago Model 821330). Pulses from the detector are linearly amplified and fed into a pulse height analyzer (Nuclear Chicago Model 8727) for differential analysis. The analyzer was set to accept radiation of  $0.66 \pm 0.02$  Mev. A six digit scaler with a pulse pair resolution of 1.5 microseconds was an integral part of the analyzer. The rate meter (Nuclear Chicago Model 8751) operated by the analyzer drove an XY millivolt recorder. A voltage regulator was used to reduce line voltage fluctuations.

Resolution time of the complete system does not exceed two microseconds. The scintillation detector and pulse height analyzer

were the limiting components in the system since they accepted all incident counts, scattered or primary, while the scaler and rate meter received only counts from the analyzer. The count rates were kept low enough by collimation to minimize coincident losses.

#### Performance of Apparatus

The system was evaluated to determine if the prediction of the linear relationship by Equation 51 could be obtained. Several tests with various collimator arrangements were evaluated by placing successive numbers of 0.635 cm thick aluminum blocks perpendicular to the gamma rays. These data showed the addition of two pin-hole collimators produced the more precise results. These collimators were simply lead blocks 5.08 cm by 5.08 cm by 6.35 cm in front of the source holder and 5.08 cm by 3.81 cm by 6.35 cm in front of the detector with a 0.317 cm hole drilled through the center of each. The blocks were held in place by the adjustable gates on the source shield and on the detector holder. Alignment of the collimators was accomplished by removal of the lead plug at the rear of the shield holder and visually sighting through the two pinholes.

The high voltage control on the pulse height analyzer was adjusted for maximum count rate at a base control setting of 0.66 Mev before every series of measurements. To center the measured voltage band as close as possible on 0.66 Mev, the base control setting was shifted. For long periods of time the peak voltage setting remained constant, however, peak amplitude variations were common and could vary from day to day, probably due to line voltage fluctuations.

## Calibration and Precision

The mass absorption coefficient for aluminum was determined by taking 15 minute counts on successive numbers of 0.635 cm thick aluminum plates. This value ( $0.077 \text{ cm}^2/\text{gm}$ ) was comparable to the theoretical value given by Grodstein (1957) (Figure 4).

Determination of the mass absorption coefficient for water was made by taking 15 minute counts on successively increasing number of 0.635 cm thick aluminum plates. Counts were taken after each plate was added to an empty plastic box and to the same box filled with distilled water. The ratio  $I/I_0$  was determined for successive thicknesses of water as it was displaced by each addition of an aluminum plate. These values were plotted and the slope of the resulting curve yielded the absorption coefficient.

Additional calibration was accomplished by packing air dry soil into containers of the same diameter and material as that used in the soil column container. A precisely measured quantity of water was added to each container of sandy loam soil to provide a range in water content. To establish equilibrium in the system several days were allowed after wetting. Counts of 15 minute duration were taken before and after wetting and used for the determination of count ratios.

The absorption coefficient of water that was determined was not significantly different from the theoretical value of 0.085 (Figure 5). The theoretical value was used in all water content determinations.

The variance of the counting ratio  $I_m/I_d$  was used to obtain an estimation of the preciseness of the soil water measurement.

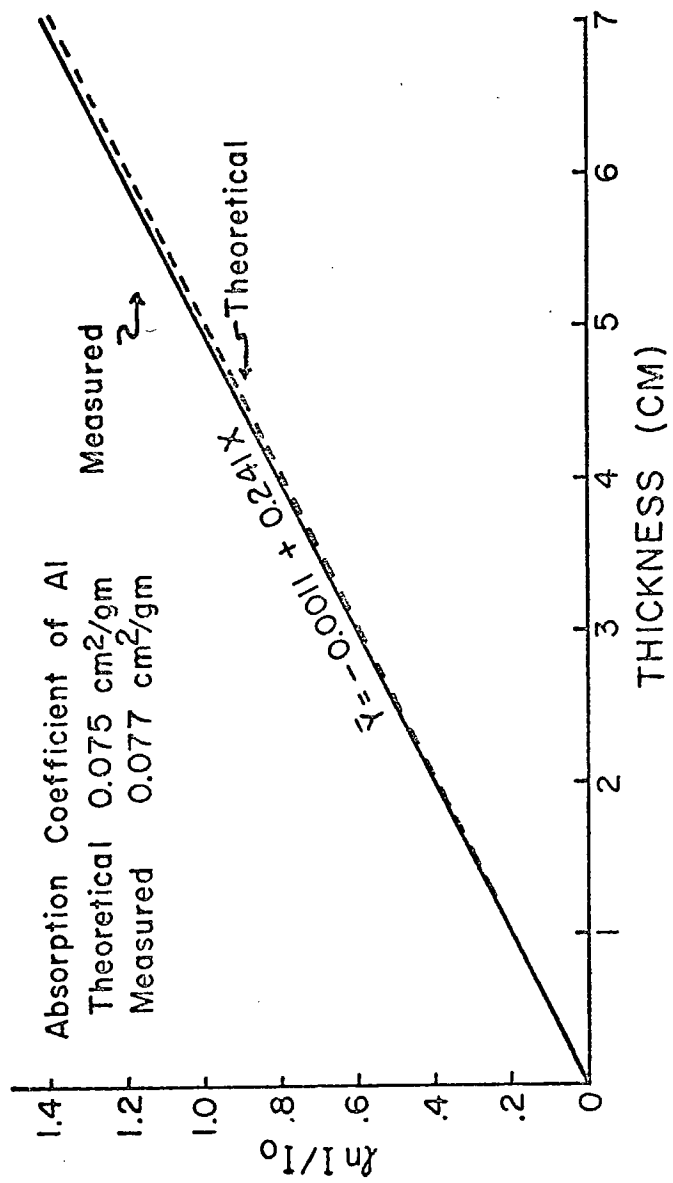
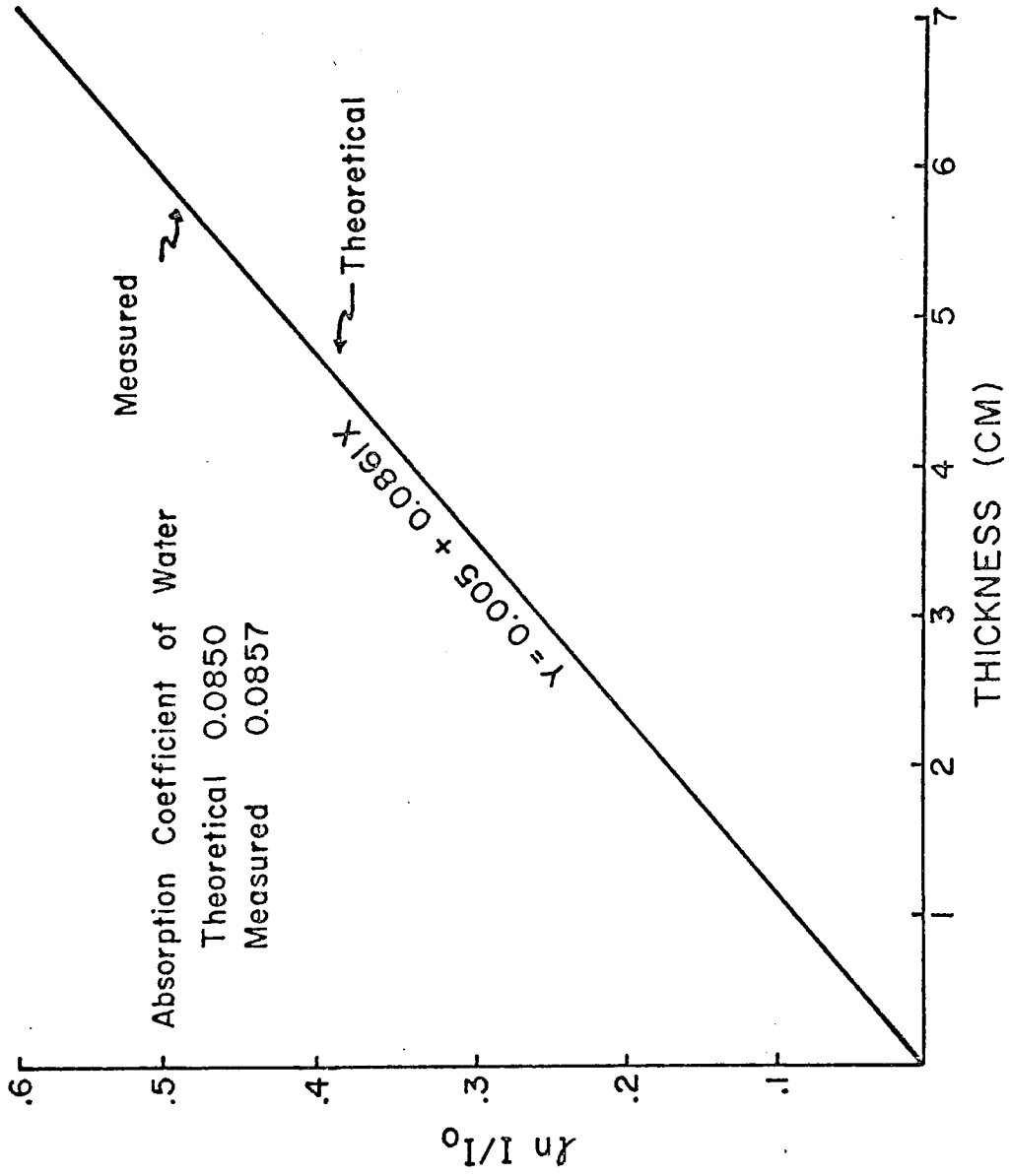


Figure 4. Relative intensity plotted against adsorber thickness (cm) to determine mass absorption coefficient for aluminum.



Absorption Coefficient of Water  
 Theoretical 0.0850  
 Measured 0.0857

Figure 5. Relative intensity plotted against adsorber thickness for determining mass absorption coefficient for water.

Considering two quantities  $x$  and  $y$  with standard deviations of  $\sigma_x$  and  $\sigma_y$ , the variance  $\sigma^2$  of a function  $(x,y)$  is (Thames, 1966)

$$\sigma^2 = \left(\frac{\partial}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial}{\partial y}\right)^2 \sigma_y^2 \quad (57)$$

Upon substitution of the partial derivatives from the equation for dry soil (55) gives

$$\sigma_{\theta}^2 = \left(\frac{\sigma_{I_m}}{\mu x_{I_m}}\right)^2 + \left(-\frac{\sigma_{I_d}}{\mu x_{I_d}}\right)^2 \quad (58)$$

Determination of soil water content and bulk density errors due to the random nature of emission of radioactive particles may be evaluated by the law of probability applicable to the emission of radioactive particles. The number of counts measured in any given time interval obeys a Poisson probability law. The standard deviation  $\sigma$  for this probability distribution equals the square root of the number of counts measured in the time interval  $\Delta t$ , therefore:

$$\sigma = \sqrt{I} \quad (59)$$

where  $I$  is the number of counts measured in the time interval  $\Delta t$ . The error in soil bulk density for a dry soil (Equation 55) may be estimated by

$$d\rho_s = \frac{\partial \rho_s}{\partial I_d} dI_d + \frac{\partial \rho_s}{\partial I_o} dI_o \quad (60)$$

and therefore

$$\Delta\rho_s \approx \frac{\partial \rho_s}{\partial I_d} \Delta I_d + \frac{\partial \rho_s}{\partial I_o} \Delta I_o \quad (61)$$

Upon substitution for the partial derivatives in Equation 61

$$\Delta\rho_s \approx \frac{\Delta I_d}{\mu_s x I_o} + \frac{\Delta I_o}{\mu_s x I_o} \quad (62)$$

Using the classical  $\pm 2 \sigma$  as the maximum statistical error for the counting process and Equation 62, an upper limit may be determined for errors in the measurement of bulk density

$$\Delta\rho_s \max \leq \frac{2}{\mu_s x} \left[ \frac{1}{\sqrt{I_d}} + \frac{1}{\sqrt{I_o}} \right] \quad (63)$$

An upper limit for the error in the determination of soil water content due to the random nature of the counting process may be defined by

$$\Delta\theta \max \leq \frac{2}{\mu_s x} \left[ \frac{1}{I_m} + \frac{1}{I_d} \right] \quad (64)$$

It was found for the purposes of this study that variation in count rates for time periods of less than one minute were too great.

#### Soil Columns

The soil material used in the soil columns was obtained from an area near Payson, Arizona. The sandy loam soil had developed under a coniferous forest. The soil material was air dried at room temperature, manually pulverized, passed through a 2-mm sieve to remove all extraneous materials, and thoroughly mixed before used as packing material. Analyses of the soil material are presented in Table 1.

Table 1. Physical and chemical analysis of the unleached soil material used in soil columns.

| Sandy Loam Soil     |        |        |                                   |       |
|---------------------|--------|--------|-----------------------------------|-------|
| Mechanical Analysis |        |        | Soil Extract<br>Chemical Analysis |       |
| Sand %              | Silt % | Clay % | Chemical                          | PPM   |
| 75                  | 19     | 6      | Calcium                           | 132.0 |
|                     |        |        | Magnesium                         | 24.0  |
|                     |        |        | Sodium                            | 9.0   |
|                     |        |        | Chlorine                          | 70.0  |
|                     |        |        | Fluoride                          | 0.3   |
|                     |        |        | Carbonate                         | 0.0   |
|                     |        |        | Bicarbonate                       | 195.0 |

The soil was randomly divided in two equal portions. One portion was leached of soluble salts with distilled water. Electrical conductivity measurements were made periodically during the leaching operation on the soil extract to determine when most of the salts had been leached.

Similar soil columns were then prepared, one of natural soil and one of leached soil, in a plexiglas column 15 cm long and 8.89 cm ID and 10.16 cm OD. Packing of the soil columns was accomplished by continuously vibrating the column while metering the soil material through a funneling attachment at a constant rate. Copper screens with 60 mesh weave were placed in four of the soil columns at 2 and 6 cm distance from the warm end. A filter paper was placed on both sides of each screen wire. This provided an artificial barrier to liquid water transfer.

Uniformity of packing throughout the length of the column was evaluated using the gamma radiation attenuation device before acceptance of any column. This required destroying a great number of packed columns before one was packed that achieved acceptable uniformity. A column was acceptable if packing of the soil material varied one percent or less for the positions where water content determinations were to be taken.

The mass absorption coefficient for the soil ( $\mu_s$ ) was determined by using the gross bulk density and calculating  $\mu_s$  for each position of count rate determination. After completion of a run the entire volume of soil material was oven dried and the gross bulk density of the soil column determined. A mean value of the mass absorption coefficient was

then used to determine soil bulk densities along the column. Bulk density values for the columns are given in Table 2.

Table 2. Bulk density values for soil columns.

|             | <u>Leached Sandy Loam</u>        | <u>Unleached Sandy Loam</u>      |
|-------------|----------------------------------|----------------------------------|
|             | Bulk Density gms/cm <sup>3</sup> | Bulk Density gms/cm <sup>3</sup> |
| No air gaps | 1.63                             | 1.60                             |
| 1 air gap   | 1.62                             | 1.64                             |
| 2 air gaps  | 1.66                             | 1.60                             |

#### Saturation of the Soil Sample

After a sample was determined as being uniform the column was ready to be saturated. The water applicator consisted of a sintered glass bead plate 0.635 cm thick and 8.89 cm in diameter mounted into a plexiglas holder. The water applicator replaced a heat exchanger on one end of a column during the wetting process. The soil column was saturated and then a suction was applied to reduce it to the desired water content.

The sample was monitored periodically for uniform water content using  $\pm 2$  standard deviations. When a uniform distribution of water was obtained in the column at the desired water content, the water applicator device was replaced by the heat exchanger. Water content profiles were obtained by counting transmitted radiation and employing Equation 56.

### Temperature Control Apparatus

The temperature difference between the ends of the soil column was maintained by placing a heat exchanger at each end of the column. The heat exchanger consisted of an aluminum plate affixed between the end of the soil column and a reservoir that water was continuously circulated through. A hot water bath was connected to the exchanger at one end of the column while a cold water bath was connected to the exchanger at the other end. The desired thermal gradient was established along the soil column by adjusting the temperatures of the two baths. Temperatures were sensed by 24 gauge copper-constantan thermocouples radially centered and spaced in the columns. Thermocouples were connected into a 12-channel millivolt recorder with a full scale span of 2 mv. The thermal gradients were adjusted so that the average temperature of the columns was equal to the temperature of the constant temperature room. Soil columns were kept in the constant temperature room for the duration of the experiment. Room temperature variations were less than  $\pm 0.5^{\circ}\text{C}$  during the course of the experiment. Heat losses radially from the system were minimized by insulating the columns with a five cm thick layer of cotton wrapped in tinfoil.

### Experimental Run

Prior to making an experimental run the soil column was aligned, leveled, and fixed in the gamma radiation apparatus. The water transport mechanisms from the two water baths were attached to the heat exchangers on each end of the column. After insulating the columns, the desired temperature gradient was applied. Temperature gradients

were monitored continuously by the 12 channel millivolt recorder. Water content measurements were made 4, 32, and 48 hours after the run had commenced. A one-minute counting time was used for count rate determinations at the same positions as the other density measurements along the soil sample. Counting rates obtained for moist soil were in the order of magnitude of  $1 \times 10^5$ .

## RESULTS AND DISCUSSION

### Temperature Distribution

The temperature differential was applied across the soil columns for 48 hours. The distribution reached an apparent steady state condition in less than two hours. This was determined by continuously monitoring the columns. Temperature profiles for four of the soil columns after a 48-hour time period are shown in Figure 6. The average thermal gradient for the soil columns was  $2.67^{\circ}\text{C}/\text{cm}$ . The temperature distributions for the leached and unleached soil material without air gaps were approximately the same with a temperature difference of  $1.0$  to  $1.5^{\circ}\text{C}$  between the columns from the 2 cm depth to the 6 cm depth. The temperature gradient was slightly steeper near the warm end of the column. This would be expected because the soil material was driest in the warm region. Also included in Figure 6 are temperature profiles for columns with one and two air gaps. If slope differences existed between the temperature curves in columns with and without air gaps, it may be explained by the differences in water content near the warm end of the columns. Also, as Cassel, Nielsen, and Biggar (1969) point out, the sigmodial shape of the final temperature curves may indicate a radial heat loss from the system where column temperatures are above ambient.

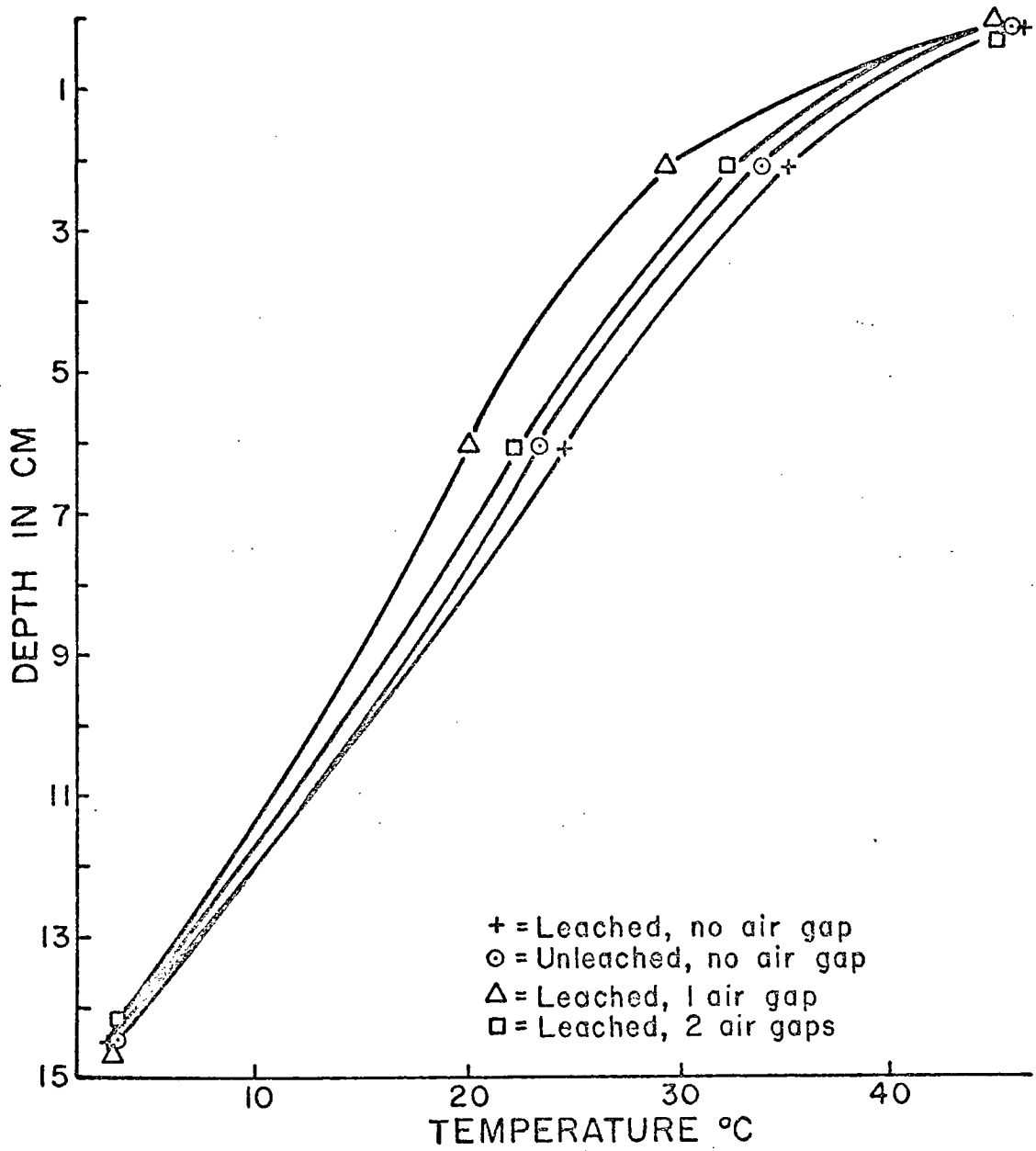


Figure 6. Temperature distribution after steady state water content distribution is reached for a leached and unleached sandy loam soil material.

### Water Content Distribution

The influence of thermal gradients on the distribution of water in soil material has been reported previously by Taylor (1962). Taylor reported that as the thermal gradient was increased, the elapsed time necessary for a steady state condition to be reached was shortened. The large temperature gradient imposed through the soil material in these experiments should cause a corresponding increase in the rapidity with which the steady state condition of no net water flow was reached.

The effect of the temperature gradients on water content distribution can be observed for this investigation in Figures 7, 8, 9, 10, 11, and 12 for the leached and the unleached soil material, with and without air gaps. These figures show the soil water content versus distance for the columns for specified times after the boundary temperatures were established. Each point represents the average of three one minute counts. The soil water distribution changed with time as indicated from the data until an apparent state condition was reached at the 48-hour time period.

The initial water content for all the columns was between the values of 0.105 and 0.115  $\text{cm}^3/\text{cm}^3$ . The final distribution of water was nearly linear for the unleached soil (Figure 8) while it was more of an S-shape for the leached soil (Figure 7).

Water loss for these two columns without barriers was calculated from the data and found to be negligible. Volume of soil water unaccounted for was 0.04 and 0.02 percent for the leached and the unleached soil, respectively, when  $\theta(x)$  was integrated over the length of the soil column.

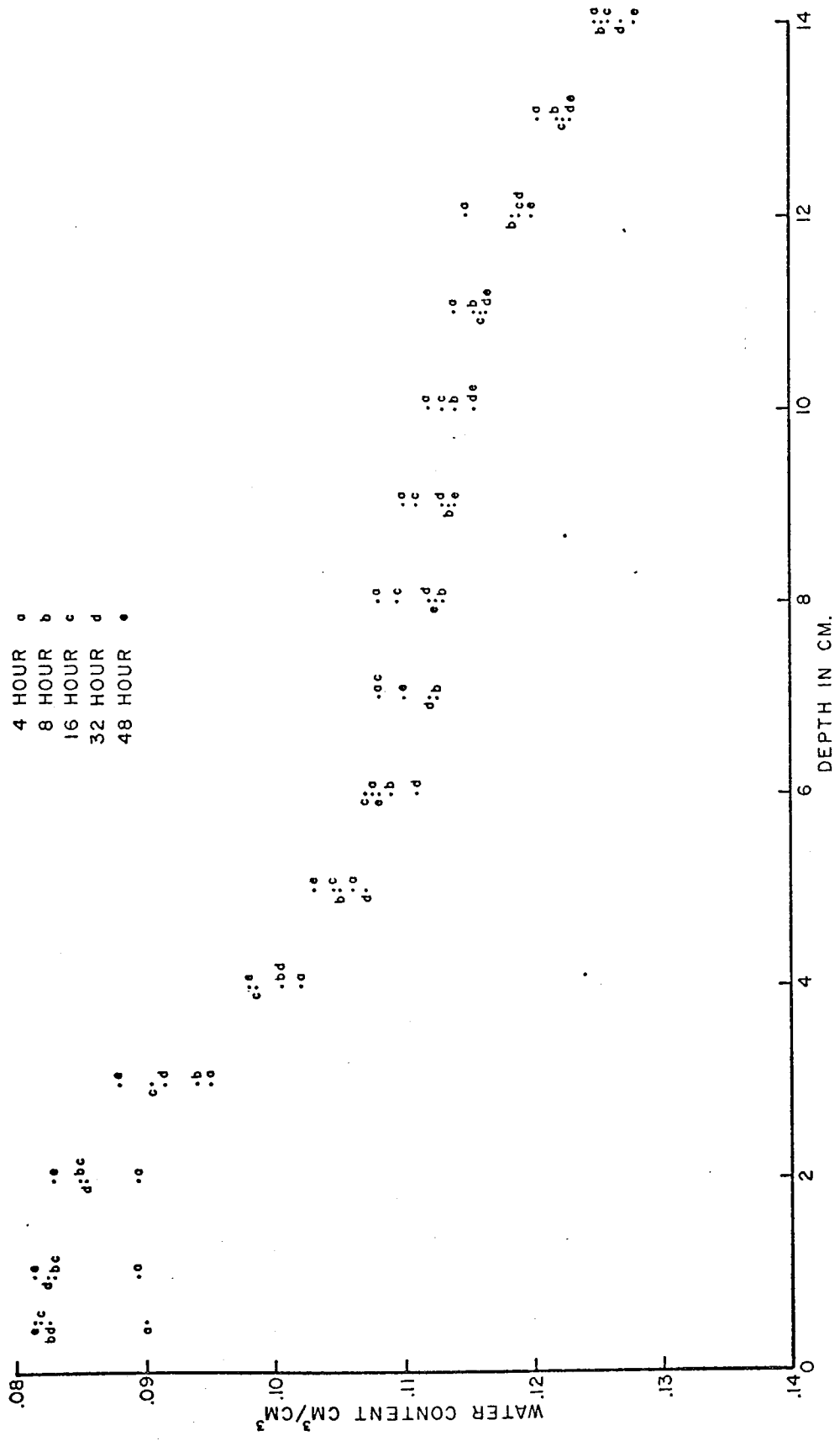


Figure 7. Water content as a function of depth for various times in the column for leached sandy loam soil material without air gaps.

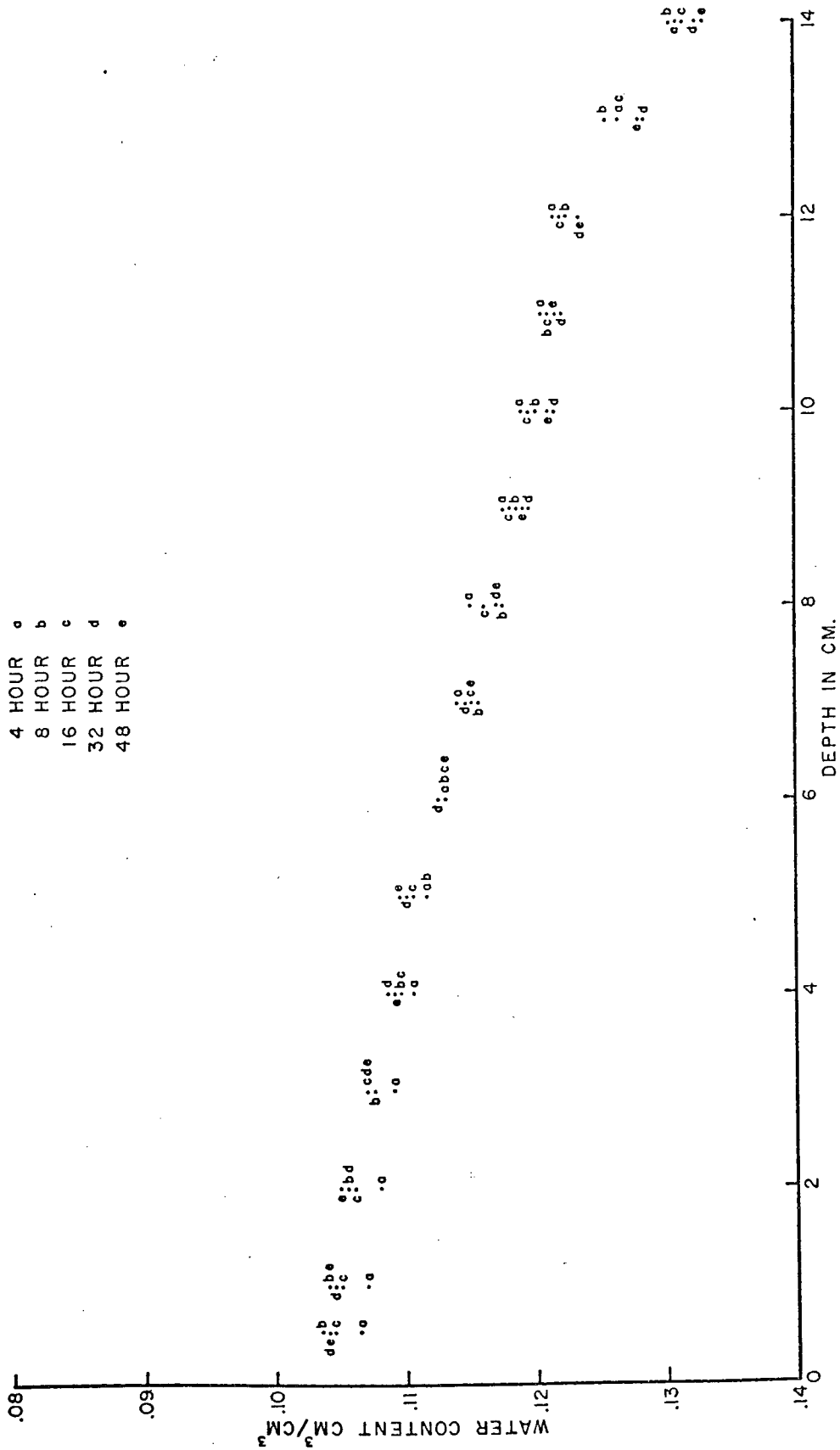


Figure 8. Water content as a function of depth for various times in the column for unleached sandy loam soil material without air gaps.

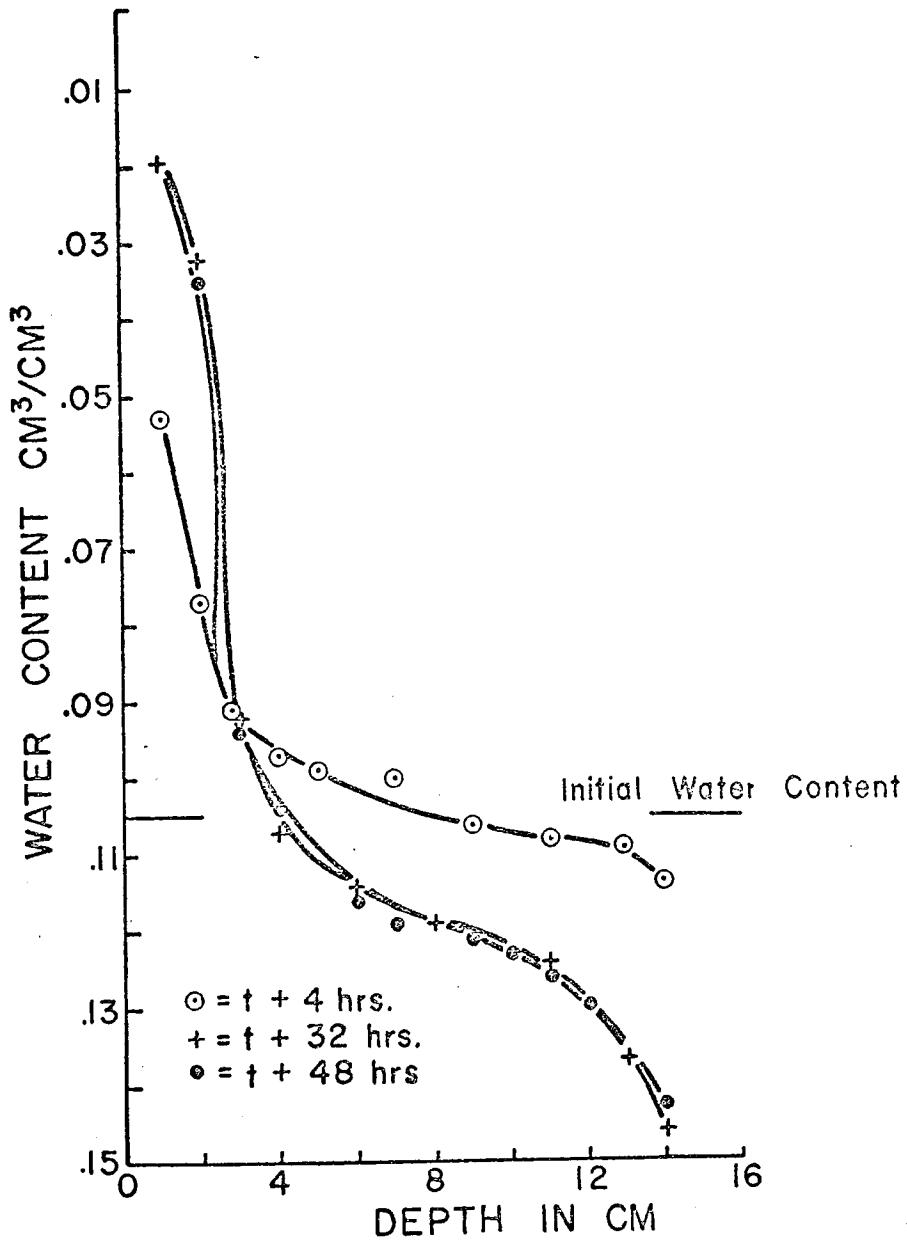


Figure 9. Water content as a function of depth for various times in the column for leached sandy loam soil material with one air gap at 2 cm.

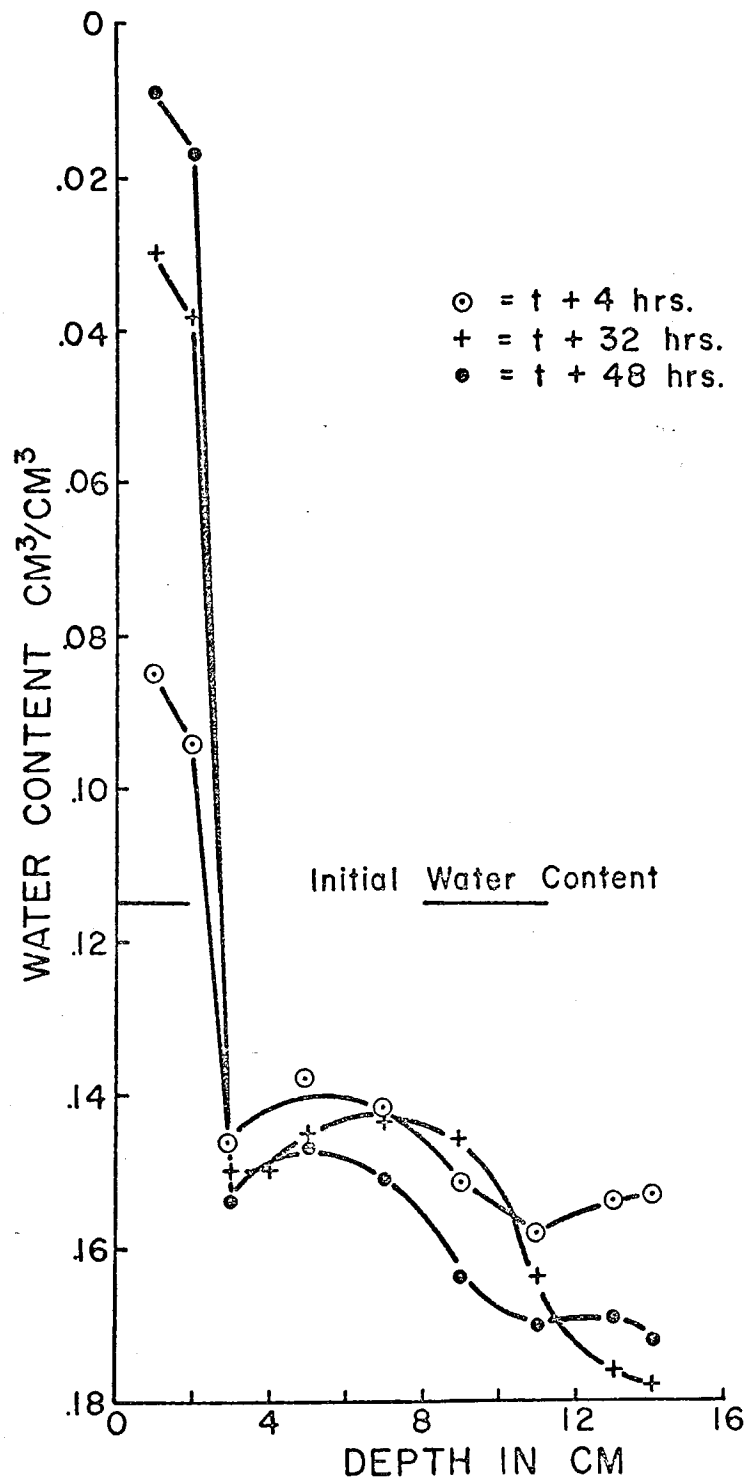


Figure 10. Water content as a function of depth for various times in the column for unleached sandy loam soil material with one air gap at 2 cm.

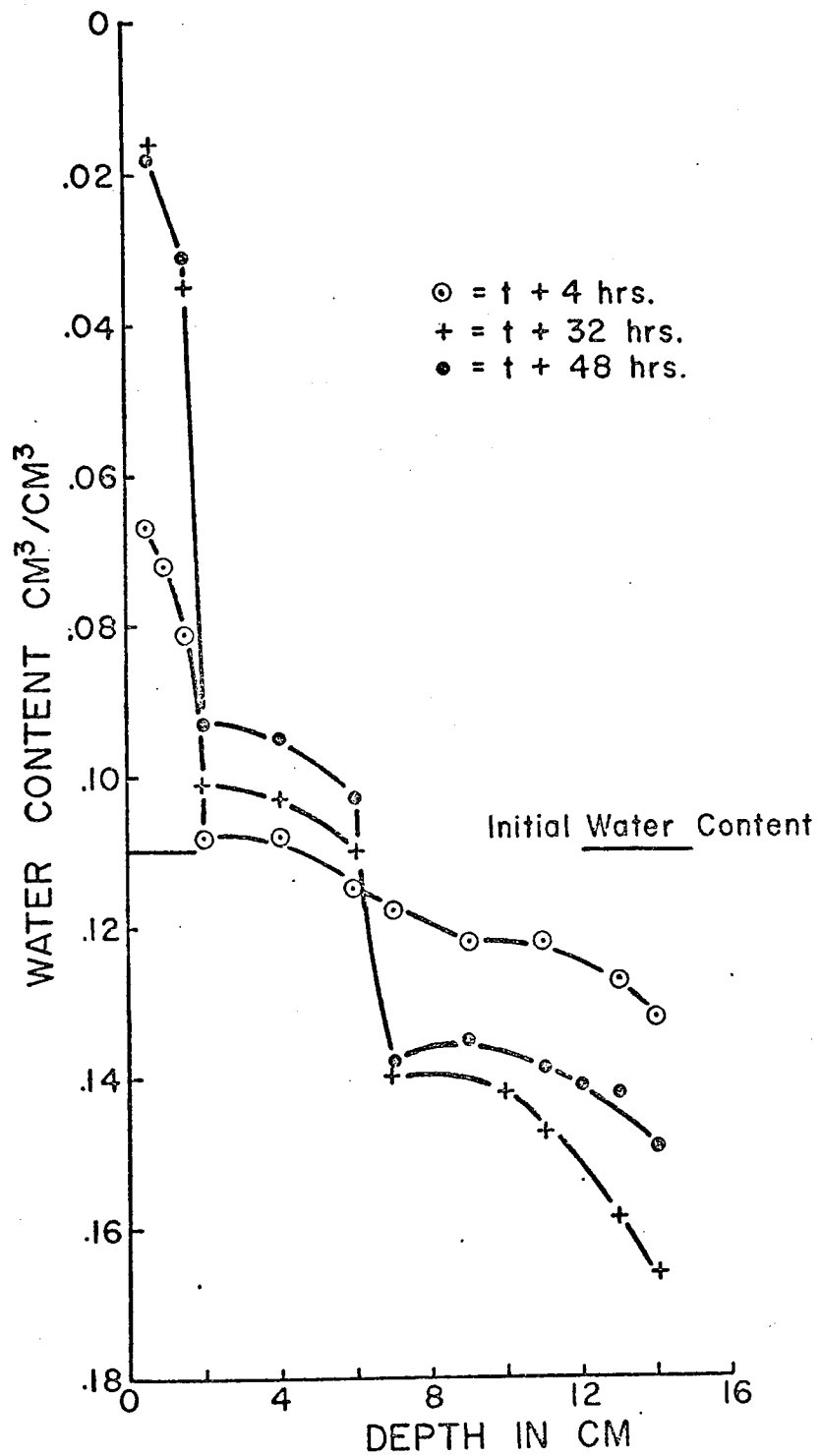


Figure 11. Water content as a function of depth for various times in the column for leached sandy loam soil material with two air gaps at 2 and 6 cm.

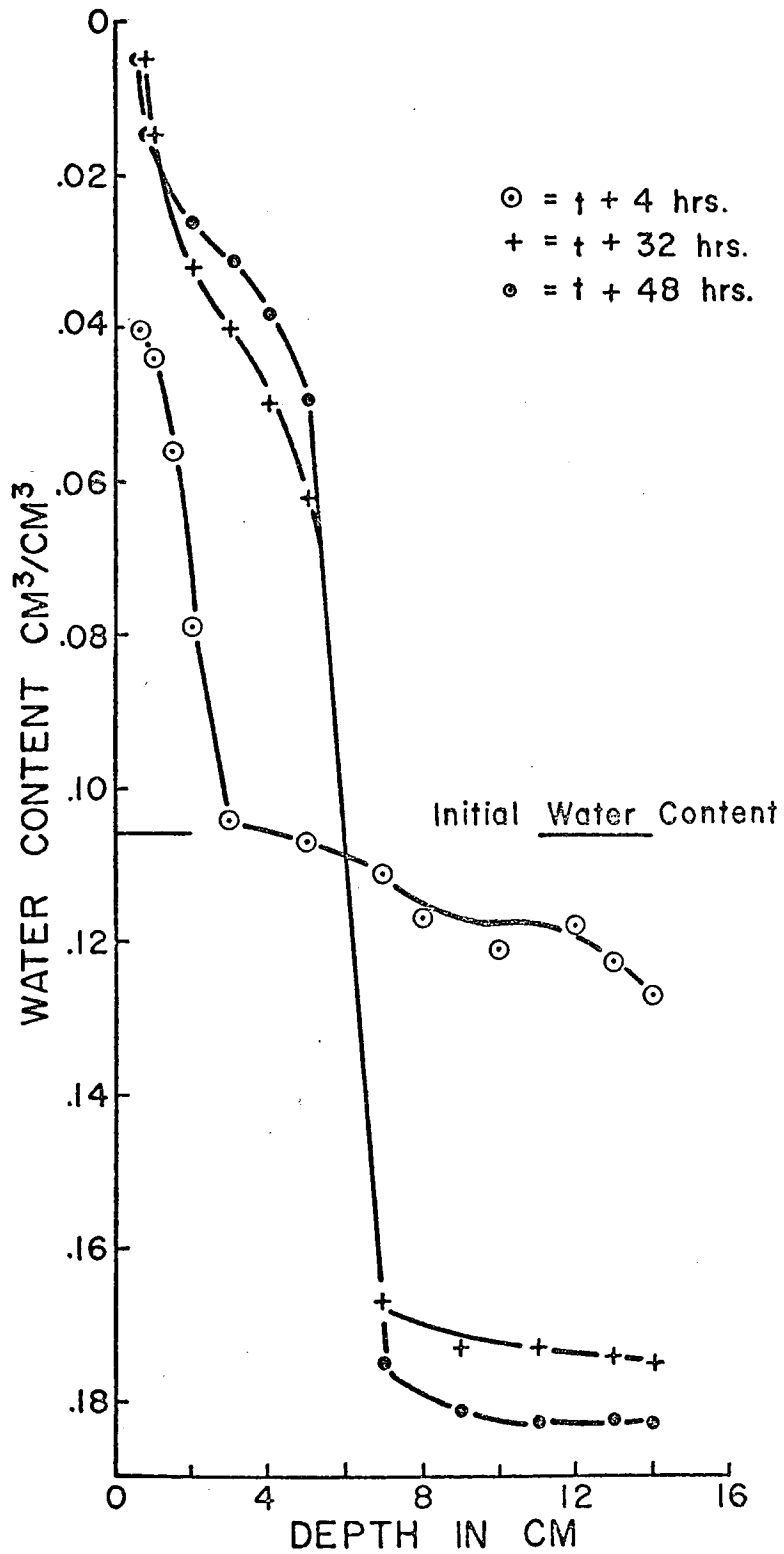


Figure 12. Water content as a function of depth for various times in the column for unbleached sandy loam soil material with two air gaps at 2 and 6 cm.

The wet water flux for this sandy loam soil material was observed to vary with water content. When a soil column with a water content of  $0.16 \text{ cm}^3/\text{cm}^3$  was subjected to temperature gradients equal to that imposed on the described columns, no measurable flux of water was observed using the gamma attenuation device for the specified time period. Although water movement did not occur at  $0.16 \text{ cm}^3/\text{cm}^3$ , it did occur at water content values of 0.105 and  $0.115 \text{ cm}^3/\text{cm}^3$ . This would indicate that initial water content values fell within the range of values where water transfer occurred as a result of temperature gradients. This would further substantiate the findings of Gurr et al. (1952), Jones and Kohnke (1952), and Cassel et al. (1969) who reported that a range in water content exists that yields a maximum net water transfer for a given soil.

Analysis and interpretation of data were primarily oriented towards the soil columns without air gaps. The soil columns with air gaps were included in this research for the purpose of detecting if the soil water was moving in a circular manner within the columns as was reported by Jackson, Rose, and Penman (1965). Air gaps influenced the movement of water as can readily be seen by differences in water content distributions in the soil columns with and without air gaps. There was an abrupt decrease in water content across each air gap with the water content being much lower on the warmer side of the air gap. The water content at the 0.5 cm depth was on the order of 0.01 to  $0.02 \text{ cm}^3/\text{cm}^3$  when there were air gaps but 0.08 to  $0.10 \text{ cm}^3/\text{cm}^3$  when there were no air gaps. The excessive drying above the air gaps may indicate the air gaps permitted water vapor flow toward the cold end

but reduced the return flow of liquid water from the cold to the hot end. A difference in the water content gradient can also be observed between the columns with one air gap and those with two air gaps. This difference, though not as pronounced as that between columns with and without air gaps, is most obvious in the region between the depths of the two air gaps, 2 and 6 cms. The second air gap at 6 cm caused a reduction in water content between 2 and 6 cm depth (Figures 11 and 12). These data indicate that water has apparently moved across the air gaps in vapor form due to a vapor pressure difference from hot to cold and the return liquid water flow due to the water concentration gradient was obstructed. The initial water content, bulk density, temperature gradient, and mean temperature are factors that would influence the magnitude of the soil water gradient in a soil material when a thermal gradient is applied according to Gurr et al. (1952).

Mean bulk density values for the leached sandy loam and the unleached sandy loam soil columns without air gaps were 1.63 and 1.60 gm/cm<sup>3</sup>, respectively. The maximum range in bulk density within the columns was 0.06 gm/cm<sup>3</sup>.

A theoretical evaluation was made to determine the portion of water flow that could be attributed to the gravity component since the columns were run in the vertical position. This was accomplished by determining the slope of the water tension vs  $\theta$  curve (Figure 13) at an average value of  $\theta$  determined for the soil columns. This can be shown mathematically by the following, assuming,

$$q = K \left[ \frac{\partial \psi}{\partial z} + 1 \right] \quad (65)$$

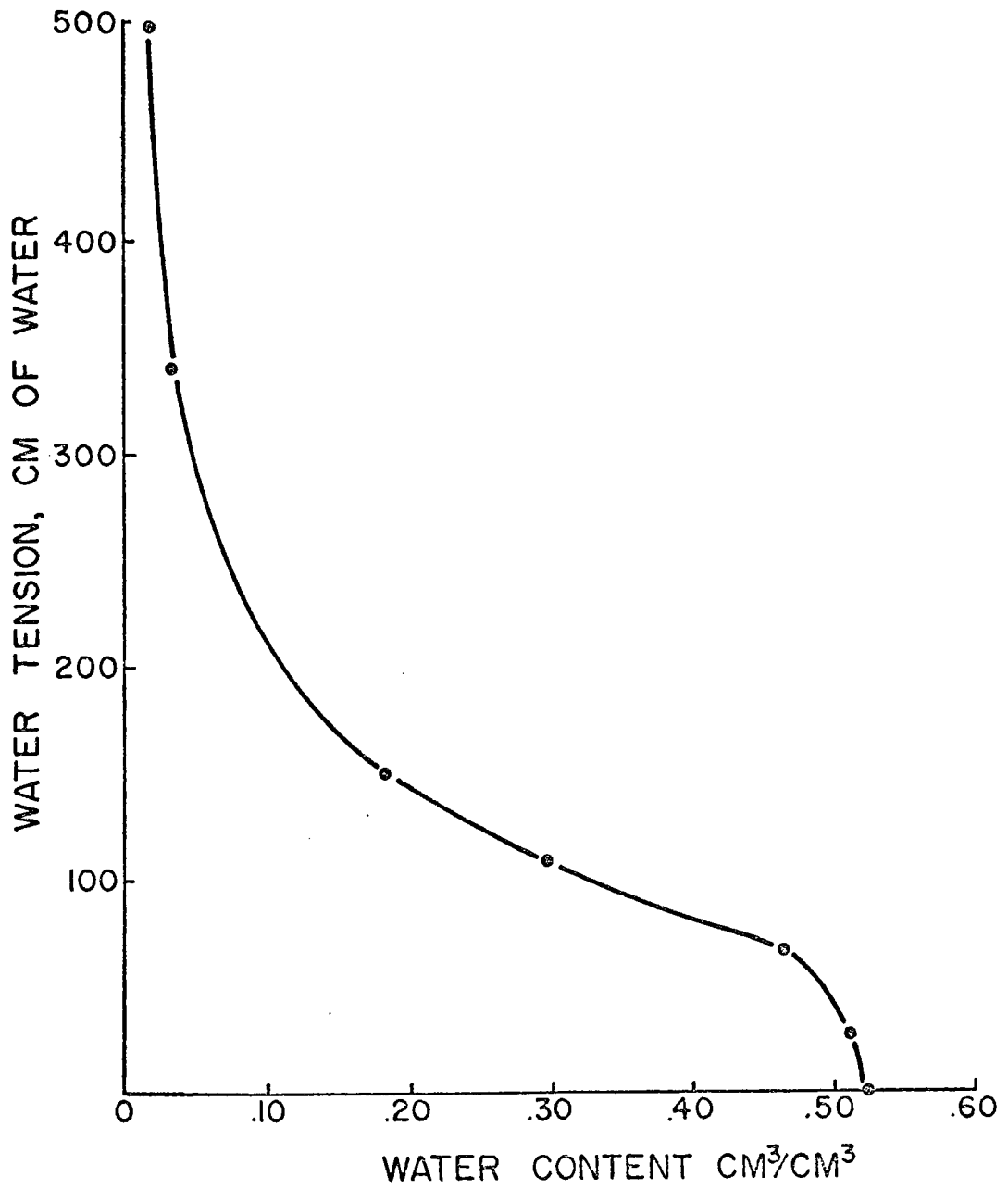


Figure 13. Water tension as a function of the water content for a sandy loam soil material.

where

$q$  = water flow rate

$K$  = conductivity

$\frac{\partial \psi}{\partial z}$  = water potential gradient

$l = \frac{\partial z}{\partial z}$  = gravitational component.

Introduction of the diffusivity term ( $D$ ) provides a means for expressing Equation 65 as

$$q = D \left[ \frac{\partial \theta}{\partial z} + \frac{K}{D} \right] \quad (66)$$

Equation 66 may be written as

$$q = D \left[ \frac{\partial \theta}{\partial z} + z \right] \quad (67)$$

The gravitational component was calculated and found to be negligible when compared to the water content gradient (1:10 ratio) at 11 percent water content. Since experimental errors might have occurred in the determination of the water tension vs water content curve or taking the slope of the curve, measurements to detect water flow in vertical column were taken using the gamma ray attenuation device. This was accomplished after attainment of the desired water content in the soil columns, by suspending them in the elevator in the same vertical position in which they would be when the temperature was applied. It was found that counts made at intervals along the soil column at the time the unit was first installed in the elevator were not significantly different from counts made 72 hours later at the same locations. Since the length of time the columns were to be subjected to a thermal

gradient was shorter than this, the assumption was made that the effect of gravity on water flow could be neglected. Apparently the amount of gravity on water flow could be neglected. Apparently the amount of water that did flow due to gravity was small and was not detectable by the gamma ray device for the period involved.

#### Salt Distribution

The electrical conductivity (EC) of a saturation extract of soil material after an imposed temperature differential for 48 hours was examined for the leached and unleached soil material. The EC value is considered proportional to the average concentration of salts in the soil column from which the sample was taken. The EC of the unleached soil material was 0.25 mmhos/cm. Other chemical and physical properties of the soil material are given in Table 1. Passing distilled water through the soil material reduced the average EC from 0.25 to 0.09 mmhos/cm.

At low water contents, liquid water movement would be small and thus move correspondingly small amount of salts. Assuming liquid water movement toward the warm end and vapor movement toward the cold end of the columns without air gaps, it would be expected that the soluble salt concentration would be greatest at the warm end after a period of time (Gurr et al. 1952; Jackson, Rose, and Penman 1965). Similarly, assuming no liquid movement across the air gaps, the salt and water concentration should be greatest in the warmer regions of the soil columns next to the air gaps. However, no increase in water content was measured. If increases in water content occurred adjacent to the air gaps it was not detectable by the gamma attenuation device.

Transfer Coefficient  $\beta^*$

Water flow data were evaluated to ascertain if the approach (page 24) of Taylor and Cary (1964) would adequately describe flow under conditions imposed by these experiments. Referring to the linear flow Equation 48, it can be seen that for one dimension when no net flow is occurring,

$$q = 0 = -D \left[ \frac{d\theta}{dx} + \beta^* \frac{d \ln T}{dx} \right] \quad (68)$$

Upon equating the terms on the right hand side of the equation and re-arranging,  $\beta^*$  may be expressed as

$$\beta^* = - \frac{d\theta}{d \ln T} = - \frac{d\theta}{2.303 d \log T} \quad (69)$$

$\beta^*$  was defined in Equation 47 as the ratio  $\frac{L}{D} \frac{wG}{D}$  (defined on page 33) and was further shown by Taylor (1963) to be identified as  $\beta \left( \frac{\partial \theta}{\partial \phi} \right)$  at constant temperature, pressure, and composition of a system, or as defined by Taylor, the diffusion transfer coefficient. If  $\beta^*$  were a constant over a wide range of water contents then by measurement of the temperature and water content distributions and evaluation of  $\beta^*$ , it would be simple to expand the flow equation to account for the influence of the temperature gradient.

If  $\beta^*$  is dependent on the water content, then the relationship  $\left( \frac{d\theta}{d \ln T} \right)$  would be described by a straight horizontal line when  $\beta^*$  is plotted against  $\theta$ . However, if  $\beta^*$  is independent of the amount of water in the system its values would change with changes in water content. Graphical analysis of the logarithm of temperature versus the water content curve provides a means for the determination of  $\beta^*$  values at selected water contents. A determination of the logarithm of

temperature versus the water content was made for the leached and unleached soil material. This determination is presented in Figure 14 using the temperatures as given in Figure 6 and the corresponding steady state water contents for 48 hours shown in Figures 7 and 8, then plotting  $\beta^*$  as a function of the water content (Figure 15) provides a means for demonstrating the dependency of  $\beta^*$  on the water content.

The dependency of  $\beta^*$  on the water content indicates that as the water content values change the values of the transfer coefficient change also. Within the low range of the observed water contents for the leached soil, the value of  $\beta^*$  decreased with increase in water content. Thus the dependency of  $\beta^*$  on water content for this range of water content can readily be seen.  $\beta^*$  continued to decrease until at approximately  $.116 \text{ cm}^3/\text{cm}^3$  water content the trend reversed and then  $\beta^*$  increased with increasing water content. Examination of  $\beta^*$  for the unleached soil shows that for  $0.102 < \theta < 0.121 \text{ cm}^3/\text{cm}^3$ ,  $\beta^*$  decreases. At  $0.121 \text{ cm}^3/\text{cm}^3$  water content,  $\beta^*$  then starts increasing with increasing water content. These data as presented here would indicate that  $\beta^*$  is not independent of water content or temperature for this soil material over the range of values evaluated since nowhere was  $\beta^*$  constant.

A plot of  $\beta^*$  versus  $\theta$  from information taken from Cassel et al. (1969) as shown in Figure 15 indicates similar results to those found in this research. As will be noted,  $\beta^*$  values of Cassel et al. are greater than those found here. Cassel used a sandy loam soil material with a lower average soil water content  $0.077 \text{ cm}^3/\text{cm}^3$  and a

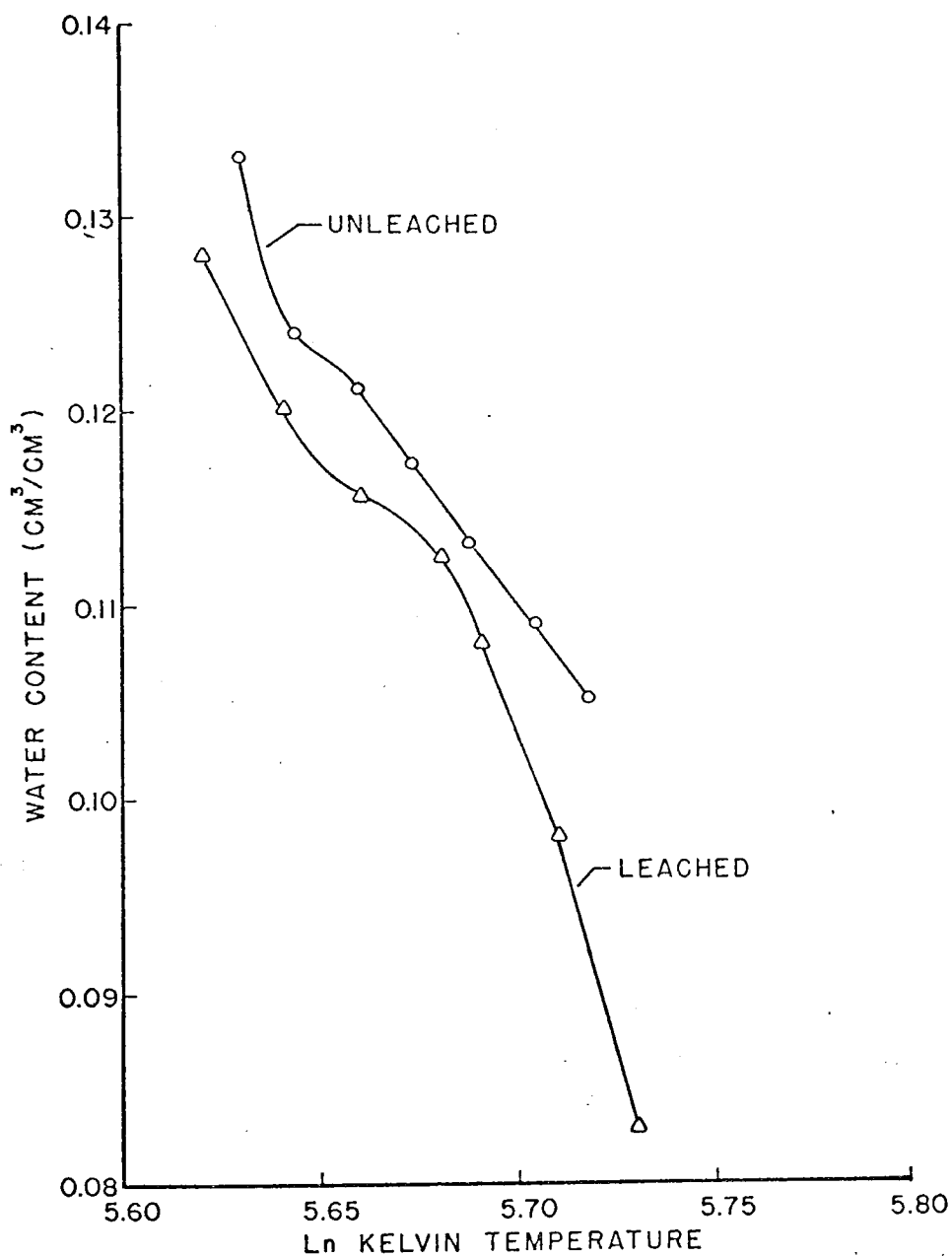


Figure 14. Relation of water content vs. ln of temperature after attainment of steady state water distribution in leached and unleached sandy loam soil material without air gaps.

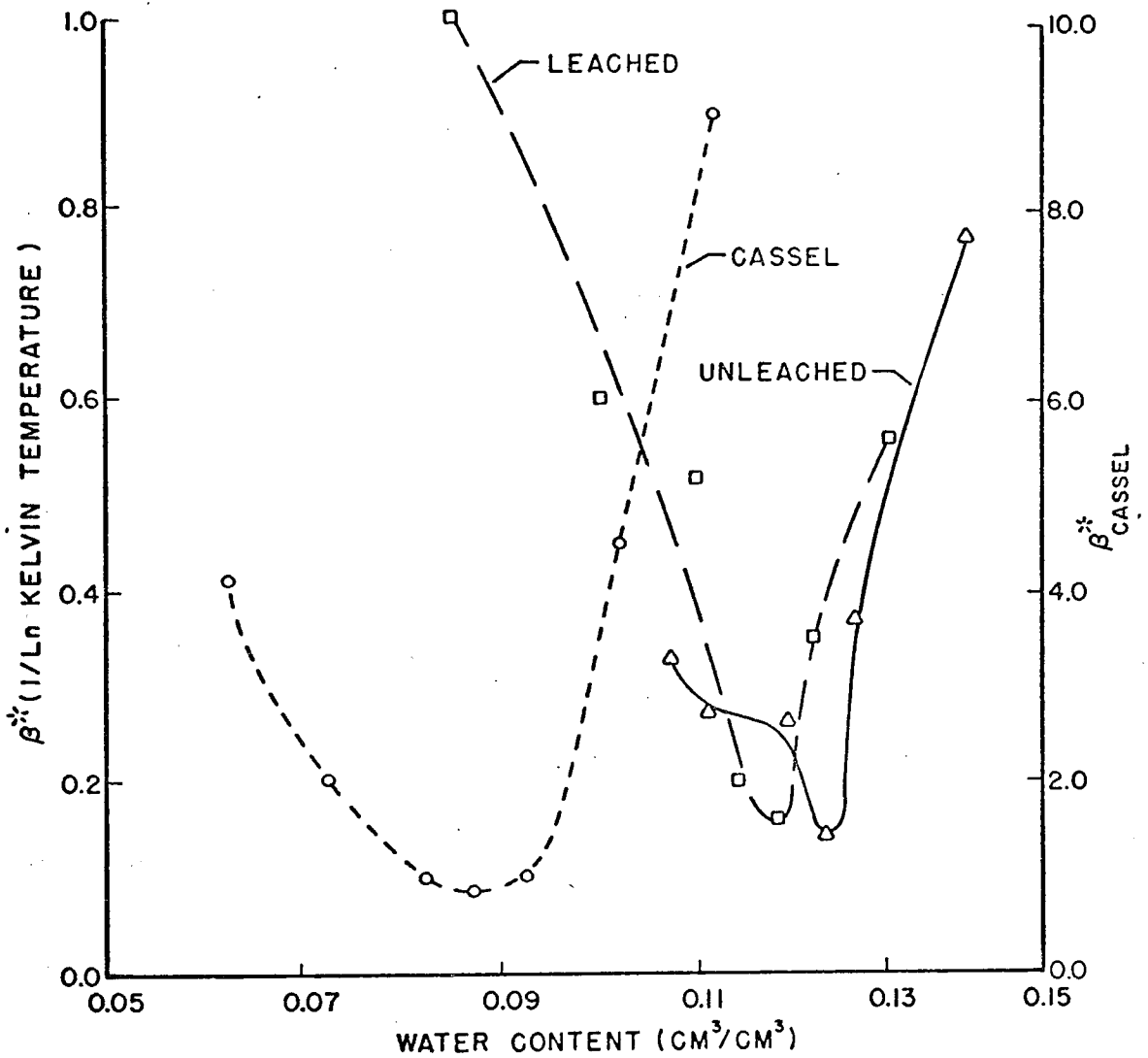


Figure 15. Transfer coefficient,  $\beta^*$ , as a function of water content for unleached and leached sandy loam soil material without air gaps.

temperature gradient of  $0.72 \text{ }^{\circ}\text{C/cm}$ . This could help to explain the differences in values obtained between the two experiments since  $\beta^*$  is a function of the temperature gradient and the water content in the soil columns. Some error may be inherent in the computation of  $\beta^*$  since values for  $\beta^*$  are derived from the slope of the curves in Figure 14. A significant factor, however, is that the curves of Cassel and those derived here are similar in shape, both being concave upwards. Nowhere in the plot of water content versus  $\ln T$  is there an indication of  $\beta^*$  being constant. The nonlinearity of  $\beta^*$  over a range of water content is further illustrated in Figure 15 wherein  $\beta^*$  versus  $\theta$  is plotted. As has been pointed out previously for the prediction of net water flux by the Taylor and Cary equations to be valid the values of the experimentally determined parameter  $\beta^*$  is assumed constant over a range in soil water content.

To explain the shape of the  $\beta^*$  versus  $\theta$  curves of Figure 15, consideration must be given to the dependence of  $D$  and  $L_{wq}$  on water content since  $\beta^*$  is the ratio of these two parameters. It is known that  $D$  decreases continuously as the water content decreases. However, the dependence of  $L_{wq}$  on water content has not been established. It is possible to discuss the relative rates of change of the two parameters in different water content ranges. Differences were found between the two soil materials, so the leached soil will be discussed first. For  $0.0815 < \theta < 0.116$ ,  $L_{wq}$  must increase more rapidly with increase in water content than does  $D$ . While for  $0.116 < \theta < 0.128$ , the rate of increase is not as great for  $L_{wq}$  as for  $D$ . It is only in the higher range of water content that  $L_{wq}$  could decrease with increase in water

content, but it is not probable since  $D$  increases rapidly with water content in this range. The curve for the unleached soil may be analyzed in a similar manner. It is possible for  $L_{wq}$  to decrease with increase in water content at low water content, but again it is not probable. It is noted that  $\beta^*$  varies from about 0.1 to 1.0 for the two soil materials. Thus,  $L_{wq}$  varies for 0.1 to 1.0 times the value of  $D$ .

An analysis of  $\beta^*$  was made for data reported by Jackson, Rose, and Penman (1965). A  $1^\circ\text{C}/\text{cm}$  temperature gradient was imposed across 10 cm long columns in which the water content was initially uniform. Two major experimental differences between these columns and those of the present experiment were a higher initial water content (0.13-0.14  $\text{cm}^3/\text{cm}^3$  by volume) and a greater bulk density (2.54  $\text{gm}/\text{cm}^3$ ). The soil used as described by Jackson, Rose, and Penman (1965), was a "local top soil" to which a sodium chloride solution was added to one-half of the soil material. Plotting water content vs  $\ln T^\circ\text{Kelvin}$  (Figure 16) reveals that for the nonsaline soil a sigmodial curve is scribed while a similar plotting for the saline soil yielded a more linear curve.

Computations made from Jackson's data of the values for the transfer coefficient,  $\beta^*$  as a function of the water content are presented in Figure 17. Since  $\beta^*$  is considered constant only at a given water content in the region where it would be shown as a straight horizontal line, it can readily be seen that nowhere does  $\beta^*$  meet this criteria on the curves. For the unleached soil  $\beta^*$  values decreased over the water content range of 10 to 12 percent while  $\beta^*$  values obtained from Jackson's data decreased over a range of 10.4 to 14.6

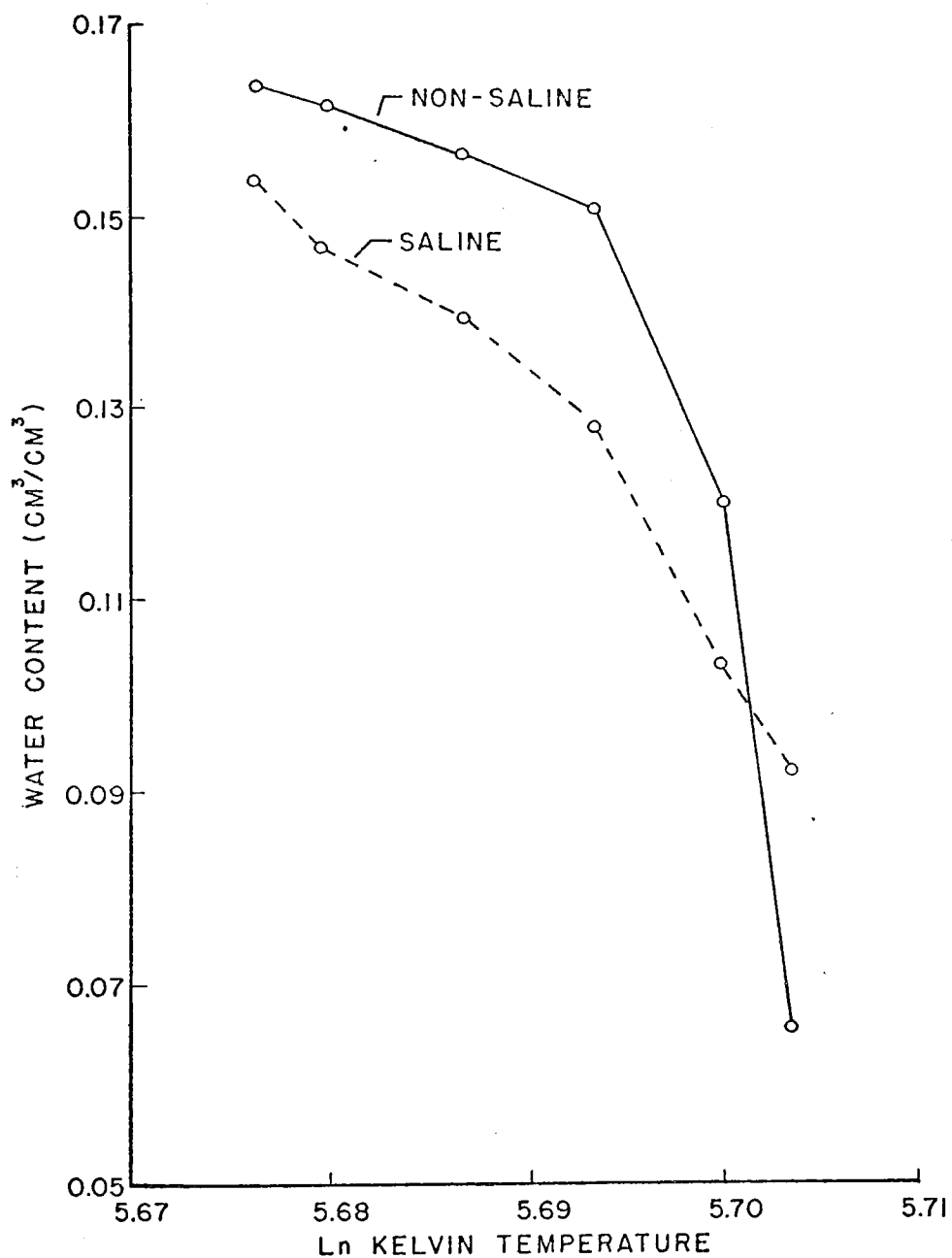


Figure 16. Relation of ln of temperature vs. water content after 12 days from the data of Jackson, Rose, and Penman (1965).

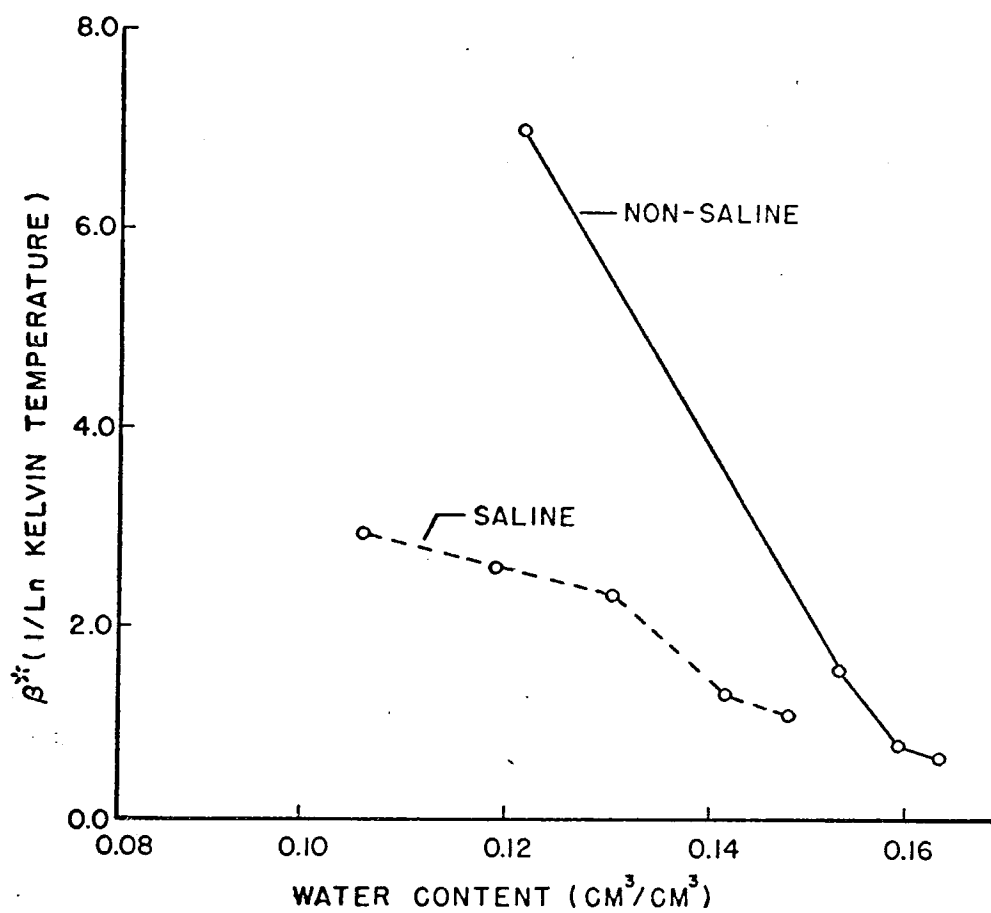


Figure 17. Transfer coefficient,  $\beta^*$ , as a function of water content from data presented by Jackson, Rose, and Penman (1965).

percent water content. The nonsaline soil and the leached soils showed decreases in  $\beta^*$  over a water content range of 12 to 16 percent and 8 to 11 percent, respectively. Although  $\beta^*$  values calculated from Jackson's data did not reach a minimum and then increase with higher water contents as did those in Figure 15, the shapes of the curves are similar to curves reported by Cassel et al. (1969). Therefore, for this soil material,  $\beta^*$  was dependent upon the water content over all ranges of water contents measured. This supports the evidence found in the present study that  $\beta^*$  is not independent but changes as water content values change.

Jackson, Rose, and Penman (1965) stated that when a temperature gradient is applied along a column in which water and salt are initially distributed uniformly, two kinds of transfer are possible. If water moves only as vapor, the ratio of salt concentration per gram of soil will remain constant, but the ratio of salt concentration per gram of water will increase in regions where evaporation occurs and decrease in regions of condensation. Conversely, if water moves only as liquid, the ratio of salt concentration per gram of water will remain constant, but that of salt concentration per gram of soil will increase in the regions to which the water flows and decrease in those from which it flows. It would also seem likely that where a combination of the two types of transfer might occur the concentrations of salts within the soil material would become increasingly more difficult to describe.

### Taylor-Cary Equation

It was observed that steady state water flow was achieved approximately 48 hours after the initial temperature cycle was started. Analysis of water flow was made on all the columns for both the leached and the unleached soil with water flow measurements being taken at the conclusion of 4-, 32-, and 48-hour time periods for the columns. These flow measurements were obtained graphically using the trapezoidal rule for approximate integration on the area under the curves at the warm end of the columns at the specified time periods.

Values for the observed water flow rates are presented in Table 3 and represent the flow rates of water that occurred for each of the specified time periods. These are compared to predicted flow rates (Table 3) as calculated by using the Taylor and Cary Equation 48. Slopes of the temperature and water content curves were determined and used with corresponding  $\beta^*$  values for calculating the predicted flow rates. Water flow rates for the various time periods are standardized to cm/sec for ease of comparison and ratios of the observed to the predicted flow rates are also shown.

Under the conditions imposed, the Taylor and Cary equation did not accurately predict the flow for all the situations tested. The best estimate was for the initial four-hour period in which the theoretically calculated value was slightly less for the column with two air gaps and slightly more for the leached columns with one gap and no air gap. In the unleached soil column the predicted value was slightly less than the observed value for the column with no air gaps,

Table 3. Comparison of calculated to measured flow rates using the Taylor and Cary equation on leached and unleached soil materials with no air gaps.

| Time<br>(hours) | Flow Rates cm/sec( $\times 10^{-4}$ ) |       |                                 |       |                                     |       |                                 |  |                                      |  |                                 |
|-----------------|---------------------------------------|-------|---------------------------------|-------|-------------------------------------|-------|---------------------------------|--|--------------------------------------|--|---------------------------------|
|                 | Observed,<br>Predicted<br>no air gaps |       | Observed:<br>Predicted<br>Ratio |       | Observed,<br>Predicted<br>1 air gap |       | Observed:<br>Predicted<br>Ratio |  | Observed,<br>Predicted<br>2 air gaps |  | Observed:<br>Predicted<br>Ratio |
| 4               | 4.829                                 | 4.885 | .989                            | 4.102 | .849                                | 5.164 | 1.069                           |  |                                      |  |                                 |
| 32              | 4.721                                 | 1.250 | .265                            | 1.021 | .216                                | 1.387 | .294                            |  |                                      |  |                                 |
| 48              | 4.662                                 | .504  | .108                            | .678  | .145                                | 1.126 | .242                            |  |                                      |  |                                 |
| Leached         |                                       |       |                                 |       |                                     |       |                                 |  |                                      |  |                                 |
| 4               | 1.517                                 | 1.628 | 1.073                           | 4.354 | 2.870                               | 7.161 | 4.721                           |  |                                      |  |                                 |
| 32              | 1.480                                 | .437  | .295                            | 1.242 | .839                                | 2.991 | 2.021                           |  |                                      |  |                                 |
| 48              | 1.349                                 | .290  | .215                            | .999  | .740                                | 2.174 | 1.611                           |  |                                      |  |                                 |
| Unleached       |                                       |       |                                 |       |                                     |       |                                 |  |                                      |  |                                 |

however, the predicted value was decidedly less than the observed value for the two remaining columns.

The predicted value from the Taylor and Cary equation overestimated the observed flow in most cases after the initial four-hour period. The only case in which observed water flow exceeded the predicted flow was for the unleached soil material with two air gaps.

The assumption was made that the presence of the two air gaps in the soil material prevented the return of liquid flow toward the hot end and thus caused a greater net water movement. However, a significantly higher net water flow did not occur in the leached soil column with two screens. This column was handled experimentally in the same manner as the other columns.

The predicted flow rates for the leached soil are approximately three times the predicted flow rates of that for the unleached soil (Table 3). The slopes of the curves in Figures 7 and 8 are only slightly different for the leached and the unleached soil material and the temperature gradient is essentially the same for the two, therefore the factor contributing measurably to the difference in the water flow appears to be the  $\beta^*$  variable. This appears to be substantiated upon examination of the  $\beta^*$  transfer coefficient in Figure 17 for the two soil materials.

#### Philip-de Vries Equation

Philip and de Vries (1957) in summarizing existing data on water transfer in soils in response to imposed temperature gradients concluded: 1) values observed for the apparent vapor diffusion are

larger than those predicted by Equation 17, 2) water transfer is negligibly small in both very wet and very dry soils, but attains a maximum value at some intermediate water content, 3) transfer of latent heat occurs by distillation, and 4) apparent vapor transfer is dependent upon the air pressure. Philip and de Vries developed an equation for describing water movement through soils in response to both water content and temperature gradients. In their development, it was assumed that osmotic and gravitational effects are small enough to be negligible. Thus,  $\psi_c$  in this case is essentially equal to the soil-water pressure  $\psi$ .

The equation which Philip and de Vries called an "approximate equation" given previously as (27) is as follows:

$$q = -D_{\theta} \nabla \theta - D_T \nabla T \quad . \quad (70)$$

The Philip-de Vries equation which may be considered as an extended treatment of the vapor diffusion equation provides for the interaction of soil solids, liquid water, and vapor water. Philip and de Vries proposed the following to explain this interaction: Consider a soil so dry that liquid continuity does not exist or if it does, assume  $K'$  negligible. The soil still contains liquid water but it occurs only in isolated pockets, filling small pores and forming wedges (islands) at the points of contact between soil particles. When a temperature gradient is applied across this soil some liquid water will evaporate at the warm side and diffuse toward the lower vapor pressure at the cool side. The liquid islands interspersed at random throughout the soil are considered regions where the rapid transport of liquid water occurs. Evaporation on the warm side and condensation

from the cool side of these islands produce alterations in the originally equal meniscus curvatures. Evaporation and condensation at the warm and cool sides of an island, respectively, continue until the meniscus curvatures become such that capillary flow through the liquid island equals the rate of condensation and evaporation. This short circuit in the vapor diffusion process tends to increase the rate of soil-water movement toward the cold end. Thus, water transfer under temperature gradients in a soil with poor liquid continuity is regarded by Philip and de Vries as a series-parallel process of flow through regions of vapor and liquid.

Since it is almost impossible to evaluate the temperature and vapor fields in air-filled pores, the vapor flux can be evaluated on the basis of certain simplifying assumptions based on the thermal conductivity of the soil-water system. Because the thermal conductivities of air (including water vapor), liquid water, and soil minerals vary greatly, temperature changes in the soil-air pores lag behind those occurring in the soil solids and water. It is reasonable, therefore, to believe that the temperature gradient across the air-filled pore is somewhat greater than the mean temperature gradient. The term  $(\nabla T)_a / (\nabla T)$  is introduced to account for this difference.

Values of  $\xi$  calculated by de Vries (1952) for different values of  $\theta$  at different values of  $(a + \theta)$  in a hypothetical soil where

$$\xi = \frac{(\nabla T)_a}{a(\nabla T)_a + \theta (\nabla T)_w + (1-a-\theta) (\nabla T)_s} \quad (71)$$

and  $(\nabla T)_a$ ,  $(\nabla T)_w$ , and  $(\nabla T)_s$  are the temperature gradients averaged

over the volumes occupied by air, water, and soil, respectively.

Values of  $\xi$  at 20 °C, as tabulated by de Vries are given in Table 4.

Thermal conductivity values at 20°C for the components used in the calculations taken from Smith (1942) and de Vries (1952) are:

$$\begin{aligned}\lambda_{\text{most soil minerals}} &= 7 \times 10^{-3} \text{ cal/cm sec} \\ \lambda_{\text{quartz}} &= 20 \times 10^{-3} \text{ cal/cm sec} \\ \lambda_{\text{air}} &= 0.0615 \times 10^{-3} \text{ cal/cm sec} \\ \lambda_{\text{water}} &= 1.42 \times 10^{-3} \text{ cal/cm sec}.\end{aligned}$$

These values should hold in the 10 to 30°C temperature range.

In soils where liquid continuity is significant, liquid phase transfer due to temperature differences becomes dominant. Vapor induced flow becomes less important due to a reduction in the number of islands and to an increase in the radii of curvature of the menisci in the remaining islands to the point where automatic adjustment to the vapor flux no longer occurs. This is treated in the equation by the introduction of the  $f(a)$  terms in the expression for  $D_{T\text{vap}}$ .

Before Equation 70 can be applied to a particular soil-water system, several physical properties of that system must be experimentally determined. Of those required, the liquid soil-water diffusivity  $D_{\theta\text{liq}}$  or the hydraulic conductivity  $K'$  is the most difficult to experimentally measure. Although several laboratory methods have been proposed for evaluating the isothermal soil-water diffusivity  $D_{\theta}$ , none of them actually eliminate the contribution of the vapor phase to soil-water movement; thus, for most soil-water contents, it is assumed that the vapor phase contributes little to the net water movement. For small  $\theta$  values, it is assumed that nearly all water movement occurs in

Table 4. Values of  $\xi$  at 20°C for different porosities and water contents.\* Values of  $\xi$  for quartz are shown in parentheses. (Taken from Philip and de Vries, 1957.)

| $\theta$<br>(cm <sup>3</sup> /cm <sup>3</sup> ) | $(a + \theta)$<br>(cm <sup>3</sup> /cm <sup>3</sup> ) |           |           |
|---|---|-----------|-----------|
|   | 0.7   | 0.5       | 0.3       |
| 0   | 1.4 (1.4)   | 1.0 (2.0) | 3.0 (3.2) |
| 0.1   | 1.3 (1.4)   | 1.7 (1.9) | 2.0 (2.7) |
| 0.3   | 1.5 (1.6)   | 1.7 (2.0) | 1.0 (2.9) |
| 0.5   | 1.5 (1.6)   | 1.8 (2.2) |           |
| 0.7   | 1.6 (1.8)   |           |           |

\* Note that for  $a = 0$ ,  $\xi$  means  $\lim_{a \rightarrow 0} \xi(a)$

the vapor phase. Similarly, various laboratory methods have been proposed for evaluating capillary conductivities. These methods are time consuming and tedious with duplicate determinations differing as much as 1 or 2 orders of magnitude. Measurements of  $D_{\theta}$  and  $K'$  are also subject to considerable error due to non-uniform soil bulk densities.

Another source of difficulty is encountered in evaluating  $\partial \theta / \partial \psi_c$  or the slope of the water content-capillary pressure curve. If a non-saline soil is used, the capillary potential  $\psi_c$  can be closely approximated by the soil-water pressure  $\psi$ . Techniques are available for measuring  $\theta$  over an extremely wide range of  $\psi$ . The  $\theta$ - $\psi$  relation, too, is sensitive to changes in soil bulk density. Hysteresis probably will be manifested in that the  $\theta$ - $\psi$  relation is usually different when  $\theta$  and  $\psi$  are measured for a desorbing soil as opposed to an adsorbing one. Even the rate at which  $\psi$  is applied to the soil water has an effect upon the final  $\theta$  for a given  $\psi$  value (Davidson, 1965).

It is thus clear that a successful application of the Philip-de Vries equation to a given soil-water system entails: 1) experimental evaluation of difficult-to-measure physical properties of the soil such as those listed above and 2) measurements of temperature and water content gradients.

The values of  $D_{\theta liq}$  for a particular soil are required for the Philip-de Vries equation. A method is not yet available for measuring  $D_{\theta liq}$ , therefore this value is approximated by  $D_{\theta}$ . Knowing that  $D_{\theta} = K' \frac{\partial \psi}{\partial \theta}$  an approximate value can be obtained once the hydraulic conductivity and the slope of the moisture tension curve are evaluated. Values for the hydraulic conductivity may be calculated using the

method described by Jackson, Reginato, and van Bavel (1965) and Millington and Quirk (1959, 1960, 1961).

$$K(\theta)_i = \frac{K_s 30 \sigma^2 \theta}{K_{sc} (\rho g \eta)^2} \sum_{j=i}^n \left[ (2j + 1 - 2i) h_j^{-2} \right] \quad (72)$$

$$i = 1, 2 \dots n$$

$K(\theta)_i$  = Calculated conductivity for a specified moisture content or pressure class (cm/min)

$K_s/K_{sc}$  = Matching factor (measured saturated conductivity/ calculated saturated conductivity).

$\sigma$  = surface tension of water (dynes/cm)

$g$  = gravitational constant (cm/sec<sup>2</sup>)

$\eta$  = water viscosity

$\theta$  = water-filled porosity (cm<sup>3</sup>/cm<sup>3</sup>)

$n$  = total number of pore classes

$h$  = potential (cm)

Evaluation of the method by Green and Corey (1971) and Kunze, Vehara, and Graham (1968) it appears that use of the calculated values for hydraulic conductivity for evaluating water flux should be satisfactory at least for order of magnitude values.

Table 5 presents the calculated hydraulic conductivity values for the soil material over a water content gradient and also corresponding values calculated for  $D_{T\text{vap}}$ ,  $D_{T\text{liq}}$ , and  $D_T$ .

Soil water hysteresis effects should be minimal for values obtained in the drying portion of the soil columns. Erratic values obtained for low soil water content may result from the inability to

Table 5. Summarization of values for hydraulic conductivity, volumetric water content, water potential and vapor and liquid water thermal diffusivity.

| Hydraulic Cond.<br>(cm/min.) | Water content<br>( $\text{cm}^3/\text{cm}^3$ ) | Suction head<br>(cm of water) | $D_T$ vap.<br>( $\text{cm}^2/\text{sec}-^\circ\text{C}$ ) | $D_T$ liq.<br>( $\text{cm}^2/\text{sec}-^\circ\text{C}$ ) | $D_T$<br>( $\text{cm}^2/\text{sec}-^\circ\text{C}$ ) |
|------------------------------|--|-------------------------------|---|---|--|
| 38.70E-02*                   | 51.48E-02                                      | 41.93E-01                     | 10.21E-09   | 33.91E-04   | 33.91E-04  |
| 25.49E-02                    | 50.44E-02                                      | 65.06E+00                     | 30.64E-09   | 34.66E-03   | 34.66E-03  |
| 23.71E-02                    | 49.40E-02                                      | 66.17E+00                     | 51.06E-09   | 32.79E-03   | 32.79E-03  |
| 22.03E-02                    | 48.36E-02                                      | 67.32E+00                     | 71.49E-09   | 30.99E-03   | 30.99E-03  |
| 20.43E-02                    | 47.32E-02                                      | 68.52E+00                     | 91.91E-09   | 29.26E-03   | 29.26E-03  |
| 18.92E-02                    | 46.28E-02                                      | 69.76E+00                     | 11.23E-08   | 27.59E-03   | 27.59E-03  |
| 17.49E-02                    | 45.24E-02                                      | 71.06E+00                     | 13.28E-08   | 25.98E-03   | 25.98E-03  |
| 16.15E-02                    | 44.20E-02                                      | 72.41E+00                     | 15.32E-08   | 24.44E-03   | 24.44E-03  |
| 14.88E-02                    | 43.16E-02                                      | 73.83E+00                     | 17.36E-08   | 22.96E-03   | 22.96E-03  |
| 13.60E-02                    | 42.12E-02                                      | 75.30E+00                     | 19.40E-08   | 21.55E-03   | 21.55E-03  |
| 12.57E-02                    | 41.08E-02                                      | 76.84E+00                     | 21.45E-08   | 20.19E-03   | 20.19E-03  |
| 11.52E-02                    | 40.04E-02                                      | 78.46E+00                     | 23.49E-08   | 18.89E-03   | 18.89E-03  |
| 10.54E-02                    | 39.00E-02                                      | 80.15E+00                     | 25.53E-08   | 17.65E-03   | 17.65E-03  |
| 96.15E-03                    | 37.96E-02                                      | 81.92E+00                     | 27.57E-08   | 16.46E-03   | 16.46E-03  |
| 87.55E-03                    | 36.92E-02                                      | 83.79E+00                     | 29.62E-08   | 15.33E-03   | 15.33E-03  |
| 79.53E-03                    | 35.88E-02                                      | 85.75E+00                     | 31.66E-08   | 14.25E-03   | 14.25E-03  |
| 72.07E-03                    | 34.84E-02                                      | 87.82E+00                     | 33.70E-08   | 13.23E-03   | 13.23E-03  |
| 65.15E-03                    | 33.80E-02                                      | 90.01E+00                     | 35.74E-08   | 12.25E-03   | 12.26E-03  |
| 58.73E-03                    | 32.76E-02                                      | 92.31E+00                     | 37.78E-08   | 11.33E-03   | 11.33E-03  |
| 52.79E-03                    | 31.72E-02                                      | 94.76E+00                     | 39.83E-08   | 10.45E-03   | 10.45E-03  |

Table 5. (Continued)

| Hydraulic Cond.<br>(cm/min.) | Water content<br>(cm <sup>3</sup> /cm <sup>3</sup> ) | Suction head<br>(cm of water) | D <sub>T</sub> vap.<br>(cm <sup>2</sup> /sec-°C) | D <sub>T</sub> liq.<br>(cm <sup>2</sup> /sec-°C) | D <sub>T</sub><br>(cm <sup>2</sup> /sec-°C) |
|------------------------------|--|-------------------------------|--|--|---|
| 47.31E-03                    | 30.68E-02  | 97.36E+00                     | 41.87E-08  | 96.25E-04  | 96.26E-04                                   |
| 42.26E-03                    | 29.64E-02  | 10.01E+01                     | 43.91E-08  | 88.42E-04  | 88.43E-04                                   |
| 37.62E-03                    | 28.60E-02  | 10.31E+01                     | 45.95E-08  | 81.03E-04  | 81.04E-04                                   |
| 33.37E-03                    | 27.56E-02  | 10.62E+01                     | 48.00E-08  | 74.07E-04  | 74.07E-04                                   |
| 29.49E-03                    | 26.52E-02  | 10.96E+01                     | 50.04E-08  | 67.52E-04  | 67.53E-04                                   |
| 25.95E-03                    | 25.48E-02  | 11.32E+01                     | 52.08E-08  | 61.38E-04  | 61.38E-04                                   |
| 22.73E-03                    | 24.44E-02  | 11.71E+01                     | 54.12E-08  | 55.62E-04  | 55.62E-04                                   |
| 19.82E-03                    | 23.40E-02  | 12.13E+01                     | 56.17E-08  | 50.23E-04  | 50.23E-04                                   |
| 17.19E-03                    | 22.36E-02  | 12.58E+01                     | 58.21E-08  | 45.19E-04  | 45.19E-04                                   |
| 14.83E-03                    | 21.32E-02  | 13.08E+01                     | 60.25E-08  | 40.49E-04  | 40.50E-04                                   |
| 12.69E-03                    | 20.28E-02  | 13.62E+01                     | 62.29E-08  | 36.11E-04  | 36.12E-04                                   |
| 10.79E-03                    | 19.24E-02  | 14.21E+01                     | 64.33E-08  | 32.04E-04  | 32.05E-04                                   |
| 90.93E-04                    | 18.20E-02  | 14.87E+01                     | 66.38E-08  | 28.25E-04  | 28.26E-04                                   |
| 75.87E-04                    | 17.16E-02  | 15.38E+01                     | 68.42E-08  | 24.39E-04  | 24.40E-04                                   |
| 62.57E-04                    | 16.12E-02  | 15.89E+01                     | 70.46E-08  | 20.79E-04  | 20.79E-04                                   |
| 50.91E-04                    | 15.08E-02  | 16.46E+01                     | 72.50E-08  | 17.51E-04  | 17.52E-04                                   |
| 40.78E-04                    | 14.04E-02  | 17.09E+01                     | 74.54E-08  | 14.56E-04  | 14.57E-04                                   |
| 32.07E-04                    | 13.00E-02  | 17.79E+01                     | 76.59E-08  | 11.93E-04  | 11.93E-04                                   |
| 24.68E-04                    | 11.96E-02  | 18.58E+01                     | 78.63E-08  | 95.87E-05  | 95.95E-05                                   |
| 18.50E-04                    | 10.92E-02  | 19.49E+01                     | 80.67E-08  | 75.38E-05  | 75.46E-05                                   |

Table 5. (Continued)

| Hydraulic Cond.<br>(cm/min.) | Water content<br>(cm <sup>3</sup> /cm <sup>3</sup> ) | Suction head<br>(cm of water) | D <sub>T</sub> vap.<br>(cm <sup>2</sup> /sec-°C) | D <sub>T</sub> liq.<br>(cm <sup>2</sup> /sec-°C) | D <sub>T</sub><br>(cm <sup>2</sup> /sec-°C) |
|------------------------------|--|-------------------------------|--|--|---|
| 13.43E-04                    | 98.80E-03  | 20.54E+01                     | 82.71E-08  | 57.66E-05  | 57.74E-05                                   |
| 93.56E-05                    | 88.40E-03  | 21.78E+01                     | 84.75E-08  | 42.58E-05  | 42.67E-05                                   |
| 61.80E-05                    | 78.00E-03  | 23.25E+01                     | 86.79E-08  | 30.03E-05  | 30.19E-05                                   |
| 37.94E-05                    | 67.60E-03  | 25.06E+01                     | 88.84E-08  | 19.88E-05  | 19.97E-05                                   |
| 20.98E-05                    | 57.20E-03  | 27.36E+01                     | 90.88E-08  | 12.00E-05  | 12.09E-05                                   |
| 98.71E-06                    | 46.80E-03  | 30.39E+01                     | 92.92E-08  | 62.70E-06  | 63.63E-06                                   |
| 36.01E-06                    | 36.40E-03  | 37.81E+01                     | 94.95E-08  | 28.46E-06  | 29.41E-06                                   |
| 89.74E-07                    | 26.00E-03  | 61.42E+01                     | 96.98E-08  | 11.52E-06  | 12.49E-06                                   |
| 12.31E-07                    | 15.60E-03  | 12.83E+02                     | 98.97E-08  | 33.02E-07  | 42.92E-07                                   |
| 54.10E-10                    | 52.00E-04  | 19.23E+03                     | 99.70E-08  | 21.74E-08  | 12.14E-07                                   |

\* E-02 = x10<sup>-2</sup>

accurately determine the small values of  $\frac{\partial \theta}{\partial x}$  from the water content distribution curves.

Having obtained the experimental values of the  $\psi$ - $\theta$  relations and  $D_\theta$ , theoretical diffusivity values may be computed. Assumptions inherent in the diffusivity calculations are: the  $\psi$ - $\theta$  relations are unique,  $\sigma = -2.09 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$  for the 10 to 30 $^\circ\text{C}$  temperature range, particle density = 2.65 gm/cm<sup>3</sup>,  $\partial \rho_o / \partial T = 1.05 \times 10^{-6} \text{ gm/cm}^3 \text{ } ^\circ\text{C}$  for 10 to 30 $^\circ\text{C}$ ,  $\alpha = 0.66$ ,  $g = 980.6 \text{ cm/sec}^2$ ,  $\rho = 1.00 \text{ gm/cm}^3$ , and  $a_k = 0.20 \text{ cm}^3/\text{cm}^3$ . The remaining physical constants used in Equation 70 were taken from the Handbook of Chemistry and Physics (1965). In general,  $D_{\text{Tliq}}$  and  $D_{\theta\text{liq}}$  increase with increasing water content and temperature;  $D_{\text{Tvap}}$  and  $D_{\theta\text{vap}}$  decrease with increasing  $\theta$  but increase with increasing T.

The observed mean net water flux (cm/sec) was calculated for the columns by determining the mass of water passing through cross sections of the column at  $x = 1, 2, \dots, 14$  by numerical integration of the area between the final and initial water content distribution curves.

The instantaneous net water flux is expected to be a maximum shortly after changing the boundary temperatures. The driving forces responsible for water movement will initially decrease as water moves from the warm to the cold side of the column. Therefore, values of instantaneous net water flux and mean net water flux will be time dependent. Fluxes predicted by the Philip-de Vries equation increase as  $\nabla T$  increases.

Recalling that mean net water flux depends upon the length of time interval over which the mean value is calculated, acceptable agreement between predicted and observed values would not always be expected. The Philip-de Vries equation consistently overestimated the flux when calculated over the duration of the run (Table 6). Cassel (1968) also reported an overestimation of water flux for 9 soil columns using the Philip and de Vries equation. Closer agreement was achieved for one column during initial time periods (Cassel et al., 1969).

It is commonly believed that soil temperature plays its greatest role in affecting water behavior near the soil surface. Extreme values of temperature and extreme temperature gradients are most commonly recorded in this region. For large temperature gradients existing near the soil surface, some investigators have predicted that water vapor movement would be downward into the soil. However, vapor pressure gradients induced by differences in soil water content in the surface layer of soil tends to move water vapor from cold to warm regions. Formation of a dry layer of soil at the surface, however, greatly reduces evaporation.

It is apparent that temperature gradients are present in in situ soils and that water movement in response to these gradients could be expected although extremely difficult to measure. Measurements made on all laboratory soil columns show that soil-water movement was from warm to cold regions in response to imposed temperature gradients.

Most previous investigators have used relatively short soil columns (10 cm or less) with temperature gradients ranging from 1.0 to

Table 6. Comparison of calculated to measured flow rates using the Philip and de Vries equation on two soil materials in columns without air gaps.

| Time<br>(Hours) | Flow Rates cm/sec ( $\times 10^{-4}$ ) |          |                                 |
|-----------------|--|----------|---------------------------------|
|                 | Predicted                              | Observed | Observed:<br>Predicted<br>Ratio |
| 4               | 18.80                                  | 4.885    | .260                            |
| 32              | 18.52                                  | 1.250    | .067                            |
| 48              | 18.40                                  | 0.504    | .027                            |
| <hr/>           |  |          |                                 |
| 4               | 21.73                                  | 1.628    | .075                            |
| 32              | 21.52                                  | .437     | .020                            |
| 48              | 21.49                                  | .290     | .014                            |

3.5 C/cm (e.g., Taylor and Cary, 1960; Taylor and Cavazza, 1954).

In field soils such extreme gradients exist for only a few hours at a time and occur only to depths of a few cm. Upon increasing the boundary temperature at one or both ends of the column the temperature distribution within the column began to change. Time intervals of 2 to greater than 5 hours were required for the soil temperature distribution to stabilize. Taylor and Cary (1960), using a 10-cm long column of Millville silt loam, reported comparable values of 1 to 3 hours, whereas the 30-cm long columns of Hutcheon (1958) required 24 to 36 hours for a stable distribution to develop under similar temperature gradients. Associated with the changes in soil temperature are changes in the soil-water vapor pressure. The consequent development of a vapor pressure gradient gives rise to a water vapor flux from warm to cold regions of the column. The rate of vapor diffusion or the water vapor flux is dependent upon the soil porosity and the soil-water content (de Vries, 1950).

Several investigators (e.g. Smith, 1943; Jones and Kohnke, 1952; Gurr et al., 1952) observed that a specific water content or range of water content existed at which the maximum net water transfer occurred for a given soil due to the temperature gradient. The results of this experiment for a fine sandy loam indicate that if a maximum net water transfer in response to temperature gradients does exist it may be for values of  $0.109 < \theta < 0.115$  since water transfer in columns with higher and lower water content was less.

In addition to vapor transfer, another mechanism thought to be responsible for water movement in response to soil temperature gradients

is that due to changes in surface tension at air-water interfaces (Cary, 1966); the water flux would again be from warm regions to cold. Since the changes in surface tension in this investigation were small and since the capillary conductivity values of the soil at  $\theta = 0.109$  and  $\theta = 0.115$  were small, it is expected that little water moved within the soil columns in response to surface tension gradients. Jackson, Rose, and Penman (1965) presented a thermodynamic model showing that when  $q = 0$ , the vapor flux and liquid flux are equal and opposite in direction but neither can equal zero when a temperature gradient exists.

Results of this investigation were analyzed assuming the following: 1) water vapor tends to move from warm to cold regions of the soil column in response to vapor pressure gradients due to temperature differences, 2) liquid water tends to move from warm to cold in response to surface tension gradients, 3) water vapor tends to move from cold to warm in response to vapor pressure gradients induced by differences in soil-water content, and 4) liquid water tends to move from cold to warm in response to soil-water pressure gradients.

It has been shown that under steady state conditions the total flow is zero (Jackson, Rose, and Penman, 1965). The net water flux at any instant in time is approximated by the observed flow which is calculated from changes in water content with time along the column. The total flow is maximum immediately after the temperature differences are imposed and knowing the total net flux is zero at a steady state condition, then the observed flow is time dependent. This time dependency of the water flux is inherent in similar investigations and may result in a sizeable error in the estimation of the total flux.

Calculated values of observed flow show this time dependence. Thus, when comparing observed flow to the instantaneous water fluxes predicted by Philip-de Vries equation, and the Taylor-Cary equation, it would be important to use values of observed flow obtained early in the experiment. Gee (1966), using neutron attenuation to determine transient water content distributions within a 10-cm long column of Palouse silt loam, calculated values of observed flow for the 0 to 25 hour interval after initiating the experimental run; water contents measured within the sample varied from  $\theta = 0.093$  to  $\theta = 0.172$ . When calculating observed flow for any given mean (initial) water content it is desirable to keep the water content range small.

If differences in bulk density had occurred between soil columns, the columns with a smaller bulk density would have greater pore space. Associated changes in pore geometry would change the water diffusivity values with a resultant increase or decrease in net water movement. However, bulk density values for the unleached ( $1.63 \text{ gm/cm}^3$ ) and leached ( $1.60 \text{ gm/cm}^3$ ) soil columns were similar and thus should have similar water diffusivity values. If as in some experiments where the different soil columns have to be dismantled and the water content determined gravimetrically, then uncertainties resulting from variations in bulk densities are incorporated in the results obtained for water movement. This factor stems from the important role played by the air-filled porosity in water vapor movement. One of the important factors when using the Taylor and Cary equation is the reliability of the

soil-water diffusivity values. These values are extremely sensitive to small variations in soil bulk density values.

The value of  $D_{\theta}$  used by Gee was experimentally determined in a separate experiment. No indication was given whether or not the bulk density values of the soil samples used in each experiment were identical. The values of  $D_{\theta}$  reported by him varied more than 10-fold for the range of  $\theta$  over which the calculations and predictions were made.

#### Water Flow Analysis

The very nature of the experimental system limits the extent of the water flow analysis. It was a closed system with one portion of the column wetting while the other portion was drying. One assumption of the general theory of irreversible thermodynamics stipulates that  $\beta^*$  can be evaluated only in regions where hysteresis is not present. Thus, only a rigorous analysis would be applicable to that part of the system in which hysteresis was not involved. Since the soil column was first saturated then drained to the initial water content, the analysis is limited to the warm end of the column where water was lost when the temperature was applied.

In a closed soil system of this type mass transfer of water may be occurring simultaneously in opposite directions. This would be true as water vapor moved from the hot end to the cold end under the influence of a thermal gradient. As the water continued to flow in this direction a build up of water at the cooler end would then cause a water content gradient toward the warm end of the soil column. If this liquid water gradient was such that the flow caused by it

would just offset the vapor water flow caused by the thermal gradient, the result would be no net flow of water in either direction. Thus, no water flow would be detected even though water was still being transported in the soil column. Therefore, the application of a temperature gradient to a uniform closed soil system, would result in a circulatory system being established and the eventual steady state water distribution would be actually a dynamic balance of opposing fluxes, predominantly liquid from cold to hot, and predominantly vapor from hot to cold. At equilibrium a static system would not exist, but rather a circulatory system would be present in the closed soil system (Jackson, Rose, and Penman, 1965).

In conjunction with the changes in soil temperature are changes in the soil-water vapor pressure. The consequent development of a vapor pressure gradient gives rise to a water vapor flux from warm to cold regions of the column. The rate of vapor diffusion or the water vapor flux is dependent upon the soil porosity and the soil-water content (de Vries, 1950).

The Taylor and Cary equation best describes the initial flow, but fails at later times. During the initial period the diffusion coefficients are constant, except at both ends of the columns. The flow system most closely meets conditions at this time which can be described in terms of linear equations with constant coefficients as required by the general theory of the equation. Flow decreases as time progresses as a result of the changing distribution of the water content and the changes in the diffusivity values associated with these water content changes. Isothermal diffusivity

values differed along the columns when the water distribution attained equilibrium. For later times in the experiment the required conditions were not met, thus a nonlinear flow equation may be required to adequately describe the flow.

Cassel (1968) found that the Taylor and Cary equation under predicted the observed flux 25 to 40 times. Gee (1966), however, found an over prediction of the water flux using the Taylor and Cary equation. A precise value for  $D_{\theta}$  is needed to calculate water flux using the Taylor and Cary equation since water flow as predicted by the equation is proportional to  $D_{\theta}$ . Thus, if an error is associated with the calculation of  $D_{\theta}$  it would be reflected in the predicted values.

Plotting of  $\beta^*$  versus  $\ln T$  will indicate whether  $\beta^*$  has a constant value over a given water content interval. Plotting the final soil water content versus  $\ln T$  was done to evaluate  $\beta^*$  and indicates that  $\beta^*$  was not a linear function.

The approximate nature of the Philip-de Vries equation may partially explain the inadequacy of the equation for predicting flow. The terms calculated by Philip and de Vries for differences between the temperature gradients occurring in pores and the overall temperature gradient were computed for average soil materials and under homogeneous conditions. These conditions may not be applicable to a complex system as used in laboratory experiments. A detailed analysis of the system's thermal properties would need to be made for proper evaluation of thermal conductivities associated with air, water, and soil properties of the overall system. The Philip-de Vries theory bases its formulation of this component upon the classical model of

the air-water interface surface tension and a temperature induced gradient of this surface tension.

Investigations of soil water movement in response to temperature gradients under field conditions are needed. Soil water fluxes could be determined from changes in soil water content in a manner similar to that used in the laboratory. It has, however, been most difficult to detect small changes in water content under field conditions.

Soil water content changes versus soil depth in the field may be determined by a portable gamma-radiation attenuation instrument. Precise soil water content values are difficult to determine because of bulk density variations throughout the soil profile.

Temperature sensors could be installed easily in the soil. This would give temperature distribution data and yield a soil temperature profile.

Field soils vary in structure and texture which increases the difficulty of making water flow investigations. Differences in particle size and bulk density result in large variations of  $K$ ,  $D_{\theta}$ ,  $\psi$ , and  $\theta$  for the same soil water content. Care must be exercised so that soil structure is not disturbed in the installation of the measuring devices.

Uncertainties are involved in measuring values of  $K$ ,  $D_{\theta}$ ,  $\psi$ , and  $\theta$  in field experiments. Mathematical models as discussed in this study are at best sufficient only for predicting trends in net soil water flux in laboratory columns. The models have not yet been examined under field conditions.

## SUMMARY AND CONCLUSIONS

Soil water redistribution within insulated columns of fine sandy loam soil material was studied in response to imposed thermal gradients. Atmospheric pressure was maintained within each soil column through a pin hole. Six soil columns were packed in plexiglas cylinders. Three columns were packed with the leached soil material and three columns with the unleached soil material. The three columns for each of the two soil materials contained no screens, one screen, and 2 screens, respectively. The two columns without screens as liquid water barriers were examined more intensively than the other columns. Initial, transient, and final water content values were taken at specified intervals along the soil columns by using gamma-radiation attenuation. Continuous measurements of temperature distribution were taken throughout the column. Values of the net water flux in the soil columns were analyzed using the Taylor-Cary irreversible thermodynamic theory and the Philip-de Vries theory of water movement.

These conclusions were reached after analyzing the data for the sandy loam soil material.

Water content and temperature influence the diffusion transfer coefficient,  $\beta^*$ , in a closed soil system, thus the transfer coefficient is not an independent entity.

Hysteresis is present in the wetter part of the system, although the magnitude of hysteresis involved is unknown. The  $\beta^*$

coefficient can be evaluated only in regions where hysteresis is not present.

The individual diffusion coefficients for liquid, vapor, and absorbed phase water are generally known to differ greatly with respect to water content and temperature. The general relationship for the combined diffusivity term would necessarily need to be a dependent one since the mechanisms for flow differ within the soil column. In the wet portion of the column the magnitude of liquid flow would be greater than in the drier end, especially when water flow approaches equilibrium. Water flow would be largely in the vapor phase in the drier end of the column.

The transfer of soil water was greater in the leached soil with no air gap than in the unleached soil with no air gap. Absence of increased water content adjacent to the air gap in the warmer region indicates that liquid water continuity did not exist throughout the column.

Application of the Taylor-Cary equation in an attempt to describe water flow reveals that for initial time periods the flow is slightly overestimated. The water flow after the initial time period for the sandy loam soil columns is not described well by the Taylor-Cary equation. Since a single relationship either linear or exponential could not be expected between the water content and diffusivities or transfer coefficient except for possibly selected portions of the water content range, the solution of the flow equation for nonisothermal conditions would be extremely difficult. Diffusivity

relationships must be known and numerical procedures used to describe the experimental flow conditions.

The Philip-de Vries theory of soil-water movement over predicted water flow when compared to the observed net soil-water flux in response to the  $2.67^{\circ}\text{C}/\text{cm}$  temperature gradient for an average water content of 10.5 to  $11.5 \text{ cm}^3/\text{cm}^3$ .

The observed change in soil-water content distribution at  $18.0 \text{ cm}^3/\text{cm}^3$  to the imposed temperature gradient was not significant for the sandy loam soil material.

The observed soil-water flux increased in response to the imposed temperature gradient as soil-water content decreased from 18.0 to  $10.5 \text{ cm}^3/\text{cm}^3$ .

No analytical procedure is presently available that will describe thermally induced flow under all conditions. The approach of Philip and de Vries requires that the physical properties of the soil must be known accurately so that correct estimates can be made of the individual diffusivities. This approach has been found to predict the flow with some success in relatively dry soils, however, it possibly would be in error when estimating thermally induced flow in the regions where liquid continuity exists and up through to a saturated system.

The Taylor-Cary equation may be adequate for describing water flow to predict trends or obtain comparative values, however, much additional work needs to be done before it will adequately describe the flow for transient conditions or steady state conditions where nonuniform water content distributions are present.

## LIST OF SYMBOLS

- $D_{atm}$  = diffusion coefficient of water vapor in air ( $\text{cm}^2/\text{sec}$ )  
 $D_{TV}$  = thermal vapor diffusivity ( $\text{cm}^2/\text{sec-}^\circ\text{C}$ )  
 $D_{Tliq}$  = thermal liquid diffusivity ( $\text{cm}^2/\text{sec-}^\circ\text{C}$ )  
 $D_{\theta v}$  = isothermal vapor diffusivity ( $\text{cm}^2/\text{sec}$ )  
 $D_{\theta liq}$  = isothermal liquid diffusivity ( $\text{cm}^2/\text{sec}$ )  
 $J_k$  = flow of component k ( $\text{gm}/\text{cm}^2\text{-sec}$ )  
 $K$  = hydraulic conductivity ( $\text{cm}/\text{sec}$ )  
 $K(\theta)_i$  = calculated conductivity for a specified moisture content or  
                   pressure class ( $\text{cm}/\text{min}$ )  
 $L$  = transmission coefficient  
 $P$  = pressure (mm Hg)  
 $S$  = internal entropy ( $\text{gm-cm}^2/\text{sec}^2\text{-}^\circ\text{C}$ )  
 $T$  = temperature ( $^\circ\text{C}$ )  
 $U$  = internal energy ( $\text{gm-cm}^2/\text{sec}^2$ )  
 $V$  = volume ( $\text{cm}^3$ )  
 $V_q$  = calorimetric heat flux ( $\text{cal}/\text{cm}^2/\text{sec}$ )  
 $X$  = driving force  
 $a$  = air content ( $\text{cm}^3/\text{cm}^3$ )  
 $h$  = potential (cm)  
 $K$  = unsaturated hydraulic conductivity ( $\text{cm}/\text{sec}$ )  
 $n$  = number of driving forces  
 $q$  = water flux ( $\text{gm}/\text{cm}^2/\text{sec}$ )

- $q_v$  = water vapor flux ( $\text{gm}/\text{cm}^2/\text{sec}$ )  
 $t$  = time (sec)  
 $z$  = gravity ( $\text{cm}/\text{sec}^2$ )  
 $\alpha$  = dimensionless factor depending on structure  
 $\beta^*$  = diffusion transfer coefficient ( $1/\ln T$ )  
 $\gamma$  = temperature coefficients of surface tension of water ( $^\circ\text{C}$ )  
 $\theta$  = water content ( $\text{cm}^3/\text{cm}^3$ )  
 $\theta_s$  = water content on a dry weight basis ( $\text{gm}/\text{cm}^3$ )  
 $\lambda$  = thermal conductivity ( $\text{cal}/\text{cm}/\text{sec}/^\circ\text{C}$ )  
 $\mu_k$  = chemical potential of the kth component ( $\text{cm}^2/\text{sec}^2$ )  
 $\nu$  = mass flow factor  
 $\xi$  = ratio of average temperature gradient in the air-filled pores to the overall temperature gradient ( $\text{cal}/\text{cm}/\text{sec}/^\circ\text{C}$ )  
 $\rho$  = density ( $\text{gm}/\text{cm}^3$ )  
 $\rho^t$  = mass of fluid per unit volume of medium ( $\text{gm}/\text{cm}^3$ )  
 $\rho_s$  = bulk density on a dry weight basis ( $\text{gm}/\text{cm}^3$ )  
 $\rho_v$  = water vapor density ( $\text{gm}/\text{cm}^3$ )  
 $\sigma$  = surface tension (dynes/cm)  
 $\Phi$  = hydraulic head gradient (cm)  
 $\psi$  = capillary potential (cm)

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