ANALYSIS OF HYDROLOGIC PERFORMANCE TESTS
IN UNCONFINED AQUIFERS

by

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STATEMENT BY AUTHOR

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ABSTRACT

Evaluation of aquifer characteristics was one of the facets of hydrological investigations started in 1954 in the Indus plains of West Pakistan. Rechna and Chaj Doabs (area between Ravi and Chenab rivers; Fig. 1) were the first to be investigated. Analysis of the pumping tests in these areas was made after methods of Theis and Jacob. The values of storage coefficients obtained indicated artesian conditions but this was in conflict with the available geological evidence. Transmissibilities were also questionably high, and there was a disparity between results calculated from time-drawdown versus distance-drawdown analyses.

The purpose of the present study was to understand the flow regime around a pumping well in the area of study; to identify the causes of inapplicability of Theis' analysis method, and to evaluate alternative approaches for analysis of pumping tests in this area.

Analysis of four specially designed long duration pumping tests indicates that vertical components of flow due to partial penetration and a variable storage coefficient due to slow drainage are the two
major factors which control the aquifer response and make Theis' method inapplicable. In the initial pumping period the effect of partial penetration on drawdown is very significant whereas the effect of delayed yield is inappreciable. In general, the effect of partial penetration is more pronounced throughout the pumping period as compared to delayed yield. However, with increasing distance from the pumping well, the partial penetration effects decrease rapidly and thereby become less important.

It was found that analysis of the first 20 - 40 minutes of pumping data, with allowance for partial penetration, gave an accurate value of "P" and artesian storage. In most cases the effective depth of aquifer could also be estimated. Observation wells at about 1.5b or farther from the pumped well, with adjustment for delayed yield, gave good values of specific yield and transmissibility. Close agreement was observed between the values of "T" from both methods. The use of data of nearer wells for the partial penetration analysis, and the farther well data at later times for the delayed yield analysis provided the best approach for aquifer characteristic determination.
In the present analysis the effects of anisotropy and vertical component of flow due to lowering water table are so small relative to the partial penetration influence that they do not affect the early data analysis. However, a method has been suggested to account for these factors if in any test their magnitude appears significant.

The average value found for permeability was 7.5 x 10^{-4} \text{ ft/sec}, the artesian storage coefficient was in the range of 0.0001 - 0.005, and specific yield was 0.2 - 0.25. The effective depth of aquifer taking part ranged from 720 feet to over 1500 feet.
INTRODUCTION

Bari Doab (doab means the land between two rivers) and the Bhawalpur area, where these tests were conducted, form a small part of the Indus Plains, West Pakistan. In order to identify the hydrologic environment, a brief description of West Pakistan seems appropriate.

Location

The State of Pakistan emerged on the world map as an independent sovereign state on August 14, 1947, as a result of the partition of the subcontinent of India. It has two parts, namely, West Pakistan and East Pakistan, separated by 1,000 miles of Indian territory. (Fig. 1)

West Pakistan occupies the westerly portion of the Indo-Pakistan subcontinent between longitudes 61 to 75 degrees east, and latitudes 24 to 37 degrees north. On the west it is bounded by Iran, on the north and northwest by Afghanistan, on the northeast by Azad-Kashmir and the disputed territory of Jammu and Kashmir,
and on the south by the Arabian Sea. Its area is 310,403 square miles and according to the 1961 census, its population is 42,968,000.

General Features

West Pakistan on its north touches the Himalayan foothills and the Hindukush mountains. The rugged mountainous region gives way to the plains and fertile land of the Indus valley and thence to barren deserts in the south and west. The Indus plains constitute the central portion of the Indus valley and have an area of about 50 million acres.

West Pakistan has three well defined seasons: winter, summer, and monsoon or rainy season. In some parts the winter is extremely cold but generally dry. The summer begins in mid-April, and during the next three months the temperature in the plains may reach 120° F. Between July and September the monsoon provides an average rainfall of about 10 inches in the plains to about 60 inches in the hills.
Location map of Pakistan.
Geology of Indus Plains

Between the Tibetan plateau, with its mighty mountain chains along its southern edge, and the Indian peninsula block, is a broad expanse of flat valleys known as the Indo-Gangetic plains, curving northerly from east to west for approximately 2,000 miles. It conceals a great rift, or fracture, many thousand feet deep in the earth's crust. This depression, over the past few million years, has become filled with alluvium that was washed down from the mountains and swept into the rift by the Indus, Ganges, and their tributaries. (A. V. Karpove, 1964)

The depth of the alluvium filled depression is not known, but is expected to be a few thousand feet at places. Test drilling has not extended beyond 1,500 feet. Often lenticular clay lenses interspersed in the aquifer are found but there is no evidence of artesian conditions in the area under study. Figure 2 gives a subsurface section of Rechna Doab, and it is expected that similar conditions occur in other doabs of the Indus plain. Deep test holes indicate a general decreasing trend in the permeability beyond a few hundred feet.
Figure 2

Fence diagram showing sub surface formations of Rechna Doab in West Pakistan. Clay lenses are indicated by dark shading.
History of Investigation

Agriculture is the mainstay of West Pakistan's economy, and is practiced in the Indus plain. Very low rainfall led to the gradual development of canal irrigation system since 1900. At present the canal system is about 38,000 miles in length and diverts 74 M. A. F. of river water annually to irrigate this area. This additional source of recharge imposed by man on the hydrologic system of the area, without providing an equivalent discharge facility, has disturbed the hydrologic balance. A rapid rise of the water table has brought the country face to face with the twin menace of waterlogging and salinity. About 100,000 acres are being lost to cultivation annually in addition to the decrease in production of the remaining land. Open drains were tried for some time but without much success.

In 1954, with the assistance of the U. S. International Co-operation Administration (now named U. S. A. I. D.), an era of hydrologic studies started. The main purpose was to study the hydrologic system of various areas, diagnose the causes of waterlogging, and search for the necessary remedy. A comprehensive program was started by "Ground Water Development Organization" (now named Water and Soil Investigation Division) to collect
the basic data needed by planners and to research some of the problems encountered in this program. Pumping tests to evaluate the aquifer characteristics is one of the many facets of its investigations.

**Initial Pumping Test Program and Preliminary Analysis**

In the initial phases of investigation more than 100 tests were performed, principally in Rechna and Chaj Doabs (area between Ravi and Chenab rivers, Fig. 1) of Indus plains. The pumping test set up in general consisted of a 300-foot pumped well, screened between 120-300 feet, and four or five observation wells about 200 feet deep, installed at different distances from the pumped well.

Arif and Rehman (1960) published the results in their preliminary analysis of these tests. The analysis was carried out after methods of Theis (1935) and Cooper and Jacob (1946). It was considered that the field conditions of these tests closely matched the assumed conditions for which these methods are applicable.

It was observed that the values of storage coefficients were very small indicating artesian conditions for which there was no geological evidence. Transmissibilities were also questionably high, and
their values as calculated from time-drawdown analysis and distance-drawdown analysis did not agree. There was no apparent cause to explain this discrepancy.

Previous Studies

Arif (1964) analyzed some of these tests with a view to locate the cause of these discrepancies and find a method to improve interpretation. He assumed that delayed yield, partial penetration, and anisotropy may be the factors affecting results. In his analysis of tests, after Boulton (1963), he observed that results of observation wells beyond 400 feet did not agree with the results of nearer wells. He also tried a solution of one of the tests on an electric analog assuming both the aquifer thickness and the ratio of $P_z/P_r$. It was observed that usefulness of the method lies in the accurate knowledge of aquifer thickness and $P_z/P_r$ ratio. These are generally unknown to start with and their incorrect estimation may affect the conclusions.

Nature of the Problem and Purpose of Present Study

The effective thickness of the Indus plain aquifers may be from 750 feet to more than 1,500 feet.
Interspersed clay lenses are often found in these aquifers (Fig. 1). It is generally difficult and also uneconomical to determine the aquifer thickness by test drilling at each pumping site, and even when determined, the effective depth of aquifer throughout the flow system may be different due to the presence of interspersed clay lenses.

Without specific information on the effective aquifer depth, the magnitude of $P_z/P_r$, and the drainage delay of the sediments, the modeling of all aquifer factors is virtually impossible. These may be the reasons that Arif (1964) observed that the analog model results did not agree with analytical results.

At present no analytical formulation is available which accounts for all of these factors. In view of these conditions the purpose of the present study is: (1) to understand the flow regime around a pumping well in the area; (2) to identify and separate the perturbations which render conventional methods inapplicable, and to estimate their magnitudes; and (3) to select the best possible analytical approach for estimating aquifer characteristics.

In order to make this study possible, special aquifer tests were designed. The pumped well, as
previously stated, was about 300 feet deep and screened between 120 feet and 300 feet. Sets of deep and shallow observation wells were used to observe the drawdown. The deep observation wells were approximately 150 feet deep with 5 - 10 feet of strainer, and the shallow wells only tapped the water table. The distance of observation wells ranged from 20 feet to 2,000 feet from the pumped well. In comparison to earlier tests which were only of six days duration, these tests were run for about 20 days. A complete layout plan of one such test site BR-T/W.9, along with constructional detail of the pumped well, the observation wells and their lithology are given in Appendix A.
FLOW REGIME OF A PUMPING WELL

General

Flow around a pumping well among other things is highly dependent upon anisotropy, partial penetration of the pumped well, confinement or unconfinement of the aquifer, and variation of the storage coefficient with time.

Many equations of flow to a pumping well are available in the literature, with each taking care of one factor or the other, but none is available which accounts for all the factors complicating the aquifer response. The selection of any given flow-system model, to represent the field conditions, is therefore a matter of judgment left to the analyst. When field conditions are complicated, and one system cannot fully represent the response, then the best procedure is to select a system which is nearest to the field conditions, and estimate thereby the aquifer characteristics. It is also possible that a given factor may be more pronounced at one time of pumping or within a certain distance from the pumping well, whereas others may have a different influence both in space and time. During the same
pumping test, therefore, the response of the aquifer may change in time and space. In view of the variety of possibilities, the knowledge of characteristics of various flow conditions around a pumping well and their mutual relation can be of great help in weighing the field observations to find a flow-system model which can represent them in the best possible manner.

Theis' (1935) equation is an idealized model conceived for a confined aquifer but is applicable to an unconfined aquifer under certain limiting conditions. A comparison between the responses of confined and unconfined aquifers has been made possible by Boulton's (1954) treatment of unconfined aquifers. Due to the paucity of models for unconfined flow, artesian flow equations are used in this analysis, but corrections wherever necessary are applied using Boulton's (1954) integral. In order to study the causes of departures and their magnitude from the Theis model, a brief summary of Theis' flow model and other flow systems which come into play due to violation of one or the other of Theis' assumptions are given in the following pages. This brief treatment will also help to understand the analysis procedure adopted later in this report.
Theis' Model

Theis (1935) using a heat analogy and Jacob (1946) from the hydraulics point of view derived the nonsteady state equation, which gives drawdown after time "t" at a point "r" feet from a well pumping continuously at constant discharge as,

\[ s = \frac{Q}{4 \pi T} \int_{0}^{\infty} \frac{e^{-u}}{u} \, du \quad (2.1) \]

where "T" is the transmissibility of aquifer, "S" is storage coefficient, and "Q" is the discharge of the pumping well. Various assumptions involved in this derivation are summarized (after J. G. Ferris) as follows:

a. **Pumping Well**
   1. Infinitesimal diameter
   2. Fully penetrates the aquifer
   3. Uniform flux distribution
   4. Constant discharge
b. The Aquifer

1. Infinite areal extent
2. Homogenous and isotropic
3. Uniform thickness

c. Flow System

1. Instantaneous release from storage
2. Radial flow
3. Darcy's law is applicable
4. No vertical flow component

Most of these assumptions are fulfilled in the field but several modify the aquifer response in some instance to such an extent that Theis' equation is not applicable.

Effect of Anisotropic Conditions

Theis' equation of drawdown around a pumping well is a particular solution of the general flow equation,

\[ Pr \left( \frac{1}{r} \frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} \right) + Pz \left( \frac{\partial^2 h}{\partial z^2} \right) = S \cdot \frac{\partial h}{\partial t} \]  \hspace{1cm} (2.2)

subject to the assumptions listed in the previous section. If all other assumptions are valid except that of isotropy, even then the term \( \frac{\partial^2 h}{\partial z^2} \) remains
zero for strictly radial flow conditions, and anisotropy would not affect Theis' flow model. However, if either partial penetration or water table conditions prevail, then vertical components of flow exist, and $\frac{\partial^2 h}{\partial z^2}$ will not be zero, and anisotropy will accentuate the response. Anisotropy magnifies the influence of vertical flow components and the nature of its effect will be discussed under partial penetration effects.

**Partial Penetration and Its Effects**

If the pumped well does not penetrate the full saturated thickness of the aquifer or if only a portion of a fully penetrating well is screened, the well is said to be partially penetrating.

Figure 3 and Figure 4 (Muskat, pp. 270-271) depict the potential distribution for two cases:

1. A non-penetrating pump-well;
2. A pump-well penetrating only 50 percent (upper half).

From the figures it follows that under the circumstances vertical flow components cannot be avoided and their magnitude increases with decreasing distance from the pumped well. The head registered in the vicinity of
Figure 3. Potential distribution around a non-penetrating well. (After Muskat, p. 270)

Figure 4. Potential distribution around a well penetrating upper half of 125 feet thick sand (after Muskat, p. 270).
the pumped well is not only a function of screen location of the pumped well but also of the screen portion and depth of the observation well. The greater the magnitude of vertical flow, the larger is the departure from the radial flow assumption and from the applicability of Theis' ideal model.

Hantush (1961) observed that the average drawdown registered by an observation well, however close it may be to a partially penetrating pumped well, is given by Theis' equation, provided the observation well is screened throughout the aquifer. The same is true in the case of an observation well located at a distance greater than \(1.5b \left(\frac{P_r}{P_z}\right)^{1/2}\) regardless of the space position of its screen.

In cases other than mentioned, the drawdown is not given by the Theis equation. During the early period of pumping and before the curve inflection appears, time drawdown curves have the same shape and general appearance as for the case of complete penetration. The Theis formula is not applicable even in the very early period of pumping, when one might assume that the aquifer ends at the bottom of the pumped well unless the geometry of flow is such that the drawdown equation
for partially penetrated case for relatively short
time reduces to the form

\[ s = \frac{Q}{4\pi PL} \cdot M(u, L/r) \]  \hspace{1cm} (2.3)

in which case the validity is assured only in the range
where "t" is less than \( \frac{LSs}{20 P_L} \).

The general equation of drawdown registered by
a piezometer at depth "z" in the case of a partially
penetrated well, screened between depth "L" and "d"
(\( L \) greater than \( d \)) is given by Hantush (1961, p. 85)
either of the two equations: (assuming \( P_z = P_r \))

\[ s = \frac{Q}{4\pi Pb} \left[ W(u) + f(u, r/b, L/b, d/b, z/b) \right] \]  \hspace{1cm} (2.4)

where

\[ f = 2b/\pi (L-d) \sum_{n=1}^{\infty} \frac{1}{n} \sin (n\pi L/b - \sin (n\pi d/b)) \cos (n\pi z/b; W(u, n\pi r/b)) \]

or

\[ s = \frac{Q}{8\pi P_r (L-d)} \left[ M(u, \frac{L+z}{r}) + M(u, \frac{L-z}{r}) + \right. \]

\[ f(u, b/r, L/r, z/r) - M(u, \frac{d+z}{r}) - \]

\[ M(u, \frac{d-z}{r}) - f(u, b/r, d/r, z/r) \]  \hspace{1cm} (2.5)
where

\[ f'(u, \frac{b}{r}, \frac{L}{r}, \frac{z}{r}) = \sum_{n=1}^{\infty} M(u, \frac{2nb + x + z}{r}) - M(u, \frac{2nb - x - z}{r}) + M(u, \frac{2nb - x + z}{r}) - M(u, \frac{2nb - x - z}{r}) \]

For relatively short time, \( t \) less than \( \frac{(2b - L - z)^2}{20P} \), the equation reduces to an approximate form as

\[ s \approx \frac{Q}{8\pi P (L - d)} \cdot "E" \quad (2.6) \]

where

\[ "E" = M(u, \frac{L + z}{r}) + M(u, \frac{L - z}{r}) - M(u, \frac{d + z}{r}) - M(u, \frac{d - z}{r}) \]

and for large times, \( t \) greater than \( bS/2P \) the equation can be written as

\[ s = \frac{Q}{4\pi Pb} \left[ W(u) + f_s(r/b, L/b, d/b, z/b) \right] \quad (2.7) \]

where

\[ f_s = \frac{4b}{(L - d)} \sum_{n=1}^{\infty} \frac{I/n K_0}{\frac{n\pi r}{b}} \cos \frac{n\pi z}{b} \]

\[ \left( \sin \frac{n\pi L}{b} - \sin \frac{n\pi d}{b} \right) \]
The effect of partial penetration resembles the effect of leakage from storage in thick semi-pervious confining layer (Hantush 1960). Also if the curve inflection is apparent, but the period of observation is not long enough to establish the ultimate straight line variation on a semi-log, the effect may resemble that of a recharge boundary.

The effect of partial penetration on drawdown is therefore to cause a rapid increase in drawdown to begin with, but with time the effect of partial penetration reaches its maximum value and the time-drawdown curve assumes a slope comparable to full penetration conditions, if there is no other influencing factor.

If anisotropy is coupled with partial penetration, its effect will be to increase the drawdown still further and much more time will be needed before partial penetration effect becomes constant; thus, much longer distance would be required before partial penetration effects could be considered negligible (Hantush 1964, p. 353; R. W. Stallman 1965).
Non-instantaneous Release Effects

Slow draining of saturated soil samples is the general phenomena noted by L. K. Wenzel (1942, W. S. P. 887) and mentioned by R. W. Stallman (1964, professional paper 411-E) with reference to sand columns of F. H. King. Slow draining phenomena is of particular significance in unconfined flow study.

L. K. Wenzel (1942) observed that its effect on the determination of "T" from the steady state formula is not very significant, but in non-steady state cases which also involve determination of "S" it does present difficulty.

As mentioned by Boulton (1963), the initial response of an unconfined aquifer resembles the artesian case until the delayed yield effect becomes significant. The phenomena of slow draining, therefore, can also be looked upon as that of increasing storage coefficient with time; and if the initial system be taken as our reference, then increasing storage coefficient is the departure of the assumed constant "S" condition from Theis' ideal model.

The effect of slow draining can be looked upon in two ways: if the initial artesian response is taken
as the reference, one can say that the effect of slow draining on departure from the Theis model is negligible in the early period of pumping. If reference is taken as the Theis water table response, then the effect can be interpreted as maximum in the early period of pumping (see curve 1 and 2, Fig. 5).

Therefore, the effect of slow draining on the drawdown in the former case can be considered as negligible in the early period of pumping, and in my analysis I selected this initial system as the reference point.

Boulton (1955, p. 475) recommended the following equation for calculating the correction to be applied to the drawdown calculated by Theis' equation with reference to the initial system:

$$s' - s = \frac{Q}{4\pi T} \left( \ln \eta - E_1 (\eta \alpha t) + E_1 (\alpha t) \right) (2.8)$$

Limitation of this formula is that it is applicable for small distances from the pumped well where

$$J_0 \left( r \sqrt{\frac{u}{a}} \right) = 1$$

and

$$a = \frac{T}{S}$$
Figure 5

Transition from initial artesian response to water table response due to delayed yield.
DATA OF HYPOTHETICAL CURVE

\[ T = 0.75, \quad Q = 3 \text{CFS}, \quad S = 0.0055 \]

\[ s_y = 0.13, \quad b = 1000, \quad T = 50, \quad \alpha = 2.3 \times 10^5 \]
Artesian Flow Equations and Their Application to Unconfined System

In the case of a confined aquifer the discharge of the pumping well is sustained by compaction of aquifer and expansion of water due to the lowering of pressure. If the well fully penetrates the aquifer and there is no dewatering, flow is radial and no vertical component of flow is involved. In an unconfined aquifer the pumping well discharge is supplied by actual dewatering of the sediments and the lowering of water table develops vertical components of head, even if the pumping well is fully penetrating.

Boulton (1954) observed that the Theis exponential integral as applied to the water table conditions is theoretically incorrect because it takes no account of the vertical component of pore water flow approaching the well. This criticism applies particularly to the early stages of a pumping test in a deep well, when the motion of water in the upper part of aquifer is more vertical than horizontal.

He observed that after a time factor $\tau = \frac{P_z t}{S_b}$ has become greater than 5, only then does Theis' exponential integral become applicable. Before that time
the drawdown at a point is given by

\[ s = \frac{Q}{2 \pi T T} V(\rho, \tau) \quad (2.9) \]

where

\[ V(\rho, \tau) = \int_0^\infty \frac{J_0 (\lambda, \rho)}{\lambda} \left[ 1 - \exp(-\tau \lambda \tanh \lambda) \right] \lambda \, d\lambda \]

This treatment provides a means for testing the applicability of various flow equations derived strictly for artesian conditions. If "\( \tau \)" is greater than 5, no correction is needed in those equations. But for smaller times the correction can be calculated as follows:

\[ s_c = \frac{Q}{4 \pi T T} \left[ W(u) - 2V(\rho, \tau) \right] \quad (2.10) \]

now \( P = \frac{P}{b} \) and \( \tau = \frac{P t}{S b} \)

\[ u = \frac{\rho^2 S}{4 T t} = \frac{\rho^2}{4 \tau} \]

For each value of "\( u \)," \( P \) being known, "\( \tau \)" can be calculated and also the value of \( V(\rho, \tau) \).
AN APPROACH TO THE PUMPING TEST ANALYSIS

General

In order to decide the best possible approach to pumping test analysis, it is of vital importance to identify and assess the magnitude of system noise. Most of Theis' assumed conditions are fulfilled in the field within practical limits. The principal departures are: the partial penetration of pumped well and observation wells, the anisotropy of the aquifer, the slow drainage of sediments, and the confined or unconfined nature of the aquifer. A brief theory of the aquifer response and the effect on Theis' flow model has been discussed in the previous section.

The perturbations relative to Theis' model as the frame of reference may be due to one or more of the following causes:

1. Anisotropy of the aquifer,
2. Vertical flow components due to partial penetration,
3. Slow drainage of sediments,
4. Vertical flow component due to water table conditions.
At present no analytical formulation is available which takes into account all of the above-mentioned factors. The problem, therefore, is one of selecting the flow regime which best represents the field conditions. The method of analysis to be followed is to select a condition where or when one factor is dominant and other influences are either absent or negligible. From a preliminary evaluation of aquifer characteristics, an effort is made to reproduce a time-drawdown curve for one of the observation wells. A method of successive trial is then followed to reduce the departure between the theoretically predicted and actually observed time-drawdown curve. The magnitude of perturbation due to each factor is calculated to evaluate the analysis approach or the probable error to be expected in results.

The flow regime changes with space and time in the same pumping test. For the long pumping duration of these pumping tests, it was possible to select data so as to satisfy different boundary conditions necessary for minimizing certain effects and thus isolate them.

A summary of the boundary conditions as given by R. W. Stallman (in his letter dated July 21, 1965) is as follows:
Theis' equation or others of artesian radial flow can be applied to the draw downs observed in vicinity of partially or fully penetrating pumped well (except where the restrictions noted) if:

i) "t" is greater than \( \frac{5bS}{P_z} \); (Boulton, 1954). At smaller times the vertical flow components due to lowering water table are significant.

ii) "r" is greater than 1.5b \( (P_r/P_z)^{1/2} \) (Jacob, 1945; Hantush, 1961, and 1964, p. 355). At smaller radii partial penetration produces vertical flow components.

iii) "Tt/r^2S" is to be greater than 0.5; "Tt/b^2S" to be greater than 5; "r/b, \( (P_z/P_r)^{1/2} \) to be greater than 0.8 for draw down at water table near a fully penetrating well. (Boulton, 1964, p. 608).

iv) "Tt/r^2S" to be greater than 1.0; "r/b, \( (P_z/P_r)^{1/2} \) to be greater than 3; approximately for draw downs anywhere in the vicinity of a partially penetrating well. (Boulton, 1964, p. 603).

v) "dS/\delta t" is to be less than 0.002P; (R. W. Stallman's letter). If water table declines at a faster rate, specific yield will likely to be in error by more than 20%.

vi) "\alpha t" greater than 1.8 + 2.6 r/B; where this condition is met, delay yield effects are negligible.

Using some of the above mentioned conditions and other flow characteristics discussed in the second chapter along with the process of successive trial, an analysis of BR-T/W 9 is given in detail in the following treatment. Results of other tests have been summarized in Table 4.
Analysis of B R - T/W 9

In the previous section under non-instantaneous release effects, it was observed that if Theis' artesian response is considered as the reference, the effect of slow draining is very small in the early part of the pumping period. Keeping this in view, early pumping data may have the disturbances due to anisotropy, partial penetration, and vertical flow component due to lowering water table.

Table 1 gives the drawdowns registered by deep and shallow observation wells after 21000 minutes of pumping. Visual observation of the data indicate the presence of partial penetration effects. As a first approximation, assume that the other two effects are comparatively small, and analyze the earlier data for partial penetration only.
TABLE 1
VERTICAL HEAD DIFFERENCE BETWEEN DEEP AND SHALLOW WELLS
@ "t" 21000 (minute)

<table>
<thead>
<tr>
<th>r (ft)</th>
<th>BR - T/W 8</th>
<th></th>
<th></th>
<th>BR - T/W 9</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s_d$</td>
<td>$s_s$</td>
<td>$s_d - s_s$</td>
<td>$s_d$</td>
<td>$s_s$</td>
<td>$s_d - s_s$</td>
</tr>
<tr>
<td>20</td>
<td>10.14</td>
<td>7.48</td>
<td>5.79</td>
<td>5.64</td>
<td>4.59</td>
<td>1.05</td>
</tr>
<tr>
<td>50</td>
<td>7.48</td>
<td>4.09</td>
<td>3.39</td>
<td>6.95</td>
<td>4.46</td>
<td>2.49</td>
</tr>
<tr>
<td>100</td>
<td>5.36</td>
<td>3.72</td>
<td>1.64</td>
<td>4.69</td>
<td>4.03</td>
<td>0.66</td>
</tr>
<tr>
<td>200</td>
<td>3.65</td>
<td>3.06</td>
<td>0.59</td>
<td>3.03</td>
<td>2.81</td>
<td>0.22</td>
</tr>
<tr>
<td>300</td>
<td>2.56</td>
<td>2.31</td>
<td>0.25</td>
<td>2.45</td>
<td>1.89</td>
<td>0.56</td>
</tr>
<tr>
<td>500</td>
<td>1.73</td>
<td>1.62</td>
<td>0.11</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1000</td>
<td>0.87</td>
<td>0.75</td>
<td>0.12</td>
<td>0.99</td>
<td>1.11</td>
<td>-0.12</td>
</tr>
<tr>
<td>2000</td>
<td>0.18</td>
<td>0.21</td>
<td>-0.03</td>
<td>0.42</td>
<td>0.44</td>
<td>-0.02</td>
</tr>
</tbody>
</table>
Partial Penetration Analysis of Early Data:

For a relatively short period of pumping the drawdown around a partially penetrating well is given by equation (2.6). According to Hantush (1961) if the semi-log time drawdown curve of the observed data exhibits an inflection, and if the number and distribution of observed points are such that the details of the curve prior to attainment of ultimate straight line are discernible, then the type curve method (as given below) can be used to determine the formation coefficients and often its thickness.

\[ s = \frac{Q}{8\pi P(L-d)} \quad "E" \]

3.1

The equation is of Theis' type in composition, wherein "E" in addition includes characteristics of the pumped well and observation wells: and is a function of "u." A type curve therefore can be plotted between "E" and "1/u" in the same manner as can be plotted between "W(u)" and "1/u." Values of "E" can be calculated by equation (2.6).
In the present test, the data used to evaluate "E" is as follows:

"L" = 180 ft; "d" = 60 ft; "Z_d" = 110 ft;

"Z_s" = 0.0 ft

Semi-log plots of the observation well data indicated that wells up to 500 feet had enough points to define the curve inflection, therefore only the nearer wells were analyzed. Figures 6 and 7 give the type curves and early data for the observation wells used in analysis. Values of "P" and the artesian storage as obtained by this analysis are given in Table 2. The average value of transmissibility is 0.52 (ft²/sec) and of artesian storage, 0.0057. The aquifer thickness taking part was calculated as 720 feet. It was observed that the value of storage coefficient as obtained from shallow wells was much greater than that calculated from the deep wells.

The method used here is applicable only when "t" is less than \( \frac{(2b - L - Z)^2 S}{20 T} \). The time calculated is about 11 minutes. The data used for analysis was within this time limit.
Figure 6. Type curves for selected observation wells taking into account partial penetration effects for analysis of early pumping test data.
Figure 7. Plots of early pumping test data of selected observation wells.
TABLE 2

RESULTS OF EARLY PUMPING TEST DATA
ANALYSIS AFTER HANTUSH (1961)

<table>
<thead>
<tr>
<th>r (ft)</th>
<th>P (ft/sec)</th>
<th>( S_s )</th>
<th>b (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deep Wells</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>( 6.6 \times 10^{-4} )</td>
<td>( 1.36 \times 10^{-5} )</td>
<td>540</td>
</tr>
<tr>
<td>100</td>
<td>( 8.6 \times 10^{-4} )</td>
<td>( 8.4 \times 10^{-6} )</td>
<td>695</td>
</tr>
<tr>
<td>200</td>
<td>( 7.2 \times 10^{-4} )</td>
<td>( 8.4 \times 10^{-6} )</td>
<td>595</td>
</tr>
<tr>
<td>300</td>
<td>( 6.6 \times 10^{-4} )</td>
<td>( 7.4 \times 10^{-6} )</td>
<td>-</td>
</tr>
<tr>
<td><strong>Avg.</strong></td>
<td>( 7.2 \times 10^{-4} )</td>
<td>( 8 \times 10^{-6} )</td>
<td></td>
</tr>
<tr>
<td><strong>Shallow Wells</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>( 7.8 \times 10^{-4} )</td>
<td>( 2.68 \times 10^{-4} )</td>
<td>590</td>
</tr>
<tr>
<td>50</td>
<td>( 7.2 \times 10^{-4} )</td>
<td>( 2.82 \times 10^{-4} )</td>
<td>655</td>
</tr>
<tr>
<td>100</td>
<td>( 6.4 \times 10^{-4} )</td>
<td>( 2.00 \times 10^{-4} )</td>
<td>790</td>
</tr>
<tr>
<td>200</td>
<td>( 6.6 \times 10^{-4} )</td>
<td>( 2.52 \times 10^{-4} )</td>
<td>750</td>
</tr>
<tr>
<td><strong>Avg.</strong></td>
<td>( 7.0 \times 10^{-4} )</td>
<td>( 2.5 \times 10^{-4} )</td>
<td>720</td>
</tr>
</tbody>
</table>
Probable Magnitude of Anisotropy and Its Effect on Early Data Analysis:

In the partial penetration analysis it was assumed that the aquifer is isotropic. Next an estimate of the probable magnitude of \( P_r/P_z \) is made and its effect on the analysis is examined.

Four special tests, whose analysis is presented here, included sets of deep and shallow observation wells up to 2000 feet. The shallow wells just penetrated the water table and deep wells were about 150 feet deep. General observation of the head difference registered by the deep versus shallow wells indicate that beyond 1000 feet the difference is negligibly small. Table 1 gives the drawdown observed at different distances from the pumping well both in the deep and shallow wells of test BR-T/W, 8 and 9. Analysis of these tests for partial penetration gave the effective depth of aquifer as 1000 feet and 750 feet respectively. If from the data it is now assumed that in both cases the vertical head difference vanishes at 1500 feet, the value of \( P_r/P_z \) can be estimated by the following equation:

\[
P_z = 2.25 \frac{P_r}{P_v} (b/r_v)^2
\]  

(3.2)
The ratio "Pr/Pz" for BR-T/W 8 appears to be one, and for BR-T/W 9, two. These values indicate that anisotropy in the area under study is of quite limited degree.

In order to study the effect of anisotropy on the early data analysis of BR-T/W 9, assume that "Pr/Pz" in this test is 4. Equation (3.1) still holds, except that the function giving the value of "E" is slightly modified. The value of "E" is given by the following equation: (Hantush, 1964, p. 353)

\[
E = M(u, a'L + Z) + M(u, a'L - Z) - \frac{r}{r} M(u, a'd + Z) - M(u, a'd - Z) \tag{3.3}
\]

where

\[
a' = (Pr/Pz)^{1/2}
\]

As before, type curve can be prepared to take account of anisotropy and the early data can be analyzed. Figure 8 gives type curves for a 50-foot well of BR-T/W 9 for "Pr/Pz" 4 and 1 respectively. It was found that the error in results was less than 15 per cent, if the effect
Figure 8. Type curves taking into account the partial penetration and anisotropic effects for a deep well 50 feet from BR-T/W 9.
of vertical permeability was neglected. The value of \( P \) would be slightly smaller and that of storage coefficient would be slightly larger than actual values.

**Effect of Water Table Conditions on Early Data Analysis:**

The effect of lowering water table upon the early data of observation wells near the pumping well is quite significant, and can be neglected only when \( t \) is greater than 5. In order to study its effect on analysis, it was assumed that \( P_r = P_e \) and the effect of delay yield was negligible in early period.

Boulton's (1954) solution for the unconfined aquifer provides a means to modify the artesian formula of partial penetration to suit the water table condition and thus estimate the effect of neglecting this correction.

Drawdown around a pumping well in a water table aquifer is given by equation (2.9), and in a confined aquifer by equation (2.1).

\[
\begin{align*}
S_{wt} &= \frac{Q}{2\pi T} \quad "V" \\
S_t &= \frac{Q}{4\pi T} \quad "W(u)"
\end{align*}
\]
The correction \( s_c \) to be applied to the drawdown given by partial penetration equation (3.1) will be:

\[
s_c = s_t - s_{wt} = \frac{Q}{4\pi T} (W(u) - 2V) \quad (2.10)
\]

Drawdown around a partially penetrating well in a water table aquifer for "t" less than \( \frac{Sb}{2b} \left(1 - \frac{L-Z}{2b}\right)^2 \) is then given by the following equation:

\[
s = s_{pp} - s_c = \frac{Q}{4\pi T} \left( \frac{D}{2(L-d)} - W(u) + 2V \right) \quad (3.4)
\]

Partial penetration analysis of the early pumping data indicates the aquifer thickness to be 720 feet, 
"b/2(L - d)" then reduces to "3" and equation (3.4) simplifies to

\[
s = \frac{Q}{4\pi T} (3E - W(u) + 2V) \quad (3.5)
\]

Now "E," and "W(u)" are all related to "u" and a modified type curve can be prepared for analysis of early pumping data observed in a partially penetrating water table aquifer.

Figure 9 gives such type curves for observation wells at 50 feet and 200 feet from BR-T/W 9. On the same figure are plotted in dotted line the type curves
Figure 9. Type curves indicating the difference if the effect of vertical component of flow due to lowering water table is neglected.
neglecting the water table conditions. From the type curves it can be seen that the departure is not very significant, and the magnitude of the partial penetration effects appear so dominant that the water table condition does not affect the results from the initial data analysis. However, in cases where the partial penetration effects are so significant, the method suggested could be used successfully. In the present analysis, however, it was neglected. A table giving values of "V" for selected values of "u" is given by R. W. Stallman (1961, U.S.G.S, prof. paper, 424-C) but they are not sufficient for detailed preparation of such curves.

Delayed Yield Analysis:

With reference to Table 1 it can be seen that the effect of partial penetration is negligible, at 1000 feet and beyond. Therefore, the analysis of data for wells at 1000 feet and 2000 feet can give an estimate of "T" and the specific yield if analyzed according to Boulton (1963) by using "Type B" curves. In the present test, the well at 2000 feet was probably registering some additional influence and could not be analyzed. Analysis of the 1000-foot well gave the following values:
"T" = 0.44  S' = 0.19  r/B = 0.8  \( \chi = 1.5 \times 10^{-6} \)

The value of Boulton's (1954) t factor for "T" 21000 minutes is 5.7 which precludes the effect of vertical components of flow due to lowering of the water table.

Distance Drawdown Analysis:

As pumping begins, the hydraulic gradient, which is essentially an equilibrium gradient or enough to transmit the required amount of water to the pumping well, is established in the immediate vicinity. As pumping continues, the equilibrium gradient continues to be established at greater distances from the well (L. K. Wenzel, W.S.G.S., Wsp. 887, 00. 98-99). Thus the portion of the cone nearer to the well reaches steady flow condition at a much earlier time, and as such, Wenzel's argument for applying the distance-drawdown analysis to reduce the delayed yield effect is applicable.

In order to apply this approach, the partial penetration correction is needed. At a relatively large pump time, the drawdown around a partially penetrating
well is given by equation (7) of Hantush (1961, p. 173). The partial penetration correction is therefore given by:

\[ s_{pc} = \frac{Q}{4\pi T} f_s \]  

(3.6)

This drawdown is to be deducted from the observed drawdown to get the drawdown free of partial penetration effects. At the very large times used in this analysis, the effect of the vertical component of flow due to lowering water table is also absent. The only effect remaining is delayed yield which as noted above according to Wenzel will not affect the results to any great extent.

Partial penetration corrections have been calculated for deep wells using values of \( M(u, \beta) \) function given by Hantush (professional paper 102, 1961). The corrections calculated are given in Table 3. The values used are:

\[ b = 720 \text{ ft.} \quad P = 7.2 \times 10^{-4} \quad L = 180 \text{ ft.} \]

\[ d = 60 \text{ ft.} \]
<table>
<thead>
<tr>
<th>r</th>
<th>$f_s$</th>
<th>$s_{pc}$</th>
<th>Observed d/d t 21000 mt</th>
<th>Corrected d/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>15.4</td>
<td>6.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>7.6</td>
<td>3.10</td>
<td>6.95</td>
<td>3.85</td>
</tr>
<tr>
<td>100</td>
<td>4.06</td>
<td>1.68</td>
<td>4.69</td>
<td>3.01</td>
</tr>
<tr>
<td>200</td>
<td>1.74</td>
<td>0.70</td>
<td>3.03</td>
<td>2.33</td>
</tr>
<tr>
<td>300</td>
<td>0.93</td>
<td>0.38</td>
<td>2.45</td>
<td>2.07</td>
</tr>
<tr>
<td>500</td>
<td>0.292</td>
<td>0.12</td>
<td>1.70</td>
<td>1.58</td>
</tr>
<tr>
<td>1000</td>
<td>-</td>
<td>0.008</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>2000</td>
<td>-</td>
<td>-</td>
<td>0.24</td>
<td>0.24</td>
</tr>
</tbody>
</table>
The corrected drawdowns have been plotted on semi-log graph (Fig. 10) and have been found to lie on a straight line. The value of transmissibility and storage coefficient calculated are as given below:

\[
\begin{align*}
&T = 0.49; \quad S_y = 0.19 \\
&\text{These values confirm the values of } T \text{ and } S_y \text{ already calculated.}
\end{align*}
\]

**Evaluation of the Magnitude of Perturbations**

Partial penetration analysis of the early data, delayed yield analysis after Boulton (1963), and the distance-drawdown analysis indicate the following values for \( T, S, S_y, \) and other constants.

\[
\begin{align*}
&T = 0.48; \quad S = 0.00575; \quad S_y = 0.19 \\
b = 720 \text{ ft.}; \quad \alpha = 1.5 \times 10^{-6}; \quad \eta = 34
\end{align*}
\]

Using the above values, the departures from Theis' response were calculated with equations 2.1, 2.6, 2.8, 2.10 and 3.6.

In figure 11, the observed and estimated drawdown for the 50-foot well from BR-T/W 9 were plotted to check
Partial penetration correction incorporated in distance-drawdown plot (BR-T/W 9).
\[ \Delta S = 2.05 \]
\[ T = \frac{2.3 \times 2.75}{2 \times 2.05} = 0.49 \]

\[ S_v = \frac{2.25 \times 0.49 \times 2 \times 1000 \times 60}{(2700)^2} = 0.19 \]
Figure 11

Observed and estimated drawdown for the deep observation well 50 feet from the pumped well.
the values of the estimated aquifer characteristics. It was observed that the estimated and actual field measurements agreed closely for times larger than 1200 - 1500 minutes. At smaller times the departure is significant and can be attributed to the approximate estimation of the partial penetration effect in the transition zone.

In Figure 12, departure from Theis' model were plotted for each cause. It can be seen from this graph that the partial penetration effect becomes significantly great, as the pumping starts, and after about 100 minutes, it tends to become almost constant. The delayed yield effect is quite small in the early periods and the storage coefficient can be assumed constant for all practical purposes during the early pumping period. However, at later times, its effect is quite significant and cannot be ignored. The vertical component of flow is quite important in the early periods, but decreases rapidly with time. After about three minutes of pumping, the effect of this relative to partial penetration is hardly 15 percent.

This analysis clearly proves that partial penetration is the most important factor in controlling the aquifer response in the early period, and at later times delayed yield becomes quite effective and important.
Figure 12. Magnitude of departures from Theis' initial artesian model with time for various conditions for the deep well 50 feet from BR-T/W 9.
Results of Selected Tests

Three other long range and specially designed aquifer tests were analyzed as per the procedure developed and their final results are given in Table 4.

It was observed that this analysis procedure gives good results and the estimated values from different portions of data match closely. However, this analysis procedure can be further improved if it is possible to evaluate the ratio between vertical and horizontal permeability. If ratio of $P_z/P_r$ is known, then type curves can be prepared taking into account both the effect of partial penetration and of anisotropy by using Hantush (equation 82, 1964, p. 353). By coupling this equation with vertical flow component effect as suggested by the author in previous section, it is possible to analyze the early data with great confidence.
## TABLE 4

RESULTS OF SELECTED TESTS

<table>
<thead>
<tr>
<th>Test Site</th>
<th>Partial Penetration Analysis of Early Data</th>
<th>Analysis of Distant Wells for Delayed Yield Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_r$</td>
<td>$S$</td>
</tr>
<tr>
<td>BR-T/W 9</td>
<td>0.52</td>
<td>$5.75 \times 10^{-3}$</td>
</tr>
<tr>
<td>BR-T/W 8</td>
<td>0.75</td>
<td>$4 \times 10^{-3}$</td>
</tr>
<tr>
<td>BR-T/W 10</td>
<td>0.95</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>BR-T/W 12</td>
<td>0.7</td>
<td>$5 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
Behaviour of Shallow Observation Wells

In the course of this analysis it was observed that the value of initial storage coefficient as calculated from early pumping data of the shallow observation wells was much higher than that calculated from the deep wells, and often was within the water table range (see Table 5). A quantitative explanation of this phenomenon was not developed, but qualitatively it may be explained as follows:

Pumped wells in general are not screened through the first 50 to 60 feet below the water table because the aquifer may be capped by fine material. This is true also in case of test wells analyzed here. In the early pumping period of a water table case the free surface boundary equation is according to Boulton (1954, p. 568)

$$S_y \frac{\partial s}{\partial t} = P_r \left( \frac{\partial s}{\partial r} \right)^2 - P_z \left( \frac{\partial s}{\partial z} \right)^2 - \frac{\partial s}{\partial z}$$ (3.7)

Initially the water table is everywhere horizontal. Before water levels begin to decline along the surface of the saturated zone, a head gradient between the water table and well face is produced in response to the reduced water level in the pumped well. Along the
water table, this gradient is directed downward initially and perpendicular to the water table. In the early period the water table may be considered virtually horizontal over most of the area, $\frac{\partial s}{\partial r}$ is almost zero, and the term $\frac{\partial s}{\partial z}$ is much larger than the term $(\frac{\partial s}{\partial z})^2$. Also in the unconfined system, the total effect on the relation between drawdown and time caused by the storage changes due to the change in the water table position is generally of a much greater magnitude than that resulting from decompression of the water below the water table. Therefore, equation 3.7 can be approximated as (R. W. Stallman, 1963, p. 215):

$$ S_y \frac{\partial s}{\partial t} = - P_z \frac{\partial s}{\partial z} \quad (3.8) $$

Now suppose that it is possible to vary the magnitude of "$P_z$" keeping "$\frac{\partial s}{\partial z}$" constant. "$S_y$" is the aquifer characteristic and is also constant; "$\frac{\partial s}{\partial z}$" therefore will vary directly as "$P_z$". The smaller the value of "$P_z$", the smaller will be the lowering rate of the water table.

According to Wyllie and Gardner (1958) the hydraulic conductivity can be estimated as "$P_z \theta^4$" where $\theta$ is the decimal fraction of the pore space filled with
liquid removable by gravity—or, the decimal fraction of specific yield held in the pore space. As an approximation it may be noted that the water held at some point in the pore space cannot join the moving water table if it is located where the hydraulic conductivity is less than the velocity of water table. Thus, above a water table moving at a constant velocity, the water content will not be lowered to a value less than

\[ \theta = \left( \frac{\Delta s/\Delta t}{P_z} \right)^{1/4} \] (3.9)

Let "S_a" be the apparent specific yield established and "S_y" be the final specific yield, then

\[ \frac{S_a}{S_y} = (1 - \theta) \] (3.10)

\[ \frac{S_a}{S_y} = 1 - \left( \frac{\Delta s/\Delta t}{P_z} \right)^{1/4} \] (3.11)

It follows that the smaller the value of the second term, the smaller is the difference between the initial and final storage values.

From equations 3.7 to 3.11, the following conclusions can be made:

1. Shallow wells in unconfined aquifers are expected to respond to water table conditions from the
beginning unlike the deep wells whose initial response is essentially artesian.

2. On account of the process of slow drainage, the value of storage coefficient from initial data may be much smaller than the final specific yield depending upon the rate of water table lowering during the initial period as compared to the final lowering rate.

3. Vertical permeability of the layer between the water table and the top of the pumping well screen does affect the lowering rate of water table and may cause variation between the ratio of initial to final specific yield from one site to another as can be noted from Tables 5 and 6.
TABLE 5

INITIAL STORAGE COEFFICIENTS AS DETERMINED FROM DEEP AND SHALLOW WELLS

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Initial Storage Coefficients</th>
<th></th>
<th></th>
<th>Sy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deep Wells</td>
<td>Shallow Wells</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BR-T/W 8</td>
<td>0.004</td>
<td>0.02</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>BR-T/W 9</td>
<td>0.00575</td>
<td>0.13</td>
<td>0.19</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 6

DRAWDOWN OF SHALLOW WELLS AFTER FIRST 10 MINUTES

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Distance From Pumped Well (ft.)</th>
<th>Pumping Rate (cfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>BR-T/W 8</td>
<td>0.48</td>
<td>0.36</td>
</tr>
<tr>
<td>BR-T/W 9</td>
<td>0.39</td>
<td>0.17</td>
</tr>
</tbody>
</table>
AUXILIARY TECHNIQUES

In addition to the methods already described, the following techniques may be used to estimate the values of "T" and "S_y." Their application, however, depends upon the duration of pumping and aquifer response and may not be used indiscriminately.

**Time Drawdown Semi-log Plots**

After a large time "t" greater than "bS/2P_z" the effect of partial penetration reaches its maximum value (Hantush 1961). Also, if "t" is greater than 1.8 + 2.6 r/B, the final specific yield has almost been established (R. W. Stailman, letter). When both these conditions are met and also "u" is less than 0.02, then the observed drawdowns should plot on semi-log graph as a straight line.

Only from the shape of the plot it then becomes possible to see whether these conditions are fulfilled or not. In the course of the present analysis semi-log plots of BR-T/W 8 (deep wells) behaved in the same
manner and after about 5000 to 6000 minutes the data plotted on a straight line for the remaining pumping period (Fig. 13).

Transmissibility being a function of slope only, can be evaluated by using Jacob's time-drawdown formula. The storage coefficient is affected because of the translation of the graph due to the partial penetration effect. The correction for this has been suggested by Hantush (1961); and the storage coefficient is calculated by the following equation:

\[ S_y = \frac{2.25 T t \exp (f_s)}{r^2} \]  \hspace{1cm} (4.1)

where \( f_s \) is evaluated from equation 2.7.

Special feature noted in the present analysis is the variation of the final slope of time-drawdown graph with distance, and also the greater slope of a shallow observation well as compared to its counterpart deep well (Table 7). The decrease of final slope with distance may be attributed to one or more of the following causes: (a) increase of transmissibility with distance due to increasing depth of effective aquifer thickness; (b) variation of storage coefficient with distance.
Figure 13

Time drawdown plot of deep and shallow observation wells at 100 feet from BR-T/W 8.
In view of the above variation it has been observed that well at 1000 feet on analysis gave "T" as 0.70, and "Sy" as 0.19. These values agree with the already calculated values. This indicates that the above method if used for farther wells can give good values.
### TABLE 7

**SLOPE PER LOG-CYCLE OF TIME-DRAWDOWN GRAPHS**

**AT LATER TIMES**

<table>
<thead>
<tr>
<th>r</th>
<th>Deep Wells Slope</th>
<th>Shallow Wells Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.6 ft/log cycle</td>
<td>1.75</td>
</tr>
<tr>
<td>50</td>
<td>1.4</td>
<td>1.8</td>
</tr>
<tr>
<td>100</td>
<td>1.35</td>
<td>1.70</td>
</tr>
<tr>
<td>200</td>
<td>1.32</td>
<td>1.6</td>
</tr>
<tr>
<td>300</td>
<td>1.0</td>
<td>1.35</td>
</tr>
<tr>
<td>500</td>
<td>1.0</td>
<td>1.3</td>
</tr>
<tr>
<td>1000</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Estimation of Probable Error in Specific Yield

Sediments take a very long time to drain completely and thus for the limited duration of most field tests the final value of specific yield is not likely to be established. R. W. Stallman (his letter 1965) suggested a method to estimate the probable error in the value of specific yield from tests of limited duration. A summary of his method follows.

Assume we could lower the water table at any desired constant velocity, and consider first an infinitesimal rate of lowering. The liquid content in the pore space at some distance above the water table is but a small fraction of the pore volume and consequently the hydraulic conductivity is small compared with the vertical permeability below the water table. If the downward velocity of the water table is small compared with the conductivity above the water table, water in and above the capillary fringe can easily follow the lowering water table and thereby contribute to storage removal instantaneously, as per the definition of specific yield. However if we were to lower the water table at a velocity much greater than the vertical permeability, most of the water would remain stranded above because there is available a maximum downward hydraulic gradient of 1, and the maximum downward velocity attainable in a gravity field is therefore $P_z$. With the latter there would be no free drainage possible as long as we maintained a rate of lowering greater than $P_z$ and the observable specific yield would be zero. Coupling unsaturated flow theory with observed drawdown rates and value of $P_z$ affords a means for estimating the magnitude of error in $S_y$ possibly arising from delayed yield. According to Wyllie and Gardner (1958) the hydraulic conductivity can be estimated as $P_z \varnothing^4$, where $\varnothing$ is...
the decimal fraction of the pore space filled with liquid removable by gravity—or, the decimal fraction of $S_y$ held in the pore space. As an approximation we can say that water held at some point in the pore space cannot join the moving water table if it is located where the hydraulic conductivity is less than the velocity of the water table. Thus above a water table moving at constant velocity, the water content will not be lowered to a value less than

$$\theta = \left( -\frac{\Delta S}{\Delta t} / P_z \right)^{1/4}$$

approximately. Also it can be seen that the apparent specific yield will be

$$S_y (1 - \theta) = S_{ya}$$

Following table gives ratio of apparent to final specific yield.

<table>
<thead>
<tr>
<th>$S_{ya}/S_y$</th>
<th>$(\Delta s/\Delta t)P_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>≥ 1</td>
</tr>
<tr>
<td>0.2</td>
<td>0.41</td>
</tr>
<tr>
<td>0.4</td>
<td>0.13</td>
</tr>
<tr>
<td>0.6</td>
<td>0.026</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0016</td>
</tr>
<tr>
<td>0.9</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
The knowledge of vertical permeability is of great value in such estimates and as pointed out before can be fairly estimated as suggested by Stallman (1963) and Weeks (1964).

Figure 14 gives the plotting of drawdown versus time for three observation wells selected from BR-T/W 8. The average lowering rate of these wells at later times is from $1.94 \times 10^{-5}$ to $2.8 \times 10^{-5}$ ft/minute.

Tests, as indicated above, show that the value of $S_y$ estimated as 0.2 may be in error as much as 33 percent. The value of vertical permeability used has been assumed equal to radial permeability which is $5.5 \times 10^{-4}$ ft/sec.

**Specific Yield Calculation on Basis of Volume of Cone of Depression**

A semi-log plot of a set of deep and shallow observation wells is given in Figure 13. From inspection it appears that both are lowering almost at the same rate. This response of the aquifer was used by Bennet, Rehman, and others (1964) to estimate the probable specific yield. A brief review and summary is given below.
Figure 14

Time drawdown plot of 300, 500, and 1000 feet observation wells (BR-T/W 8) for estimation of "ds/dt."
Deep wells initially show an artesian response until the slow draining becomes significant. From the behaviour of deep and shallow wells it is clear that the artesian response of deep wells begins to change slowly towards water table response in accord with what has been termed as delayed yield by Boulton (1954, 1963). It is believed that the cause of change of the response from artesian to water table in effect relates to vertical percolation through the strata lying between the water table and the top of pumped well screen and its effect is registered by shallow wells. When the rate of lowering of deep and shallow wells becomes almost the same, it can be concluded that the vertical percolation from upper strata is supplying the pumped water and the contribution from artesian response is negligible.

If the cone of depression is plotted on semi-log graph paper for various times, then volume of the cone of depression is given by:

\[
\text{"V" } = \frac{\pi}{4.6} \left\{ (K_1 - K_2) r_1^2 + (K_2 - K_3) r_2^2 + \ldots \right. \\
\left. + (K_{n-1} - K_n) r_{n-1}^2 + K_n r_n^2 \right\} 
\]

(4.2)

where \( K \) is the slope of the various segments of the cone
and "r" gives the various radii at which the slopes change (Bennet and others, 1964). If the cumulative dewatered volume thus calculated for different times is plotted against the cumulative pumping, the limiting slope of the graph gives an estimated value of the specific yield achieved during the pumping period. Such an estimate has been made for BR-T/W 8, as shown in Figure 15, and gives an estimate of $S_y$ as 0.21.

This method has a decided advantage over that suggested by Ramsahoye and Lang (1961) in the sense that at latter times when delayed yield effect is almost vanishing, the slope gives the final yield established during the last period. The effect of partial penetration is also eliminated automatically because at later times its effect is constant and by taking the slope its effect vanishes from the results. However, the main drawback of the method lies in the fact that the determination of zero drawdown radius is subjective and may lead to appreciable error. Despite this drawback, if wells at great enough distances are used, it can serve as an additional test for the value of $S_y$ already estimated.
Figure 15. Plot of cumulative pumping and the volume of the cone of depression for estimation of specific yield.
CONCLUSIONS

1. In the course of analysis it has been observed that the effect of vertical components of flow is the single most dominant factor which controls the drawdown around the pumping well and seriously affects the determination of transmissibility and artesian storage from the earlier pumping data.

2. Boulton's delayed yield curves are the most appropriate for determination of specific yield but can be used only where there is no other dominating factor influencing the observed drawdown. As such, their application to wells in the vicinity of partially penetrating wells will always lead to inconclusive results.

3. Hantush's (1961) partial penetration analysis can be used quite successfully in determination of permeability from early pumping data.

4. The general value of the permeability in the area under analysis appears to be $7.5 \times 10^{-4}$ and the artesian storage is in the range of 0.0001 to 0.005; the specific yield is 0.2 to 0.25. The depth of aquifer taking part is 750 feet to 1500 feet.
5. The analog solution may prove to be successful but at the present moment it has not been possible to model the delayed yield (Stallman, 1965). The other major unknowns are the depth of aquifer and ratio of vertical permeability to radial permeability.

In view of this analysis, the following points, if noted during performance and analysis of tests in unconfined aquifer, may improve the estimation:

(i) Design of aquifer test should provide at least three or four observation wells in the range of 800 feet to 1500 feet. Wells between 50 to 200 feet can serve as a check on results and may permit evaluation of effective depth of aquifer taking part. Wells nearer than 50 feet may not be very useful.

(ii) Lithologic samples collected from each test should be carefully analyzed for permeability and specific yield determination and compared with this analysis to support it. In the past no such effort has been made and I hope this can improve understanding of the aquifer behaviour.

(iii) The axiom of analysis is to remain outside the area of trouble whenever possible and the various limits have already been given on page 28.
(iv) According to Stallman, if all plots are made on the same graph paper using \( t/r^2 \), then many difficulties can be avoided.
APPENDIX A

DETAILED DATA OF TEST WELL
Diameter of drilling hole is 22 inches.

- 97 ft housing pipe, 14 inches diameter
- 120 ft strainer, 12 inches diameter
- 5 ft bail plug
- 315 ft
## GROUND WATER SURVEY
### Sheet No. 1
#### WELL LOG (LITHOLOGICAL)

<table>
<thead>
<tr>
<th>Depth IN Feet</th>
<th>Graphic LOG</th>
<th>Thickness IN Feet</th>
<th>Formation</th>
<th>Geological Description</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>CLAY</td>
<td>Sandy and silty clay</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td>CLAY</td>
<td>Silty clay, Earthy brown</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>35</td>
<td></td>
<td>GRAVEL</td>
<td>Gravel of sandstone, silty clay 4-5%</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>2</td>
<td>3</td>
<td>SAND</td>
<td>Very fine, Silty, 40% gravel, sandabout 50%</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>21</td>
<td>2</td>
<td></td>
<td>Fine, light grey, sub-rounded, sub-rounded, fairly well assorted, gravel 4-5%</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td>sand as above, granules 10-15%</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td>sand as above, granules 20-35%</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td>6</td>
<td>CLAY</td>
<td>Hard Silty Clay, Blackish</td>
<td></td>
</tr>
</tbody>
</table>

**Area:** Bari disturbing

**Well NO:** BRT/W. 9

**Type:** Test Well

**Location:** Near Lake 60, 4 miles from Naikawa post house, Way to Naikawa from Honofino.

**Coordinates:**

**Drilling commenced:** At 4:32 AM on 6/6/2

**Drilling completed:** At 7 AM on 7/6/2

**Ground Level Elevation:**

**Geologist:** Bashuul Haque

**Driller:** Mr. Jag
<table>
<thead>
<tr>
<th>Depth</th>
<th>Formation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>235</td>
<td>CLAY</td>
<td>Silty clay, fally gray.</td>
</tr>
<tr>
<td>245</td>
<td>SAND</td>
<td>Fine light gray, subrounded, subangular, well sorted. Gravel 238-10-15%.</td>
</tr>
<tr>
<td>255</td>
<td>SAND</td>
<td>Medium, fine, light gray, subrounded, subangular, fairly well sorted.</td>
</tr>
<tr>
<td>265</td>
<td>SAND</td>
<td>Fine medium, light gray, subrounded, subangular, fairly well sorted.</td>
</tr>
<tr>
<td>285</td>
<td>CALY</td>
<td>Medium light gray, subrounded, subangular, well sorted.</td>
</tr>
<tr>
<td>295</td>
<td>SAND</td>
<td>Fine light gray, subrounded, subangular, well sorted.</td>
</tr>
<tr>
<td>305</td>
<td>GRAY</td>
<td>Medium, light gray, subrounded, subangular, well sorted.</td>
</tr>
<tr>
<td>315</td>
<td>SAND</td>
<td>Fine light gray, subrounded, subangular, well sorted, Quartz, green, yellow, mica. Gravel 35-41%. Gravel, Sandstone, and siltstone.</td>
</tr>
</tbody>
</table>
Plan Showing the Location of Observation Well Around BR T/W 9.
<table>
<thead>
<tr>
<th>Level</th>
<th>4-S</th>
<th>4-D</th>
<th>5-S</th>
<th>5-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Silty clay with some sandy clay and granules.</td>
<td>Silty clay with some sandy silty clay and granules. Some very fine sand from 26-28. Some gravel in last few feet.</td>
<td>Very fine sand with rare clay and some gravel.</td>
<td>Very fine sand with rare clay and some gravel.</td>
</tr>
<tr>
<td>2</td>
<td>Very fine sand with rare clay and some gravel.</td>
<td>Fine sand with about 15% gravel &amp; clay. About 15% gravel. 35% clay from 102-114.</td>
<td>Fine-medium sand with 5% clay and gravel.</td>
<td>Fine-medium sand with gravel and clay. 30% gravel from 45-122.</td>
</tr>
<tr>
<td>15</td>
<td>Sand very fine with rare gravel &amp; clay. 15% gravel from 156-160.</td>
<td>Fine-medium sand with rare gravel and clay.</td>
<td>Fine-medium sand with rare gravel and clay.</td>
<td>Med. coarse sand with some clay 30% gravel in first 2 feet.</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td>Fine medium sand with 40% gravel &amp; clay.</td>
</tr>
</tbody>
</table>
Silty clay with some sandy very fine sand from 34-22 to 20-32.

Very fine sand with rare gravel.

Silty clay with some sandy very fine sand from 26-23 and 20-32.

Very fine sand with rare gravel + clay.

Very fine sand with rare gravel + clay.

Silty clay with some sandy very fine sand.

Some granules in first few feet.

Very fine sand with some gravel + rare clay.

Fine sand with gravel. Some clay from 34-22.

Fine medium sand. Some gravel. 15%

Clay 8% gravel from 20-12.

Fine sand with about 5% gravel + rare clay.

Vertical Scale: 1' = 40'
APPENDIX B

GLOSSARY OF SYMBOLS
GLOSSARY OF SYMBOLS

\begin{itemize}
    \item \textbf{a} = \frac{T}{S}
    \item \textbf{b} = \text{Effective aquifer thickness taking part in flow (ft)}
    \item \textbf{d} = \text{Depth of top of pumpwell screen below water table (ft)}
    \item \textbf{f_s} = \frac{4b}{(L - d)} \sum_{n=1}^{\infty} \frac{1}{n} K_0 \left( \frac{n \pi r}{b} \right) \cos \frac{n \pi z}{b} \left( \sin \frac{n \pi L}{b} - \sin \frac{n \pi d}{b} \right)
    \item \textbf{n} = 1, 2, 3, \ldots \ldots \ldots \ldots \text{n}
    \item \textbf{r} = \text{Distance of observation well from pumped well (ft)}
    \item \textbf{r_v} = \text{Distance from pumped well (ft) at which vertical head difference is negligible}
    \item \textbf{s} = \text{Drawdown (ft)}
    \item \textbf{s_1} = \text{Drawdown for } \eta = 1
    \item \textbf{s_{p-p}} = \text{Drawdown considering only partial penetration effects}
    \item \textbf{s_t} = \text{Drawdown due to Theis artesian response}
    \item \textbf{s_{wt}} = \text{Drawdown considering water table case after Boulton (1955)}
    \item \textbf{t} = \text{Time in minutes since the start of pumping}
    \item \textbf{u} = \frac{r^2 S}{4Tt} \text{ (a dimensionless parameter)}
    \item \textbf{w} = \frac{z}{2b}
\end{itemize}
\[ B = \sqrt{\frac{T}{aS'}} \]

\[ E = \text{It's a function same as } W(u) \]

\[ J_0(X) = \text{Bessel function of first kind and zero order} \]

\[ L = \text{Depth of pumpwell below water table (ft)} \]

\[ M.A.F. = \text{Million Acre-feet} \]

\[ M(u,\beta) = \int_{u}^{\infty} \frac{e^{-y}}{y} \text{erf} (\beta \sqrt{y}) \, dy \]

\[ P = \text{Isotropic permeability} \]

\[ P_r = \text{Radial permeability} \]

\[ P_{\theta} = \text{Permeability along } 0 \text{ direction} \]

\[ P_z = \text{Vertical permeability} \]

\[ Q = \text{Pumping rate of well (cfs)} \]

\[ S_y = \text{Specific yield} \]

\[ S_{ya} = \text{Apparent specific yield} \]

\[ S = \text{Instantaneous storage coefficient} \]

\[ S' = \text{Delayed storage coefficient} \]

\[ S_s = \text{Specific storage } (S/b) \]

\[ T_s = \text{Transmissibility of aquifer (ft}^2/\text{sec)} \]

\[ W(u) = \text{Well function of } "u"; \int_{u}^{\infty} \frac{e^{-u}}{u} \, du. \]

\[ Z_d = \text{Depth of piezometer (deep) below water table (ft)} \]

\[ Z_s = \text{Depth of shallow piezometer below water table (ft)} \]

\[ = \text{Reciprocal of delay index (see Boulton, 1963)} \]
\[ \theta = \frac{4 \, Tt/r^2 S}{r^2 S} \]
\[ \theta' = \frac{4 \, Tt/r^2 S'}{r^2 S'} \]
\[ \eta = \frac{S + S'}{S'} \]
\[ \tau = \frac{Pt}{S \, b} \]
REFERENCES


1964, Discussion of "Analysis of data from non-equilibrium pumping tests allowing delayed yield from storage": Inst. Civil Engineers Proc., London, v. 28, p. 603-610.


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Kirkham, Don, 1959, Exact theory of flow into a partially penetrating well: J. G. R., v. 64, no. 9, p. 1317-1327.


