EVAPOTRANSPIRATION FROM A STAND OF SALTCEDARS

by

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A Thesis Submitted to the Faculty of the

DEPARTMENT OF HYDROLOGY

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

In the Graduate College

THE UNIVERSITY OF ARIZONA

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ACKNOWLEDGMENTS

The author wishes to thank Dr. William D. Sellers under whose direction this report has been prepared. The author is also thankful to Dr. Eugene S. Simpson who made arrangements for the procurement of data for this report. The author also gratefully acknowledges the generous cooperation extended by Mr. T. E. A. Van Hylckama, Research Officer, Buckeye Project, U. S. G. S. Department.

SYMBOLS

Symbol	Units	Description
a	cm^2	an area
a_1		a proportionality constant
b	cm	a small height
c_p	$cal/gm/c^o$	specific heat of air at constant pressure
d	cm	thickness of laminar layer
e	$g\ cm^{-1}\ sec^{-2}$	vapour pressure
e_a	$g\ cm^{-1}\ sec^{-2}$	vapour pressure of air
e_o	$g\ cm^{-1}\ sec^{-2}$	vapour pressure at $z = o$
e_d	$g\ cm^{-1}\ sec^{-2}$	vapour pressure in the laminar layer
e_z	$g\ cm^{-1}\ sec^{-2}$	vapour pressure at $z = z$
e_s	$g\ cm^{-1}\ sec^{-2}$	saturation vapour pressure at temperature T_s
g	$cm\ sec^{-2}$	acceleration due to gravity
h	cm	thickness of sublayer in Budyko's formula
k		crop coefficient in Blaney-Criddle method
k_o		Von Karman's constant, generally considered equal to 0.40
l_s	cm	mixing length in stable air

SYMBOLS--Continued

Symbol	Units	Description
l	cm	i, mixing length in Prandtl's theory ii, length of layer considered
l_θ	cm	mixing length in unstratified air
m		dimensionless quantity in Budyko's formula
n		a number
o		used as subscript to indicate the value of a quantity at the water surface
p		monthly percentage of daytime hours in a year
q		specific humidity
r		used as a subscript to indicate the value of the quantity in flow over a rough surface
s		a stability parameter
t	sec	time
t	c^o	monthly mean temperature in Blaney-Criddle method
u	cm sec^{-1}	wind speed
$\bar{u}$	cm sec^{-1}	average wind speed
u_*	cm sec^{-1}	the friction velocity
$\bar{u}_a$	cm sec^{-1}	average wind velocity at height, a
u_{*r}	cm sec^{-1}	the friction velocity for a rough surface

SYMBOLS--Continued

Symbols	Units	Description
v	$cm^2\ sec^{-1}$	kinematic viscosity of the air
x		a horizontal co-ordinate axis
z	cm	height above surface (and vertical coordinate axis)
z_o	cm	roughness parameter
A_1	$c^o\ cm^{-1}$	adiabatic lapse rate
B_1		a constant equal to $\frac{1}{a_1}$
C_1		slope of e versus ln z curve
C_s	sec^{-1}	vertical wind shear in stable air
D	$cm^2\ sec^{-1}$	molecular vapour diffusivity
D_1, D_2	$cm^2\ sec^{-1}$	molecular diffusion coefficients for water vapour and heat energy through air
D_w, D_h, D_m	$cm\ sec^{-1}$	transfer coefficients of water vapour and heat and momentum
E	$g\ cm^{-2}\ sec^{-1}$	evaporation per unit area
E_d	$g\ cm^{-2}\ sec^{-1}$	evaporation in laminar layer
E_o	mm/month	potential evapotranspiration
G	$cal.\ cm^{-2}\ min^{-1}$	energy stored in soil
G_o	$cal.\ cm^{-2}\ min^{-1}$	energy stored in wet soil
H	$cal.\ cm^{-2}\ min^{-1}$	energy conducted away as sensible heat
H_o	$cal.\ cm^{-2}\ min^{-1}$	sensible heat from a moist surface

SYMBOLS--Continued

Symbol	Units	Description
I		heat index in Thornthwaite method
K_M	$cm^2\ sec^{-1}$	exchange coefficient in Budyko's formula
K_z	$cm^2\ sec^{-1}$	(kinematic) eddy vapour diffusivity in z-direction
L	cal. gm^{-1}	latent heat of evaporation
LE	cal. $cm^{-2}\ min^{-1}$	energy used for evaporation
LE_o	cal. $cm^{-2}\ min^{-1}$	energy for evaporation from moist surface
P	$g\ cm^{-1}\ sec^{-2}$	atmospheric pressure
Q	cal. $cm^{-2}\ min^{-1}$	Bowen notation of energy used by evaporation
Q_a	cal. $cm^{-2}\ min^{-1}$	energy advected into or out of body of soil
Q_b	cal. $cm^{-2}\ min^{-1}$	effective back radiation
Q_H	cal. $cm^{-2}\ min^{-1}$	Bowen notation of sensible heat energy
Q_r	cal. $cm^{-2}\ min^{-1}$	reflected solar radiation
Q_s	cal. $cm^{-2}\ min^{-1}$	solar radiation
R	cal. $cm^{-2}\ min^{-1}$	net radiation
R_B		Bowen ratio
R_o	cal. cm^{-2}/year	radiation balance for a wet surface
$\overline{T}$	c^o	mean temperature
T	A^o c^o	i, absolute temperature of layer ii, temperature in climatic methods

SYMBOLS--Continued

Symbol	Units	Description
T_a	A^o	absolute temperature of air
U	mm/month	consumptive use
τ	g cm^{-1} sec^{-2}	shearing stress
γ_a		the resistant coefficient at height a
μ_1, μ_2		experimental constants
ρ	g cm^{-3}	density of air
σ_1		Holzman's proportionality constant
ϕ, ϕ_1, ϕ_2		densities of specific heat energies at the two faces of a space of length l
Θ, Θ_1, Θ_2		energy per unit volume in the form of latent heat at the two faces of a space of length l
Γ_E		evaporation coefficient
Γ_H		sensible heat transfer coefficient
$\sum$		summation sign

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

ABSTRACT

A comprehensive study to determine the application, reliability and efficiency of aerodynamic and energy budget methods in computing evapotranspiration during September and October 1962 over a stand of saltcedars at Buckeye has been done. Computed evapotranspiration is evaluated against evapotranspiration measured through water budget control and compared with that obtained by climatological methods.

Conclusions:

1. Average evapotranspiration rates computed by aerodynamic, energy budget and climatological methods are respectively +20%, -3 to +8% and ±5% of the measured evapotranspiration rates.

2. Mass-transfer equations which are generalized for stability and instability cannot be applied in the present situation.

3. The Thornthwaite-Holzman equation in adiabatic form promises good results for calculating evapotranspiration over a period of a day or so.

4. The Bowen ratio method has given the best results with $\frac{K_H}{K_V} = 0.53$.

5. The energy budget method yields more consistent results than the aerodynamic method.

Recommendations:

1. Hourly measurements of the water consumption should be made and a range of the values of 'constants' B_1 and σ_1 in Rossby and Deacon's approaches should be examined against stratified conditions.

2. Using hourly measurements of water consumption the ratio $\frac{K_H}{K_V}$ under different conditions of stability should be further examined.

3. Climatological methods have yielded good results and should be tried as a control.

CHAPTER I

INTRODUCTION

In arid as well as humid regions where phreatophyte and non-phreatophyte plants grow on a substantial scale a study about their occurrence and their annual consumption of ground water becomes a basic requirement in the framing of any ground water development project.

A phreatophyte gets its water requirements directly from the water table and thus reduces the amount of water available to man. The effect of phreatophytes, particularly on the hydrology of arid regions is distressing when it is realized that most of the phreatophytes have a very low economic value, are heavy users of water and occur mostly on the floors of valleys and the floodplains of streams where ground water is readily available and where it can be utilized most effectively. Our field observations have so far shown that there are three important factors which exert a controlling influence on the occurrence and growth of phreatophytes. They are: climate, depth to water table or capillary fringe and quality of the ground water. The character of the soil may be a fourth important factor in some localities. Quite a few of these phreatophytes, among which saltcedar is one, can thrive under a wide range of climatic, depth to water table and quality of ground water conditions. A warm climate is, however, quite conducive to its growth. Use of water by saltcedars follows very closely the seasonal changes in temperature, increasing as the maximum air temperature increases and decreasing as the maximum air temperature decreases.

The effect of humidity on the use of ground water is the opposite of temperature. The rate of use decreases as the relative humidity increases and increases as the relative humidity decreases. Thus a low relative humidity combined with high air temperature, a combination which is usually found in arid regions, is very favorable for high rate of transpiration and hence a high rate of loss of ground water. Wind movement is effective in increasing water use by keeping the relative humidity low by replacing air made humid by transpiration with drier air from the adjacent areas. Rainfall causes a reduction in the use of ground water by increasing soil moisture as well as by increasing air humidity and decreasing air temperature.

Altitude also affects the use of ground water because of its effect on temperature and length of the growing season and daytime hours. Based upon field observations, it has been inferred that as depth to water table under a stand of saltcedars increases the plants become scattered and less vigorous and gradually diminish in size until they cease to exist altogether. Though the range of water table depth under which this plant can exist has not been mentioned conclusively in any literature, yet it is realized that the plant is capable of developing a deep root system and can envelop a huge volume of soil and subsoil strata. Saltcedars have been observed to grow in quite a range of dissolved solids content of the water and exhibit a good range of salt tolerance. Though this plant will grow very well where the salt content of the ground is very low, more commonly and perhaps due to the degree of competition offered by other plants, these are seen growing in areas of moderate to high salt concentration especially where the ground water is rich in common salt.

Ground water that is consumptively wasted by saltcedars can, however, be available for salvage or can be put to beneficial consumptive use. The degree or the extent to which this salvage can be done is an economic problem and much study is needed on any individual situation to develop methods by which salvage can be accomplished economically. Lowering of the water table beyond the reach of saltcedars by pumping or drainage and subsequently using this water economically or by substitution of plants of high economic value can be possible ways of effecting salvage, but either operation requires a knowledge of the occurrence and habits of these plants and their annual consumption of the ground water.

As will be clear from the preceding discussion, consumptive waste determined for the saltcedars or any other such plant in a certain area cannot be applied to a stand of saltcedars or such plants in some other area. There can be large differences in the annual rate of use depending upon the position of water table, the quality of ground water and the climatic conditions. In addition to all these factors the density of growth is also very important in affecting the consumptive use of water. This further illustrates the nonfeasibility of attempting to apply water use or evapotranspiration data of one situation to another. Thus there is obviously a need to develop methods to estimate it, which are not only very reliable but can also provide results in the least possible time and with a reasonable amount of expenditure. The Buckeye Project aimed at measuring water-use by saltcedars near the Gila River just southwest of Buckeye has an elaborate design to measure water-use by all possible means. The main purpose of this report is to use the Buckeye data to study the application, reliability and efficiency of various methods. These methods for estimating evapotranspiration can be:

1. The aerodynamic method in which the evapotranspiration is computed from an analysis of the changes in water vapour, temperature and wind velocities with height over the vegetation;

2. The energy budget method in which the evapotranspiration is computed using the heat balance equation for the required surface;

3. The water budget method in which the evapotranspiration can be determined by measuring inflow, outflow and the changes in storage, and

4. The climatological methods in which climatological data are used to determine evapotranspiration.

Although it is theoretically possible to use the water budget method for the determination of evapotranspiration, it is usually impracticable to do so because of the effects of errors in measuring the various items. Evapotranspiration as determined by this method is a residual, and therefore may be subject to considerable error if it is small relative to other items. The aerodynamic and the energy budget methods have come into existence as recent developments in oceanographic and meteorological theories. These methods have more or less a theoretical significance so far as these have not been widely used except over water surfaces or in some rare cases over smooth land and vegetation. In this study these methods are used to estimate evapotranspiration from a rough growth of saltcedars. Keeping in view the unusual environments as well as the complex nature of the meteorological and heat dissipation phenomena which control the evapotranspiration it is possible that neither of these two methods will produce sufficiently accurate results. Under such a situation the evapotranspiration thus computed will be evaluated against the evapotranspiration computed from the lysimeter measurements as well as from the climatological data.

The details of the instrumentation will not be included in this report. To provide the necessary data, the various meteorological variables measured are: air temperature, humidity, and wind speed at 2, 4 and 8 meters above the vegetation, and also rainfall and net solar radiation. The air temperatures and humidity are measured with dry and wet thermocouples. The wind speeds are measured with standard 3-cup Robinson-type contact anemometers. The rainfall is measured with standard types of rain gauges, one within and one outside the vegetation. The net solar radiation is measured with a Beckman Net Radiometer. There are six lysimeters, each 30' x 30' in cross-section and 14 feet deep, fitted with automatic water level maintaining and recording arrangement. To measure the flux of heat into the soil two of these six tanks are provided with four thermocouples in each, at depths of less than an inch, 3 feet, 6 feet and 12 feet below ground level.

CHAPTER II

THE WATER BUDGET CONTROL

The depth to water table in the lysimeter tanks is not the same in each case. In two the water table is 9 feet below the ground surface. Two others have water tables at 7 feet and the remaining two have water tables at 5 feet below the natural land surface. The quality of water provided in each case is categorized as "fresh." The filling and measuring arrangement in each case is such that it does not provide a dependable minute by minute record of the consumption of ground water by evapotranspiration by saltcedars. As such, evapotranspiration for short periods of time, like an hour or so, cannot be ascertained. However, from the recorder charts an idea of daily evapotranspiration can be made with a reasonable amount of precision. Thus, the average rates of evapotranspiration, which have been computed from the recorder charts for all six lysimeters as shown in Table 1 are believed to be correct within $\pm$ 5 percent. It may be mentioned that some data in the latter part of September had to be rejected due to leakage in the tanks.

Table 1. Average evapotranspiration measured by six lysimeters at Buckeye (cm/day).

Date	September Evapotranspiration (cm/day)	October Evapotranspiration (cm/day)
1	0.54	0.34
2	0.57	0.34
3	0.60	0.48
4	0.58	0.44
5	0.60	0.43
6	0.54	0.47
7	0.55	0.47
8	0.54	0.46
9	0.52	0.45
10	0.53	0.42
11	0.54	0.42
12	0.59	0.44
13	0.60	0.44
14	0.58	0.46
15	0.60	0.42
16	0.59	0.45
17	0.63	0.44
18	0.58	0.41
19	0.59	0.41
20	0.60	0.42
21	0.45	0.42
22	0.28	0.45
23	Doubtful due to leakage	0.45
24	"	0.41
25	"	0.43
26	"	0.42
27	"	0.46
28	"	0.46
29	"	0.51
30	"	0.41
31		0.41

CHAPTER III

AERODYNAMIC METHODS

In the case of flow of air over land the lower part of the atmosphere can be divided into three layers. Beginning at the surface: the laminar boundary layer, the turbulent boundary layer and the outer layer of no frictional influence. In the laminar layer, which is never more than a few millimeters thick and present only over a very smooth surface, temperature, humidity and wind speed vary linearly with height and the transfer of heat, water vapour and momentum take place through molecular processes. In the turbulent boundary layer, in the absence of temperature gradient, wind speed and water vapour vary linearly with the logarithm of height and their transfer takes place through turbulent processes. The thickness of this layer varies from about 15 meters under very stable conditions to some 150 meters under very turbulent conditions.

In evaporation the basic problem is the eddy diffusion of water vapour. It is generally assumed that the eddy diffusivity of water vapour is equal to the eddy viscosity of momentum. Theoretical and experimental evidence shows that this is a valid assumption.

A review of the evaporation literature shows that the problem of obtaining evaporation by using mass transfer concepts has been attacked along two lines. One approach, the "continuous mixing" approach, is based upon O. G. Sutton's theory (1932 and 1934) which is in fact an extension of Taylor's statistical theory of turbulence (1922). The other, the "mixing length" approach, is based on the turbulence theory developed by Prandtl and Schmidt. The distinction between the two theories is physical as well as mathematical. Physically, the mixing length approach to atmospheric diffusion is based on an analogy with molecular motion. Prandtl visualizes mixing as a discontinuous process, whereby a mass of fluid called an eddy leaves its environment and moves to a new level, there mixing with its new surroundings and eventually losing its identity. The mean distance traveled by the eddy during this process is regarded as the mixing length. The theory requires that the eddy be vertically at rest, relative to its environment, at the beginning and end of the mixing process. This approach recognizes the existence of three layers, the laminar layer, the intermediate layer and the turbulent layer, while the continuous mixing approach considers only the turbulent layer extending down to the very surface. The continuous mixing theory considers an eddy as a mass of fluid which, moving as a separate body, blends continuously with its surroundings. There is no waiting before the mixing takes place. As the surroundings change there is a continuous sharing of the transferable property and a gradual change in the excess or deficit originally possessed by the eddy, until it is undistinguishable from its surroundings and mixing is complete. Prandtl's theory has been successful in explaining observed wind structure in the turbulent layer for adiabatic stratification and is, therefore, suitable for application to the transfer of momentum under such conditions. Since evidence exists that near the ground the diffusion of water vapour is identical with the diffusion of momentum, the theory may also be successful in treating water vapour transfer under the same conditions.

The significant physical difference between the two approaches is that the continuous mixing equations give the decrease of evaporation with distance downwind, while the mixing length equations give point evaporation only. Another important physical difference is that the former is automatically adjusted for varying stability while the latter necessitates complicated computations for such effects. However, both approaches assume that the eddy diffusivities for momentum and water vapour are identical. In this report only the former approach will be tried, because the wind data are too insufficient and unsatisfactory to confidently handle the continuous mixing approach.

There are quite a few methods under the mixing length approach for the evaluation of water vapour flux. So far these methods have been used mainly to compute evaporation from water surfaces. The adoption of a particular method or methods to compute evaporation in the present case will be very much subject to the physical conditions prevailing over the saltcedar stand at Buckeye and the following discussion is included to justify the basis of the selection of a particular method.

For an adiabatic lapse rate, the vertical component of the eddy diffusivity varies linearly with height. Using,

$$\frac{\tau}{\rho} = K_z \frac{\partial \overline{u}}{\partial z} = \text{constant}$$

with

$$\frac{\tau}{\rho} = l^2 \left(\frac{\partial \overline{u}}{\partial z}\right)^2$$

and

$l = k_o\, z$ for a smooth surface

and $l = k_o\, (z + z_o)$ for a rough surface

the following equations result

$K_z = k_o^2\, z^2 \dfrac{\partial \overline{u}}{\partial z}$ for a smooth surface

and $K_z = k_o^2\, (z + z_o)^2 \dfrac{\partial \overline{u}}{\partial z}$ for a rough surface

Using the wind laws of Von Karman and Prandtl for unstratified flow

$\dfrac{\overline{u}}{u_*} = 5.5 + 5.75 \ln \dfrac{u_* z}{v}$ for a smooth surface

and $\overline{u} = \dfrac{u_*}{k_o} \ln \dfrac{z + z_o}{z_o}$ for a rough surface

the following equations result

$K_z = k_o\, u_* z$ for a smooth surface

and $K_z = k_o\, u_* (z + z_o)$ for a rough surface.

But these equations are applicable to an adiabatic atmosphere. For stable or unstable conditions, the eddy diffusivity is no longer a linear function of height. However, the variation is such that if measurements are made below one meter, a linear relation can be assumed. The limitations of instrumentation usually make this procedure impracticable. It is necessary, therefore, to make the observations at higher levels and account for the effects of variable stratification. Since eddy diffusivity is a function of wind shear, the problem is to extend the wind laws to other than adiabatic atmospheres.

It has long been accepted that the influence of stability is reflected in the balance between buoyant and dynamic forces as expressed by the Richardson number (1920),

$$Ri = \frac{\frac{g}{T}\left[\frac{\partial T}{\partial z} + A_1\right]}{\left(\frac{\partial u}{\partial z}\right)^2}$$

Rossby (1935) has attempted to generalize Prandtl's logarithmic wind law. He has assumed that in a stable atmosphere turbulent kinetic energy is proportional to $l_s^2\ C_s^2$ and in the case of an adiabatic atmosphere having the same shear the energy is proportional to $l_\theta^2\, C_s^2$. The difference between the two is equal to the work done in displacing eddies under stable conditions, or,

$$a_1 l_\theta^2 C_s^2 - a_1 l_s^2 C_s^2 = \frac{g}{T} l_s^2 \left[\frac{\partial T}{\partial z} + A_1\right]$$

[He has not explained why he has used the same proportionality coefficient for both terms on the left.] Rearranging,

$$l_\theta^2 C_s^2 = l_s^2 C_s^2 + B_1 \frac{g}{T}\left[\frac{\partial T}{\partial z} + A_1\right] l_s^2 \qquad \text{where } B_1 = \frac{1}{a_1}$$

The equation for the stable mixing length then becomes,

$$l_s = \frac{l_\theta}{\left[1 + \frac{B_1 \frac{g}{T}\left(\frac{\partial T}{\partial z} + A_1\right)}{C_s^2}\right]^{1/2}}$$

Then by using complex mathematics and assuming a constant lapse rate and that the Ri = ∞ when $l_s = 0$, he gives a solution:

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k_o z}\left[1 + B_1 \text{ Ri}\right]^{1/2}$$

Sverdrup (1936) has tried to give a value of B_1. He assumes that both wind and temperature follow the same logarithmic law of the type

$$\frac{\partial \bar{u}}{\partial z} = \mu_1 f(z); \quad B_1 \frac{g}{T}\frac{\partial T}{\partial z} = \mu_2 f(z)$$

where μ_1 and μ_2 are experimentally determined constants. He shows that for great stability

$$\left(\frac{\partial \bar{u}}{\partial z}\right)_s \propto (z + z_o)^{-2/3}$$

and for neutral equilibrium and instability

$$\left(\frac{\partial \bar{u}}{\partial z}\right)_s \propto (z + z_o)^{-1}$$

and

$$C_s = \left(\frac{\partial \bar{u}}{\partial z}\right)_s \propto (z + z_o)^{\frac{(1-\mu_1)}{\mu_1}} \qquad 3 < \mu_1 < \infty$$

and gives $B_1 = 11$

The writer has tried the equation given by Rossby using the value of $B_1 = 11$ as given by Sverdrup. In quite a number of cases the term B_1 Ri is less than -1 thus giving imaginary values for $\frac{\partial \bar{u}}{\partial z}$.

Deacon (1948) has shown that the value of B_1 varies from 2.0 for very unstable conditions to 20.0 for very stable conditions. An examination of the wind data as well as the temperature data at 8- and 4-meter levels shows that an average value of the difference of velocity ranges from 20 to 40 centimeters per second and that of the air temperatures from 0.4 to 0.6 degrees centigrade. Substituting these average values in the equation,

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{z k_o}\left[1 + B_1 \, Ri\right]^{1/2}$$

and using the minimum value of $B_1 = 2$ the term B_1Ri works out to be again less than -1 thus giving imaginary values for $\frac{\partial \bar{u}}{\partial z}$.

Holzman (1943) has suggested a change in the Rossby's equation for stable mixing length and has proposed

$$l_s = l_\theta \left[1 - \frac{\sigma_1 \frac{g}{T}\left(\frac{\partial T}{\partial z} + A_1\right)}{C_s^2} \right]^{1/2}$$

where σ_1 is Holzman's proportionality constant.

And based upon it and using the assumptions made by Sverdrup regarding wind and temperature profiles, he proposes,

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k_o \, z \, (1 - \sigma_1 Ri)^{1/2}}$$

Though no further indication has been given about σ_1 it is perhaps apparent that the evaluation of σ_1 will have to be done in each case keeping in view the critical value of Ri.

An equation for the stable as well as unstable conditions, similar to the aforementioned equation in form but somewhat different in its approach has also been proposed by Budyko (1948). According to his approach the exchange coefficient K_M in the surface layer of height 'h' can be given by,

$$K_M = k_o u_* m z$$

where,

$$m = \left[1 - \frac{g k_o h \Delta T}{T u_*^2 \ln \frac{z_1}{z_2}} \right]^{-1/2}$$

which gives,

$$K_M = k_o^2 z \frac{\Delta u}{\ln \frac{z_1}{z_2}} \left[1 + \frac{gh \Delta T}{T (\Delta u)^2} \ln \frac{z_1}{z_2} \right]$$

where,

$$\Delta T = T_1 - T_2$$

$$\Delta u = u_2 - u_1$$

or solely from the wind profile data,

$$K_M = \frac{k_o^2 u_1 z}{\ln \frac{z_1}{z_2}} \frac{(z_o)^2}{(z_1)^2} \left[\frac{z_1}{z_2} \right]^{\frac{2u_1}{u_2 - u_1}}$$

where Δu is the difference in wind velocities u_1 and u_2 at heights z_1 and z_2 respectively, above the evaporating surface. Both the equations were tried for the analysis of data but the results were not satisfactory. The evapotranspiration calculated ranged from 2 to 200 times greater than that calculated by other methods. The values obtained by using the wind profile data were miserably low and were discarded for the obvious reasons that wind profile data were not good. The apparent reasons for the failure of this equation are the mild wind velocities and factor 'h' representing the thickness of the surface layer. Budyko has recommended a value of 25 to 35 meters for 'h' which according to his numerous observations remains fairly stable under different meteorological conditions. The very low value of Δu makes the negative term within brackets very large and thus causes the whole relationship to crumble down.

Sverdrup in 1936 developed an evaporation equation assuming that the roughness parameter is proportional to the wind speed and that the thickness of the boundary layer is independent of wind speed. Since this assumption was quite contrary to experimental evidence he later revised his approach (1937) using Rossby's results (1935). Using the basic equation for point evaporation E,

$$E = - \frac{0.623 \rho K_z}{P} \frac{\partial e}{\partial z} \qquad (1)$$

and integrating it through the thickness of laminar layer, where all properties were linear with height, he obtained

$$E_d = \frac{0.623 \rho D}{P} \left[\frac{e_o - e_d}{d} \right] \qquad (2)$$

And for the turbulent layer using wind laws

$$\frac{\bar{u}}{u_*} = 5.5 + 5.75 \ln \frac{u_* z}{v} \qquad \text{for a smooth surface (3)}$$

and

$$\bar{u} = \frac{u_*}{k_o} \ln \frac{z + z_o}{z_o} \qquad \text{for a rough surface (4)}$$

and assuming eddy diffusivity to be identical with eddy viscosity and to vary linearly with height in unstratified flow

$$K_z = k_o\, u_* z \qquad \text{for a smooth surface}$$

and $$K_z = k_o\, u_* (z + z_o) = \frac{k_o^2 (z + z_o)\bar{u}}{\ln \frac{z + z_o}{z_o}} \qquad \text{for a rough surface}$$

Since at $z = d$, $K_z = v$ it follows that

$$K_z = k_o\, u_* (d + z_o) = v$$

and that,

$$z_o = \frac{v}{k_o\, u_*} - d \qquad \text{for a smooth surface (5)}$$

For a rough surface it was assumed that z_o equals 0.6 centimeters, independent of wind speed.

Integrating equation (1) between d and z and using values of eddy diffusivity from equation (4) Sverdrup obtained for a smooth or rough surface,

$$E = \frac{0.623\, \rho k_o\, u_* (e_d - e_z)}{\ln \frac{z + z_o}{d + z_o}} \tag{6}$$

And on the basis of results given by Montgomery (1940) taken on the Atlantis showed that,

$$e = e_o - C_1 \ln \frac{z + z_o}{z_o} = e_d - C_1 \ln \frac{z + z_o}{d + z_o} \tag{7}$$

or $$E = \frac{0.623\, \rho k_o u_* C_1}{P} \tag{8}$$

And by using equations (2), (6) and (7) Sverdrup obtained the following equation for point evaporation from a smooth or rough surface:

$$E = \frac{0.623\rho k_o u_*}{P \ln \frac{z + z_o}{d + z_o} + u_*} \qquad (9)$$

And from equations (3), (4) and (5) the approximate values of u_* and z_o for smooth and rough surfaces could be obtained. The value of was given by

$$d = \frac{D}{k_o u_*} \left(\frac{e_o - e_z}{C_1} - \ln \frac{z + z_o}{d + z_o} \right)$$

Using equation (9) and assuming alternately that the surface was smooth and rough Sverdrup concluded that evaporation from a rough surface is twice that for a smooth surface. But Sverdrup has used same value of C_1 from equation (7) for both rough and smooth surface, whereas C_1 itself depends on the nature of the surface. Also the friction velocity for rough flow is approximately twice that for smooth flow and the use of same value of C_1 assumes the same type of surface and its effect on equation (8) is eliminated. Therefore the results show rough surface evaporation to be twice that for smooth surface. Actually, it is evident that values of C_1 computed from the measurements were almost all for smooth flow. Hence Sverdrup's conclusion is not justified and the equation cannot be used for the case of rough flow over saltcedars.

Thornthwaite and Holzman (1939) using the Holzman equation, $\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k_o z (1 - \sigma_1 Ri)^{1/2}}$ as a general wind law generalized the point evaporation equation,

$$E = -\frac{0.623\rho K_z}{P}\frac{\partial e}{\partial z} \tag{10}$$

for thermal stratification by putting $K_z = l_s u_*$, so that

$$E = -\frac{0.623\rho u_* l_s}{P}\frac{\partial e}{\partial z}$$

and by substituting, $l_s = l_\theta \left[1 - \frac{\sigma_1 \frac{g}{T}\left(\frac{\partial T}{\partial z} + A_1\right)}{C_s^2}\right]^{1/2}$

$$E = -\frac{0.623\rho u_* k_o z}{P}(1 - \sigma_1 Ri)^{1/2}$$

where z_o has been ignored as explained below. Using the relationship

$\frac{\partial u}{\partial z} = \frac{\bar{u}_*}{k_o z\,(1 - \sigma_1 Ri)^{1/2}}$ and eliminating u_*

$$E = -\frac{0.623\rho k_o^2 z^2}{P}(1 - \sigma_1 Ri)\frac{\partial \bar{u}}{\partial z}\frac{\partial e}{\partial z}$$

Following Sutton (1936) that near the ground properties vary logarithmically with height they state:

$$E = -\frac{0.623\rho k_o^2 z\,\mu_1}{P}\left[1 - \frac{\sigma_1 \frac{g}{T}\left(\frac{\mu_2}{z}\right) + A_1}{\frac{\mu_1{}^2}{z^2}}\right]\frac{\partial e}{\partial z}$$

The adiabatic lapse rate is small compared to other terms in the numerator of the stability term and can be neglected. Defining

$$s = \frac{\sigma_1 \frac{g}{T}\mu_2}{\mu_1{}^2}$$

and using equations $\mu_1 = \frac{\bar{u}_2 - \bar{u}_1}{\ln\frac{z_2}{z_1}}$ and $\mu_2 = \frac{T_2 - T_1}{\ln\frac{z_2}{z_1}}$

and then integrating the last equation for E from z_1 to z_2

$$E = \frac{0.623\rho k_o^2 (e_1 - e_2)(\bar{u}_2 - \bar{u}_1)}{P \ln \frac{z_2}{z_1}} \left[\frac{1}{\ln \left[\frac{z_2 (1 - s z_1)}{z_1 (1 - s z_2)} \right]} \right]$$

The integration interval was so chosen that z_o is very very less than z_1, and z_o was therefore ignored. For neutral stability (adiabatic conditions), 's' is approximately zero and the equation reduces to,

$$E = \frac{0.623\rho k_o^2 (e_1 - e_2)(\bar{u}_2 - \bar{u}_1)}{P \left[\ln \frac{z_2}{z_1} \right]^2}$$

This adiabatic form, with s = 0 has been widely accepted and recommended for computing evaporation from land surfaces where z_o is a constant. Substituting values for the Buckeye as:

$\rho = 10^{-3}$ gm cm^{-3}

$P = 950$ mb

$z_2 = 8$ meters

$z_1 = 4$ meters

$k_o = 0.4$

$$E = 2.180 \times 10^{-5} \ \Delta e \Delta u = E_A \text{ for adiabatic case} \tag{11}$$

(neglecting z_o which is 4-6 cm and negligible)

where

$\Delta e = e_4 - e_8$ in millibars

$\Delta u = u_8 - u_4$ in meters per second

The subscripts 4, 8 denote the respective observation levels of 8 meters and 4 meters above the average vegetation. And for the stratified atmosphere

$$E = E_A \frac{0.693}{\ln\left[\frac{2-800s}{1-800s}\right]} = E_s \text{ or } = E_A F_s$$

where

F_s = Factor for stratification

$$= \frac{0.693}{\ln\left[\frac{2-800s}{1-800s}\right]}$$

and

$$s = \sigma_1 \frac{g}{T} \frac{\mu_2}{\mu_1^2}$$

which on substituting the values reduces to

$$s = \sigma_1 \; 0.07 \frac{\Delta T}{T\,(\Delta u)^2}$$

Now to check whether this formula can give tangible results under the range of conditions existing at Buckeye the factor 'F_s' has been examined. Under increasing stable conditions the factor 'F_s' would decrease. This decrease is possible if the denominator increases or (2-800s) increases which means (1-800s) should decrease. In the limiting case,

$$800s = 1$$

or

$$\sigma_1 \; 0.07 \frac{\Delta T}{T(\Delta u)^2} = \frac{1}{800}$$

To find the lower limit for σ_1 substituting the maximum observed values for $\Delta T = 2.5^o$ and minimum observed value for $\Delta u = 0.1$ and assuming an average $T = 300^oA$

$$\sigma_1 \approx 0.02$$

Under increasing unstable conditions the factor 'F_s' should increase. This increase is possible if the denominator decreases or the factor (2-800s) decreases. In the limiting case

$$800s = 2$$

or

$$\sigma_1 \; 0.07 \frac{\Delta T}{T \Delta u^2} = \frac{2}{800}$$

To find the upper limit for σ_1 substituting the minimum observed value for $\Delta T \approx 0.1$ and maximum observed value for $\Delta u \approx 1.0$ and assuming an average $T = 300^oA$

$$\sigma_1 = 100$$

Thus under the prevailing temperature and wind conditions the range of σ_1 is 0.02 to 100. Such a wide range of σ_1 is difficult to handle under the existing circumstances in the absence of a very precise actual hourly water consumption data.

Montgomery (1940), Sverdrup (1946) and Norris (1948) agree on evaporation from a smooth surface. For a rough surface the latter two differ from Montgomery by a factor of four. For the atmospheric layer in which shearing stress may be considered constant, all three writers used the wind laws for unstratified flow, expressed by equations 3 and 4. In the derivation a slightly different form for eddy diffusivity has been used.

Since Z — d (the thickness of boundary layer)

K_z — D (molecular vapour diffusivity) and continuity was maintained by using

$$K_z = D + k_o u_* z \qquad \text{for a smooth surface}$$

and $$K_z = D + k_o u_* (z + z_o) \qquad \text{for a rough surface.}$$

Montgomery's equations indicate that the evaporation coefficient for a rough surface is one half that for a smooth surface. Since the friction velocity for a rough surface is almost twice that for a smooth surface, he concludes that evaporation from a rough surface is nearly the same as for a smooth surface. From a physical point of view it is difficult to accept Montgomery's conclusions that evaporation is independent of surface roughness. Examination of Sverdrup and Norris development of equations for a rough surface reveals conclusive evidence that Montgomery's approach is inconsistent. Sverdrup writes that:

$$K_z \text{ (vapour)} = D + k_o u_{*r} (z + z_o)$$

where u_{*r} = Friction velocity for a rough surface

$$= v + k_o u_{*r} (z + z_o) = K_z \text{ (momentum)}$$

and then by integrating the basic equation,

$$E = - \frac{0.623 \rho K_z}{P} \frac{\partial e}{\partial z}$$

and integrating it from zero to any height, and ignoring D he obtains,

$$E = \frac{0.623 \rho k_o u_{*r} (e_o - e_z)}{P \ln \frac{z + z_o}{z_o}}$$

If integrating between any two levels an equation identical with Thornthwaite-Holzman equation will result. The Norris equation though quite good for both stratified and unstratified flow is based upon certain assumptions which are good for flow over open water. Hence the examination of his equation for the present field conditions is not done.

Thus a review of the foregoing mass-transfer equations shows that almost all of them, which have been generalized for stable as well as unstable atmosphere, can hardly be considered satisfactory for analyzing the present data. It is impracticable to determine the values of B_1 and σ_1 in the Sverdrup and Holzman equations under different conditions of stability because the actual water consumption data are not available for periods of less than 24 hours duration.

Sverdrup (1936) recommends that the wind law for adiabatic case is a good approximation for instability. H. L. Penman (1956) also recommends the use of adiabatic equations for computing daily evaporation for "days on which atmospheric stability is such as to justify use of the (adiabatic) equation." Guided by these indications a detailed study of the entire available data for the hourly differences of temperatures $(T_8 - T_4)$, vapour pressures $(e_4 - e_8)$ and the wind velocity has been done. Quite unexpectedly, not for a single day over the entire period do the vapour pressure and wind data show any correlation with the stability or otherwise of the atmosphere. Average daily $(T_8 - T_4)$ figures were also compared with the actual daily water consumption data. It was found that, whereas there was a correlation between the mean daily air temperatures and the actual water consumption and that the lapse rate was predominantly stable, there was practically no increase or decrease of evaporation with the decrease or increase of average $(T_8 - T_4)$. Prompted by this indication the writer attempted to find out the evapotranspiration using Thornthwaite-Holzman equation for neutral stability, i.e.

$$E = \frac{0.623 \rho k_o^2 (e_1 - e_2)(u_2 - u_1)}{P \left[\ln \frac{z_2}{z_1} \right]^2}$$

which reduces to the following simple form, as already detailed,

$$E = 2.180 \times 10^{-5} \Delta u \Delta e$$

Sample calculations are shown in Table 2 and the computed evapotranspirations for each day are in Table 3. For a quick comparison the ratio of the computed and the water budget evaporation is also written. Rejecting a few extremely low and high values which are obvious anomalies the calculated daily evapotranspiration ranges from 0.6 to 1.8 times the measured one and on the average the calculated evapotranspiration is 1.20 times the measured evapotranspiration. Apparently in the present case these higher results may be due to the fact that the effects of stability have not been properly accounted for. In order to check this an examination of the average daily atmospheric stratification as indicated by the differences of air temperatures between the two levels ΔT and the evapotranspiration has been carried out. The comparison shows no correlation between them. The absence of any

Table 2. Sample calculations used in aerodynamical approach.

Sept. 5 Hours	$e_4 - e_8$ $\triangle e$ mb	$u_8 - u_4$ $\triangle u$ m/sec	$T_8 - T_4$ $\triangle T$ C°	$\triangle u \triangle e$	Evapo-transpiration cm/sec x 10^{-5}
1	0.075	0.3	0.1	0.023	0.050
2	0.002	0.7	0.0	0.0014	0.003
3	0.855	0.6	0.2	0.513	1.114
4	0.341	0.4	0.2	0.136	0.295
5	0.612	0.3	0.3	0.184	0.399
6	0.631	0.4	0.6	0.252	0.547
7	0.342	0.3	0.2	0.103	0.224
8	1.617	0.1	-0.2	0.162	0.351
9	0.349	0.1	0.2	0.035	0.076
10	1.086	0.1	0.0	0.109	0.237
11	1.545	0.3	-0.3	0.464	1.007
12	0.857	0.1	-0.2	0.086	0.187
13	1.058	0.35	-0.8	0.370	0.803
14	1.084	0.35	-0.6	0.379	0.823
15	0.476	0.3	-0.3	0.143	0.310
16	1.243	0.2	-0.5	0.249	0.541
17	0.049	0.35	0.4	0.017	0.037
18	0.00	1.1	0.0	0.000	0.000
19	0.578	0.7	0.1	0.405	0.879
20	0.842	0.5	0.0	0.421	0.914
21	0.636	0.45	0.0	0.286	0.621
22	0.648	0.4	0.0	0.259	0.562
23	1.190	0.1	0.5	0.119	0.258
24	0.728	0.25	0.8	0.182	0.395
Average					0.38 cm/day

Table 3. Measured daily water consumption compared with evapotranspiration calculated by aerodynamical method.

Sept. 1962	Measured daily water consumption (cm/day)	Evapotranspiration by aerodynamic method (cm/day)	Computed evapotranspiration/Measured evapotranspiration (ratio)
1	0.54	--	--
2	0.57	--	--
3	0.60	--	--
4	0.58	0.76	1.31
5	0.60	0.38	0.63
6	0.54	0.86	1.60
7	0.55	0.67	1.20
8	0.54	0.32	2.17
9	0.52	0.59	1.13
10	0.53	0.73	1.39
11	0.54	0.60	1.11
12	0.59	0.65	1.10
13	0.60	0.70	1.17
14	0.58	0.44	0.76
15	0.60	0.86	1.43
16	0.59	data missing	--
17	0.63	"	--
18	0.58	1.55	2.67
19	0.59	0.92	1.56
20	0.60	1.19	1.97
21	0.45	2.56	5.69
22	0.28	0.54	1.97
23	doubtful	0.64	--
24	"	0.61	--
25	"	0.74	--
26	"	0.44	--
27	"	0.29	--
28	"	0.53	--
29	"	1.18	--
30	"	0.79	--

Table 3. (Continued)

Oct. 1962	Measured daily water consumption (cm/day)	Evapotranspiration by aerodynamic method (cm/day)	Computed evapotranspiration/Measured evapotranspiration (ratio)
1	0.34	wind data missing	--
2	0.34	"	--
3	0.48	"	--
4	0.44	"	--
5	0.43	"	--
6	0.47	"	--
7	0.47	"	--
8	0.46	"	--
9	0.45	"	--
10	0.42	"	--
11	0.42	"	--
12	0.44	"	--
13	0.44	"	--
14	0.46	"	--
15	0.42	"	--
16	0.45	"	--
17	0.44	"	--
18	0.41	0.78	1.88
19	0.41	0.37	0.90
20	0.42	0.15	0.35
21	0.42	0.16	0.39
22	0.45	--	--
23	0.45	0.33	0.73
24	0.41	0.35	0.85
25	0.43	0.77	1.78
26	0.42	0.36	0.84
27	0.46	0.43	0.91
28	0.46	0.40	0.86
29	0.51	1.47	2.88
30	0.41	1.58	3.81
31	0.41	1.52	3.67
Average			1.2

correlation between the hourly values of ΔT and Δe has already been mentioned. All these indications go a long way in pointing out that the effect of ignoring stability is at least not the only reason for higher computed figures of evapotranspiration. The other possibilities can be incorrect measurements due to defects in instrumentation or the advection of moisture from the adjoining fields. A perusal of the entire record of wind speeds, dry and wet bulb temperatures shows that quite often the respective instruments have not worked either regularly or properly. The wind speed data are not only missing for the greater part of the entire period but are also very unsatisfactory and unreliable. The possibility of errors due to advection of moisture from the adjoining areas can also be significant for the reasons that observations of humidity are being taken at levels which are too high above the average vegetation.

Conclusions on Aerodynamic Approach

1. Computed evapotranspiration is 1.20 times measured evapotranspiration.

2. Equations which are in the generalized form break down in the present situation.

3. The Thornthwaite-Holzman equation in the adiabatic form promises to yield good results, but a more rigorous application of this equation needs to be done with more reliable data to arrive at some definite understanding.

4. The high values resulting from the use of this approach are more due to incorrect measurements and inappropriate location of instruments than due to neglecting the stability of the atmosphere.

5. The presence of non-linear relationships between the different parameters involved in this approach restricts the application of this method to short intervals of time. An interval of an hour or so can be most appropriate to get more precise results.

CHAPTER IV

ENERGY BUDGET METHODS

The energy equation which is based upon the principle of conservation of energy valid for a column of soil can be written as:

$$Q_s - Q_r - Q_b - LE - H + Q_a - G = 0 \qquad (12)$$

i.e., the sum of solar radiation (Q_s), the reflected solar radiation (Q_r), the effective long wave back radiation (Q_b), the energy used for evaporation (LE), the energy conducted away as sensible heat (H), the energy advected into or out of the body of soil (Q_a), and the energy stored in the body of soil (G), is equal to zero. The resultant of the first three terms in the equation is the net radiation and can be denoted by R. Further the component Q_a is insignificant and is always neglected except over water. Thus the equation becomes

$$R = H + G + LE \qquad (13)$$

From which we can express evaporation as:

$$E = \frac{1}{L}\left[\frac{R - G}{1 + R_B}\right] \qquad (14)$$

where

R_B = Bowen Ratio

$$= \frac{H}{LE} \qquad (15)$$

I. S. Bowen after whose name the ratio of sensible heat and the latent heat has been named as "Bowen Ratio" (1926) attempted to relate R_B to easily measurable quantities by starting from two fundamental equations assuming molecular processes for diffusion as well as conduction. He writes, using his notations,

$$Q_e = D_1 \, a \, \frac{(\Theta_1 - \Theta_2)}{l} \tag{16}$$

and

$$Q_h = D_2 \, a \, \frac{(\phi_1 - \phi_2)}{l} \tag{17}$$

In developing the value of his ratio Bowen considered the following three cases of which the first two represent limiting conditions:

Case I: Suppose the wind speed has a constant value from the surface to a small height b, above the water surface. If the body of water is large and the wind speed above height b is small relative to that below, then the layer of air from the surface to b will have its temperature changed to that of the water surface and will become saturated. In the limiting case there will be no transfer of heat or water vapour across b. The specific heat of the air below b will change from ϕ_2 to ϕ_1 and its latent heat per unit volume from Θ_2 to Θ_1. For the water body, the ratio of heat loss by conduction to that lost by evaporation will be, $R_B = \dfrac{\phi_1 - \phi_2}{\Theta_1 - \Theta_2}$

since this is the ratio of the two kinds of energy taken up by each cubic centimeter of air passing over the water body. This case represents a condition where the whole quantity of air below height b is completely changed in temperature and water content to that of the layer of air in contact with the water, so that the diffusion across b is not a factor. However, if the limiting case is not met with, that is, there is some transfer across b, such transfer will occur in a way which increases the loss by the process having the larger diffusion coefficient relative to the loss by the other method. Hence, under any wind condition above b, since D_1 is greater than D_2.

$$R_B \leqslant \frac{\phi_1 - \phi_2}{\Theta_1 - \Theta_2}$$

Case II: Suppose there is no wind between the water surface and height b. If the area of the water surface is small and wind speed above b is large, the water vapour and heat diffused through the stationary layer from the water surface to height b will be carried away immediately, and at height b, $\phi = \phi_2$ and $\Theta = \Theta_2$. Under these conditions these rates at which heat and water vapour leave the water surface are determined only by the rate of molecular diffusion below b, and the ratio is:

$$R_B = \frac{D_2 \, (\phi_1 - \phi_2)}{D_1 \, (\Theta_1 - \Theta_2)}$$

This case represents conditions where diffusion is the dominating factor, with the heat and water vapour being carried away after diffusing through the stationary layer. However, if the limiting conditions are not quite met with and heat and water vapour are not immediately carried away, then the temperature and vapour density will build up on the top side of the layer and thus decrease the gradient in the stationary layer. This further slows down diffusion. The process with the larger diffusion coefficient will build up faster and hence be retarded more than the other. Therefore, $R_B \geqq \frac{D_2 (\phi_1 - \phi_2)}{D_1 (\Theta_1 - \Theta_2)}$

Case III. Using the above two limiting equations, Bowen derived two differential equations that are valid for all points above the water. These equations were solved by assuming that the wind speed was proportional to any positive power, n, of the distance above the surface. This led to the following results:

$$R_B = \left[\frac{D_2}{D_1}\right]^{\frac{n+1}{n+2}} \left[\frac{\phi_1 - \phi_2}{\Theta_1 - \Theta_2}\right]$$

Bowen thus showed that the value of the ratio under any conditions was between the values given by Case I and Case II. Making use of well known relations of D_1 and D_2 and temperature Bowen found that:

$$R_B = 0.61 \frac{T_o - T_a}{e_o - e_a} \frac{P}{1000}$$

where T_o and T_a are absolute temperatures of the surface and air.

As it is evident, the fundamental equations from which Bowen starts apply to transfer of heat and vapour by processes of molecular diffusion. It might, therefore, be concluded that the computed Bowen ratio is valid only for conditions of laminar flow. However, as Bowen states, it can be expected "that heat losses by evaporation and diffusion will follow the same laws and will be affected in the same way by convection." This gives an idea that the ratio between the heat losses will be independent of the state of turbulence.

In order to establish the validity of the Bowen ratio and discover its limitations Cummings and Richardson (1927) undertook to test the theory and concluded that "Bowen's theoretical conclusions respecting this ratio were found to be consistent with observations."

D. W. Pritchard has also derived the Bowen ratio from mass transfer concepts, using Sverdrup's equations as a basis. Sverdrup derived that (1946) for latent heat and sensible heat transfer:

$$LE = L\rho k_o \frac{0.623}{P} \gamma_a \Gamma_E (e_o - e_a) \bar{u}_a$$

and

$$H = c_p k_o \gamma_a \Gamma_H (T_o - T_a) \bar{u}_a$$

where Γ_E and Γ_H are coefficients for evaporation and sensible heat transport and γ_a the resistance coefficient at height a_1 and using c_p = 0.241 cal/gm/c^o and L = 585.0 cal/gm (the specific heat of air at constant pressure of 1 atmosphere and 100^{o}C and the latent heat of evaporation at 20^{o}C) and for rough flow $\Gamma_E = \Gamma_H$, derived that:

$$R = \frac{0.66P}{1000} \frac{(T_o - T_a)}{(e_o - e_a)}$$

The various developments of the ratio of heat losses by convection to those be evaporation depend fundamentally on the assumption that the eddy diffusivities for these two quantities are identical. There is evidence indicating that this assumption is not always valid.

Pasquill (1949) made an experimental study of the various factors involved in the turbulent transfer of water vapour and heat in the lowest layer of the atmosphere over open grasslands. From these experimental data he demonstrated that the eddy diffusivity for heat is reasonably equivalent to that for water vapour under stable conditions, but is substantially increased for unstable conditions and may be twice as great.

In a treatment of convection near the ground, Sutton (1948) has given theoretical evidence for the existence of a difference in heat and momentum transfers.

Priestley and Swinbank (1947) recently put forward a modification of the theory of turbulent transfer of heat, taking into account buoyancy effects. This study indicates that the eddy diffusivity for heat differs from that for other physical properties.

Recently Rider (1954) gave individual values leading to mean ratios:

$$\frac{K_H}{K_V} = 1.14 \pm 0.06; \quad \frac{K_H}{K_M} = 1.48 \pm 0.27; \quad \frac{K_V}{K_M} = 1.23 \pm 0.17$$

In the heat balance equation:

$$R = LE + H + G$$

or

$$LE = \frac{R - G}{1 + \frac{K_H}{K_V} \frac{c_p}{l} \frac{\Delta T}{\Delta q}}$$

where

$$H = -\rho K_H c_p \frac{(T_2 - T_1)}{z_2 - z_1} = \rho K_H c_p \frac{\Delta T}{\Delta z}$$

and

$$LE = -\rho K_V L \frac{q_2 - q_1}{z_2 - z_1} = \rho K_V L \frac{\Delta q}{\Delta z}$$

where $\Delta T = T_1 - T_2$, $\Delta q = q_1 - q_2$ and $\Delta z = z_2 - z_1$ and q denotes specific humidity, or substituting for, $c_p = 0.241$ cal/gm/c°, $L = 585.0$ cal/gm and replacing Δq the difference of specific humidity by the respective vapour pressure difference the equation reduces to,

$$LE = \frac{R - G}{1 + \frac{K_H}{K_V} 0.63 \frac{\Delta T}{\Delta e}} \tag{18}$$

Now if the studies conducted by the aforementioned writers be considered and K_H be assumed greater than K_V rather than equal, the magnitude of the term $\left[1 + \frac{K_H}{K_V} 0.63 \frac{\Delta T}{\Delta e}\right]$ in the denominator of the above equation (18) decreases when T_1 is less than T_2 thereby increasing the value of LE. Thus any attempt to compute LE assuming $K_V = K_H$ should have the effect of giving comparatively lower evaporation rates under stable conditions. Keeping this in mind, and recalling that the lapse rate was predominantly stable at Buckeye in which case K_H and K_V should be about equal, a few days data at the Buckeye saltcedar stand were first quickly analyzed using daily averages. Quite unexpectedly the results, on the average, were higher than the actual measured water consumption figures. In order to verify the applicability or otherwise of the Bowen ratio method the entire available data were analyzed and the evapotranspiration was computed in each case and is shown in Table 4. The computed evaporation ranges mainly between $\pm$ 50 percent of the measured evaporation. The average evaporation is 22% higher than measured one. The wide range of variation as well as a significant high average apparently indicates that Bowen ratio method is not very successful in existing circumstances if K_H is taken equal to K_V.

Trial calculations of the hour by hour evapotranspiration were also done for a period of eight days in the middle of September assuming $K_H = K_V$. The results of these computations are shown in Table 5. The evaporation values are significantly higher than the actual figures. The average computed evapotranspiration is 207% of the measured evapotranspiration. This also indicates that Bowen ratio method does not prove successful in the present situation if K_H is taken equal to K_V.

A review of the evaporation equation

$$LE = \frac{R - G}{1 + \frac{K_H}{K_V} 0.63 \frac{\Delta T}{\Delta e}}$$

gives an indication that under the condition, T_1 less than T_2, LE will decrease if the second term in the denominator decreases. This is possible if K_H is less than K_V under stable conditions or if otherwise the terms accounting for the sensible heat tend to zero. Assuming the latter conditions, the heat balance equation reduces to,

$$LE = R - G$$

This equation has been applied to the daily averages of net radiation and flux of heat into the soil. The results are shown in Table 6. The average evapotranspiration is 83% of the measured evapotranspiration. Considering these two cases together, evapotranspiration can be equal to measured evapotranspiration if in the Bowen ratio method the average value for the ratio R_B is -0.17. But in the equation,

$$LE = \frac{R - G}{1 + \frac{K_H}{K_V} 0.63 \frac{\Delta T}{\Delta e}}$$

substituting, $LE = 1.22$

$$R - G = 0.83$$

$$0.63 \frac{\Delta T}{\Delta e} = R_B \qquad \text{(For } K_H = K_V\text{)}$$

$$= -0.32$$

These two values of R_B will agree if $\frac{K_H}{K_V} = 0.53$.

Thus under the conditions prevailing at Buckeye the general assumption of taking $K_H = K_V$ under stable conditions does not hold good.

The coefficient of thermal diffusion is probably less than the coefficient of vapour diffusivity. Though a determination of a precise range of the ratio $\frac{K_H}{K_V}$ is not possible in the absence of hourly water consumption data, yet, an average value of 0.53 for conditions similar to those analyzed herein is likely to give good results.

In order to verify the application and efficiency of the ratio thus found, the evapotranspiration rates for the entire period, both on hourly basis as well as daily average basis, are recalculated and shown in Tables 4 and 5. The results are very consistent and mostly range between ± 8% of the actual figures. The average evapotranspiration in the case of hourly calculation is 8% higher than the measured evapotranspiration and that in the case of daily average basis is 3-4% lower than the actual measured figure. These results are well within the acceptable range of accuracy and prove that the ratio of $\frac{K_H}{K_V}$ adopted herein is reasonable. Thus a common understanding that under normal circumstances the coefficient of thermal diffusivity is equal to that of vapour diffusion is not applicable in the present situation. With the proper evaluation of this ratio the Bowen ratio method can be safely applied on daily averages to get satisfactory results. Sample calculations are shown in Table 7.

In the calculations the flux of heat into the soil has been computed by the relationship:

$$G = \rho_s c_s \frac{\Delta \overline{T}}{\Delta t} \Delta z \tag{19}$$

The density of the soil has been taken as 1.38 gms/cm^3. This value is the result of analysis of numerous soil samples which are taken from different locations at different depths. A value of 0.18 has been taken for the heat capacity of the soil which is generally between 0.18-0.20. The average temperature of the soil column was calculated graphically for a depth of 12 feet below natural surface.

Table 4. Measured evapotranspiration rate and evapotranspiration calculated by energy budget method (Bowen ratio method).

Month Sept.	Measured evapotranspiration cm/day	Evapotranspiration by Bowen ratio method with $K_H = K_V$ cm/day	with $K_H = 0.53\ K_V$ cm/day	Computed Evapotranspiration/Measured Evapotranspiration when $K_H = K_V$ ratio	when $K_H = 0.53\ K_V$ ratio
1	0.54	radiation data missing		--	--
2	0.57	"	"	--	--
3	0.60	"	"	--	--
4	0.58	"	"	--	--
5	0.60	"	"	--	--
6	0.54	"	"	--	--
7	0.55	"	"	--	--
8	0.54	"	"	--	--
9	0.52	"	"	--	--
10	0.53	"	"	--	--
11	0.54	"	"	--	--
12	0.59	0.71	0.60	1.20	1.01
13	0.60	0.65	0.51	1.08	0.85
14	0.58	0.81	0.64	1.39	1.11
15	0.60	0.78	0.65	1.30	1.08
16	0.59	0.42	0.55	0.71	0.93
17	0.63	0.68	0.57	1.08	0.90
18	0.58	0.55	0.50	0.95	0.85
19	0.59	0.74	0.56	1.25	0.95
20	0.60	0.40	0.40	0.67	0.66
21	0.45	0.52	0.51	1.15	1.12
22	0.28	0.62	0.61	--	--
23	doubtful	0.56	0.55	--	--
24	"	0.55	0.55	--	--
25	"	0.43	0.41	--	--
26	"	0.35	0.34	--	--
27	"	0.64	0.58	--	--
28	"	0.54	0.53	--	--
29	"	0.52	0.50	--	--
30	"	0.60	0.54	--	--

Table 4. (Continued)

Month Oct.	Measured evapotranspiration	Evapotranspiration by Bowen ratio method		Computed Evapotranspiration/Measured Evapotranspiration	
		with $K_H = K_V$	with $K_H = 0.53\ K_V$	when $K_H = K_V$	when $K_H = 0.53\ K_V$
	cm/day	cm/day	cm/day	ratio	ratio
1	0.34	0.54	0.49	1.59	1.44
2	0.34	0.91	0.59	2.68	1.74
3	0.48	0.51	0.48	1.06	1.00
4	0.44	0.45	0.44	1.02	1.00
5	0.43	0.45	0.44	1.05	1.02
6	0.47	0.50	0.45	1.06	0.96
7	0.47	0.54	0.47	1.15	1.00
8	0.46	0.47	0.43	1.02	0.92
9	0.45	0.71	0.51	1.58	1.13
10	0.48	0.71	0.51	1.48	1.06
11	0.42	1.67	0.77	3.98	1.83
12	0.44	0.41	0.36	0.93	0.81
13	0.44	0.30	0.43	0.68	0.97
14	0.47	--	--	--	--
15	0.42	--	0.81	--	1.93
16	0.44	0.39	0.38	0.89	0.86
17	0.44	0.13	0.19	0.29	0.42
18	0.40	0.21	0.28	0.52	0.68
19	0.41	0.24	0.26	0.59	0.63
20	0.42	--	--	--	--
21	0.42	1.27	0.53	3.02	1.27
22	0.45	0.34	0.32	0.76	0.72
23	0.45	2.74	0.47	6.08	1.04
24	0.41	1.28	0.49	3.12	1.20
25	0.43	0.40	0.33	0.93	0.76
26	0.42	0.35	0.30	0.83	0.72
27	0.47	0.58	0.33	1.23	0.69
28	0.47	0.31	0.28	0.66	0.59
29	0.51	0.32	0.24	0.63	0.46
30	0.41	0.24	0.21	0.59	0.51
31	0.41	0.48	0.36	1.17	0.88
Average				1.22	0.97

Table 5. Measured evapotranspiration rate and evapotranspiration by energy budget methods.

Month Sept.	Measured evapo-transpi-ration	Evapotranspiration by Bowen ratio method (daily averages)		Evapotranspiration by Bowen ratio method (hourly basis)		Evapotranspiration by Bowen ratio method (daily)/ Measured evapotranspiration		Evapotranspiration by Bowen ratio method (hourly)/ Measured evapotranspiration	
		when $K_H = K_V$	when $K_H = 0.53\ K_V$	when $K_H = K_V$	when $K_H = 0.53\ K_V$	when $K_H = K_V$	when $K_H = 0.53\ K_V$	when $K_H = K_V$	when $K_H = 0.53\ K_V$
	cm/day	cm/day	cm/day	cm/day	cm/day	ratio	ratio	ratio	ratio
12	0.59	0.71	0.59	1.29	0.73	1.20	1.01	2.19	1.23
13	0.60	0.65	0.51	2.12	0.70	1.08	0.85	3.53	1.16
14	0.58	0.81	0.64	1.51	0.62	1.39	1.11	2.61	1.06
15	0.60	0.78	0.65	0.83	0.63	1.30	1.08	1.38	1.05
16	0.59	0.42	0.55	0.79	0.62	0.71	0.93	1.34	1.05
17	0.63	0.68	0.57	0.99	0.63	1.08	0.90	1.57	1.00
18	0.58	0.55	0.50	1.10	0.69	0.95	0.85	1.90	1.18
19	0.59	0.74	0.56	1.20	0.67	1.25	0.95	2.03	0.93
Average ratios						1.12	0.96	2.07	1.08

Table 6. Measured evapotranspiration rate and evapotranspiration by energy budget method by ignoring component of sensible heat.

Month Sept.	Measured evapotranspiration cm/day	Evapotranspiration by energy budget method cm/day	Computed evapotranspiration/Measured evapotranspiration ratio
1	0.54	radiation data missing	--
2	0.57	"	--
3	0.60	"	--
4	0.58	"	--
5	0.60	"	--
6	0.54	"	--
7	0.55	"	--
8	0.54	"	--
9	0.52	"	--
10	0.53	"	--
11	0.54	"	--
12	0.59	0.58	0.98
13	0.60	0.41	0.70
14	0.58	0.54	0.93
15	0.60	0.54	0.90
16	0.59	0.48	0.81
17	0.63	0.48	0.77
18	0.58	0.45	0.78
19	0.59	0.69	1.18
20	0.60	0.39	0.64
21	0.45	0.49	1.10
22	0.28	0.60	--
23	doubtful	0.54	--
24	"	0.54	--
25	"	0.41	--
26	"	0.35	--
27	"	0.53	--
28	"	0.52	--
29	"	0.48	--
30	"	0.48	--

Table 6. (Continued)

Month Oct.	Measured evapotranspiration cm/day	Evapotranspiration by energy budget method cm/day	Computed evapotranspiration/Measured evapotranspiration ratio
1	0.34	0.48	1.41
2	0.34	0.46	1.36
3	0.48	0.45	0.93
4	0.44	0.43	0.98
5	0.43	0.43	1.00
6	0.47	0.41	0.89
7	0.47	0.41	0.89
8	0.46	0.39	0.85
9	0.45	0.39	0.87
10	0.42	0.39	0.92
11	0.42	0.48	1.14
12	0.44	0.32	0.73
13	0.44	0.37	0.84
14	0.47	0.38	0.81
15	0.42	0.41	0.96
16	0.44	0.37	0.84
17	0.44	0.35	0.78
18	0.40	0.45	1.13
19	0.41	0.29	0.69
20	0.42	0.34	0.80
21	0.42	0.32	0.76
22	0.45	0.32	0.71
23	0.45	0.27	0.60
24	0.41	0.29	0.71
25	0.43	0.28	0.64
26	0.42	0.27	0.63
27	0.47	0.36	0.78
28	0.47	0.26	0.56
29	0.51	0.18	0.36
30	0.41	0.19	0.46
31	0.41	0.29	0.69
Average ratio			0.83

Table 7. Sample calculations to compute evapotranspiration by energy budget methods, hourly basis, daily basis, and by ignoring H.

Sept. 12 Hours	Net radiation cal./ $cm.^2 min.$ R	Difference of vapour pressures millibars Δe	Difference of air temperatures c^o ΔT	Difference of soil temperatures c^o ΔT_s	Soil heat flux cal./ $cm.^2 min.$ G	Bowen ratio when $K_H = K_V$	Evapotranspiration cal/ $cm.^2 min.$ when $K_H = K_V$	when $K_H = 0.53 K_V$
1	-0.060	1.363	-1.5	-0.13	-0.197	-0.694	0.447	0.217
2	-0.069	0.776	-1.2	-0.07	-0.106	-0.974	1.423	0.764
3	-0.072	1.455	-1.0	-0.08	-0.121	-0.433	0.087	0.064
4	-0.068	1.484	-1.7	-0.00	-0.121	-0.722	0.190	0.086
5	-0.041	1.027	-1.5	-0.08	-0.121	-0.922	1.025	0.157
6	-0.009	1.013	-1.2	-0.05	-0.076	-0.745	0.262	0.110
7	0.009	0.374	-0.9	-0.15	-0.227	-1.517	-ve	
8	0.185	1.494	-0.4	-0.13	-0.197	-0.169	0.460	0.420
9	0.441	1.566	-0.6	-0.05	-0.076	-0.242	0.682	0.594
10	0.654	2.449	-1.1	-0.25	-0.379	-0.283	1.440	1.220
11	0.811	1.893	-0.5	0.45	0.682	-0.166	0.154	0.141
12	0.650	3.652	-0.3	0.23	0.348	-0.052	0.318	0.311
13	0.920	1.515	-0.3	0.02	0.030	-0.125	1.017	0.944
14	0.974	2.700	+0.3	0.18	0.272	+0.930	0.363	0.471
15	0.707	1.578	-0.4	0.12	0.182	-0.160	0.625	0.574
16	0.297	1.242	-0.2	0.01	0.015	-0.101	0.313	0.298
17	0.207	1.282	-0.3	0.34	0.515	-0.147	-ve	-ve
18	0.009	1.514	-0.3	0.12	0.182	-0.125	-ve	-ve
19	-0.047	2.761	-0.7	0.07	0.106	-0.159	-ve	-ve
20	-0.052	1.362	-1.4	0.02	0.030	-0.647	-ve	-ve
21	-0.049	2.989	-1.6	-0.15	-0.227	-0.337	0.268	0.217
22	-0.039	2.449	-1.5	-0.07	-0.106	-0.387	0.109	0.084

Table 7. (Continued)

Sept. 12 Hours	Net radiation cal./ cm.^{2}min. R	Difference of vapour pressures millibars Δe	Difference of air temperatures c^o ΔT	Difference of soil temperatures c^o ΔT_s	Soil heat flux cal./ cm.^{2}min. G	Bowen ratio when $K_H = K_V$	Evapotranspiration cal/ cm.^{2} min. when $K_H = K_V$	when K_H = 0.53 K_V
23	-0.036	1.292	-1.0	-0.10	-0.152	-0.497	0.231	-ve
24	-0.081	1.140	-0.5	-0.03	-0.046	-0.276	-ve	

1. Hence E [average of hourly] = 1.29 cm/day (when $K_H = K_V$)
   0.73 cm/day (when $K_H/K_V = 0.53$)

2. Net R = 314 ly/day

   G = 20 ly/day

$$LE = \frac{R - G}{1 + \frac{c_p}{L}\frac{\Delta T}{\Delta q}\frac{K_H}{K_V}} = 422 \text{ ly/day}$$

   Evapotranspiration on daily basis = 0.71 cm/day (when $K_H = K_V$)
   = 0.60 cm/day (when $K_H/K_V = 0.53$)

3. By ignoring H in daily basis computations

   Evapotranspiration = 0.58 cm/day

Conclusions from Energy Budget Studies

1. The computed evapotranspiration is 3% less than the measured evapotranspiration.

2. The average ratio K_H/K_V under stable conditions is 0.53. The usual assumption of taking $K_H = K_V$ does not hold good in the present case.

3. The average value for Bowen ratio comes out as -0.17.

4. Contrary to the general opinion, the component of sensible heat is not only significant but is in a downward direction under the present circumstances and cannot be ignored in the calculations of evapotranspiration.

5. When computing over daily intervals of time the component flux of heat into the soil can be safely neglected.

6. During early morning hours or in the later part of the night or otherwise when the quantity R - G is either zero or negative the method fails. However, no significant error is caused if the evaporation at these moments is neglected as it is invariably very small.

7. Under stable as well as unstable conditions as the difference in air temperatures at the two levels increases and the difference in vapour pressure decreases the method breaks down.

8. Under stable conditions when the difference ΔT increases, the results get rapidly farther from the actual measured figures.

9. Though no optimum period could be ascertained over which the energy budget methods be applied to yield results within reasonable percentage of accuracy, yet, the computations show that when energy budget method is applied over a period of a day or so the results are satisfactory provided due attention is given to the variation of K_H/K_V with stability.

10. A comparison of the results from energy budget methods and the aerodynamic method shows that the former give more consistent results.

CHAPTER V

CLIMATOLOGICAL METHODS

Thornthwaite (1948) has pointed out that, when the root zone of the soil is well supplied with water, the amount used by the vegetation depends more on the amount of solar energy received by the surface and the resultant temperature than on the kind of vegetation growing in the area. The water loss under optimum conditions of soil moisture, E_o, thus appears to be determined principally by climatic conditions and especially by the radiative energy input. A few of the more widely used methods for calculating evapotranspiration under such conditions can be grouped under two categories: (1) those based on air temperature, and (2) those based on all components of the energy balance equation. Consumptive use by saltcedars at Buckeye growing in six lysimeter tanks with water table maintained at 5 to 9 feet below land surface can very well be expected to be equal to that consumed under optimum conditions of soil moisture. In any case the use of these methods can furnish us the upper limit for the loss of water and can serve as a control.

A. Temperature Methods

Thornthwaite.--As a result of his studies on 21 irrigation projects in the western United States he gives an empirical relationship between potential evapotranspiration, E_o, and the mean monthly temperature T, in ^oC.

$$E_o = 16\left[\frac{10\,T}{I}\right]^a \quad \text{mm/month} \tag{20}$$

where $a = [0.675\ I^3 - 77.1\ I^2 + 17920\ I + 492390]\ 10^{-6}$

and $I = \sum_{1}^{12} \left[\frac{T}{5}\right]^{1.514}$

This relation is only valid when $0 \leq T \leq 26.5^oC$.

When the mean monthly temperature exceeds 26.5^oC, the potential evapotranspiration is considered to be a function of temperature only. The mean temperatures for the months of September and October at Buckeye are 29.2^oC and 22.2^oC. Since the mean monthly temperatures for the remaining months are not available evaluation of I to compute evapotranspiration for the month of October (which is less than 26.5^oC) is not possible directly. However, using the nomogram supplied in his paper, evapotranspiration during the month of September (which is greater than 26.5^oC) works out to be 0.54 cm/day against the measured average for the month as 0.58 cm/day.

The daily evapotranspiration during September is set in Table 8 against the actual daily consumption measured. It is noticed that for a mean daily temperature between 30^o to 34^oC the evapotranspiration

Table 8. Measured evapotranspiration rate and evapotranspiration by the Thornthwaite method.

Month Sept.	Measured evapotranspiration cm/day	Mean daily temperature °C	Computed evapotranspiration cm/day	Computed evapotranspiration/ Measured evapotranspiration ratio
1	0.54	29.60	0.47	0.86
2	0.57	30.80	0.57	1.00
3	0.60	31.60	0.59	0.99
4	0.58	31.40	0.59	1.01
5	0.60	30.00	0.55	0.91
6	0.54	28.70	0.53	0.98
7	0.55	30.10	0.55	1.00
8	0.54	29.70	0.55	1.02
9	0.52	27.60	0.49	0.45
10	0.53	28.00	0.51	0.97
11	0.54	28.80	0.53	0.98
12	0.59	31.40	0.59	1.00
13	0.60	30.40	0.56	0.94
14	0.58	30.50	0.57	0.99
15	0.60	32.00	0.60	0.99
16	0.59	32.60	0.60	1.03
17	0.63	33.80	0.61	0.97
18	0.58	33.50	0.61	1.06
19	0.59	33.90	0.62	1.06
20	0.60	33.10	0.60	1.00
21	doubtful	--	--	--
22	"	--	--	--
23	"	--	--	--
24	"	--	--	--
25	"	--	--	--
26	"	--	--	--
27	"	--	--	--
28	"	--	--	--
29	"	--	--	--
30	"	--	--	--
Average	0.58 cm/day		0.54 cm/day	0.94

given by the Thornthwaite method is very nearly equal to the measured one. At temperatures less than 29°C the computed evapotranspiration is within 86-98% of the measured one. On the average the results given by the Thornthwaite method are within 94% of the actual figures.

Sums of Temperature.--Budyko has noted that the annual radiation balance for a wet surface R_o, with an assumed albedo of 0.18 is closely related to the sum of all daily mean temperatures greater than 10°C (ΣT). He finds that

$$R_o = 10 \, \Sigma T \quad \text{Ly/year} \tag{21}$$

Neglecting heat storage and the sensible heat in the annual heat balance equation he recommends that

$$E_o \simeq \frac{R_o}{L} \simeq 0.17 \, \Sigma T \quad \text{mm/year.} \tag{22}$$

For rough estimates of potential evapotranspiration on daily or monthly basis, the following approximation may be used

$$E_o = 0.2 \, \overline{T} \quad \text{mm/day} \tag{23}$$

$$= 5 \, \overline{T} \quad \text{mm/month} \tag{24}$$

Where $\overline{T}$ is the mean temperature for the appropriate period in degrees centigrade. Using this simple method the evapotranspiration has been computed for daily and monthly basis and has been set against the actual values in Table 9. As will be noticed the values are within a close proximity of the measured evapotranspiration. The computed evapotranspiration is within 5% of the actual measured evapotranspiration.

Table 9. Measured evapotranspiration rate and evapotranspiration by Budyko's sums of temperature method.

Month Sept.	Measured evapotranspiration cm/day	Computed evapotranspiration cm/day	Computed evapotranspiration/ Measured evapotranspiration ratio
1	0.54	0.60	1.10
2	0.57	0.61	1.08
3	0.60	0.63	1.06
4	0.58	0.63	1.09
5	0.60	0.60	1.00
6	0.54	0.58	1.08
7	0.55	0.60	1.09
8	0.54	0.60	1.09
9	0.52	0.55	1.07
10	0.53	0.56	1.07
11	0.54	0.58	1.08
12	0.59	0.63	1.07
13	0.60	0.61	1.03
14	0.58	0.61	1.06
15	0.60	0.64	1.06
16	0.59	0.66	1.12
17	0.63	0.67	1.07
18	0.58	0.67	1.16
19	0.59	0.68	1.16
20	0.60	0.67	1.10
21	doubtful	0.63	--
22	"	0.50	--
23	"	0.48	--
24	"	0.53	--
25	"	0.49	--
26	"	0.49	--
27	"	0.52	--
28	"	0.52	--
29	"	0.54	--
30	"	0.52	--

Table 9. (Continued)

Month Oct.	Measured evapotranspiration cm/day	Computed evapotranspiration cm/day	Computed evapotranspiration/ Measured evapotranspiration ratio
1	0.34	0.54	1.62
2	0.34	0.54	1.62
3	0.48	0.54	1.13
4	0.44	0.52	1.18
5	0.43	0.41	0.94
6	0.47	0.41	0.87
7	0.47	0.42	0.91
8	0.46	0.43	0.94
9	0.45	0.44	0.98
10	0.42	0.45	1.06
11	0.42	0.46	1.08
12	0.44	0.45	1.02
13	0.44	0.46	1.04
14	0.47	0.48	1.04
15	0.42	0.47	1.10
16	0.44	0.44	1.00
17	0.44	0.37	0.84
18	0.40	0.38	0.96
19	0.41	0.36	0.88
20	0.42	0.37	0.88
21	0.42	0.41	0.98
22	0.45	0.43	0.96
23	0.45	0.42	0.94
24	0.41	0.42	1.02
25	0.43	0.41	0.96
26	0.42	0.44	1.04
27	0.47	0.42	0.91
28	0.47	0.42	0.91
29	0.51	0.47	0.92
30	0.41	0.48	1.15
31	0.41	0.47	1.13
Average ratio			1.05

<u>The Blaney-Criddle Method.</u>--In this method, disregarding many influencing factors, it has been considered that the consumptive use varies directly with the temperature, length of the day and the available moisture. If ample water supply is available the monthly consumptive use is given by:

$$U = kp \left[\frac{45.7t + 813}{100}\right] \quad \text{mm/month} \tag{25}$$

where U = monthly consumptive use in millimeters

t = mean monthly temperature, in degrees centigrade

p = monthly percentage of the daytime hours of the year

k = empirical consumptive use crop coefficient for irrigation season or growing period. This factor has been found to be reasonably constant for all areas. Though there is no value for this factor in the literature for saltcedars yet it is noticed that for the months under consideration the value for this factor for alfalfa and other plants ranges from 0.7 to 1.0.

Hence for the month of September using a value of 8.36 for p, 1.0 for k and the mean monthly temperature as 29.2°C, the evapotranspiration works out to be 0.60 cm/day against a measured figure of 0.58 cm/day or within 2% of the measured. Similarly for the month of October using a value of 7.85 for p, 0.9 for k and the mean monthly temperature as 22.2°C, the evapotranspiration works out to be 0.41 cm/day against a measured rate of 0.43 cm/day or within 5% of the measured.

B. Energy Balance Methods

Budyko. --He makes use of the basic equation to compute potential evaporation E_o,

$$E_o = 0.622 \frac{\rho D_w}{P} (e_s - e) \tag{26}$$

where e_s is the saturation vapour pressure at the temperature T_s of the evaporating surface, e is the vapour pressure of the air, D_w is the transfer coefficient, ρ is the density of the air, P the atmospheric pressure. In place of E_o Budyko introduces the heat balance equation,

$$R_o = LE_o + H_o + G_o$$

where the subscript 'o' indicates values for a moist surface of net radiation R, sensible heat flux H from a surface of temperature T_s, flux of heat into the ground G.

Since $H_o = c_p D_h (T_s - T)$

where T = air temperature at about 2 meters level

and D_h = heat transfer coefficient

Assuming $D_h = D_w = D$ he writes

$$E_o = R_o - G_o - \rho c_p D (T_s - T) = 0.622L \frac{\rho D}{P} (e_s - e) \tag{27}$$

In this report this equation has been tested in an indirect way. The values of vapour pressures of the air at 8 meters level (Table 10) have been tried against the saturation vapour pressure for the temperatures of leaves. Setting the differences in each case against the evapotranspiration actually measured in the lysimeters the values of 'D' have

Table 10. Checking measured evapotranspiration by Budyko and Penman energy balance methods.

Air Temperature at z = 0 T_s	Saturation vapour pressure at T_s e_s	Vapour Pressure at 8 meters level e	Difference $e_s - e$	Measured evapotranspiration cm/sec	Exchange coefficient cm/sec D_w	Wind Speed at 8 meters level m/sec u
27.80	37.36	8.0	29.36	0.63×10^{-5}	0.298	1.43
29.60	41.47	11.70	30.47	0.66	0.299	1.75
30.50	43.66	19.40	24.26	0.69	0.395	
31.50	46.22	23.00	23.22	0.67	0.399	2.44
30.10	42.67	18.20	24.47	0.70	0.397	2.01
28.20	38.24	17.60	21.64	0.62	0.397	2.01
29.40	40.99	14.80	26.19	0.64	0.339	2.69
28.80	39.59	12.10	27.49	0.69	0.348	2.46
25.85	33.30	9.90	23.40	0.60	0.355	1.55
36.40	34.41	10.5	23.91	0.61	0.354	2.02
26.90	35.44	11.3	24.14	0.62	0.382	1.56
30.40	43.41	15.1	28.31	0.68	0.333	1.20
29.70	41.71	20.1	21.61	0.60	0.385	1.45
30.10	42.67	20.1	22.57	0.67	0.412	1.50
30.30	43.17	16.5	26.67	0.70	0.364	1.30
31.40	45.96	16.9	29.06	0.68	0.324	
32.00	47.55	15.6	31.95	0.73	0.317	1.60
31.30	45.70	15.7	30.00	0.67	0.310	1.40
31.70	46.75	15.4	31.35	0.68	0.301	1.46
32.40	48.64	17.1	31.54	0.70	0.308	1.19

been calculated and plotted against the average wind velocities. The resulting Figure 1 shows no indication that the general linear relationship between 'D' and the wind velocity of u of the form, $D = a + bu$ exists under these circumstances.

Penman. --Penman's method is simply a modification of the preceding Budyko's method. He eliminates the factor $(T_s - T)$ using the finite difference form of the Clausius Clapeyron equation

$$T_s - T = \frac{R_d\, T^2}{0.622 L e_s} (e_s - e_{sa}) = \frac{1}{\Delta}(e_s - e_{sa})$$

where $\Delta = \dfrac{0.622\, L e_s}{R_d\, T^2}$

and $R_d =$ gas constant for dry air

$e_{sa} =$ saturation vapour pressure at the air temperature on substitution in the heat balance equation ultimately reduces to

$$E_o = \frac{L^{-1}\,(R_o - G_o) + \gamma E}{\Delta + \gamma} \tag{28}$$

where $L =$ latent heat of evaporation

$\gamma = \dfrac{c_p P}{0.622\, L}$

and $E = 0.622 \dfrac{\rho D}{P}(e - e_{sa})$

This method which is an approximation of Budyko's method should work well so long as $T_s - T$ is small. Since in the present case as Budyko's method has not yielded good results Penman method is not being attempted. The main reason for the failure of these methods is the factor D. It is still not very much confirmed whether the assumption that $D_h = D_w = D_m$ is correct or not. Secondly the factor D_w is linearly connected with 'u' the wind velocity the data of which are apparently very much doubtful in the present case. Another cause can be the failure to assess correct leave temperature.

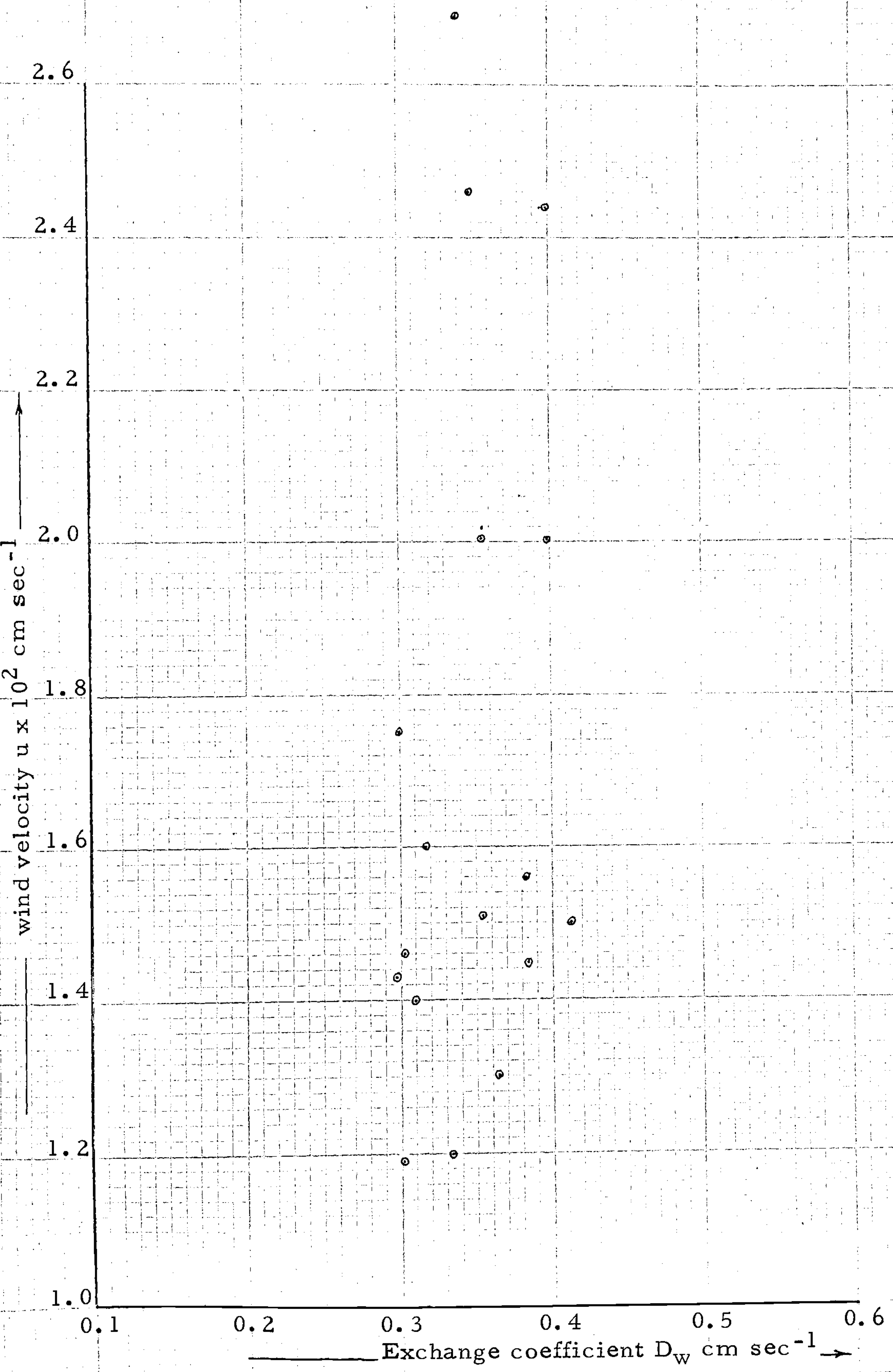

Figure 1. A plot between exchange coefficient D_W and wind speed U at 8 meters level, in Budyko and Penman Energy Balance Methods.

Table 11. Summary of results showing ratios of evapotranspiration computed by different methods, to that measured in lysimeters

	Energy Budget Methods		Climatological Methods				
Aero-dynamic Method	Bowen Ratio Method	Heat balance equation ignoring component of sensible heat	Thorn-thwaite Method	Blaney-Criddle Method	Budyko's Method of Sums of Temperatures	Budyko's Energy Method	Penman's Method
1.20	0.97 - 1.08	0.83	0.94	0.96	1.05	Not proved successful	Not proved successful

CHAPTER VI

CONCLUSIONS OF THE PRESENT STUDY AND RECOMMENDATIONS

Conclusions

1. No single evaporation experiment can be conclusive particularly with regards to aerodynamic approach. The major cause for the aerodynamic method to prove unreliable is the inaccurate observation of the wind speeds at different levels.

2. The instrumentation developed for measuring and recording the factors needed in aerodynamic and energy budget approaches has not proved reliable and practicable. The average data yield during the period under analysis is hardly 40%.

3. In the aerodynamic approach, only the mixing length approach has been tried and the computed evapotranspiration was evaluated against the water budget figures. The results of this study are as follows:

   a. Almost all the evaporation equations in the generalized form which were tried broke down.

   b. Thornthwaite-Holzman equation for neutral conditions has been applied to compute evapotranspiration by this method. Though this equation has yielded higher results yet it is probable that the higher results are due to unsatisfactory wind data and the incorrect location of the equipment and not because of ignoring the stability effects.

c. The presence of nonlinear correlations between the various parameters involved in this approach restricts the application of the method over longer periods of time. The best results can be obtained only if the period over which evapotranspiration is computed is one hour or so.

4. In the energy budget method the Bowen ratio approach has been stated in detail to give a more complete understanding of the physical processes involved. The following results are obtained:

a. The Bowen ratio is approximately -0.17.

b. The evapotranspiration obtained by the Bowen ratio method has been found as 3% less than the measured evapotranspiration.

c. The effect of stability makes this ratio negative which in turn tends to give higher computed figures for evapotranspiration unless an appropriate value of K_H/K_V is used. The effect of instability is just the reverse.

d. As the difference of vapour pressures between the two levels tends to decrease below a certain limit the Bowen ratio approach breaks down.

e. The usual assumption of taking the coefficients of thermal diffusivity and vapour diffusivity as equal does not seem to hold good in the present case under stable conditions. Though no attempt can be made to determine an exact relationship between these two coefficients in the absence of hourly measured water consumption data yet it is evident from the computations that under stable conditions K_H tends to be considerably less than K_V. The usual stable period in a day is that of 18-20 hours duration or 75-85% and the average ratio of K_H/K_V is equal to 0.53.

f. As in the case of aerodynamical methods, in the energy budget methods, too, though the complex relationship between the various parameters restricts the period over which the method can be applied to yield accurate results yet it is shown that reasonable results can be obtained if evapotranspiration is computed over a period of a day or so.

g. In the present situation no error will be caused in the daily computed evapotranspiration figures if the component of flux of heat into the ground is ignored. However, the component of sensible heat is quite significant in the present situation.

5. The often tried old climatological methods have their own merits. They are very simple, inexpensive and in no way less accurate. Thornthwaite, Blaney-Criddle and Budyko's methods where applied in

the present situation have yielded results within 4 to 6% of the measured figures and have thus proved to be the quite reliable and efficient methods.

Recommendations

1. Rigorous improvement in the technique of measuring various parameters in the aerodynamic and energy budget approaches is required.

2. In both the "continous mixing length" as well as "mixing length" approaches the basic wind law for smooth and rough surfaces is that which is only valid for unstratified atmosphere. A more general wind law needs to be investigated which is valid for stratified flow also.

3. With sufficient data at hand the continuous mixing length approach should also be tried.

4. Accurate hourly measurements of the water consumption should be made which should then be compared with the hourly computed evapotranspiration figures to evaluate the effects of stability and unstability of the atmosphere on the evapotranspiration.

5. Since calculation of evapotranspiration on hourly basis with $\frac{K_H}{K_V} = 0.53$ has given good results it will be worthwhile to apply these hourly results to compute σ_1 and B_1 in aerodynamic methods.

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APPENDIX

GENERAL DESCRIPTION OF BUCKEYE PROJECT AREA

Buckeye Project for measuring water use by saltcedars is located southwest of Buckeye about three-quarters of a mile from Gila River. It is about 27 miles west of Phoenix (33°N, 112°W), in the State of Arizona. The project comprises an area of 10 acres and is a part of a long, thick stretch of saltcedars growing along the river.

The area is subject to extreme variations in temperature during the year. Temperatures as high as 120° have been recorded during summer. The average annual rainfall is about 5 inches. Depth to water table ranges from 20 to 25 feet below land surface and the concentration of salts in ground water is of the order of 4000 parts per million.