# A GROUND WATER MODEL OF THE WILLIAMS LAKE WATERSHED HUBBARD COUNTY, MINNESOTA

by

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## ABSTRACT

Two-dimensional ground-water models were used to simulate ground-water flow in the Williams Lake watershed. The models were used to estimate the seepage rates to and from Williams Lake, and to determine model sensitivity to several parameters governing the flow systems. The models are based on a two-dimensional, block-centered, finite-difference scheme. A steady-state model was developed for a period in early July 1979, and this model provided initial conditions for a transient simulation through June 1981. The results of the modeling and sensitivity analysis showed that the model is most sensitive to the thickness, and hydraulic conductivity of the lake bottom sediments.

# INTRODUCTION

Lake and ground water interchange is one of several processes that affect the chemical composition of lakes. To gain understanding of this process numerical ground water models are being used to help define the time and space relationships of lake and ground water interchange.

Numerical simulation of hypothetical lake and ground water settings has been done in the past (Winter 1976, 1978) and these studies indicated the need for experimental field sites to determine realistic estimates and combinations of various components of the hydrologic system.

Williams Lake in north-central Minnesota was selected by the U.S. Geological Survey (U.S.G.S.) as one of several field sites in various climatological and geological settings to help define and quantify the relationship of lakes with all other components of the hydrologic cycle. One phase of the Williams Lake project, and the purpose of this study, is the use of two-dimensional numerical models to evaluate the magnitude of ground water flow through the lake, the sensitivity of ground-water flow through the lake to variations in parameter values, and the sensitivity of hydrualic head values in the watershed to variations in parameter values.

Field work was conducted in 1980 and 1981 to define hydrologic parameters and boundary conditions of the models in this study. Many climatological parameters used in this study were obtained by direct and indirect methods at the Williams Lake site since field work began in 1977.

The models used were first calibrated at steady state. Parameter sensitivities were then analysed. The objective of the sensitivity analysis is to guide data collection in similar studies. The response of lakes to nutrient loading, and the response of lake levels to variations in climatic conditions are the main focuses of many studies on lake and ground water relationships. By finding the most sensitive components of the lake and ground water relationship time and money can be saved.

The calibrated steady state model provided initial conditions for a transient model which covered the period from July 1979 to June 1981. The purpose of the transient model was to obtain estimates of the magnitudes of seepage to and from the lake on a seasonal basis, and to help verify the steady-state model.

# HYDROLOGY OF THE STUDY AREA

## Location and Extent of Area

Williams Lake is in Hubbard County in north-central Minnesota, about 150 miles north northwest of Minneapolis and 105 miles west of Duluth (Figure 1). It lies mainly in the S 1/2 , Section 12, Township 140 N., Range 32 W.

Williams Lake is approximately 3000 feet north of Crystal Lake and 1300 feet east of Mary Lake and Doe Lake. The water surface of Crystal Lake is about 14 feet higher than Williams Lake. Mary Lake and Doe Lake are about 13 feet and 10 feet lower than Williams Lake, respectively (Figure 2). The area modeled in this study is approximately 2600 acres.

## Previous Investigations

Erickson (1981) used Williams Lake as a site to study the use of seepage meters to calculate lake seepage rates.

Erickson used two methods to calculate seepage. In the first method he utilized piezometers to define the horizontal hydraulic head gradient near the lake, and combined this information with Darcy's Law and estimates of the hydraulic conductivity to obtain horizontal littoral seepage rates. The other method utilized a seepage meter designed by Lee (1972). This method involves emplacing a cylinder with one end open, 57 cm in diameter by 15 cm in depth, into the lake bottom. The exposed top of the cylinder was fit with a stopper and tubing so that any flow into

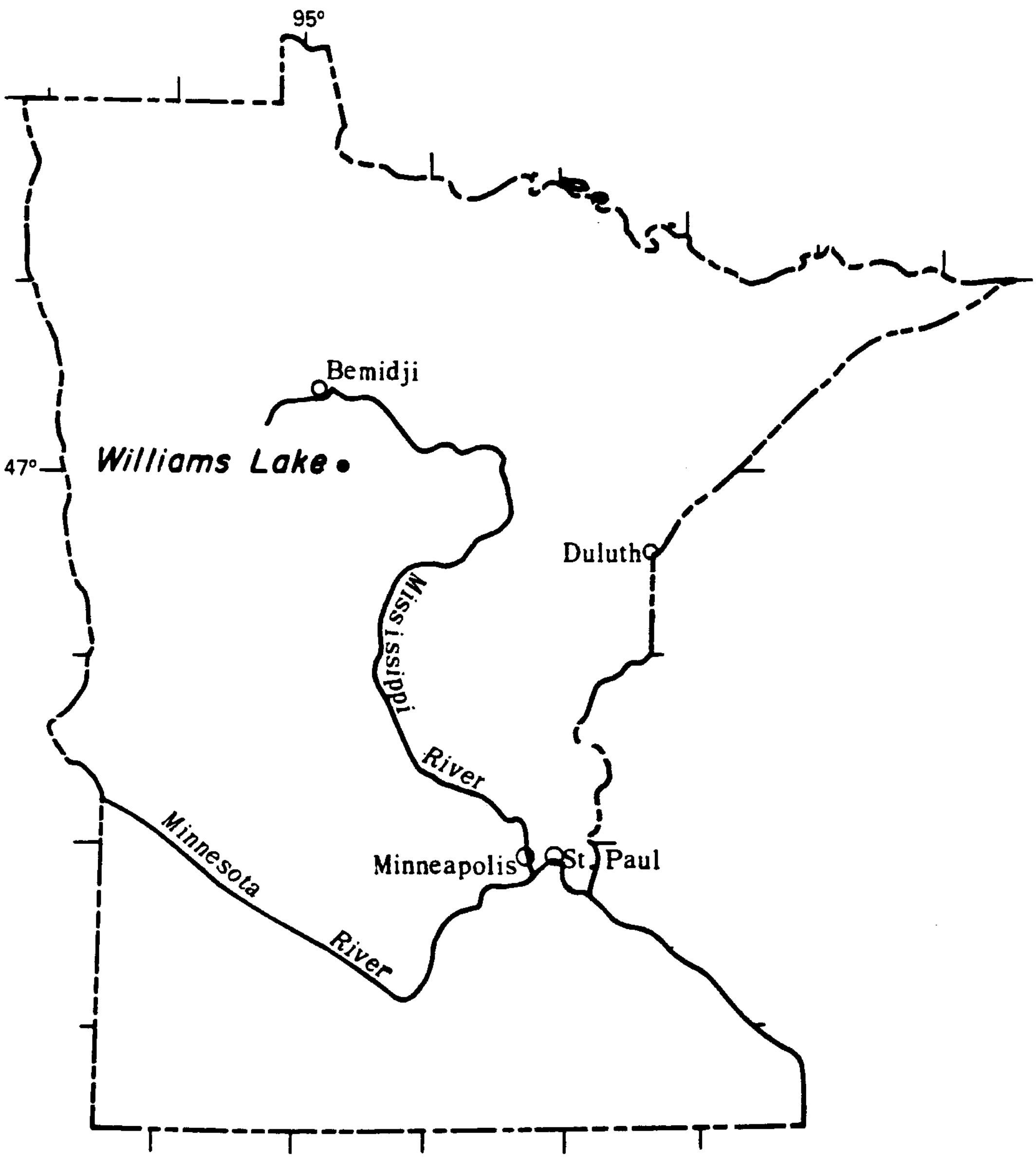



Figure 1. Location Map

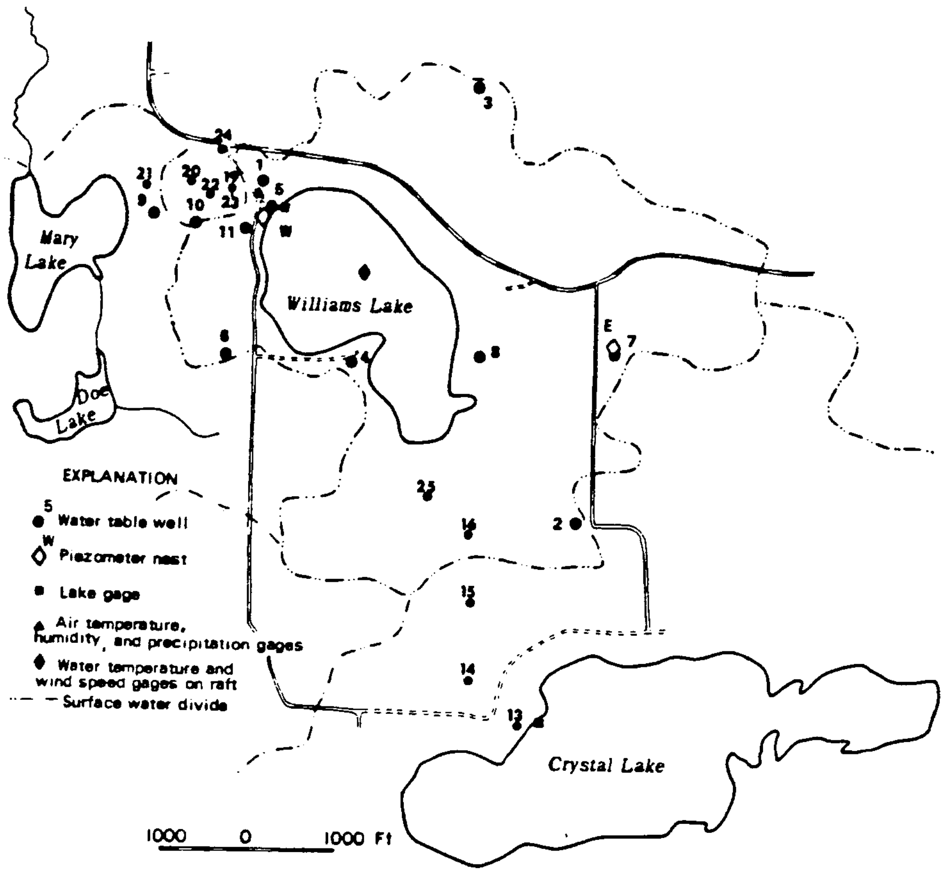



Figure 2. Site Map

or out of the area of the lake bottom covered by the cylinder would pass through the tubing. A plastic bag was then partially filled with a known amount of water and secured over the projecting end of the tubing. This apparatus was left for varying amounts of time, and then the bag was removed and the quantity of water in the bag was measured. This provided a qualitative measurement of the direction of flow (in seepage or out seepage) and hopefully a quantitative measurement of the magnitude of the flow.

Erickson concluded that the seepage meters were useful in determining areas of inflow and outflow, but that the techniques for quantifying the amount of inflow and outflow were not refined enough to make their results reliable.

Siegel and Winter (1980) prepared a progress report on the first year of the U.S.G.S. Williams Lake Study. This report described the physiography, soils, vegetation, hydrologic and climatic settings, the work done in 1978, and an example of the long-term study approach. Their report also dealt with the problems of measuring precipitation and evaporation and regionalization and computational techniques for determining these parameters.

Siegel and Winter used data from the first field season to prepare initial two dimensional vertical-plane (cross-sectional) models of the study area. These models, showed the benefits of using modeling to guide future work.

Groschen (1981) studied the major hydrogeochemical processes that occur in the Williams Lake watershed. He determined that the watershed is dominated by carbonate water geochemistry, and that the lake serves as a sink for $CaCa_3$.

Munter (1979) used finite-difference ground water flow models to simulate the flow systems around three lakes set in glacial terrain in Wisconsin. Munter used two-dimensional cross-sectional models on Snake, Bass and Nepco Lakes, and a three-dimensional model on Nepco Lake to estimate seepage rates to and from the lakes and to evaluate the parameters governing the flow systems.

Munter demonstrated that flow rate estimations in two-dimensions are superior to one-dimensional Darcy's Law calculations, and that the magnitude of lake seepage rates was found to be a function of the presence or absence of fine-grained littoral sediments, and the vertical hydraulic conductivity of the sediments.

Anderson and Munter (1981) used a two-dimensional horizontal ground water flow model to simulate ground water flow system around Snake Lake, Wisconsin, and to analyse the formation of a stagnation point beneath Snake Lake which has the effect of reducing or eliminating out-seepage from the lake. Anderson and Munter found that a ground water mound on the down-gradient side of the lake in the regional flow system was necessary for the formation of a stagnation point.

Larson, McBride and Wolf (1975) constructed two-dimensional areal and cross-sectional finite-difference models of the Pearl-Sallie Lakes area in west-central Minnesota. The models were used to obtain flow rates to and from the lakes, and to define areas where more data collection was necessary. Data input for the cross-sectional model was more difficult than for the areal model because of a lack of information on the stratigraphic relationships of the geologic materials, and the lack of information on vertical hydraulic gradients.

The Pearl-Sallie Lakes study demonstrated the interaction of the lakes with the ground water flow system, and indicated the need for further research on lake and ground-water interaction.

## Climate

The average annual precipitation in the Williams Lake area is 26 inches (Baker and Strub, 1967) and about 3 inches is usually in the form of snowfall (Watson, 1974). Annual lake evaporation is about 26 inches based on averages from 1946-1955 (Kohler et al, 1959).

North-central Minnesota experiences a wide range in temperature. The highest temperatures are in July with an average monthly maximum of 80$^{o}$F. The lowest temperatures are in January with an average monthly maximum of less than 20$^{o}$F (Baker and Strub, 1965).

## Physiographic Setting

The Williams Lake watershed has the physiographic characteristics of ice contact deposits. The lake is 10 miles east of the Itasca moraine which was formed by the Wadena Lobe of Wisconsin glaciation that moved into Minnesota from the northwest. The Itasca moraine consists largely of silty, calcareous, buff colored till.

The St. Croix moraine is about 3 miles east of the lake. The Superior lobe of Wisconsin glaciation formed the St. Croix moraine, which consists of sandy, brown till that is generally not calcareous. The

Superior Lobe moved into the area from the east out of the Lake Superior Basin.

Rolling hills produce relief that is greater than 100 feet. Williams lake has no streams entering or leaving it, and the terrain is hummocky in areas of undrained depressions. Soils in the area are sandy, light colored, and poorly drained (Arneman, 1969).

Williams Lake has a surface area of 90 acres and a maximum depth of about 35 feet. Organic, gelatinous sediments are about 20 feet thick near the center of the lake. Along the north and west shore about 3 feet of marly sediment overlies medium to coarse sand (Siegel and Winter, 1980; Groschen, 1981). The extent of the marly sediments has not yet been determined. The morphometric characteristics of the lake are shown in Table 1 and Figure 3.

## Ground Water Flow System

The surficial geologic materials in the watershed are mostly sand and gravel, but clay lamina and lenses can be found near the surface in areas north and east of the lake.

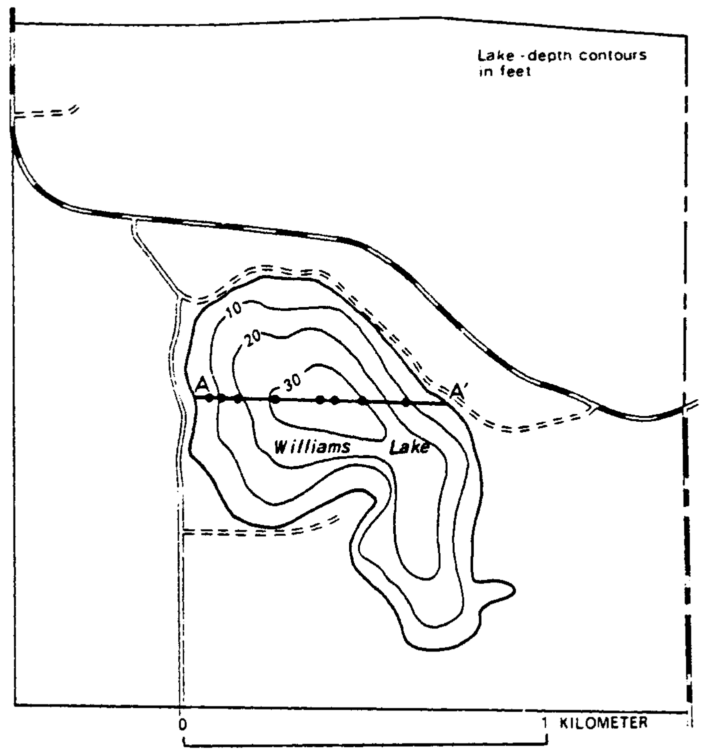
The test hole logs (Appendix 1) show a system of sand, silt and clay layers that vary in total thickness from about 30 feet near Williams Lake observation well 8 (WL-8), to about 157 feet near WL-12. Thin clay lamina (less than 1 foot) are difficult to detect when test drilling with an auger type drilling rig. Drilling done in the summer of 1980 suggested the presence of such lamina, and road cut inspections in the area confirmed several such lamina near the surface. These clay layers

Table 1. -- Morphometric characteristics of Williams Lake

| | | |
|---|---|---|
| Surface area | = | 90 acres |
| Drainage basin area | = | 560 acres |
| Maximum depth | = | 35 $\pm$ feet |
| Mean depth | = | 17 feet |
| Maximum length | = | 3,220 feet |
| Orientation | = | NW - SE |
| Maximum width | = | 1,740 feet |
| Mean width | = | 1,240 feet |
| Volume | = | $6.7 \times 10^7$ cubic feet |
| Length of shoreline | = | 9,430 feet |
| Mean Surface Elevation | = | 1383 $\pm$ |

Definitions:

$$\text{Mean depth} = \frac{\text{lake volume}}{\text{lake area}}$$



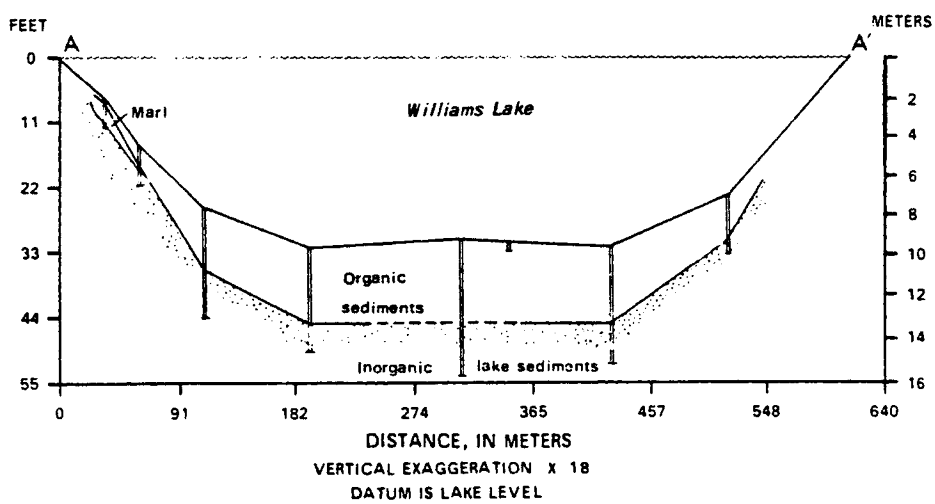




Figure 3. Williams Lake Cross-Section

are lenticular in cross section and are probably quite irregular in plan view. They are generally less than 2 feet in thickness and tens of feet in the horizontal dimension.

The clay layers are important hydrologically because they cause the system to be anisotropic with the horizontal hydraulic conductivity ($K_H$) commonly being several orders of magnitude greater than the vertical hydraulic conductivity ($K_V$). Winter (1976) used ratios of horizontal hydraulic conductivity to vertical hydraulic conductivity ($K_H$:$K_V$) of 1:1 to 10,000:1 in modeling theoretical flow systems similar to Williams Lake.

The horizontal hydraulic conductivity of the aquifer is more important in a horizontal flow model, such as this study, than the vertical hydrualic conductivity. This is because a fundamental assumption in the model used in this study is that flow is taking place only in the horizontal plane with no vertical flow components. This is discussed further in a later section.

Values of hydraulic conductivity for materials found in the study area were obtained by particle size estimation of drilling logs in conjunction with Table 2, and by piezometer tests.

Bouwer and Rice (1976) presented a method for determining the in situ hydraulic conductivity values for partially penetrating piezometers in an unconfined aquifer.

The normal technique in this method is to withdraw a volume of water from the well instantaneously and then measure the response of the water table with time. The hydraulic conductivity can then be calculated from the equation:

$$K = \frac{r_c^2 \ln(R_e/r_w)}{2L} \frac{1}{t} \ln \frac{y_o}{y_t}$$

where

$K$ = hydraulic conductivity of the aquifer.
$r_c$ = inside radius of the well.
$R_e$ = effective radius over which the head difference is dissipated.
$r_w$ = radial distance between the well center and the undisturbed aquifer.
$L$ = vertical distance of the perforated section of the casing.
$y_o$ = total amount that the head in the well was reduced.
$y_t$ = vertical distance between water level in well at equilibrium and water level at time t.
$t$ = time since the instantaneous lowering of the water level.

Bailers of various diameters and lengths were used to remove water from the wells. The volume of water removed was measured with a

Table 2 -- Range of values of hydraulic conductivity (after Morris and Johnson, 1967)

| Predominant material | Hydraulic Conductivity (ft/day) |
|---|---|
| Clay or silt . . . . | Less Than 1 |
| Sand, fine . . . . | 1-20 |
| Sand, medium . . . | 20-100 |
| Sand, coarse to very coarse; . . . . | Greater Than 100 |

graduated cylinder, and the amount the water level was lowered was calculated from the known dimensions of the piezometer. The response of the water level in the well was measured with a graduated electric tape and a stop watch.

The results of these tests were only partially satisfactory. For piezometers completed in units with a conductivity value less than approximately 10 feet/day the conductivity obtained from the test is believed to be fairly representative. For piezometers completed in materials with conductivity values larger than about 10 ft/day the results are probably low. This may be due to such factors as a partially clogged or undeveloped well screen, rapid flow through the well screen with inertial forces becoming important near the well, and a response to stress in the well that is too fast to measure without pressure transducers. The results of the piezometer tests are given in Table 3.

Hvorslev (1951) presented a method of calculating the horizontal hydraulic conductivity using piezometer tests based on "basic time lag theory". Most of the screened portions of the piezometers in the Williams Lake watershed are 2 to 7 feet below the phreatic surface. Hvorslev's method assumes that the aquifer is continuous in the horizontal and vertical directions near the well screen. The proximity of the well screens to the phreatic surface at this site violates Hvorslev's assumptions.

The values presented in Table 3 represent values of the hydraulic conductivity for a single point in the aquifer. Piezometer tests, such as the bailer method used here, stress only a very small volume around the piezometer and the effect of heterogeneities on the microscale such as pebble orientation will affect the results while heterogeneities on the macroscale such as individual clay lamina will not affect the results.

Table 3 - Results of Piezometer Tests

| Well | Hydraulic Conductivity (feet/day) |
|---|---|
| WL-2 | > 8 |
| 4 | > 6 |
| 6 | > 16 |
| 7 | > 10 |
| 8 | > 17 |
| 13 | > 8 |
| 14 | > 20 |
| 16 | > 16 |
| 19 | 8 |
| 20 | 6 |
| 22 | 1 |
| 24 | > 30 |
| 25 | > 27 |

To help define the configuration of the water table in the area, and the horizontal and vertical gradients present, 36 piezometers were installed by the U.S.G.S. (Appendix 1). These include seven in a piezometer nest that vary in depth from 24 feet to 299 feet and two in a piezometer nest that are 128 feet and 344 feet deep.

The piezometric surface (Figure 4), based on observation well records, is quite flat with an average horizontal gradient of 0.005 feet/foot. Flow generally is from Crystal Lake north-northwest to Williams Lake and then westward to the Doe and Mary Lake watersheds.

Figure 5 is a cross-section the location of which is shown on Figure 4. For the purposes of this study the ground-water models will simulate only the upper sand layer above the first major till unit shown in the cross section.

Freeze and Witherspoon (1967) have shown that a layer only 1 order of magnitude lower in permeability than the surface layer can form an effective lower boundary to a near surface flow system. In the Williams Lake area, the underlying till is composed of clay and is easily several orders of magnitude lower in permeability than the surficial sands.

The vertical gradients in the surficial sand layer are measured only at the west piezometer nest and are less than 0.002 (Siegel and Winter, 1980). The west piezometer nest also has two deeper piezometers that measure the head in two deeper sand layers. The east piezometer nest consists of two piezometers, one completed in the surficial sand layer, and the other completed in a buried sand layer.

Cross-sectional models of the Williams Lake area (Siegel and Winter, 1980) indicated that the upper till unit in Figure 5 isolates the upper sand unit from the lower sand unit. A cross-section illustrating

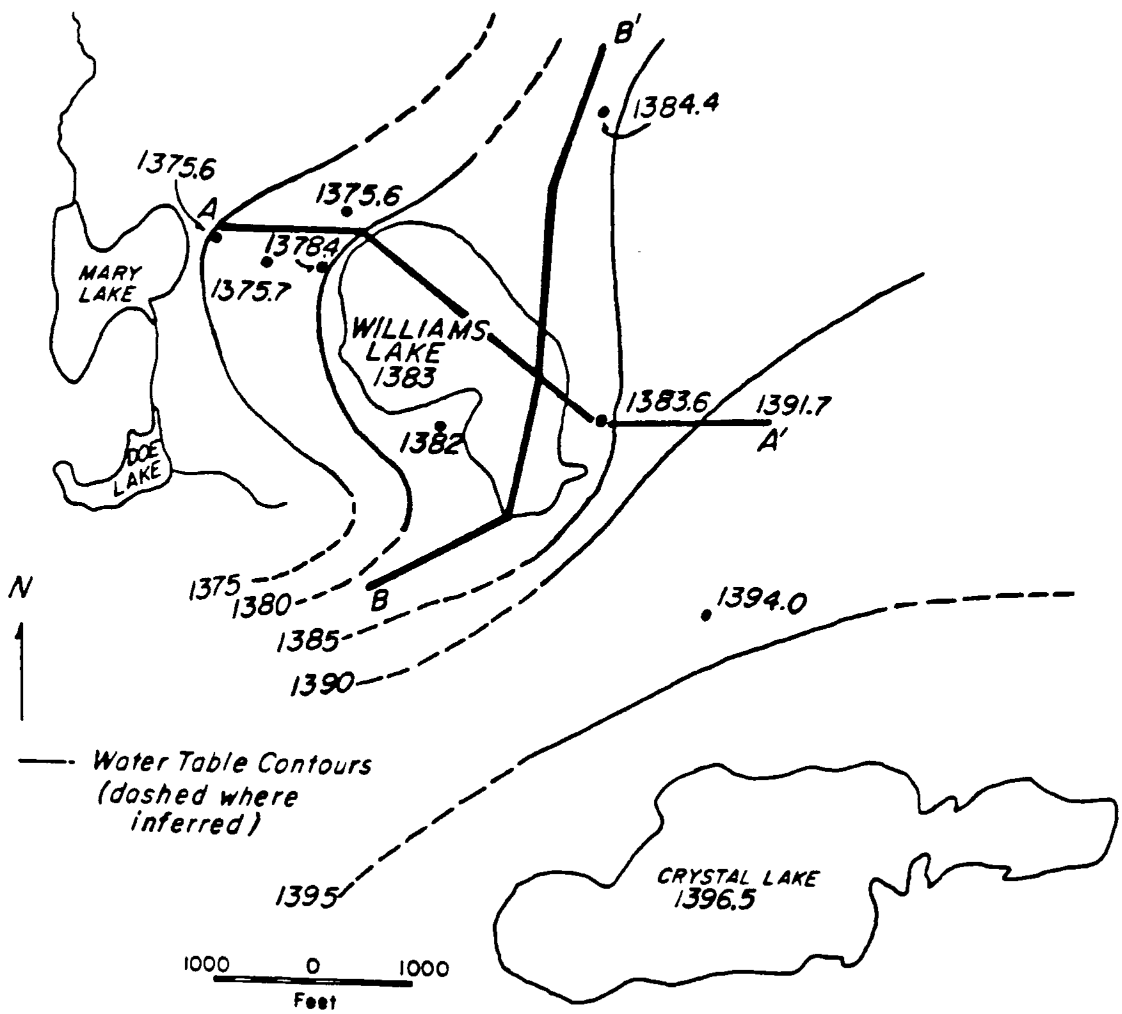



Figure 4. Water Table Contour Map

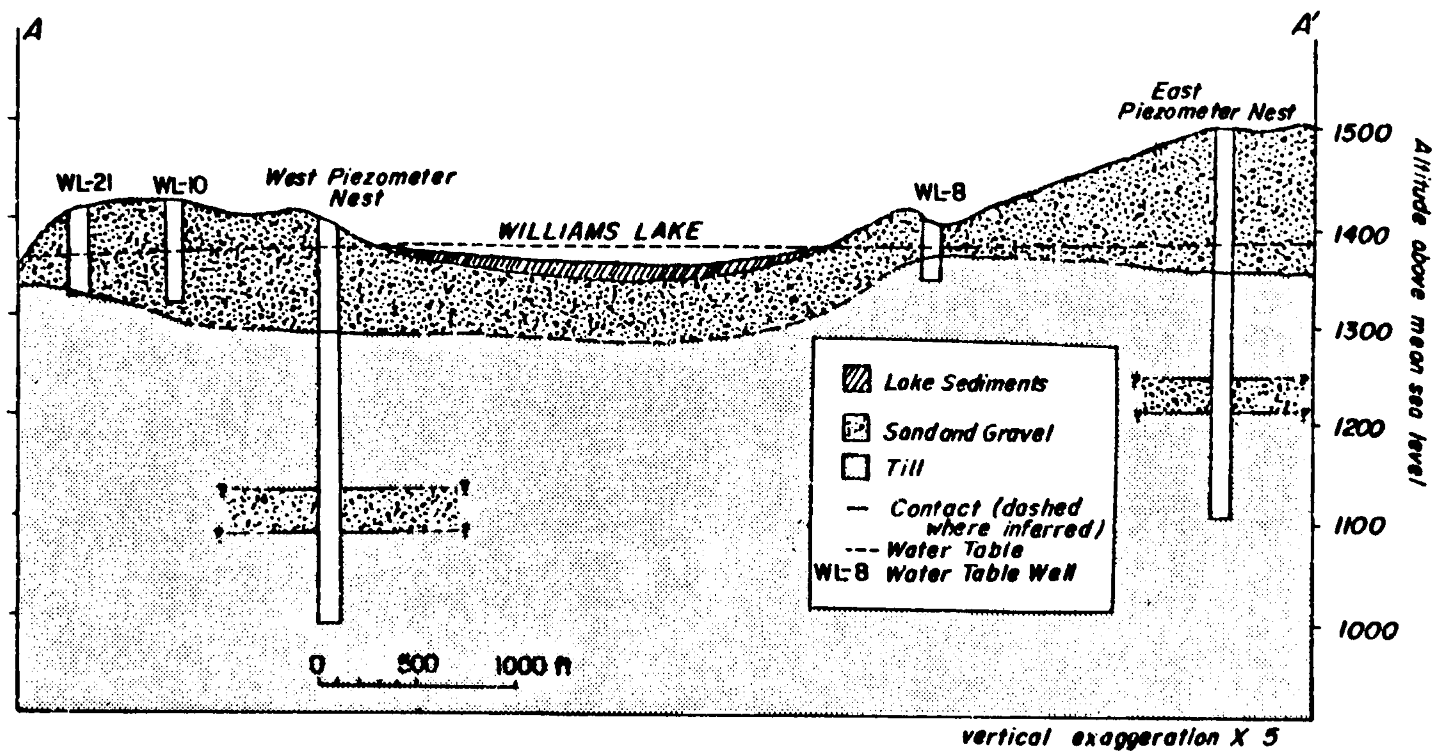



Figure 5. Geologic Cross-Section

ground-water flow near Williams Lake as simulated by Siegel and Winter is shown in Figure 6. The near orthogonality of the equipotentials with the top of the till surface indicates that flow is along the base of the surficial sand and not across the boundary between the till and the sand. The nearly vertical equipotentials in the surficial sand indicate that flow is horizontal in that layer. The vertical gradients near the right and left margins of Figure 6 are caused by the selection of boundary conditions for this model.

Figure 7 is cross-section B-B' that is shown in Figures 4 and 6. This plane of section is along the surface expression of the 1383 foot equipotential in Figure 4, and intercepts the 1382 foot equipotential at point "X" in Figure 6. This cross-section illustrates the vertical gradients in the lake sediments, with flow becoming horizontal (into the plane of the diagram) rapidly after leaving the lake sediments.

The rapid transition to horizontal flow can be demonstrated theoretically by analysing the refraction of a streamline as it passes from an area of low hydraulic conductivity (lake sediments), to an area of higher hydraulic conductivity (aquifer materials). The refraction occurs according to the equation (Davis and DeWiest, 1966):

$$\frac{K_1}{\tan \theta_1} = \frac{K_2}{\tan \theta_2}$$

where

$K_1$ = hydraulic conductivity of the lake sediments

$K_2$ = hydraulic conductivity of the aquifer

$\theta_1$ and $\theta_2$ are the angles illustrated in Figure 8

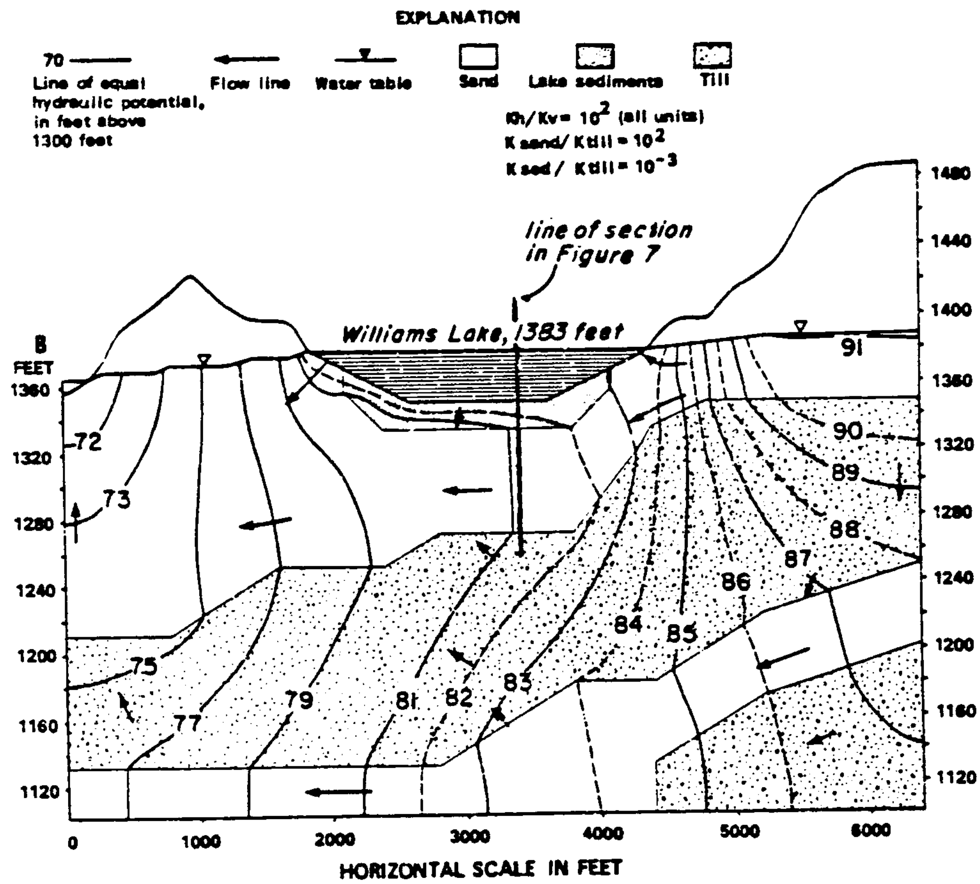



Figure 6. Cross-Section Along a Stream Line

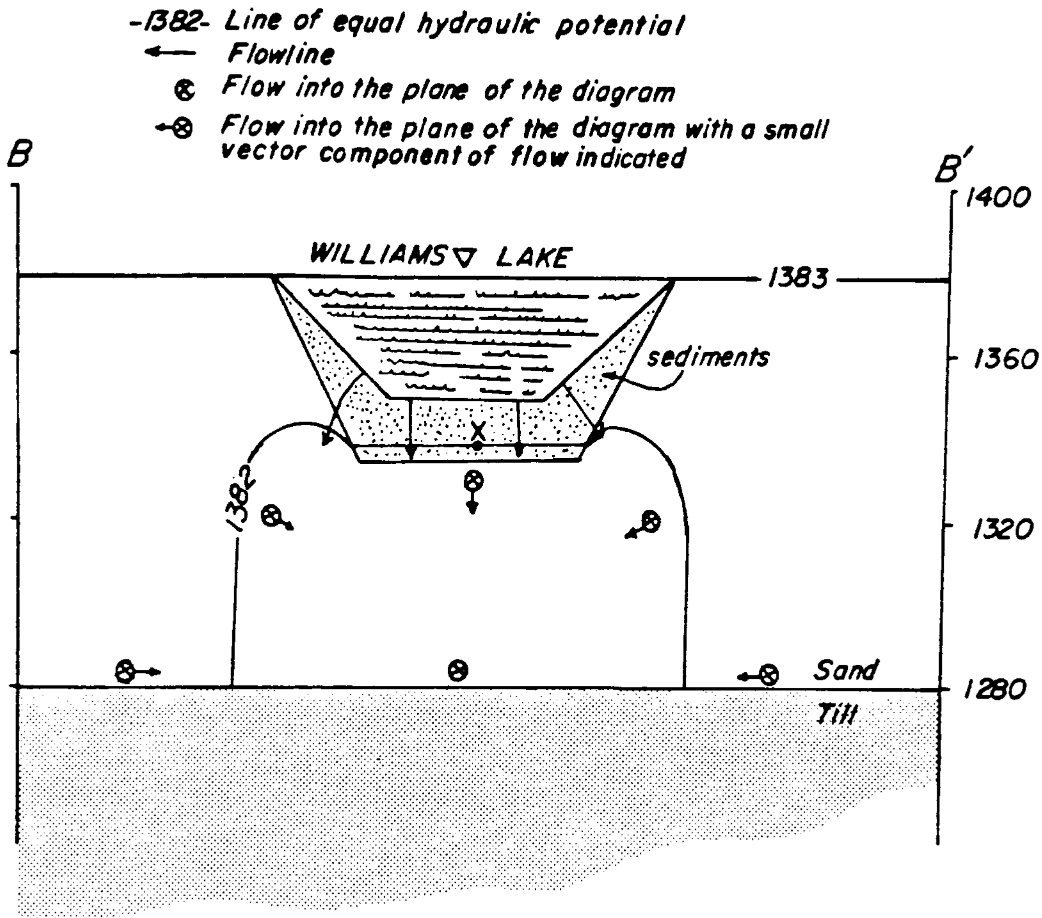
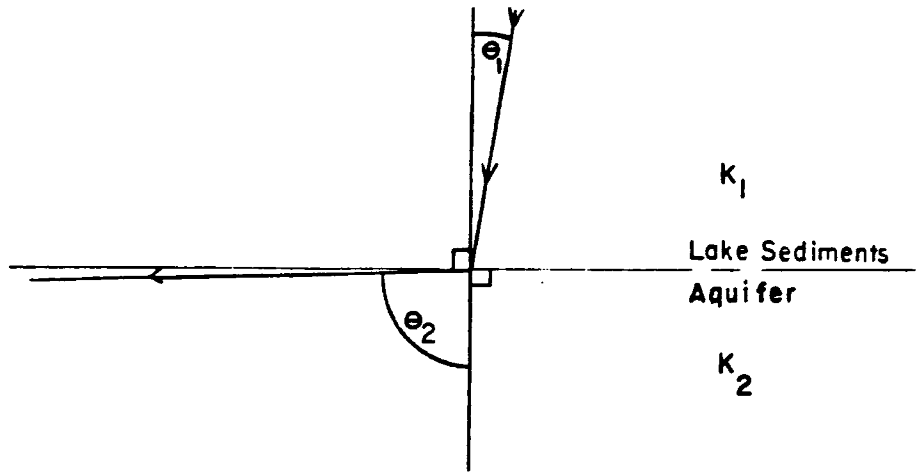



Figure 7. Cross-Section Along an Equipotential

If we assume that the contrast in hydraulic conductivity between the materials is three orders of magnitude, and that flow through the lake sediments is sub-vertical ($\theta_1 = 10^0$) the result is a refraction to nearly horizontal flow in the aquifer. An example of this calculation is given in Figure 8.

The cross-sections in Figures 6 and 7, and the water table map in Figure 4, give a three-dimensional view of the flow system in the Williams Lake watershed. They show that flow is horizontal except in the lake sediments where flow is vertical. By using the leakage option of the program used to model flow at this site, the vertical flow of water through the lake sediments is simulated, and the rest of the system can be modeled assuming two-dimensional horizontal flow.




$$\frac{K_1}{K_2} = 10^{-3} = \frac{\tan\theta_1}{\tan\theta_2}$$

$$\frac{\tan\theta_1}{10^{-3}} = \tan\theta_2$$

$$\frac{\tan(10°)}{10^{-3}} = \tan\theta_2$$

$$89.67° = \theta_2$$

Figure 8. Refraction of a Stream Lines

## DESCRIPTION OF GROUND WATER FLOW MODEL

### General Aspects of the Program

A two-dimensional model was constructed over the Williams Lake watershed and surrounding flow boundaries. The program used was a finite difference ground water flow model developed by Trescott, Pinder and Larson (1976) for two dimensions.

The code was modified (Appendix 2) to allow the model to simulate the problem more closely. In the transient simulation the head at the lake, recharge, and evapotranspiration were changed between stress periods (seasons of the year) to allow for climatic and seasonal variations. The stress periods are discussed in the section on transient simulation.

The partial differential equation of non-steady ground water flow in two dimensions may be written as (Bredehoeft and Pinder, 1970).

$$\frac{\partial}{\partial x}\left(K_{xx}\, b\, \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\, b\, \frac{\partial h}{\partial y}\right) = S\, \frac{\partial h}{\partial t} \pm W(x,y,t)$$

in which

$K_{xx}$, $K_{yy}$ are the principal components of the hydraulic conductivity tensor ($Lt^{-1}$);

$S$ is the specific yield when the aquifer is unconfined, and the storativity when the aquifer is confined (dimensionless);

$b$ is the saturated thickness of the aquifer ($L$);

$W(x,y,t)$ is the volumetric flux of recharge or withdrawal per unit surface area of the aquifer ($Lt^{-1}$).

This equation assumes the principal axes of the conductivity tensor are aligned with the x and y coordinates.

A computer is used to solve the above differential equation by approximating it with a set of linear algebraic equations, and solving for the unknown head distribution. The Strongly Implicit Procedure (SIP) solution algorithm was used for all the simulations in this study.

At each node in the model, the following aquifer properties and characteristics are specified.

1. Hydraulic conductivity, (L/T)
2. Aquifer base altitude, (L)
3. Initial estimate of the head distribution, (L)
4. Storage Coefficient
5. Specific Yield
6. Hydraulic conductivity of confining bed, (L/T)
7. Head on the other side of the confiding bed, (L)
8. Thickness of the confining bed, (L)
9. Land surface elevation, (L)
10. Recharge rate, (L/T)
11. Lateral and longitudinal dimensions of the area represented by each node, (L)

In addition to aquifer properties, any sources or sinks, if present, are included. In this study, evapotranspiration and seepage to the lake were the only sink terms included. Wells in the study area do not withdraw a significant amount of water. Precipitation and seepage from the lake were the only source terms.

Steady-state conditions are achieved when the amount of discharge from the system is equal to the amount of recharge and no change of head occurs with time. For this study the period from late June to early July 1979 was selected to represent a quasi-steady state period. The hydrologic systems in this area are rarely in true balance with each other because the extreme climatic variations cause seasonal fluctuations in piezometer and lake hydrographs. The period chosen for steady-state simulation in this model represents an inflection point in most of the hydrographs obtained in this study (Appendix 3). At an inflection point the system should be near a balance between recharge and discharge.

A 1,170 node (39 x 30) variable grid was used to model the surficial flow system in the Williams Lake area (Figure 9).

Node size varied from 540,000 square feet near the boundary of the model, to 40,000 square feet in the interior portion of the model. The variable grid sizes were used to insure adequate definition of the geometry of the lake and the head distribution near the lake.

## Boundary Conditions

Hydrologic characteristics along the aquifer boundaries are specified by selecting one of the following conditions.

1. No-flow across the boundary.
2. Constant flux across the boundary.
3. Constant hydraulic head at the boundary.

A no-flow boundary is inserted around the border of the model as a computational expediency, and constant head or constant flux boundaries are placed inside this border.

Streamlines have been drawn on Figure 9 to indicate the approximate direction of flow near the boundaries of the study area based on the observed head distributions. Because the streamlines are tangent to the velocity vector at every point in the flow field there can be no flow across a streamline. These streamlines are then modeled as no flow boundaries as shown in Figure 9. This is a valid approximation as long

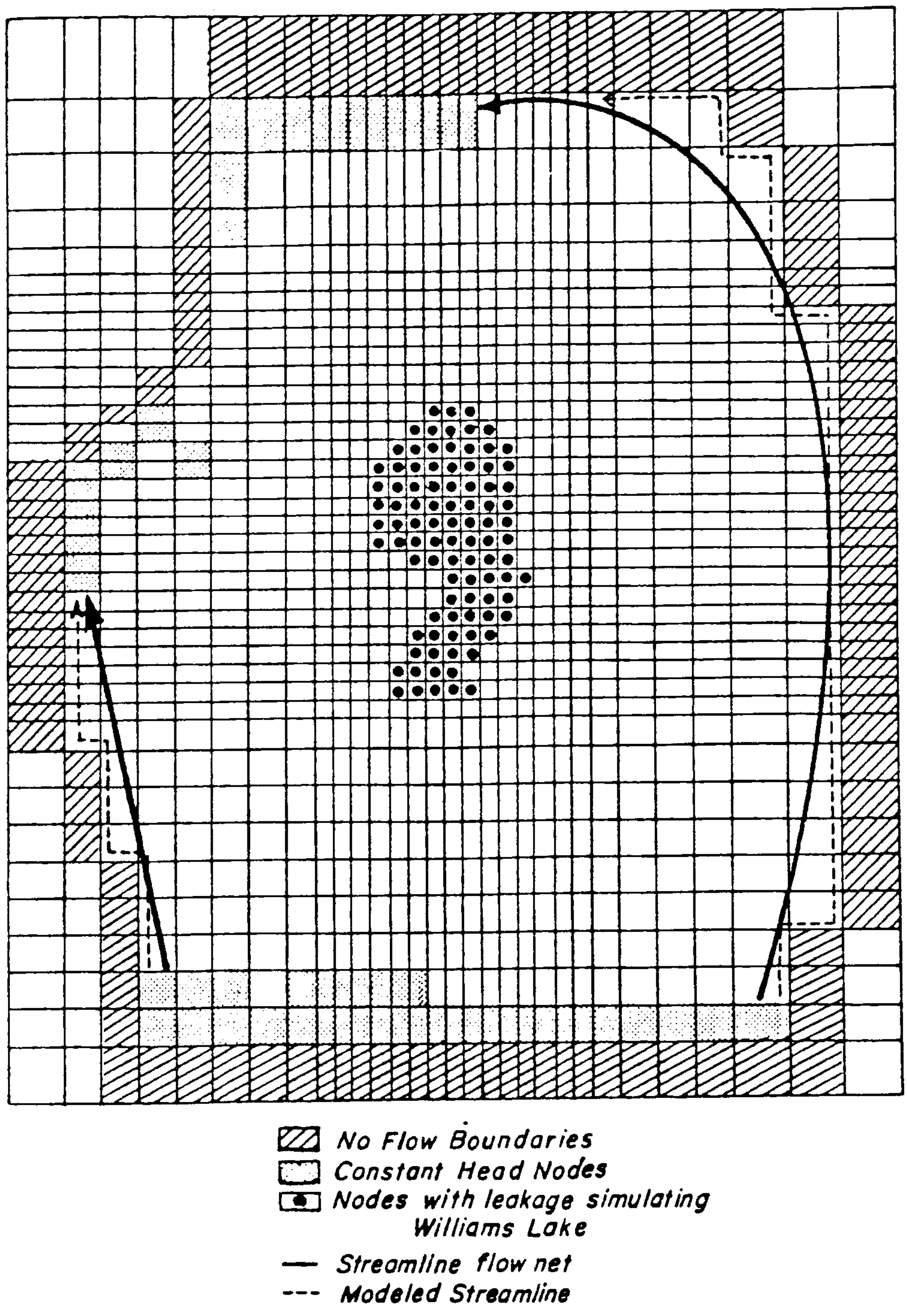


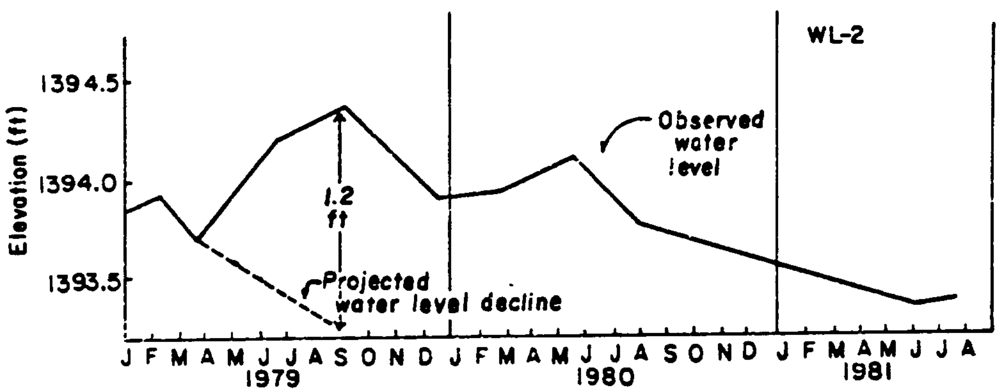

Figure 9. Finite Difference Mesh

as these boundaries are kept away from areas near Williams Lake where the head distribution is critical in determining fluxes into and out of the lake. Constant head nodes were assigned when the aquifer extended beyond the boundary of the model and did not parallel a stream line, and when a lake was lying directly on the boundary.

## Initial Conditions

The specific yield ($S_y$) of the aquifer was assumed to be constant, except for the confined portion of the system under the lake. A value of 0.2 was used by Helgeson (1977) in the Pineland Sands study which was just to the west of the Williams Lake area. Helgeson determined this value through interpretation of pumping test data. The 0.2 value was used in this study for all unconfined portions of the aquifer. The portion of the aquifer under the lake is confined by lake sediments, and is assumed to have a storativity (S) of 0.02. The specific yield and storativity values used in this study are assumed values, and sensitivity analysis of these parameters in future transient simulations is recommended.

The major source of recharge to the aquifer in this area is snowmelt and rain during the spring. Most rain entering the soil during June through October is evaporated and transpired (Helgeson, 1977). Determination of seasonal recharge rates were made using 1979-1981 water level records from observation wells in the study area. An example hydrograph and computation of recharge is shown in Figure 10. The largest error in this calulation is the estimation of specific yield. A recharge rate of 0.24 ft/year was used for the steady-state simulation.




Spring recharge = (water-level rise) x (specific yield)

= 1.2 feet x 0.20

= 0.24 feet

Figure 10. Hydrograph with Recharge Calculation

Recharge rates for the transient simulations were calculated in this manner and are presented in the section on transient simulation.

The following contour maps were used to determine model values for the various parameters: 1) hydraulic conductivity (Figure 11), 2) land surface elevation (Figure 12), 3) aquifer base altitude (Figure 13) and 4) Williams Lake sediment thickness (Figure 14).

Potential evaporation rates were calculated using mass transfer methods. The mass transfer calculations for 1979 were done by the author, and those for 1980 and 1981 were done by U.S.G.S. personnel of the Lakes Hydrology Group in Denver, Colorado.

The mass transfer methods relate the difference in water vapor pressures for the air temperature and that calculated for the water temperature to potential evaporation. The equation is:

$$\text{Evaporation} = Nu\,(e_o - e_a)$$

where

$u$ = wind speed 2 meters above the water surface;

$e_o$ = saturation water vapor pressure at the temperature of the water surface;

$e_a$ = water vapor pressure of the air 2 meters above the water surface;

$N$ = a constant for a given lake

The coefficient N was determined for Williams Lake using the method proposed by Harbeck (1972). Details of the mass transfer method are beyond the scope of this paper but are reviewed by Sturrock (1977) and Winter (1981). Potential evapotranspiration rates were assumed to be

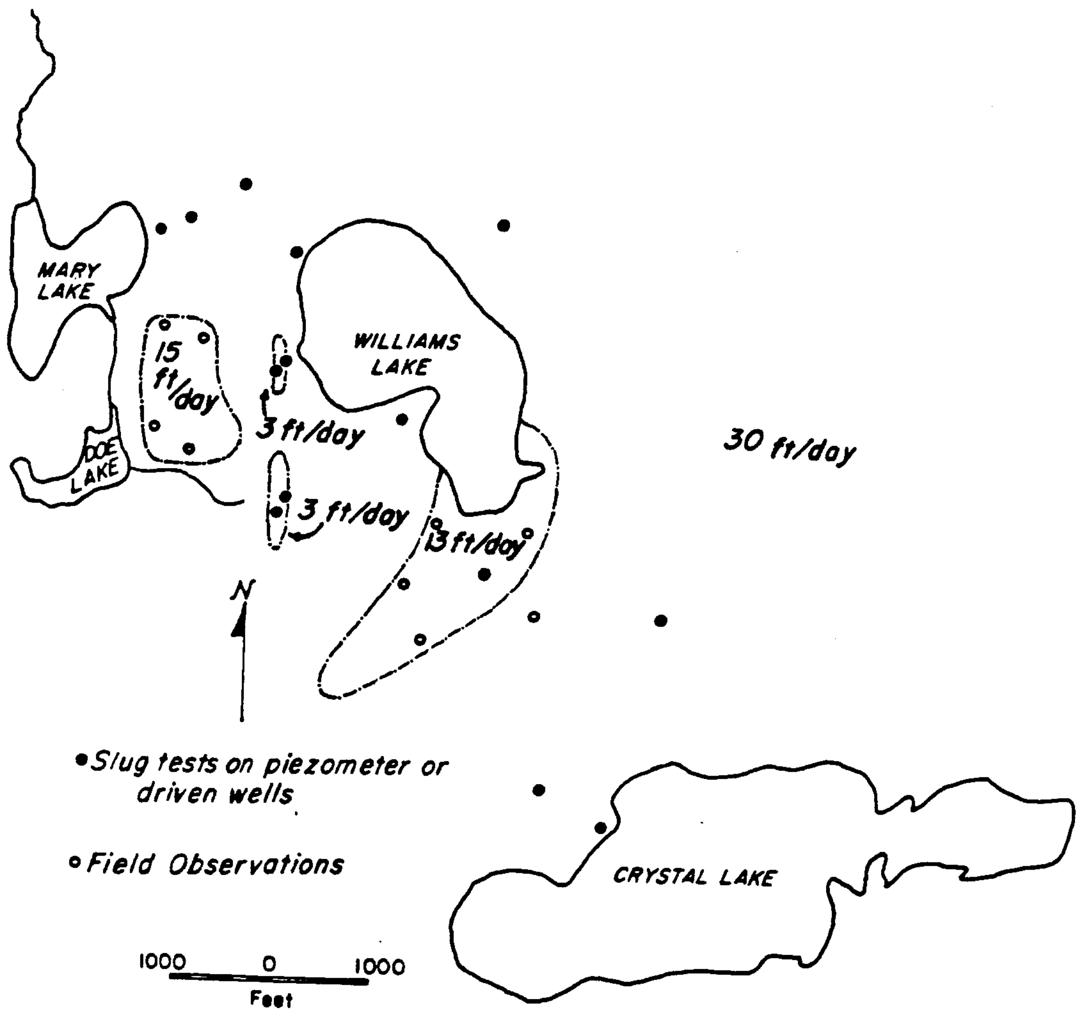



Figure 11. Hydraulic Conductivity Distribution

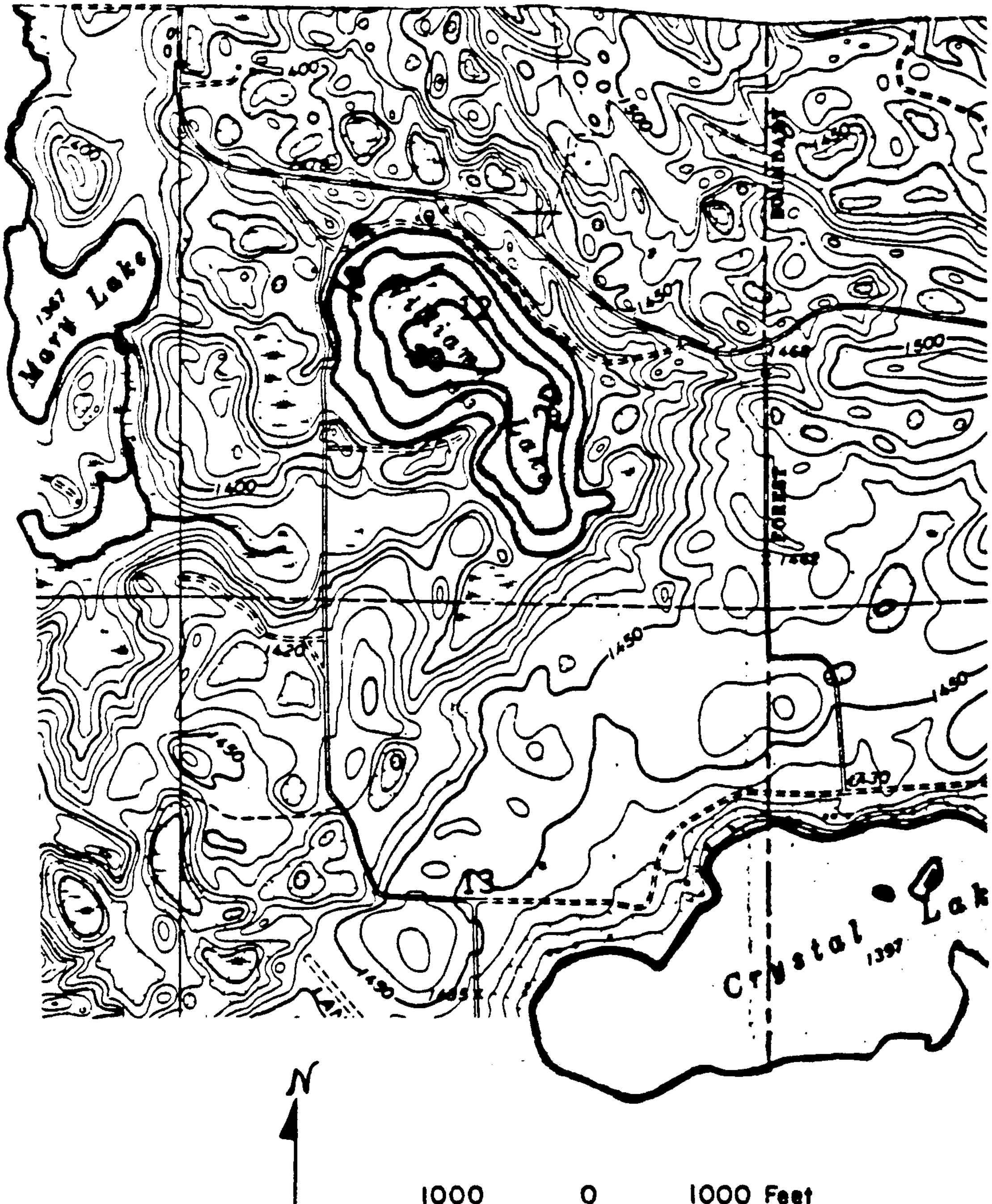



Figure 12. Land Surface Elevation

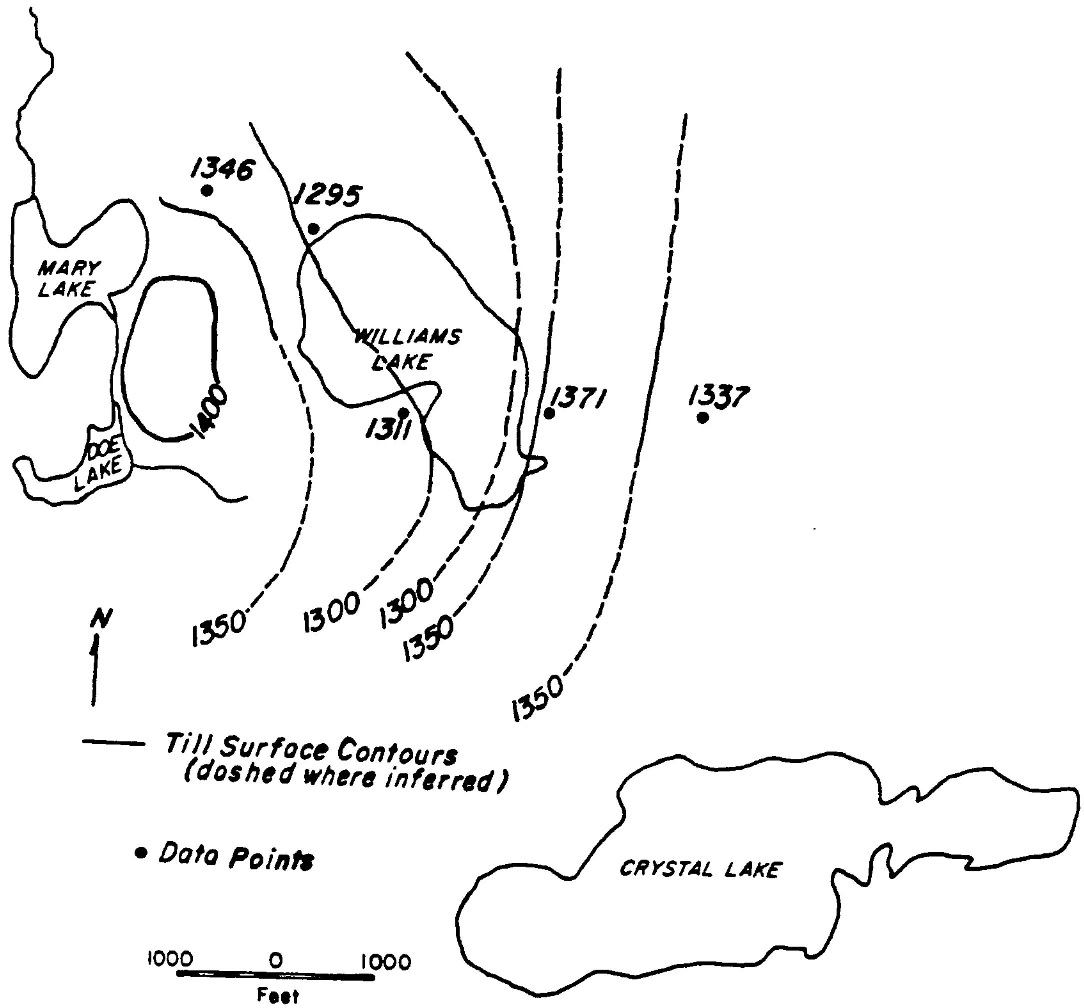



Figure 13. Aquifer Base Altitude

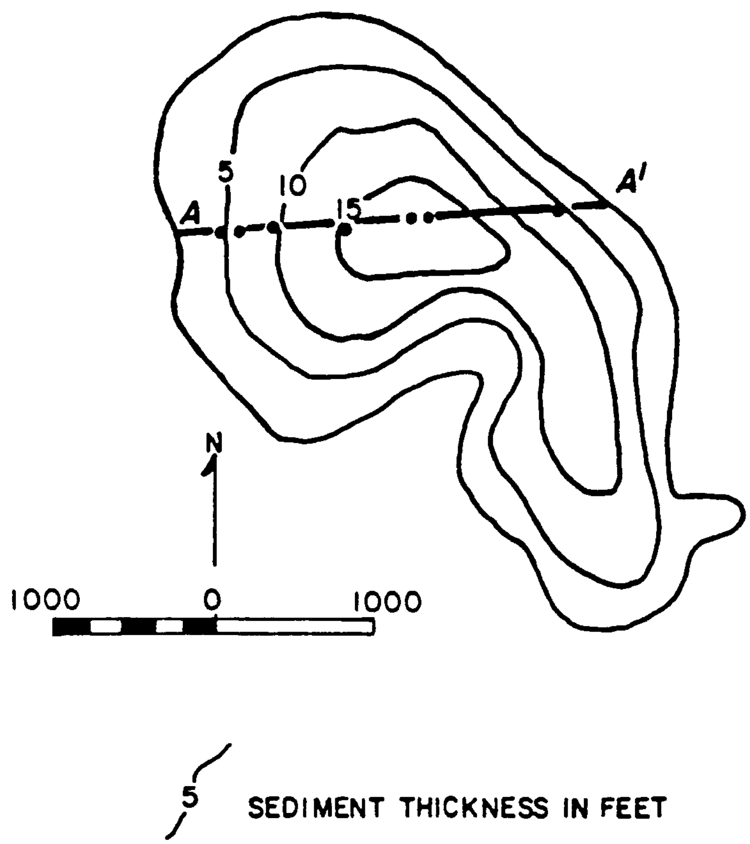



Figure 14. Lake Sediment Thickness

equal to the evaporation rates determined by the method described above. The rate computed for the steady-state simulation was 2.52 ft./year.

# MODELING PROCEDURES

## Steady State Calibration and Results

Observed and simulated water table contours are shown in Figure 15 for the initial and boundary conditions described in the previous section. The head values compare favorably except in the area northwest of Williams Lake where the simulated heads are generally 1 foot higher than observed heads.

Calibration of the steady state model was accomplished by fixing the boundary conditions and adjusting only the values of hydraulic conductivity. The hydraulic conductivity was increased in the northwestern part of the study area relative to that obtained by slug tests along the northern edge of Williams Lake to obtain an acceptable fit between simulated and measured values. This adjustment was well within a reasonable range for the types of materials present, and the suspected underestimation of hydraulic conductivity by the slug testing methods as mentioned previously. The hydraulic conductivity obtained from calibration in the northwest area ranged from 48 ft/d to 58 ft/d. The hydraulic conductivity distribution for the area is shown in Figure 16.

A comparison between observed head values and the simulated head distribution for the steady state model is shown in Figures 17 and 18. The average difference between simulated and measured water levels at 10 points within the system is about 0.81 ft.

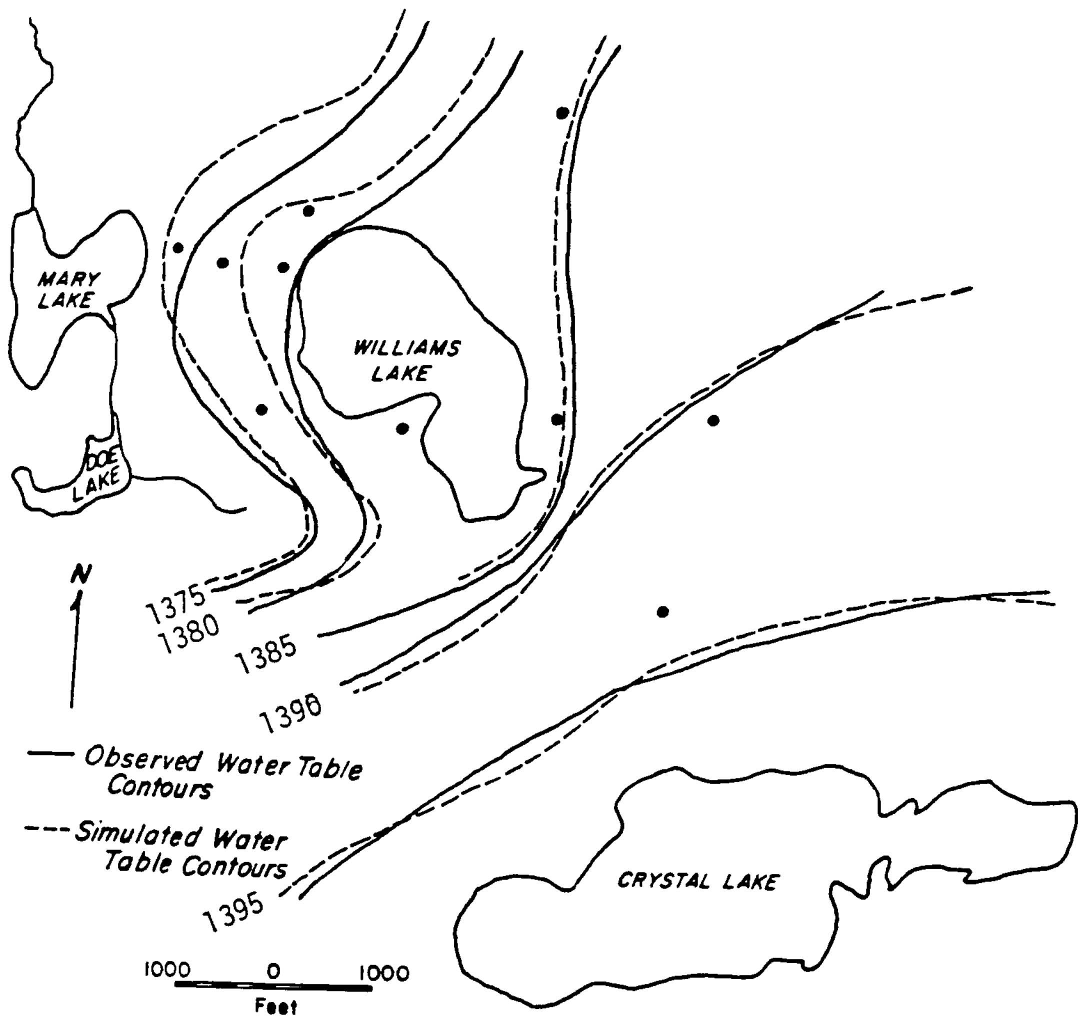



Figure 15. Observed and Simulated Water Table Contours

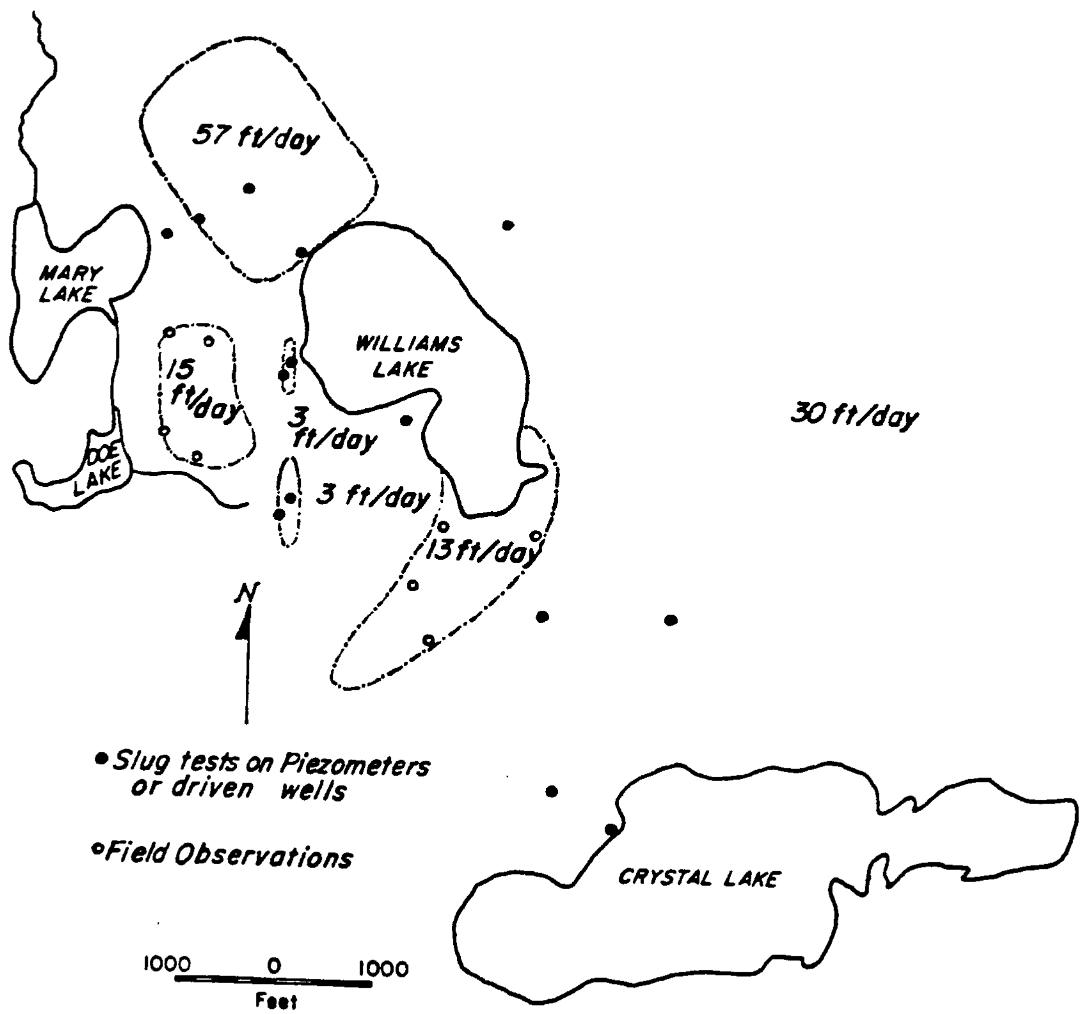



Figure 16. Calibrated Hydraulic Conductivity Distribution

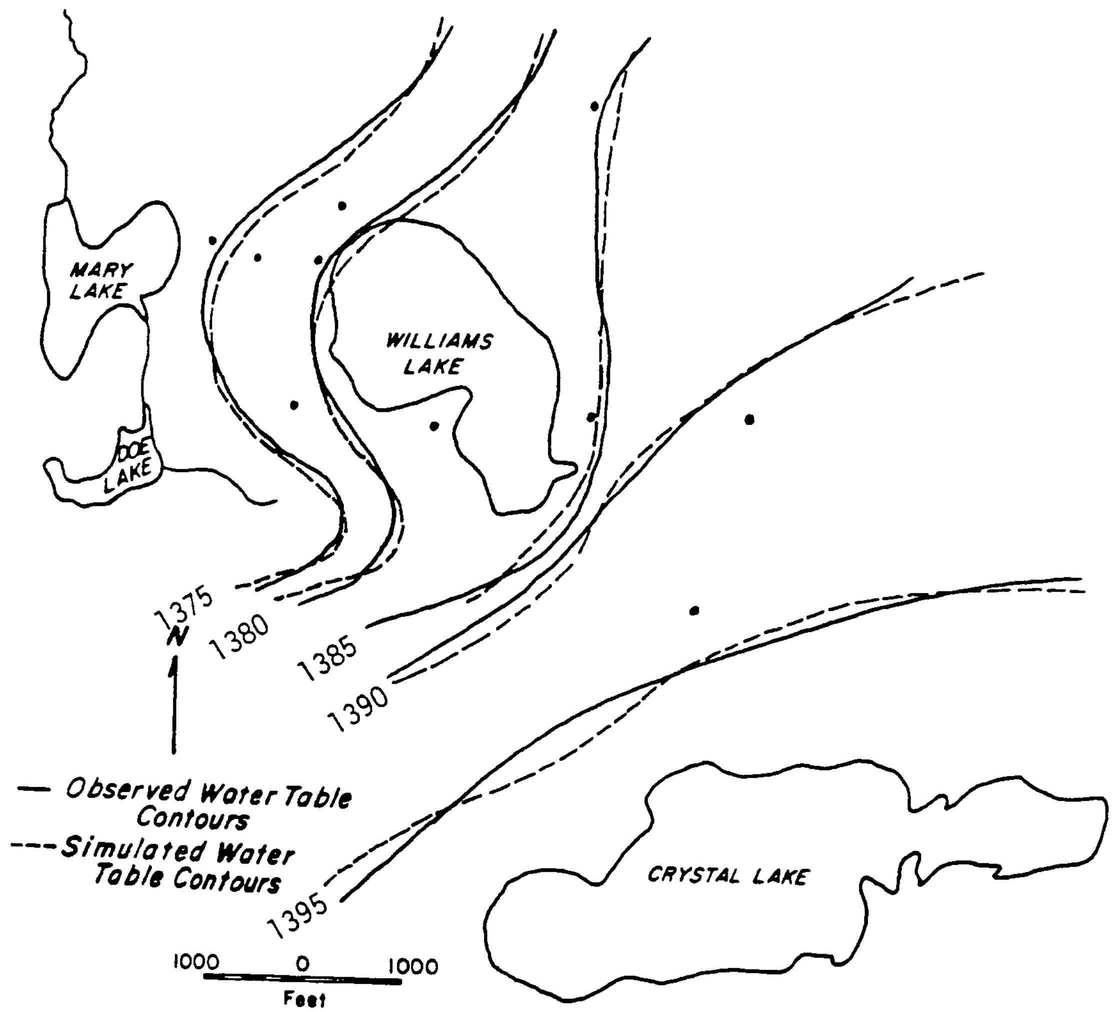



Figure 17. Observed Head and Calibrated Simulated Head Comparison -- Steady State

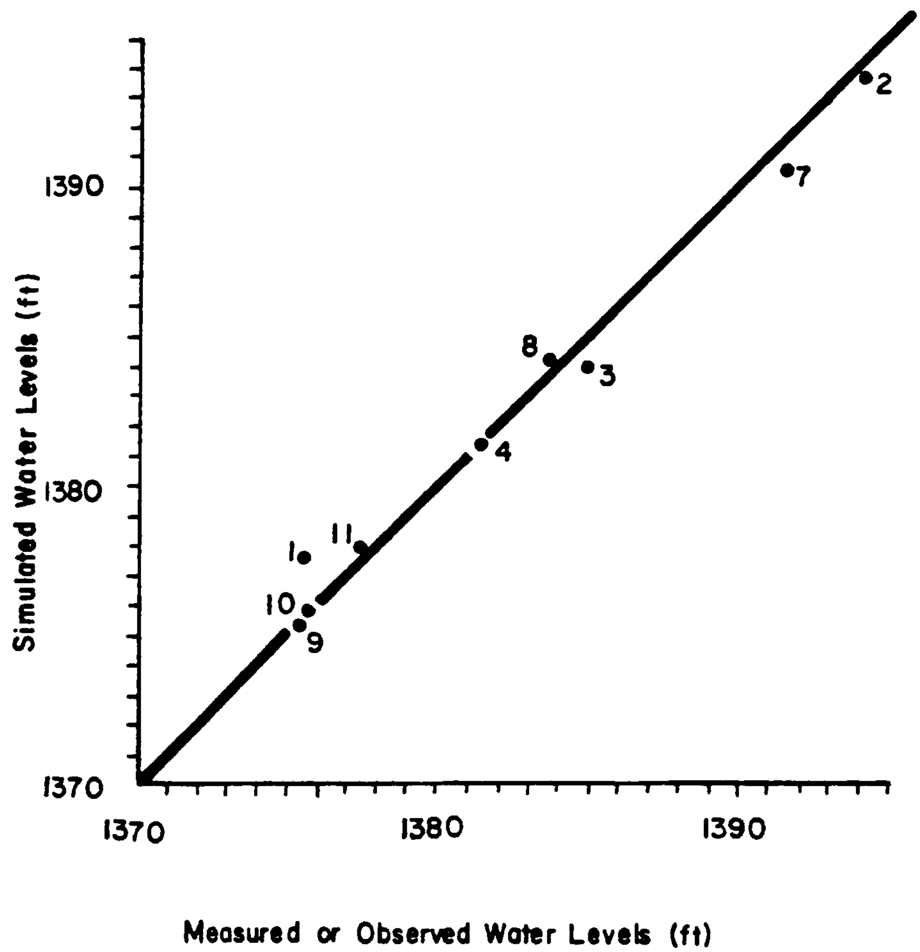



Figure 18. Observed Head and Simulated Head
-- Graphical Comparison

The results of the calibration described above could have been achieved by locally increasing the recharge rate, or changing the boundary conditions in that area. It was necessary to reduce the recharge rate by 75 percent to achieve results similar to those obtained by increasing the hydraulic conductivity as described above. There is no reason obvious to the author that would cause recharge to be reduced in this area. Changing the boundary conditions in that area to simulate the observed head distribution required the constant head boundaries to be lowered to an unreasonably low level (about 1360 feet).

It is possible that the proper conditions may be a combination of a lower recharge rate, altered boundary conditions and an increased hydraulic conductivity, but the increase in hydraulic conductivity had the most substantiation in field data. The piezometer tests in WL-24 indicated a hydraulic conductivity larger than 30 ft/day (Table 3), so the calibrated values of 48 ft/day to 58 ft/day are not unreasonable in that area.

A closer approximation between the simulated head distribution and the measured head distribution is possible if the input parameters are adjusted in a systematic manner until the desired effect is achieved. The adjustment of parameters or "dial twisting" produces a combination of parameter values that are not unique to a particular solution. The author feels that rather than making unfounded adjustments of parameters, any other discrepancies in simulated and observed data should be left to indicate the need for further investigation in the areas where the discrepancies occur.

Williams Lake is simulated by using the leakage option of the model. This option allows the user to specify a confining layer of variable thickness at any node in the model and a constant head which represents the lake level at that node. The flux to or from the lake is computed using a constant head value for the lake and the thickness and hydraulic conductivity of the lake sediments at each node along with the head value in the aquifer at that point. The value of $1.3 \times 10^{-4}$ ft/day was used for the hydraulic conductivity of the lake sediments. Siegel and Winter (1980) estimated that the hydraulic conductivity of the lake sediments was 5 orders of magnitude lower than that of the surficial aquifer. The sensitivity of the model to this parameter is analysed in the next section. This value represents the vertical hydraulic conductivity of the sediments. The cross-sectional analyses presented in an earlier section of this study indicate that most seepage through the lake sediments at Williams Lake is vertical.

Sediment thicknesses were estimated for the lake based on the cross-section shown in Figure 5 and the contour map in Figure 10. Sediments are regarded as absent in the wave washed littoral zone of the lake where wave action removes fine materials.

## Steady-State Sensitivity Analysis

An analysis to determine the relative sensitivity of calibrated head values and fluxes to changes in input parameters was performed for the steady-state model. As mentioned in the introduction, the results of this analysis are useful in guiding future field investigations.

The method of analysis is similar to that of Gillham and Farvolden (1974), Kohberger, Scavia and Wilkinson (1978), Pickens and Lennox (1976), Hearne (1980), Boggs (1980), and Matlock (1982). The sensitivity criterion are defined to represent a change in head or flux from some calibrated state resulting from a change in parameter values.

A generalized example of this method is

$$\sigma = B \, || h^* - h ||$$

where

$\sigma$ is the sensitivity coefficient,

B is a constant,

h is the calibrated head value, and

h* is the perturbed head value.

McElwee and Yukler (1978), Coleman and DeCoursey (1976), and Bathala, Rao and Spooner (1980) use a sensitivity criterion that relates a change in head or flux from some calibrated state to a change in parameter values.

A generalized example of this method is

$$\sigma = B \, \frac{|| h^* - h ||}{|| x^* - x ||}$$

where

$\sigma$ is the sensitivity coefficient,

B is a constant,

h is the calibrated head value,

h* is the perturbed head value,

X is the calibrated parameter value, and

X* is the perturbed parameter value.

The first method is used in this study because several parameters will be varied by a uniform fixed percentage and the resulting sensitivity coefficients will be calculated for various parameters and compared. If the magnitude of the change in the parameters entered into the equations, as in the second method, parameters with large magnitudes and hence large changes in the parametric values after variation by a uniform percentage, would have larger values in the denominator and the sensitivity coefficients would be underestimated.

A number of zones of equal size and shape were distributed over the modeled area (Figure 19). The design of the finite difference mesh controlled the size and to some extent the placement of the sensitivity zones. The variable node sizing used in the mesh resulted in a number of zone sizes that could be used. The zone had to contain an integer number of nodes to facilitate calculation of the sensitivity coefficients. Zones smaller than those used in this analysis require a large number of zones to insure adequate coverage of the area, and these smaller zones had a very small affect on heads in the aquifer and fluxes to and from the lake. Zones larger than those used intersected each other or the boundaries resulting in erroneous sensitivity coefficients. The zones were placed in areas that would allow analysis of the system both up-gradient and down-gradient from Williams Lake, and in area where the variation of node sizes would accommodate them.

A steady-state simulation was performed in which an individual parameter from group A of Table 4 was decreased by 50% at all nodes within a given zone while all other parameters were held constant at

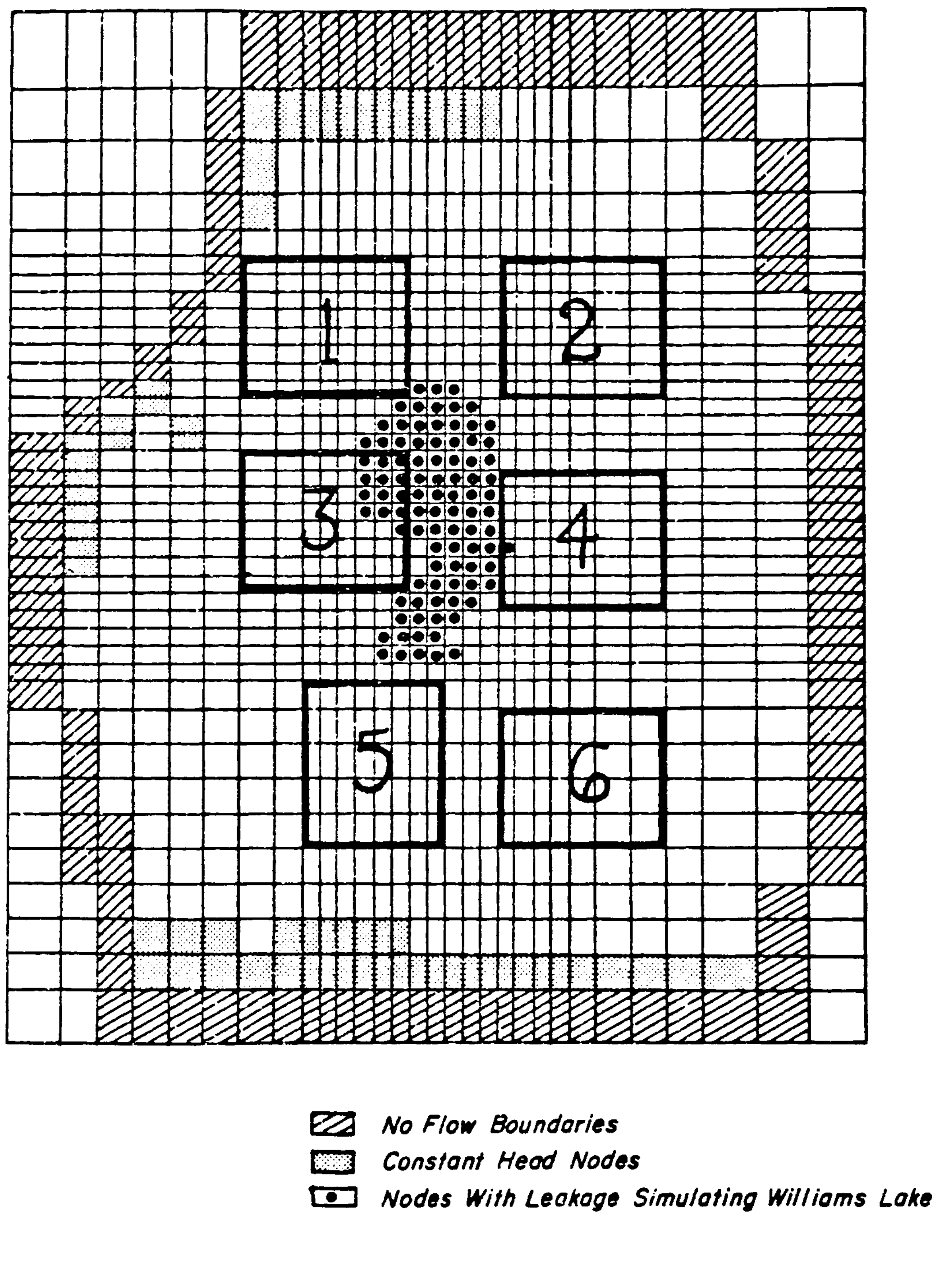



Figure 19. Sensitivity Zones

their initially calibrated values.

Table 4 - Group A and B Parameters

| | | |
|---|---|---|
| Group A | - | Hydraulic Conductivity |
| | - | Recharge Rate |
| Group B | - | Lake Sediment Hydraulic Conductivity |
| | - | Lake Sediment Thickness |

The weighted average head change, or sensitivity coefficient ($B_n$), was then calculated for all the zones by the equation (Matlock, 1982):

$$B_n = \frac{\sum_{i=1}^{k} h_i \, A_i}{S_n} \qquad \text{for } n = 1,2,\ldots N$$

where

$h_i$ is absolute value of the resulting head change at node i,

$A_i$ is the area of the cell associated with node i,

$S_n$ is the area associated with sensitivity zone n,

K is the number of nodes in sensitivity zone n, and

N is the total number of sensitivity zones.

This resulted in a sensitivity array (an individual row in Tables 5 and 6). The same parameter was then altered in another sensitivity zone and the process repeated. The final result of the sensitivity analysis on Group A parameter is a sensitivity "matrix" as shown in Tables 5 and 6.

The matricies in Tables 5 and 6 are averaged horizontally by rows and vertically by columns with the zone in which the parameter change occurred being excluded. The horizontal averages indicate the average magnitude of head changes caused by a parametric change in just one zone. The vertical average indicates the average magnitude of head changes in just one zone as a result of parametric changes in all the other zones.

In Table 5, zones 4 and 6 have the largest averages vertically indicating that they are effected by changes in hydraulic conductivity in other areas more than the other zones are. Zone 4 has the largest horizontal average indicating that it has the largest effect over the rest of the zones with respect to hydraulic conductivity. It is important to note that Zone 2 has the largest single $B_n$, but Zone 2 is not the most sensitive zone on the average, and it does not have the largest effect on the other zones.

Figure 20 illustrates the effect of hydraulic conductivity variation on the fluxes into and out of the lake. Zone 1 has the largest effect on out-seepage, reducing it by 2600 ft$^3$/day (13 percent of the calibrated value). Zone 5 has the largest effect on in-seepage, reducing it by 1400 ft$^3$/day (26 percent of the calibrated value).

Table 5 -- Hydraulic Conductivity Sensitivity

| Zone of Parameter Variation | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ | Horizontal Average |
|---|---|---|---|---|---|---|---|
| 1 | 0.271 | 0.059 | 0.085 | 0.033 | 0.012 | 0.014 | 0.041 |
| 2 | .075 | .481 | .017 | .298 | .033 | .120 | .109 |
| 3 | .032 | .015 | .242 | .052 | .108 | .043 | .050 |
| 4 | .010 | .026 | .011 | .461 | .107 | .435 | .118 |
| 5 | .008 | .003 | .138 | .018 | .341 | .100 | .053 |
| 6 | .010 | .067 | .022 | .309 | .063 | .267 | .094 |
| Vert Avg | 0.027 | .034 | .055 | .142 | .065 | .141 | |

Table 6 -- Recharge Rate Sensitivity

| Zone of Parameter Variation | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ | Horizontal Average |
|---|---|---|---|---|---|---|---|
| 1 | 0.100 | 0.026 | 0.009 | 0.007 | 0.003 | 0.003 | 0.009 |
| 2 | .026 | .182 | .007 | .052 | .007 | .020 | .022 |
| 3 | .013 | .009 | .048 | .007 | .013 | .004 | .009 |
| 4 | .009 | .060 | .012 | .206 | .030 | .090 | .040 |
| 5 | .003 | .008 | .021 | .031 | .157 | .052 | .023 |
| 6 | .005 | .027 | .009 | .099 | .055 | .194 | .039 |
| Vert. Avg | 0.011 | .026 | .012 | .039 | .022 | .034 | |

Some results of the hydraulic conductivity sensitivity analysis could have been predicted before the simulations were conducted, but the sensitivity analysis quantifies the results. By decreasing the hydraulic conductivity in a zone down-gradient from the lake (Zones 1, 2 and 3), the out-seepage from the lake is reduced, and the inflow is increased because the lake stage is held constant in the model and the boundary conditions up-gradient from the lake remain the same. Slight increases in the hydraulic head under the lake cause more water to flow through the lake bottom sediments toward the constant head representing the lake. This can be seen if Figure 6 and the approximate cross-section of the modified flow system in Figure 22 are compared. The area of in-seepage along the bottom has increased and the area of out seepage has decreased.

An increase in the constant head value representing Williams Lake of 0.15 feet to 1383.67 feet during the hydraulic conductivity sensitivity analysis of Zone 3 brought the inflow and outflow of Williams Lake back to their original steady state values. Similar small changes in the lake stage brought the inflow and outflow back to their original steady state values for other zones as well.

This is an indirect measurement of the sensitivity of the fluxes into and out of the lake as a result of changes or uncertainties in lake stage. A small error of 0.15 $\pm$ feet in measuring lake stage would have as large an effect on the in-seepage and out-seepage as an error of 50 percent in estimating the hydraulic conductivity over an area the size of a sensitivity zone (69 acres, or 73 percent of the size of Williams Lake).

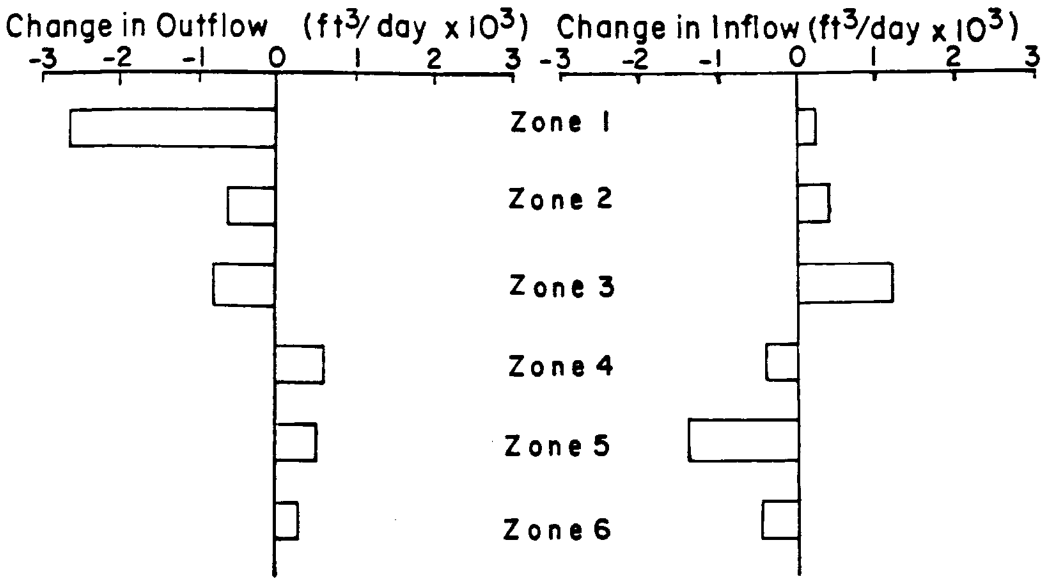



Figure 20. Sensitivity of flux to a 50% reduction of hydraulic conductivity in individual zones.

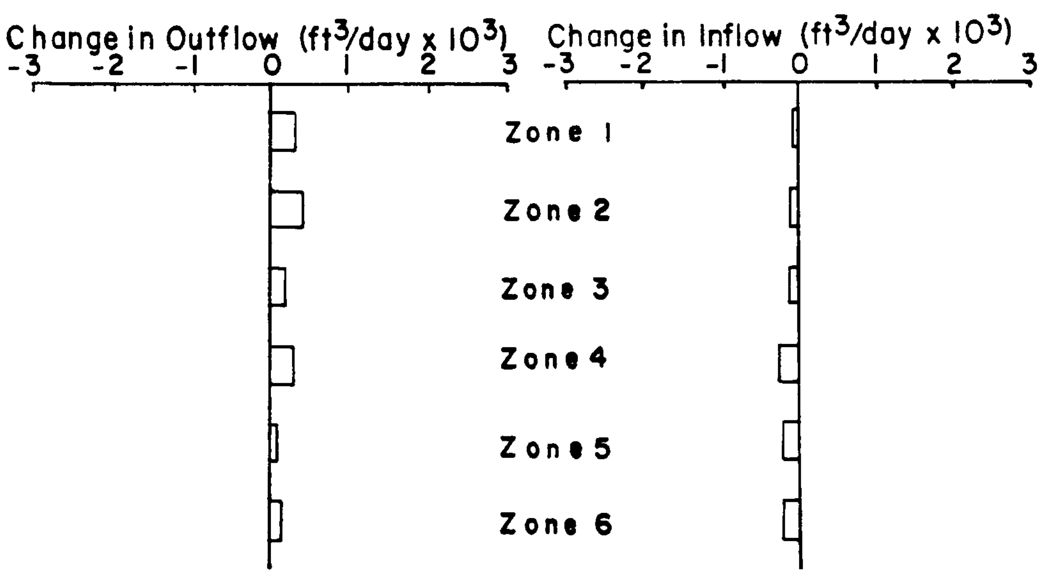



Figure 21. Sensitivity of flux to a 50% reduction in recharge rates in individual zones.

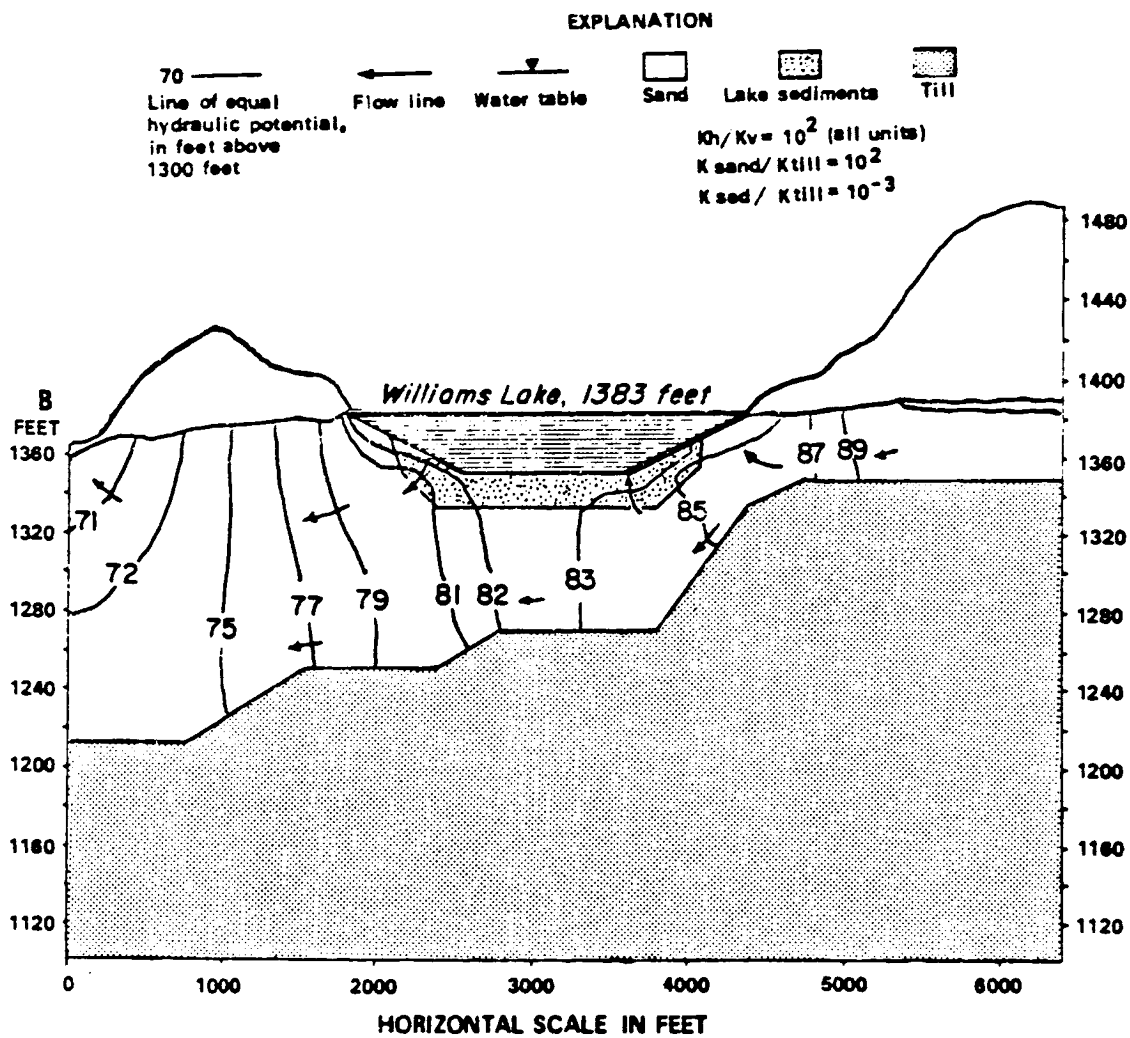



Figure 22. Modified Cross-Section

Zone 4 which has determined to have the largest overall effect on head values in the watershed did not have as large an effect on seepage as Zone 5. This is probably because Zone 5 is directly between Crystal Lake, which is represented by a constant head boundary, and Williams Lake (Figure 19). A reduction in the hydraulic conductivity of Zone 5 affects the shortest flow path between the two lakes. Similarly, Zone 1 which was previously mentioned for its relatively large effect on out-seepage, lays directly between Williams Lake and the constant head boundary of Mary Lake.

Zone 4 in Table 6 has the largest vertical and horizontal averages indicating that this zone is most sensitive to recharge, and it has the most influence over the other zones. Zone 4 also has the largest $B_n$ (0.206). Figure 21 indicates little variation in the fluxes into and out of the lake with changes in recharge. This steady state model is for a period in early July when recharge is no longer near the peak which occurs in April and May. Recharge rates may be an order of magnitude or more higher in April or May and there may be a significant change in the flux in and out of the lake in response to changes in the recharge rate at these times. Transient sensitivity analysis would be required due to the unsteady state flow system during this period, and further work in this area is recommended.

The Group B parameters are lake bottom parameters. An individual parameter from Group B was decreased by 50 per cent over the entire lake bottom and a steady state simulation was performed. The effect of the parameter variation resulted in a sensitivity "array" for the sensitivity zones, and a change in the flux into and out of the lake.

Even though the parametric values in the sensitivity zones were not changed, their response to changes in the lake bottom parameters is important since the development of ground water mounds, particularly on the down gradient side in the lake control the development of stagnation points which can eliminate out-seepage from lakes (Winter 1976, 1978).

The resulting sensitivity "arrays" are presented in Table 7 and the change in fluxes in Figure 23.

Reducing the vertical hydraulic conductivity (K') of the lake sediments by 50 per cent reduces the amount of in-seepage and out-seepage and has the largest effect on Zones 1, 2 and 3 (Table 7). These zones are in the down gradient portion of the flow system and receive the out-seepage from the lake. The out-seepage from the lake is approximately 4 times the in-seepage so a large effect on the zones down gradient from the lake is expected.

Table 7 -- Lake Bottom Sensitivity

| | Sensitivity Coefficient | | | | | |
|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ |
| Sediment Conductivity | 0.413 | 0.452 | 0.220 | 0.131 | 0.031 | 0.024 |
| Sediment Thickness | 0.405 | 0.407 | 0.174 | 0.071 | 0.067 | 0.017 |

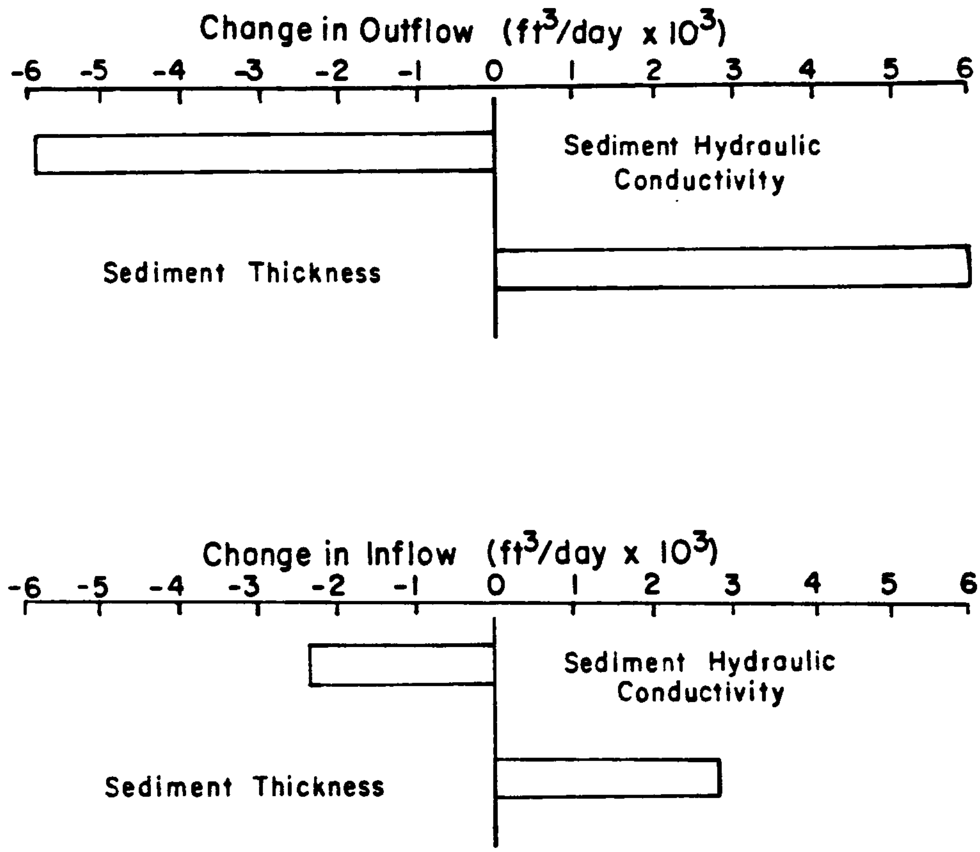



Figure 23. Sensitivity of flux to a 50% reduction in the value of the lake bottom parameters.

Reducing the thickness of the lake bottom sediments by 50 percent increases the in-seepage and out-seepage, and Zones 1, 2, and 3 again show the largest response for reasons similar to those given for their response to changes in K'.

Figure 24 shows the head changes as a result of varying the hydraulic conductivity of Zone 4 which was the zone whose parameters had the largest effect on the other areas. The changes are small and do not significantly change the overall flow pattern.

Changes in head (the magnitude of the sensitivity coefficients), resulting from parametric changes during the sensitivity analysis, appears to be primarily a function of the specific discharge. Zone 4, which is the most sensitivity zone when Group A parameters are changed, is near WL-8 in Figure 5. This is an area of decreased cross-sectional flow due to the higher till surface in the area, and as the cross-sectional area is reduced the specific discharge increases. Zones 1, 2, and 3 are the most sensitive zones when Group B parameters are varied. The aquifer characteristics are not being varied in this case but the specific discharges are, since the amount of water is being increased or decreased as a function of the out-seepage from the lake.

The sensitivity analysis indicates that the lake bottom characteristics have a larger effect on the fluxes into and out of the lake than do changes in the Group A parameters in any discrete zone.

Proper calibration of the model to the fluxes into and out of the lake is not possible if the hydraulic conductivity and thickness of the lake sediments is not known. The thickness of the sediments in this

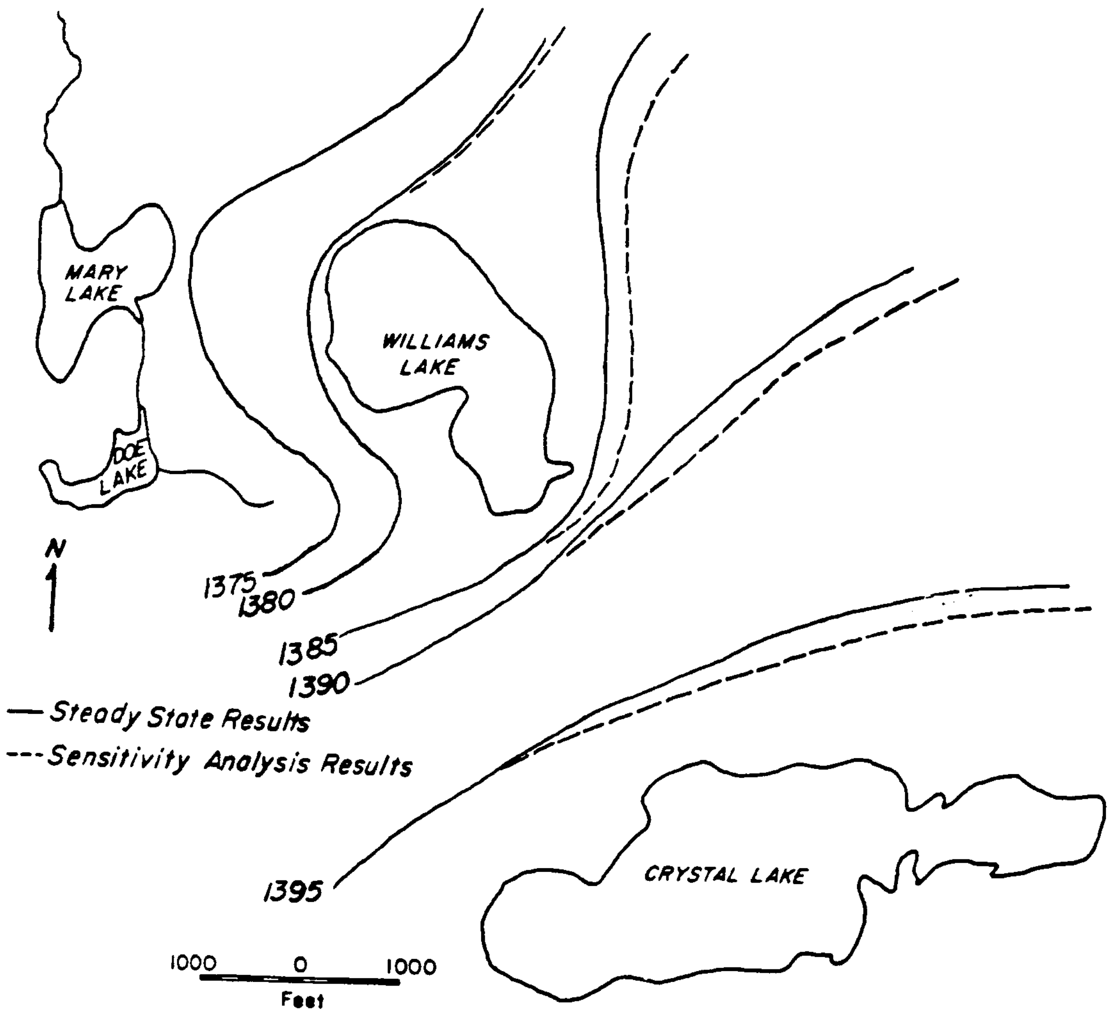



Figure 24. Head Changes Resulting From Sensitivity Analysis

study were obtained from cores along one line of section which is shown in Figure 3, and estimates were made for the rest of the lake based on that profile. The hydrualic conductivity of the sediments is an estimate, as stated previously, and is assumed to be constant over the entire lake bottom. Considering the results of the sensitivity analysis, and the assumptions and estimates that were made about the lake sediments, considerable doubt must be placed on the estimates of lake seepage rates determined by the model. The sensitivity analysis has shown that small head changes in areas directly between the lake and constant head boundaries has a significant effect on seepage rates. Calibration of a model to measured head values must be detailed and exacting to accurately determine seepage rates to and from a lake. This requires sufficient data from piezometers and reliable estimates of parameters such as hydraulic conductivity. Models should be constructed with a grid fine enough to accurately simulate steep gradients near lake shores. A grid finer than the one used in this study with the boundary conditions shown in Figure 9 would require an unreasonable number of nodes. Finite-element methods would significantly reduce the number of nodes needed. The boundaries of the model could be moved closer to the lake, and the boundary conditions changed accordingly, to minimize the number of nodes needed if finite-difference methods are used in subsequent modeling studies.

If the scale of the model is reduced the assumption of horizontal flow must be checked. If vertical flow becomes important in the aquifer, cross-sectional or three-dimensional models may give a more accurate representation of the flow system.

These sensitivity analyses are only as good as the data that were used to construct the model. Sensitivity analyses should be conducted at all phases of a modeling project to redefine sensitive areas of the model as more data is collected and the calibration of the model is continued.

## Transient Simulation

The steady state conditions described in an earlier section were used as the starting point for a transient simulation from July 1979 to June 1981. The purpose of the transient simulation is to analyse the changing flux of water into and out of the lake with seasonal and climatic changes, and to help verify the steady state model.

A problem with the model was that is assumed the lake stage, recharge rate, evapotranspiration rate and head at boundaries remained constant with time. The code was modified (Appendix 2) to allow the lake stage, recharge rate and evapotranspiration rate to vary as a step function with the seasonal changes of these parameters. The constant head nodes are far from the area influencing flux into and out of the lake and their changes are relatively small so they were considered insignificant.

The time steps for seasonal changes were modeled as 120 day periods. This results in 3 periods each year. The period from March 1 to June 30 is a period of high recharge to the ground water system and moderate evapotranspiration. The period from July 1 to October 30 has low recharge to ground water and high evapotranspiration. The period from November 1 to February 28 has little or no recharge to ground water and no evapotranspiration. Climatic parameters are given in Table 8.

Table 8 -- Transient Climatic Parameters

| Period | Recharge Rate (feet/second) | Evapotranspiration Rate (feet/second) | Lake Stage (ft) |
|---|---|---|---|
| 1 July - Oct 1979 | $1.46 \times 10^{-8}$ | $9.43 \times 10^{-8}$ | 83.13 |
| 2 Nov - Feb 1980 | $1.46 \times 10^{-11}$ | $9.43 \times 10^{-11}$ | 82.97 |
| 3 March - June 1980 | $2.92 \times 10^{-10}$ | $6.96 \times 10^{-8}$ | 82.79 |
| 4 July - Oct 1980 | $1.46 \times 10^{-11}$ | $9.49 \times 10^{-8}$ | 82.42 |
| 5 Nov - Feb 1981 | $1.45 \times 10^{-14}$ | $9.49 \times 10^{-11}$ | 82.02 |
| 6 March - June 1981 | $8.18 \times 10^{-8}$ | $5.86 \times 10^{-8}$ | 81.96 |

Recharge rates were calculated by the method described in the section on initial conditions. The bulk of the recharge to the system is during the period from March 1 to June 30 when the snow-melt occurs and the vegetation has not yet reached its maximum transpiration rate due to the cooler more humid weather, and the emergence of foliage as late as early May.

Preliminary studies of an infiltration model being developed by Richard Naff of the U.S.G.S. indicate that very little recharge occurs during the growing season in the Williams Lake area (Winter, 1982).

Little or no recharge to the aquifer was assumed to take place during the period from November 1 to February 28. The soil horizon is frozen during most of this period eliminating the possibility of vertical flow to recharge the aquifer.

Evapotranspiration was calculated by the method described in the section on initial conditions. Oaks and Bidwell (1968) estimated the annual potential evapotranspiration in this portion of Minnesota to be 19.98 inches by using the method of Thornthwaite and Mather (1957). This compares well with the rates used in this model which resulted in 20.44 inches of potential evapotranspiration in 1980. The weighting of the evapotranspiration was done using mass transfer evaporation records and evaporation pan estimates of free surface evaporation. The period from November 1 to February 28 was assumed to have little evapotranspiration because the weather is cold with average temperatures below freezing for all four months (Watson, 1974), vegetation is dormant, and the soils are frozen. Discharge from the ground water system by evapotranspiration occurs where the water table is within the root zone of vegetation, which is at a depth of 8 feet in the study area, based on values of root zone depth given by Thornthwaite and Mather (1957). Therefore, evapotranspiration from the aquifer is considered to occur where the water table is less than 8 feet below the land surface (Figure 25), increasing linearly to where the water table is at the land surface (wetland areas) and the actual evapotranspiration equals the potential evapotranspiration.

A comparison between measured water levels and those simulated for each time period are shown in Figures 26a through 26f.

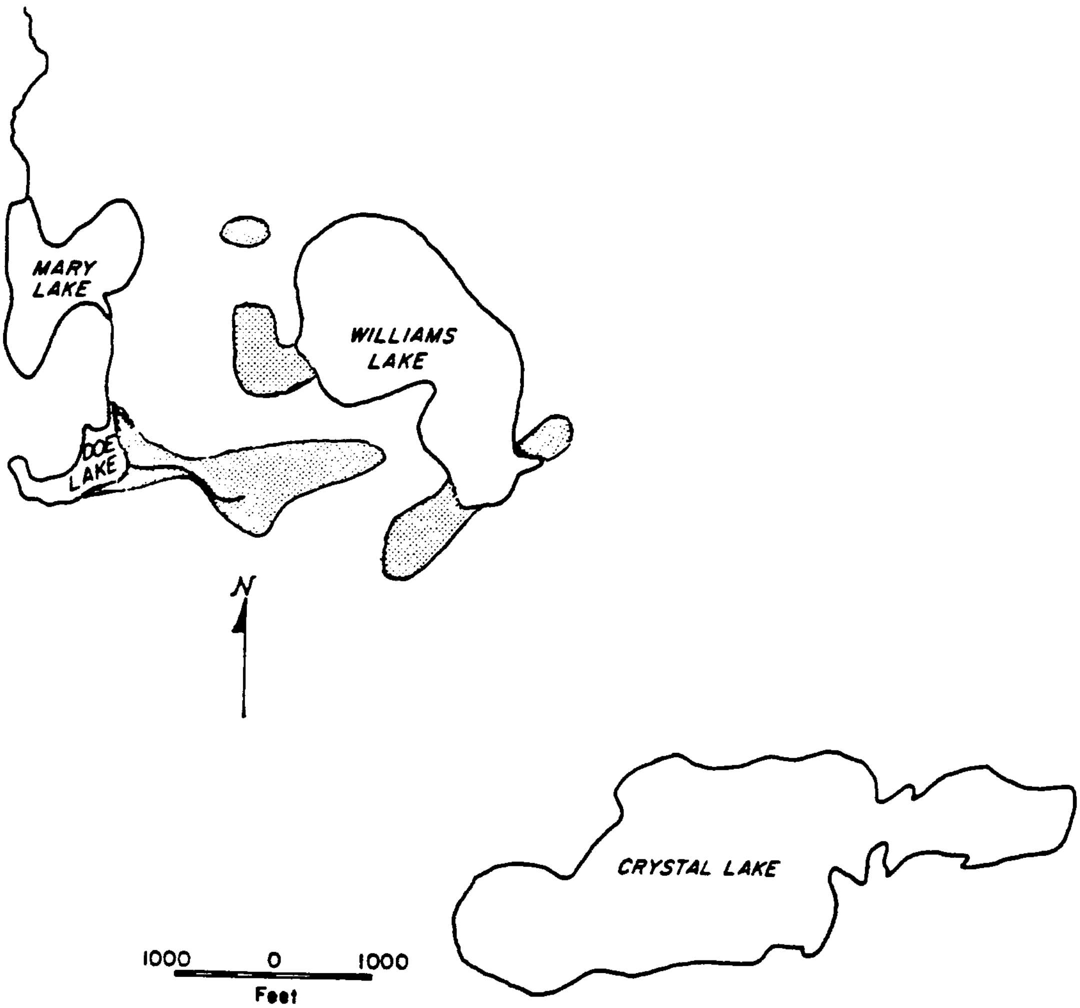



Figure 25. Areas of simulated evapotranspiration

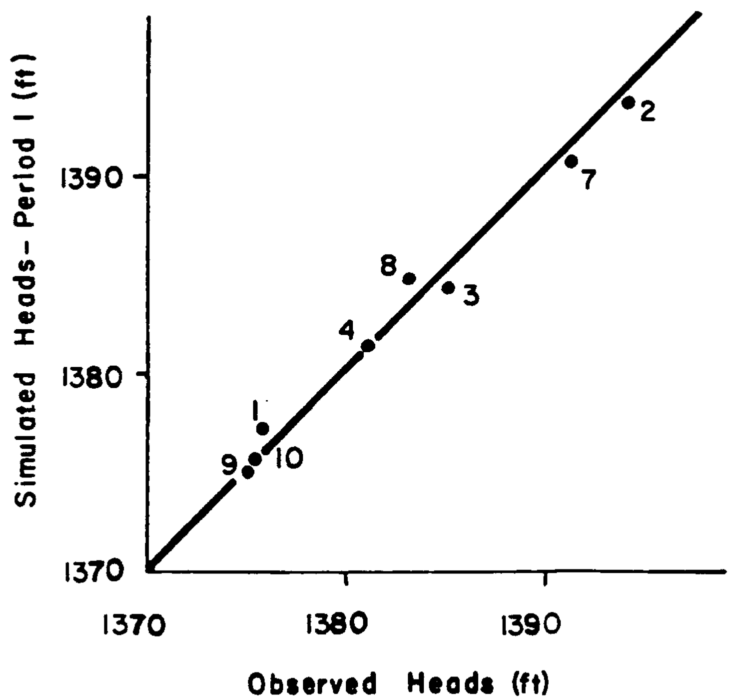



Figure 26a. Observed and Simulated Head Comparison -- Period 1.

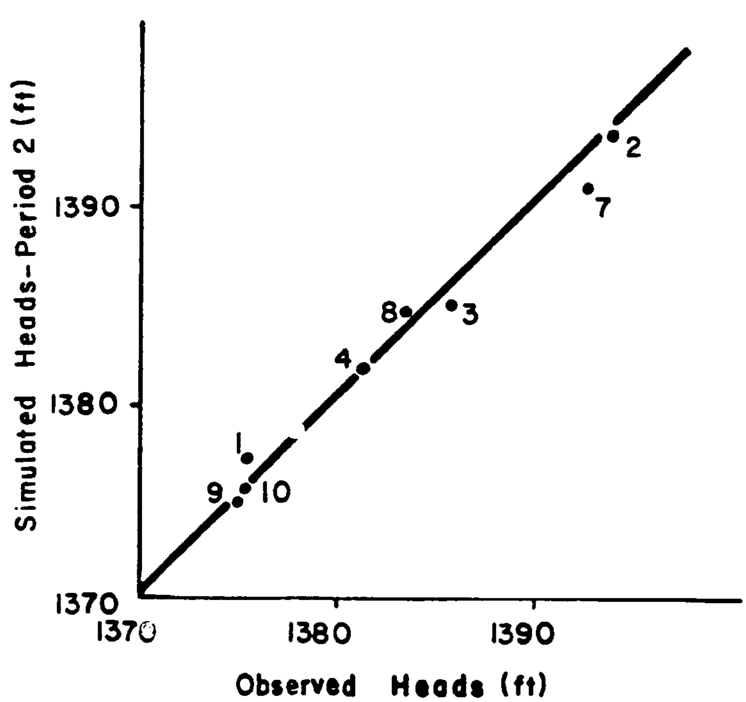



Figure 26b. Observed and Simulated Head Comparison -- Period 2.

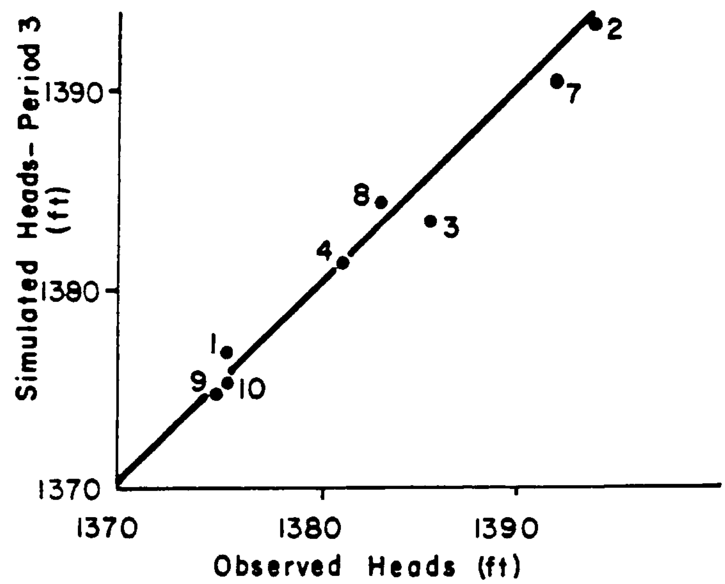



Figure 26c. Observed and Simulated Head Comparison -- Period 3.

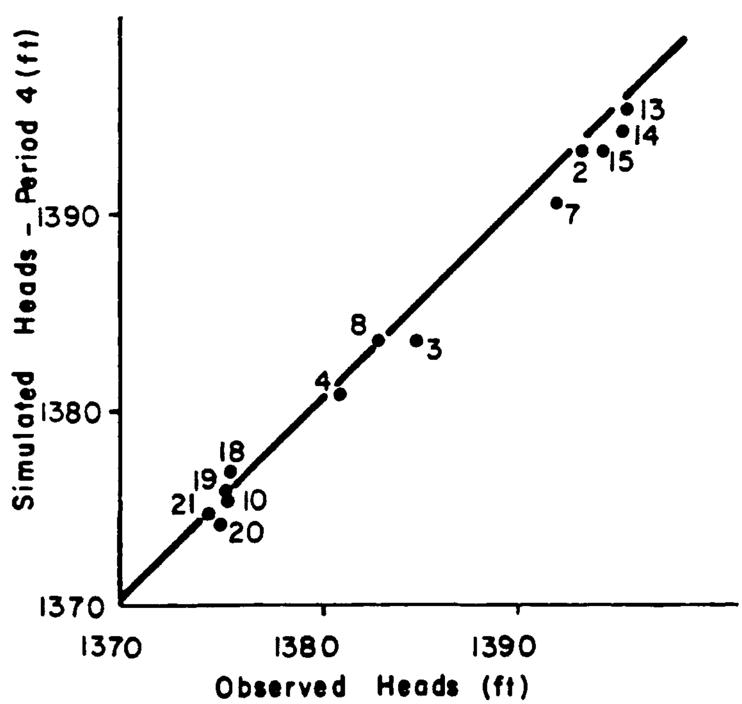



Figure 26d. Observed and Simulated Head Comparison -- Period 4.

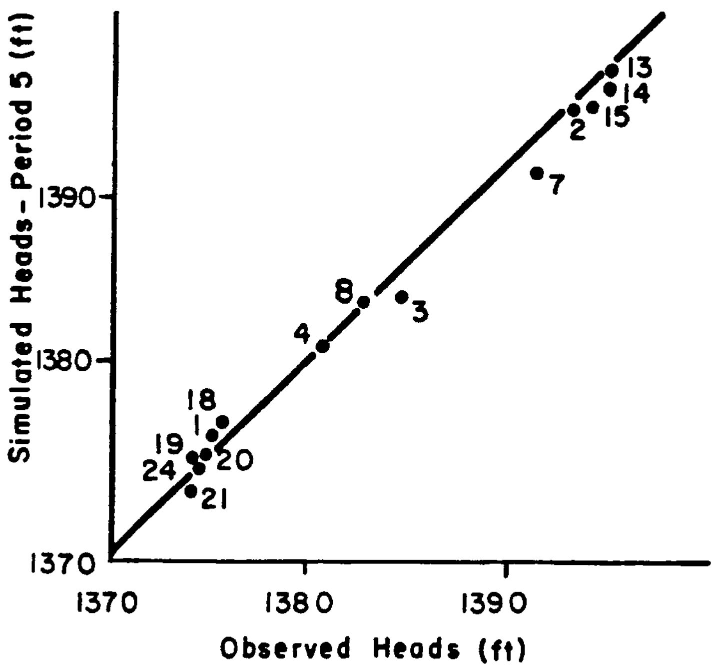



Figure 26e. Observed and Simulated Head Comparison -- Period 5.

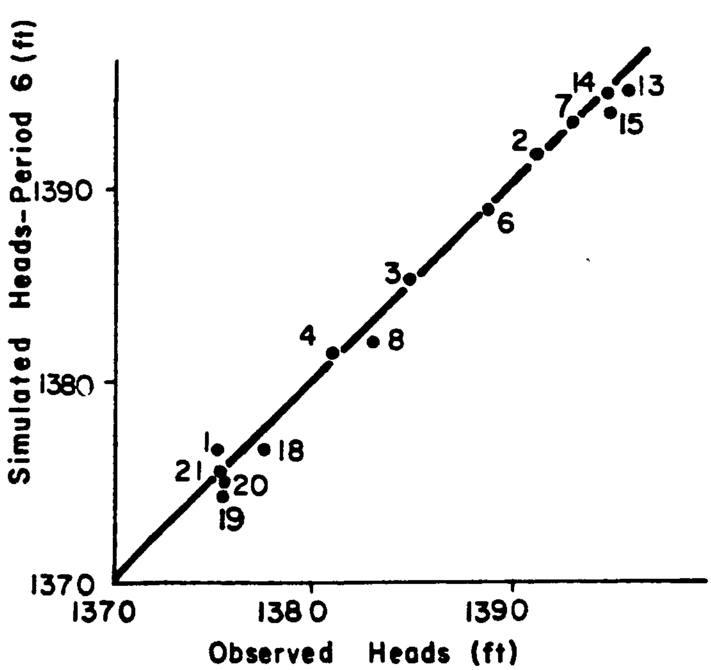



Figure 26f. Observed and Simulated Head Comparison -- Period 6.

The differences between observed and simulated heads is generally less than a foot. Some typical hydrographs are given in Appendix 3. Exceptions to this are simulated head values near the lake (observation wells 1, 8 and 18) where the nodes in the model represent the average head over an area of relatively steep gradients and the observation wells represent the head at a single point along the relatively steep gradients. Observation wells 3 and 7 have measured heads that are between 1 and 2 feet higher than simulated heads. This may be due to an overestimation of hydraulic conductivity, improper boundary conditions, improper piezometer emplacement or a combination of these and other factors which make further calibration of the model to match observed measurements rather ambiguous. These discrepancies indicate the need for further field investigations in these areas.

A comparison of observed and simulated head distributions for period 5 which represented the largest deviation from the steady state conditions is shown in Figure 27.

The flux of ground water into and out of Williams Lake is of primary importance in this study. Through the use of the leakage option of the model these fluxes can be simulated. Verification of the model cannot be achieved for these fluxes as simply as the head distribution can be verified since these fluxes are difficult to measure in the field (Winter, 1981).

Water budget can be calculated for the lake using precipitation, evaporation, and lake stage data to calculate a budget, with the net ground water flow being a residual term. The lake has no surface water

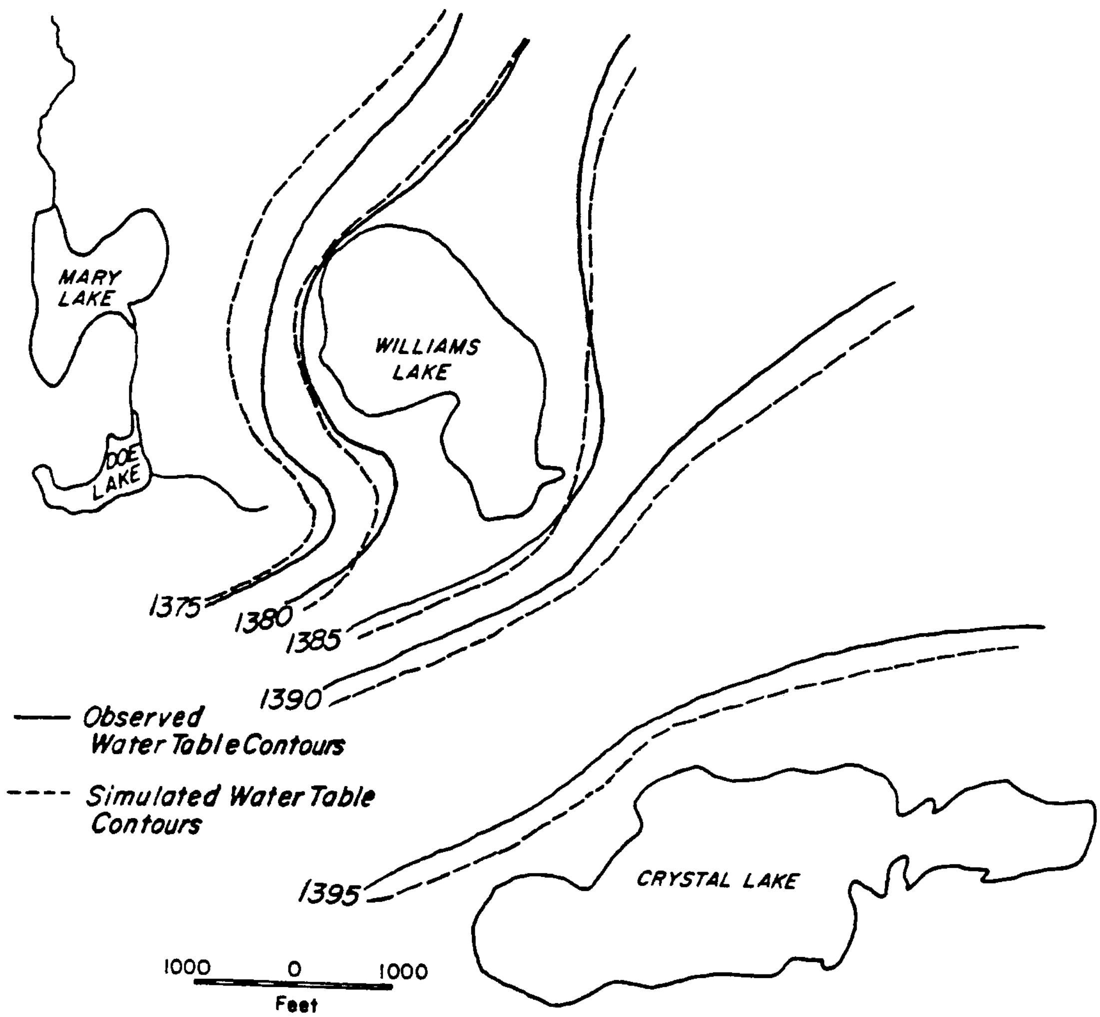



Figure 27. Observed and Simulated Head Contour Comparison -- Period 5.

inlets or outlets and overland flow is considered insignificant at Williams Lake, owing to the permeable sandy soil and vegetation types of the watershed (Groschen, 1981).

The water budget equation is:

$$P - E - \Delta S = NGW$$

where

$P$ = precipitation,
$E$ = evaporation,
$GI$ = ground water inflow,
$GO$ = ground water outflow,
$\Delta S$ = change in storage of lake, and
$NGW$ = net ground water flow (GO-GI).

Precipitation was measured at the field site (Figure 2) with a tipping bucket rain gauge and was assumed to be uniform over the entire lake. Errors associated with this are about 5 to 15 per cent (Winter, 1981), but Williams Lake is relatively small (90 acres) so these errors are probably 5 percent or less.

Evaporation was calculated by mass transfer methods, as described earlier. Air temperature, water temperature, humidity and wind speed were measured at the shore station or at the raft station near the center of the lake (Figure 2), and are required in the mass transfer computations. Winter (1981) estimated the errors associated with this method range from 15 to over 25 per cent.

Lake stage was monitored with a continuous recorder in a stilling-well on the lake shore. Errors associated with measurements of the lake stage are less than 5 per cent. The lake stage hydrograph is in Appendix 3.

Values for precipitation, evaporation, and change in lake storage are given in Table 9. Little or no precipitation or evaporation was assumed to have occurred on the lake surface during the period November 1 through February 28. An exception to this is in November 1979 when heavy rains occurred early in the month before ice covered the lake.

Comparisons between the calculated net ground water fluxes and the simulated ground water fluxes are also shown in Figure 28.

The percent difference is less than 10% except during period 4 when it was 17.6%. The magnitudes of precipitation and evaporation are greatest during period 4 and their respective errors as mentioned previously could have a large effect on the net ground water flux determined by the water balance method.

Ground water inflow and outflow could be computed using piezometers to define local horizontal gradients on the in-seepage and out-seepage sides of the lake, and estimates of horizontal hydraulic conductivity in a Darcy's law computation. This method is inaccurate because most of the flow into and out of the lake occurs vertically and mainly in the littoral zone of the lake. There are no piezometer nests in the littoral zone of Williams Lake to define the vertical gradients.

Discrepancies that exist in these net fluxes can be reduced by further hydrogeological investigations, utilizing energy budget methods to calculate evaporation, installing nested piezometers in the littoral zone of the lake, and by analyzing the spatial variability of precipitation over the lake surface.

Table 9 -- Mass Balance Parameters and Simulation Comparison

| Period | Precipitation $\times 10^3$ ft$^3$ | Evaporation $\times 10^3$ ft$^3$ | Change in Lake Storage $\times 10^3$ ft$^3$ | Mass Ballance Net Groundwater Flux[1] $\times 10^3$ ft$^3$ | Simulated Net Groundwater Flux[1] $\times 10^3$ ft$^3$ | % Diff.[2] |
|---|---|---|---|---|---|---|
| 1. July - Oct 1979 | 2,497 | 4,146 | -3,131 | 1,482 | 1,577 | 6.4 |
| 2. Nov - Feb 1980 | 1,025 | --- | - 198 | 1,223 | 1,106 | 9.5 |
| 3. March - June 1980 | 1,546 | 2,241 | -1,999 | 1,304 | 1,270 | 2.6 |
| 4. July - Oct 1980 | 3,964 | 3,571 | -1,189 | 1,582 | 1,304 | 17.6 |
| 5. Nov - Feb 1981 | --- | --- | - 793 | 793 | 860 | 8.4 |
| 6. March - June 1981 | 3,964 | 2,400[3] | +2,525 | - 961 | -1,047 | 8.9 |

Notes:

1. A positive value indicates a net flux out of the lake, and a negative value indicates a net flux into the lake.
2. $\frac{\text{(Water Budget Flux - Modeled Flux)}}{\text{Water Budget Flux}} \times 100\%$
3. Estimated

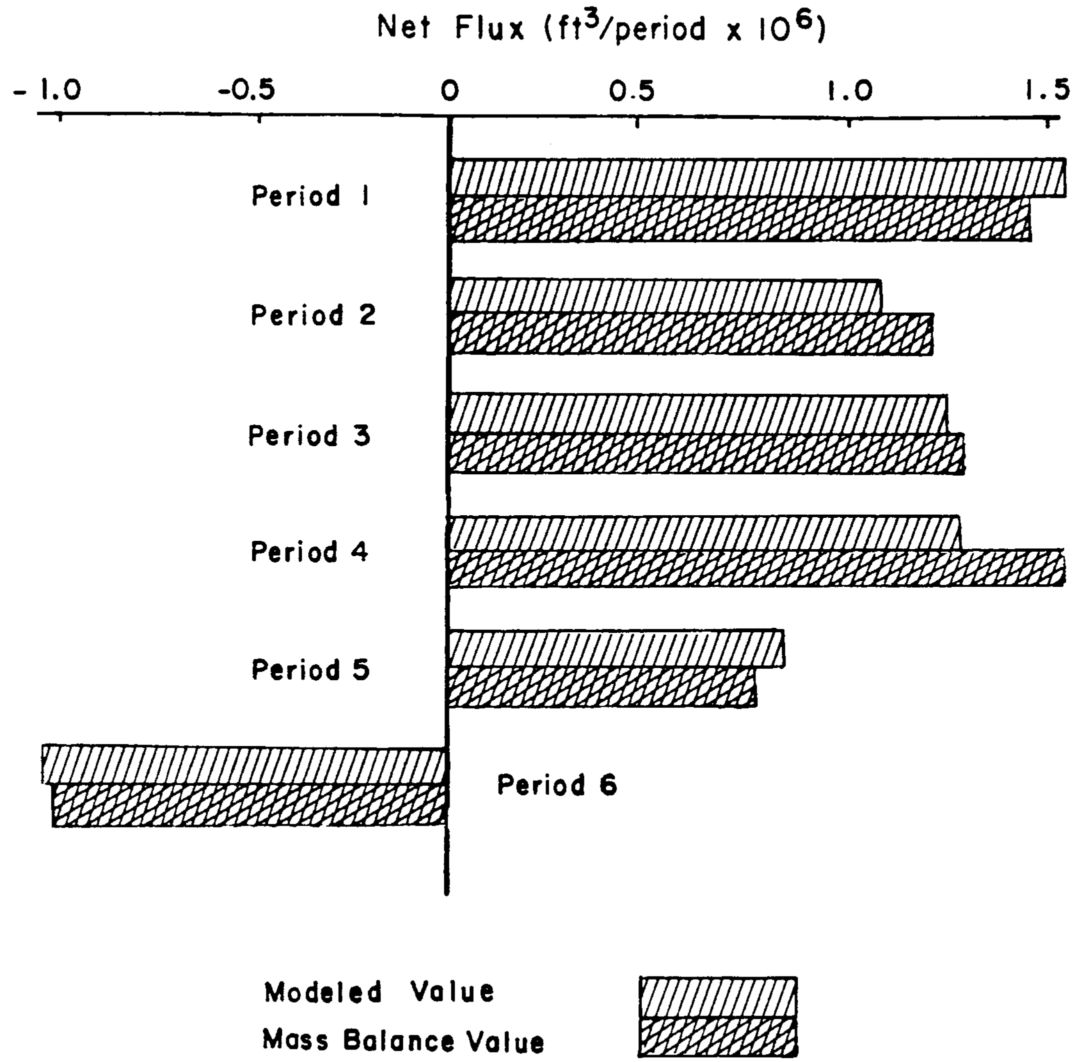



Figure 28. Comparison Between Simulated Net Fluxes and Mass Balance Computed Fluxes.

The magnitudes of the in-seepage and out-seepage simulated by the model are given in Table 10. For comparison, the values for period 1 reflect rates approximately twice those obtained by Erickson (1981) for a similar period using seepage meters, and an order of magnitude higher than the rates determined by Groschen (1981) using Darcy's law calculations based on horizontal flow.

Table 10 -- Magnitudes of In-seepage and Out-seepage

| Period | In-Seepage | Out-Seepage |
|---|---|---|
| 1. July - Oct 1979 | 604,417 | 2,181,487 |
| 2. Nov - Feb 1980 | 902,135 | 2,008,361 |
| 3. March - June 1980 | 822,704 | 2,092,794 |
| 4. July - Oct 1980 | 768,973 | 2,072,557 |
| 5. Nov - Feb 1981 | 1,001,988 | 1,861,719 |
| 6. March - June 1981 | 2,061,783 | 954,416 |

The difference between the modeled results and Erickson's seepage meter results may be due to (1) the difficulties in obtaining good quantitative results from seepage meters as mentioned in the section on previous investigations; (2) errors in modeling the system; or (3) from a combination of 1 and 2. The main reason for the large discrepancy between these results and Geroschen's, is probably his assumption of horizontal flow. The use of cross-sections and refraction methods in the section on the ground water flow system have shown that vertical flow occurs through the lake sediments.

The transient simulations show variations in the magnitudes of in-seepage and out-seepage that would play an important part in determining the chemical balance of the lake.

# CONCLUSIONS AND RECOMMENDATIONS

## Conclusions

The reliability of the model is limited, in general by possible errors from (1) inaccurate input data; and (2) numerical difficulties in representing a continuous three dimensional flow system with a discrete two dimensional model.

The input data used were the best available, and the sensitivity analysis has been used to examine the reliability of the model considering the assumptions that have been made. The characteristics of the lake sediments have been found to exert the largest effect on seepage rates, and unfortunately are parameters for which little field data has been collected.

The use of the leakage option in the model to simulate the lake has reduced the errors arising from the assumption of two-dimensional flow. Cross-sectional analysis of the flow system has shown that flow is predominantly vertical through the lake sediments, and the leakage option has been used to simulate this vertical flow. Refraction equations have shown that flow becomes horizontal after crossing the lake sediment/aquifer boundary.

The sensitivity analysis has shown that the lake seepage rates are sensitive to small changes in the hydraulic gradients near the lake. The scale of future modeling efforts may have to be reduced to allow the model to simulate these gradients, and cross-sectional models would be needed to examine the effects of vertical flow when the scale is reduced.

The use of a computer model to simulate the Williams Lake watershed has allowed the author to examine a variety of conditions and combinations of parameters that would have been difficult and time consuming to analyse using other approaches such as flow nets.

The models developed in this study have indicated that the amounts of inflow and outflow are greater than had previously been estimated. There is probably some error in the actual amount of inflow and outflow calculated by the model arising from uncertainties in input data, but the good reproduction of net ground water fluxes by the transient model suggest that the actual seepage rates are close to the computed rates.

This study is centered on the interaction of Williams Lake with the ground water flow system, but results such as the importance of the lake sediments, and the vertical flow through the sediments may be applicable to other lakes in similar situations.

## Recommendations For Further Research

A survey should be made of the lake bottom sediments, studying their thickness and distribution. A series of cores should be obtained to estimate the vertical hydraulic conductivity of the sediments. A study of this type might be conducted most easily through the winter ice which would provide a stable working surface.

The marl unit along the north and west shore of the lake should be explored and its extent and characteristics determined.

Piezometer nests should be completed in the littoral areas on the in-seepage and out-seepage sides of the lake to determine the magnitude of the vertical gradients present in these areas. Proper construction of these piezometers would be difficult because they need to be sealed and grouted thoroughly to prevent any vertical leakage.

Additional drill holes should be completed on the northern and eastern sides of the modeled area to further define the boundary conditions.

Additional test holes should be completed in sensitivity Zones 1 and 5 to provide more information on the thickness and characteristics of the materials present. Next to the characteristics of the lake sediments, these areas had the largest influence on seepage into and out of Williams Lake because of their position between the lake and the boundaries of the system.

Future modeling efforts could include transient sensitivity analyses if specific yield and recharge rates to analyse their effect on the system.

These recommendations will provide data in the areas of greatest uncertainty, and will allow the estimation of seepage with increased reliability.

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APPENDIX 1

Test Hole and Water Table Well Construction Data

| Well number | Total depth of well below land surface, in feet[1] | Lithology | Depth below land surface, in feet |
|---|---|---|---|
| WL-1 | 62 | Sand; brown, fine to medium, well sorted. | 0-14 |
| | | Sand; brown, fine to medium, silty, some gravel. | 14-17 |
| | | Sand; brown, fine. | 17-42 |
| | | Sand; brown, fine, silty, some gravel. | 42-62 |
| | | Sand; gray, medium to fine, some coarse sand, uniform and clean. | 52-107 |
| WL-2 | 82 | Sand; brown, fine to very fine, some medium to coarse, some gravel and silt. | 0-45 |
| | | Sand; brown, fine to very fine, some medium to coarse, some gravel. | 45-82 |

[1] Test hole usually deeper than well hole.

| Well number | Total depth of well below land surface, in feet | Lithology | Depth below land surface, in feet |
|---|---|---|---|
| WL-3 | 124 | Sand; brown, medium to coarse. | 0-22 |
| | | Sand; brown, medium to coarse, some large gravel. | 22-52 |
| | | Sand; brown, medium to coarse, some gravel and pebbles. | 52-142 |
| WL-4 | 52 | Sand; brown, medium to coarse, silty, some lenses slightly coarser. | 0-112 |
| | | Till; contains silty clay. | 112 |
| WL-5 | 15.8 | Sand; gray, medium to coarse, medium to well sorted. | 0-12 |
| | | Sand; greenish gray, medium to coarse, "slushy", poorly sorted, silty. | 12-97 |
| | | Till; gray, clayey, silty, sharp contact. | 97-102 |
| WL-6 | 32 | Sand; brown, silty, fine. | 0-27 |
| | | Sand; gray, silty, some clay. | 27-42 |

| Well number | Total depth of well below land surface, in feet | Lithology | Depth below land surface, in feet |
|---|---|---|---|
| WL-7 | 128.3 | Sand; brown, some pebbles. | 0-22 |
| | | Sand; brown, medium to coarse, well sorted. | 22-57 |
| | | Sand; brown, medium to coarse, some coarse to very coarse, pebble size. | 57-72 |
| | | Sand; brown, medium to coarse, silty. | 72-137 |
| WL-8 | 25 | Sand; brown, medium to coarse, clean. | 0-25 |
| | | Sand; gray, medium to coarse, silty. | 25-34 |
| | | Till; sandy silt mixed with granules, more clayey with depth. | 34-57 |
| WL-9 | 46.1 | Sand; dark brown, silty, medium tol coarse, some pebbles at top. | 0-12 |
| | | Sand; light brown, very well sorted, medium to fine. | 12-27 |
| | | Sand; brown to gray, coarse to granule sized, little silt. | 27-74 |

| Well number | Total depth of well below land surface, in feet | Lithology | Depth below land surface, in feet |
|---|---|---|---|
| WL-10 | 54.3 | Sand; light brown, medium to fine, well sorted. | 0-17 |
| | | Sand; light brown, medium to fine. | 17-47 |
| | | Sand; gray, coarse to very coarse, no silt. | 47-132 |
| WL-11 | 36.4 | Sand; dark brown, medium, silty, some clay. | 0.7 |
| | | Silt; dark brown, clayey. | 7-22 |
| | | Sand; gray, medium to fine. | 22-37 |
| WLN-299 | 299 | Sand and gravel; brown and gray. | 0-133 |
| | | Till; gray, clayey. | 133-150 |
| | | Sand and gravel; gray. | 150-178 |
| | | Till; gray, clayey. | 178-248 |
| | | Sand and gravel; gray. | 248-253 |
| | | Till; gray, clayey. | 253-269 |
| | | Sand and gravel; gray. | 269-312 |
| | | Till; gray clayey, interbedded with lenses of sandy till. | 312-418 |

| Well number | Total depth of well below land surface, in feet | Lithology | Depth below land surface, in feet |
|---|---|---|---|
| WL-12 | 344 | Sand and gravel, brown to gray, clean. | 0-157 |
| | | Till; gray, silty. | 157-193 |
| | | Till; gray to brown, sandy. | 193-237 |
| | | Sand; brown, medium to coarse. | 237-242 |
| | | Till; gray, silty. | 242-250 |
| | | Sand; gray, medium to very coarse. | 250-273 |
| | | Till; gray, clayey. | 273-285 |
| | | Sand; gray, silty. | 285-300 |
| | | Till; gray, silty. | 300-345 |
| | | Till; gray, clayey. | 345-360 |
| | | Till; red, clayey. | 360-374 |
| | | Till; gray, sandy. | 374-390 |
| | | Till; gray, clayey. | 390-400 |
| WL-13 | 17.2 | Sand; brown, medium. | 0-8 |
| | | Sand; brown, silty | 8-11 |
| | | Sand; gray, coarse clean. | 11-17 |

| Well number | Total depth of well below land surface, in feet | Lithology | Depth below land surface, in feet |
|---|---|---|---|
| WL-14 | 41.5 | Sand; brown, medium to coarse, silty. | 0-22 |
| | | Sand; gray, medium to coarse, clean. | 22-47 |
| WL-15 | 55.4 | Sand; brown, medium some gravel and silt. | 0-13 |
| | | Boulders, gravel and cobbles greater than 4 inches. | 13-17 |
| | | Sand; brown, medium to coarse, clean. | 17-40 |
| | | Sand; gray, fine to medium. | 40-57 |
| WL-16 | 69.5 | Sand; brown, medium gravel to 1/4 inch. | 0-12 |
| | | Gravel; cobbles greater tan 2 inches. | 12-15 |
| | | Sand; medium to coarse, brown. | 15-17 |
| WL-18 | 25 | Sand; brown, medium. | 0-22 |
| | | Sand; gray, medium. | 22-27 |
| WL-19 | 27 | Sand; brown, silty. | 0-11 |
| | | Clay; brown, smeary. | 11-22 |
| | | Sand; gray, coarse well sorted. | 22-47 |

| Well number | Total depth of well below land surface, in feet | Lithology | Depth below land surface, in feet |
|---|---|---|---|
| WL-20 | 36.9 | Clay; brown, sandy. | 0-12 |
| | | Sand; brown, fine to medium, silty. | 12-46 |
| | | Sand; gray, silty medium. | 46-72 |
| WL-21 | 17.5 | Sand; brown medium. | 0-15 |
| | | Sand; gray, silty fine to medium. | 15-41 |
| | | Till; gray, clay with some fine sand. | 41-47 |
| WL-22 | 10 | Sand; brown, medium. | 0-2 |
| | | Clay; brown, smeary, some sand. | 2-10 |
| WL-24 | 46.5 | Sand; brown, medium. | 0-27 |
| | | Sand; gray, medium to coarse, clean. | 27-52 |
| WL-25 | 47.5 | Sand; brown, silty to clayey. | 0-48 |

## APPENDIX 2

### Modifications to the Computer Codes

The lines of code on the following page must be modified or added as indicated to change the lake stage, recharge rate, and evapotranspiration rate with each time period.

Factors must be calculated to change the rates and values to their proper rates and values for the next time period. These factors are then added to the end of the Group IV data (Trescott et al, 1976) in the format (3F10.7).

The following lines must be modified to read as follows:

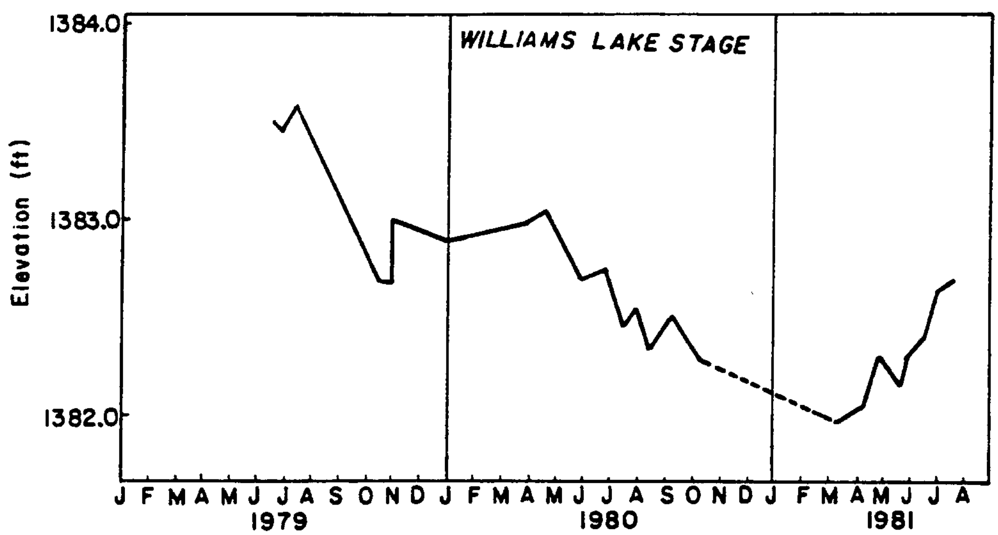
```
DAT1950
    READ(P,493) KP,KPM1,NWEL,TMAX,NUMT,CDELT,DELT,FACTQ          DAT1950
FACTE,FACTR

DAT2260
370 IF(NWEL.EQ.0) GO TO 405                                      DAT2260

DAT2620
493 FORMAT(3I10,F10.0,I10,2F10.0/8F10.0)                         DAT2620
```

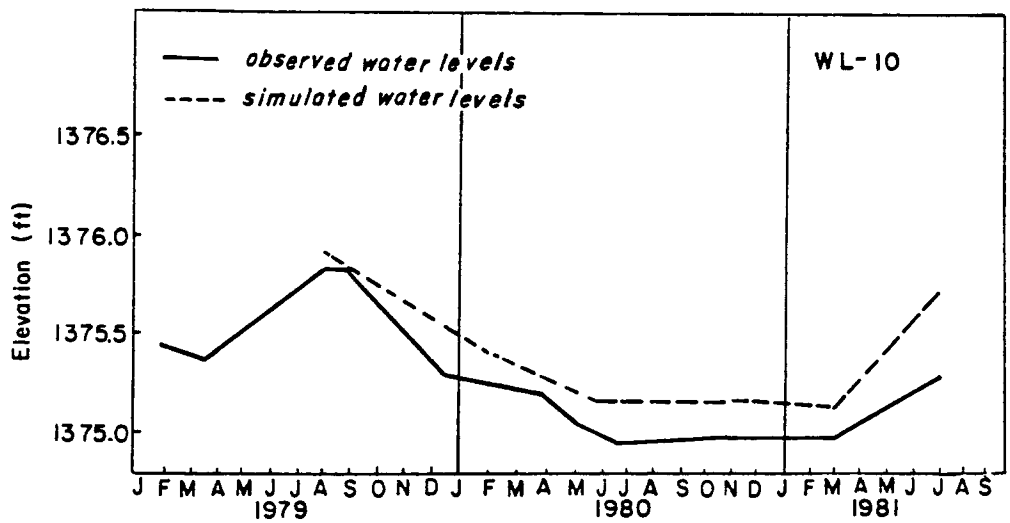
The following lines must be added where indicated:

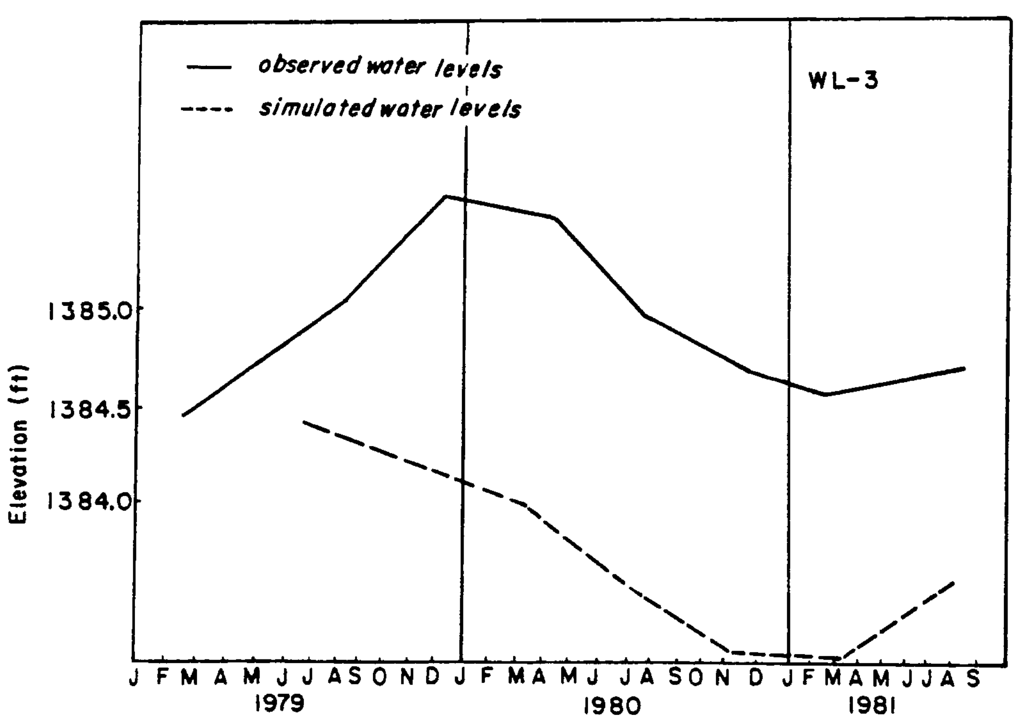
```
DAT2145
    QET=QET*FACTE                                                DAT2145

DAT2163
    QRE(I,J)=QRE(I,J)*FACTQ                                      DAT2163

DAT2167
    RIVER(I,J)=RIVER(I,J)*FACTR                                  DAT2167

DAT2425
    WRITE(P,620)FACTR,FACTE,FACTQ                                DAT2425

DAT2865
620 FORMAT(*0*,2X,*FACTR*,F10.7/3X,*FACTE*,F10.5/                DAT2865
4X,*FACTQ*,F9.5)
```

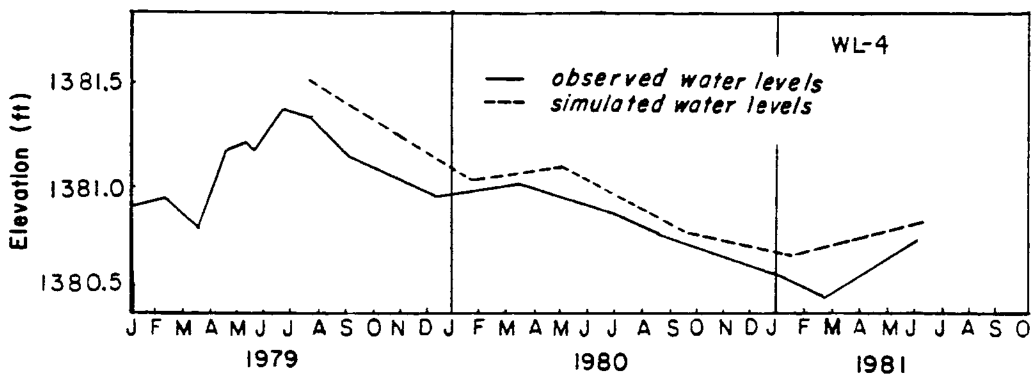
## APPENDIX 3

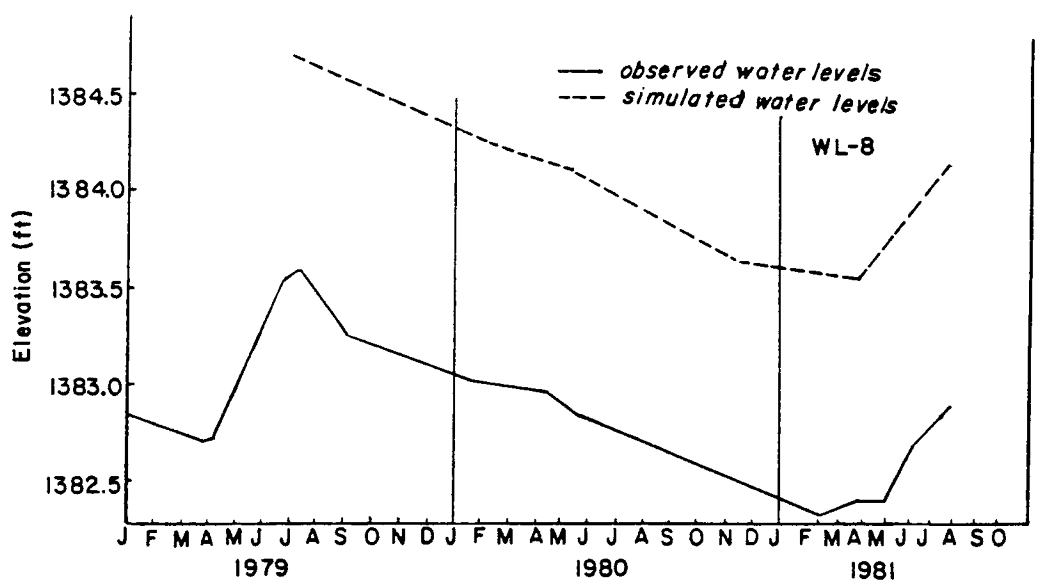
### Hydrographs

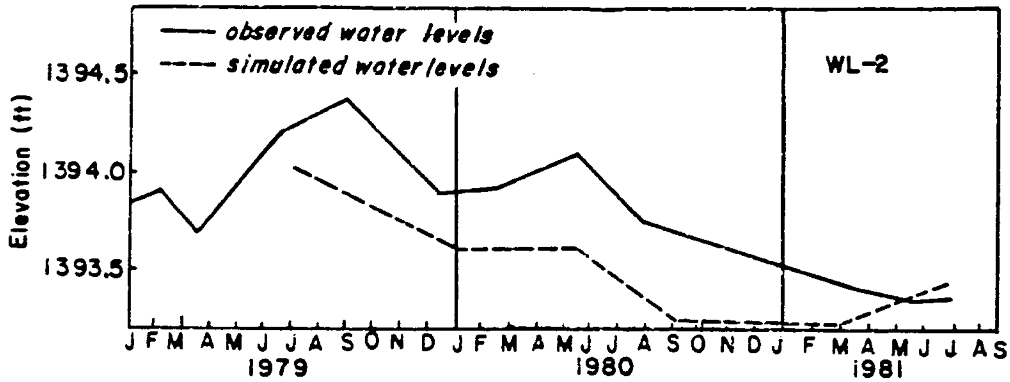

WILLIAMS LAKE STAGE
Elevation (ft)
1384.0
1383.0
1382.0
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A
1979
1980
1981


observed water levels
simulated water levels
WL-10
1376.5
1376.0
1375.5
1375.0
Elevation (ft)
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S
1979
1980
1981

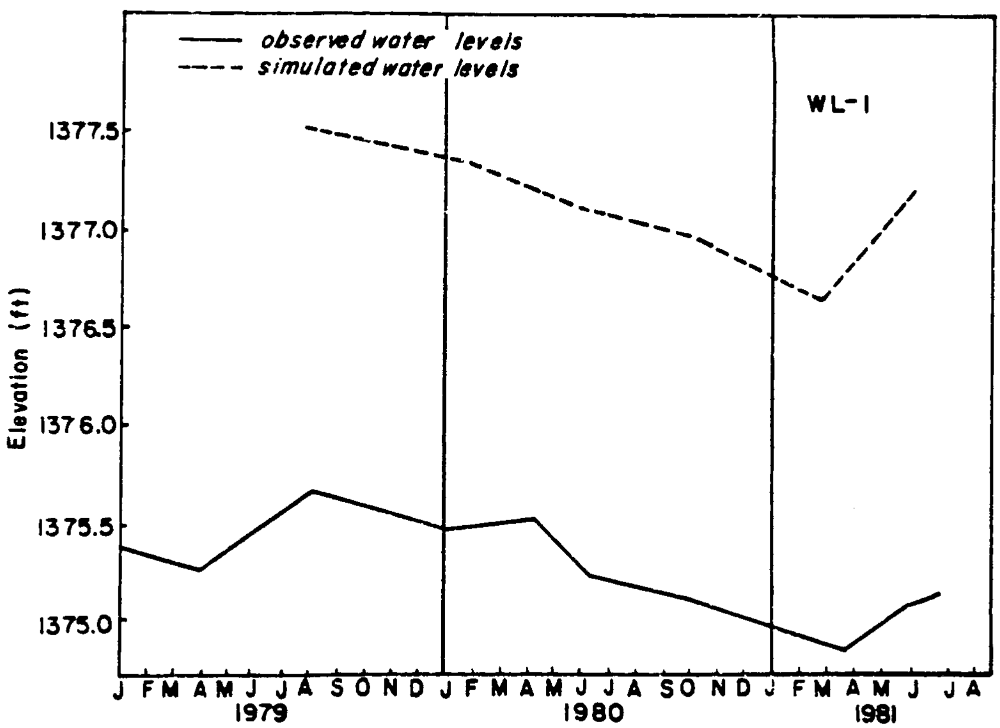

observed water levels
simulated water levels
WL-3
1385.0
1384.5
1384.0
Elevation (ft)
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S
1979
1980
1981


WL-4
observed water levels
simulated water levels
Elevation (ft)
1381.5
1381.0
1380.5
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O
1979
1980
1981


observed water levels
simulated water levels
WL-8
Elevation (ft)
1384.5
1384.0
1383.5
1383.0
1382.5
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O
1979
1980
1981



observed water levels
simulated water levels
WL-2
1394.5
1394.0
1393.5
Elevation (ft)
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S
1979
1980
1981

observed water levels
simulated water levels
WL-1
1377.5
1377.0
1376.5
1376.0
1375.5
1375.0
Elevation (ft)
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A
1979
1980
1981