TRANSVERSE DISPERSION

THROUGH NON-UNIFORM POROUS MEDIA

by

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STATEMENT BY AUTHOR

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_____   _7 May 1968_

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Professor of Hydrology

ACKNOWLEDGMENTS

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ABSTRACT

A laboratory experiment was devised to measure the effect on transverse dispersion of a permeability interface. A layer of coarse sand was interposed within a matrix of fine sand at two different orientations: (1) at right angles, and (2) at an angle of 60°, to the direction of flow. Dispersion was first measured in fine sand alone in order to obtain a base measurement unaffected by permeability changes. Calculations for the non-uniform cases were made assuming that increased dispersion could be accounted for by changes in medium particle size. Thus in the calculations it is assumed that the permeability interface does not affect transverse dispersion.

With the coarse sand layer at right angles to the flow field, the standard deviation of the observed dispersion was about 27% greater than predicted. With the layer at a 60° angle to the flow field, the standard deviation of the observed dispersion was 79% greater than predicted.

Thus it appears that a permeability interface in a porous medium does have an effect on dispersion and that the magnitude of the effect depends, in part, on the orientation of the interface to the direction of mean flow.

INTRODUCTION

On this planet, water is essential to the very existence of life in every form, whether plant or animal. Civilized man has added to his basic needs an increasing demand for high quality water for municipal, industrial, recreational and agricultural uses. While the supply of such water available today from surface flow and ground water storage is sufficient in most places to meet these demands, increasing population pressures and increasing volume requirements per individual will in the future put a great strain on our water resources.

In the past, careless exploitation of water resources has been the rule. Haphazard disposal of low quality fluid waste material has, in many cases, been allowed to contaminate surface and ground water supplies. At present, disposal of bacterial, chemical, caloric and radioactive liquid waste is a difficult problem; it will become more so as the demands of our industrial civilization create an even greater volume of these waste products.

Water stored by nature or man in subsurface reservoirs (aquifers) is an element in regional water resources management which will become more important as time goes on. In the past, aquifers have been used to store imported water and to dispose of liquid contaminants. In the future, they will be used increasingly for these duties.

To be able to use an aquifer efficiently, its various hydrologic properties (permeability, storage capacity and dispersive properties) must be evaluated. Permeability and storage capacity give a measure of the quantity of water economically available. The dispersive properties of an aquifer may affect the quality of the water available. Since the quality of water is an important factor relating to water use, the dispersive property must be evaluated.

Research by Ogata (1961, 1964) and Simpson (1962), among others, investigated the processes which cause mixing, and Hubbert (1940) investigated the mechanics of ground water flow. Theory and the results of laboratory studies provide several reasonably accurate predictive models for homogeneous, uniform porous media, but these results cannot be transferred to field conditions which are neither homogeneous nor uniform. Studies by Skibitzke (1964) have shown that non-uniformities greatly affect the magnitude of mixing observed.

The effect of a permeability interface, i.e., the boundary between two regions of different permeability, on the magnitude of dispersion is one factor still to be investigated. This thesis will consider the question: What is the effect, if any, of a permeability interface on the magnitude of transverse dispersion in flow through porous media?

OUTLINE OF THEORY

Dispersion as considered in this report is the phenomenon by which two or more miscible fluids are mixed during displacement through a porous medium. Diffusion is the mixing of miscible liquids due to concentration gradients alone. In general, the effects of dispersion are greater than those of simple diffusion. Dispersion has been divided into longitudinal and transverse components although this is somewhat arbitrary and has been done to facilitate mathematical treatment.

Longitudinal dispersion is dispersion parallel to the direction of flow and results from the fact that there are: (a) fluid velocity gradients within individual pore spaces, (b) variations in the length of individual flow paths per unit time, and (c) molecular diffusivity. Transverse dispersion is mixing perpendicular to the line of flow and is due to: (a) molecular diffusivity and (b) intertwining of flow paths.

Fluid velocity gradients result from the fact that at the center of an individual pore space the fluid is moving at a velocity greater than the fluid velocity at the pore boundary due to the viscosity of the fluid. Individual flow paths are of varying lengths due to the geometry of the medium, and as a result some flow paths make a more complicated transit through the medium than other flow paths.

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Molecular diffusivity is due to the thermo-kinetic energy of fluid molecules, while intertwining of flow paths (secondary rotation of the flow paths) probably occurs to an appreciable degree only in non-uniform media.

Several investigators have demonstrated that for longitudinal dispersion in a uniform medium molecular diffusivity is minor compared to other possible mechanisms and for practical purposes may be ignored. For transverse dispersion at low velocities in a uniform medium it has been shown by Simpson (1962) that molecular diffusivity is a significant aspect of the dispersion. Shamir and Harleman (1967) analysed dispersion in a layered medium assuming that the permeability interface is not a factor in the dispersion. They showed experimentally that this is true in longitudinal dispersion where the layers are at right angles to the flow path. Skibitzke (1964) states that a layered medium is anisotropic and that permeability must be considered as a tensor quantity. Thus, there should be a noticeable increase in dispersion in a layered medium. Simpson (1962) states that in the presence of non-uniformities, intertwining of flow paths will occur and that an increase in dispersion should be observed.

Development of Transverse Dispersion Equations

When discussing transverse dispersion in a homogeneous isotropic medium where a uniform flow parallel to the axis of the frame of reference exists, the approach developed by Ogata (1961)

utilizing the Fickian assumptions has been used. Thus the distribution of an introduced tracer material can be obtained using the conservation of mass equation:

$$D \, \nabla^2 \, C = \partial C/\partial t + u\partial C/\partial x \tag{1}$$

where:

D = Coefficient of Bulk Dispersion

C = tracer concentration

u = fluid velocity in x direction

x = length of travel in x direction

t = time since tracer introduced

or,

$$\partial C/\partial t = D\partial^2 C/\partial^2 \xi \tag{2}$$

where:

$\xi = x - ut.$

It is assumed that the concentration gradient which results is normal in character and that it is possible to obtain a plot of tracer concentration versus distance from the centerline of the tracer band using solutions provided by Crank (1956), and that the standard deviation of dispersion will be given by Einstein's (1956) equation:

$$\sigma = \sqrt{2Dt} \tag{3}$$

where:

σ = standard deviation of dispersion.

For a tracer source of finite width with the boundary conditions:

$$C(-h < y < +h, 0) = C_o \tag{4}$$

$$C(\infty, t) = 0$$

where

C = ion concentration at measurement point

C_o = initial ion concentration

h = initial half width of tracer source

y = distance from centerline of tracer band to measurement point

D = coefficient of transverse dispersion

t = time since tracer entered medium

a solution of equations (2) and (4) is given by Crank (1956) as follows:

$$C/C_o = (1/2) \{ \text{erf} [(h - y) / \sqrt{2Dt}] + \text{erf} [(h + y) / 2\sqrt{Dt}] \}. \tag{5}$$

It is possible to modify equation (5) with equation (3) to yield a new equation whose parameters are more convenient for our purpose:

$$C/C_o = (1/2) \{ \text{erf} [(h - y) / \sqrt{2\sigma}] + \text{erf} [(h + y) / \sqrt{2\sigma}] \}. \tag{6}$$

Since the tracer source is finite in width, equation (6) gives a relationship which is actually the summation of an infinite number of normal distributions within the limit of the source width. Since h, y and C/C_o may be measured, it is possible to solve for σ.

A relationship based on theoretical considerations and supported by experimental data (Harleman, Melhorn and Rumer 1963, for example), is as follows:

$$D = \beta d \bar{u}^n \tag{7}$$

where:

D = coefficient of dispersion

$\bar{u}$ = average fluid velocity

β = coefficient dependent upon pore shape

d = mean grain diameter

n = experimentally determined exponent.

In the case of longitudinal dispersion n is usually found to be slightly greater than one, whereas for transverse dispersion it is slightly less than one. For purposes of this report it will be sufficiently exact to assume that n equals one. Assuming n = 1, and substituting equation (7) into equation (3), we have:

$$\sigma^2 = 2\beta d \bar{u} t \tag{8}$$

$$= 2\beta dx$$

or,

$$\sigma = k\sqrt{dx} \tag{9}$$

where:

$k = \sqrt{2\beta}$ = dimensionless constant depending on pore shape

$x = \bar{u}t$ = average flow distance in time t.

The two sands of differing grain size used in the experiment were different size fractions of the same crushed quartz, and their

grain size distributions are quite similar, as shown in Figures 8 and 9. If we assume an identical k value for each (implying that pore diameter changes, but not pore shape), the following ratios may be derived for a given flow distance or for a given medium:

$$\sigma_A \big/ \sigma_B = \sqrt{d_A/d_B} \tag{10}$$

where:

σ_A = standard deviation of dispersion at a

given flow distance in medium A

d_A = the average grain diameter in medium A

σ_B and d_B = similar quantities in medium B

and

$$\sigma_{x_1} \big/ \sigma_{x_2} = \sqrt{x_1 / x_2} \tag{11}$$

where:

σ_{x_1} = standard deviation of dispersion in medium A or B

at a distance x_1 from the origin

σ_{x_2} = standard deviation of dispersion in medium A or B

at a distance x_2 from the origin.

Having measured σ in the uniform fine-grained medium and using relationships (10) and (11) it was possible to obtain values of σ which would occur in the coarse layer. The grain diameter was obtained by mechanical sieve analysis while the length of travel was measured directly from the sides of the tank. A possible check on the validity of equation (9) may be made by plotting the increase in σ as a function of flow distance x. The initial width of the band was estimated to be
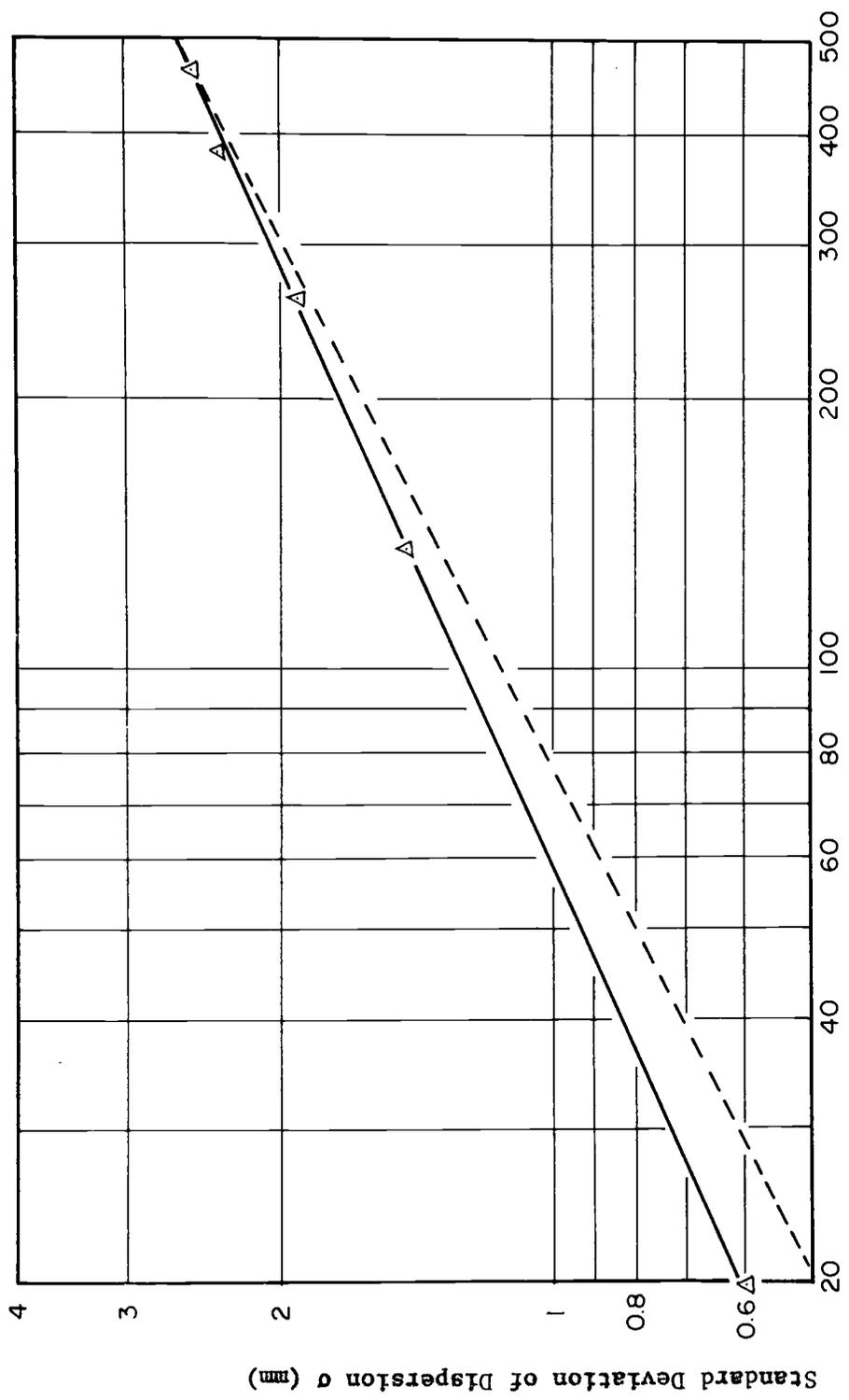
Figure 1. Log-Log Plot of Standard Deviation of Dispersion σ versus Flow Distance x.

2.0 mm based on measurement of the distance between the color change

boundaries as explained in detail in a later section. When plotted on

log-log paper, σ should be a straight line function of x with a slope

of 0.5 if equation (9) is valid. As seen in Figure 1, the data points

are linear but the slope is 0.45. Increasing the initial width to

something slightly greater than 2.0 mm would result in a slope of 0.5.

Hence it is possible that the initial width of the tracer band was

underestimated. However, for purposes of calculation, the initial

width was assumed to have been 2.0 mm.

Prediction of σ in a Layered Medium

By first measuring dispersion in the uniform matrix, it is

possible to use equations (6), (9) and (11) to compute the dispersion

that should occur in a matrix with an intervening coarse layer

assuming that the permeability interfaces do not affect dispersion.

The procedure (see Figure 2 for location of position points), is as

follows:

1. For the experiment with uniform fine-grained matrix only,

    measure at position 3 the distance from the center line to

    the location of the color change where $C/C_o = 0.0044$ (as

    explained in a later section). Using equation (6),

    calculate σ. This value is denoted as σ_{A_3}.

2. For the experiment with the intervening coarse layer,

    compute σ at position 1 using equation (11). This value

    is denoted as σ_{A_1}.

3. Imagine that the magnitude of σ_{A_1} was produced by flow through medium B. Compute the flow distance that would occur in medium B to produce the same σ. This is done by modifying equation (9) to obtain the relationship:

$$x_{B_1} = x_{A_1} (d_A / d_B).$$

4. Compute the increase in σ between positions 1 and 2 through the coarse layer using equation (11). In this case:

$$\sigma_{x_1} = \sigma_{A_1}$$

$$x_1 = x_{B_1}$$

and,

$$x_2 = x_{B_1} + \text{width of coarse layer.}$$

In equation (11) solve for σ_{x_2} and denote it as $\sigma_{A_1B_2}$.

5. Imagine that the magnitude of $\sigma_{A_1B_2}$ was produced by flow through medium A only. Using the procedure outlined in Step 3, compute the flow distance that would occur.

6. Compute the increase in σ between positions 2 and 3 of the fine-grained matrix using equation (11). Denote this as $\sigma_{A_1B_2A_3}$.

It should be noted that: (a) this procedure may be expanded to include any number of layers, and (b) it is possible to reduce the number of steps by adding the flow lengths in layers of the same kind together and treating each kind as one continuous layer.
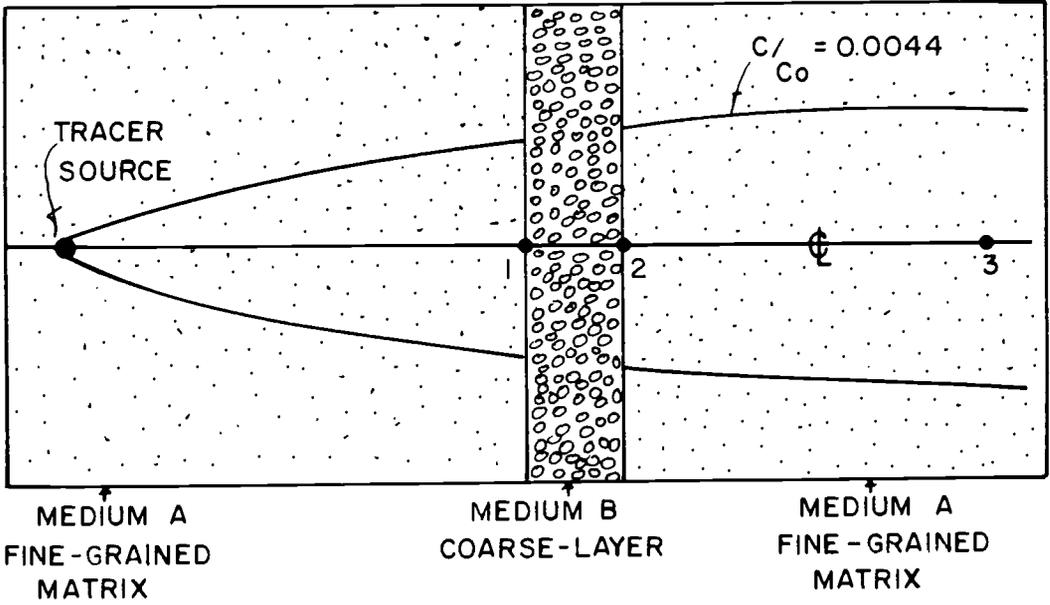
Figure 2. Values Needed to Compute σ for Layered Case

General Remarks

Skibitzke (1964) compared the differences and similarities between isotropic and anisotropic media. He showed that in an anisotropic medium, permeability can no longer be considered to be a scalar quantity. If permeability is a scalar property of the medium, fluid motion must be at right angles to the equipotential lines. With permeability as a tensor property it is possible to have fluid motion which is not at right angles to the equipotential lines, and bending and intertwining of the flow lines will result. He asserts that in a non-uniform medium where the non-uniformity is randomly distributed a tracer source must move a considerable distance down the direction of flow before a normal distribution of the tracer material will be achieved. He gives no equations for predicting this distance but does show qualitatively what type of distribution to expect with varying lengths of travel.

Shamir and Harleman (1967) developed equations to describe longitudinal dispersion in a layered medium with all of the layers at right angles to the flow paths. They obtained an equation assuming that the permeability interface has no effect on the longitudinal dispersion which is observable at the far end of the column. Their equation predicts and they observed that the location of a layer has no effect on longitudinal dispersion and that it is possible to add up all the lengths of the various layers and hence simplify calculations.

In the same article, a discussion of transverse dispersion in a layered porous medium where the layers are parallel to the flow lines is presented. The boundary between the two layers is placed at several locations with respect to the line source.

In this thesis, transverse dispersion is studied in relation to layers of different permeability interposed at right angles and at $60°$ to the flow direction. Hence the results are not directly comparable to those of Shamir and Harleman (1967). However, it is possible to test the assumption that transverse dispersion does not "see" the interface.

In the experiment under consideration the uniform medium has a smaller grain size than the interposed layer, and it is expected that σ obtained using the steps outlined above will give a value of transverse dispersion greater than that obtained for the uniform medium alone. Since it was assumed in the theoretical computations that the permeability interface played no role in the dispersion, it should be possible to test this assumption by comparing observed results with the predicted values.

DESCRIPTION OF APPARATUS

The purpose of the laboratory model was to measure transverse dispersion in a layered medium and to compare the results with predicted values as outlined in the previous section. Provision was made to interpose layers of a different permeability at various angles to the fluid flow path. Dispersion measurements were made only after steady state conditions were reached.

The model shown in Figures 3, 4, 5, 6 and 7 consisted of three major components: (a) the sand tank; (b) external accessories which feed bulk and tracer fluids to the sand tank; and (c) the accessories which were used to measure transverse dispersion and head differences.

The Sand Tank

The overall dimensions of the sand tank were 88.8 cm x 41.6 cm x 7.5 cm. It was constructed of 1.27 cm thick plexiglas, sealed to prevent leakage from the ends and the bottom, and mounted on a tilt-able base. The tank as shown in Figure 4 was divided into three sections, two of the three being reservoirs for the influent and effluent fluids. The entire tank was open at the top and filled with liquid to a level several centimeters higher than the upper surface of the sand. Flow above the sand surface and for 3 to 5 cm below the surface of the sand was essentially prevented by a series of nine metal baffles inserted across the flow direction.

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The influent tank (A), shown in Figure 4, was fed by a 24-liter Mariotte flask at a flow rate which was nearly constant, with any pulses in the rate smoothed out by the large surface area and volume of tank (A).

The tracer solution was injected into the sand through a slotted hollow brass cylinder (B), shown in Figures 4 and 5. The ends of the cylinder were coated with teflon and it was spring loaded to prevent fluid leakage out of the model. When initially filled, the slot was pointed upwards to allow the escape of all entrapped air; then the cylinder was rotated to bring the slot into contact with the sand.

The bulk solution flowed into the sand (C) from influent tank (A) across the full length of a slotted partition covered with wire gauze. The bulk solution plus the contained tracer solution from (B) flowed out of the sand across a similar partition into effluent tank (D). Overflow tubes in both tanks (A) and (D) permitted the adjustment and maintenance of a constant head difference.

To permit the emplacement of the layer of contrasting permeability, grooves were cut on the inside of the plexiglas to hold temporary metal plates in place. After the sand was in place, the plates were withdrawn. The grooves are shown in Figure 4.

External Accessories

External constant head tank (E) was separated from the porous medium reservoir. It was a cylinder constructed from 0.32 cm thick plexiglas for the sides and 1.27 cm thick plexiglas for the base.

The tracer solution was fed from tank (E) into the tracer source
cylinder (B) in the sand tank by syphon action. Tank (E) in turn
was fed by a 4-liter Mariotte flask, the overflow from tank (E)
being recirculated and used again (Figures 3, 6 and 7).

Measurement of Dispersion and Head Differences

Dispersion was measured by means of a pH-controlled color
change of the tracer solution as described in the next section. Here
it is sufficient to note that the width of the dye band as defined by
the color change was measured by eye with a scale placed along a scored
line at right angles to the flow direction.

The problem of measuring the piezometric head difference was
simplified by the free water surface of the model. The head dif-
ference between tanks (A) and (D) was measured by a manometer, and
this was used as the head difference for the model. In this way it
was possible to avoid special fittings in the sand matrix, and the
head loss caused by the wire mesh cage was small enough to be
neglected.

General Remarks

As previously stated, a free water surface and stagnation
of the water in the top few centimeters of the sand matrix was
achieved by placing nine metal baffles at the top of the medium.
The effectiveness of this set-up was checked by placing $KMNO_4$
crystals into the sand matrix during the first experimental

sequence. The $KMNO_4$ flow lines showed there was virtually no flow
in the region of the baffles once the medium was fully saturated and
steady state flow obtained.

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┌─────────────┐              ┌──────────────┐
│ TAP  WATER  │─────────────▶│ EXCESS VAPOR │
│   BOILED    │              │  AND  GASES  │
└─────────────┘              └──────────────┘
       │
       ▼
┌──────────────────┐
│ DEGASSED  WATER  │───────────────────┐
│     COOLED       │                   │
└──────────────────┘                   │
       │                               │
       ▼                               ▼
┌──────────┐  ┌──────────┐      ┌──────────┐  ┌──────────┐
│ HCl      │─▶│  BULK    │      │  TRACER  │◀─│ NaOH     │
│INDICATOR │  │ SOLUTION │      │ SOLUTION │  │INDICATOR │
│ NaCl     │  └──────────┘      └──────────┘  └──────────┘
└──────────┘       │                 │
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─ ─ ─ ─ ─ ─ ─ ─ ─ ─│─ ─ ─ ─ ─ ─ ─ ─ ─│─ ─ ─ ─ ─ ─ ─
                   ▼                 ▼
            ┌──────────┐      ┌──────────┐
            │ MARIOTTE │      │ MARIOTTE │
            │  FLASK   │      │  FLASK   │
            └──────────┘      └──────────┘
                 │                 │       RECIRCULATE
┌────────┐  ┌──────────┐      ┌──────────┐  EXCESS
│ EXCESS │◀─│ TANK (A) │      │ TANK (E) │
└────────┘  └──────────┘      └──────────┘
                 │                 │
                 │            ┌──────────┐
                 │            │ TANK (B) │
                 │            └──────────┘
                 ▼     ┌──────────┘
            ┌──────────┐
            │ TANK (C) │
            └──────────┘
                 │
            ┌──────────┐
            │ TANK (D) │
            └──────────┘
              │        │
        ┌────────┐ ┌──────────────┐
        │ WASTE  │ │ PERMEABILITY │
        └────────┘ │  APPARATUS   │
                   └──────────────┘
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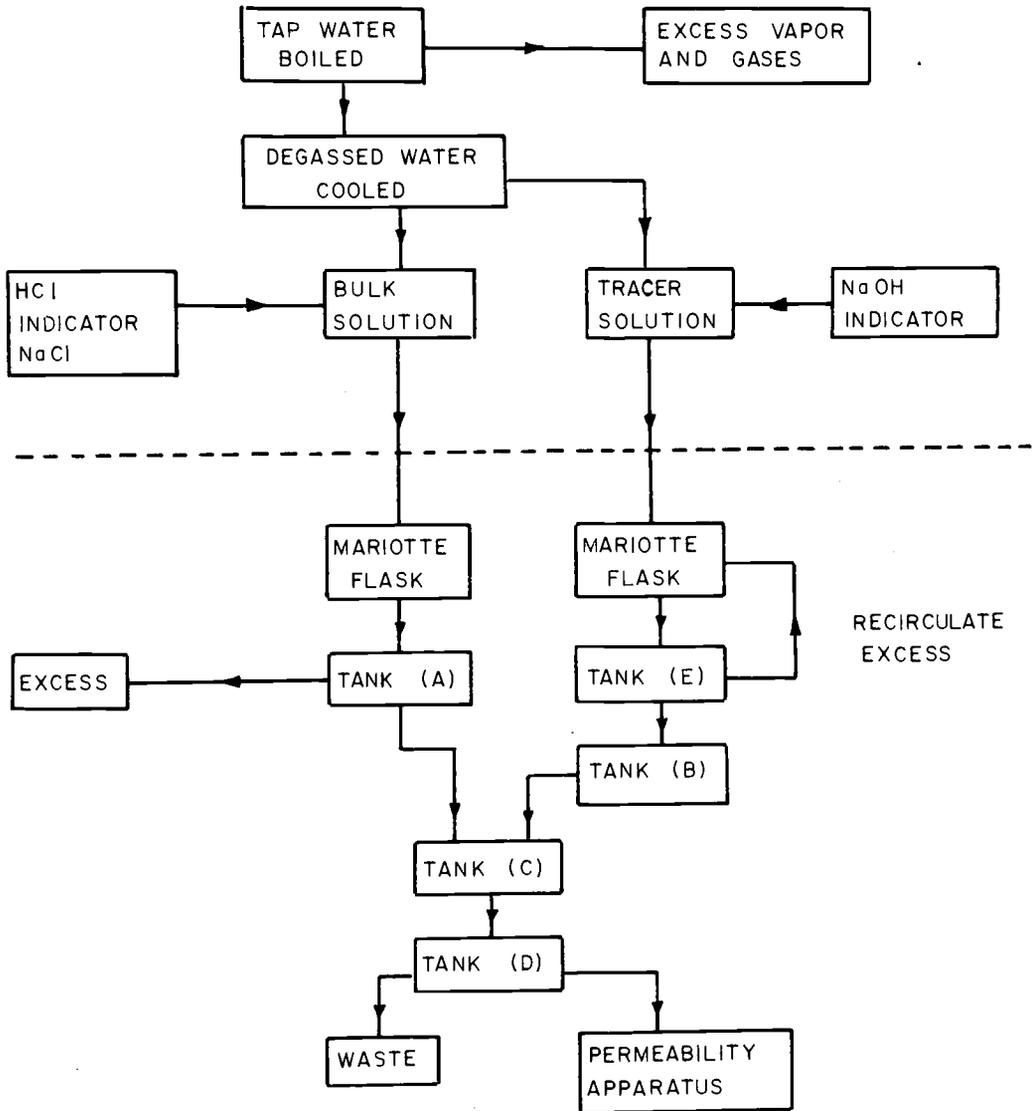
FLOW   DIAGRAM

Figure 3. Flow Diagram of Experimental Procedure
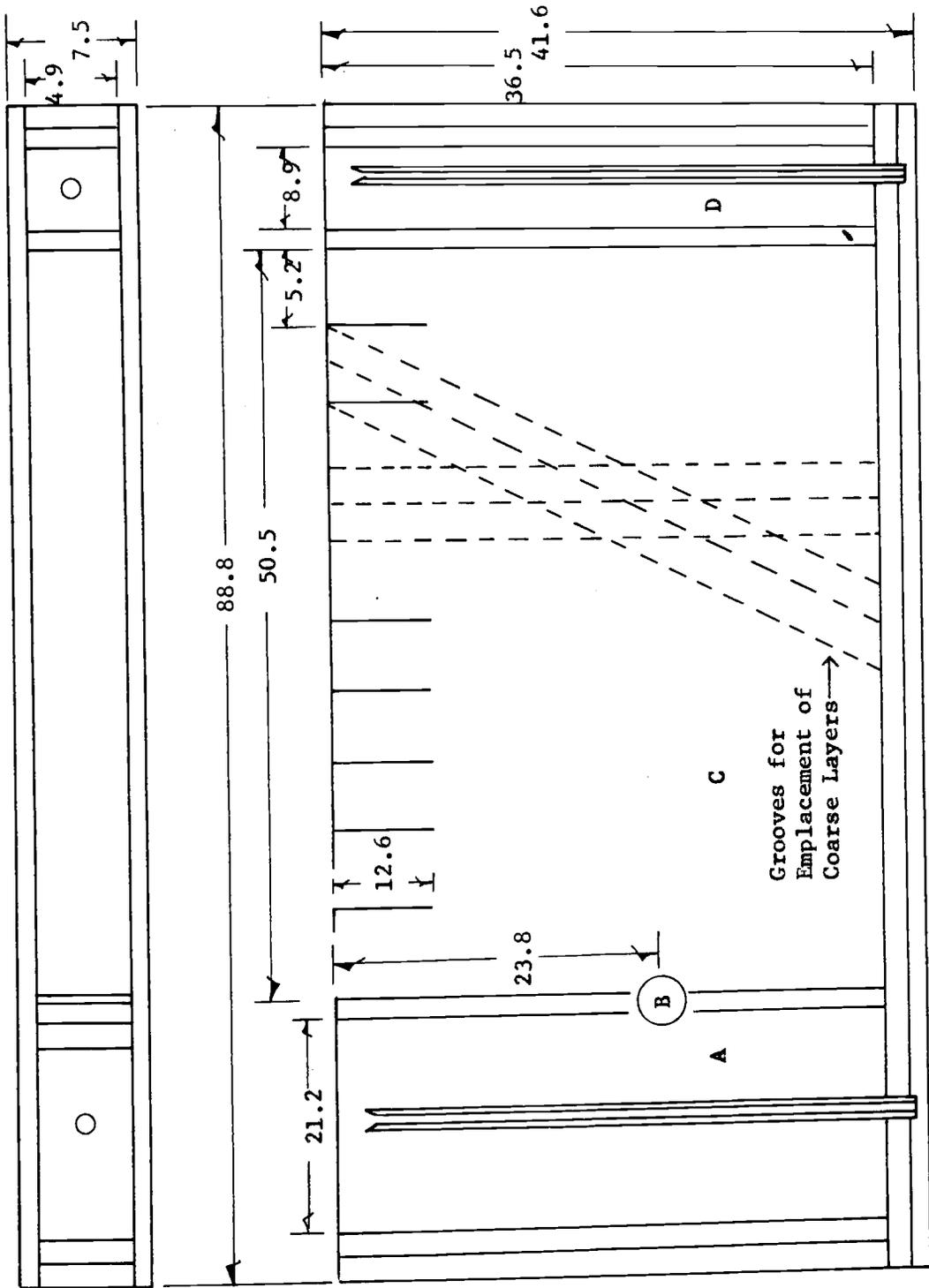
Figure 4. Sand Tank Showing Detail of Groove Location
(all dimensions in cm)

Bolt

Washer

Rubber Seal

Plexiglas Side

Fitting to Allow
Rotation of Cylinder

Threads

Teflon Seal

Spring

Brass Cylinder

Threads

Fitting to Allow
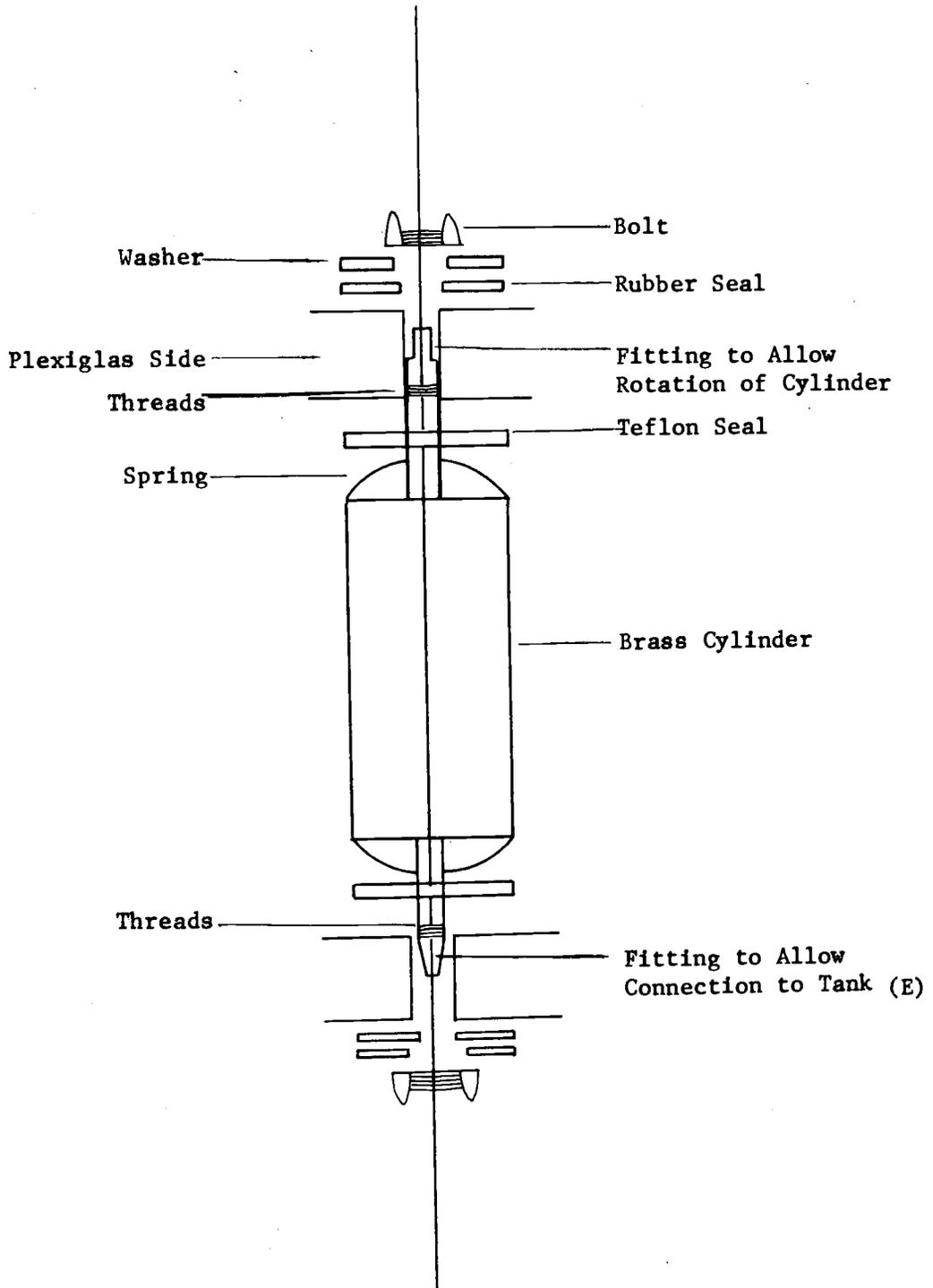Connection to Tank (E)

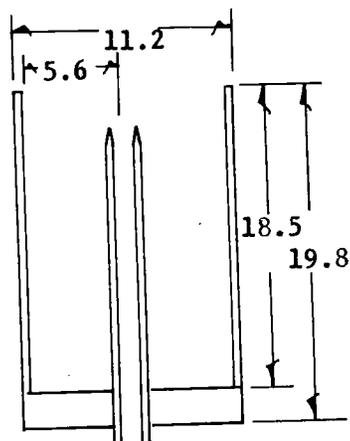Figure 5. Detail of Tracer Source Cylinder (B)

Figure 6. External
Constant Head Tank (E)
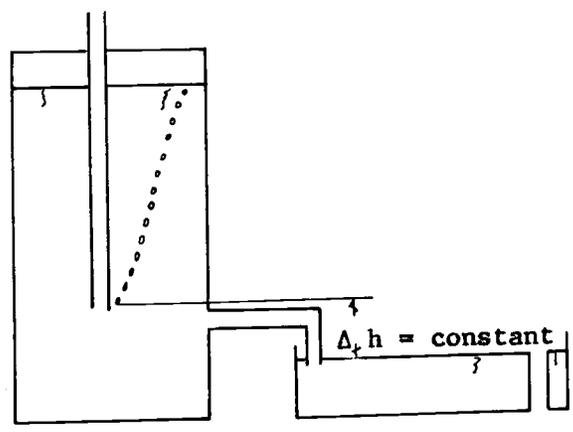(all dimensions in cm)

Figure 7. Principle of the
Mariotte Flask
(all dimensions in cm)

EXPERIMENTAL PROCEDURE

The experimental procedure consisted of: (1) preparation of
the sand matrix and solutions used in the model; (2) placing the
solution containers and constant head apparatus in the proper rela-
tionship to the sand tank; and (3) running the fluids through the
various connections and taking the experimental measurements.

Sand

The sand used as the fine-particle matrix was a 20-mesh sand
sold by the Crystal Sand Company, Oceanside, California. It consisted
of angular quartz particles graded by the manufacturer, but containing
a considerable amount of fine material. Most of the fine material was
removed by washing, after which the product was used in the model with-
out further preparation. The results of a gravimetric sieve analysis
of the 20-mesh sand after washing is shown in Figure 8.

The material used for the coarse layer was a 12-mesh sand
obtained from the same source. This material was prepared in the same
manner as was the 20-mesh sand. The results of the sieve analysis are
shown in Figure 9.

To insure that the sand would be as similar as possible for
each of the different runs, enough sand was washed at one time for all
subsequent requirements. The model was packed with sand on three
separate occasions, each representing one of the three conditions
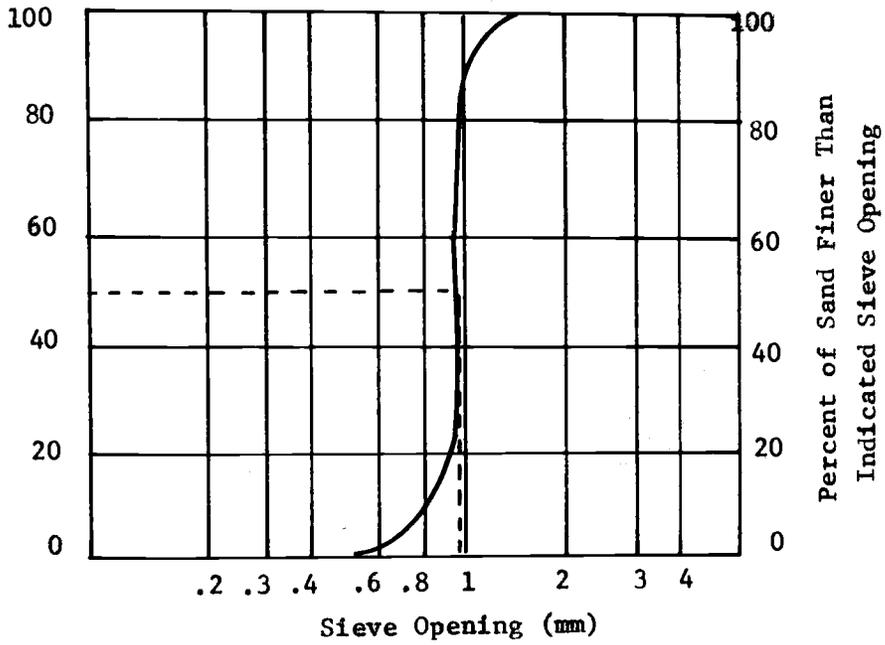which were investigated: (1) uniform medium of fine sand; (2) layered
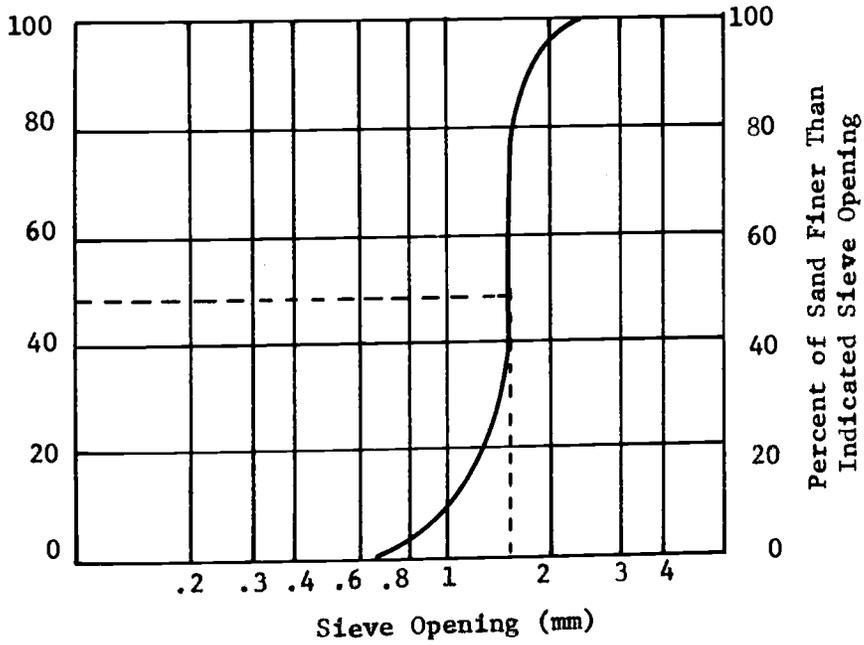
Figure 8. Gravimetric Analysis 20-Mesh Sand

Figure 9. Gravimetric Analysis 12-Mesh Sand

medium with coarse-sand layer at 90° to flow direction; (3) same as (2) but with coarse layer at 60° to flow direction.

The packing of the sand for the uniform medium was straight-forward. The sand was dampened, a thin layer placed in the bottom was tamped to maximum density. More sand introduced into the model was in turn packed. This process continued until the model was filled to within 5 cm of the top of the tank. Nine baffles were inserted and C-clamps applied at the top to assure a good seal between the baffles and the tank sides.

When layered media were used, placing the sand matrix into the model proceeded in much the same manner as that used for the uniform medium but with the temporary insertion of thin metal barriers; the 20-mesh sand outside the metal barriers and the 12-mesh sand between them. After the model was filled with sand, the metal barriers were slowly and carefully removed, and any gaps were eradicated by packing the matrix several times as the barriers were slipped out. The baffles were inserted; the C-clamps applied.

In the uniform medium the tracer stream was nearly straight, while in the layered media any perturbations due to uneven packing were much smaller than those caused by the layering itself.

Solutions

All solutions used in the experimental runs were prepared from water degassed by heating under a partial vacuum until boiling was initiated. It was then allowed to cool to room temperature still under

a partial vacuum. Reheating a sample of the solution showed that bubbles did not form until a temperature of 90 degrees Centigrade was reached. There were no problems of air entrapment noted in the apparatus. The bulk and tracer solutions for this experiment were prepared according to the method given by Hiby (1962).

The tracer solution was a 5×10^{-2} N solution of sodium hydroxide into which phenothaline indicator had been added giving it a red color. This solution was prepared in 5-liter batches and the overflow in the constant head tank (E) was recirculated during each experimental run. A new batch of tracer solution was prepared at least twice a week during the experimental runs and in no case was it used for more than three experimental runs.

The bulk solution was a 5×10^{-4} N solution of hydrochloric acid to which sodium chloride was added to equalize its density to that of the tracer solution. Phenothaline indicator was also added to the bulk solution but no color change resulted. The bulk solution was prepared in 24-liter batches, and the sand model was kept saturated with this solution at all times. Twenty-four liters of solution was sufficient to permit one run of the experiment to be performed and then to flush all of the tracer solution out of the system.

The ion concentration of the tracer solution in the mixture at the color change was determined to be 2.2×10^{-4} N (OH⁻). Using

the relationship $C = 2.2 \times 10^{-4}$, $C_o = 5 \times 10^{-2}$, or $C/C_o =$

$\dfrac{2.2 \times 10^{-4}}{5 \times 10^{-2}} = 0.0044$, a value of C/C_o at a measured distance y from

the centerline could be established. This made it possible to cal-

culate the standard deviation of the dispersion by equation (6) and

assuming a normal distribution.

Reservoir Placement

The placement of the fluid containers, the external constant

head tank (E), and the slope of the model were factors which could be

adjusted. The slope of the hydraulic gradient was controlled by the

base which was designed to allow the sand tank to have several dif-

ferent slopes. The first step in running the experiment was placing

the model at the desired slope, usually approximately one to nine, on

the work bench. It was then necessary to place tank (E) so that its

overflow tube was at the same elevation as the overflow tube in tank

(A). Thus it was possible to assure that the hydraulic head of the

tracer solution and the bulk solution were the same as they entered

the sand matrix.

Before fluids were allowed to flow through the system, the

Mariotte flasks were positioned. To assure that tank (A) was

functioning, a head difference of 15 cm was maintained between

the top of its overflow tube and the large Mariotte flask.

Experimental Run and Measurements Taken

Saturating the porous medium had to be done slowly to insure that all of the air in the sand matrix was forced upwards. Before the tracer solution was introduced, the bulk solution was passed through the porous medium for twenty minutes to allow the flow lines to straighten. In the interum, the tracer source reservoir (B) was filled and rotated into position.

After the twenty-minute period, the connection between external tank (E) and tracer source (B) was established by opening the connecting valve, and the actual experimental run began. The width of the tracer stream entering the model, controlled by adjusting the transmission resistance in the connecting tubes, was maintained the same for each of the experimental runs.

There were several measurements taken after the start of each run. The air and water temperature, the head difference across the porous media tank (C), and the fluid discharge per unit time were all noted.

The width of the tracer stream was measured at five-minute intervals at reference lines scratched on the exterior of tank (C). Thus it was possible to observe the transverse dispersion and note the progress of the tracer front as it passed through the porous sand medium.

EXPERIMENTAL RESULTS

The measurements of transverse dispersion made under the three different experimental conditions are summarized in Table I and in Figures 10, 11, 12, 13, 14 and 15. In Table I are also tabulated the standard deviations of the measured dispersion and the standard deviations of the predicted dispersion assuming the interface, as such, had no effect on dispersion. On the face of it, it appeared that the measured dispersion was significantly greater than predicted dispersion and, hence, that the interface did affect dispersion. Also, the effect of the interface on dispersion seemed to vary with the angle at which the interface intersected the average direction of flow. However, before going more deeply into the role of the interface, let us examine possible sources of experimental error and their effect on the validity of the results.

Possible Sources of Experimental Error

Experimental error could have been introduced into the results through two factors: (1) extraneous effects on the mixing process due to model design, and (2) measurement errors.

1. The extraneous effects consisted of: (a) grooves cut on the inside of the plexiglas tank, (b) the boundary conditions present which would be absent in an infinite aquifer, and (c) non-homogenities present in the sand matrix and compaction of the sand matrix with time.

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Table I

Tracer Band Width and Standard
Deviation 480mm. from Injection Point

Medium	Run	Band Width at 480mm.	Standard Deviation Calculated from Band Width[a]	Standard Deviation Predicted[b]
Uniform	1	16mm.		
"	2	18mm.		
"	3	17mm.		
	Ave.	17.0mm.	2.5mm.	
Layer 90°	1	21mm.		
"	2	22mm.		
"	3	22mm.		
	Ave.	21.7mm.	3.3mm	2.6mm.
Layer 60°	1	32mm.		
"	2	29mm.		
"	3	28mm.		
	Ave.	29.7mm.	5.0mm.	2.8mm.

a. Standard deviation computed according to equation (6), assuming initial band width equals 2.0mm.

b. Standard deviation based on equations (9) through (11), assuming standard deviation of uniform medium equals 2.5mm. at 480mm. from injection point.
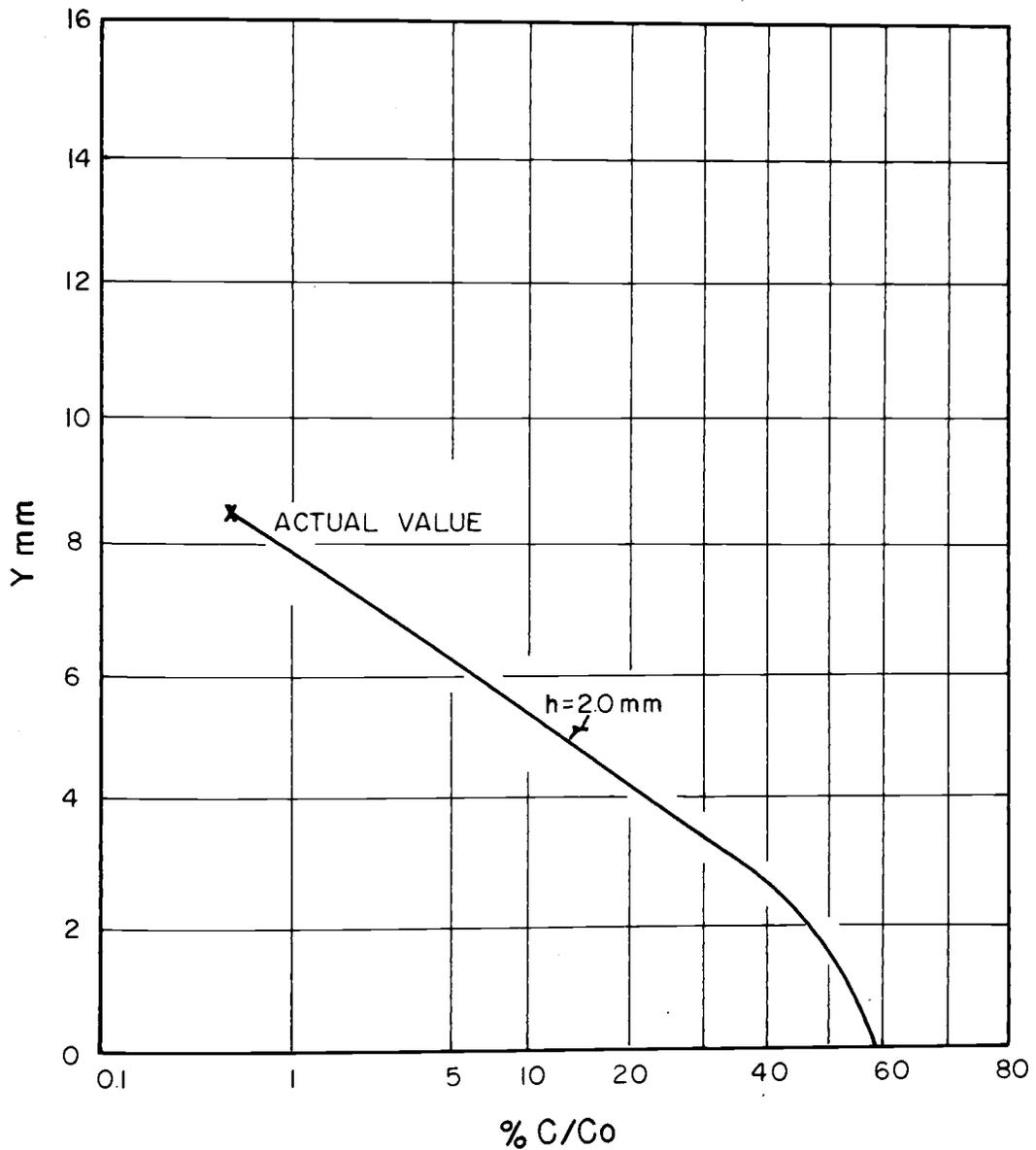
31

Figure 10. Plot on Arithmetic Probability Paper of
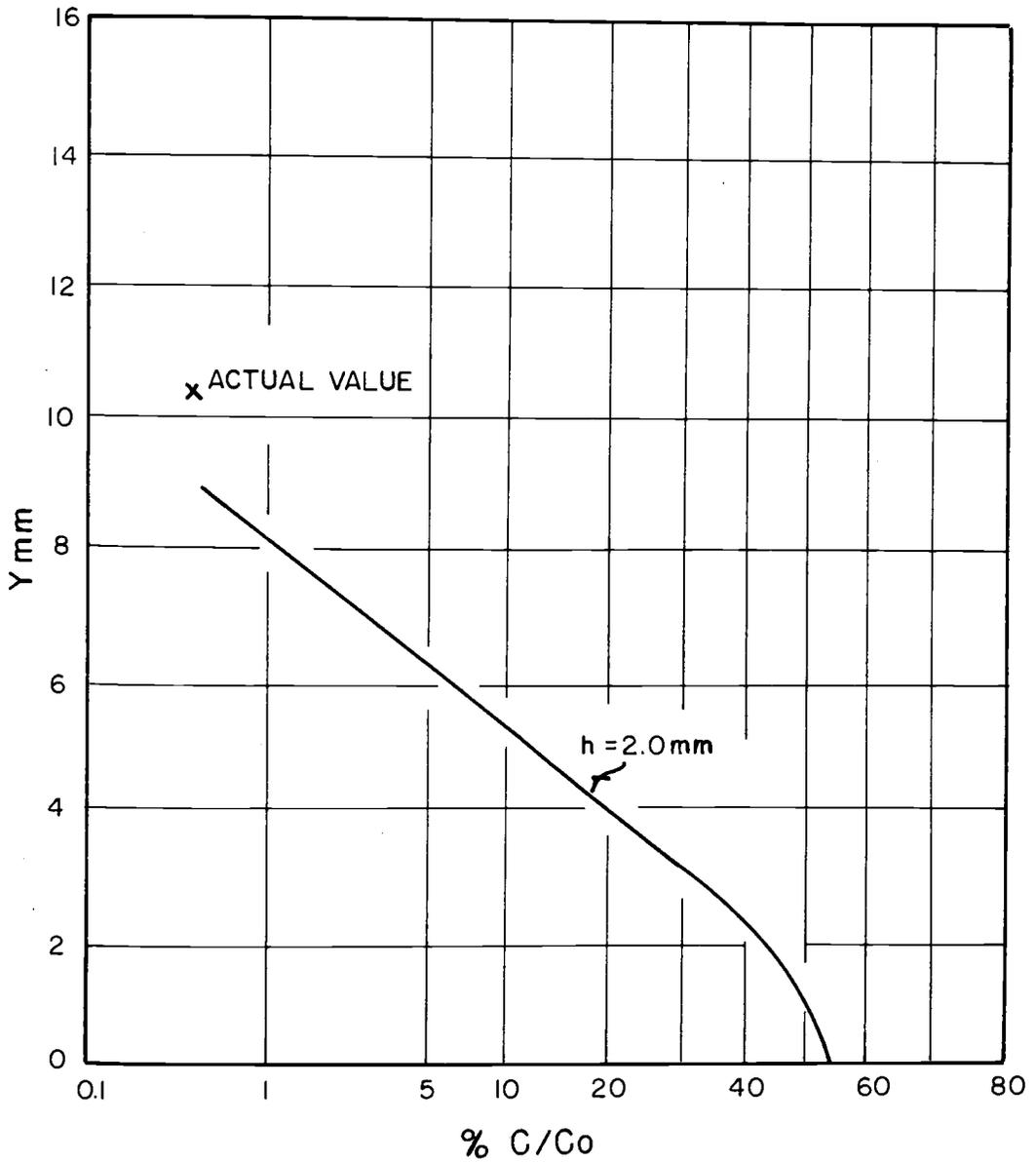C/C_o versus y – Uniform Medium

Figure 11. Plot on Arithmetic Probability Paper of
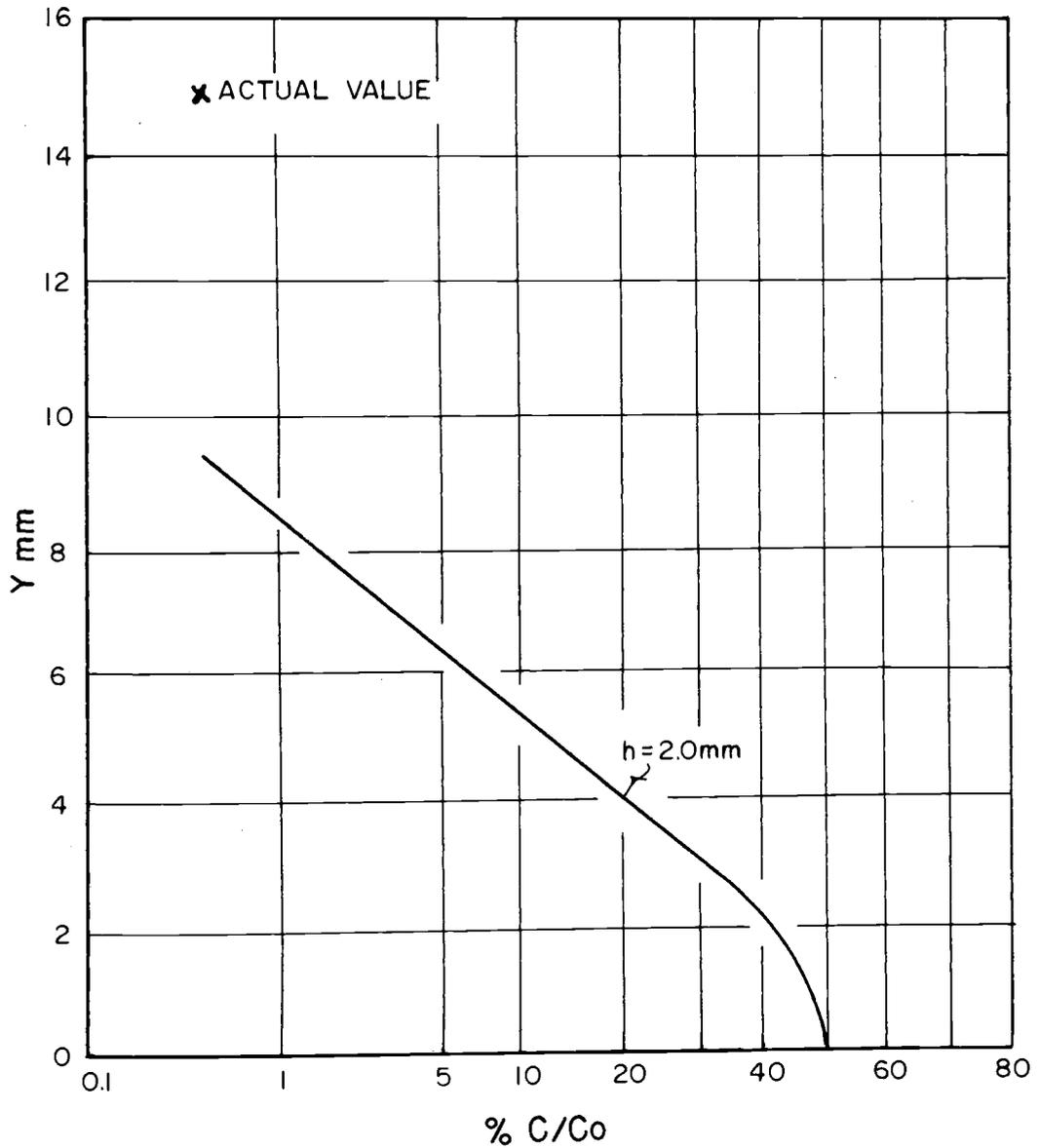C/C_o versus y – layer at right angles

Figure 12. Plot on Arithmetic Probability Paper of
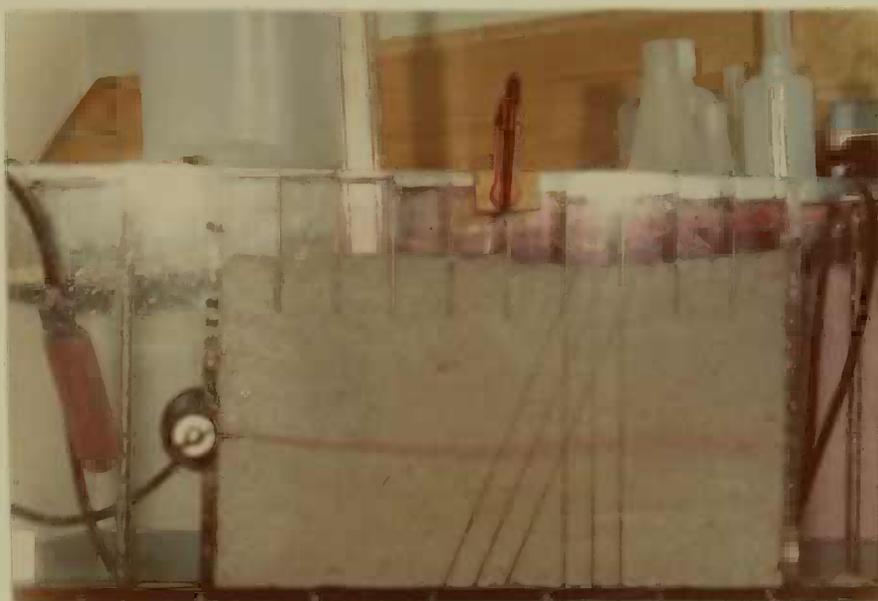C/C_o versus y - layer at 60 degrees

Figure 13. Tracer in the Uniform Medium

Figure 14. Tracer Band with
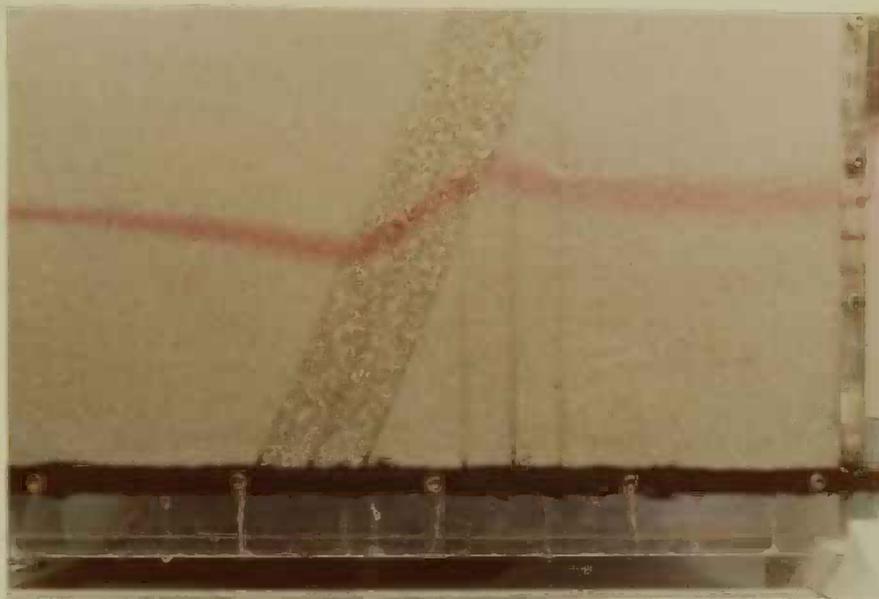Coarse Layer at Right Angles

Figure 15.   Tracer Band with
Coarse Layer at 60 Degree Angle

a. Grooves which were cut into the plexiglas sides of the model to guide the metal plates into position allowing layers of contrasting permeability to be interposed into the model tended to act as a "short circuit" to the flow lines. The effect was not constant throughout the three experimental conditions explored; it was most pronounced when the interposed layer was at an angle of 60 degrees to the average flow paths in the model. In the uniform medium and when the coarse layer was at a 90 degree angle, the flow paths were at right angles to the grooves, and less disturbance of the fluid flow by the grooves was observed. It was impossible to determine exactly what percentage of the dispersion observed was caused by the grooves, but its magnitude was small compared to the total mixing observed.

b. The effects of the boundary conditions were investigated using an electric analog model to get a qualitative picture of the conditions which existed. The results obtained from this method are presented in Figures 16, 17 and 18.

An electrically conductive Teledeltos paper was utilized and differences in permeability were simulated by punching holes in regions of low permeability and leaving untreated the regions where high permeability was desired. The flow diagrams which resulted showed that in the uniform medium and when the permeable layer was at a 90 degree angle the flow pattern was quite simple. When the highly permeable layer was at an angle of 60 degrees, the flow lines became more complex, but in the central region
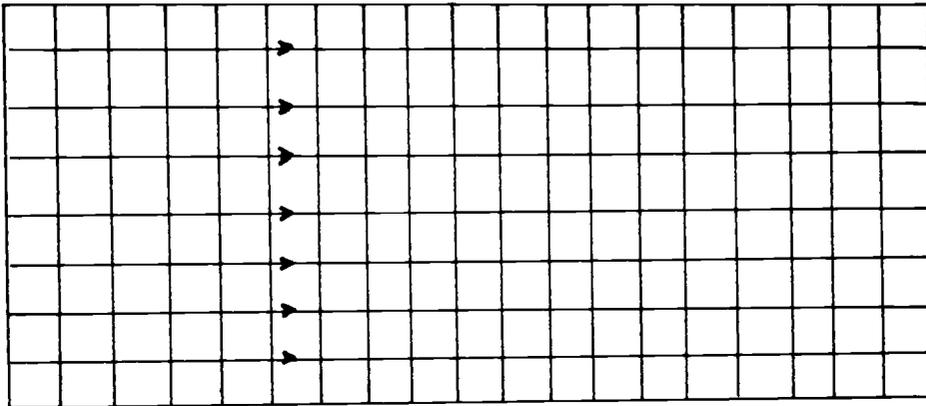
38

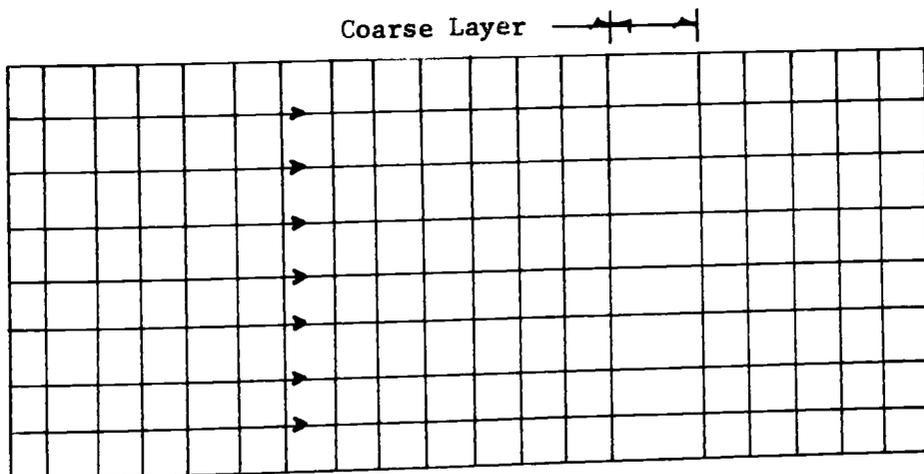Figure 16. Indicated Flow Paths - Uniform Medium

Coarse Layer

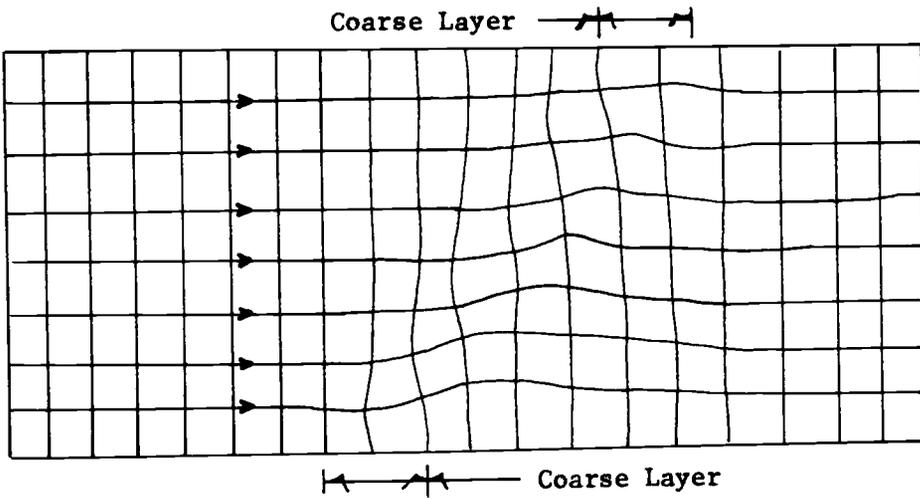Figure 17. Indicated Flow Paths - Layer at Right Angles

Figure 18. Indicated Flow Paths – Layer at 60 Degree Angle

of the model where the tracer stream was located, it was observed

that the refraction was similar to what would be expected accord-

ing to the equation by Hubbert (1940):

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{P_1}{P_2} \tag{12}$$

where:

θ_1 = angle between incident streamline and a normal

to the interface

P_1 = permeability in medium 1

θ_2 and P_2 = similar quantities in medium 2

See Figures 19 and 20.

The flow pattern which resulted along the bottom and top

boundaries also presented no difficulties. In both cases, the

boundary caused the flow lines to straighten out; i.e., as the

boundaries were approached the angle of refraction became less.

The tracer stream was placed in the region of maximum angle of

refraction and the boundary conditions were such that, if anything,

they tended to reduce the angle of refraction and reduce the dis-

turbance caused by the more permeable layer. Thus it can be said

that the boundary conditions on the top and bottom of the model

tended to reduce the effect of differences in permeability and

to make the flow lines straighter than if the aquifer were

infinite in these directions.

The boundary conditions at the upstream and downstream

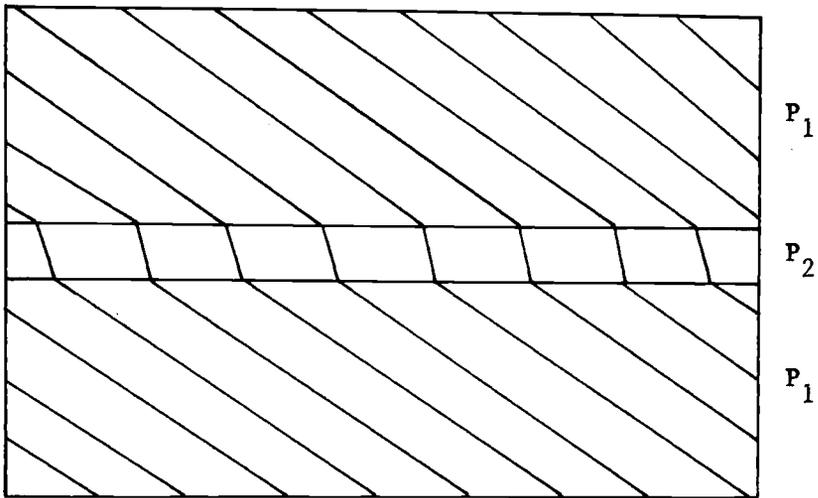ends of the model may have had some influence on the flow paths

Figure 19.   Refraction at Boundary
P_2 less than P_1
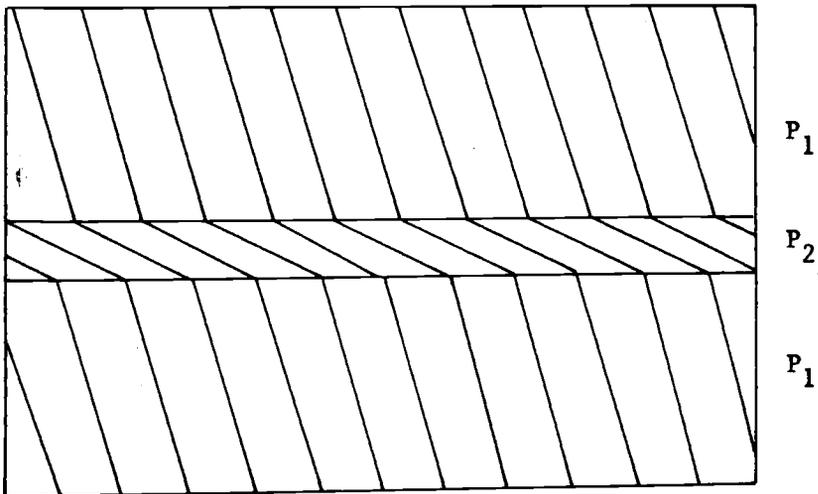

Figure 20.   Refraction at Boundary
P_1 less than P_2

because structural strength was required to hold the sand in place. Thus the flow paths were squeezed into the small openings in the upstream and downstream regions of the sand media. To minimize this effect, measurements were made approximately 2.5 cm before the end of the porous matrix rather than at the boundary between the media and the downstream constant head tank. This seemed to be a satisfactory solution since there was no noticeable effect until 1.0 cm before the boundary.

 c. Non-homogenities such as differences in grain size and shape, variations in permeability with time, and the inability to reproduce exactly the same conditions each time the model was filled with sand could have affected the results of the experiment. It was noted that compaction of the porous media with time was a minor problem with only a 1% decrease in permeability observed among experimental runs of the same type.

 2. The possible measurement errors consisted of two different types. They were: (a) reliability of the color change as an indicator, and (b) parallax or other errors in reading the scale.

 a. The reliability of the color change as an indicator rested in part upon how many persons took the data for a particular series of experiments because personal judgment must be used when determining the location of the color change. Between the initial concentration C_o and the concentration C where the indicator became clear, there was a gradual loss of color

intensity of the indicator probably due to its dilution making the exact location of the change to zero color difficult to spot. Resulting errors, mainly systematic, would affect all measurements to the same degree.

The water used in making all the solutions came from a tap connected to the University water supply. Its pH and chemical composition may have varied slightly from day to day, thus the ratio C/C_o at the color change may actually have varied from the value of 0.0044 which was initially measured. This effect should be quite minute since the variation of pH of the tap water was very small compared to the pH difference between the tracer and bulk solutions.

b. The thickness of the plexiglas sides could affect the apparent location of the change because of parallax and also possibly affect the visibility of color change since the original calibration was carried out in a glass beaker.

Discussion of Mathematical Model

After taking possible sources of error into consideration, the writer does not feel that they adequately account for the difference between experimental results and theory. This leads to the conclusion that the theory is incorrect. The theory may be incorrect either in terms of the mathematical model used to calculate the dispersion as given in the "Outline of Theory," or in the assumption that the interface played no role in the dispersion, or both.

It is recognized that the mathematical model is not rigorous, but there is reason to believe it is sufficiently accurate for the present need. The mathematical model rests on the basic assumption that the standard deviation of transverse dispersion is proportional to the square root of the product of average grain diameter and average flow distance. It is further assumed that other factors such as grain shape and packing may be considered to have equal effect in the two different grain sizes used. For the flow through the uniform medium, with grain diameter constant, this relationship was shown to be essentially valid by the measurements of σ made at various distances from the point of tracer injection (Figure 1).

For the measurements of flow through the layered media, a change in the dispersion was assumed to be caused by a change in average grain diameter and in the change of flow distance. With the layer at right angles to the flow direction, there was no change in flow distance. When the layer was at an angle of 60 degrees to the flow direction, there was a change in average flow distance resulting from refraction of flow lines. The distance used in calculating the various values of x was in all instances measured along the centerline of the tracer band. This was straightforward in the uniform medium and when the layer was at 90 degrees. When the angle was at 60 degrees, because the dye stream entered the layer at an angle, the dye stream was not symetrical about its centerline. This complication was not considered in the calculations and may introduce some error into the results.

The assumption made that it was possible to neglect the effect of possible differences in grain shape and packing between layers of different grain size is defensible since both sand sizes came from the same source, being merely two different size fractions.

The Role of the Interface

We now face the problem of the role of the interface as such. In other words, does something occur at the interface which results in a change in dispersion over and above that which can be accounted for by grain size and flow distance?

The role of the interface can be explained in two ways: (1) pore size variations great enough to cause intertwining, or (2) refraction of the flow lines at the interface resulting in loss of resolution.

1. Pore size variations at the interface could cause mixing because at the interface there are locations where a large pore opening connects onto several small pores or several small pores open into a large pore. Under either circumstance there is an increased possibility for flow lines to become intertwined and for an increase in dispersion to occur.

2. When a flow tube of heat, light or sound is refracted, the resolution of the flow tube is reduced. Sound waves when refracted generate many secondary waves and the character of the refracted waves becomes less distinct. It was felt that this same type of effect could be possible in incompressible fluid flow.

Intertwining could be introduced by refraction of flow lines or it could be introduced by pore size variation. When a layer is at right angles to the flow field, intertwining can be caused by pore size variations only; while if the layer is at an oblique angle to the flow, both mechanisms are possible.

General Remarks

Placement of a layer of highly permeable material at right angles to the flow lines had but a small effect upon transverse dispersion in the model. This resulted from the fact that the flow lines were only slightly more disturbed than they would be in a homogeneous medium. As flow lines crossed the boundary they were not bent since the law of refraction indicates that when water flows toward a boundary at right angles it will leave the boundary at right angles. Thus, all that happened was an increase or decrease in resistance to flow as the tracer flowed across the boundaries. This was noted in a drop in head loss per unit length when moving across the boundary, without change in the quantity of flow. In the equation Q = PIA, Q and A remained constant, with any changes in P compensated for by changes in I. The flow pattern was slightly affected by this change, dependent only upon the homogenity of the various layers, but there was no change in flow velocity because the conservation of mass dictates that the quantity of water flowing normally to the boundary must be the same on both sides of the boundary. Thus the flow of the fluid was not greatly disturbed.

Where the flow lines were at a 60 degree angle to the inter-
face between the zones of different permeability, the law of refrac-
tion predicted that the flow lines would bend. In addition, as a
result of the fact that the conservation of mass law must be ful-
filled, there was a change in fluid velocity at the boundary. There
was less resistance to flow in the region of high permeability. In
the coarse layer the width of the flow tube decreased, reflecting the
increase in fluid velocity. Skibitzke and Robinson (1963) indicates
that in non-uniform media, intertwining of flow paths ensues. The
observed results indicate that a larger increase in transverse
dispersion occurred.

CONCLUSIONS

1. The observed increases in transverse dispersion over what is predicted by the method outlined by equations (9) to (11) are 27% when a coarse layer is at right angles to the flow paths and 79% when a coarse layer is at a 60 degree angle to the flow paths. Thus it may be concluded that in transverse dispersion in a layered media that the permeability contrast between the various layers does not account for all of the increased dispersion.

2. In view of the work by Skibitzke (1964), the most logical place to look to determine what would cause an increase in transverse dispersion are mechanisms which exist only at the interface. Thus when predicting the amount of transverse dispersion which will occur in a layered media, the magnitude of the permeability contrast and the orientation of the interfaces must be considered.

3. The mechanisms which may take place at an interface are mixing, refraction and channeling of the flow paths. Mixing of flow paths will be independent of the orientation of the layer, while refraction and channeling of the flow paths will be dependent upon the orientation of the layer.

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