SATURATED FLOW OF WATER THROUGH CLAY LOAM SUBSOIL MATERIAL OF THE BROLLIAR AND SPRINGERVILLE SOIL SERIES

by

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ABSTRACT

The water transmission characteristics of subsoil material from the Brolliar and Springerville soil series of northern Arizona were determined. The flow behavior of the soils was ascertained using a system that measured the hydraulic head difference across a soil sample when known flow rates through the soil sample were being produced. Current theories for describing non-Darcy behavior together with quantitative expressions were used to describe and offer possible explanations for the hydrologic behavior of the soil materials. Extremely low permeability rates were found for both soils at all porosities and hydraulic gradients. The flow-hydraulic gradient relationship was found to be non-linear. Non-linearity was more pronounced at very low gradients for both soils. The Springerville soil materials with higher clay and salt content exhibited the least departure from linearity. As porosity increased, flow behavior became more Darcy-like for both soils. The flow behavior of these two major soil series have important implications to management of the watersheds where these soils are found.

INTRODUCTION

The northern central part of Arizona, above the Mogollon Rim, supplies a vital source of water to the state from the relatively large amount of precipitation that falls in the area. The climate is semi-arid and continental with cold winters and warm summers at the higher elevations and mild weather year around in the lower areas. The annual precipitation is 18-27 inches of which 40-45% is snow. The topography of the area is characterized by gentle slopes which have permitted moderately deep weathering and soil profile development.

The major soils of the area are derived from basalt parent material. The Springerville and Brolliar soils are typical of the soils in the region. The soils of the Springerville series are characteristically vertisols developed from the weathering basalt and cinders. The Brolliar soil series is characterized by deep to moderately deep profiles weathered from porous basalts. Both series cover extensive areas above the Mogollon Rim. The clay fraction of these soils is almost entirely montmorillonite which causes pronounced shrinking and swelling. Upon wetting, the subsoils swell and become exceedingly tight and very slowly permeable. This condition greatly affects the hydrologic behavior of the watershed areas where these soils are found. It has been suggested that if the soils were not of a shrinking nature the flow characteristics of the streams in these areas would be altered considerably.

The controlling influence on the regional hydrology of basaltic subsoil material has been surmised only in general qualitative terms. Little or no quantitative data are available on the water transmission characteristics of these important soil materials.

Objectives

The study had two major objectives:

1. To determine the water transmission characteristics of subsoil material from two major soil series of northern Arizona.

2. To investigate quantitative expressions and present theories for describing the hydrologic behavior of these materials.

Scope

The study was made under controlled conditions in the laboratory on prepared subsoil material. The soil material was a heavy montmorillonic clay loam which had been collected from the Brolliar and Springerville soil series by methods described in A Study to Determine the Hydrologic and Physical Properties of Some Beaver Creek Soils (Watershed Management Dept. 1968). Flow characteristics of the soil materials were determined with a system that measured the hydraulic head difference across a soil sample when known flow rates through the soil sample were being produced. A range of bulk densities, which are normally found under field conditions, was used in the study to determine the effect porosity would have on the flow characteristics of the soil material.

LITERATURE REVIEW

The purpose of the literature review has been to cover those articles which have a direct bearing on the study rather than to cover all aspects of saturated flow.

The classic equation of water movement set forth by Henry Darcy (1856) has occupied a unique place in the study of fluid flow through porous media. Darcy's flow may be written as

$$v = Ki \tag{1}$$

where v is the macroscopic flow velocity, K is the hydraulic conductivity and i is the hydraulic gradient. Following Richards (1952), K itself is written as

$$K = kgp/\eta \tag{2}$$

where k is the intrinsic permeability, η is the fluid viscosity, p is the fluid density and g is the acceleration of gravity.

The proportionality between flow velocity and hydraulic gradient [equation (1)] is the condition of linearity which combines with the law of conservation of mass to yield the Lapace heat-type or diffusion type equations. This enables conventional methods of mathematical physics to be used for describing flow through porous media.

Darcy recognized that his relationship was not valid for high fluid velocities. During the past forty years, much research has been

concerned with the nature of this deviation, which occurs at large hydraulic gradients (Muskat 1946, Scheidegger 1960). It seems well established that when the hydraulic gradient exceeds a critical value, the flow velocity is no longer proportional to the hydraulic gradient but increases less rapidly than the gradient. This could be accounted for since viscous forces no longer mask the inertia and turbulence forces and not all of the driving force of the hydraulic gradient is used to overcome viscous resistance.

Compared with the extensive study conducted on deviations at large gradients, much less work has been directed toward the testing of Darcy's law for liquid flow at low gradients, even in soils where such flow is common. Hubbert (1940) indicated that there was no apparent reason to suspect failure at low gradients. However, Scheidegger (1960) recognized the possibility of deviations arising from the so-called "boundary effect," from ions in solution and from non-Newtonian fluids.

Until recently, Darcy's law was believed to be valid over the complete range of hydraulic gradients. During the fifties, the reports of Lutz and Kemper (1959) and Schmid (1957) suggested that Darcy's law is not valid at low hydraulic gradients. Instead of velocity being directly proportional to hydraulic gradient i, a non-linear relationship of the kind shown in Figure 1 was found at the low hydraulic gradients. A feature of these reports was that there exists a magnitude of the hydraulic gradient i_c, referred to as the critical hydraulic gradient, such that the discharge velocity is zero unless the applied hydraulic gradient is greater than i_c.

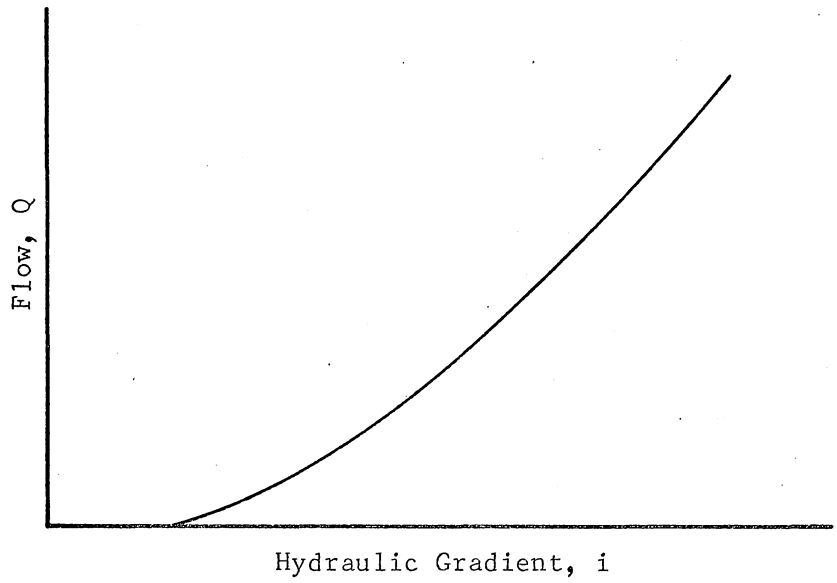

Figure 1. Non-linear relationship at low hydraulic gradients.

(after Kraft and Yaakobi 1966)

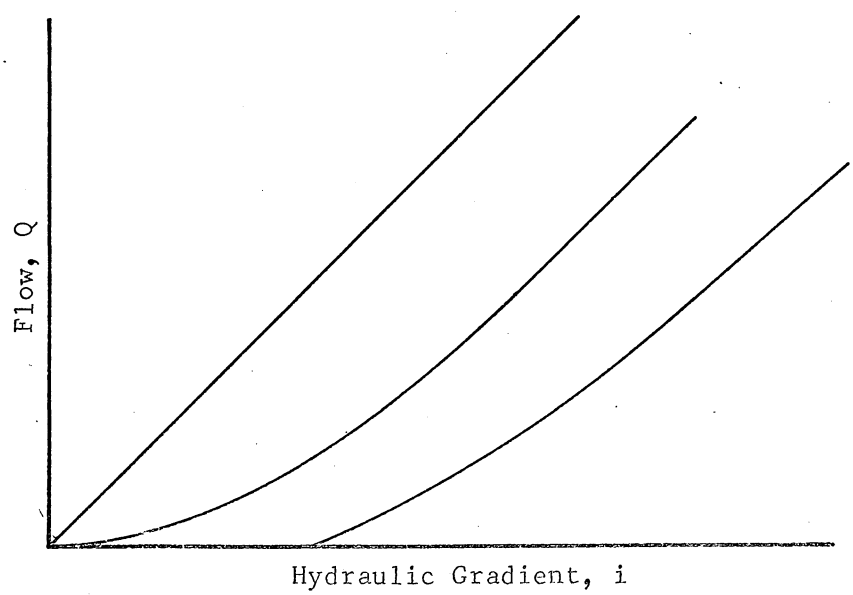

Figure 2. Discharge curves at low hydraulic gradients.

(after Kraft and Yaakobi 1966)

Mathematical work started before 1963 to incorporate non-Darcy behavior into the mathematical equations of fluid flow through a porous media (Gheorghitza 1963). An examination of the recent work concerning non-Darcy behavior revealed that its existence is not firmly supported by experiments. In addition, there is little agreement as to the theoretical explanation for non-Darcy phenomena.

All of this is seen in experiments which were directly aimed at investigating flow in clay soils at low hydraulic gradients (Hansbo 1960, Miller and Low 1963, Li 1963, and Olsen 1965). The type of discharge curve that each of the investigators found is shown in Figure 2. Besides the seemingly inconsistency of their findings, there are several aspects of these experiments which indicate the need for further improvement in experimental details and more clarification as to which physical quantities are to be measured.

Olsen (1966) reviewed the problems associated with the conventional measuring technique that is used in determining the hydraulic conductivity of saturated clays. He also described a new technique in which these problems were avoided. With the new technique, a known flow rate through the clay sample was produced while the head difference induced across the sample was measured. Known constant flow rates were produced by driving the plunger of a syringe at constant rates with a multispeed syringe pump. The applied flow rates and the measured head differences were not affected by the air-water meniscuses using the new technique and the time intervals needed to obtain head difference measurements were shortened. Data obtained with this new

technique showed the validity of Darcy's law in kaolinite over a wide range of porosities at low gradients.

The conventional laboratory technique for determining the hydraulic conductivity consists essentially of (1) placing a clay sample in a test cell connected in series with a capillary tube containing an air-water meniscus or an air bubble, (2) producing flow through the clay with a known hydraulic head difference across the test cell-capillary tube system, and (3) determining the induced flow rate from measurements of the rate at which the meniscus or air bubble moves through the capillary tube (Olsen 1966). The hydraulic conductivity (K) is calculated from the applied head difference (Δh), the induced flow rate (Q/t), length (l), and the cross-sectional area (A) of the sample, using Darcy's law

$$Q/t = KiA \tag{3}$$

where i is the hydraulic gradient and equals $\Delta h/l$.

The principal difficulties with this technique arise from contaminants present in the apparatus and from the long times and/or high gradients required to obtain measurable flow rates. Contamination gives rise to an error in the calculation of the head difference across the clay sample. Olsen (1965) concluded that atmospheric contamination in flow rate measuring capillary tubes can cause false deviations in the data of Hansbo (1960). Hansbo's data therefore appear to be evidence of Darcy behavior in confined samples of natural illitic clays that were tested at gradients ranging from 1-10.

Miller and Low (1963) observed deviations in confined samples of montmorillonite that occurred over both low and high ranges of gradients. The principal deviations appear at gradients in the range from 0 to 100. Olsen's (1965) study showed that only part of these deviations can be attributed to the atmospheric contamination error that accounted for Hansbo's deviations. This error results from inaccurate corrections for pressure differences across air-water meniscuses in contaminated capillary tubes. The contamination error was negligible in comparison with the observed deviations in the unconfined clay and sandstone samples of Lutz and Kemper (1959) and Von Englehardt and Tunn (1955). Another possible error that could be present in the test cell-capillary tube system is in the inaccurate flow rate measurements due to the movement of liquid around the air bubbles of the capillary tubes.

When hydraulic conductivities are determined using low applied gradients, periods of hours or days are often required to obtain measurable movements of meniscuses in the capillary tube. Precautions must be taken to avoid false movements of the meniscuses due to volume changes in the sample, the capillary tube, connecting channels, fittings, and tubing. The sample must be rigidly confined to avoid creeping with subsequent consolidation or expansion. Rigorous temperature control is needed to avoid expansion and contraction of the apparatus.

Some investigators have minimized these difficulties by employing relatively high gradients to obtain more rapid flow rates. Although this method shortens measurement time and reduces the temperature control requirement, it introduces another complication. Large seepage

forces may alter the arrangement of particles within a clay sample in ways that do not occur under the much smaller seepage forces associated with the usual low gradients in clay bearing sediments.

King (1898) reported for low gradients in various porous media that the flow velocity was not proportional to the gradient. Instead, the velocity increased more than proportionally with the gradient. This was in contrast to the less-than-proportional increases at high gradients of which King was well aware.

The flow media used by King included tube-confined brass-wire gauze discs, sand, sandstone, and ordinary glass capillaries. For all of these he reported greater-than-proportional increases for flow velocity versus gradient. His evidence for the capillary tubes is perhaps the least convincing. Not only were the deviations small, but they essentially disappeared when evaporation from the outflow water droplets was prevented. When King's results are expressed in terms of hydraulic gradient, Darcy's law, for all practical purposes, is verified even though King's detailed calculations indicated a departure. In much of King's data there is evidence that, at constant gradient and temperature, his systems were subject to flow reduction with time. As reported by Gupta and Swartzendruber (1962) this response might have been caused by bacterial activity.

Kraft and Yaakobi (1966) gave a theoretical deviation of Darcy's law. The derivation was a guide as to what physical quantities should be measured in experiments used to determine permeability.

Klausner and Kraft (1966) proposed a model for non-Darcy behavior that assumes an ordinary Newtonian fluid is imagined to be flowing in an ensemble of parallel capillaries of circular cross section with steady Poiseuille flow. The wall of each capillary exerts a force (Chapman-Gouy crystal force) on the fluid (Kryt 1952). This force F is perpendicular to the flow velocity which is along the tube axis and therefore cannot do work on the fluid. F is taken into account by assuming that it brings into action a second plastic type force f (wall force) acting along the tube axis and opposing the flow. With these assumptions it is shown mathematically that, depending on the porosity of the model and the wall force magnitude, the model's discharge curves can have any of the shapes shown in Figure 2 (Kraft and Yaakobi 1966). A group of discharge curves for a Chapman-Gouy type wall force is shown for varying porosity in Figure 3. The model shows a remarkable similarity to the intuitive description of non-Darcy flow given by Miller and Low (1963).

Kraft and Yaakobi (1966) suggested that when future permeability tests are made they cover a wide range of different porosities and clay types with varying Chapman-Gouy potentials.

Three types of flow behavior are demonstrated in Figures 4 and 5. Newtonian flow is shown as a straight line passing through the origin. This type of flow is present when the viscosity of the liquid is constant with respect to different hydraulic gradients. For non-Newtonian flow, the curve passes through the origin but is no longer

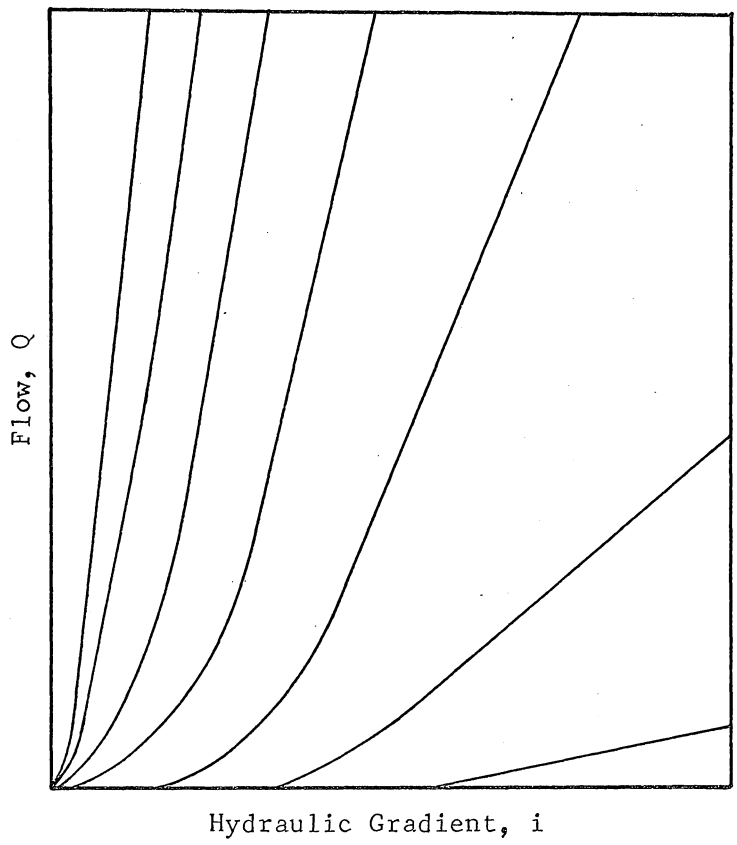

Figure 3. Discharge curves for Chapman-Gouy force.

(after Kraft and Yaakobi 1966)

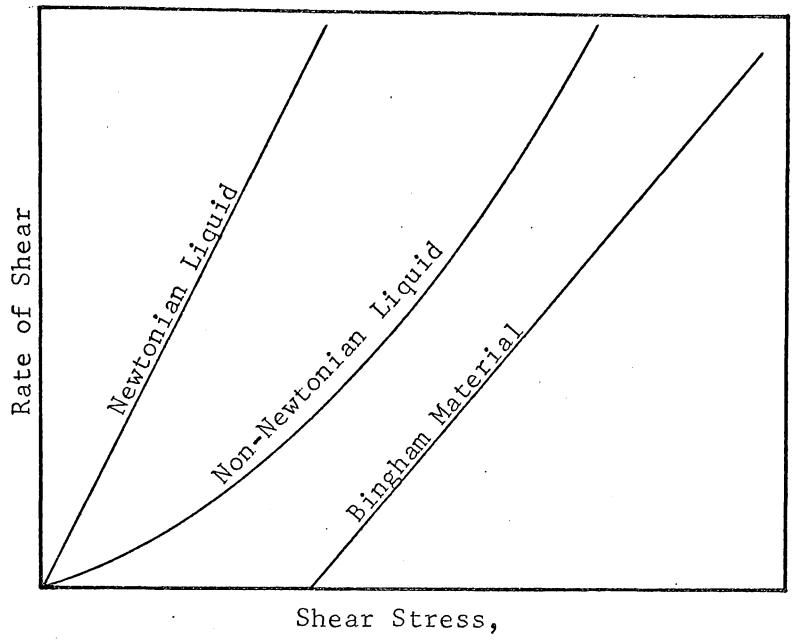

Figure 4. Shear rate versus shear stress.

(after Swartzendruber 1962a)

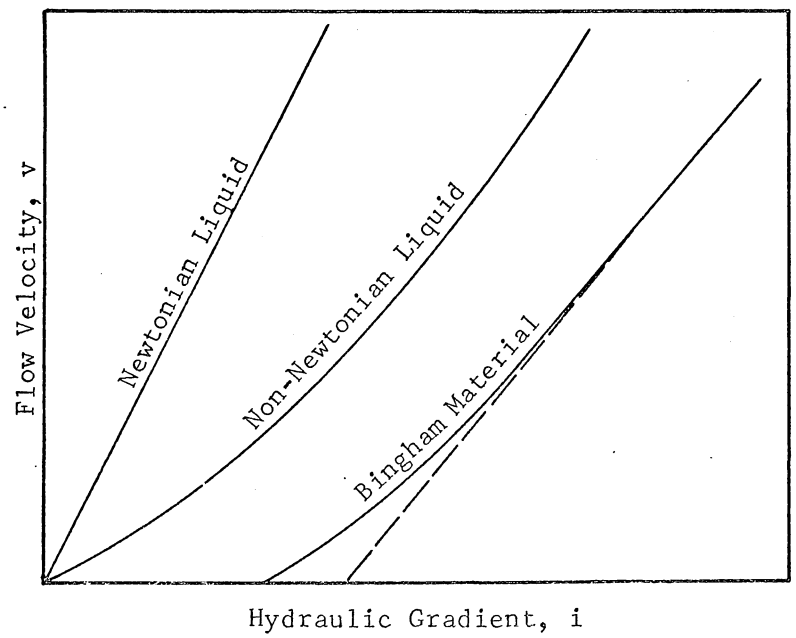

Figure 5. Flow velocity versus gradient.

(after Swartzendruber 1962a)

straight. For Bingham flow, the curve is linear but only for shear stress in excess of the yield stress.

The assertion of Darcy's law is analogous to the graph of the Newtonian liquid (Figure 5). If the flow is not Newtonian, it is then either non-Newtonian or Bingham. Von Englehardt and Tunn (1955) and Low (1961) suggested the possibility of non-Newtonian behavior but they did not propose a mathematical expression.

The straight lines produced by sand and sandstone in King's (1898) work are suggestive of the straight-line asymptote of the Bingham curve in Figure 5. These lines could conceivably bend through the origin if the necessary experimental points were available. The data of Von Englehardt and Tunn (1955), Lutz and Kemper (1959), and Hansbo (1960) indicate that such curvature does occur, which is suggestive of non-Newtonian flow (Figure 5). While recognizing that threshold gradients have been reported for ceramic and charcoal filters, Derjaguin and Krylou (1944) and Swartzendruber (1962a) felt that adequate experimental proof of such gradients for sandstones and clays is, as yet, lacking. It is then conjectured that the qualitative aspects of Hansbo's (1960) approach are, at present, the most reasonable (Swartzendruber 1962a). These are shown in Figure 6 by the solid-line curve which begins at the origin, bends upward and then approaches a straight line which extrapolates to an i-intercept designated by I. This choice of curve combines the origin and curvature aspects of non-Newtonian flow with the straight-line aspect of Bingham flow. By the gradient range in Figure 6, it is assumed that deviations due to inertia and turbulence do not appear.

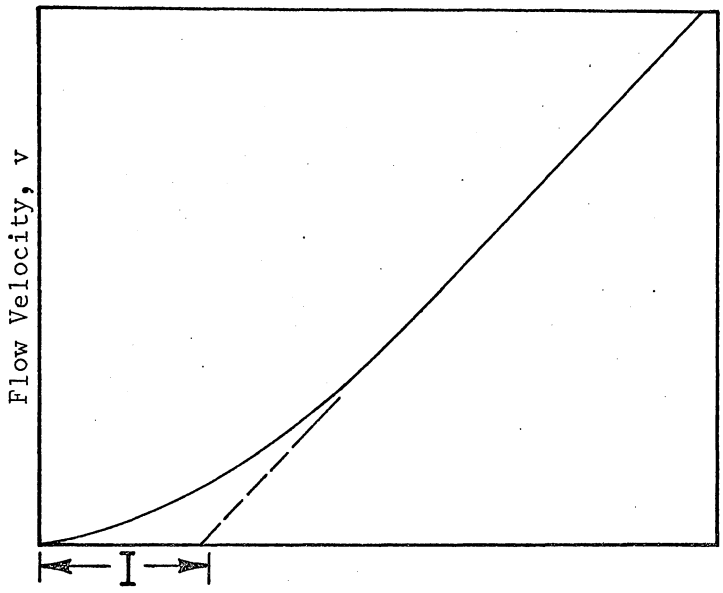

Figure 6. General aspects of the non-linear flow relationship.

(after Swartzendruber 1962a)

Swartzendruber (1962b) gives three reasons for reconsidering the question of the validity of Darcy's law. First, there is the data of Von Englehardt and Tunn (1955, Lutz and Kemper (1959), and Hansbo (1960) which do not obey Darcy's law, at least from the phenomenological standpoint represented by velocity-gradient curves. Secondly, there is increasing evidence that water properties near clay surfaces are not the same as those of normal bulk water (Low 1960, 1961). Finally, the work of King (1898), who considered such deviations more than sixty years ago, seems largely to have been ignored.

Hansbo suggested that the curved part of Figure 6 is represented mathematically by a power function and that this be replaced by a linear function when the curve has straightened out. Three independent parameters or constants are required with this approach. While the two mathematical forms are a complication, a worse drawback is that the three parameters cannot be evaluated unless data are available all the way from i = 0 out to and including an appreciable part of the linear portion of the velocity-gradient curve.

To circumvent these difficulties, Swartzendruber (1962a) proposed the relationship

$$dv/di = M(1-e^{-1/I}) \tag{4}$$

where M and I are constants, v is the macroscopic flow velocity, and i is the hydraulic gradient. Integrating and using v = 0 at i = 0 (for non-Newtonian flow) leads to

$$v = M[i - I(1 - e^{-i/I})] \tag{5}$$

which then is the modified flow equation that appears to fit the published data of several investigators (Von Englehardt and Tunn 1955, Lutz and Kemper 1959, Hansbo 1960, and Low 1960).

As i increases in equation (5), $e^{-i/I}$ approaches zero, with the result

$$v = M(i - I) \tag{6}$$

which is the equation of a straight line of slope M and i-intercept I.

Due to the similarity between M and K [equations (1) and (6)], M is partitioned in the manner of Richards (1952) by writing

$$M = mpq/\eta \tag{7}$$

where m is determined by particle geometry and η is the normal bulk viscosity of the liquid. In order to avoid confusion with Darcy's law, M = K could be called the hydraulic conductance and m = k, the permeance.

For the sandstones of Von Englehardt and Tunn (1955), equation (7) was used to calculate m-values. Swartzendruber values (1962a) always exceeded the liquid k_{max} values of Darcy's law as would be expected since Darcy's law presumes a straight line through the origin for v versus i. The m-values were always less than the air permeability since some particle rearrangement might be expected when the clay-bearing sandstone was wetted with water or salt solution. In every case, m increased with salt concentration. This could be expected since, for higher salt concentrations, the clay should be less subject to deflocculation and dispersion.

With one exception, I decreased as the salt concentration increased. This implied that pure water exhibited the greatest degree of non-Newtonian behavior since I is a measure of non-Newtonian behavior (Swartzendruber 1962a).

METHODS

The water transmission characteristics of confined montmorillonite clay subsoil material of the Springerville and Brolliar soil series were investigated. The soil materials were obtained from the 24-30 inch depth. A general description of the soils is given by Williams and Anderson (1967). Physical and chemical analyses of the soil materials were made as described in A Study to Determine the Hydrologic and Physical Properties of Some Beaver Creek Soils (Watershed Management Dept. 1968).

A description of the system used to measure the flow of water through saturated, confined clay soil is given in Figure 7. The apparatus consisted of a mercury manometer (17), pressure transducer (18), multispeed syringe pump, deairing flask (16), and confined soil core (7).

The manometer was used to measure the hydraulic pressure when a steady flow rate had been obtained. A pressure transducer with a strip chart recorder was used to determine when steady state conditions were reached. The transducer was a Sanborn model 267 BC which was operated with a Sanborn model 311A Transducer Amplifier-Indicator. The output was monitored with a Leeds and Northrup Company model W recorder.

A Harvard Apparatus Company model 600-901 syringe pump was used to produce fixed flow rates through the soil core (7) by driving the plunger of a hypodermic syringe at constant rates with the multispeed

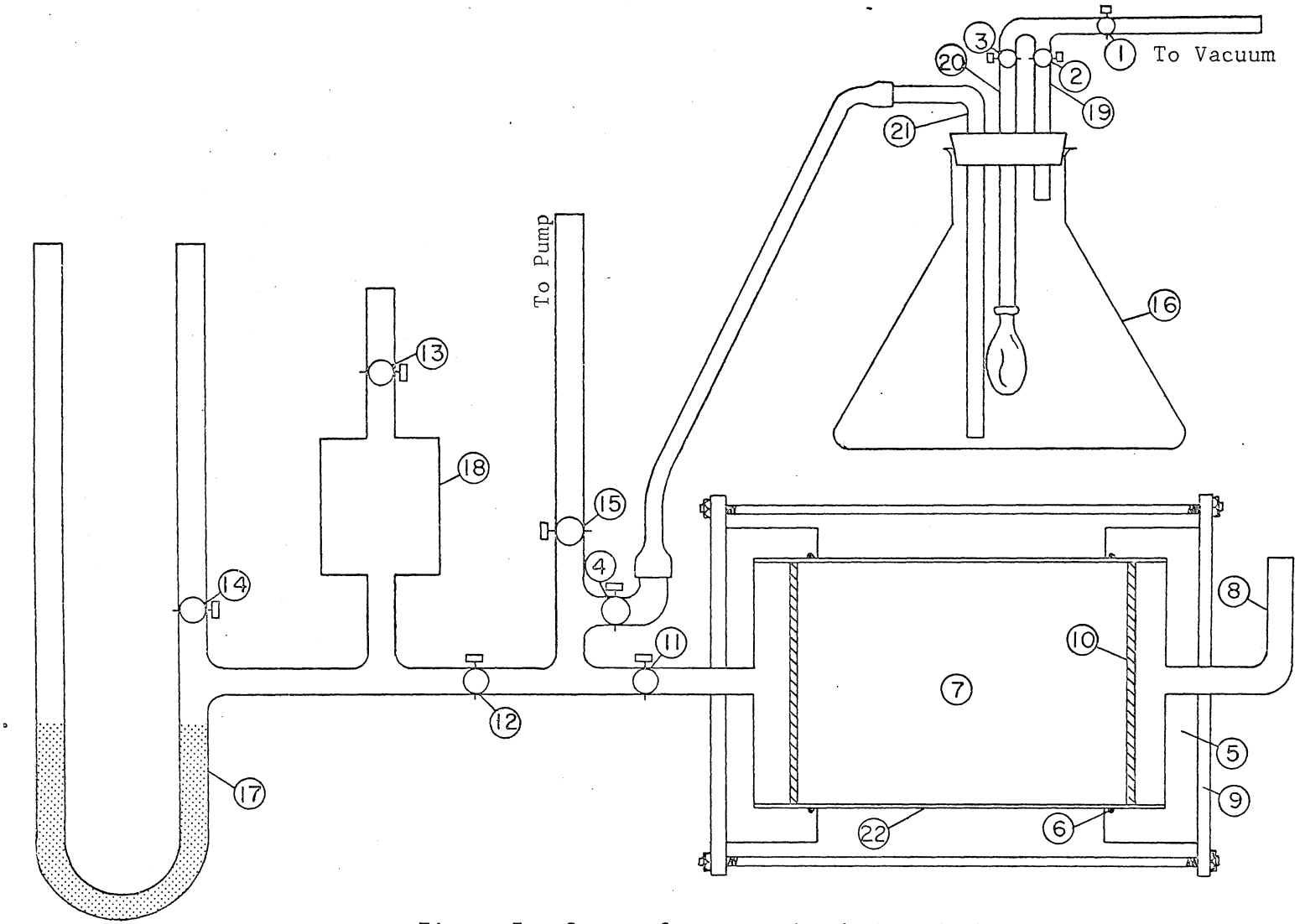

Figure 7. System for measuring hydraulic head.

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in the top of the flask. Tube (19), which was connected to the water source, was used for filling the flask. Tube (20) was connected to a rubber bladder within the flask, and tube (21) was connected to the outlet for the deaired water. Tubes (19) and (20) were connected to the vacuum pump by way of valve (1).

Water was deaired under a vacuum of 65 mm of mercury. To deair the water in the flask, the bladder was first evacuated and then valve (3) was closed. Valve (2) was then opened and a vacuum was applied and maintained until mo more bubbles issued from the water. Valve (2) was then closed. Valve (1) was kept open and valve (4) closed during these operations. If water was not to be taken from the flask within a short period of time, the bladder and the flask were re-evacuated to prevent air from dissolving in the water.

The amount of air in the deaired water was not determined. However, the fact that at the first evacuation air bubbled out profusely, while subsequent applications of vacuum created few or no bubbles, shows that the low air content in the water was maintained.

The flow assembly consisted of two aluminum caps (5) with "0" rings (6) to seal the brass sample ring (22). The brass ring was 3.0 cm long and 7.3 cm in diameter. Fritted glass bead plates (10) were used to insure even distribution of water across the core ends. One layer of coarse filter paper was placed between the porous plate and core ends to prevent soil particles from clogging the porous plates. The top of the outlet tube (8) was at the same height as the top of the brass ring to keep the soil completely saturated. The cell was

mounted in a channel aluminum frame (9) and clamped to maintain soil core volume.

In order to pack the soil in the cell, the brass ring was first installed in one of the aluminum caps. The air-dried soil was packed in the ring at as uniform a bulk density as possible. By knowing the weight of the soil placed in the brass ring and the volume of the ring, the bulk density of the soil core could be estimated. The porosity of the soil core was calculated by assuming a soil particle density of 2.72 g/cm^3.

The frame and cell were placed in a horizontal position to eliminate the effects of gravity on the flow of water through the cell. The soil was saturated and air was flushed from the cell by pumping de-aired water through the soil core and cell at the rate of 0.294×10^{-3} cm/min per unit cross section of the soil core. It took about 7 days to deair and saturate the soil core. The actual time depended on the bulk density of the core and the amount of entrapped air within the cell. When the pressure remained constant for 48 hours, the soil was assumed to be saturated and the swelling pressure constant.

Before starting a run, the distilled water was deaired and then allowed to come to room temperature. To transfer deaired water from the flask (16) to the pump, air was allowed to enter the bladder by opening valves (1) and (3). Valves (4) and (15) were then opened and (11) and (12) were closed. Valves (13) and (14) were open to allow air to escape from the transducer and manometer, respectively. They were closed when a run was in progress. Valve (4) was closed and valves

(11), (12), and (15) were open during a run. When the pressure for a certain flow rate had become constant, the pressure was noted and the next higher flow rate was started. The pressure for each flow rate was multiplied by a correction factor of 0.93 which took into account the pressure effect that the water had on the level of mercury in the manometer. The relationship

$$1 - \frac{\text{density of water}}{\text{density of mercury}} \tag{8}$$

was used to determine the correction factor. Four runs were made for each soil at each given bulk density. The hydraulic pressure was divided by the length of the soil core in order to determine the hydraulic gradient necessary to produce a fixed flow rate.

RESULTS AND ANALYSIS

The average hydraulic gradient for the fixed flow rates of the four runs for each soil at each porosity are given in Tables 1-A and 2-A of the Appendix. The data are plotted in Figures 8 through 19. The experimental points are shown in the figures as circles. The solid-curve line is the data fitted to equation (5). Two features of the relationship of flow to hydraulic gradient are evident in these figures: (1) the extremely low permeability rates for both soils at all porosities and hydraulic gradients, and (2) the non-linearity of the flow-hydraulic gradient relationship.

Although both soils have low permeability, the Brolliar soil material at the higher porosity was considerably more permeable than the Springerville material. As porosity decreased, the difference in permeability between the two soils decreased. The low permeabilities found in these materials have important implications to watershed management. For example, a soil column three feet thick with a porosity of 49.6%, which had a one-half inch of water maintained on its surface, such as might occur during snow melt, would transmit only about 4.4 gallons of water per day per acre of subsoil material. Only about 7.1 gallons of water per day per acre would be transmitted by the more permeable Brolliar subsoil material. This points up the controlling influence that the low permeability of these subsoils play in the hydrologic behavior of watersheds where they are located. The contribution to

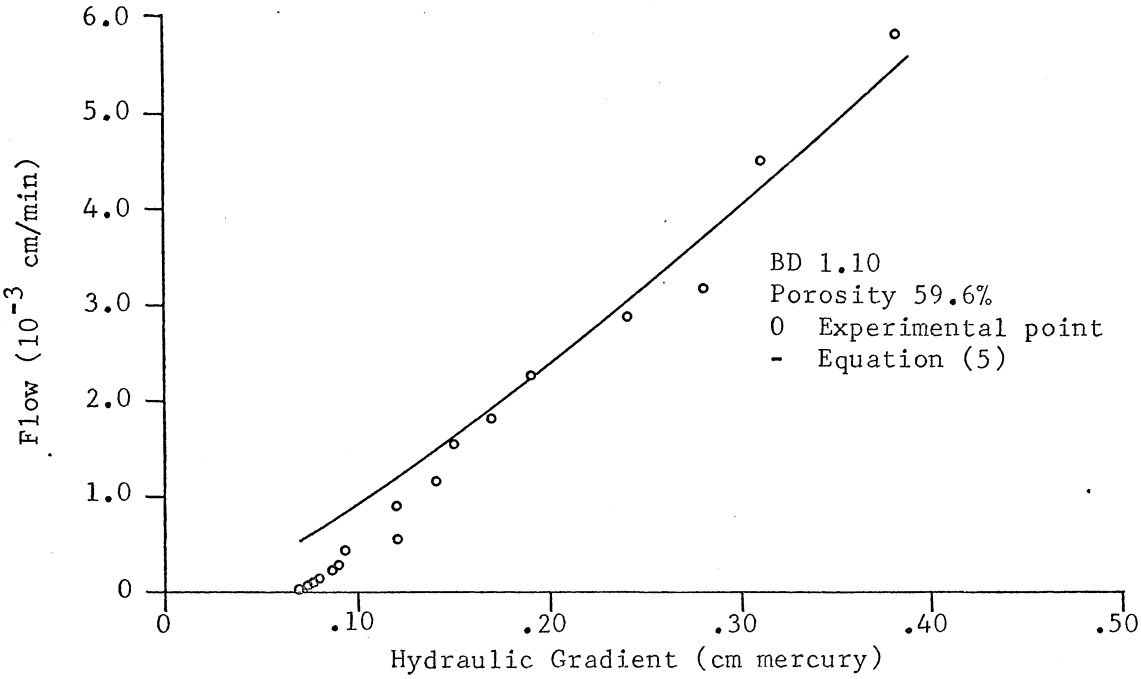

Figure 8. Flow rate versus hydraulic gradient for Springerville soil (BD 1.10).

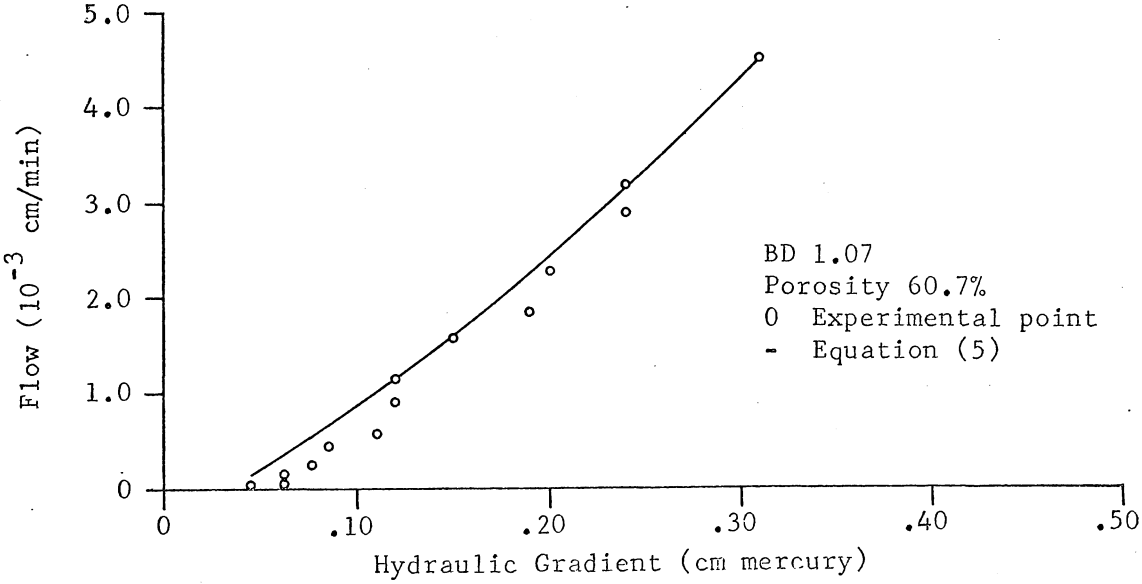

Figure 9. Flow rate versus hydraulic gradient for Brolliar soil (BD 1.07).

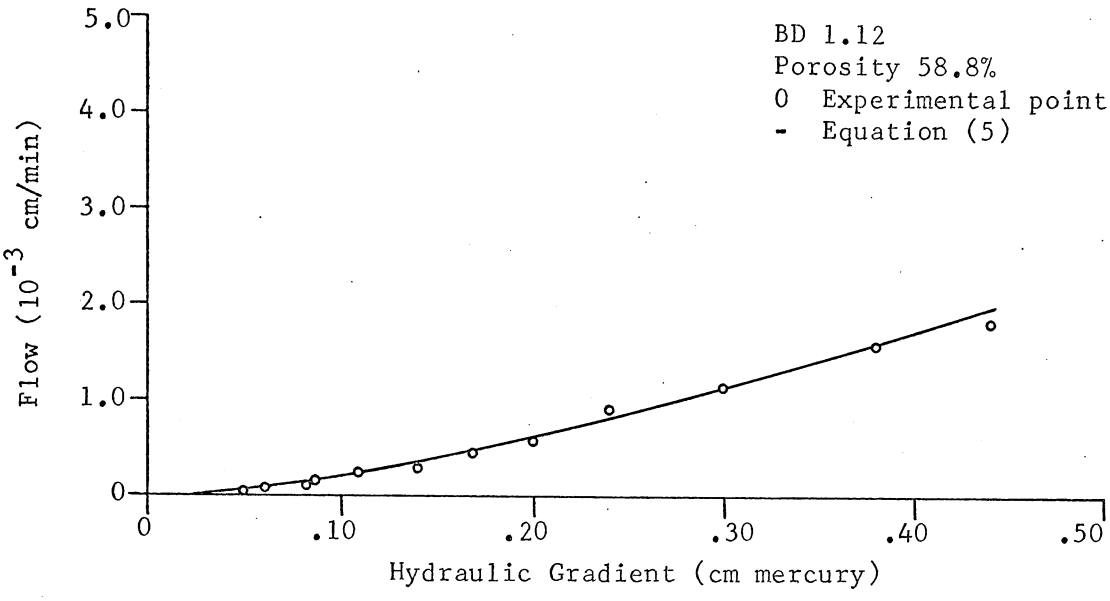

Figure 10. Flow rate versus hydraulic gradient for Brolliar soil (BD 1.12)

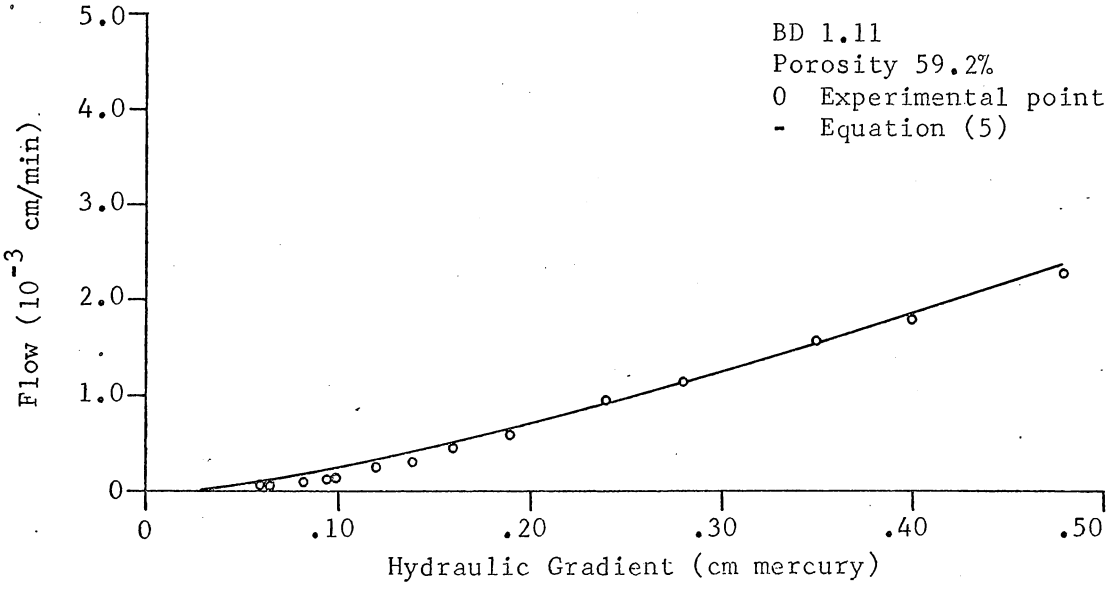

Figure 11. Flow rate versus hydraulic gradient for Brolliar soil (BD 1.11)

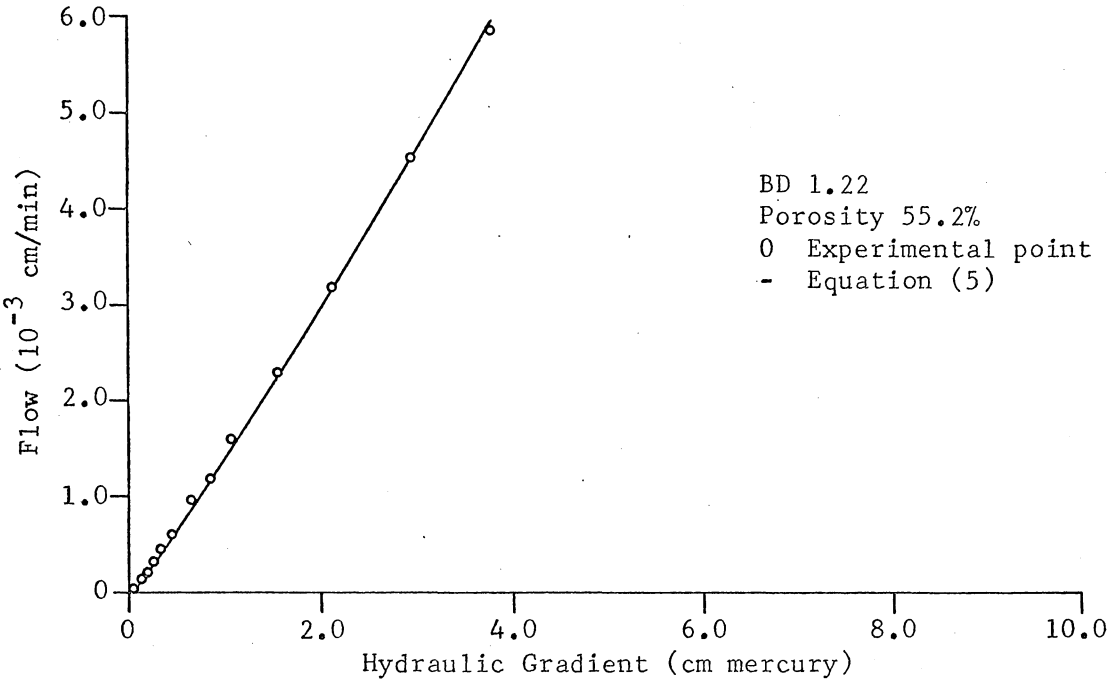

Figure 12. Flow rate versus hydraulic gradient for Springerville soil (BD 1.22).

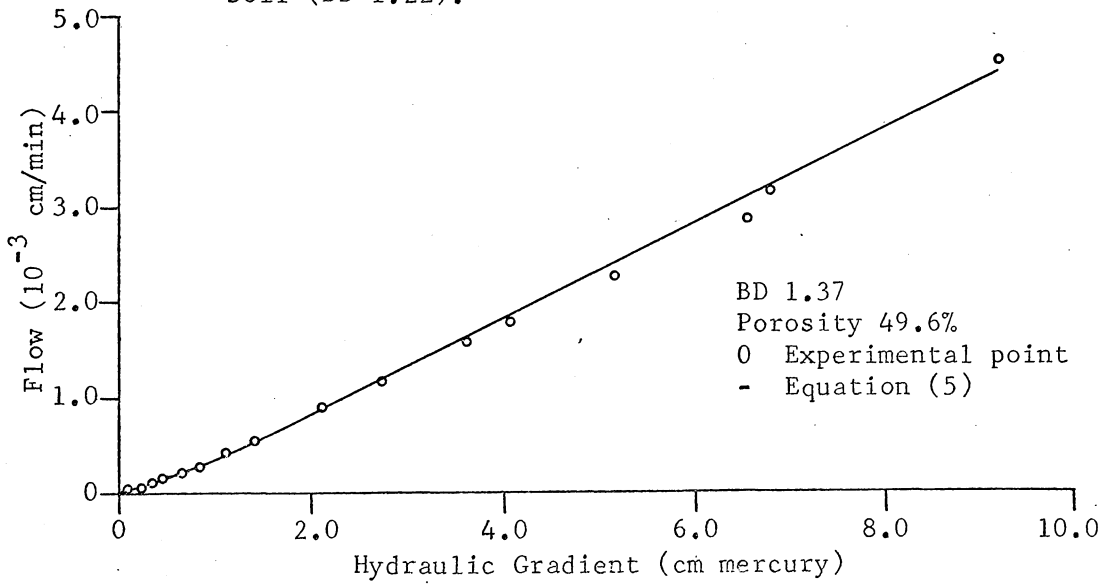

Figure 13. Flow rate versus hydraulic gradient for Brolliar soil (BD 1.37).

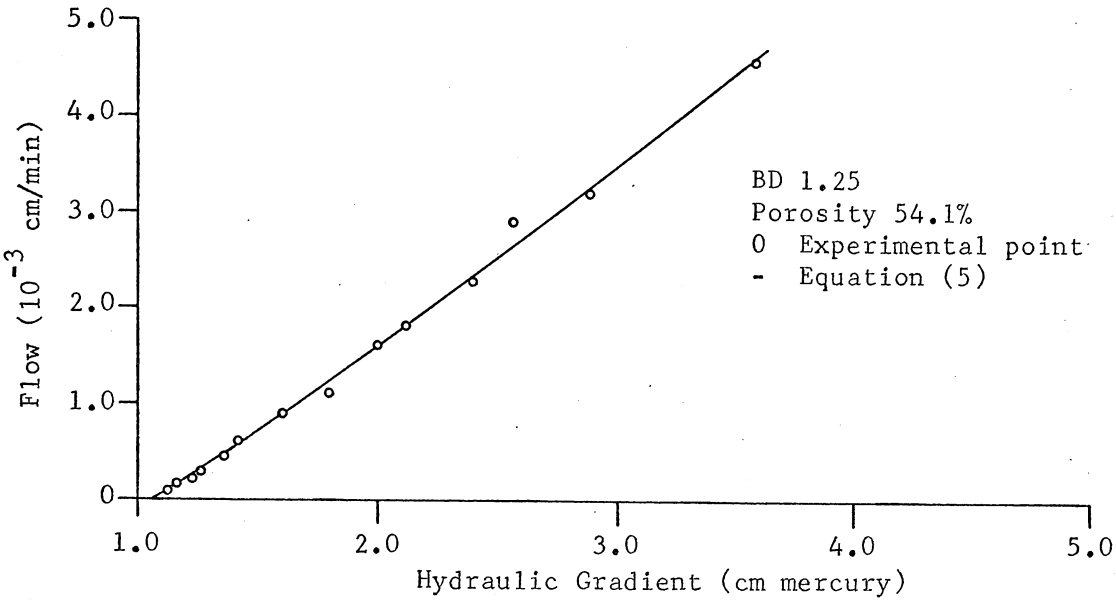

Figure 14. Flow rate versus hydraulic gradient for Brolliar soil (BD 1.25).

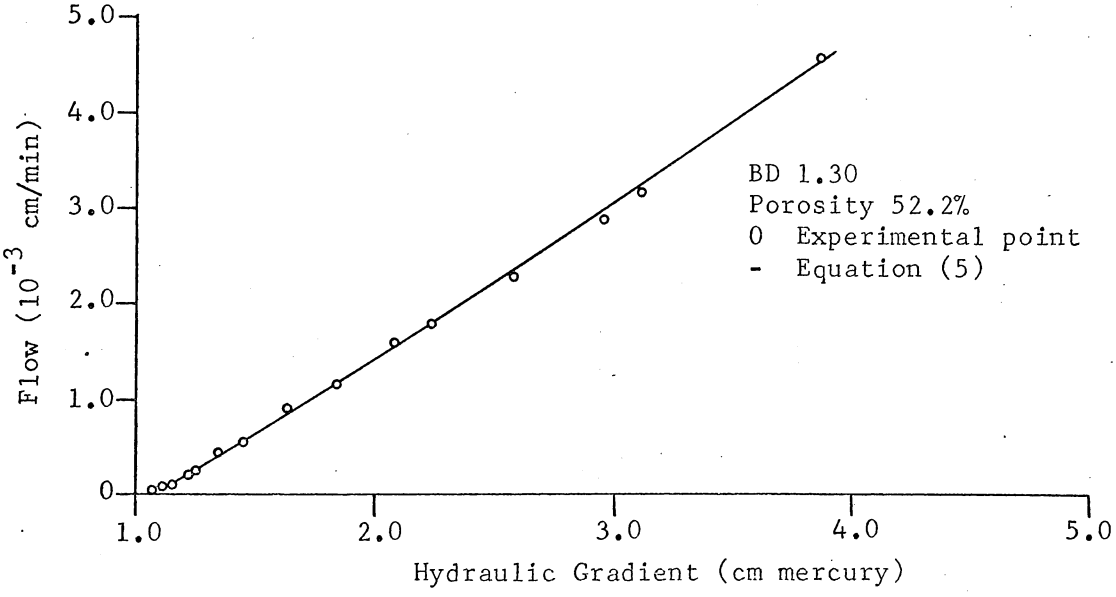

Figure 15. Flow rate versus hydraulic gradient for Springerville soil (BD 1.30).

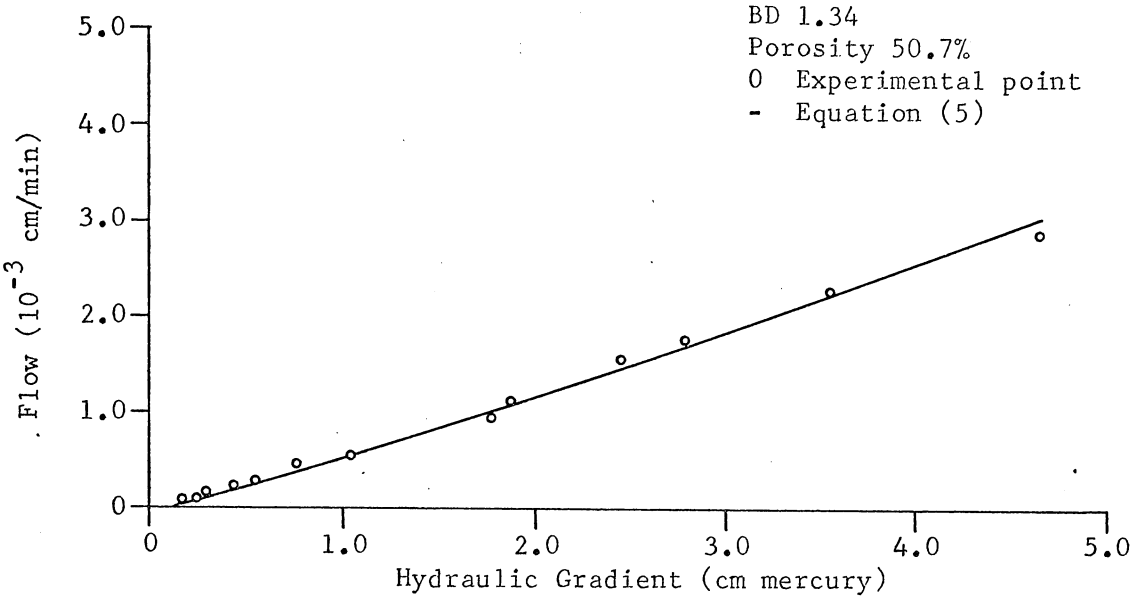

Figure 16. Flow rate versus hydraulic gradient for Springerville soil (BD 1.34)

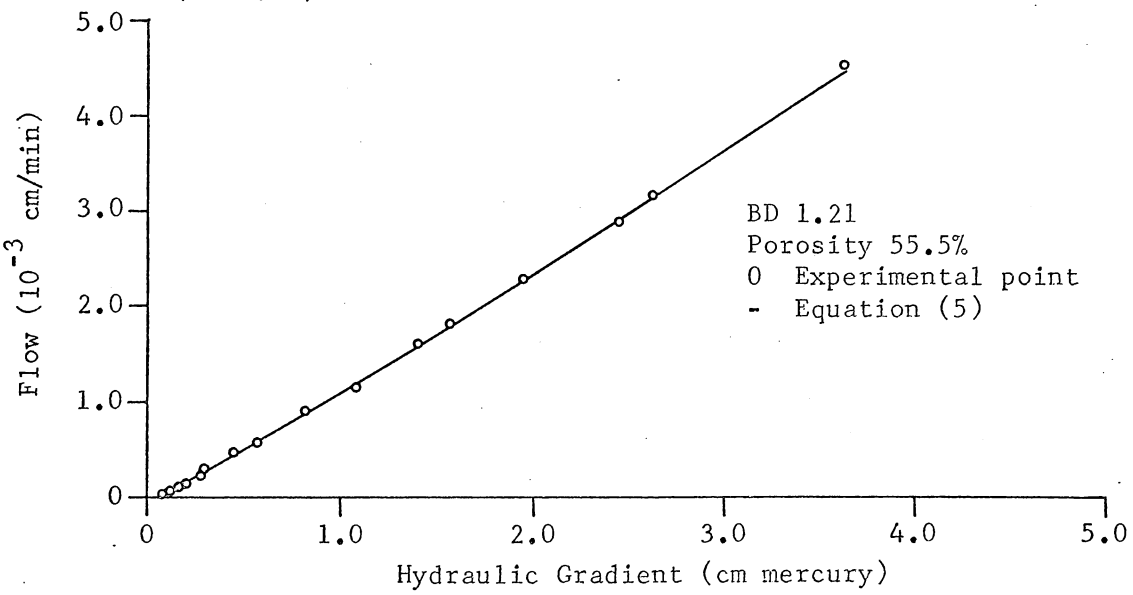

Figure 17. Flow rate versus hydraulic gradient for Springerville soil (BD 1.21).

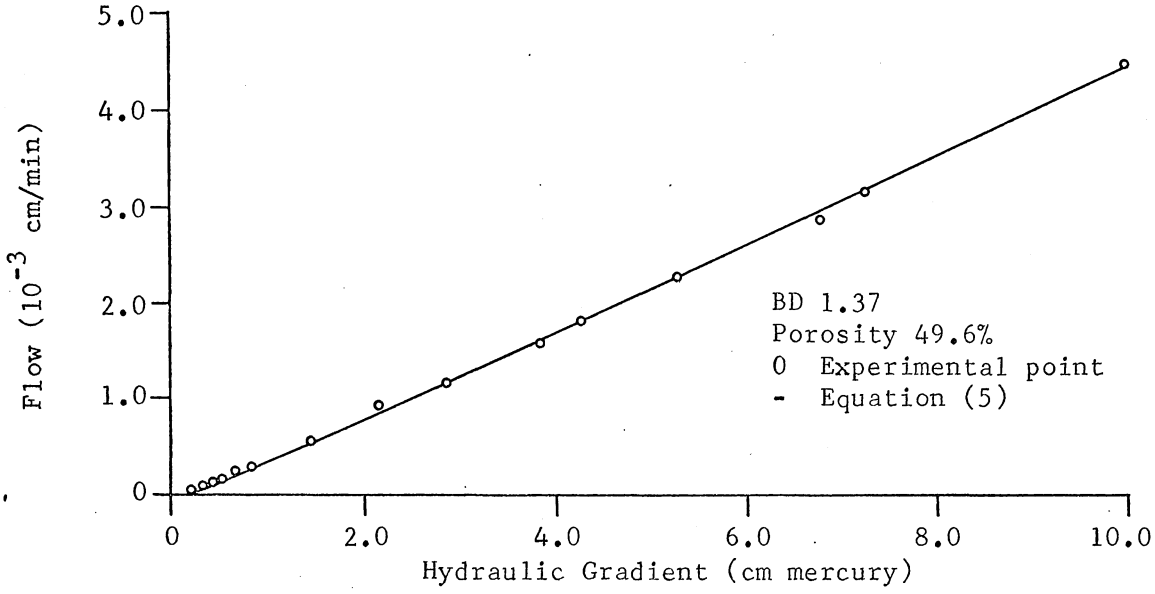

Figure 18. Flow rate versus hydraulic gradient for Springerville soil (BD 1.37)

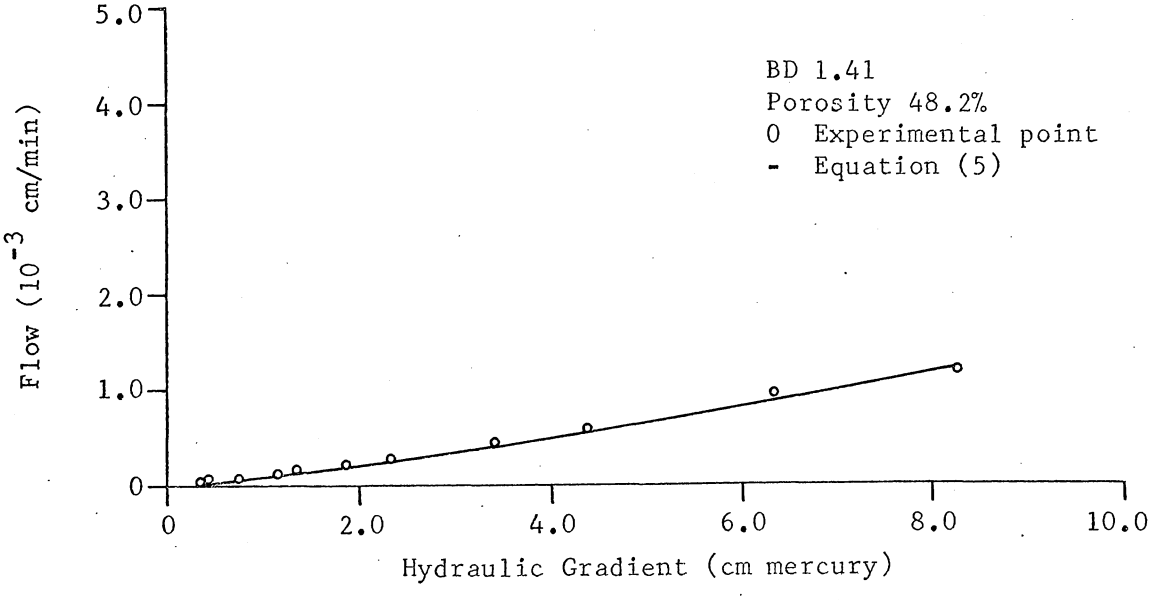

Figure 19. Flow rate versus hydraulic gradient for Springerville soil (BD 1.41)

base flow from the watersheds would be very small and delayed only over a short period of time. The predominance of ephemeral streams of the Beaver Creek area are evidence of this effect.

Non-linearity between flow and hydraulic gradients were pronounced at very low gradients. Flow rates increased at a more than proportional increase with the hydraulic gradient for both soils. A linear or straight line relationship between flow rates and hydraulic gradients existed only at the higher gradients. Thus, Darcy's equation does not strictly apply in describing the flow of water through these soil materials.

Because Darcy's equation, namely, equation (1), does not apply to the conditions of this study, the data were fitted to Swartzendruber's (1962a) modified flow equation

$$v = M[i-I(1-e^{-i/I})] \tag{5}$$

Swartzendruber (1962a) assumed that the hydraulic gradient range involved is below that at which kinetic or turbulence effects appear. As i increases in equation (5), $e^{-i/I}$ approaches zero, with the result

$$v = M(i-I) \tag{6}$$

which is the equation of a straight line of slope M and i-intercept I.

Equation (5) is a two-parameter expression which apparently possesses the behavior of Figure 6. It must be noted that I is not a threshold gradient. Instead it is an extrapolated intercept and is a measure of non-Newtonian or non-Darcy behavior. It is also the only

measurement of departure from proportionality in equation (5). When I = 0, the behavior is Newtonian, and equation (5) reduces to Darcy's law where M is equivalent to K in equation (1). It imposes the somewhat restrictive requirement that the slope of the velocity-gradient curve always be zero at i = 0, regardless of the size of I, unless I = 0. Thus, Swartzendruber's equation is a compatible modification of Darcy's equation in that Darcy's law is included as a special case when the flow behavior is linear or Newtonian. It provides two parameters to describe the flow of water and ionic solutions in the presence of clay. In the past, all such description has of necessity been forced into the single Darcy parameter.

The I parameter in equation (5) is defined as the straight-line asymptote of Swartzendruber's (1962a) modified flow equation. I was determined for each soil by running a linear regression of the form

$$Y = b_o + b_1 X_1 \tag{9}$$

on the data obtained for the higher hydraulic gradients and solving for the hydraulic gradient (X) at zero flow (Y = 0).

Equation (6) was used to determine the value of the M parameter. The equation was rearranged in the form

$$M = \frac{\bar{v}}{(\bar{i} - I)} \tag{10}$$

where $\bar{v}$ and $\bar{i}$ are the mean flow velocity per unit cross section of soil core and mean hydraulic gradient, respectively. They were determined from the linear portion of the flow-hydraulic gradient relationship.

The values for I were calculated as above. After the two parameters (I and M) had been determined, they were adjusted as suggested by Swartzendruber (1962a), in order that equation (5) would give a greater emphasis to the experimental points derived at the lower gradients. The values for I and M are given in Tables 1 and 2.

The relationship of I to soil porosity for both soils is shown in Figures 20 and 21. Both linear and quadratic functions were fitted to the data by regression analysis. Equation (9) was used for linear regression and the equation

$$Y = b_o + b_1 X_1 + b_2 (X_1)^2 \qquad (11)$$

was used for curve linear regression. The R^2 values for linear regression were .80 and .92 for the Springerville and Brolliar subsoil materials, respectively. For curve linear regression the R^2 values were .94 and .98, respectively. The quadratic equation fitted the data significantly better than did the linear equation.

Figure 20 indicates that as porosity increases, flow behavior becomes more nearly Darcy-like since the I parameter becomes increasingly smaller as porosity increases. The rate at which I approaches zero varies between the two soil materials. Interestingly, the I values for the Brolliar soil, which has the generally higher permeability, are greater than the I values for the less permeable Springerville material. Thus, the Brolliar material has a flow behavior that is more non-Darcy than the Springerville materials. These contrasting features could be caused by the sodium and clay content of the soils presented in the Discussion section.

Table 1. I and M parameters for Springerville subsoil material.

BD (g/cm^3)	Porosity (%)	I	M
1.10	59.6	.040	.0154
1.21	55.5	.067	.00125
1.22	55.2	.083	.0016
1.30	52.2	.100	.00164
1.34	50.7	.130	.00069
1.37	49.6	.177	.00046
1.41	48.2	.267	.00015

Table 2. I and M parameters for Brolliar subsoil material.

BD (g/cm^3)	Porosity (%)	I	M
1.07	60.7	.067	.0183
1.11	59.2	.097	.0064
1.12	58.8	.083	.0056
1.25	54.1	.133	.0019
1.37	49.6	.267	.0005

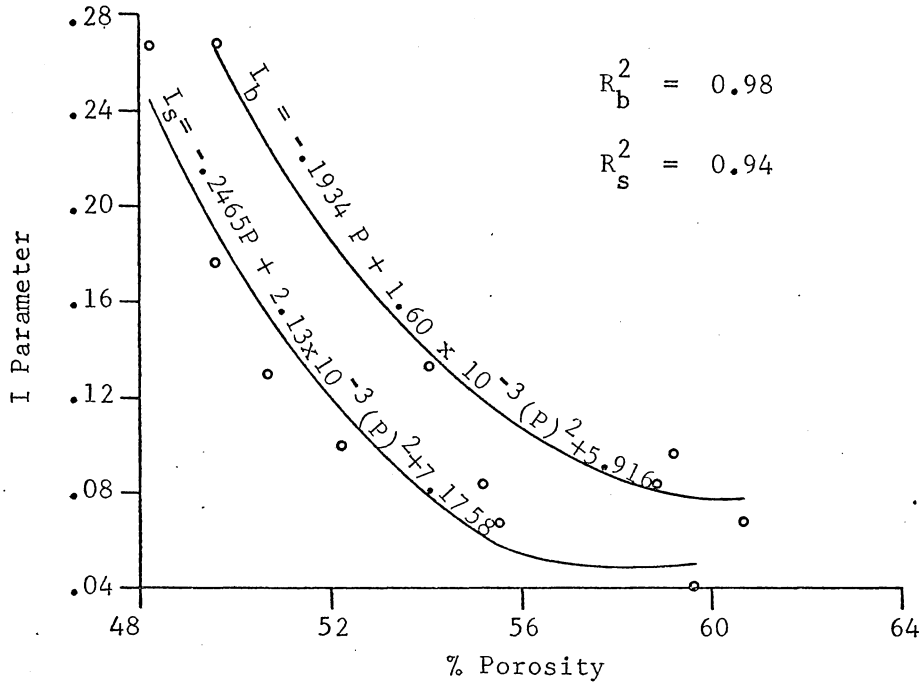

Figure 20. Curve linear relationship of I parameter to porosity.

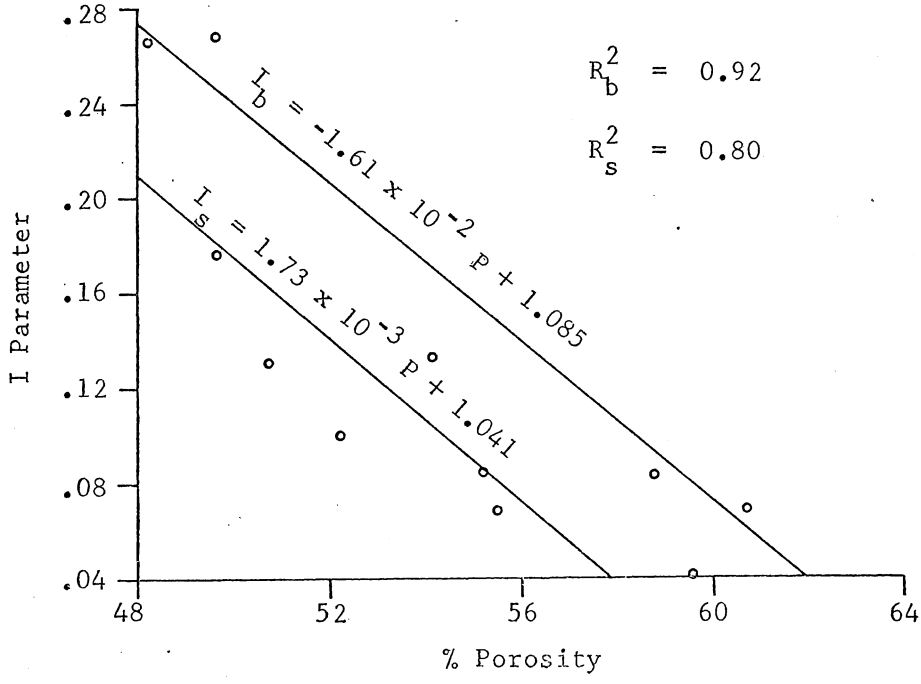

Figure 21. Linear relationship of I parameter to porosity.

The Darcy equation (1) states that the function v(i) has a slope K, which is the conductivity of the soil, and an intercept of zero. A linear equation of the form

$$Y = b_o + b_1 X_1$$

was fitted to the data from all runs (Table 3). In equation (9), Y is equivalent to v and b_1 is equivalent to K. If the data obeyed equation (1), then the intercept b_o should not differ from zero. The parameter differed significantly from zero at the 1% level for all runs. Furthermore, the intercept was negative in all cases. Thus, the equation is essentially useless at low gradients despite the high values obtained for the coefficients of determination. There was only one instance in which less than 99% of the variation was accounted for. This occurred in the Springerville subsoil material which had a porosity of 59.6%.

The linear equation was not applicable to the results of this study. Although Swartzendruber's (1962a) equation (5) better described the relationships found, it did not fit all runs equally well (see Figures 8, 9, and 11). An empirical equation written as

$$v = Ci^B \qquad (12)$$

was fitted to the flow data by regression analysis after readjustment to the form

$$\ln v = \ln C + B \ln i \, . \qquad (13)$$

Table 3. Coefficients for $Y = b_o + b_1X_1$ model.

BD	% Porosity	Regression Coefficients b_o	Regression Coefficients b_1	Partial Correlation Coefficients Y vs X	Coefficients of Determination R^2
		Springerville Soil			
1.10	59.6	-.06858	.02159	.9898	.9797
1.21	55.5	-.004957	.001783	.9996	.9992
1.22	55.2	-.007383	.02292	.9998	.9996
1.30	52.2	-.005118	.02232	.9996	.9992
1.34	50.7	-.001460	.009023	.9974	.9947
1.37	49.6	-.003605	.006403	.9997	.9993
1.41	48.2	-.002197	.002167	.9988	.9976
		Brolliar Soil			
1.07	60.7	-.04165	.2337	.9961	.9922
1.12	58.8	-.01616	.07480	.9982	.9963
1.25	54.1	-.006384	.02518	.9995	.9989
1.37	49.6	-.004728	.006968	.9985	.9970

The equation is reminiscent of the Darcy equation (1) where C is analogous to K. It was thought that flow might be proportional to some power (B) of the hydraulic gradient and that the B parameter would therefore index non-Darcy behavior. Although the equation appeared suitable at lower porosities, it deviated considerably at the higher porosities where the flow relationship became more linear (Table 4).

Non-Darcy behavior was further investigated using a quadratic function fitted to the curve-linear regression equation of

$$\ln Y = b_o + b_1 \ln X_1 + b_2(\ln X_1)^2 \qquad (14)$$

where Y is the flow and X_1 is the hydraulic gradient. The parameters for the equation together with the partial correlation coefficients and coefficients of determination (R^2) for all soils are given in Table 5. Equation (14) fit the data extremely well. There were only two cases in which less than 99% of the variation in flow could be accounted for. This occurred at the higher porosities where most of the variation was encountered.

Although the equation does not hold at the origin, it shows that zero is rapidly approached at hydraulic gradients in the domain of 0-1 cm of mercury. The low gradients in this domain of the function are exactly those that exist most often in the Beaver Creek area. However, the equation clearly shows, by virtue of its close fit to the data over the range of the flow rates observed, the non-linearity of the flow relationship. The relationships differed significantly from a straight line proportional relationship at all but the lowest porosities for

Table 4. Coefficients for $\ln Y = b_o + b_1 \ln X_1$ model.

BD	% Porosity	Regression Coefficients b_o	Regression Coefficients b_o	Partial Correlation Coefficients Y vs X	Coefficients of Determination R^2
			Springerville Soil		
1.10	59.6	-1.415	2.3698	.9575	.9168
1.21	55.5	-4.460	1.2133	.9971	.9942
1.22	55.2	-4.239	1.1928	.9965	.9931
1.30	52.2	-4.248	1.2586	.9925	.9851
1.34	50.7	-4.646	.8893	.9825	.9653
1.37	49.6	-5.513	1.1569	.9989	.9978
1.41	48.2	-6.586	1.1235	.9997	.9994
			Brolliar Soil		
1.07	60.7	-1.2497	2.391	.9623	.9260
1.11	59.2	-3.0119	1.7628	.9835	.9674
1.12	58.8	-3.0670	1.6158	.9891	.9782
1.25	54.1	-4.1284	1.2503	.9966	.9932
1.37	49.6	-5.353	1.1115	.9994	.9988

Table 5. Coefficients for $\ln Y = b_o + b_1 \ln X_1 + b_2(\ln X_1)^2$ model.

BD	% Porosity	Regression Coefficients			Partial Correlation Coefficients		Coefficients of Determination
		b_o	b_1	b_2	Y vs X		R^2
				Springerville Soil			
1.10	59.6	-1.405	.8733	-1.1793	.9575	-.9790	.9758
1.21	55.5	-4.3647	1.2944	-.07225	.9971	.6728	.9983
1.22	55.2	-4.208	1.2390	-.02768	.9965	.7569	.9940
1.30	52.2	-4.0816	1.3686	-.1268	.9925	.5799	.9949
1.34	50.7	-4.894	.87217	.09450	.9825	.2761	.9930
1.37	49.6	-5.5538	1.2927	-.04475	.9989	.9344	.9994
1.41	48.2	-6.589	1.1288	.001403	.9997	.9636	.9994
				Brolliar Soil			
1.07	60.7	-1.643	.6888	-.9389	.9623	-.9825	.9721
1.11	59.2	-2.742	1.5144	-.4065	.9835	-.7514	.9968
1.12	58.8	-2.875	1.4396	-.2471	.9891	-.7384	.9942
1.25	54.1	-4.0613	1.2729	-.0453	.9966	.3629	.9949
1.37	49.6	-5.3526	1.1133	-.0006897	.9994	.8994	.9988

both soils. This is illustrated in Figures 22 and 23. Three typical runs for each of the two soils at similar porosities were fitted to equation (14) and plotted on log-log scale. It can be noted in both soils that as porosity decreases or bulk density increases, the departure from non-Darcy flow becomes greater.

The comparison of equations (5), (9), and (14) to the actual data is given in Tables 3-A and 4-A of the Appendix. Equation (9), the linear model represented the worst fit of all the models used. The equation predicts negative flow rates at and below 0.12×10^{-3} and 0.0492×10^{-3} cm/min for the Springerville and Brolliar subsoil materials, respectively. This feature makes the model meaningless at flow rates below these values. Swartzendruber's semi-empirical equation (5) predicts more realistic flow rates at the lower flows than did equation (9). However, the magnitude of departure from the actual flow rates for both soils is large at the lower flows and small at the higher flow rates. The departure was greatest at the higher porosities for each soil. The empirical equation (14) very closely predicted the actual flow rates over the entire range of flows for both soils at all bulk densities. The data point out the inaccuracy of Darcy's equation which can be developed from rational considerations. Thus, it appears that after fitting the data to four rather simple functions a more universal function will have to be developed in order to describe the flow behavior in clay subsoil material at low flow rates and hydraulic gradients.

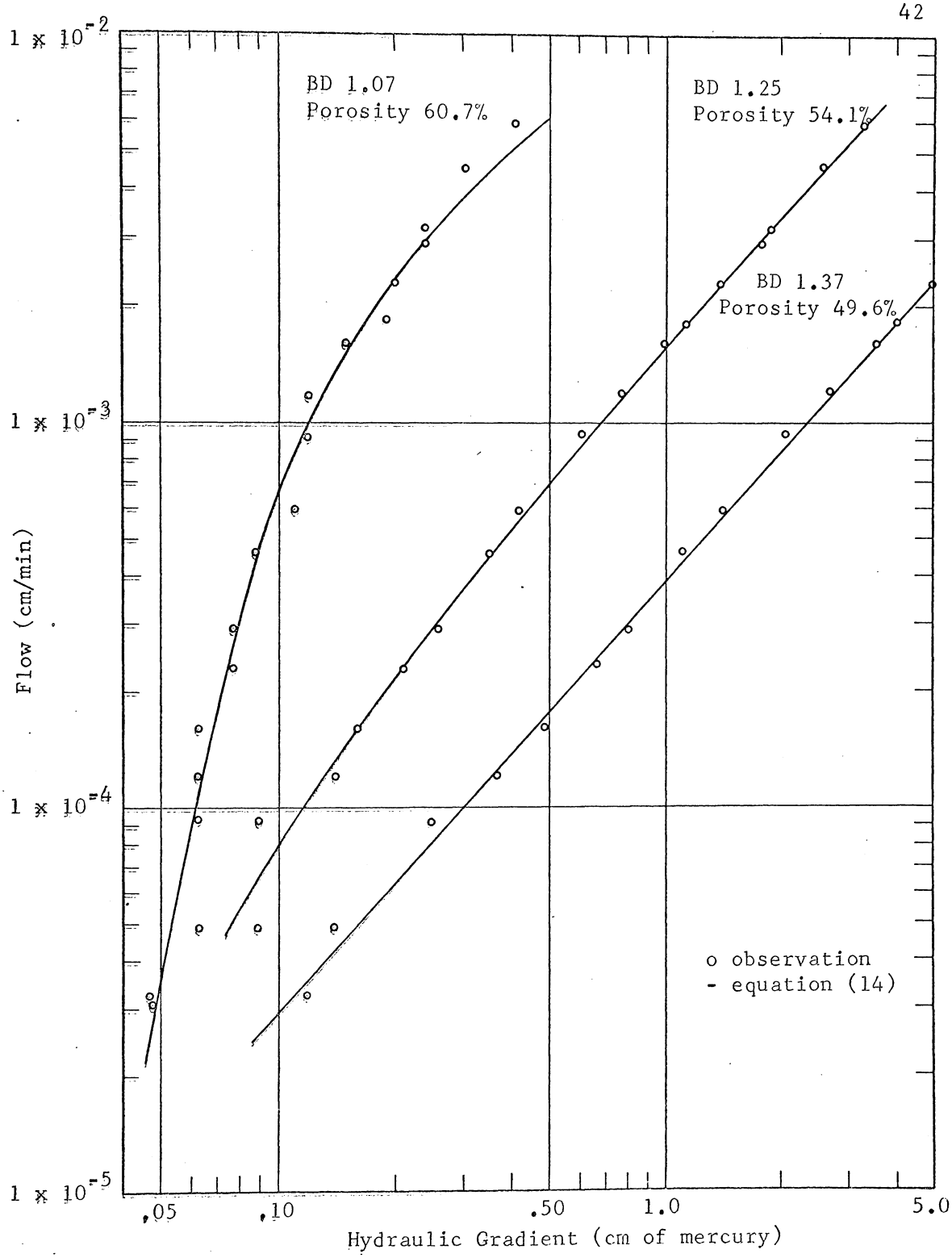

Figure 22. Log-log relationship of flow rate versus hydraulic gradient for Brolliar soil.

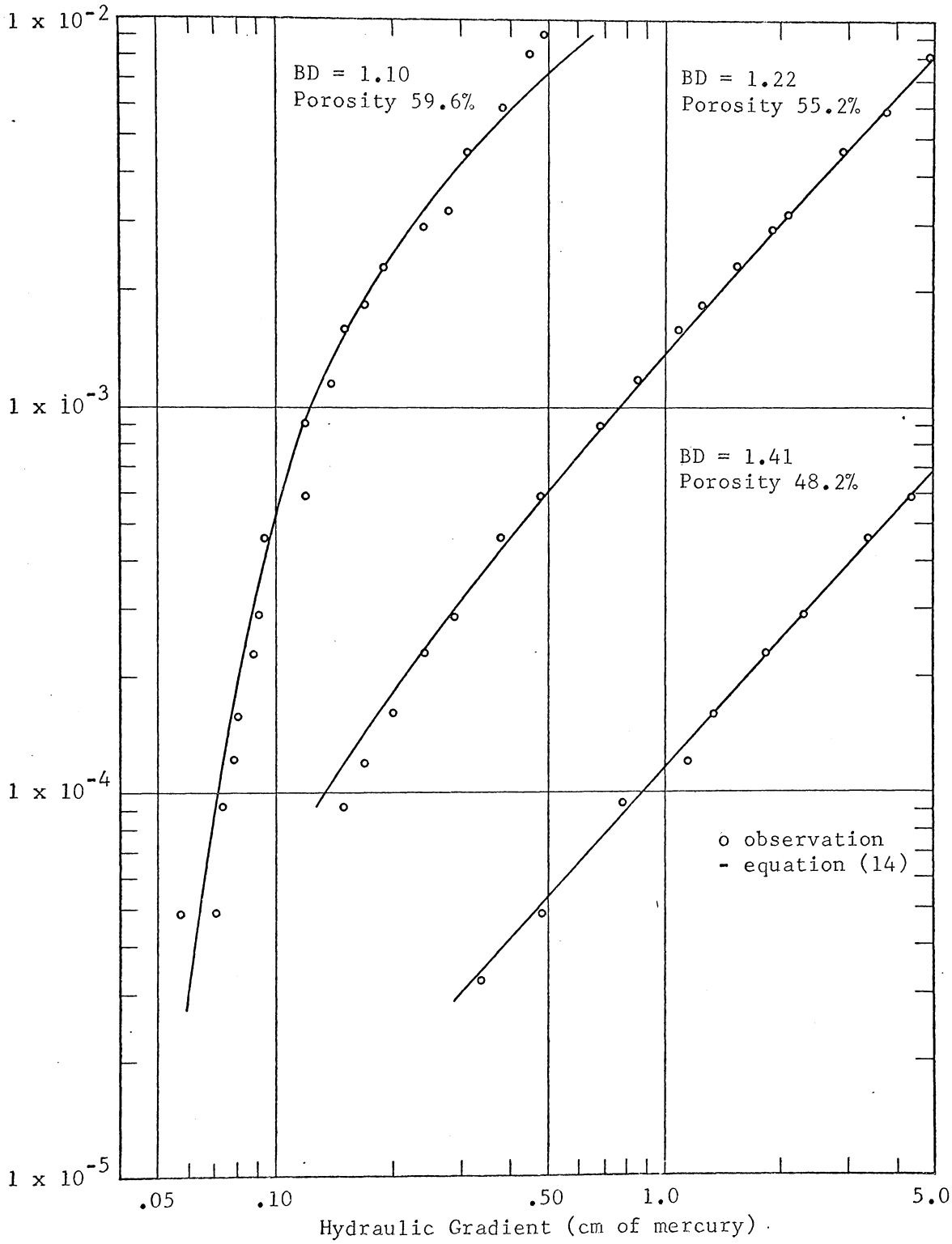

Figure 23. Log-log relationship of flow rate versus hydraulic gradient for Springerville soil.

Swartzendruber (1968) gives an example of the effect of non-Darcy behavior in which surface sealing during raindrop splash is considered. If a threshold gradient, such as discussed by Low (1961), existed with a magnitude of 55 cm of water, then the development of a clay film 0.1 mm thick over the soil surface could support a water depth of 5.4 mm before water movement through the soil could begin. Values for hydraulic gradients in the present study are expressed in cm of mercury per cm length of soil core (Tables 1-A and 2-A of the Appendix) and must be multiplied by 13.6 to be expressed in units of water column height. This would give a threshold value of about 3.68 cm of mercury. Tables 3-A and 4-A show that these values did not occur until high bulk densities and flow rates were obtained. This would indicate that a threshold gradient similar to Swartzendruber's would not be obtained in most instances. Due to this aspect of non-Darcy behavior, flow through the soil of the Beaver Creek area would virtually cease since the soil's surface can be easily sealed by even a slight disturbance.

DISCUSSION

The purpose of the discussion is to review the current theories for describing the hydrologic behavior of clay materials with respect to non-Darcy behavior. All of the theories have merit, but they cannot be completely reconciled with the present state of knowledge. Since they have not been resolved, specific explanations for the hydrologic behavior for the Beaver Creek soils cannot be offered. Since the theories were derived from work on clay soil material, their pertinence to the Beaver Creek area is evident.

Three criteria must be met in a saturated clay sample for Darcy's law to be valid (Olsen 1966). First, the interstitial liquid must exhibit Newtonian behavior. In other words, at every point in the liquid, the viscosity must remain constant with respect to the hydraulic gradients imposed on the sample. Secondly, the clay particles must be arranged in a skeleton sufficiently rigid to prevent seepage forces from modifying the architecture of the pore geometry. Finally, the assumptions for Darcy's law must be met. These assumptions are (1) laminar flow through the porous medium, (2) isotropic permeability, (3) homogeneous porous medium, (4) low hydraulic gradients, and (5) no clay-water interaction.

In order to explain the observed deviations from Darcy's law, several mechanisms have been proposed that provide possible ways in which these criteria can be violated. The mechanisms are quasi-crystalline water, electrokinetic or electroviscous effects, particle

reorientation, and a range in pore sizes. Two of the mechanisms allow for non-Newtonian liquid behavior. One is derived from the postulated existence of a quasi-crystalline structure in the water near clay surfaces. The other mechanism involves the electroviscous component of resistance to liquid movement.

Quasi-Crystalline Theory

Von Englehardt and Tunn (1955), Hansbo (1960), Lutz and Kemper (1959) and Miller and Low (1963) accounted for the non-linearity they observed by invoking the idea that clay surfaces alter water structure. This altered water is thought to be more ordered because of surface-induced hydrogen bonding and is most pronounced at the clay-water interface. The amount of hydrogen-bonded water decreases with distance from the clay surface until normal water is reached. The distance to normal water depends on the surface, its charge, and the exchangable ions present. When low hydrostatic heads are applied, water flows in the center of the pores only where the least altered water exists. With the crystalline structure mechanism at work, presumably a certain yield must be exceeded before the water structure can be deformed and thereafter the rigidity or viscosity of the water structure would decrease with increasing shearing force or gradient (Low 1961). With each higher pressure increment, additional layers of water are sheared, giving increased permeabilities or flow rates. This continues with further increase in head until the whole pore is conducting water. This theory does not provide any information on the behavior of discharge velocity under change of porosity or crystallinity of the clay.

Low (1961) maintains that this theory is not valid at high salt concentrations.

Low suggested that the presence of a threshold gradient would be a critical test for the presence of quasi-crystalline water on clay surfaces. Below the threshold, no water would flow and thus electroviscous or plugging effects could be discounted. Miller and Low (1963) argued that if clay surfaces do order water, then at some clay concentrations this quasi-crystalline water would extend across the entire pore. A finite energy, or head, would be needed to break down this structure barrier before flow would begin. On the basis of the threshold gradient they reported and from experiments which indicated that the activation energy for water flow in clay pastes increased as the applied gradient decreased, Miller and Low concluded that a quasi-crystalline water structure does exist on clay surfaces.

Olsen (1966) observed deviations that are consistent with deviations expected from Darcy's law that postulates a quasi-crystalline structure in the clay pore water.

Miller and Low (1963) investigated the flow of water in clay soils at low hydraulic gradients. The occurrence of deviations in their system is not inconsistent with the absence of deviations in the samples of kaolinite and natural illitic clays of Olsen (1962). Montmorillonite is very fine-grained compared with kaolinite and illite. The influence of quasi-crystalline water could be substantial for flow in the extremely small pores of montmorillonite and negligible for flow in the much larger pores of illite and kaolinite. Thus, according to

Olsen (1966), Miller and Low's data may be interpreted as evidence that there are exceptions to the validity of Darcy's law in very fine-grained natural clays and clay sediments.

Swartzendruber (1962a) presented some conditions within a porous medium which could give rise to the behavior specified by equations (5) and (7). In non-Newtonian flow, the viscosity, taken as a point function, is no longer constant throughout the liquid region, but decreases with the shear rate. This is seen in Figure 4 by taking the slope of the non-Newtonian curve to be the reciprocal of the variable point viscosity.

The reduction in viscosity with shear rate leads consequently to the more-than-proportional increase in the velocity-gradient curve as seen from Figure 5. According to Swartzendruber, it would appear reasonable that in a porous medium the reduction point viscosity with shear rate gives rise to the early, upward-curving aspect of equation (5). If, for increasing gradients, the variable point viscosity is eventually reduced to a uniform and constant value, the linear portion of equation (5) could result. Its slope M could then involve the normal viscosity η and the permeance m as expressed by equation (7).

With the limit of the variable point viscosity presumed to be the normal bulk viscosity, the concept just outlined provides for liquid domains in the porous medium where the point viscosity is higher than the normal bulk viscosity. Such a condition could be accounted for by a water structure as postulated by Low (1961). This would allow for the reduction of the non-Newtonian index I with added

electrolyte since ions usually break down the water's structure. As can be seen from Tables 1 and 2 of the text and Table 5-A (Appendix), the value of I for a given porosity decreased as the salt concentration of the soil increased. This could be expected since for higher concentrations the clay should be less subject to deflocculation and dispersion. This implies that a clay soil with a low sodium content would probably exhibit the greatest degree of non-Newtonian behavior. Adding salt to the soil could cause the flow behavior to be more in the direction of Newtonian flow. The salt concentration had a similar effect on the M parameter. As the salt concentration increased, the value of M decreased. This is to be expected since M is derived in part from the I term of equation (10). There appears to be a general decrease in the M parameter of equation (10) with a decrease in porosity (see Tables 1 and 2). The parameter M could be called the hydraulic transmissibility or hydraulic conductance of the soil.

If the water in the vicinity of clay is quasi-crystalline, the closer the particles are together, the greater the proportion of ordered water. If movement of different layers of this water is shear dependent, the greater the clay concentration, the greater the non-Darcy flow. The Brolliar subsoil material with its 20% clay content would tend to support this, but not the Springerville soil material with its 50% clay content. It is possible that when the clay particles are very close together as in the case of the Springerville material, the force fields are so great that the altered water structure is destroyed and Darcy's flow characteristics appear again.

Electroviscosity Theory

When the cations associated with the negative exchange site on a clay mineral in water are not specifically adsorbed, they distribute themselves in a diffuse layer under the influence of electrostatic and thermal forces. The electrostatic forces of attraction tends to concentrate the cations in the vicinity of the clay particle. Thermal motion or diffusion of these cations creates a tendency for ions to move from the region of higher concentration near the clay mineral surface to regions of low concentration out in solution (Kemper 1960).

When the water moves in thin films on or between negatively charged clay particles, there is an initial tendency for more cations than anions to be carried to the low pressure side of the gradient because of the larger number of cations as compared to free anions in the system. This tendency causes a rapid buildup of positive charge at the low pressure end of the gradient and a decrease in the positive charge at the high pressure end of the gradient. The resulting electrostatic potential gradient exerts a force on the ions in the system, most of which is in turn transmitted to the solution in the immediate vicinity of the ions.

Kemper (1960) determined that the presence of ions associated with a charged clay surface causes a reduction in solution velocity and electroviscous drag decreases with increasing hydraulic gradient. The reduction in velocity was generally more marked as the liquid film became thinner and the ions of the diffuse layer occupied a proportionately greater part of the liquid volume. Under these conditions the hydrated ions exert a large portion of the total drag on the liquid

movement. When the films become very thin the drag on the liquid motion caused by the proximity of the solid surface becomes so large that the drag offered by the ions is a smaller part of the total resistance to flow.

Reduction of the external electrolyte concentration causes the resistance to flow to be greater by allowing the ions associated with the solid surface to extend further out into the central part of the film. Cations in the central part of the film, where the velocity of the solution is greatest, are moved rapidly with the flow, tending to build up large streaming potentials. Since the streaming potential tends to reverse the direction of cation movement, increase the rate of anion movement and there are more cations than anions in the liquid, the net effect of the larger streaming potential is to reduce the velocity of liquid flow (Kemper 1960).

An interesting aspect of the effect of ions associated with the charged surface on the velocity of a fluid is the actual reversal of direction of flow next to the solid surface. If a thin film (<30 Å) were associated in parallel with a thicker film (>150 Å), the liquid flow in the thin film would be against the pressure gradient (Kemper 1960). It is apparent that the flow pattern of liquid in media with extensive diffuse layers of associated ions may be very complicated.

Kemper (1960) determined that when the liquid film is not uniform and constrictions exist in the flow channel, his equation for determining average solution velocity in the thin film is probably not valid for systems with low permeability and extensive diffuse layers of

associated ions. Large pressure gradients will not only cause the flow through the constriction to be proportionately greater, but the more rapid flow will sweep hydrated ions associated with the clay particle out of the constrictions and actually increase the intrinsic permeability. Lutz and Kemper (1959) found that increased pressure gradients cause increases in the intrinsic permeability in systems where the diffuse double layers would be expected to extend to an appreciable distance into the flow channels.

A gradient-dependent electroviscous effect might possibly account for equation (5). It would seem necessary, however, to restrict this to water and relatively dilute electrolyte solutions because in concentrated solutions the effect is usually negligible (Swartzendruber 1962a).

Micheals and Lin (1954) postulated an electroviscous resistance to flow which decreased with increasing head as the flow restricting cations were swept into the larger pores. They concluded that these effects were probably small or of no major importance since electrokinetic theory and experimental results suggest that there is a linear relationship between the electroviscous effect and applied pressure.

Particle Reorientation

Two mechanisms provide ways in which seepage forces can cause irreversible changes in the pore geometry. First, seepage forces can consolidate a clay sample so that the pores are irreversibly diminished in size (Lutz and Kemper 1959). Secondly, if some of the finer clay particles exist free within the pores of the load-carrying skeleton of

clay and coarser particles, seepage forces may cause these mobile particles to plug or unplug flow channels in the load-carrying skeleton (Martin 1962).

It has been proposed that reversible pore geometry changes may result from reversible or elastic particle reorientations caused by seepage forces. The pore geometry might be altered in two ways. The seepage forces might realign particles along stream-lines and thereby reduce the tortuosity of the flow paths (Miller and Low 1963). The seepage forces might also reorient particles in such a way that the size distribution of the flow channels is altered (Terzaghi 1925). Reversible particle reorientations would occur with increasing gradients in such a manner that the effective cross section for flow is increased. Some of the larger channels might be enlarged at the expense of the smaller channels. However, essential reversibility of reorientations is required since the careful replications of Von Englehardt and Tunn (1955) rule out the likelihood of irreversible particle reorientations or movements. Von Englehardt and Tunn (1955) reported no data unless the initial results of the flow experiment could be duplicated at the end of the experiment. The reversible reorientations herein considered are analogous to the behavior of the water flow system made up of very expansible rubber tubes as described by Terzaghi (1925).

Micheals and Lin (1954), Martin (1962), and Mitchell and Younger (1967) suggest that particle reorientation is the most important effect in non-Darcy flow. These matrix effects need not be just a simple

movement of particles, but may include bending and the flexing of particles or the breaking of edge-to-surface bonds to permit particle orientation with the flow path.

Lutz and Kemper (1959) and Von Englehardt and Tunn (1955) data may be interpreted as evidence that deviations from Darcy's law can occur in systems that consist partially or entirely of unconfined clays, owing to alterations in the clay fabric in response to large seepage forces. These data raise the possibility that similar deviations may occur under natural occurring gradients in shallow unconfined clays and also in granular sediments containing small amounts of clay.

A certain orientation of the grains in the clay skeleton of the subsoil will be caused by the pressure of the top soil. The results of the consolidometer tests of Hansbo (1960) suggest that the pore water was allowed to escape in a horizontal direction (i.e., parallel to the clay strata) and the coefficient of consolidation was greater than in the case where the pore water was allowed to escape in the vertical direction.

In a saturated soil, the space between the mineral grains is assumed to be filled with pore water, consisting of ions and water molecules, which are more or less rigidly bound to the mineral surface. The pore water seems to be fixed in a rigid lattice and may be regarded as part of the mineral phase. For the water phase, the viscosity was shown by Hansbo (1960) to increase considerably when approaching the mineral surface, from 24 centipoises at 30% to 153 centipoises at 10% water content for the clay in question.

When sedimented clay is gradually loaded by overlying sediments, not all grains appear to become part of the load-carrying network (the rigid phase or clay skeleton). Consequently, within this network, the space may be filled not only with pore water in its present sense but also with mobile particles of colloidal or greater size which may be bound by sorption and hydrodynamical forces (the mobile phase). Hansbo (1960) suggested this last statement as one explanation for the phenomenon observed in his permeability tests. During his tests, the direction of flow through the specimen was often altered. His system behaved as if the mobile particles were being meshed during their travel through the pore space, thus clogging part of the pores. The pores would be re-opened by an increase in flow velocity or by a change in flow direction, or in some cases, after some lapse of time.

From results obtained by Hansbo (1960) it was found that Darcy's law of permeability is not always valid for small pore pressure gradients. The forces which bind the mobile phase gradually become stronger when the distance to the rigid phase decreases. It was also suggested that within certain limits, the clay becomes increasingly porous or more permeable with increasing pore pressure gradient. This deviation from Darcy's law of permeability ought to depend on the interaction between skeleton grains and pore water. With this in mind, Hansbo (1960) studied the results obtained by pH-metric titration of the samples used in his permeability tests. The titer curves showed considerable reactions between the clay and hydrochloric acid for all

of the samples deviating from Darcy's law of permeability. Little reaction between the clay and acid was obtained on samples which were found to obey Darcy's law.

Because of its characteristics, montmorillonite would be expected to be more subject to matrix rearrangements. It would have a large component of particles that can bend, flex, and move, whereas the other kinds of clays do not have as many small particles to move or shift. Because of their low surface area, the clay minerals of kaolinite and illite are usually thought of as minimizing water structure, if they do in fact alter water structure. Montmorillonite, having both a large surface area and associated exchange capacity because it is an expanding mineral, would be expected to maximize any water effect that is a result of surface, exchangable cations or charge-induced phenomena.

It would be expected that if non-Darcy behavior is a result of either water or matrix effects, the subsoil material with the high clay content would have large values of I since this is a measure of non-Darcy behavior. Tables 1 and 2 of the text and Table 5-A of the Appendix indicate that for a given porosity the Springerville soil material with the high clay content produces lower values of I than does the Brolliar material with the lower clay content.

One explanation for this interesting feature is the effects of high clay concentrations on the subsoil material. As the montmorillonite becomes more concentrated in the soil material, the particles are in closer proximity and the swelling pressure between the particles

increases. At some clay concentration, apparently above 20%, these forces could become great enough to stabilize the particle, preventing or reducing matrix effects at the low flow rates used. At the clay concentration of 50%, no particle rearrangement took place or it was so slow that more Darcy flow resulted as is shown in Tables 1 and 2.

Although no evidence indicating a pressure dependent relaxation of the containing cells was found, this possibility cannot be ruled out. In the Brolliar soil material with its low clay content, the permeability and swelling pressures may be such that small changes in cell volume could occur and cause non-linear pressure dissipation. The swelling pressure in the Springerville soil material could be large enough to eliminate cell volume changes.

Range in Pore Sizes

Miller and Low (1963) suggested that non-linear flow could be caused by a range in pore sizes. As the hydraulic gradient was increased, the threshold gradient was exceeded in smaller pores increasing the flow rate. A threshold gradient is necessary for this explanation.

Various authors (Hansbo 1960, Kemper 1960, Olsen 1966) have shown that most of the data explain the two dominant characteristics of the deviations from Darcy's law, namely, the more-than-proportional increase in flow velocity with increasing gradient and the increasing departures from direct proportionality with decreasing electrolyte concentrations in the permeant liquid. Some features of the data are inconsistent with electroviscous drag and irreversible pore geometry

changes as dominant mechanisms (Low 1961). The most likely causes of non-Darcy flow behavior appear to be the quasi-crystalline structure of water near clay surfaces and reversible clay fabric changes induced by seepage forces (Olsen 1966).

Two characteristics of saturated clay systems should govern the extent to which these mechanisms can cause deviations from Darcy's law. They are the sizes of the flow channels and the rigidity of the clay fabric. The influence of the quasi-crystalline structure in clay pore water should increase with decreasing clay pore sizes because, presumably, the rigidity of the water structure decreases with distance from clay surfaces. The clay's fabric susceptibility to alteration by seepage forces should decrease with increasing degrees of consolidation and confinement.

If the water in a porous medium can be modified by the influence of particle surfaces, the classical non-Newtonian or Bingham concepts must be applied with caution. For material flowing in a cylindrical capillary pore, the shear stress is zero at the axis of the pore and maximal at the wall (Reiner 1960). This means that any non-Newtonian effect, be it an elevated shear-rate dependent viscosity or a yield-value Bingham plasticity, will be broken down first at the wall and will persist longest in the central axial portion of the pore. Neither of these effects, however, appears directly comparable with surface-induced changes in water properties, for it would seem that these should be minimal at the center of a pore and maximal at the walls. The approach of Powell and Eyring (1944), involving two types

of bonds, might lead to further explanation of the water properties.

Von Englehardt and Tunn (1955) also recognized an alternative mechanism consisting of adsorbed, immobile water layers on the particle surfaces with Newtonian conditions prevailing elsewhere. Increased shear stress would need to decrease the thickness of the immobile water layers so that the effective cross-section for flow would increase with the gradient.

SUMMARY

The present study has been concerned entirely with liquid-saturated porous media. The same non-Newtonian effects, perhaps to an even greater degree, might well be expected in partially saturated porous media, since liquid movement must then take place in closer proximity to the particle surfaces. Because of the relationship of saturated water flow to soil drainage, the present study has clear relevance in watershed management for conditions of excess precipitation. Also, if non-Newtonian flow manifests itself in partially saturated soils, then further important implications arise on the question of unsaturated flow to the evapotranspiration stream. Problems introduced by unsaturated drainage as a source of base flow also become pertinent.

If existing greater-than-proportional behavior were ignored when determining hydraulic conductivity by a laboratory test, Figures 8 through 19 illustrate the resulting implications. If the test were conducted at a high gradient and the flow condition of interest occurred at a low gradient, the effective value of K under the flow condition of interest would be overestimated. The reverse would be true if the gradient were less than the gradient at the flow condition of interest. The magnitude of discrepancy would depend on the velocity-gradient relationship and on the spread between the test gradient and the gradient under the flow condition of interest.

APPENDIX

STATISTICAL DATA

Table 1-A. Actual flow rates and hydraulic gradients for the Brolliar subsoil material.

Actual Flow (10^{-3} cm/min)	Hydraulic Gradient (cm mercury)				
	BD 1.07	1.11	1.12	1.25	1.37
	Porosity 60.7%	59.2%	58.8%	54.1%	49.6%
.0325	.047	.063	.047	.047	.12
.0492	.063	.067	.063	.09	.14
.0927	.063	.087	.083	.09	.25
.12	.063	.093	.083	.14	.37
.16	.063	.10	.087	.16	.49
.23	.077	.12	.11	.21	.67
.294	.077	.14	.14	.26	.81
.456	.087	.16	.17	.35	1.13
.590	.11	.19	.20	.42	1.44
.913	.12	.24	.24	.61	2.10
1.18	.12	.28	.30	.79	2.74
1.60	.15	.35	.38	1.00	3.60
1.83	.19	.40	.44	1.12	4.06
2.3	.20	.48	.52	1.39	5.13
2.9	.24	.60	.66	1.77	6.52
3.2	.24	.63	.68	1.87	6.80
4.56	.31	.84	.90	2.58	9.20
5.9	.41	1.07	1.14	3.32	11.66

Table 2-A. Actual flow rates and hydraulic gradients for the Springerville subsoil material.

Actual Flow (10^{-3} cm/min)	Hydraulic Gradient (cm mercury)						
	BD 1.10	1.21	1.22	1.30	1.34	1.37	1.41
	Porosity 59.6%	55.5%	55.2%	52.2%	50.7%	49.6%	48.2%
.0325		.077		.077	.027		.33
.0492	.07	.087	.057	.093	.035	.20	.48
.0927	.073	.13	.15	.12	.19	.34	.79
.12	.077	.17	.17	.13	.25	.44	1.15
.16	.08	.20	.20	.16	.32	.52	1.37
.23	.087	.27	.24	.22	.44	.69	1.88
.294	.09	.31	.29	.26	.55	.83	2.31
.456	.093	.45	.38	.35	.76	1.18	3.40
.590	.12	.57	.48	.46	1.03	1.43	4.39
.913	.12	.83	.67	.65	1.77	2.19	6.38
1.18	.14	1.08	.86	.83	1.87	2.85	8.27
1.60	.15	1.38	1.09	1.09	2.42	3.81	10.66
1.83	.17	1.57	1.26	1.25	2.79	4.24	11.76
2.3	.19	1.94	1.53	1.58	3.56	5.25	
2.9	.24	2.46	1.94	1.96	4.65	6.75	
3.2	.28	2.63	2.11	2.14		7.21	
4.56	.31	3.62	2.91	2.87		9.98	
5.9	.38	4.65	3.78	3.73			
8.1	.45		4.94				
9.13	.49		5.60				
12.0			7.56				

Table 3-A. Actual and predicted flow using the Swartzendruber, linear and log-log model on the Brolliar soil material.

Actual Flow (10^{-3} cm/min)	Porosity 60.7%			Porosity 59.2%			Porosity 58.8%		
	Eq. (5)	Eq. (9)	Eq. (14)	Eq. (5)	Eq. (9)	Eq. (14)	Eq. (5)	Eq. (9)	Eq. (14)
.0325	.242	-.213	.0315	.106	-.0824	.0406	.061	-.136	.0306
.0492	.406	.0661	.110	.119	-.0628	.0456	.105	-.0464	.0624
.0927	.406	.0661	.110	.189	.0547	.0956	.171	.0608	.114
.12	.406	.0661	.110	.212	.0939	.116	.171	.0608	.114
.16	.406	.0661	.110	.24	.1331	.138	.185	.0787	.124
.23	.470	.289	.221	.327	.270	.229	.275	.204	.202
.294	.470	.289	.221	.422	.348	.289	.405	.365	.320
.456	.70	.457	.33	.522	.505	.428	.547	.526	.456
.590	1.02	.848	.679	.683	.642	.557	.647	.686	.606
.913	1.17	1.071	.92	.967	.956	.896	.905	.883	.800
1.18	1.17	1.071	.92	1.2	1.21	1.19	1.23	1.22	1.15
1.60	1.65	1.574	1.54	1.6	1.60	1.66	1.7	1.65	1.63
1.83	2.3	2.13	2.26	1.95	1.90	2.00	2.0	1.99	1.99
2.3	2.5	2.36	2.56	2.4	2.37	2.53	2.5	2.4	2.46
2.9	3.2	3.08	3.39	3.2	3.05	3.25	3.23	3.1	3.2
3.2	3.2	3.08	3.39	3.4	3.23	3.39	3.3	3.24	3.3
4.56	4.46	4.31	4.44	4.75	4.48	4.35	4.58	4.46	4.42
5.9	6.3	5.93	5.14	6.2	5.85	5.18	5.9	5.7	5.47
8.1									
9.13									
12.0									

Table 3-A.--Continued

Actual Flow (10^{-3} cm/min)	Porosity 54.1%			Porosity 49.6%		
	Eq. (5)	Eq. (9)	Eq. (14)	Eq. (5)	Eq. (9)	Eq. (14)
.0325	.0148	-.0683	.0282	.0117	-.0530	.0363
.0492	.0464	.00991	.0719	.0158	-.0414	.0442
.0927	.0464	.0159	.0757	.0437	.0135	.0834
.12	.1	.1002	.132	.0851	.0702	.126
.16	.13	.142	.162	.132	.133	.175
.23	.20	.232	.231	.212	.220	.25
.294	.277	.311	.294	.278	.292	.30
.456	.43	.479	.437	.433	.450	.454
.590	.56	.612	.557	.587	.605	.576
.912	.909	.955	.879	.916	.938	.877
1.18	1.25	1.28	1.2	1.24	1.25	1.18
1.6	1.65	1.65	1.57	1.66	1.68	1.6
1.83	1.88	1.88	1.81	1.90	1.91	1.82
2.3	2.4	2.4	2.30	2.43	2.45	2.36
2.9	3.1	3.0	3.0	3.13	3.14	3.08
3.2	3.3	3.2	3.22	3.3	3.28	3.22
4.56	4.65	4.50	4.61	4.47	4.48	4.52
5.9	6.0	5.8	6.0	5.7	5.71	5.8
8.1						
9.13						
12.0						

Table 4-A. Actual and predicted flow using the Swartzendruber, linear and log-log model on the Springerville soil material.

Actual Flow (10^{-3} cm/min)	Porosity 59.6%			Porosity 55.5%			Porosity 55.2%		
	Eq. (5)	Eq. (9)	Eq. (14)	Eq. (5)	Eq. (9)	Eq. (14)	Eq. (5)	Eq. (9)	Eq. (14)
.0325				.0390	-.0204	.0387			
.0492	.569	-.175	.0848	.0478	-.0076	.0456	.0251	-.0833	.0363
.0927	.608	-.106	.1046	.0908	.0435	.0812	.129	.070	.130
.12	.66	-.0361	.127	.14	.103	.131	.16	.0974	.146
.16	.699	.0335	.153	.17	.137	.154	.20	.152	.187
.23	.793	.173	.213	.25	.227	.230	.26	.212	.238
.294	.835	.242	.248	.304	.278	.277	.335	.300	.299
.456	.876	.312	.287	.479	.457	.444	.476	.448	.418
.590	1.26	.80	.64	.63	.614	.60	.636	.607	.552
.913	1.26	.87	.70	.954	.943	.932	.939	.919	.83
1.18	1.56	1.22	1.05	1.27	1.26	1.26	1.24	1.23	1.11
1.6	1.71	1.57	1.46	1.6	1.64	1.65	1.6	1.61	1.48
1.83	2.01	1.84	1.82	1.88	1.89	1.90	1.88	1.89	1.75
2.3	2.31	2.26	2.38	2.3	2.36	2.38	2.3	2.34	2.21
2.9	3.1	3.31	3.78	3.0	3.03	3.03	3.0	3.0	2.89
3.2	3.7	4.21	4.85	3.2	3.24	3.25	3.2	3.28	3.18
4.56	4.16	4.84	5.47	4.44	4.51	4.42	4.5	4.6	4.59
5.9	5.2	6.30	6.45	5.7	5.83	5.59	5.9	6.0	6.12
8.1	6.3	7.7	6.83				7.8	7.95	8.22
9.13	6.9	8.39	6.88				8.83	9.02	9.39
12.0							12.0	12.25	13.0

Table 4-A.--Continued

Actual Flow (10^{-3} cm/min)	Porosity 52.2%			Porosity 50.7%		
	Eq. (5)	Eq. (9)	Eq. (14)	Eq. (5)	Eq. (9)	Eq. (14)
.0325	.0382	.00038	.0411	.00158	-.0176	.0361
.0492	.0531	.0270	.0576	.00289	-.0133	.040
.0927	.0822	.0644	.0834	.0622	.0858	.111
.12	.0938	.0804	.0953	.096	.125	.139
.16	.132	.139	.142	.14	.172	.173
.23	.22	.224	.219	.22	.247	.228
.294	.274	.294	.284	.291	.319	.282
.456	.415	.438	.432	.435	.459	.394
.590	.590	.619	.624	.62	.631	.540
.913	.902	.912	.946	1.1	1.11	.999
1.18	1.20	1.21	1.27	1.2	1.11	1.06
1.6	1.6	1.62	1.71	1.58	1.53	1.46
1.83	1.89	1.87	1.97	1.84	1.77	1.75
2.3	2.4	2.40	2.51	2.37	2.27	2.41
2.9	3.0	3.0	3.06	3.1	2.98	3.44
3.2	3.3	3.3	3.3			
4.56	4.54	4.47	4.52			
5.9	6.0	5.85	5.26			
8.1						
9.13						
12.0						

Table 4-A.--Continued

Actual Flow (10^{-3} cm/min)	Porosity 49.6%			Porosity 48.2%		
	Eq. (5)	Eq. (9)	Eq. (14)	Eq. (5)	Eq. (9)	Eq. (14)
.0325				.0211	-.0007	.033
.0492	.0369	.0057	.0473	.0385	.0216	.049
.0927	.0872	.0684	.0937	.080	.0702	.087
.12	.13	.117	.133	.13	.126	.132
.16	.16	.152	.163	.16	.16	.16
.23	.24	.232	.23	.24	.24	.23
.294	.30	.296	.292	.306	.306	.289
.456	.461	.454	.44	.470	.473	.449
.590	.576	.57	.55	.618	.629	.597
.913	.926	.92	.90	.917	.938	.91
1.18	1.23	1.23	1.21	1.2	1.23	1.22
1.6	1.7	1.66	1.66	1.6	1.6	1.62
1.83	1.87	1.86	1.86	1.72	1.77	1.80
2.3	2.3	2.32	2.3			
2.9	3.0	3.01	3.0			
3.2	3.2	3.22	3.22			
4.56	4.51	4.5	4.47			
5.9						
8.1						
9.13						
12.0						

Table 5-A. Physical and chemical properties of subsoil material.

	pH	Na	Sol. Salts		NO_3
	Paste	meq/l	$EC_e \times 10^3$	ppm	ppm
Springerville	7.4	2.1	.60	420	8
Brolliar	5.4	.78	.40	280	10

	PO_3	CEC	N	Ca	Mg
	ppm	meq/100 g	KJELDAHL %	meq/l	Sat. Ext.
Springerville	.5	41.2	.26	2.1	.8
Brolliar	5.0	60.0	.35	2.3	1.7

	Sand	Silt	Clay	O.M.
	%	%	%	%
Springerville	9	41	50	1.47
Brolliar	30	50	20	5.39

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