DETERMINATION OF UNIT WATERSHED SIZE
FOR USE IN SMALL WATERSHED
HYDROLOGICAL MODELING

by

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STATEMENT BY AUTHOR

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This thesis is dedicated to my parents and, in general, my family. During all my life, it was their moral and financial support that offered me a happy home and the opportunity I got in my education. I always feel the love and care from them no matter where I am. If there could be any achievement in my life, it no doubt owes to them.
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ABSTRACT

Since the uniform rainfall over the watershed is the most fundamental assumption in small watershed modelling, the limitation on watershed size should be investigated. This study defines the unit watershed size as a dimensional criterion which is associated with the storm size, and the extent and frequency of storm exclusion (called spatial and temporal errors).

Two approaches of determining average storm cell radius were proposed. One is related with the spatial variation in storm rainfall (DSIP), while another considers both spatial variation and storm exclusion events (RVIP). Both analytical and empirical solutions are obtained and the effect of multiple-storm events is discussed. The storm radius for Walnut Gulch is determined as 4.6 miles which is close to others' results.

Given storm radius, a relationship between unit watershed size and the spatial and temporal errors is developed analytically. Based on this relationship, both selection and evaluation of unit watershed size are made possible. If the error levels are known, then the proper watershed size can be selected and if the watershed size is given, then the error levels can be evaluated. By using unit watershed size, the models of small watersheds may be extended to those of large watersheds.
CHAPTER 1

INTRODUCTION

This study is one segment of a larger project between the University of Arizona and three sponsoring agencies -- the Salt River Project, Arizona Department of Water Resources, and United States Forest Service. The overall objective of the project was to assess the hydrologic impact and performance of stock-watering ponds in Arizona.

Past Work

The project began to be making impact and performance assessments for single watersheds at three locations. The first location considered was in the pinyon-juniper cover type on the Beaver Creek Experimental Watersheds in north-central Arizona, near Flagstaff (Almestad, 1983). Kiyose (1984) subsequently analyzed data from the Walnut Gulch Experimental Watershed in Southeast Arizona, near Tombstone for desert scrub watersheds. Currently, the Whitespar Experimental Watersheds in central Arizona, near Prescott, is being studied by McDowell (1985).

In these point studies, a coupled stochastic-deterministic model of the rainfall-runoff-routing processes occurring on the watershed was used with Monte Carlo computer simulation techniques to develop probabilistic results.
on pond performance. The results from these studies were favorable, but the applicability to other locations in the state was limited. The large amount of time needed to analyse data from each location separately further limited general application of the point model approach. In addition, the point model was not capable of considering multiple or coupled watersheds and ponds.

Objectives

Project Objective

In an attempt to generalize the point model, the final phase of the project was to develop an interactive, regional computer model capable of handling multiple ponds and watersheds (Long and Henkel, 1985). Development of such a model required three basic techniques. The first needed was a methodology for quickly and indirectly specifying input precipitation distribution parameters for any point in an extended region. In this project, a technique developed by Henkel (1985) for Southeast Arizona was used. The second technique required was a method for estimating channel transmission losses between coupled watersheds. A model proposed by Lane (1982) was employed for this purpose. The third requirement was to determine an acceptable watershed size that could be modeled as a single, homogeneous unit. Implicit in this determination was the estimation of an average storm cell size and its probabilistic relationship
to uniform coverage of the (unit) watershed. This third task was the subject of this study.

Study Objective

The objective of this study was to develop a methodology for estimating the size of a unit watershed and to apply that methodology to data from Southeast Arizona. The methodology consisted of two basic components. The first was the estimation of an "average" storm cell radius, and the second was the determination of the relationship between the frequency and extent of partial coverage with unit watershed size. The goal was to develop a decision-making criteria for choosing watershed unit sizes or evaluating exclusion errors in pre-selected watershed sizes.

Study Organization

To model storm cell sizes and watershed exclusion errors, it was necessary to make a number of simplifications of actual processes. Chapter 2 summarizes these storm model assumptions. In chapter 3, two analytical approaches for analyzing storm cell sizes are introduced. The first of these, the Regionalized Variation Index of Precipitation (RVIP), is considered in detail in chapter 4. Chapter 5 focuses on a second, simplified approach: the Difference Square Index of Precipitation (DSIP). In chapter 6, the approaches are used with data from Southeast Arizona to
estimate an average storm cell radius. Chapter 7 then considers the second part of the study -- the relationship between unit watershed size and errors due to partial storm coverage. A procedure for selecting watershed sizes or evaluating exclusion errors is presented. Chapter 8 provides a summary and a discussion of the conclusions and applications of the study. An outline of the study organization is shown in figure 1.
Figure 1 Thesis Organization
CHAPTER 2

STORM MODEL

In this chapter, the basis for determining storm size and the size of the related watershed unit is presented. Five major simplifying assumptions are needed to develop the storm model. Following a brief literature review, each of these assumptions is discussed in detail.

Related Literature

In Southeast Arizona, precipitation is typically of three basic types: frontal, air mass and frontal convective thunderstorm (Sellers, 1960, 1972, and Petterssen, 1956). However, about 70 percent of the annual rainfall and 90 percent of the annual runoff results from air mass thunderstorms (Osborn and Hickok, 1968; Osborn and Laursen, 1973 and Osborn et al, 1979). Therefore, the storm model in this study was developed to describe the characteristics of air mass thunderstorms.

Fogel (1968) summarized that thunderstorms are highly variable in time and are limited in areal extent. Several studies have attempted to describe the spatial extent of thunderstorms, most of which were based on the area-depth approach (Woolhiser and Schwalen, 1960; Court,
In these studies, the concept of a storm "cell" was introduced. Isohyetals recorded from a storm cell were described as elliptical in shape. The spatial distribution of storm isohyetals were fit to smooth, symmetrical bell-shaped curves. The occurrence of storms over Southeast Arizona appeared random.

Court (1961) reasoned that a realistic model of thunderstorms should have the following characteristics:

1) Any realistic representation of the distribution of rainfall depth about the storm center should be smooth and rounded at the center;

2) Rainfall depth should approach zero asymptotically as distance from storm center increases. The use of a bivariate Gaussian distribution to describe the observed (elliptical) isohyetals of storm rainfall was also proposed.

Fogel and Duckstein (1969) also found that the isohyetals of a storm cell "exhibited a marked tendency towards an elliptical shape" after studying the pattern of nearly 200 convective storms on the Atterbury and Walnut Gulch experimental watersheds. They also noted that the ratio of the major axis to the minor axis of the ellipses, on the average, was about 1.5 to 1.0. Based on regression analysis, they also concluded that spatial distribution of isohyetals from a storm cell could be described by one of the bell-shaped equations.
Based on rainfall data from Walnut Gulch, Osborn and Lane (1972) proposed another depth-area relationship, and further assumed symmetry around the storm center depth. The storm cell shape was modeled as a circle, with radius assumed to be constant.

In the absence of observable orographic effects, Osborn and Reynolds (1963) concluded that the thunderstorms appeared to be random in the Southwestern United States.

**Storm Model**

The five simplifying assumptions made in the storm model used in this study are detailed below.

**Assumption 1**

The first and most fundamental assumption in the model is that storms occur in the form of individual cells. The assumption appears to be acceptable for Southeast Arizona, since most runoff-producing rainfall results from (summer) air mass thunderstorms. The assumption does not hold for frontal rainfall in the winter season; in this case, a frontal event may be statistically treated as several storm cells coupled together. Besides, frontal rainfall has a low volume compared with that of air mass thunderstorm rainfall in Southeast Arizona.
Assumption 2

The second assumption is that the storm cell isohyetal cover a circular area on the ground. There are several arguments that support this assumption.

First, although previous work has shown that the storm shape is closer to an ellipse, the ratio of major to minor axes was close to one. The size approximation is best if a geometric average of the major and minor axes is used as the radius, since the area of the circle and ellipse would be the same.

A second argument for using a circle involves the statistical connotation of a storm radius. Since the shape of a particular storm is almost impossible to predict, and since few storms have simple or regular shapes, a smooth, rounded shape such as a circle fits the long-term average for many storms adequately. Since radar studies have found that storms seldom move farther than one mile in the span of their lifetime (Braham, 1958 and Battan, 1982), the isohyetal pattern is expected to be stable and similar in shape to a circle.

Assumption 3

The third assumption is that the storm radius can be modeled as a constant; that is, all storm cells can be adequately represented by a mean or mode radius.
There are two reasons for this assumption. First, the radius is defined as the radius of an "averaged" storm cell. It should be stable for a proper region. If the individual storm dimension follows a symmetric distribution, the storm event with a dimension close to the mean radius would most likely occur. Otherwise, the mode radius will give the greatest probability to the events with a dimension close to the mode. If the constant does not represent the size of all storm cells, it will represent the most likely events among them. If it is impossible to model all events, the modeling of most likely events may be a reasonable approximation. This study follows this consideration. In this sense, the constant radius is interpreted as the mode or mean radius.

Second, some empirical evidence in Southeast Arizona also shows that the storm cell radius, \( R \), is relatively invariant with changing storm center depths. For instance, Osborn and Lane (1972) obtained the following storm area-depth formula with data from the Walnut Gulch Experimental Watersheds:

\[
PPT(r) = PPT_0*(0.9-0.2*ln(3.14*r^2))
\]

where \( PPT_0 \) is the depth of precipitation at the storm center, and \( PPT(r) \) is the depth of precipitation at a distance \( r \) from the storm center. If \( PPT(r) \) is equal to 0.01 inches (i.e., the smallest unit of measurable precipitation), then \( r \) is equal to the storm cell radius, \( R \). At this limit, \( R \) is
a function of only the storm center depth, \( PPT_0 \). When \( PPT_0 \) is varied over the most common range of 0.5 to 3.0 inches, for instance, \( R \) only changes from 5.11 to 5.35 miles. Since very few events have storm center depths greater than 3.0 inches (Osborn and Lane, 1972), the storm cell radius can be modeled adequately as a constant with respect to storm center depth.

Assumption 4

The fourth assumption is that storm occurrence is random; that is, every point has an equal probability of being covered by the storm center. The influences of topography on storm occurrence is ignored. Most researchers in Southeast Arizona seemed to have accepted such an assumption, primarily because of the small area covered by individual storms as compared with the much larger area of a region. Furthermore, convective storms in the Southwest travel only one or two miles, and there is a lack of statistical evidence that they follow consistent passways.

Assumption 5

The final major assumption is that the spatial distribution of rainfall depths about the storm center decays exponentially; specifically,

\[
PPT(r) = PPT_0 \exp\left(-A\frac{r^2}{R^2}\right), \quad r < R
\]

\[
= 0, \quad \text{elsewhere}
\]
where $A$ is a factor which describes the shape of this distribution. This assumption implies that storm rainfall is symmetrically distributed about the storm center and that, for a given value of $a$, the rainfall depth depends only on the relative distance from the storm center, $r/R$.

This assumption satisfies the two requirements proposed by Court in 1961 (i.e., smooth, rounded and symmetrical distribution). It is also comparable to the formula presented by Fogel and Duckstein (1969):

$$PPT(r) = PPT_0 \cdot \exp(-3.14 \cdot B \cdot r^2)$$

where $B(PPT_0) = 0.27 \cdot \exp(-0.67 \cdot PPT_0)$. The equation 2-3 does not assume $PPT(r)$ is directly related to storm radius, $R$, and does not restrict the range of $r$.

If it is assumed that

$$3.14 \cdot B = A/R^2$$

then equation 2-3 is equivalent to equation 2-2. In fact, equation 2-4 is used in this study to estimate the factor $A$. If let $PPT(r) = 0.01$ inches and therefore $r=R$ (i.e., the boundary of storm cell), then data from Fogel and Duckstein (1969) on storm center depths $(PPT_0)$ can be used to calculate $R$ and $B$ with equation 2-3. Equation 2-4 can then be employed to determine a factor $A$ corresponding to each $PPT_0$ value. Through regression analysis, the relationship between
A and PPT₀ was determined in this study as:

\[ A(\text{PPT}_0) = 4.051 \times \exp(0.131 \times \text{PPT}_0) \]

Since PPT₀ is multiplied by a small number (i.e., 0.131), then

\[ \exp(0.131 \times \text{PPT}_0) \to e^0 = 1.0. \]

As such, the factor A is relatively insensitive to PPT₀. In this study, A was assumed to be equal to 4.051.

All five assumptions above constitute the basis of the storm model developed in this study. As with other assumptions, they are simplifying approximations of actual phenomena and have limitations which should be tested further. Special testing was not carried out directly in this study.
CHAPTER 3

APPROACHES FOR DETERMINING STORM RADIUS

This chapter introduces the two basic approaches used for statistically estimating the "average" radius of a storm cell. The approaches are given detailed consideration in chapters 4 and 5.

Different Methods

There are several possible ways to determine storm size. The most direct method is to use radar on a storm by storm basis; however, the cost and time required by the method and the difficulties in extrapolating information on ground coverage ruled out its use in this study. Another method that has been extensively employed is to use recorded rainfall totals in an area-depth analysis. With this method, precipitation data from a dense raingauge network are plotted on isohyetal maps for single storm events. The storm size is then measured and the measurements averaged to obtain statistical information on storm size. The method requires considerable time and was not used in this study. Instead, a statistical analysis was made directly (i.e., without the use of graphics) with daily rainfall data from a dense raingauge network. The main advantage of the method is
that analyses can be made easily and quickly on the computer. The following outlines two different approaches to the statistical analysis.

Statistical Determination

The assumption behind the statistical analysis is that the spatial variation in precipitation as measured at different gauges reflects the important characteristics concerning storm cell sizes. There are at least two kinds of information which can be obtained from measurements made at several points on the ground. The first is the difference in precipitation amounts between points, and the second is the determination of when certain points are excluded from storm coverage.

The difference in precipitation amounts is generally a function of the location of the points. If the storm radius is a constant, as assumed, the closer that two points are to each other, the smaller is the difference in their measurement of precipitation for a single storm. Conversely, the greater the distance between points, the larger will be the difference. With the assumption that storm occurrence is a random process at a point, the spatial variation in precipitation is only a function of the displacement between points, and not the location of the points. Since each point has the same chance to receive rainfall as others, there is no special reason to expect differences in precipitation
catch to be different for the same distances (i.e., displacements) at different locations over the long-term.

**Approach 1: Without Storm Exclusion**

The variation in precipitation with distance depends on whether or not consideration is given to events where one of the two points is not covered by the storm. In the first approach, such storm exclusion events are not considered; that is, differences are only calculated for events where both points receive measurable rainfall. An illustration of these types of events is shown in figure 2.

A graph of the theoretical variation in precipitation catch with displacement between points for events without storm exclusion is shown in figure 3. Spatial variation is defined as the expected value of the differences squared: $E(\Delta \text{PPT})^2$. For small displacements, the expected variation is also small (figure 3a). As the displacement between points increases, the variation in precipitation increases. At some distance, the variation reaches a maximum (figure 3c). This maximum variation is referred to as the peak variation and it occurs at a displacement labelled as the peak distance. Due to the assumed bell-shaped distribution of precipitation amounts over space and the small extreme or tail values, the variation decreases for displacements larger than the peak distance (figure 3b). The
Figure 2. Event Types: Exclusion and Non-exclusion
(a) Generally small $\Delta$PPT, while small d.

(b) Small $\Delta$PPT, while large d.

(c) Large $\Delta$PPT, when mediate d.

Figure 3. Precipitation Difference and Distance
Distance between Control and Reference points

$E[\Delta Pf^2(d)]$

Figure 4. Expected Precipitation Difference
overall feature of the precipitation variation verse the
displacement, \( d \), is expected in figure 4.

The so-called peak distance is thought to be highly
related to the size of the storm. It can be expected that
the larger the storm radius, the larger the peak distance.

Detailed consideration of approach 1 and what is
called the Difference Square Index of Precipitation (DSIP)
is provided in chapter 5.

Approach 2: With Storm Exclusion

In considering events where one of two points is not
covered by the storm (figure 2b), the variation in precipi-
tation with increased displacement does not decline for very
large displacements. Instead, the variation reaches a
maximum and remains stable (constant). This is the result of
considering displacements that are larger than the actual
storm diameter. With the assumption of random occurrence of
storms, the variation should remain constant over the long-
term for all displacements beyond the storm diameter.

The displacement at which the variation becomes
stable is referred to as the critical distance. Like the
peak distance in approach 1, the critical distance is also
thought to be highly related to the storm diameter.

Detailed consideration of approach 2 and what is
called the Regionalized Variation Index of Precipitation
(RVIP) is provided in chapter 4. The RVIP approach is
considered before the DVIP approach since it was found that a more natural treatment of the storm model assumption was possible.
CHAPTER 4

REGIONALIZED VARIATION INDEX OF PRECIPITATION

This chapter develops the Regionalized Variation Index of Precipitation (RVIP) approach for estimating the average storm radius. The RVIP approach considers events where both gauges receive precipitation, as well as events where only one of two gauges receive precipitation (i.e., storm exclusion events). The discussion is focussed strictly on the theoretical aspects and analytical solutions of the approach.

Definitions

In estimating the average storm radius, it is useful to define a fixed control point or gauge \( X_c \) and a series of reference points \( X_r \) at increased distances along a line from the control point (figure 5). It is necessary to specify a control point in order to distinguish between the two types of RV events: storm exclusion and non-storm exclusion events. An RV event is said to occur only when the selected control point, \( X_c \), receives precipitation. For this RV event, a given reference point, \( X_r \), at a given displacement, \( d \), may or may not receive precipitation; if it does not, storm exclusion is said to occur.
RV Event: Control Point is Covered by Storm

Non-RV Event: Control Point is Excluded by Storm

Figure 5. Calibrating Points and RV Event
Precipitation Variation

The concept of the variation in precipitation between a control and a reference point is developed differently for storm exclusion and non-storm exclusion events. In the non-storm exclusion case (i.e., both where the control and reference points receive rainfall), the variation is defined as the expected value of the difference in precipitation squared; that is, \( E(\Delta PPT^2(d)) \), where \( \Delta PPT^2(d) = (PPT(X_c) - PPT(X_r))^2 \) and \( d \) is the displacement between \( X_c \) and \( X_r \). In the storm exclusion case (i.e., where only the control point receives rainfall), the difference in precipitation, \( \Delta PPT(d) \), is defined as the storm center depth \( (PPT_0) \). This definition was meant to exaggerate the difference in precipitation and thereby emphasize the occurrence of storm exclusion. It is thus clear that this definition of variation is not that of variance in statistics and it includes extra information about nearby region (i.e., the storm exclusion). This variation is called "regionalized" to emphasize this characteristic. The total regionalized variation in precipitation, \( TRV \), is calculated by combining the variation from both storm exclusion and non-storm exclusion events. The probability of a non-storm-exclusion event, \( p \), is used to weight the two components in the total variation. Specifically,
\[ \text{TRV}(x_c, x_r) = (p \cdot \text{E}(\text{PPT}(x_c) - \text{PPT}(x_r))^2 + (1 - p) \cdot \text{E}(\text{PPT}_0)^2 \]

Both \( p \) and \( \text{E}(\text{PPT}(x_c) - \text{PPT}(x_r))^2 \) are, in general, a function of the location of \( x_c \) and \( x_r \) within a region. Under the assumption of random storm occurrence, however, TRV is reduced to a function of the displacement, \( d \), between \( x_c \) and \( x_r \); that is,

\[ \text{TRV}(d) = p \cdot \text{E}(\Delta \text{PPT}^2(d)) + (1 - p) \cdot \text{E}(\text{PPT}_0^2) \]

**Indexing Precipitation Variation**

The variation in precipitation, as defined, is largely a function of the mean storm center depth in a region. In an effort to generalize the precipitation variation and enable comparison between different regions, an indexed variation, called the Regionalized Variation Index of Precipitation (RVIP), is introduced. By dividing the precipitation variation by the expected value of the mean storm center depth squared, \( \text{E}(\text{PPT}_0^2) \), and taking the square root, RVIP is a measure of the precipitation variation scaled between 0 and 1. Specifically,

\[ \text{RVIP}(d) = \left\{ \frac{\text{TRV}(d)}{\text{E}(\text{PPT}_0^2)} \right\}^{1/2} \]

**Properties of RVIP**

Several simple properties of RVIP can be shown. First, since RVIP is defined with the square root operator,
and since TRV and $E(PPT_0^2)$ are never negative, it is evident that $RVIP \geq 0$.

Second, $RVIP$ is $\leq 1$. This can be shown by rewriting equation 4-3 as

$$RVIP(d) = \left\{1 - P(d) \ast [E(PPT_0^2) - E(\Delta PPT^2(d))] / E(PPT_0^2)\right\}^{1/2}$$

and noting that $\Delta PPT^2(d) \leq PPT_0^2$, and, thus, $E(\Delta PPT^2(d))$ is less than or equal to $E(PPT_0^2)$.

Third, as the displacement between $X_c$ and $X_r$, $d$, goes to zero, then $RVIP$ also goes to zero. This is illustrated by examining equation 4-2 and noting that, as $d$ goes to zero, then $p(d)$ goes to 1 and $E(\Delta PPT^2(d))$ goes to zero.

A fourth property of $RVIP$ is that, where $d \geq$ storm diameter, then $RVIP=1$. Since no storm can cover two points separated by a displacement greater than its own dimensions, then the probability of no storm exclusion, $p(d)$, equates to zero, when $p(d)=0$ is substituted into equation 4-4, $RVIP=1$.

The third and fourth properties imply that somewhere between $d=0$ and $d=2r$, $RVIP=1$. The value of $d$ at which this occurs is said to be the critical distance, $D_c$. This critical distance is thought to be highly related to an average storm radius. This turns out to be the key for determining storm radius in the RVIP approach.
Estimating RVIP

In practice, it is necessary to be able to estimate RVIP from observed data. The following formula is proposed as an estimator of RVIP with discrete observations:

\[
RVIP(d) = \left\{ \frac{1}{N} \sum_{i=1}^{N} \frac{\Delta \text{ppt}_i(d)^2}{\text{ppt}_0^2} \right\}^{1/2}
\]

where \(N\) is the number of observed RV events and the precipitation difference between \(X_C\) and \(X_R\) for the event, is

\[
\Delta \text{ppt}_i(d) = \begin{cases} 
\text{ppt}_i(X_C) - \text{ppt}_i(X_R), & \text{ppt}_i(X_R) > 0; \\
\text{ppt}_0, & \text{ppt}_i(X_R) = 0.
\end{cases}
\]

Equation 4-5a can be shown to have equivalent form as the originally defined by equation 4-1. If \(N\) is the total number of RV events, let \(m\) be the number of storm exclusion events. Then, \((N-M)\) is the number of non-storm exclusion events. Therefore, the probability of no storm exclusion, \(p(d)\), can be estimated by

\[
p(d) = \frac{N-M}{N} \quad \text{and} \quad 1-p(d) = \frac{M}{N}.
\]

Therefore,

\[
\sum_{i=1}^{N} \text{ppt}_i^2(d) = \sum_{i=1}^{N-M} (\text{ppt}_i(X_C) - \text{ppt}_i(X_R))^2 + \sum_{i=1}^{M} \text{ppt}_0^2
\]

\[
= (N-M)\Delta \text{ppt}^2(d) + M \text{ppt}_0^2
\]

and equation 4-5a can be written as
\[
\hat{RVIP}(d) = \left\{ \frac{N-M}{N} \Delta \text{ppt}^2(d) + \frac{M}{N} \text{ppt}_0^2 \right\}^{1/2}
\]

\[
= \left\{ \hat{p}(d) \Delta \text{ppt}^2(d) + (1-\hat{p}(d)) \text{ppt}_0^2 \right\}^{1/2}
\]

4-5b

where sample means of \( p(d) \), \( E(\text{PPT}^2(d)) \) and \( E(\text{PPT}_0^2) \) are substituted in equation 4-5a. However, this form equivalence does not imply that this RVIP estimate is unbiased.

Equation 4-5b was used with actual data to estimate RVIP in this study. Results are presented in Chapter 6.

**Analytical Solution For RVIP**

It is possible to solve for RVIP analytically using only geometric relationships and the simplifying assumptions of the storm model as presented in chapter 2. To solve for \( RVIP(d) \), it is necessary first to evaluate the probability of non-storm exclusion, \( p(d) \), and the expected value of the precipitation variation, \( E(\Delta \text{PPT}^2(d)) \).

**Probability of Non-Exclusion Storms \( p(d) \)**

If the area of coverage for a single storm is denoted by ASC, then the probability of no storm exclusion (i.e., that both \( X_C \) and \( X_R \) receive rainfall) is given by the following equation:

\[
p(d) = \text{prob}\{ X_R \in \text{ASC} \mid X_C \in \text{ASC} \}
\]

\[
= \text{prob}\{ X_R \in \text{ASC} \mid \text{RV event} \}
\]

4-6.
In order to determine $p(d)$, the following notations are introduced. Let $ASC_c$ represent the geometric area in which a potential storm center may be located and still have rainfall cover the control point, $X_c$. The area $ASC_r$ is similarly defined for the reference point, $X_r$. Under the assumption of random storm occurrence, these areas correspond to probabilities of occurrence. Since storm cells are assumed to be circular in shape with a fixed radius, $R$, the following relationships are found:

$$ASC_c = \pi R^2 \quad \text{and} \quad ASC_r = \pi R^2$$

(4-7)

Where $ASC_c$ and $ASC_r$ intersect, both $X_c$ and $X_r$ receive rainfall and no storm exclusion occurs. This area of intersection which corresponds to the probability of non-exclusion storms is illustrated in figure 6 and is defined as follows:

$$ASC_c \cap ASC_r = 2R^2\left(\cos^{-1}\left(\frac{d}{D}\right) - \frac{d}{D}\left(1 - \left(\frac{d}{D}\right)^2\right)^{1/2}\right)$$

(4-8)

where $d$ is the distance between $X_c$ and $X_r$ and $D$ is the diameter of storm ($D=2R$).

The probability of no storm exclusion, $p(d)$, is therefore a conditional probability and can be written as

$$p(d) = \text{prob}\{ X_r \in ASC \mid X_c \in ASC \} \quad \text{or}$$
Figure 6. Graph for Determining the Probability of Non-exclusion Storms
p(d) = \frac{\text{prob}\{ X_r \in \text{ASC and } X_c \in \text{ASC} \}}{\text{prob}\{ X_c \in \text{ASC} \}}

= \begin{cases} 
2[\cos^{-1}(d/D) - (d/D)(1-(d/D)^2)^{1/2}] / \pi, & d < D, \\
0, & d > D. 
\end{cases}

Expected Precipitation Variation \(E(\Delta PPT^2(d))\)

The expected value of the square of the difference in precipitation between \(X_c\) and \(X_r\) when both points receive rainfall (i.e., \(E(\Delta PPT^2(d))\)) can also be expressed analytically.

In figure 7, let \(r\) and \(r'\) be the distances from the storm center, \(X_s\), to the control point, \(X_c\), and reference point, \(X_r\), respectively. The distance between \(X_c\) and \(X_r\) is \(d\), and the angle between \(r\) and \(d\) is \(w\).

Using probability theory, \(E(\Delta PPT^2(d))\) can be expressed as

\[
E(\Delta PPT^2(d)) = \int_0^R E(\Delta PPT^2(d|r)) \ast f_r(r) \, dr
\]

where \(E(\Delta PPT^2(d|r))\) is the expectation of \(\Delta PPT^2(d)\) over all possible angles \(w\) for a given \(r\), and \(f_r(r)\) is the probability density function of the random variable \(r\).

Since storm occurrence is assumed to be random, and, given the occurrence of an RV event, the probability that a storm center occurs within a distance less than \(r\) to \(X_c\) is proportional to the area of \((\pi \ast r^2)\) (figure 7). The
\[ r'^2 = r^2 + d^2 - 2rd \cos(w) \]

where, \( 0 < w < W_r \) and

\[ W_r = \cos^{-1}\left[\frac{(R^2 - r^2 - d^2)}{2rd}\right]. \]

Figure 7. Graph for Determining \( E(\Delta PT^2(d)) \)
probability of an RV event (i.e., a storm covers $X_C$) is proportional to the area $ASC = \pi R^2$. Therefore, the conditional probability is given as

$$\text{prob\{ distance }< r \mid \text{ RV event }\} = \frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2},$$  

4-11.

By definition, this probability is the cumulative probability function of $r$; that is

$$F_r(r) = \frac{r^2}{R^2},$$  

4-12.

The probability density function, $F_r(r)$, is then the first derivative of $F_r(r)$:

$$f_r(r) = \frac{dF_r(r)}{dr} = \frac{2r}{R^2},$$  

4-13.

To evaluate $E(\text{PPT}^2(d|r))$, it is necessary to determine the distribution of $\text{PPT}(d)$ conditioning on $r$. Given a particular $r$, the conditional distribution of the angle between $r$ and $d$, $w$, less than $w$, is equal to the ratio of the arc length $w*r$ to that of $W_r*r$. Specifically,

$$F_{W|r}(w|r) = \text{prob\{ angle }< w \mid r \} = \frac{w*r}{W_r*r} = \frac{w}{W_r},$$  

4-14.

where $W_r$ is the angle corresponding to where $r'=R$. Therefore, the conditional probability density function is
By the definition of expectation,

\[
E(\Delta \text{PPT}^2(d|\rho)) = \left. \int (\text{PPT}(\rho) - \text{PPT}(\rho'))^2 f_{w|\rho}(w|\rho) \right|_0^{\rho} \, dw
\]

where \( PPT(\rho) \) and \( PPT(\rho') \) correspond to the rainfall depths at \( X_c \) and \( X_r \), respectively. It can be shown geometrically that \( \rho \) and \( \rho' \) are related as

\[
\rho'^2 = \rho^2 + d^2 - 2rd\cos(w)
\]

and that \( w \) is the only independent variable in 4-17.

Substituting equation 4-13 for \( f_{\rho}(\rho) \) and equation 4-16 for \( E(\Delta \text{PPT}^2(d|\rho)) \) into equation 4-9 yields the following relationship for \( E(\Delta \text{PPT}^2(d)) \):

\[
E(\Delta \text{PPT}^2(d)) = \int \int (\text{PPT}(\rho) - \text{PPT}(\rho'))^2 \left. \frac{4\rho}{\rho R^2} \right|_0^{\rho} \, dr \, dw
\]

Generally, the storm center depth \( \text{PPT}_0 \) is also needed to determine \( E(\Delta \text{PPT}^2(d)) \), since \( \text{PPT}(\rho) \) and \( \text{PPT}(\rho') \)
are also a function of PPT₀ by the assumption of a bell-shaped spatial distribution of storm rainfall (the fifth assumption in chapter 2). In fact, ΔPPT(d) can be expressed as

\[ \Delta PPT(d) = PPT(r) - PPT(r') \]

\[ = PPT₀ \left[ \exp\left(-A*r^2/R^2\right) - \exp\left(-A*r'^2/R^2\right) \right] \]

and thus,

\[ ΔPPT^2(d) = PPT₀^2 G(r, r') \]  

4-19

where \( G(r, r') = \exp(-A*r^2/R^2) - \exp(-A*r'^2/R^2) \). If equation 4-19 is rewritten in Taylor series form and only the first order approximation is considered, \( E(ΔPPT^2(d)) \) can be determined (Benjamin and Cornel, 1970):

\[ E(ΔPPT^2(d)) \approx E(PPT₀^2) * E(G(r, r')^2) \]

4-20

where

\[ E(G(r, r')^2) = \int \int G(r, r')^2 \, u \, du \, dv \]

0 0

4-21.

A numerical solution to the integral equation in 4-21 was used to develop the graph in figure 8. The distribution of \( E(G(r, r')^2) \) (the major part of \( E[ΔPPT^2(d)] \)) versus d is essentially bell-shaped (due to the initial assumptions) with a clearly definable peak that could be correlated with the storm cell boundary.
Figure 8. $E[G(r,r')^2]$
Analytical RVIP(d)

With the analytical solutions for $p(d)$ and $E(\Delta \text{PPT}^2(d))$, it is possible to develop an analytical solution for the complete Regionalized Variation Index of Precipitation. The result is as follows:

$$RVIP(d) = \left\{ \frac{p(d) \cdot E(\Delta \text{PPT}^2(d)) + (1-p(d)) \cdot E(\text{PPT}_0^2)}{E(\text{PPT}_0^2)} \right\}^{1/2}$$

where $p(d)$ and $E(G(r,r')^2)$ are given in equations 4-9 and 4-21 and $E(\text{PPT}_0^2)$ cancels from the numerator and denominator. RVIP(d) is shown plotted against distance in figure 9. In accordance with the storm model assumptions, the graph goes through the origin and reaches 1.0 at a distance of 2*R.

**RVIP Distortion by Multiple-Cell Storms**

In the preceding discussions, it was assumed that a given storm was composed of a single cell; that is, if both the control and reference points received rainfall, it was due to coverage by one cell. However, if the points were covered by two or more different cells occurring at the same time, then the RVIP(d) estimate of a storm cell radius would be distorted.

Let $P_m(d)$ be the probability that both the control point, $X_c$, and reference point, $X_r$, receive rainfall from
Figure 9. Analytical Solution of RVIP

Figure 10. Expected RVIP Distortion by Multiple-Cell Storms
multiple or different storm cells, and let $E(\Delta \text{PPT}_m(d))$ represent the mean square of the difference in precipitation at $X_c$ and $X_r$ for such multiple-cell events. Non-storm-exclusion events resulting from a single storm cell can still be represented with $p(d)$ and $E(\Delta \text{PPT}^2(d))$.

The original definition of $\text{RVIP}(d)$ given in equation 4-3 as

$$\text{RVIP}(d) = \sqrt{\frac{p(d) E(\Delta \text{PPT}^2(d)) + (1-p(d)) E(\text{PPT}_0^2)}{E(\text{PPT}_0^2)}}_{1/2}$$

is modified to account for multiple-cell storms in the following fashion:

$$\text{RVIP}_m(d) =$$

$$= \frac{p(d) E(\Delta \text{PPT}^2(d)) + p_m E(\Delta \text{PPT}_m^2(d)) + (1-p(d) - p_m(d)) E(\text{PPT}_0^2)}{E(\text{PPT}_0^2)}_{1/2}$$

$$= \frac{p(d) E(\Delta \text{PPT}^2(d)) - (1-p(d)) E(\text{PPT}_0^2)}{E(\text{PPT}_0^2)}$$

$$- \frac{p_m(d) [E(\text{PPT}_0^2) - E(\Delta \text{PPT}_m^2(d))]}{E(\text{PPT}_0^2)}_{1/2}$$

$$= \{ \text{RVIP}(d)^2 - \delta \}^{1/2}$$

and

$$\delta = \frac{p_m(d) [E(\text{PPT}_0^2) - E(\Delta \text{PPT}_m^2(d))]}{E(\text{PPT}_0^2)}$$

4-23.

Since $E(\text{PPT}_0^2(d)) \geq E(\Delta \text{PPT}_m(d))$, and $p_m(d)$ and $E(\Delta \text{PPT}_m^2(d))$ are $\geq 0$, then $\delta$ is $\geq 0$ and $\text{RVIP}_m(d) \leq \text{RVIP}(d)$. The effect of multiple cell storms is, therefore, to lower $\text{RVIP}$. 


Similar to single cell events, $p_m(d)$ and $E(\Delta \text{PPT}_m^2(d))$ approach a constant level as $d$ becomes greater than the diameter of a single storm cell. As such, it can be shown that $\delta$ approaches a constant level for $d >= 2R$; that is, since $RVIP_m(d) = \left[ RVIP(d)^2 - \delta(d) \right]^{1/2}$, and

$$RVIP(d) \rightarrow 1,$$
$$p_m(d) \rightarrow \text{constant}, \text{ and}$$
$$E(\Delta \text{PPT}_m^2(d)) \rightarrow \text{constant},$$

then $RVIP_m(d) \rightarrow (1 - \text{constant})^{1/2} \rightarrow \text{constant}$.

The lowering of RVIP by multiple-cell events will also cause the critical distance, $D_c$, to be lowered (figure 10). To determine quantitatively the extent of the lowering, computer simulation was employed.

**Critical Distance and Probability of Single Cells**

Although the $RVIP_m(d)$ depends on the probabilities $p(d)$ and $p_m(d)$, this does not imply that the critical distance, $D_c$, will directly depend on these probabilities. Conceptually, $p(d)$ and $p_m(d)$ are meaningful only for given two points (i.e., $X_C$ and $X_r$) and critical distance is implied in the relationship between $RVIP_m(d)$ and distance $d$. Generally, the critical distance, $D_c$, depends on the probabilities of 1, 2, 3, etc. storm cells occurring in the range near $X_C$. To make the critical distance useful in determining storm radius, it is necessary to study the
relationship between critical distance and the probabilities of multiple-cell occurrence which are related with a given area. The following is a discussion of a computer simulation performed to determine such a relationship.

Simulation Assumptions

With the exception of the multiple cell consideration, the simulation is based on the initial assumptions: (1) storm cell are circular in shape, (2) the storm cell radius is constant, (3) the spatial distribution of rainfall is bell-shaped, and (4) storm occurrence is random.

For the areal probabilities related to multiple cell events, the following simplifications were made. Because of limited cell size, the maximum number of cells at one time is limited for an area near the control point $X_c$. Since only a few cells can occur at one time, the probability of a single cell near $X_c$, $P_{as}$, can be used to describe the multiple cell events. In fact, $1-P_{as}$ is the cumulative probability of multiple cell events near $X_c$. By considering a range of probability for $P_{as}$, the simulation will produce a corresponding critical distance. In this manner, a relationship between critical distance and areal effect of multiple cell events was determined.
Simulation Organization

In the simulation, storm centers are generated from a uniform distribution over the area of interest. Precipitation amounts for all other points are then determined from the bell-shaped distribution. The probability of multiple-cell events is equal to 1 minus the assumed probability of single cell events, and subsequently generated with a $U(0,1)$ distribution. For multiple cell events, precipitation totals are summed at all points and $RVIP_m(d)$ calculated as in equation 4-5a. Simulated RVIP curves can therefore be plotted and critical distances calculated for various single cell probabilities. In this manner, a numerical solution can be obtained to express the relationship between the critical distance and probability of single storm occurrence.

For purposes of the simulation, the control point is placed at the origin. Since storms occur randomly, one direction is sufficient to represent the variation in precipitation. Therefore, the X axis was chosen as the direction of interest. In addition, twenty computer reference gauges were considered along the X axis. Since RVIP is stable when the distance between the control and reference points is larger than the storm diameter, the twenty reference gauges distributed within one storm diameter from the control gauge at an equal spacing of one tenth of a storm radius. RV
events were the only events of interest; that is, events that covered the control point. Therefore, only simulated storm events that had at least one storm cell centered in the area ASC (figure 11) were considered.

Storms were simulated in the following manner. First, a storm was generated in the circle ASC. The probability of single storm cell occurrence was then used to determine if a multiple-cell event occurred. If it did not, the next event was simulated. If there were more than one cell, the following procedure was used. If the first storm center was generated in area I, then a second storm was generated in areas II+III+IV. If the second storm center was in area IV, a third storm was generated in area II+III. However, if the first storm center was located in area II, then a second storm was generated in area III+IV. In this case, a third storm was not generated.

This procedure for generating storms was based on the consideration that partial overlap in storm coverage can occur. The spatial distribution of the first two cells determined whether or not a third cell could occur without almost complete overlap. The limited area essentially ruled out the probability of the occurrence of more than three storm cells.

Six simulation runs were performed for different probabilities of single storm occurrence. For each simulation run, 500 RV events were generated to estimate RVIP for
Figure 11. Simulation Raingauge Network
each distance from the control gauge to the reference gauges. The mean storm center depth of precipitation used was 1.433 inches, in accordance with Fogel's data (1969). Another simulation was performed with the center depth exponentially distributed. In this case, RVIP estimates seemed to be shifted parallel with a "noise" level, although the shape of RVIP curve was not changed (see figure 12). Because the determination of the critical distance depends only on the shape of the RVIP curve, the constant center depth was used instead of the distributed depth.

Simulation Result

The simulation results are presented in figure 13. From this figure, several features can be seen. First, without the distortion of multi-occurrence storms, the RVIP curve \( P_{as}=1.0 \) is similar to the analytical solution derived from the storm model in the previous section (figure 9). Second, the multiple-cell events lower the RVIP(d) curve; that is, the curves with \( P_{as} \) less than 1 are shifted down. Third, the critical distance becomes shorter with the distortion from multiple-cell events. This result suggest the critical distance is a function of the probability of single storm occurrence, \( P_{as} \), given a particular storm radius.

From figure 13, it appears that the critical distance can be expressed in terms of some factor times the
Figure 12. Simulated RVIP: Fixed Storm Center Depth and Random Center Depth

Figure 13. Simulated RVIP with Respect to the Areal Probability of Single Storms
Table 1. Factor K as a Function of the Areal Probability of Single Storms

<table>
<thead>
<tr>
<th>Probability</th>
<th>K value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.70</td>
</tr>
<tr>
<td>0.2</td>
<td>0.81</td>
</tr>
<tr>
<td>0.4</td>
<td>0.92</td>
</tr>
<tr>
<td>0.6</td>
<td>1.18</td>
</tr>
<tr>
<td>0.8</td>
<td>1.60</td>
</tr>
<tr>
<td>1.0</td>
<td>2.00</td>
</tr>
</tbody>
</table>

Figure 14. Factor K as a Function of the Areal Probability of Single Storms
storm radius $R$. If the factor is denoted by $K$, the relationship between $D_c$ and $R$ is given by $D_c = K \cdot R$. However, $K$ is actually a function of the probability of single storm occurrence, $P_{as}$. This results in the following relationship:

$$D_c(R, P_{as}) = K(P_{as}) \cdot R$$  \hspace{1cm} 4-25

The function $K(P_{as})$ was determined numerically from the simulation analysis, and is summarized in table 1 and plotted in figure 14.

The result in equation 4-25 was used with observed data to estimate $R$ as described in chapter 6.
CHAPTER 5

DIFFERENCE SQUARE INDEX OF PRECIPITATION

It is possible to index the variation in precipitation between the control point and reference points without considering storm exclusion events. This essentially amounts to considering only the first part of RVIP. This simplified approach is labelled the Difference Square Index of Precipitation (DSIP) and is described in this chapter.

Definitions

The displacement, $d$, at which the graph of the mean square of the difference in precipitation or variation, $E(\Delta PPT^2(d))$, peaks is called the peak distance, $D_p$, and forms the basis for estimating the storm radius (figure 8). Due to the bell shape, the peak distance is easily discernible on the graph.

Indexing Precipitation Variation

As before, the variation in precipitation is sensitive to the mean storm center depth of a region. To generalize the variation, the Difference Square Index of Precipitation (DSIP) is defined as

$$DSIP(d) = \left(\frac{E(\Delta PPT^2(d))}{E(PPT_0^2)}\right)^{1/2}$$
where \( E(PPT_0^2) \) is a scaling factor. Due to the assumption of random occurrence of storms, DSIP is reduced to a function of the displacement between (i.e., and not the location of ) \( X_c \) and \( X_r \).

**Properties of DSIP**

By reviewing the properties of RVIP and noting that DSIP does not consider storm exclusion events, it can be shown that DSIP has the following properties:

1. \( 0 \leq DSIP \leq 1 \)
2. As \( d \) approaches 0, DSIP approaches 0
3. For \( d \geq 2R \), DSIP = 0
4. \( D_p \) exists between \( d=0 \) and \( d=2R \)

**Estimating DSIP**

Similar to the estimator of RVIP(d), the following is proposed as an estimator of DSIP(d) from discrete observations:

\[
\hat{DSIP}(d) = \left\{ \frac{1}{N} \sum_{i=1}^{N} \left[ ppt_i(X_c) - ppt_i(X_r) \right]^2 / ppt_0^2 \right\}^{1/2}
\]

where \( ppt_i(X_c) \) and \( ppt_i(X_r) \) are the rainfall depths recorded for the \( i \)th RV event at the control point and reference point, respectively, and \( ppt_0 \) is the storm center depth. It should be noted that, unlike the case with RVIP(d), both \( ppt_i(X_c) \) and \( ppt_i(X_r) \) are \( \geq 0 \) for all \( i \), since storm exclusion events are not considered in DSIP(d).
Analytical Solution For DSIP

DSIP(d) can also be analytically solved for the storm model assumed in chapter 2. The basis for the solution was developed earlier in equations 4-20 and 4-21 and graphed in figure 8. If equation 4-20 is divided by \( E(PPT_0^2) \), the solution is given as follows:

\[
DSIP(d) = \left\{ \frac{E(\Delta PPT^2(d))}{E(PPT_0^2)} \right\}^{1/2} = \left\{ \frac{E(PPT_0^2)E(G(r,r')^2)}{E(PPT_0^2)} \right\}^{1/2} = E(G(r,r')^2)^{1/2}
\]

5-3.

The analytical solution for DSIP(d) is shown plotted in figure 15. From this graph, the peak distance, \( D_p \), is estimated as

\[
D_p = 0.85*R,
\]

where \( R \) is the storm radius.

If the absence of multiple cell storms is assumed, and if \( D_p \) is estimated from actual data, then equation 5-4 can be used to estimate the storm radius.

DSIP Distortion by Multiple Cell Storms

The DSIP approach to estimating the storm radius will be distorted if recorded rainfall events are the result of multiple-cell storms. The nature of the distortion is considered in this section.
Figure 15. Analytical Solution of DSIP
Let \( m \) be the number of events out of the total number non-storm exclusion events, \( n \), that result of multiple-cell storms and are signified with the subscript \( m \) (e.g., \( PPT_m(d) \)). Equation 5-2 is therefore modified as follows:

\[
DSIP^\wedge_m(d) = \left\{ \frac{1}{n} \sum_{i=1}^{m} \frac{\Delta PPT^2_i(d)}{ppt_0^2} \right\}^{1/2}
\]

\[
= \left\{ \frac{1}{n} \left[ \sum_{i=1}^{m} \frac{\Delta PPT^2_i(d)}{ppt_0^2} + \sum_{i=m+1}^{n} \frac{\Delta PPT^2_i(d)}{ppt_0^2} \right] \right\}^{1/2}
\]

\[
= \left\{ \frac{m}{n} \frac{\Delta PPT^2(d)}{ppt_0^2} + \frac{n-m}{n} \frac{\Delta PPT^2_m(d)}{ppt_0^2} \right\}^{1/2}
\]

5-5.

By noting that \( m/n \) is the probability of no storm exclusion, \( p \), and that \( DSIP(d)^2 = (ppt^2(d)/ppt_0^2) \), equation 5-5 can be written as

\[
DSIP^\wedge_m(d) = \left\{ \frac{\Delta PPT^2(d)}{ppt_0^2} \right\}^{1/2}
\]

\[
\approx \left\{ [p(d)^\wedge * \Delta PPT^2(d)/ppt_0^2] + (1-p(d)^\wedge) * \Delta PPT^2_m(d)/ppt_0^2 \right\}^{1/2}
\]

\[
= \left\{ \frac{\Delta PPT^2(d)}{ppt_0^2} \right\}^{1/2}
\]

5-6.

As defined in equation 5-6, \( DSIP^\wedge_m(d) \) has the following properties:

1. \( DSIP^\wedge_m(d) \) approaches 0 as \( d \) approaches 0, since as \( d \) approaches 0, \( p(d)^\wedge \) approaches 1 and \( DSIP(d)^\wedge \) approaches 0.
(2) \( \hat{\text{DSIP}}_m(d) = \text{constant} (\geq 0) \) for \( d \geq 2R \), since for \( d \geq 2R \), \( \hat{\text{p}}(d) = 0 \) and \( \frac{\text{ppt}_m^2(d)}{\text{ppt}_0^2} = \text{constant} (\geq 0) \).

These two properties and the overall distortion of DSIP due to multiple-cell storms is evident in the graph of \( \text{DSIP}_m(d) \) in figure 16.

Although it is clear from the \( \text{DSIP}_m(d) \) graph that the peak has been lowered, the tail raised, and the curve skewed to the right, the important question as to whether or not the peak distance, \( D_p \), has been changed is not clear. In a manner similar to that for \( D_c \), \( D_p \) can be formulated as follows:

\[
D_p = C(p_{as}) \times R 
\]

where \( C(p_{as}) \) is a function of the areal probability of single-cell storms, \( p_{as} \). \( C(p_{as}) \) can be analyzed with computer simulation and an approximate numerical solution developed as in table 2 and figure 17. The results of such analyses indicate that \( C(p_{as}) \) is relatively insensitive to \( p_{as} \), as it varies from 0.80 to 0.90 as \( p_{as} \) goes from 0.0 to 1.0. Therefore, \( C(p_{as}) \) can be represented by a constant, with the coefficient of 0.85 from the analytical solution being adequate. Thus, equation 5-7 can be written as

\[
D_p = 0.85 \times R 
\]

If \( D_p \) can be estimated from data, equation 5-8 can be used to estimate the storm radius, \( R \).
Figure 16. Expected DSIP Distortion by Multiple-Cell Storms
Table 2. Factor C as a Function of the Areal Probability of Single Storms

<table>
<thead>
<tr>
<th>Probability</th>
<th>C value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.80</td>
</tr>
<tr>
<td>0.2</td>
<td>0.81</td>
</tr>
<tr>
<td>0.4</td>
<td>0.82</td>
</tr>
<tr>
<td>0.6</td>
<td>0.82</td>
</tr>
<tr>
<td>0.8</td>
<td>0.83</td>
</tr>
<tr>
<td>1.0</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Figure 17. Simulated DSIP with Respect to the Areal Probability of Single Storms
Since it was found that $D_p$ was largely independent of the probability of single-cell storms, $P_{as}$, whereas $D_c$ in the RVIP approach was dependent on $P_{as}$, it can be concluded that multiple-cell storms have the greatest (distorting) influence on storm exclusion events.
CHAPTER 6

EMPIRICAL EVALUATION OF STORM CELL RADIUS

In this chapter, the equations developed previously for estimating the critical distance, $D_c$, and peak distance, $D_p$, are employed with data from Southeast Arizona to estimate an average storm radius, $R$.

Data Used

The data used in this study were from a selected network of precipitation gauges on the Walnut Gulch Experimental Watersheds. Daily precipitation totals for the period of 1967-1975 were collected for a total of 16 gauges located along two approximately perpendicular lines running nearly North and East. The data base collected consisted of 420 RV events. The spacing between individual gauges was variable, ranging from 1 to 2 miles. Gauge #386 was located of the crosspoint of the two transects and was selected as the control point (figure 18).

Empirical RVIP and DSIP Curves

Equations 4-5a and 5-2 were used to estimate DSIP(d) and RVIP(d) from the data; specifically,
Figure 18. Selected Raingauges at Walnut Gulch
\[ \text{RVIP}(d) = \left\{ \frac{1}{N} \sum_{i=1}^{N} \frac{(\Delta \text{ppt}_i(d))^2}{\text{ppt}_0^2} \right\}^{1/2} \]

where \( N \) is the number of observed RV events and the precipitation difference between \( X_C \) and \( X_R \) for event, is

\[ \Delta \text{ppt}_i(d) = \left\{ \begin{array}{ll} \text{ppt}_i(X_C) - \text{ppt}_i(X_R), & \text{ppt}_i(X_R) > 0; \\ \text{ppt}_0, & \text{ppt}_i(X_R) = 0, \end{array} \right. \]

and,

\[ \text{DSIP}(d) = \left\{ \frac{1}{N} \sum_{i=1}^{N} \frac{(\text{ppt}_i(X_C) - \text{ppt}_i(X_R))^2}{\text{ppt}_0^2} \right\}^{1/2} \]

where \( \text{ppt}_i(X_C) \) and \( \text{ppt}_i(X_R) \) are the rainfall depths recorded for the \( i \)th RV event at the control point and reference point, respectively, and \( \text{ppt}_0 \) is the storm center depth.

The data were initially divided into summer and non-summer seasons, and analyzed in two parts. This approach was meant to emphasize the importance of the summer, convective thunderstorm season that produces much of the rainfall and most of the surface runoff in Southeast Arizona. In accordance with a study by Henkel (1985), the starting and ending times for the summer season were selected as julian days 177 and 263, respectively.

The results of the DSIP(d) and RVIP(d) calculations are shown plotted in Figures 19, 20 and 21, respectively. The empirical curves have discernible peak and critical distances for the northern and eastern directions in both the
Figure 19. Empirical RVIP: Seasonal Difference

Figure 20. Empirical RVIP: Spatial Difference
Figure 21. Empirical DSIP: Spatial Difference

(a) Estimated DSIP along the northern direction

(b) Estimated DSIP along the eastern direction
summer and non-summer seasons. Distortions due to multiple-cell storms are also evident, in that the RVIP curves do not reach 1.0 and the DSIP curves are skewed to the right and raised up. An unexpected result is the closeness of the summer and non-summer curves. The differences between the curves are so slight that subsequent analysis were simplified by considering the entire year without the seasonal divisions. Of course, this is not meant to imply that there are no physical differences in the nature of storms in the summer and non-summer seasons, but rather to indicate that, from a statistical standpoint, the differences are not large. The simplification facilitated the eventual determination of watershed size from the storm cell radius.

Estimated Critical and Peak Distances

From the graphs in figure 20, the critical distances in the RVIP approach were estimated as \( D_{c,E} = 3.1 \) miles for the northern direction, and \( D_{c,E} = 4.4 \) miles for the eastern direction.

These estimates were made graphically, and were limited in precision by the spacings between gauges and the noise in the data set. The critical point corresponds to where RVIP begins to increase at a rate of less than 10 percent.

The peak distances in the DSIP approach were estimated from the graphs in figure 21 as \( D_{p,N} = 4.5 \) miles
for the eastern direction. With a limited sampling scheme, estimating the point where the bell-shaped curves of DSIP(d) peaked was found to be more difficult than estimating where the RVIP(d) curve flattened out. As such, the above estimates of $D_p$ may only be accurate within ± 0.5 miles.

Estimating the Storm Radius

The storm radius, $R$, can be calculated with the above estimates of $D_p$ and $D_c$ and with the analytical solutions developed in equations 5-8 and 4-25. In the DSIP approach, equation 5-8 can be rewritten as

$$ R = \frac{D_p}{0.85} \quad 6-1 $$

and in the RVIP approach, equation 4-25 can be rewritten as

$$ R = \frac{D_c}{K(P_{as})} \quad 6-2. $$

Since $K$ in equation 6-2 is not a constant, the latter approach requires an additional estimation of the probability of single-cell events, $P_{as}$. To estimate $P_{as}$, simple frequency analysis was performed on 8 gauges separated by at least 4 miles and scattered over the Walnut Gulch Experimental Watershed (figure 22). The displacements between gauges were assumed large enough to assure that no two gauges could be covered a single storm cell. Events where rainfall was recorded at only one of the eight gauges were assumed to be the result of single cell storms; events
Figure 22. Selected Rain gauges for Estimation of the Area Probability of Single Storms
where rainfall was recorded at more than one gauge were assumed to be the result of multiple-cell storms, since no two gauges could be covered by a single storm cell. In this manner, the probability or fraction of single-cell storms, \( P_{as} \), was estimated as 0.18. From the graph in Figure 14, \( K(P_{as}) \) was subsequently approximated as 0.8.

Equation 6-2 can therefore be written as follows:

\[
R = D_c/0.8
\]

6-3.

For the northern direction,

\[
R_N = D_{c,N}/0.8 = 3.1/3.88 = 3.9 \text{ miles.}
\]

For the eastern direction,

\[
R_E = D_{c,E}/0.8 = 4.4/0.8 = 5.5 \text{ miles.}
\]

The ratio of the radii was therefore

\[
R_E/R_N = 1.42,
\]

and the geometric average of \( R_N \) and \( R_E \) was calculated as

\[
R = (R_N \times R_E)^{1/2} = 4.62 = 4.6 \text{ miles.}
\]

Using the DSIP approach, the corresponding results for \( R \) were calculated with equation 6-1. Specifically, for the northern direction,

\[
R_N = D_{p,N}/0.85 = 3.3/0.85 = 3.70 = 3.7 \text{ miles,}
\]
and for the eastern direction,

\[ R_E = \frac{D_{PE}}{0.85} = \frac{4.5}{0.85} = 5.0 \text{ miles}. \]

The ratio of the radii was

\[ \frac{R_E}{R_N} = 1.35, \]

and the geometric average of \( R_N \) and \( R_E \) was calculated as

\[ R = (R_N \cdot R_E)^{1/2} = 4.29 \approx 4.3 \text{ miles}. \]

**Discussion of Storm Radius Results**

**Shape of Isohyetals**

The empirical evidence supports the contention that ground coverage of rainfall totals is generally in the shape of an ellipse. The data indicate that the eastern direction may be aligned closely with the major axis, and the northern direction with the minor axis.

There are at least two explanations for the elliptical shape of the storm coverage. The first of these has to do with the movement of storm cells. Since radar studies by Braham (1958) have shown that more than 90 percent of all convective storms in Arizona move between 1 to 2 miles, migration of a circular storm cell with the prevailing wind direction would produce an elliptical ground coverage of rainfall. Since the difference between \( R_n \) and \( R_e \) at Walnut Gulch was found to be approximately 1.5 miles, the movement
hypothesis seems plausible. A second explanation for the elliptical shape was proposed by Court (1961) and is statistical in nature. Court argued that a bivariate normal distribution of rainfall amount and center location could also produce an ellipitical pattern of isohyetal.

In this study, the assumption that storm cells can be modeled as a circle is justified on an area versus area basis. That is, if the geometric mean of the major and minor axes of an ellipse is taken as a radius, the area within a circle so defined is equal to the area within the ellipse. Since the project ultimately considers only "total" rainfall and runoff volumes, this approximation seems to be justified.

Comparison of DSIP and RVIP

The estimates of storm radius from the DSIP and RVIP approaches are comparable. The RVIP estimates were adopted for further application in this study for two basic reasons. First, the RVIP approach is conceptually more general in its consideration of both storm exclusion and no storm exclusion events. And second, the critical distance, \( D_c \), was more clearly discernible from the empirical RVIP curves than was the peak distance, \( D_p \), from the empirical DSIP curves. This was primarily due to the more sharply changing curves (i.e., slopes) in the DSIP graph. Therefore, the adopted value for the radius of an equivalent circular storm cell at Walnut Gulch was 4.6 miles.
The final result obtained for storm radius compares favorably with results obtained by other researchers. Osborn and Lane (1972) described a precipitation depth-area formula for Walnut Gulch. When the relation is solved for 0.01 inches of rainfall (i.e., the smallest measurable depth) and an equivalent circular shape is assumed, a storm radius of 5.35 miles is obtained. This result was relatively insensitive to changes in the assumed storm center depth. An additional depth-area relation developed by Fogel and Duckstein (1969) for the Atterbury and Walnut Gulch Experimental Watersheds can also be solved in a similar fashion for storm radius. This results in a storm radius that varies from 2.95 to 5.50 miles as the assumed storm center depth is varied from 0.5 to 3.0 inches. In both studies, an elliptical pattern of isohyetal s was observed with a ratio of major to minors found to be approximately 1:1.4. If the results from the previous studies are thought to be accurate, then the lower ratio calculated in this study may be due to a slight deviation between the eastern and northern axes of the raingauge transects and the true major and minor axes of the elliptical cells. A simple appraisal of the geometry of an ellipse reveals that the ratio of any two perpendicular axes other than the true major and minor axes will lead to a smaller ratio (Figure 23).
Because $R_2 > R_{\text{min}}$ and $R_1 < R_{\text{max}}$, thus $R_1/R_2 < R_{\text{max}}/R_{\text{min}}$.

Figure 23. Changes in the Axis Ratio within an Ellipse
CHAPTER 7

DETERMINATION OF UNIT WATERSHED SIZE

The goal of this study was to delineate a unit watershed size over which uniform rainfall from a single storm cell could be assumed for application in a point rainfall-runoff-routing model. In the previous chapters, the average radius of an assumed circular storm cell was calculated to aid in this determination. In this chapter, a methodology for relating the storm cell size with the unit watershed size is developed.

Defining a Unit Watershed

A given storm cell will deliver measurable precipitation to a particular area on the land surface. In the previous chapter, it was shown that this area is typically elliptical in shape, with the geometric mean of the major and minor axes being approximately 4.6 miles in South-east Arizona. In modeling rainfall-runoff processes occurring on an actual watershed, however, the problem is to determine how large of an area can be treated as receiving relatively uniform precipitation inputs over the long-term. If every storm center was located at the center of a circular watershed, and if variability within a storm was neglected, this unit watershed size would be equal to
the storm cell size. Since this is not the case in nature, and storm occurrence and center location is, instead, assumed to be random, the unit watershed size should represent some fraction of the storm cell size.

Strictly speaking, the uniform rainfall assumption is only valid for a given point and a storm/watershed ratio that approaches zero. In a practical sense, however, it is necessary to accept a certain degree of error or approximation in selecting an area of workable dimensions. The error introduced concerns the frequency and extent of partial coverage of the unit watershed by a storm. The larger the unit watershed size selected, the more likely it is that a portion of the watershed will not be covered by a given storm.

In this study, the shape of the unit watershed is assumed to be circular. Since most watersheds are not circular, the circular shape is applied as a conservative, upper bound; that is the smallest circle capable of fully inscribing a section of land is used to characterize the size of the watershed. By using a circle, the watershed size problem is reduced to consideration of one dimension or parameter: a radius. In addition, consistency is maintained with the storm cell sizes.

**Defining Exclusion Errors**

In considering the error introduced by partial coverage of the unit watershed by a storm, it is convenient
to consider a spatial and a temporal component. The spatial component refers to the "extent" or magnitude of the watershed exclusion, and the temporal component refers to the frequency or probability of exclusion at a given extent. In particular, the spatial error is denoted by $\beta$ and is defined as the percentage of the unit watershed area excluded from storm coverage. The temporal error is denoted by $a$ and is defined as the probability that a fraction of the watershed $\leq (1-\beta)$ is covered by a storm. The two errors are, of course, interrelated. It should also be noted that errors due to the variability of amounts over space within a storm are neglected. This assumption is considered acceptable for small unit watershed/storm cell size ratios.

To elaborate on $a$ and $\beta$, consider the following definitions. Let $A$ be defined as an event in which any part of the watershed receives rainfall. Let $B$ be defined as an event in which a fraction of the watershed less than $(1-\beta)$ receives rainfall. Since $A$ implies $B$, then $P(B) = P(A \cap B)$. For a given $\beta$, $a$ is therefore defined as:

$$a = \text{the probability that } (1-\beta) \text{ of the watershed is covered, given that the watershed receives rainfall.}$$

That is,

$$a = \text{prob}\{ B|A \} = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)}$$

7-1.
Conversely, it is possible to consider the complement of event \( B, B' \); that is, the event that a fraction of the watershed \( \geq (1-\beta) \) is covered by a storm, given that the watershed receives rainfall. Specifically,

\[
P(B'|A) = 1 - P(B|A) = 1 - a
\]

For a given extent, \( \beta \), the higher the frequency of exclusion, \( a \), the larger the error in the assumption of complete unit watershed coverage. Alternatively, for a given frequency, \( a \), the larger the corresponding extent of exclusion, \( \beta \), the larger the errors.

**Relationship Between \( a, \beta \), and Unit Watershed Size**

Assuming that storms occur randomly and that storm cells and unit watersheds are circular, a geometric relationship between \( a, \beta \), and the watershed size can be found.

**Spatial Error**

Let \( r \) denote the unit watershed radius and \( R \) denote the storm cell radius, as shown in figure 24. The area of the watershed excluded from storm coverage is labelled \( A_B \). In calculating \( A_B \), it is necessary to consider angles \( x \) and \( y \), as defined with the center of the watershed and the storm cell, respectively. If the partial sections of the watershed and storm cell are denoted by \( A_{WS} \) and \( A_{ST} \) as in figure 24, then the area of storm exclusion, \( A_B \), is defined as follows:
Exclusion area $A_B = A_{WS} - A_{ST}$

$A_{WS} = 2rx - (1/2)r^2 \sin(2x)$

$A_{ST} = 2Ry - (1/2)R^2 \sin(2y)$

Figure 24. Graph for Determining Spatial Error
\[ A_B = A_{WS} - A_{ST} \]

where

\[ A_{WS} = \frac{1}{2} r^2 (2x) - \frac{1}{2} r^2 \sin(2x) \]
\[ = x \cdot r^2 - r^2 \cos(x) \sin(x) \quad 7-4, \]

and

\[ A_{ST} = \frac{1}{2} R^2 (2y) - \frac{1}{2} R^2 \sin(2y) \]
\[ = y \cdot R^2 - R^2 \cos(y) \sin(y) \quad 7-5. \]

Since the spatial error, $\beta$, represents the portion of the watershed excluded by storm coverage, it is calculated as follows:

\[
\beta = \frac{A_B}{A} = \frac{A_B}{\pi r^2} = \{ x - y (R/r)^2 - [\sin(x) \cos(x) - (R/r)^2 \sin(y) \cos(y)] \}
\quad 7-6
\]

where $0 < y < x < \pi/2$.

The limits on the half angles $X$ and $Y$ are set at $\pi/2$ due to the assumptions of symmetry and random storm occurrence.

Temporal Error

To solve for the temporal error, consider figure 25. For event A to occur (i.e., for the watershed to receive rainfall), a storm center would have to be located in the circular area, $S_r$ with radius $(r + R)$. Similarly, for event B
Figure 25. Graph for Determining Temporal Error

\[ z = R \cos(y) - r \cos(x) \]
\[ h = r \sin(x) = R \sin(y) \]
to occur (i.e., for a fraction $\leq (1-\beta)$ of the watershed to be covered), a storm center would have to be located in the donut-shaped area, $Q$. Specifically, using probabilities to correspond to the areas,
\[
P\{B\} = \frac{\pi(R+r)^2 - \pi(R\cos(y) - r\cos(x))^2}{\pi(R+r)^2}
\]
and
\[
P\{A\} = \frac{\pi(R+r)^2}{\pi(R+r)^2}
\]
Therefore,
\[
a = \frac{P\{B\}}{P\{A\}} = \frac{\pi(R+r)^2 - \pi(R\cos(y) - r\cos(x))^2}{\pi(R+r)^2}
\]
\[
= 1 - \frac{(R\cos(y) - r\cos(x))^2}{(R+r)^2}
\]
\[
= 1 - \frac{\cos(y) - (r/R)\cos(x)]^2}{[1+(r/R)]^2}
\]
where $x$ and $y$ satisfy equation 7-6.

Ratio of Unit Watershed to Storm Cell Size

From equations 7-5 and 7-8, $\alpha$ and $\beta$ appear to be functions of $x$, $y$, and the ratio $r/R$. However, $r/R$ is not independent of $x$ and $y$; instead,
\[
\sin(y) = h/R,
\]
\[
\{\sin(x) = h/r,
\]
and therefore,
\[
r/R = \frac{\sin(y)}{\sin(x)}
\]
As result,

\[ a = F_1(x, y) \]
\[ \beta = F_2(x, y) \] \hspace{1cm} 7-10a,

or

\[ x = G_1(a, \beta) \]
\[ y = G_2(a, \beta) \] \hspace{1cm} 7-10b.

Therefore, substitution of equation 7-10b into equation 7-9 reveals that the ratio of unit watershed size to storm cell size is a function of \( a \) and \( \beta \) only:

\[ r/R = \frac{\sin[G_2(a, \beta)]}{\sin[G_1(a, \beta)]} \]

or

\[ r/R = F(a, \beta) \] \hspace{1cm} 7-10c.

An implicit relationship between \( r/R \) and the error levels, \( a \) and \( \beta \), is given by the set of equations 7-6, 7-8 and 7-9. An explicit relationship is not obtainable, except in the special case where \( \beta = 0 \). In this case, \( x=0 \) and \( y=0 \) in equation 7-6, and equation 7-8 reduces to

\[ a = 1 - \frac{[1-(r/R)]}{[1+(r/R)]} \]

or

\[ r/R = \frac{1 - (1-a)^{1/2}}{1 + (1+)^{1/2}} \] \hspace{1cm} 7-11.
This relationship considers the frequency of exclusion of at least one point on the unit watershed; that is, the frequency of all extents (from 0% to 100%) of exclusion are taken together.

When $\beta \neq 0$, a numerical technique is needed to solve for the roots of the system of non-linear equations. The computer program of the so-called generalized Newton method (Szidarovszky and Yakowitz, 1978) employed in this study is provided in the appendix. The solution is summarized in table 3 and plotted in figure 26. This representation allows for the easy examination of the frequencies of storm exclusion at different extents for a given ratio of unit watershed size to storm cell size. For example, with a ratio of $r/R = 0.04$, 20 percent of the watershed is excluded 10 percent of the time, one percent of the watershed is excluded 15 percent of the time, etc.

When the $r/R$ ratio becomes large (i.e., the unit watershed size gets closer to the size of the storm cell), errors due to the spatial variability of precipitation amounts within a single storm become more important. For this reason, values for large $a$ and $\beta$ are not included in table 3. The selected error range from 0.0 to 0.40 for $a$ and $\beta$ is thought to provide the most useful and meaningful information for characterizing the extent and frequency of storm exclusion.
Table 3. Ratio of Unit Watershed Radius to Storm Cell Radius as a Function of Spatial and Temporal Error Levels
(Unit in \(10^{-2}\))

<table>
<thead>
<tr>
<th>(r/R)</th>
<th>0.010</th>
<th>0.025</th>
<th>0.050</th>
<th>0.100</th>
<th>0.150</th>
<th>0.200</th>
<th>0.250</th>
<th>0.300</th>
<th>0.350</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
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<td>0.2513</td>
<td>0.2513</td>
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</tr>
<tr>
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<td>18.79</td>
<td>19.96</td>
<td>22.34</td>
<td>25.10</td>
<td>28.68</td>
<td>33.98</td>
<td>44.96</td>
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</table>

Figure 26. The Ratio of Unit Watershed Radius to Storm Cell Radius as a Function of Spatial and Temporal Error Levels
Alternative Measure of the Significance of \( r \)

It is possible to consider the statistical correlation in daily precipitation amounts for points within unit watersheds of various sizes. A study of the Walnut Gulch Experimental Watershed by Osborn (1982) yielded the following empirical relationship for the correlation coefficient, \( r \), as a function of displacement, \( d \), between two points:

\[
 r_d = 1.030e^{0.187d} + 0.142
\]

7-12

with a standard error equal to 0.052. This empirical result is presented as one possible alternative for evaluating or interpreting the \( a \) and \( \beta \) error levels at the Walnut Gulch site. When the unit watershed diameter, \( 2r \), is substituted for \( d \) into equation 7-12, a conservative correlation coefficient can be calculated for the two most widely separated points in the watershed. There may be other statistical measures suited for interpreting \( a \) and \( \beta \) levels.

A Basis For Selection or Evaluation

The procedures presented for determining storm cell and unit watershed size in a region can be used in two ways. First, they afford a methodology for selecting an upper limit on the size of watersheds that can be modeled as a single unit. In accordance with the objectives of the particular study, different types and magnitudes of error may be acceptable.
In problems associated with delineating an appropriate scale of consideration for watersheds, \( a \) and \( \beta \) are decision-making criteria that can be used, for instance, to determine how finely a large watershed should be subdivided in modeling. Secondly, the procedures offer a methodology for evaluating the level and type of errors incurred on existing or preselected watersheds. If uniform rainfall coverage is assumed in a particular study, it is useful to quantify precisely the validity or invalidity of those assumptions.

In the latter case of evaluating a given watershed, the radius of the smallest circular unit watershed capable of completely inscribing the physical watershed is known and \( a \) and \( \beta \) are subsequently calculated. In the case of selecting an appropriate unit watershed size, acceptable \( a \) and \( \beta \) are identified, and \( r \) is then calculated. In either case, it is necessary first to determine the average storm cell radius in a region with the DSIP or RVIP approach.

**Example Calculation**

Since the interest in the project that prompted this thesis is to select a unit watershed size, an example calculation for this purpose is provided. In the first part of this study, the so-called average radius of a storm cell in Southeast Arizona was found to 4.6 miles. For this and most similar watershed studies, the error levels are most easily
selected by considering one fixed $\alpha$ level of 0.30 is suggested; that is, it is suggested to consider how frequently one is willing to accept exclusion of 30 percent or less of the watershed area. This is equivalent to selecting an acceptable frequency for coverage of more than 70% of the watershed area by storm cells. For this project of pond simulation, the suggested $\beta$ is 0.20. According to table 3, the $r/R$ ratio is therefore 0.1132, and the unit watershed radius is given as

$$r = 0.1132*R = 0.1132*4.6 \approx 0.52 \text{ miles}.$$ 

This radius corresponds to a unit watershed area of 0.852 square miles or 545.2 acres. The largest section of a watershed considered as a single unit should therefore be able to be circumscribed by a circle of this size.
CHAPTER 8

SUMMARY, CONCLUSION AND APPLICATION

This chapter summarizes and concludes this study and recommends the possible applications of this work.

Summary

This study presents two related methods of determining storm cell size and a procedure for evaluating effective watershed unit size for small watershed modeling.

The RVIP and DSIP approaches are essentially statistical approaches for determining storm cell size, based on rainfall data from several points. Both assume that the spatial information (storm size) can be revealed from the spatial variation of storm rainfall. In fact, this consideration is the basis of the DSIP method. In constrast, RVIP considers the effects of the storm exclusion events as well as that of spatial variation.

The characteristic distances in RVIP and DSIP (i.e. critical and peak distances) are key elements in determining storm cell size. A proportional relationship was developed between these characteristic distances and storm radius. The coefficient factors in this relationship were determined via simulation studies. The coefficient factor
was modeled as a function of the areal probability in RVIP while in DSIP the coefficient factor is almost a constant.

The analytical and empirical results obtained in this study are consistent to that of other workers. The storm radius for Walnut Gulch is determined to be 4.6 miles, using this method. To facilitate application, the formula estimating RVIP and DSIP are proposed.

To determine watershed unit size, a relationship between the watershed size and attendant errors was developed in this study. This implies there is no absolute watershed unit size for which the purposes of the uniform rainfall assumed. This size evaluation is a decision-making process based on the relationship developed in this study. That is, the watershed unit size only depends upon how often a watershed unit is excluded and how much great partial storm coverage is.

The errors identified in this study are temporal and spatial types (a and β). The former is related to the how often the watershed unit is within the coverage of a storm while the latter represents how much of the watershed is excluded by the storm. Both of these errors are the probabilistic errors. The analytic solutions for this relationship between watershed unit size and errors is one of the principle results in this study. Based on this relationship, a procedure of watershed unit size evaluation is presented.
Conclusions

The consistency between the analytic and empirical solutions presented here, and the comparability of these results with those of other studies seems to support the methodology developed. Although the study was empirical and performed at Walnut Gulch, the methodology itself is a general one. It can be applied to any region where the basic assumptions of the storm model hold and a raingauge network is available. The coefficients in the critical and peak distance equations provide the flexibility to the methodology. Studies could be carried out to determine the necessary coefficients for application of the method to other areas.

It is believed that the method could provide a useful tool for both the selection and evaluation of watershed size for modeling the hydrology of small watersheds. In small watershed modelling, watershed size is the critical factor which affects accuracy. With this methodology, the watershed size can be evaluated probabilistically (e.g. given a watershed size, the error level combination can be obtained). On the other hand, the method can be used to select watershed size in modelling work with some control on error allowances (e.g. given an error level combination, it can determine the corresponding watershed size). This
evaluation and selection process is a decision-making process and appears realistic because of the random nature of watershed size.

However, the method does have some limitations. The basic limitations are the five major assumptions of the storm model. That is, 1) cellular storms; 2) circular cells; 3) fixed storm radius; 4) random occurrence of storms, and 5) bell-shaped precipitation distribution within storm cells. Even though these assumptions appear artificial, most researchers have adopted them when working with convective storms in southeast Arizona.

Two more assumptions were also employed in this study. One is the validity of generating multiple occurrence storms in the simulation study. This assumption needs to be modified in further studies. The second is the circular shape of unit watershed.

The watershed unit size (radius) is a conservative measure for evaluating watershed size in watershed modelling. That is, in evaluating the size of an actual watershed with the concerns of uniform rainfall, the most significant dimension of the actual watershed should be used as the diameter (2R) of watershed unit (see figure 8-1). In this manner, the actual watershed can be evaluated conservatively.
Applications

In general, the method developed in this study can be applied to the following problems:

(1) to help in evaluating the performance and results of modelling small watersheds, most of which assume uniform rainfall (e.g. point rain data is applicable to whole watershed).

(2) to provide a basic unit for subdividing an irregular large watershed into smaller subwatersheds where the assumption of uniform rainfall would not be greatly violated. This may also help to extend some useful small watershed models to larger watersheds.

(3) to study spatial rainfall distribution for a large weather system. For example, a large storm event might be handled as several small storm cells.

In the project which prompted this study, the final goal was to develop a simulation model which could analyse the performance and impact of a stock pond system on several watersheds, based on the existing model which deals with a single pond and a single watershed. The strategy used in this final model is: (1) to develop a methodology which could quickly obtain the parameters required by the simulation model. (2) to evaluate the watershed size within which the previous single pond and single watershed is still applicable; (3) to subdivide a large watershed with several
ponds into several sub-watersheds each of which has at most one pond. This allows modeling the subwatersheds individually with the existing model; (4) to determine the distribution and number of sub-watersheds receiving rain in a day. Then, use the distribution to generate storm events for those sub-watersheds; (5) to use Lane's Transmissions Loss model to link the subwatersheds and route channel flows through the whole watershed. Combining all five aspects, the performance and impact of stock ponds can be evaluated for a larger watershed with multiple ponds.

This study developed a method to implement tasks two and three above. A methodology for estimating rainfall process parameters (task 1) was developed by Arthur Henkel(1985). Tasks (4) and (5) were accomplished by Long and Henkel (1985).

Although the final model has not been tested in detail, it allows the analysis of existing or proposed ponds on several subwatersheds and evaluate the cumulative impacts of those ponds on downstream water yields. The method may be used to extend the models developed for small watersheds to the models of larger watersheds in a structural manner.
APPENDIX

PARTIAL LISTING OF FORTRAN PROGRAMS

Multiple-cell Storm Event Simulation Program MULTSM

00001 FTH7X.L ; HP ONLY: COMPILER DIRECTIVES
00002 $FILE1,1 ; HP ONLY: COMPILER DIRECTIVES
00003
00004 *****************************************************
00005 *
00006 *
00007 *
00008 *
00009 *
00010 *
00011 * This program simulates RVIP and DSIP of a number of computer
00012 * rainfall gauges along one direction (x-axis) with respect to the
00013 * areal probability of single storms. The basic assumptions and
00014 * simulation organization can be referred to the corresponding sections
00015 * in chapter 4.
00016 *
00017 * The control gauge locates at the origin and a number of reference gauges up to a maximum of 40 are arranged along x-axis
00018 * with an equal spacing. The radius of storm is used as a basic unit.
00019 *
00020 * In order to estimate RVIP and DSIP reliably, a number of simulation runs (defaulted as 10) are performed for each gauge at the
00021 * same condition (i.e., each value of the areal probability of single
00022 * storms). The results from simulation runs are averaged to obtain
00023 * RVIP and DSIP estimates.
00024 *
00025 * This program is interactive to the user. Five inputs must be
00026 * supplied by the user. The inputs are:
00027 *
00028 *
00029 * 1) Name of the output file
00030 * 2) Number of RV events or storms
00031 * 3) Number of simulation reference gauges
00032 * 4) Areal probability of single storms
00033 * 5) Mean precipitation depth at storm center
00034 *
00035 * The output are tabulated both in the output file and on the screen.
00036 *
00037 *
00038 * Two subroutines are called in this program. PPTCT is a function of storm center depth of precipitation. RAIN is a function
00039 * of storm rainfall depth at one point which is from the storm center by a distance of d.

91
This program was written in standard FORTRAN 77, except a few statements restricted to HP 1000 system. These restricted statements follow by the phrase, 'HP ONLY'. When this program is transported to other computer systems other than HP 1000, such statements may be needed to be modified accordingly.

Major parameters and variables are described below:

### Parameters:
- **IFILE**: logical unit number for output file (40)
- **ISCRN**: logical unit number for screen, HP ONLY (1)
- **NGMAX**: maximal number of simulation reference gauges (40)
- **NUMRUN**: number of simulation runs (10)
- **RADIUS**: storm radius (1)

### Variables:
- **DELTAX**: spacing between gauges in units of storm radius
- **DIST(i)**: x-coordinates of reference gauge i
- **DPPTNX(i)**: precipitation difference counter for non-exclusion storms at reference gauge i
- **DPPTEX(i)**: precipitation difference counter (including storm exclusion events) at reference gauge i
- **DSIP(i)**: DSIP counter for reference gauge i
- **FNAME**: name of output file (maximal 12 characters)
- **IRV**: flag of RV storm. If RV storm is in area I or II, IRV is set to 1 or 2 respectively.
- **ISC**: flag of second storm. If the storm is in area II, III or IV, it is set to 2, 3 or 4 respectively
- **NGAGES**: number of simulation reference gauges
- **NRV**: number of RV events or storms per simulation run
- **NSTORM**: number of storm cells in one RV event
- **PAS**: areal probability of single storms
- **PPT0**: mean precipitation depth at storm center
- **RVIP(i)**: RVIP counter for reference gauge i
- **XCT,YCT**: coordinates of control gauge
- **XNEGEXCL(i)**: number of non-exclusion events in one run
- **XNEXCL(i)**: number of above runs
- **XRV,YRV**: coordinates of the first (RV) storm
- **XSEC,YSEC**: coordinates of the second storm
- **XTD,YTD**: coordinates of the third storm

This program was written by Junsheng Long, 1984.
PROGRAM MULTSM

* PARAMETERS AND VARIABLES

PARAMETER (ISCRN=1, IFILE=40, NGMAX=40, NUHRUN=10, RADIUS=1.)

REAL DIST(NGMAX), DPTEX(NGMAX), DPTEXX(NGMAX), DSIP(NGMAX)

REAL RVIP(NGMAX), XNEXCL(NGMAX), XNR(NGMAX)

CHARACTER COMMAND#1, FNAME#12

* INITIALIZATIONS

* OUTPUT FILE

WRITE(ISCRN,* ) ' Enter output file name '
READ(ISCRN,10) FNAME

10 FORMAT(A)

OPEN(IFILL,FILE=FNAME,STATUS='NEW')

* INPUT PARAMETERS.

WRITE(ISCRN,* ) 'Enter number of simulated RV storms '
READ(ISCRN,9) NRV

WRITE(ISCRN,* ) 'Enter number of reference gauges '
READ(ISCRN,9) NGAGES

WRITE(ISCRN,* ) 'Enter single storm probability '
READ(ISCRN,9) PAS

WRITE(ISCRN,* ) 'Enter mean storm center PPT '
READ(ISCRN,9) PPT0

* SIMULATION INITIALIZATION

IRUNCN7=0

DELTX = 2.*RADIUS/FLOAT(NGAGES)

DO 1=1,NGAGES

DIST(I)=DELTX*FLOAT(I)

DSIP(I)=0.

RVIP(I)=0.

XNR(I)=0.

END DO

* RANDOM NUMBER GENERATOR SEEDING

CALL SSEED(12345) ; HP ONLY

* SIMULATION RUN LOOP

DO IRUN=1, NUHRUN

* INITIALIZATIONS OF COUNTERS
DO I=1,NGAGES
  DPPTEX(I)=0.
  DPPTNX(I)=0.
  XNEXCL(I)=0.
END DO

* STORM EVENT GENERATION LOOP
DO ISTORM=1, NRV
  GENERATE FIRST (RV) STORM: SINCE THE CONTROL GAUGE MUST BE COVERED,
  THE DISTANCE BETWEEN STORM CENTER (XRV,YRV) AND CONTROL POINT (XCT, YCT) HAS TO BE LESS THAN STORM RADIUS.

  D = 2.*RADIUS
  DO WHILE (D.GT.RADIUS)
    XRV = RADIUS*(-1.+2.*URAN())
    YRV = RADIUS*(-1.+2.*URAN())
    D = (XRV-XCT)**2+(YRV-YCT)**2
  END DO

  SLT 1RV FLAG: IF XRV < 0, THEN THE STORM CENTER IS IN AREA 1 AND, THUS,
  IRV=1; OTHERWISE, THE STORM IS IN AREA II, AND, THEREFORE, IRV=2.

  UPDATE NUMBER OF STORMS IN THIS RV EVENT
  IF (XRV.LE.0.) THEN
    IRV=1
  ELSE
    IRV=2
  ENDIF
  NSTORM=1

  MULTIPLE STORM GENERATION: IF THE GENERATED PROBABILITY IS LARGER THAN
  PAS. THEN THERE IS A MULTIPLE-STORM EVENT
  U = URAN()
  IF ( U.GT.PAS ) THEN
    SECOND STORM GENERATION: SINCE ONLY PARTIAL OVERLAP IS ALLOWED, THE
    STORM SHOULD BE FAR FROM THE FIRST STORM BY SQRT(1/2) OF STORM RADIUS
    MEANWHILE, THE SECOND STORM HAS TO COVER THE LAST REFERENCE GAUGE.

    D = 0.
    DO WHILE (D.LE.RADIUS/2.).OR.(DD.GT.RADIUS)
      XSC = RADIUS*(0.+3.*URAN())
      YSC = RADIUS*(-1.+2.*URAN())
      D = (XSC-XRV)**2+(YSC-YRV)**2
      DD = (XSC-DIST(NGALES)**2+YSC**2
    END DO
SET ISC FLAG: IF XSC < RADIUS, THEN IT IS IN AREA II (ISC=2). IF XSC RADIUS AND < 2*RADIUS, THEN IT IS IN AREA III (ISC=3). OTHERWISE, IT IS IN AREA IV (ISC=4). UPDATE NUMBER OF STORMS IN THIS EVENT.

IF (XSC.LE.RADIUS) THEN
  ISC=2
ELSE
  IF (XSC.LE.2.*RADIUS) THEN
    ISC=3
  ELSE
    ISC=4
  ENDIF
ENDIF
NSTORM=NSTORM+1

GENERATE THE THIRD STORM: IF THE FIRST AND SECOND STORMS ARE IN AREA II, THE THIRD STORM IS GENERATED IN AREA III AND IV. IF THE FIRST IS IN AREA I AND THE SECOND IS IN AREA IV, THEN THE THIRD IS GENERATED IN AREA II AND III.

IF (IRV.EQ.2.AND.ISC.EQ.2) THEN
  D = 2.*RADIUS
  DO WHILE (D.GT.RADIUS)
    XTD = RADIUS*(1.+2.*URAN())
    YTD = RADIUS*(-1.+2.*URAN())
    D = (XTD-XNGAGES)**2+YTD*YTD
  END DO
  NSTORM = NSTORM+1
ELSE
  IF (IRV.EQ.1.AND.ISC.EQ.4) THEN
    XTD = RADIUS*(2.*URAN())
    YTD = RADIUS*(-1.+2.*URAN())
    NSTORM = NSTORM+1
  ENDIF
  ENDIF
ENDIF

GENERATE STORM CENTER PPT DEPTHS FOR EACH STORM.

PPT1 = PPTCT(PPTO)
IF (NSTORM.GT.1) PPT2 = PPTCT(PPT0)
IF (NSTORM.GT.2) PPT3 = PPTCT(PPT0)

GENERATE PPT AMOUNT AT CONTROL GAUGE. SINCE CONTROL GAUGE IS LOCATED AT ORIGIN, XGAGE=0 AND YGAGE=1.

PTCNTR=RAIN(XRV,YRV,PPT1,RADIUS,0,0)
IF (NSTORM.GT.1) PCNTR = PCNTR+RAIN(XSC,YSC,PPT2,RADIUS,0,0)
IF (NSTORM.GT.2) PTCNTR = PTCNTR+RAIN(XTD,YTD,PPT3,RADIUS,0.,0.)

* GENERATE PPT AMOUNT AT REFERENCE GAUGES AND CALCULATE PPT DIFFERENCE BETWEEN CONTROL AND REFERENCE GAUGES TO DETERMINE RVIP AND DSIP. SINCE REFERENCE GAUGES ARE ALONG X-AXIS, YGAGES = 0.

DO I=1,NGAGES
   DISTI=DISI(I)
   PPTI=RAIN(XRV,YRV,PPT1,RADIUS,DISTI,0.)
   IF (NSTORM.GT.1) PPTI=PPTI+RAIN(XSC,YSC,PPT2,RADIUS,DISTI,0.)
   IF (NSTORM.GT.2) PPTI=PPTI+RAIN(XTD,YTD,PPT3,RADIUS,DISTI,0.)

* IF PPT AT REFERENCE GAUGE I (PPTI) < 0.01 INCH, THERE IS STORM EXCLUSION EVENT SUCH THAT FOR RVIP, THE PPT DIFFERENCE IS STORM CENTER DEPTH SQUARE AND, FOR DSIP, THIS EVENT IS DISCARDED.

* OTHERWISE, CALCULATE THE DIFFERENCE AND UPDATE NUMBER OF NON-
* EXCLUSION STORM EVENTS.

   IF (PPTI.LT.0.01) THEN
      DIFRVIP = PPTO*PPTO
      DIFDSIP = 0.
   ELSE
      DIFRVIP = (PPTI-PTCHTK)*(PPTI-PTCNTR)
      DIFDSIP = DIFRVIP
      XNEXCL(I)=XNEXCL(I)+1.
   ENDIF

* UPDATE STATISTIC COUNTERS

   DPPTEX(I) = DPPTEX(I)+DIFRVIP
   DPPTNX(I) = DPPTNX(I)+DIFDSIP

END DO

* END OF RV EVENT SIMULATION LOOP

* END DO

* OUTPUT SIMULATION RESULTS

* PRINT SIMULATION RUN TITLE AND PARAMETERS.

   IRUNCNT = IRUNCNT+1
   WRITE(ISCRN,20) IRUNCNT, PPTO, PAS, NRV
20 FORMAT(///5X,'THE ','12,'TH RUN OF SIMULATION'/
   1 5X,'============='/
   1 5X,' Center PPT depth: ',F5.3,' inch'/
   1 5X,' Single storm prob: ',F5.3.'/
   1 5X,' No. of storms simulated: ',F5.0'/
   15X,' DISTANCE RVIP DSIP NO. NEXCL')
WRITE(IFILE,20) IKUNCNT, PP10, PAS, NRV

* GENERATE RVIP AND DSIP. PRINT OUT THE RESULTS.

DO 1=1,NGAGES
  DISTI = DIST(I)
  RVIP(I) = SQRT(DPPTEX(I)/FLOAT(NRV))/PP10
  IF (XNEXCL(I).GT.0.0) THEN
    DSIP(I) = SQRT(DPPTNX(I)/XNEXCL(I))/PP10
    XNR(I)=XNR(I)+1.
  ELSE
    DSIP(I)=0.
  END IF

END DO

UPDATE SIMULATION RUN STATISTIC COUNTERS FOR GAUGE I

RVIP(I) = RVIP(I)+RVIP(I)
DSIP(I) = DSIP(I)+DSIP(I)

END DO

OUTPUT FINAL STATISTIC RESULTS OF NUMRUN RUNS.

IF(XNR(I).GT.0.0)
  DSIP(I) = DEP(I)/XNR(I)
ELSE
  DSIP(I)=0.
ENDIF

WRITE(ISCRN,40) PPT0, PAS, NRV, NUMRUN
WRITE(IFILE,40) PPT0, PAS, NRV, NUMRUN

FORMAT(///5X,' SIMULATION SUMMARY'/
  1 5X,' PPT0 = ',F5.3,' inch'/
  1 5X,' Single storm prod. = ',F5.3/
  1 5X,' Number of storms = ',F5.0/
  1 5X,' No. of applications= ',F5.0/
  1 5X,' Distance RVIP DSIP ')

REPORT RVIP AND DSIP FOR EACH REFERENCE GAUGE.

DO 1=1,NGAGES
  RVIP(I) = KVIP(I)/FLOAT(NUMRUN)
  IF(XNR(I).GT.0.0) THEN
    DSIP(I) = DSIP(I)/XNR(I)
  ELSE
    DSIP(I)=0.
  END IF

END DO

END OF SIMULATION RUN LOOP

END DO

END OF

SIMULATION

RUN LOOP

END DO

OUTPUT FINAL STATISTIC RESULTS OF NUMRUN RUNS.

HEADING THE REPORT

WRITE(ISCRN,40) PPT0, PAS, NRV, NUMRUN
WRITE(IFILE,40) PPT0, PAS, NRV, NUMRUN

FORMAT(///5X,' SIMULATION SUMMARY'/
  1 5X,' PPT0 = ',F5.3,' inch'/
  1 5X,' Single storm prod. = ',F5.3/
  1 5X,' Number of storms = ',F5.0/
  1 5X,' No. of applications= ',F5.0/
  1 5X,' Distance RVIP DSIP ')

REPORT RVIP AND DSIP FOR EACH REFERENCE GAUGE.

DO 1=1,NGAGES
  RVIP(I) = KVIP(I)/FLOAT(NUMRUN)
  IF(XNR(I).GT.0.0) THEN
    DSIP(I) = DSIP(I)/XNR(I)
  ELSE
    DSIP(I)=0.
  END IF

END DO

END OF SIMULATION RUN LOOP

END DO

OUTPUT FINAL STATISTIC RESULTS OF NUMRUN RUNS.

HEADING THE REPORT

WRITE(ISCRN,40) PPT0, PAS, NRV, NUMRUN
WRITE(IFILE,40) PPT0, PAS, NRV, NUMRUN

FORMAT(///5X,' SIMULATION SUMMARY'/
  1 5X,' PPT0 = ',F5.3,' inch'/
  1 5X,' Single storm prod. = ',F5.3/
  1 5X,' Number of storms = ',F5.0/
  1 5X,' No. of applications= ',F5.0/
  1 5X,' Distance RVIP DSIP ')

REPORT RVIP AND DSIP FOR EACH REFERENCE GAUGE.

DO 1=1,NGAGES
  RVIP(I) = KVIP(I)/FLOAT(NUMRUN)
  IF(XNR(I).GT.0.0) THEN
    DSIP(I) = DSIP(I)/XNR(I)
  ELSE
    DSIP(I)=0.
  END IF

END DO

END OF SIMULATION RUN LOOP

END DO

OUTPUT FINAL STATISTIC RESULTS OF NUMRUN RUNS.

HEADING THE REPORT

WRITE(ISCRN,40) PPT0, PAS, NRV, NUMRUN
WRITE(IFILE,40) PPT0, PAS, NRV, NUMRUN

FORMAT(///5X,' SIMULATION SUMMARY'/
  1 5X,' PPT0 = ',F5.3,' inch'/
  1 5X,' Single storm prod. = ',F5.3/
  1 5X,' Number of storms = ',F5.0/
  1 5X,' No. of applications= ',F5.0/
  1 5X,' Distance RVIP DSIP ')

REPORT RVIP AND DSIP FOR EACH REFERENCE GAUGE.

DO 1=1,NGAGES
  RVIP(I) = KVIP(I)/FLOAT(NUMRUN)
  IF(XNR(I).GT.0.0) THEN
    DSIP(I) = DSIP(I)/XNR(I)
  ELSE
    DSIP(I)=0.
  END IF

END DO
00347     DSIP(I)=0.
00348     END IF
00349
00350     WRITE(ISCRN,50) DIST(I), RVIP(I), DSIP(I)
00351     WRITE(IFILE,50) DIST(I), RVIP(I), DSIP(I)
00352     50     FORMAT(10X,F8.5,8X,F8.6,8X,F8.6)
00353
00354     END DO
00355
00356     *     RE RUN OR STOP OPTIONS
00357
00358     500     WRITE(ISCRN,*) ' Enter R to re-run or E to exit ....... '
00359
00360     READ(ISCRN,10) COMMAND
00361
00362     IF(COMMAND.EQ.'R') GOTO 100
00363     IF(COMMAND.NE.'E') THEN
00364     WRITE(ISCRN,*) ' Your entry is wrong! Please try again. '
00365     GOTO 500
00366     ENDIF
00367     WRITE(ISCRN,60)
00368
00369     60     FORMAT(/////////')
00370     STOP
00371     END
00372
00373     *****************************************************************************
00374     *     FUNCTION PPTC1
00375     *
00376     *     PPTC1 determines the storm center depth of precipitation. The
00377     *     only argument is the mean center depth. In this study, a constant
00378     *     center depth was used. If other methods are needed, this function
00379     *     should be modified accordingly.
00380     *
00381     *****************************************************************************
00382     Function PPTC1(PPTO)
00383     *
00384     *     THE METHOD USED IN THIS STUDY. CONSTANT PPTO.
00385     *
00386     PPTCT=PPTO
00387     *
00388     *     EXAMPLE OF OTHER METHOD TO GENERATE CENTER DEPTH: EXPONENTIAL
00389     *
00390     *     DISTRIBUTED
00391     C
00392     C
00393     C
00394     PPTCT=-(PPTO-0.8)*ALOG(U)+0.8
00395     C
00396     K RETURN
00397     END
FUNCTION RAIN

RAIN determines the rainfall depth at reference gauges.

The underlying model of storm rainfall was described in assumption five, chapter two. Specifically, the precipitation depth at a point which is d from storm center is given by:

\[ PPT(d) = PPT0 \times \exp(-A \times d^2 / R^2) \]

where \( A = 4.052 \times \exp(0.131 \times PPT0) \) and PPT0 is the center depth. R is storm radius.

Five arguments used in this function are described below:

- XSTORM, YSTORM: coordinates of storm center
- PPTSTM: center depth of precipitation
- R: storm radius
- XGAGE, YGAGE: coordinate of the reference gauge

Since the reference gauges are arranged along x-axis, the v-coordinate of the gauge is zero.

FUNCTION RAIN(XSTORM, YSTORM, PPTSTM, R, XGAGE, YGAGE)

CALCULATE THE DISTANCE BETWEEN STORM CENTER AND THE GAUGE.

\[ DD = (XSTORM - XGAGE)^2 + (YSTORM - YGAGE)^2 \]

CALCULATE THE PRECIPITATION DEPTH AT THE GAUGE.

IF(DD > R^2) THEN
    \[ A = 4.052 \times \exp(0.131 \times PPTSTM) \]
    \[ RAIN = PPTSTM \times \exp(-A \times DD / R^2) \]
ELSE
    RAIN = 0.
END IF
RETURN
END
This calculates the ratio of unit watershed radius to that of storm, given alpha and beta levels (chapter 7). Equations 7-6, 7-6 define above relationship in implicit manner. Thus, two intermediate variables, named by x and v, are introduced to ease the calculation of the ratio from above equations system. Detailed equations can be referred in Long's thesis.

The method used to solve the system of equations is the generalized Newton's method which could be found in any textbook of numerical analysis (for example, Yakowitz, 1978).

This program is interactive to the user. The user inputs the levels of alpha and beta, and the initial estimates of angles x and v. Then the program finds the correct angles x and v through above method. From x and v, the ratio is determined by equation 7-9.

The output is both stored in a file and shown on screen.

One subroutine, ROOT, is called by this program. ROOT is used to implement the Newton's method. This program was written in standard FORTRAN 77 on HP 1000 system. The restricted statements to this system follow by the phrase, "HP ONLY", in order help in the modifications of the program on other systems.

Major parameters and variables are described below:

Parameters:
- IFILE: logical unit number of the output file (40)
- ISCRN: logical unit number of screen, HP ONLY (1)
- DEGREE: conversion factor from degree to arc degree
Variables:

A level of alpha
B level of beta
FNAME name of output file
IEROR error flag of subroutine ROOT
RATIO ratio of watershed radius/storm radius
X value of angle x
Y value of angle y

This program was written by Junshen Long, 1984.

PROGRAM RATIO

PARAMETERS AND VARIABLES

PARAMETER ( IFILE=40, ISCRN=1, DEGREE=3.1415926/180. )
CHARACTER COMMAND*1, FNAME*12

OUTPUT FILE INITIALIZATION

WRITE(ISCRN,a)
READ(ISCRN,10) FNAME
10 FORMAT(A)
OPEN(1FILE, FILE=FNAME, STATUS='NEW')

INPUT ALPHA, BETA AND INITIAL ESTIMATES OF ANGLE X, Y

WRITE(ISCRN,a)
READ(ISCRN,AB)
WRITE(ISCRN,t)
READ(ISCRN,*) XDEGREE,YDEGREE

CONVERT X AND Y INTO ARC DEGREE

X=XDEGREE*DEGREE
Y=YDEGREE*DEGREE

ENTRY ERROR PROOF: X MUST BE LARGER THAN Y
IF (Y.GT.X) THEN
WRITE(ISCRN,a) 'Since x must be larger than y, please try again!'
GOTO 2000
ENDIF

CLEAR ERROR FLAG AND USE GENERALIZED NEWTON METHOD TO THE PROBLEM
102

00093  IERROR=0
00094  CALL ROOT(A,B,Y,X,IERROR)
00095
00096  *  IF NO ERRORS, OUTPUT SOLUTIONS OF RATIO
00097 00098  IF (IERROR.EQ.0) THEN
00099
00100  RATIO=SIN(Y)/SIN(X)
00101
00102  WRITE(ISCRN,30) A,B,Y,X,RATIO
00103  30 FORMAT(5X,'GIVEN ALPHA =',F10.7,2X,'BETA =',F10.7,
00104  1     2X, AND INITIAL Y =',F10.7,2X,', X =',F10.7,','
00105  1/
00106  5X,'WATERSHED/STORM RATIO =',F10.8,2X,F10.8)
00107
00108  ELSE
00109  WRITE(ISCRN,*) ' ERROR MESSAGE WITH ESTIMATES X=',X,' Y=',Y
00110
00111  END IF
00112  *  PROMPT THE OPTION MENU
00113
00114  40 WRITE(ISCRN,50)
00115  50 FORMAT(//////'OPTION MENU'///
00116  1     2X, Enter Alpha and beta levels'/
00117  1     2X, Enter initial X and Y'/
00118  1     2X, Stop execution' /
00119  1     2X, PLEASE ENTER YOUR CHOICE')
00120  READ(ISCRN,10) COMMAND
00121
00122  *  EXECUTE SELECTED OPTION
00123
00124  IF(COMMAND.EQ.'A') GOTO 1000
00125  IF(COMMAND.EQ.'X') GOTO 2000
00126  IF(COMMAND.NE.'S') GOTO 40
00127
00128  *  STOP EXECUTION
00129
00130  CLOS(FILE)
00131  STOP
00132  END
00133
00134
00135  ******************************************
00136
00137  SUBROUTINE ROOT
00138
00139  ROOT solves the equation system of the relationship between
00140  alpha, beta and unit watershed size. The method employed is called
00141  generalized NEWTON method (Yakowitz, 1978). Five arguments are
00142  described as follows:
SUBROUTINE ROOT(A,B,ZETA,PHI,IERROR)

REAL JK11,JK12,JK21,JK22,F(6)

SET ERROR ALLOWANCE DD AND USE INITIAL ESTIMATES AS THE SOLUTION

DD=1.E-20
Y1=ZETA
Y2=PHI

ITERATION CALCULATION LOOP BEGINS

DO WHILE (DD.GT.1.E-8)

INITIALIZE THE OLD ESTIMATE VECTOR X0

X1=Y1
X2=Y2

DETERMINE FUNCTION VECTOR AND THEIR JK MATRIX

CALL FUNCT(X1,X2,A,B,F)

JK MATRIX

JK11=F(3)
JK12=F(4)
JK21=F(5)
JK22=F(6)

FUNCTION VECTOR

F11=F(1)
F22=F(2)

CONSTANT VECTOR: XI=JK*X0

XI1=JK11*X1+JK12*X2
XI2=JK21*X1+JK22*X2

DETERMINE DETERMINANT OF JK MATRIX
DELTA=JK11*JK22-JK12*JK21

* IF DELTA=0, THEN ERROR CONDITION: SET FLAG AND RETURN

IF (DELTA.EQ.0.) THEN
  IERROR=1
  WRITE(ISCRN,*) ' ERROR TYPE: DELTA = 0. IN JK MATRIX'
  RETURN
ENDIF

CALCULATE THE NEW ESTIMATES OF SOLUTION

Y1=(X11*JK22-X22*JK12)/DELTA
Y2=(X22*JK11-X11*JK21)/DELTA

ESTIMATE ERROR TERM OF THIS SOLUTION

DD=(X1-Y1)*(X1-Y1)+(X2-Y2)*(X2-Y2)

* IF Y1 OR Y2 IS NEGATIVE, THEN ERROR CONDITION

IF (Y1.LT.0.) THEN
  WRITE(ISCRN,*) 'ERROR TYPE NEGATIVE SOLUTION ON Y'
ELSE
  WRITE(ISCRN,*) 'ERROR TYPE NEGATIVE SOLUTION ON X'
ENDIF

* IF Y1 > Y2, THEN ERROR CONDITION: INTERMEDIATE RESULTS ERROR

IF (Y1.GT.Y2) THEN
  IERROR=1
  WRITE(ISCRN,*) 'ERROR TYPE: INTERMEDIATE Y > X'
ENDIF

END OF THE ITERATIONS

END DO

RETURN THE SOLUTION

ZE1A=Y1
PH1=Y2

RETURN

END
SUBROUTINE FUNCT

FUNCT calculates the values of the equation system of the relationship between alpha, beta and unit watershed size, given alpha, beta, x, and v. It also returns the JK matrix of the equation system.

Five arguments are:

1) X angle 1
2) Y angle v
3) A alpha level
4) B beta level
5) F array of returned values.

F(1), F(2): values of equation system
F(3), F(4), F(5), F(6) are elements of the JK matrix, jkim, kijm, jk2m, jk2i, jk22 respectively

SUBROUTINE FUNCT(X,Y,A,B,F)

REAL F(6)

CONSTANT AND INTERMEDIATE TERMS EVALUATION

FA1=3.14159

SINX=SIN(X)
SINY=SIN(Y)
SINX2=SINX*SINX
SINY2=SINY*SINY
SINXY=SINX*SINY
COSX=COS(X)
COSY=COS(Y)
R=SINY/SINX
R2=1./(R*R)
A1=SQR1(1.-A)
B1=B*FA1

EVALUATION OF EQUATION SYSTEM
\[ F(1) = \sin Y( \cos X - A1) - \sin X( \cos Y + A1) \]

\[ F(2) = \sin X( \sin Y - \sin Y \cos Y) - \sin Y( X - \sin X \cos X) - B1 \sin X2 \]

* \[ \text{EVALUATION OF JK MATRIX} \]

\[ F(3) = -((A1 + \cos Y) \cos X + \sin Y) \]

\[ F(4) = (\cos X - A1) \cos Y + \sin Y \]

\[ F(5) = 2.\sin X \cos Y( \sin Y - \sin Y \cos Y) - \sin Y \sin X2 - \sin X2 \cos Y \]

\[ F(6) = 2.\sin X2 \sin Y2 - \sin Y \cos Y \]

* \[ \text{RETURN TO CALLING ROUTINE} \]

\[ \text{RETURN} \]

\[ \text{END} \]
REFERENCES


