MATHEMATICS WORD PROBLEMS SOLVING BY ENGLISH LANGUAGE LEARNERS AND WEB BASED TUTORING SYSTEM

By

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ABSTRACT

The goal of the study was to investigate the impact of English text difficulty on English learners' math word problem solving. Booklets containing eight word problems and 5 point Likert-type rating scale items were given to 41 students. The students were asked to solve 8 math word problems written in English, and varying in grade level readability (vocabulary and grammatical complexity) as well as in the math operation (e.g., arithmetic, simple algebra). The researcher provided the students with hints as needed to ensure that students found the correct solution. The results showed that both English difficulty and Math level difficulty contributed to the students' poor achievements. Based on the results, some suggestions for improvements to an existing Web based math instructional software aimed at helping ELL students (called Animal Watch) have been made.

CHAPTER I

INTRODUCTION

Motivation for this Study

The process of learning mathematics represents a complex school activity, which involves acquiring a new vocabulary, manipulating new symbols and concepts in a specific context, developing new thinking and reasoning skills as well as communicating the results and the steps of the deductive processes to the outside world. Some authors would even consider that “the language of mathematics can be as challenging as a foreign language” (Freeman & Crawford, 2008, p. 11). For the case of English language learners (ELLs), the difficulties associated with learning a specific mathematical vocabulary would be corroborated with the difficulties appearing from an incomplete knowledge of English.

The current need for a better understanding of the challenges facing the ELL students in learning mathematics comes as the result of consistently lower performance of English learner students compared with native English speakers. In regard to this issue, Cuevas (1984) points out that “An inadequate grasp of the language of instruction is a major source of underachievement in schools” (Cuevas, 1984, p. 134). The same study cites, as a landmark in recognizing this situation, the Supreme Court case of Lau v. Nichols (1974) which concluded that “students who do not understand English are effectively foreclosed from any meaningful education” (Cuevas, 1984, p. 134). In 1980, the estimated number of ELL students of ages 5 to 14 learning in the US schools was about 2.4 million, while two decades later, a more recent study (Freeman & Crawford, 2008) finds this number to be about 5 million, with 80% of them being Latino. The latter study also estimates that by 2030, the number of ELL students enrolled in American schools will be about 40% of the entire school population. The situation becomes even more complicated if one is required to differentiate among English proficiency levels of the ELL students. As Moschkovich (1999) points out, a Latino student who is a recent arrival in the US and who missed 3 years of school in her country would have different instructional needs from a Latino student who was born in the US and followed a regular school schedule. Significant differences exist also among the ELL students' background, socio-economic status and motivation for learning.

General Socio-Cultural Context

In an attempt to a improve the level of mathematics education for all students, the National Council of Teachers of Mathematics (NCTM) published a comprehensive set of guidelines for teaching and learning mathematics at each grade level (NCTM, 2000). The central part of the Standards is represented by six educational principles: a focus on equity and excellence for all students, necessity of developing a coherent curriculum, the need to present students with teaching and learning experiences that build on students' existing knowledge base, careful assessment and a seamless technology implementation. The interpretation and implementation of NCTM's recommendations was done independently by every state and in many cases contradictions and discrepancies exist.

Moreover, the overall result of these efforts did not lead to the expected improvement in ELL mathematics performance: “Nationwide, 82% of Hispanic fourth-grade students are bellow proficient in mathematics (56% of whom are bellow basic), increasing to 88% of Hispanic eight-grade students (50% of whom are below basic) “ (Freeman, 2008, p.11)

In an attempt to address the root causes of this achievement gap, different authors tried to understand various aspects of ELL students’ mathematics education. The plethora of articles published in the field spanned different research directions and methods, from simple case studies to careful investigations of the psychological process involved in learning of mathematics and the role of English language structures used in the learning process. As a result of these investigations, different principles and teaching methodologies have been proposed and implemented as it will be reviewed in detail in the subsequent sections of this thesis.

Purpose of the Study

The purpose of this study is two-fold. First I will try to understand the nature and the reasons of the most frequent errors made by the ELL middle school children when faced with arithmetic word problems. For this investigation we will draw on two well defined areas of research:

(a) arithmetic word problem errors made by native English speaking students and

(b) specific language problem faced by the ELL students.

This approach is motivated by the fact that previous research (Newman, 1977; Casey, 1978; Clements, 1980) determined that poor mathematics results at middle school level can be understood as being produced by a lack or an incorrect understanding and interpretation of the word problems as well as by poor computational skills. In the case of ELL students we expect that the number of failures generated by incorrect problem comprehension to be higher than in the case of native English speaking students, and in order to understand the factors generating these discrepancies, we will draw on previous research performed in this area.

Second, we will make use of this information in order to improve an existing web-based tutoring system for middle school math word problem solving, called Animal Watch (Cohen, Beal, & Adams, 2008). The purpose of this software is to help struggling ELL students to better understand the algebra specific language and the methodologies used in solving them.

Questions of the Study

The questions this investigation will try to answer are the following:

1. What types of mathematical errors are made by ELL students while solving arithmetic word problems and what is the frequency of these errors?
2. How well do Measure of Academic Progress (MAP) scores predict the mathematical performance of the ELL students faced with word problems?

To answer these questions, this study asked 41 students from three middle schools in the southwestern United States to complete booklets with eight word problems and

5 point Likert-type rating scales printed on the page below each problem, one per page (see appendix D). The students were asked to solve 8 math word problems written in English, and varying in grade level readability (vocabulary and grammatical complexity) as well as in the math operation required to solve the problem (e.g., arithmetic, simple algebra). A more detailed description of the study can be found in chapter 3. In the end, I use the findings of this study in order to make suggestions to improve the functionality of the Web-based Math tutoring system called Animal Watch.

CHAPTER II

LITERATURE REVIEW

The first section of this review will present general considerations concerning the nature and origin of mathematical errors made by ELL students when solving arithmetic word problems. The second section will research the relationship between learning language and learning mathematics. The third section will identify the specific difficulties met by ELL students in solving arithmetic word problems and possible remedies. The fourth section will describe the role of language structures in teaching and learning mathematics and implications for the ELL students. The final section will present a classification of arithmetic errors when performing elementary operations.

General Considerations Concerning the Nature and Origin of Mathematical Errors

The mathematical errors that ELL students make when presented with arithmetic word problems is the result of both language difficulties and lack of appropriate mathematical skills (Clements, 1980). Scientific research has determined that even in the case when the tested students were native English speakers, a significant percentage of their errors can be attributed to an incorrect understanding of the problems' text. As mentioned before, we expect that this percentage would increase in the case of ELL students. Newman (1977) attempted to structure the errors made by middle school students by creating a hierarchy of the cognitive levels involved in the problem solving process which contained five separate levels:

1. Reading
2. Comprehension
3. Transformation
4. Process Skills
5. Encoding.

These levels correspond to the succession of the steps that a student must take when presented with a simple arithmetic word problem: first the students must read the text, then they need to comprehend what they have read, transform this information into an appropriate mathematical model, apply the necessary process skills and execute the necessary computations. At the end, the result of the computation must be interpreted (encoded) in the form required by the problem. An error performed at any step of this process would lead to an incorrect answer for the problem. Closely intertwined with these levels the author considers the effect of two external factors: “Motivation” and “Carelessness”. Both these factors have the potential to affect negatively the results at every level of the above hierarchy. The hierarchy structure proposed by Newman was modified and expanded by Casey (1978). In contrast to Newman hierarchy, Casey considers that the question form plays an important part in the success of problem solving attempt and she places it as the first step in the newly proposed hierarchy. The steps in Casey’s hierarchy are:

1. Question Form

2. Question reading (done by the student himself, the teacher or another student in the group)
3. Question comprehension (the manner in which the question is understood by the student and how it relates to his/her existing knowledge base)
4. Strategy selection (including the steps towards solution)
5. Skills selection (transformation, encoding, derivation etc.)
6. Skills manipulation (in general related to the technical aspects of the solution)
7. Answer presentation

While each of these steps builds on previous ones, the general process of solving a word problem requires the student to return to any of the previous steps in the case when the final result is in contradiction with the expected value. Casey (1978) also expands the external categories that can influence every step of the hierarchy. Where Newman considers only the “Motivation” and “Carelessness”, Casey refers to these factors as “Known block” and “Unknown block”.

To correctly identify the reasons for students’ errors, Clements (1980) suggests that a simple evaluation of written tests is not usually enough. The author suggests that a much better understanding can be obtained from follow up interviews with the students who gave the wrong answer. He also presents an evaluation of the number of errors in each category outlined above for children in grades 5-7 (Figure 1, after Clements 1980, p. 11) and from grades 7 (Figure 2, after Clements 1980, p.12)

Error category	Grade 5 (*n* = 55)		Grade 6 (*n* = 207)		Grade 7 (*n* = 280)	
	No. of errors	Percentage of errors	No. of errors	Percentage of errors	No. of errors	Percentage of errors
Reading	53	8	121	5	50	2
Comprehension	86	14	201	8	187	9
Transformation	170	27	636	25	538	27
Process skills	173	27	794	32	523	26
Encoding	12	2	40	2	19	1
Carelessness or motivation	143	22	707	28	705	35
Total	637	100	2499	100	2022	100

Figure 1 Error categories for students in the grades 5-7 (after Clements, 1980, p. 11)

Error category	Low achievers (*n* = 92)		Average achievers (*n* = 92)	
	No. of errors	Percentage of errors	No. of errors	Percentage of errors
Reading	117	8	18	3
Comprehension	225	16	32	6
Transformation	401	28	150	28
Process skills	351	24	126	23
Encoding	37	3	12	2
Carelessness or motivation	306	21	206	38
Total	1437	100	544	100

Figure 2. Error categories for students in the grades 7 (after Clements, 1980, p. 12)

These data show that most of the errors made by middle school students when solving word problems can be included in one of the following three categories: reading, comprehension and transformation. As a result, these students are much more likely to select an inappropriate process skill in order to solve the problem than to apply incorrectly a correctly chosen skill. A second conclusion that can be inferred from this data is that compared with the fifth graders, the seventh graders are less prone to make errors due to faulty reading or miscomprehension. This conclusion is especially important for the case of the ELL students, because it clearly demonstrates the direct correlation that exists between a good understanding of the mathematical content written in English and the success in Mathematics.

Relationship between Language and Mathematics

Previous research has separated the process of solving arithmetic word problems into two main components: problem comprehension and problem solution (Mayer, 1985, 1986). Problem comprehension is loosely defined as the activity of associating the text statements with previously known representations or structures and establishing meaningful relationships between them. For example, a statement: “a car is moving at a constant speed of 40 mph…” would require an internal representation of the general concept of “car” and an associated action (relationship) associated with it for “moving at a constant speed”. The second component, the problem solution, can be also seen as consisting of two major activities: (a) variable assignment and (b) mathematical computations. For the above statement the assignment requires the association of an abstract variable (speed) with a numerical quantity (40 mph). Typical computational situations would require the students to use the definition of the speed in order to determine the time necessary for the car to travel a certain distance or to determine the distance traveled in a certain amount of time. The determination of the correct solution requires that all the cognitive steps involved in solving the problem are executed correctly. While this is hardly a new idea, an important body of research has been dedicated to trying to understand how different errors at the comprehension or mathematical computational level can influence the general outcome of problem solving activity. In parallel, a number of researchers also tried to create a general model to describe students' reasoning as they solve arithmetic problems. Nathan et al. (1992) summarized this activity in the following manner "To comprehend a problem, the student must make a correspondence between the formal equations and the student's own informal understanding of the situation described in the problem" (Nathan et al., 1992, p. 330).

Cummins et al. (1988) differentiate among the following six miscomprehension error types:

a. structure-preserving transformations (SP) – the wording of the problem was changed during the recall but the mathematical relations among sets was preserved
b. structure-violating transforms (SV) when the original problems were transformed into other legitimate problems but with different mathematical conditions or requirements than the original ones
c. non-sense problems (NP) in which the recalled problems did not contain a logical set of mathematical relationships
d. two references to a superset in a problem (2S) which included either a double assignment for the same structure or bypassed the logical

computation process by requiring information about an already assigned structure

e. zero reference supersets (0S) in which references were not properly assigned, and

f. all other recalls that could not be classified as one of the first five categories

Not surprisingly, the authors found that the type of miscomprehension errors that were associated with the most successful outcome in problem solving was the structure-preserving transformations (SP). A student capable to recall correctly the mathematical structures of a problem turned out to have a 64% chance of finding a correct answer and a 25% chance of making a computational error. While these percentages can vary widely among different students and schools, the overall conclusion remains valid – a correct problem recall generate a higher rate of success in solving the problem. Among the students who performed a structure-violation type error (SV), the vast majority of wrong answers were in fact correct answers to the incorrectly transformed problem. The good correlation between the ability of remembering the correct logical mathematical requirements of the problem and the correct answers can be understood in terms of a common underlying mechanism for these two processes. The correct problem recollection demonstrates the fact that the internal conceptualization of the problem was successful – the students successfully associated the new problem with an already known pattern of relationships. In other words, the student had some a priori knowledge which he could relate to the informational content of the problem. The same pre-existing pattern of relationships can also explain the higher rate in achieving a correct final answer – the pattern could have been formed only if the students have been previously exposed to a situation identical or close to the one presented in the problem. Interestingly enough, the same study determined that the zero reference supersets (0S) type of errors were most often associated with correct solutions. The authors found that in 61% the cases when students would not be capable to recall the problem’s assignments or requirements, they were still able to find the correct solution. While the study does not provide an explanation for these findings, it can be only assumed that, since the cognitive processes involved in solving the problem were different than those involved in its correct recollection, a very good correlation between these two types of activities might not exist.

The role played by the internal pattern of relationships in correctly understanding the problem formulation can also explain the results obtained by Lewis and Mayer (1987). They pointed out that “the consistency hypothesis is that problem solvers have a preference for the order in which problem information will be presented. This preferred format corresponds to the order of information presentation in consistent language problems but is in conflict with the order of information presentation in the inconsistent language problems. A straightforward prediction of the consistency hypothesis is that comprehension errors will be more likely to occur when the structure of the presented information does not correspond to the problem solver's preferred format.” (Lewis & Mayer, 1987, p. 364). The authors differentiate between consistent language problems and inconsistent language problems. While both types of language problems express logical structures, the consistent language problems require the least amount of information reorganization in order to obtain the final answer. According to their study, “the sentences in word problems can be classified as *assignments*, *relations,* or *questions*”(Lewis & Mayer, 1987, p. 364). Based on this classification, an example of a consistent language problem would present the following structure (Lewis & Mayer, 1987, p. 364):

(Set A) = (Value a)

(Set B) = (Value b) (relation) (Set A)

(For example: Joe has 3 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?)

while an inconsistent language structure would be:

(Set A) = (Value a)

(Set A) = (Value b) (relation) (Set B)

(For example: Joe has 3 marbles. He has 5 less marbles than Tom. How many marbles does Tom have?)

It can be easily seen that the second structure requires mental rearrangement in order to be brought to the form of a consistent language problem. The students’ preference for the consistent language problems can be understood from the same perspective of a previously established internal pattern of relationships. In this case, the established pattern is having the unknown variable as the subject of the statement rather than being the object of it. The students were more likely to understand a statement of the form: “The amount of money that person A has is 5 dollars more than 10 dollars” than a statement of the form ”10 dollars represent an amount which is 5 dollars less than the amount of money that person A has”. At the grammatical level, the first form can be associated with active voice while the second one with the passive voice. In everyday language, active voice expressions like “I close the door” are more frequent than their passive voice counterparts “the door is being closed by me”. It is not surprising then to see the brain’s preference for storing and retrieving information in a manner similar with the consistent language. The study also found that, when presented with an inconsistent language problem, the students would make more mistakes if additions or multiplications were required, compared with the case when only subtractions or divisions were involved. This second result was attributed to the particular format of the problems and the authors do not conclude that this behavior represents a general characteristic.

The role played by the incorrect mathematical reasoning in obtaining wrong solutions to the word problems has also been investigated from the perspective of the processes involved in the reasoning. Resnik specified that: “…there are two domains of reference for mathematical language: the world of abstract mathematical entities and world of mathematizable situations”(Resnik, 1988, p.35). Abstract mathematical entities do not lend themselves very easily to everyday language characterization. For example, when children were required to express in words the cognitive processes involved in a simple multiplication process (for example why 2x3=3x2) they attempted to do so by describing the existing relationships between numbers (2 is less than 3 but is taken three times while 3 is larger than 2 but is taken only 2 times). In fact, this should come as no surprise: an abstract concept or operation has no well established definition in the real world language – it can only be presented by describing its projections onto the real world. For example, the concept of speed is an abstract entity; nobody can take a piece of "speed" and hand it over. We can only describe it in relation to other physical, measurable concepts (speed and time).

The mathematizable situations referred to in the paper present an equally challenging problem – that of mapping the physical situation onto the abstract space of mathematical concepts and relationships. In most of the cases it turns out that the preexisting mathematical structure does not support a direct mapping – it takes proper content reorganization in order to achieve a correct representation. Resnik (1988) discusses as an example the subtraction operation: it can represent a "Change" situation when a certain quantity is modified by subtracting another quantity from it, a "Combine/Separate" situation in which a quantity is split into 2 components and a "Compare" situation when two quantities are subtracted in order to find the difference. In each of these cases the difficulty does not consist in performing the subtraction – it is assumed that the students could master relatively quickly this technicality – as much as is to understand what is required of them, how to assign the problem's data to the correct abstract structure (what to subtract from what) and how to interpret the results. Since all these steps – building abstract structures, assigning problem data to them, performing mathematical operations and interpreting the results – must be done correctly in order to obtain a correct final answer, it is easy to estimate that errors in overall mathematical reasoning might have a comparable rate of appearance as the errors in understanding the word problems content.

Specific Difficulties Met by ELL Students in Solving Arithmetic Word Problems

The activity of learning, in general, and that of knowledge formation, in particular, is a complicated psychological process that has been actively researched in cognitive psychology within the last three decades. The manner in which new knowledge is added to the already existing knowledge is of crucial importance for understanding the process of mathematics learning in school. The new information becomes knowledge if it can be connected in a logical manner to the information already stored in the brain. At elementary level, the process of learning mathematics can be seen as made up of two fundamental blocks: a) learning new concepts (or definitions) and b) establishing new connections (proofs, relationships, theorems) at cognitive level between concepts. The mathematical knowledge can not be understood or committed to the long-term memory in isolated pieces; the new concepts must be related to the existing ones in a logical manner, and based on a set of rules. In the case of ELL students, it is expected that an additional level of complexity will exist: that of translating the newly provided information from English to the native language. The existing knowledge can be both formal and informal, and using it as a basis for further scientific accumulation is one of NCTM standards recommendations. Gutstein (1997) mentions that "Evidence confirms that helping teachers build on children's informal knowledge in mathematics classroom helps children use their intellect well, make meaning out of mathematical situations, learn mathematics with understanding, and connect their informal knowledge to school mathematics" (Gutstein et al., 1997, p.711).

Closely related to the idea of building on the children's informal knowledge is the idea of testing that relies on the existence of this knowledge. Campbell et al. (2007) present the case of an ELL preservice elementary school teacher who had difficulties in solving a math problem formulated as a baseball problem. The authors concluded that the teacher, in fact, possessed the necessary mathematical skills for solving the problem, but she did not posses the informal knowledge about the baseball game, knowledge taken for granted for a person born and raised in the US. This particular example illustrates the complexity of the ELL situation; even if the students happen to understand the English words, in some cases they might not have a clear representation of their meaning.

Starting from the presented case, the authors also point out another issue related to instruction that relies heavily on existing student knowledge: the limited amount of memory available for storing new information. Assuming that the teacher is aware of the limitations in informal knowledge of the ELL students and that she is willing to compensate for it with additional explanations, these students will have to understand and memorize an additional amount of information compared to their English speaking peers. By introducing this extraneous linguistic information, not necessarily connected to the mathematical concept under investigation, the ELL students might run the risk of not being able to properly absorb the critical information. Under these circumstances, it's imperative that the teacher finds an informal common ground for all students before attempting to make use of this preexisting knowledge.

The issue of extraneous linguistic information overload is present not only in the teaching process, but also in the current standardized testing process. The authors mention that: “There is prima facie evidence that test writers are not linguistically or culturally aware of the difficulties that particular wording and phrasing in word problems cause students, especially those taking the tests in second or additional language.” (Campbell, 2007, p. 13)

The key to a correct understanding of the cognitive processes involved in teaching and learning mathematics is the association between cognitive load theory and the reflective abstraction processes. The cognitive load theory is centered on the attributes of memory, either long or short-term, while the reflective abstraction represents the process of active reenactment of the learned concepts, re-thinking of the problem and committing the remodeled structures to the long-term memory. The common liaison between these two entities is represented by the memory and we can not understand the process of mathematics learning by analyzing them separately.

> Without attention to issues of working memory, students are doomed to suffer inefficient and unproductive problem-solving techniques… Also, unless students have a stimulus to abstract by reflecting on the operations used in the solution of a problem, they risk never seeing a more inclusive picture. (Campbell, 2007, p. 15)

To better understand the issues faced by English learners as they solve mathematics problems Campbell (2007) draws our attention not just to the mathematical content and students’ cognition, but also to the role of students’ culture and language in the teaching and learning process, They suggest that “reflection on culture, language and socially situated prior experiences, in addition to reflection on mathematical content and on students’ cognitive processes and understanding, be incorporated into models of mathematics teaching” (Campbell, 2007, p. 16). This framework is considered to be especially beneficial for the ELL students: in the case when the teacher and the students are coming from different cultures, establishing a common ground requires continuous analysis and lesson planning.

In essence, the framework proposed by Campbell et al. (2007) has four components: " (a) academic content; (b) mathematical and cognitive processes; (c) mathematical and contextual language; and (d) cultural/life experiences" (p. 20). The academic content brings to attention the mathematical knowledge of the student – the richer this basis, the more the students will be capable to process and analyze the new information. The second component, mathematical and cognitive processes is concerned with identification and development of the cognitive processing skills needed for learning mathematics. Cognitive and metacognitive skills can be taught especially by using well chosen examples, but also by questioning, planning, or drawing conclusions. The goal of the instruction, in the view of the authors "becomes one of enabling students to take control over their own learning through the practice and development of increasingly complex processes modeled by the teacher in activities and demonstrations, and in texts and materials" (p. 22). In the case when students have difficulties in transferring strategies learned in one problem to another problem, the preferred method is the reduction of the goal specificity. According to this strategy, the students must be directed towards understanding the situation presented in the problem instead of focusing on the goal required by the problem

The third component of the proposed framework is represented by the relationship between mathematics and contextual language. In essence, the researchers are concerned with the degree in which the language used in problems' statement corresponds to the level of the English language of the ELL students. One suggestion made in the literature was for the teachers to enhance the role of the natural (or ordinary) language in instruction. It was argued that the natural language helps in mediation between mental processes, symbolic expressions and logical organization as well as in “finding counterexamples and in developing arguments of validity” (Campbell, 2007, p. 23). The last of the components proposed by the authors, namely the cultural/life experiences is, in essence, concerned with the informal knowledge base that a student needs to posses in order to understand the mathematical concepts. Since many of the ELL students are coming from non dominant cultural groups, their informal knowledge basis can not be taken for granted. In the aforementioned example, not knowing details about the baseball game can prove a real impediment in understanding or application of simple mathematical rules.

The Role of Language Structures in Teaching and Learning Mathematics

In the process of transmitting the new information to the students, the major vehicle is represented by spoken and written language. A model for the role of the first and second language in mathematical activity is presented by Cuevas (1984) in the form of a complex diagram which shows that the language is situated at the intersection between concepts, mathematical notations, diagrams and inspiration. The language is involved in all the major activities of the learning process: representation, definition, creation, discussion, instruction, description, and verbalization. The same paper, citing previous research, emphasizes that “Researchers have found high positive correlations (.40 to .86) between mathematics achievements and reading ability… In addition, there appears to be a direct relationship across various school subjects between instruction in the student’s native language and high achievement in the subject.” (p. 138).

Starting from this established relationship between mathematics learning and language ability, the early recommendations for the ELL students were that emphasis should be placed on vocabulary and comprehensive skills.

Today the research community agrees that a successful result for the ELL students in mathematics learning can only happen if the process is seen in a holistic way (Moschkovich, 2002).

In a study published in 2002, Moschkovich investigated three perspectives of the role of the language on the learning process: “acquiring vocabulary, constructing meanings and participating in discourses” (Moschkovich , 2002, p. 191). Each of these perspectives is based on a gradually larger encompassing concept. Acquiring vocabulary was founded on the concept of lexicon - the student must learn the correct meaning of the mathematical words and symbols. Constructing meanings was founded on the concept of mathematics register (Halliday, 1978) and was understood by Moschkovich as “a language variety associated with a particular situation of use” (Moschkovich , 2002, p. 194). The concept of register was contrasted to the concept of lexicon from the perspective of non-verbal and contextual inclusions; for example, a “quarter” was seen as 25 cents in a particular money problem or as one fourth of a whole in common situations. The ELL student participation in mathematical discourses was situated on both these practices (acquiring vocabulary and constructing meanings). The concept of "discourse" represents more than words (lexicon) and meanings (registers), because it refers also to models of action, thinking, reasoning, and communicating: "mathematical Discourse practices can be understood in general as talking and acting in the ways that mathematically competent people talk and act" (Moschkovich , 2002, p. 199). While the vocabulary and the registry are in general stable, the mathematical discourse as a situated-sociocultural perspective will depend on variety of factors. The author suggests that teachers, by shifting the focus of instruction from simple vocabulary and registry learning to a mathematical discourse can help the ELL students to focus on mathematics learning.

The issues of vocabulary and registry are also considered by Campbell et al., (2007). In terms of vocabulary, the authors mention that in many cases, the ELL students' language might be one to three levels below that of the mathematics level. Even in the case when the language and the mathematics levels are compatible, the teachers still should pay close attention to the registry. The meaning of words in the natural language might be completely different in mathematics language. Words like "table" or "division" are pointed out by the authors as examples for illustrating this problem.

Moreover, Adler (1998) argues that "even within a mathematical register, meanings shift" (p. 28). The example analyzed by the author is the word "most" which, in early mathematics learning, is argued to be associated with the term "more" or, equivalently with the expression "greater than". But in some specific cases, for example when used in the construct “at most”, the same word has exactly the opposite meaning and becomes the negation of “greater than”.

While Moschkovich built the concept on learning mathematical vocabulary in an inclusive manner (from vocabulary to registry and from there to the role of mathematical discourse), other authors considered a leveled approach to mathematics instruction for the ELL students with respect the language. In a study published in 1997, Garrison presented four equally important perspectives in learning mathematics: (a) mathematics as problem solving, (b) mathematics as communication, (c) mathematics as reasoning and (d) mathematical connections. The NCTM standards emphasizing the language development for the ELL students are seen to be in agreement with the current teaching strategies in bilingual education. The author suggests that the complexity of the language in a lesson can be reduced by using a combination of strategies such as: “providing a rich context environment, using manipulatives, preteaching vocabulary, and allowing students to work in cooperative groups” (Garrison, 1997, p. 136). Among these techniques, the one concerned with providing a rich context environment is seen as a major help for ELL students to infer the meaning of unfamiliar words. Moreover, the author suggests that groups of related words can be taught through mathematics. The idea of teaching English language through mathematics was also explored by Campbell et al. (2007). Starting from Cummins’ assumption that an ELL student who masters the mathematical concepts in his/her own language would use them for English learning, a lot of research had been dedicated to the practical implementation of this idea. But the authors warn that “one of the limitations of a language-content approach to lesson and material analysis is that the language analysis tends to be at the surface level" (Campbell, 2007, p. 19) and that the ELL student would be best served if the main focus of the lesson was on mathematical content.

From Freeman and Crawford (2008) perspective, "a distinction between social language (basic interpersonal communication) and cognitive academic language … is essential to understanding why teachers need to take a special approach toward the teaching of mathematics to ELLs" (Freeman & Crawford, 2008 p. 12). The study points out that the use of a simplified language in mathematics classes showed positive results, but the authors warn against a simplification of the mathematical concepts that must be taught. They also argue that mathematics teaching must be also associated with vocabulary building.

Classification of Arithmetic Errors When Performing Elementary Operations

The type and frequency of mathematical errors that the students make when solving arithmetic word problems has been extensively investigated, and various attempts have been made to classify and correct them in a structured manner. In analogy with the terminology used in Computer Science, these errors have been named "bugs" (Brown& Burton, 1978) and the starting premise of these investigations was that, by performing a careful classification of these errors, the teachers and students can learn how to identify their root cause and use this understanding as a springboard for a deeper understanding (Borasi, 1994) of the concepts that generated them. While the feasibility and the success of such ambitious projects (Brown & Burton, 1978; Brown & VanLehn 1980; Young & O'Shea, 1981) is still open for debate, the work done in this direction helped improve our understanding in how the children learn and apply new knowledge.

The simplest problems to describe and catalogue in terms of possible errors were additions and subtractions. Brown and Burton (1978) proposed a diagnostic modeling system which would allow, besides the error recognition, an automatic identification of the causes that generated that error. The fundamental premise of such a system is that the computational errors are repeatable and, regardless of their manifestation, they are usually rooted in students' misunderstanding of the applied algorithm. The proposed technique, called "procedural networks", allows constructing diagnostic models "that capture a student's common misconceptions or faulty behavior as simple changes to (or mistakes in) a correct model of the underlying knowledge base" (p. 156). According to the authors, it is much more difficult to diagnose "what is wrong with a student's method of performing a task – to form a diagnostic model – than it is to perform the task itself" (p. 156). This conclusion should come as no surprise – in Mathematical language the task of such a system is to deal with an inverse ill posed problem or, in Computer Science parlance we have a many-to-one relationship, since the same symptoms (error) can have multiple causes or be generated by a combination of causes. Contrary to the assumption of most teachers that the erratic errors are caused by the fact that the students do not follow the correct procedure, the authors believe that in fact, the students are very good procedure followers, but often they choose to follow the incorrect method. If most or all of these incorrect procedures can be identified, a computer system could then be used in order to associate them with different error types. The procedural network model proposed by the authors consists of (a) a collection of procedures appropriate for solving the task at hand representing the nodes of the network, and (b) the relationships between these procedures representing the branches of the network. For most of the mathematical tasks, the procedural network would contain more than one correct method for solving the problem. These methods would then be decomposed in elementary skills (procedures) and the success of the solution would depend upon the correct application of these steps and of relationships between them. In the case of addition for example, these primary procedures can be selected as: recognizing a digit, writing a digit, adding the numbers from a column, etc. The efficiency of such approach in identifying the correct root for a bug is directly proportional with the number of known bugs identified a priori.

Once the error has been identified, Borasi (1994) suggested that educators and teachers could take advantage of it in order to increase the mathematical understanding of the analyzed concept. In author's view, simply correcting the bug is not enough; finding and discussing the roots that generated its appearance offers more support for learning. Most educator and students do not see mathematical errors in a positive light and, from behaviorist perspective, the learning is enhanced when correct responses are rewarded (positive reinforcement) and when the incorrect ones are punished (negative reinforcement) or ignored. From this perspective, paying too much attention to the errors is considered dangerous, as it could generate confusion in young student's mind between the correct and the wrong answer. On the other hand, one step ahead is done in the framework of constructivist learning where, the errors are seen both as an inevitable part of the instructional process and as an important source of feedback about the learning process itself. With a right strategy, the instructors can discover what students really know and operate on the very misconceptions that triggered the formation of those errors. But this perspective is, in author’s view, only half of the process as it does not encourage the students themselves to engage in recognizing and analyzing the errors. The most significant advantage of such an approach would be the fact that it can stimulate the inquiry during the learning process. By asking questions and trying to determine the roots of the conflicting results, it is expected that the students would start seeing the mathematical process as evolutionary discipline and would help them think in a manner similar to the one used in mathematical research. For such an approach to be useful, the following aspects of an error should be taken into account “(a) the nature of the error, (b) the context in which the error activity developed, (c) the source of the error, (d) the students' level of participation in the error activity, (e) the educational goals of the activity, and (f) major results of the activity” (Borasi , 1994, p. 174). The experiments performed by the author showed that, besides the intended effect of understanding the roots of students’ misconceptions and correcting them, a more subtle effect happened: it successfully changed the students’ perspective about the mathematics from “cut-and-dried and impersonal” science to a “humanistic” form and thus the participants in the experiments became more confident that they could do mathematics.

The hypothesis formulated by Brown and Burton (1978) that mathematical errors do not happen at random in most cases, but they are the result of consistent application of misunderstood rules was largely embraced by the scientific community. Blando et al. (1989), for example, determined the following 4 categories of errors associated with the elementary arithmetic operations: (1) Precedence errors, (2) Substitutions errors (3) Nonmodeled errors and (4) Other errors. Each of these categories includes specific algorithmic errors and they are shown in Figure 3 bellow (Blando et. al. ,1989, p. 303)

Descriptive label	Example
Precedence Errors (P- -)	
Add before multiplying	$4 + 2 \times 3 \rightarrow 6 \times 3$
Add before dividing	$10 / 2 + 3 \rightarrow 10 / 5$
Subtract before multiplying	$9 - 2 \times 3 \rightarrow 7 \times 3$
Subtract before dividing	$8 - 6 / 2 \rightarrow 2 / 2$
Add before subtracting	$6 - 4 + 3 \rightarrow 6 - 7$
Ignore parentheses	$8 - (2 + 4) \rightarrow 6 + 4$
Substitution Errors (S- -)	
Divide when task calls for addition	$5 + 2 \rightarrow 2$ r 1
Subtract when task calls for addition	$5 + 1 \rightarrow 4$
Multiply when task calls for addition	$2 + 3 \rightarrow 6$
Add when task calls for multiplication	$6 \times 2 \rightarrow 8$
Divide when task calls for multiplication	$6 \times 2 \rightarrow 3$
Subtract when task calls for multiplication	$3 \times 2 \rightarrow 1$
Multiply when task calls for subtraction	$6 - 1 \rightarrow 6$
Add when task calls for subtraction	$3 - 1 \rightarrow 4$
Divide when task calls for subtraction	$8 - 2 \rightarrow 4$
Multiply when task calls for division	$10 / 5 \rightarrow 50$
Subtract when task calls for division	$10 / 2 \rightarrow 8$
Add when task calls for division	$6 / 3 \rightarrow 9$
Other Error (O- -)	
Negate task solution	$8 - (2 + 4) \rightarrow -2$
Nonmodeled Errors[a] (N- -)	
Careless addition	$6 + 4 \rightarrow 9$
Careless subtraction	$15 - 13 \rightarrow 4$
Careless multiplication	$4 \times 12 \rightarrow 45$
Careless division	$24 / 4 \rightarrow 8$
Undiagnosed	$12 \times 5 \rightarrow 26$

Figure 3. Typical errors for elementary arithmetic (Blando et al., 1989, p. 303)

This categorization allowed the authors to determined that most of the errors were tied closely to the format of the items. For example, after the students were shown how to apply the precedence rules in the case of the following format:

$$a + (b \times c)$$

they could correctly identify and apply the rule for similar formats, but they had difficulty in applying the precedence rules in closely related expression with a different format:

$$(a + b) \times c$$
$$a \times b \times c$$
$$(a - b) \times c$$

Equally important, the study determined that the errors were the result of one or more incorrect steps performed in the application of the precedence rules.

CHAPTER III

METHODOLOGY

This chapter provides information about the methodology used in this study that investigates the impact of English text difficulty on English learners' math word problem solving. It provides information about the research context as well as the methods used to collect and analyze the data.

Research Questions

This study investigated the following research questions:

1. What types of mathematical errors are made by the ELL students while solving arithmetic word problems and what is the frequency of these errors?
2. How good predictors are the official MAP scores for the mathematical performance of the ELL students faced with word problems?

Research Context

The study was conducted at two XY public middle schools from the same district and one Z charter middle school in the southwestern US. At the time of the study, the district was the third largest public school district in the city in terms of enrollment, with over 16,000 students. The school X had 814 students and 56 teachers with a teacher: student ratio of 1:15. The school Y had 739 students and 45 teachers with a teacher:

student ratio of 1:16. Both schools were of type charter and they were (a) dedicated to college preparation of middle and high school students with an emphasis on sustained inquiry in science and mathematics; (b) focused especially on attracting diverse population of low-income students; (c) grounded in a close connection to the resources and expertise of the community; (d) envisioned as a resource locally and nationally for educational demonstration, research, and innovation; and (e) devoted to helping students have authentic leadership experiences applicable to success in living, learning, and working in modern society.

The schools were diverse as well as shown in Figure 4. The percentage of female students ranged from 44% to 53%. The Hispanic population ranged from 57% to 76%.

School	X	Y	Z
# of Students	814	739	187
Male	52%	47%	56%
Female	48%	53%	44%
Native American	4%	2%	*
African American	8%	4%	*
Asian American	2%	4%	*
Hispanic	61%	57%	76%
White	25%	33%	*

*data not available

Figure 4. Student Demographics by School

Methods of Data Collection

This study was a descriptive study with an experimental design.

The student population sample included 41 middle school English Learners (forty students Spanish- primary, and one student Chinese-primary; 14 girls and 27 boys). Students were eligible for participation in the study if they were identified by the district as an English Learner (based on students' background), were enrolled in the state-mandated English instruction period, and had received parental consent to participate.

Each EL student was individually interviewed in English during a 30- minute session. We have used eight problems for each interview with each problem falling in one of the following four categories: (a) Easy-Math (EM) & Easy-English (EE), (b) Easy-Math (EM) & Hard-English (HE), (c) Hard-Math (HM) & Easy-English (EE) or (d) Hard-Math (HM) & Hard-English (HE) (see appendix G). Easy-math problems were considered those that required only one single-digit addition; the hard math problems contained a combination of multi-digit multiplications and additions. The language difficulty was established by using the REAP readability assessment software developed at Carnegie Mellon University http://reap.cs.cmu.edu . The same mathematical problem was expressed in both easy-language and hard-language such that we created 2 different problems with exactly the same mathematical content. The increased language difficulty was obtained by changing the vocabulary and the grammar structure while keeping the word count constant. On the difficulty scale provided by REAP software (from 1 to 10), the easy-language problems were rated at level 3, while the difficult-language problems were rated at level 8.

The protocol for the interview was the following:

1. Explain process. In the very beginning the interviewer introduced the booklet to the student and specified that the problems' level of difficulty will be variable from one problem to the other;
2. Read problem. The researcher would read each problem (in English) to the student and make sure that each student has access to the same problem information;
3. Mark words. Ask the student to mark any words or sentences that are confusing or unfamiliar, using a highlighter;
4. Solve problem. Ask the student to solve the problem working out the solution on the booklet page. The interviewer followed the progress closely in order to make sure that he/she could detect any wrong steps. When an incorrect step happened the researcher would make a record of it and then point out the error to the student. The purpose was to help the student to solve each of the problems correctly while noting how much help he/she needed during the process. If the answer was correct, the researcher would provide feedback and move on to next problem.

5. Rate problem. During the last step of the process the student had to indicate on a 5-part Likert-type rating scale (1 “very easy” to 5 “very hard”) the language difficulty and mathematical difficulty of the problem. At the end of the interview, after solving all eight problems, the student was required to indicate the most difficult problem from booklet.

Analysis Process

As mentioned before, the main objective of the interview was to obtain clear evidence of all difficulties (mathematical or linguistic) encounter by the student during the entire process of solving a problem. We drew conclusions from 3 different sources:

(a) notes made during the interview concerning the errors

(b) questions and comments made by the student (and written by the interviewer) while solving each problem, including his/hers indication of unfamiliar English words and sentences and

(c) overall evaluation of the most difficult problem made at the end of the interview and after reaching the correct solution for each problem.

In analyzing the results I have used the *triangulation* analysis method because the logic of triangulation is based on the premise that no single method ever adequately solves the problem of rival explanations. According to Patton (2002), four kinds of triangulation can contribute to verification and validation of qualitative analysis: *methods* *triangulation* (checking out the consistency of findings generated by different data collection methods), *triangulation of source* (checking out the consistency of different data sources within the same method), *analyst triangulation* (using multiple analysts to review findings) and *theory/perspective triangulation* (using multiple perspectives or theories to interpret the data). In this study I used *the triangulation of qualitative data sources* that means comparing and cross-checking the consistency of information derived at different times and by different means within qualitative methods. Combinations of interviewing, observations during the interview and document analysis containing the students' actual work are presented in this study.

In order to quantify the mathematical performance of the students, we have graded each problem with one point when the correct operation has been identified and with zero points when not. We have also given one point for each problem solved correctly, without any assistance and zero points in all other situations. We have recorded the number of hints necessary for a student to solve the problem and we catalogued them as errors; in the case when hints were necessary to identify the correct operation or to set up the correct equation the error was classified as an operational error (see appendix E). When hints were necessary during solving the problem, the error was considered to be computational. As mentioned before, the students would rate each problem from two perspectives: perceived language difficulty and perceived mathematics difficulty – the scale for each of these ratings was from 1 (very easy) to 5 (very hard).

CHAPTER IV

FINDINGS

This chapter provides an analysis of the data that were collected from the student booklets and during the student interviews. In the first part of the chapter I report the computational errors made by the students and their frequency. The mathematical operations considered were (addition, multiplication and division). The second part of the chapter discusses the operational errors. In the third part I discuss the correlation with the MAP scores for different categories and in the end I show how English and Math difficulty levels were perceived by the students.

Computational Errors

This category of errors presents the advantage of being a well defined category, interacting very little with other types of errors. The computational stage happens after the problem conditions and requirements have been understood (or misunderstood) and the equations for the problem have been set up. At this stage, the student needs only to apply a well defined algorithm for performing the specific mathematical operation(s) (addition, subtraction….) and the success or failure of this step is entirely determined by its correct application.

As shown during the literature review, a complete analysis of computational errors and their causes is a very complicated task, even for the simplest mathematical operations and an exhaustive classification of these errors is beyond the scope of this work. Instead, I will limit my presentation to discussing the specific errors committed by the participants in this study.

The total number of elementary mathematical operations the students had to perform was 328 out of which 123 were additions, 144 were multiplications and 61 were divisions. The overall number computational mistakes were 68 (20.7%) out of which 3 were addition errors, 38 were multiplication errors and 27 were division errors. A quick inspection shows us that the highest rate of errors was for division (27 out of 61 or 44.3%) followed by multiplications (38 out of 144 or 26.4%) and addition (3 out of 123 or 2.4%). In the following we will inspect closer the type of errors for each operation.

Addition errors:

All 3 addition errors involved addition of a one digit number with a two digit number (3 + 11) and in all cases the wrong answer was 13 (instead of 14). When asked to verify the calculation all 3 students corrected the mistake, which suggests that this error was due to lack of attention.

Multiplication errors

The multiplication errors could be divided in 6 distinct categories:

a. Misplaced decimal point (19 errors or 50%). These errors were the most frequent and they included the cases of misplaced decimal point and the cases when students simply forgot to add a decimal point.

b. Wrong column alignment (11 errors or 29%). The second most frequent multiplication error happened when multiplying numbers made up of 2 or more digits either containing decimal points or not. In the case when one of the numbers contained decimal points the wrong alignment errors were also associated with misplaced decimal point errors. In this case, each error was counted in the corresponding category.

c. Single digit multiplication error when performing multiplications of numbers made up of 2 or more digits (5 errors or 13%). This category involved errors when multiplying a multidigit number by a single digit number and it included two cases: when the multiplication between two single digit numbers was carried out wrong as in

2 x 3 = 7

and the case when the multiplication of two digits was carried out correctly, but then the student forgot to add the number of tens into the tens place for the following multiplication as in

15 x 3 = 35

d. Final addition error (1 error or 2.6%). This error occurred when multiplying 2 numbers made up of 2 or more digits. Each of the multiplications by a single digit was performed correctly, but the final addition of the resulting number was wrong as in

```
  23 x
  47
 -----
 161
 92
-------
1071
```

e. Forgot to multiply by all digits (1 error or 2.6%). This error happened in one case when the student who had to multiply 2 three digits numbers used only two digits of the second number for multiplication, as in:

```
  123 x
  147
 -----
  861
 492
 -------
 5781
```

f. Complete multiplication error (1 error or 2.6%), when the student essentially had no strategy for multiplying two 3-digit numbers.

Division errors

The division error can be classified in four distinct categories

a. Decimal point errors (6 errors or 22.2%). The errors in this category include misplacing the decimal point or forgetting about it altogether.

b. Wrong one digit multiplication (3 errors or 11.1%). This type of error involved an incorrect multiplication of the quotient by the single digit determined in the final answer as in the following example.

```
      34
    -----------
23 | 782
     68  <-------- 3x23=69
    -----------
     102
      92
    -----------
      10
```

c. Wrong subtraction (3 errors or 11.1%). In this case, the error was done when the new number was subtracted from the remainder as in the following example:

```
      33
    -----------
23| 782
    69
    -----------
      82  <-------- 78-69 = 9
      69
    -----------
      13
```

d. Complete division error (15 errors or 55.6%) when essentially the student had no idea how to divide numbers made or 2 or 3 digits. These situations represented more than half of the division error cases and the students, even with assistance from the interviewer were not able to carry out the calculation correctly.

General observations

The error distribution was consistent with the level of difficulty involved by the mathematical operation. Division of numbers with 2 or 3 digits was the most complex mathematical operation included in the test since it involves multiplication, subtraction as well as correct column alignment and so it was more probable that students would make an error in this case.

Operational Errors

Error Description

This type of errors consists in wrong identification of the mathematical operation necessary to solve the problem (e.g. addition instead of subtraction). As mentioned before, after hearing the entire problem (in English) the students had to indicate what mathematical operation would be required in order to solve it (each problem required exactly one arithmetic operation for the solution): if the student could correctly identify the operation, the interviewer would allow him/her to continue, if a mistake was made, the student would be given hints, mainly in the form of additional explanations for the critical terms in the problem. This stage gave the interviewer the possibility to decide if the operational error was made because of the insufficiencies in the student's vocabulary (for example the student would not know what "double" means) or because the student could not associate a familiar term with a specific mathematical operation. In some cases, when the problems could be solved by using addition or multiplication (as in the case of determining the "double" of a quantity), both options were considered correct operations.

Results

The number of operational errors (in %) made by the students in each of the 4 categories (Easy Math – Easy English, Easy Math – Hard English, Hard Math – Easy English, Hard Math – Hard English) is presented in Figure 5.

	EM-EE	EM-HE	HM-EE	HM-HE
Op Errors (%)	18%	24%	43%	38%

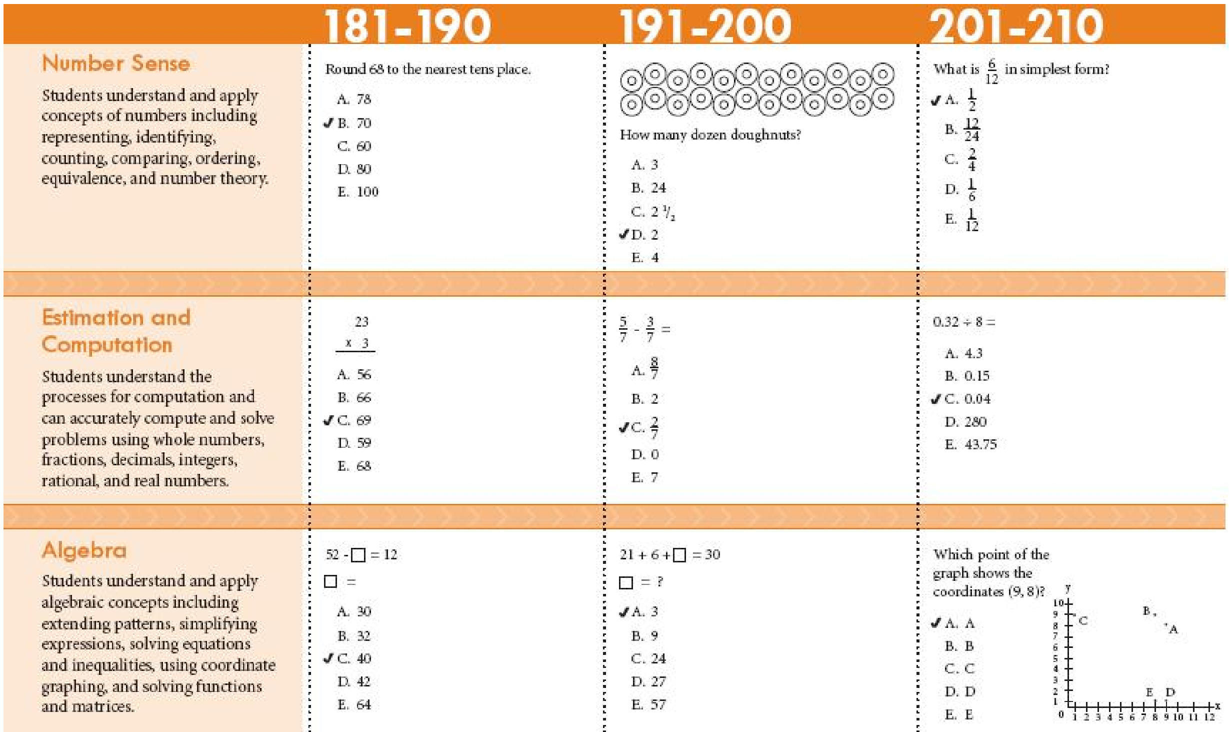
Figure 5. Operational error distribution (in %)

Interestingly enough, the number of operational errors seems to be less affected by the English difficulty of the problem, but increases considerably with the math difficulty. This finding supports the idea that correct English understanding is not a sufficient condition for success in mathematical problem solving.

MAP Scores Correlation

- One useful instrument in monitoring the students' development over the academic year is represented by the Measures of Academic Progress (MAP) test. Students take the MAP test to assess instructional level and measure academic growth. This is one- hour –computer- based examination designed to adapt the next question's difficulty based on the student's answers to previous questions. In this manner, no student will be able to answer correctly all questions and there will be no student who could not answer any question. The score of this test is reported in Rausch unITs (RIT) and can assist teachers with future instructional decisions. The RIT Scale is a curriculum scale that uses individual item difficulty values to estimate student achievement. An advantage of the RIT scale is that it can relate the numbers on the scale directly to the difficulty of items on the tests. In addition, the RIT scale is an equal interval scale. Equal interval means that the difference between scores is the same regardless of whether a student is at the top, bottom, or middle of the RIT scale, and it has the same meaning regardless of grade level. RIT scales, like scales underlying most educational tests, are built from data about the performance of individual examinees on individual items.

The Math and Reading sections of the test can be administered up to 4 times a year, the Science part of the test up to 3 times a year. An example of MAP problems for 3 different Math sections (Numbers, Computation and Algebra) and 3 different RIT scores (between 180 and 210) are shown in Figure 6 bellow.



	181-190	191-200	201-210
Number Sense Students understand and apply concepts of numbers including representing, identifying, counting, comparing, ordering, equivalence, and number theory.	Round 68 to the nearest tens place. A. 78 ✔B. 70 C. 60 D. 80 E. 100	How many dozen doughnuts? A. 3 B. 24 C. $2\frac{1}{2}$ ✔D. 2 E. 4	What is $\frac{6}{12}$ in simplest form? ✔A. $\frac{1}{2}$ B. $\frac{12}{24}$ C. $\frac{2}{4}$ D. $\frac{1}{6}$ E. $\frac{1}{12}$
Estimation and Computation Students understand the processes for computation and can accurately compute and solve problems using whole numbers, fractions, decimals, integers, rational, and real numbers.	23 x 3 A. 56 B. 66 ✔C. 69 D. 59 E. 68	$\frac{5}{7} - \frac{3}{7} =$ A. $\frac{8}{7}$ B. 2 ✔C. $\frac{2}{7}$ D. 0 E. 7	0.32 ÷ 8 = A. 4.3 B. 0.15 ✔C. 0.04 D. 280 E. 43.75
Algebra Students understand and apply algebraic concepts including extending patterns, simplifying expressions, solving equations and inequalities, using coordinate graphing, and solving functions and matrices.	52 - ☐ = 12 ☐ = A. 30 B. 32 ✔C. 40 D. 42 E. 64	21 + 6 + ☐ = 30 ☐ = ? ✔A. 3 B. 9 C. 24 D. 27 E. 57	Which point of the graph shows the coordinates (9, 8)? ✔A. A B. B C. C D. D E. E

Figure 6. Sample of MAP problems and scores.

The objective of this investigation was determine if at least one of the MAP sections (Number, Computation or Algebra) would constitute good predictors for the results obtained in our study. If such a predictor could be found, the results of the MAP test could be used in the future to gage the changes made in order to improve the “Animal Watch” tutoring system. In figure 5 bellow are presented the correlation results (R^2) between MAP scores (Number and Algebra sections) obtained in 2009 and the present experiment results. The “overall” result gives the score obtained by each students when solving the each problem (1 point for a correct, unaided solution, 0 points otherwise). The “computation”’ result gives the score obtained by each student in performing the computational part of the problem, regardless of correct identification of the operation involved. It can be noticed (Figure 7) that the level of correlation is very small in both cases, but it's better for the MAP "Numbers" section. The low correlation between the MAP scores and the present experiment is not entirely surprising: on one hand, the level of correlation between MAP scores obtained in 2008 and 2009 by the same students is relatively small ($R^2 = 0.35$) and on the other hand, the type of problems we have used for the test were substantially different from the MAP standards.

	MAP Numbers (2009)	MAP Algebra (2009)
R2 overall	0.19	0.024
R2 computation	0.1	0.04

Figure 7. Correlation with MAP scores.

Word and Math Rating Correlation

In order to understand better the nature of the difficulties encountered by the students in solving the problems, they were required to indicate on a Likert-type scale (1-5) the perceived Word and Math difficulty of each problem. The results, shown in the figure bellow

	Mean Word Rating	Mean Math Rating
EMEE	1.82	1.37
EMHE	2.41	1.37
HMEE	1.87	2.1
HMHE	2.6	2.54

Figure 8. English and Math difficulty reported by students

It can be seen from this table that, on average, the students correctly the difficulty levels of English and Math. On average, the students indicated the same Math difficulty (1.37) regardless of the English difficulty when the Math was easy, but their perception of the Math difficulty was different (2.1 vs. 2.54) when the Math was harder, but the English level changed from Easy to Hard (see appendix F).

CHAPTER V

CONCLUSIONS

The first part of this chapter summarizes the study and uses the findings of the study to answer the research questions. In the second part we use the findings of the study in order to make suggestions for improvements to the functionality of the "Animal Watch" Web-based software designed to assist ELL students in learning Math. The third part of this chapter discusses the limitations of the study.

Summary of the study

The goal of this study was to understand the type and frequency of errors made by the ELL students when faced with arithmetic word problems. To this end we have interviewed 41 students from two different schools in Southwestern USA. Each of the students was required to solve a total of eight arithmetic word problems divided in 4 distinct categories: Easy English – Easy Math, Easy English – Hard Math, Hard English – Easy Math and Hard English – Hard Math. During the interview, the students were monitored for possible errors and hints were given when they could not find the correct solution. The conclusions of this study will be used in order to improve the Animal Watch Web based software for student learning.

Summary for Research Question One

The first question we tried to answer during the study was related to the types of errors made by the ELL students faced with arithmetic word problems. The results of our analysis showed that out of the 4 elementary operations, by far the most frequent computational errors were made when the students had to divide number made up of 2 or more digits (44.3%). At the elementary level, division errors could be separated in errors made in placing the decimal point, errors in multiplying by a single digit, or errors in subtraction. All these type of errors have been well documented in literature before (Brown & Burton, 1978; Brown & Van Lehn 1980; Young & O'Shea, 1981); in addition we also found out that an important part of the division errors (55.6%) were made because the students had no idea how to perform such divisions and no amount of help from the examiners could lead to a positive result.

Besides the standard computational errors, the students also performed an important number of operational errors (31%). The distribution for this type of errors supports the hypothesis that they are more often associated with difficulty of the computation involved than with the difficulty of the English language used in formulating the problem. In other words, the students didn't seem to have major problem in understanding the meaning and the requirements of the problem, but they seemed to be uneasy about how a specific word task could be translated into a mathematical operation. The same phenomenon has been discussed by other authors before (Moschkovich 1999, 2002, 2005) and it can be summarized as the difficulty of translating the everyday language into meaningful mathematical thinking. This kind of difficulties is expected to be larger in the case of ELL students, as in the present study.

Summary for Research Question Two

The second objective of this investigation was to determine how good predictors are the official MAP scores for the mathematical performance of the ELL students faced with word problems. The purpose of this analysis was to see how well the MAP scores can be used as a measure of success in implementing the necessary changes to the Animal Watch Web based software. To this end we have tried to determine a correlation between two official MAP scores (for Numeric calculation and for Algebra) and two categories of operations the students needed to perform during the interviews (a complete solution to the problem with no external help and a measure of computational achievement, regardless if they have selected the correct operation or not). The degree of correlation between each of these categories was rather small, mainly due to the differences between the official MAP types of problems and the ones presented in the interviews. We conclude from this low correlation levels and from the result obtained for the operational errors that the interviewed students were only partial familiar with the elementary mathematical operations and they had difficulties in expressing the familiar word concepts into meaningful mathematical operations.

Recommendations for "Animal Watch" Web-based software improvements

As mentioned before, the final objective of this investigation was to determine a set of principles that would enhance the efficacy of the Animal Watch Web based software for the benefit of the ELL students struggling with Mathematics. From the present observations, we can recommend:

1. An increased emphasis on the mechanics of elementary computations. The software should provide examples of additions, subtractions, multiplications and divisions with one, two and three digit numbers.
2. That ELL students be helped to translate into standard mathematical operations some of the everyday keywords (like double, more, less, half etc.) by providing multiple visual examples and reinforcements. Examples of math keywords should include but not limited to the following list:

Addition:	**Subtraction:**	**Multiplication:**	**Division:**	**Equality:**
The Sum of Plus Total Increased by More	Decreased by Less than Subtracted from Difference between	Times Multiplied by Of The Product of Twice	Divide(s) Divided by Divided into Half of Third of	Equals Is/was/are Is equal to What is left What remains

More than Added to Exceeds Expands Greater than Gain Profit Longer Older Heavier Wider Taller Add And Raise Both Combined In all Altogether Additional As well as Entire	Diminished by Takes away Reduced by Less than Minus Loss/Lost Lower Shrinks Smaller than Younger Slower Fewer Deduct Dropped Change Have left Remain Fell How much more	Double Triple Half As much By Every Pair At (sometimes) Total (sometimes)	Per Quotient Divisor Between Distribute Split Dividend Ratio Parts In fourths Average Cut Evenly Each Among Equal Pieces Every Out of Shared	The same as Gives/Giving Makes Leaves

3. The hints offered to students should be introduced gradually, based on their feedback regarding the unknown words or sentences. Since some parts of the problems were not essential for the correct problem understanding, the software should be capable to distinguish between critical (keywords) and non-critical information (regular English words).
4. For a good understanding of the problem, the student may listen to the text of the word problem. In this way he/she will develop a good pronunciation as well as a good comprehension of the problem.

Limitations of the study

One of the main limitations of the present investigation is represented by the relatively small number of students examined. The number of problems faced by the ELL student learning Mathematics is large and in order to cover the entire spectrum of difficulties a large number of schools from different geographic regions of the US should be selected.

As a second limitation for the study, we mention the limited number of levels for the two factors investigated (English difficulty and Mathematical difficulty). The present study used only 2 levels for each factor and did so in order to limit the number of problems presented for each child. Increasing the number of levels would require longer interviews and maybe multiple sessions with each student.

Finally, the third limitation of the study is represented by its narrow area of investigation. The students have been tested only in arithmetic problems and these problems did not cover a very large area as the standard MAP tests do. In order to obtain meaningful correlation with students' previous achievements, a test should be modeled after these standards.

Even though the study does have some limitations, its findings and conclusions are still valuable. It provides a picture of impact of English text difficulty on English learners' math word problem solving that had not been examined in the literature to date.

APPENDIX A

CONSENT FORM FOR STUDENTS

MINOR'S ASSENT FORM

Title of Project: Mathematics word problems solving by English Language Learners and web based tutoring system

Your mother/father has told me it was okay for you to participate in this study. You will work one session of no more than 30 minutes with a University of Arizona mathematics tutor who will be a graduate student in Mathematics Education. The tutor will show you 8 mathematics word problems, written in English. The mathematics word problems involve additions, subtractions, multiplications, divisions and fractions. The tutor will talk about each problem with you, helping you to go from the words to the mathematics equations and providing hints to make sure you know how to find the answer. The purpose of this project is to develop a computer program to help students learn to solve mathematics word problems. When the program is ready, it will be available for free to students in Arizona and other states with large numbers of English Language Learners.

Your participation in this study is voluntary. You may decide to not begin or to stop the study at any time. Your refusing to participate will have no effect on your student status.

____
Child's Name

____
Child's Signature **Date**

____
Presenter's Signature **Date**

Otilia Barbu______________________4/10/2009_____________________
Investigator's Signature **Date**

APPENDIX B

PARENT/LEGAL GUARDIAN PERMISSION FORM

Project Title: Mathematics word problems solving by English Language Learners and web based tutoring system

You are being asked to read this form so that you know about this research study. Federal regulations require that you know about the study and the risks involved. If you decide that you want your child to participate in this study, signing this form will say that you know about this study. If you decide you do not want your child to participate, that is okay. There will be no penalty to you or your child and your child will not lose any benefit which s/he would normally have.

WHY IS THIS STUDY BEING DONE?
You have the choice for your child to participate in this project.

The purpose of this project is to develop a computer program to help students learn to solve mathematics word problems. Mathematics word problems can be especially hard for students who are learning to read English. We want to provide tutoring to English Language Learners to find out what is most helpful for them. We will use the results to guide the design of our computer program. When the program is ready, it will be available for free to students in Arizona and other states with large numbers of English Language Learners.

WHY IS YOUR CHILD BEING ASKED TO BE IN THIS STUDY?
To be in this study, your child must be an English Language Learner enrolled in Grade 6 or 7 and a student in the Amphitheater School District.

WHAT ARE THE ALTERNATIVES TO BEING IN THIS STUDY?
The alternative is that your child does not have to participate in the study.

WHAT WILL YOUR CHILD BE ASKED TO DO IN THIS STUDY?
Your child’s participation in this study will include one session of no more than 30 minutes. The Principal Investigator will work with your child’s mathematics teacher to find the best time for the session. Your child will work with a University of Arizona mathematics tutor who will be a graduate student in Mathematics Education. The tutor will show your child 8 mathematics word problems, written in English. The mathematics word problems involve additions, subtractions, multiplications, divisions and fractions. The tutor will talk about each problem with your child, helping him or her go from the words to the mathematics equations and providing hints to make sure your child knows how to find the answer.

ARE THERE ANY RISKS TO MY CHILD?
Students sometimes get frustrated if they cannot solve a math problem. However, the tutor will help the student learn to solve each problem.

ARE THERE ANY BENEFITS TO MY CHILD?
We hope that your child may learn some useful strategies for solving word problems in English by working with the University of Arizona tutor.

WILL INFORMATION FROM THIS STUDY BE KEPT CONFIDENTIAL?
We do not need to record any identifying information about your child. For this project, the only information that will be collected will be your child's comments about how difficult each problem seems to him or her, any words in the problem that he or she does not know, and the types of hints that he or she needs to get the correct answers.

People who have access to your child's information include the Principal Investigator and study personnel. In addition, representatives of regulatory agencies (including the University of Arizona Human Subjects Protection Program) may access your child's records to make sure the study is being run correctly and that information is collected properly. The agency that funds this study (sponsor) may also see your child's information. However, any information they have will be coded with a number so that they cannot tell who your child is. If there are any reports about this study, your child's name will not be in them. This consent form will be filed in an official area.

WILL I OR MY CHILD BE PAID TO BE IN THIS STUDY?
There is no cost to you or your child for being in this study except your/your child's time. You/your child will not be paid for being in this study

WHO CAN BE CONTACTED ABOUT THIS STUDY?
You or your child can call the Principal Investigator to ask questions or tell her about a concern or complaint about this research study. The Principal Investigator is Otilia Barbu, and you can call her on her cell phone at (520) 975-7694.
If you have questions about your child's rights as a research subject you or your child may call the University of Arizona Human Subjects Protection Program office at (520) 626-6721. If you or your child have questions, complaints, or concerns about the research and cannot reach the Principal Investigator; or want to talk to someone other than the Investigator, you or your child may call the University of Arizona Human Subjects Protection Program office. (If out of state use the toll-free number 1-866-278-1455.) If you or your child would like to contact the Human Subjects Protection Program via the web (this can be anonymous), please visit http://www.irb.arizona.edu/contact/.

STATEMENT OF CONSENT
The procedures, risks, and benefits of this study have been told to me and I agree for my child to be in this study and sign this form. My questions have been

answered. I may ask more questions whenever I want. My child can stop participating in this study at any time and there will be no bad feelings. My child's medical care will not change if s/he quits. The researcher can remove my child from the study at any time and tell me why my child has to stop. New information about this research study will be given to me/my child as it is available. My child and I do not give up any legal rights by signing this form. A copy of this signed consent form will be given to me.

___________________________________ ________________________________
Subject's Name Date

___________________________________ ________________________________
Parent/Legal Guardian Date

___________________________________ ________________________________
Parent/Legal Guardian Date

INVESTIGATOR'S AFFIDAVIT:
Either I have or my agent has carefully explained to the subject the nature of the above project. I hereby certify that to the best of my knowledge the person who signed this consent form was informed of the nature, demands, benefits, and risks involved in his/her participation.

___________________________________ ________________________________
Signature of Presenter Date

___Otilia Barbu_______________ 4/10/2009______________________
Signature of Investigator Date

Please sign one copy and return it to your child's mathematics teacher. You can keep one copy so that you have all the information in case you have questions later on.

APPENDIX C

SITE AUTHORIZATION LETTER

Dear Otilia Barbu:

I have reviewed your request regarding your study and am pleased to support your research project entitled “Mathematics word problems solving by English Language Learners and web based tutoring system”. Your request to use the Amphitheater School District as a research site is granted. Potential participants are English Language Learners enrolled in Grade 6 or 7, students in classes taught by teachers who have volunteered in this study. The research will include individual sessions of interviews with a sample of English Language Learners, audiotaping and class observations. Consent materials will be sent to students and parents of the students nominated by their math teachers.
This authorization covers the time period of 4/10/2009 to 08/01/2009. We look forward to working with you.

Sincerely,

Name
Title

APPENDIX D

ACTUAL TEST SAMPLE

Problem _______

People are learning that White Sharks can swim a long way. One Shark swam from California to Hawaii. The trip was 2,800 miles. The Shark swam 43 miles every day. How many days did it take the Shark to make the trip?

WORDS …………………………….	Very Easy	Easy	OK	Hard	Very Hard
MATH …………………………	Very Easy	Easy	OK	Hard	Very Hard

ANSWER:

Equation

APPENDIX E

MATHEMATICAL MISTAKES SUMMARY

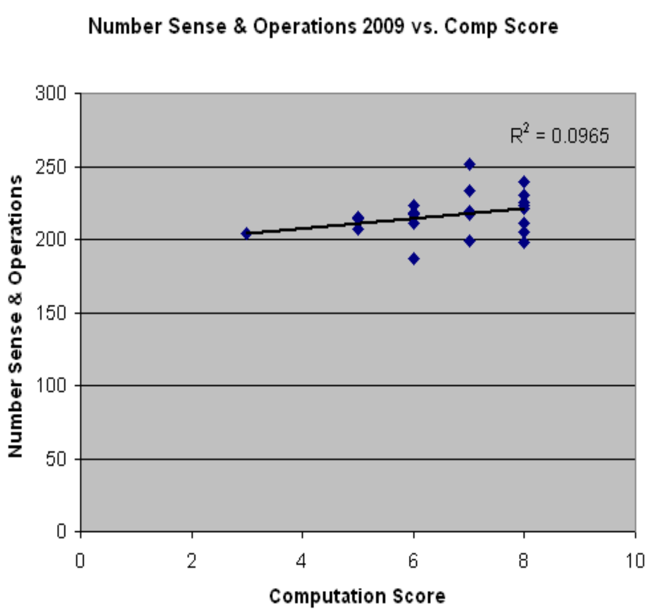
Mathematical errors	Abbreviations	
missing or misplaced decimal points	MP	Student forgot or misplaced the decimal points
computation errors	CE	Student makes computation errors (e.g. addition, subtraction, multiplication or division errors)
operations errors	OE	Student identifies wrong operation for the problem (e.g. addition instead of subtraction)
non-identified key words	NW	Student did not identify the math key words (e.g. double, average...)
set up errors	SE	Student is not able to set up problem without help (e.g. divides denominator by nominator,...)
mentally solved	MS	Student solved problem mentally without help
correct	C	Student solved problem correctly without help
missing information	-1	Information is missing due to lack of time to complete the problem

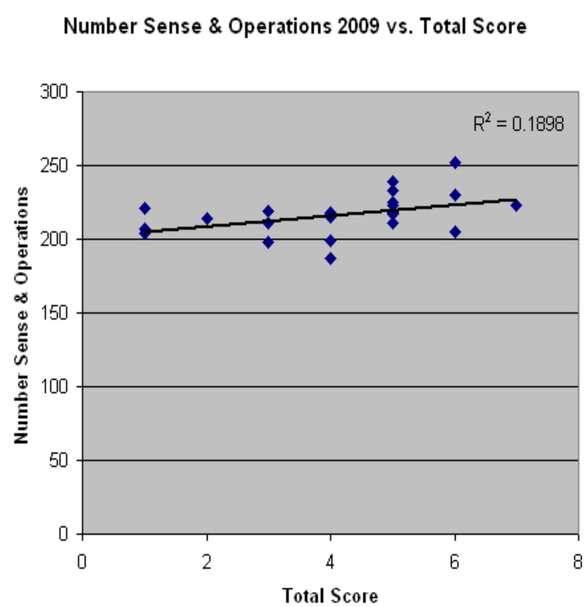
Gender	School	Booklet	EMEE1	EMEE2	EMHE1	EMHE2	HMEE1	HMEE2	HMHE1	HMHE2	Total C, MS
0	WC	1	NW	C	C	C	OE	OE	OE	C	4
0	WC	1	NW	NW, MS	NW, OE	NW	OE, CE	OE, CE	OE, CE	OE, CE	1
0	WC	1	NW, OE	C	C	NW, OE	SE, CE	SE, CE	CE	OE, CE, MP	2
0	WC	1	NW, OE	MS	C	C	CE	CE	C	CE	4
0	WC	1	NW, OE	MS	NW	C	MP, CE	OE, CE	C	CE	3
0	AMS	1	C	C	NW	NW	MP, CE	CE	C	C	4
0	AMS	1	C	C	NW, OE	NW	MP	C	OE	OE	3
1	AMS	1	C	NW	NW, OE,MS	NW, OE	MP,OE	MP, CE	CE	OE, CE, MP	2
		1	NW, OE	C	NW	NW, OE	MP,OE	MP	C	MP	2
0	WC	2	MS, OE	MS	MS,OE	MS	C	OE, MP	OE, MP	OE	5
0	WC	2	MS,NW	C	C	MS	CE	OE, CE	C	NW	5
1	AMS	2	C	C	C	C	NW	OE, CE, MP	OE, MP	C	5
0	AMS	2	C	C	C	C	C	C	OE, MP	C	7
1	AMS	2	NW, OE	CE	NW, OE	C	CE	OE, CE	OE, CE	OE, CE, NW	1
1	AMS	2	CE	C	C	C	C	OE, CE, MP	OE, CE, MP	CE	4
0	LC	2	NW	SE	NW, OE, SE	SE	SE, CE	CE, MP	SE	C	1
0	LC	2	OE, CE	NW, MS	OE, NW	MS	C	OE, CE, MP	MP	OE, CE, NW	3
1	LC	2	OE	C	C	C	C	C	MP	C	6
0	LC	2	C	C	NW	C	C	C	MP	OE, CE	5
1	LC	2	C	NW	C	OE	OE	OE, MP	C	OE	3
0	WC	3	C	C	C	MS	-1	-1	C	MP	5
1	WC	3	NW	NW	C	C	OE, NW	-1	OE, MP,NW	OE, NW, SE, MP	2
0	WC	3	C	MS	NW, MS	MS	CE	CE, MP	NW, OE, MP, CE	MP	4
0	WC	3	C	C	OE	NW	C	CE	CE	C	4
1	WC	3	NW	NW	OE, NW	CE, NW	OE, NW	OE, CE, NW	SE, CE, MP	OE	0
0	AMS	3	OE, NW	OE, NW	OE	OE, NW	OE	C	OE	OE, MP	1
1	AMS	3	NW, MS	C	MS	NW	C	OE, SE, MP	CE, MP	OE, NW, MP	4
1	AMS	3	C	C	OE, NW	NW	C	OE, NW, MP	CE, MP	OE, CE, MP	3
0	LC	3	NW, OE	C	C	C	OE, NW	OE, NW, MP	OE, NW, MP	C	4
0	LC	3	NW	C	C	C	OE	MP	OE	C	4
0	WC	4	NW, OE	C	OE, NW	C	OE, NW	C	C	OE, NW,CE	4
0	AMS	4	C	C	OE	NW	OE, CE	CE	OE	CE, MP	2
0	AMS	4	C	C	C	C	MP	C	C	CE, MP	6
1	AMS	4	C	C	C	OE	OE	C	C	CE,MP	5
0	LC	4	C	NW	C	C	C	C	NW,CE	NW	5
0	LC	4	C	MS	C	NW, MS	C	CE	C	CE, MP	6
0	LC	4	C	C	C	C	OE, MP, NW	CE	C	CE, MP	5
0	LC	4	C	C	C	C	OE, MP, NW, CE	SE, CE	C	SE, CE, MP	5
1	LC	4	C	C	OE	C	C	OE	C	C	6
0	LC	4	C	C	C	C	OE	OE	OE	OE, MP	4
1	LC	4	C	C	OE	C	OE, MP, NW	OE	C	OE, MP, NW	4

APPENDIX F

CORRELATION DATA

Comp Errors	Comp Score	MAP data ID	Algebra&Functions	Number Sense & Operations	Test RIT score	Total C, MS
5	3	NSF09_AP13	210	204	206	1
3	5	NSF09_AP27	221	207	212	1
0	8	NSF09_AP32	239	221	225	1
3	5	NSF09_AP26	220	214	218	2
0	8	NSF09_LC12	207	198	198	3
1	7	NSF09_AP23	226	219	222	3
0	8	NSF09_LC09	213	211	209	3
2	6	NSF09_AP24	203	211	209	3
1	7	NSF09_LC29	204	199	200	4
2	6	NSF09_AP34	234	217	227	4
3	5	NSF09_LC16	216	215	206	4
2	6	NSF09_AP33	215	187	209	4
1	7	NSF09_AP29	210	218	213	4
2	6	NSF09_LC18	198	211	212	5
1	7	NSF09_LC21	213	233	224	5
2	6	NSF09_AP17	225	223	219	5
2	6	NSF09_LC37	231	218	217	5
0	8	NSF09_AP21	233	225	233	5
0	8	NSF09_LC07	247	239	240	5
1	7	NSF09_LC10	244	217	233	5
0	8	NSF09_LC08	196	205	198	6
1	7	NSF09_LC17	247	252	238	6
0	8	NSF09_LC36	222	230	220	6
0	8	NSF09_AP19	229	223	222	7





APPENDIX G

WORD PROBLEMS

Easy Addition

Many zoos around the world are trying to raise Wild Horses. One zoo had 11 Wild Horses. By the end of the year, 3 baby horses had been born. How many wild horses did the zoo have then?

Word count: 38
REAP readability: Grade 3

Many zoos around the world are breeding Wild Horses. Initially, one zoo owned 11 Wild Horses. At the year's end, 3 foals had been born. What was the total population of Wild Horses in residence at the zoo then?

Word count: 39
REAP readability: Grade 6

Pandas look like black and white bears. But pandas are different. Girl bears can have babies when they are only 3 years old. Girl pandas are not ready to have a baby for 3 more years. How old is a panda when she can have a baby?

Word count: 47
REAP readability: Grade 2

Pandas look like piebald bears. However, pandas are a different species. Female bears can become pregnant when they are 3 years old. Female pandas are not ready to gestate for 3 more years. How old is a panda when she has the ability to bear a cub?

Word count: 47
REAP readability: Grade 3

The Snow Leopard has a very long tail. It wraps its tail around its face to keep warm. Its tail and its body are the same length. If its tail is 2 feet long, how many feet long is the Snow Leopard in all?

Word count: 44
REAP readability: Grade 4

The Snow Leopard has an unusually long tail, which it often uses to shield its face for warmth. Its tail and body are equivalent in length. If its tail is 2 feet long, what is the total length of the Snow Leopard?

Word count: 42
REAP readability: Grade 9

A Mountain Lion living in a zoo will eat 5 pounds of meat in one day. How many pounds of meat will it eat in 2 days?

Word count: 27

REAP readability: Grade 3

A Mountain Lion living in captivity will consume 5 pounds of meat in a single day. How many pounds of meat will it consume in 2 days?

Word count: 27

REAP readability: Grade 6

Easy Multiplication

Pandas eat mostly bamboo. A Panda in a zoo will eat 20 pounds of bamboo in one day. A Panda living in the wild eats twice as much. How many pounds of bamboo will the wild Panda eat in one day?

Word count: 41

REAP readability: Grade 3

The Panda's diet consists primarily of bamboo. A Panda in a zoo can consume 20 pounds of bamboo in a single day. A wild Panda will consume double that amount. How many pounds of bamboo will the wild Panda consume in a day?

Word count: 43

REAP readability: Grade 4

The Snow Leopard is a very good hunter. It can kill an animal that is three times its own weight. If a Snow Leopard weighs 20 pounds, what is the weight of the biggest animal that it could kill?

Word count: 39

REAP readability: Grade 6

The Snow Leopard is a skilled predator. It can successfully attack a prey animal that is three times its weight. If a Snow Leopard weighs 20 pounds, what is the weight of the largest prey that it could kill?

Word count: 39

REAP readability: Grade 12

When it is born, a Right Whale baby is 14 feet long. On its first birthday, it will be twice as long. How long will it be then?

Word count: 28

REAP readability: Grade 3

At parturition, a Right Whale infant is 14 feet long. Twelve months later, its length will have doubled. How long will it be on its first birthday?

Word count: 27

REAP readability: Grade 3

A Wild Horse baby weighs 55 kilograms when it is first born. Its mother's rich milk helps it grow very quickly. Its weight will double in one month. How much does the baby horse weigh when it is a month old?

Word count: 41

REAP readability: Grade 4

A Wild Horse foal has an average weight of 55 kilograms at delivery. The high nutritional content of maternal milk supports rapid growth. The foal's weight will double within a month. How much will the foal weigh one month after delivery?

Word count: 41

REAP readability: Grade 8

Hard Multiplication

Many people think that Pandas are cute. But Pandas are big and can be dangerous. An adult male Panda can weigh 123 kilograms. One kilogram equals 2.2 pounds. What is the Panda's weight in pounds?

Word count: 35

REAP readability: Grade 1

The Panda's cute appearance can be deceptive; these animals are large and can be dangerous. An adult male Panda can weigh 123 kilograms. If one kilogram is equivalent to 2.2 pounds, what is the Panda's weight in pounds?

Word count: 38

REAP readability: Grade 6

The Condor is a very big bird. It has strong wings that help it fly very high in the sky. One Condor flew 14,750 feet above the ground. One foot equals 12 inches. How many inches above the ground was the Condor?

Word count: 42

REAP readability: Grade 2

The Condor is an enormous bird with powerful wings that allow it to soar to extraordinary heights. One Condor was documented flying at an altitude of 14,750 feet. If one foot is equivalent of 12 inches, what was the Condor's altitude in inches?

Word count: 43

REAP readability: Grade 12

Pandas are called "cloud bears" because they live high in the mountains. Bamboo grows there. One panda lived on a mountain 3,800 meters above sea level. One meter is 3.28 feet. What was the height of the mountain in feet?

Word count: 40

REAP readability: Grade 4

Pandas are sometimes referred to as "cloud bears" because their preferred habitat is mountain slopes, where bamboo is plentiful. Scientists documented one Panda living on a mountain 3,800 meters above sea level. One meter is 3.28 feet. What was the mountain's elevation in feet?

Word count: 44

REAP readability: Grade 4

Right Whales eat tiny shrimp. The Whale has special teeth that catch the shrimp in the sea water. The Whale needs to eat 2,625 pounds of shrimp every day. How many pounds of shrimp does a Whale eat in 7 days?

Word count: 41

REAP readability: Grade 5

The Right Whale's diet consists primarily of tiny plankton, which are filtered from the sea water by its baleen. The Whale must consume 2,625 pounds of plankton every day. What is the Whale's total consumption of plankton in pounds over 7 days?

Word count: 42

REAP readability: Grade 6

Hard Division

People are learning that White Sharks can swim a long way. One Shark swam from California to Hawaii. The trip was 2,800 miles. The Shark swam 43 miles every day. How many days did it take the Shark to make the trip?

Word count: 42

REAP readability: Grade 5

Scientists studied one White Shark that swam all the way from California to Hawaii, with a total distance of 2,800 miles. The scientists recorded that the shark swam 43 miles every day. How many days did it take the Shark to complete the journey?

Word count: 44

REAP readability: Grade 5

Many people are scared of the White Shark because it is very big. An adult White Shark weighs 5,000 pounds. One pound is the same as 2.2 kilograms. What is the average weight of a White Shark in kilograms?

Word count: 39

REAP readability: Grade 3

One reason that many people are frightened of the White Shark is that these creatures can be extremely big. An adult White Shark typically weighs 5,000 pounds. One pound equals 2.2 kilograms. What is the average weight of a White Shark in kilograms?

Word count: 43
REAP readability: Grade 4

There are no Wild Horses living in the wild any more. But there are 2,100 Wild Horses living in zoos. There are 175 zoos around the world that have Wild Horses. How many horses does each zoo have, on average?

Word count: 40
REAP readability: Grade 3

The Wild Horse species has been extinct in the wild for several decades. However, there are 2,100 Wild Horses in captivity in zoos. If 175 zoos have Wild Horses in their collections, what is the average number of horses per zoo?

Word count: 41
REAP readability: Grade 12

Mountain lions are good hunters. They are big enough to kill a deer. One mountain lion will kill a deer for food every 8 days. There are 365 days in one year. How many deer does a mountain lion kill in one year?

Word count: 43
REAP readability: Grade 2

Mountain lions are powerful hunters capable of killing a deer. Scientists estimate that one mountain lion will kill and consume a deer every 8 days. There are 365 days in one year. How many deer will a mountain lion kill in one year?

Word count: 43
REAP readability: Grade 6

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