ADAPTABILITY OF A DAILY DISAGGREGATION MODEL TO THE MIDWESTERN UNITED STATES

by

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ABSTRACT

A daily disaggregation model developed with summer rainfall data from southeastern Arizona was analyzed for its utility at two midwestern locations, Hastings, Nebraska, and McCredie, Missouri. The model required modification. For example, two distributions required modification and one was replaced. An addition to the model was made to describe the diurnal variation in shower depth for days with one shower. With modifications, the disaggregation model accurately described the daily intermittant rainfall process for the two midwestern locations.

Parameter values developed from the Hastings and McCredie's (primary stations) data sets were used to simulate the daily intermittent rainfall at three nearby locations (test stations). Statistical testing indicated that the primary stations' parameter values adaquately simulated the test stations rainfall if the stations ~~were within the same precipitation climatologic zone~~.

CHAPTER 1

INTRODUCTION

Background

Watershed simulation models have proven to be useful tools in simulating the transport of chemicals, water and sediment from field-size areas (c.f. Knisel, 1980) and in estimating runoff (c.f. Rovey et al., 1977); and sediment yield (c.f. Smith, 1981) from urban and rural watersheds. These models use or have the capabilities to use, physically-based infiltration submodels that partition rainfall into surface runoff and infiltration. The submodels require rainfall data with a high temporal resolution (approximately 5 minutes). The use of physically-based infiltration submodels has been limited in part, by their need for rainfall data with a high temporal resolution. Daily precipitation data, the most abundant and readily available form of precipitation data do not provide the degree of resolution needed. Hourly precipitation data - not as readily available - still do not provide the time resolution needed. Data with the required degree of resolution are the least available because of the high cost and large amount of time required to collect and process it.

Current research has developed methods that use the more available daily rainfall data in the stochastic simulation of shorter time period rainfall. The methods may be used to disaggregate daily rainfall into the intermittent rainfall process, and further disaggregate the significant storms into short period rainfall intensities. In this manner, short-period rainfall can be simulated at locations where only daily rainfall data are available.

Literature Review

At least two methods of disaggregating daily rainfall into shorter time periods exist in the literature (Woolhiser and Goodrich, 1987). They are the discrete-discrete (D-D) method, and the discrete-continuous (D-C) method. The two methods will be described below.

The D-D method consists of a submodel to disaggregate the discrete process of daily rainfall into the discrete process of hourly rainfall, and a second submodel to disaggregate hourly rainfall into a shorter time period rainfall. This method was used by Betson et al. (1980), and Srikanthan and McMahon (1985). Betson et al. (1980) used transition probabilities to determine if the hours were wet or dry. The transition probabilities of a wet hour following a wet hour and a dry hour following a dry hour were described by:

$$P = \exp(-aR^{b}) \tag{1.1}$$

where:

P is the transition probability.

R is the daily rainfall, and

a, b are coefficients.

The coefficients in the expression for the transition probability of a wet hour following a wet hour was allowed to vary seasonally with a Fourier function. Hourly precipitation amounts were independently generated from the Weibull distribution, and were linearly adjusted to sum to the daily total. Five minute rainfall amounts were generated in a similar fashion (R in equation 1.1 is equal to the hourly rainfall), however, the transition probabilities did not vary seasonally. The five minute rainfall amounts were added together to produce amounts for durations evenly divisible into an hour (i,e., 5, 10, 15, 20, and 30 minutes). The Betson model did not account for the diurnal variability of rainfall occurrence.

The model used by Srikanthan and McMahon (1985) is more complex. To generate hourly rainfall from a daily rainfall depth two types of days with rainfall were considered. Type 1 wet days were days with rainfall amounts less than a threshold, and type 2 wet days were those greater than or equal to the same threshold. Parameter values were calculated separately for each type of wet day. To determine occurrence during an hour a two-state second order Markov chain was used. To account for the diurnal variability of occurrence - the day was grouped into six units, each four hours in duration. Markov chain probabilities were estimated for each unit, the probabilities were considered to be constant within the 4-hour period. A transitional probability matrix (TPM) calculated for each hour, was used to generate rainfall amounts for the hour. To generate six minute rainfall amounts, wet hours were grouped into four types based on hourly rainfall depth. The six minute model consisted of four sets of TPM's, one set for each type of wet hour. The Srikanthan and McMahon model accounted for the diurnal variability of occurrence, however, it required a multitude of parameters.

The D-C method consists of two submodels. The first submodel disaggregates daily rainfall into a continuous process of showers and dry periods within the day, the second submodel disaggregates the showers into short-term intensities.

The technique of stochastically breaking down a daily rainfall depth into multiple showers and dry periods was developed by Hershenhorn (1984). The sum of the multiple showers' depths must equal the daily depth. Each shower is characterized by a unique depth, duration and time of beginning. Several statistical properties of the rainfall process are to be preserved by the daily disaggregation model: (1) the number of showers per day given daily rainfall; (2) the distribution of shower amounts; (3) the distribution of event starting times; (4) the joint distribution of shower amounts and duration and (5) the diurnal fluctuation of depth for one per day showers. The individual showers may be disaggregated into intensity patterns as described by Woolhiser and Osborn (1985).

The D-C method appears to be more parsimonious than the D-D method. Srikanthan and McMahon's (1985) technique used to disaggregate daily rainfall into 6 minute intervals requires over two thousand parameters. The D-C method using Hershenhorn's (1984) model to generate shower statistics from daily rainfall, and the technique developed by Woolhiser and Osborn (1985) to simulate intra-storm intensities requires 45 parameters for summer rainfall in southeastern Arizona.

A different approach to modeling short time period rainfall is rainfall aggregation. Various authors have utilized this technique

(Grace and Eagleson, 1966; Todorovic and Yevjevich, 1969; Bras and Rodriguez-Iturbe, 1976; Nguyen and Rouselle, 1981; Rodriques-Iturbe, (1986), however, this approach does not maintain the daily statistics or diurnal variations of the data and it would be very difficult to estimate parameters from daily precipitation data, so that the model could be easily transferred to a site without breakpoint data.

Objectives

Hershenhorn's disaggregation model (1984) was developed with precipitation data collected from the Walnut Gulch Watershed in southeastern Arizona during the months of July and August. One of the objectives of this study was to determine if the disaggregation technique could be adapted for use in the midwestern United States, specifically Hastings, Nebraska and McCredie, Missouri. The second objective was to determine the portability of the parameter values - whether model parameters developed at one station could be transferred to another location without a significant loss in model accuracy.

CHAPTER 2

DATA SET

The precipitation data used in the study were collected on the Agricultural Research Service's Experimental Watersheds and were obtained on magnetic tape from the U.S.D.A.'s Water Data Laboratory. The precipitation data was in breakpoint form. Breakpoint data is the record of the time patterns of rainfall accumulation. The months of May through August were studied for five midwestern locations. This period of the year was chosen because this is when peak runoff from the small watersheds in the midwest usually occurs (Linsley, et al., 1949). Two of the stations - Hastings, Nebraska and McCredie (now Kingdom City), Missouri, were used in determining the feasibilty of using the daily disaggregation model in the midwestern United States. The three remaining locations were used to determine if the parameters obtained at Hastings, or McCredie could be used as inputs into the model to disaggregate rainfall at other locations. The three locations used for this purpose were Fennimore, Wisconsin, Monticello, Illinois, and Treynor, Iowa.

The climate for the midwestern region is considered to be continental, with an annual precipitation cycle that exhibits a winter minimum and a summer maximum. The monthly precipitation maximum usually occurs during June, and over half of the total annual precipitation occurs during the months of May through September.

Approximately 80 percent of the summer precipitation in the region has been classified as frontal in nature (Rudd, 1961). In contrast, most of the rainfall during the July and August season in southeastern Arizona is a result of local, convective, air mass type thunderstorms (Trewartha, 1981). Air mass thunderstorms are little affected by frontal activity. The maximum daily thunderstorm activity during the summer (June - August) in the region which McCredie and Hastings lie, occurs between 8 pm and 1200 am (Easterling and Robinson, 1985). The time of maximum summer thunderstorm occurrence throughout the United States is shown in figure 2.1. In table 2.1 some precipitation statistics for Hastings and McCredie are listed. Figure 2.2 is an illustration of the different precipitation regions in North America based on the characteristics of the annual march of precipitation. All of the locations used fall in region 3, Hastings is in subregion 3c, the remaining four locations fall in subregion 3d. Mean monthly precipitation at McCredie and Hastings is shown in figure 2.3.

Hastings' precipitation exhibits a typical monthly variation for subregion 3c. Total rainfall is strongly concentrated in the early summer, June is the wettest month - some locations in this subregion may reveal a May maximum. The early summer maximum is a result of the continental influence resulting in increasing temperatures and advection of moisture from the Gulf of Mexico, coupled with still fairly active spring storms (Trewartha, 1981). The crest of the precipitation profile is characteristically sharp. It is common for the profile to halt its decline in August-September, so

Table 2.1. Selected precipitation statistics for Hastings, Nebraska and McCredie, Missouri.

Location	Period of Record	Mean Annual Precip (in.)	Mean Monthly Precipitation (in.) May	June	July	August
Hastings	1938-1967	22.62	3.58	4.93	2.88	2.59
McCredie	1941-1974	35.81	4.39	4.58	3.55	2.94

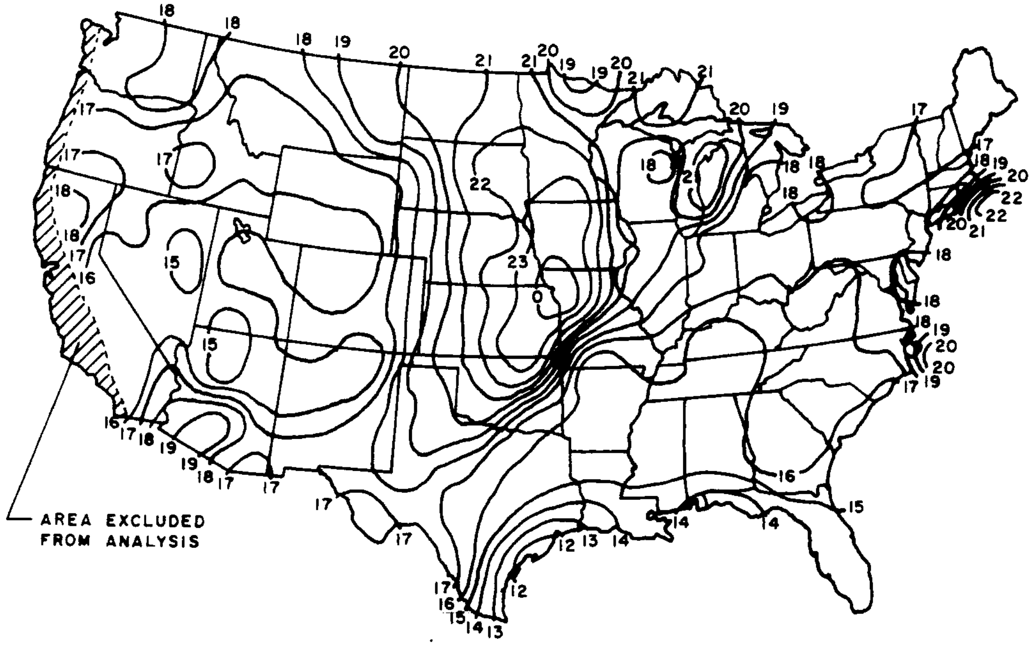

Figure 2.1. Summer Pattern of Time of Maximum Thunderstorm Activity in the United States

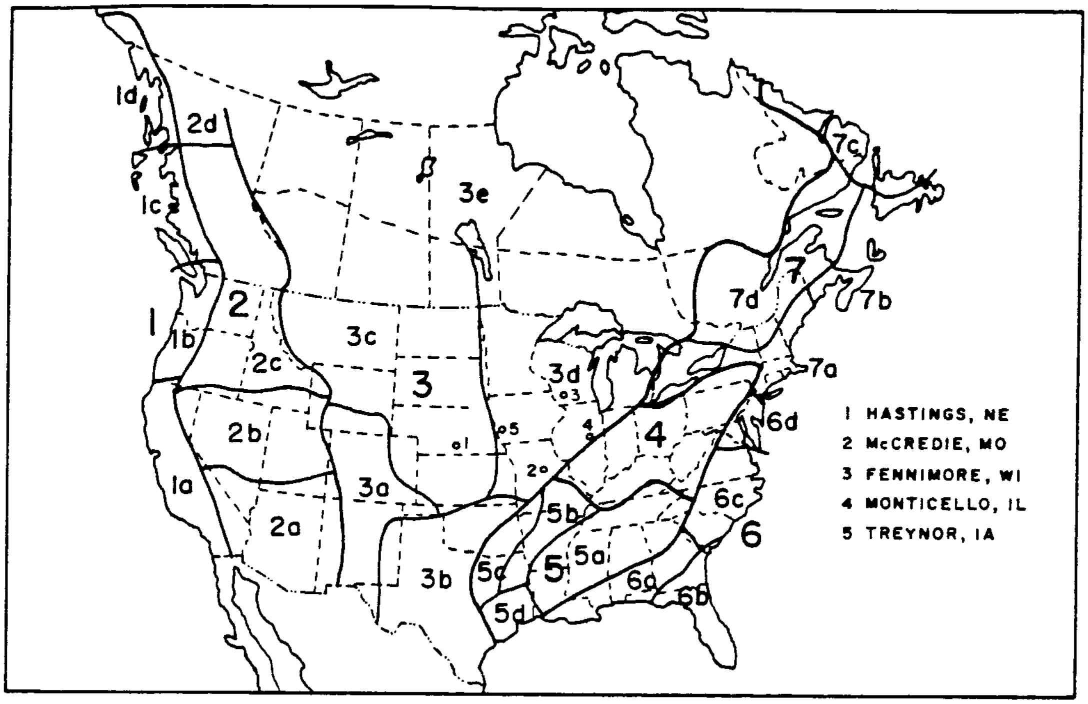

Figure 2.2. Precipitation Regions of North America Based on the Annual March of Precipitation

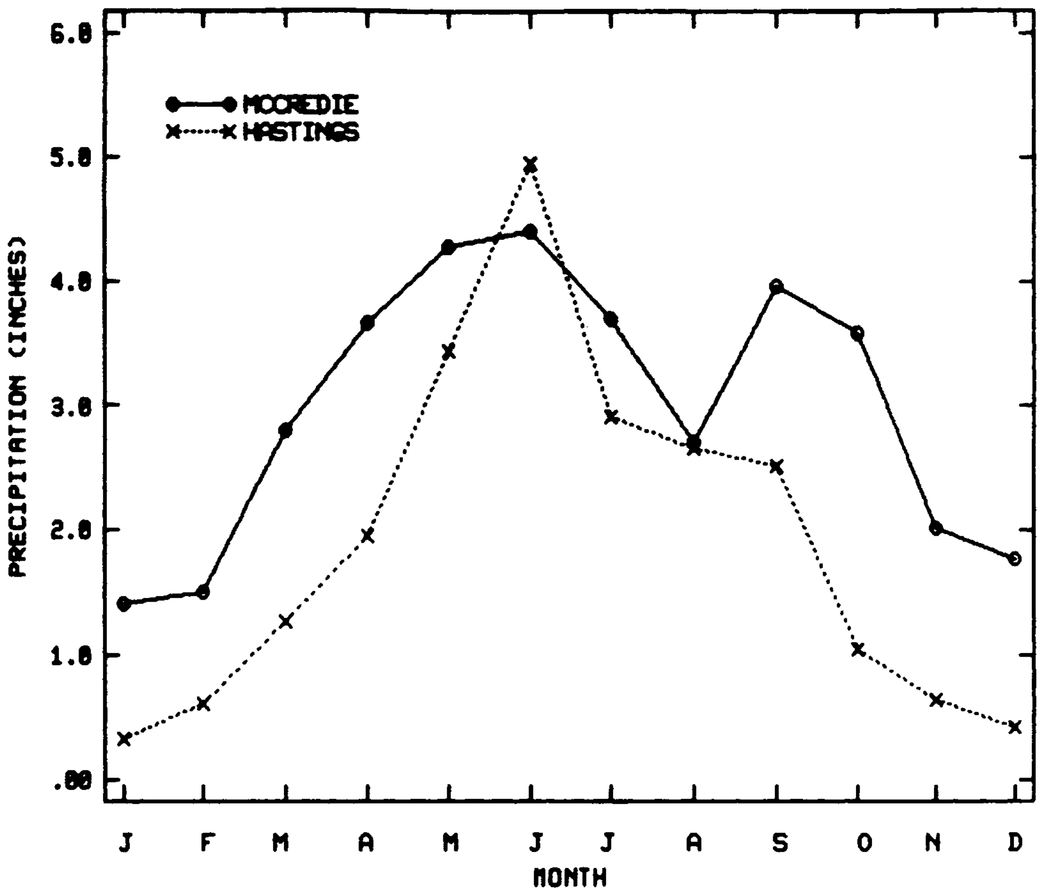

Figure 2.3. Mean Monthly Precipitation at Hastings, Nebraska and McCredie, Missouri

that a bench or even a hint of a slight secondary September maximum may be evident. The possible explanation for this phenomenon will be explained below.

The monthly precipitation profile at McCredie is typical of the profile of a location in subregion 3d. Stations in this subregion occasionally have a double maximum in the summer. The first peak usually occurs in June (occasionally May or July) and the second peak occurs most commonly in September. The June maximum is the result of the same climatic influences present in subregion 3d. A partial explanation for the decreased rainfall in the July-August period may be that a dry northernly tongue of air associated with the anticyclonic cell over the interior of the country reaches its maximum development during July and August. This dry current appears to a cause a decrease of rainfall in mid and late summer in an area extending from the Great Lakes southward through the Mississippi Valley to Texas.

CHAPTER 3

MATHEMATICAL MODEL

The mathematical disaggregation model used in this study, was described in detail by Hershenhorn (1984) and Hershenhorn and Woolhiser (1987), and will be described briefly here. Let $\xi(t)$ denote the continuous process of precipitation intensity at a point in space. $\xi(t)$ is non-negative and will take on positive values over random intervals of time. A possible realization of this function in an arbitrary period of time is illustrated in figure 3.1.

Let a positive $\xi(t)$ pulse which is bounded on both sides by $\xi(t)$ equal to zero be defined as a shower. The total depth in the nth shower X(n) can be determined by:

$$X(n) = \int_{\tau(n)}^{\tau(n*)} \xi\ (t)dt \tag{3.1}$$

where:

$\tau(n)$ = the starting time of the nth storm, in cumulative hours from some arbitrary time, t=0.

$\tau(n*)$ = the ending time of the nth storm.

The duration of the nth shower, D(n)= $\tau(n*)$ - $\tau(*)$. A complete shower is defined as one that occurs within one day, and a partial

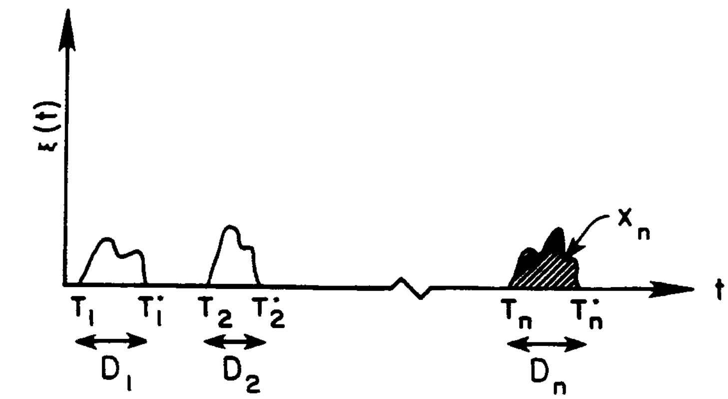

Figure 3.1. Possible Sample Function of the Rainfall Intensity Process

shower is one that begins on day j and ends on day j+1. The total number of showers in a day, k is determined by:

$$k = N(c) + N(p) \tag{3.2}$$

where:

N(c) - is the number of complete showers.

N(p) - is the number of partial showers.

The rainfall process is assumed to be stationary, and may have seasonal cycles. The sample functions of the rainfall process for each year are assumed to be repeated samples of an underlying process.

To describe the starting time for a shower within the day, the variable U(t) is described such that:

$$U(t) = t - \{[t/24] \times 24\} \tag{3.3}$$

where:

t - is time from an arbitrary midnight (in hours).

[t/24] - represents represents the integer part of the number t/24.

The total rainfall occurring on day j of year i is represented by Z(i,j). Then Z(i,j) can be determined by:

$$Z(i,j) = \int_{24(j-1)}^{24j} \xi(t)dt \tag{3.4}$$

Each day with an observable amount of precipitation is accompanied by random variables which describe the number of complete and partial showers, the depths and durations for the showers, and the starting times for the showers. The model developed by Hershenhorn (1984), provides a procedure for simulating these random variables from a daily precipitation amount.

CHAPTER 4

DATA ANALYSIS

Data analysis was performed on two bimonthly periods (May-June and July-August) of the Hastings and McCredie data. The analysis was performed to determine if the model developed by Hershenhorn could be used for these time periods and locations. If the model did not appear to fit well new techniques were developed in order to describe the disaggregation process at the midwestern stations.

The analysis for the disaggregation of daily rainfall into individual showers requires the following steps:

1. Determine what special procedures are required for those showers which continue over midnight (partial showers).
2. Develop the appropriate form for the joint distribution of amount of rainfall per day, and the number of individual showers. Estimate the required parameters.
3. Devise a procedure to disaggregate the daily rainfall amount Z into N showers (of depth Y(i) and duration D(i)), and estimate the required parameters.
4. Determine the form of the distribution function of the time of shower beginning T(i), and estimate the required parameters.

5. Determine the form of the joint distribution of shower amount Y(i) and duration D(i), for each of the showers, and estimate the required parameters.

Complete Versus Partial Showers

A shower is defined as a fall of rain greater than 0.01 inch over a indefinite time period, and does not contain a period of ten minutes or more without rainfall. Two types of showers are defined - a complete shower is one that begins and ends within the same day, and a partial shower is one that begins on one day and ends on the next. Table 4.1 lists the number of two consecutive days with precipitation and the percentage of those containing a partial event. The non-parametric rank sum test (Hoel, 1971) was used to determine if the distributions of the sums or products of the daily depth amounts of the consecutive wet days with partials and those without partials were significantly different. At the five percent level, the rank sum test could not determine any significant differences between the consecutive wet days with partials and those without partials.

A partial shower can be separated into two smaller showers which will be assigned to the day on which they occurred - one shower is that part which occurred before midnight; and the second is that part which occurred after midnight. For the remainder of this text a partial shower is to be considered one of these smaller showers. The mean, standard deviation, and the coefficient of skew of the showers' duration and depth for the complete and partial showers are listed in

Table 4.1. Consecutive wet days and percent partials.

Location	Period	No. of two consecutive wet days	Percentage with partials
Hastings	May and June	188	34
Hastings	July and August	107	34
McCredie	May and June	272	24
McCredie	July and August	164	21
Walnut Gulch	July and August	217	17

4.2 and 4.3, respectively. The partial showers tend to have larger depths and longer durations than the complete showers. The May and June period at both locations have the greatest differences between the partial and complete showers' mean depths.

Hershenhorn (1984) with the use of the likelihood ratio test found that the complete and partial depths could be described by the same distribution for the Walnut Gulch data. To determine whether this was the case with the Hastings and McCredie data sets the likelihood ratio test (Hoel, 1971) was also used. The null hypothesis (Ho) tested was whether the distributions of partial and complete showers may be described by one parameter set from a distinct distribution or two parameter sets of that same distribution. The test statistic λ which approaches being a Chi-Square variable as the population sizes increases may be written as:

$$\lambda = -2 \operatorname{Log}\{L(1 + 2)/(L(1)+L(2))\} \tag{4.1}$$

where:

L (1 + 2) is the maximum likelihood value obtained from parameter estimation over the populations of 1 and 2 combined.

L(1) is the maximum likelihood value obtained from parameter estimation of population 1.

L(2) is the maximum likelihood value obtained from parameter estimation of population 2.

Table 4.2. Mean, standard deviation, and coefficient of skew for complete and partial showers' durations.

Location	Period	Shower Type	Mean (min)	Std. Dev. (min.)	Coef. of Skew
Hastings	May and June	Complete	157.2	177.4	2.415
		Partial	175.4	203.2	2.568
	July and August	Complete	137.5	137.5	1.725
		Partial	137.9	156.5	1.713
McCredie	May and June	Complete	93.76	112.0	2.659
		Partial	186.3	218.6	2.336
	July and August	Complete	91.08	100.4	2.113
		Partial	150.8	148.7	1.221

Table 4.3. Mean, standard deviation, and coefficient of skew for complete and partial showers' depths.

Location	Period	Shower Type	No. of Obs.	Mean (ins.)	Std. Dev. (ins.)	Coef. of Skew
Hastings	May and June	Complete	540	.332	.434	2.68
		Partial	129	.429	.688	3.94
	July and August	Complete	432	.318	.412	3.08
		Partial	72	.349	.585	3.12
McCredie	May and June	Complete	1107	.219	.282	2.65
		Partial	129	.323	.426	2.19
	July and August	Complete	770	.250	.343	3.27
		Partial	69	.307	.390	1.61

Maximum likelihood techniques were used to determine the parameter values for four distributions. The distributions were the Exponential, Gamma, Mixed Exponential, and the Weibull (equations 4.4 - 4.7). The Akaike Information Criteria (AIC) (Akaike, 1974), to be explained in further detail later in the text, was used to determine which distribution could be considered to have the best fit to the complete and partial showers individually and collectively (three data sets in all). If a distribution resulted in the best fit for all three data sets that distribution was used in the likelihood ratio test, otherwise, the distribution that fit two of the three sets the best was used. The three distributions that revealed the best fits were the Gamma, Mixed Exponential and the Weibull. In tables 4.4 through 4.7, the parameter values for these distributions and their associated AIC values are listed. The likelihood ratio test could not reject Ho, at the one percent level, for any the data sets except the McCredie - May and June. The results of the likelihood ratio test are listed in table 4.8. Also, the two-sample Kolmogorov-Smirnov (KS) test, a less rigorous but distribution free test, could not distinguish between the distribution of complete shower depths or partial shower depths for any of the midwestern data sets.

It is logically apparent that if a location experienced a diurnal fluctuation in shower depth, the depths of the partial showers (i.e., those occurring close to midnight) would be different than the complete showers (i.e., showers not occurring close to midnight). The same may hold true for the shower durations. However, to make the

Table 4.4. Parameter values and AIC for the distributions of daily depth[1] at Hastings - May and June.

Shower Type	Distri-bution	Parameters θ_1	θ_2	θ_3	AIC
Partial	Gamma	.579	1.38	---	5.16
	Mix. Exp.	.397	.064	.653	-.214
	Weibull	.320	.689	---	-.600
Complete	Gamma	.759	2.34	---	-154.1
	Mix. Exp.	.563	.137	.562	-198.5
	Weibull	.286	.817	---	-179.8
All	Gamma	.710	2.08	---	-166.1
	Mix. Exp.	.522	.122	.581	-194.6
	Weibull	.292	.783	---	-175.7

[1]See text for functional forms.

Table 4.5. Parameter values and AIC for the distributions of daily depth[1] at Hastings - July and August.

Shower Type	Distri-bution	Parameters θ_1	θ_2	θ_3	AIC
Partial	Gamma	.606	1.78	---	-22.11
	Mix. Exp.	.800	1.57	1.02	-31.84
	Weibull	.261	.702	---	-27.57
Complete	Gamma	.799	2.59	---	-162.0
	Mix. Exp.	.510	.130	.494	-183.5
	Weibull	.279	.842	---	-171.3
All	Gamma	.760	2.42	---	-184.1
	Mix. Exp.	.587	.140	.560	-216.2
	Weibull	.274	.813	---	-198.8

[1]See text for functional forms.

Table 4.6. Parameter values and AIC for the distributions of daily depth[1] at McCredie - May and June.

Shower Type	Distri-bution	Parameters θ_1	θ_2	θ_3	AIC
Partial	Gamma	.501	1.61	---	-92.85
	Mix. Exp.	.341	.025	.462	-93.53
	Weibull	.226	.630	---	-91.98
Complete	Gamma	.626	2.96	---	-1413.
	Mix. Exp.	.349	.039	.305	-1423.
	Weibull	.174	.732	---	-1427.
All	Gamma	.606	2.71	---	-1494.
	Mix. Exp.	.354	.038	.324	-1505.
	Weibull	.179	.714	---	-1511.

[1]See text for functional forms.

Table 4.7. Parameter values and AIC for the distributions of daily depth[1] at McCredie - July and August.

Shower Type	Distri-bution	Parameters θ_1	θ_2	θ_3	AIC
Partial	Gamma	.501	1.71	---	-57.39
	Mix. Exp.	.397	.029	.468	-59.85
	Weibull	.210	.625	---	-57.65
Complete	Gamma	.663	2.75	---	-750.7
	Mix. Exp.	.454	.068	.385	-777.4
	Weibull	.201	.753	---	-769.8
All	Gamma	.643	2.62	---	-806.6
	Mix. Exp.	.440	.063	.389	-835.3
	Weibull	.202	.742	---	-826.6

[1]See text for functional forms.

Table 4.8. Likelihood ratio test results for the distributions of shower depths.

Station	Period	Dist. Used in Test	Degrees Freedom	Test Stat.	Sig. Level Ho is Not Rejected At
Hastings	May and June	Mix. Exp.	3	10.16	.01
	July and August	Mix. Exp.	3	5.10	.05
McCredie	May and June	Weibull	2	12.75	*
	July and	Mix. Exp.	3	7.82	.01

* - Ho is rejected at the one percent significance level.

description of the rainfall process more tractable, and because of the results of the tests described in the previous paragraph, the partial and complete depths, will be assumed to be samples from the same distribution. This assumption is needed in order to distribute the daily depth into multiple shower depths. The durations of the partial and complete showers will be described on the basis of their respective shower depths.

Distribution of Number of Showers Given Daily Precipitation

The joint distribution of the number of showers per day, and the daily amount can be written as the product of the conditional and marginal distributions:

$$H_{N,Z'}(n,z) = G_{N|Z'}(n|z)F_{Z'}(z) \tag{4.2}$$

where:

N = the number of showers.

Z'= the daily precipitation depth minus a threshold.

Because the lower limit of observation is 0.01 inch, the threshold was set to 0.009 inch. The threshold was set at this value so all small depth values would remain in the data set.

Marginal Distribution

In this study, the marginal distribution of daily rainfall is developed so we can test the goodness of fit of the joint distribution with Chi-Square goodness of fit test (Appendix 1). When the overall model is used, the daily depth will be obtained from historical data or perhaps a climate generating model, and the conditional distribution will be used to generate the number of showers within a day.

Various distributions have been used to model precipitation amounts on wet days. The theoretical distributions used include the Exponential (Woolhiser, et al., 1973), the Gamma (Ison, et al., 1971), the Log-Normal (Brakensiek, 1958), the Kappa (Mielke and Johnson, 1973), the Mixed Exponential (Smith and Schreiber, 1974) and the Weibull (Hershenhorn, 1984). In this study, the four distributions investigated were the Exponential, the Gamma, the Mixed Exponential, and the Weibull. The probability density functions are as follows:

Exponential Distribution

$$f(z) = \frac{1}{\theta_1} \exp(-\frac{z}{\theta_1}) \tag{4.3}$$

Gamma Distribution

$$f(z) = \left[\theta_2^{\theta_1} \, z^{(\theta_1 - 1)} \, e^{-\theta_1 z} \right] / \Gamma(\theta_2) \tag{4.4}$$

Mixed Exponential

$$f(z) = \frac{\theta_1}{\theta_2} \exp\left(\frac{-z}{\theta_2}\right) + \frac{(1-\theta_1)}{\theta_3} \exp\left(\frac{-z}{\theta_3}\right) \tag{4.5}$$

Weibull Distribution

$$f(z) = \frac{\theta_2}{\theta_1} \left(\frac{z}{\theta_1}\right)^{\theta_2 - 1} \exp\left[-\left(\frac{z}{\theta_1}\right)^{\theta_2}\right] \tag{4.6}$$

Parameter values for the four different statistical distributions were obtained by maximum likelihood techniques with modifications as used by Buishand (1977). Buishand used a modification of the maximum likelihood method which ignored the value of rainfall amounts less than or equal to EPS and merely used the number of these observations. The modification is made because the solution of maximum likelihood is very sensitive to the accuracy of the small observations. The likelihood equation without the modification is of the general form:

$$L(\theta|X(1),X(2),\ldots,X(m+n)) = \prod_{i=1}^{m+n} f(X(i)|\theta) \tag{4.7}$$

With modifications the likelihood equation becomes:

$$L(\theta|X(1),X(2),\ldots,X(m+n)) = \prod_{i=1}^{m} f(X(i)|\theta) \prod_{i=1}^{n} \int_{0}^{EPS} f(X(i)|\theta)dx \tag{4.8}$$

where:

m is the number of observations greater than EPS.

n is the number of observations less than or equal to EPS.

θ is the parameter set.

In the analysis EPS was set to be equal to 0.016 inches, so all showers less than 0.02 inches were considered as small observations. The parameter values for the distributions were obtained by maximizing the likelihood function described above with the Simplex algorithm (Nelder and Mead, 1965).

The Akaike Information Criteria (Akaike, 1974) was used to identify the best of the alternative models. The model for which the maximum likelihood estimate of the parameters results in the minimum AIC is considered to be the best model. AIC is defined by:

$$AIC = -2\log L(y,\theta(k)) + 2k \qquad (4.9)$$

where:

$L(y,\theta(k))$ is the maximum likelihood value for the model.

k is the number of independently adjusted parameters within the model.

According to the AIC, the Mixed Exponential distribution had the best fit for three of the four data sets, and the Gamma distribution obtained the best fit with the McCredie, July and August data set. Hershenhorn (1984) found, without using the modifications as suggested by Buishand, that the Weibull distribution resulted in the best fit for the Walnut Gulch data. The optimized parameter values for all four distributions along with their corresponding AIC are listed in table 4.9. The null hypothesis (Ho) was that all four data sets were from the Mixed Exponential distribution with parameters values obtained from each data set, respectively. The KS and Chi-Square goodness of fit tests were used to test Ho. The one-sample KS test could not reject Ho at the five percent significance level for any of the four data sets. The Chi-Square test at the same

Table 4.9. Parameter values and AIC for the marginal distributions of daily depth[1].

Station	Period	Distribution	Parameters θ_1	θ_2	θ_3	AIC
Hastings	May and June	Exponential	.446	---	---	382.3
		Gamma	.739	1.66	---	351.6
		Mix. Exp.	.425	.149	.665	334.5
		Weibull	.395	.811	---	341.9
	July and August	Exponential	.393	---	---	153.3
		Gamma	.795	2.03	---	140.7
		Mix. Exp.	.483	.159	.610	121.8
		Weibull	.355	.840	---	131.8
McCredie	May and June	Exponential	.417	---	---	357.4
		Gamma	.751	1.80	---	332.0
		Mix. Exp.	.226	.082	.515	329.0
		Weibull	.377	.828	---	329.1
	July and August	Exponential	.424	---	---	665.4
		Gamma	.667	1.59	---	594.6
		Mix. Exp.	.187	.041	.507	596.2
		Weibull	.365	.782	---	598.1

[1]See text for functional forms.

significance level only accepted the two McCredie data sets as from the distribution and rejected the Hastings data sets. Based on these tests the Mixed Exponential was used to test the goodness of fit of the joint distribution.

Conditional Distribution

Hershenhorn (1984) found that the Shifted Negative Binomial (SNB) distribution provided a good fit for the conditional distribution of the number of showers given daily rainfall. The probability mass function of the SNB is written as:

$$P(N=n) = \binom{n+r-2}{n-1} p^{r}(1-p)^{n-1};\ n=1,2,\ldots,6 \qquad (4.10)$$
$$r \geq 0;\ 0 < p \leq 1$$

Variables p and r were allowed to vary with daily depth. The Walnut Gulch data indicated that the number of showers given Z asymptotically approached a limiting value (figure 4.1). However, the McCredie data suggested that the expected number of showers asymptotically approached a straight line with a positive slope (figure 4.1). Therefore, a new functional form was developed for p and r. The new functional form that accounts for this factor and includes the Walnut Gulch function as a special case. The functional form for p and r is:

$$p = \exp(-A_1 Z') \qquad (4.11)$$

$$r = (E - 1.0)\ p/(1.0 - p) \qquad (4.12)$$

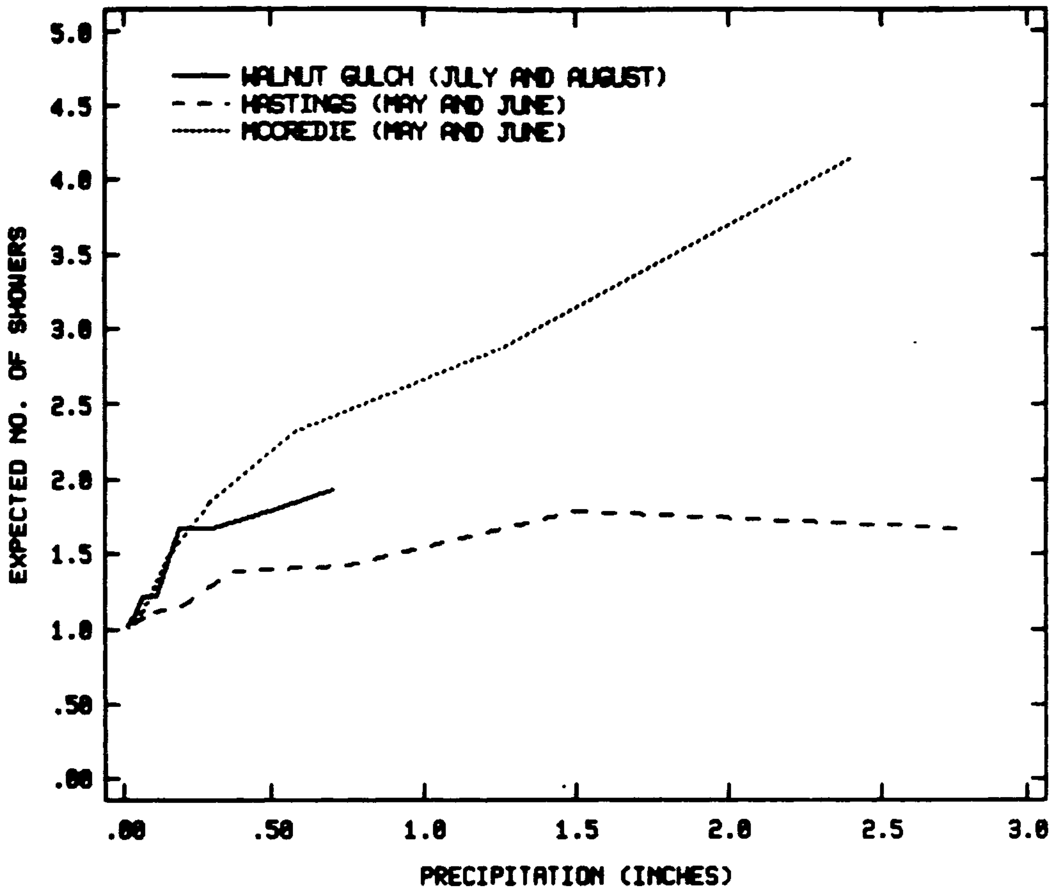

Figure 4.1. Expected Number of Showers Versus Daily Rainfall Amount

$$E = A_2 + A_3 \times Z' + (1.0 - A_2) \times \exp(-A_4 \times Z') \qquad (4.13)$$

where:

E is the expected number of showers given a daily precipitation amount.

Z' is daily precipitation depth minus a threshold of 0.009 inches.

A_2, A_3 and A_4 are fitting parameters.

Parameter values were obtained for the midwestern data by numerical maximum likelihood techniques. Figures 4.2, and 4.3 illustrate possible shapes that p, r, and E may assume. Table 4.10 contains the optimized parameter values for the data sets.

Bivariate Distribution

The bivariate distribution function can be written by utilizing the expressions for the Mixed Exponential and Shifted Negative Binomial distributions.

$$H_{N,Z'}(n,z) = \sum_{n=1}^{N} \int_0^z \left[\binom{n+r-2}{n-1} p^r (1-p)^{n-1} \right]$$

$$\left[\frac{\alpha}{\beta} \exp \frac{-z}{\beta} + \frac{1-\alpha}{\theta} \exp \frac{-z}{\theta} \right] dz \qquad (4.14)$$

where:

N is the number of events.

Table 4.10. Parameter values for the Shifted Negative Binomial distribution[1].

Station	Period	Parameters A1	A2	A3	A4
Hastings	May and June	.2685	1.387	.0731	1.250
	July and August	.3600	1.623	.2464	1.167
McCredie	May and June	1.136	4.162	.3547	1.575
	July and August	.8131	4.677	.2407	1.499

[1]See text for functional form.

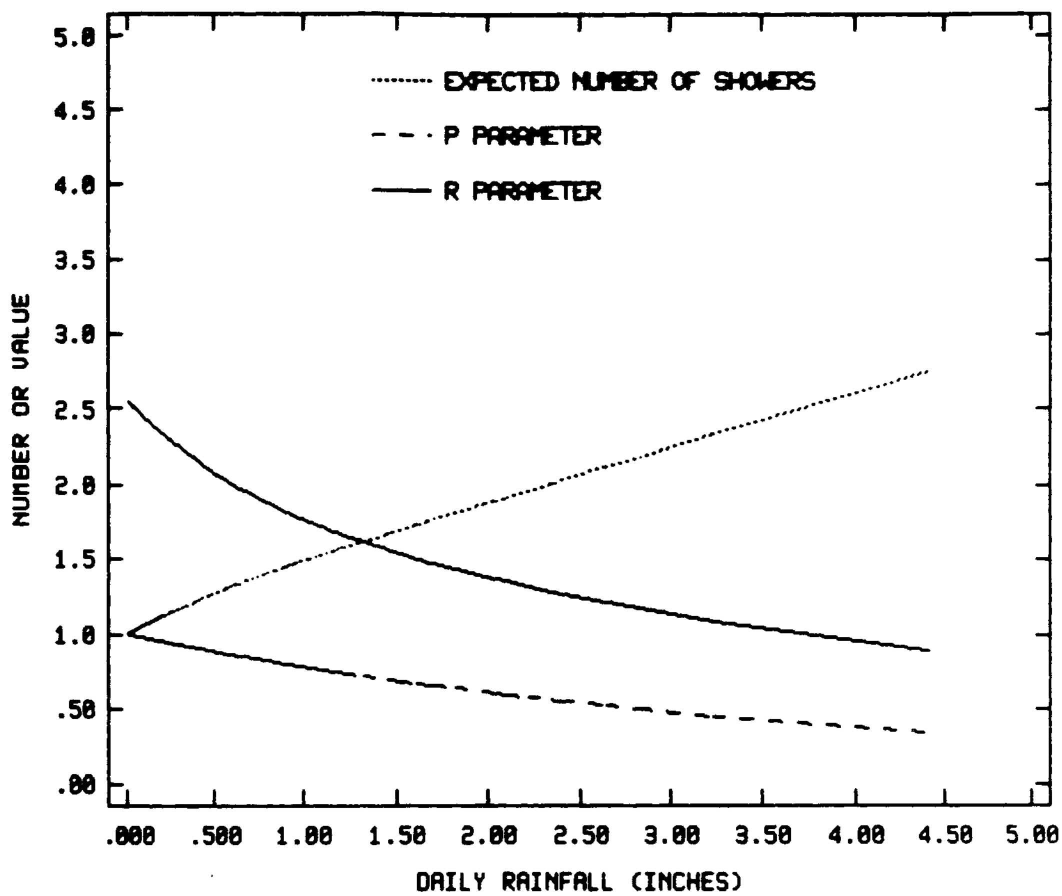

Figure 4.2. Expected Number of Showers, p and r as Functions of Daily Rainfall at Hastings, July and August

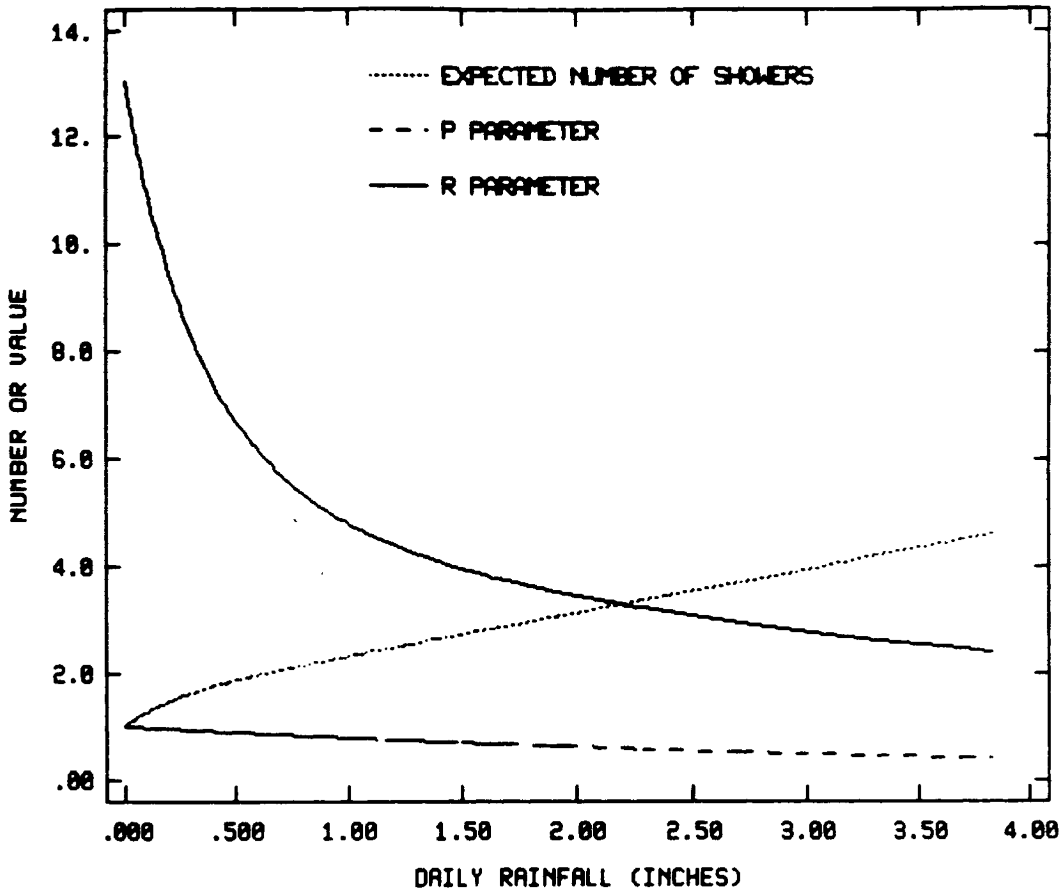

Figure 4.3. Expected Number of Showers, p and r as Functions of Daily Rainfall at McCredie, July and August

A Chi-Square goodness of test was used to test the hypothesis (Ho) that the sample data were taken from the identified bivariate distribution. To perform the test the sample data were subdividedinto classes of 1 shower/day, 2 showers/day, and greater than or equal to 3 showers/day. Each of these subclasses was divided into eight more subclasses which were dependent upon depth so there were a total of 24 subclasses. The count of the number of observations in each subclass and the calculated joint probability of an observation falling into a subclass were used in the calculation of the Chi-Square statistic. According to a Chi-Square goodness of fit test, at the one percent level, Ho could not be rejected for both of the McCredie data sets. The distributions of shower occurrences for simulated and historical data were also compared using the Chi-Square test (Little and Hills, 1978), (see Appendix 1). The comparisons revealed that for all the simulated distributions from the 4 data sets, the null hypothesis that the distributions came from the same parent distribution could not be rejected, at the one percent level. Since the marginal distributions of daily rainfall depth for the Hastings data did not pass significance testing, but the simulation of number of showers per day passed significance testing, the bivariate fit for the Hastings' data was considered acceptable.

Individual Shower Depths

Given a daily rainfall depth, Z, and the number of showers, N, the next requirement would be to generate individual shower depths, Y_1, Y_2, . . Y_n. The summation of all the individual shower depths which occur within a day must equal that day's rainfall depth. A ratio technique developed for the Walnut Gulch data (Hershenhorn, 1984), also worked well with the midwestern locations. The technique uses various ratios developed between shower depth(s) and daily depth. The ratios are defined in the following expressions:

a. Two showers per day:

R1 = (depth of first shower)/(daily total) = Y(1)/Z. (4.15)

b. Three showers per day:

R2 = (Y(1) + Y(2))/ Z ; R3 = Y(2) /(Y(2)+Y(3)). (4.16)

c. Four showers per day:

R4 = (Y(3) + Y(4))/ Z ; R5 = Y(1) /(Y(1)+Y(2)); (4.17)

R6 = Y(3) /(Y(3)+Y(4)). (4.18)

Hershenhorn (1984) used the Beta Fourier distribution to model R(1) and R(2) and used the uniform distribution to model the remaining ratios.

The Beta Fourier used to model R(1) and R(2) has the following form:

$$f(r) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \; r^{(\alpha-1)}(1-r)^{(\beta-1)} + \theta \; \mathrm{COS}(2\pi r) \tag{4.19}$$

$0.0 < r < 1.0$; α, $\beta > 0$; and θ must be defined so that

$$|\,\theta \; \mathrm{COS}\; 2\pi r| < \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \; r^{\alpha-1} \; (1-r)^{\beta-1}$$

The parameters for the Beta Fourier were obtained by maximum likelihood techniques for R1 and R2, individually. Parameter values were also obtained from R1 and R2 combined into one data set. The likelihood ratio test was used to test the hypothesis that R1 and R2 came from the same parent distribution. At one percent level, the hypothesis is rejected only for the May and June season at Hastings. The one-sample KS test was used to test the hypothesis that R1 combined with R2 can be described by the Beta-Fourier distribution. At five percent level, for all four data sets the hypothesis could not be rejected. The parameter values optimized over R1 and R2 are listed in table 4.11.

The one-sample KS test was used to test the goodness of fit of the uniform distribution to the ratios R3 through R6. At the five percent level, the only data subset that was rejected was that for R5 at Hastings - July and August, the hypothesis for this subset could not be rejected at the one percent level.

Hershenhorn (1984) developed ad hoc procedures for disaggregating the depths for 5 and 6 showers per day because of the low frequency of occurrence in the Walnut Gulch data set. The Hastings data were similar to the Walnut Gulch data in that there was only one day with five events and none with six events. The McCredie data had a higher frequency of days with five and six events per day, if fact one day in the May and June period had up to 13 events. The frequency of the number of days with over six showers was approximately one percent for the entire McCredie record. Because of

Table 4.11. Parameter values for the Beta-Fourier distribution[1].

Station	Period	Parameters α	β	θ
Hastings	May and June	1.192	1.054	.3003
	July and August	1.390	.9230	.5228
McCredie	May and June	1.419	1.019	.2959
	July and August	1.224	.7954	.0490

[1]See text for functional form.

the low frequency of days with more than six showers it was decided to ignore them in depth disaggregation.

To re-emphasize what was stated previously, the complete and partial shower depths could be considered to be from the same distribution for the Walnut Gulch data but this was not strictly the case with the midwestern locations. However, one-sample KS statistics do not indicate any significant differences between the historical distributions of shower depth and theoretical distributions developed using the ratio technique.

Starting Times For Complete Showers

The starting time of a shower is important because of its affect on antecedent moisture conditions. The starting time of the complete showers will be simulated. Because a partial shower is defined as one that ends or begins at midnight, once the durations have been determined, the time of the start of the shower can be determined. The starting time for a partial shower that occurs before midnight is calculated by subtracting the duration from midnight. The partial that occurs on the next day begins at midnight.

Hershenhorn (1984) used the Mixed Beta distribution to describe normalized starting times for the complete showers. The starting times were normalized by defining the starting times as time (in hours) from midnight and dividing by 24. The Mixed Beta distribution can written as:

$$f_{Tc}(t) = \omega \left[\frac{\Gamma(\alpha_1 + \beta_1)}{\Gamma(\alpha_1)\Gamma(\beta_1)} t^{(\alpha_1-1)} (1-t)^{(\beta_1-1)} \right]$$

$$+ (1-\omega) \left[\frac{\Gamma(\alpha_2 + \beta_2)}{\Gamma(\alpha_2)\Gamma(\beta_2)} t^{(\alpha_2-1)} (1-t)^{(\beta_2-1)} \right] \qquad (4.20)$$

$\alpha_1,\ \alpha_2,\ \beta_1,\ \beta_2 > 0;\quad 0 < \omega < 1;\quad 0 \leq t \leq 1$

The Mixed Beta distribution has some undesirable features. The distribution is not necessarily periodic. However, the process that we wish to describe is periodic. The distribution can not be explicitly integrated. If numerical integration is performed care must be taken at the tails where the Mixed Beta distribution may take on infinite values. To overcome the problems with the Mixed Beta, a Fourier distribution with a mean of one and with up to two harmonics was tested to determine if it could replace the Mixed Beta. The Fourier distribution is periodic and can be explicitly integrated. The Fourier distribution can be written as:

$$f_{T_c}(t) = 1.0 + \alpha_1 \mathrm{COS}(2\pi t + \gamma_1) + \alpha_2 \mathrm{COS}(4\pi t + \gamma_2) \qquad (4.21)$$

$$0 \leq t \leq 1; \text{ and } \alpha_1 \mathrm{COS}(2\pi t + \gamma_1) + \alpha_2 \mathrm{COS}(4\pi t + \gamma_2) \leq 1.0$$

Parameter values for the Mixed Beta distribution and the Fourier distribution for the individual data sets (i.e., McCredie - May and June) were obtained by using numerical maximum likelihood techniques. The parameter values for the Mixed Beta distribution are listed in table 4.12 and those for the Fourier distribution in table 4.13. The data sets were divided into subclasses, which were grouped according to the number of showers per day, up to 6 showers per day. All days with 6 or more showers were grouped together. One-sample KS tests were performed to determine whether the historical CDF's of starting times were significantly different from the CDF's calculated from the parameter values optimized over the entire data set. At the one percent level, all the historical CDF's for each subclass were not considered significantly different from the Fourier CDF's. The Hastings, July and August - 1 event per day data was the only subclass that was significantly different, at the one percent level, from the Mixed Beta CDF's. In figures 4.4 and 4.5, graphical illustrations of the two distributions and their fit to the historical data are shown.

The hypothesis of the independence of multiple starting times on a day with more than one event, and whether they can be described by order statistics was tested for both the Mixed Beta and the Fourier distributions. Let $F_{tr}(t)$ represent the CDF of the starting time of the r^{th} event on day which n showers occur (Kendall and Stuart, 1977). The general form is:

$$F_{Tr}(t) = \frac{n!}{(r-1)!(n-r)!} \left[\sum_{i=0}^{n-r} \frac{\binom{n-r}{r}}{i+r} (-1)^i \left(F_{T_c}\right)^{i+r} \right] \tag{4.22}$$

where F_{T_c} is the CDF of all event starting times as given by the CDF of the Mixed Beta or Fourier distribution.

Table 4.12. Parameter values for the Mixed Beta distribution[1].

Station	Period	α_1	β_1	α_2	β_2	θ
Hastings	May and June	6.845	1.198	.8320	1.968	.3187
	July and August	6.078	1.236	.8134	2.697	.3585
McCredie	May and June	1.101	1.073	.2107	.6373	.9481
	July and August	1.049	1.241	.2799	.5680	.9474

[1]See text for functional form.

Table 4.13. Parameter values for the Fourier distribution[1].

Station	Period	Parameter α_1	γ_1	α_2	γ_2
Hastings	May and June	.4986	-.3784	-.1180	.1227
	July and August	.6511	-.3446	-.1924	1.223
McCredie	May and June	.0454	-.3188	*	*
	July and August	-.2551	1.563	-.1236	.9312

* - Not significant according to AIC

[1] $f(x) = 1.0 + \alpha_1\cos(2\pi x + \gamma_1) + \alpha_2\cos(4\pi x + \gamma_2)$

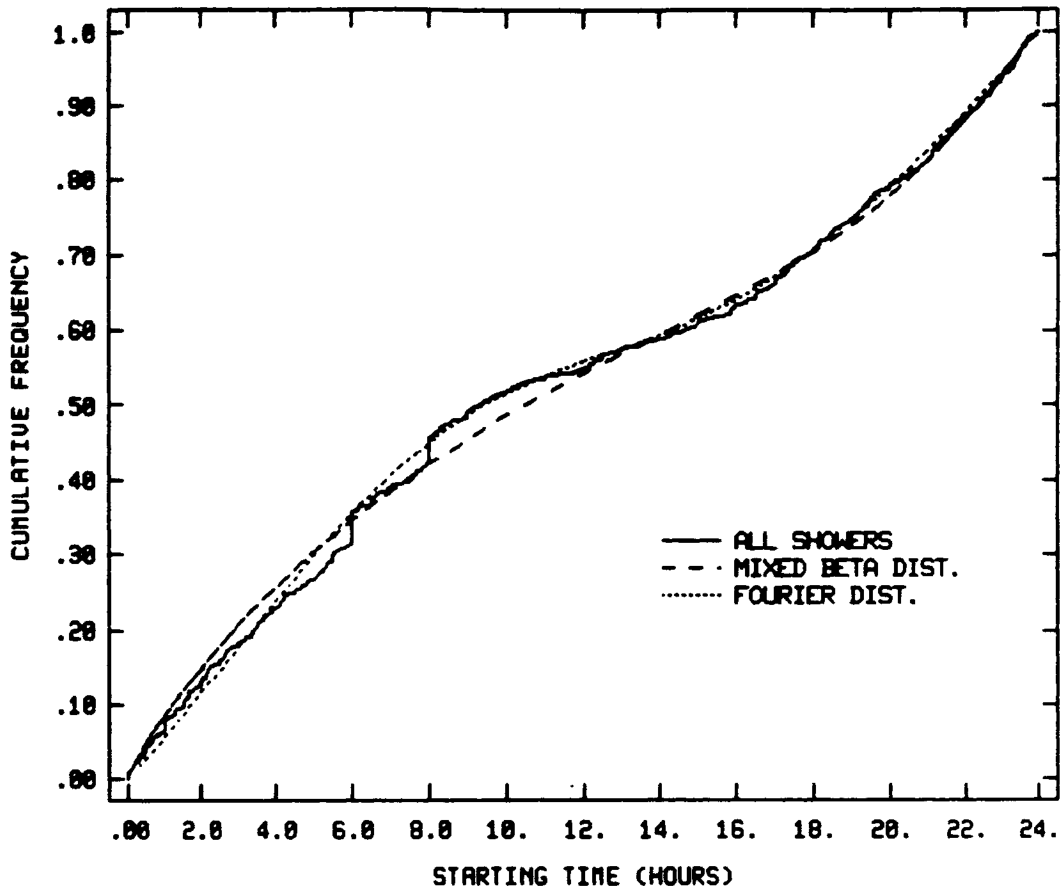

Figure 4.4. Distributions of Starting Times for Hastings, May and June

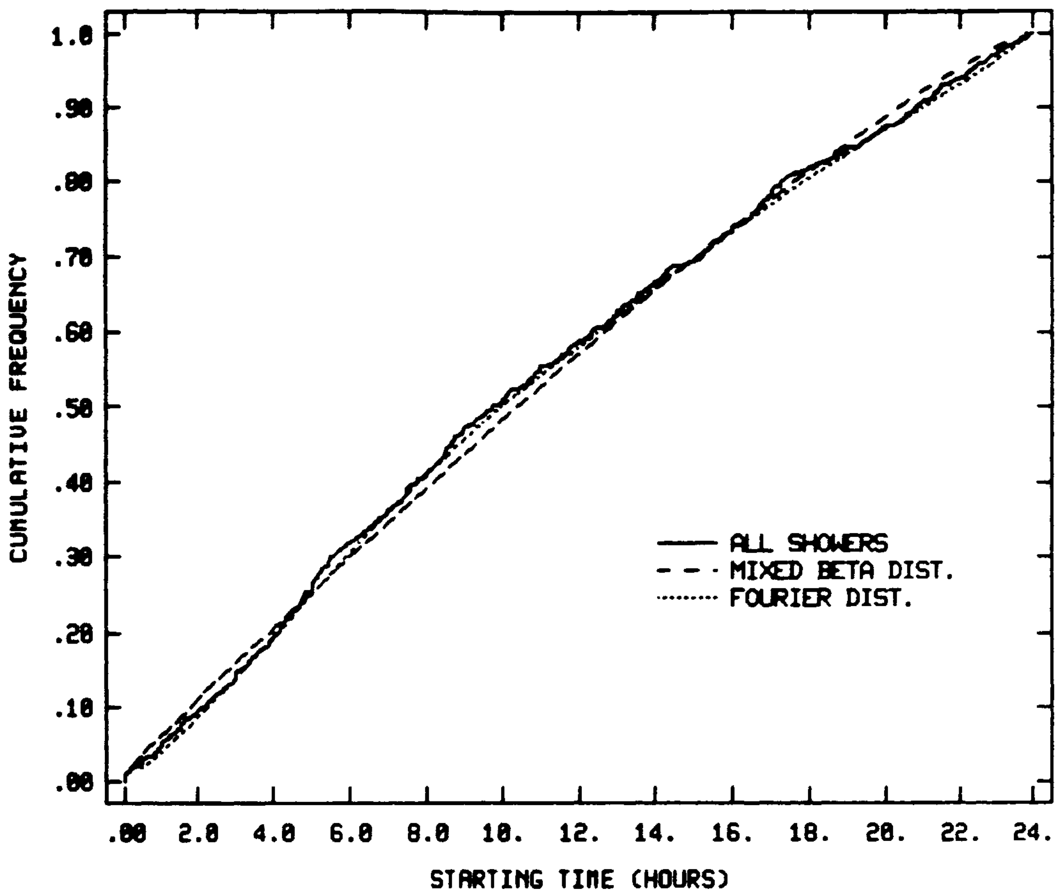

Figure 4.5. Distributions of Starting Times for McCredie, July and August

The use of order statistics would simplify the generation of the starting times on multiple shower days. For example, if N showers are to be simulated within a day, the simulation scheme would generate N starting times from the Mixed Exponential or the Fourier distribution, and order the starting times from earliest to latest.

The empirical distribution functions were compared with the theoretical distributions given the assumption of independence (equation 4.22). For example, the historical distribution for the first shower from the three showers per day data set were compared with F(t) with r=1 and n=3. The one-sample KS test was used to test the hypothesis that the event starting times were independent. When the order statistics were calculated from the Mixed Beta distribution there were 12 instances out of 58 where the distribution were considered significantly different at the one percent level (tables 4.14 through 4.19). Tests with the Fourier distribution revealed 11 instances out of 58 where the distributions were significantly different (tables 4.14 through 4.19)

The Mixed Beta and the Fourier distribution appear to fit the starting times equally as well. Therefore, because of its favorable properties the Fourier distribution was chosen as the distribution to use to describe the starting times of complete showers, instead of the Mixed Beta as used by Hershenhorn (1984).

Starting Times for One Shower Per Day

Kincer (1916) calculated that approximately 65 percent of the warm season (April-September) precipitation, in a region which

Table 4.14. Goodness of fit of the Fourier and Mixed Beta distributions to Order Statistics. Hastings, May and June, two to four showers per day.

Number of Showers Per Day	Occurrence of Shower	Is Ho accepted for Fourier	or Mixed Beta?
2	1st	Yes	Yes
	2nd	Yes	Yes
3	1st	Yes	Yes
	2nd	Yes	Yes
	3rd	Yes	No
4	1st	Yes	Yes
	2nd	Yes	Yes
	3rd	Yes	Yes
	4th	Yes	Yes

Table 4.15. Goodness of fit of the Fourier and Mixed Beta distributions to Order Statistics. Hastings, July and August, two to four showers per day.

Number of Showers Per Day	Occurrence of Shower	Is Ho accepted for Fourier	or Mixed Beta?
2	1st	Yes	Yes
	2nd	Yes	Yes
3	1st	Yes	Yes
	2nd	Yes	Yes
	3rd	Yes	Yes
4	1st	Yes	Yes
	2nd	Yes	Yes
	3rd	Yes	Yes
	4th	Yes	Yes

Table 4.16. Goodness of fit of the Fourier and Mixed Beta distributions to Order Statistics. McCredie, May and June, two to four showers per day.

Number of Showers Per Day	Occurrence of Shower	Is Ho accepted for Fourier or	Mixed Beta?
2	1st	Yes	Yes
	2nd	Yes	Yes
3	1st	Yes	No
	2nd	Yes	Yes
	3rd	Yes	Yes
4	1st	Yes	Yes
	2nd	Yes	Yes
	3rd	Yes	Yes
	4th	Yes	No

Table 4.17. Goodness of fit of the Fourier and Mixed Beta distributions to Order Statistics. McCredie, May and June, five to six showers per day.

Number of Showers Per Day	Occurrence of Shower	Is Ho accepted for Fourier or	Mixed Beta?
5	1st	Yes	Yes
	2nd	Yes	Yes
	3rd	Yes	Yes
	4th	Yes	Yes
	5th	No	Yes
6	1st	No	No
	2nd	Yes	Yes
	3rd	Yes	Yes
	4th	Yes	Yes
	5th	No	Yes
	6th	No	No

Table 4.18. Goodness of fit of the Fourier and Mixed Beta distributions to Order Statistics. McCredie, July and August, two to four showers per day.

Number of Showers Per Day	Occurrence of Shower	Is Ho accepted for Fourier or	Mixed Beta?
2	1st	Yes	Yes
	2nd	No	No
3	1st	Yes	No
	2nd	Yes	Yes
	3rd	No	No
4	1st	No	Yes
	2nd	Yes	Yes
	3rd	Yes	No
	4th	Yes	No

Table 4.19. Goodness of fit of the Fourier and Mixed Beta distributions to Order Statistics. McCredie, July and August, five to six showers per day.

Number of Showers Per Day	Occurrence of Shower	Is Ho accepted for Fourier or	Mixed Beta?
5	1st	No	No
	2nd	Yes	Yes
	3rd	Yes	Yes
	4th	No	Yes
	5th	No	No
6	1st	Yes	Yes
	2nd	Yes	Yes
	3rd	Yes	Yes
	4th	Yes	Yes
	5th	Yes	Yes
	6th	No	Yes

includes Hastings, fell between 8:00 p.m. and 8:00 a.m. 52 percent of the warm season rainfall at McCredie fell during the same time period. Figures 4.6 - 4.9 are illustrations of the diurnal variation in shower depths at Hastings and McCredie. In the figures, the two hour mean is the mean depth of all the showers which started in that two hour time period. The two hour time periods were 0 - 2, 2 - 4, and so on. The mean shower depth is the mean of all showers. Inspection of all four graphs reveals a midday period where all the two hour shower depths are less than the mean shower depth, and a time period approximately between 1600 (military time) and 400 where the two hour mean shower depth is greater than the mean shower depth. The Hastings data, especially the May and June data have the most pronounced diurnal variation.

To reflect the relation between depth and starting time, the shower starting times were conditioned upon depth. However, only one shower per day will be described in this manner because the statistical description of more than one event per day becomes intractable. Also, the technique used to describe the starting times and depths for two or more showers in a day, partially describes the diurnal fluctuation. For example, if a diurnal fluctuation is present on the days with two showers, with the later showers exhibiting larger rainfall depths than the earlier showers, R1 - the ratio of the depth of the first shower to the daily depth - would reflect this situation.

The conditional distribution of shower starting time given depth can be found by determining the joint distribution of shower

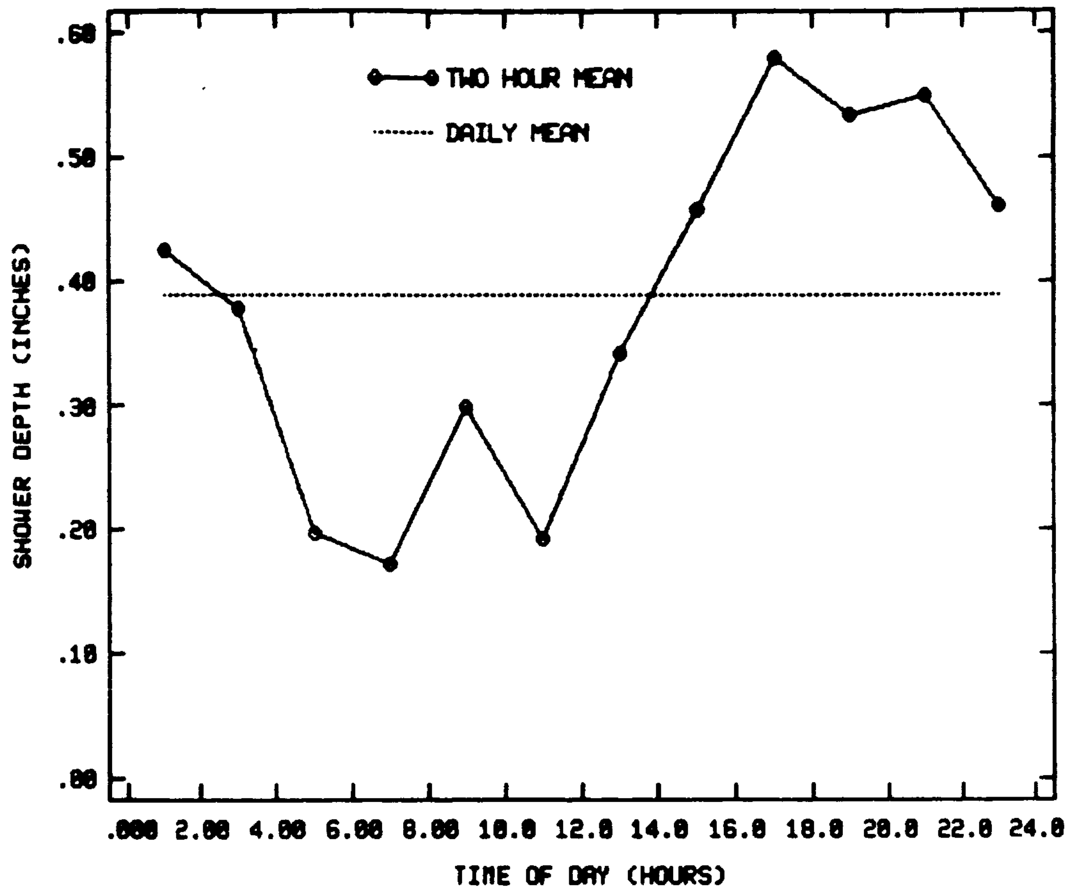

Figure 4.6. Diurnal Variation of Shower Depth at Hastings, May and June

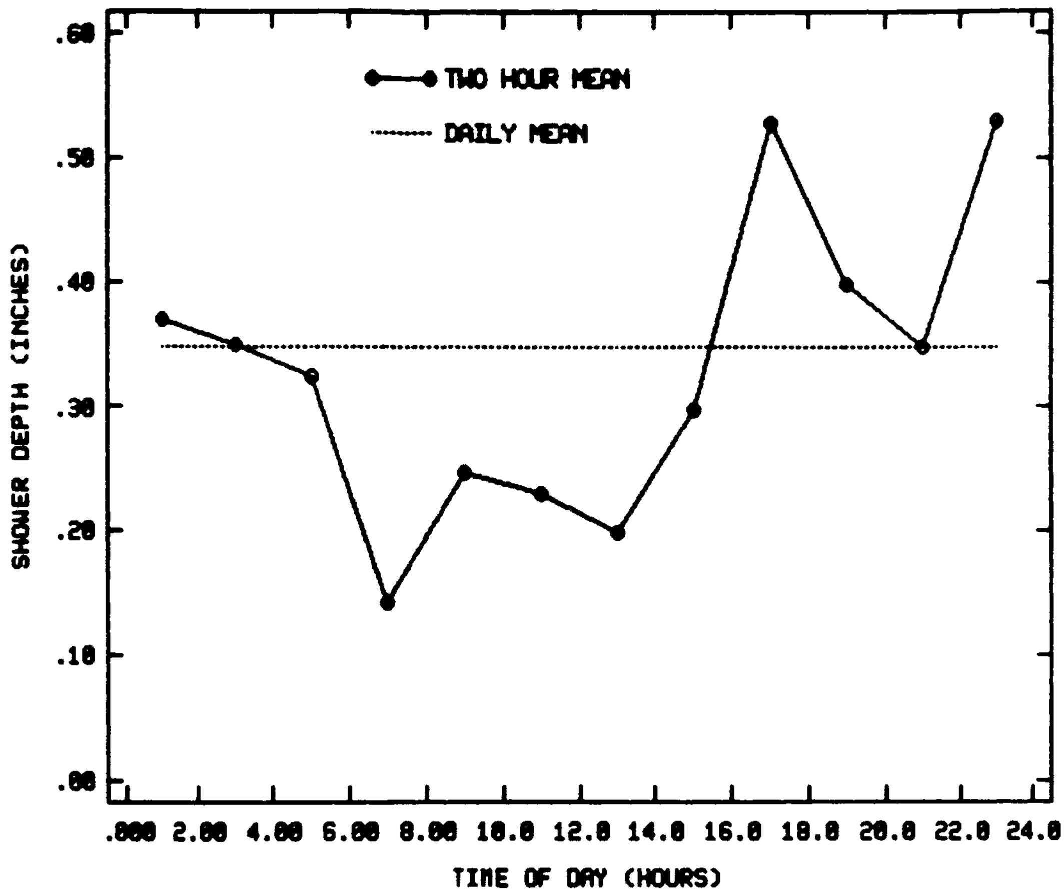

Figure 4.7. Diurnal Variation of Shower Depth at Hastings, July and August

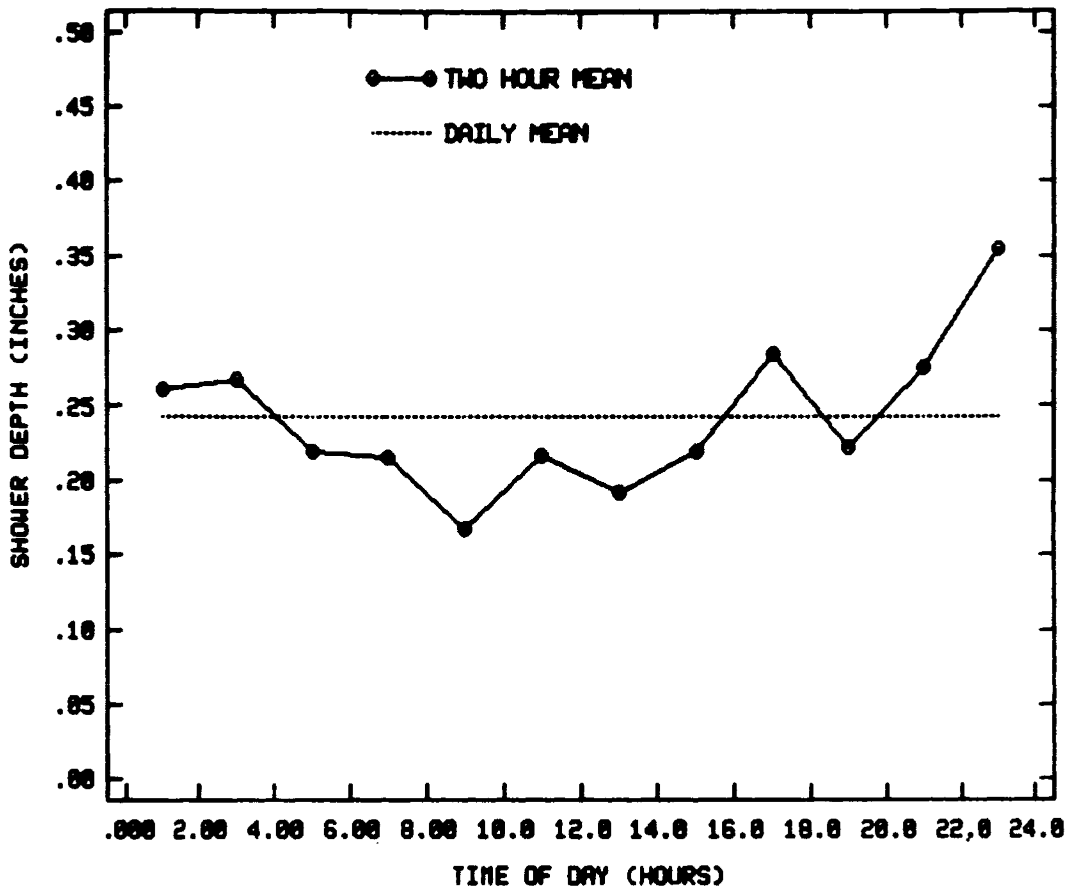

Figure 4.8. Diurnal Variation of Shower Depth at McCredie, May and June

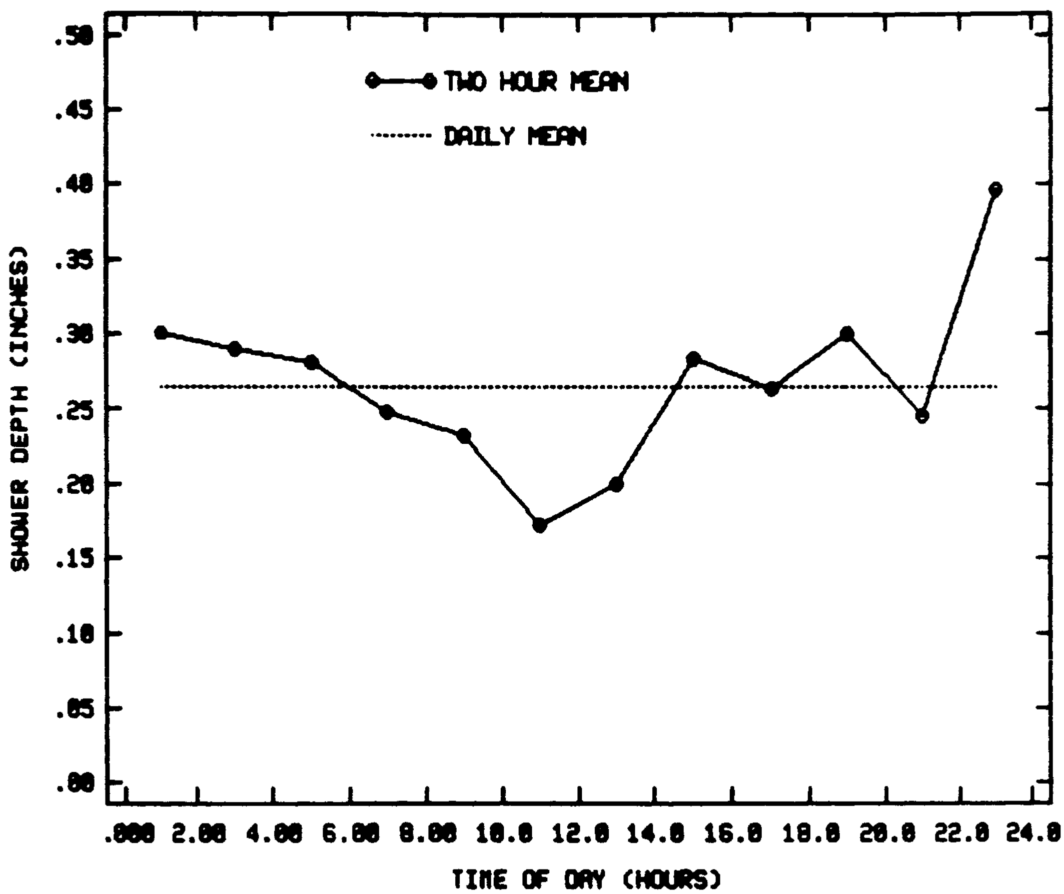

Figure 4.9. Diurnal Variation of Shower Depth at McCredie, July and August

starting time and depth and then solving for the conditional distribution of shower starting time given depth. The joint distribution of storm depth and time of day may be written as:

$$f_{Y,T_c}(y,t) = f_{Y|T_c}(y|t)\ g_{T_c}(t) \tag{4.23}$$

where:

$f_{Y|T_c}(y|t)$ is the conditional distribution of depth given time of day.

$g_{T_c}(t)$ the marginal distribution of starting times - the Fourier distribution.

The marginal distribution for depth may be obtained by integrating the joint distribution over time:

$$g_Y(y) = \int g_{T_c}(u)\ f_{Y|T_c}(y|u)du \tag{4.24}$$

By Bayes' theorem the conditional distribution of time given depth y given is :

$$f_{T_c|Y}(t|y) = [g_{T_c}(t)\ f_{Y|T_c}(y|t)]\ /\ g_Y(y) \tag{4.25}$$

The cumulative probability of the conditional time given depth is:

$$P(T \leq t \mid Y=y) = \int_0^y f_{T_c|Y}(u|y)\,du \tag{4.26}$$

Preliminary investigations revealed that the Exponential distribution may be used to describe the distribution of storm depths within two hour intervals. Therefore, an Exponential distribution with a time varying parameter was chosen for the conditional distribution of depth of a shower given time of day. The conditional distribution may be written as:

$$f_{Y|T_c}(y|t) = \lambda(t)^{-1}\exp(-y|\lambda(t)) \tag{4.27}$$

where the mean $\lambda(t)$, is represented by a Fourier series with two terms:

$$\lambda(t) = \tau + \alpha_1 \mathrm{COS}(2\pi t + \gamma_1) + \alpha_2 \mathrm{COS}(4\pi t + \gamma_2) \tag{4.28}$$

The parameters τ, α_1, α_2 γ_1 and γ_2 are obtained by numerical optimization using maximum likelihood techniques. To improve the fit of this distribution to the data set and to reduce the number of showers to be simulated, only showers greater than 0.10 of an inch were included. The resultant parameter values are in table 4.20.

The Chi-Square goodness of fit test was used to determine whether the theoretical joint distribution was significantly different from the historical distribution of shower depths and starting times.

Table 4.20. Parameter values for $\lambda(t)$[1].

Station	Period	Parameters τ	α_1	γ_1	α_2	γ_2
Hastings	May and June	.3765	.1122	1.233	.0881	.0441
	July and August	.3585	.1441	1.177	*	*
McCredie	May and June	.2808	.0673	.4221	*	*
	July and August	.2888	.0659	-.4976	*	*

* Not significant according to AIC.

[1] $\lambda(t) = \tau + \alpha_1\cos(2\pi x + \gamma_1) + \alpha_2\cos(4\pi x + \gamma_2)$

At the one percent significance level, the only data set that was considered significantly different is the July and August data set from Hastings. Inspection of the cumulative plot of starting times (figure 4.10) for the one per day showers reveals a high frequency of points at 6:00 am. The occurrence of this many points at one time probably is an artifact due to the techniques used to convert analog rainfall data into a digital form. The number of points at 6:00 am makes it difficult to fit a continuous distribution to this data.

<u>Joint Distribution of Shower Duration and Amount</u>

Shower durations were determined from the corresponding shower depth. The joint distribution of shower duration and amount may be written as:

$$h_{Y,D}(y,d) = g_{D|Y}(d|y)\, f_{Y}(y) \tag{4.29}$$

Where d is the shower duration and y is the shower depth minus a threshold amount. The shower depth is obtained from the shower ratio technique.

Rojiani et al. (1985) used a method based on least square regression to predict storm duration given storm depth, and described the spread about a regression line by the normal distribution with a mean of zero and standard deviation equal to the standard error of estimate. Grace and Eagleson (1966) used a similar approach, but their approach differs in that depth is predicted from duration and the showers were classified into three different types based on duration

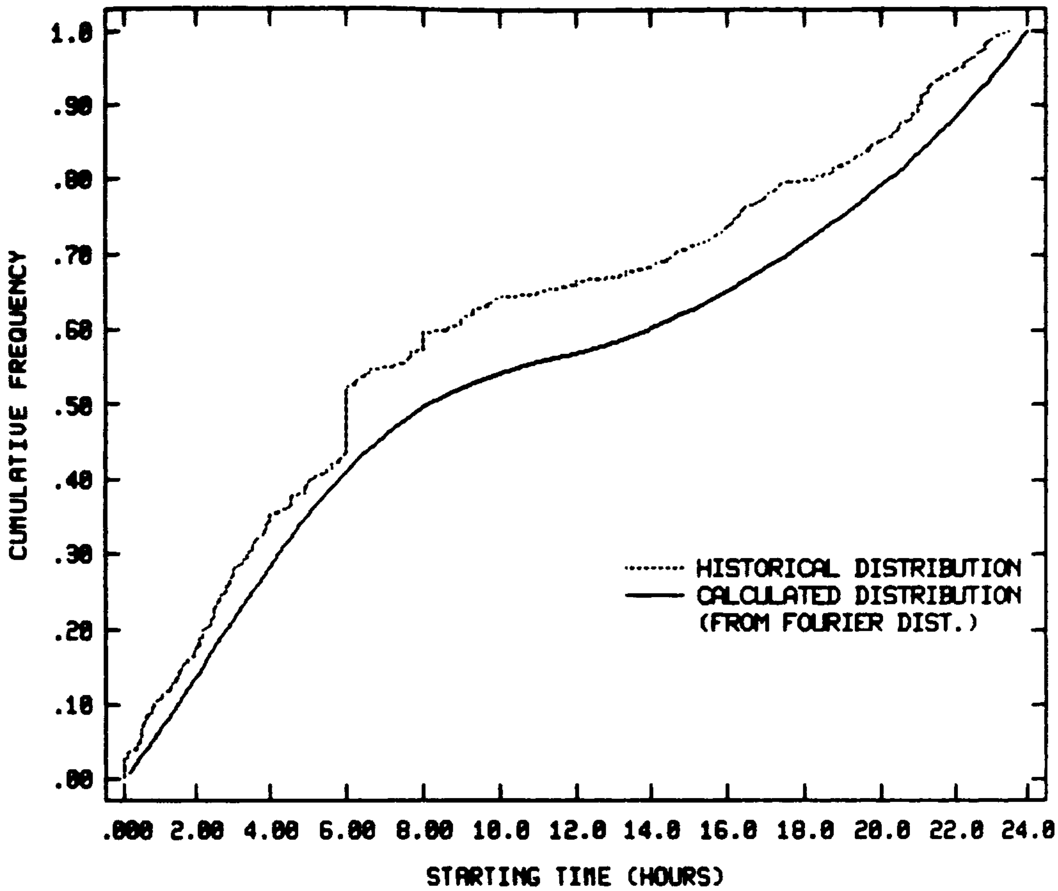

Figure 4.10. Historical Distribution of Time of Occurrence for the One Per Day Complete Showers

and depth. They described the spread about the regression line by fitting a Beta distribution to to the residual per cent. The percent residual can be written as:

$$\%H = (\text{depth residual})/(\text{depth from regression equation}). \qquad (4.30)$$

The approach used in this study was to use a regression line to predict duration from depth, and to use the normal distribution with mean zero and the standard deviation equal to the standard error of estimate as the predictor of spread about the regression line. The first step in this approach was to find the functional relationship that resulted in the greatest linear dependency between depth and duration. For the Walnut Gulch data the best linear relationship occurred when the depth and duration were transformed to their natural logs (Hershenhorn, 1984). In this study the following functional relationships were studied to determine their linearity: duration vs. depth; natural log of duration vs. depth; natural log of duration vs. natural log of depth; natural log of duration vs. square root of depth; and natural log of duration vs. cubed root of depth. The second step was to test the data sets to determine if the depth-duration relationship was indeed linear. The final step was to check the spread of the residuals about the prediction line to insure that they were normally distributed with a mean of zero and a standard deviation equal to the standard error of estimate. The residuals for the Walnut Gulch data tested as such (Hershenhorn, 1984). The complete and partial showers were analyzed separately.

Complete Showers

For three out of the four data sets, the functional relationship which resulted in the largest coefficient of determination for the complete showers with a threshold equal to 0.009 inches, was the natural log of duration versus depth to the one-third power. The highest coefficient of determination for the Hastings, July and August data set resulted when both the depths and durations were transformed to their natural logs.

The natural log of duration-depth to the one-third power functional relationship was tested for linearity and the residuals were tested for normality. The hypothesis of linearity of the functional relationship was tested by using a correlation ratio test (Kendall and Stuart, 1979). All four data sets passed the hypothesis of linearity. Significance testing was performed on the four data sets to determine if the residuals about the regression were from a normal distribution with a mean of zero and a standard deviation equal to the standard error of estimate. Visual inspection of the residuals revealed that the residuals appeared to decrease in magnitude with increased depth. To determine whether the standard deviation of the residuals was constant the Chi-square test was performed on four subclasses of each data set (16 subclasses in total). The subclasses were set up so each subclass had approximately the same number of observations. The hypothesis that the residuals came from a normal distribution with a mean equal to zero and a standard deviation equal to the standard error of estimate could not be accepted by the Chi-Square test, at the one percent significance level, for 8 of the 16 subclasses. When the residuals were not subdivided into subclasses and tested for normality over the entire four data sets, individually, just the residuals from the Hastings and McCredie, July and August data sets could be considered normally distributed, at the one percent significance level.

The threshold was increased to 0.099 inches, to reduce the number of data points, and to facilitate the fit of the regression line. In future simulations, showers of less than 0.10 inches will be treated in a simpler fashion than showers greater than or equal to 0.10 inches. The highest coefficient of determination for the McCredie, May and June and the Hastings, July and August data sets resulted when the durations were transformed to their natural logs and the depths were not transformed. The remaining data sets highest coefficient of determination resulted when the duration was transformed to its natural log and the depth was transformed to its square root.

The hypothesis of linearity was tested for both of the functional relationships described in the previous paragraph. When all of the four data sets' durations were transformed to their natural logs and the depths were not transformed, all four transformed data sets passed the linearity test, at the five percent level. When the data sets durations' were transformed to their natural logs and the depths were transformed to their square roots, three of the data sets passed the test at the five percent level, and the Hastings, May and June data passed at the one percent level.

Testing was performed to determine if the residuals from the regressions were from a normal distribution with a mean of zero and a standard deviation equal to the standard error of estimate. Each of the four data sets were divided into four approximately equal subclasses, and the residuals in each subclass were tested for normality. Four of the total of sixteen subclasses' residuals could not be considered from a normal distribution, at the one percent level, for the log duration vs. depth relationship. At the one percent level, four of the sixteen subclasses' residuals could not be considered from a normal distribution for the log duration vs square root of depth relationship. The residuals for all of the four midwestern data were also tested for normality without breaking up the data sets into subclasses. At the one percent level, just the McCredie July and August data set's residuals could not be considered as from a normal distribution for the log duration vs depth relationship. Only the Hastings, May and June residuals did not pass the significance testing when the log duration vs. square root of depth relationship was examined, at the same significance level.

The log duration vs. depth relationship was chosen for use in simulations. This relationship requires one less transformation than that of the log duration vs. square root of depth relationship. The intercept, slope and standard deviation for this relationship are tabulated in table 4.21.

Table 4.21. Regression statistics for ln(duration) vs. depth relationship for complete and partial showers[1].

Station	Period	Complete Showers Y-Inter.	Complete Showers Slope	Complete Showers SEE	Partial Showers Y-Inter.	Partial Showers Slope	Partial Showers SEE
Hastings	May and June	.3656	.6208	1.052	.1774	.6622	1.230
	July and August	.4098	.3158	1.056	-.1734	.9305	1.159
McCredie	May and June	.04183	.8111	.9468	.3087	.7524	1.081
	July and August	-.1183	.9251	.9643	.1841	.4650	1.228

[1]Threshold for the complete showers is 0.099 inches, and the threshold for the partial showers is 0.009 inches.

Partial Showers

The partial showers were also studied to determine which transformations resulted in the highest coefficient of determination. The threshold for the partials was equal to 0.009 inches. Two of the four midwestern data sets had the highest coefficient of determination when both the durations and depths were transformed to their natural logs. If transformed in this manner all four of the data sets passed the linearity test, at the five percent level. However, when the residuals from the entire data sets were tested for normality using the Chi-Square goodness of fit test, the Hastings, July and August data did not pass at the one percent level. The log duration, depth transformation used for the complete data sets was tested for linearity and normality, with the partial data sets. At the five percent level, the data sets transformed in this manner passed all the tests. The log duration vs. depth transformation was chosen for the partial showers as well as the complete showers, to keep the overall simulation model as simple as possible and to keep the number of calculations to a minimum.

The log of duration vs. depth relationship resulted in the highest degree of normality and linearity for both the complete and partial showers. The slope, intercept, and standard error of estimate for the partial showers transformed in this manner are in table 4.21.

CHAPTER 5

SIMULATION

Figure 5.1 is a simplified flow chart of the daily disaggregation simulation. Some modifications were made in the model developed by Hershenhorn (1984). The most significant modification was in the simulation of starting times for the one per day complete showers, which are conditioned upon depth in this study. The second modification was the replacement of the Mixed Beta distribution with the Fourier distribution. The Fourier distribution should decrease simulation time and increase model accuracy because it can be explicitly integrated. The functional relationship between depth and duration was changed from the ln duration vs. ln depth to ln duration vs. depth. Durations of complete showers less than 0.10 inches were just set equal to the mean duration of the showers from this class and not simulated in the same manner as all the other shower durations. Hershenhorn's (1984) model did not allow the duration of a shower to exceed 8 hours. The model now allows showers up to 24 hours in duration, however, if the sum of the durations of two or more showers exceed 24 hours, the showers are reduced in duration proportional to their original duration.

One Per Day Showers

An improvement made upon the previous disaggregation model is the incorporation of the diurnal variation of depths for days on which

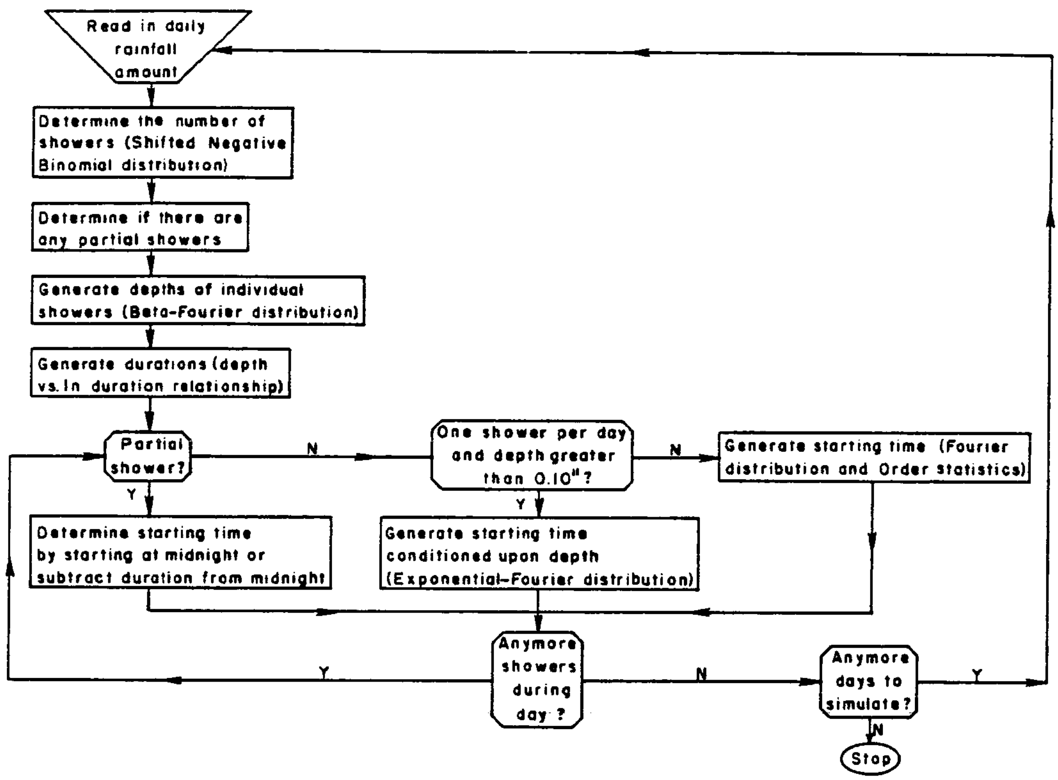

Figure 5.1. Simplified Flowchart of Disaggregation Model

only one complete shower occurs. The one per day shower's depth is identical to the daily rainfall amount. The CDF of the conditional distribution of time of occurrence and depth may be written as:

$$F_{T|Y}(t|y) = \int_{o}^{t} g_T(t)\ f_{Y|T}(y|t)/g_Y(y)dt \tag{5.1}$$

since g(y) is constant with respect to time equation 5.1 may be rearranged to:

$$F_{T|Y}(t|y)g_Y(y) = \int_{o}^{t} g_T(t)\ f_{Y|T}(y|t)\ dt \tag{5.2}$$

$g_Y(y)$ may be calculated by:

$$g_Y(y) = \int_{o}^{24} g_T(t)\ f_{Y|T}(y|t)\quad dt \tag{5.3}$$

The expression for $g_T(t)$ is the Fourier distribution (equation 4.16) and the distribution for the conditional distribution is the Exponential-Fourier (equation 4.27).

The inverse transformation procedure is used to generate time described by the joint distribution. The steps are as follows:

1. Generate a uniform random variable distributed between 0 and 1. Substitute its value for $F_{T|Y}(t|y)$ into equation 5.2.

2. Because the expression for $g_Y(y)$ can not be analytically integrated equation 5.3 is numerically integrated. The rainfall depth, is known. The numerical integration technique divides the function up into enough subintervals that the difference in the value of the integrated function for N subintervals and the function value for 2N sub-intervals is less than a specified tolerance.
3. Solve the right hand side of equation (5.2) with respect to time. To speed up the simulation, each integration value for the subintervals are stored in a table, along with the corresponding time during step 2. All that remains is to find where the value of the left hand side fits in the table and find the corresponding time.

Simulation at Hastings and McCredie

The daily disaggregation model as developed by Hershenhorn (1984) with modifications (as described above) was used to simulate the intermittent rainfall process at Hastings and McCredie for the May and June, and July and August periods. Various simulated distributions were statistically compared with the historical distributions from the stations. Statistics were calculated from the simulated data and were compared with the statistics from the historical data. In tables 5.1-5.4 the statistics skew for the shower depths, durations (just mean and standard deviation calculated), starting times, and number of occurrences of the historical and simulated data are listed.

Table 5.1. Statistics of historical and simulated depths[1] from Hastings and McCredie.

Station	Period	Type of Data	Mean (in.)	Stand. Dev. (in.)	Coeff. of Skew
Hastings	May and June	Historical	.350	.495	3.61
		Simulated	.382	.493	3.39
	July and August	Historical	.323	.441	3.21
		Simulated	.330	.449	3.55
McCredie	May and June	Historical	.230	.302	2.69
		Simulated	.233	.303	2.92
	July and August	Historical	.254	.347	3.08
		Simulated	.267	.345	2.82

[1]All depths (complete and partials).

Table 5.2. Statistics of historical and simulated durations[1] from Hastings and McCredie[2].

Station	Period	Type of Data	Mean (min.)	Stand. Dev. (min.)
Hastings	May and June	Historical	181.3	193.8
		Simulated	184.4	233.8
	July and August	Historical	153.9	145.1
		Simulated	171.0	226.9
McCredie	May and June	Historical	131.7	141.4
		Simulated	136.2	171.7
	July and August	Historical	117.5	112.5
		Simulated	124.2	166.5

[1] 11 partial durations and complete durations from complete showers with 0.10 inches of rainfall or more.

[2] Ten repetitions.

Table 5.3. Statistics of historical and simulated starting times[1] of showers from Hastings and McCredie.

Station	Period	Type of Data	Mean (hrs.)	Stand. Dev. (hrs.)	Coeff. of Skew
Hastings	May and June	Historical	11.40	7.80	.161
		Simulated	11.63	7.72	.139
	July and August	Historical	10.74	8.00	.272
		Simulated	9.73	7.45	.492
McCredie	May and June	Historical	11.73	7.02	.025
		Simulated	11.99	6.98	.003
	July and August	Historical	10.69	6.76	.266
		Simulated	11.15	6.85	.154

[1]All complete shower starting times and those partials which occur before midnight.

Table 5.4. Statistics of historical and simulated number of daily shower occurrences for Hastings and McCredie.

Station	Period	Type of Data	Mean	Stand. Dev.	Coeff. of Skew
Hastings	May and June	Historical	1.30	.603	2.25
		Simulated	1.19	.490	2.86
	July and August	Historical	1.24	.604	2.80
		Simulated	1.22	.538	3.10
McCredie	May and June	Historical	1.87	1.34	2.53
		Simulated	1.84	1.20	1.70
	July and August	Historical	1.69	1.09	1.89
		Simulated	1.60	.958	2.07

All (complete and partial) shower depths are used in the comparisons. The durations from all the partial showers and all the complete showers with depths greater than 0.099 inches are used to compared simulated versus historical durations. No attempt was made to simulate durations from complete showers with less than 0.10 inches of rainfall, therefore, they were excluded from the comparisons. Since the standard deviation of the shower durations were so large the mean and standard deviation were calculated over ten repetitions of simulations, each repetition simulated the intermittant rainfall process for the period of record of the daily rainfall. The starting times for the partial showers which occur at midnight were excluded from the comparisons as well.

A two-sample KS goodness of fit statistic was used to test the null hypothesis (Ho) that the simulated distributions of shower starting times, depths and durations came from the same parent distribution as the corresponding historical distributions. For the distribution of the number of showers within a day, Ho was tested with the use of the Chi-Square statistic. In table 5.5, the levels at which Ho cannot be rejected for each distribution at the two locations are listed.

The Hasting's simulations did well, Ho, at the five percent level, could not be rejected for all of the comparisons between the distributions, with the one exception of the comparison between the distributions of the number of showers during May and June. However, Ho could not be rejected at the one percent level, for this comparison. The McCredie simulations did not do as well, with the

Table 5.5. Results of statistical tests between the distributions of historical and simulated showers at Hastings and McCredie.

		Level which Ho[1] is not rejected for distributions of:			
Station	Time Period	Starting Times[2]	Depths[2]	Durations[2]	No. of Showers Per Day[3]
Hastings	May and June	.05	.05	.05	.01
	July and August	.05	.05	.05	.05
McCredie	May and June	.05	.01	.05	.01
	July and August	.05	.01	.01	.05

[1]Ho is the hypothesis that the simulated and historical distributions come from the same parent distribution.

[2]Two-sample KS test was used.

[3]Chi-Square test was used.

exception of the simulations for the starting times, for which at the five percent level, Ho could not be rejected for either of the time periods. Ho from the comparisons of the distributions of shower depths for both of the time periods was rejected at the five percent level, but not at the one percent level. The May and June simulated mean depth was within 0.003 inches of the historical depth, the July and August's simulated depth was 0.013 inches greater. The coefficients of skew, and standard deviations for the historical and simulated data were very similar. The comparisons of the distributions of duration were mixed with Ho being accepted, at the five percent level, for the May and June record, but at the one percent level, for July and August. The simulated means of the durations were close to the historical mean, however, the standard deviations were considerably different. Because of the transformation involved in the natural log vs. depth relationship the historical statistics of the duration are not necessarily maintained. The comparisons between the distributions of the number of showers per day, resulted with Ho being accepted at the five percent level, for the May and June record, but only at the one percent level, for July and August. The simulated mean number of showers per day were slightly less than the historical mean.

Simulations for both locations and periods resulted in fewer simulated showers (complete and partial) than were historically present (table 5.6). The percent differences range from 8.1 less for Hastings, May and June to 1.8 percent less for McCredie, July and August.

Table 5.6. Total number of simulated and historical showers.

Station	Period	Type of Data	Total No. of Shwrs	Percent Difference[1]
Hastings	May and June	Historical	669	-8.1
		Simulated	615	
	July and August	Historical	504	-3.0
		Simulated	489	
McCredie	May and June	Historical	1236	-1.8
		Simulated	1213	
	July and August	Historical	839	-5.4
		Simulated	794	

[1]Percent difference = (Simulated - Historical)/Historical x 100

The ratios of the number of partial showers to the total number of complete and partial showers ratios ranged from being five percent larger than the historical ratios at Hastings, May and June to three percent less than the historical ratios at Hastings, July and August.

The duration of time between showers is important in time-varying infiltration models because it determines antecedent moisture conditions. The duration between showers is not explicitly accounted for by the simulation model. Factors in the simulation model which contribute to the duration between showers are the simulation of number of showers per day, shower starting times and durations. The historical and simulated statistics for the the duration between all showers are listed in table 5.7. Inspection of table 5.7 reveals that the statistics for the simulated and historical data are in close agreement. Statistics for the duration of times between showers over 0.10 inch were also calculated. In addition to all the components which contributed to the simulation of the duration between all showers, the simulation of shower depths in also an important contributor. Examination of table 5.8, reveals close agreement in the statistics from the historical and simulated data sets.

Simulation of One Per Day Showers

A Chi-Square test as described by Little and Hills (1978) (Appendix 1) was used to test the simulation of the one per day shower starting times which were conditioned upon shower depth. The hypothesis (Ho) tested was whether the simulated joint distribution of

Table 5.7. Statistics of historical and simulated time between consecutive showers, threshold equals 0.009 inches, at Hastings and McCredie.

Station	Period	Type of Data	Mean (hrs.)	Stand. Dev. (hrs.)	Coeff. of Skew
Hastings	May and June	Historical	60.32	73.92	2.16
		Simulated	65.28	74.39	2.13
	July and August	Historical	81.28	89.70	1.64
		Simulated	86.18	89.22	1.69
McCredie	May and June	Historical	35.31	58.98	2.85
		Simulated	36.72	58.79	2.82
	July and August	Historical	52.79	79.75	2.07
		Simulated	57.15	81.48	2.01

Table 5.8. Statistics of historical and simulated time between consecutive showers, threshold equals 0.10 inches, at Hastings and McCredie.

Station	Period	Type of Data	Mean (hrs.)	Stand. Dev. (hrs.)	Coeff. of Skew
Hastings	May and June	Historical	87.63	95.42	1.75
		Simulated	87.58	94.04	1.73
	July and August	Historical	114.4	122.0	1.89
		Simulated	118.5	121.2	1.85
McCredie	May and June	Historical	65.44	89.29	2.54
		Simulated	62.05	86.91	2.67
	July and August	Historical	88.92	114.3	2.20
		Simulated	88.58	117.1	2.35

Table 5.9. Chi-Square statistics (test of proportionality) for simulated one per day showers.

		χ^2 Statistic[1]	
Station	Period	Conditioned One Per Day	Not Conditioned One Per Day
Hastings	May and June	16.58	7.58
	July and August	5.17	7.84
McCredie	May and June	6.61	10.79
	July and August	6.32	6.67

[1]8 degrees of freedom.

$\chi^2_{.05} = 15.507$; $\chi^2_{.01} = 20.09$

starting time and depth could be considered as from the same parent distribution as the historical joint distribution. All one per day complete showers (historical and simulated) greater than 0.10 inch were grouped into classes according to starting time and depth of shower. The results of the tests are listed in table 5.9. The only data set for which Ho was rejected was Hastings, May and June.

Simulations were also done in which the starting time of the one per day showers were not conditioned upon depth. The same statistical testing as described above was used to test Ho. For all the data sets Ho could not be rejected at the five percent level. However, for three of the data sets, all except Hastings (May and June), the test statistic for this comparison was greater than the test statistics for the comparison which included the conditioned starting times. This indicates that the conditioned showers better described the historical starting times.

CHAPTER 6

TRANSFERABILITY OF MODEL PARAMETERS

To be useful the parameter values for the disaggregation model should be applicable over climatologically similar areas. The transferability of the model parameters from Hastings and McCredie to other locations in the midwest will be discussed in this chapter.

To determine whether the parameters developed from the Hastings, and McCredie data could be used in other locations in the midwest, daily precipitation data from three locations were disaggregated into showers using the Hasting and McCredie parameter values (obtained earlier in the study). The locations used were Fennimore, Wisconsin, Treyor, Iowa, and Monticello, Illinois. These locations will be referred to as the test locations, whereas McCredie and Hastings will be referred to as the primary locations.

In table 6.1, the period of record, and precipitation statistics are summarized for each test location. The distance from each test stations to the primary station are listed in table 6.2. The monthly profiles of precipitation are illustrated in figure 6.1. All the test locations appear to be in the same precipitation subtype as McCredie (figure 2.2). Monticello's precipitation is uniform during the months of May through August, with the peak monthly rainfall occurring during July. Monticello may be a transitional station between subtype 3d and subtype 4 (Ohio Valley Region). The flatter precipitation profile at Monticello is characteristic of

Table 6.1. Selected Precipitation Statistics for the Test Stations.

Station	Period of Record	Mean Annual Precip (in.)	Monthly Precipitation (in.)			
			May	June	July	August
Fennimore	1939-1968	32.05	3.70	4.92	4.17	3.75
Monticello	1941-1974	32.25	3.35	3.83	3.99	3.47
Treynor	1964-1977	31.94	4.82	4.63	3.58	3.71

Table 6.2. Distances from test stations to primary stations.

Station	Distance in miles to Hastings	McCredie
Fennimore	420	292
Monticello	480	200
Treynor	280	132

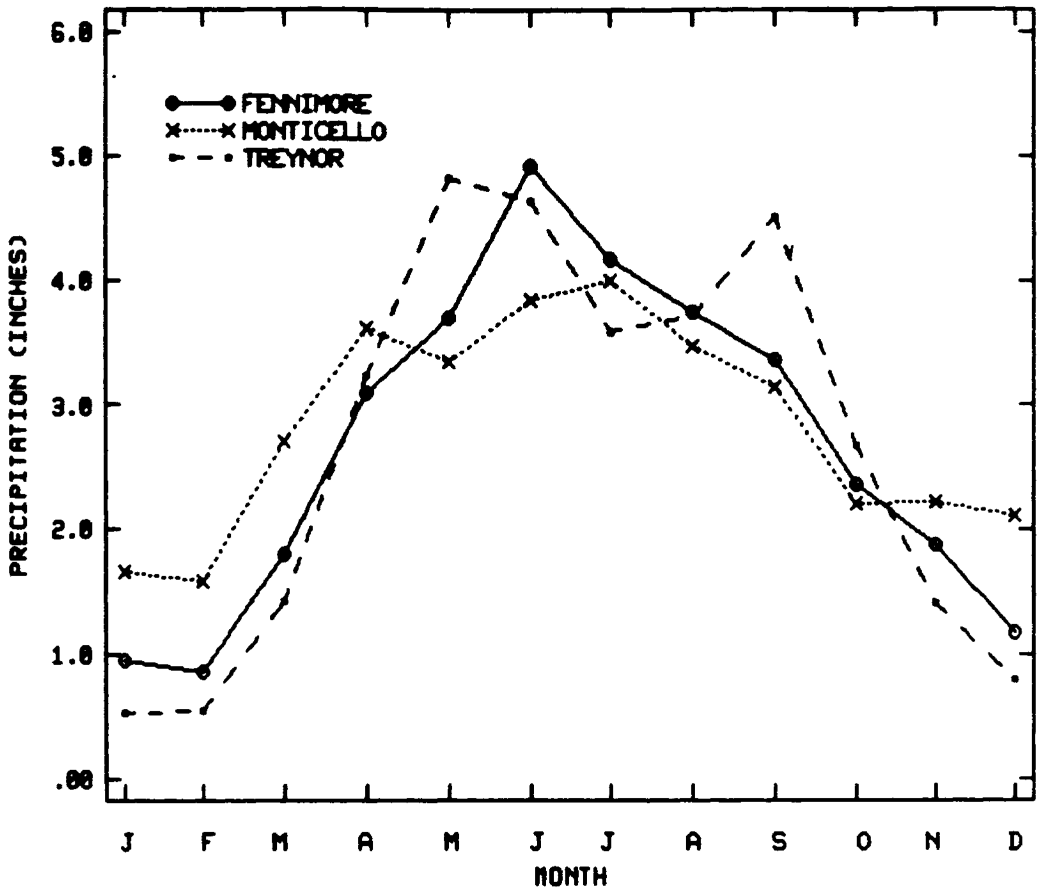

Figure 6.1. Mean Monthly Precipitation at the Test Stations

subtype 4, and it's limited cool season precipitation is a characteristic of subtype 3d. Treynor exhibits the double peak profile, similar to McCredie's. The primary maximum at Treynor is during the month of May and the secondary maximum occurring in September. The double peaked profile at Treynor along with the mean annual precipitation of 31.9 inches places Treynor in the 3d subtype. Fennimore has a steep profile, with the month of June having the maximum monthly rainfall. The lack of a twinned peak profile does not make Fennimore an anomaly within the 3d subtype, Trewartha (1981) emphasizes that even within the 3d core region not all of the stations exhibit a twinned summer maximum.

Comparison of Historical Data at the Test and Primary Stations

In order to determine whether the test and primary stations had similar shower rainfall statistics, comparisons of various precipitation statistics from the test and primary stations were made. The distributions of shower duration, shower depth, starting time of a shower, and shower occurrence were tested for similarity. The means, standard deviations and coefficient of skew were calculated for shower duration, shower depth, and starting time of a shower for the test locations, and are listed in tables 6.3-6.14.

The historical CDF's of duration, depth, and starting time, at the test locations were compared to the corresponding historical CDF at each of the primary locations. The two-sample KS statistic was used to test the hypothesis that the historical and simulated CDF's came from the same parent distribution (Ho). The distributions of number

Table 6.3. Statistics of historical and simulated shower depths[1] at Fennimore.

Period	Historical Data or Parameter Set Used in Simulation	Mean (in.)	Stand. Dev. (in.)	Coeff. of Skew
May and June	Historical	.216	.316	3.38
	Hastings	.334	.400	3.17
	McCredie	.237	.306	3.29
July and August	Historical	.276	.455	4.20
	Hastings	.382	.520	2.88
	McCredie	.294	.408	3.28

[1]All complete and partial showers.

Table 6.4. Statistics of historical and simulated shower depths[1] at Monticello.

Period	Historical Data or Parameter Set Used in Simulation	Mean (in.)	Stand. Dev. (in.)	Coeff. of Skew
May and June	Historical	.246	.328	2.64
	Hastings	.372	.427	2.17
	McCredie	.247	.315	2.98
July and August	Historical	.323	.432	2.50
	Hastings	.417	.488	2.41
	McCredie	.307	.408	3.53

[1]All complete and partial showers.

Table 6.5. Statistics of historical and simulated shower depths[1] at Treynor.

Period	Historical Data or Parameter Set Used in Simulation	Mean (in.)	Stand. Dev. (in.)	Coeff. of Skew
May and June	Historical	.278	.434	3.54
	Hastings	.402	.517	2.51
	McCredie	.256	.375	4.14
July and August	Historical	.273	.414	3.65
	Hastings	.352	.448	2.85
	McCredie	.276	.381	3.44

[1]All complete and partial showers.

Table 6.6. Statistics of historical and simulated shower durations[1] at Fennimore[2].

Period	Historical Data or Parameter Set Used in Simulation	Mean (mins.)	Stand. Dev. (mins.)
May and June	Historical	124.0	118.3
	Hastings	177.4	221.6
	McCredie	137.3	173.9
July and August	Historical	114.9	101.6
	Hastings	170.2	221.4
	McCredie	134.0	187.8

[1]All partial showers and those complete showers with 0.10 inches or more of rainfall.

[2]Ten repetitions.

Table 6.7. Statistics of historical and simulated shower durations[1] at Monticello[2].

Period	Historical Data or Parameter Set Used in Simulation	Mean (mins.)	Stand. Dev.. (mins.)
May and June	Historical	138.6	143.6
	Hastings	182.2	229.6
	McCredie	134.6	168.4
July and August	Historical	125.3	129.2
	Hastings	172.3	220.6
	McCredie	127.1	172.0

[1]All partial showers and those complete showers with 0.10 inches or more of rainfall.

[2]Ten repetitions.

Table 6.8. Statistics of historical and simulated shower durations[1] at Treynor[2].

Period	Historical Data or Parameter Set Used in Simulation	Mean (mins.)	Stand. Dev. (mins.)
May and June	Historical	114.0	105.7
	Hastings	191.1	238.5
	McCredie	144.4	184.8
July and August	Historical	101.0	92.5
	Hastings	161.8	209.5
	McCredie	124.8	160.8

[1]All partial showers and those complete showers with 0.10 inches or more of rainfall.

[2]Ten repetitions.

Table 6.9. Statistics of historical and simulated shower starting times[1] at Fennimore

Period	Historical Data or Parameter Set Used in Simulation	Mean (hrs.)	Stand. Dev. (hrs.)	Coeff. of Skew
May and June	Historical	11.59	6.99	0.014
	Hastings	11.48	7.77	0.179
	McCredie	12.21	7.13	-0.045
July and August	Historical	11.23	7.10	0.080
	Hastings	10.72	7.84	0.257
	McCredie	10.75	7.09	0.229

[1]All complete showers and those partial showers which occur before midnight.

Table 6.10. Statistics of historical and simulated shower starting times[1] at Monticello.

Period	Historical Data or Parameter Set Used in Simulation	Mean (hrs.)	Stand. Dev. (hrs.)	Coeff. of Skew
May and June	Historical	11.53	7.04	0.004
	Hastings	11.82	7.66	0.114
	McCredie	11.82	7.15	-0.006
July and August	Historical	11.88	7.17	-0.005
	Hastings	10.84	7.88	0.232
	McCredie	10.76	6.77	0.234

[1]All complete showers and those partial showers which occur before midnight.

Table 6.11. Statistics of historical and simulated shower starting times[1] at Treynor.

Period	Historical Data or Parameter Set Used in Simulation	Mean (hrs.)	Stand. Dev. (hrs.)	Coeff. of Skew
May and June	Historical	11.37	7.57	0.085
	Hastings	11.31	7.92	0.095
	McCredie	11.74	7.24	0.038
July and August	Historical	11.03	7.64	0.183
	Hastings	11.57	8.13	0.132
	McCredie	11.30	6.91	0.230

[1]All complete showers and those partial showers which occur before midnight.

Table 6.12. Statistics of historical and simulated number of showers per day[1] at Fennimore.

Period	Historical Data or Parameter Set Used in Simulation	Mean	Stand. Dev.	Coeff. of Skew
May and June	Historical	1.91	1.25	1.62
	Hastings	1.23	.528	2.52
	McCredie	1.74	1.08	1.69
July and August	Historical	1.72	1.20	2.10
	Hastings	1.23	.576	3.48
	McCredie	1.60	.989	2.15

[1]Complete and partial showers.

Table 6.13. Statistics of historical and simulated number of showers per day[1] at Monticello.

Period	Historical Data or Parameter Set Used in Simulation	Mean	Stand. Dev.	Coeff. of Skew
May and June	Historical	1.83	1.18	2.01
	Hastings	1.21	.473	2.46
	McCredie	1.82	1.19	1.78
July and August	Historical	1.65	1.08	2.32
	Hastings	1.28	.676	3.62
	McCredie	1.74	1.12	1.83

[1]Complete and partial showers.

Table 6.14. Statistics of historical and simulated number of showers per day[1] at Treynor.

Period	Historical Data or Parameter Set Used in Simulation	Mean	Stand. Dev.	Coeff. of Skew
May and June	Historical	1.74	1.20	2.22
	Hastings	1.20	.524	3.35
	McCredie	1.88	1.26	1.55
July and August	Historical	1.58	.885	1.61
	Hastings	1.19	.526	3.77
	McCredie	1.52	.940	2.24

[1]Complete and partial showers.

of showers per day were compared with the Chi-Square statistic. The results of the significance tests are listed in tables 6.15 and 6.16.

The historical distributions at McCredie are fairly similar to the historical distributions at the test stations (table 6.16). The only comparison that rejected Ho, at the one percent level, was the comparison between the historical and simulated distributions of starting times at Monticello, July and August season.

The historical distributions at Hastings are almost completely different from those at the test stations (table 6.15). The simulated distributions of starting times at Treynor and Hastings, for both time periods, can be considered as from the same parent distributions as the historical data, at the five percent level. Also, the distributions of depths compare favorably, at the five percent level, at Monticello, July and August.

The depth statistics for the test locations (tables 6.3-6.5) during May and June correspond well with the depths at McCredie during the same period. Hasting's statistics of depth do not do as well for May and June, the mean depth and the standard deviation are generally greater, and the coefficient of skew are similar to those of the test stations. Examination of July and August's depth statistics reveals that the Hastings' statistics are slightly closer to the test station's statistics than are McCredie's.

The mean, skew, and May and June's standard deviation, of the test stations' durations (tables 6.6-6.8) are like those from the McCredie station. McCredie's July and August standard deviation is

Table 6.15. Results of statistical tests between the historical distributions from Hastings and the test stations.

		Level which Ho[1] is not rejected for distributions of:			
Station	Time Period	Starting Times[2]	Depths[2]	Durations[2]	No. of Showers Per Day[3]
Fennimore	May and June	*	*	*	*
	July and August	*	*	*	*
Monticello	May and June	*	*	*	*
	July and August	*	.05	*	*
Treynor	May and June	.05	*	*	*
	July and August	.05	*	*	*

* - not accepted at the one percent significance level.

[1]Ho is the hypothesis that the simulated and historical distributions come from the same parent distribution.

[2]Two-sample KS test was used.

[3]Chi-Square test was used.

Table 6.16. Results of statistical tests between the historical distributions from McCredie and the test stations.

Station	Time Period	Level which Ho[1] is not rejected for distributions of: Starting Times[2]	Depths[2]	Durations[2]	No. of Showers Per Day[3]
Fennimore	May and June	.05	.05	.05	.05
	July and August	.05	.05	.05	.05
Monticello	May and June	.05	.05	.05	.05
	July and August	*	.01	.05	.05
Treynor	May and June	.01	.05	.05	.05
	July and August	.01	.05	.05	.05

* - not accepted at the one percent significance level.

[1]Ho is the hypothesis that the simulated and historical distributions come from the same parent distribution.

[2]Two-sample KS test was used.

[3]Chi-Square test was used.

larger than all the test stations standard deviation. Only the coefficients of skew of the Hastings'shower durations are similar to the test stations'.

The test stations' May and June showers have shower starting time statistics (tables 6.9-6.11) which are similar to the showers from the Hastings and McCredie stations. Hastings' coefficient of skew is slightly larger indicating a mild skewness on the high side of the mean. The mean shower starting time at McCredie and Hastings, during the months of July and August, is earlier than those at the test stations. The standard deviation of the shower starting times are generally the same at the test and primary stations, the primary stations' starting times have larger coefficients of skew.

The mean number of showers per day at the test locations (tables 6.12-6.14) are close to the mean number at McCredie. The means at Treynor are the most different from McCredie's. Treynor has .13 less showers per day during the May and June period and .09 less showers per day during July and August. The mean number of showers at Hastings are not similar to the mean number at the test locations.

Comparison of Simulations

To determine if the parameter values from the primary locations were transferable, their parameter values were used to disaggregate the daily rainfall depths which were obtained at each of the test locations. Then similar comparisons as used for the historical distributions were made. However, the comparisons were between the historical distributions from the test location versus the simulated distribution which required that test location's daily rainfall record as input. The results of these comparisons are listed in tables 6.17 and 6.18. Statistics of duration, depth, starting times and occurrence were calculated from the simulated data (tables 6.3-6.14).

For most of the comparisons between the simulated distributions - using McCredie's parameter values - and the historical distributions, Ho, the null hypothesis that the two distributions came from the same parent distribution could not be rejected, at the five percent level. The comparisons of the distributions of depth for both time periods at Fennimore, failed at the five percent level. If all showers that were less than 0.10 inches are ignored then Ho could not be rejected. The simulated distributions of durations could not be considered as from the same distribution as the historical distributions for the Fennimore - July and August data set. Please recall that when all the historical distributions from Fennimore were compared to McCredie's historical distributions Ho could not be rejected, at the five percent level. The distributions of starting time at Monticello - July and August do not pass significance testing, but neither did the historical distribution.

The statistics obtained from the simulated depths (using McCredie's parameters) were like those from Fennimore's historical depths. The mean depth at both time periods at Fennimore is within .021 inches of the mean depth at McCredie for the respective time period. The standard deviations and and the May and June coefficient of skew are very similar, the coefficient of skew for the July and August period is larger for the historical data. The statistics obtained from the simulated starting times (using McCredie's

Table 6.17. Results of statistical tests between the simulated distributions using Hastings' parameters and the historical distributions at the test stations.

		Level which Ho[1] is not rejected for distributions of:			
Station	Time Period	Starting Times[2]	Depths[2]	Durations[2]	No. of Showers Per Day[3]
Fennimore	May and June	*	*	*	*
	July and August	*	*	*	*
Monticello	May and June	*	*	.05	*
	July and August	*	*	.05	*
Treynor	May and June	.05	*	*	*
	July and August	*	*	.05	*

* - not accepted at the one percent significance level.

[1]Ho is the hypothesis that the simulated and historical distributions come from the same parent distribution.

[2]Two-sample KS test was used.

[3]Chi-Square test was used.

Table 6.18. Results of statistical tests between the simulated distributions using McCredie's parameters and the historical distributions at the test stations.

		Level which Ho[1] is not rejected for distributions of:			
Station	Time Period	Starting Times[2]	Depths[2]	Durations[2]	No. of Showers Per Day[3]
Fennimore	May and June	.05	*	.05	.05
	July and August	.05	*	*	.05
Monticello	May and June	.05	.05	.05	.05
	July and August	*	.05	.05	.05
Treynor	May and June	.05	.05	.05	.05
	July and August	.05	.05	.05	.05

* - not accepted at the one percent significance level.

[1]Ho is the hypothesis that the simulated and historical distributions come from the same parent distribution.

[2]Two-sample KS test was used.

[3]Chi-Square test was used.

parameters) were not like those from Monticello's July and August historical starting times.

The simulated number of showers per day has an impact on shower depth and duration. For example, if the model is simulating too many showers per day given a daily depth, the simulated mean shower depth will be less than the historical, the converse also holds true. The historical mean number of showers at the test locations are like those simulated with McCredie's parameters.

For most of the comparisons between the simulated distributions - using Hasting's parameter values - and the historical distributions, Ho was rejected at the 1 percent level. The comparisons that did not reject Ho, were between the distribution of starting times at Treynor (May and June), and the distributions of duration at Treynor (July and August), and Monticello for both time periods. Please recall that for most of the comparisons between the historical distributions at Hastings and the test stations Ho was rejected.

Because of the large standard deviation of the shower durations the statistics of shower duration are the statistics calculated from ten repetitions. Each repetition is the equivalent to one simulation run which requires daily rainfall data for the period of record at each test station as input. The means and standard deviations for the simulated and historical data are listed in tables 6.6 - 6.9. All of the mean shower durations obtained by simulation with Hastings' parameters are considerably larger than the historical means of the test stations. The mean durations from simulations using McCredie's parameters are in closer agreement to the historical means.

All the standard deviations from the simulated data sets are greater than those from the historical data sets. As mentioned previously, since the duration was transformed to its natural log in the simulation procedure, the historical statistics of duration are not necessarily maintained.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The conclusions to follow will be broken down into two subsections. First, the adaptability of the rainfall disaggregation model as developed by Hershenhorn (1984) will be discussed. The changes which were needed to be made in the model will be summarized in this subsection. In the second subsection, the transferability of the model parameters that were obtained from Hastings and McCredie data sets will be summarized.

Adaptability of Disaggregation Model

The daily disaggregation technique developed by Hershenhorn (1984) from summer precipitation data collected on the Walnut Gulch watershed in southeastern Arizona required modification in order to describe late spring and summer precipitation for two midwestern stations.

Two of the required modifications improved the tractability of the model. The Fourier distribution replaced the Mixed Beta distribution for the description of the starting times of the showers. This replacement made the description of the starting times more theoretically correct because the Fourier distribution is periodic where the Mixed Beta is not. The Fourier density distribution is also much easier to integrate for use in simulations. Also, the functional forms used by Hershenhorn (1984) for the p and r parameters of the Shifted Negative Binomial (SNB) distribution required modification. The SNB distribution is used as a conditional distribution of the number of showers given daily depth. The modification made in the functional forms of p and r allowed the SNB distribution to describe situations where the expected number of showers given a daily depth asyptotically approaches a straight line with a positive (ie. Hastings and McCredie) and zero (i.e., Walnut Gulch) slope.

To model shower duration, a regression line is used to predict duration from shower depth, and a normal distribution with mean zero and a standard deviation equal to the standard error of estimate is used describe the spread about the regression. Hershenhorn (1984) found that the largest coefficient of determination was obtained when both the depth and duration were transformed to their natural logs. This relationship was used for both the complete and partial showers and passed normality and linearity testing. At the midwestern stations, the largest coefficient of determination for the majority of the complete shower (threshold equal to 0.009 inches) data sets resulted when the depth was transformed to its one-third power and duration was transformed to its natural log. However, the residuals from this functional relationship did not pass as being normally distributed. The threshold for the complete showers were increased to 0.099 inches. This change of threshold should not reduce the applicability of the model since the intensities of showers of less than 0.10 inch are rarely needed in models which use time varying infiltration techniques. When the depths were not transformed and the durations were transformed to their natural logs, the coefficient of determination was the greatest for half of the data sets, all data sets passed linearity testing and the residuals for the majority of them proved to be normal. The partial showers were transformed in the same manner and passed all relevant testing.

An addition was made to the disaggregation technique to account for the diurnal variation in depth. The starting times of the one complete shower days were conditioned upon depth of the shower. The earlier model definitely did not account for the diurnal variation of depth for the one per day showers, although it implicitly did so for more than one per day showers. Bayes' thereom was used to transform the conditional distribution of depth given time and the marginal distribution of time to the conditional distribution of starting time given depth. The conditional distribution of depth given starting time was a Exponential distribution with a time varying mean. The Fourier distribution was used to discribe the marginal distribution of starting time.

Various simulated and historical distributions were compared to determine how well the disaggregation model was working. The one-sample KS test was used to test the hypothesis (Ho) that the historical and simulated distributions of shower starting time, depth, and duration were from the same parent distributions. At the one percent level, Ho could not be rejected for any of the comparisons. The Chi-Square test was used to test the null hypothesis for the simulated distributions of historical and simulated number of showers per day. Ho could not be rejected at the one percent level. The Chi-Square statistic was also used to test Ho for the joint distributions of shower amount and starting time of the one per day complete showers (with depths greater than 0.099 inches) from the simulated and historical data sets. At the five percent level, the null hypothesis was rejected only for the Hastings, May and June data.

Transferability of Model Parameters

To test whether the model parameter values developed from McCredie and Hastings data could be used in disaggregating rainfall at nearby locations, the model parameters were used to disaggregate the daily rainfall from three test stations (Fennimore, Wisconsin, Monticello, Illinois, and Treynor, Iowa). Statistical tests were used to test the hypothesis (Ho) that the historical distributions from the test stations came from the same parent distribution as the simulated distributions which were derived from that test station's daily rainfall. The distributions of shower depth, starting time, duration and occurrence were compared. To determine if the test stations' precipitation data were similiar to that from Hastings and McCredie, statistical tests were also used to test the hypothesis (Ho) that the test stations' historical distributions came from the same parent distributions as the historical distributions from Hastings or McCredie.

Most of the historical distributions at Hastings were considered significantly different, at the one percent level, from the test stations' historical distributions. Ho could not be rejected, at the five percent level, for the distributions of starting time from

Treynor, and the distribution of depth from Monticello, July and August. Most of the simulated distributions obtained when the test stations' daily rainfall was disaggregated using Hastings' parameters were considered different from the historical distribution from the respective test station, at the one percent level. Ho could not be rejected, at the five percent level, for the comparisons between the simulated and historical distributions of starting time at Treynor, May and June, duration at Montcello, both time periods, and duration at Treynor, July and August.

The comparisons between the historical distributions from McCredie and the test stations resulted in Ho not being rejected at the five percent level, for most of the comparisons. The only comparison which was rejected at the one percent level, was with the starting times at Monticello, July and August. Ho was rejected for four comparisons between the simulated distributions and the historical distributions. The historical distributions of depth from Fennimore, for both time periods, could not be considered the same as the simulated distributions of depth at the one percent level. The greatest difference in the simulated and historical mean shower depths from Fennimore was .021 inches. When only shower depths greater than 0.10 inch were considered, Ho could not be rejected at the five percent level. This indicates that the simulations of depth at Fennimore work well for showers greater then 0.10 inches. Ho was rejected, at the five percent level for the durations from Fennimore, July and August. Since shower durations are simulated from shower depths, shower durations at Fennimore may be rejected as a consequence of the depth's rejection. The fourth comparison which was rejected, at the one percent level, was with the starting times from Monticello, July and August, which were considered significantly different from the historical distribution from McCredie.

Recommendations

More experimental work needs to be performed to determine the transferability of the parameter values. This study indicated that the parameter values may have a degree of regionality. Whether this was true just for the particular subtype studied needs to be determined. The temporal variation of the parameter values were not studied, and needs to be.

Throughout the study a recurring question arose - What kind of resolution do we need to model? The answer to the question was always dependent upon what the model was to be used for. If the disaggregation model was to determine maximum flood rates over long time periods - the showers comprised of small depths with long durations could be ignored. But if the disaggregation was being used in conjunction with other models to predict moisture availabilty for crop growth the small depth showers with long durations take on a increased importance. Grace and Eagleson (1966) in their study of stochastic modeling of rainfall data classify storms into three types: trace storms, moderate storms; and peaked storms, and subsequently predicted duration from depth in a different manner for each type. Further research is needed to determine the degree of resolution needed to be provided by daily disaggregation. Once the degree of resolution is determined then vast improvements in model speed and accuracy can be made.

APPENDIX A

THE CHI-SQUARE STATISTIC AND THE KOLMOGOROV-SMIRNOV TEST

In the preceeding study, the Chi-Squared statistic and the Kolmorogov-Smirnov test were used to check the goodness of fit of various distributions to a data set. They were also used to test if the simulated joint distribution of shower starting times and depths and the historical joint distribution could be considered from the same parent distribution. The various properties of the Chi-Square and the Kolmorogov-Smirnov test will be discussed in this appendix.

Chi-Square Statistic

Chi-Squared (χ^2) with ν degrees of freedom is defined as the distribution of the sum of squares of ν independent normal deviates. If Z(1), Z(2),...Z(N), are independent normal deviates then the quantity

$$\chi^2 = Z(1)^2 + Z(2)^2 + Z(3)^2 + \ldots + Z(\nu)^2 \tag{A.1}$$

follows the χ^2 distribution with ν degrees of freedom.

Goodness of Fit

An important use of the Chi-Squared distribution is in its use in testing the goodness of fit of a theoretical distribution to a sample data set. When used as a measure of the extent the observed and theoretical distribution agree, the statistic is:

$$\chi^2 = \sum_{i=1}^{k} (f(i) - F(i))^2 / F(i) \tag{A.2}$$

where:

f(i) is the observed frequency for the ith class

F(i) is the expected frequency for the ith class

k is the number of classes.

The χ^2 statistic gives a test of the null hypothesis: Can the observations be regarded as randomly drawn from the theoretical distribution? As can be seen from the equation, large values of χ^2 correspond to increasingly poor experimental agreement. Pearson (1900) showed that if the null hypothesis holds true then the test criterion for large samples is distributed approximately as a Chi-Square. This is a one-sided test. The degrees of freedom are determined by:

$$\tau = (k-1-n) \tag{A.3}$$

where:

n is the number of fitted parameters in the theoretical distribution.

Suppose a goodness of fit test resulted in a χ^2 value of 21.0 with 12 degrees of freedom. The χ^2 value along with the degress freedom are then used in determining the corresponding exceedance probability from a χ^2 table. For this example, the probability is approximately 0.05, which means that if the null hypothesis is true a value as large or greater than 21.0 has a five percent chance of occurring.

The χ^2 test is specifically designed for use with categorical data. However, it may be used with continuous variables where it is necessary to divide the region of definition for the variable into a finite number of intervals, count the number of observations in the intervals and calculate the expected number. The Chi-Square statistic is affected by the number of classes and their width, which are chosen by the experimenter. It is desirable for the experimenter to seek intervals with equal expected frequency which are equal to or greater than five (Mann and Wald, 1942).

Test of Proportionality

The Chi-Square statistic can also be used to test the proportionality of classes in two or more samples. The functional form of the Chi-Square statistic is the same:

$$\chi^2 = \sum_{i=1}^{k} \sum_{j=1}^{n} (f(i,j) - F(i,j))^2 / F(i,j) \qquad \text{(A.4)}$$

where:

k is the number of classes.

n is the number of samples.

f(i,j) is the observed frequency in sample j, class i.

F(i,j) is the expected frequency in sample j, class i, if the null hypothesis were true.

If the size of the samples are equal F(i,j) equals the mean of the j samples in class i. If the size of the samples are not equal then F(i,j) is weighted porportionally according to the size of sample j (Snedecor and Cochran, 1980). The number of degrees freedom is found by:

$$\tau = (N - 1)(C-1) \qquad \text{(A.5)}$$

where:

N is the number of classes.

C is the number of samples.

This is also an one-sided test, the null hypothesis is that the samples are from the same distribution. The χ^2 statistic has the same meaning as for goodness of fit. This test was used for comparing the historical number of shower occurences at the test stations and primary stations; comparing the simulated number of shower occurrences with the historical number; and the historical and simulated joint distribution of shower depth and starting time for the one per day complete showers.

<u>Kolmogorov-Smirnov Test</u>

The Kolmogorov-Smirnov (KS) test was also used for the same purpose as the χ^2test. The one-sample KS was used to test goodness of fit of a theoretical distribution, and the two-sample KS was used to test the hypotheiss that the two samples came from the same parent distribution.

Goodness of Fit

The one-sample KS test measure of the vertical deviations between the observed and expected cumulative distribution functions of continuous functions, whereas the Chi-Square is the measure of the vertical deviation between the observed and expected histograms. Yet, both types of deviations are useful in determining goodness of fit and probably are equally as informative for data that is not categorically collected (Gibbons, 1985). The KS test allows for the goodness of fit of the n observations, instead of K classes as for the Chi-Square. In this sense, the KS makes more complete use of the available data. The KS test is more useful than the the Chi-Square with small samples, otherwise there is no basis for a choice for which is the best test to use (Gibbons, 1985).

The statistic the KS test is based on is:

$$D(n) = \max_{i=1}^{n} \left(\left| F(x(i) - F^*(x(i)) \right| \right) \qquad (A.6)$$

where

$F^*(x(i))$ is the hypothesized cumulative distribution function.

$F(x(i))$ is the observed cumulative distribution function.

The distribution of the KS statistic is independent of the hypothesized distribution and has one parameter N which is the sample size. The null hypothesis is accepted when D(n) is less than the critical value.

Two-Sample KS

The two-sample KS test is similar to the one-sample KS test but in the two-sample test D(n) is the measure of the vertical deviations between two observed cumulative distribution functions. The null hypothesis is that the two samples are drawn from the same distribution. The distribution of the test statistic has two parameters, N(1) the size of sample one, and N(2) the size of sample two. The null hypothesis is rejected if the test statistic is greater than the critical value.

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