COMPARISON OF RAINFALL SAMPLING SCHEMES USING A CALIBRATED STOCHASTIC RAINFALL GENERATOR

by

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STATEMENT BY AUTHOR

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ABSTRACT

Accurate rainfall measurements are critical to river flow predictions. Areal and gauge rainfall measurements create different descriptions of the same storms. The purpose of this study is to characterize those differences.

A stochastic rainfall generator was calibrated using an automatic search algorithm. Statistics describing several rainfall characteristics of interest were used in the error function. The calibrated model was then used to generate storms which were exhaustively sampled, sparsely sampled and sampled areally with 4X4 km grids. The sparsely sampled rainfall was also kriged to 4X4 km blocks. The differences between the four schemes were characterized by comparing statistics computed from each of the sampling methods. The possibility of predicting areal statistics from gauge statistics was explored.

It was found that areally measured storms appeared to move more slowly, appeared larger, appeared less intense and have shallower intensity gradients.
CHAPTER ONE
INTRODUCTION

1.1 Introductory Remarks

1.1.1 Introduction

Because precipitation is the most important input for runoff prediction, it is important for hydrologists to understand the methods used to quantify the natural rainfall process. Two ways of quantitatively describing a 2-dimensional rainfall field are 1) a grid of areal averages and 2) point measurements at several locations. Unfortunately both methods introduce distortions into the field description. A rainstorm delineated with points may appear different from the same rainstorm delineated with areal averages. Statistical measures of the rainfall, the median, the variance, or the maximum for example, as well as the size and shape of the rainstorm may differ depending on the way the storm is represented. This thesis addresses the differences between point measured and areally averaged rainfall fields.

1.1.2 Definition of Areal Rainfall and Point Rainfall

Areal rainfall values may be either measured using radar and satellite\(^1\) technology, or interpolated from gauge measurements using spatial interpolation techniques. In this thesis all areal rainfall has been computed for square grids, not

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\(^1\) Though satellite rainfall measurements are not widely available at this time, it is likely that they will be available in the future.
over irregular regions such as a watershed. Point rainfall refers to rainfall measured at a gauge. The words point and gauge are used interchangeably.

1.1.3 Objectives

There is one main and two secondary objectives in this thesis. The main objective is to explore the relationships between point derived and areally derived statistics of rainfall, focusing on rainfall intensity distributions. The secondary objective, to which these explorations may lead, is the description of a method for evaluating areal rainfall estimates. The third objective is to gain insights into the calibration and use of stochastic rainfall models.

1.1.4 Motivation

Rainfall characteristics are highly significant in hydrologic studies. Rainfall measured in terms of intensity and/or volume is the most important forcing input to hydrologic models. Changes to the rainfall input can have dramatic affects on predicted runoff (Michaud and Sorooshian, in press 1994; Pessoa et al., 1993; Baldini et al., 1992; Woolhiser and Goodrich, 1988; Hamlin, 1983; Wilson, 1979; Larson, 1974). For example, in the semi-arid regions of the U.S. southwest, it is the high intensity rain that generally produces runoff via infiltration excess mechanisms. When rainfall is measured with areal averages, the high intensity rainfall is often smoothed, so that the rainstorm appears less intense. When the smoothed rainfall is used as input to a runoff model the amount of runoff may be substantially altered.
With the National Weather Service Next Generation Radar system (NEXRAD) making radar rainfall data more available, the issues surrounding the use of areal rainfall as opposed to gauge measured rainfall will become more and more important. Analysis of rainfall data, however, is complicated by measurement error. Error is present in gauge, radar and satellite measurements. While a gauge measurement is generally considered more accurate than an areal measurement, gauges cannot provide exhaustive spatial sampling.

The search for a method to evaluate areal rainfall values with gauge data is also driven by the coming era of NEXRAD. Comparing point and areal rainfall is complicated because there is no measure of the true areal rainfall; radar measurements of rainfall cannot be compared to some objective truth for validation. Radar measurements must be compared in some way with gauge measurements. Unfortunately, as the first portion of this study elucidates, gauge measurements and areal measurements in the same area are not likely to be exactly the same. This thesis describes a modelling approach to the problem of comparing areal data and point data.

1.1.5 Two Possible Methods

One method for comparing rainfall based on point measurements and rainfall based on areal measurements is to select one or more spatial interpolation techniques, interpolate gauge data to areal values and compare the interpolated fields with areal measurements. Because the interpolated fields will always contain
error, comparisons of areal measurements to areal values interpolated from point measurements do not definitively determine the validity of the areal measurements.

To circumvent the incompatibility between point and areal measurements, this thesis used a rainfall generating model to produce a reference rainfall field. The model generates rainfall fields that can be sampled with points and with areal averages. Statistics from the areal and the point fields were then computed and compared without complications arising from measurement error. A relationship between the point statistics and the areal statistics was developed. This modelled relationship was then used to predict areal statistics from gauge measured statistics. The predicted areal statistics can then be compared to the measured areal statistics to evaluate the quality of the measured fields.

1.1.6 A Modelling Approach

In order to determine the nature of the differences between areal and point rainfall, the total rainfall field must be known. This information is not available. Gauge measured data at a very fine grid are not available on a large scale and the new large scale measurement techniques, such as radar, do not provide the areal resolution required. One alternative method for accessing information about total rainfall fields is to produce synthetic fields using a numerical model.

Using a model has two advantages over simply using a statistical interpolation technique. First, using a model permits the generation of a large data set from which to draw the statistics required for comparison. In regions with short periods of
record, or unreliable data this ability will be very useful. A second advantage is that by using a model a new perspective, different from the traditional interpolation techniques, is gained. There are many skeptics, of course, who consider numerical modelling as reliable as rolling dice. Further justification for using a numerical model will be discussed below.

Rainstorms are complicated events and no model can recreate a rainstorm in all its complexity. It may be possible, however for a model to recreate some of the important characteristics of a rainfall regime. A stochastic rainfall generator is unlikely to recreate a particular measured storm, but it may, on the average, recreate intensity and depth patterns. Model error cannot be eliminated, and the use of a model in this circumstance must be considered in the light of the use of models in general.

Using a model to fill out spatially sparse data is not new. The so called 4-DDA, 4-Dimensional Data Assimilation output used in numerical weather prediction is model interpolated data. It is widely used and the basis for much scientific work. The primary defense of its use is that it can fill data gaps otherwise unavailable. The same may be said of the modelled rainfall in this study.

It is important to note that the major assumption made in this work is not that the rainfall process is perfectly replicated by the model, but that the model can generate rainfall as well as spatial interpolation techniques commonly in use today can interpolate to an ungauged location. Considering the sophistication of rainfall models today, such an assumption does not seem unreasonable. This study does not
attempt to replace other spatial interpolation techniques but suggests that combined with other methods, this study may enhance our understanding of rainfall.

1.1.7 Scope

When runoff is produced by the Hortonian infiltration excess mechanism, intensity has greater control over runoff than does the storm depth, therefore this study focused on intensity measurements. More specifically, the focus was on extreme convective rainfall of the desert southwest. Data from the ARS Walnut Gulch experimental watershed, near Tombstone Arizona, were used and a rainfall model written by Krajewski et al. (1993), that has its origins in Walnut Gulch, was used.

1.2 Description of Differences Between Points and Areas

1.2.1 Notation

Rainfall can be considered a random process and a rainstorm a single realization of that random process. A rainstorm over some region of interest $W$ is then $R(X,t)$. The rainstorm may be sampled with either gauges, $R(X_g,t)$ or areas $R(X_a,t)$. Some statistic of interest derived from gauge measurements is noted as $G$ while some statistic of interest derived from areal averages is denoted $A$. Modelled storms are noted with $MG$, and $MA$ to indicate modelled gauges and modelled areas respectively. Subscripts may be added to $G$ and $A$ or to $MG$ and $MA$ to indicate, for example, if the areal values have been interpolated from gauges as in $A_{kriged}$. 
This notation is used in all the following sections. It is important to remember that A, G, MG and MA refer to statistics derived from a rainstorm that has been measured with either points or areas. A, G, MG and MA do not refer to the measurements themselves.

1.2.2 Sources of Difference Between Areal (A) and Gauge (G)

There are five causes for the difference between the areal and gauge rainfall descriptions: 1) areal integration; 2) gauge spatial sampling error; 3) gauge measurement error; 4) areal measurement error; and (if the areal values are not measured but interpolated from gauges) 5) spatial interpolation error.

Areal integration error, denoted $\epsilon_{\text{integration}}$, refers to differences between statistics calculated from areal measurements and statistics calculated from point values that are caused by the process of areal integration. It is independent of the technique by which the areal average is found. If $A_{\text{true}}$ is the true statistic derived from some areal measurements and $G_{\text{true}}$ is the true statistic derived from some gauge measurements, then

$$A_{\text{true}} = G_{\text{true}} + \epsilon_{\text{integration}}$$

Gauge measurement error, $\epsilon_{\text{g-measurement}}$, refers to the errors caused by splash in, splash out, turbulence and any other factors that cause gauges to mis-measure rainfall. Gauge spatial sampling error, $\epsilon_{\text{sample}}$ refers to incomplete gauge coverage of the region of interest, $W$ over which the rainfall is being measured. If $G_{\text{measured}}$ is
some statistic from a gauge measured field then

\[ G_{\text{true}} = G_{\text{measured}} + \epsilon_{g\text{-measurement}} + \epsilon_{\text{sample}} \] (1.2)

Areal measurement error, \( \epsilon_{a\text{-measurement}} \), refers to the errors associated with an areal measurement technique, radar or satellite. This is noted as

\[ A_{\text{true}} = A_{\text{measured}} + \epsilon_{a\text{-measurement}} \] (1.3)

Spatial interpolation error, \( \epsilon_{\text{interpolation}} \), refers to the biases introduced into an areal average by the interpolation technique used to calculate that areal average. It is noted

\[ A_{\text{true}} = A_{\text{interpolated}} + \epsilon_{\text{interpolation}} \] (1.4)

The model was used to evaluate \( \epsilon_{\text{integration}} \), \( \epsilon_{\text{sample}} \), and \( \epsilon_{\text{interpolation}} \) from which it may be possible to develop relationships to evaluate \( \epsilon_{a\text{-measurement}} \).

1.3 Contributions to Hydrology

This thesis makes three contributions to hydrology: first, it closely examines the relationships between point measured and areally measured rainfall. Second, it provides insights into the parameterization of a rainfall generator. Third, it presents a modeling approach to estimate areal interpolation errors and areal measurement errors.

The close examination of the relations between areal and point rainfall is useful for several reasons. By exploring the differences between point and area
rainfall, a clearer notion of exactly what drives runoff models was gained. This may enhance the model development and parameterization processes. As operators switch from using gauge measured to radar measured rainfall data, it may be useful for them to understand the differences between the two types of data. It may provide them with insight into the necessity of reparameterizing their models.

Quantifying the differences between points and areas may also indicate whether or not it is appropriate to use gauge measured data as a substitute for areally measured data, or areally measured for gauge measured.

The second contribution is the examination of using an automatic method for parameterizing a rainfall model. The difficulties encountered in the automatic parameterization may be of interest to others needing to find optimal parameters for a rainfall generator. It was also useful to parameterize the model to a specific rainfall regime and then to evaluate the model's performance and the suitability of the model for use in future studies.

The third contribution, providing a new method for evaluating areal interpolation and areal measurement errors provides a useful second perspective on the dilemma of selecting between the numerous areal interpolation methods. Currently the most common way of comparing and validating spatial interpolation techniques is to use the fictitious point method. One point is suppressed in the interpolation; values for that point are interpolated from the remaining values and the interpolated and the actual values are compared. (See Creutin and Obled 1984 and Tabios and Salas 1987.) The fictitious point method strongly favors techniques,
such as kriging, that are designed to produce point values while methods, such as Thiessen polygons, which are designed to produce only areas are strongly disadvantaged. Using a finely resolved model output rainfall field to evaluate spatial interpolation techniques does not favor the point producing methods over the area producing methods and so provides a useful second perspective on the evaluation problem.

Another common way of evaluating the quality of interpolated rainfall is to compute the error variance with a small variance indicating an accurate interpolation. Unfortunately it is possible to find that the variance is low but that the value is far from the true value as may be seen in Creutin and Obled (1984). Again, another perspective would be useful.

If the proposed method for estimating areal measurement error is successful, this contribution is useful in a number of ways. As the NEXRAD system comes on line, validating the rainfall measurements will be important. The proposed method may be able to provide a perspective on the NEXRAD rainfall output that existing interpolation techniques do not provide.

Both radar measured and (in the future) satellite measured rainfall values suffer from errors associated with a uniform multiplicative bias (Ahnert 1986, Chiu 1990). Determining a relationship between areal and point statistics may provide another method of evaluating this multiplicative bias.
CHAPTER 2
LITERATURE REVIEW

2.1 Areal Average Rainfall in Hydrology

2.1.1 Rainfall Aggregation Affects Predicted Hydrographs

Hydrologists have been aware for a long time that hydrologic models are sensitive to the rainfall input. In 1974 Larson declared that "one of the most important factors in successful hydrologic modelling is reliable and representative precipitation data." A few years later Wilson (1979) concluded that the "spatial distribution of rain and the accuracy of the precipitation input have a marked influence on the outflow hydrograph from a small catchment."

Hamlin (1983) pointed out that the reason accurate rainfall measurements are important in hydrology is that poor rainfall estimates inevitably lead to poor runoff estimates and so to poor decisions. Woolhisler and Goodrich (1988) determined that when runoff is produced by Hortonian overland flow, the rainfall input to a runoff model will affect the hydrograph to some degree unless the system is very highly damped. However, they also found that the spatial variation of infiltration may reduce some of the variation in hydrographs due to temporal rainfall intensity variability.

Baldini et al. (1992) simulated rainfall measured with gauges and radar and then modelled runoff. They found that with only a few rain gauges on a watershed, the runoff is underestimated, while the simulated radar, when properly calibrated
with gauge data will produce more accurate results. They also found that if the
gauge measurements sample the storm poorly, calibrating the radar with the gauges
will result in overestimated hydrographs.

Pessoa et al. (1993) used frontal storms measured with radar to examine the
effects of spatially and temporally aggregating rainfall on hydrographs. They found
that the time average did not significantly affect the predicted hydrographs although
they point out that for high intensity convective rainfall, the time average may be
significant. They also found that lumping the rainfall over the whole basin did not
significantly affect the hydrographs.

More recently Michaud and Sorooshian (in press) have demonstrated that
interpolating gauge measured convective rainfall to 4 km grids will substantially
reduce the amount of predicted runoff in the semi-arid Southwest. The authors also
speculated that averaging to a 2 km grid is less detrimental to runoff prediction and
that time averaging may cause greater changes in predicted runoff than spatial
averaging.

The differences between the findings of Pessoa et al. and Michaud and
Sorooshian may be a result of the differences in the environments they studied. In
the semi-arid Southwest studied by Michaud and Sorooshian, high intensity rainfall
excess is the dominant runoff generating mechanism. Pessoa et al., on the other
hand, used the Seive river basin near Florence, Italy, where a rising water table is
likely to be the dominant runoff generating mechanism.
2.1.2 Finding Areal Averages

With accurate rainfall measurements ranking high in hydrologic importance, much effort has been spent on developing methods for determining areal average rainfall. In 1911, Thiessen (1911) described his now widely known method of Thiessen polygons. Later the U.S. Weather Bureau developed the method of Areal Reduction Factors. In 1974 Rodriguez-Iturbe (1974) revisited the problem, and described a more statistically rigorous method for finding areal averages. Delfiner and Delhomme (1975) proselytized the word of kriging to hydrologists in the 1970s and kriging has gained wide acceptance since then. Several authors have explored and refined the use of kriging in hydrology: Hughes et al. (1981) provided some guidelines for minimum data requirements in the use of kriging; Bastin et al. (1984) discussed the use of kriging for selecting raingauge locations and producing areal averages; Lebel et al. (1987) developed the "climatic variogram"; and Barancourt et al. (1992) contributed a two step process involving indicator kriging to determine whether rain fell at a location and ordinary kriging to find the amount.

Pegram et al. (1993) suggested using the multi-quadric technique of Hardy (1971) to interpolate rainfall. Several authors have contributed to the so called threshold method (Kedem 1990, Atlas 1990, Braud 1993 and Short 1993). For a discussion of additional methods Singh et al. (1986) provided a table listing 13 interpolation techniques with appropriate references.
2.1.3 Evaluating Interpolation Errors

With the large number of interpolation techniques available it is inevitable that there will be comparison studies. The most common method used for evaluating an interpolation technique is to suppress a single sample point or set of sample points, then interpolate to those points and analyze the differences between the observed and the interpolated values. Creutin and Obled (1982) computed correlation coefficients between the observed and calculated values then performed correspondence analysis on a matrix of binned observed and calculated values. They concluded that optimal interpolation and kriging are similar in accuracy.

Tabios and Salas (1985) also used the fictitious point method computing several measures of difference between the observed and calculated values. In addition, they used the standard deviation of the error of the interpolation. They concluded that optimal interpolation runs a very close second to kriging.

Lebel et al. (1987) took a more democratic approach. When they intended to evaluate the accuracy of areal rainfall estimates they were faced, as is everyone, with the problem of having no truth by which to judge. To solve this problem they interpolated gauge data to areal averages with kriging, spline surface fitting and Thiessen polygons using a dense raingauge network approximately 1 gauge per 42 km². They found that the three interpolation techniques produced similar areal averages and that the error variance was low for all three so they used the averages produced by the Thiessen polygons as the true field. They then compared this "true
field" to areal averages produced with less dense networks measuring the same storms; the authors concluded that kriging is the best interpolation method.

2.1.4 Evaluating Measured Areal Rainfall Errors

Associated with the development of the NEXRAD system have been several studies to define the mean field bias of a radar field. Ahnert et al. (1983) pointed out that while some of the errors associated with radar-measured rainfall "will be localized or range dependent, others will often produce a uniform multiplicative bias in the radar estimated precipitation." Hudlow (1991) later reiterated this point.

Ahnert et al. (1986) described a method for predicting the mean field bias based on Kalman filtering. Smith and Krajewski (1991) developed an alternative method which they claim is less cumbersome than Ahnert et al.'s (1986) method. Smith and Krajewski assumed, as many others have, that rainfall and reflectivity are lognormally distributed. In addition, Smith and Krajewski assumed that the errors in the radar measurements not associated with the mean field bias are also lognormally distributed. They then used maximum likelihood estimation to predict the true bias from a calculated sample bias. The sample bias is a ratio of gauge and radar measured rainfall. Smith and Krajewski found that the sample bias ranged from 1.8 to 2.5. They also found that the mean field bias estimated from the sample bias depended heavily on the parameters used for the estimation, but that the estimated bias also ranged around 2. When NEXRAD precipitation products are available for use, they will include bias estimates (Klazura 1993).
One additional error check will be made with the NEXRAD system. Manual editing of rainfall fields by the NEXRAD operators will also be done to improve the accuracy of the NEXRAD precipitation products (Shedd 1991).

Chiu et al. (1993) developed a method for estimating the mean field bias of satellite measured rainfall using ship based radar measurements as the true rainfall. The authors found that the mean field bias was highly correlated to the mean rainfall rate. Chiu et al. described the bias as a function of the rainfall characteristics and the relationship used to relate rainfall to measured sensor temperatures. Chiu et al. found the bias to range from 1.25 to 1.5 depending on the type of rainfall. The bias estimates provided by the method of Chiu et al. (1993) do not vary in time. However, the bias estimates provided by the method of Krajewski et al. (1991) do vary throughout the duration of a storm.

Graves et al. (1993) used a rainfall model by Waymire et al. (1984) to evaluate satellite measured rainfall. They concluded that the model "is useful for estimating sampling errors over regions and periods when sufficient records do not exist for conventional techniques."

2.2 Rainfall Models

2.2.1 The Rainfall Generator Used in This Study

Eagleson et al. (1987) noted that rainfall models fall into two broad categories: 1) those that are deterministic in nature and 2) those which rely on statistics and which provide generalized space-time descriptions of a rainfall regime.
The model used in this work is of the second type. It has been through several evolutionary steps beginning with Rodriguez-Iturbe et al. (1986) then Rodriguez-Iturbe and Eagleson (1987) and finally the coded form used in this thesis was written by Krajewski et al. (1993). Rodriguez-Iturbe et al. (1986) began with a model of total depth for a stationary storm which Rodriguez-Iturbe and Eagleson (1987) modified to provide a description of the decay of the storm in time. Krajewski et al. (1993) made further modifications so that the rainstorm grows and then decays in time. The model creates clusters of rain cells according to a Neyman-Scott process; the clusters then move across the region of interest. The cells around the cluster center are born at different times, they grow to a peak intensity and then decay. A more complete description of the model is provided in the Methods section.

2.2.2 Parameterizing Rainfall Models

Smith and Karr (1985) discussed the use of maximum likelihood and method of moments to find parameters for a stochastic rainfall model. The authors concluded that maximum likelihood estimation becomes intractable quickly and they prefer the method of moments. Smith and Karr estimated the mean cell radius from previous studies. Then they used analytic expressions for the distribution moments and a 2 step iterative procedure to estimate the remaining parameters. In the first iteration the authors estimated the spatial parameters from the mean and variance of the measured rainfall depths while holding the temporal parameters at their values
from the previous iteration. In the second step they estimated the temporal parameters from the sequence of wet and dry days holding the spatial parameters at the values obtained in the first step. Smith and Karr pointed out that if the minimum distance between gauges is greater than the storm cell diameter the cell radius parameter may not be identifiable.

On the other hand, Valdes et al. (1985) concluded that the method of moments using analytic expressions is not adequate for finding all the parameters of the Waymire et al. (1984) model. The Waymire et al. model is also a stochastic rainfall generator. Valdes et al. (1985) used theoretical expressions for the mean, variance, and autocorrelation of the rainfall intensity process to estimate three of the model parameters. They found that the modelled autocorrelation function was insensitive to all but three of the model parameters which led them to conclude that alternative methods were required for finding the remaining parameters.

Eagleson et al. (1987) used the Rodriguez-Iturbe et al. (1986) total storm depth model to analyze daily precipitation at Walnut Gulch. Eagleson et al. fit the models to daily rainfall at Walnut Gulch using analytic expressions for the mean, variance and the correlation function of the model. They found a satisfactory fit to the daily rainfall. The authors validated their efforts by comparing observed and modelled fractions of the catchment area that were covered with a storm depth less than or equal to depths varying from 0 to 50 mm.

Islam et al. (1988) explored the parameterization problem using the intensity model of Waymire et al. (1984) and Walnut Gulch rainfall. Islam et al. (1988)
pointed out that models hold promise for aggregating and disaggregating rainfall. Islam et al. determined three of the parameters from the physical nature of the rainfall -- the velocity of the storms $U=20$ km/hr, the cell diameter $D=2.0$ km and the parameter controlling the spread of cells around a cluster, $\sigma = 3.5$. The authors found the other six by solving analytic expressions for various moments using the Davidon-Fletcher-Powell automatic search algorithm.

Jacobs et al. (1988) parameterized the Rodriguez-Iturbe (1987) model of storm intensities for Walnut Gulch rainfall. Interestingly, Jacobs et al. assumed that the velocity of the storms is zero, $U=0$ km/hr, while Islam et al. (1988) assumed $U=20$ km/hr. Jacobs et al. used analytic expressions for the moments of the model and fit these to the sample moments from the rainfall time series. Jacobs et al. found that the model did not recreate all the complexity of the spatial correlation of the rainfall. The authors noted that improving the model will require more parameters and more complex analytic expressions for the model moments. This thesis will address the issues of rainfall sampling and stochastic rainfall model calibration using the methods described in the next chapter.
CHAPTER THREE

METHODS

3.1 Introduction

As noted earlier, the goals of this study are 1) to characterize the differences between gauge measured and areally measured rainfall; 2) to develop relationships between the statistics of gauge measured rainfall and the statistics of areally measured rainfall; and 3) to demonstrate how these characteristics and relationships may be used to evaluate the quality of areal rainfall measurements. The method consists of using a rainfall generator to provide information about rainfall fields on a finer mesh than is usually available. From this fine mesh areal averages of the rainstorm were computed and point measurements taken. These point and areal measurements were then compared.

The steps taken to complete this study are the following: 1) find optimal parameters for the rainfall model using an automatic search algorithm; 2) generate storms using those optimal parameters; 3) sample the storms with a very fine network of points, with a sparse network of points and with areal averages computed from the fine network; 4) interpolate the sparse point to areal averages using kriging; 5) calculate sample statistics from the different sampling techniques for all the storms; 6) determine relationships between the model generated point and areal statistics; and 7) apply the modelled relationships to Walnut Gulch.
3.2 Description of the Stochastic Rainfall Generator

Before describing the method used in this work, the model and the automatic search routine are presented. The model generates rain within a user defined Large Meso-Scale Area (LMSA). Rain storms are clusters of rain cells that grow and decay in time and that move through the LMSA.

To create rain, the model begins by generating the number of cluster centers from a Poisson distribution with the model parameter $\rho$ indicating the average number of clusters/km$^2$. The initial coordinates for the cluster locations are generated from uniform distributions.

After generating the cluster centers, the model generates the number of cells around each cluster, again from a Poisson distribution. The model parameter $\lambda$ is the average number of cells/cluster. The initial locations of the cells around the cluster center are generated from a multivariate-normal distribution. The user provides a 2X2 variance-covariance matrix, called $cov$, that controls the initial locations of the cell centers. Each cell is then assigned a maximum intensity at its center from an exponential distribution with the mean intensity provided by the model parameter $\gamma$. A time of birth for each cell is then assigned also from an exponential distribution with the model parameter $\beta$ providing a mean time of birth. The cells are then assigned a time to peak from a uniform distribution that is scaled by the model parameter TMAX, the maximum time to peak. The velocity vectors for each cell are assigned from lognormal distributions with the mean velocities
provided by u1m and u2m and with the standard deviations of the velocities provided by u1s and u2s.

The intensity at any point in a cell is a function of both the cell's age and the distance from the center of the cell. The cell first grows to its peak intensity and then decays exponentially in time, with the model parameter \( \alpha \) controlling the speed of the decay. The cell also decays exponentially in space with the parameter \( D \) controlling the size of the cells. To compute the intensity at a point, the intensities of all the cells overlying that point are combined and a lognormal jitter is added to the rainfall to make the rainfall patterns more random. The model parameters are listed in Table 3.1.

### 3.3 Description of the Optimization Routine

The Shuffled Complex Evolution, SCE-UA Duan et al. (1992), is a random search algorithm that searches an error function surface for the minimum. The SCE-UA searches a feasible parameter space that is defined by the user as possible ranges for the parameters.

To begin, the feasible parameter space is randomly sampled. This random sample is then divided into complexes of \( 2n+1 \) points, where \( n \) is the number of parameters. Each complex then moves independently along the surface according to the competitive complex evolution (CCE) algorithm for a user specified number of steps.
<table>
<thead>
<tr>
<th>NAME</th>
<th>DISTRIBUTION</th>
<th>EXPLANATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Poisson</td>
<td>Intensity of cluster occurrence in space (clusters/km(^2))</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Poisson</td>
<td>Intensity of cell occurrence in a cluster (cells/cluster)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Exponential</td>
<td>Mean rainfall intensity at the center of a cell (mm/min)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Exponential</td>
<td>Parameter controlling the birth process in time (cells/min)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td></td>
<td>Decay parameter of spread function in time (min(^{-1}))</td>
</tr>
<tr>
<td>( \text{cov} )</td>
<td>Multivariate Normal</td>
<td>2x2 variance covariance matrix controlling location of cells around a cluster center.</td>
</tr>
<tr>
<td>( x_{\text{dom}}, y_{\text{dom}} )</td>
<td></td>
<td>Dimensions of large mesoscale area (km)</td>
</tr>
<tr>
<td>( D )</td>
<td></td>
<td>Cell radius (km)</td>
</tr>
<tr>
<td>( t_{\text{pmax}} )</td>
<td>Uniform</td>
<td>Maximum time for cells to reach their full maturity (min)</td>
</tr>
<tr>
<td>( u_{1m}, u_{2m} )</td>
<td>Lognormal</td>
<td>The mean velocity vector components (km/hr)</td>
</tr>
<tr>
<td>( u_{1s}, u_{2s} )</td>
<td>Lognormal</td>
<td>The standard deviation of the velocity vector components (km/hr)</td>
</tr>
<tr>
<td>( \text{stdjit} )</td>
<td>Lognormal</td>
<td>Standard deviation of the lognormal jitter process</td>
</tr>
<tr>
<td>( dx, dy )</td>
<td></td>
<td>Mesh size of grid (km)</td>
</tr>
<tr>
<td>( t_{\text{max}} )</td>
<td></td>
<td>Maximum time of interest (min)</td>
</tr>
<tr>
<td>( \text{deltim} )</td>
<td></td>
<td>time step (min)</td>
</tr>
</tbody>
</table>

Table 3.1 Parameter Names and Functions
The CCE algorithm is used to evolve each complex independently. From each complex a simplex of $n+1$ points is selected. The points are selected according to a triangular probability distribution so that the points with the lowest error function value are the most likely to be selected. After the simplex moves along the error surface for a user specified number of iterations, the points generated from the simplex rejoin the unselected points from the complex. A new simplex is then selected as before, the new simplex moves along the error surface, and so on for a user specified number of iterations.

After evolving independently the points in the individual complexes are shuffled together and repartitioned into new complexes. These new complexes then evolve and are shuffled and new complexes evolve until the convergence criteria are met.

3.4 Kriging Points to Areas

As one part of this study areal values interpolated from point values were compared to the various other description techniques. Estimates of areal rainfall values were made with the block kriging method. Block kriging was selected because of its wide popularity and the general acclaim of the methods greatness, the comments of some (Davis, D., personal communication, 1993) notwithstanding. The method of block kriging is well described in the literature (see Isaaks and Srivastava 1989) and only the points specific to this study will be mentioned here.
Kriging is an interpolation technique that utilizes information about the structure of the rainfall field expressed as a semi-variogram, covariance function or correlation function. The correlation function used here is from Michaud (1992), and is

\[ \rho = \exp(-\text{DIST}/4.61) \]

where DIST is in km. In Michaud (1992), this correlation function is shown to be very similar to one found by Eagleson et al. (1987) for daily Walnut Gulch data.

Block estimates were found using sixteen points within each 4km X 4km block. Only those gauges within a 7.3 km window from the center of the block were used to calculate the block averages. At 7.3 km the correlation is reduced to 0.20.

3.5 The Walnut Gulch Data

For this study the model was parameterized to simulate extreme storms on the ARS Walnut Gulch watershed. Walnut Gulch covers an area of 150 km² with a length of 25 km and a width of 10 km.

Nine storms were used in the parameterization. The rainstorms being used are nine extreme events that occurred over the Walnut Gulch experimental watershed. They are storms v1, v3, v4, v5, v7, v11, v12, v13, and v17 that occurred on the following days: 8/17/57, 8/12/72, 9/10/67, 7/22/64, 8/10/71, 8/14/58, 7/27/73, 9/26/77, 8/25/63. These events are considered the largest because they produced large peak flows at the outlet of the watershed.
Although there are 96 gauges on Walnut Gulch, 58 gauges were used for this analysis. The same 58 gauges were used for this work as in Michaud (1992) and Michaud and Sorooshian (in press, 1994). The 58 gauges provide a uniform coverage of the watershed and most of the gauges are operational for each storm. At least 56 gauges measure each storm except the storm v1. The storm v1 occurred prior to the completion of the network installation. Only 30 gauges are used to measure storm v1.

The gauge density with 96 gauges is 1 gauge per 1.6 km$^2$ with an average distance between nearby gauges of 1.3 km. However, with only 55 gauges the gauge density becomes 1 gauge per 2.7 km$^2$ with an average distance between gauges of 1.6 km. With only thirty gauges the gauge density is 1 gauge per 5.0 km$^2$ and the average distance between gauges is 2.4 km. The rainfall timestep used for this study was 12 minutes.

3.6 Model Calibration

3.6.1 Introduction

Because several factors characterize a rainstorm, finding optimal parameters for a rainfall generator is a multi-objective optimization problem. The approach taken here is to transform the multi-objective problem into a single-objective problem and to then use an automatic random search on the single objective reformulation. The random search technique by Duan et al. (1992), the SCE-UA
1) Find Alternatives

Select storm characteristics to match.

Form error functions to evaluate those characteristics based on statistics of actual storms.

Select a trial set of parameters $\Theta'$.

Generate numerous storms with $\Theta'$.

Calculate values of statistics from modelled storms.

Compare to actual storms using means and standard deviations of sequence of generated statistics.

Determine if difference from actual is ok.

no

yes

$\Theta' \Rightarrow \Theta_i$

$\Theta_i$ is an alternative.

2) Select the Best $\Theta_i$

Examine parameters for groupings into ranges.

Evaluate distance of alternative from (0, 0, ..., 0).

Select optimal parameter set.

These steps are completed by the SCE-UA routine.

Figure 3.1  The Method for the Parameter Search
routine, was used for the parameterization. The calibration method is described in
detail below and a flow chart is provided in Figure 3.1.

3.6.2 Calibration Technique

Because the model being calibrated is a stochastic one, matching individual
modelled events to individual actual events is not possible. It is possible however,
to compare the modelled intensity distributions to the measured intensity
distributions. Unfortunately, the measurements of actual events do not make a
sample distribution to which the modelled events may be compared. The intensity
measurements cannot be combined, because although the rain events are
independent the measurements within the events are not.

The approach taken here is to choose certain storm characteristics that are of
interest and to use statistics computed from the intensity measurements to describe
those characteristics. The characteristic statistics sampled from a series of storms are
independent samples and form a distribution. The distribution of statistic values
calculated from the actual events may then be compared to a similar distribution
calculated from modelled storms. The modeled and actual distributions are
compared using means and standard deviations.

In other words, as before $G$ is the value for some statistic calculated from a
rainstorm measured with gauges. A group of storms produces a group of statistics,

$$\{G_i\} \ i=1 \ to \ NS$$

where $NS$ is the number of storms. Because the $G_i$ are independent, they may
constitute a sample distribution F(G). The same may be done with a group of modelled storms from which statistics MG\textsubscript{i} are derived,

\{MG\textsubscript{i}\} \textit{i=1 to NMS}.

\textit{NMS} is the number of modelled storms and the number of MG\textsubscript{i} in the sample distribution F(MG).

For some statistic of interest, F(G) is compared with F(MG) using means and standard deviations. The mean and standard deviation of G may be found with,

\[
\overline{\Gamma} = \frac{1}{N} \sum_{i=1}^{N} \Gamma_i \\
\sigma_{\Gamma} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (\Gamma_i - \overline{\Gamma})^2} 
\]

\( \Gamma = G \text{ or } MG \text{ and } N = NS \text{ or } NMS. \)

An error function for a single statistic is

\[
f = \left( \frac{\Pi_{\text{model}}}{\Pi_{\text{actual}}} - 1 \right)^2
\]

where \( \Pi_{\text{actual}} = (\overline{\Gamma} \text{ or } \sigma_{\Gamma})_{\text{actual}} \text{ and } \Pi_{\text{model}} = (\overline{\Gamma} \text{ or } \sigma_{\Gamma})_{\text{model}} \)

The equation is normalized by \( \Pi_{\text{actual}} \) to facilitate the multi to single objective transformation described in the next section.

3.6.3 Multi to Single Objective Transformation

It is desirable to include several rainfall characteristics in the calibration
process. For each characteristic of interest (storm length, maximum intensity and so on) error functions as described above may be formed. Therefore, there are $nf$ error functions called $f_i$, each of which evaluates a particular aspect of a rainstorm. The error functions, $f_i$, are combined as a weighted sum to form the error function, $F$. This summation is written as

$$F = \sum_{i=1}^{nf} a f_i$$

(3.3)

The weights could be used to emphasize some aspect of a storm that is of particular interest to the model user. This single objective function is then minimized using the SCE-UA automatic search routine.

3.6.4 Selection of Parameters

If several automatic searches are done, possibly with different combinations of weights or starting seeds there is a set of optimal parameters,

$$\{\theta_i\} \text{ i}=1 \text{ to } nas$$

where $nas$ is the number of automatic searches. The $\theta_i$ are called alternatives.

To select the best $\theta$, two methods were used. First, if a particular parameter tends to lodge in a range for a majority of the parameter searches, then it was assumed that the best parameter value lay in that range.

Second, for each alternative $\theta$, there is an associated set of error function, $f_i$, values. The ideal point in the error space is $(0,0,...0)$, and it is there that one would like to find one set of $f_i$ located. No set of $f_i$ was located at $(0,0,...0)$ so the minimum
3.7 Storm Generation

3.7.1 Introduction

After estimating the model parameters, generating storms is a straight-forward process of running the model. There are two matters to consider prior to the model runs: selecting raingauges and selecting blocks.

3.7.2 Selecting Point Locations for the Modelled Storms

While the particular gauge locations used to measure a storm do not affect the rainfall field, the network geometry may affect the description of the field provided by the gauges. In order to preserve the influence a particular gauge configuration has had on a real storm, the locations, $\mathbf{X}_{Mg}$, used to measure the modelled storms were selected from the locations $\mathbf{X}_g$ used to measure the real storms.

To select point measurement locations for the modelled storms one of the gauge location vectors used to measure a real storm is randomly paired with a simulated storm. For the sequence of real storms there is a set of gauge locations $\{\mathbf{X}^i_g\} i=1$ to $NS$.

For a modelled storm the point measurement locations used to measure the modelled storm, $\mathbf{X}_{Mg}$ are

$$\mathbf{X}_{Mg} = \mathbf{X}^n_g,$$
This selection process is also used in the calibration.

Several other methods exist for selecting $X_{M_g}$: 1) using all the Walnut Gulch gauge locations to measure the modelled storm 2) generating a random selection of the Walnut Gulch gauge locations and 3) generating random points in the region of interest to serve as point measurement locations. These methods were rejected because they do not preserve the peculiarities of the gauge measurements in the real storms. Option one does not preserve the peculiarities of the gauge measurements because not all the Walnut Gulch gauges were used to measure the actual storms.

### 3.7.3 Selecting Blocks

There are no predetermined sampling blocks in the real storms, so there is nothing to match. Therefore, a grid was arbitrarily placed on the watershed and it was used for the modelled areal averages. This same grid was used later in the comparison phase.

### 3.8 Calculating Statistics

The following statistics were used to characterize the differences between the areal and point intensity measurements: the sample median, the sample 90th percentile, and the sample maximum. The statistics are taken across area and time for a given storm.
The sample median is

\[
\hat{X} = \begin{cases} 
X_{(n+1)/2} & \text{if } n \text{ is odd} \\
\frac{X_{n/2} + X_{(n/2)+1}}{2} & \text{if } n \text{ is even}
\end{cases}
\]

where \( X \) is arranged in increasing order.

The sample 90\(^{th} \) percentile is similar to the median.

\[
X_{90} = \text{int}(n \cdot 0.9)
\]

where \( \text{int} \) means to take only the integer part of the product
and \( X \) is arranged in increasing order.

The maximum is

\[
X_{\text{max}} = \max_{i=1}^{n}(X_i)
\]

The median was selected because it is a standard measure of rainfall distributions. The 90\(^{th} \) percentile and the maximum were selected because in the semi-arid flow regime of Walnut Gulch it is the high intensity rainfall that produces runoff.

In addition, the median percent wet area taken across time and the median duration at a gauge taken across the gauges were used to provide information about the storm area and persistence. The median duration at a gauge for a storm is found by counting the number of timesteps that each gauge measures rain. This was a
measure of the local persistence of the rainfall. At the end of each storm the median duration is found.

The median percent wet area is found by counting the number of gauges measuring rain in each timestep. This is a measure of the areal extent of the rainstorm. The median of these gauge counts is taken at the end of the storm. Because the number of gauges changes from storm to storm the median is then divided by the number of gauges so that the wet area is expressed as a fraction of the total.

3.9 Making the Comparisons

3.9.1 Comparing Different Sampling Schemes

The procedure for comparing the different sampling schemes is straightforward. First, the model is run and the generated fields are sampled with a very fine mesh of 100m. If each point in the mesh is considered a gauge then statistics calculated from the fine mesh are called $MG_{\text{fine}}$. Areal averages are computed from the fine mesh by taking the arithmetic average of the points within a grid area. In this work the grid area is a 4X4 km square. From these areal averages statistics $MA_{\text{fine}}$ may be calculated. Because the 100m mesh is so fine it is considered to sample the rainfall well enough to make $MG_{\text{fine}} = MG_{\text{true}}$ and $MA_{\text{fine}} = MA_{\text{true}}$.

The modelled rainfall is also sampled with the sparser gauge networks found on Walnut Gulch. This sparser sampling leads to statistics $MG_{\text{sparse}}$. Interpolating the sparse gauge network to areas with block kriging leads to statistics $MA_{\text{interpolated}}$. 
The following comparisons are made to evaluate the error terms listed in chapter one. To characterize the differences between areal and point measurements,

\[ \epsilon_{\text{integration}} \] compare \( MA_{\text{fine}} \) to \( MG_{\text{fine}} \).

To evaluate the errors associated with incomplete gauge sampling,

\[ \epsilon_{\text{sample}} \] compare \( MG_{\text{fine}} \) to \( MG_{\text{sparse}} \).

To evaluate the errors associated the interpolation of sparse data,

\[ \epsilon_{\text{interpolation}} \] compare \( MA_{\text{fine}} \) and \( MA_{\text{interpolated}} \).

3.9.2 Method for Evaluating Areal Measurements

Evaluating the quality of areal rainfall measurements can be done by developing a relationship between the modelled statistics \( MA_{\text{fine}} \) and \( MG_{\text{sparse}} \). This modelled relationship is then used to predict the \( A_{\text{true}} \) value for an actual storm from the actual measured \( G_{\text{sparse}} \) statistic. The assumption is that the modelled storms recreate the rainfall process on the actual watershed well enough for the modelled \( MA_{\text{fine}}-MG_{\text{sparse}} \) relationship to be true for the actual \( MA_{\text{true}}-MG_{\text{sparse}} \) relationship.

The modelled \( MA_{\text{fine}}-MG_{\text{sparse}} \) relationship is determined by plotting \( MA_{\text{fine}} \) vs \( MG_{\text{sparse}} \) and fitting a line to the plotted points.

3.9.3 Validating the Evaluation Method

The soundness of this evaluation technique was examined by computing the percent difference between the modelled \( MA_{\text{fine}} \) values and the predicted \( MA_{\text{fine}} \)
values. The percent difference was calculated as

$$\text{DIFF} = \frac{(\text{MA}_{\text{predicted}} - \text{MA}_{\text{output}})}{\text{MA}_{\text{output}}}$$

\(\text{MA}_{\text{output}}\) refers to the model output \(\text{MA}_{\text{fine}}\). The coefficient of determination, \(r^2\) was also computed.

In order to test the applicability of modelled relationships to actual data, the method was used to predict actual kriged statistics. Rather than developing a relationship between the modelled \(\text{MA}_{\text{fine}}\) and \(\text{MG}_{\text{sparse}}\) statistics, a relationship between the modelled \(\text{MA}_{\text{interpolated}}\) and \(\text{MG}_{\text{sparse}}\) was developed. The modelled \(\text{MA}_{\text{interpolated}}\) is computed from modelled gauge rainfall that was kriged to areas. The modelled relationship was then applied to the actual Walnut Gulch storms and the predicted \(\text{A}_{\text{interpolated}}\) was then compared to the actual \(\text{A}_{\text{interpolated}}\). The actual \(\text{A}_{\text{interpolated}}\) is computed from the gauge measured Walnut Gulch storms kriged to areas. The limitations of this test will be discussed in the results.
CHAPTER FOUR

RESULTS

4.1 Introduction

In addition to relating the results of this study, this chapter will provide information regarding computer codes and elaborate on some of the points from the section on Methods. The results of several optimization runs are presented and compared. Comparisons of several schemes for measuring the modelled intensities are also described. The results of an evaluation of the kriging method of interpolating gauges to areas are related.

4.2 Model Calibration

4.2.1 Rainfall Model SCE-UA Linkage

The rainfall model (Krajewski et al., 1993) and the SCE-UA optimal parameter search routine (Duan et al., 1992) were linked by making the model code a subroutine in the SCE-UA code. The error functions were also added as subroutines to the SCE-UA code. Minor modifications to the model were required to facilitate this link. These are explained in the following paragraphs.

The model generates storms within some large area. The size of the area for this study is 32km X 20km with the watershed in the middle of this area. This size was selected to reduce any edge effects on the watershed by providing a border around it.
In its original form the model evaluated the rainfall intensity at points on a regular grid within the area of interest. The mesh size selected for this study was 0.1 km. This size was selected so that the rainfall would be well sampled for the areal averages and so that raingauges could be arranged in the model space as they are arranged on Walnut Gulch. The large number of points in this mesh slowed the model to such an extent that using the SCE-UA routine became impractical. Subroutines were added to the model code so that rainfall was evaluated only at those points that are co-located with the gauge locations. One of the Walnut Gulch gauge location vectors was selected at random.

Subroutines were also added to impose temporal coherence on the storms. A storm begins when any gauge records rain and ends when no rain is recorded at any gauge for one twelve minute timestep.

4.2.2 Detailed Description of the Error Functions

The SCE-UA finds the minimum of some error function. For this study a normalized sum of the square errors was used for the error function. It is

\[ P = \sum_{i=1}^{nf} a_i \left( \frac{MG_i}{G_i} - 1 \right)^2 \]

with \( nf \) the number of rainfall characteristics,

\( a_i \) a user specified weight

\( MG \) referring to modelled statistics, and \( G \) referring to observed statistics.
The error function computation proceeded as follows. 200 or 500 storms are generated for a particular set of parameters. For each generated storm all six statistics (reviewed below) were computed and then the mean of each statistic was computed by averaging across all the storms. For example there might be say 500 median storm intensities generated for a single set of parameters. The mean of these 500 medians was then taken, and it was the mean that was used in the error function. This process was followed for each statistic. For the next set of parameters to be evaluated a different set of 500 storms was generated and so on.

The standard deviation of the maximum intensities was also used. It is computed as the standard deviation of the maximum intensities found over the ensemble of storms generated to evaluate a parameter set.

4.2.3 More Details About the Six Statistics

This section is a reminder of what has been written in section 3.8. The maximum intensity, the 90th percentile intensity, and the median intensity for a single storm are computed across time and space. The maximum is the maximum intensity measured at any gauge at any time step, and the same for the 90th percentile and the median intensities. Thus for any storm there is only one maximum, one 90th percentile and one median.

The median duration at a gauge for a storm is found by counting the number of timesteps that each gauge receives rain. This will be a measure of the local persistence of the rainfall. At the end of each storm the median duration is found.
The median fraction wet area is found by counting the number of gauges measuring rain in each timestep. This is a measure of the areal extent of the rainstorm. The median of these gauge counts is taken at the end of the storm. Because the number of gauges changes from storm to storm the median is then divided by the number of gauges measuring the storm so that the wet area is expressed as a fraction of the total.

4.2.4 Parameter Searches - General Information

In the first parameter searches, several of the model parameters were fixed. Initial tests of the model showed it to be relatively insensitive to the parameter $cov$. $cov$ is a variance-covariance matrix that controls the location of the births of cells around a cluster center. The initial values for $cov$ were extracted from an example input file. The velocity vectors were also fixed. $u1m$ and $u2m$ were set to 6km/hr because that was considered a reasonable average speed of Walnut Gulch storms (Michaud, 1993, personal communication). The parameters $u1s$ and $u2s$ are the standard deviation of the velocities. In early tests of the model they appeared to have little effect on the model output so they were fixed. The $stdjit$ parameter controls the jitter process. The jitter adds a little random noise to the rainfall fields. It was found that values of $stdjit$ that were higher than 0.01 made the model output very erratic. Lower values made the rainfall very smooth so $stdjit$ was fixed at 0.01.

After a number of searches it was noticed that the wet area statistic was consistently too high. It was thought that the variance covariance matrix, $cov$, might
contribute to the control of this characteristic of the rainfall so runs permitting the 
cov parameter to vary were included. As a part of the cell location generation 
procedure the cov matrix is inverted. A check for invertability was included in the 
search routine; if cov is singular the error function is set to 100 and the model is not 
run to generate an error function value.

The parameter ranges were selected on the basis of many exploratory runs of 
the model. They were made broader than the initial examination indicated to avoid 
having the ranges adversely affect the parameter searches. The final parameters lie 
near the center of the ranges. Table 4.1 lists the model parameters and their values 
if they were fixed and the search range if they were not fixed. These are the ranges 
used for the final parameter searches. The initial parameter searches had slightly 
different ranges.

4.2.5 Parameter Searches -- Specific Discussion

The following discussion will refer to Tables 4.2, 4.3, and 4.4. These tables 
list the final results of 8 parameter searches -- Table 4.2 lists information regarding 
the error functions, Table 4.3 lists the parameter values, and Table 4.4 lists the 
statistics generated by the parameter sets. The column of Table 4.2 titled "SSQ" is 
just a calculation of the unweighted error function using the statistics in Table 4.4. 
The column titled "SSQ*" is the same calculation excluding the mean and standard 
deviceation of the maximum intensities. The "SSQ*" column was included because the
<table>
<thead>
<tr>
<th>NAME</th>
<th>VALUE or RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Not Used</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>40.0 - 100.0 cells/cluster</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.1 - 0.4 mm/min</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.001 - 0.4 cells/min</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.001 - 0.02 min(^{-1})</td>
</tr>
<tr>
<td>VAR(1)</td>
<td>VAR(2)</td>
</tr>
<tr>
<td>D</td>
<td>1.0 - 6.0 km</td>
</tr>
<tr>
<td>t( \text{pmax} )</td>
<td>10.0 - 100.0 min</td>
</tr>
<tr>
<td>u1m u2m</td>
<td>6.0 6.0 km/hr</td>
</tr>
<tr>
<td>u1s u2s</td>
<td>0.1 0.1 km/hr</td>
</tr>
<tr>
<td>stdjit</td>
<td>0.01</td>
</tr>
<tr>
<td>x( \text{dom} ) y( \text{dom} )</td>
<td>32.0 20.0</td>
</tr>
<tr>
<td>dx dy</td>
<td>0.1 0.1 km</td>
</tr>
<tr>
<td>t( \text{max} )</td>
<td>400 min</td>
</tr>
<tr>
<td>delt( \text{im} )</td>
<td>12 min</td>
</tr>
</tbody>
</table>

Table 4.1 Parameter Names and Values or Ranges
## ERROR FUNCTION VALUES

<table>
<thead>
<tr>
<th>PAR-SET</th>
<th>WEIGHTS</th>
<th>NUMRUN</th>
<th>SSQ</th>
<th>SSQ*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>all weights = 1</td>
<td>200</td>
<td>0.9523</td>
<td>0.3032</td>
</tr>
<tr>
<td>2</td>
<td>all weights = 1</td>
<td>200</td>
<td>0.6160</td>
<td>0.0741</td>
</tr>
<tr>
<td>3</td>
<td>all weights = 1</td>
<td>200</td>
<td>0.6083</td>
<td>0.0752</td>
</tr>
<tr>
<td>4</td>
<td>$a_{\text{max}}=2$, $\sigma_{\text{max}}=2$</td>
<td>200</td>
<td>0.6995</td>
<td>0.1290</td>
</tr>
<tr>
<td>5</td>
<td>$a_{\text{max}}=2$, $\sigma_{\text{max}}=2$</td>
<td>500</td>
<td>0.8068</td>
<td>0.2262</td>
</tr>
<tr>
<td>6</td>
<td>$\sigma_{\text{max}}=2$</td>
<td>200</td>
<td>0.6184</td>
<td>0.0914</td>
</tr>
<tr>
<td>7</td>
<td>$\sigma_{\text{max}}=2$</td>
<td>200</td>
<td>0.9270</td>
<td>0.3441</td>
</tr>
<tr>
<td>8</td>
<td>all weights = 1</td>
<td>200</td>
<td>2.8140</td>
<td>0.0670</td>
</tr>
</tbody>
</table>

**Notes:**
- Set 1 has fixed values for the variance covariance matrix, $cov$.
- Sets 2 and 3 have different seeds for the search routine.
- Set 7 had a limited range for the cell diameters.
- Set 8 did not include the maximum intensity in the error function.

**Table 4.2** Results of Eight Parameter Searches
### MODEL PARAMETERS

<table>
<thead>
<tr>
<th>PAR-SET</th>
<th>$\lambda$</th>
<th>$\gamma$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$D$</th>
<th>TMAX</th>
<th>VAR(1)</th>
<th>VAR(2)</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.899</td>
<td>2.783</td>
<td>0.163</td>
<td>0.032</td>
<td>4.156</td>
<td>64.051</td>
<td>159.92</td>
<td>0.92</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>65.993</td>
<td>1.170</td>
<td>0.342</td>
<td>0.006</td>
<td>3.379</td>
<td>68.654</td>
<td>102.77</td>
<td>54.36</td>
<td>51.44</td>
</tr>
<tr>
<td>3</td>
<td>70.362</td>
<td>1.138</td>
<td>0.245</td>
<td>0.010</td>
<td>3.956</td>
<td>58.808</td>
<td>142.13</td>
<td>82.25</td>
<td>-93.94</td>
</tr>
<tr>
<td>4</td>
<td>18.005</td>
<td>1.604</td>
<td>0.210</td>
<td>0.005</td>
<td>4.326</td>
<td>82.598</td>
<td>159.92</td>
<td>0.92</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>19.043</td>
<td>1.663</td>
<td>0.126</td>
<td>0.034</td>
<td>4.480</td>
<td>65.489</td>
<td>159.92</td>
<td>0.92</td>
<td>2.0</td>
</tr>
<tr>
<td>6</td>
<td>57.508</td>
<td>1.026</td>
<td>0.232</td>
<td>0.006</td>
<td>4.024</td>
<td>33.288</td>
<td>143.46</td>
<td>2.48</td>
<td>-7.37</td>
</tr>
<tr>
<td>7</td>
<td>55.484</td>
<td>1.693</td>
<td>0.251</td>
<td>0.037</td>
<td>1.998</td>
<td>98.838</td>
<td>173.42</td>
<td>154.89</td>
<td>158.23</td>
</tr>
<tr>
<td>8</td>
<td>78.102</td>
<td>2.078</td>
<td>0.299</td>
<td>0.017</td>
<td>2.037</td>
<td>55.193</td>
<td>115.18</td>
<td>59.47</td>
<td>-30.06</td>
</tr>
</tbody>
</table>

Notes: $\lambda$ in cells/cluster; $\gamma$ in mm/min; $\beta$ in cells/min; $\alpha$ in min$^{-1}$; $D$ in km; TMAX in min

Table 4.3 Eight Optimal Parameter Sets

### MODELLED STATISTICS

<table>
<thead>
<tr>
<th>PAR-SET</th>
<th>TIME-STEP</th>
<th>MED-INTENS</th>
<th>90%-INTENS</th>
<th>MAX-INTENS</th>
<th>STD-INTENS</th>
<th>MED-DUR</th>
<th>WET-AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACTUAL</td>
<td>20.3</td>
<td>0.56</td>
<td>15.5</td>
<td>129.4</td>
<td>32.0</td>
<td>10.2</td>
<td>0.48</td>
</tr>
<tr>
<td>1</td>
<td>16.9</td>
<td>0.48</td>
<td>9.0</td>
<td>33.0</td>
<td>48.4</td>
<td>10.1</td>
<td>0.62</td>
</tr>
<tr>
<td>2</td>
<td>17.6</td>
<td>0.57</td>
<td>14.8</td>
<td>35.6</td>
<td>41.8</td>
<td>10.4</td>
<td>0.59</td>
</tr>
<tr>
<td>3</td>
<td>17.6</td>
<td>0.50</td>
<td>16.3</td>
<td>37.7</td>
<td>43.5</td>
<td>10.3</td>
<td>0.54</td>
</tr>
<tr>
<td>4</td>
<td>17.2</td>
<td>0.45</td>
<td>13.2</td>
<td>35.4</td>
<td>44.7</td>
<td>10.9</td>
<td>0.58</td>
</tr>
<tr>
<td>5</td>
<td>16.2</td>
<td>0.51</td>
<td>10.8</td>
<td>34.6</td>
<td>44.8</td>
<td>10.3</td>
<td>0.63</td>
</tr>
<tr>
<td>6</td>
<td>16.8</td>
<td>0.56</td>
<td>14.8</td>
<td>36.7</td>
<td>41.4</td>
<td>10.7</td>
<td>0.59</td>
</tr>
<tr>
<td>7</td>
<td>16.1</td>
<td>0.52</td>
<td>7.4</td>
<td>34.4</td>
<td>44.8</td>
<td>10.0</td>
<td>0.54</td>
</tr>
<tr>
<td>8</td>
<td>17.8</td>
<td>0.53</td>
<td>16.3</td>
<td>75.0</td>
<td>96.4</td>
<td>10.5</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Notes: Intensities in mm/hr; Med-Dur and Storm Length in min; Median Wet Area, fraction of the total. Actual refers to observed Walnut Gulch Storms

Table 4.4 Statistics Computed for the Eight Parameter Sets
maximum was always badly reproduced so the SSQ* values permit evaluation of the model parameters without the maximum.

A comparison of the values presented in Tables 4.2, 4.3, and 4.4 demonstrates the effects on the parameter searches of 1) excluding the maximum intensity statistic from the error function, 2) the matrix cov, and 3) the number of model runs, numrun. The comparison will also be used to evaluate 4) the convergence of the SCE-UA routine by using a different seed to initiate the search routine, and 5) the possibility of modelling the storm spatial variability by limiting the size of the cell diameter, $D$. Finally a comparison may 6) provide some information about the characteristics of Walnut Gulch rainfall.

The maximum intensity for a storm is the statistic which is most poorly reproduced. The magnitude of the error caused by the maximum intensity is always at least twice the size of the error caused by all the other statistics combined. This fact may be observed by comparing the SSQ and SSQ* values. The error function for parameter set 8 did not include the maximum intensity. Searching without the maximum does not appear to significantly improve the error for the other statistics. The SSQ* values for the searches 2 and 3 are not much different from the SSQ* value for the search number 8. It does not appear that excluding the maximum is very beneficial.

Not shown are parameter searches in which only one statistic is matched. These searches had very positive results, matching the single statistic value to within less than 1% of its value. The other statistics, however, were not close at all.
The supposition that the covariance matrix partially controls the wet area appears to be true. In parameter sets 2 and 3, in which \( \text{cov} \) was not fixed, the SSQ value is quite a bit lower than the SSQ value for set 1 in which \( \text{cov} \) was fixed. The wet area statistics are also much closer to the true value when \( \text{cov} \) is not fixed. There is a difference of 0.14 (30% of the actual wet area) between the modelled and actual wet areas with the fixed \( \text{cov} \), and only 0.06 (13%) with \( \text{cov} \) not fixed in parameter set 3.

Increasing the number of model runs from 200 to 500 seems to have a detrimental effect on the model output. Parameter sets 4 and 5 have the same error function just a different number of model runs per error function evaluation. As the number of model runs increase from 200 to 500, the SSQ value goes up from 0.69 to 0.80 and the SSQ* from 0.12 to 0.22. Whether or not this increase indicates a trend is not known i.e. would \( \text{numrun} = 1000 \) have even worse values. The statistics for parameter sets 4 and 5 in Table 4.4 make a similar indication. The reason for this behavior can only be that with more storms there is more variability in the model output. It is somewhat disconcerting to observe such a change in the model output. This effect implies that the model parameters cannot be used to generate more storms than \( \text{numrun} \), the number of storms with which the model was parameterized.

Parameter sets 2 and 3 use different seeds to initiate the search routine. Using different seeds to initiate the search routine is one way of evaluating the effectiveness of the SCE-UA search routine. If the searches are effective they will
converge to nearby points though begun with different seeds. The error function values are pleasingly close with a difference of less than 2%. An examination of the parameters themselves is more ambiguous. There are no hard and fast rules by which to determine if two points which are close are "close enough". One way of making this judgement is to calculate the distance between the two parameters as a percent of the range over which the parameters were optimized. Six of the parameters converge to within 15% of the range while three converge to within 25% of the range. These values indicate that the parameters are not unique and that there may be many local optima on the error surface. The non-uniqueness of the parameters is important when considering the problems associated with subjective as opposed to objective model calibration.

Another way of looking at the convergence is to plot the parameter values, normalized by the range, for all of the error function evaluations in the final loop of the SCE-UA routine. This plot is provided in Figure 4.1 and similar conclusions may be drawn from it. The $\gamma$ parameter is well defined as is the $\text{cov}$ parameter. One of the variances is better defined than the other and the remaining parameters are reasonably well defined. The discrete appearance of the $\alpha$ parameter is probably an artifact of the output format of the SCE-UA routine and is not a result of the shape of the error surface.

The correlations between the parameters of the final optimization loop were computed. The graph of the correlations is presented in Figure 4.2. As one would expect the $\lambda$ (cells/cluster) is negatively correlated with the TMAX (time to peak
Preferred Region at Final Loop

Figure 4.1 SCE-UA Output
Cross Correlations

Figure 4.2 Parameter Cross Correlations
intensity) parameter. If the model generates many cells they must live only a short
time, while few cells must live a long time to generate similar intensity statistics. The
$\beta$ (birth of cells in time) parameter is also negatively correlated with the $\text{TPMAX}$
parameter. If cells live a long time they must be born quickly so that they may die
before they cause the storm to be too long. The negative correlation between the
$\lambda$ parameter and the $\beta$ is less easy to explain. It may be that with many cells it is
necessary to spread the cell births in time so that there are always cells able to
generate rain. The parameters are in general not very correlated which is a positive
indication for the model.

The actual Walnut Gulch storms are highly variable in space while the
modelled storms have a much smoother appearance. An attempt to recreate the
spatial irregularities of the actual storms was done by limiting the size of $D$, the cell
diameter. The notion was that numerous small cells being born and dying at
different times might reproduce the spatial chaos of the real storms. This search is
parameter set 7 and it may be compared to parameter set 6 which is identical except
for the limitations on $D$. The SSQ and SSQ$^*$ values are very different; with the
limited $D$ the SSQ value is increased from 0.6 to 0.9. This change may be attributed
largely to a reduction in the high intensity rainfall, the 90th percentile intensity and
the maximum are both lower with the limited $D$. One explanation for this reduced
intensity is that the cells did not live long enough, the parameter $\text{TPMAX}$ is near the
top of its range. With a higher $\text{TPMAX}$ value, the model may generate higher
intensities. It would seem then that limiting the size of the D parameter may be one way of parameterizing the spatial variability of rainfall.

If the model parameters have any realistic physical meaning, then a physical description of Walnut Gulch rainstorms may be inferred from the parameter values. It would appear that cells within a storm have an average diameter of 4 km (D) and that cells within a storm have an average peak intensity of about 60-70 mm/hr (\( \gamma \)). It takes a cell less than an hour after it is born to reach maturity (TPMAX). After reaching its peak intensity, a cell has an average lifetime of 1.5 hours (\( \alpha \)). Cells tend to be born around a cluster center in an elliptical formation (VAR1, VAR2 and cov). About 12 cells are born every hour (\( \beta \)) until all the cells for a particular cluster are born. In addition the parameters imply that each cluster has on average about 70 cells (\( \lambda \)). The number of cells per cluster is the least physically understandable.

4.4.4 Parameter Selection

Parameter set 3 was selected as the final set according to the following criteria. First the parameters are examined to see if parameters are grouped into ranges. Not all the listed parameter sets can be considered in this comparison because some had either fixed cov (sets 1, 4, 5) or a limited parameter range (set 7). The parameters do show a tendency to group themselves, in spite of the fact that they have slightly different error functions. Parameter set 3 tends to fall toward the middle of the ranges.
The second criterion is the normalized distance from the point \((0,0,...0)\). The normalized distance is the same as the unweighted error function, the SSQ values. Again parameter set 3 appears to be the best. The SSQ* value is not used because the maximum intensity will be used in the latter analyses.

After selecting the parameters an independent test of the output was done. The ranges of the output statistics were examined. The number of storms that had all of the six statistics in the range of the actual Walnut Gulch statistics was found. With the maximum included only 1 of the 200 storms satisfied these criteria. However, without the maximum 65% of the storms satisfied these criteria. From this comparison, it appears that the model is reproducing the type of storm found on Walnut Gulch, except in the maximum intensities.

4.3 Storm Analysis

4.3.1. Introduction

The Walnut Gulch storms and the modelled storms were examined both visually and statistically.

4.3.2 Walnut Gulch Storm Analysis

The Walnut Gulch storms were analyzed to determine qualitative spatial and temporal characteristics. A computer program using the MATLAB software package and the GRASS GIS package was written to display the gauge measured storm intensities and the kriged areal intensities at each timestep. The program also
displays the cumulative frequency distribution curves of the gauge measured and interpolated areal intensities. The program displays an outline of the watershed and then the gauge measured rainfall intensities are overlaid as X's at the gauge locations. The X's are color coded to indicate intensity with a color table that begins at grey for zero rainfall then uses shades of green, blue and red. Only the gauges which recorded rainfall are displayed. The interpolated areal average intensities are displayed with a color table that uses light to dark shades of blue for low to high intensity. C language programs were used to bin the intensities into intervals of 10 mm/hr. The gauges were interpolated to areas using a C program that performs block kriging.

Upon viewing the maps of the gauge measured intensities for the Walnut Gulch storms, it becomes clear that the storms have different shapes and sizes and that they are travelling in different directions at different speeds. It also becomes clear that the rainfall fields are frequently highly disorganized -- the intensity patterns appear almost random. High intensities do not move smoothly across the watershed nor are they surrounded by rings of lower intensities. Rather the high intensities seem to simply appear haphazardly over the watershed irrespective of the general storm movement. This randomness may be difficult to model. As the storm decays, and the intensities lower, the fields do become smoother and less irregular. In addition, the highest intensities are very local and short lived usually recorded at only one or two gauges and for one or two time periods. Figure 4.3a is an example of a map of one timestep of actual gauge measured intensities.
Storm V5 on Walnut Gulch

Modelled Storm 103

Figure 4.3 Actual and modelled intensities for one timestep
Upon comparing the display of the gauge intensities with the displays of the areal intensities, it becomes clear that areal averaging smears the gauged measured intensities. If no limits are set on the kriging interpolation, that is the windowing method is not used, then large sections of the watershed are covered with the interpolated rain, where no gauge measured rain occurs. With the windowing method, the areal redistribution is less apparent and the main effect of the interpolation is to reduce the rainfall intensity. The frequency distribution plots show the same effect -- areal averaging reduces the storm intensity. Examples of the cumulative frequency distribution plots are provided in Figures 4.4a and 4.4b.

Plots of the individual timesteps in general show the same reduced intensities for the interpolated areas. The differences between the areal and gauge frequency plots are usually more dramatic at the shorter timesteps.

In order to make a more quantitative assessment of the differences between the gauge measured intensities and the kriged areal intensities, several Shell and AWK scripts were written. These programs compute the maximum intensity, the 90th percentile, the median, the median duration of rain measured in each gauge for a storm, the median number of gauges measuring rain at each timestep for a storm and the storm lengths. These programs operate on both the gauge measured and interpolated storm data.

Before computing the grid statistics, the grid squares to be included in the computation had to be determined. The maps of a typical gauge network and the
Figure 4.5 Gauge and Grid Configurations

a) Typical 58 Gauge Network on Walnut Gulch

b) 4 km Grid on Walnut Gulch
4 km grids are in Figures 4.5a and 4.5b. The grids were selected to cover the watershed as well as possible. The selected 4 km grids cover an area of 208 km².

The same phenomenon observed qualitatively with the maps is seen in these statistics, which are presented in Tables 4.5 and 4.6. The maximum intensity is lower for the areal averages than for the gauges, yet the median intensities are higher for the areas than for the gauges. The areal averaging smears the tails of the intensity distributions. The 90th percentile intensities of the gauges and areas tend to be close in value, indicating that the areal and gauge distributions cross in this region. The areal smearing seen in the maps is evidenced in the wet fraction and the median duration statistics. Both statistics are consistently larger for the interpolated areal intensities than for the gauge measured intensities.

4.3.3 The Modelled Storms

Displaying the modelled storms exposed a needed change in the rainfall generator parameters. The model produces numerous cluster centers, and then cells around the cluster centers. The result is several non-contiguous storms. The rainfall fields on Walnut Gulch are contiguous though they appear random. The parameter \( p \) was eliminated so that only one cluster center was generated for each model run. This modification produced the desired contiguous fields.

As expected, after a review of several of the modelled storms, it becomes obvious that the modelled storms do not reproduce the random character of the actual storms. The modelled storms have intensity patterns that are generally
### GAUGE STATISTICS FOR WALNUT GULCH STORMS

<table>
<thead>
<tr>
<th>STORM</th>
<th>MAX INTENS</th>
<th>90th% INTENS</th>
<th>MEDIAN INTENS</th>
<th>MED-DUR</th>
<th>MED-WET</th>
<th>STORM LENGTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>v1</td>
<td>164.6</td>
<td>16.7</td>
<td>0.7</td>
<td>168</td>
<td>0.80</td>
<td>228</td>
</tr>
<tr>
<td>v10</td>
<td>96.5</td>
<td>2.0</td>
<td>0.0</td>
<td>144</td>
<td>0.43</td>
<td>336</td>
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<tr>
<td>v12</td>
<td>102.0</td>
<td>19.0</td>
<td>0.4</td>
<td>108</td>
<td>0.75</td>
<td>180</td>
</tr>
<tr>
<td>v13</td>
<td>154.9</td>
<td>14.6</td>
<td>1.0</td>
<td>264</td>
<td>0.77</td>
<td>396</td>
</tr>
<tr>
<td>v17</td>
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<td>11.0</td>
<td>0.0</td>
<td>60</td>
<td>0.15</td>
<td>192</td>
</tr>
<tr>
<td>v3</td>
<td>154.9</td>
<td>22.4</td>
<td>2.8</td>
<td>156</td>
<td>0.98</td>
<td>192</td>
</tr>
<tr>
<td>v4</td>
<td>154.0</td>
<td>7.4</td>
<td>0.0</td>
<td>72</td>
<td>0.14</td>
<td>300</td>
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<tr>
<td>v5</td>
<td>162.3</td>
<td>26.7</td>
<td>0.0</td>
<td>48</td>
<td>0.35</td>
<td>120</td>
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<tr>
<td>v7</td>
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<td>0.0</td>
<td>84</td>
<td>0.14</td>
<td>252</td>
</tr>
<tr>
<td>Mean</td>
<td>129.40</td>
<td>15.50</td>
<td>0.56</td>
<td>122.4</td>
<td>0.48</td>
<td>243</td>
</tr>
</tbody>
</table>

Notes: Intensities in mm/hr; Med-Dur and Storm Length in min; Median Wet Area, fraction of the total.

Table 4.5 Gauge Measured Statistics for Actual Storms

### 4KM BLOCK STATISTICS FOR KRIGED WALNUT GULCH STORMS

<table>
<thead>
<tr>
<th>STORM</th>
<th>MAX INTENS</th>
<th>90th% INTENS</th>
<th>MEDIAN INTENS</th>
<th>MED-DUR</th>
<th>MED-WET</th>
</tr>
</thead>
<tbody>
<tr>
<td>v1</td>
<td>90.07</td>
<td>18.41</td>
<td>1.12</td>
<td>216</td>
<td>1.0</td>
</tr>
<tr>
<td>v10</td>
<td>53.69</td>
<td>3.03</td>
<td>0.33</td>
<td>264</td>
<td>0.92</td>
</tr>
<tr>
<td>v12</td>
<td>63.89</td>
<td>19.70</td>
<td>0.67</td>
<td>168</td>
<td>1.0</td>
</tr>
<tr>
<td>v13</td>
<td>63.94</td>
<td>14.05</td>
<td>2.26</td>
<td>384</td>
<td>1.0</td>
</tr>
<tr>
<td>v17</td>
<td>39.65</td>
<td>13.76</td>
<td>0.04</td>
<td>108</td>
<td>0.69</td>
</tr>
<tr>
<td>v3</td>
<td>97.80</td>
<td>21.68</td>
<td>3.18</td>
<td>180</td>
<td>1.0</td>
</tr>
<tr>
<td>v4</td>
<td>90.77</td>
<td>8.49</td>
<td>0.02</td>
<td>156</td>
<td>0.46</td>
</tr>
<tr>
<td>v5</td>
<td>98.18</td>
<td>26.18</td>
<td>0.18</td>
<td>60</td>
<td>0.53</td>
</tr>
<tr>
<td>v7</td>
<td>71.71</td>
<td>17.33</td>
<td>0.01</td>
<td>120</td>
<td>0.61</td>
</tr>
<tr>
<td>Mean</td>
<td>74.41</td>
<td>15.84</td>
<td>0.86</td>
<td>184</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Notes: Intensities in mm/hr; Median Duration in minutes; Median Wet area as a fraction of the total area.

Table 4.6 Kriged Statistics for Actual Storms
concentric with high intensities near the center and lower intensities near the outer edges of the storm. See Figure 4.3b for a typical example of the modelled storm intensities.

Forcing the model to skip all but the gauge points (the modified model) has an interesting consequence. The model statistics taken from the storms generated in the parameter searches are not the same as the statistics from storms generated later with the rainfall model standing on its own. The reason for this was traced to the lognormal jitter that is added to the rainfall. Each time certain criteria about distance from a cell center are met a random number is generated from a lognormal distribution. For the unmodified model this means that the lognormal generator is run many thousands of times while for the modified model the generator is run less than a thousand times. As a result, the numbers produced by the lognormal generator are different which in turn produces slightly different storm statistics. Fixing this problem would slow the model down to its original slow speed. Because the rainfall produced by the modified and the unmodified model are only different realizations of the same process the parameter optimization completed for the modified model should hold for the unmodified model.

A comparison of the statistics for the modified and the unmodified model indicates that the low intensities are affected more than the high intensities. The statistics are presented in Table 4.7. The maximum is not changed; the standard deviation of the maximum intensities actually goes down with the unmodified model; and the 90th percentile is slightly raised. However, the median is much higher for the
<table>
<thead>
<tr>
<th>MODEL</th>
<th>STORM LENGTH</th>
<th>MED-INTENS</th>
<th>90%-INTENS</th>
<th>MAX-INTENS</th>
<th>STDD-MAX</th>
<th>MED-DUR</th>
<th>WET-AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACTUAL</td>
<td>243.6</td>
<td>0.56</td>
<td>15.50</td>
<td>129.40</td>
<td>32.00</td>
<td>122.4</td>
<td>0.48</td>
</tr>
<tr>
<td>MODIFIED</td>
<td>196.8</td>
<td>0.56</td>
<td>10.20</td>
<td>32.00</td>
<td>42.40</td>
<td>124.8</td>
<td>0.63</td>
</tr>
<tr>
<td>UNMODIFIED</td>
<td>174.0</td>
<td>0.81</td>
<td>11.23</td>
<td>34.24</td>
<td>29.21</td>
<td>127.44</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Notes: Intensities in mm/hr; Med-Dur and Storm Length in min; Median Wet Area, fraction of the total. Actual refers to the observed Walnut Gulch storms.

Table 4.7 Comparison of Modified and Unmodified Model Statistics
unmodified than for the modified model. Another indication that the low intensities are affected more than the high intensities is that the wet area is higher for the unmodified model than for the modified.

4.3.4 Discussion of Both Walnut Gulch and Modelled Storms

The frequency distributions of the actual and modelled storms are in general similar. The effects of the selection of statistics for the calibration may be seen in the modelled distributions. In the actual distributions the intensity begins to increase significantly after about the 70th percentile while the modelled distributions begin to increase at about the median. This is in fact the single biggest difference in the two sets of frequency curves. This may indicate that central tendency in characterizing the Walnut Gulch convective storms is less important than the higher percentiles.

Sample hyetographs of the actual and modelled storms also show that there is less variability in the timing of the modelled storms than there is in the actual storms. The hyetographs are presented in Figure 4.6. The hyetographs are plots of the intensities measured at the gauge location at which the highest intensity occurred. For comparisons of statistics the timing of the storms may not be that important, but for runoff generation the timing of the storms can be of great importance.

4.4 Comparing Measurements

4.4.1 Differences Between Modelled True Points and Areas

To evaluate the difference caused by areal integration, the statistics from the
Figure 4.6 Hyetographs for a Single Gauge or Point
very fine network of points and the 4X4 km areal averages computed from the very fine network were compared. A map of the very fine gauge network is provided in Figure 4.7. The six statistics, the maximum, 90th percentile, the median, the storm length, the median duration, and the fraction wet area were compared graphically. A regression line was fit to the points. The sample coefficients of determination, \( r^2 \), for the points were also computed. The sample coefficient of determination is calculated from

\[
r^2 = \frac{S_{AG}^2}{S_G S_A}
\]

with \( S_{AG} \) the sample covariance of the areal and gauge statistics, \( S_G \) and \( S_A \) are the sample variances of \( A \) and \( G \).

The trends and the \( r^2 \) values are provided in Tables 4.8 and 4.9. The trends are also summarized in Table 4.10.

The maximum intensity was always reduced when the storm was measured with areas as opposed to a very fine mesh of gauges. This comes as no surprise. The median intensity on the other hand was higher when measured with the 4 km blocks and 90th percentiles are about equal for the gauges and areas. This is the same type of behavior seen with the frequency distributions and may be attributed to the areal smearing of rain across the watershed.

Interestingly the storm length is also decreased when the storm is measured with areas. One gauge may measure some trace of rain that when averaged across
### SUMMARY OF TRENDS

<table>
<thead>
<tr>
<th></th>
<th>MAX-INTENS</th>
<th>90%-INTENS</th>
<th>MED-INTENS</th>
<th>MED-DUR</th>
<th>% WET AREA</th>
<th>STORM LENGTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{\text{fine}} ) ( P_{\text{fine}} )</td>
<td>AF &lt; PF</td>
<td>AF = PF</td>
<td>AF &gt; PF</td>
<td>AF &gt;= PF</td>
<td>AF &gt; PF</td>
<td>AF &lt;= PF</td>
</tr>
<tr>
<td>( P_{\text{sparse}} ) ( P_{\text{fine}} )</td>
<td>PS &lt; PF</td>
<td>PS = PF</td>
<td>PS &gt; PF</td>
<td>PS &gt; PF</td>
<td>PS = PF</td>
<td>PS &lt; PF</td>
</tr>
<tr>
<td>( A_{\text{fine}} ) ( P_{\text{sparse}} )</td>
<td>AF &lt; PS</td>
<td>AF = PS</td>
<td>AF &lt;= PS</td>
<td>AF &gt;= PS</td>
<td>AF &gt; PS</td>
<td>AF &gt; PS</td>
</tr>
<tr>
<td>( A_{\text{fine}} ) ( A_{\text{kriged}} )</td>
<td>AF &lt; AK</td>
<td>AF = AK</td>
<td>AF &lt; AK</td>
<td>AF &gt; AK</td>
<td>AF = AK</td>
<td>AF &gt; AK</td>
</tr>
</tbody>
</table>

Notes: These trends refer to modelled storms.  
\( A_{\text{fine}} \) and AF refer to statistics derived from modelled rainfall measured with areal averages computed from the very fine grid.  
\( P_{\text{fine}} \) and PF refer to statistics derived from modelled rainfall measured with the very fine grid of points.  
\( P_{\text{sparse}} \) and PS refer to statistics derived from modelled rainfall measured with the 58 gauge networks.  
\( A_{\text{kriged}} \) and AK refer to statistics derived from areal average modelled rainfall that has been kriged from the sparse networks.

Table 4.8 Trends in the Modelled Storms

### SAMPLE COEFFICIENTS OF DETERMINATION

<table>
<thead>
<tr>
<th></th>
<th>MAX-INTENS</th>
<th>90%-INTENS</th>
<th>MED-INTENS</th>
<th>MED-DUR</th>
<th>% WET AREA</th>
<th>STORM LENGTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{\text{fine}} ) ( P_{\text{fine}} )</td>
<td>0.52</td>
<td>0.95</td>
<td>0.95</td>
<td>0.97</td>
<td>0.78</td>
<td>0.94</td>
</tr>
<tr>
<td>( P_{\text{fine}} ) ( P_{\text{sparse}} )</td>
<td>0.50</td>
<td>0.99</td>
<td>0.77</td>
<td>0.97</td>
<td>0.44</td>
<td>0.95</td>
</tr>
<tr>
<td>( A_{\text{fine}} ) ( P_{\text{sparse}} )</td>
<td>0.84</td>
<td>0.96</td>
<td>0.85</td>
<td>0.98</td>
<td>0.41</td>
<td>0.97</td>
</tr>
<tr>
<td>( A_{\text{fine}} ) ( A_{\text{kriged}} )</td>
<td>0.82</td>
<td>0.95</td>
<td>0.89</td>
<td>0.97</td>
<td>0.76</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Notes: These trends refer to modelled storms.  
\( A_{\text{fine}} \) refer to statistics derived from modelled rainfall measured with areal averages computed from the very fine grid.  
\( P_{\text{fine}} \) refer to statistics derived from modelled rainfall measured with the very fine grid of points.  
\( P_{\text{sparse}} \) refer to statistics derived from modelled rainfall measured with the 58 gauge networks.  
\( A_{\text{kriged}} \) refer to statistics derived from areal average modelled rainfall that has been kriged from the sparse networks.

Table 4.9 \( r^2 \) Values
Summary of Trends

- Areal average and gauge CDF's are not the same.
- 90% is a stable measure of intensity CDF's
- Areal averaging has a similar effect on intensity statistics as sparse sampling.
- Sparse sampling error is slightly larger than the areal averaging.
- Interpolation error follows the differences between the fine areas and sparse points indicating that the interpolation error may be largely controlled by the sampling error.
- In general, storms measured with areas 1) appear to move more slowly, 2) appear larger, 3) appear less intense and 4) have shallower intensity gradients.

Table 4.10 Summary of Sampling Trends
a 4 km block becomes small enough to be rounded to zero. The median duration at a single gauge or grid square, however, shows a slight tendency to increase with the areal averages. This appears almost contradictory to the previous comments. However, the median duration is increased because the rain has a greater opportunity to fall in the large grid square than on the small gauge and the very low intensities in the tails of the distribution will not affect the median duration as much as the low intensities will affect the length of the storm. The wet fraction also increases with areal measurements. The areally measured storm appears larger, to move more slowly and to be less intense, and with shallower intensity gradients than the gauge measured storm.

4.4.2 The Effects of Reduced Gauge Density

The errors caused by insufficient gauge sampling were evaluated using the statistics from the very fine gauge network and sparser gauge networks. The sparse gauge network is the 58 gauge network used to measure the Walnut Gulch storms. Sparse sampling acts on the intensity distribution in the same way as areal averaging. The sparse network maximum is less than the fine network maximum, the 90th percentiles are almost equal and the sparse median is too high. The storms also appear shorter with the sparse network and the duration at a gauge is higher for the sparse network. As with areal averaging, there is less information provided from the sparse network than from the fine network so the tails of the distribution are lost with the sparse sampling.
4.4.3 Comparing MA_{true} and MG_{sparse}

The modelled sparse gauge statistics were compared next. In general, the sparse network preserves the maximum intensities better than the areal averages but not in every case. The errors caused by the sparse gauge sampling result in some of the storms registering higher maximums with the areal averages than with the sparse gauges. Therefore, the sparse gauge errors will make it that much more difficult to create realistic rainfall fields by interpolating gauge measurements. On the other hand the areal averages preserve the median intensities better than the sparse gauges with the median intensities being lower for the areal measurements than the sparse gauges.

4.4.4 Evaluating the Interpolation Error

The interpolation error is evaluated using statistics generated from the 4 km block averages of the very fine mesh and statistics calculated from kriged 4 km blocks. The effects of kriging reflect the sparse gauge basis for the kriging. Every kriged statistic, except the wet area, is different from the areal statistic in the same way as the sparse gauges. The interpolation error is therefore more dependent upon the errors in the gauge measurements than upon the kriging method.

4.4.5 Evaluating Areal Measurements

One of the stated goals of this study was to apply the relationships between the point and area measurements in order to evaluate areal rainfall values. The
method consists of developing a predictive equation between the modelled areal averages and the modelled sparse gauge network, and then applying that equation to actual storms.

For each of the six statistics there is a different regression equation that can be used to predict the areal statistic, $MA_{\text{fine}}$, from the sparse gauge statistic, $MG_{\text{sparse}}$. Before proceeding with an example, two validation steps are made.

The first validation step was to determine if the modelled relationships can in fact predict actual statistics. In other words, can the modelled relationships translate to the real world. To make this assessment, a regression equation between the modelled sparse gauges and the areas kriged from the modelled sparse gauges was computed. This equation was then used to predict the actual Walnut Gulch kriged statistics from the Walnut Gulch gauge measurements. The predicted statistics were then compared to the actual kriged statistics. Because of the linearity of the kriging method, and the fact that the same correlation function was used to krig the Walnut Gulch storms, the modelled relationships should be able to predict the Walnut Gulch statistics. This assessment can only indicate a problem, and not make any definitive validation. The predictions for the median and 90th percentile are very good and adequate for the maximum and median duration. This is the expected positive result. Graphs of the four statistics are presented in Figure 4.8. The storm length and percent wet area are not included because the storm length for a kriged storm will always be the storm length of the gauge measured storm and the wet area has a very low $r^2$ value.
Notes: Actual kriged statistics are statistics derived from actual Walnut Gulch storms kriged to 4X4 km blocks. Predicted kriged statistics refer to statistics predicted with the regression equations developed with model output.
The next validation step is used to evaluate the strength of the $\text{MA}_{\text{true}}$ and $\text{MG}_{\text{sparse}}$ relationships. In addition to the $r^2$ percent differences are computed. All of the tested statistics have reasonably low mean percent errors. The graphs of the actual vs predicted statistics and the % difference are in Figures 4.9 and 4.10. The graphs indicate that the wide ranges are caused by only a few points in the low intensity regions. The unusual shape of the median duration may be caused by the discrete nature of the blocks. This assessment is also positive.

4.4.6 An Example: Analysis of Kriging Walnut Gulch Storms

Using the information gleaned from the modelled statistics, the block kriging method of areal interpolation was examined. The gauge measured rainfall for the actual storms was kriged to 4km blocks. The grids used are the same ones used in the modelling exercise. The general trends of the kriged data versus the gauge data were examined first. Then the relationships developed with the modelled statistics are used to compute predicted areal statistics. These predicted areal statistics are then compared to the kriged statistics.

The actual kriged statistics follow the pattern found with the model. The areally measured maximum intensity is reduced while the median, the median duration and the percent wet area all increase with the areal measurements. The 90th percentile shows no trend.

Predicted areal statistics are calculated from the actual Walnut Gulch gauge measured statistics using the relationships developed with the model. The predicted
Figure 4.9  Modelled and Predicted Areal Average Statistics

Notes: Modelled areal average statistics are statistics derived from areal averages computed from the very fine mesh.
Predicted areal average statistics refer to statistics predicted with the regression equations developed with model output.
Figure 4.10 Percent Difference — Modelled and Predicted Areal Average Statistics

Notes: Modelled areal average statistics are statistics derived from areal averages computed from the very fine mesh. Predicted areal average statistics refer to statistics predicted with the regression equations developed with model output.
areal statistics and the kriged areal statistics are then compared. The graphs of the maximum, the 90th percentile, the median intensity and the median duration are presented in Figure 4.11. At first glance these graphs and those in Figure 4.7 appear to be the same but they are not. The kriging method appears to under predict the maximums and to under predict the median durations. However on the whole the predicted and the kriged statistics are in fairly good agreement.
Figure 4.11 Actual Kriged and Predicted Areal Average Statistics

Notes: Actual kriged statistics are statistics derived from actual Walnut Gulch storms kriged to 4X4 km blocks. Predicted areal average statistics refer to statistics predicted with the regression equations developed with model output.
CHAPTER FIVE
DISCUSSION

5.1 Introduction

This section will be a discussion of the results presented in Chapter Four. General conclusions of this study and recommendations for future work are also included.

5.2 Discussion About the Model Calibration

5.2.1 Introduction

The calibration of rainfall models is a topic which can bear further study. Previous calibration methods were not adequate for the model of Krajewski et al. (1993) because they relied on analytic expressions for the model moments. Relying on the analytic expressions limits the rainfall characteristics which can be used in the optimization. An alternative automatic method was described in this work. General comments about formulating an error function and the subjective-objective calibration dilemma will be made.

5.2.2 Formulating an Error Function

The purpose of the error function is to provide a quantitative measure of the difference between the observed and modelled rainfall. The Krajewski et al. (1993) model is a stochastic model and therefore is not designed to reproduce particular historical events. Consequently, the differences between the general characteristics
of observed and modelled rainfall must be measured to assess the differences between the observed and modelled storms. In general, statistics derived from the rainfall are compared.

The particular statistics selected for use in the error function will depend on the use for which the model is being calibrated. For this work, the intensity distributions and areal averaging were of interest. The maximum, the 90th percentile and the median were selected to describe the intensity distributions. The wet area and duration at a gauge were used to assess a storm's areal and temporal characteristics.

For a different study a different set of statistics might be appropriate. For example in a study involving runoff generation, time-to-peak might also be important.

The next step in developing an error function is to select a method by which to compare the observed and modelled statistics. Again, it is not possible to exactly match modelled statistics with historical storms. The modelled statistics are only required to fall into some reasonable region which is circumscribed by the historical storms. In this study, means and standard deviations were used to compare the groups of historical statistics to the groups of modelled statistics. Other comparison methods include non-parametric statistical tests, or some calculation involving the ranges of the historical statistics.

A wide variety of rainfall characteristics may be accommodated in the error function. In fact, the freedom to select the rainfall characteristics which will be used in the error function is one of the benefits of the calibration method described in this
study. There are, however, a few limitations on what may be included in the error function. The rainfall statistics and the method of comparison must be coded onto a computer and be quantifiable without any operator interference. Lengthy calculations should be avoided because the statistics must be calculated thousands of times. In addition, it is a good idea to avoid contradicting statistical assumptions. For example, rainfall is modelled as a random process and rainstorms are independent realizations of that random process. However, the gauge measurements of a rainstorm are not independent from one another. Because the rainfall measurements are not independent, the error function must be made up of statistics derived from the rainstorm, not the intensities themselves.

5.2.3 The Subjective vs Objective Calibration Dilemma

One advantage objective calibration has over subjective calibration is that parameters found through objective means are usually more widely applicable than parameters found through subjective means. In other words, objective methods find better parameters than subjective methods.

The method of Islam et al. (1988) is the most popular alternative to the method used here and it is considered objective. (At least the authors define it as such.) Islam et al. developed analytic expressions for several model moments, the mean, the variance, lag 1 covariance and lag 0 cross covariance. They then used an automatic search algorithm to solve the system of equations. Jacobs et al. (1988) used a method similar to that of Islam et al. (1988) and found that the resulting
parameters reproduced only the moments used in the calibration. For a calibrator who is interested in rainfall characteristics which cannot be described with analytic expressions derived from the model distributions, the method of Islam et al may not be satisfactory.

One additional consideration is parameter uniqueness. As was seen when examining the convergence of the SCE-UA routine, (see page 50) there was not a unique set of parameters. One reason that the parameters are not unique is that it might be possible to reproduce the same storm characteristics by using a lot of small cells or a few larger ones, or produce the same characteristics with cells of similar size arranged in different patterns. There are numerous possibilities. Because no unique parameter set exists, the selection of the best parameter set must lie with the calibrator.

As rainfall models become more sophisticated it will become more difficult to find analytic expressions with which to parameterize the models. Automatic search techniques, such as the one used here, will play an increasingly important role.

5.3 Discussion About the Model

5.3.1 Introduction

In general the Krajewski et al. (1993) model performed well. However, the simulated rainfall differed from the observed rainfall in two important ways: 1) the observed intensity fields appeared more random than the modelled fields and 2) the observed maximum intensities were much higher than the modelled maximums.
Causes of these discrepancies will be discussed below as a means of offering insight into the model structure and the calibration process. Several other topics related to the model and the model's overall performance will also be discussed. It should be noted that reproducing maximum intensities was one objective of the calibration, whereas reproducing the spatial structure (the degree of randomness) was not.

5.3.2. Modelling the Random Appearance

The model may have difficulty simulating the observed randomness of the intensity fields because of the model's origins. The model of Krajewski et al. (1993) evolved from a total depth model to its present form which simulates time varying rainfall intensities. Rainfall has a different appearance depending upon the time and space scales at which it is modelled. Islam et al. (1988) noted that the calibration of a model is specific to a particular space time scale. They pointed out that the calibration at one space time scale may not be valid for generating storms with a different space time scale.

On the Walnut Gulch watershed, the storms look well organized when measured as total depths. It is not until intensity maps for short time periods are examined that the random character of the convective rainfall becomes obvious. This random appearance is also seen in total depth measurements made on a fine spatial grid.1

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1Unpublished data collected by the author at Gates Pass, Arizona.
The assumption that rainfall intensities decrease outward from a high intensity center may not hold for convective events in the U.S. Southwest. For total storm depths on the other hand, this assumption may be quite reasonable. Because the model of Krajewski et al. (1993) was developed from a total depth model, the assumption of a high intensity center is implicit in the model structure. The spatial arrangement of the modelled intensities reflects this assumption.

Krajewski et al. (1993) addressed the issue of smooth fields by adding a lognormal jitter to the intensity patterns to make them more variable. However, the jitter is simply noise added to an existing intensity pattern; it does not completely alter the intensity pattern generated by the cell clusters. The jitter is generated from a lognormal distribution, and consequently it will always be positive. Increasing the jitter standard deviation (the model parameter stdjit) will allow more variation in the amount of noise added to the initial pattern. However, the initial cell generated intensity pattern will always underlie the final output intensity patterns. The jitter alone cannot reproduce the randomness of the observed intensity patterns.

5.3.3 Modelling the Maximum Point Intensities

The model was unable to reproduce the maximum intensities of the observed storms when the maximum was included in the ensemble of statistics. It is worth noting that the model matched all the other statistics except the maximum to within 12% of the observed value. The mean of the modelled maximums is 70% too low and almost no modelled storms had a maximum intensity in the range of the
observed values. However when only the maximum was being matched the model reproduced the observed maximum to within 1%. It would seem then that some interaction in the ensemble of statistics disrupted the maximums. Several possible explanations are considered here.

The simplest explanation is that there is some interaction between the selected statistics that the model cannot resolve. Even when the maximum was excluded from the error function, mean of the modelled maximums was still about 40% too low. The observed cumulative frequency distributions show a very sharp curve with the distance between the points in the tail increasing toward the maximum. This type of long and highly variable tail is very difficult to model. It may simply be unrealistic to expect the model to reproduce this behavior.

Another possible explanation is a conflict between the calibration data and the model structure. The model generates storms that begin in different places around the watershed and then travel in different directions. It is likely that many of the modelled storms will simply touch only a small part of the watershed. The calibration storms, on the other hand, all tracked across the whole watershed, covering most of the watershed area.

This conflict can be understood as a sampling problem. All the calibration storms were extensively sampled but not all the modelled storms were well sampled. Incomplete sampling of the modelled storms would lead to a lowered average maximum intensity and to a higher standard deviation of the maximums. A modelled storm that happened to cross the watershed would register a high maximum intensity,
while the same storm that only crossed a corner of the watershed would register a very low maximum intensity. In addition, the sampling problem is consistent with the short average length and the high percent wet area of the modelled storms. The modelled storms would appear shorter because they might frequently cross only a portion of the modelled watershed. The percent wet area might appear high because in order to insure that the generated storms always crossed a portion of the watershed the generated storms would have to be large.

5.3.4 The Problem of Excessive Intensities

One peculiar behavior of the model is that when the model samples all of the points in the very fine grid, as opposed to only the gauge locations, it has a tendency to generate a few impossibly high intensities. (The low maximums just discussed were measured by the sparse 58 gauge network.) One early parameter set generated 50 out of 500 storms with intensities >500 mm/hr when measured with the fine grid. However, with the same parameter set no intensities >500 mm/hr were recorded when the storm was measured with the sparser 58 gauge network. Using the final parameter set selected in this study only 1 storm out of 200 registered an intensity >500 mm/hr when the storms were measured with the fine mesh. That point occurred outside of the Walnut Gulch watershed.

There are several possible reasons for this behavior. The use of a parameter set calibrated with gauges that have a larger space scale than the fine grid may cause this behavior. As noted earlier, Islam et al. (1988) demonstrated that parameters for
one space time scale are not appropriate for generating storms at a different space
time scale.

A second possible cause is the sparse vs. intensive sampling. The exhaustive
sampling may be revealing a phenomenon that is not noticed with the sparser
sampling. Because the number of excessive intensities was significantly reduced with
the second parameter set it appears that this behavior may be controlled by judicious
parameter selection.

5.3.5 Three Additional Comments

First, the model may or may not recreate the storm characteristics to which
it is not explicitly calibrated. The parameters selected for use in this study should
not be viewed as the parameter set for generating Walnut Gulch storms. The
calibration of this model must depend on the goals of the study in which the model
will be used.

Second, the model was parameterized, and then run using a specific sequence
of random seeds. The use of these parameters with different seeds may not be
appropriate.

Third, this model does not attempt to model the inter-arrival time of storms.
The model is designed to generate meso-scale rainfall systems, but not a sequence
of events over a series of days. The Waymire et al. (1984) model is designed to
model the inter-arrival time of storms.
5.3.6 Overall Assessment of the Model

It is difficult to assess exactly "how well" the model of Krajewski et al. fits the Walnut Gulch regime. The model simulates maximum point intensities very poorly. In addition, the smoothly varying fields it produced, rather than the haphazardly constructed fields seen in the Walnut Gulch storms, make it appear that some fundamental aspect of the Walnut Gulch storms has not been modelled.

However, the problem with the maximum intensities indicates a problem with only the tail of the distribution, the most difficult region of any distribution to model. In addition, just as the low maximums and the smooth fields produced in the model output indicate a poor fit, the modelled statistics, except the maximum, do fall within the range of the actual Walnut Gulch statistics. Again the only problem appears to lie in the tail of the distribution.

In summary, the Krajewski et al. (1993) model matches the statistics (except the maximum) of the observed Walnut Gulch storms well enough to serve the purposes of this study. The Krajewski et al. (1993) model is a valuable research tool and its use in different rainfall regimes and in different types of studies should be explored.

5.3.7 Possible Ways to Improve the Model Fit

Three suggestions will be made that may (or may not) improve the model's performance. First, it may be possible to parameterize the model so that it recreates the randomness of the Walnut Gulch intensities. Initial tests of limiting the cell
diameter done in this study indicate that such a parameterization may produce the desired random appearance. However, many small cells may also create a problem with the model generating a few impossibly high intensities in a storm. The very high intensities can only be produced when the several cells overly a single point.

Second, three possible solutions to the sampling problem discussed in the section on maximum intensities are: 1) to force the modelled storms to cross the modelled watershed, 2) to calibrate the model to a more general data set and 3) to develop some criteria by which to pre-select storms from a larger data set automatically during the calibration. Each of these methods has its own disadvantages. If the modelled storms are forced to cross the watershed during the calibration then the generated parameters will only be valid for the particular storm tracks defined in the calibration. Calibrating the model to a more general data set will require the operator to select storms from a larger set if a particular type of storm is desired. Preselecting storms during the calibration will allow only a particular portion of the modelled distribution into the calibration process.

In the comparison of the cumulative frequency distributions the use of the median in the calibration is clearly evident (Figure 4.3). The frequency distribution begins to curve right after the median in the modelled distributions, while the actual distributions hold off until the 60%. The model calibration might benefit from using the 60th percentile in addition to the median.
5.4 Comparison of Measurement Techniques

5.4.1. Model Caveats

As always conclusions drawn from a model must be treated with caution. The first portion of the comparison phase of this study, in which the effects of different measurement techniques were examined, does not depend heavily on the individual values of statistics. Therefore the conclusions regarding the effects of the different measurement schemes may be viewed with some confidence for all the statistics. In the second portion of the comparison phase, the attempt to predict actual statistics from modelled relationships, particular values of statistics may be more important. Because the model had some difficulty in recreating the observed maximum intensities, the application of the modelled relationships to the real world for the maximum intensities must be treated with extreme caution. The application for the other statistics may be considered to be more robust.

5.4.2. Discussion of Trends

Measuring rainfall accurately is very difficult. Even the dense raingauge network of Walnut Gulch cannot completely capture rainstorm characteristics and causes distortions in the storm description. This was seen in the comparison of the fine network of points and the sparser 58 point network. A network that is less dense than Walnut Gulch will cause an even greater distortion in the measured rainfall.
Both the areal integration and the sparser point sampling of rainfall do not completely capture the rainfall characteristics. However, the distortions caused by areal integration may have a more detrimental effect on runoff prediction than the sparser sampling in Hortonian driven runoff regimes. With the sparse sampling the highest intensities are better preserved than with the areal measurements.

Another useful result of this comparison may be the realization that the 90th percentile is well preserved in both the areal and the 58 point network. The 90th percentile would appear to be a very stable measure of a rainfall distribution. In addition, because the 90th percentile is preserved, the loss of the maximum intensities may be mitigated.

The results of this study indicate that for some, but not all rainfall statistics there exist simple linear relationships between the gauge and areally derived statistics. This fact has been implicitly used since the development of the Areal Reduction Factor (ARF) which converts point rain (at the storm "center") to areal averages. It may be possible to develop a new set of Areal Reduction Factors (ARF-UA), based on model derived equations.

5.4.3. Future Comparisons

There are endless possible comparisons that could be made. 1) It would be useful to compare sparser gauge networks more similar to the operational networks. There may be a threshold at which the correlation between the gauges and areas becomes too small to make comparisons useful. 2) It would also be useful to extend
this study to larger areas to determine if the relationships between the gauges and areas break down at some threshold distance. 3) A comparison of gauges to different sized areas would also be interesting; there may be some optimum area that captures the rainfall characteristics very well. The need to know more about the relationship between areal and point rainfall measurements will grow as more research is done to develop areal measurement techniques. Studies such as this one can provide useful insights into the gauge-area relationship.
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