PREDICTION OF CATION DISTRIBUTION, GYPSUM REQUIREMENT, AND INFILTRATION RATE IN DYNAMIC SOIL-WATER SYSTEMS

by

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A Thesis Submitted to the Faculty of the

DEPARTMENT OF AGRICULTURAL CHEMISTRY AND SOILS

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

In the Graduate College

THE UNIVERSITY OF ARIZONA

1966

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ACKNOWLEDGMENT

The author wishes to express his gratitude to Dr. Gordon R. Dutt. The love Dr. Dutt bears toward his work was a constant source of inspiration. Daily contact with Dr. Dutt afforded both intellectual and personal pleasure.

A special note of thanks is due Mr. George Draper for his help with the laboratory analyses.

The author wishes to thank the members of the University community and the people of Tucson for having opened new vistas of experience to him. He will never forget the year he spent amidst them.

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ABSTRACT

A computer program for prediction of the cation distribution resulting from irrigation of a layered soil was prepared. Half scale models of a ten-layer soil profile were constructed from chemically unaltered Gila Loam and Gila Silt Loam soil samples. A synthetic Colorado River water was the percolating solution. Comparison of predicted and experimental results showed satisfactory agreement.

A gypsum requirement prediction loop was inserted into the program. This loop predicted the quantities necessary to reduce the exchangeable sodium percentage of a given layer to a desired level. Intensive experimental investigation proved this method of prediction accurate.

The dependence of the hydraulic conductivity of a soil upon its exchangeable sodium percentage was taken advantage of in an attempt at prediction of infiltration rates during irrigation. Despite elaborate precautions against interference by extraneous factors, the attempt was not successful. The source of error was probably the formation of a soil film of high exchangeable sodium percentage at the interface between the soil column and the percolating solution.

INTRODUCTION

The maintenance of optimum infiltration rates is a major concern in irrigated agriculture and ground water recharge. As the soil moisture supply is depleted in arid regions, it must be possible to put water into the soil. If this is not accomplished crops suffer from lack of water. The worth of a ground water recharge site is solely determined by the rate at which it can put water into the underground aquifer system. Any physical or chemical changes which hinder infiltration lessen the value of a ground water recharge operation.

The chemical entity often responsible for lowered infiltration rates is the sodium ion. When present on the soil exchange complex in sufficient quantity, this ion can create a soil surface virtually impervious to water. As many waters in arid regions contain a large proportion of sodium, decreasing infiltration rates are a constant problem. Many lands, once productive, are now desert wasteland due to the effects of sodium on the soil complex (12). This is not an inevitable occurrence.

Water for domestic use is preferably high in sodium. Such water is called "soft water." The problems encountered in recharge and irrigation operations with this water are grave. These waters destroy the water transmitting properties of the soil and are, therefore, avoided in recharge and irrigation operations.

As the effects of sodium ion are a result of its adsorption by the soil complex, the addition of calcium will alleviate difficulties encountered during infiltration. The most common source of calcium used for this purpose is gypsum, $CaSO_4 \cdot 2 H_2O$.

The intention of this research was to provide accurate methods for prediction of the cation distribution resulting from irrigation of a layered soil, the gypsum required to reclaim a sodium infested layered soil, and the infiltration rates exhibited by a homogeneous soil being irrigated.

LITERATURE REVIEW

Excessive exchangeable sodium has an adverse effect upon the rate of water movement into and through the soil (e.g., 6, 12, 14, 24). Henderson (15) and Quirk and Schofield (25) percolated waters of varying sodium per cent and salinity through soil columns. Both studies found that for a water of a given salinity, the hydraulic conductivity of the soil column decreased as the per cent sodium of the water increased. This effect was particularly noticeable with waters of low salinity.

Doneen (7) has taken the differences in permeability between soils into account in a permeability index (PI); units are meq/L:

$$PI = \frac{(Na^{+} + (HCO_3^{-})^{\frac{1}{2}}) \times 100}{Na^{+} + Ca^{++} + Mg^{++}} \quad . \qquad [1]$$

This is a revision of the possible sodium per cent formula (12). The bicarbonate is added as a square root to the numerator instead of being subtracted from the denominator. Doneen relates the index to soil properties by establishing different standards for light, medium, and heavy textured soils. Waters of progressively higher index are permitted as the texture of the soil lightens.

The permeability index is based upon the line of thought previously pursued by Eaton (12, 13) and Doneen et al. (6). It generalizes the relations between soil texture and infiltration rates.

These relations are quite unpredictable. The permeability index may therefore lead to large errors.

A common amendment to soils afflicted with low infiltration rates, due to the presence of excessive sodium on the exchange complex, is gypsum. Several methods (21, 28, 30) have been devised to predict the quantity of gypsum required to reduce the exchangeable sodium per cent to an acceptable level.

McGeorge and Breazeale (21) treated each of several soil samples with one of a series of solutions containing varying amounts of gypsum. The resultant soil-water mixtures were filtered and the filtrates tested for gypsum. The smallest gypsum supplement which yielded gypsum in the filtrate was considered to be the gypsum requirement.

Schoonover (28) based his method upon the same principle as had McGeorge _et al_. (21). Instead of using a series of solutions of differing gypsum concentration, he equilibrated one soil sample with a saturated gypsum solution. This mixture was filtered and the calcium and magnesium concentrations determined. The difference between the calcium concentration of the gypsum solution and the calcium and magnesium concentrations of the filtrate was called the gypsum requirement.

The methods of McGeorge _et al_. (21) and Schoonover (28) determine the quantity of gypsum necessary to equilibrate a saturated gypsum solution with the exchangeable and soluble ions in the soil. These methods suffer from a lack of relation to conditions existent in the field. The static systems used in the experimental determination of the gypsum requirement fail to take into account the leaching conditions prevalent in reclamation procedures involving gypsum. The relative adsorption of sodium and calcium varies with the moisture content of the soil (26). The water-soil ratios used in these methods (21, 28) are invariant and higher than those found in the field. McGeorge _et al_. (21) and Schoonover (28) used distilled water in their experiments, thereby neglecting the influence of irrigation water quality upon the cation exchange processes in soil. Reclamation is often carried to depths exceeding the one-foot depth with which these methods are concerned. Neither method enables the user to choose the resultant exchangeable sodium per cent reduction.

The U. S. Salinity Laboratory (30) proposed a method based upon the assumption that the calcium furnished by a gypsum addition will replace an equal amount of adsorbed sodium. For example, if the exchangeable sodium per cent is thirty and the cation exchange capacity is twenty meq/100 g, it would take a gypsum addition of three meq/100 g to reduce the sodium per cent to fifteen. The authors were cognizant of the inadequacy of their assumption. The effects of the exchangeable sodium per cent and the salinity of the soil solution upon the relative adsorption of calcium and sodium were noted as primary sources of error. To account for these factors the authors recommended multiplication of the gypsum requirement, reached by their method, by a correction coefficient of 1.25.

The gypsum requirement method of the U. S. Salinity Laboratory (30) does offer a way to vary the gypsum requirement to meet differing degrees of sodium per cent reduction. Otherwise, it is subject to the same criticisms as the methods of McGeorge _et al_. (21) and Schoonover (28).

McGeorge (20) compared the gypsum requirement tests of McGeorge et al. (21), Schoonover (28), and the U. S. Salinity Laboratory (30). He found no significant disagreement between the values yielded by the three methods. No distinct pattern appeared to relate the results obtained from these methods.

Eaton (13) based a gypsum requirement upon the composition of the irrigation water. He divided the gypsum requirement into three categories: (1) that required to lower the sodium per cent of the water to an acceptable value, which the author chose as seventy per cent; (2) that necessary to offset precipitation of carbonate and bicarbonate compounds of calcium and magnesium; (3) that which accounts for plant uptake of calcium. Units are meq/L:

$$(1) = Na^{+} \times 0.429 \tag{2}$$

$$(2) = (HCO_3^{-} \times (100 - \%\ \text{leaching}))/100 \tag{3}$$

$$(3) = (0.30 \times (100 - \%\ \text{leaching}))/100\ . \tag{4}$$

Eaton offers no theoretical support or experimental evidence in favor of these equations.

A better method for determining a gypsum requirement would be to evaluate the changes in the cation distribution which such an addition would cause. This can be done only if a system which predicts the reactions between the exchangeable ions and the ions in the soil solution is available. Several such systems have been devised.

Rible and Davis (27) adapted the chromatographic theory of De Vault (5) to ion exchange in soil columns. The basic equations of the theory are:

$$(1) \qquad X = \frac{V}{A + Mf'(c_1)} \qquad [5]$$

where X is the depth, A is the pore volume per unit length of column, V is the volume of the effluent, M is the amount of exchanger per unit length of column, and $f'(c_1)$ is the first derivative of the relation between the replacing ion and the displaced ion.

$$(2) \qquad \frac{C}{C_o} = (C_o - (\,(QKC_oX)/(V-ex)\,)^{\frac{1}{2}})/\;C_o\;(1-K) \qquad [6]$$

where Q is the exchange capacity, e is the pore volume per unit length, x is the length of the column, C_o is the concentration of ion in the influent, C is the concentration of ion in the effluent, and K is the equilibrium exchange constant.

Four experiments were performed on a homogeneous soil. A calcium water was passed through a sodium soil, a magnesium water was passed through a calcium soil, and the reverse of these two procedures was also run. The theory predicted the cation distribution most successfully for the case of sodium replacing calcium. With the other three ion pairs the theory underestimated the size of the zone of partial replacement. For the case of calcium replacing sodium the theory predicted no such zone whatsoever.

The experimentally found zones of partial replacement were not very large though. Thus the predicted values rarely deviated from the experimental values by as much as twenty-five per cent.

There are inherent weaknesses in the method tested by Rible and Davis. It is extremely difficult to arrive at the mathematical solutions of the equations [5, 6] predicting the cation distribution. The authors simplified the task by using mono-cationic waters and soils. Were a layered soil used a unique solution to the equation would be required for each layer. Inclusion of slightly soluble salts, such as gypsum or calcium carbonate, would present almost insurmountable difficulties.

The experiments did reveal the modes by which calcium moves in the soil. When displaced it tends to be partially replaced throughout the entire profile. When it is the replacing cation it is extensively adsorbed to a given depth, beneath which it is present to a minor degree.

A method for prediction of cationic exchange processes by use of second order reaction kinetics was developed by Bower, Gardner, and Goertzen (3). This work was an extension of the theories of Hiester and Vermeulen (16). The equation relating the concentration of the ion in the influent to that in the effluent is as follows:

$$\frac{C}{C_o} = J(S/K,t)/(J(S/K,t) + \exp((1-K)(t-S)/K)(1-J(S,t/K)))\ . \qquad [7]$$

where t is the solution capacity parameter, S is the column capacity parameter, J is a complex integral involving a modified Bessel function of the first kind, and K, C, and C_o are as used previously.

Experiments were conducted with mono-cationic waters and soils as well as with di-cationic waters and soils. In all cases excellent agreement between predicted and experimental results was found. The discrepancies between the theoretical and actual results were negligible.

It is lamentable that this system is extremely hard to use in large scale operations. The equation [7] is difficult to solve. Extensive laboratory work not normally done in routine analysis is necessary. For a tri-cationic system in a layered soil containing gypsum and calcium carbonate, the problems posed by the methods of Bower _et_ _al_. (3) are virtually insoluble.

An attempt at partial simplification of the equation [7] used by Bower _et_ _al_. (3) was made by Brooks, Goertzen, and Bower (4). The concentration of sodium in the soil solution, C_{Na}, and the concentration of the sodium adsorbed by the soil, Q_{Na}, can be calculated from the following equations:

$$C_{Na} = \frac{1}{1-K} \left(C_o - \left(\frac{C_o KQpZ}{(D-fZ)}\right)^{\frac{1}{2}}\right) \qquad [8]$$

$$Q_{Na} = \frac{1}{1-K} \left(-KQ + \left(\frac{C_o KQp(D-fZ)}{(Z)}\right)^{\frac{1}{2}}\right) \qquad [9]$$

where p is the bulk density of the soil, f is the soil porosity, D is the depth of irrigation water applied, Z is the depth within the soil, and the other symbols are as previously used.

The authors concede that the equations [8, 9] have varying accuracy. These equations are approximations which are most accurate when the per cent sodium of the applied water is over eighty five and the per cent sodium of the initial soil solution is over twenty five. Thus, the simplifications offered by Brooks _et_ _al_. (4) are of greatest value with waters classified as a major sodium hazard by prevalent standards (6, 30). The successful application of these approximations by the authors is therefore of only academic value.

Nielsen and Biggar (23) have found extensive evidence of dispersion of solutes in the soil moisture stream. These findings are of pertinence to systems which try to predict ion exchange. Most models of ion exchange in dynamic soil-water systems presume water flow to proceed through the soil in a piston-like fashion. This precludes consideration of dispersion. Biggar and Nielsen (2) tested the theories of Rible and Davis (27), Bower <u>et al.</u> (3), and Lapidus and Amundson (19). The equation obtained from the theory of Lapidus and Amundson was:

$$\frac{C}{C_o} = \tfrac{1}{2}(\mathrm{erfc}((x(Q+eC_o)-C_oV)/(4DVC_o(Q+eC_o)/v)^{\frac{1}{2}})$$

$$+ \exp(vx/D)\ \mathrm{erfc}((x(Q+eC_o)+C_oV)/4DVC_o(Q+eC_o)/v)^{\frac{1}{2}}). \quad [10]$$

All variables are as previously defined, except that v is the average flow velocity, and D the apparent diffusion coefficient. It should be noted that this equation [10] does consider diffusion.

The authors found breakthrough by magnesium, which was displacing calcium, in advance of that predicted by all three theories. As the velocity of flow decreased the deviations increased. Decreased concentration of both replacing and displaced ions accentuated the discrepancies, as did unsaturated flow.

The authors attributed the difficulties encountered by the theories to their insufficient consideration of the effects of dispersion. Biggar and Nielsen are too critical, however, when they claim that this defect renders the theories inadequate. The greatest error exhibited by the predictions based on the theories of Rible and Davis (27) and Bower et al. (3) is twenty-five per cent in the saturated case, for which the theories were designed. The Lapidus and Amundson theory (19), which does contain a diffusion factor, errs by as much as forty-five per cent and must be considered unsatisfactory.

The greatest dispersion effects occur during unsaturated flow of water through soil. Biggar and Nielsen (2) advise the use of this type of flow to move a replacing cation further down the profile than would occur under conditions of saturated flow. In particular, they note the difficulties encountered when deep leaching of calcium is desired to correct a sodium problem and state the possible pertinence of their findings to reclamation procedures.

Dutt and Tanji (9) have derived a method from chemical thermodynamics for the prediction of ion exchange in dynamic soil-water systems. Six equations have been combined to form the general case for a computer program. These equations are:

Calcium-Magnesium Exchange:

$$\frac{a_{Ca^{++}}}{a_{Mg^{++}}} = K \frac{N_{Ca}}{N_{Mg}} \qquad [11]$$

Sodium-Calcium Exchange:

$$\frac{a^2_{Na^+}}{a_{Ca^{++}}} = K' \frac{N^2_{Na}}{N_{Ca}(N_{Na} + 1.5\ N_{Ca} + 1.5\ N_{Mg})} \qquad [12]$$

Solubility of Gypsum:

$$a_{Ca^{++}} \, a_{SO_4^{=}} = K_{SP} = 2.4 \times 10^{-5} \qquad [13]$$

Undissociated Calcium Sulfate in Solution:

$$C_{CaSO_4} = \frac{a_{Ca^{++}} \, a_{SO_4^{=}}}{K_d} = \frac{a_{Ca^{++}} \, a_{SO_4^{=}}}{4.9 \times 10^{-3}} \qquad [14]$$

Ionic Strength:

$$u = \frac{1}{2} \sum_{i=1}^{n} C_i Z_i^{\,2} \qquad [15]$$

Extended Debye-Hückel Theory:

$$\log \gamma_j = \frac{-0.509 \, Z_j^2 \, (u)^{\frac{1}{2}}}{1 + (u)^{\frac{1}{2}}} \qquad [16]$$

In these equations, a refers to the activity of an ion, C to the concentration of an undissociated entity in solution, N to the concentration of an ion on the soil exchange complex, K to the calcium-magnesium equilibrium exchange constant, K' to the sodium-calcium equilibrium exchange constant, γ to the activity coefficient, u to the ionic strength, and C and Z to the concentration and valence, respectively, of an ion species i or j present in n concentration in the solution.

It should be noted that the activity of an ion is equal to the product of its concentration in solution and its activity coefficient. The extended Debye-Hückel theory, devised for pure solutions, has been found applicable to soil solutions (10).

The most recent accomplishment with this program was the correct prediction of the cation distribution in a homogeneous calcium soil treated with a sodium-magnesium water.[1] Previously the solute composition of the effluent from a column containing calcium, magnesium, sodium, and gypsum had predicted with good accuracy (11).

In contrast to the systems previously discussed (3, 4, 19, 27) the method of Dutt requires quantities found in routine laboratory analysis, except that the exchange constants have to be determined. The only mathematical operation necessary is the preparation of the data cards for the computer program. This is in marked contradistinction to the complex operations noted in the other systems mentioned.

Dutt has worked exclusively with soils saturated with calcium prior to insertion in the filtration tube. These soils have also been leached with distilled water to free them of soluble salts. These simplifications have helped avoid interference from dispersion. The computer program assumes piston fashion movement of water through the soil column. Were large quantities of salt present in the soil prior to the passage of water through the soil column, there would be increased likelihood of discrepancies between predicted and experimentally found values due to dispersion.

The waters used by Dutt have contained only sulfate and chloride anions. Waters containing carbonates and bicarbonates have been avoided, as there is no means in the program of treating the

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[1] Gordon R. Dutt, Cation Exchange in Soil Columns. Paper in Preparation for Soil Sci. Soc. Am. Proc.

precipitation of the carbonates of calcium and magnesium. If the program were applied to a soil-water system containing magnesium, calcium, carbonate, and bicarbonate, difficulty might be encountered.

THEORY

Chemical Thermodynamics

It is the aim of this research to provide an easily used method for prediction of the cation distribution of a dynamic soil-water system, closely approximating those found in everyday work. It is hoped that gypsum requirements, and hydraulic conductivities during infiltration, may be predicted by similar means. The method proposed by Dutt and Tanji (9) offers the most promising approach.

The cations on the soil exchange complex may exist in a state of thermodynamic equilibrium with the cations in the soil solution. Many equations have been written to describe this condition (18). All yield the same result when cations of equal valence are being considered:

$$\frac{a_A}{a_B} = K \frac{N_A}{N_B} \qquad [17]$$

where the activities of the ions in solution are a and the molar concentrations of the adsorbed ions are N. K is the equilibrium exchange constant and has been found to generally range from 0.62 to 0.70 (18, 26) when calcium is A and magnesium is B.

With ions of different valence many different equations result from the various approaches (18). An equation which has a good thermodynamic basis is a modified Kerr equation.[2]

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[2] Hassan Hajrah, The Effect of Cation Exchange on Gypsum Requirement of Soils - 1965. Unpublished M.S. Thesis, University of Ariz.

$$\frac{a_A^2}{a_B} = K \frac{N_A^2}{N_B} \times \frac{1}{W} \qquad [18]$$

where the symbols are as used above and W is the clay content of the soil expressed as a decimal. K equals 1.385 x 10^{+4} when sodium is A and calcium is B. The sodium-magnesium constant may be obtained by multiplication of the calcium-magnesium and sodium-calcium constants. It is equal to 0.93 x 10^{+4}.

When gypsum is present in the soil a condition of thermodynamic equilibrium may also exist between the solid form of gypsum, undissociated calcium sulfate in solution, and calcium and sulfate ions in solution,

$$CaSO_{4(S)} \rightleftarrows CaSO_{4(Undiss.)} \rightleftarrows Ca^{++} + SO_4^{=} \quad . \qquad [19]$$

The relation between the solid phase and the ions in solution is given in [13]. That existing between the ions in solution and undissociated calcium sulfate is [14]. When gypsum is present in the solid state this equation [14] becomes:

$$C_{CaSO_4} = \frac{K_{SP}}{K_D} = 4.9 \times 10^{-3} \quad . \qquad [20]$$

The extended Debye-Hückel theory [16] has been found accurate for soil solutions (10). This is because of the small influence of the forces exerted by the soil upon the behavior of ions in the soil solution. However, in very dilute soil solutions it has been found that the solubility product of gypsum does vary. This is explained by Dutt (8) as being a result of the unusually high soil forces present in such solutions. In soil solutions normally encountered there is probably little interference from the soil forces.

Adaptation of Thermodynamic Equations to Conditions Extant in Soil and to Computer Techniques

In order to use these equations [13, 14, 15, 16, 17, 18, 20] in programs (A, B, C, D), it is necessary to modify them to fit the dynamic conditions extant in the soil. If gypsum is initially present in a quantity sufficient to maintain a saturated gypsum solution at equilibrium, the following modifications are made.

STEP ONE:

$$C_{Ca} = C^o_{Ca} + X \qquad [21]$$

$$C_{SO_4} = C^o_{SO_4} + X \qquad [22]$$

$$N_{GYP} = N^o_{GYP} - X/BT \qquad [23]$$

where C^o is the initial ionic concentration and C the equilibrium ionic concentration in the soil solution, N^o is the initial concentration, and N the equilibrium concentration in the soil. X represents the changes in the system as it goes from initial to equilibrium conditions, and BT is the grams of soil per liter of solution. These equations [15, 16, 21, 22] are substituted into [13].

$$X^2 + (C^o_{Ca} + C^o_{SO_4})\ X + (C^o_{Ca}\ C^o_{SO_4} - K_{SP}\ (\exp(EX))\ ) = 0 \qquad [24]$$

where

$$EX = (9.366 \times u)/(1 + u) \qquad [25]$$

and all other terms are as previously defined.

The program then considers undissociated $CaSO_4$.

STEP TWO:

$$CAS1 = 4.987 \times 10^{-3} - CASO^{o} \qquad [26]$$

where $CASO^{o}$ is the initial concentration of undissociated $CASO_4$, and CAS1 is the quantity needed to make the concentration of undissociated $CaSO_4$ equal 4.987×10^{-3} moles/L [20]. Then,

$$CASO = CASO^{o} + CAS1 \qquad [27]$$

and from [23]

$$N'_{GYP} = N_{GYP} - CAS1/BT \quad . \qquad [28]$$

In this case CASO is the equilibrium concentration of undissociated $CaSO_4$, N' is the final equilibrium concentration in the soil, and other terms are as before. All terms used in equations [29] through [47] will also be as previously defined.

If N'_{GYP} is negative, insufficient gypsum is present to form a saturated solution of undissociated $CaSO_4$. In this case, and if insufficient gypsum were present prior to Step One, Step Two Alternate is necessary.

STEP TWO ALTERNATE:

$$X = N^{o}_{GYP} \times BT \qquad [29]$$

$$N'_{GYP} = 0.0 \qquad [30]$$

$$C^{a}_{Ca} = C^{o}_{Ca} + X \qquad [31]$$

$$C^{a}_{SO_4} = C^{o}_{SO_4} + X \qquad [32]$$

and then

$$C_{Ca} = C^{a}_{Ca} - X1 \qquad [33]$$

$$C_{SO_4} = C^{a}_{SO_4} - X1 \qquad [34]$$

$$CASO = CASO^{o} + X1 \quad . \qquad [35]$$

X1 is found thusly:

$$AA = \exp(EX) \qquad [36]$$

and then

$$(AA)X1^2 - (K_d + AA \times C^{a}_{Ca} + AA \times C^{a}_{SO_4})\, X1 +$$

$$(AA \times C^{a}_{SO_4} \times C^{a}_{Ca} - K_d \times CASO^{o}) = 0 \quad . \qquad [37]$$

The program then considers sodium-calcium ion exchange. Had no gypsum, undissociated $CaSO_4$, or $SO_4^{=}$, been present prior to Step One, this step would have been immediately proceeded to:

STEP THREE:

$$C_{Na} = C^{o}_{Na} - 2BT \times X \qquad [38]$$

$$C_{Ca} = C^{o}_{Ca} + BT \times X \qquad [39]$$

$$N_{Na} = N^{o}_{Na} + 2X \qquad [40]$$

$$N_{Ca} = N^{o}_{Ca} - X \qquad [41]$$

When substituted in [18] a third degree equation results:

$$(-4BT\ (K'EX+BT)\ )\ X^3 + (4((BTxN^o_{Ca}+C^o_{Na}-K'EX\ N^o_{Na})\ BT$$

$$-\ K'C^o_{Ca}EX)\ )\ X^2 - (C^o_{Na}((4BTx\ N^o_{Ca} + C^o_{Na}) - (K'EX\ N^o_{Na})$$

$$(BTxN^o_{Na}+4xC^o_{Ca}))\ )\ X + (C^o_{Na}C^o_{Na}N^o_{Ca} - K'C^o_{Ca}EX\ N^o_{Na}N^o_{Na})$$

$$= 0\ . \qquad [42]$$

Next to be considered is Ca-Mg ion exchange.

STEP FOUR:

$$N_{Mg} = N^o_{Mg} + Y \qquad [43]$$

$$N_{Ca} = N^o_{Ca} - Y \qquad [44]$$

$$C_{Mg} = C^o_{Mg} - BT \times Y \qquad [45]$$

$$C_{Ca} = C^o_{Ca} + BT \times Y \qquad [46]$$

where Y is the change undergone by the system in passing from the initial state to equilibrium state. Substitution of [43, 44, 45, 46] into [17] creates a second degree equation.

$$(BT\ (1-K)\ )Y^2 + (C^o_{Ca} + BT\ (N^o_{Mg} + KN^o_{Ca}) + KC^o_{Mg})Y$$

$$+ (C^o_{Ca}N^o_{Mg} - KC^o_{Mg}N^o_{Ca}) = 0\ . \qquad [47]$$

When the values of C_{Ca} from Step Four of the previous cycle, and Steps Two or Two Alternate and Three of the present cycle, are within 1.0×10^{-5} of the value of C_{Ca} from Step Four of the present cycle, the system is considered to be at equilibrium and the computer begins the calculation for the next section of the column.

When the gypsum requirement is determined (program B) additional equations [48 - 50] are placed within Step Two. Equations [23] from Step One, [28] from Step Two, and [33 - 37] from Step Two Alternate are avoided. The following is the cycle when the solution in the top soil layer is not saturated with gypsum:

STEP ONE [21, 22, 24, 25] → STEP TWO [27] → STEP TWO GYPSUM:

$$\mathrm{GYP} = \mathrm{X/BT} \qquad [48]$$

$$\mathrm{GZP} = \mathrm{CAS1/BT} \qquad [49]$$

$$\mathrm{GXP} = \mathrm{GYP} + \mathrm{GZP} + \mathrm{GXP}^{o} \qquad [50]$$

where GYP and GZP are the gypsum required to saturate the solution with Ca^{++} and $SO_4^{=}$, and undissociated $CaSO_4$, respectively, GXP^{o} is the cumulative gypsum requirement from prior cycles, and GXP is the cumulative gypsum requirement after addition of this cycles' requirement.

Should Step Two Alternate be entered into, the normal route is followed through [32]. However, upon completion of [32] the computer returns to the beginning of Step One. It then follows the cycle leading to Step Two Gypsum.

Assumptions About Behavior of Water and Anions in Soil

Some assumptions must be made regarding the behavior of water in a soil column. Consider a soil column to be divided into ten sections, corresponding to a soil profile consisting of ten layers; furthermore, water is moved down the profile in discrete quanta called aliquots. The pore volume of the first layer determines the size of the aliquot. The aliquots are considered to react with each layer until they attain equilibrium. These aliquots proceed down the column until they leave it or reach a predetermined layer. The movement of the water is assumed to be piston fashion. It is thus presumed that no mixing between aliquots occurs. In addition, the soluble salts present in the soil prior to the entry of the wetting solution are considered to be moved through the column solely by the action of the first aliquot of water to enter the column.

Heretofore, the moisture contents fed to the computer have been those actually found in the column. If this were required in practical operations, it would be necessary to determine the field capacity of each soil sample submitted for analysis by the computer program. It was therefore decided to test an approximation of the field capacity. The saturation percentage of a soil is commonly found during the standard analytical procedures. One-half of the saturation percentage is a rough estimate of the field capacity. It was seen whether this moisture content could be considered to be the moisture content within the column.

The only anion individually treated in these programs is sulfate. Although the reactions of carbonate and bicarbonate are of importance in soil-water systems, it is not yet possible to account for them. In this model these two ions are treated as if they were chloride ions. Also, the pH value and the exchange capacity of the soil are both considered to be invariant.

The Cation Distribution Program

The cation distribution program (A) passes the desired number of aliquots into the soil column. When the necessary calculations have been completed, the solute composition of the effluent and soil solution and the exchangeable cations are printed out.

The Gypsum Requirement Program

A program for computer prediction of gypsum requirements (program B) has been based upon the cation distribution program (A). This program calculates the gypsum addition required in the top layer of the soil to bring the exchangeable sodium percentage down to a desired level in a designated layer of the soil. The desired level may be chosen to fit the needs of the particular soil. In these experiments it was chosen as fifteen per cent. The designated layer may also be varied. One additional assumption in this program (B) is that the gypsum supplement is evenly distributed throughout the first soil layer.

The gypsum requirement loop commences operation when a saturated gypsum solution is not present in the top soil layer. The program internally adds gypsum so as to reconstitute a saturated gypsum solution. This loop operates only when the water is in the top soil layer; otherwise the program is identical to the cation distribution program (A). When the exchangeable sodium per cent in the designated layer has been reduced to the desired level, the computer prints out the gypsum requirement in moles of gypsum required per gram of soil and as tons of gypsum per acre, the necessary water as aliquots and as acre feet, and the resultant exchangeable sodium per cent in the designated layer. If the cation distribution resulting from this gypsum addition is of interest, it is an extremely simple matter to transfer the data from the gypsum requirement program (B) to the cation distribution program (A).

The Sodium-Infiltration Rate Program

The hydraulic conductivity of a soil is determined by the sizes of the pores and the tortuosity of the pore pathways. These are part of the soil structure. The undesirable effects of sodium are due to the influence of the ion upon soil and water structure. Every soil has a unique relation between hydraulic conductivity and exchangeable sodium per cent. Fifteen per cent sodium is the generally recognized maximum allowable exchangeable sodium per cent (30). However, there are some soils which can maintain satisfactory infiltration with higher sodium percentages and some which cannot tolerate even this level of exchangeable sodium. It is therefore of importance that a method be developed which will determine the maximum allowable sodium per cent of an individual soil.

When a solution which differs from the soil solution enters the soil, the infiltration rate of the soil will be modified. If all other factors are controlled, this may be a discrete function of the exchangeable sodium percentage of the soil. If a method which determines the exchangeable sodium per cent of a soil being wetted is combined with the mathematical relationship between the exchangeable sodium per cent and the hydraulic conductivity for the particular soil in question, it may be possible to compute the changing hydraulic conductivity as the process of infiltration progresses.

The hydraulic conductivity encountered by an aliquot as it passes through a section of soil is assumed to be that determined by the equilibrium exchangeable sodium per cent between the aliquot and the section.

The hydraulic conductivity of the entire column is a function of the hydraulic conductivities of the sections comprising the column.[3]

$$\frac{L_T}{K_T} = \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} \cdots\cdots\cdots \qquad [51]$$

where L_T is the total length of the column, $L_{1,2,3,\ldots}$ are the lengths of the sections, K_T is the hydraulic conductivity of the entire column, and $K_{1,2,3,\ldots}$ are the hydraulic conductivities of the sections. A computer program (C) has been prepared which finds the hydraulic conductivities of the sections and then uses [51] to find the hydraulic conductivity of the entire column.

The Exchangeable Cation Determination and Effect of Moisture Variation Program

Values which must be found experimentally prior to the running of programs (A,B,C) are the soluble salts, clay content, saturation percentage, and exchange capacity of the soil, and the ionic composition of the water. A supplementary program (D) finds the exchangeable ions if it is fed the above information. This program makes use of a relationship between the Sodium Adsorption Ratio (SAR) of a water and the

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[3] Personal Communication from Dr. D. D. Evans.

exchangeable sodium per cent (ESP) of a soil at equilibrium with the water (30):

$$\mathrm{SAR} = \frac{\mathrm{Na}^{+}}{(\mathrm{Ca}^{++} + \mathrm{Mg}^{++})^{\frac{1}{2}}}$$ [52]

where units are mmole/L, and

$$\mathrm{ESP} = 100(-0.0126+0.01475\ \mathrm{SAR})/1+(-0.0126+0.01475\ \mathrm{SAR}) \quad .$$ [53]

Use of this program eliminates the need for laboratory determination of the exchangeable ions, an often inaccurate procedure. If the program (D) is given the aforementioned experimental information at one moisture content, it will compute what the soluble salts and exchangeable ions would be at any other moisture content, such as the field capacity. This makes it possible to use any convenient water-soil ratio when analyzing a soil. Thus, the laboratory work required for use of these programs is comparable to that done in a routine soil-water analysis.

METHODS AND MATERIALS

The soils used in the cation distribution and gypsum requirement studies were a Gila Loam and a Gila Silt Loam, respectively. Both are alluvial soils from the South Gila Valley in southwestern Arizona. They were collected as a series of six-inch deep layers to a depth of five feet. The integrity of each layer was maintained throughout the experiment.

The soils were air dried and then put through a two mm sieve. Soluble salts were determined from a saturation extract (Table 1). The moisture content of each saturation extract was noted. Clay content was measured by the Bouyoucos process. Exchange capacities were found by the ammonium acetate procedure. Gypsum content of the soils was taken to be equal to the quantity of sulfate appearing in a 1:5 extract in excess of that which appeared in the saturation extract (Table 2).

The filtration tubes used in the experiments were made of one-inch inner diameter lucite plastic. The soil was packed in them to a depth of two and one-half feet, with about a three-inch section alloted to each layer. This was accomplished by pouring the soil into the filtration tube and rapping the tube until the soil occupied the desired volume. The layers in the column were arranged as found in the field. Each layer weighed fifty-five grams. Thus, 555 grams of soil were in each column. The only chemical change imposed upon the soils was the addition of gypsum to the top layer of the Gila Silt Loam used in the gypsum requirement study. The quantity of gypsum

Table 1. - Saturation Percentages and Soluble Salts of Saturation Extracts of Soils for Cation Distribution and Gypsum Requirement Prediction Columns

Soil	Layer	Moisture (%)	Mg^{++} (meq/L)	Ca^{++} (meq/L)	Na^{+} (meq/L)	$SO_4^{=}$ (meq/L)	$HCO_3^- + Cl^-$ (meq/L)
Gila Loam	1	36.9	15.9	30.0	45.6	16.3	75.2
	2	36.9	15.1	39.6	94.5	60.0	89.2
	3	39.3	33.8	45.0	147.5	70.9	155.4
	4	39.3	36.0	43.0	170.0	70.9	178.1
	5	37.1	33.6	39.8	192.5	82.1	183.8
	6	33.6	33.0	42.4	188.5	70.9	193.0
	7	40.7	54.0	61.2	280.0	62.5	332.7
	8	43.1	42.4	54.0	239.5	62.5	273.4
	9	44.6	40.8	57.0	225.0	56.8	266.0
	10	47.6	48.8	65.2	239.5	47.1	306.4
Gila Silt Loam	1	40.0	39.1	52.6	174.0	67.5	198.2
	2	40.0	72.8	62.8	338.0	83.7	389.9
	3	53.7	75.2	78.8	422.0	72.5	503.5
	4	49.2	82.0	97.2	435.0	58.3	555.9
	5	38.5	82.0	106.0	390.0	39.6	538.4
	6	44.6	71.6	111.2	410.0	35.8	557.0
	7	53.7	53.0	87.6	397.0	50.0	487.6
	8	50.0	50.8	89.2	447.0	41.6	545.4
	9	61.6	35.2	59.6	348.0	39.1	403.7
	10	46.1	35.2	54.8	328.0	44.1	373.9

Table 2. - Properties of Soils for Cation Distribution and Gypsum Requirement Prediction Columns

Soil	Layer	Calcium (meq/100g)	Magnesium (meq/100g)	Sodium (meq/100g)	Sodium (%)	C.E.C. (mmoles/100g)	Gypsum (mmoles/100g)	Clay (%)
Gila Loam	1	14.6	5.2	2.0	9.2	21.8	0.95	16
	2	15.5	7.6	3.2	12.2	26.3	1.30	16
	3	10.8	6.2	5.4	24.1	22.4	1.65	16
	4	13.2	7.0	7.2	26.3	27.4	1.70	21
	5	9.5	5.8	6.0	28.1	21.3	1.25	16
	6	8.7	4.9	5.7	29.5	19.3	1.25	12
	7	9.4	5.8	6.6	30.3	21.8	1.15	16
	8	10.5	6.6	7.0	29.0	24.1	1.10	22
	9	8.8	4.5	7.3	35.4	20.6	1.05	24
	10	12.5	6.8	9.7	33.4	29.0	0.70	33
Gila Silt Loam	1	9.7	5.0	3.4	18.2	18.1	3.45	16
	2	6.7	5.3	7.2	37.5	19.2	2.30	15
	3	9.3	6.8	12.1	42.9	28.2	1.10	20
	4	9.5	5.6	10.5	41.0	25.6	0.00	21
	5	6.6	3.4	6.7	40.1	16.7	0.00	18
	6	8.5	3.9	7.6	38.0	20.0	0.00	20
	7	12.3	5.3	13.0	42.5	30.6	0.00	32
	8	10.4	4.5	12.1	44.8	27.0	0.00	26
	9	12.7	5.5	14.7	44.7	32.9	0.00	34
	10	10.7	4.4	11.0	42.1	26.1	0.00	30

added to the two columns in this study was 455 mg, mixed thoroughly with the top layer.

The columns were placed in a vertical position. The constant head of solution applied was within one inch of five and one-half feet. The water used in the experiments was a simulated Colorado River water, such as is actually used upon the fields of the South Gila Valley. This water contained 3.42 meq/L Ca^{++}, 5.50 meq/L Na^{+}, 1.78 meq/L Mg^{++}, 5.40 meq/L $SO_4^{=}$, 3.00 meq/L HCO_3^{-}, and 2.30 meq/L Cl^{-}. No direct temperature control was exercised, but the temperature rarely varied more than three degrees fahrenheit from seventy-seven degrees. Glass wool was placed at all locations where evaporation could occur. Aluminum foil was wrapped around the columns to inhibit the growth of algae.

Cation Distribution Prediction: Two columns containing Gila Loam, hereafter referred to as column 1 and column 2, were filled with water until the wetting front had arrived at the boundary between the sixth and seventh layers. It was of interest whether the procedures adopted would be successful when applied to a column only partially filled with water. When the proper wetting had occurred, the columns were cut apart at the divisions between layers with a sabre saw.

A portion of each soil layer wetted was oven-dried at 105°C. The moisture contents of the layers in the column were thereby determined. The remainders were air-dried and then analyzed for soluble salts in a 1:5 extract. This data were fed into computer program (D), which modified the data to fit the moisture contents of the layers in the columns. The data yielded by program (D) were considered to be an accurate representation of the experimental results.

Gypsum Requirement Prediction: The three columns containing Gila Silt Loam were divided into two groups. Two columns had the gypsum requirement recommended by computer program (B), plus a control quantity, added to the top layer. The other column was not altered. The control section chosen to have its exchangeable sodium percentage reduced to fifteen was the third layer. The computer program (B) advised the passage of 1.6 column pore volumes into the column. This advice was followed in the operation of the unaltered column and one gypsum treated column. The second gypsum treated column was wet with 1.5 column pore volumes. These irrigations allowed 0.6 column pore volume to leave the former two columns and 0.5 to leave the latter. This effluent was collected and analyzed in approximately 0.05 column pore volume quantities. When the proper amount of effluent had left the columns, they were cut apart.

Program D has no means of accounting for precipitation of calcium and magnesium carbonates. This would be a serious defect in a case where these compounds contribute a preponderance of the calcium and magnesium present in the soil solution. A different method was used to determine soluble cations present in the soil solutions of the three columns of the gypsum requirement study, where the aforementioned conditions were found. The concentrations of the soluble cations were measured in 1:5, 1:2, and saturation extracts. The curves thus obtained were extrapolated to the actual moisture contents within the columns (Figs. 1, 2). The concentrations at the actual moisture contents were taken to be the actual values.

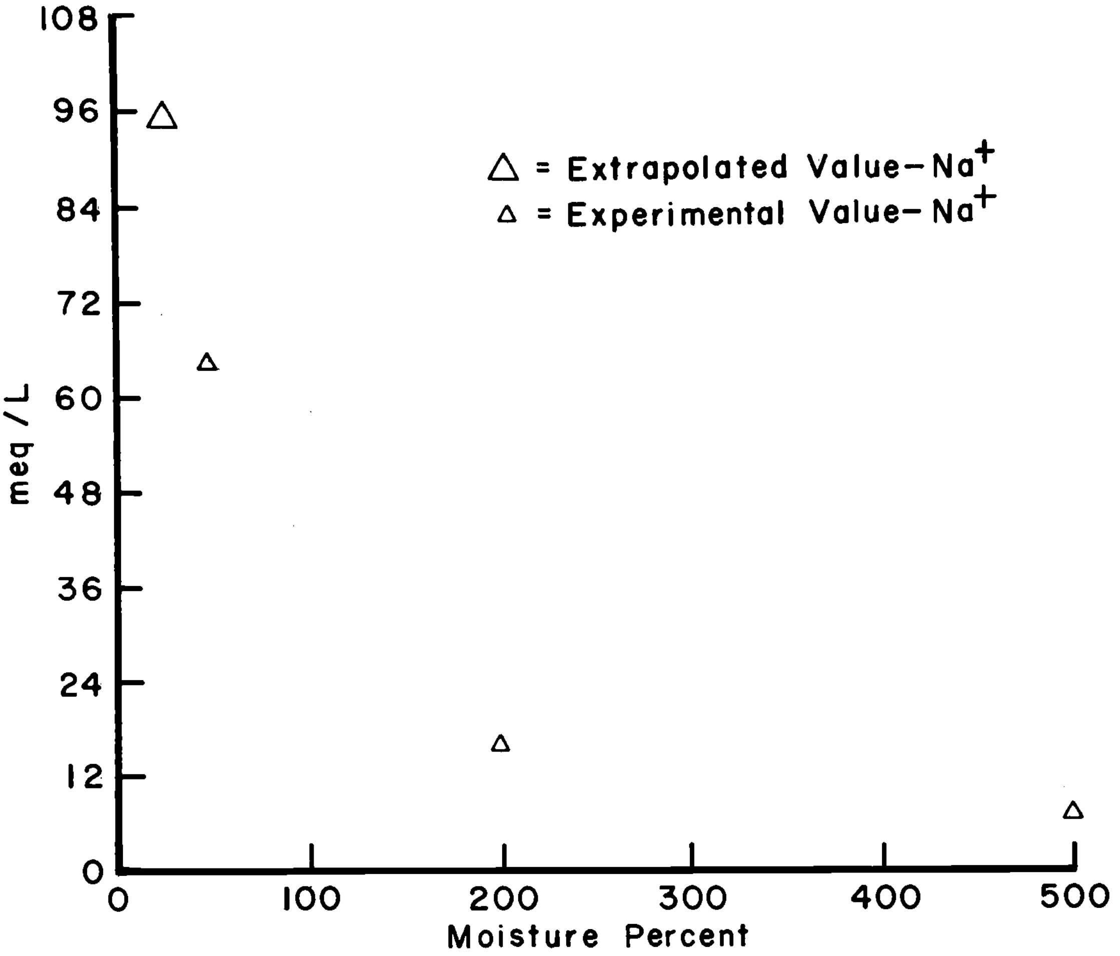

Fig. 1–Derivation of Soluble Sodium Concentration in Layer 4 of Unaltered Column

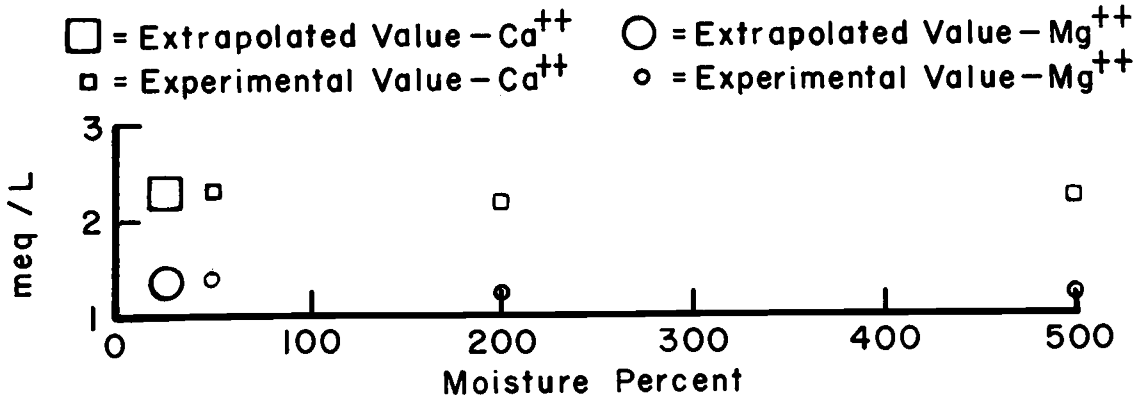

Fig. 2–Derivation of Soluble Calcium and Magnesium Concentrations in Layer 4 of Unaltered Column

Infiltration Rate Prediction: The soil used in the infiltration study was a Gila Silt Loam. This is an alluvial soil taken from the same location as those previously described. It was obtained from the six-inch to twelve-inch layer and was thoroughly mixed during the procedures preparing it for laboratory use. It was, therefore, considered to be homogeneous. The soil was first put through a two mm sieve. A calcium saturated soil was then formed by repeated washing with 0.5 normal calcium chloride solution. This soil was treated with 0.1 normal hydrochloric acid until the pH value of the soil was 4.5. It was hoped that this would eliminate the calcium and magnesium carbonate content of the soil. Excess salts were removed by washing with distilled water until the electrical conductance of the soil solution was beneath 0.1 mmhos/cm^2. The soil was again air dried and put through a two mm sieve. As uniformity of packing was a necessity in this experiment, the soil was then put through a 0.42 mm sieve. The clay content, saturation percentage, and exchange capacity of the soil were then determined by methods previously described. They were 22%, 48.7%, and 22.5 meq/100g, respectively. The soluble salts in a 1:5 extract were 1.2 meq/L Ca^{++} and 0.2 meq/L Na$^+$, both as chlorides.

The soil was then packed as previously described, into 2.54 cm inner diameter lucite filtration tubes. Two lengths of tube were used. The short columns were utilized to determine the relation of hydraulic conductivity to exchangeable sodium per cent. They contained 11.91 grams of soil, packed to a depth of 1.91 cm. The filtration tubes used to test the applicability of this relationship to dynamic soil-water systems were packed to a depth of 13.0 cm with 81.1 grams of soil.

The bulk density of the soil was virtually identical in both sizes of column.

The columns of both sets were vertically positioned in a vacuum dessicator and evacuated for ten to twelve hours. While still in the dessicator, the columns were filled from the bottom with a 9.75 meq/L solution of calcium chloride. These steps were taken to minimize the effects of entrapped air upon water movement; also to have an initial salinity in the soil solution equal to that of the water to be passed through the columns.

Each of the short columns was then wet with a different solution of calcium and sodium chlorides (Table 3). The large columns were treated with a sodium chloride solution. The columns were wetted until the daily changes in the hydraulic conductivity were minimal. Both sets were maintained in a vertical position throughout the experiment. The experiment was performed in an air bath which maintained a temperature of $25^{\circ} \pm 1^{\circ}C$.

The effluent from the third of the three large columns was collected in approximately forty ml portions. It was analyzed for sodium and calcium.

Table 3. - SAR of Waters Used in Hydraulic Conductivity Study and Resultant Equilibrium ESP of Soil

SAR	ESP
0	0
4	4.5
5	6
7.5	9
11.5	13.5
15	17
19	21
25	26
37	35
52	43
71	51

ANALYTICAL PROCEDURES

CATIONS

Calcium - 0.01 N CDTA titration with KOH buffer and Calcein indicator (17, 30)

Magnesium - 0.01 N CDTA titration with NH_4OH buffer and methyl red and calmagite indicators - this titration minus the calcium titration equaled magnesium (17)

Sodium - Beckman DU spectrophotometer (30)

ANIONS

Sulfate - 2mg/cc Barium Chloride titration with ethyl alcoholic thorin indicator (29)

Chloride and Bicarbonate - Sum of cations minus Sulfate

All ions were measured as meq/L.

RESULTS AND DISCUSSION

Cation Distribution Prediction

The cessation of percolation of Colorado River water through soil columns 1 and 2, prior to breakthrough of the wetting front, does not appear to have introduced any unforeseen factors. The predicted and experimental values for the exchangeable and soluble cations are in good agreement (Tables 4, 5). Notice in Figure 3 that the experimental values for the sixth layer are lower than the predicted value. The water within this layer was assumed by program (A) to contain all of the soluble salts initially present in layers one through six. Dispersion appears to have caused a minor lag in the downward movement of these soluble salts. As these salts were present to a degree found only in highly saline soils, it is doubtful whether dispersion would ever cause a major discrepancy between predicted and experimental values.

The exchange process taking place was primarily a replacement of sodium by calcium. The exchangeable magnesium percentages throughout the soil column were not greatly modified by the reaction of the soil with the synthetic Colorado River water (Fig. 4). The sodium displacement was most evident in the uppermost three layers (Fig. 5). By the conclusion of the wetting, exchangeable calcium had substantially increased in these layers (Fig. 6).

The calcium in the water seems to have reacted primarily with a specific layer until the sodium percentage in that layer was reduced beneath approximately ten (Fig. 5). Only after this reduction had

Table 4. - Distribution of Exchangeable Cations in Soil Columns 1 and 2

Layer	Sodium Per Cent			Calcium Per Cent			Magnesium Per Cent		
	Pr.	Column 1 Exp.	Column 2 Exp.	Pr.	Column 1 Exp.	Column 2 Exp.	Pr.	Column 1 Exp.	Column 2 Exp.
1	4.0	5.3	4.0	72.1	69.3	66.7	23.3	26.7	29.3
2	6.0	5.3	6.0	67.2	71.4	70.7	27.5	24.7	22.1
3	18.6	15.3	14.7	54.7	66.0	58.6	27.5	21.4	29.3
4	30.0	28.4	24.6	44.7	47.9	51.4	24.7	26.0	23.3
5	38.6	35.3	34.0	36.7	38.6	34.7	24.0	27.9	30.7
6	43.3	42.8	39.3	32.1	31.4	35.5	24.0	27.4	24.8

Table 5. - Distribution of Soluble Cations in Soil Columns 1 and 2

Layer	Sodium (meq/L)			Calcium (meq/L)			Magnesium (meq/L)		
	Pr.	Column 1 Exp.	Column 2 Exp.	Pr.	Column 1 Exp.	Column 2 Exp.	Pr.	Column 1 Exp.	Column 2 Exp.
1	9.10	26.3	26.9	1.80	25.8	19.6	0.80	14.4	13.4
2	38.0	28.3	33.4	16.6	16.8	17.8	9.80	8.80	7.20
3	89.8	62.3	60.8	14.6	16.4	16.2	11.0	8.00	11.8
4	154	142	145	14.0	14.8	19.8	11.8	11.0	13.4
5	212	213	205	14.2	17.2	15.8	14.4	17.2	21.2
6	1070	967	907	364	282	332	400	354	346

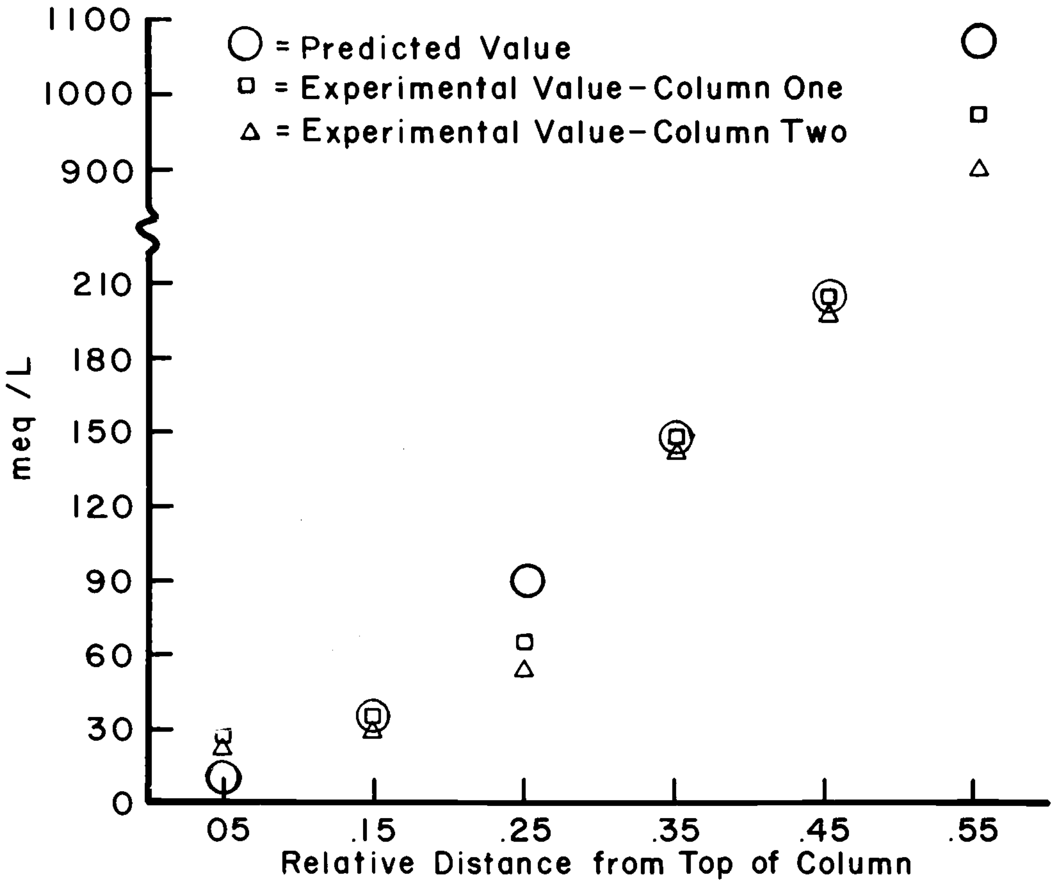

Fig. 3–Distribution of Soluble Sodium in Soil Columns One and Two

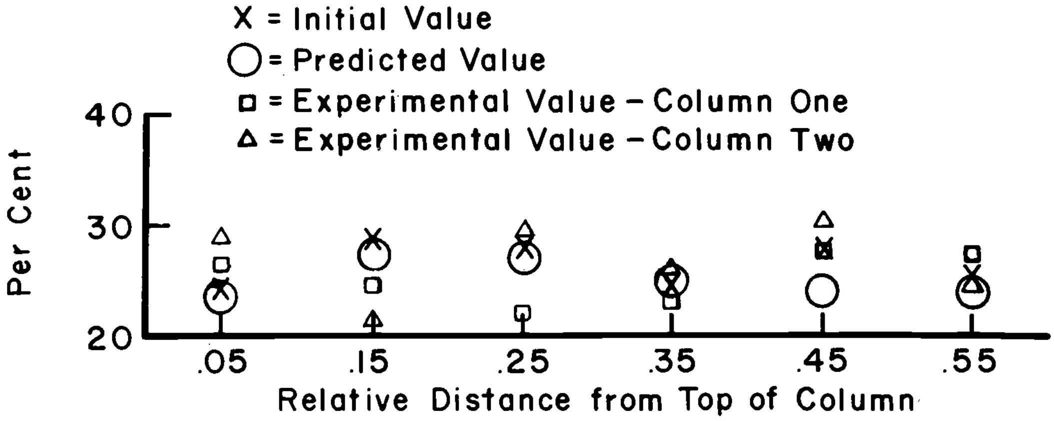

Fig. 4–Distribution of Exchangeable Magnesium in Soil Columns One and Two

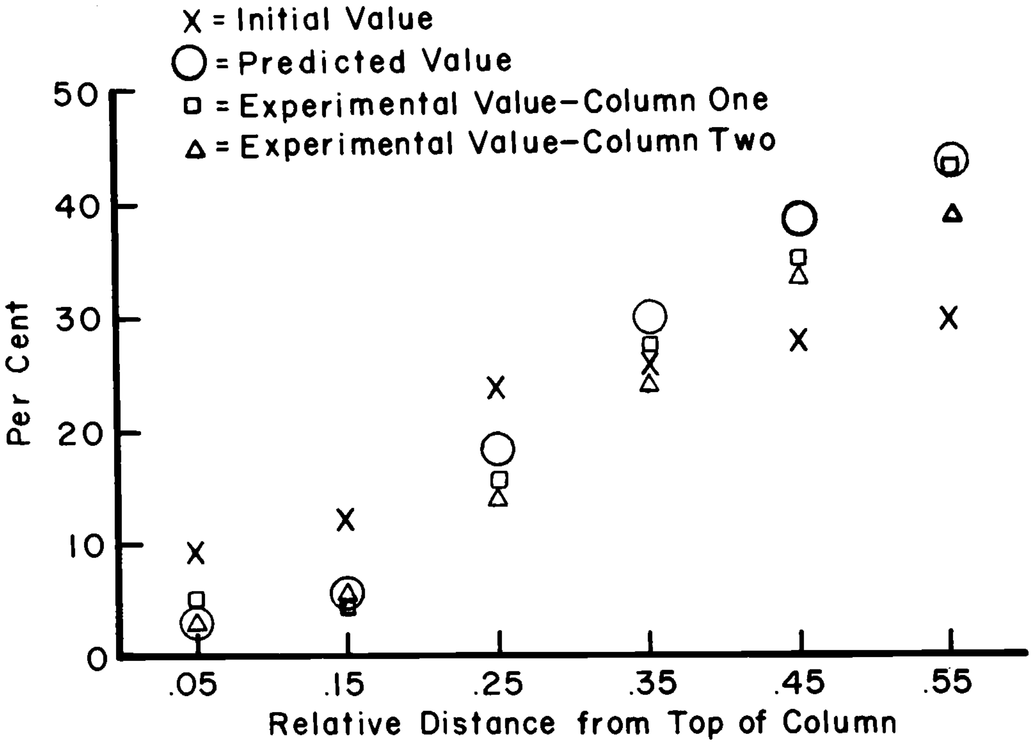

Fig. 5 – Distribution of Exchangeable Sodium in Soil Columns One and Two

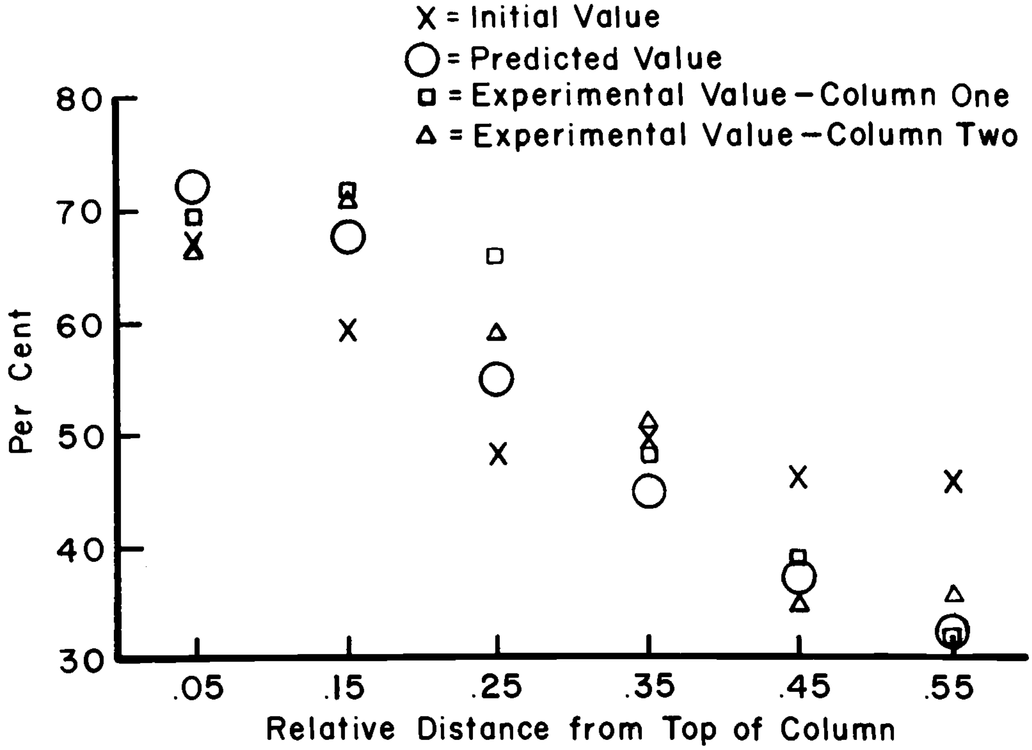

Fig. 6 – Distribution of Exchangeable Calcium in Soil Columns One and Two

occurred was the sodium percentage in the layer immediately beneath reduced significantly. This accounts for the sharp slopes seen in Figures 5 and 6. By extrapolation of the data presented in Figure 5 it can be seen that the exchangeable sodium percentage in the third layer ranged from eleven at the top to twenty-one at the bottom. At the halt of water flow into the column this layer must have been the primary zone of displacement of sodium by calcium. It would be expected that the agreement between predicted and experimental values would not be as good in this layer as elsewhere. Both this discrepancy and that caused by dispersion are nonetheless smaller than the probable variation in cation distribution at different locations in a field.

Test of Assumption Regarding Moisture Content

The predicted values mentioned above were obtained using the assumption that the field capacity of a soil is equal to one-half of its saturation percentage. To test the effect of this assumption upon the values yielded by program (A), the actual moisture contents present in column 1 of the cation distribution experiment were used in the calculation of a separate set of predicted results. All of the actual moisture contents, save that of the third layer, were higher than one-half of the saturation percentage (Table 6). Figures 7 and 8 show the discrepancies produced in prediction of the soluble sodium and exchangeable magnesium distributions, by assuming that the actual moisture contents were equal to one-half of the saturation percentage. Only within the second layer, where the actual moisture content was

Table 6. - Actual and Assumed Moisture Contents of Soil Column 1

Layer	Saturation Percentage	Assumed Moisture Percentage	Actual Moisture Percentage
1	36.9	18.45	23.3
2	36.9	18.45	26.0
3	39.3	19.65	17.9
4	39.3	19.65	23.1
5	37.1	18.55	23.9
6	33.6	16.80	19.7

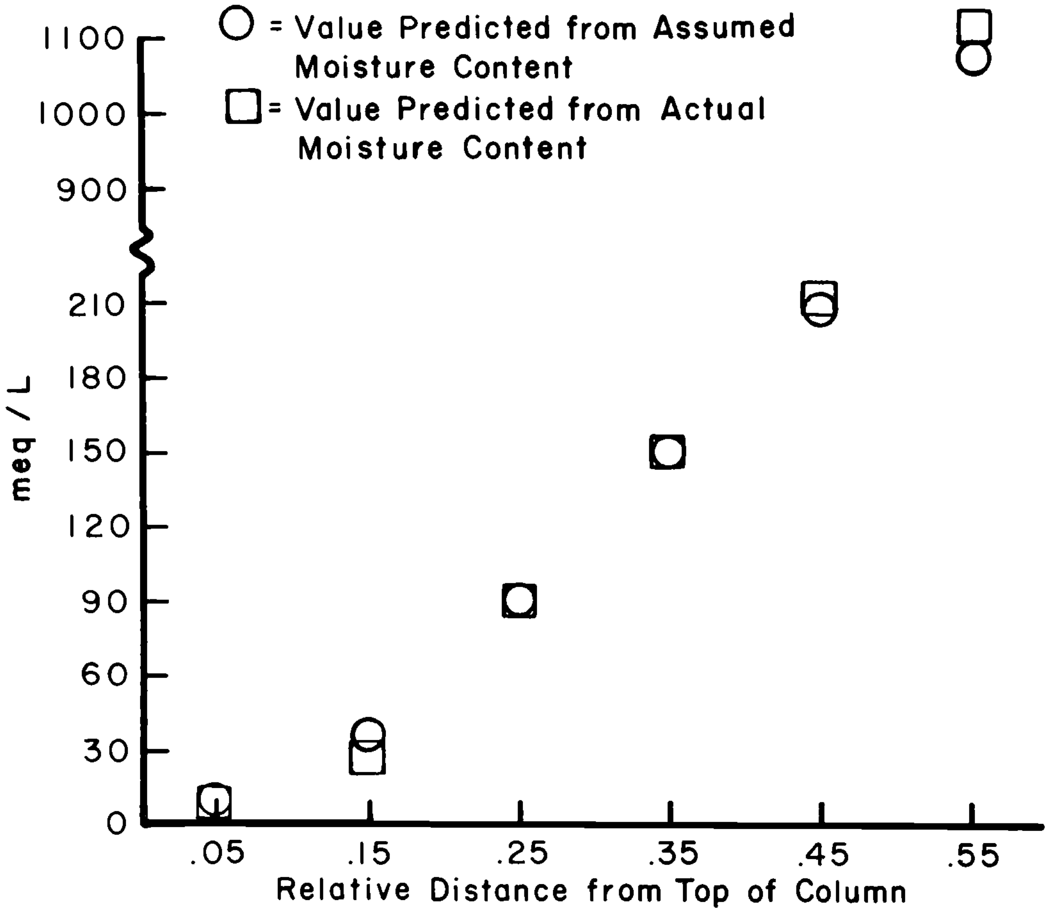

Fig. 7—Effect of Moisture Content Assumption upon Prediction of Soluble Sodium Distribution in Soil Columns One and Two

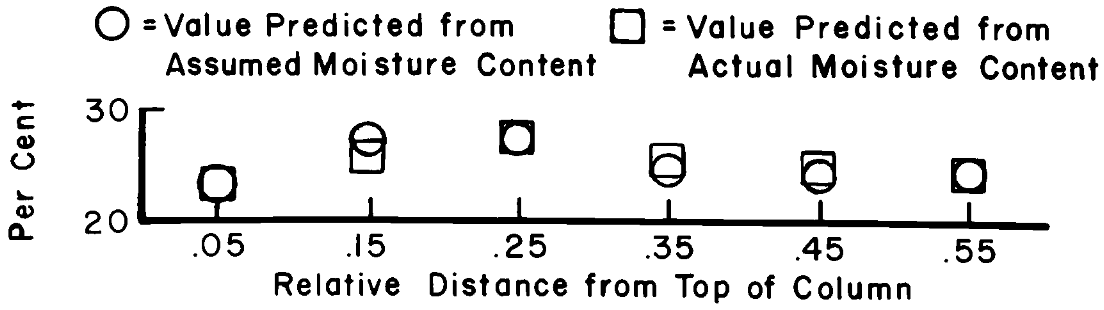

Fig. 8—Effect of Moisture Content Assumption upon Prediction of Exchangeable Magnesium Distribution in Soil Columns One and Two

equal to 70.5 per cent of the saturation percentage, were significant deviations produced. Otherwise, the errors were not serious, even where the actual moisture content was as high as 64.6 per cent of the saturation percentage. Comparison of the predicted values obtained by use of the actual moisture contents (Figs. 7, 8) with the experimental results (Figs. 3, 4) indicates that no tangible benefit would ensue were the actual moisture contents used instead of one-half of the saturation percentages.

Gypsum Requirement Prediction

The concentration of gypsum which program (B) said would reduce the exchangeable sodium percentage beneath fifteen in the third layer was $.230 \times 10^{-4}$ moles/g soil in the first layer. The program predicted reduction of the sodium per cent to 13.8 in the third layer were this addition made. The required number of aliquots of water was predicted to be 16. This treatment is equal to a gypsum addition of 4.56 tons/acre and leaching with 2.67 acre feet of water.

The reduction in exchangeable sodium per cent in the third layer which program (B) said would occur, were its instruction followed, was indeed found (Table 7). The predicted value was within 2.2 per cent of the actual value. Figure 9 illustrates the precision with which program (B) predicted the required gypsum addition.

The gypsum initially present in the soil was also instrumental in the reduction of the sodium percentage of the uppermost three layers. The exchangeable sodium per cent of the third layer of the unaltered column was reduced from 42.9 to 30.2. However, the similarity

Table. - Comparison of Sodium Percentages of Gypsum Treated and Unaltered Columns

Layer	Sodium Per Cent			
	Gypsum Treated Column		Unaltered Column	
	Pr.	Exp.	Pr.	Exp.
1	0.7	1.9	2.8	6.7
2	2.0	1.2	5.4	8.3
3	13.8	13.5	27.9	30.2
4	48.7	55.0	53.1	57.1
5	60.7	67.0	60.7	63.3
6	63.3	70.2	63.3	66.0
7	65.4	71.8	65.4	63.3
8	67.4	70.6	67.4	68.6
9	70.0	66.9	70.0	62.9
10	72.1	72.1	72.1	72.1

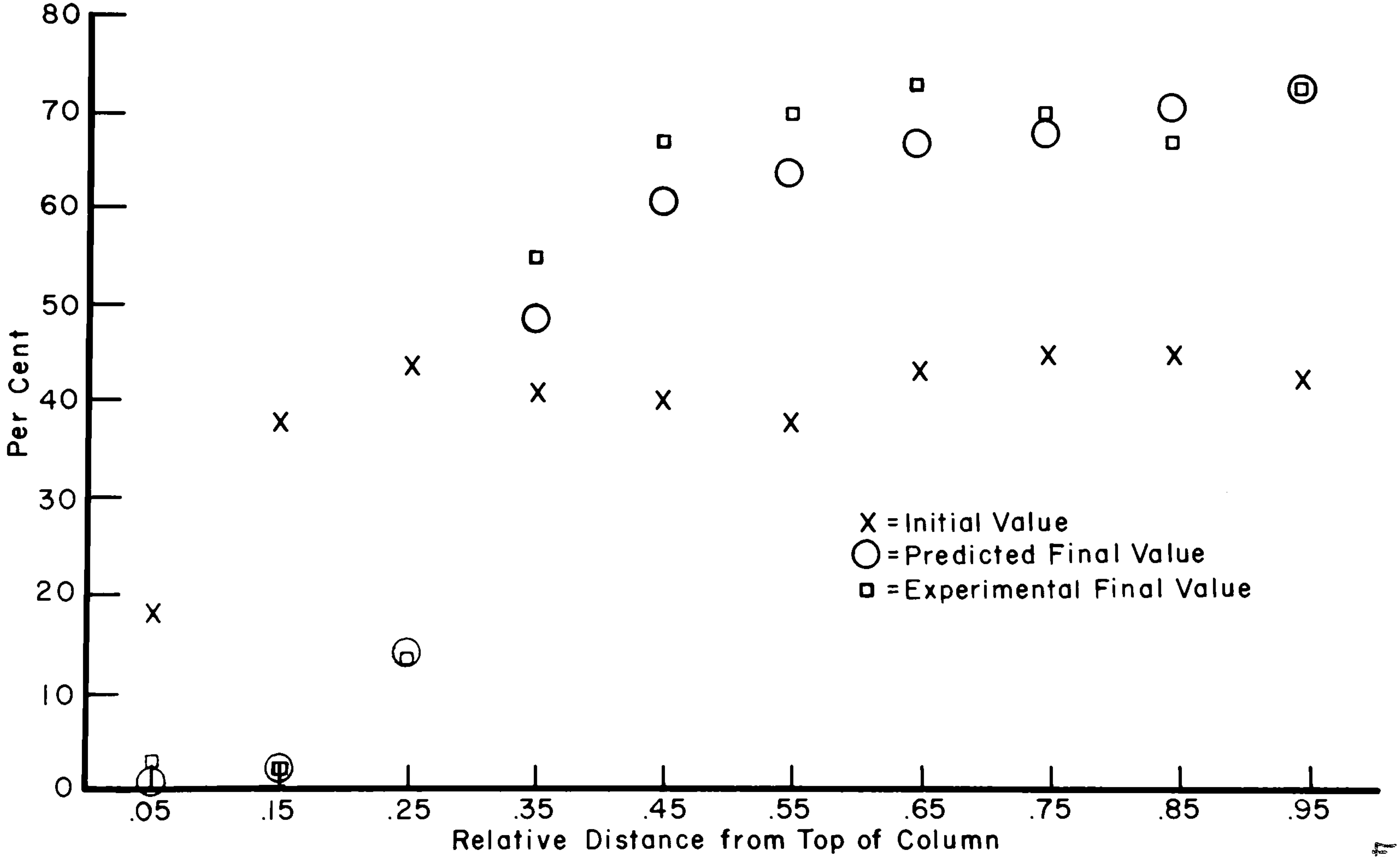

Fig. 9— Initial and Final Exchangeable Sodium Distribution in Gypsum Treated Column

of predicted and experimental values of soluble cations in the effluent and soil solution of the bottom layer of the unaltered and gypsum treated columns (Table 10, see p. 53) indicates that the native gypsum supply was dissolved in the first seven aliquots to pass through the columns. Further decreases in the sodium content of the unaltered column would, therefore, have occurred at a diminished rate. The benefit of the gypsum addition may be deduced from the fact that the depth of soil reduced to a sodium per cent beneath fifteen was increased 44.4 per cent. That this reduction was indeed a result of the gypsum addition can be seen by comparing the exchangeable calcium percentages of the top four layers of the gypsum treated and unaltered columns (Table 8).

The behavior of the gypsum supplement may be observed by comparing the calcium concentrations in the uppermost three layers of the unaltered and gypsum treated columns (aliquots 14, 15, 16 in Figs. 10 and 11). The soluble calcium concentration in the uppermost three layers of the gypsum treated column is higher than that found in the corresponding layers of the unaltered column by factors of 6.25, 6.00, and 9.09, respectively; whereas, in both columns the concentration of soluble calcium in layers four through ten was virtually the same. Apparently most of the calcium emanating from the solution of gypsum was adsorbed by the soil complex of the first three layers.

The average sodium concentration of the soil solution present in the lower seven layers of the gypsum treated column was 18.0 meq/L higher than that found in the corresponding soil solution of the

Table 8. - Comparison of Calcium and Magnesium Percentages of Unaltered and Gypsum Treated Columns

Layer	Calcium Per Cent				Magnesium Per Cent			
	Unaltered Column		Gypsum Treated Column		Unaltered Column		Gypsum Treated Column	
	Pr.	Exp.	Pr.	Exp.	Pr.	Exp.	Pr.	Exp.
1	75.3	68.7	82.8	73.5	23.3	25.0	16.8	24.6
2	67.4	66.7	73.3	72.2	25.4	24.5	24.8	26.6
3	49.3	50.1	60.0	68.1	23.3	20.0	26.0	18.4
4	32.8	31.5	34.6	33.8	15.4	10.8	16.6	11.2
5	27.3	29.3	27.3	24.1	12.8	8.6	12.8	8.9
6	26.0	24.0	26.0	22.5	12.1	9.4	12.1	7.5
7	25.0	27.4	25.0	21.3	8.6	9.2	8.6	6.7
8	24.0	24.7	24.0	21.9	7.3	6.7	7.3	8.1
9	22.1	28.6	22.1	23.1	6.7	8.6	6.7	9.9
10	21.4	22.8	24.1	19.0	6.0	5.4	6.0	9.0

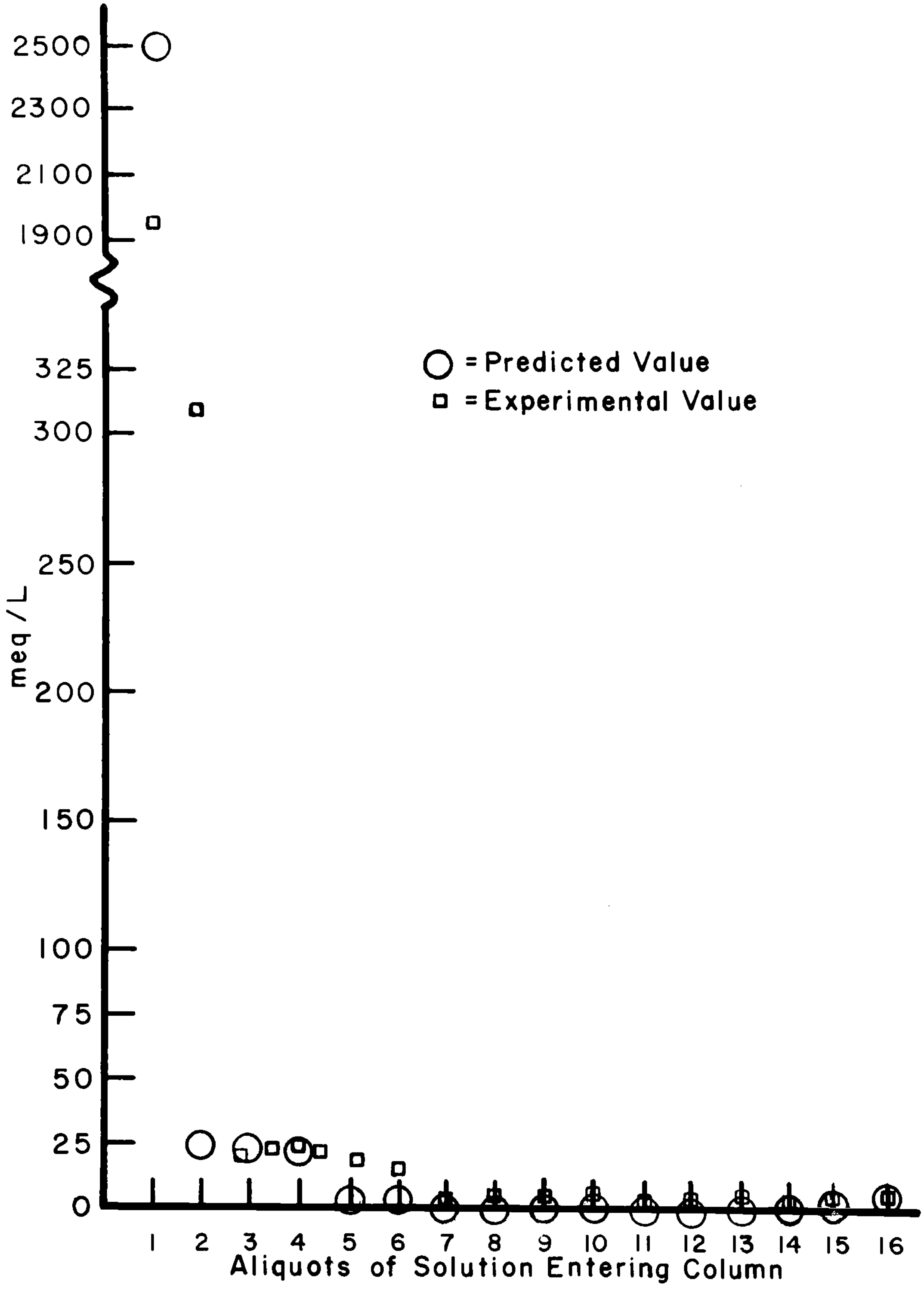

Fig. 10 — Unaltered Column — Soluble Calcium Distribution

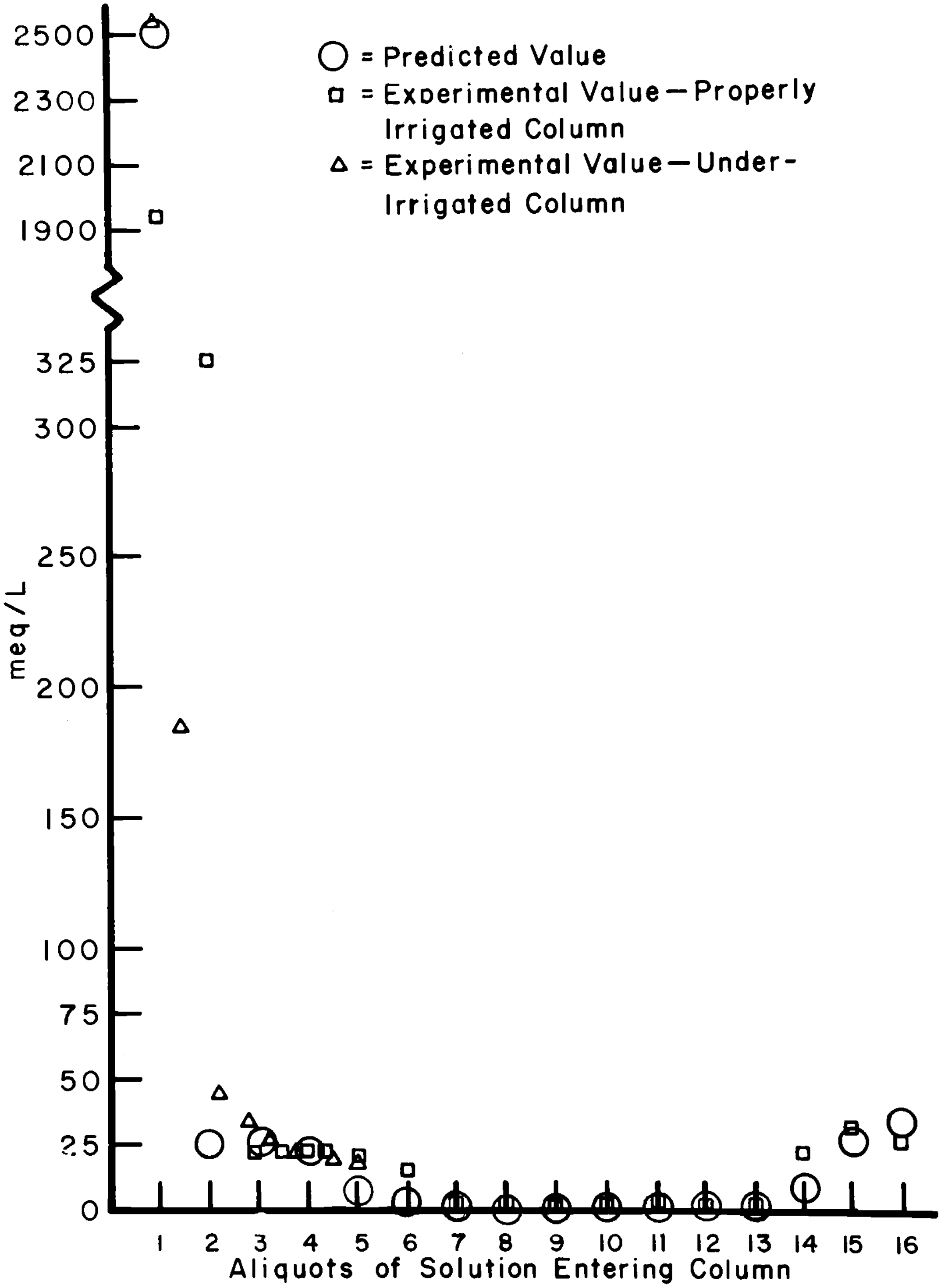

Fig. 11—Gypsum Treated Columns—Soluble Calcium Distribution

unaltered column (Table 9). The additional sodium is that which was displaced by the calcium originating from gypsum solution. It was present in such intense concentrations that the sodium percentages of these layers of the gypsum treated column went up (Fig. 9). Note in Table 7 that the sodium percentages of the corresponding layers of the unaltered column are almost as high. The increased sodium percentages probably did not cause much additional physical deterioration of these soil layers. As indicated by Figure 12, most of the adverse effects of sodium are accomplished once the sodium percentage reaches the level at which these layers were originally. The water applied during farming operations would eventually remove much of the sodium remaining in the lower part of the profile.

The flow velocities manifested by the columns of the gypsum requirement study were between .005 cm/hr and .01 cm/hr. It is under such conditions of low flow velocity that maximum dispersion occurs (2). The quantity of soluble salt initially present in the soil was about 100 times that found in the synthetic Colorado River water. Figures 10 and 11 show that most of the calcium was leached by the first aliquot of water to pass through the column. A sharp decrease in the salt content of the water was predicted by computer program (A). The lag in salt removal created by dispersion tended to smooth out this decrease to some extent. Similar effects are noticeable with soluble sodium and magnesium (Table 10).

In many areas where reclamation by gypsum addition is practiced, maximum utilization of the water supply is critical. The accuracy of

Table 9. - Soluble Sodium Distribution in Unaltered and Gypsum Treated Columns

Layer	Sodium (meq/L)	
	Unaltered Column	Gypsum Treated Column
1	15.7	9.40
2	24.4	7.50
3	71.0	19.6
4	96.0	83.5
5	139.7	140.8
6	180.3	152.3
7	142.8	169.3
8	120.1	174.0
9	117.6	170.1
10	192.5	225.7

Table 10. - Solute Composition of Effluent from Unaltered Column and Gypsum Treated Columns

Volume of Effluent	Sodium (meq/L)				Calcium (meq/L)				Magnesium (meq/L)			
Ml.	Pr.	1*	2**	3***	Pr.	1*	2**	3***	Pr.	1*	2**	3***
11.0	-	-	-	5893	-	-	-	2514	-	-	-	2200
13.0	4730	4666	4800	-	2550	1920	1940	-	1884	1350	1350	-
19.6	-	-	-	2550	-	-	-	184	-	-	-	210
25.0	-	2180	-	-	-	309	-	-	-	288	-	-
26.0	497	-	2200	-	25.3	-	322	-	10.6	-	335	-
28.2	-	-	-	1345	-	-	-	42.9	-	-	-	200
35.0	-	-	-	934	-	-	-	33.0	-	-	-	131
36.0	-	640	-	-	-	21.5	-	-	-	40.0	-	-
39.0	466	-	770	-	25.2	-	26.0	-	9.4	-	59.0	-
41.4	-	-	-	736	-	-	-	28.4	-	-	-	76.8
44.0	-	-	479	-	-	-	24.6	-	-	-	27.2	-
44.8	-	540	-	-	-	24.2	-	-	-	56.0	-	-
48.5	-	-	-	525	-	-	-	25.0	-	-	-	43.0
50.6	-	-	367	-	-	-	24.0	-	-	-	19.0	-
52.0	370	377	-	-	21.9	23.6	-	-	6.2	36.0	-	-
57.2	-	-	320	-	-	-	23.4	-	-	-	14.0	-
58.0	-	346	-	-	-	23.2	-	-	-	29.2	-	-
59.5	-	-	-	300	-	-	-	21.5	-	-	-	32.5
63.6	-	-	287	-	-	-	21.2	-	-	-	13.4	-
65.0	110	-	-	185	1.4	-	-	21.0	0.4	-	-	10.4
66.0	-	282	-	-	-	18.4	-	-	-	16.7	-	-
70.7	-	-	166	-	-	-	20.0	-	-	-	5.4	-
78.0	92.1	210	115	-	1.0	16.3	15.9	-	0.3	15.0	2.7	-

*1. Experimental Value - Unaltered Column. **2. Experimental Value - Properly Irrigated Gypsum Treated Column. ***3. Experimental Value - Under-Irrigated Gypsum Treated Column.

the water predicting segment of program (B) was tested by terminating the wetting of the duplicate gypsum treated soil column one aliquot of water short of the number recommended by the program (B). The water saving would be equal to 0.17 acre feet were the reduced quantity of water sufficient to dissolve the gypsum addition. However, this quantity of water did not reduce the sodium percentage as well as did the recommended quantity (Table 11). It may, therefore, be presumed that it was not sufficient to dissolve the supplemental gypsum. Program (B) appears to predict accurately the water necessary to dissolve the gypsum addition it calculates.

Relation of Infiltration Rate to Exchangeable Sodium Percentage

The relation between the hydraulic conductivity and the exchangeable sodium percentage of the Gila Silt Loam used in this experiment is shown in Figure 12. It has the shape common to such curves (15, 25). This relation was used in an attempt to predict the hydraulic conductivity of a dynamic soil-water system. This attempt did not yield accurate results (Fig. 13).

Calcium carbonate, which was still present in the soil despite removal of a preponderance of the native quantity, would have decreased the rate at which sodium replaced calcium. This would have slowed the rate of fall of the hydraulic conductivity. As this is the reverse of the actual occurrence, calcium carbonate may be discounted as the cause of failure.

Table 11. - Comparison of Proper Irrigation and Under-Irrigation of Gypsum Treated Soil

Layer	Sodium Per Cent				Calcium Per Cent			
	Properly Irrigated Gypsum Treated Column		Under-Irrigated Gypsum Treated Column		Properly Irrigated Gypsum Treated Column		Under-Irrigated Gypsum Treated Column	
	Pr.	Exp.	Pr.	Exp.	Pr.	Exp.	Pr.	Exp.
1	0.7	1.9	1.1	1.4	82.8	73.5	81.6	69.0
2	2.0	1.2	1.9	1.4	73.3	72.2	73.0	69.0
3	13.8	13.5	16.4	18.7	60.0	68.1	58.2	61.7
4	48.7	55.0	49.6	58.0	34.6	33.8	34.2	30.2
5	60.7	67.0	60.7	65.1	27.3	24.1	27.3	21.0
6	63.3	70.2	63.3	58.0	26.0	22.5	26.0	25.2
7	65.4	71.8	65.4	61.4	25.0	21.3	25.0	26.9
8	67.4	70.6	67.4	59.7	24.0	21.9	24.0	25.4
9	70.0	66.9	70.0	59.5	22.1	23.1	22.1	31.2
10	72.1	72.1	72.1	75.3	21.4	19.0	21.4	17.8

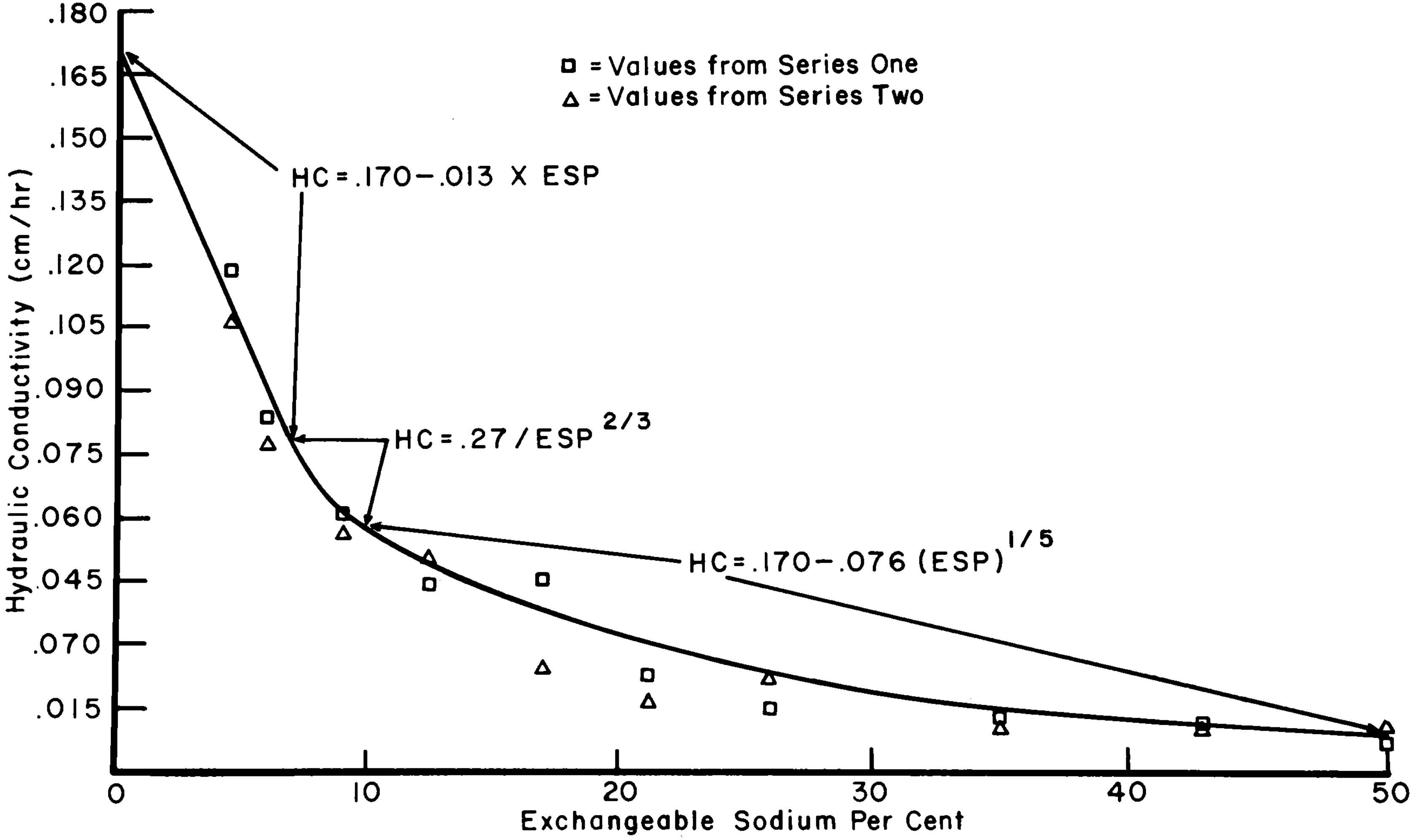

Fig. 12 — Relation between Hydraulic Conductivity and Exchangeable Sodium Percentage of Gila Silt Loam

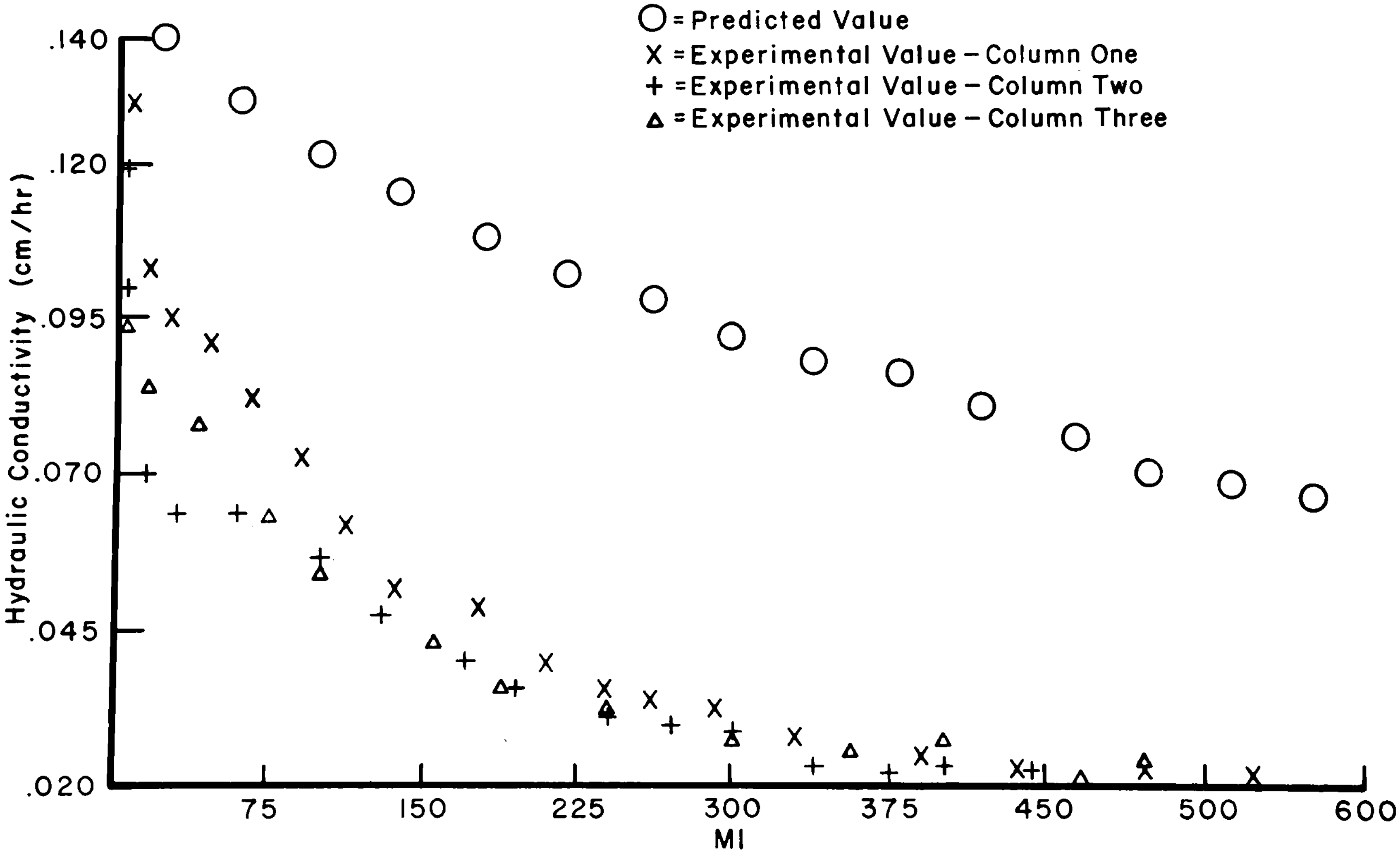

Fig. 13 – Hydraulic Conductivity of Gila Silt Loam Soil Columns

The reproducibility of the experimental results indicates that the source of error was of a chemical nature, as it is doubtful whether a physical factor, such as clay swelling, would have interfered in the exact same fashion in all three columns. The slopes of the curves of the predicted and experimental values are similar after approximately the 60 ml mark in Figure 13. This would indicate that some chemical occurrence which had taken place in a small portion of the column prior to the passage of approximately 60 ml of solution, was masking the reactions in the rest of the column. Formation of a thin film of molecular thickness with very high exchangeable percentage could produce such an effect. This film may well have formed at the interface between the soil column and the percolating solution.[4] The rate of reaction is faster at such an interface and a relatively impermeable layer could have developed rapidly. Though this layer would have a thickness of a few molecules, it would have a large effect upon the hydraulic conductivity of the soil column. This disproportionate influence would indeed overshadow the effects of the hydraulic conductivities of the other sections of the column in determining the hydraulic conductivity of the entire column. The formation of such a layer is quite possibly the cause of the failure of program (C).

The effluent from the third of the columns of this experiment was analyzed. The results of the analysis are shown in Figure 14. Two plateaus appear in the curves of sodium and calcium. One extends from

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[4] In a personal communication, Kenneth L. Dyer stated that such films have actually been found to occur.

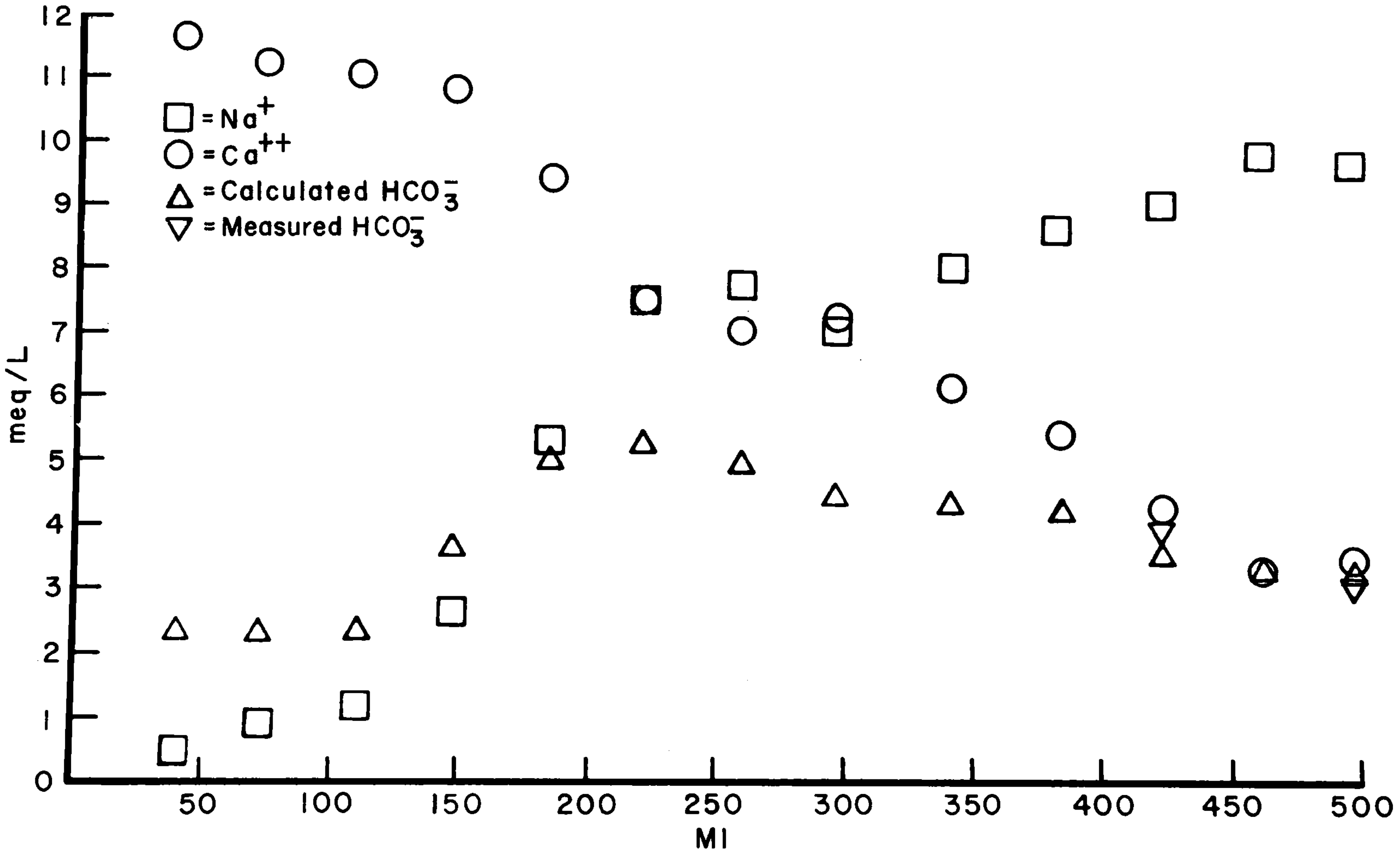

Fig. 14 — Effluent Composition of Gila Silt Loam Soil Column

225 ml to 300 ml and the other from 465 ml to the conclusion of the experiment at 500 ml. The second plateau may have continued had not the experiment terminated at that point. These plateaus are similar in shape to those reported by Dutt et al. (11) for gypsum. The appearance of two plateaus is possibly due to the dependence of the solubility of calcium carbonate upon the pH value. As the pH value increases the solubility of calcium carbonate decreases. The initial pH value of the solution was 5.0 and the final value was 6.6.

If the soil had been truly salt free, the salt concentration of the effluent would have been equal to that of the influent. As most salts were removed by washing the soil prior to the inception of the experiment, the bulk of the increased salt concentration of the effluent must be from a sparingly soluble salt, such as calcium carbonate or calcium sulfate. The values for bicarbonate plotted on Figure 14 were obtained by subtracting the salt concentration of the influent from that of the effluent. As the effluent showed no sulfate this is a valid procedure. The effluent was tested for bicarbonate at the points indicated on Figure 14. These show good agreement with the calculated values. The calcium present in the effluent after the 450th ml had passed was probably almost entirely from the solution of calcium carbonate.

SUMMARY AND CONCLUSIONS

Sodium is the chemical entity most often responsible for reduction of infiltration rates. Its removal from the soil exchange complex often entails application of a chemical amendment. A common additive is calcium sulfate, gypsum. Heretofore, no method has been available which would accurately predict the gypsum required to lower the exchangeable sodium percentage to a desired level in a given layer.

A computer program was developed to predict the cation distribution in a layered soil upon irrigation of the soil. The method was developed from that of Dutt _et al._ (9) for similar reactions in a homogeneous soil. The soil upon which the program was tested consisted of ten layers, reaching to a depth of five feet and containing gypsum, calcium carbonate, and magnesium carbonate. Half scale models of the soil profile were constructed in lucite filtration tubes of inner diameter of one inch. The soil was not altered chemically prior to insertion into the tubes. A synthetic Colorado River water was percolated into two tubes until the wetting front had reached the boundary between the sixth and seventh layers. The columns were then cut apart at the boundaries between the layers and analyzed. Agreement between the predicted and experimental values was satisfactory.

For simplicity of data collection prior to use of all programs in this work, it was assumed that the actual moisture content of a layer was equal to one-half of its saturation percentage. The validity of this assumption was tested by using the actual moisture contents present in one of the columns of the above experiment in the prediction of its cation distribution. No significant benefit resulted from the use of the actual moisture contents.

A loop was then added to the cation distribution program to predict the gypsum required to lower the sodium percentage in a given layer to a desired level. The loop internally adds gypsum so as to maintain a soil solution saturated with gypsum in the top layer. The program prints the gypsum requirement, water necessary to dissolve this supplement, and the sodium percentage which will result from this treatment. The case chosen was reduction of the sodium percentage in the third layer from 42.9 to 15.0.

Three soil columns were used to test the accuracy of the predictions. One was run according to the directions of the computer, a second had the proper gypsum addition but was under-irrigated, and a third was properly irrigated but received no gypsum addition. The results indicated that the gypsum and water requirements were being accurately predicted.

It is noteworthy that the effluent composition of these columns was adequately predicted by the cation distribution program. This was despite the presence of conditions conducive to dispersion.

An attempt was then made to predict infiltration rates during an irrigation. This was based upon the relation between the hydraulic conductivity of a soil and its equilibrium exchangeable sodium percentage. The particular relation for the soil used in the experiment was found and inserted into a program which predicted the sodium percentage in a dynamic soil-water system. The program was tested upon three soil columns. This soil had been calcium saturated and freed of much of its calcium carbonate and soluble salts. Despite evacuation of the columns for twelve to fourteen hours, presaturation with a calcium chloride solution of concentration equal to that of the sodium chloride solution applied afterward, and maintenance in a constant temperature air bath, the predictions were unsuccessful. Formation of a thin film of high exchangeable sodium percentage, at the interface between the soil column and the percolating solution, was probably the source of error.

The effluent from the third of the above soil columns was analyzed and yielded information regarding the reactions of calcium carbonate in soils. Its presence was believed responsible for the formation of two plateaus in the concentration curves of the cations in the effluent. It was concluded that much of the calcium present in the effluent throughout the experiment, and almost all in the latter part, was probably from solution of calcium carbonate.

APPENDIX

PROGRAM A

```
*    COMPILE FORTRAN, EXECUTE FORTRAN

C     CATION DISTRIBUTION-LAYERED SOIL
C     RICHARD TERKELTOUB
  3030DIMENSION C(10), E(10), XX(10), SA(10), A(10), G(10),
     1CAS(10), S(10), B(10), W(10), H(10), SOS(10),
     2AMG(10), CL(10), SO(10), CA(10), R(10)
   30 FORMAT (E10.3, 2XE10.3, 2XE10.3, 2XE10.3, 2XE10.3)
   41 FORMAT (1HO, E10.3, 2XE10.3, 2XE10.3, 2XE10.3, 2XE10.3, 2XE10.3)
  241 FORMAT (1HO, E10.3, 2XE10.3, 2XE10.3, 2XE10.3)
   20 READ 30, A5, S5, R5, H5, G5
      READ 30, CAS5, DA
   31 FORMAT (I2,I3,I2)
   99 READ 31, L,JS,JR
   21 READ 30, SA5, E5, C5, XX5
      READ 30, SO5, CL5, B5, CA5, AMG5
      READ 30, SOS5, W5
      IIIII=1
      J=0
   22 J=J+1
      SA(J)=SA5
      E(J)=E5
      C(J)=C5
      XX(J)=XX5
      W(J)=W5
      SO(J)=SO5
      CL(J)=CL5
      B(J)=B5
      CA(J)=CA5
      AMG(J)=AMG5
      SOS(J)=SOS5
      IF (J-JR)  222,500,500
  222 GO TO (23, 25, 26, 27, 28, 29, 32, 33, 37, 500), IIIII
   23 IIIII=2
      READ 30, SA5, E5, C5, XX5
      READ 30, SO5, CL5, B5, CA5, AMG5
      READ 30, SOS5, W5
      GO TO 22
   25 IIIII=3
      READ 30, SA5, E5, C5, XX5
      READ 30, SO5, CL5, B5, CA5, AMG5
      READ 30, SOS5, W5
      GO TO 22
```

```
 26 IIIII=4
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 27 IIIII=5
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 28 IIIII=6
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 29 IIIII=7
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 32 IIIII=8
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 33 IIIII=9
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 37 IIIII=10
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
500 J=0
    IS=2
    J7=JS
    D2=1.385E4
    RT=R5
    GR=G5
    HR=H5
    AR=A5
    SK=S5
    CASO=CAS5
201 J=J+1
    CT=C(J)
    ET=E(J)
    SAT=SA(J)
    XXT=XX(J)
    BT=B(J)
```

```
    WT=W(J)
    D=D2/WT
    J=J-1
    GO TO (24,202), IS
202 J=J+1
    HR=HR+2.0*CL(J)
    GR=GR+2.0*SO(J)
    SK=SK+2.0*SOS(J)
    AR=AR+2.0*CA(J)
    RT=RT+2.0*AMG(J)
    J=J-1
    GO TO 24
 39 J=0
    SK=S5
    AR=A5
    HR=H5
    RT=R5
    GR=G5
    CASO=CAS5
    GO TO 201
  8 J=J+1
    E(J)=ET
    C(J)=CT
    SA(J)=SAT
    XX(J)=XXT
 16 IF (JS-J)  299,299,75
 75 IF (J-L) 201,400,400
400 PRINT 41, AR,SK, RT, HR, GR, CASO
    GO TO 203
299 S(J)=SK
    A(J)=AR
    H(J)=HR
    G(J)=GR
    R(J)=RT
    CAS(J)=CASO
203 JS=JS-1
    IS=1
208 IF (JS) 177,177,39
177 IF (J7-L) 277,77,77
277 L=J7
 77 PRINT 41, (A(J), S(J), R(J), H(J), G(J), CAS(J), J=1,L)
    PRINT 241,(SA(J), C(J), E(J), XX(J), J=1,L)
    GO TO 99
 24 A1=AR
    IF (XXT) 4,4,42
  4 U=SQRTF(2.0*(RT+AR+GR)+0.5*SK+HR)
```

```
      AA=EXPEF(-9.366*U/(1.0+U))
      IF (2.4E-5-AR*GR*AA) 42,18,18
   42 X=0.0
      U=SQRTF(2.0*(RT+AR+GR)+0.5*SK+HR)
      BB=AR=GR
      EX=(9.366*U)/(1.0+U)
      CC=AR*GR-(2.4E-5)*EXPEF(EX)
      T=SQRTF(BB*BB-4.0*CC)
      X=(-BB+T)/2.0
      CAS1=4.987E-3-CASO
      DEL=BT*XXT-CAS1
      IF (DEL-X) 127,128,128
  127 X=XXT*BT
      XXT=0.0
      CAS1=0.0
      AR=AR+X
      GR=GR+X
      U=SQRTF(2.0*(RT+GR+AR)+0.5*SK+HR)
      AA=EXPEF(-9.366*U/(1.0+U))
    7 BB=-(4.9E-3+AA*AR+AA*GR)
      CC=AA*AR*GR-4.9E-3*CASO
      X1=(-BB-SQRTF(BB*BB-4.0*AA*CC))/(2.0*AA)
      CASO=CASO+X1
      AR=AR-X1
      GR=GR-X1
      GO TO 44
   18 IF (GR) 1,1,6
    6 IF (AR) 1,1,7
    1 IF (CASO) 44,44,7
  128 AR=AR+X
      GR=GR+X
      XXT=XXT=X/BT
      CASO=CASO+CAS1
      XXT=XXT-CAS1/BT
   44 A2=AR
      IF (AR-RT) 45,46,46
   45 IJ=1
      AZ=AR
      AR=RT
      RT=AZ
      CTZ=CT
      CT=ET
      ET=CTZ
      DZ=D
      D=D*DA
      GO TO 5
   46 IJ=2
```

```
  5 Z=1.0E-12
    U=SQRTF(2.0*(AR+GR+RT)+0.5*SK+HR)
    EX=EXPEF((-2.341*U)/(1.0+U))
    AA=-4.0*BT*(D*EX+BT)
    BB=4.0*(BT*CT+SK-D*EX*SAT)*BT-D*AR*EX)
    CC=-SK*(4.0*BT*CT+SK)-D*EX*SAT*(BT*SAT+4.0*AR)
    DD=SK*SK*CT-D*AR*EX*SAT*SAT
115 ZZ=-(((AA*Z+BB)*Z+CC)*Z+DD)
    ZZZ=((3.0*AA*Z+2.0*BB)*Z+1.0*CC)
    ZZ=ZZ/ZZZ
    Z=Z*(1.0+ZZ/Z)
    IF (ZZ/Z+1.0E-3)  115,116,116
116 IF (ZZ/Z-1.0E-3)  178,178,115
178 AR=AR+BT*Z
    SK=SK-2.0*BT*Z
    CT=CT-Z
    SAT=SAT+2.0*Z
    GO TO (17,13) , IJ
 17 AZ=AR
    AR=RT
    RT=AZ
    CTZ=CT
    CT=ET
    ET=CTZ
    D=DZ
 13 A3=AR
    BB=AR+BT*(ET+DA*CT)+DA*RT
    AA=BT*(1.0-DA)
    CC=(AR*ET-DA*RT*CT)
    T=SQRTF(BB*BB-4.0*AA*CC)
    Y=(-BB+T)/(2.0*AA)
    IF (1.0-DA)  3,2,3
  2 Y=-(AR+ET-DA*CT+RT/BB)
  3 AR=AR+BT*Y
    RT=RT-BT*Y
    CT=CT-Y
    ET=ET+Y
    DEL=AR-A1
    IF (DEL+1.0E-5) 24,48,48
 48 IF (DEL-1.0E-5) 49,49,24
 49 DEL=AR-A2
    IF (DEL+1.0E-5) 24,50,50
 50 IF (DEL-1.0E-5) 51,51,24
 51 DEL=AR-A3
    IF (DEL+1.0E-5) 24,52,52
 52 IF (DEL-1.0E-5) 8,8,24
    END
```

PROGRAM B

```
*     COMPILE FORTRAN, EXECUTE FORTRAN

C      GYPSUM REQUIREMENT
C      RICHARD TERKELTOUB
   3030DIMENSION C(10), E(10), XX(10), SA(10), A(10), G(10),
      1CAS(10), S(10), B(10), W(10), H(10), SOS(10), AMG(10),
      2CL(10), SO(10), CA(10), R(10)
    30 FORMAT (E10.3, 2XE10.3, 2XE10.3, 2XE10.3, 2XE10.3)
   141 FORMAT (1H0, E10.3, 2XE10.3, 2XI5, 2XE10.3, 2XE10.3)
    20 READ 30, A5, S5, R5, H5, G5
       READ 30, CAS5, DA, WH
    21 IIIII=1
       J=0
       READ 30, SA5, E5, C5, XX5
       GY=XX5
       READ 30, SO5, CL5, B5, CA5, AMG5
       READ 30, SOS5, W5
    31 FORMAT (I2)
       READ 31, JR
    22 J=J+1
       SA(J)=SA5
       E(J)=E5
       C(J)=C5
       XX(J)=XX5
       W(J)=W5
       SO(J)=SO5
       CL(J)=CL5
       B(J)=B5
       CA(J)=CA5
       AMG(J)=AMG5
       SOS(J)=SOS5
       IF (J-JR)  222,500,500
   222 GO TO (23,25,26,27,28,29,32,33,37,500) , IIIII
    23 IIIII=2
       READ 30,SA5, E5, C5, XX5
       READ 30, SO5, CL5, B5, CA5, AMG5
       READ 30, SOS5, W5
       GO TO 22
    25 IIIII=3
       READ 30, SA5, E5, C5, XX5
       READ 30, SO5, CL5, B5, CA5, AMG5
       READ 30, SOS5, W5
       GO TO 22
    26 IIIII=4
       READ 30, SA5, E5, C5, XX5
       READ 30, SO5, CL5, B5, CA5, AMG5
       READ 30, SOS5, W5
       GO TO 22
```

```
 27 IIIII=5
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 28 IIIII=6
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 29 IIIII=7
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 32 IIIII=8
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 33 IIIII=9
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
 37 IIIII=10
    READ 30, SA5, E5, C5, XX5
    READ 30, SO5, CL5, B5, CA5, AMG5
    READ 30, SOS5, W5
    GO TO 22
500 J=0
    N=0
    BK=0.0
    IS=2
    D2=1.385E4
    GXP=0.0
    RT=R5
    SK=S5
    GR=G5
    HR=H5
    AR=A5
    CASO=CAS5
201 J=J+1
    CT=C(J)
    ET=E(J)
    SAT=SA(J)
    XXT=XX(J)
    BT=B(J)
    WT=W(J)
    D=D2/WT
    J=J-1
    GO TO (24,202) , IS
```

```
202 J=J+1
    HR=HR+2.0*CL(J)
    GR=GR+2.0*SO(J)
    RT=RT+2.0*AMG(J)
    SK=SK+2.0*SOS(J)
    AR=AR+2.0*CA(J)
    J=J-1
    GO TO 24
  8 J=J+1
    E(J)=ET
    C(J)=CT
    SA(J)=SAT
    XX(J)=XXT
    IF (J-JR)  201,39,39
 39 J=0
    RT=R5
    HR=H5
    AR=A5
    GR=G5
    SK=S5
    CASO=CAS5
    GO TO 201
 24 A1=AR
    IF (XXT) 4,4,42
  4 U=SQRTF(2.0*(RT+AR+GR)+0.5*SK+HR)
    AA=EXPEF(-9.366*U/(1.0+U))
    IF (2.4E-5-AR*GR*AA)  42,103,103
103 IF (J) 105,105,18
105 U=SQRTF(2.0*(AR+GR+RT)+0.5*SK+HR)
    BB=AR+GR
    EX=(9.366*U)/(1.0+U)
    CC=AR*GR-(2.4E-5)*EXPEF(EX)
    T=SQRTF(BB*BB-4.0*CC)
    X=(-BB+T)/2.0
    AR=AR+X
    GR=GR+X
    GYP=X/BT
    CAS1=4.987E-3-CASO
    GZP=CAS1/BT
    GXP=GYP+GZP+GXP
    CASO=CASO+GZP*BT
    GO TO 44
 42 X=0.0
    U=SQRTF(2.0*(RT+AR+GR)+0.5*SK+HR)
    BB=AR+GR
    EX=(9.366*U)/(1.0+U)
    CC=AR*GR-(2.4E-5)*EXPEF(EX)
    T=SQRTF(BB*BB-4.0*CC)
```

```
      X=(-BB+T)/2.0
      CAS1=4.987E-3-CASO
      DEL=BT*XXT-CAS1
      IF (DEL-X) 123,128,128
  123 IF (J) 129,129,127
  129 X=XXT*BT
      XXT=0.0
      CAS1=0.0
      AR=AR+X
      GR=GR+X
      GO TO 105
  127 X+XXT*BT
      XXT=0.0
      CAS1=0.0
      AR=AR+X
      GR=GR+X
      U=SQRTF(2.0*(RT+AR+GR)+0.5*SK+HR)
      AA=EXPEF(-9.366*U/(1.0*U))
    7 BB=-(4.9E-3+AA*AR+AA*GR)
      CC=AA*AR*GR-4.9E-3*CASO
      X1=(-BB-SQRTF(BB*BB-4.0*AA*CC))/(2.0*AA)
      CASO=CASO+X1
      AR=AR-X1
      GR=GR-X1
      GO TO 44
   18 IF (GR) 1,1,6
    6 IF (AR) 1,1,7
    1 IF (CASO) 44,44,7
  128 AR=AR+X
      GR=GR+X
      XXT=XXT-X/BT
      CASO=CASO+CAS1
      XXT=XXT-CAS1/BT
   44 A2=AR
      IF (AR-RT) 45,46,46
   45 IJ=1
      AZ=AR
      AR=RT
      RT=AZ
      CTZ=CT
      CT=ET
      ET=CTZ
      DZ=D
      D=D*DA
      GO TO 5
   46 IJ=2
    5 Z=1.0E-12
      U=SQRTF(2.0*(RT+AR+GR)+0.5*SK+HR)
      EX=EXPEF((-2.341*U)/(1.0+U))
```

```
      AA=-4.0*BT*(D*EX+BT)
      BB=4.0*((BT*CT+SK-D*EX*SAT)*BT-D*AR*EX)
      CC=-SK*(4.0*BT*CT+SK)-D*EX*SAT*(BT*SAT+4.0*AR)
      DD=SK*SK*CT-D*AR*EX*SAT*SAT
  115 ZZ=-(((AA*Z+BB)*Z+CC)*Z+DD)
      ZZZ=((3.0*AA*Z+2.0*BB)*Z+1.0*CC)
      ZZ=ZZ/ZZZ
      Z=Z*(1.0+ZZ/Z)
      IF (ZZ/Z+1.0E-3) 115,116,116
  116 IF (ZZ/Z-1.0E-3) 178,178,115
  178 AR=AR+BT*Z
      SK=SK-2.0*BT*Z
      CT=CT-Z
      SAT=SAT+2.0*Z
      GO TO (17,13),IJ
   17 AZ=AR
      AR=RT
      RT=AZ
      CTZ=CT
      CT=ET
      ET=CTZ
      D=DZ
   13 A3=AR
      BB=AR+BT*(ET+DA*CT)+DA*RT
      AA=BT*(1.0-DA)
      CC=(AR*ET-DA*RT*CT)
      T=SQRTF(BB*BB-4.0*AA*CC)
      Y=(-BB+T)/(2.0*AA)
      IF (1.0-DA) 3,2,3
    2 Y=-(AR+ET-DA*CT+RT/BB)
    3 AR=AR+BT*Y
      RT=RT-BT*Y
      CT=CT-Y
      ET=ET+Y
      DEL=AR-A1
      IF (DEL+1.0E-5) 24,48,48
   48 IF (DEL-1.0E-5) 49,49,24
   49 DEL=AR-A2
      IF (DEL+1.0E-5) 24,50,50
   50 IF (DEL-1.0E-5) 51,51,24
   51 DEL=AR-A3
      IF (DEL+1.0E-5) 24,52,52
   52 IF (DEL-1.0E-5) 151,151,24
  151 IF (J-(JR-1)) 8,152,8
  152 N=N+1
      BK=BK+1.0
      IS=1
      Y2=SAT/(SAT+2.0*CT+2.0*ET)
```

```
   IF (Y2-.15) 53,53,8
53 J4=N-1
   XS=BK-1.0
   XS=XS+WH
   JS=J4+JR
   J=1
   GXP=GXP-XX(J)
   GP RQ=198.0E3*GXP
   SF=(237.6*(GXP+GY)*B(J))/XS
   AC FT=(GP RQ+(GY*198.0E3))/SF
   PRINT 141, GXP, Y2, JS, GP RQ, AC FT
   STOP
   END
```

PROGRAM C

```
*    COMPILE FORTRAN, EXECUTE FORTRAN

C     SODIUM-INFILTRATION RATE
C     RICHARD TERKELTOUB
      DIMENSION C(10), SA(10), A(10), S(10), H(10)
   21 READ 30, A5, S5, C5, SA5, BT
      READ 30, D, H5, X2, X3, W
      READ 30, A6, S6
   30 FORMAT (E10.3, 2XE10.3, 2XE10.3, 2XE10.3, 2XE10.3)
   41 FORMAT (1H0, E10.3, 2XE10.3)
  141 FORMAT (1H0, E10.3, 2XE10.3, 2XE10.3)
   31 FORMAT (12)
      READ 31, JR
      YT=0.0
      IS=JR
      JS=1
      SF=10.0
   32 RT=0.0
      GR=0.0
      D=D/W
   33 DO 34 J=1,JR
   34 A(J)=A6
  233 DO 234 J=1,JR
  234 S(J)=S6
  333 DO 334 J=1,JR
  334 C(J)=C5
  433 DO 434 J=1,JR
  434 SA(J)=SA5
   35 J=JR
      GO TO 301
    8 J=J-1
      C(J)=CT
      SA(J)=SAT
      A(J)=AR
      S(J)=SK
      H(J)=HR
      Y2=SAT/(SAT+2.0*CT)
      IF (Y2-.50)  128,128,999
  999 PRINT 41, (SA(J), C(J), J=1,JR)
      STOP
  128 IF (Y2-.07) 100,100,228
  100 SI=.170-.013*100.0*Y2
      GO TO 400
  228 IF (Y2-.10) 200,200,300
  200 SI=.27/(100.0*Y2)**.67
      GO TO 400
  300 SI=.170-.076*(100.0*Y2)**.2
  400 Y=X2/SI
      YT=Y+YT
      IF ((IS+1)-J) 201,302,302
```

```
302 GO TO (301,38) , JS
301 Y=SF/.170
    YT=Y+YT
    IS=IS-1
    SF=SF-1.0
    IF (IS) 501,501,38
501 JS=2
 38 SJ=X3/YT
    YT=0.0
    J=1
    PRINT 141, SJ, A(J), S(J)
419 DO 16 J=2,JR
 16 ZEBRA=A(J)
    J=J-1
    A(J)=ZEBRA
 49 DO 50 J=2,JR
 50 DUCK=S(J)
    J=J-1
    S(J)=DUCK
 39 J=JR+1
    SK=S5
    HR=H5
    AR=A5
201 J=J-1
    CT=C(J)
    SAT=SA(J)
    J=J+1
 19 IF (J-(JR+1)) 17,24,24
 17 J=J-1
    SK=S(J)
    AR=A(J)
    J=J+1
 24 Z=1.0E-12
    U=SQRTF(2.0*(RT+AR+GR)+0.5*SK+HR)
    EX=EXPEF((-2.341*U)/(1.0+U))
    AA=-4.0*BT*(D*EX+BT)
    BB=4.0*((BT*CT+SK-D*EX*SAT)*BT-D*AR*EX)
    CC=-SK*(4.0*BT*CT+SK)-D*EX*SAT*(BT*SAT+4.0*AR)
    DD=SK*SK*CT-D*AR*EX*SAT*SAT
115 ZZ=-(((AA*Z+BB)*Z+CC)*Z+DD)
    ZZZ=((3.0*AA*Z+2.0*BB)*Z+1.0*CC)
    ZZ=ZZ/ZZZ
    Z=Z*(1.0+ZZ/Z)
    IF (ZZ/Z+1.0E-3) 115,116,116
116 IF (ZZ/Z-1.0E-3) 178,178,115
178 IF (SK-2.0*BT*Z) 8,278,278
278 AR=AR+BT*Z
    SK=SK-2.0*BT*Z
    CT=CT-Z
    SAT=SAT+2.0*Z
    IF (Z/CT+1.0E-4) 24,22,22
 22 IF (Z/CT-1.0E-4) 8,8,24
    END
```

PROGRAM D

```
*     COMPILE FORTRAN, EXECUTE FORTRAN
C     MODIFICATIONS IN SYSTEM DUE TO VARIANCE OF MOISTURE CONTENT
C     RICHARD TERKELTOUB
   30 FORMAT (E10.3, 2XE10.3, 2XE10.3, 2XE10.3, 2XE10.3)
   41 FORMAT (1H0, E10.3, 2XE10.3, 2XE10.3, 2XE10.3, 2XE10.3, 2XE10.3)
   21 READ 30, CA, AMG, SOS, CL, SO
      READ 30, B, B1, EX, W
      B=1.E5/B
      B1=1.E5/B1
      SAR=SOS/(SQRTF((CA+AMG)/2.))
      SI=-0.0126+0.01475*SAR
      ESP=SI/(1.+(-0.0126+0.1475*SAR))
      SAT=ESP*EX
      RJ=EX-SAT
      SAT=SAT/1000.
      SJ=CA/(AMG*.67)
      ET=RJ/(SJ+1.)
      CT=RJ-ET
      ET=ET/2000.
      CT=CT/2000.
      XXT=0.0
      SO=SO/(2000.*B1)
      CL=CL/(2000.*B1)
      SOS=SOS/(1000.*B1)
      CA=CA/(2000.*B1)
      DA=0.67
      D2=1.384E4
      D=D2/W
      AMG=AMG/(2000.*B1)
  201 AR=B*CA
      GR=B*SO
      RT=B*AMG
      HR=B*CL
      SK=B*SOS
      GO TO 24
   32 PRINT 41, AR, SK, RT,HR,GR,CASO
   34 PRINT 41, SAT,CT,ET,XXT
      GO TO 21
   24 A1=AR
      IF (XXT) 4,4,42
    4 U=SQRTF(2.0*(RT+GR+AR)+0.5*SK+HR)
      AA=EXPEF(-9.366*U/(1.0+U))
      IF (2.4E-5-AR*GR*AA) 42,18,18
   42 X=0.0
```

```
      U=SQRTF(2.0*(RT+GR+AR)+0.5*SK+HR)
      BB=AR-GR
      EX=(9.366*U)/(1.0+U)
      CC=AR*GR-(2.4E-5)*EXPEF(EX)
      T=SQRTF(BB*BB-4.0*CC)
      X=(-BB+T)/2.0
      CAS1=4.987E-3-CASO
      DEL=B*XXT-CAS1
      IF (DEL-X) 127,128,128
  127 X=XXT*B
      XXT=0.0
      CAS1=0.0
      AR=AR+X
      GR=GR+X
      U=SQRTF(2.0*(AR+GR+RT)+0.5*SK+HR)
      AA=EXPEF(-9.366*U/(1.0+U))
      BB=-(4.9E-3+AA*AR+AA*GR)
      CC=AA*AR*GR-4.9E-3*CASO
      X1=(-BB-SQRTF(BB*BB-4.0*AA*CC))/(2.0*AA)
      CASO=CASO+X1
      AR=AR-X1
      GR=GR-X1
      GO TO 44
   18 IF (GR) 1,1,6
    6 IF (AR) 1,1,7
    1 IF (CASO) 44,44,7
  128 AR=AR+X
      GR=GR+X
      XXT=XXT-X/B
      CASO=CASO+CAS1
      XXT=XXT-CAS1/B
   44 A2=AR
      IF (AR-RT) 45,46,46
   45 IJ=1
      AZ=AR
      AR=RT
      RT=AZ
      CTZ=CT
      CT=ET
      ET=CTZ
      DZ=D
      D=D*DA
      GO TO 5
   46 IJ=2
    5 Z=1.0E-12
      U=SQRTF(2.0*(GR+AR+RT)+0.5*SK+HR)
      EX=EXPEF((-2.341*U)/(1.0+U))
```

```
      AA=-4.0*B*(D*EX+B)
      BB=4.0*((B*CT+SK-D*EX*SAT)*B-D*AR*EX)
      CC=-SK*(4.0*B*CT+SK)-D*EX*SAT*(B*SAT+4.0*AR)
      DD=SK*SK*CT-D*AR*EX*SAT*SAT
  115 ZZ=-(((AA*Z+BB)*Z+CC)*Z+DD)
      ZZZ=((3.0*AA*Z+2.0*BB)*Z+1.0*CC)
      ZZ=ZZ/ZZZ
      Z=Z*(1.0+ZZ/Z)
      IF (ZZ/Z+1.0E-3) 115,116,116
  116 IF (ZZ/Z-1.0E-3) 178,178,115
  178 AR=AR+B*Z
      SK=SK-2.0*B*Z
      CT=CT-Z
      SAT=SAT+2.0*Z
      GO TO (17,13) , IJ
   17 AZ=AR
      AR=RT
      RT=AZ
      CTZ=CT
      CT=ET
      ET=CTZ
      D=DZ
   13 A3=AR
      BB=AR+B*(ET+DA*CT)+DA*RT
      AA=B*(1.0-DA)
      CC=(AR*ET-DA*RT*CT)
      T=SQRTF(BB*BB-4.0*AA*CC)
      Y=(-BB+T)/(2.0*AA)
      IF (1.0-DA) 3,2,3
    2 Y=-(AR+ET-DA*CT+RT/BB)
    3 AR=AR+B*Y
      RT=RT-B*Y
      CT=CT-Y
      ET=ET+Y
      DEL=AR-A1
      IF (DEL+1.0E-5) 24,48,48
   48 IF (DEL-1.0E-5) 49,49,24
   49 DEL=AR-A2
      IF (DEL+1.0E-5) 24,50,50
   50 IF (DEL-1.0E-5) 51,51,24
   51 DEL=AR-A3
      IF (DEL+1.0E-5) 24,52,52
   52 IF (DEL-1.0E-5) 32,32,24
      END
```

TERMINOLOGY OF PROGRAMS

SYMBOL	MEANING	UNITS	PROGRAM
A5	Ca^{++} in irr. water	m/L	A,B,C
A6	Ca^{++} in presat. solution	m/L	C
A(J)	Ca^{++} in soil sol., effluent	m/L	A,C
AC FT	Water Requirement	acre-feet	B
AMG	Mg^{++} in soil sol.	meq/L	D
AMG5	Mg^{++} in soil sol.	m/L	A,B
AR	Ca^{++} in effluent, soil sol.	m/L	A,D
B	Moisture content, final	%	D
B1	Moisture content, initial	%	D
B5	Moisture content	g/L	A,B
BT	Moisture content	g/L	C
C5	Exchangeable Ca^{++}	mole/g	A,B,C
C(J)	Exchangeable Ca^{++}	mole/g	A,C
CA	Ca^{++} in soil sol.	meq/L	D
CA5	Ca^{++} in soil sol.	mole/L	A,B
CAS5	Undiss. $CaSO_4$ in irr. water	mole/L	A,B
CAS(J)	Undiss. $CaSO_4$ in soil sol.	mole/L	A
CASO	Undiss. $CaSO_4$ in eff., soil sol.	mole/L	A,D
CL	$(Cl^- + HCO_3^-)/2$ in soil sol.	meq/L	D
CL5	$(Cl^- + HCO_3^-)/2$ in soil sol.	mole/L	A,B
CT	Exchangeable Ca^{++}	mole/g	D
D	$Na^+ - Ca^{++}$ exchange K	-	C
DA	$Ca^{++} - Mg^{++}$ exchange K	-	A,B
E5	Exchangeable Mg^{++}	mole/g	A,B
E(J)	Exchangeable Mg^{++}	mole/g	A
ET	Exchangeable Mg^{++}	mole/g	D
EX	Exchange Capacity	meq/g	D
G5	$SO_4^{=}$ in irr. water	mole/L	A,B
G(J)	$SO_4^{=}$ in soil sol.	mole/L	A
GP RQ	Gypsum Requirement	tons/acre	B
GR	$SO_4^{=}$ in effluent, soil sol.	mole/L	A,D
GXP	Gypsum Requirement	mole/g	B
H5	$(Cl^- + HCO_3^-)/2$ in irr. water	mole/L	A,B
H(J)	$(Cl^- + HCO_3^-)/2$ in soil sol.	mole/L	A
HR	$(Cl^- + HCO_3^-)/2$ in eff., soil sol.	mole/L	A,D
JR	Number of layers wetted	-	A,C
JR	Control layer	-	B
JS	Water Requirement	aliquots	B
L	Number of layers in soil	-	A
R5	Mg^{++} in irr. water	moles/L	A,B
R(J)	Mg^{++} in soil sol.	moles/L	A
RT	Mg^{++} in effluent, soil sol.	moles/L	A,D

SYMBOL	MEANING	UNITS	PROGRAM
S5	Na^+ in irr. water	mole/L	A,B,C
S6	Na^+ in presat. solution	mole/L	C
S(J)	Na^+ in soil sol., effluent	mole/L	A,C
SA5	Exchangeable Na^+	mole/g	A,B,C
SA(J)	Exchangeable Na^+	mole/g	A,C
SAT	Exchangeable Na^+	mole/g	D
SI	Eq. of hydraulic cond.	-	C
SJ	Hydraulic conductivity	-	C
SK	Na^+ in effluent, soil sol.	mole/L	A,D
SO	$SO_4^=$ in soil sol.	meq/L	D
SO5	$SO_4^=$ in soil sol.	mole/L	A,B
SOS	Na^+ in soil sol.	meq/L	D
SOS5	Na^+ in soil sol.	mole/L	A,B
W	Clay content	decimal	C,D
W5	Clay content	decimal	A,B
WH	Control layer	-	B
X2	Length of delta section	cm	C
X3	Length of column	cm	C
XX5	Gypsum content of soil	mole/g	A,B
XX(J)	Gypsum content of soil	mole/g	A
XXT	Gypsum content of soil	mole/g	D
Y2	Exchangeable sodium per cent	decimal	B

Note: In programs A and B the quantities CAS5 and XX5 may be fed to the computer as 0.0. They will be calculated by the computer during the solution of the problem.

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