FORWARD OSMOSIS EXTRACTORS: THEORY, FEASIBILITY
AND DESIGN OPTIMIZATION

by

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PREFACE

With the development of modern membrane materials, forward osmosis can be expected to play a significant role in separation technology. Water-solute separation by forward osmosis is not new, but is a necessary process in the most primitive life forms. However, while biological osmotic systems have had time to develop optimum designs through evolutionary selection, engineered forward osmosis applications require cost analyses and mathematical models containing coupled non-linear differential equations in order to select optimum designs.

This thesis discusses the design theory of forward osmosis extractors and applies this theory in detail in evaluating and optimizing two water desalting applications: Forward Osmosis Survival (FOS) for drinking water production on lifeboats, and Fertilizer-Driven Forward Osmosis (FDFO) for irrigation water production. Similar analyses can be used in evaluating and optimizing other forward osmosis applications. FOS appears to be feasible, and a FOS device development proposal has been submitted to the Army and Navy for evaluation. A final evaluation of FDFO feasibility requires more information on low-pressure membrane transport properties, costs, and lifetimes. Further research is also needed on membrane geometry optimization in order to minimize concentration polarization while maintaining low pressure drop in the fluid distribution system.
For pure water production from seawater, FO can employ an easily removable and recyclable driving solute such as sulfur dioxide. Development of this system or similar ones will revolutionize water desalting technology.
ACKNOWLEDGMENTS

It has been my pleasure and good fortune to have worked with Professor J. O. Kessler, Department of Physics, in developing the material contained in this dissertation. Professor Kessler originated the applied forward osmosis concept, and in 1971 we conducted an initial experimental investigation of membrane flow rates. Since then we have jointly authored the five forward osmosis articles listed in the reference section. I have enjoyed immensely the opportunity to work in the applied science atmosphere involving the School of Renewable Natural Resources, Department of Physics, and Department of Chemical Engineering.

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ABSTRACT

Osmosis occurs when two solutions of differing osmolar concentrations are separated by a membrane permeable to the solvent but not to the solutes. In osmosis, water flows spontaneously from the low concentration source solution to the high concentration driving solution. This dissertation examines forward osmosis as a low-technology, low-energy use process for hydration and dehydration of aqueous solutions.

The fundamental mechanical device is a continuous counterflow extractor which incorporates a semipermeable membrane separating the source and driving solutions. The counterflow design permits maximum water recovery from the source solution and maximum dilution of the driving solution. The nonlinear differential equations describing the water and solute flows in the extractor are solved using analytical and numerical techniques. The resulting mathematical models contain design equations which can be used to determine the optimum membrane transport characteristics, optimum membrane size, and the asymmetric membrane orientation which minimizes concentration polarization. Theoretical and experimental results compare well.

Two applications discussed in detail are the production of potable water from seawater using human nutrients, and fertilizer-driven forward osmosis (FDFO) for converting saline water to irrigation water. In these applications, the final desalted product is not pure but contains the human or plant nutrient used to drive the process. For
extracting drinking water from seawater, 1 kilogram of nutrient powder can extract 6 kilograms of fresh water from one osmolal seawater, thus reducing the storage weight for food and water aboard a lifeboat by a factor of seven. The product water contains approximately 14 weight percent nutrients. Other driving solutions can be used as well. One kilogram of ethanol can extract approximately 20 kilograms of drinking water from seawater with the alcohol concentration of the resulting drinking water product being four to five weight percent. For converting saline water to irrigation water, FDFO can economically extract 80 kilograms of water per kilogram of fertilizer from 3200 mg/l (0.1 osmolal) brackish water and 14 kilograms of water per kilogram of fertilizer from seawater. For open greenhouses, these quantities of water represent 24 and 4 percent of the total irrigation requirements, respectively. A final evaluation of the economic feasibility of FDFO requires more information on low-pressure membrane transport properties, costs, and lifetimes.

For pure water production from seawater, the forward osmosis extractor can employ an easily removable and recyclable driving solute such as sulfur dioxide. The ten kilocalories per kilogram of water low temperature (100°C) energy for removing and recycling the sulfur dioxide can be supplied by waste heat, by solar heating, or by burning crop wastes.
CHAPTER I

INTRODUCTION

The free energy of a solution is lower than the combined free energies of its pure constituents. Formation of solutions is therefore spontaneous, whereas a definite minimum amount of work is required for partial or complete separation of solvents and solutes (1,2). The usual motives for such separations are recovery of solvents, or reduction of the volume of a solution (increase of its concentration by removing solvent). Standard methods are available for aqueous solutions. They include evaporation/recondensation, separation by freezing, electrodialysis, and reverse osmosis. All require expenditure of at least the minimum energy of separation and also relatively sophisticated equipment. Descriptions and an excellent bibliography of these water desalting methods are contained in Delyannis and Delyannis (3). This dissertation considers another separation method (see also 4-22) which depends on forward, or direct, osmosis.

Osmosis occurs spontaneously when two solutions of different osmotic pressure are separated by a membrane which is permeable to water but not to the dissolved solutes. At zero hydrostatic pressure difference water flows from the low osmotic pressure solution (source solution) to the higher osmotic pressure solution (driving solution). This process is called "Forward Osmosis" (FO) in this dissertation to
distinguish it from reverse osmosis (RO), where a hydrostatic pressure difference is employed to produce a flow in the direction opposite to FO. Appendix A contains the thermodynamic background of these osmotic phenomena. FO requires no hydrostatic pressure difference, but cannot provide an output solution less concentrated than the source solution. In the case of a seawater source, the output is slightly greater than one osmolal. Some solutes also pass through the membrane at rates that depend on the membrane properties.

Forward osmosis is a natural process in living organisms. While biological osmotic systems have had time to develop optimum designs through evolutionary selection, engineered forward osmosis applications require cost analyses and mathematical models containing coupled non-linear differential equations in order to select optimum membrane transport characteristics, membrane area, membrane thickness, and system hydrodynamics. A unique contribution of this dissertation is the development of these mathematical models and cost analyses.

Forward osmosis may be employed in several situations. For producing pure water from seawater, Murray (10) suggests an ammonium carbonate driving solution. The ammonium carbonate would be removed from the product water by stripping. Muller (11) suggests a sucrose driving solution. After extraction of water from seawater, the sugar would be combined into larger molecules, thereby lowering the osmotic pressure of the product solution. The solution could then be dewatered by ultrafiltration. The pure water permeate is the final product, and the solute molecules would be subjected to enzymatic hydrolysis to become sucrose
again for recycling. A process using forward osmosis to provide the hydrostatic driving pressure for reverse osmosis desalination is described in Popper et al. (12). For power production, Loeb (19,20) has proposed a "Pressure-Retarded Osmosis" power plant which uses osmosis to produce a solution under a high hydrostatic pressure. The high pressure of the solution can be recovered as electricity with a hydroturbine-generator.

Forward osmosis may be employed in situations which do not require the separation of pure solvent. One application is the concentration or dewatering of a solution where the fate of the solvent is irrelevant. Concentration of fruit juices has been suggested (2, p. 389; 14). Preliminary experimental results on fruit juice concentration have been reported by Popper et al. (15) and Mizutani et al. (16). Wang (17) used reverse osmosis followed by forward osmosis for recovery of proteins from whey. For simultaneous dehydration and hydration requirements, Randal et al. (18) have proposed osmosis as an energy-saving method for concentrating sugar beet extract while hydrating the molasses by-product.

Another type of situation occurs in irrigation where water containing salt solutes deleterious to plants may be readily available. The forward osmosis method can be used to transfer water from such a brackish source solution to a harmless or useful one -- containing fertilizer, for instance -- thereby reclaiming a resource that would otherwise be lost (4,5). In an analogous situation, forward osmosis can produce emergency potable water for humans in small ocean vessels such as lifeboats. In that case, the process transfers water from the sea into a concentrated nutrient solution (9,21).
Although application of the FO method in this context is new, it requires few technical advances; the membranes can be identical with, or very like, the ones currently used for RO. The principal advantages of FO over other purification processes currently available are:

1. The only energy required is in the initial construction, in maintenance, and sometimes in the distribution pumping of fluids. This low-energy feature is important in remote installations, such as lifeboats or primitive agriculture, or where the energy costs are unusually high.

2. The elimination of hydrostatic pressure differences results in advantages over the popular reverse osmosis method.

   a. There is no membrane compaction problem.
   
   b. For extracting water from an infinite source (e.g., seawater), it is not necessary to increase the concentration of the source solution. This advantage both minimizes the work required to extract the water and reduces the pretreatment requirements.
   
   c. The mechanical construction may utilize lightweight plastics which would be inexpensive and not subject to corrosion.
   
   d. There is no need for expensive and potentially dangerous high pressure pumping systems which require extensive operator training and maintenance checks.

The chief disadvantage of forward osmosis for fresh water production is that the water produced contains dissolved solutes used to drive the process. Unless these solutes are to be consumed anyway, as is the case
for plant fertilizers and human nutrients, the cost of these driving solutes or the cost to remove the driving solutes must be considered in evaluating the economic feasibility of FO.

An ultimate objective assessment of the utility of the proposed process will depend on the theoretical results given by the mathematical models and economic analysis presented in this thesis, experimental results in the field, and the availability of new membranes which may be designed especially for forward osmosis.

This thesis presents forward osmosis as a low-technology, low-energy use process for hydration and dehydration of aqueous solutions. The thesis is divided into six chapters. The general theory of forward osmosis extractors is presented in Chapter II, where three mathematical models of increasing complexity describe the continuous counterflow extractor. Concentration polarization and the optimum orientation for asymmetric membranes are discussed. In Chapter III, the theoretical models are compared with experimental results. Two applications discussed in detail in Chapters III and IV are the production of potable water from seawater using human nutrients and fertilizer-driven forward osmosis for converting saline water to irrigation water. For extracting drinking water from seawater, the mathematical models are used in Chapter III to select the optimum cellulose acetate membrane permeabilities which maximize the mass of water extracted per mass of nutrient for a stated allowable salt concentration in the drinking water and for a given membrane area. For fertilizer-driven forward osmosis, marginal cost analysis in Chapter IV determines the optimum mass of water which
can be extracted per osmole of fertilizer with available cellulose acetate membranes. Chapter V discusses other possible forward osmosis desalting applications including the use of removable driving solutes for the production of almost pure water from saline water. Each of these chapters is intended to be self contained. Although some repetition is thereby introduced, it is hoped that by this method a particular chapter can be understood without having read the previous chapters.
CHAPTER II

MATHEMATICAL MODELING OF THE FORWARD OSMOSIS EXTRACTOR

Summary

Osmosis occurs when two solutions of differing osmolar concentrations are separated by a membrane permeable to the solvent but not (or nearly not) to the solutes. This chapter derives the relationship between the kinetics and design parameters of systems designed for the purpose of applying this process to problems such as agricultural water reclamation, dehydration of solutions, and the production of potable nutrient solutions from seawater. Three mathematical models that include increasingly complex fundamental process assumptions are presented. This chapter discusses the predicted effects of concentration polarization on membrane water and solute fluxes. In all cases, the fundamental mechanical device is a continuous flow extractor that incorporates a semipermeable membrane.

Objective

The objective of this chapter is to establish a mathematical framework that can be used in determining design criteria which will maximize water transfer rate, minimize solute transfer, optimize the utilization of driving solute, minimize the physical size of the extractor, and optimize any other pertinent factors.
Physical Model

A forward osmosis continuous flow extractor in countercurrent mode is shown schematically in Fig. 1. $\pi_d(x)$ is greater than $\pi_s(x)$, and solvent flows through the membrane from right to left from the source solution into the driving solution. If the membrane is also permeable to either solute, then there are also solute flows across the membrane (not shown in the schematic).

The feed rates, $Q_d(0)$ and $Q_s(h)$, can be varied in order to achieve different degrees of hydration of the driving solution and dehydration of the source solution.

Mathematical Models

In Model 1, the membrane is assumed completely impermeable to the solutes, and the concentration polarization at the membrane is neglected. These assumptions are made so that the one differential equation for water flow in the countercurrent extractor can be solved analytically. In Model 2, the membrane is assumed permeable to the solutes and solvent. The membrane solute and water flows are assumed independent of each other, concentration polarization at the membrane is neglected, and the source solution concentration is assumed constant.

1. The osmotic pressure, $\pi$, of an aqueous solution can be calculated by the equation $\pi = acRT$. The osmotic coefficient, $a$, reflects the degree of dissociation of the dissolved solute ($a = 1.86$ for NaCl); $c$ is the solute concentration in moles per kg of water; $R$ is the universal gas constant; and $T$ is the temperature. The combination of $ac$ defines the osmolality, $C$, of the solution (osmoles/kg of water). For example, seawater contains about one osmolal ($C = 1$) of NaCl. Since $RT = 24.5$ liter atm mole$^{-1}$ at 25°C, $\pi = 24.5$ atm. Appendix A contains a derivation of the osmotic pressure equation and a table of the osmotic coefficients for selected solutes.
Fig. 1. Schematic Exploded View of the Countercurrent Extractor.

The schematic illustrates the conservation of solute current (dashed arrows) and the change of solvent current (solid arrows of changing length) due to the addition and subtraction of the membrane solvent current $Q_m$. The width of the extractor and membrane (into the paper) is $L$. The height is $h$, the height dimension being specified by $0 < x < h$. The differential membrane current in the increment $\Delta x$ is $q_m x \Delta x$, where $q_m$ is the membrane flux. The subscripts $d$ and $s$ stand for "driving" and "source," respectively. The dots indicate the membrane halves in this "exploded" diagram.
The three differential equations which describe the membrane solute and water flows may be solved by numerical integration. Model 3 presents the same situation as Model 2 but, in addition, the concentration polarization on both sides of the membrane is taken into account in order to approximate the effect of countercurrent fluid dynamics on the membrane transfer rates.

Model 1: Perfect Membrane with No Concentration Polarization

The simplest mathematical description of the forward osmosis countercurrent extractor is based on the assumptions listed below. These assumptions are made in order to derive an analytic solution to the differential equation governing the countercurrent water flow. Assumptions 1 and 2 may represent serious oversimplifications of the physical system:

1. The membrane is completely impermeable to all solutes.
2. Perfect mixing exists perpendicular to the counterflows so that, at a given location $x$, the solution is homogeneous on each side of the membrane (i.e., neglect concentration polarization at the membrane).
3. There exists no convective or diffusive back mixing in the direction of the counterflows.
4. The temperature and the osmotic coefficients, $\alpha_d$ and $\alpha_s$, are constant.
5. Any hydrostatic pressure difference across the membrane is too small to affect the membrane flux.
The differential equation for the membrane water flux in the extractor is

\[
\frac{dQ_m(x)}{dx} = q_m(x) = L[\pi_d(x) - \pi_s(x)],
\]

where \( \ell \) is the width of the membrane and is assumed constant (\( \ell \) corresponds to the circumference of tubular membranes). \( q_m(x) \) is the local membrane flux in \( \text{gm/cm}^2 \cdot \text{sec} = \text{cm/sec} \) for water; and \( Q_m(x) \) in \( \text{gm/sec} \) is the total flow through the membrane in the interval \((0,x)\). \( L \) is the membrane water permeability constant in \( \text{gm cm}^{-2} \text{atm}^{-1} \text{sec}^{-1} \). The osmotic pressures appearing on the right-hand side of Eq. 1 are determined by

\[
\pi_i(x) = \alpha_i RT c_i(x)
\]

and

\[
c_i(x) = 1000 \frac{n_i}{Q_i(x)}.
\]

Appendix A contains values of \( \alpha_i \) for selected solutes. By assumption 1, the solute currents on each side of the membrane are conserved so that \( n_i \) is constant.

With Eqs. 1, 2, and 3, one can derive Eqs. 4, 5, and 6 which determine the feed rates per membrane area, \( Q_d(0)/2\ell \) and \( Q_s(h)/2\ell \), and the average membrane flux, \( \bar{q}_m(h) \), as functions of the membrane permeability constant, \( L \), the temperature, and the boundary conditions: the given feed concentrations, \( c_d(0) \) and \( c_s(h) \), and the specified product concentrations, \( c_d(h) \) and \( c_s(0) \):
\[
\frac{Q_d(0)}{\gamma h} = \frac{\ln_d(0)\beta}{Q^*_m(h)^2 + \frac{(1-K^* + K^*\beta)Q^*_m(h) + (K^*\beta)^2(1-\beta)\gamma n(1-\frac{Q^*_m(h)}{1-K^*/\beta})}{2}}
\]

(4)

\[
\frac{Q_s(h)}{\gamma h} = \frac{Q_d(0)}{\gamma h} \cdot \frac{[c_d(0)/c_d(h) - 1]}{[1 - c_s(h)/c_s(0)]}
\]

(5)

\[
\tilde{q}_m(h) = \frac{Q_d(0)}{\gamma h} \cdot [c_d(0)/c_d(h) - 1]
\]

(6)

where the dimensionless coefficients \(K^*, \beta,\) and \(Q^*_m(h)\) are given by the following equations:

\[
K^* = \left[1 - \frac{c_d(0)\cdot c_s(h)}{c_d(h)\cdot c_s(0)}\right]/\left[1 - \frac{c_s(h)}{c_s(0)}\right]
\]

(7)

\[
\beta = 1 - \frac{\alpha_s c_s(h)\cdot [c_d(0)/c_d(h) - 1]}{\alpha_d c_d(0)\cdot [1 - c_s(h)/c_s(0)]}
\]

(8)

\[
Q^*_m(h) = \frac{c_d(0)}{c_d(h)} - 1
\]

(9)

For dehydration of the source solution only or for hydration of the driving solution only, it is simpler to rewrite the differential equation, Eq. 1, instead of evaluating Eqs. 4, 5, and 6 at the limits of \(K^*\) and \(\beta\) \((K^* = \beta = 1.0\) for dehydration; \(K^* = \beta = -\infty\) for hydration). For dehydration only, \(c_d\) is constant and Eq. 1 becomes²

2. The factor 1000 relates grams and liters or kg water.
\[
\frac{dQ_m(x)}{dx} = L[\pi_d - 1000 \frac{\alpha_n RT}{Q_s(x)}]. \tag{10}
\]

Integrating Eq. 10 and rearranging the result yields the operating solvent feed rate of the source solution as a function of the given control variables: \(c_d\), \(c_s(h)\), \(L\), and \(T\), and the specified product concentration of the source solution, \(c_s(0)\):

\[
\frac{Q_s(h)}{\lambda h} = \frac{L\pi_d}{1 - \frac{c_s(h)}{c_s(0)} - \frac{\pi_s(h)}{\pi_d} \ln \left[ \frac{\pi_d/\pi_s(h) - 1}{\pi_d/\pi_s(0) - 1} \right]} . \tag{11}
\]

Constant \(c_d\) requires that \(Q_d(0)/\lambda h \to +\infty\). The average membrane water flux is then

\[
\bar{q}_m(h) = \frac{Q_s(h)}{\lambda h} \left[ 1 - \frac{c_s(h)}{c_s(0)} \right] . \tag{12}
\]

For hydration only, \(c_s\) is constant and Eq. 1 becomes

\[
\frac{dQ_m(x)}{dx} = L[1000 \alpha_d n_d RT/Q_d(x) - \pi_s]. \tag{13}
\]

Integrating Eq. 13 and rearranging the result yields the operating solvent feed rate of the driving solution as a function of the given control variables: \(c_s\), \(c_d(0)\), \(L\), and \(T\), and the specified \(c_d(h)\):

\[
\frac{Q_d(0)}{\lambda h} = \frac{L\pi_s}{1 - \frac{c_d(0)}{c_d(h)} + \frac{\pi_d(0)}{\pi_s} \ln \left[ \frac{1 - \pi_s/\pi_d(0)}{1 - \pi_s/\pi_d(h)} \right]} . \tag{14}
\]
Constant $c_s$ requires that $Q_s(h)/\lambda h \rightarrow +\infty$. The average membrane water flux is

$$\bar{q}_m(h) = \frac{Q_d(0)}{\lambda h} \left[ \frac{c_d(0)}{c_d(h)} - 1 \right]. \quad (15)$$

In Fig. 2, Eqs. 4, 5, and 6 are used to plot $Q_d(0)/\lambda h$, $Q_s(h)/\lambda h$, and $\bar{q}_m(h)$ versus $c_d(h)$ for the case of transferring fresh water from brackish water into a fertilizer solution. For the specific case of transferring fresh water from seawater into a nutrient solution for humans, Eqs. 4 and 6 are used to plot $\bar{q}_m(h)$ and the kilograms of fresh water obtained per kilogram of nutrient supplied (FRESH) versus $\lambda h/Q_d(0)$ in Fig. 3.

Model 2: Imperfect Membrane with No Concentration Polarization

In this section, the mathematical model includes the passage of solute through the membrane. Concentration polarization is still neglected. In addition, the source solution concentration is assumed constant so that only hydration of the driving solution occurs. This constant source solution concentration restriction is made for mathematical simplicity and for applying to the case where the extractor is used to transfer water from an unlimited seawater or brackish water source.

The new assumptions are:

1. Solute rejection by the membrane is less than 100%.
2. Water and solute membrane flows are independent.
3. Concentration polarization at the membrane surfaces is neglected.
Fig. 2. Simulation by Model 1 of a Forward Osmosis Extractor for the Production of Fertilizer Water from Brackish Water.

c_d(h) is the product concentration of the ammonium sulfate fertilizer water; c_d, limit = (α_s/α_d)·c_s(h); A is the total membrane area in cm²; Q_d(0) is the fertilizer solvent feed rate in grams/sec; Q_s(h) is the solvent feed rate of the brackish source solution; q_m(h) is the average membrane water flux. Fixed parameters are: \( L = 1.37 \times 10^{-5} \) grams-cm⁻²-s⁻¹-atm⁻¹, \( c_d(0) = 5.75 \) molal \((\text{NH}_4)_2\text{SO}_4\), \( c_s(h) = 0.0513 \) molal NaCl (3000 mg/liter), \( c_s(0) = 0.6 \) molal NaCl (35,000 mg/liter), \( α_d = 2.30 \), \( α_s = 1.86 \), and RT = 24.5 liter-atm mole⁻¹. Model 1 assumes zero membrane solute flux and no concentration polarization.
Fig. 3. Simulation by Model 1 of a Forward Osmosis Extractor for the Extraction of Drinking Water from Seawater.

Fresh is liters of water extracted per kg of nutrients; \( q_m(h) \) is the average membrane water flux in grams-cm\(^{-2}\)-sec\(^{-1}\); \( \theta \) is the membrane area in cm; \( Q_d(0) \) is the nutrient solvent feed rate in grams/sec. Fixed parameters are: \( L = 3.05 \times 10^{-5} \) grams-cm\(^{-2}\)-sec\(^{-1}\)-atm\(^{-1}\), \( c_d(0) = 10.0 \) molal glucose-fructose, \( c_s = 0.6 \) molal NaCl, \( \alpha_d = 1.04 \), \( \alpha_s = 1.86 \), and \( RT = 24.5 \) liter-atm-mole\(^{-1}\). Model 1 assumes zero membrane solute flux and no concentration polarization.
4. The source solution concentration, $c_s$, is constant. The change of $\pi_s$ due to loss of source solute molecules and due to gain of driving solute molecules is neglected.

5. There exists no convective or diffusive back mixing in the direction of the counterflow.

6. The temperature and the osmotic coefficients, $\alpha_s$ and $\alpha_d$, are constant.

7. The hydrostatic pressure difference across the membrane is negligible.

There are three differential equations for the membrane flows of water and solutes. For membrane water flow into the driving solution,

$$\frac{dQ_d(x)}{dx} = q_m(x) = L[\pi_d(x) + \pi_{ds}(x) - \pi_s]. \quad (16)$$

For loss of driving solute from the driving solution,

$$\frac{dn_d(x)}{dx} = -\frac{w_d c_d(x)}{1000}. \quad (17)$$

For gain of source solute into the driving solution,

$$\frac{dn_{ds}(x)}{dx} = \frac{w_s [c_s - c_{ds}(x)]}{1000}. \quad (18)$$

The algebraic equations for the extractor are:

$$\pi_i = \alpha_i c_i(x)RT, \quad (19)$$
\[ c_d(x) = 1000 \frac{n_d(x)}{Q_d(x)} \quad (20) \]

\[ c_{ds}(x) = 1000 \frac{n_{ds}(x)}{Q_d(x)} \quad (21) \]

\[ c_p(x) = \frac{w_s[c_s - c_{ds}(x)]}{q_m(x)} \quad (22) \]

\[ Q_m(x) = Q_d(x) - Q_d(0) \quad (23) \]

Alternatively, one differential equation may be eliminated by replacing Eq. 16 with the algebraic expression obtained by combining Eqs. 16, 17, and 18 and integrating:

\[ Q_d(x) = Q_d(0) \frac{1 + L[\pi_d(0)/w_d + \pi_{ds}(0)/w_s]}{1 + L[\pi_d(x)/w_d + \pi_{ds}(x)/w_s]} \quad (24) \]

The boundary conditions at \( x = 0 \) for Eqs. 16, 17, and 18 are:

\[ n_d(0) = c_d(0) \frac{Q_d(0)}{1000} ; \]

\[ n_{ds}(0) = c_{ds}(0) \frac{Q_d(0)}{1000} \quad (= 0 \text{ if the driving feed solution contains no source molecules}) ; \]

\[ Q_d(0) = 1.0 \text{ gm/sec so that } \& \text{h has units of cm}^2 \text{ in the computer simulation } [Q_d(0)/\&\text{h is the important solvent feed parameter}] . \]

See Appendix B for the Model 2 simulation program.
The membrane transport coefficients \((w_s, w_d, \text{ and } L)\) are related to each other by the state of the art in membrane technology. Manufacturers' attempts to decrease the solute permeability generally result in decreasing the water permeability. For example, Sourirajan (2, p. 229) presents data for sodium chloride and water permeability of cellulose acetate membranes. For membranes which were initially treated at 20.4 atm, \(w_{NaCl}\) and \(L\) are related by the empirical equation \(L = 9.38 \times 10^{-4} (w_{NaCl})^{-29}\). The additional information provided by such empirical equations between \(L, w_s, \text{ and } w_d\) can be used in conjunction with Model 2 in order to select the optimum membrane for a given application. These results are presented in Chapter 3.

Model 3: Imperfect Membrane with Concentration Polarization

By the two-film theory approach developed by Whitman (23) for interfacial mass transfer, Model 3 approximates the effect which the fluid dynamics of the counterflow has on the membrane transfer rate by assuming the existence of an unstirred film or concentration boundary layer on each side of the membrane. The thicknesses are defined by the distances \(\delta_d\) and \(\delta_s\) from the membrane to the point where the concentrations equal the bulk solution concentrations, \(c_d(x)\) and \(c_s\). As in Model 2, the concentration of the source solution is taken to be constant at some distance from the membrane.

The assumptions for Model 3 are:

1. The unstirred film thicknesses, \(\delta_s\) and \(\delta_d\), are independent of \(x\) or local changes of concentration and flow rate.
2. Assumptions 1, 2, 4, 5, 6, and 7 of Model 2.

In response to water flowing through the membrane, the concentration of the source solute increases near the membrane surface on the source side of the extractor, and the concentration of the driving solute decreases near the membrane surface on the driving side of the extractor. This nonlinear phenomenon, which decreases membrane water flux, is called concentration polarization. In order to calculate the water and solute fluxes through the membrane, it is necessary to know the solute concentrations at the membrane surfaces.

For a binary system, the mole flux of the solute A relative to stationary coordinates is (24)

\[ N_{Ay} = - c^* D_{AB} \frac{dx_A}{dy} + c^* A U, \]  

(25)

where \( y = \) the direction normal to the membrane;
\( c^* = \) the total number of moles per \( \text{cm}^3 \) of solution;
\( c^*_A = \) the number of moles of solute A per \( \text{cm}^3 \) of solution;
\( D_{AB} = \) the diffusion coefficient of solute A in solvent B;
\( x_A = \) the mole fraction of solute A; and
\( U = \) the mole average velocity relative to stationary coordinates.

Inserting into Eq. 25 the equation for \( U, U = (N_{Ay} + N_{By})/c^* \), yields

\[ N_{Ay} = - c^* D_{AB} \frac{dx_A}{dy} + x_A (N_{Ay} + N_{By}). \]
Rearranging terms,

\[ dy = \frac{c^* D_{AB} dx_A}{-N_{Ay} + x_A (N_{Ay} + N_{By})} \]  

(26)

In addition, \( c^* \) is a function of \( x_A \),

\[ c^* = \frac{1}{x_A V_A + x_B V_B} = \frac{1}{V_B [1 + x_A(n-1)]} \]

where \( V_A, V_B \) are the partial mole volumes of A and B, and \( n \equiv V_A/V_B \).

Defining \( \varepsilon = N_{Ay}/N_{By} \), and inserting the above equation for \( c^* \) into Eq. 26,

\[ dy = \frac{D_{AB} dx_A}{q_m \{ -\varepsilon + x_A [1 - \varepsilon(n-2)] + x_A^2 (n-1)(1 + \varepsilon) \}} \]  

(27)

where \( q_m = V_B N_{By} \).

For constant \( D_{AB}, q_m, \varepsilon, \) and \( n \), Eq. 27 can be integrated with the boundary conditions: at \( y = 0 \), \( x_A = x_{A_0} \); and at \( y = \delta \), \( x_A = x_{A_\infty} \).

\[ \frac{q_m (1 + \varepsilon n) \delta}{D_{AB}} = 2n \left\{ \frac{[(1 + \varepsilon)x_{A_\infty} - \varepsilon][(n - 1)x_{A_\infty} + 1]}{[(1 + \varepsilon)x_{A_\infty} - \varepsilon][(n - 1)x_{A_\infty} + 1]} \right\} . \]  

(28)

The membrane is chosen and the extractor is operated so that \( \varepsilon \ll (x_{A_\infty} \) and \( x_{A_\infty} \)). If \( (n - 1)x_{A_\infty} \ll 1 \), then Eq. 28 can be approximated by
\[ \frac{q_m \delta}{D_{AB}} = \ln \frac{x_{A\infty}}{x_{Aw}}. \]

The mole fraction of solute A at the membrane surface is

\[ x_{Aw} = x_{A\infty} e^{-q_m \delta/D_{AB}}. \]

Making the approximation, \( c_{Aw}/c_{A\infty} = x_{Aw}/x_{A\infty} \), yields

\[ c_{Aw} = c_{A\infty} e^{-q_m \delta/D_{AB}}. \]

For non-negative \( q_m \), \( \delta_s \), and \( \delta_d \), the molal concentrations at the membrane surfaces are:

\[ c_{dw} = c_d e^{-q_m \delta_d/D_d}; \] (29)

\[ c_{sw} = c_s e^{q_m \delta_s/D_s}. \] (30)

Dainty (25) uses these concentration profiles in discussing permeability measurements of biological membranes. Loeb (20,22) also employs Eqs. 29 and 30 to describe concentration polarization although he does not include the porosity of the membrane support backing (see below) in his calculations.

If the unstirred film is created primarily by a porous membrane support, then \( \delta_1 \) denotes the effective film thickness. The effective film thickness is the thickness of the porous support divided by \( \delta^2 \),
where $\theta (0 < \theta < 1)$ is the porosity of the supporting material. This increased $\delta_i$ results because in the porous support the local velocity equals $q_m/\theta$ instead of $q_m$, and the unoccluded area for solute diffusion is the fraction $\theta$ of the total membrane area. An additional increase in $\delta_i$ results from the tortuosity of the porous material.

The membrane water flux into the driving solution is

$$q_m(x) = L[\pi_{dw}(x) + \pi_{ds}(x) - \pi_{sw}(x)] .$$  \hspace{1cm} (31)

$\pi_{ds}(x)$ in Eq. 31 may be more realistically represented by a value intermediate between $\pi_{ds}(x)$ and $\pi_p(x)$. The source solute osmotic pressure on the driving side of the membrane surface is

$$\pi_{dsw} = \pi_{ds} \cdot e^{-q_m \delta_d/D \cdot ds} + \pi_p (1 - e^{-q_m \delta_d/D \cdot ds}) .$$

For a conservative approximation, $\pi_{ds}(x)$ is used instead of $\pi_{dsw}$.

Inserting the relationships for $c_{dw}$ and $c_{sw}$ from Eqs. 29 and 30, Eq. 31 becomes

$$q_m(x) = L[\pi_d(x) e^{-q_m \delta_d/D \cdot ds} + \pi_{ds}(x) - \pi_s e^{-q_m \delta_s/D \cdot s}] .$$  \hspace{1cm} (32)

$\delta(x)$ is the diffusion coefficient of the driving solution averaged in the $y$ direction and is defined by:
Taking the derivative of Eq. 32 results in Eq. 34 which, although more complicated than Eq. 32, yields a linear equation for $\frac{dq_m(x)}{ldx}$. The first-order differential equation for $\frac{dq_m(x)}{ldx}$ can be numerically integrated to obtain $q_m(x)$, and then $q_m(x)$ can be numerically integrated to yield $Q_d(x)$.

For Model 3, the six coupled differential equations are (although not so noted, $c_d$, $c_{ds}$, $c_{dw}$, $\bar{D}$, $Q_d$, $q_m$, and $\tau_d$ are functions of the location $x$):
The algebraic equations for the partial derivatives are:

\[
\frac{\partial \tilde{D}_d}{\partial x} = \frac{\partial \tilde{D}_d}{\partial c_d} \frac{dc_d}{dx} + \frac{\partial \tilde{D}_d}{\partial c_{dw}} \frac{dc_{dw}}{dx};
\]

\[
\frac{dc_{ds}}{dx} = \frac{w_d}{Q_d} \left( c_s e^m \delta_s / D_s - c_{ds} \right) - \frac{q_m}{Q_d} c_{ds}.
\]

The algebraic equations for the partial derivatives are:

\[
\frac{\partial c_{ds}}{\partial q_m} = \left( \frac{1}{L} + \alpha_s RT \frac{\delta_s}{D_s} c_s e^m \delta_s / \alpha_d RT \right).
\]

\[
\frac{\partial c_{ds}}{\partial c_{ds}} = - \frac{\alpha_s RT}{\alpha_d RT}.
\]

Evaluation of \( \frac{\partial \tilde{D}_d}{\partial c_d} \) and \( \frac{\partial \tilde{D}_d}{\partial c_{dw}} \) requires an empirical equation for the diffusion coefficient, \( \tilde{D}_d \), as a function of the molal concentration, \( c_d \).

The values of \( q_m, Q_d, c_d, c_{dw}, \tilde{D}_d, \) and \( c_{ds} \) at \( x = 0 \) are:

\( Q_d(0) = 1.0 \) gm/sec;

\( c_d(0) = \) feed concentration of the driving solution; and

\( c_{ds}(0) = 0.0 \) (\( c_{ds}(0) \) is non-zero only if the driving feed solution contains source molecules).

\( q_m(0), c_{dw}(0), \) and \( \tilde{D}_d(0) \) can be obtained by successive approximations with the first two terms of the Taylor series expansion for \( q_m(0) \) where \( q_m(0) \) is defined by Eq. 32, \( \tilde{D}_d(0) \) is defined by Eq. 33, and \( c_{dw}(0) \) is obtained from Eq. 42.
\[ c_{dw}(0) = \frac{1}{\alpha_d RT} \left\{ \frac{q_m(0)}{L} + \alpha_s RT [c_s e^{-\frac{q_m(0) \delta_s}{D_s}} - c_{ds}(0)] \right\}. \]  

(42)

Eqs. 32 through 42 require estimates of the unstirred film thicknesses, \( \delta_s \) and \( \delta_d \) (see Appendix C for the Model 3 simulation program). In Chapter III, \( \delta_s \) and \( \delta_d \) for a laboratory forward osmosis extractor are estimated by minimizing the sum of squares deviation of the experimental results and the results predicted by Model 3.

**Concentration Polarization**

Concentration Polarization Approximation

The numerical integration required in Model 3 for describing the continuous flow extractor with concentration polarization yields numerical results which are tedious to use for general optimization of the FO extractor. This section presents a simple analytic approximation of the extractor with concentration polarization.

The three models (Models 1 and 2, which neglect concentration polarization; and Model 3, which includes it) estimate approximately the same average membrane water fluxes except at high flux (compare Figs. 3 and 8, pp. 16 and 43; and Figs. 10 and 11, pp. 51 and 53). The estimated average membrane flux can be used to calculate the average driving solute concentration, \( \bar{c}_d(h) \), where

\[ \bar{c}_d(h) = \frac{\int_0^h c_d(x)dx}{\delta h}. \]  

(43)

Using the no concentration polarization relationship that
\[ C_d(x) = q_m(x)/LRT + C_s(x) , \quad (44) \]

and assuming constant \( C_s \), Eq. 43 yields

\[ \tilde{C}_d(h) = \tilde{q}_m(h)/LRT + C_s . \quad (45) \]

For hydration only, Model 1 (refer to Eqs. 14 and 15) estimates the average membrane water flux

\[ \tilde{q}_m(h) = \frac{LRT \ C_s}{[C_s/C_d(h) - C_s/C_d(0)] - 1 \ln \left[ \frac{1 - C_s/C_d(0)}{1 - C_s/C_d(h)} \right] - 1} . \quad (46) \]

Thus, \( \tilde{C}_d(h) \) for the extractor with and without concentration polarization is approximately

\[ \tilde{C}_d(h) = C_s \{ 1 + \frac{1}{[C_s/C_d(h) - C_s/C_d(0)] - 1 \ln \left[ \frac{1 - C_s/C_d(0)}{1 - C_s/C_d(h)} \right] - 1} \} \]

\[ (47) \]

Because Eq. 47 is derived from Model 1, \( C_d(h) \) corresponds to the osmolar concentration of the extractor product assuming no solute loss.

The other approximation assumes that the membrane water flux with concentration polarization is relatively constant for the entire membrane. Thus, average values can be inserted into the local non-linear water flux equation (refer to Eq. 32),
\[ q_m(x) = LRT[C_d(x)e^{-q_m(x)\delta_d/D_d} - C_s e^{-q_m(x)\delta_s/D_s}], \quad (48) \]

to yield the approximation for the average membrane water flux,

\[ \bar{q}_m(h) = LRT[\bar{C}_d(h)e^{-\bar{q}_m(h)\delta_d/D_d} - \bar{C}_s e^{-\bar{q}_m(h)\delta_s/D_s}], \quad (49) \]

where \( \bar{C}_d(h) \) is given by Eq. 47. Eq. 49 can be solved by successive approximations. With Newton's method, convergence to four significant figures rarely requires more than three iterations and never more than eight.

The source solute gained per kilogram of extracted water is approximately

\[ C'_d(h) = \bar{q}_m(h)^{-1} w_s e^{-\bar{q}_m(h)\delta_s/D_s}. \quad (50) \]

The driving solute lost per kilogram of extracted water is approximately

\[ \text{Driv. Loss} = \bar{q}_m(h)^{-1} w_d \bar{C}_d(h)e^{-\bar{q}_m(h)\delta_d/D_d}. \quad (51) \]

Because of the loss of driving solute, the product concentration of the driving solute is

\[ C_d(h) = \eta C_d(h) \text{model 1}, \quad (52) \]

where \( \eta \) is the driving solute fraction conserved and can be calculated by
\[ \eta = 1 - \text{Driv. Loss} \left[ \frac{1}{C_d(h)} \text{model 1} - \frac{1}{C_d(0)} \right]. \] (53)

See Appendix D for the complete Fortran program.

In Figs. 4 and 5, these approximations for \( \bar{q}_m(h) \), \( C'_d(h) \), and \( \eta \) are compared with the predicted results from Model 3. For a film thickness of 0.05 cm on one side of the membrane the difference between the two methods is negligible. For unstirred film thickness of 0.05 cm on both sides of the membrane the analytic approximation underestimates the predicted water flux by 50% (not shown). Nevertheless, asymmetric membranes which have porous backings less than 0.005 cm thick (which including porosity creates an effective film thickness less than 0.05 cm) can be adequately described with the above approximation.

In Figs. 4 and 5, \( \delta = 0.05 \) cm corresponds to the effective film thickness created by a porous backing. Fig. 5 has a larger water flux, lower source salt contamination, and lower driving solute loss than Fig. 4 because in Fig. 5 the porous backing is oriented toward the solution with the higher diffusivity, in this case the NaCl source solution. Estimated results with solutions of different diffusivities indicate that if the diffusivities differ by more than 20% the optimum orientation of the porous support is towards the solution with higher diffusivity. If diffusivities are equal, water and solute fluxes are maximized with the backing toward the source solution. The optimum orientation for a diffusivity difference less than 20% requires a detailed analysis using the methods of Chapter IV and either Model 3 or the approximation presented in this section.
Fig. 4. Comparison of the Predicted Results by Model 3 (Denoted by x) with the Concentration Polarization Approximation (Solid Line); Membrane Backing Faces the Driving Solution.

η is the driving solute fraction conserved (not lost through the membrane). $C'_ds(h)$ is the source salt concentration of the extracted water. $\bar{q}_m$ is the average membrane water flux in units of $10^{-3}$ grams-cm$^{-2}$-sec$^{-1}$. To convert from grams-cm$^{-2}$-sec$^{-1}$ to U. S. gallons-day$^{-1}$-ft$^{-2}$, multiply by $2.12 \times 10^4$). The operating conditions are: $C_s = 1.12$ osmolal NaCl; $D_s = 1.5 \times 10^{-5}$ cm$^2$/sec; $C_d(0) = 10.4$ osmolal glucose-fructose; $D_d = 0.67 \times 10^{-5}$ cm$^2$/sec; $L = 3.05 \times 10^{-5}$ grams-cm$^{-2}$-sec$^{-1}$-atm$^{-1}$.
Fig. 5. Comparison of the Predicted Results of Model 3 (Denoted by x) with the Concentration Polarization Approximation (Solid Line); Membrane Backing Faces the Source Solution.

Refer to Fig. 4 for definition of terms and units used.
Polarization Due to Membrane Backing

One objection to forward osmosis is the possibility that the unstirred film created by the membrane backing in asymmetrical membranes will reduce the membrane water flux to an uneconomical level. Although the membrane backing thickness enhances concentration polarization and thus decreases the water flux and increases the salt flux, concentration polarization can be reduced by using thin, high-porosity membrane backings. An acceptable thickness is the 22 micron wall thickness of DuPont Permasep B-9 hollow fine fibers (42 micron i.d., 85 micron o.d.) (26). Fibers with a larger inside diameter would be necessary to minimize pressure drop in distribution pumping. It may be possible to produce fibers with a wall thickness less than 22 microns. A 25 micron (1 mil) backing thickness with a porosity of 0.5 will create an effective unstirred film thickness $\delta$ of approximately 0.01 cm [0.0025 cm (.5)$^{-2}$].

Low-flux membranes have lower concentration polarization than high-flux membranes. Fig. 6 compares the relative water flux calculated by Model 3 for two membranes with and without a membrane backing ($\delta_s = 0.01$ cm). The backing faces a 1.12 osmolal seawater source solution. The high-flux membrane ($L = 3.05 \times 10^{-5}$ g cm$^{-2}$ sec$^{-1}$ atm$^{-1}$) is membrane D in Table 2 (p. 45). The low-flux membrane ($L = 1.6 \times 10^{-5}$ g cm$^{-2}$ sec$^{-1}$ atm$^{-1}$) is the optimum RO membrane for seawater fertilizer-driven forward osmosis in Table 5 (p. 70). For the high-flux membrane, the average water flux with the backing is approximately 60% of the flux with no backing. For the low-flux membrane, this ratio is approximately 75%.
Fig. 6. Predicted Effective Driving Pressures with and without a Membrane Backing Facing the Source Solution.

\( \delta_s = 0.01 \) cm corresponds to a 25 micron (1 mil) thick backing. Dimensions for the membrane water constant \( L \) are \( 10^{-5} \) gm-cm\(^{-2}\)-sec\(^{-1}\)-atm\(^{-1}\). \( C_s = 1.12 \) osmolal seawater; \( C_d(0) = 10.4 \) osmolal; \( \delta_d = 0.0 \) cm; and \( D_s = 1.5 \times 10^{-5} \) cm\(^2\)/sec.
Although 25% is a sizable reduction in water flux, such a reduction can be included in the extractor design.

The backing increases the salt flux into the driving solution by a factor of approximately \(\exp[\tilde{a}_m(h)\delta_s/D_s]\), which for \(\delta_s = 0.01\) cm and 
\[D_s = 1.5 \times 10^{-5}\ cm^2/sec\]
is 
\[\exp[\tilde{a}_m(h)(g\ cm^{-2}\ sec^{-1})/1.5\times10^{-3}]
\[\exp[\tilde{a}_m(h)(gpd/ft^2)/32].\]
Thus, for an average water flux of 10 gpd/ft\(^2\)
\((4.7\times10^{-4}\ g\ cm^{-2}\ sec^{-1})\), which occurs for the above two membranes at \(C_s/C_d(h)\) of 0.64 and 0.78, the backing increases the salt flux by 37%.

This increased salt flux coupled with a 25 to 40% decrease in water flux results in an increased salt concentration in the driving solution of 80 to 120% as compared to the membrane with no backing and no concentration polarization. For the low-flux membrane at \(C_s/C_d(h) = 0.86\), the average water flux with the backing is 5 gpd/ft\(^2\) or 22% less than the water flux with no backing. The resulting salt concentration increase due to the backing is approximately 50%.

**Review**

Three mathematical models for the forward osmosis extractor have been presented. For the ideal case where there exists no membrane solute flow and no concentration polarization at the membrane surfaces, Model 1 describes the extractor. The analytic solution of the governing differential equation for membrane water flow can be used without recourse to a computer. For the more general case where the membrane is slightly permeable to the solutes and with the assumption that no concentration polarization exists at the membrane surfaces, Model 2 describes the extractor's potential for a given membrane. Model 3
contains the complete design characteristics for the forward osmosis extractor based on the membrane transport properties, the diffusion coefficients of the two solutions, and average values of the unstirred film thicknesses at the membrane surfaces.

Concentration polarization in the continuous-flow extractor is also described using average values for concentrations and membrane water flux estimated by Model 1. For film thicknesses less than 0.05 cm, this analytic approximation estimates the average membrane water flux and salt contamination within 10% of the results produced by numerical integration with Model 3. The simple approximation has the advantage that it can be more easily used in the optimization and cost analysis described in the following two chapters. For membranes with porous backings, maximum water flux and minimum solute fluxes occur for the backing oriented towards the solution with higher diffusivity if the diffusivities differ by more than 20%. Low-flux membranes with thin backings minimize the detrimental effects of concentration polarization. For a membrane water constant of $L = 1.6 \times 10^{-5} \text{ g cm}^{-2} \text{ sec}^{-1} \text{ atm}^{-1}$, a 25 micron thick porous membrane backing can be expected to reduce the fresh water flux from seawater by 20 to 25%. The membrane backing thickness has a less detrimental effect on the performance of low-flux membranes than on high-flux membranes.
CHAPTER III

DRINKING WATER FROM SEAWATER BY FORWARD OSMOSIS

Summary

Potable water can be extracted from a saline source, such as the ocean, using a concentrated solution of nutrients and a semipermeable membrane. This chapter applies the previously presented theory of the forward osmosis extractor to this case and compares that theory with experiment. Theory and experiment match well. Calculated and observed magnitudes of water extraction rate and nutrient utilization indicate that practical devices may be constructed. The volume of water obtained per mass of nutrient meets the human requirement for fluids and nutrient. Optimization of the system is discussed.

Introduction

Forward osmosis can produce emergency potable water supplies in situations where there is little storage capacity available, as in lifeboats or other small craft. The process can be used to transfer water from the sea into a concentrated nutrient solution (8,14) placed in a forward osmosis extractor. The weight of the only stored consumable required, the soluble food used to extract water, is expected to be less than one-sixth of the weight of the normally carried food plus water. The results in this chapter apply to a 1.12 osmolal seawater source solution. For 1.0 osmolal seawater, the results will be slightly improved.

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The theory of forward osmosis extraction has been discussed in the previous chapter. The mathematical models presented there are used in this chapter to furnish quantitative predictions relating to the transfer of fresh water from an appropriate infinite supply of seawater into a concentrated solution of appropriate nutrients such as various low molecular weight sugars and essential amino acids. The model predictions are then compared with experimental results. Another objective of this chapter is to develop insights into the choice of physical components which will optimize the water extraction process, i.e., which will yield the most potable water for the smallest amount of solute, the smallest extraction apparatus, and at the highest transfer rate.

**Allowable Salt Content in Drinking Water**

No practical perfectly salt rejecting membranes exist. For a given membrane type and fabrication technique, the membrane water permeability coefficient increases with salt permeability, i.e., poor salt rejection correlates with high water transfer rate. It is therefore necessary before proceeding further to state the amount of salt that can be assumed tolerable in an efficient potable water supply.

The minimal daily requirement of drinking fluids is given in Table 1 (27). It is seen that at 90°F the change in the water volume requirement rises nearly exponentially with salt content. The concentration of .17 molal will be taken as an arbitrary guide. The rather substantial amount of metabolic water from the nutrients will be ignored for the sake of conservatism.
Table 1. Minimum Daily Requirements of Drinking Fluids. -- Reference (27).

<table>
<thead>
<tr>
<th>Electrolyte Osmolality of Drinking Fluid ( \alpha_s c_{ds} )</th>
<th>Minimal Human Fluid Requirements (liters/day)</th>
<th>( T_{Air} = 80^\circ F )</th>
<th>( T_{Air} = 90^\circ F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>2.4</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>2.6</td>
<td>4.9</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>2.7</td>
<td>5.1</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>2.8</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>3.1</td>
<td>6.5</td>
<td></td>
</tr>
</tbody>
</table>

Description of the Forward Osmosis Extractor Relations

A schematic flow diagram of an experimental forward osmosis extractor is shown in Fig. 7. Fig. 9 (p. 47) shows a possible practical configuration. A differentially permeable membrane, either flat or as a tube, separates the nutrient driving solution from the seawater source solution. The nutrient solution enters at the inlet with concentration \( c_d(0) \) and solvent feed rate \( Q_d(0) \). The seawater enters with concentrations \( c_s(h) = c_s \) (\( c_s = 0.6 \text{ molal NaCl} = 1.12 \text{ osmolal in this chapter} \)) and solvent feed rate \( Q_s(h) \). An infinite supply of seawater is assumed. \( Q_s(h) \) is set much larger than the membrane flux so that the concentration of the seawater leaving the extractor at \( x = 0 \) is set at \( c_s \).
Fig. 7. Schematic of an Experimental Forward Osmosis Countercurrent Extractor.

For extracting fresh water from seawater, the source solution is seawater and the driving solution contains human nutrients. For practical applications, a small fraction \( \frac{Q_d(0)}{Q_d(h)} \approx \frac{c_d(h)}{c_d(0)} \) of the drinking fluid at the outlet would be used to dissolve additional nutrient powder.
Water flows through the membrane from the seawater into the nutrient solution due to the higher osmotic pressure of the nutrient solution. Osmotic pressure is proportional to concentration by the van't Hoff relation, \( \pi_i = a_i c_i RT \), so that as the nutrient solution proceeds downstream and its volume is augmented by incoming membrane water, \( c_d \) decreases and so does the local membrane water flow rate, \( q_m(x) \). The product drinking fluid emerges with a concentration of \( c_d(h) \) much less than \( c_d(0) \) and with a water flow rate \( Q_d(h) \) equal to \( Q_d(0) \) plus the water extracted through the membrane, \( Q_m(h) \).

**Optimum Membrane Selection**

Generally, the membrane is slightly permeable to the solutes so that sea salt passes through the membrane into the nutrient solution. The salt concentration in the nutrient solution is denoted by \( c_{ds} \). As explained in a previous section, we shall require

\[
    c_{ds}(h) \leq .15 c_s \quad (= 0.17 \text{ osmolal electrolytes})
\]

in the produced drinking fluid. In addition, the extractor must be operated so that everywhere the salt concentration of the membrane permeate, \( c_p(x) \) is less than \( c_s \). Other factors to consider are the loss of nutrients through the membrane and the physical size of the extractor.

There is a wide range of membrane transport properties to consider in the selection of the optimum membrane, one which will produce

1. Otherwise one could just add seawater.
the highest transfer rate of water with the lowest transfer rates of sea salt and nutrient solute. For a given membrane type, the permeabilities to water, sodium chloride, and nutrients \((L, w_{NaCl}, \text{ and } w_d)^2\) vary simultaneously. Using data from Sourirajan (2) for cellulose acetate membranes which were initially treated at a hydrostatic pressure of 20.4 atm (300 psi), the water and sodium chloride permeabilities vary according to the empirical relation \(L = 9.38 \times 10^{-4} (w_{NaCl})^{0.29}\). There are less data in the literature for the cellulose acetate rejection for human nutrients such as glucose. Lonsdale et al. (29) give reverse osmosis rejections of NaCl and glucose for two membranes. Using these

2. The membrane transport properties \((L, w_d, \text{ and } w_{NaCl})\) are defined to relate fluxes to the gradient causing the flux,

Solute flux \((\text{moles sec}^{-1} \text{cm}^{-2}) = w \text{ (cm sec}^{-1})\)

\[= \text{x (concentration difference across membrane; moles cm}^{-3}\)\]

Solvent flux \((\text{gm sec}^{-1} \text{cm}^{-2}) = L \text{ (gm cm}^{-2} \text{ sec}^{-1} \text{ atm}^{-1})\)

\[= \text{x (effective pressure difference across membrane; atm)}\]

The relation \(w/L = (1 - SR_{50}) \times 50 \text{ atm}\) was used to convert reverse osmosis solute rejections to values for \(w\) and \(L\). \(SR_{50}\) is the reverse osmosis solute rejection at 50 atm hydrostatic and osmotic pressure difference across the membrane. The reverse osmosis solute rejection is defined by Merten (28) as:

\[SR = \frac{c_f - c_p}{c_f} = 1 - \frac{w}{L(\Delta p - \Delta \pi)}\].

\(\Delta p - \Delta \pi\) is the effective pressure difference at which \(SR\) is evaluated, and \(c_f\) and \(c_p\) are the concentrations of the feed and permeate solution, respectively.
two data points and assuming that \( w_{\text{glucose}} \) equals zero when \( w_{\text{NaCl}} \) equals zero yields the approximate empirical relationship \( w_{\text{glucose}}/L = 0.24(w_{\text{NaCl}}/L)^{0.42} \). Although the basis for the equation is limited, the two points correspond to the region of interest, and the actual value of \( w_{\text{glucose}} \) is not crucial at this time so long as \( w_{\text{glucose}} \) is small.

Assuming the permeability relations given, one may compute the operating characteristics of an extractor utilizing a cellulose acetate membrane. Assuming transfer of solutes and solvent, but ignoring polarization effects (Model 2), the calculation was performed using the Runge-Kutta Merson integration technique contained in version 3 of DARE (30) (see Appendix B for the simulation program).

Five different NaCl permeabilities were assumed; the results for case D are shown in Fig. 8. FRESH is the number of liters of water extracted per kilogram of nutrient solute supplied. \( c'_{ds}(h) \) is the salt concentration in the drinking fluid extracted. \( c_p(h) \) is the salt concentration of the local membrane permeate. \( n \) is the fraction of nutrient which remains on the driving side of the extractor. \( q_m(h) \) is the average membrane water flux in cm/sec. (To convert from cm/sec to U. S. gallons per day per square foot, multiply by \( 2.12 \times 10^4 \).)

If \( c'_{ds}(h)/c_s \) is less than 0.15, then seawater can be added to the extracted water until its concentration is 0.15 \( c_s \). The total water thus obtained per kilogram of nutrient can be calculated by the equation

\[
\text{Total water/nutrient} = \text{FRESH} \times \frac{[1 - c'_{ds}(h)/c_s]}{(1 - 0.15)}.
\]
Fig. 8. Results Predicted by Model 2 of a Forward Osmosis Extractor for the Extraction of Drinking Water from Seawater.

c_p(h) and c'_d_s(h) are the local and total salt concentrations of the membrane flux; FRESH is liters of water extracted per kg nutrient; \( \lambda h \) is the membrane area in cm\(^2\); \( \bar{q}_m(h) \) is the average membrane water flux in grams-cm\(^{-2}\)-sec\(^{-1}\); \( Q_d(0) \) is the nutrient solvent feed rate in grams/sec; \( \eta \) is the nutrient fraction conserved. The membrane properties are: \( L = 3.05 \times 10^{-5} \) grams-cm\(^{-2}\)-sec\(^{-1}\)-atm\(^{-1}\), \( w_{NaCl} = 7.62 \times 10^{-6} \) cm/sec, \( w_d = 4.0 \times 10^{-6} \) cm/sec. The operating parameters are: \( c_d(0) = 10.0 \) molal glucose-fructose, \( c_s = 0.6 \) molal NaCl, \( \alpha_d = 1.04 \), \( \alpha_s = 1.86 \), \( RT = 24.5 \) liter-atm-mole\(^{-1}\). Model 2 neglects concentration polarization.
Table 2 lists the outputs which result from employing the conditions: $c'_d s(h)/c_s \leq 0.15$, $c_p(h)/c_s \leq 1.0$, and total water/\&h/time $\geq 2.7 \times 10^{-4}$ g cm$^{-2}$sec$^{-1}$. (The last condition is arbitrary at this point and is used solely for the purpose of comparing the predicted results of membranes C, D, and E).

It can be seen from Table 2 that membrane D produces the largest volume of 0.17 osmolal water per mass of nutrient, 5.2 liters per kg, based on the above three output conditions and the assumption of no concentration polarization.

The assumption of no concentration polarization used above over-estimates the membrane water flux. However, because of the absence of membrane compaction, the water permeability coefficient, $L$, can be expected to be larger for membranes used in the forward osmosis mode than for membranes used in reverse osmosis, given a particular sodium chloride permeability. One may, therefore, expect $L > 9.4 \times 10^{-4}$ ($w_{NaCl}^{-0.29}$ for forward osmosis and higher water fluxes than calculated above. These factors may well balance each other out.

**Survival at Sea: Summary of Theory**

For every kilogram of nutrient powder stored on a liferaft (1.5 kg of solution if stored as a 64% nutrient solution), approximately 5.2 liters of drinking water containing 0.17 osmolal electrolytes can be obtained from seawater containing 1.12 osmolal electrolytes at a rate of $2.7 \times 10^{-4}$ cm/sec (5.7 gpd/ft$^2$). 520 grams of nutrient and 116 cm$^2$ of membrane (0.12 ft$^2$) can supply 2000 food calories and 2.7 liters of drinking water, the daily fluid requirement at 80°F, which indicates
Table 2. Predicted Results for Five Membranes with No Concentration Polarization Based on Model 2.

Operating parameters are: $c_a = 0.6$ molal NaCl, $c_d(0) = 10.0$ molal glucose-fructose, $\alpha_S = 1.86$, $\alpha_d = 1.04$, $T = 25^\circ C$. The membrane transport properties are:

<table>
<thead>
<tr>
<th>Membrane</th>
<th>$S_{R50}$ of NaCl (dimensionless)</th>
<th>$w_{NaCl}$ (cm/sec)</th>
<th>$w_d$ (cm/sec)</th>
<th>$L$ (g cm$^{-2}$ sec$^{-1}$ atm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.96</td>
<td>$1.43 \times 10^{-4}$</td>
<td>$2.26 \times 10^{-5}$</td>
<td>$7.15 \times 10^{-5}$</td>
</tr>
<tr>
<td>B</td>
<td>.98</td>
<td>$5.38 \times 10^{-5}$</td>
<td>$1.27 \times 10^{-5}$</td>
<td>$5.38 \times 10^{-5}$</td>
</tr>
<tr>
<td>C</td>
<td>.99</td>
<td>$2.02 \times 10^{-5}$</td>
<td>$7.13 \times 10^{-6}$</td>
<td>$4.05 \times 10^{-5}$</td>
</tr>
<tr>
<td>D</td>
<td>.995</td>
<td>$7.62 \times 10^{-6}$</td>
<td>$4.00 \times 10^{-6}$</td>
<td>$3.05 \times 10^{-5}$</td>
</tr>
<tr>
<td>E</td>
<td>.9975</td>
<td>$2.87 \times 10^{-6}$</td>
<td>$2.25 \times 10^{-6}$</td>
<td>$2.29 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Membrane</th>
<th>$t_{h}/q_{d}(0)$ (sec/cm)</th>
<th>$c_{ds}(h)/c_s$</th>
<th>$c_{ps}(h)/c_s$</th>
<th>$t_{d}(h)/c_s$</th>
<th>$q_{m}(h)$ (cm/sec)</th>
<th>$(1 - n)$ Nutrient Fraction Lost</th>
<th>$\theta c_{ds}/c_s = 0.15$</th>
<th>Liter H$_2$O (kg nutrient$^{-2}$)</th>
<th>Total Water per Nutrient</th>
<th>Total Water per Area per Time (g cm$^{-2}$ sec$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$9.5 \times 10^{-3}$</td>
<td>.15</td>
<td>.64</td>
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<td>$8.8 \times 10^{-4}$</td>
<td>.03</td>
<td>4.64</td>
<td>8.8$\times 10^{-4}$</td>
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<td></td>
</tr>
<tr>
<td>B</td>
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<td>.14</td>
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<td>.04</td>
<td>4.99</td>
<td>3.8$\times 10^{-4}$</td>
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<td></td>
</tr>
<tr>
<td>C</td>
<td>$3.44 \times 10^{-4}$</td>
<td>.08</td>
<td>1.0</td>
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</tr>
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<td></td>
</tr>
<tr>
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<td>.04</td>
<td>1.0</td>
<td>4.64</td>
<td>$1.7 \times 10^{-4}$</td>
<td>.02</td>
<td>5.23</td>
<td>1.9$\times 10^{-4}$</td>
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<td>.11</td>
<td>4.42</td>
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<td>.01</td>
<td>5.13</td>
<td>2.7$\times 10^{-4}$</td>
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<tr>
<td></td>
<td>$6.88 \times 10^{-4}$</td>
<td>.02</td>
<td>1.0</td>
<td>4.60</td>
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<td>.02</td>
<td>5.28</td>
<td>1.4$\times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
that nutrient and liquid output are well matched for the assigned application. Fig. 9 contains a schematic of a possible practical geometry for the forward osmosis water supply device.

Experiment

Description

In the experiment reported here, the extractor shown schematically in Fig. 7 is oriented with the flat membrane in a horizontal position. In order to provide passive mixing by free convection, the glucose-fructose driving solution is above the membrane, and the sodium chloride source solution is below the membrane. The smoother, active side of the cellulose acetate membrane faces the driving solution. The source solution concentration, $c_s$, is kept constant along the length of the extractor by maintaining the source solution feed rate large relative to the membrane water flux. The output tube of the driving solution is higher than the output of the source solution so that a slight hydrostatic pressure opposes the membrane water flow. This hydrostatic pressure difference assures that the membrane water flow is due only to osmotic flow and is not due to bulk flow through possible pinholes in the membrane. Under the membrane are seven 1/8-inch diameter stainless steel rods to reduce bowing of the membrane under the slight hydrostatic pressure.

The membrane transport characteristics were measured under well-stirred conditions in a commercial spiral continuous flow dialysis cell. The values determined were $L = 1.4 \times 10^{-5} \text{ gm cm}^{-2} \text{ sec}^{-1} \text{ atm}^{-1}$ and
Fig. 9. Forward Osmosis Survival (FOS): A Compact Automatic Water Supply Device for Use in Lifeboats (Schematic).

The extractor with hollow fiber membranes is held beneath the ocean surface. The concentrated nutrient solution enters the extractor at a constant rate from a hand pressurized cylinder. The drinking water produced exits at the top. A small fraction of the drinking water can be used to dissolve more dry nutrient for replenishing the concentrated solution in the feed cylinder.
\[ W_{NaCl} = 3.3 \times 10^{-5} \text{ cm/sec}. \] The membrane thickness, measured with a micrometer, was found to be approximately fifty microns. The cellulose acetate membrane was supplied through the courtesy of Fluid Systems Division/UOP, Inc.

The ten molal driving feed solution consisted of five moles of glucose and fructose in one kilogram of water. Due to the high osmotic pressure of the solution, it is not subject to microbial attack while in storage. However, four grams of benzoic acid were added in order to prevent microbial attack on the membrane, and twenty-one grams of citric acid and six grams of sodium hydroxide were added to lower the pH to 4.1.

A refractometer and a conductivity bridge were used to measure the sugar and sodium chloride concentrations in the driving solution product, which simulates the potable output of the extractor. The refractive index of the driving solution product was measured using a temperature compensated hand refractometer (American Optical 10430). \( c_{ds}(h) \) was measured with a conductivity bridge. \( c_{d}(h) \) was calculated based on the measured \( c_{ds}(h) \) and the refractometer reading. \( Q_{d}(0) \) was monitored with a No. 2 Gilmont flowmeter. Experimental results are listed in Table 3. The fraction of glucose and fructose lost is calculated by the equation

\[ 1 - \eta = 1 - \frac{c_{d}(h)Q_{d}(h)}{c_{d}(0)Q_{d}(0)}. \]
Table 3. Experimental Results of a Laboratory Forward Osmosis Extractor.

\[ L = 1.4 \times 10^{-5} \text{gram cm}^{-2}\text{sec}^{-1}\text{atm}^{-1} \]
\[ \ell h = 436 \text{ cm}^2 \]
\[ w_s = 3.3 \times 10^{-5} \text{ cm/sec} \]
\[ \gamma_d = 0.5 \text{ cm}; \gamma_s = 0.4 \text{ cm} \]
\[ c_d(0) = 10 \text{ molal glucose-fructose} \]
\[ c_s = 0.6 \text{ molal sodium chloride} \]
\[ \text{Average source solution velocity} = 1 \text{ cm/sec} \]
\[ \text{Maximum driving solution velocity} = 0.024 \text{ cm/sec} \]

<table>
<thead>
<tr>
<th>( \ell h/Q_d(0) )</th>
<th>( Q_d(0)/\ell h )</th>
<th>( Q_d(h)/\ell h )</th>
<th>( Q_m(h)/\ell h )</th>
<th>( c_{ds}(h)/c_s )</th>
<th>( c_d(h) ) (moles/liter H_2O)</th>
<th>Fraction of Nutrient Lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{cm}^2\text{-sec} )</td>
<td>( \text{gram} ) \text{cm}^{-2}\text{sec}^{-1} )</td>
<td>( \text{gram} ) \text{cm}^{-2}\text{sec}^{-1} )</td>
<td>( \text{gram} ) \text{cm}^{-2}\text{sec}^{-1} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 1.91 \times 10^4 )</td>
<td>( 5.24 \times 10^{-5} )</td>
<td>( 2.51 \times 10^{-4} )</td>
<td>( 1.99 \times 10^{-4} )</td>
<td>( .217 )</td>
<td>( 1.67 )</td>
<td>( .20 )</td>
</tr>
<tr>
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<td>( 2.70 \times 10^{-5} )</td>
<td>( 1.81 \times 10^{-4} )</td>
<td>( 1.54 \times 10^{-4} )</td>
<td>( .262 )</td>
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<td>( .16 )</td>
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<td>( 1.06 \times 10^{-4} )</td>
<td>( .313 )</td>
<td>( .965 )</td>
<td>( .18 )</td>
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<tr>
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<td>( 5.35 \times 10^{-6} )</td>
<td>( 7.07 \times 10^{-5} )</td>
<td>( 6.54 \times 10^{-5} )</td>
<td>( .445 )</td>
<td>( .701 )</td>
<td>( .07 )</td>
</tr>
</tbody>
</table>
Design Error

The seven membrane support rods which run the length of the extractor are an unfortunate source of error in the design of the experimental extractor. The rods occlude sections of the membrane and also create a thick unstirred region in their vicinity where one may expect an atypically high concentration of sodium chloride at the membrane surface. This local increase in sodium chloride concentration not only decreases the membrane water flux but greatly increases the membrane salt flux in the area along the rods.

Comparison of Predicted and Experimental Results

In Fig. 10 are the results for the experimental extractor as predicted by Model 2. The average membrane water flux, \( \tilde{q}_m(h) \), is predicted reasonably well except at high flow rates. Because concentration polarization is neglected, the model overestimates the membrane water flux at high flow rates and underestimates the membrane salt flux. Both of these factors contribute to an underestimation of \( c_{ds}(h)/c_s \), the salt concentration in the nutrient solution.

Model 3, discussed in Chapter II, attempts to include the fluid dynamics of the driving and source solutions by assuming the existence at the membrane of two unstirred films or concentration boundary layers of thicknesses \( \delta_d \) and \( \delta_s \). The concentrations are assumed to equal the bulk solution concentrations, \( c_d(x) \) and \( c_s \), at the distances \( \delta_d \) and \( \delta_s \) from the membrane. The concentrations near the membrane surfaces are given by: \( c_{dw}(x) = c_d(x) \exp[-q_m(x)\delta_d/\tilde{D}_d(x)] \) for the driving side and \( c_{sw}(x) = c_s \exp[q_m(x)\delta_s/D_s] \) for the source side where \( D_d \) and \( D_s \) are the
$c_{ds}(h)/c_s$ is the normalized salt concentration in the nutrient solution. $q_m(h)$ is the average membrane water flux in grams-cm$^{-2}$sec$^{-1}$. Model 2 neglects concentration polarization. Refer to Table 3 for the operating parameters.
diffusivities (cm$^2$/sec) of the driving and source solutes. Although one of the unstirred film thicknesses consists partially of a fixed thickness membrane backing, both $\delta_d$ and $\delta_s$ can be expected to change with fluid velocities, viscosity, and turbulence. Nevertheless, as a first approximation, averages values of $\delta_d$ and $\delta_s$ were assumed for the Model 3 calculation.

Based on the eight experimental values of $c_{ds}(h)/c_s$ and $\tilde{q}_m(h)$, $\delta_d = 0.0029$ cm and $\delta_s = 0.0453$ cm were estimated by nonlinear least squares analysis with the Marquardt algorithm (31) contained in the FORTRAN model building package XTRACTR (32). The estimated effective film thickness $\delta_s$ of 0.0453 cm is the expected result for a porous backing thickness of 50 microns (.0050 cm) and a porosity $\theta = 0.33$ [refer to Model 3; (.005 cm)(.33)$^{-2} = 0.0460$ cm]. The residual sum of squares [normalized by the respective means of the experimental values, $c_{ds}(h)/c_s$ and $\tilde{q}_m(h)$] is $3.4 \times 10^{-3}$. The model explains 99.6% (corresponding to $r^2$) of the variance of the experimental values. The predicted results are shown in Fig. 11.

**Conclusion**

Both theoretical and experimental results demonstrate that a forward osmosis extractor can transfer fresh water from seawater into a solution containing low molecular weight nutrients. The five liters of water obtained per kilogram of nutrient, the potential transfer rate of $\tilde{q}_m(h)$ is $3.4 \times 10^{-3}$. The model explains 99.6% (corresponding to $r^2$) of the variance of the experimental values. The predicted results are shown in Fig. 11.

3. Assuming $D_d$ is inversely proportional to $\mu$ (24), the diffusivity of glucose and fructose in water may be approximated by the relation: $D_d$ (cm$^2$/sec) = $6.73 \times 10^{-5}$ exp(-.44 $c_d$). $D_d$ is the diffusivity of the driving solute averaged in the direction normal to the membrane.
Fig. 11. Comparison of Experimental and Model 3 Predicted Results of an Experimental Forward Osmosis Extractor.

$c_{ds}(h)/c_s$ is the normalized salt concentration in the nutrient solution. $\overline{q}_m(h)$ is the average membrane water flux in grams-cm$^{-2}$-sec$^{-1}$. $\delta_s$ and $\delta_d$ are the estimated thicknesses of the concentration boundary layers. The estimates were obtained by a nonlinear least squares analysis. Refer to Table 3 for the operating parameters.
water through the membrane of greater than 5 gpd/ft$^2$, the low loss of nutrients, and the simplicity inherent in the forward osmosis process indicate that forward osmosis can be used to obtain emergency water for humans at sea.
CHAPTER IV
CONVERSION OF SALINE WATER FOR IRRIGATION BY FERTILIZER-DRIVEN FORWARD OSMOSIS (FDFO): ECONOMIC FEASIBILITY AND DESIGN OPTIMIZATION

Summary

Irrigation water can be extracted from a salt water source by fertilizer-driven forward osmosis (FDFO). The optimum design and economic utility of FDFO extractors depend on the salt concentration of the available water; the transport properties of available semipermeable membranes; the cost of membranes, associated equipment, and operating personnel; the salt tolerance of crops; and the current local value of purchased irrigation water. The operating point of an extractor is determined by setting equal the marginal cost of an increase in FDFO output and the market price of locally purchased water. This optimum operating point determines the overall average cost of the water extracted by FDFO, the quantity of water per fertilizer unit that can be extracted, and the fraction of the irrigation requirement that can be supplied. The results show that FDFO can economically extract approximately 80 kg of water per kg of fertilizer from 3200 mg/l brackish water and 14 kg of water per kg of fertilizer from seawater. The feasibility of applications to low-evapotranspiration greenhouses and to the Yuma desalting plant is discussed.

55
Agricultural development of arid lands in the vicinity of seashores or brackish aquifers still awaits an inexpensive and reliable method of desalination. The currently available processes, solar, or fossil fuel-powered distillation, electrodialysis, and reverse osmosis (RO) are all expensive. RO is the least costly process in general use. It is simple and uses little operating energy; it does require trained personnel; corrosion-resistant, high-pressure equipment; and high-pressure membranes.

Forward osmosis is expected to be cheaper than RO. It is a low-energy use, low-pressure, low-technology process employing harmless or useful solutes to provide the osmotic pressure to extract fresh water from salt water through a semipermeable membrane. Forward osmosis, therefore, produces an aqueous solution containing solutes different from those in the source solution. For agricultural applications, the solute may be dissolved fertilizer. The process will, therefore, be referred to as FDFO, fertilizer-driven forward osmosis.

In San Diego, California, the cost of agricultural water is $70 per acre-foot. Two hundred miles east, in Yuma, Arizona, the U. S. Department of the Interior is testing a desalting plant which will produce fresh water from brackish water at an estimated cost (1976) of $214 per acre-foot. The salt concentration of this source is only one-tenth that of seawater. These water costs are too high for most open field irrigated agriculture. Using these two practical examples, this chapter will demonstrate that FDFO can improve the water cost situation:
slightly for the open field case, and significantly for greenhouses. Methods for finding the most economical FDFO extractor design for a given water source and application will be considered.

Objectives and Assumptions

The objective of this chapter is to derive economic criteria for evaluating FDFO applications to realistic agricultural situations. These economic criteria lead to optimization of FDFO based on agricultural conditions and membrane costs. This chapter uses preliminary membrane cost estimates based on RO membrane costs in order to obtain numerical results for water costs and optimum FDFO design. For this chapter, the principal FDFO and agricultural assumptions are:

1. Imperfect membranes slightly permeable to solutes.
2. No unstirred boundary layers at the membrane surfaces.
3. Plants with a limited appetite for fertilizer and a stated tolerance for salt.
4. Controlled growth environments (greenhouses) which reduce evapotranspiration in comparison with open field systems.
5. An unlimited quantity of salt water that may be considered as a source for FDFO-derived irrigation water.

The two salt water source concentrations discussed in this chapter are 35,000 mg/l seawater and 3200 mg/l brackish water, the irrigation effluent which the U. S. Department of the Interior plans to desalt in Yuma.
A FDFO extractor is better adapted for supplying water to greenhouses than to open field agriculture because enclosed growth areas use less water than open field agriculture due to protection from wind and sun, and the recycling (reflux) of water. The fertilizer-charged output of the FDFO extractor can be used, therefore, to make up greenhouse water losses rather than to supply the total crop evapotranspiration. Fig. 12 illustrates the concept. The notation is summarized in the List of Symbols. Dry fertilizer is continuously dissolved to its maximum solubility $C_d(0)$ in a small flow of water $Q_d(0)$ derived from the extractor output. Within the FO extractor, the water flows from the source solution through the semipermeable membrane into the fertilizer solution and increases the flow of the output solution to $Q_d(h)$. Assuming no leaks, a small fraction $f$ of the fertilizer is lost through the membrane by diffusion. A small amount of salt from the salt water similarly diffuses through the membrane, resulting in a final salt concentration in the fertilizer solution of $C_{ds}(h)$. The fraction $Q_d(0)/Q_d(h)$ of the output is returned to the bottom level of the extractor to dissolve more fertilizer for continuing the process. The net extracted flow $Q_m = Q_d(h) - Q_d(0)$ goes to the greenhouse after possible further dilution with purchased fresh water $Q_l$. The evaporation flow from plants and growth medium is $Q_v$. The portion $rQ_v$ of $Q_v$ that is refluxed (see section on greenhouse irrigation) is an additional fresh water supply.

FDFO is practical if it can reduce substantially the cost of irrigation water. If the fraction of the irrigation requirement produced
Fig. 12. Flow Schematic of fertilizer-Driven Forward Osmosis Extractor and Greenhouse.
by FDFO is defined as \( v \), then the fraction of purchased water is \( 1-v \).

For a FDFO average cost \( AC \) per kg of water and a purchased water price \( k_1 \), the cost \( k_v \) per kg of irrigation water is

\[
k_v = (1 - v)k_1 + vAC \quad (v \leq 1)
\]

or

\[
k_v = k_1 - v(k_1 - AC) \quad (v \leq 1).
\]

For a given \( k_1 \), the cost of irrigation water is lowest for large \( v \) and small \( AC \).

**FDFO Contribution to the Water Supply**

The fraction \( v \) of irrigation water which FDFO can supply is calculated by considering mass balances of water and fertilizer between the FDFO extractor and the greenhouse. Assuming zero drainage, the vaporized water flow equals the water supplied,

\[
Q_v = Q_m + Q_1 + rQ_v.
\]

The units of \( Q \) are kg of water sec\(^{-1}\). If \( Q_m \geq Q_v(1 - r) \), no purchased water is required and \( Q_1 = 0 \). Fertilizer is consumed by the crop at the average rate of \( n \) osmoles sec\(^{-1}\); its concentration \( C_N \) in the irrigation water is \( C_N = n/Q_v \) (osmoles/kg water). The amount of water extracted by the fertilizer supplied to the greenhouse is \( 1/C_d(h) \) in kg/osmole. Thus, fertilizer balance indicates that the fraction of total evapotranspiration that can be supplied by FDFO is
\[ \frac{Q_m}{Q_v} = \frac{C_N}{C_d(h)}. \]  

(57)

Because the FDFO output solution cannot be less concentrated than the source, the maximum extractable mass of water per osmole of fertilizer is \(1/C_s\). Defining the FDFO efficiency as

\[ e = \frac{C_s}{C_d(h)}. \]  

(58)

Eq. 57 becomes

\[ \frac{Q_m}{Q_v} = e \frac{C_N}{C_s}. \]  

(59)

The fraction \(\nu\) of external irrigation water supplied by FDFO is

\[ \frac{Q_m}{(Q_m + Q_1)}. \]  

Eqs. 56 and 59, combined with the definition of \(\nu\), yield

\[ \nu = \frac{eC_N}{(1 - r)C_s} \leq 1. \]  

(60)

The inequality arises from the restriction that \(Q_1 \geq 0\). If fertilizer consumption \(n\) and evaporation \(Q_v\) are known, replacing \(C_N\) by \(n/Q_v\) in Eq. 60 yields

\[ \nu = \frac{en}{(1 - r)Q_v C_s} \leq 1. \]  

(61)

The values of \(\nu\) displayed in Table 4 use values of \(C_N\), \(r\), and \(e\) which are discussed subsequently. The table indicates, for instance, that FDFO can supply 4% of the irrigation requirement for a seawater source and no reflux, or 47% from the brackish water source for a somewhat optimistic
Table 4. The Fraction $\nu$ of Greenhouse Irrigation that FDFO Can Supply.

<table>
<thead>
<tr>
<th>$C_s$ Source Salt Concentration (osmole/kg)</th>
<th>$C_N$ Fertilizer Concentration in the Irrigation Water (osmole/kg)</th>
<th>$r$ Reflux Coefficient</th>
<th>FDFO Fraction $\nu^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(e = .19 .47 1.0)</td>
</tr>
<tr>
<td>.10 (3200 mg/l, Yuma brackish water)</td>
<td>.10 (2800 mg/l)</td>
<td>.05 0.0</td>
<td>.10 .24 .50</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.75</td>
<td>.19 .47 1.0</td>
</tr>
<tr>
<td></td>
<td>.75</td>
<td>.76 1.0</td>
<td>.19 .47 1.0</td>
</tr>
<tr>
<td>.10 0.0</td>
<td>.5</td>
<td>.38 1.0</td>
<td>.19 .47 1.0</td>
</tr>
<tr>
<td>.75</td>
<td>.76 1.0</td>
<td>.19 .47 1.0</td>
<td>.19 .47 1.0</td>
</tr>
<tr>
<td>.5</td>
<td>.38 1.0</td>
<td>.19 .47 1.0</td>
<td>.19 .47 1.0</td>
</tr>
<tr>
<td>.75</td>
<td>.76 1.0</td>
<td>.19 .47 1.0</td>
<td>.19 .47 1.0</td>
</tr>
<tr>
<td>(e = .63 .84 1.0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0 (35000 mg/l seawater)</td>
<td>.05 0.0</td>
<td>.03 .04 .05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.06 .08 .10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.75</td>
<td>.13 .17 .20</td>
<td></td>
</tr>
<tr>
<td>.10 0.0</td>
<td>.5</td>
<td>.13 .17 .20</td>
<td></td>
</tr>
<tr>
<td>.75</td>
<td>.25 .34 .40</td>
<td>.13 .17 .20</td>
<td></td>
</tr>
</tbody>
</table>

* $A$ = optimum efficiencies with RO membrane at RO equipment costs.
  $B$ = optimum efficiencies with hypothetical low-pressure membranes at half the cost of RO equipment.
  $C$ = maximum e.
membrane assumption, 50% reflux, and the standard value of $C_N = .05$.
The table is included here for general orientation.

**Average Cost AC**

The primary costs of the FDFO process are the depreciation cost of the physical apparatus, the size of which is provisionally taken as proportional to the membrane area (33); the cost of fertilizer lost by diffusion through the membrane, and the cost of salt contamination of the irrigation water due to salt diffusion through the membrane.

**Average Membrane Cost**

The average membrane cost per kg of FDFO water is

$$AMC = k_2 \frac{\varphi h}{Q_m}$$

(62)

where $k_2$ is the cost per cm$^2$ of membrane per sec to operate the extractor. $\varphi h/Q_m$ is the average membrane water flux $\dot{q}_m$ in kg cm$^{-2}$ sec$^{-1}$. For small salt gain and nutrient loss, Model 1 (refer to Eqs. 14 and 15 in Chapter II) relates $\dot{q}_m$ to the membrane properties, the concentration of the salt water, and the feed and product concentrations of the driving solution by

$$\dot{q}_m = \frac{LRT C_s}{[C_s/C_d(h) - C_s/C_d(0)]^{-1} \ln[\frac{1 - C_s/C_d(h)}{1 - C_s/C_d(0)}] - 1}$$

(63)
Average Fertilizer Cost

The average cost of fertilizer loss per kg of FDFO water is

\[ AFC = k_3 w_d \Omega_m^{-1} \int_0^h C_d(x) dx, \quad (64) \]

where \( k_3 \) is the cost per osmole of fertilizer and \( w_d \) is the permeability of the membrane to fertilizer. The local water flux is

\[ q_m(x) = LRT[C_d(x) - C_s]. \quad (65) \]

When \( C_d(x) \) from Eq. 65 is inserted into Eq. 64, and observing that

\[ \ell \int_0^h q_m(x) dx = Q_m, \]

one obtains

\[ AFC = k_3 w_d (1/LRT + C_s/q_m). \quad (66) \]

Average Salt Cost

Assuming that the only cost associated with contamination by the source salt is the cost \( k_1 \) of the fresh water which is wasted as drainage water to continuously remove the salt from the growth medium (i.e., that the disposal costs of the drainage water and any salt damage to plants are negligible), the average cost of salt contamination per kg of FDFO water is

\[ ASC = k_1 Q_{\text{drain}} / Q_m. \quad (67) \]
The drainage water salt concentration is assumed to be $C_{50}$, where $C_{50}$ is the salt concentration of the soil extract solution that reduces yields by 50%.\(^1\) If $C_{50}/C_s < 1$, then it is practical to reclaim the fraction $1 - C_{50}/C_s$ of the drainage water by FDFO. For these economic calculations, the savings gained by reclaiming the drainage water is neglected. Salt balance implies

$$\frac{Q_{\text{drain}}}{Q_m} = \frac{C_{ds}(h)}{C_{50}}.$$  \hspace{1cm} (68)

Thus, Eq. 67 becomes

$$ASC = k_1 C_{ds}(h)/C_{50}. \hspace{1cm} (69)$$

$C_{ds}(h)$ is the total gain of salts through the membrane, divided by the total water flow. Thus,

$$C_{ds}(h) = Q_{m}^{-1} \int_0^h [C_s - C_{ds}(x)] dx.$$  \hspace{1cm} (70)

Since $C_{ds}(x) \ll C_s$, and $C_s$ is assumed constant,

$$C_{ds}(h) \approx w_s C_s (q_m)^{-1}. \hspace{1cm} (71)$$

Combining Eqs. 69 and 71 yields

---

1. $C$ is related to the electrical conductivity by

$$C = 0.013 \, EC_e^{1.07},$$

where $EC_e$ (34) is in millimhos cm$^{-1}$.\(1\)
\[ \text{ASC} = \frac{k_{w d} C_s}{q_m C_{50}}. \] (72)

The average cost per kg of FDFO water is the sum of the average costs given by Eqs. 62, 66, and 72. Thus,

\[ AC = \frac{k_{3d}}{LRT} + \left( k_2 + k_{3d} C_s + k_{1w} C_s / C_{50} \right) (\bar{q}_m)^{-1}. \] (73)

Inserting Eq. 63 for \( \bar{q}_m \) and \( e = C_s / C_d(h) \) yield

\[ AC = \frac{k_{3d}}{LRT} + \left( \frac{k_2}{LRTC_s} + \frac{k_{3d}}{LRT} + \frac{k_{1w}}{LRTC_{50}} \right) \ln \left[ \frac{1 - C_s / C_d(0)}{e - C_s / C_d(0)} \right] - 1 \] (74)

Minimizing Cost \( k_V \) of Irrigation

Marginal Costs

The cost \( k_V \) of irrigation water is minimized at that FDFO efficiency where the marginal cost of FDFO water equals the selling price of irrigation water. It is assumed that \( AC < k_1 \), \( MC \) increases monotonically with \( e \) and that all of the FDFO produced water can be used, i.e., \( \nu < 1 \). The marginal or incremental cost \( MC \) of FDFO water is the sum of the marginal costs of the membrane, fertilizer lost, and salt contamination. The calculation of \( MC \) is similar to that of \( AC \) except that instead of integrated values \( MC \) uses local fluxes and concentrations at \( x = h \), the output end of the extractor where the lowest water flux occurs (the key equations are 62, 64, and 72). Thus,
The local water flux at the output end of the extractor is 

\[ q_m(h) = \frac{LRT[C_d(h) - C_s]}{C_s} \]

Using this equation and the definition \( e = C_s / C_d(h) \), Eq. 75 becomes

\[
MC = \frac{e(k_2/C_s + k_1 w_s/C_{50}) + k_3 w_d}{LRT(1 - e)}.
\]

Thus, \( MC = MR \) (where \( MR = k_1 \)) yields the optimum operating efficiency, \( e_{\text{OPT}} \), as

\[
e_{\text{OPT}} = \frac{1 - \frac{w_d k_3}{LRT k_1}}{1 + \frac{w_s}{LRT_{50}} + \frac{k_2}{LRT_{s} k_1}}.
\]

Eqs. 60, 73, and 76 are used to plot \( \nu, AC, \) and \( MC \) versus \( e \) in Fig. 13 under one set of conditions. For a particular price \( k_1 \) of purchased water, one can graphically determine \( e_{\text{OPT}} \) by the intersection of the MC curve with \( k_1 \) and then locate the corresponding \( AC \) and \( \nu \) which minimize the cost \( k_\nu \) of irrigation water in Eq. 55.

Optimum Membrane Selection

Eqs. 55, 60, 74, and 77 indicate that minimum water cost occurs for large \( L \) and small \( w_d \) and \( w_s \). Unfortunately, \( w_s \) and \( w_d \) vary with \( L \). For cellulose acetate membranes pressurized to 20.4 atm (2), \( w_{NaCl} \) and \( L \) are related by the empirical equation,

\[
w_{NaCl} = 6.43 \times 10^{17} L^{3.45}.
\]
Fig. 13. Optimum Operating Point of FDFO.

The optimum efficiency $\epsilon_{OPT}$ is located at the crossing of the FDFO marginal cost curve MC with the market price (MR = $214/a.f.) of purchased water. The corresponding average cost AC of FDFO water and the irrigation fraction $\nu$ are indicated (*). For the above graphs, $k_1 = $214/ac-ft, $C_s = .1$ osmole/kg (3200 mg/l brackish water), $C_N = .05$ osmole/kg, $r = 0.5$. The membrane permeabilities are listed in Table 5-B.
Assuming \( w_d = w_s = w_{\text{NaCl}} \), and using Eq. 77 to find the optimum \( e \) for a given \( L \), a single-variable search on \( L \) finds the optimum membrane permeabilities that maximize \( e_{\text{OPT}} \) and thus minimize \( k_v \). The results are listed in Table 5, row A.

For that case, the membrane and equipment cost contributes 70% of the average cost of the FDFO water. Since FDFO requires no high-pressure equipment, membrane and equipment costs can be greatly reduced, compared to RO. In addition, the hydrostatic pressures less than 1/3 atm characteristic of FDFO avoid the RO membrane compaction problem, so FDFO membranes have a higher \( L \) for given \( w_{\text{NaCl}} \). Based on pressurization data for cellulose acetate membranes (2),

\[
 w_{\text{NaCl}} = 9.15 \times 10^{15} L^{3.45} P_{I}^{1.41} ,
\]

(79)

where \( P_I \) is the initial pressure treatment of the membrane in atm.

Extrapolating to \( P_I = 5 \) atm yields a hypothetical relation between \( w_{\text{NaCl}} \) and \( L \) for low-pressure cellulose acetate membranes. Assuming membrane and equipment cost is half that for RO, a single-variable search on \( L \) using Eq. 77 to find \( e_{\text{OPT}} \) for a given \( L \) finds the optimum membrane permeabilities listed in Table 5, row B.

The mass of water produced per osmole of fertilizer is \( e/C_s \).

Using values for \( e_{\text{OPT}} \) and \( AC \) in Table 5, row B, FDFO can produce 4.7 kg/osmole (78 kg water/kg fertilizer, .06 acre-feet/ton) from 3200 mg/l brackish water at a cost of $7.7 x 10^{-5}/kg ($95/acre-foot). From seawater FDFO can produce .84 kg/osmole (14 kg water/kg fertilizer, .01 acre-feet/ton) at a cost of $4.3 x 10^{-5}/kg ($53/acre-foot). Although
Table 5. FDFO: Optimum e and Membrane Permeabilities; Corresponding Average Costs AC*.

A = with RO membrane and at RO membrane cost of $k_2 = 2.6 \times 10^{-11} \text{ cm}^{-2} \text{sec}^{-1}$ ($\$740/320 \text{ ft}^{2/3}$ year life of UOP spiral wound #8150) not including costs of salt water pretreatment, mechanical equipment, or overhead.***

B = with hypothetical low-pressure cellulose acetate membranes at half RO membrane cost.

<table>
<thead>
<tr>
<th>C_s</th>
<th>Source Salt Concentration (osmole/kg)</th>
<th>L** Water Permeability</th>
<th>W_d' W_s Solute Permeability</th>
<th>e_OPT FDFO Efficiency</th>
<th>AC = Average FDFO Cost $10^{-5}$ $/$kg</th>
<th>AMC Membrane Cost $10^{-5}$ $/$kg</th>
<th>+ AFC Fertilizer Cost $10^{-5}$ $/$kg</th>
<th>+ ASC Salt Cost $10^{-5}$ $/$kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.1</td>
<td>1.8</td>
<td>.13</td>
<td>.19</td>
<td>10.</td>
<td>7.1</td>
<td>2.8</td>
<td>.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($120/ac-ft)</td>
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<tr>
<td></td>
<td>1.0</td>
<td>1.6</td>
<td>.08</td>
<td>.63</td>
<td>7.6</td>
<td>5.4</td>
<td>2.0</td>
<td>.2</td>
</tr>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>($94/ac-ft)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>.1</td>
<td>3.6</td>
<td>.19</td>
<td>.47</td>
<td>7.7</td>
<td>5.4</td>
<td>2.1</td>
<td>.1</td>
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<td></td>
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<td></td>
<td>($95/ac-ft)</td>
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<tr>
<td></td>
<td>1.0</td>
<td>2.6</td>
<td>.06</td>
<td>.84</td>
<td>4.3</td>
<td>3.1</td>
<td>1.0</td>
<td>.2</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>($53/ac-ft)</td>
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</tbody>
</table>

*Assuming $\nu < 1$ and under the following conditions: $k_1 = 1.7 \times 10^{-4}/$kg water ($\$214/acre-foot$); $k_3 = 1 \times 10^{-2}/$osmole fertilizer ($\$150/short ton for an osmole weight of 60 grams)$; $C_{50} = .12$ osmole/kg ($E_{0e} = 8.0$ millimhos/cm); $C_d(0) = 5.0$ osmole/kg.

**Units of $L$ are $10^{-8}$kg cm$^{-2}$sec$^{-1}$atm$^{-1}$; units of $w$ are $10^{-8}$kg cm$^{-2}$sec$^{-1}$.

***A schematic view of a spiral wound membrane module is shown in Fig. 14.
Fig. 14. Spiral Wound Forward Osmosis Extractor Configuration (Schematic).

Each channel is bounded on both sides by a membrane sheet (solid lines). The membranes are separated by a spacer screen (dashed lines) so that the width of the channel is the thickness of the screen. For countercurrent flow, the driving solution (shaded channel) enters at the inner pipe and exits at the top. The source solution (unshaded channel) enters at the top and exits at the inner pipe.
the cost of water extracted from a seawater source is cheaper than water extracted from a brackish water source, the net profit per ton of fertilizer (or savings in water cost compared to purchasing water at $214/ac-ft) is greater for the brackish water case ($7/ton) than for the seawater case ($2/ton).

**Greenhouse Irrigation Cost kv**

Calculating $k_v$ (refer to Eq. 54) and $v$ (refer to Eq. 60) requires values for $C_N$ and $r$.

$C_N$

The concentration $C_N$ of fertilizer in the greenhouse irrigation water varies, depending on the crop, the growing conditions, and the horticulturist. Jensen and Eisa (35) used 1800 mg/l (0.03 osmole/kg) for tomatoes at Puerto Peñasco, Mexico; Ellis et al. (36) recommend 2800 mg/l (0.05 osmole/kg); and Douglas (37) suggests 5000 mg/l (0.07 osmole/kg) for hydroponic agriculture. In addition, the value of the extracted FDEFO water may suggest use of maximum $C_N$ up to the limits of the crop's fertilizer and salt tolerance. In Table 4 and Fig. 15, $C_N = 0.05$ osmole/kg is used as a typical fertilizer concentration, and $C_N = 0.10$ osmole/kg is listed as a possibility.

Reflux $r$

The amount of water needed to grow a particular crop depends primarily on the air temperature, solar radiation, relative humidity, wind speed, and $CO_2$ concentration in the greenhouse. In order to include growth variations, the water requirement of a particular crop can be
Fig. 15. FDFO Irrigation Fraction $\nu$ and Greenhouse Water Cost $k_V$.

Water cost includes purchased and FDFO-derived water, assuming low-cost, low-pressure membranes. Minimum $k_V$ for $\nu \leq 1$ defines the optimum setting for the FDFO efficiency, $e$.  

---

---
measured in terms of the transpiration ratio, $t_r$, in kg of water used per kg of crop dry weight produced. In two experiments at The University of Arizona, the transpiration ratio varied greatly under different humidity and CO$_2$ environments. The $t_r$ of bean plants grown at three humidity levels was 1,960 at 35-40% R.H., 1,194 at 70-75% R.H., and 350 at 95-100% R.H. (38). Increasing the relative humidity from 45% to 80-100% reduced the transpiration ratio of tomato plants from 155 to 70; and increasing the CO$_2$ concentration from 300 to 1500 ppm in the high humidity greenhouse further reduced $t_r$ to 33 (39).

Reflux is the fractional reduction of the transpiration ratio, taking into account both actual condensation within the greenhouse and water not transpired as a result of maintained CO$_2$ and humidity. The reflux is defined equal to zero for the growing conditions and $Q_v$ at which $C_N$ is specified. Thereafter, any reduction in externally supplied irrigation water is attributed to an increase in $r$ at constant $Q_v$ and $C_N$. For the experiments cited above, $r$ increases from 0 to .39 to .82 for the bean plants, and from 0 to .55 to .79 for the tomatoes. To avoid $r$ optimization at this early stage of the work, three values of $r$ are used in Table 4 and Fig. 15 for calculating $\nu$ and $k_v$.

Fig. 15 plots $\nu$ and $k_v$ versus $e$ for low-pressure, low-cost membranes and for the values of $C_N$ and $r$ discussed above. Depending on the optimum values for $C_N$ and $r$, the cost of greenhouse irrigation water may be as low as $5x10^{-5}$/kg ($60/acre$-foot) for extraction from brackish water, and as low as $31x10^{-5}$/kg ($160/acre$-foot) for extraction from
seawater, where the cost of purchased fresh water from other sources is $214/acre-foot.

**Impact on the Yuma Desalting Plant**

Colorado River water reaching Imperial Dam (18 miles upriver from Yuma, Arizona) has an average annual flow of about six million acre-feet and contains about 850 ppm (1.2 tons/acre-foot) dissolved solids. This water provides irrigation for over one million acres of land of which 169,000 acres are in the Yuma area, 486,000 acres are in the Imperial and Coachella Valleys, and 350,000 acres are in Mexico (40).

By current irrigation practices, the undesirable salts, which are carried into the soil with the irrigation water and are then concentrated in the root zone by evapotranspiration, are removed from the root zone by the application of extra water which percolates the salts down away from the root zone. When this extra water causes the groundwater table to rise high enough to threaten the root zone with waterlogging, it must be removed by a drainage system.

In the Yuma area, brackish drainage water returned to the Colorado River from the 62,000 acre Wellton-Mohawk Irrigation and Drainage District has increased the salinity of the Colorado River water delivered to Mexico at Morelos Dam. The salty Colorado River water has caused the Mexican farmers to complain that the salt is accumulating in the soil and is making the land unfit for farming. To alleviate the politically embarrassing situation, a reverse osmosis and electrodialysis desalting plant is being planned to desalt part of the 175,000 acre-feet of the 3200 ppm drainage water from the Wellton-Mohawk
Irrigation and Drainage District. The treatment of Wellton-Mohawk drainage water will result in the following annual outputs:

(a) 101,000 acre-feet/year -- desalted water.
(b) 43,000 acre-feet/year -- brine water.
(c) 31,000 acre-feet/year -- untreated 3200 ppm water.

Fractions (a) and (c) are returned to the Colorado River for use in Mexico. Fraction (b) is channeled sixty miles south where it is dumped into Santa Clara Slough in Mexico (41). The 1976 estimated cost for the desalted water is $214/acre-foot (42).

The economic contribution of FDFO to the Yuma desalting plant can be calculated with Eqs. 55, 61, and 74 where \( n \) is the rate of fertilizer consumption by crops in the area and \( Q_v \) is the volume of reclaimed water. With the estimated 250,000 tons of fertilizer used on the one million acres in the Yuma, Imperial, and Coachella Valleys and in the Mexican border area (5), FDFO can reclaim part of the brackish water from each irrigation district in several local FDFO extractors. By releasing to Mexico an equal volume of Colorado River water, the local FDFO extractors perform the same objective as the Yuma desalting plant, i.e., delivering good quality (850 to 965 mg/l) water to Mexico. By Eq. 61 with \( r = 0 \), the FDFO fraction of the 100,000 acre-feet of reclaimed water is

\[
\nu = \frac{en}{Q_v C_s} \tag{80}
\]
\[ v = e \left( \frac{250,000 \text{ tons/yr} \times 1.5 \times 10^4 \text{ osmole/ton}}{100,000 \text{ ac-ft/yr} \times 1.23 \times 10^6 \text{ kg/ac-ft} \times 0.1 \text{ osmole/kg}} \right) \]

(81)

\[ v = 0.3e . \]

(82)

With this equation for \( v \) and Eqs. 55 and 74, the cost per acre-foot to reclaim 100,000 acre-feet of brackish water using the desalting plant and FDFO is shown in Fig. 16 as a function of \( e \). At \( e_{\text{opt}} = 0.47 \), \( v = 0.14 \); thus, 14,000 acre-feet of irrigation water can be produced at a cost of $95/ac-ft. The FDFO contribution lowers the cost \( k_v \) by $16/ac-ft, or $1.6 million a year.

Although FDFO appears to be economically competitive with the projected costs of the Yuma Desalting Plant, this does not imply that FDFO is an economical method for reclaiming brackish water in the Lower Colorado River Region. On the contrary, the low price of irrigation water ($6 to $8/acre-foot) (40) precludes the use of any existing desalting method as an economical alternative source of water. More economical ways to supply the required 130,000 acre-feet of river water each year would be to improve the irrigation efficiency of the 700,000 acres of U. S. land by only 3% or reduce by 11% the Colorado River water flow (1.2 million ac-ft/year) intended for the proposed Central Arizona Project.

**Conclusion**

The main result of this chapter is the economic analysis developed for ascertaining the optimum fertilizer-driven forward osmosis (FDFO) design parameters and economic feasibility. This analytic method
Fig. 16. Cost $k_v$ of Desalting Yuma Irrigation Effluent and the Fraction $\nu$ of the 100,000 Acre-Feet/Year which FDFO Can Produce Assuming Low-Pressure, Low-Cost Membranes.
provides the framework for evaluating forward osmosis systems in any application. In order to provide specific numbers for the economic analysis and design optimization, two water sources were assumed: a typical irrigation effluent containing 3200 mg/l salts and seawater having ten times that salt concentration. With a projected FDFO membrane system, FDFO can extract 80 kg water per kg fertilizer (.06 acre-feet/ton) from 3200 mg/l brackish water at a cost of $95/acre-foot, where a marginal water value of $214/acre-foot is assumed. With the same marginal value of water and a similar membrane system, FDFO can extract 14 kg water per kg fertilizer at $53/acre-foot from seawater. This FDFO cost estimate includes the costs of the membrane equipment, salt gained through the membrane, and fertilizer lost through the membrane. The FDFO cost does not include the value of the fertilizer in the product water because normal fertilizer consumption was assumed.

The membrane system cost and performance projections are intended to be conservative; other assumptions can be inserted by the methods developed. Because the quantity of water which FDFO can produce is limited by the fertilizer consumption of the crop, FDFO can supply only a small fraction of the irrigation requirement for open field agriculture due to the high transpiration rates encountered. The .03 feet of water which can be produced from brackish water by an annual fertilizer consumption of 1/2 ton per acre is insignificant compared to the high crop transpiration rates of 4 to 6 feet per year. Nevertheless, with the fertilizer

2. The 1976 estimated cost of 386 mg/l RO-reclaimed water from the projected Yuma desalting facility, using 3200 mg/l source water.
normally consumed in the Yuma, Imperial Valley, and Mexicali areas, FDFO can reclaim 14,000 acre-feet of irrigation effluent. Thus, FDFO would save about 1.6 million dollars per year, or about 8 percent of the 1976 cost estimates for the proposed Yuma desalting plant. Distribution problems with the fertilizer charged water can be avoided by installing numerous small FDFO systems in each of the irrigation districts. The projected low maintenance requirements of FDFO systems resulting from atmospheric pressure operation allows such decentralized installation.

FDFO may be feasible for controlled environment agriculture because enclosed growth areas use less water than open field agriculture due to protection from wind and sun, and the recycling of water. It is shown that FDFO can supply economically 24 to 100 percent of this type of irrigation requirement from 3200 mg/1 brackish water and 4 to 34 percent from seawater. The low range represents the irrigation fraction which FDFO can supply to existing open greenhouses; the high range applies to high humidity, water saving greenhouses, the feasibility of which requires further research. At the high range, the total irrigation water cost, including both the cost of FDFO water and the cost of any necessary additional fresh water purchased at $214/acre-foot, is $60/acre-foot from a brackish water source and $160 from a seawater source.

To proceed beyond the present conclusions requires further studies to find or develop optimum FO membranes with lowered cost and enhanced operational characteristics. Costs and operating lifetimes of low pressure membrane equipment incorporating these improved, specially
developed membranes are not known, and neither are the possible savings
on water pretreatment in FDFO, relative to RO systems. In addition,
because the quantity of water which FDFO can produce is ultimately
limited by the amount of fertilizer consumed by the crop, it may be
appropriate to search for or develop fast-growing plants with high
fertilizer requirements to be utilized in closed systems in areas where
there is little or no fresh water supply. In any cases, the most
economical values of fertilizer concentration remain to be determined
for particular plants. That information and the maximum possible water
recycling inside the greenhouse are needed for improved estimates of the
FDFO irrigation fraction.
CHAPTER V

OTHER FORWARD OSMOSIS POSSIBILITIES FOR PRODUCING
FRESH WATER FROM SALINE WATER

Introduction

Two physical requirements of FO are that the osmolar concentration of the driving solution be greater than the concentration of the source solution, and that the membrane separating the two solutions be impermeable or nearly impermeable to the dissolved solutes. Subject to these conditions, FO can employ any soluble substance in the driving solution. Other driving solution properties, such as corrosiveness, viscosity, toxicity, and cost, also influence the practicality of a driving solute, but these properties can be dealt with in a detailed analysis of the particular FO application. For example, the cost of the glucose-fructose-amino acid nutrients, which are employed to produce drinking water from seawater, is irrelevant to the cost of the extracted water (except for the fraction lost through the membrane) because these nutrients would be consumed. This chapter presents other FO possibilities, including the use of driving solutes which can be removed and then either eliminated or recycled, for the production of fresh water from saline water.
Ethanol

Drinking Water from Seawater

Ethanol or a mixture of ethanol and human nutrients can be used to produce drinking water from seawater. Existing poly(ether/amide) membranes reject 90% ethanol at 64 atm hydrostatic pressure (43). Better rejection of ethanol can be expected with membranes specifically designed for this purpose.

Preliminary calculations indicate that FO can extract 20 kg of water per kg of ethanol from one osmolal (35,000 mg/l) seawater, compared to 6 kg of water which FO can extract per kg of glucose-fructose-amino acid nutrient. The final ethanol concentration is approximately 4.5% by weight. The reduction in weight of stored water supplies for emergency water at sea is thus 95%. A 1:2 mixture of ethanol and the glucose-fructose-amino acid nutrients in the driving solution can extract 10 kg of water per kg of driving solutes from seawater. The drinking water produced contains approximately 3% ethanol and 6% nutrients, and the reduction in weight of stored water supplies is 90%. Design optimization of the alcohol-driven forward osmosis extractor can employ the same methods presented in Chapters III and IV.

Removal by Distillation

If ethanol (or any other harmless or beneficial solute) were cheap enough, it could be left in the water. The ethanol would eventually leave solution by decomposing into acetic acid and then to carbon dioxide and water. But if ethanol sells for $.50/gallon, then the cost
of the ethanol allowed to remain in the water produced from seawater would be about 2 1/2 cents/gallon of water or $8000/acre-foot. Thus, the value of the ethanol and the goal of producing pure water dictate the removal of the ethanol from the product water except, perhaps, for the emergency water supply situation discussed above.

In order to produce pure water, the ethanol can be removed from the extracted water with a distillation column. For a 5% by weight ethanol feed concentration, 0.5% concentration in the bottoms product, and 40% concentration in the distillate, the minimum number of stages is 2.5 and the minimum reflux ratio is 0.21 as determined by the McCabe-Thiele method (44). For a reflux ratio of 0.27, 13.4 stages are required. For every kg of water produced at this reflux, 66 kcal of energy at 100°C are needed to vaporize 6.9 moles of water and ethanol. The 100°C bottoms water product can be used to preheat the FO product entering the column to 95°C so that liquid heating requires approximately 5 kcal per kg of water. The total energy use per kg water is approximately 70 kcal at 100°C. Although low-temperature energy is used, the high capital cost of the distillation column would appear to make this system more expensive than conventional desalination methods.

Driving Solute Removal for a Pure Water Product (SO2)

In order to remove the driving solute from the extracted water, it may be practical to employ a driving solute whose solubility is a strong function of pH or temperature. By changing the pH or temperature, the solute is removed from solution either by precipitating as a solid,
by vaporizing as a gas, or by forming a liquid which is immiscible with water. In Fig. 17 are three examples of materials with solubilities that are temperature dependent. The osmolalities of Cl₂ and SO₂ are calculated from freezing point depression data (46).

The operational schematic for sulfur dioxide-driven forward osmosis is shown in Fig. 18. The membrane must be able to tolerate a pH of 1.0. Poly(ether/amide) membranes have demonstrated this property (47). After extracting water from seawater, the 5% SO₂ solution is heated to approximately 95°C. At this temperature, less than 0.5% SO₂ remains in solution (see Table 6). The SO₂ gas is then cooled, redissolved in water, and recycled to extract more fresh water from seawater by forward osmosis. To minimize the heating requirement, the hot water (with 0.5% SO₂) preheats the cool extractor product to 90°C. Neglecting the heat of vaporization of evaporating water, the approximate heating requirement per kg of water is 5 kcal for vaporizing the SO₂ (50 grams SO₂/kg x 95 cal/g) plus 5 kcal for heating the preheated water from 90°C to 95°C, for a total of 10 kcal. This is equivalent to 44 kwh/1000 gallons and is slightly greater than the 30-35 kwh/1000 gallons (48) required by reverse osmosis in desalting seawater. The electrical energy used in RO, however, is at least three times more expensive than the 100°C heat used in the sulfur dioxide-driven forward osmosis. (If fossil fuel is used in both cases, for heat generation for FO and for electrical power generation for RO, three times as much fuel is needed for generation of an equal amount of electrical energy due to the approximately 33% energy conversion efficiency from fuel to electricity).
Fig. 17. Three Solutes with Temperature-Dependent Aqueous Solubilities.

Reference (45,46). Only the SO₂ case is discussed in the text; chlorine and cream of tartar are included as additional illustrative examples.
The incoming seawater temperature is $T_0$, usually about 15°C. The desalted water stream is heated by heat exchangers so that $T_0 = T_1 < T_2 < T_3 < 95°C$, the temperature at which $SO_2$ is recovered in an atmospheric pressure gas removal device. The $SO_2$ makeup gas is required to account for the loss of the 0.5% (weight) $SO_2$ in the product water. A fraction (a) of the main desalted water stream (b) is used for redissolving the $SO_2$. 

Fig. 18. Sulfur Dioxide-Driven Forward Osmosis: Operational Schematic.
Table 6. Solubility of Sulfur Dioxide.

Reference (45; 46, 4:258).

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>Grams SO$_2$/kg Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>193</td>
</tr>
<tr>
<td>10</td>
<td>162</td>
</tr>
<tr>
<td>20</td>
<td>113</td>
</tr>
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<td>30</td>
<td>78</td>
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<td>70</td>
<td>17</td>
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<tr>
<td>80</td>
<td>9.8</td>
</tr>
<tr>
<td>90</td>
<td>5.8</td>
</tr>
<tr>
<td>95</td>
<td>4.2</td>
</tr>
</tbody>
</table>
The low-temperature energy can be waste heat or can be supplied by burning fossil fuels and crop wastes or by solar heating. Because mining and coal-burning industries produce an abundance of \( \text{SO}_2 \) as an undesirable air-polluting by-product, the cost of the small amount of \( \text{SO}_2 \) remaining in the water is expected to be low. The remaining \( \text{SO}_2 \) may be either deleterious or beneficial for irrigated agriculture, depending on the soil pH. In the arid Southwest where alkaline soils prevail, the slightly acidic water might be beneficial. In addition, the low pH water would prevent the gaseous loss of ammonia fertilizer from the irrigation water.

**Utilizing Crop Wastes**

In order to produce irrigation water, crop waste-derived glucose and alcohol can drive the forward osmosis process. Assuming a yearly production of 10 tons of cellulose per acre and a 60% conversion to glucose (49), the 6 lbs of water extracted from seawater per lb of glucose (refer to Chapter III) yields 9000 gallons/acre or 0.03 acre-feet/acre. If cellulose-derived alcohol is used, three to four times as much water can be produced, approximately 0.1 acre-feet/acre. The low quantities of water produced appear to make this application impractical.

It might be practical to burn the crop wastes to provide energy for separating \( \text{SO}_2 \) from water in the sulfur dioxide-driven forward osmosis. If burned, 10 tons of waste yields \( 36 \times 10^6 \) kcal which can be used in producing approximately \( 3.6 \times 10^6 \) liters or 3 acre-feet of water.
Conclusion

Of the forward osmosis possibilities presented in this chapter, the use of ethanol to extract drinking water from seawater appears to be promising for small-scale, intensive applications. On a larger scale, sulfur dioxide-driven forward osmosis appears promising for producing almost pure water from seawater due to expected lower costs of energy and equipment as compared to reverse osmosis and the possibility of by-product utilization. Crop wastes can be burned to generate the 10 kcal/kg (44 kwh/1000 gallons) of 100°C energy used in removing and recycling the SO$_2$ driving solute where the process is applied to agricultural water production.
CHAPTER VI

CONCLUSION

This thesis presents forward osmosis as a low-technology, low-energy use process for hydration and dehydration of aqueous solutions. Two applications discussed in detail are fertilizer-driven forward osmosis for converting saline water to irrigation water, and the production of potable water from seawater using human nutrients. Three mathematical models of increasing complexity describe the continuous counter-flow extractor. These theoretical models compare well with experimental results. For extracting drinking water from seawater, the models are used to select the optimum cellulose acetate membrane permeabilities which maximize the mass of water extracted per mass of nutrient for a stated allowable salt concentration in the drinking water and for a given membrane area. For fertilizer-driven forward osmosis, marginal cost analysis determines the optimum mass of water which can be extracted per osmole of fertilizer with available cellulose acetate membranes. A concentration polarization approximation for film thicknesses less than 0.05 cm can also be used in the optimizations to include the effect of unstirred films at the membrane surfaces. The possibility of using a removable, reusable driving solute, such as sulfur dioxide, for the production of almost pure water from seawater is also presented.
The principal practically applicable results of the mathematical analyses for the basic processes and for the economic optimizations are:

1. **Concentration polarization**: This effect, which is principally caused by the membrane backing providing mechanical support, can be minimized by using thin, high-porosity backings. For a membrane water permeability constant of $1.6 \times 10^{-5} \text{ gm cm}^{-2}\text{sec}^{-1}\text{atm}^{-1}$, and at a driving solute utilization efficiency of 86% in extracting fresh water from seawater, Model 3 indicates that a 25 micron backing reduces the water flux 22% (to 5 gpd/ft$^2$) compared to water flux with no backing and no concentration polarization. The resulting salt concentration increase due to the backing is approximately 50%. If the diffusivities of the two solutions differ by more than 20%, maximum water and minimum solute fluxes occur with the membrane backing oriented toward the solution with the higher diffusivity.

2. **Production of drinking water from seawater**: One kg of nutrient powder can extract 6 kg of water from one osmolal seawater. The drinking water produced contains 14 to 15% dissolved nutrients. One kg of ethanol can extract approximately 20 kg of water. The product contains 4 to 5% alcohol. FO can also use a combination of nutrients and ethanol. Use of the nutrient powder results in a factor of 7 weight reduction in stored supplies of food and water. Use of ethanol results in a factor of 20 weight reduction in stored food and water supplies.
3. **Production of irrigation water by fertilizer-driven forward osmosis (FDFO):** With 1976 costs and a projected FDFO membrane system, FDFO can extract 80 kg water/kg fertilizer (.06 acre-feet/ton) from 3200 mg/l brackish water at a cost of $95/acre-foot, and 14 kg water/kg fertilizer from seawater at $53/acre-foot. For open field agriculture, FDFO can contribute only a small fraction, approximately 1%, of the crop irrigation requirement. Depending on the optimum values of fertilizer concentration in the irrigation water and of greenhouse reflux, the cost of greenhouse irrigation water may be as low as $60/acre-foot for extraction from brackish water and $160/acre-foot for extraction from seawater. The marginal cost analysis utilizes the assumed price of purchased water of $214/acre-foot to determine the point of diminishing returns. More incisive future evaluations utilizing the methods developed will require further research to determine the most economical fertilizer concentration in the irrigation water, to determine the water requirement for closed and semiclosed greenhouses, and to discover optimal crops.

4. **Production of almost pure water from seawater, by use of a recyclable driving solute such as sulfur dioxide:** To remove and recycle the sulfur dioxide from the product water requires approximately 10 kcal/kg water (44 kwh/1000 gallons) of 100°C energy. This energy can be produced by burning crop wastes.
Burning ten tons of crop waste can purify three acre-feet of water.

In order to proceed further with the analyses, additional investigation is needed on the membrane transport properties, lifetimes, substrate thickness, substrate porosity, and cost of low-pressure membranes. Final design also requires a detailed fluid flow analysis to determine the physical configuration of the extractor which minimizes concentration polarization without creating excessive pressure drop and dissipation of energy in distribution pumping.

It is recommended that work be continued on the development of a prototype nutrient and ethanol-driven forward osmosis extractor for the production of emergency drinking water from seawater. This application should be of utility to the Coast Guard, Navy, and individual boat owners. In addition, the design information gained in fabricating and operating the prototype can be used in making a more accurate economic evaluation of fertilizer-driven forward osmosis, sulfur dioxide-driven forward osmosis, and other FO applications.
APPENDIX A

THERMODYNAMICS

The chemical potential $\mu_1$ of component $i$ in a mixture is the partial molal Gibbs free energy. For liquid solutions (50),

$$\mu_1 = \mu_1^o + \bar{V}_i(P-P_o) - \bar{S}_i(T-T_o) + RT \ln \gamma_i x_i \quad (A)$$

where $\mu_1^o =$ chemical potential of component $i$ at some reference temperature ($T_o$), pressure ($P_o$), and concentration;

$\bar{V}_i =$ partial molal volume of component $i$;

$\bar{S}_i =$ partial molal entropy of component $i$;

$T =$ temperature;

$R =$ universal gas constant;

$\gamma_i =$ activity coefficient of component $i$ ($\gamma_i x_i$ is the activity);

and

$x_i =$ mole fraction of component $i$.

The difference in chemical potential of a component in two phases or in two solutions separated by a membrane determines the direction in which the component will diffuse spontaneously. The flow is from high to low chemical potential. Eq. A indicates that for two aqueous solutions at equilibrium separated by a rigid membrane which is permeable to water, increasing the hydrostatic pressure, $P$, of one
solution raises its chemical potential. If \( P \) is raised sufficiently, water diffuses out through the membrane from the high to the low pressure solution. This phenomenon is known as "reverse osmosis." Lowering the temperature also changes the chemical potential, and water may diffuse by "thermoosmosis." Lowering the mole fraction of water by adding dissolved solutes lowers the chemical potential of water, since \( \ln \gamma_w x_w \) becomes more negative, so that water diffuses into the solution by "osmosis" (referred to as forward osmosis in this dissertation).

Defining the osmotic pressure, \( \pi \), of the solution by

\[
\pi = -\frac{RT}{V_w} \ln \gamma_w x_w ,
\]

and inserting this relationship into Eq. A, the chemical potential of the water component is

\[
\mu_w = \mu_w^0 + \bar{v}_w (P-P^0) - \bar{S}_w (T-T^0) - \bar{V}_w \pi .
\]

For dilute solutions, the logarithmic term in the osmotic pressure can be approximated by

\[
\ln \gamma_w x_w \approx \ln x_w ;
\]

\[
= \ln \left( 1 - \frac{N_s}{N_s + N_w} \right) ;
\]

where \( w = \text{water} \), \( s = \text{solute} \), and \( N = \text{number of moles of component in system} \).
Because \( \ln(1 - x_s) \approx -x_s \), for \( |x_s| \ll 1 \), the above equation can be approximated by

\[
\ln \gamma_w x_w \approx -\frac{N_s}{N_s + N_w},
\]

\[
= -\frac{N_s}{N_w},
\]

\[
= \frac{N_s \bar{V}_w}{\bar{V}_w N_w}
\]

\[
= -c \bar{V}_w,
\]

where \( \bar{V}_w N_w \) is the volume of water in liters. A liter of water has approximately one kg mass so that \( c \) is the solute molality in moles/kg \( H_2O \). Thus, from Eqs. B and D,

\[
\pi = cRT.
\]

For electrolytes which dissociate into \( i \) or more ions per molecule, Eq. E is the van't Hoff equation

\[
\pi = icRT
\]

where \( i \) is an integer.

Robinson and Stokes (51) insert an empirical correction factor, the osmotic coefficient \( \phi \), to account for nonlinearities in the mathematical approximation for concentrated solutions, deviations from ideal
solutions due to solute-solvent interactions, and incomplete electrolyte
dissociation. Thus,

\[
\pi = i\phi cRT 
\]

is an exact empirical equation into which tabular values of \( \phi \) may be
inserted.

In this dissertation, the two coefficients \( i\phi \) are condensed into
one osmotic coefficient, \( \alpha \). A useful osmotic concentration independent
of temperature is the osmolality \( C \) in osmoles/kg water.

\[
C = i\phi c = \alpha c 
\]

Table A1 contains the osmotic coefficients \( \alpha \) of selected solutes.

The minimum separation work for a volume \( V \) of pure water from a
solution of osmotic pressure \( \pi \) is \( \pi V \), assuming \( \bar{\nu}_w = V_w \), where \( \bar{\nu}_w \) is the
partial molal volume of water in the solution, \( V_w \) is the molal volume of
pure water, and \( \pi \) is the isothermal, isobaric change in the Gibbs free
energy. The stiff membrane allows for different pressures in the pure
and solution compartments in the "isobaric" case. Typical seawater con-
tains 35,000 mg/l of dissolved salts, mostly NaCl. Its osmotic pressure
is approximately 24 atm at 25°C, and the minimum separation work is
therefore 24 l-atm/l, or 0.6 kcal/l, or 2.6 kwh/1000 gal.

The separation work is supplied mechanically in reverse osmosis,
and by a change in free energy (chemical potential) of the driving
solution in forward osmosis. To obtain finite flow rates in the latter
case, the chemical potential of the driving solution must be lower than
Table A1. Osmotic Coefficients of Selected Solutes.

<table>
<thead>
<tr>
<th>Solute</th>
<th>Molecular Weight (gm/mole)</th>
<th>Osmotic Coefficient $\alpha$</th>
<th>Concentration in Equilibrium with 1.0 Osmolal Seawater (moles/kg water)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$c &lt; 1.0$</td>
<td>$c &gt; 1.0$</td>
</tr>
<tr>
<td>NaCl</td>
<td>58.4</td>
<td>1.9</td>
<td>1.9-2.6</td>
</tr>
<tr>
<td>KCl</td>
<td>74.6</td>
<td>1.8</td>
<td>1.8-2.0</td>
</tr>
<tr>
<td>$\text{H}_2\text{SO}_4$</td>
<td>98</td>
<td>2.1</td>
<td>2.4-7.2</td>
</tr>
<tr>
<td>CaCl$_2$</td>
<td>111</td>
<td>2.7</td>
<td>3.3-9.6</td>
</tr>
<tr>
<td>Sucrose</td>
<td>342.3</td>
<td>1.05</td>
<td>1.1-1.5</td>
</tr>
<tr>
<td>KI</td>
<td>166</td>
<td>1.8</td>
<td>1.9</td>
</tr>
<tr>
<td>KNO$_3$</td>
<td>101</td>
<td>1.6</td>
<td>1.4-1.2</td>
</tr>
<tr>
<td>$\text{KH}_2\text{PO}_4$</td>
<td>136</td>
<td>2.4</td>
<td>2.1</td>
</tr>
<tr>
<td>$\text{NH}_4\text{Cl}$</td>
<td>53.5</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>$\text{NH}_4\text{NO}_3$</td>
<td>80</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>MgCl$_2$</td>
<td>95.2</td>
<td>3.0</td>
<td>3.3-9.0</td>
</tr>
<tr>
<td>Mg(NO$_3$)$_2$</td>
<td>148.4</td>
<td>3.0</td>
<td>3.3-7.2</td>
</tr>
<tr>
<td>Ca(NO$_3$)$_2$</td>
<td>164</td>
<td>2.5</td>
<td>2.6-4.2</td>
</tr>
<tr>
<td>MnCl$_2$</td>
<td>126</td>
<td>2.7</td>
<td>3.0-5.1</td>
</tr>
<tr>
<td>FeCl$_2$</td>
<td>127</td>
<td>2.7</td>
<td>3.0-4.2</td>
</tr>
<tr>
<td>K$_2$SO$_4$</td>
<td>173</td>
<td>2.1</td>
<td>-</td>
</tr>
<tr>
<td>(NH$_4$)$_2$SO$_4$</td>
<td>132</td>
<td>2.1</td>
<td>2.0</td>
</tr>
<tr>
<td>MgSO$_4$</td>
<td>120</td>
<td>1.0</td>
<td>1.4</td>
</tr>
<tr>
<td>CuSO$_4$</td>
<td>160</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Table A1, Continued.

<table>
<thead>
<tr>
<th>Solute</th>
<th>Molecular Weight (gm/mole)</th>
<th>Osmotic Coefficient α</th>
<th>Concentration in Equilibrium with 1.0 Osmolal Seawater (moles/kg water)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Al}_2(\text{SO}_4)_3$</td>
<td>342</td>
<td>2.0-4.5</td>
<td>.44</td>
</tr>
<tr>
<td>$\text{K}_2\text{HPO}_4$</td>
<td>174</td>
<td>2.1</td>
<td>.47</td>
</tr>
<tr>
<td>$\text{Na}_3\text{PO}_4$</td>
<td>164</td>
<td>2.4</td>
<td>.45</td>
</tr>
<tr>
<td>$\text{K}_3\text{PO}_4$</td>
<td>212</td>
<td>2.8</td>
<td>.38</td>
</tr>
<tr>
<td><strong>B. Values from Weast (52)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acetic acid</td>
<td>60</td>
<td>1.0</td>
<td>.94</td>
</tr>
<tr>
<td>Citric acid</td>
<td>192</td>
<td>1.05</td>
<td>.94</td>
</tr>
<tr>
<td>Ethanol</td>
<td>46</td>
<td>1.0</td>
<td>1.0-6</td>
</tr>
<tr>
<td>Ethylene glycol</td>
<td>62</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>Fructose</td>
<td>180</td>
<td>1.0</td>
<td>.98</td>
</tr>
<tr>
<td>Glucose</td>
<td>180</td>
<td>1.0</td>
<td>.97</td>
</tr>
<tr>
<td>Glycerol</td>
<td>92</td>
<td>1.0</td>
<td>.97</td>
</tr>
<tr>
<td>Methanol</td>
<td>32</td>
<td>1.0</td>
<td>1.0-8</td>
</tr>
<tr>
<td>Urea</td>
<td>60</td>
<td>1.0</td>
<td>1.0-7</td>
</tr>
</tbody>
</table>


that of the source \([\pi(\text{driving}) > \pi(\text{source})]\). Thus, the minimum work for separating pure water from the driving solution is greater than that required for separation from the source. The ratio is the ratio of the osmotic pressures and is the inverse of the FO extractor efficiency, \(e\). For extraction from seawater, the design analyses in this dissertation indicate that the extractor may have an optimum \(e\) in the range of 0.80 to 0.90. Therefore, although the minimum energy of separation of water is increased 10 to 25 percent, from 2.6 kwh/1000 gallons to 3.2 kwh/1000 gallons, this increase is less than 2% of the 30 to 35 kwh/1000 gallons (48) used by the energy efficient reverse osmosis process. As discussed in Chapter V, favorable separation properties of the driving solute may permit the use of lower quality energy and possibly less total energy than reverse osmosis for producing pure water from seawater.
APPENDIX B

MODEL 2: DARE-P SIMULATION PROGRAM
QM=PL*(ADRT*CD+ASRT*(CD-S))
QD=-US

*** THE FOLLOWING ARE THE NECESSARY EQUATIONS FOR THE PROGRAM.***
G AND SDS ARE THE MOLE FLOWS OF GLUCOSE AND SALT RESPECTIVELY

* IN THE DRIVING SOLUTION.
G = WD*CD
SDOT=WS*(CS-CDS)
SDS=SDOT

* CD IS THE NUTRIENT CONC. IN THE NUTRIENT SOLN.
CD=G/QD

* SDS IS THE NUTRIENT CONC. IN THE NUTRIENT SOLUTION.

CDS=SDS/QD

*** THIS CONSERVATION EQ. PROVIDES A CHECK FOR QD IN TERMS OF CD AND CDS.
QDA = (1.+PL*ADRT/WD+CDS)/(1.+PL*(ADRT/WD*CD+ASRT/WS+CDS))

*** THESE EQU. PROVIDE USEFUL INFORMATION FOR LISTING.
SDOT = SDS/(QD-.999999999)/CS

* CP IS THE CONC. OF THE PERFiBERATE SOLN. IF CP IS GREATER THAN CS
THEN IT WOULD BE BETTER SIMPLY TO ADD SEAWATER TO THE NUTRIENT

* SOLUTION.
CP=SDOT/QM
CPCS=C/CS
QMN = (QD-1.)/(T+.0000001)

* GRAT IS THE NUTRIENT FRACTION CONSERVED.
GRAT=G/QD

* FRESH IS LITERS OF WATER EXTRACTED PER KG. OF NUTRIENT SUPPLIED.
FRESH = (QD-1.)/(GO*.18016)

* T2 IS THE TOTAL WATER OBTAINED AFTER ADDING SUFFICIENT SEAWATER

* TO RAISE CDS TO 1.5*CS.
TB=FRESH*(1.-SBDRT)/(1.-15)

*** VARIABLES ARE DIMENSIONED TO PLOT FROM 0 TO 10.
SBDRT=10.*SDOT
CPD=10.*CP
GRAT=10.*GRAT

***

SL
WRITE(6,10)
10 FORMAT(9X,1HF,14X,2HFD,14X,2HFL,14X,2HMS,14X,2HMD)
V=2.
DO 6 K = 1,5
PL = .0000538*F**.41
FD = .236*F**.423
WS=PL*F
WD=PL*FD
WRITE(6,11) F,FD,PL,WS,WD
11 FORMAT(1X,5E16.5)
CALL SAVE
CALL RUN
CALL RESET
6 F=F/2
END

TMAX=80000.
ADRT=25.44, ASRT=45.51, QD=1.0, SDS=0.0, CS=0.6, CDO=10., GO=CDO, G=G

END

WARNING - FOLLOWING STATE/UNDEFINED VARIABLES NOT INITIALIZE
PL
WD
WS

INITIAL CONDITION WILL BE ZERO FOR THESE NAMES

*F=50*(1-59) IN ANM P1=2, P2=1, P3=.5, P4=.25, P5=.125
LIST(1) SBDRT, CPCS, QMN, GRAT, FRESH, TB, CD
LIST(2) SBDRT, CPCS, QMN, GRAT, FRESH, TB, CD
APPENDIX C

MODEL 3: DARE-P SIMULATION PROGRAM
SD1

- THE FOLLOWING ARE THE NECESSARY EQUATIONS FOR THE PROGRAM.

\[ Q_{\text{DOT}} = \left( (CD+TH2)*QM/\text{FICKD} + \text{FICKD} + QM + CDDOT \right) * \text{ADRT}/\text{ADRT} + \text{CDDOT} \]

\[ \$ = \text{FICKD} + \text{ASRT} + \text{CDDOT} + \text{CDDOT} \]

\[ \text{CDW} = \text{FICKD} \quad \text{CDW} = \text{FICKD} \quad \text{CDW} = \text{FICKD} \]

\[ \text{CDW} = \text{FICKD} \quad \text{CDW} = \text{FICKD} \quad \text{CDW} = \text{FICKD} \]

- THE FOLLOWING EQUATIONS PROVIDE A CHECK ON SOME OF THE ABOVE EQUATION

\[ \text{CDW} = \text{CDW} \quad \text{CDW} = \text{CDW} \quad \text{CDW} = \text{CDW} \]

\[ \text{CDW} = \text{CDW} \quad \text{CDW} = \text{CDW} \quad \text{CDW} = \text{CDW} \]

- DETAILED INFORMATION FOR LISTING IS OBTAINED IN THE FOLLOWING.

\[ \text{SDF}=\text{SDF} \quad \text{SDF}=\text{SDF} \quad \text{SDF}=\text{SDF} \]

\[ \text{SDF}=\text{SDF} \quad \text{SDF}=\text{SDF} \quad \text{SDF}=\text{SDF} \]

- SCALING FACTORS ARE ADDED TO OUTPUT VARIABLES FOR PLOTTING BETWEEN 0 AND 10.

\[ \text{SDFATD}=10.*\text{SDFAT} \]

\[ \text{SDFATD}=10.*\text{SDFAT} \]

- THE LOGICAL SUBROUTINE DETERMINES THE INITIAL VALUES FOR QM, FICKD,

\[ \text{IF(K1.EQ.1)QM=0.0001} \]

\[ \text{IF(K1.EQ.2)QHM=0.005} \]
IF(K1.EQ.3) TH1=.010
IF(K1.EQ.4) TH1=.050
IF(K2.EQ.3) TH2=0.001
IF(K2.EQ.2) TH2=.005
IF(K2.EQ.3) TH2=.010
IF(K2.EQ.4) TH2=.050
610 FORMAT(1X,4HPL, WS, WD, TH1(SOURCE BL), TH2(DRIVING BL) 5E16.5)
N=0
4 CONTINUE
USFAC = 1.0
IF(N.LE.5) USFAC=5
IF(N.LE.2) USFAC=.2
CDW = (QM/PL+ASB* (CS*EXP(QM*TH1/FICKD) - CDS))/ABST
IF(CDQ.CDQ) GO TO 17
FICKD = -FICKDO/.44*EXP(-.44*CD)/CD-CDW
GO TO 10
17 FICKD = FICKD*EXP(-.44*CD)
18 CONTINUE
QMCALC= PL*(ABST*CD*EXP(-QM*TH2/2)*FICKD) - ASB* (CS*EXP(QM*TH1/FICKD)) -
S - CDS)
CDWQ = (1.0/PL + ASB* TH1/ICKDO*CS*EXP(QM*TH1/FICKD) - CDS)/ABST
FCKDD = FICKDO/.44*EXP(-.44*CD)/CD-CDW
GO TO 10
600 FORMAT(14,*QM, QMCALC, FKB, CDW, FICKD, QMNEW)
QM=QMNEW
IF(ABS((QMCALC-QM)/QM).LT.0.00001) GO TO 5
IF(N.GT.70) GO TO 1
GO TO 4
5 CONTINUE
CDW = (QM/PL+ASB* (CS*EXP(QM*TH1/FICKD) - CDS))/ABST
FICKD = -FICKD/.44*EXP(-.44*CD)/CD-CDW
CALL RUN
CALL SAVE
CALL RESET
59 CONTINUE
1 CONTINUE
END
TRAX=100000.
DTMN=4., EMN=1.0E-06, NPOINTER=51
FICKDO=6.73E-06, FICKD=1.52E-05, CD=10., CD=CDW, ABST=25.44, ASB=45.51
CD=0.0.
PL=1.04750-05
WS=7.61852E-06, WD=4.00112E-06
CS=.6
G=CD, SDS=CDS
END
WARNING - FOLLOWING STATE/UNDEFINED VARIABLES NOT INITIALIZED
QM
CDW
FICKD
TH2
TH1
INITIAL CONDITION WILL BE ZERO FOR THESE NAMES
* PL=.05E-5 CM/SEC/ATM, WS=.62E-8 CM/SEC, WD=4.00E-6 CM/SEC * B L A Y E
B
LIST FICKD, FICKDA, CDWA, CA, TB, PIDT, CSW
LIST QH, QO, CD, CDWAA, CDWA, VS
LIST QMN, CDW, CP, CD, FRESH, QHAT, SDBOT
PLOT CDWAD, GRAD, SGRAD, FRESH, CPD
LIST(1) FICKD, FICKDA, CDWA, CA, TB, PIDT, CSW
| List (1) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (1) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (1) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (2) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (2) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (2) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (2) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (3) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (3) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (3) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (3) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (4) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (4) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (4) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (4) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (5) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (5) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (5) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (5) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (6) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (6) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (6) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (6) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (7) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (7) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (7) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (7) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (8) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (8) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (8) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (8) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (9) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (9) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (9) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (9) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (10) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (10) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (10) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (10) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (11) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (11) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (11) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (11) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (12) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (12) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (12) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (12) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (13) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (13) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (13) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (13) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (14) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (14) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (14) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (14) | CDEATD, GRATD, SDBATD, FRESH, CPD |
| List (15) | FICKD, FICKDA, CDS, CDA, TB, PINT, CSW |
| List (15) | QM, QD, CDW, CDWA, CDWA, VD, V5 |
| List (15) | QM, VD, CD, CP, CD, FRESH, GRAT, SDBOT |
| Plot (15) | CDEATD, GRATD, SDBATD, FRESH, CPD |

**Usage: Iblock Names Table**

1605/**** 147/2100 79/2000
APPENDIX D

CONCENTRATION POLARIZATION APPROXIMATION (FORTRAN PROGRAM)
PROGRAM QMAG

THE BASIC EQUATION IS...

WHERE COBAR = CS*(1+AA) AND AA IS LISTED BELOW.

OMEGA = 1.0
RT = 24.47
CS = 1.116
CDG = 10.4
OMNEW = 4.0E-04
AL = 3.0475E-05
WS = 7.60186E-05
WD = 4.0011E-05
FICKS = 1.5E-05
FICKD = 6.73E-05
DO 100 KS = 1,4
DO 100 KD = 1,4
100 IF(KS.EQ.11) DELTAS = 0.0
IF(KS.EQ.12) DELTAS = 0.005
IF(KS.EQ.13) DELTAS = 0.01
IF(KS.EQ.14) DELTAS = 0.05
IF(KD.EQ.11) DELTAS = 0.0
IF(KD.EQ.12) DELTAS = 0.005
IF(KD.EQ.13) DELTAS = 0.01
IF(KD.EQ.14) DELTAS = 0.05
WRITE (6,580) RT,WS,WD,DELTA,D,CS,CDG,RT
20 WRITE (6,619)
DFD = DELTA/FICKD
DFS = DELTAS/FICKS
ALRTCS = AL*RT*CS
EFFO = CS/CDG
30 EFF = EFFO
10 EFF = EFFO*0.02
40 COM = CS/EFF
IF(EFF.EQ.1.0) GO TO 100

HEMOLYSIS METHOD SOLVES THE TRANSCENDENTAL EQ. FOR WAVE AVERAGE.

QMODL = OMEGW
I = 1,1
A = EXP(QMODL*DFD)/AA
B = EXP(-QMODL*DFD)
C = EXP(QMODL*DFS)
F = ALRTCS*(A+B*C) - QMODL
FPRIME = ALRTCS*(-DFD*A)
B = -DFD + DFS*C - 1.
QMODL = QMODL - OMEGA*F/FPRIME
IF(ABS(QMODL-QMODL)/QMODL.LE.0.0001) GO TO 30
IF(KD.LT.30) GO TO 15
50 CONTINUE
60 CONTINUE
CWSKAR = CS*EXP(QMNEW*DFS)
CDSH = WS*CWSKAR/QMNEW
COBAR = CS*(1+AA)
PROGRAM GMAVG  73/74  OPT=0 TRACEN  FTN 4.6+428  07/08/77  16:25:25  PAGE  2

CDWBAR * COBAR*8
FERT = WD*CDWBAR/GMNEW
GRAT = 1. - WD*CDWBAR*(1./CDH-1./CD0)/GMNEW
CDOUT = CDH*GRAT/1.04

THE FOLLOWING NEGLECT CONC. POLARIZATION.

OMWDF = AA*ALRTCS
COSWDF = WS*CS/OMWDF
FTWDF = WD*CLEAR/OMWDF
GRAF = 1. - WD* COBAR*(1./CDH-1./CD0)/OMWDF
CDUF = CDH*GRAF/1.04

WRITE(6,620) 1,EFF,CDH,CDWBAR,CSWBAR,GMNEW,COSH,FERT,GRAT,CDOUT,
1COBAR,OMWDF,COSWDF,FTWDF,GRAF,CDUF
GO TO 10
100 CONTINUE
580 FORMAT(1H)
* THIS SIMPLE PROGRAM USES AVERAGE VALUES OF GM AND CD0(X) TO ESTIMA
1TE/
* THE EFFECT OF UNSTIRRED FILMS NEXT TO THE MEMBRANE SURFACES. */
* NEWTON'S METHOD FOR SUCCESSIVE APPROXIMATIONS IS USED TO SOLVE*/
* FOR THE AVERAGE MEMBRANE WATER FLUX. Eqs. 10 AND 13 IN */
* CONVERSION OF SALINE WATER FOR IRRIGATION By FDF0... ARE USED*/
31 TO ESTIMATE CD0(X) AVERAGE.***/
590 FORMAT(1X)  *E12.4* WS = *E12.4* WD = *E12.4* DELTA = *F9.6*
* DELTA = *F9.6* /
* CS = *F9.6* SMOLAL, CD(0) = *F8.4* SMOLAL, RT = *F6.2* LITER-
3ATP/SMOLAL*/
85 *619 FORMAT(22X, *WITH CONC. POLARIZATION* 39X, *WITHOUT CONC. POLARIZ-
ATION* 39X, */

11Y.  *I  EFF CD0H) CDWBAR CSWBAR OMBAR COSH) NUT LO
155 GRAT CDOUT) CD0HBAR OMBAR COSH) NUT LOSS GRAT C
100OUT/#
90 213X.* OSM/KG OSM/KG OSM/KG CM/SEC OSM/KG OSM/KG
1 MOLE/KG OSM*
2/KG CM/SEC OSM/KG OSM/KG MOLE/KG*/
620 FORMAT(1X, 12.4F8.5,12.5*E12.5*F8.5,2X,F8.5,E12.5*F6.5)
150 END
APPENDIX E

FERTILIZER-DRIVEN FORWARD OSMOSIS: OPTIMUM MEMBRANE SELECTION (FORTRAN PROGRAM)
PROGRAM CMOPT (INPUT, OUTPUT, TAPE 6 = OUTPUT)

THIS OPTIMIZATION PROGRAM MINIMIZES THE OBJECTIVE FUNCTION BY
PERFORMING A SINGLE DIMENSION SEARCH FOR EACH VARIABLE OF THE MODEL.

SSQBAL IS THE USER SUPPLIED SUBROUTINE WHICH CALCULATES THE OBJECTIVE
FUNCTION, SSQ.

EACH SINGLE VARIABLE SEARCH IS CONDUCTED BY FINDING THREE VALUES OF SSQ
OF VAR(I) WHICH BRACKET THE MINIMUM. A QUADRATIC CURVE CAN BE DEFINED
BY THESE THREE POINTS, IE. SSQ = A + B*VAR(I) + C*VAR(I)**2. THE MINIMUM
OF THIS CURVE IS X=-B/(2*C). THE SPECIFIC SINGLE VARIABLE SEARCH
IS TERMINATED AND VAR(I)=X-B/(2*C) IS USED TO CALCULATE THE APPROXIMATE
MINIMUM VALUE OF SSQ. IF THE OBJECTIVE FUNCTION IS
GREATER THAN A SECOND ORDER POLYNOMIAL OF VAR, THE CALCULATED VAR
SINGLE VARIABLE SEARCH FINDS THE EXACT MINIMUM BEFORE PROCEEDING TO THE
NEXT VARIABLE.

APPROXIMATES THE MINIMUM POINT FOR THE INDEPENDENT VARIABLE, VAR. THE

DIMENSION VAR(2), VOLD(2)
COMMON/COUNT/ICALL, MAXCAL
COMMON/SALT/CS
MAXGLB = 5
MAXCAL = 30

N = 1
DO 50 NSALT = 1, 2
IF (N.SALT.EQ.1) CS = 0.1
IF (N.SALT.EQ.2) CS = 1.0
DO 500 NMAX=1, 7
30 IF (NMEN.EQ.1) VAR(1) = 1.0E-09
IF (NMEN.EQ.2) VAR(1) = 5.0E-09
IF (NMEN.EQ.3) VAR(1) = 1.0E-08
IF (NMEN.EQ.4) VAR(1) = 5.0E-08
IF (NMEN.EQ.5) VAR(1) = 1.0E-07
IF (NMEN.EQ.6) VAR(1) = 5.0E-07
IF (NMEN.EQ.7) VAR(1) = 1.0E-06
ICALL = 0
ITGLOB = 0
SSNEW = 1.
40 SSTOL = 0.0000001
VIOI = 0.0001
CALL SSQBAL(VAR, SSQ), RETURNS (500)
CONTINUE

DV LESS THAN .25 restricts the single variable search to values of the
same sign (either positive or negative) of the initial value of that
variable (initial value for the given global iteration).

THE ESTIMATED MINIMUM VAR IS NOT SO RESTRICTED.

DV = 0.025
DO 60 I = 1, N
60 VOLD(I) = VAR(I)
DO 400 I = 1, N
S1 = SSQ
V1 = VAR(I)
VAR(I) = (1.0 - DV) * VAR(I)
V2 = VAR(I)
CALL SSQBAL (VAR, SSQ), RETURNS (500)
S2=SSQ

60 IF (S2-S1) 100, 100, 200
100 VAR (I) = (1. - 2. * DV) * VAR (I)

V3=VAR (I)
CALL SSQBAL (VAR, SSQ), RETURNS (500)
S3=SSQ

65 IF (S3-S2) 110, 300, 300
110 VAR (I) = (1. - 4. * DV) * VAR (I)

V3=VAR (I)
CALL SSQBAL (VAR, SSQ), RETURNS (500)
S1=SSQ
V1=VAR (I)

70 IF (S1-S3) 120, 300, 300
120 CONTINUE

VAR (I) = (1. - 4. * DV) * VAR (I)
CALL SSQBAL (VAR, SSQ), RETURNS (500)
S2=SSQ

75 IF (DV.GT.0.25) DV = 0. * DV

IF (S2-S1) 100, 300, 300
200 VAR (I) = (1. + 4. * DV) * VAR (I)

V3=VAR (I)
CALL SSQBAL (VAR, SSQ), RETURNS (500)
S3=SSQ

80 IF (S3-S2) 300, 300, 250
250 VAR (I) = (1. + 4. * DV) * VAR (I)

S2=SSQ
V2=VAR (I)

85 IF (S3-S2) 300, 300, 260
260 CONTINUE

VAR (I) = (1. + 6. * DV) * VAR (I)
V1=VAR (I)

90 CALL SSQBAL (VAR, SSQ), RETURNS (500)
S1=SSQ

261 DV=4. * DV
GO TO 200

95 D = (V2-V1) * (V3-V1) * (V3+V1) + (V1-V3) * (V2-V1) * (V2+V1)
E = (V2-V1) * (S3-S1) + (V1-V3) * (S2-S1)
C = E/D
B = (S2-S1) / (V2-V1) - (V2+V1) * C

100 VAR (I) = - .5 * B / C
CALL SSQBAL (VAR, SSQ), RETURNS (500)
IF (S1.GT.S2.AND.S1.GT.S3) 310, 315
310 S1=SSQ
V1=VAR (I)
IF (S2.LT.S3) 312, 313
312 SMIN=S2
GO TO 350
313 SMIN=S3
GO TO 350

105 IF (S2.GT.S3) 320, 330
315 S2=SSQ
V2=VAR (I)
IF (S1.LT.S3) 321, 323
320 S2=SSQ
V2=VAR (I)
IF (S1.LT.S3) 321, 323
321 SMIN=S1
GO TO 350
323  SMIN=S3
   GO TO 350
330  S3=SSQ
   Y3=VAR(1)
   IF(S1.LT.52) 331,332
120  331  SMIN=S1
   GO TO 350
332  SMIN=S2
350  IF(ABS((SSQ-SMIN)/SSQ).GT.SSTOL) GO TO 300
400  CONTINUE
125  ITL.OB = ITL.OB+1
   WRITE(6,410) ITL.OB,SSQ
410  FORMAT(5X,"AFTER IN" GLOBAL ITERATIONS THE OBJECTIVE FUNCTION TO BE
12  MINIMIZED = " 12.6 /")
130  SSOLD=SSNEW
   SSNEW=SSQ
   IF(ITL.OB.GT.MAX.IB) GO TO 500
   IF(ABS((SSNEW-SSOLD)/SSNEW)-SSTOL).GT.50,50,50
450  INC=0
   DO 460  I=1,IV
135  460  IF(ABS((VAR(I)-VOLD(I))/VAR(I)).LE. VTOL) INC=INC+1
   IF(INC-EV) 50,550,550
500  CONTINUE
   PROFIT = -SSNEW
   WRITE(6,500) ITL.OB,ICALL,PROFIT,(VAR(I),I=1,IV)
500  CONTINUE
140  600  FORMAT(5X,27H NO. OF GLOBAL ITERATIONS = I5//
15X,24H NO. OF FUNCTION CALLS = I5//
15X,18H MAXIMUM PROFIT IS E10.11 //
15X,12H OPTIMUM L = E10.11 )
145  END
SUBROUTINE SQGBAL VAR,SSQ),RETURNS(A1)
DIMENSION VAR(2)
COMMON/COUNT/ICALL,MAXCAL
COMMON/SALT/CS
IF(ICALL.GT. MAXCAL) RETURN A1
IF(ICALL).EQ.0 RETURN

TO CONVERT FROM DOLLARS/KG TO DOLLARS/ACRE FOOT, MULTIPLY BY 1.233E06.

AK1=1.736E-04

K2 = 2.63E-11 $ PER SQ CM PER SEC CORRESPONDS TO UOF SPIRAL MODULE
NO. 9150 AT $740 FOR 220 SQ FT FOR 3 YEARS.
AK2 = 2.63E-11

TO CONVERT FROM COST/Osmole TO COST/TON OF FERTILIZER, MULTIPLY BY
1.585 FOR Osmole WEIGHT = 60 GRAMS.
AK3 = 1.08E-02
CD0 = 5.0

TO CONVERT FROM Osmole/KG TO EC IN MILLIMOS/CM, EC=57.6*C50**0.935, OR
C50=0.0133*EC**1.07. SEE USDA EDBK. NO. 60, PAGE 15.

C50 = 0.12
T=298.15
R=0.0821
WRITE(6,510) AK1, AK2, AK3, CS, C50, CD0, T
WRITE(6,511)
WRITE(6,512)
WRITE (6,620)
CONTINUE

ICALL = ICALL+1
AL = YAF(1)
ALST = ALST+1

RELATION OF W TO L IS FROM
SQUIRABAJAS, PAGE 229, FOR LOW PRESSURE CELLULOSE ACETATE MEMBRANES.
PRESS = 20.4
WS = 1.0E-6*(1.4E-08*AL/11.34)**3.45*PRESS**1.41
WD = WS
CDH = (CS*ALST+WS/C50) * AK2/AK1)/(ALST - WD*AK3/AK1)
QMAB = ALST*CS/(ALST*(1-CS/C50)/(1-CS/CDH)+(CS/CDH-C50/C50) - 1)
F = WD*(1/CDH-1/CD0)*1/ALST + CS*QMAB)
E = CS/CDH
ANC=AK2/QMAB
APC = AK3/F/(1/CDH-1/CD0)
ASC = AK1*WS*CS/(QMAB*C50)
AC = ASC + APC + ASC
PROFST = E*(AK1-AC)/CS
CDON = WS*CS/QMAB
WRITE(6,600) ICALL, AL, WD, WS, CDH, QMAB, F, CDON, E, ANC, APC,
ASC, AC, PROFST
IF(CDON.GE.CD0.OR.CDON.LE.CS) GO TO 25
SSG = -PROFST
RETURN

WRITE (6,525)

WRITE FORMAT (11,"OPTIMUM CD(W) IS GREATER THAN CD(0) OR LESS THAN CS...
11. FOR THE STATED COSTS, THE MEMBRANE CHARACTERISTICS ARE UNCONSOL
2ICAL ./")
RETURN A1
SUBROUTINE SSQBAL  73/74  OPT=0 TRACE   P2N 4.6+428  07/05/77  09.15.05  PAGE  2

600 FORMAT(1X,1W,12E9.3,  E14.8)
510 FORMAT(1X,1A,1X,5X,6H1FERTILIZER DFIVEN FORWARD OSMOSES(FDO)  PROFIT
1X MAXIMIZATION
  25X,27HEY C. MOODY AND J. KESSLER /
  25X,21HUNIVERSITY OF ARIZONA /
  25X,1HJUNE 1977

1/5X,67LHI'THIS PROGRAM MAXIMIZES PROFIT BY SEARCHING FOR THE OPTIMUM

1/5X,311CHARACTERISTICS L, WD, AND KS. ///
  1 5X," K1 = "E10.4"/KG OF IRRIGATION WATER."
  1 5X," K2 = "E10.4"/SEC/SQ CM OF MEMBRANE."
  1 5X," K3 = "E10.4"/OSMOLES OF FERTILIZER."

70
15X," CS = "F7.4 " OSMOLES/KG IS THE SALT CONC. IN THE SALT WATER.""  
1 5X," C50 = "F7.4 " OSMOLES/KG IS THE SALT CONC. IN THE DRAINAGE
1 WATER." 
15X,"CD(0)="F7.4 " OSMOLES/KG IS THE FERTILIZER SOLUBILITY." 
15X," T = "F7.2 " DEGREES KELVIN."/

511 FORMAT(5X,/
  15X,38HIL IS THE MEMBRANE WATER PERMEABILITY IN KG/SQ CM/SEC/ATM. /
  15X, "WD AND KS ARE THE MEMBRANE PERMEABILITIES TO FERTILIZER AND
 1HACL IN KG/SQ CM/SEC. " /
  15X,66HRLOW PRESSURE(20 ATM) CELLULOSE ACETATE MEMBRANES ARE U
1SED.---//
  15X,65EC(T) IS THE OPTIMUM CONCENTRATION OF THE FERTILIZER IN THE
1FDOU /
  15X,67HPRODUCT IN OSMOLES PER KG. WATER. CD(T) IS DETERMINED BY MA
3EQUAL /
  15X,15HCOST ANALYSIS. ///
  15X,58HGMAR IS THE AVERAGE MEMBRANE WATER Flux IN KG/SQ CM/SEC. /
  15X,59HRP IS THE FRACTION OF FERTILIZER LOST THROUGH THE MEMBRANE. /
  15X,67HCCS(T) IS THE SALT CONCENTRATION OF THE FDO PRODUCT IN OSMO
1LES/KG. 
  15X,40HT IS THE OPTIMUM FERTILIZER EFFICIENCY. ///

512 FORMAT(5X/
  15X,64HMAC IS THE AVERAGE MEMBRANE COST IN US DOLLARS PER KG FDO W
1ATER. 
  15X,65HACF IS THE AVERAGE COST OF FERTILIZER LOST IN US DOLLARS PER
1 KG. 
  15X,12HDFOO WATER. 
  15X,67HASC IS THE AVERAGE COST OF SAL T CONTAMINATION IN US DOLLARS
1PER KG 
  15X,13HDFOO WATER. 
  15X,63HAC = AMC + APC + ASC IS THE AVERAGE FDO COST PER KG OF WATE
R. 
  15X,64HPROFIT IS THE TOTAL FDO PROFIT IN US DOLLARS PER OSMOLE PER
1TILIZER. 
  15X,"IF PROFIT IS GREATER THAN K3, THEN ECONOMICS DICATES AN INC
CREASE IN"/
  15X,"FERTILIZER USAGE UNTIL CROP TOXICITY RESULTS."///)
620 FORMAT(1X,3H8O.,6X,1HL,7X,2HWD,7X,2HWS,6X,5HCD(T),X),
  1 5X,5DWAR,5X,1EF,5X,6HCCS(T),X,6X,
  1 1HE,7X,2HANC,6X,2HACF,6X,3HASC,7X,2MAC,7X,6HPROFIT )
END
NO. OF GLOBAL ITERATIONS = 2
NO. OF FUNCTION CALLS = 13
MAXIMUM PROFIT IS .611687378762E-04
OPTIMUM L = .15646922508E-07
APPENDIX F

FERTILIZER-DRIVEN FORWARD OSMOSIS: AVERAGE AND MARGINAL COSTS (FORTRAN PROGRAM)
PROGRAM FOCST 73/74  OPT=0  TRACE  PTH 4.6*428  07/05/77  09.16.22  PAGE 1

1  PROGRAM FOCST(INPUT,OUTPUT, TAPE 5=INPUT, TAPE 6=OUTPUT)
   PZERO = 0.0
   PHAX = 5.0
   FINC = (PHAX-PZERO)/20.
  10  CONTINUE
     READ(5,15) AL,CS
  15  FORMAT(2310.4)

   C   RELATION OF W TO L IS FROM
   C   SOURISAJAN, PAGE 229, FOR LOW PRESSURE CELLULOSE ACETATE MEMBRANES.
   C   PRESS = 20.4
   C   WS = 1.0E-08*(1.0E+08*AL/11.34)**3.45*PRESS**1.41
   C   WD=WS
   C   IF(DOP(5).NE.0) STOP

   C   TO CONVERT FROM DOLLARS/KG TO DOLLARS/ACRE FOOT, MULTIPLY BY 1.233E06.
   C   AK1=1.736E-04
   C   K2 = 2.632E-11 $ PER SQ CM PER SEC CORRESPONDS TO DOP SPIRAL MODULE
   C   NO. 8150 AT $740 FOR 320 SQ FT FOR 3 YEARS.
   C   AK2 = 2.632E-11
   C   TO CONVERT FROM COST/OSMOLE TO COST/TON OF FERTILIZER, MULTIPLY BY
   C   1.585 FOR OSMOLE WEIGHT = 60 GRAM.
   C   AK3 = 1.0E-02
   C   CDO = 5.0

   C   TO CONVERT FROM OSMOLE/KG TO EC IN ALLIMHOCS/CE, EC=57.6*CS0**0.935, OR
   C   CS0=0.0133*EC**1.07. SEE USDA HSBR. NO. 60, PAGE 15.
   C   CS0 = 0.12
   C   T=298.15
   C   B=0.0821
   C   ALRT = AL*E*T
   C   WHITE(6,310) AK1, AK2, AK3, CS, CS0,CDO,T
   C   COPP=CS*{(ALRT*WS/CS0) + AK2/AK1)/(ALRT - WD*AK3/AK1)}
   C   WHITE(6,628)
   C   WRITE(6,628) WHITE(6,619) AL, WS, WD, COPP
   C   WRITE(6,619) PARAM = PZERO
   C   PARAM = PARAM + FINC
   C   CDIM = EXP(-PARAM)
   C   CDM = (CDO-CS)*CDIM + CS
   C   CALCULATE AVERAGE VALUES.
   C   QMBAR = ALRT*CS/(ALOG((1-CS/CDH)/(CS/CDH-CS/CDG)) = 1)
   C   F = WD*(1/CDH-1/CDG)/(1/ALRT+CS*QMBAR)
   C   E = CS/CDH
   C   ANC=AK2/QMBAR
   C   AFC = AK3/F/(1/CDH-1/CDG)
   C   ASC = AK1*WS/CS/(QMBAR*CS0)
   C   AC = ABC + AFC + ASC
   C   PROFIT = P*(AK1+AC)/CS
   C   CDSH = WS*CS/QMBAR
   C   CALCULATE MARGINAL VALUES.
   C   QNH = ALRT*(CDH-CS)
   C   Xmnc = AK2/QNH
X1FC = AK3*W+CDH/QM
X5SC = AK1*W+CS/C50/QM
XMC = XMC + X1FC + X5SC
WRITE(6,630) PARAM, CDIM, CDH,QMBAE,F,CDSH,E,AC,PROFIT,QM,XMCC,
XMFC,X5SC,XMC
IF(PARAN.IL.PRAI) GO TO 20
GO TO 10

510 FORMAT(1H1,/
15X,"FERTILIZER DELIVER FORWARD OSMOSIS(FDP) MARGINAL AND AVERAGE"
2COSTS" /)
25X,2MHY C. MOODY AND J. KESLER /
25X,21UNIVERSITY OF ARIZONA /
25X,3M-FEBRUARY 1977 //
1 5X," K1 = E10.4/KG OF IRRIGATION WATER."
1 5X," K2 = E10.4/SEC/SQ CM OF MEMBRANE."
1 5X," K3 = E10.4/OUGLE OF FERTILIZER."
1 5X," CS = F7.4 " OSMOLES/KG IS THE SALT CONC. IN THE SALT WATER."
2/
1 5X," CS = F7.4 " OSMOLES/KG IS THE SALT CONC. IN THE MARGINAL"
1 WATER."/
1 5X," CD(0) = F7.4 " OSMOLES/KG IS THE FERTILIZER SLOBILITY."
1 5X," T = F7.2 " DEGREES KELVIN."/

80 618 FORMAT(
15X," L = E10.4 KG/(SQ CM-SEC-ATM)"
25X," WS = E10.4 KG/(SQ CM-SEC)"
35X," WD = E10.4 KG/(SQ CM-SEC)"
65X," CD(H) = F6.4 " IS THE OPTIMUM FERTILIZER PRODUCT CONCENTRATION"
5"/"5X, 14X, " AS DETERMINED BY MARGINAL COST ANALYSIS."/)

605 FORMAT(
15X," PARAM... IS -LOG(E) OF CDIM."
25X," CDIM... IS THE DIMENSIONLESS PRODUCT CONC., CDIM=(CD(H)-CS)/(CD"
3(0)-CS)
45X, "CD(H) IS THE PRODUCT FERTILIZER CONC. IN OSMOLES/KG."
45X, "QMBAR... IS THE AVERAGE MEMBRANE WATER Flux IN KG/SQ CM/SEC."
45X, "H....... IS THE FERTILIZER FRACTION LOST."/
45X, "CS(H) IS THE SALT CONC. IN THE PRODUCT."
45X, "E....... IS THE AVERAGE FPERLIZER EFFICIENCY."
45X, "AC....... IS THE AVERAGE FDP COST IN $/KG."
45X, "AC....... IS THE AVERAGE FERTILIZER PRICE IN $/OSMOLE."
45X, "EP(H) IS THE MARGINAL MEMBRANE WATER Flux IN KG/SQ CM/SEC."
45X, "MCC.... IS THE MARGINAL MEMBRANE COST IN $/KG."
95X, "EFCC..... IS THE MARGINAL COST OF FERTILIZER LOST THROUGH THE MEM"
100 "BANE IN $/KG."/
25X, "MCC... IS THE MARGINAL COST OF SALT GAINED THROUGH THE MEMBRAN"
3E IN $/KG."/
45X, "MC...... IS THE TOTAL MARGINAL COST IN $/KG."/)

628 FORMAT(5X," PARAM CDIM CD(H) QMBAE F E
CSD(H) 2 E AC PROFIT QM(H) M MCC MFC MSC
M 3C ")

END
FERTILIZER DRIVEN FORWARD OSMOSIS (FDFO) MARGINAL AND AVERAGE COSTS
BY C. NODDLE AND J. KESSLER
UNIVERSITY OF ARIZONA
FEBRUARY 1977

K1 = $0.1342-0.03/KG OF IRRIGATION WATER.
K2 = $0.2630-10/SEC/SQ CM OF MEMBRANE.
K3 = $0.0000-0/SQ CM OF FERTILIZER.

CS = 1.0000 OSMOLES/KG IS THE SALT CONC. IN THE SALT WATER.
C50 = 1.2000 OSMOLES/KG IS THE SALT CONC. IN THE DRAINAGE WATER.
CD(0) = 5.0000 OSMOLES/KG IS THE FERTILIZER SOLUBILITY.

T = 294.15 DEGREES KELVIN.

PARAM. IS THE LOG(2) OF CDIM.
CDIM IS THE DIAMETERLESS PRODUCT CONC., CDIM=CDM/(CDM CS)/(CDM CD(0)).
CDM IS THE PRODUCT FERTILIZER CONC. IN OSMOLES/KG.

QMBAR IS THE AVERAGE MEMBRANE WATER FLUX IN KG/SQ CM/SEC.
F IS THE FERTILIZER FRACTION LOST.

DCD(0) IS THE SALT CONC. IN THE PRODUCT.
E IS THE FDFO FERTILIZER EFFICIENCY.
AC IS THE AVERAGE FDFO COST IN $/KG.
PROFIT IS THE FDFO PROFIT IN $/SQ CM OF FERTILIZER.

QMN IS THE MARGINAL MEMBRANE WATER FLUX IN KG/SQ CM/SEC.
NMN IS THE MARGINAL MEMBRANE COST IN $/KG.
NFC IS THE MARGINAL COST OF FERTILIZER LOST THROUGH THE MEMBRANE IN $/KG.
MSC IS THE MARGINAL COST OF SALT GAINED THROUGH THE MEMBRANE IN $/KG.
MC IS THE TOTAL MARGINAL COST IN $/KG.

L = 0.15655-07 KG/(SQ CM-SEC-ATH)
W = 0.75677-09 KG/(SQ CM-SEC)
WD = 0.75677-09 KG/(SQ CM-SEC)

CD(H) = 1.5934 IS THE OPTIMUM FERTILIZER PRODUCT CONCENTRATION AS DETERMINED BY MARGINAL COST ANALYSIS.

<p>| PARAM | CDIM (CM) | CD(H) | QMBAR | F | CDS/H | E | AC | PROFIT | QMN/H | NMN | NFC | MSC | MC |
|-------|-----------|-------|-------|---|-------|---|----|--------|--------|-----|-----|-----|-----|-----|
| 0.2500 | .77840   | 4.11520 |       |   |       |   |    |        | .1189-05 | .2120-04 | .2618-04 | .9168-06 | .4966X-04 |
| 0.5000 | .77840   | 4.11520 |       |   |       |   |    |        | .1293-05 | .2207-04 | .2797-04 | .9168-06 | .5738X-04 |
| 1.0000 | .77840   | 4.11520 |       |   |       |   |    |        | .1398-05 | .2306-04 | .3126-04 | .9168-06 | .6606X-04 |
| 1.5000 | .77840   | 4.11520 |       |   |       |   |    |        | .1495-05 | .2407-04 | .3457-04 | .9168-06 | .7731X-04 |
| 2.0000 | .77840   | 4.11520 |       |   |       |   |    |        | .1593-05 | .2509-04 | .3800-04 | .9168-06 | .9047X-04 |
| 2.5000 | .77840   | 4.11520 |       |   |       |   |    |        | .1692-05 | .2613-04 | .4156-04 | .9168-06 | .1052X-04 |
| 3.0000 | .77840   | 4.11520 |       |   |       |   |    |        | .1791-05 | .2719-04 | .4525-04 | .9168-06 | .1210X-04 |
| 3.5000 | .77840   | 4.11520 |       |   |       |   |    |        | .1891-05 | .2827-04 | .4908-04 | .9168-06 | .1381X-04 |
| 4.5000 | .77840   | 4.11520 |       |   |       |   |    |        | .2093-05 | .3049-04 | .5719-04 | .9168-06 | .1769X-04 |
| 5.0000 | .77840   | 4.11520 |       |   |       |   |    |        | .2195-05 | .3162-04 | .6147-04 | .9168-06 | .1987X-04 |</p>
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Average cost of FDFO water in US$/kg \ (AC = AMC + AFC + ASC)</td>
</tr>
<tr>
<td>AMC, AFC, ASC</td>
<td>Average membrane, fertilizer, and salt cost of FDFO water; US$/kg</td>
</tr>
<tr>
<td>c*</td>
<td>Total mole concentration in moles/cm³ = c_A + c_B</td>
</tr>
<tr>
<td>c_A, c_B</td>
<td>Concentration of the solute and solvent, respectively, in moles/cm³</td>
</tr>
<tr>
<td>c_{ds}(h), c_{ds}(h)</td>
<td>Molal and osmolaral concentration of source solute in driving solution product</td>
</tr>
<tr>
<td>c'<em>{ds}(h), c'</em>{ds}(h)</td>
<td>Average source solute concentration of membrane flux; ( C'<em>{ds}(h) = \frac{c</em>{ds}(h)Q_d(h)}{Q_m(h)} )</td>
</tr>
<tr>
<td>c_i, c_i</td>
<td>Concentration of solute i in moles/kg H₂O and osmoles/kg H₂O ( (C_i = a_i c_i) )</td>
</tr>
<tr>
<td>C_N</td>
<td>Fertilizer concentration in the irrigation water; osmole/kg</td>
</tr>
<tr>
<td>c_p, c_p</td>
<td>Source solute concentration of the membrane permeate in moles/kg and osmoles/kg</td>
</tr>
<tr>
<td>C_{50}</td>
<td>Salt concentration of the soil extract solution that reduces yields by 50%; osmole/kg</td>
</tr>
<tr>
<td>D_i</td>
<td>Diffusion coefficient for solute i in water in cm²/sec</td>
</tr>
</tbody>
</table>
FO efficiency, defined as the mass of water extracted per osmole of driving solute divided by the maximum mass of water which can be extracted per osmole; 

\[ e \equiv \frac{C_s}{C_d(h)} \]

\( e_{\text{OPT}} \) Optimum FO efficiency

\( f \) Fraction of the driving solute which is lost by diffusion through the membrane

FDFO Fertilizer-driven forward osmosis (refer to Chapter 4)

FO Forward osmosis

FOS Forward osmosis survival (refer to Chapter 3)

\( j_s \) Mole flux of source solute through the membrane in moles-cm\(^{-2}\)-sec\(^{-1}\)

\( K^* \) Dimensionless matching coefficients for the solvent counterflow on the driving side and the source side of the forward osmosis extractor

\( k_1 \) Price (= marginal revenue) of irrigation water in US$/kg (to convert to $/acre-foot, multiply by 1.23x10^6)

\( k_2 \) Cost of membrane in US$ cm\(^{-2}\)sec\(^{-1}\)

\( k_3 \) Cost of fertilizer in US$/osmole (to convert to $/ton, multiply by 1.5x10^4 assuming an osmole weight of 60 grams)
$k_v$ Average cost of irrigation water including purchased (or desalted) water at a cost $k_1$ and FDFO water at a cost AC; $$/kg or $$/acre-foot

$L$ Membrane water permeability in kg cm$^{-2}$ sec$^{-1}$ atm$^{-1}$ or grams cm$^{-2}$ sec$^{-1}$ atm$^{-1}$. When the latter units are used, a factor of 1000 (grams/kg or grams/liter) appears in some equations.

$2h$ Membrane area in cm$^2$ ($l$ is the width or circumference; $h$ is the length)

$MC$ Marginal cost of the FDFO water in US$/kg (MC = MMC + MFC + MSC)

$MMC,MFC,MSC$ Marginal membrane cost, fertilizer cost, and salt cost of the FDFO water in US$/kg

$MR$ Market price (= marginal revenue) of purchased water; $MR = k_1$

$N_A, N_B$ Mole flux of solute and solvent, respectively, in moles cm$^{-2}$ sec$^{-1}$

$n_i$ Solute current for solute $i$ in moles/sec

$n$ Rate of fertilizer consumption by the crop in osmoles/sec

$Q_i$ Solvent current for solution $i$ in grams/sec or kg/sec

$Q_{drain}$ Agricultural drainage water for removing salt gained in the FDFO water; kg/sec

$q_m$ Local membrane flux in grams cm$^{-2}$ sec$^{-1}$ or kg cm$^{-2}$ sec$^{-1}$

$ar{q}_m$ Average membrane water flux ($ar{q}_m = Q_m/2h$)
$Q_m$ Total membrane water flow in grams/sec or kg/sec
$Q^*_m$ Dimensionless membrane water flow
$r$ Greenhouse reflux coefficient (see page 72)
$R$ Universal gas constant = .0821 liter-atm/mole-degree
$SR_{50}$ Reverse osmosis solute rejection at a total pressure difference of 50 atm
$T$ Temperature in degrees Kelvin
$\bar{w}_i$ Membrane permeability coefficient for solute $i$ in cm/sec or kg cm$^{-2}$ sec$^{-1}$ (to convert from kg cm$^{-2}$ sec$^{-1}$ to cm/sec, multiply by the solution volume in cm$^3$ per kg water, approximately $10^3$)
$V_A, V_B$ Partial molar volume of components A and B in solution (cm$^3$/mole)
$x_A, x_B$ Mole fraction of the solute and solvent, respectively
$\alpha_i$ Osmotic coefficient of solute $i$ (= 1.86 for NaCl), $C_i = \alpha_i c_i$
$\beta$ Dimensionless matching coefficient for the counter-current osmolar flows of the driving solution and the source solution
$\delta_i$ Unstirred film thickness of the concentration polarization on side $i$ of the membrane; cm
$\mu$ Absolute viscosity (grams sec$^{-1}$ cm$^{-1}$)
$\theta$ Porosity of membrane backing or supporting material in asymmetric membranes
\[ \pi_i \] Osmotic pressure in atmospheres, due to solute i,
\[ \pi_i = C_i RT \]
\[ \rho \] Density of water \( \text{gm/cm}^3 \), taken as unity
\[ v \] FDFO fraction of the irrigation water
\[ (0) \] Denotes property at the high concentration end of the counterflow extractor (where the driving solution enters and the source solution exits)
\[ (h) \] Denotes property at the low concentration end of the counterflow extractor (where the driving solution exits and the source solution enters)
\[ (x) \] Denotes property at any location \( x \) (between 0 and h) in the extractor

**Subscripts**

\[ d \] Driving solution
\[ ds \] Source solute which has passed through the membrane into the driving solution
\[ dw \] Driving solute property at the membrane surface
\[ i \] General solution property; any subscript may be inserted
\[ p \] Source solute property of the local membrane permeate
\[ s \] Source solution
\[ sw \] Source solute property at the membrane surface
REFERENCES


